A Multifactor Perspective on Volatility-Managed Portfolios*

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Abstract

A fundamental insight in finance is that there is a strong risk-return tradeoff. Moreira and Muir (2017) challenge this by showing that investors can increase Sharpe ratios by reducing exposure to risk factors when their volatility is high. However, Cederburg, O'Doherty, Wang, and Yan (2020) show these strategies fail out of sample and Barroso and Detzel (2020) show they do not survive transaction costs. We propose a novel conditional *multifactor* portfolio whose weights on each factor change with market volatility and outperforms its unconditional counterpart even out of sample and net of costs, and during both low- and high-sentiment periods. Our results demonstrate that the breakdown of the risk-return tradeoff is even more puzzling than previously thought.

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1 Introduction

A fundamental insight in finance is that there is a strong risk-return tradeoff. Moreira and Muir (2017) challenge this by showing that investors can increase Sharpe ratios by reducing exposure to risk factors when their volatility is high. The intuition underlying their findings is that, in the absence of a strong risk-return tradeoff for factor returns, factor exposure can be scaled back during times of high volatility without a proportional reduction in returns. This is a challenge to structural models of time-varying expected returns, which typically predict that the risk-return tradeoff improves during periods of high volatility. However, Cederburg et al. (2020) show that the utility gains from volatility management are not achievable out of sample, and Barroso and Detzel (2020) show that transaction costs erode these gains for every risk factor except the market. Moreover, Barroso and Detzel (2020) show that the volatility-managed market portfolio outperforms only during high-sentiment periods.

While the aforementioned papers focus on volatility-managed *individual-factor* portfolios, we provide a *multifactor* perspective by proposing a novel conditional mean-variance *multifactor* portfolio whose weights on each factor change with market volatility. We show that this strategy outperforms the unconditional multifactor portfolio even out of sample and net of costs, and during both low- and high-sentiment periods. Our findings show that estimation error, transaction costs, and sentiment do not explain the gains from volatility management, and hence, the breakdown of the risk-return relation is even more puzzling than previously thought.

Our approach to volatility management differs in four ways from those in the existing literature. First, we focus on *multifactor* portfolios, whereas the existing literature focuses on volatility-managed portfolios of *individual factors*. For instance, the bulk of the analysis by Moreira and Muir (2017, sections I.B and I.D) focuses on a portfolio of only an individual factor and its volatility-managed counterpart. Second, our conditional multifactor portfolios allow the relative weights on the different factors to *vary* with market volatility. In contrast, Moreira and Muir (2017, section I.E) consider a conditional *fixed-weight multifactor portfolio* whose relative weight on each factor does not vary with volatility and Barroso and Detzel (2020, ftn. 12) consider a portfolio that assigns an equal relative weight to each factor. Third, we calculate conditional multifactor portfolios that are optimized accounting for transaction costs. Fourth, we account for the reduction in transaction costs associated with the netting of trades across the different factors that are combined in the multifactor portfolio, a phenomenon termed *trading diversification* by DeMiguel, Martin-Utrera, Nogales, and Uppal (2020).

Our conditional mean-variance *multifactor* portfolio outperforms in terms of out-of-sample and net-of-costs Sharpe ratio the unconditional mean-variance multifactor portfolio by about 16% and the conditional fixed-weight multifactor portfolio of Moreira and Muir (2017) by around 10%. There are three main drivers of the favorable performance of our portfolio. The first driver is trading diversification. In particular, although both the unconditional and conditional multifactor portfolios benefit from the netting of trades across multiple factors, the benefits are larger for the conditional portfolios because the transaction costs of the volatility-managed factors are much larger than those of the unmanaged factors. For instance, while the net mean return of four of the nine managed factors is negative if one ignores trading diversification, all nine managed factors have positive net mean returns with trading diversification. Our analysis shows that accounting for trading diversification is necessary for the conditional multifactor portfolio to outperform its unconditional counterpart.

The second driver of the performance of our portfolios is that optimizing the conditional multifactor portfolios taking transaction costs into account significantly improves their performance relative to the unconditional multifactor portfolio. Again, even though the performance of both the conditional and unconditional portfolios improves when they are optimized taking transaction costs into account, the benefits are larger for the conditional portfolios because the transaction costs of trading the managed factors are relatively larger.

The third driver of the performance of our conditional portfolios is that they allow the relative weight on each factor to vary with market volatility, which offers two benefits. One, our conditional portfolio optimally assigns a relative *average* weight to each factor that differs substantially from that of the unconditional and conditional fixed-weight portfolios.[1] For instance, our conditional multifactor portfolio has a much larger average exposure to the value (HML), momentum (UMD), and betting-against-beta (BAB) factors than the unconditional and conditional fixed-weight portfolios. Two, our conditional multifactor portfolio optimally times some of the factors aggressively (HML, UMD, and BAB), while assigning a stable weight to others (MKT, SMB, and CMA). To understand the rationale for this differential timing strategy, we regress the monthly returns of each risk factor on realized market volatility and find that the relation between risk and return varies substantially across factors, with some of the factors (HML, UMD, and BAB) exhibiting a negative relation, and others exhibiting a flat or weakly positive relation. Consequently, our conditional portfolio takes advantage of the opportunity to time factors differentially, which is ruled out for the conditional fixed-weight portfolio.

Our results contrast with the findings of Cederburg et al. (2020) and Barroso and Detzel (2020) that the gains from volatility managing *individual* factors do not survive out of sample or in the presence of transaction costs.[2] We find that the favorable performance of the conditional *multifactor* portfolio compared to that of the volatility-managed *individual-factor* portfolios is driven mainly by the benefits of trading and risk diversification across *multiple* factors. In particular, we find that while trading diversification across an individual factor and its managed counterpart leads to only a modest reduction in transaction costs, trading diversification across multiple factors leads to a substantial reduction in the transaction costs of the conditional multifactor portfolio. We also find that the market and size factors are negatively correlated to the other seven factors, and thus, multifactor portfolios benefit from risk diversification across factors.

Barroso and Detzel (2020) show that the volatility-managed *market* portfolio outperforms the market during high-sentiment periods, but it underperforms the market during low-sentiment periods. This suggests that, consistent with Yu and Yuan (2011), the performance of the volatility-managed market portfolio is driven by sentiment traders who undermine the positive risk-return tradeoff during high-sentiment periods. Moti-

[1]Note that the conditional fixed-weight multifactor portfolios assign the same relative weight to each factor as the unconditional multifactor portfolio.

[2]Liu, Tang, and Zhou (2019) also highlight the problems in achieving the gains from volatility-managed portfolios out of sample.

vated by this finding, we assess the performance of the conditional mean-variance multifactor portfolio in low- and high-sentiment periods. Consistent with Barroso and Detzel (2020, panel E of table 8), we find that sentiment also explains the in-sample performance of the conditional multifactor portfolio, which significantly outperforms its unconditional counterpart only during high-sentiment periods. In contrast, the *out-of-sample* performance of the conditional multifactor portfolio is significantly better than that of the unconditional portfolio during *both* high- and low-sentiment periods. We conclude that sentiment does not explain the out-of-sample and net-of-costs performance of our proposed multifactor strategy.

Our work is related to the literature on factor timing. Ehsani and Linnainmaa (2019) and Gupta and Kelly (2019) study the performance of factor-momentum strategies, which rely on the positive autocorrelation of factor returns. Gómez-Cram (2021) shows that the market can be timed using a business-cycle predictor derived from macroeconomic data. There are also papers that, like ours, study the timing of *combinations* of factors. For instance, Miller, Li, Zhou, and Giamouridis (2015) develop a dynamic portfolio approach using classification-tree analysis. Bass, Gladstone, and Ang (2017), Hodges, Hogan, Peterson, and Ang (2017), Amenc, Esakia, Goltz, and Luyten (2019), and Bender, Sun, and Thomas (2019) study factor portfolios conditional on macroeconomic state variables. Blin, Ielpo, Lee, and Teiletche (2018) study alternative risk premia conditional on macroeconomic regimes identified using Nowcasting. De Franco, Guidolin, and Monnier (2017) consider a multivariate Markov regime-switching model for the three traditional Fama-French factors. Haddad, Kozak, and Santosh (2020) time the market and the first five principal components of a large set of equity factors using the book-to-market spread of the principal components as the timing variable. In contrast to these papers, our focus is on multifactor portfolios whose weights change with market volatility, which allows us to examine the risk-return tradeoff.

Our work is also related to the literature on the relation between *market* risk and return across time. While a number of papers find a positive relation between market risk and return (French, Schwert, and Stambaugh, 1987; Campbell and Hentschel, 1992), others find a negative relation (Breen, Glosten, and Jagannathan, 1989; Nelson, 1991; Glosten, Jagannathan, and Runkle, 1993). We study the risk-return relation for all nine factors in our dataset and find that it varies substantially across factors. Our conditional multifactor portfolio exploits this by timing some of the factors more aggressively than others.

The rest of the paper is organized as follows. Section 2 describes our data and methodology for constructing conditional mean-variance multifactor portfolios. Section 3 reports performance gains from using conditional mean-variance multifactor portfolios. Section 4 investigates the source of the gains from using the conditional mean-variance multifactor portfolios. Section 5 concludes. The Internet Appendix reports the following robustness checks and additional results: evaluating performance during periods of high market volatility, excluding the market factor from the conditional multifactor portfolios, constraining the conditional multifactor portfolio leverage, timing each factor using its own volatility instead of market volatility, exploiting each factor's value spread in addition to market volatility as a conditioning variable, exploiting business-cycle variables in addition to market volatility as conditioning variables, using a less parsimonious conditional multifactor portfolio, evaluating performance using alternative measures of risk, considering the out-of-sample factor weights to explain performance, and studying the correlation of returns across individual factors.

2 Data and methodology

In this section, we first describe the data used for our empirical analysis and then explain how we construct conditional multifactor portfolios and account for transaction costs.

2.1 Data

We combine data from CRSP and Compustat for every stock traded on the NYSE, AMEX, and NASDAQ exchanges from January 1967 to December 2020.[3] We then drop stocks for firms with negative book-to-market ratio.

[3] Moreira and Muir (2017) use data from 1926 to 2015 for MKT, SMB, HML, and MOM, from 1963 to 2015 for RMW and CMA, and from 1967 to 2015 for ROE and IA. Our multifactor analysis exploits all nine factors, so in order to ensure that we have data for all the factors over the entire sample period, our sample spans from 1967 to 2020.

We download from the authors' websites gross returns for the market (MKT), small-minus-big (SMB), high-minus-low (HML), robust-minus-weak (RMW), and conservative-minus-aggressive (CMA) factors of Fama and French (2015), the momentum (MOM) factor of Carhart (1997), the profitability (ROE) and investment (IA) factors of Hou, Xue, and Zhang (2015), and the betting-against-beta (BAB) factor of Frazzini and Pedersen (2014).[4] We then replicate these nine value-weighted long-short portfolios in order to estimate the transaction costs required to trade the stocks comprising the factor portfolios. In unreported results, we confirm that the returns of our factors have a correlation of more than 90% with those of the original factors.

For the out-of-sample analysis, we use an expanding-window approach, with the first estimation window consisting of 120 months starting from January 1967. Thus, the out-of-sample results are for the period January 1977 to December 2020. In order to ensure a fair comparison with the out-of-sample results, the in-sample results are reported for the same period, January 1977 to December 2020.

2.2 Conditional mean-variance multifactor portfolios

An *individual* volatility-managed factor can be defined as

$$f_{t+1}^{\sigma} = \frac{c}{(\sigma_t^f)^2} f_{t+1}, \tag{1}$$

where f_{t+1} is the unmanaged factor return in month $t+1$, $(\sigma_t^f)^2$ is the monthly variance of the factor at time t estimated using the daily returns of month t, and c is a scalar that ensures that the volatility of the managed factor f_{t+1}^{σ} coincides with that of the unmanaged factor f_{t+1}.

Although the bulk of their analysis focuses on individual factors, Moreira and Muir (2017) also consider timing the unconditional mean-variance multifactor portfolio. In particular, they construct the optimal combination of the unconditional mean-variance multifactor portfolio and its managed counterpart, obtained by scaling the unconditional portfolio by the inverse of its past-month return variance. The resulting portfolio assigns

[4] All factors are returns of long-short portfolios. This is true also for MKT because we use returns on the market in excess of the risk-free return.

the same relative weight to each factor as the unconditional mean-variance multifactor portfolio, and thus, we refer to it as the "conditional fixed-weight multifactor portfolio."

In contrast to timing individual factors or a conditional fixed-weight multifactor portfolio, we consider a conditional mean-variance multifactor portfolio that allows the relative weights of the different factors to vary over time and that uses inverse market volatility as the conditioning variable. We employ a common conditioning variable across all factors for simplicity, but our results are robust to using the inverse volatility of each factor as its own conditioning variable, as shown in Section IA.4 of the Internet Appendix.[5] Also, we use market volatility instead of market variance as our conditioning variable because Moreira and Muir (2017, section II.B) and Barroso and Detzel (2020, section 3.3) point out that using volatility can help reduce the transaction costs of volatility-managed factor portfolios.[6]

A conditional multifactor portfolio at time t, $w_t(\theta_t) \in \mathbb{R}^{N_t}$, can be expressed as

$$w_t(\theta_t) = \sum_{k=1}^{K} x_{k,t}\theta_{k,t}, \tag{2}$$

where $x_{k,t} \in \mathbb{R}^{N_t}$ is the long-short portfolio associated to the kth factor at time t. For parsimony, we parameterize each factor weight, $\theta_{k,t}$, as an affine function of the inverse of market volatility,

$$\theta_{k,t} = a_k + b_k \frac{1}{\sigma_t}, \tag{3}$$

where σ_t is the market volatility estimated from daily market returns realized over the previous month and $a_k, b_k \in \mathbb{R}$ for $k = 1, 2, \ldots, K$. Note that this specification allows for the weight of each factor to vary differently with market volatility because, in general, $b_i \neq b_j$ for $i \neq j$. Defining $r_{k,t+1} \equiv x_{k,t}^{\top} r_{t+1} \in \mathbb{R}$ to be the return of the kth long-short

[5] This finding is consistent with Moreira and Muir (2017), who find that "because realized volatility is highly correlated across factors, normalizing by a common volatility factor does not drastically change our results." Indeed, we find that in our dataset the first principal component of the nine factor variances explains 75% of the total variability, and the correlations of the different factor variances with the first principal component are all above 35%, with the market variance having a correlation of 96% with the first principal component.

[6] Moreover, Cejnek and Mair (2021) show that using volatility also reduces leverage.

portfolio at time $t+1$, the return of a conditional multifactor portfolio is

$$r_{p,t+1}(\theta_t) = \sum_{k=1}^{K} r_{k,t+1}\theta_{k,t} = \sum_{k=1}^{K} r_{k,t+1}\left(a_k + b_k \frac{1}{\sigma_t}\right), \tag{4}$$

where the second equality follows from substituting (3) into (2).

For convenience, we also define the "extended" long-short portfolio-weight matrix $X_{ext,t}$, factor-return vector $r_{ext,t+1}$, and factor-weight vector η as:

$$X_{ext,t} \equiv \begin{bmatrix} x_{1t}^\top \\ x_{2t}^\top \\ \vdots \\ x_{Kt}^\top \\ x_{1t}^\top \times \frac{1}{\sigma_t} \\ x_{2t}^\top \times \frac{1}{\sigma_t} \\ \vdots \\ x_{Kt}^\top \times \frac{1}{\sigma_t} \end{bmatrix}^\top, \quad r_{\text{ext},t+1} \equiv \begin{bmatrix} r_{1,t+1} \\ r_{2,t+1} \\ \vdots \\ r_{K,t+1} \\ r_{1,t+1} \times \frac{1}{\sigma_t} \\ r_{2,t+1} \times \frac{1}{\sigma_t} \\ \vdots \\ r_{K,t+1} \times \frac{1}{\sigma_t} \end{bmatrix}, \quad \text{and} \quad \eta \equiv \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_K \\ b_1 \\ b_2 \\ \vdots \\ b_K \end{bmatrix}, \tag{5}$$

respectively. Then, the conditional *mean-variance* multifactor portfolio is given by the extended factor-weight vector, η, that optimizes the mean-variance utility net of transaction costs of an investor with risk-aversion parameter γ:

$$\max_{\eta \geq 0} \quad \widehat{\mu}_c^\top \eta - \text{TC}(\eta) - \frac{\gamma}{2}\eta^\top \widehat{\Sigma}_c \eta, \tag{6}$$

in which $\widehat{\mu}_c^\top \eta$ and $\eta^\top \widehat{\Sigma}_c \eta$ are the mean and variance of the conditional multifactor portfolio return, respectively, $\widehat{\Sigma}_c$ and $\widehat{\mu}_c$ are the sample covariance matrix and mean of the extended factor-return vector, and $\text{TC}(\eta)$ is its transaction cost. To alleviate the impact of estimation error, we discipline the conditional multifactor portfolios by assigning a nonnegative weight to each unmanaged factor $a_k \geq 0$ and a higher weight to each factor when volatility is low $b_k \geq 0$; that is, we impose the constraint that $\eta \geq 0$.

2.3 Modeling transaction costs

We now explain how we model the transaction costs of a conditional multifactor portfolio. Given an estimation window with T historical observations of stock returns and factor long-short portfolios, the average transaction cost incurred by rebalancing the conditional multifactor portfolio can be estimated as

$$\text{TC}(\eta) = \frac{1}{T-1}\sum_{t=1}^{T-1} \|\Lambda_t(w_{t+1}(\eta) - w_t(\eta)^+)\|_1, \tag{7}$$

where $\|a\|_1 = \sum_{i=1}^{N} |a_i|$ denotes the 1-norm of the N-dimensional vector a, $w_t(\eta)^+$ is the conditional multifactor portfolio before rebalancing at time $t+1$, that is

$$w_t(\eta)^+ = w_t(\eta) \circ (e_t + r_{t+1}), \tag{8}$$

e_t is the N_t-dimensional vector of ones, and $x \circ y$ is the Hadamard or componentwise product of vectors x and y. The transaction-cost matrix at time t, Λ_t, is the diagonal matrix whose ith diagonal element contains the transaction cost parameter $\kappa_{i,t}$ of stock i at time t. Note that the transaction-cost term in Equation (7) accounts for the netting of the rebalancing trades across multiple factors; that is, for the trading diversification effect identified by DeMiguel et al. (2020). In particular, the transaction-cost term is computed by first aggregating the rebalancing trades of the K factor portfolios and then charging the transaction cost at the individual-stock level.

To disentangle the benefit from trading diversification, we also compute the transaction costs ignoring the netting of trades across factors. In this case, in contrast to (7), we estimate the transaction costs of the conditional multifactor portfolio by charging for the transaction cost *before* aggregating the rebalancing trades across the K factors:

$$\text{TC}(\eta) = \frac{1}{T-1}\sum_{t=1}^{T-1}\sum_{k=1}^{K} \|\Lambda_t\left(x_{k,t+1}\theta_{k,t+1} - x_{k,t}^+\theta_{k,t}\right)\|_1, \tag{9}$$

where $x_{k,t}^+ = x_{k,t} \circ (e_t + r_{t+1})$.

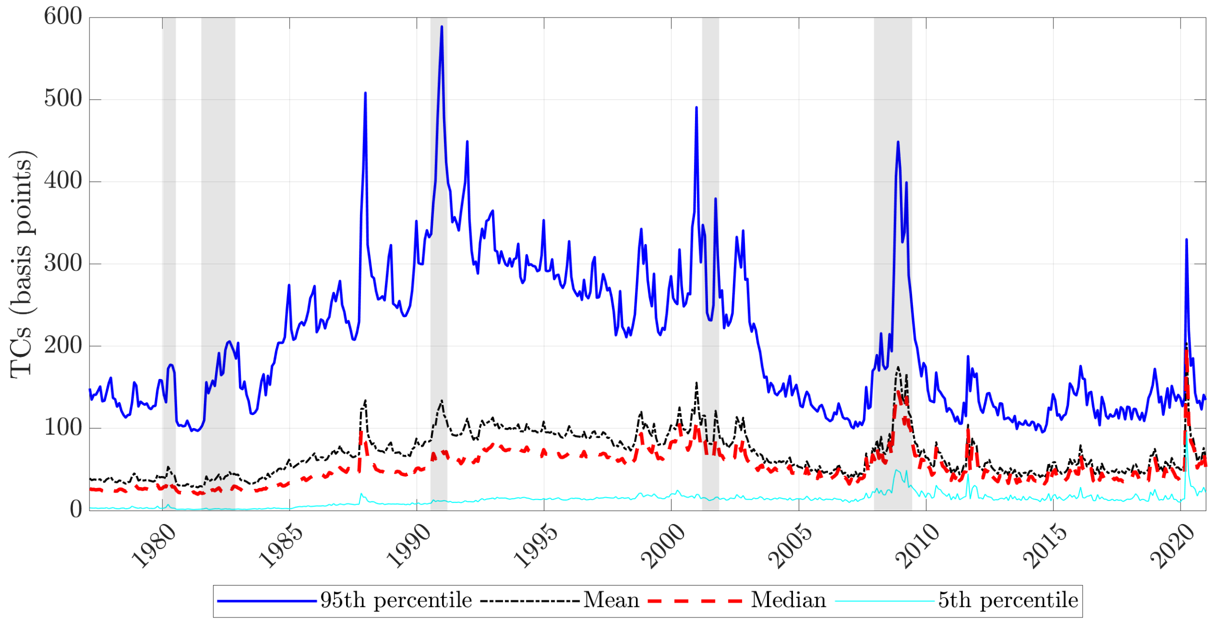
To estimate the stock-level transaction costs parameter $\kappa_{i,t}$, we use the *two-day corrected* method proposed in Abdi and Ranaldo (2017) to estimate the monthly bid-ask spread of the ith stock as:

$$\widehat{s}_{i,t} = \frac{1}{D}\sum_{d=1}^{D}\widehat{s}_{i,d}, \quad \widehat{s}_{i,d} = \sqrt{\max\{4(\text{cls}_{i,d} - \text{mid}_{i,d})(\text{cls}_{i,d} - \text{mid}_{i,d+1}), 0\}}, \tag{10}$$

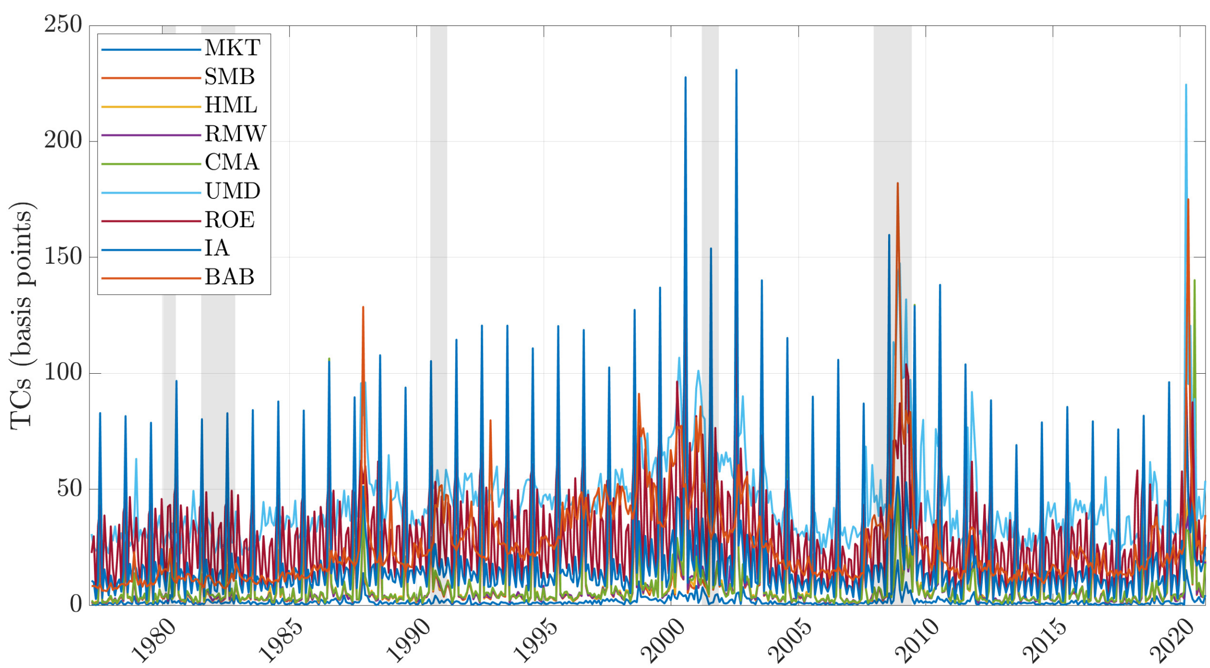
Figure 1: Proportional transaction costs

This figure depicts several descriptive statistics of the transaction costs for stocks (Panel A) and factors (Panel B) estimated using the method of Abdi and Ranaldo (2017) described in equation (10) for the out-of-sample period January 1977 to December 2020.

Panel A: Individual-stock level



Panel B: Factor level



where D is the number of days in month t, $\widehat{s}_{i,d}$ is the *two-day* bid-ask spread estimate, $\text{cls}_{i,d}$ is the closing log-price on day d, and $\text{mid}_{i,d}$ is the mid-range log-price on day d; that is, the mean of daily high and low log-prices. Finally, because the effective trading cost is half the bid-ask spread, the transaction-cost parameter for the ith stock is $\kappa_{i,t} = \widehat{s}_{i,t}/2$.

Figure 1 depicts transaction costs for the period January 1977 to December 2020 at the individual-stock level (Panel A) and at the factor level (Panel B). There are several interesting observations from these two panels. First, transaction costs of individual stocks are highly time varying, with the variation being particularly strong for less liquid stocks. The transaction cost of equity factors is also time varying. The most prominent example is momentum (UMD), whose average transaction cost is the highest among the factors we consider. We also observe that the transaction costs of the accounting-based factors, such as CMA and RMW, are cyclical because we update the accounting-based factors when new accounting information becomes available, which leads to substantial rebalancing of the portfolio.

3 Performance gains from volatility management

In this section, we study the economic gains from volatility management. Section 3.1 evaluates the performance of the volatility-managed individual-factor portfolios, which are the focus of the existing literature. Section 3.2 evaluates the performance of our proposed conditional mean-variance multifactor potfolio. Finally, Section 3.3 studies whether sentiment explains the out-of-sample and net-of-costs gains from the conditional multifactor portfolios.

3.1 Volatility-managed individual-factor portfolios

To set the stage for the analysis of our multifactor portfolios, we first evaluate the performance of the volatility-managed *individual-factor* portfolios, which are the focus of Moreira and Muir (2017). We then evaluate the performance of these strategies net of transaction costs and out of sample, which allows us to confirm the findings of Barroso and Detzel (2020) and Cederburg et al. (2020), respectively.

Table 1: Performance of volatility-managed individual-factor portfolios

For each of the nine factors we consider, this table reports the annualized Sharpe ratios of the unmanaged factor, SR(f), and the volatility-managed individual-factor portfolio, SR(f, f^σ), which is the mean-variance combination of the unmanaged factor with its managed counterpart given in (1), and the p-value for the difference in Sharpe ratios. We consider an investor with risk-aversion parameter $\gamma = 5$. Panel A reports performance in-sample and ignoring transaction costs, Panel B in-sample and net of costs but ignoring trading diversification, Panel C out-of-sample and ignoring costs, Panel D out-of-sample and net of costs but ignoring trading diversification, and Panel E out-of-sample and net of costs considering trading diversification. Our sample spans January 1967 to December 2020 and we evaluate out-of-sample performance using an expanding window with the first estimation window containing the first 120 months of data. Thus, the out-of-sample results are for the period January 1977 to December 2020. The out-of-sample return of each volatility-managed individual-factor portfolio is evaluated for the month following the last month of each estimation window. In order to ensure a fair comparison with the out-of-sample results, the in-sample results are reported for the same period, January 1977 to December 2020.

	MKT	SMB	HML	RMW	CMA	UMD	ROE	IA	BAB
Panel A: In-sample without transaction costs									
SR(f)	0.530	0.208	0.170	0.506	0.399	0.474	0.722	0.508	0.880
SR(f, f^σ)	0.585	0.246	0.215	0.739	0.419	1.088	1.153	0.621	1.397
p-value(SR(f, f^σ)-SR(f))	0.244	0.376	0.338	0.038	0.308	0.000	0.001	0.099	0.000
Panel B: In-sample net of transaction costs but without trading diversification									
SR(f)	0.519	0.126	0.054	0.357	0.162	0.117	0.313	0.109	0.627
SR(f, f^σ)	0.521	0.126	0.054	0.357	0.162	0.261	0.335	0.109	0.740
p-value(SR(f, f^σ)-SR(f))	0.464	0.500	0.500	0.500	0.500	0.223	0.389	0.500	0.127
Panel C: Out-of-sample without transaction costs									
SR(f)	0.530	0.208	0.170	0.506	0.399	0.474	0.722	0.508	0.880
SR(f, f^σ)	0.408	0.068	0.194	0.527	0.355	1.035	1.094	0.605	1.321
p-value(SR(f, f^σ)-SR(f))	0.900	0.933	0.390	0.452	0.897	0.000	0.001	0.063	0.000
Panel D: Out-of-sample net of transaction costs but without trading diversification									
SR(f)	0.519	0.126	0.054	0.357	0.162	0.117	0.313	0.109	0.627
SR(f, f^σ)	0.325	−0.292	−0.038	−0.442	−0.043	0.204	0.274	−0.122	0.727
p-value(SR(f, f^σ)-SR(f))	0.979	1.000	0.879	0.999	1.000	0.324	0.672	1.000	0.249
Panel E: Out-of-sample net of transaction costs with trading diversification									
SR(f)	0.519	0.126	0.054	0.357	0.162	0.117	0.313	0.109	0.627
SR(f, f^σ)	0.432	0.033	0.090	0.229	0.155	0.216	0.328	0.196	0.773
p-value(SR(f, f^σ)-SR(f))	0.925	0.873	0.256	0.965	0.561	0.091	0.402	0.031	0.059

For each of the nine factors we consider, Table 1 reports the annualized Sharpe ratio of the unmanaged factor, SR(f), and the volatility-managed individual-factor portfolio, SR(f, f^σ), which is the mean-variance combination of the unmanaged factor with its managed counterpart given in (1), and the p-value for the difference in Sharpe ratios. We consider an investor with risk-aversion parameter $\gamma = 5$. Panel A reports performance in-sample and ignoring transaction costs, Panel B in-sample and net of costs but ignoring trading diversification, Panel C out-of-sample and ignoring costs, Panel D out-of-sample and net of costs but ignoring trading diversification, and Panel E out-of-sample and net of costs with trading diversification. Our sample spans January 1967 to December 2020 and, similar to the base-case analysis in Cederburg et al. (2020), we evaluate out-of-sample performance using an expanding window with the first estimation window containing the first 120 months of data.[7] Thus, the out-of-sample results are for the period January 1977 to December 2020. The out-of-sample return of each volatility-managed individual-factor portfolio is evaluated for the month following the last month of each estimation window. In order to ensure a fair comparison with the out-of-sample results, the in-sample results are reported for the same period, January 1977 to December 2020.

Panel A of Table 1 confirms the main finding by Moreira and Muir (2017): in-sample and ignoring transaction costs, the Sharpe ratio of each volatility-managed individual-factor portfolio, $\text{SR}(f, f^\sigma)$, is greater than that of the unmanaged factor, $\text{SR}(f)$, for all nine factors, with the difference being statistically significant at the 10% level for five of the factors (RMW, UMD, ROE, IA, and BAB).[8]

Panel B reports the performance in-sample and net of transaction costs but, consistent with Barroso and Detzel (2020), ignoring the trading-diversification benefits from combining the unmanaged and managed factors. Comparing Panels A and B, we observe that transaction costs greatly diminish the performance of the volatility-managed individual-factor portfolios. In fact, the transaction cost of the managed factor is so large for five of the nine factors—SMB, HML, RMW, CMA, and IA—that when considering the optimal combination of the unmanaged and the volatility-managed factors, the investor assigns a zero weight to the managed factor, which explains why the Sharpe ratio of the individual-factor portfolio is equal to that of the unmanaged factor. For the other four factors, the improvement in Sharpe ratio from volatility management is not statistically significant. Thus, we conclude that even in sample the gains from volatility

[7] Cederburg et al. (2020) report that their results are not sensitive to the length of the estimation window: "We therefore consider specifications with 20-year ($K = 240$) and 30-year ($K = 360$) initial estimation periods. These designs produce roughly the same number of positive Sharpe ratio and CER differences that the base case does." They also report that their results are not sensitive to the value chosen for the risk aversion parameter: "Using a lower ($\gamma = 2$) or higher ($\gamma = 10$) risk aversion parameter leads to almost identical results to the base case with $\gamma = 5$."

[8] In unreported results, we also observe that the alphas of the nine volatility-managed individual-factor portfolios with respect to their unmanaged counterparts are positive, and they are statistically significant for the same five factors.

management are completely eroded by transaction costs, confirming the result in Barroso and Detzel (2020).

Panel C shows that the *out-of-sample* Sharpe ratio of the volatility-managed individual-factor portfolios in the absence of transaction costs is lower than the in-sample Sharpe ratio in Panel A for all nine factors. We also observe that the optimal combination of the unmanaged and volatility-managed factors delivers an out-of-sample Sharpe ratio, $\mathrm{SR}(f, f^{\sigma})$, that can be smaller than that of even the unmanaged factor, $\mathrm{SR}(f)$; this is the case for the MKT, SMB, and CMA factors.[9] The out-of-sample gains from volatility management are statistically significant at the 10% level for only four of the nine factors (UMD, ROE, IA, BAB). Overall, our results show that, consistent with Cederburg et al. (2020), estimation error diminishes the gains from volatility management.

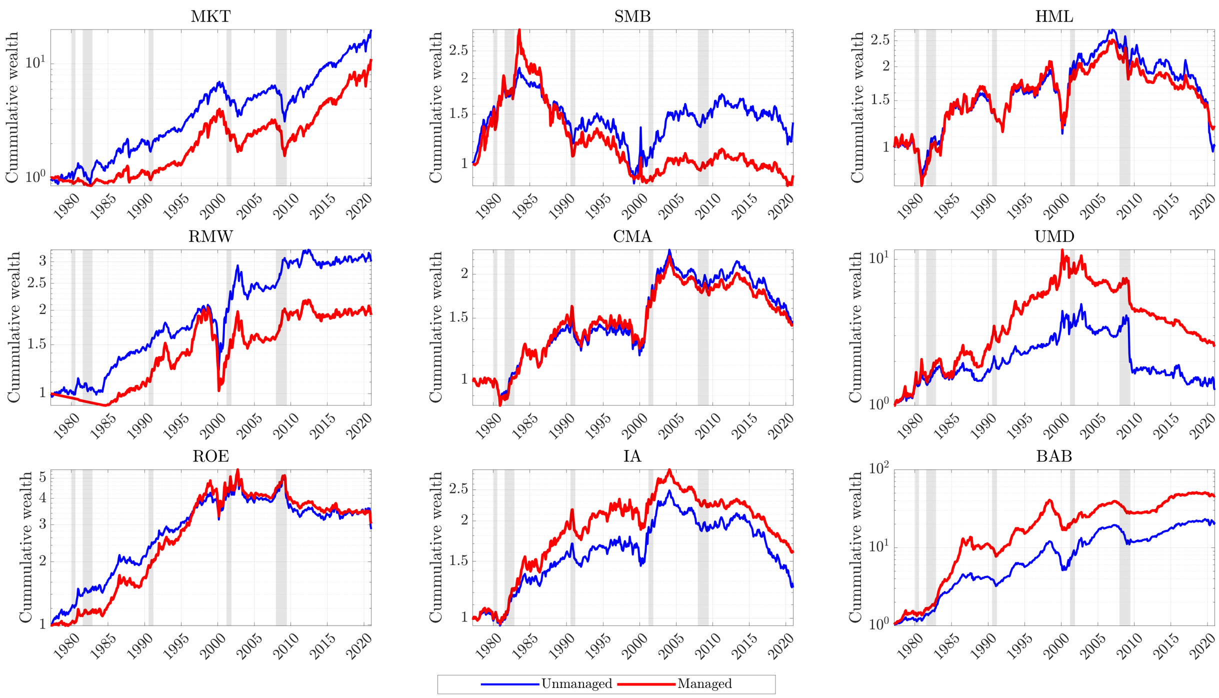
Panel D shows that transaction costs erode the out-of-sample performance further. In particular, once we account for *both* transaction costs and estimation risk, the Sharpe ratio for five out of the nine volatility-managed individual-factor portfolios becomes negative. Moreover, the Sharpe ratio of the optimal combination of the unmanaged and volatility-managed factor is lower than that of the corresponding unmanaged factor for all factors except UMD and BAB, with neither being statistically significant, which drives home the point that estimation risk and transaction costs erode entirely the gains from volatility-managing individual factors.

Comparing the Sharpe ratios of the volatility-managed individual-factor portfolios, $\mathrm{SR}(f, f^{\sigma})$, in Panels D and E, we find that accounting for the netting of trades across the unmanaged and managed factors improves the performance of all nine portfolios. Moreover, with trading diversification, volatility management improves the performance for five out of the nine factors, with the improvement being statistically significant at the 10% level for three factors (UMD, IA, BAB). Thus, trading diversification alleviates partially the concerns raised by Barroso and Detzel (2020) and Cederburg et al. (2020), but it does not fully resurrect the gains from volatility managing individual factors.

[9] Note that the unmanaged factors do not require any estimation and thus their in-sample and out-of-sample Sharpe ratios coincide. Moreover, there are no trading-diversification benefits from trading the unmanaged factors in isolation, and thus, their Sharpe ratio with costs is the same whether accounting for trading diversification or not.

Figure 2: Cumulative returns of individual factors

The nine graphs in this figure depict the out-of-sample cumulative returns net of transaction costs of each unmanaged factor (blue line) and its associated volatility-managed individual factor portfolio (red line) scaled to have the same volatility as the unmanaged factor. The cumulative returns are depicted on a logarithmic scale for the out-of-sample period from January 1977 to December 2020.



To illustrate these results, Figure 2 depicts the out-of-sample cumulative returns net of transaction costs with trading diversification of each unmanaged factor (blue line) and its associated volatility-managed individual-factor portfolio (red line) scaled to have the same volatility as the unmanaged factor over the out-of-sample period January 1977 to December 2020. These plots show again that, with trading diversification, volatility management improves the performance for five out the nine factors, although (as shown in Table 1) the difference is statistically significant for only three factors.

We conclude from the evidence presented above that, consistent with the findings of Barroso and Detzel (2020) and Cederburg et al. (2020), a volatility-managed portfolio based on an *individual* factor typically fails to significantly outperform its unmanaged counterpart when performance is measured out of sample and net of transaction costs.

3.2 Conditional mean-variance multifactor portfolio

In the previous section, we evaluated the performance of the volatility-managed individual-factor portfolios, which are the focus of the existing literature. In this section, we provide a multifactor perspective on volatility management by evaluating the benefits from volatility management for an investor who has access to multiple factors. To do this, we compare the out-of-sample and net-of-costs performance of two portfolios: the conditional mean-variance multifactor portfolio (CMV) obtained by solving problem (6) and the unconditional mean-variance multifactor portfolio (UMV) obtained by solving problem (6) under the additional constraint that $b_k = 0$ for $k = 1, 2, \dots, K$; that is, under the constraint that its weights on the K factors do not vary with market volatility.

For each multifactor portfolio, Table 2 reports the out-of-sample annualized mean, standard deviation, Sharpe ratio of returns net of transaction costs, and p-value for the difference between the Sharpe ratios of the conditional and unconditional portfolios.[10] For completeness, the table also reports the alpha of the time-series regression of the conditional portfolio out-of-sample returns net of transaction costs on those of the unconditional portfolio, the Newey-West t-statistic for the alpha, and the out-of-sample transaction costs accounting for trading diversification of the unconditional and conditional portfolios. The portfolios are constructed exploiting all nine factors in our dataset. We use an expanding-window approach and the out-of-sample period spans from January 1977 to December 2020.

Table 2 shows that the conditional mean-variance multifactor portfolio delivers an out-of-sample Sharpe ratio of net returns that is significantly larger than that of the unconditional portfolio. In particular, the conditional portfolio achieves a Sharpe ratio of 1.126, which is around 16% higher than that of the unconditional portfolio, with the difference being statistically significant at the 1% level. The conditional portfolio also has an annualized alpha of 8.195%, with a significant t-statistic of 4.416. The table

[10] Consistent with the conditional multifactor portfolio problem (6), we compute the annualized net mean return as the difference between the out-of-sample gross mean return and the transaction cost, $E[r_{p,t+1}] - \text{TC}$, the standard deviation as $\text{stdev}(r_{p,t+1})$, and the Sharpe ratio as the ratio of these two quantities. We construct one-sided p-values for the difference in Sharpe ratios from 10,000 bootstrap samples using the stationary block-bootstrap method of Politis and Romano (1994) with an average block size of five and the procedure of Ledoit and Wolf (2008, Remark 3.2) to produce the resulting p-values.

Table 2: Performance of conditional mean-variance multifactor portfolio

This table reports the out-of-sample and net-of-costs performance of two multifactor portfolios: the conditional mean-variance multifactor portfolio (CMV) obtained by solving problem (6) and the unconditional mean-variance multifactor portfolio (UMV) obtained by solving problem (6) under the additional constraint that $b_k = 0$ for $k = 1, 2, \ldots, K$; that is, under the constraint that its weights on the K factors are constant over time. For each multifactor portfolio, the table reports the out-of-sample annualized mean, standard deviation, Sharpe ratio of returns net of transaction costs accounting for trading diversificaqtion, and p-value for the difference between the Sharpe ratios of the conditional and unconditional portfolios. The table also reports the alpha of the time-series regression of the conditional portfolio out-of-sample returns net of transaction costs on those of the unconditional portfolio, alpha Newey-West t-statistic, and out-of-sample transaction costs of the unconditional and conditional portfolios. The portfolios are constructed exploiting all nine factors in our dataset. Our sample spans January 1967 to December 2020 and we evaluate out-of-sample performance using an expanding window with the first estimation window containing the first 120 months of data. Thus, the out-of-sample results are for the period January 1977 to December 2020. The out-of-sample return of each multifactor portfolio is evaluated for the month following the last month of each estimation window.

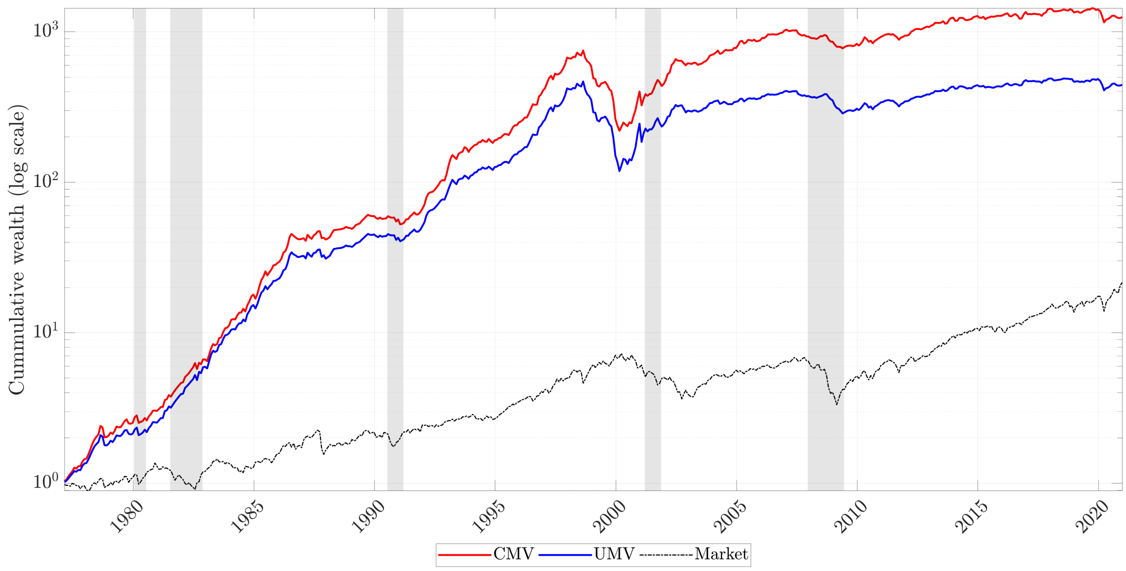
	UMV	CMV
Mean	0.446	0.507
Standard deviation	0.459	0.450
Sharpe ratio	0.971	1.126
p-value(SR_{CMV}-SR_{UMV})	–	0.001
α	–	8.195
$t(\alpha)$	–	4.416
TC	0.149	0.188

shows that the conditional portfolio has a slightly lower volatility than its unconditional counterpart and, although the conditional portfolio incurs larger transaction costs, its gross mean return more than compensates for the additional trading costs associated with factor timing.

Figure 3 plots the cumulative out-of-sample net returns of three portfolios: (1) the market, (2) the unconditional mean-variance multifactor portfolio (UMV), and (3) the conditional mean-variance multifactor portfolio (CMV). The returns are reported in logarithmic scale and the conditional and unconditional portfolios are standardized so that their returns have the same volatility as the market return. Figure 3 shows that the strategies that exploit all nine factors dramatically outperform the market. In addition, the conditional multifactor portfolio outperforms the unconditional portfolio. Investing \$1 in January 1977 generates about \$21 for the market portfolio (equivalent to a per annum return of 7.16%), \$445 for the unconditional mean-variance multifactor portfolio

Figure 3: Cumulative returns of multifactor portfolios

This figure depicts the return from investing in the market portfolio and the out-of-sample cumulative returns net of transaction costs of the unconditional and conditional mean-variance multifactor portfolios over the out-of-sample period January 1977 to December 2020. The returns are reported in logarithmic scale and the conditional and unconditional portfolios are standardized so that their returns have the same volatility as the market return.



(equivalent to a per annum return of 14.87%), and about $1,253 for the conditional mean-variance multifactor portfolio that exploits market volatility (equivalent to an annual return of 17.60%). Overall, there are substantial economic gains from investing in the conditional mean-variance multifactor portfolio relative to its unconditional counterpart.

Finally, comparing the performance of the conditional mean-variance *multifactor* portfolio in Table 2 with the performance of the volatility-managed *individual*-factor portfolios in Panel E of Table 1, we see that, not surprisingly, the conditional multifactor portfolio also outperforms substantially the volatility-managed individual-factor portfolios out of sample and net of transaction costs.

3.3 Does sentiment explain performance?

Barroso and Detzel (2020) document that the volatility-managed *market* portfolio outperforms the market only following months of high sentiment, and it *underperforms* the market during low-sentiment periods. This suggests that, consistent with Yu and

Yuan (2011), the performance of the volatility-managed market portfolio is driven by sentiment traders who undermine the positive risk-return tradeoff during high-sentiment periods. Motivated by this finding, we now assess the performance of the conditional mean-variance multifactor portfolio in low- and high-sentiment regimes.

Table 3 compares the Sharpe ratios of the unconditional and conditional versions of the market portfolio and the multifactor portfolio for the entire sample, for high-sentiment periods, and for low-sentiment periods. Panels A and B, respectively, report the in-sample and out-of-sample Sharpe ratios of returns net of transaction costs accounting for trading diversification. We consider the Baker and Wurgler (2006) sentiment index orthogonalized to economic conditions and, like Barroso and Detzel (2020), we define high-sentiment (low-sentiment) years as those for which the sentiment index at the end of the prior year is above its median value for the entire sample. Our sample spans January 1967 to December 2018, for which the sentiment index of Baker and Wurgler (2006) is available, and we evaluate out-of-sample performance using an expanding window with the first estimation window containing the first 120 months of data. Thus, the out-of-sample results are for the period January 1977 to December 2018. In order to ensure a fair comparison with the out-of-sample results, the in-sample results are reported for the same period, January 1977 to December 2018.

Consistent with Barroso and Detzel (2020, panel E of table 8), Panel A of Table 3 shows that the in-sample performance of the volatility-managed market portfolio is better than that of the unmanaged market factor only during high-sentiment periods, with a p-value that is significant at the 10% confidence level. Panel A shows that sentiment also explains the in-sample gains from the conditional multifactor portfolio, which significantly outperforms its unconditional counterpart during high-sentiment periods but not low-sentiment periods. However, the out-of-sample results in Panel B are markedly different. The volatility-managed market portfolio underperforms out of sample the unmanaged market factor during both low- and high-sentiment periods. In contrast, the out-of-sample performance of the conditional multifactor portfolio is significantly better than that of the unconditional multifactor portfolio during both high- and low-sentiment periods. We conclude that sentiment does not explain the out-of-sample and net-of-costs performance of our conditional multifactor portfolio.

Table 3: Performance for high- and low-sentiment periods

This table compares the Sharpe ratios of the unconditional and conditional versions of the market portfolio and the multifactor portfolio for the entire sample, for high-sentiment periods, and for low-sentiment periods. Panels A and B, respectively, report the in-sample and out-of-sample Sharpe ratios of returns net of transaction costs accounting for trading diversification. We consider the Baker and Wurgler (2006) sentiment index orthogonalized to economic conditions and, like Barroso and Detzel (2020), we define high-sentiment (low-sentiment) years as those for which the sentiment index at the end of the prior year is above its median value for the entire sample. Our sample spans January 1967 to December 2018, for which the the sentiment index of Baker and Wurgler (2006) is available, and we evaluate out-of-sample performance using an expanding window with the first estimation window containing the first 120 months of data. Thus, the out-of-sample results are for January 1977 to December 2018. In order to ensure a fair comparison with the out-of-sample results, the in-sample results are reported for the same period, January 1977 to December 2018.

	Entire sample			High sentiment			Low sentiment		
	Uncond.	Cond.	p-val.	Uncond.	Cond.	p-val.	Uncond.	Cond.	p-val.
Panel A: In sample									
Market	0.519	0.532	0.295	0.178	0.217	0.093	0.954	0.940	0.656
Multifactor	1.130	1.339	0.000	1.250	1.594	0.000	1.102	1.150	0.282
Panel B: Out of sample									
Market	0.519	0.449	0.889	0.178	0.082	0.887	0.954	0.952	0.496
Multifactor	0.971	1.126	0.001	1.403	1.600	0.003	0.412	0.569	0.016

4 Understanding conditional multifactor portfolios

The results in the previous section demonstrate that the conditional mean-variance multifactor portfolio significantly outperforms its unconditional counterpart out of sample and net of transaction costs, and during both low- and high-sentiment periods. In this section, we undertake various experiments to understand the sources of the favorable performance of the conditional multifactor portfolio.

4.1 Disentangling the source of the gains

Each of the four panels of Table 4 reports the performance of a different method for *choosing* multifactor portfolio weights: Panel A for the conditional fixed-weight multifactor portfolio that ignores transaction costs, as considered by Moreira and Muir (2017);[11] Panel B for the conditional mean-variance multifactor portfolio that ignores transaction costs, which is obtained by solving problem (6) ignoring the transaction-cost term $\mathrm{TC}(\eta)$;

[11]Specifically, this portfolio is the optimal combination of the unconditional mean-variance multifactor portfolio and its managed counterpart, obtained by scaling the unconditional portfolio by the inverse of its past-month return variance, ignoring transaction costs.

Panel C for the conditional fixed-weight multifactor portfolio considered by Moreira and Muir (2017) extended to account for transaction costs and trading diversification; Panel D for the conditional mean-variance multifactor portfolio obtained by solving problem (6) accounting for transaction costs and trading diversification.

For each panel, each of the four columns reports the performance of the chosen weights *evaluated* in a different way: (1) in sample ignoring transaction costs, (2) out of sample ignoring transaction costs, (3) out of sample net of transaction costs but ignoring trading diversification, and (4) out of sample with transaction costs and accounting for trading diversification.

Column (1) in Panel A of Table 4 confirms the finding in Moreira and Muir (2017) that, in-sample and ignoring transaction costs, timing the unconditional mean-variance portfolio leads to a substantial increase in the Sharpe ratio and an economically and statistically significant alpha.[12] Column (2) shows that these gains are significant even out of sample, if one ignores transaction costs, although they are smaller than those in sample. This is in contrast to the result in Table 1 that the volatility-managed *individual-factor* portfolios typically fail to significantly outperform the unmanaged factor out of sample. However, Column (3) shows that accounting for transaction costs while ignoring trading diversification eliminates the gains from volatility-managing the unconditional mean-variance portfolio. Finally, Column (4) shows that if one accounts for trading diversification by netting out the rebalancing trades across the multiple factors, then timing the unconditional mean-variance portfolio leads to performance gains even out of sample and net of transaction costs. Thus, a key takeaway from Panel A is that it is crucial to net out trades when accounting for the transaction costs of volatility-managed multifactor portfolios.

In Panel B of Table 4, instead of using the conditional *fixed-weight* multifactor portfolio of Moreira and Muir (2017), we consider the conditional mean-variance multifactor portfolios obtained by solving problem (6), but ignoring transaction costs. Unlike the conditional fixed-weight multifactor portfolios in Panel A, these portfolios allow the

[12] Cederburg et al. (2020, ftn. 17) point out that the positive unconditional alphas of the individual-factor volatility-managed portfolios can be explained by the volatility-timing effects discussed by Lewellen and Nagel (2006) and Boguth, Carlson, Fisher, and Simutin (2011). A similar argument can be made for the unconditional alpha of the conditional fixed-weight multifactor portfolio.

Table 4: Understanding the performance of the multifactor portfolios

Each of the four panels of this table reports the performance of a different method for *choosing* multifactor portfolio weights: Panel A for the conditional fixed-weight multifactor portfolio that ignores transaction costs, as considered by Moreira and Muir (2017); Panel B for the conditional mean-variance multifactor portfolio that ignores transaction costs, which is obtained by solving problem (6) ignoring the transaction-cost term TC(η); Panel C for the conditional fixed-weight multifactor portfolio considered by Moreira and Muir (2017) extended to account for transaction costs and trading diversification; Panel D for the conditional mean-variance multifactor portfolio obtained by solving problem (6) accounting for transaction costs and trading diversification. For each panel, each of the four columns reports the performance of the chosen weights *evaluated* in a different way: (1) in sample ignoring transaction costs, (2) out of sample ignoring transaction costs, (3) out of sample net of transaction costs but ignoring trading diversification, and (4) out of sample with transaction costs and trading diversification. The sample period and quantities reported for each portfolio are the same as in Table 2.

	(1) In-sample without TC		(2) Out-of-sample without TC		(3) Out-of-sample with TC without trad. div.		(4) Out-of-sample with TC with trad. div.	
	UMV	Cond.	UMV	Cond.	UMV	Cond.	UMV	Cond.
Panel A: Conditional fixed-weight multifactor portfolio ignoring transaction costs								
Mean	0.415	0.602	0.753	0.783	0.413	0.316	0.531	0.530
Standard deviation	0.288	0.347	0.580	0.520	0.580	0.520	0.580	0.520
Sharpe ratio	1.441	1.735	1.299	1.506	0.713	0.608	0.916	1.019
p-value(SR_{CMV}-SR_{UMV})		0.000		0.000		0.985		0.012
α		18.724		13.986		−3.338		7.806
$t(\alpha)$		5.738		5.012		−1.529		3.322
TC					0.340	0.467	0.222	0.253
Panel B: Conditional mean-variance multifactor portfolio ignoring transaction costs								
Mean	0.415	0.680	0.753	0.925	0.413	0.354	0.531	0.615
Standard deviation	0.288	0.369	0.580	0.569	0.580	0.569	0.580	0.569
Sharpe ratio	1.441	1.844	1.299	1.625	0.713	0.622	0.916	1.080
p-value(SR_{CMV}-SR_{UMV})		0.000		0.000		0.922		0.023
α		21.293		23.855		−2.352		13.030
$t(\alpha)$		6.684		5.797		−0.631		3.479
TC					0.340	0.571	0.222	0.310
Panel C: Conditional fixed-weight multifactor portfolio optimizing transaction costs								
Mean	0.301	0.315	0.595	0.628	0.349	0.345	0.446	0.473
Standard deviation	0.219	0.223	0.459	0.445	0.459	0.445	0.459	0.445
Sharpe ratio	1.379	1.416	1.296	1.410	0.761	0.775	0.971	1.061
p-value(SR_{CMV}-SR_{UMV})		0.005		0.001		0.333		0.001
α		0.957		5.938		1.240		4.782
$t(\alpha)$		3.075		3.831		0.970		3.507
TC					0.246	0.283	0.149	0.155
Panel D: Conditional mean-variance multifactor portfolio optimizing transaction costs								
Mean	0.301	0.436	0.595	0.695	0.349	0.337	0.446	0.507
Standard deviation	0.219	0.252	0.459	0.450	0.459	0.450	0.459	0.450
Sharpe ratio	1.379	1.729	1.296	1.543	0.761	0.748	0.971	1.126
p-value(SR_{CMV}-SR_{UMV})		0.000		0.000		0.682		0.001
α		10.914		12.882		0.357		8.193
$t(\alpha)$		7.625		6.433		0.193		4.416
TC					0.246	0.358	0.149	0.188

relative weight of each factor to vary with market volatility. Comparing the Sharpe ratios of the conditional strategies in Panels A and B, we see that for all four evaluation methods the conditional mean-variance multifactor portfolio in Panel B outperforms the conditional fixed-weight multifactor portfolio in Panel A. For instance, under the most realistic evaluation method, which is the one given in Column (4), the conditional mean-variance multifactor portfolio achieves a Sharpe ratio that is around 6% higher than that of the conditional fixed-weight multifactor portfolio. Thus, the takeaway from Panel B is that allowing the relative weight of the different factors to vary with market volatility allows one to improve performance.[13]

In Panel C of Table 4, we consider the transaction-cost-optimized conditional fixed-weight multifactor portfolio. Comparing the results in Column (4) for Panels A and C, we observe that, optimizing the conditional fixed-weight multifactor portfolios for transaction costs increases the out-of-sample and net-of-costs Sharpe ratio by around 4%.

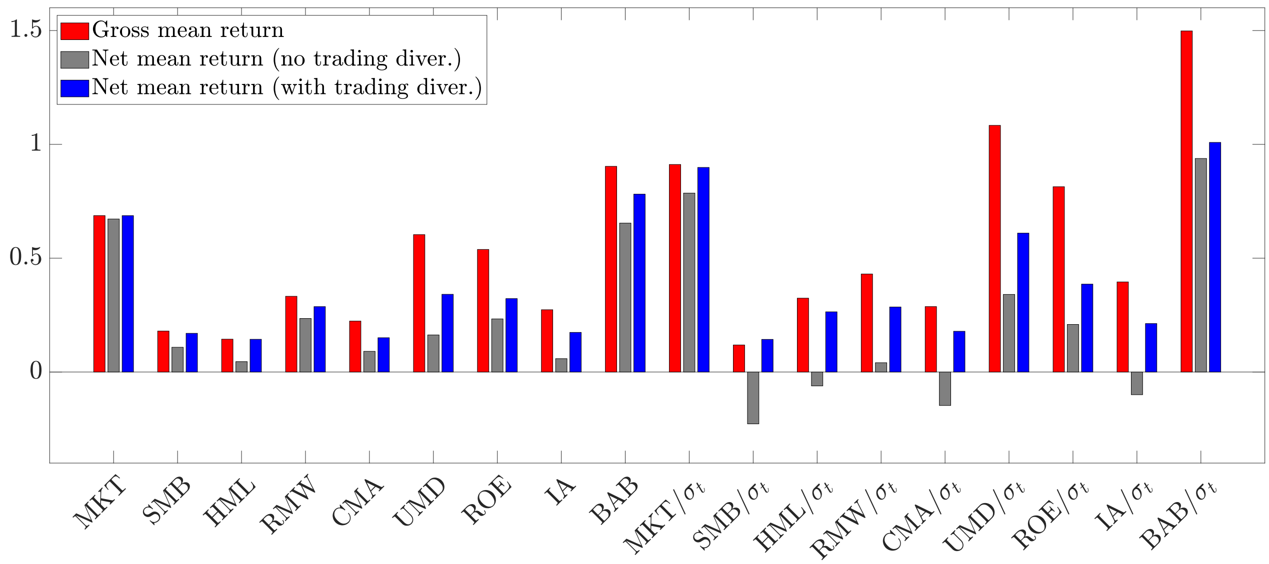
In Panel D of Table 4, we consider the conditional mean-variance multifactor portfolios whose weights are optimized accounting for transaction costs. Column (4) shows that the conditional mean-variance multifactor portfolio that accounts for transaction costs achieve the best out-of-sample and net-of-cost performance, with a Sharpe ratio that is around 10% higher than that of the conditional fixed-weight multifactor portfolio that ignores transaction costs and 5% higher than the conditional fixed-weight multifactor portfolio extended to account for transaction costs. The conditional portfolio in Panel D also incurs a lower transaction cost than the corresponding strategies in Panels A and B, where the factor weights are optimized ignoring transaction costs.

Summarizing, Table 4 shows that the favorable performance of our conditional multifactor portfolio compared to the unconditional and fixed-weight portfolios is explained by three elements: (i) taking trading diversification into account when evaluating performance, (ii) accounting for transaction costs and trading diversification when

[13] Note also that Cederburg et al. (2020, table 5, panel B) find that adding each managed factor to a portfolio that already includes its unmanaged counterpart plus the market, size, and value factors of Fama and French (1993) is often harmful. In contrast, we find that the conditional mean-variance portfolios that combine all nine managed factors with their unmanaged counterparts outperform the unconditional mean-variance portfolio.

Figure 4: Gross and net-of-costs mean factor returns

This barplot depicts the monthly average factor returns (in percentage) of the nine unmanaged and volatility-managed factors over the period January 1977 to December 2020. For each factor, we depict the gross mean return, the net mean return when the factor is exploited in isolation, that is, ignoring trading diversification, and the net mean return when the factor is exploited in combination with all the unmanaged and managed factors. For the case when the factors are exploited in combination, we use the factor weights that solve problem (6) in sample, that is, optimized accounting for transaction costs and trading diversification.



optimizing portfolio weights, and (iii) allowing the relative weights on different factors to vary with market volatility. In the rest of this section, we examine these elements.

4.2 Trading diversification of multifactor portfolios

We now investigate the source of the trading-diversification benefits that are one of the key drivers of the favorable performance of the conditional multifactor portfolios. To do this, Figure 4 compares three quantities for each factor: (i) its mean gross return, (ii) its mean return net of transaction costs when the factor is exploited in isolation, that is, ignoring trading diversification, and (iii) its mean return net of transaction costs when the factor is exploited in combination with all other factors, that is, taking trading diversification into account. For the case when the factors are exploited in combination, we use the factor weights that solve problem (6) in sample, that is, optimized accounting for transaction costs and trading diversification.

We highlight four findings from Figure 4. First, comparing the mean gross return of each factor (red bar) with its mean net return when considered in isolation (grey bar), we observe that transaction costs substantially reduce mean returns. For instance, the mean net returns when trading in isolation of three of the unmanaged factors (HML, UMD, and IA) are less than half their mean gross returns.

Second, transaction costs are even more critical for the profitability of the managed factors, with four of them (SMB, HML, CMA, and IA) having *negative* mean net returns when traded in isolation.

Third, trading diversification helps to explain why the conditional multifactor portfolio outperforms the unconditional multifactor portfolio even in the presence of transaction costs. In particular, although both multifactor portfolios benefit from the netting of trades across factors, the benefits are relatively larger for the conditional portfolios because they exploit managed factors that are expensive to trade in isolation.[14]

Fourth, most of the benefits from trading diversification arise from the netting of trades across different factors rather than across just the managed and unmanaged versions of each individual factor. To demonstrate this, Table 5 reports the out-of-sample performance of the conditional multifactor portfolio evaluated in three different ways: (1) taking trading diversification fully into account (that is, netting trades across all unmanaged and managed factors), (2) taking trading diversification into account only partially (netting trades only across the unmanaged and managed versions of each individual factor, but not across different factors), and (3) ignoring trading diversification altogether.

Column (3) in Table 5 shows that the out-of-sample Sharpe ratio of the conditional multifactor portfolio when ignoring trading diversification is 0.748, which is smaller than that of the unconditional multifactor portfolio shown in Column (3) of Panel D in Table 4, 0.761. Allowing for trading diversification just across the unmanaged and managed versions of each individual factor, increases the Sharpe ratio of the conditional multifactor portfolio only marginally from 0.748 to 0.790. However, allowing for trading diversification across all unmanaged and managed factors substantially increases the Sharpe ratio of the conditional multifactor portfolio from 0.790 to 1.126, making it significantly

[14] Note that the managed SMB factor achieves a mean net return when traded in combination that is larger than its mean gross return. This is because the rebalancing trades of the conditional mean-variance multifactor portfolio are negatively correlated with those of the managed SMB factor, and thus one can effectively exploit the managed SMB factor at a negative transaction cost.

Table 5: Sources of trading-diversification benefits

This table reports the out-of-sample and net-of-costs performance of the unconditional (UMV) and conditional (CMV) mean-variance multifactor portfolios. We evaluate the performance of the conditional multifactor portfolio in three different ways: (1) taking trading diversification fully into account (that is, netting trades across all unmanaged and managed factors), (2) taking trading diversification into account only partially (netting trades only across the unmanaged and managed versions of each individual factor, but not across different factors), and (3) ignoring trading diversification altogether. For each multifactor portfolio, we report the annualized mean, standard deviation, Sharpe ratio of out-of-sample net returns, and p-value for the difference between the Sharpe ratios of the unconditional and conditional multifactor portfolio. The table also reports the alpha of the time-series regression of the conditional portfolio out-of-sample net returns on the returns of the unconditional portfolio, the Newey-West t-statistic for this alpha, and the out-of-sample transaction costs of the conditional portfolios. The portfolios are constructed exploiting all nine factors in our dataset. For the UMV and CMV portfolios, we use an expanding-window approach and the out-of-sample period spans from January 1977 to December 2020.

	(1) with full trading div. within & across factors		(2) with trading div. only within factors	(3) without any trading div.
	UMV	CMV	CMV	CMV
Mean	0.446	0.507	0.356	0.337
Standard deviation	0.459	0.450	0.450	0.450
Sharpe ratio	0.971	1.126	0.790	0.748
p-value(SR_{CMV}-SR_{UMV})		0.000	1.000	1.000
α		8.193	−7.028	−8.879
$t(\alpha)$		4.416	−3.664	−4.589
TC	0.149	0.188	0.339	0.358

higher than that of the unconditional portfolio, 0.971. One can make a similar inference by comparing instead the alphas or the transaction costs of these three portfolios.

In summary, most of the trading-diversification benefits enjoyed by the conditional multifactor portfolio arise from the netting of trades across different factors. Thus, the favorable performance of the conditional *multifactor* portfolio compared to the volatility-managed *individual-factor* portfolios is explained partly also by the benefits of trading diversification across *multiple* factors.[15]

[15] Of course, another reason that the multifactor portfolio outperforms the individual-factor portfolios is that it takes advantage of the risk-diversification benefits from combining multiple factors. Section IA.10 of the Internet Appendix shows that the market and size factors are negatively correlated to the other seven factors and thus, multifactor portfolios benefit from risk diversification across factors.

4.3 Time variation of multifactor portfolio weights

In this section, we study how the conditional multifactor portfolios benefit from the ability to time the various factors differentially, which is ruled out for the conditional fixed-weight portfolios. Figure 5 plots the in-sample weights from January 1977 to December 2020 of the unconditional mean-variance multifactor portfolio (UMV, blue line), the conditional mean-variance multifactor portfolio (CMV, solid red line), and the conditional fixed-weight multifactor portfolio (CFW, solid black line) that account for transaction costs and trading diversification.[16] The figure also depicts the *average* weights of the conditional mean-variance multifactor portfolio (E[CMV], dashed red line) and conditional fixed-weight multifactor portfolio (E[CFW], dashed black line).

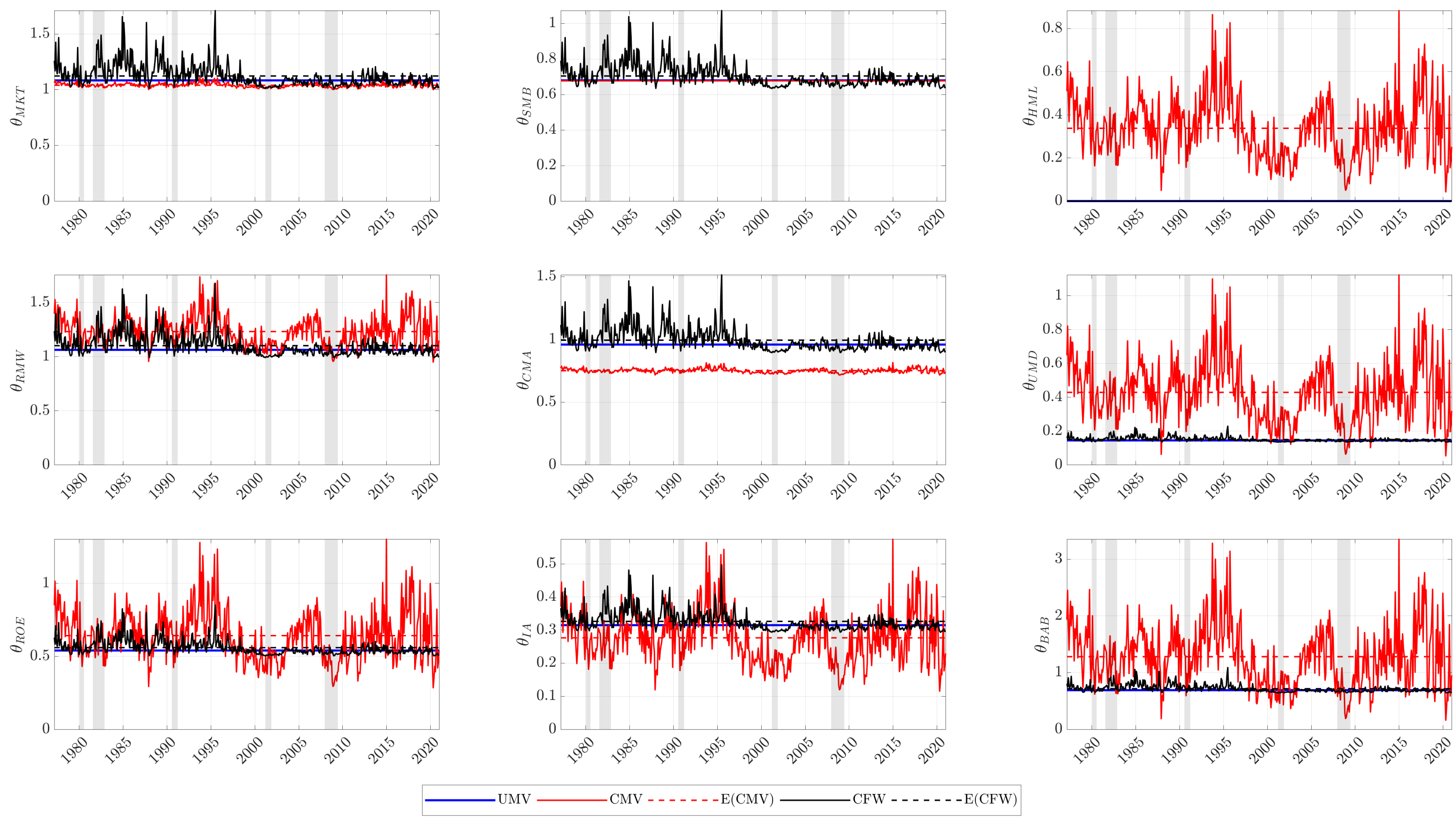
Figure 5 shows that the unconditional mean-variance portfolio assigns a strictly positive weight to every factor except value (HML), to which it assigns a zero weight. This is not surprising given that Table 1 shows that the Sharpe ratio of net returns of the HML factor is only 5.2%, the smallest across the nine factors. As explained before, the conditional fixed-weight portfolio is obtained by timing the unconditional portfolio, and thus, its relative weight on each factor coincides with that of the unconditional portfolio. Consequently, the conditional fixed-weight portfolio has zero weight on HML, just like the unconditional portfolio. For the rest of the factors, the average weight of the conditional fixed-weight portfolio is only slightly higher than that of the unconditional portfolio. Therefore, the gains from using the conditional fixed-weight multifactor portfolio do not arise from having a much larger *average* exposure to the factors, or from assigning a different relative weight to the factors, but rather from timing the unconditional mean-variance portfolio as a function of its variance.

Figure 5 also shows that the average weight on each of the factors of the conditional mean-variance portfolio differs substantially from those of the unconditional and conditional fixed-weight portfolios. In particular, the conditional mean-variance portfolio assigns a much higher average weight to the value (HML), momentum (UMD), and

[16] We consider in-sample weights in this section so that the weights of the unconditional mean-variance portfolio are constant over time, which allows us to interpret the time variation of the conditional multifactor portfolio weights. However, we show in Section IA.9 of the Internet Appendix that our insights are robust to considering the out-of-sample weight of the conditional and unconditional multifactor portfolios.

Figure 5: Weights of unconditional and conditional multifactor portfolios

This figure depicts the in-sample weights of the unconditional mean-variance multifactor portfolio (UMV, blue line), the conditional mean-variance multifactor portfolio (CMV, solid red line), and the conditional fixed-weight multifactor portfolio (CFW, solid black line) that account for transaction costs from January 1977 to December 2020. The figure also depicts the average weights of the conditional mean-variance multifactor portfolio (E[CMV], dashed red line) and the conditional fixed-weight multifactor portfolio (E[CFW], dashed black line). Each of the nine graphs depicts the weights for a particular factor.



betting-against-beta (BAB) factors than the unconditional and conditional fixed-weight portfolios. Interestingly, allowing the relative weight of each factor to vary with market volatility "resurrects" the value (HML) factor, to which the conditional mean-variance portfolio assigns a substantial average weight of 0.34.[17] However, the conditional mean-variance portfolio assigns a substantially lower average weight to the investment factors (CMA and IA), compared to the unconditional and conditional fixed-weight portfolios. Thus, the optimal average exposure to the different factors changes when the relative weight of each factor is allowed to vary with market volatility.

Finally, Figure 5 shows that the conditional mean-variance portfolio times some of the factors aggressively (HML, UMD, BAB), while assigning a stable weight to other factors (MKT, SMB, CMA). Thus, our conditional portfolio takes advantage of the opportunity to time factors differentially, which is ruled out for the conditional fixed-weight portfolio. Interestingly, the weight of the conditional mean-variance portfolio on the HML, UMD, and BAB factors drops dramatically during the Great Recession and after the Early 2000's Recession, but it increases substantially during periods of low market volatility such as the 1992–1997 period.

4.4 Market volatility and factor returns

Moreira and Muir (2017) show that the favorable performance of the volatility-managed individual-factor portfolios is explained by a weak risk-return tradeoff at the individual-factor level. In particular, they provide empirical evidence that the expected returns of individual factors do not significantly increase following an increase in the realized factor volatility. In this section, we study the tradeoff between realized *market volatility* and the returns of the the nine individual factors in our dataset as well as the unconditional multifactor portfolio.

[17] This is consistent with Panel A of Table 1, which shows that the in-sample Sharpe ratios of gross returns of the managed HML, UMD, and BAB factors are much higher than those of their unmanaged counterparts. Note that the in-sample Sharpe ratio of *net* returns of the managed HML factor is negative, however the transaction costs of trading the managed HML factor are much smaller when combined with the rest of the factors in the conditional multifactor portfolio.

Table 6: Predictive regressions of factor returns on market volatility

This table reports the intercept and slope coefficients of predictive regressions of factor returns net of transaction costs on realized market volatility. In particular, we report the slope coefficients of model (11). We consider nine factors: market (MKT), size (SMB), value (HML), profitability (RMW), investment (CMA), momentum (UMD), profitability (ROE), investment (IA) and betting-against-beta (BAB). In addition, in the last column we also report the results for the unconditional mean-variance multifactor portfolio (UMV) . The numbers in square brackets correspond to Newey-West t-statistics.

Factor	MKT	SMB	HML	RMW	CMA	UMD	ROE	IA	BAB	UMV
Intercept	0.709	−0.221	0.567	0.127	0.108	1.248	0.622	0.188	1.471	2.670
t-stat	[1.311]	[−1.046]	[2.200]	[0.735]	[0.741]	[2.472]	[2.547]	[1.434]	[4.965]	[4.578]
Slope	−0.041	0.361	−0.569	0.118	−0.018	−1.202	−0.426	−0.141	−0.904	−0.742
t-stat	[−0.064]	[1.593]	[−1.953]	[0.554]	[−0.121]	[−1.861]	[−1.389]	[−1.055]	[−2.503]	[−1.145]

To examine the tradeoff between market volatility and future factor returns, we estimate the following regression:

$$r_{k,t+1} = \alpha + \beta\, \sigma_t + \epsilon_{t+1}, \tag{11}$$

where $r_{k,t+1}$ is the return net of proportional transaction costs of factor k during month $t+1$, and σ_t is the realized market volatility for month t. We also run a similar regression with the return of the unconditional multifactor portfolio as the dependent variable.

Table 6 reports the results from the above regressions for the nine individual factors and also the unconditional mean-variance multifactor portfolio (UMV). Our first observation is that the slope for the unconditional multifactor portfolio is negative with a t-statistic of −1.145. This explains why the conditional fixed-weight multifactor portfolio outperforms the unconditional multifactor portfolio; there is a *negative* tradeoff between the return of the unconditional multifactor portfolio and market volatility. Moreover, none of the positive slope coefficients for the individual factors are significant, indicating that an increase in market volatility does not lead to an increase in the next-month return of any of the considered *individual* factors, consistent with the findings of Moreira and Muir (2017). Furthermore, the slope coefficients for the HML, UMD, and BAB factors are negative with t-statistics of −1.953, −1.861, −2.503, respectively. Thus, there is a significant or nearly significant *negative* tradeoff between the returns of these three factors and market volatility, which explains why the conditional multifactor portfolios time HML, UMD, and BAB aggressively. Because of this aggressive timing, our conditional multifactor portfolio also assigns a higher average relative weight to these three factors compared to the unconditional and conditional fixed-weight multifactor portfolios.

5 Conclusion

We develop a new strategy that exploits market volatility to time investment in popular asset-pricing factors. Instead of timing an individual equity factor conditional on its variance or timing a *fixed* combination of factors conditional on the variance of that combination, we consider a conditional mean-variance *multifactor* portfolio whose relative weight on each factor varies with market volatility. We show that the conditional multifactor portfolio outperforms volatility-managed individual-factor portfolios, conditional fixed-weight multifactor portfolios, and unconditional multifactor portfolios. The performance gains of the conditional multifactor portfolio are present even out of sample, net of transaction costs, and for both low- and high-sentiment periods.

There are three main drivers of the favorable performance of the conditional multifactor portfolio. First, we find that accounting for trading diversification is necessary for the conditional multifactor portfolios to outperform their unconditional counterparts. Second, we find that optimizing the conditional multifactor portfolios taking transaction costs into account significantly improves their performance relative to the unconditional multifactor portfolio. Third, the conditional multifactor portfolio allows the relative weight on each factor to vary with market volatility. As a result, our portfolio optimally assigns a relative *average* weight to each factor that differs substantially from that of the unconditional mean-variance portfolio. Moreover, our portfolio optimally times some of the factors more aggressively than others.

The success of our volatility-managed multifactor portfolio even out of sample and net of transaction costs, and for both low- and high-sentiment periods, suggests that the breakdown of the most fundamental relation in finance, that between risk and return, is even more puzzling than originally thought.

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Internet Appendix to

A Multifactor Perspective on Volatility-Managed Portfolios

This Internet Appendix reports the following robustness checks and additional results: (i) evaluating performance during periods of high market volatility, (ii) excluding the market factor from the conditional multifactor portfolios, (iii) constraining the conditional multifactor portfolio leverage, (iv) timing each factor using its own volatility instead of market volatility, (v) exploiting each factor's value spread in addition to market volatility as a conditioning variable, (vi) exploiting business-cycle variables in addition to market volatility as conditioning variables, (vii) using a less parsimonious conditional multifactor portfolio, (viii) evaluating performance using alternative measures of risk, (ix) considering the out-of-sample factor weights to explain performance, and (x) the risk-diversification benefits that arise because the returns on the nine factors are less than perfectly correlated.

IA.1 Performance in high-volatility periods

Table IA.1 reports annualized mean return, standard deviation, and Sharpe ratio of the unconditional mean-variance multifactor portfolio (UMV), the conditional fixed-weight multifactor portfolio (CFW), and the conditional mean-variance multifactor portfolio (CMV) during high market volatility and crises periods. We report the performance of these three multifactor portfolios evaluated in three different ways: in-sample ignoring transaction costs, out-of-sample ignoring transaction costs, and out-of-sample net of transaction costs and taking trading diversification into account. We consider periods with market volatility above the 80th, 85th, and 90th percentiles, as well as the Early 2000's Recession, the Great Recession of 2007–09, and the COVID-crisis periods.[18]

Table IA.1 shows that the performance of the conditional mean-variance multifactor portfolio is robust during periods of high market volatility. In particular, the out-of-sample Sharpe ratio net of transaction costs of the conditional mean-variance multifactor portfolio is higher than that of the unconditional portfolio by about 13%, 18%, and 16% for subperiods where volatility is greater than the 80th, 85th, and 90th percentile, respectively, and higher than that of the conditional fixed-weight portfolio by about 6%, 24%, and 27%, respectively. The conditional mean-variance portfolio also outperforms the unconditional and conditional fixed-weight portfolios for the Early 2000's

[18]We define the Early 2000's Recession as spanning from February to November 2001, the Great Recession from December 2007 to June 2009, and the COVID crisis from January to December 2020.

Table IA.1: Performance during high-volatility periods

This table reports annualized average return, standard deviation, and Sharpe ratio of the unconditional mean-variance multifactor portfolio (UMV), the conditional fixed-weight multifactor portfolio (CFW), and the conditional mean-variance multifactor portfolio (CMV) during high market volatility and crises periods. We report the performance of these three multifactor portfolios evaluated in three different ways: (1) in-sample ignoring transaction costs, (2) out-of-sample ignoring transaction costs, and (3) out-of-sample net of transaction costs and taking trading diversification into account. We consider periods with market volatility above the 80th, 85th, and 90th percentiles, as well as the 2000-Recession, the Great-Recession, and the COVID-Crisis periods.

	(1) In-sample without TC			(2) Out-of-sample without TC			(3) Out-of-sample with TC & trad. div.		
	UMV	CFW	CMV	UMV	CFW	CMV	UMV	CFW	CMV
Panel A: Mean									
σ_t >80th percentile	0.353	0.415	0.455	0.531	0.501	0.613	0.251	0.243	0.260
σ_t >85th percentile	0.327	0.367	0.390	0.380	0.348	0.450	0.130	0.111	0.139
σ_t >90th percentile	0.288	0.304	0.347	0.367	0.315	0.402	0.124	0.100	0.125
2000 Recession	0.893	0.437	1.239	1.373	0.762	1.731	0.914	0.631	1.088
Great Recession	−0.310	−0.165	−0.221	−0.322	−0.145	−0.075	−0.441	−0.327	−0.285
COVID Crisis	−0.138	−0.211	−0.086	−0.326	−0.345	−0.316	−0.241	−0.205	−0.309
Panel B: Standard deviation									
σ_t >80th percentile	0.336	0.331	0.381	0.664	0.528	0.588	0.525	0.479	0.482
σ_t >85th percentile	0.352	0.331	0.390	0.684	0.527	0.590	0.543	0.485	0.492
σ_t >90th percentile	0.372	0.313	0.393	0.704	0.528	0.574	0.558	0.493	0.487
2000 Recession	0.526	0.248	0.508	0.951	0.530	0.713	0.692	0.481	0.568
Great Recession	0.223	0.181	0.269	0.369	0.278	0.306	0.257	0.193	0.220
COVID Crisis	0.387	0.237	0.433	0.354	0.252	0.447	0.364	0.249	0.400
Panel C: Sharpe ratio									
σ_t >80th percentile	1.050	1.253	1.194	0.799	0.950	1.043	0.478	0.508	0.540
σ_t >85th percentile	0.930	1.109	1.000	0.555	0.661	0.763	0.240	0.228	0.283
σ_t >90th percentile	0.773	0.970	0.883	0.521	0.597	0.699	0.222	0.203	0.257
2000 Recession	1.698	1.763	2.437	1.444	1.438	2.427	1.320	1.314	1.916
Great Recession	−1.389	−0.911	−0.823	−0.873	−0.521	−0.245	−1.721	−1.693	−1.293
COVID Crisis	−0.356	−0.890	−0.200	−0.922	−1.367	−0.708	−0.661	−0.824	−0.772

Recession and the Great Recession. For instance, during the Early 2000s Recession, the out-of-sample Sharpe ratio net of transaction costs of the conditional mean-variance multifactor portfolio is around 45% higher than those of the unconditional and conditional fixed-weight portfolios. During the Great Recession, all three multifactor portfolios attain a negative out-of-sample Sharpe ratio of returns net of costs, but the conditional mean-variance portfolio has a substantially larger mean return than the unconditional and conditional fixed-weight portfolios, with a standard deviation of returns that is between those of the unconditional and conditional fixed-weight portfolios. An exception to the overall favorable performance of the conditional mean-variance portfolio is the COVID-crisis period, where its out-of-sample and net-of-costs performance is worse than that of the unconditional and fixed-weight conditional portfolios.

IA.2 Excluding the market in multifactor portfolios

Barroso and Detzel (2020) find that the volatility-managed market portfolio outperforms its unmanaged counterpart even net of transaction costs. Therefore, a legitimate concern one may have is whether it is just the performance of the market factor that drives entirely the good performance of the conditional mean-variance multifactor portfolio, especially because our conditioning variable is market volatility. To address this concern, Table IA.2 reports the performance of the conditional mean-variance multifactor portfolio that excludes the market factor and exploits only the other eight factors.

The results in Table IA.2 show that the conditional mean-variance multifactor portfolio delivers economic gains even when it does not exploit the market factor. Column (4) in Panels A and C show that if one takes advantage of trading diversification, then the conditional fixed-weight multifactor portfolio improves the out-of-sample net-of-transaction-costs Sharpe ratio by about 6%, with an alpha that is marginally significant. Panels B and D show, for the same experimental setting, that the conditional mean-variance multifactor portfolio delivers an improvement in Sharpe ratio of about 20% and an annual alpha that is statistically significant.

IA.3 Leverage constraints

In the main body of the manuscript, our conditional mean-variance multifactor portfolio is optimized to invest in nine equity factors that require one dollar of shorting (i.e. leverage) to fund each dollar invested in the factor's long leg. One concern is that our conditional mean-variance portfolio requires a much larger degree of leverage than the unconditional mean-variance multifactor portfolio to be profitable. We address this concern by rescaling the conditional multifactor portfolio so that the dollar investment on its short and long legs is at most 20% higher than that of the unconditional multifactor

Table IA.2: Performance without the market factor

This table reports the performance of the multifactor portfolios constructed using every factor in our database except the market factor. Each of the four panels of this table reports the performance of a different method for *choosing* multifactor portfolio weights: Panel A for the conditional fixed-weight multifactor portfolio that ignores transaction costs as considered by Moreira and Muir (2017); Panel B for the conditional mean-variance multifactor portfolio that ignores transaction costs, which is obtained by solving problem (6) ignoring the transaction-cost term TC(η); Panel C for the conditional fixed-weight multifactor portfolio considered by Moreira and Muir (2017) extended to account for transaction costs; Panel D for the conditional mean-variance multifactor portfolio obtained by solving problem (6) accounting for transaction costs. For each panel, each of the four columns reports the performance of the chosen weights *evaluated* in a different way: (1) in sample ignoring transaction costs, (2) out of sample ignoring transaction costs, (3) out of sample net of transaction costs but ignoring trading diversification, and (4) out of sample with transaction costs with trading diversification. The sample period and quantities reported for each portfolio are the same as in Table 2.

	(1) In-sample without TC		(2) Out-of-sample without TC		(3) Out-of-sample with TC without trad. div.		(4) Out-of-sample with TC with trad. div.	
	UMV	Cond.	UMV	Cond.	UMV	Cond.	UMV	Cond.
Panel A: Conditional fixed-weight multifactor portfolio ignoring transaction costs								
Mean	0.268	0.417	0.674	0.690	0.357	0.245	0.465	0.445
Standard deviation	0.232	0.289	0.572	0.512	0.572	0.512	0.572	0.512
Sharpe ratio	1.158	1.443	1.180	1.348	0.625	0.478	0.814	0.868
p-value(SR_{CMV}-SR_{UMV})		0.002		0.000		0.999		0.163
α		14.852		10.693		−6.365		4.161
$t(\alpha)$		5.772		4.479		−2.725		1.877
TC					0.317	0.446	0.209	0.246
Panel B: Conditional mean-variance multifactor portfolio ignoring transaction costs								
Mean	0.268	0.564	0.674	0.863	0.357	0.324	0.465	0.563
Standard deviation	0.232	0.336	0.572	0.559	0.572	0.559	0.572	0.559
Sharpe ratio	1.158	1.679	1.180	1.543	0.625	0.579	0.814	1.007
p-value(SR_{CMV}-SR_{UMV})		0.000		0.000		0.768		0.007
α		22.756		24.887		−0.035		14.022
$t(\alpha)$		7.550		6.298		−0.010		3.867
TC					0.317	0.539	0.209	0.299
Panel C: Conditional fixed-weight multifactor portfolio optimizing transaction costs								
Mean	0.174	0.227	0.526	0.606	0.297	0.261	0.385	0.427
Standard deviation	0.159	0.188	0.449	0.469	0.449	0.469	0.449	0.469
Sharpe ratio	1.090	1.209	1.172	1.292	0.661	0.557	0.857	0.911
p-value(SR_{CMV}-SR_{UMV})		0.005		0.000		0.998		0.086
α		2.904		6.404		−4.479		3.001
$t(\alpha)$		3.722		4.636		−2.792		2.200
TC					0.230	0.344	0.141	0.179
Panel D: Conditional mean-variance multifactor portfolio optimizing transaction costs								
Mean	0.174	0.328	0.526	0.633	0.297	0.292	0.385	0.450
Standard deviation	0.159	0.207	0.449	0.439	0.449	0.439	0.449	0.439
Sharpe ratio	1.090	1.581	1.172	1.441	0.661	0.665	0.857	1.026
p-value(SR_{CMV}-SR_{UMV})		0.000		0.000		0.514		0.000
α		12.427		13.550		1.137		8.643
$t(\alpha)$		8.249		6.734		0.614		4.632
TC					0.230	0.341	0.141	0.182

Table IA.3: Performance with leverage constraints

This table reports the out-of-sample and net-of-costs performance of the conditional multifactor portfolio (CMV) subject to the constraint that its leverage is at most 20% higher than that of the unconditional multifactor portfolio (UMV). For each multifactor portfolio, the table reports the annualized mean, standard deviation, Sharpe ratio of out-of-sample net returns, and p-value for the difference between the Sharpe ratios of the conditional and unconditional portfolios. The table also reports the alpha of the time-series regression of the conditional portfolio out-of-sample net returns on those of the unconditional portfolio, alpha Newey-West t-statistic, and out-of-sample transaction costs of the unconditional and conditional portfolios. The portfolios are constructed exploiting all nine factors in our dataset. We use an expanding-window approach and the out-of-sample period spans from January 1977–December 2020.

	UMV	CMV
Mean	0.446	0.506
Standard deviation	0.459	0.446
Sharpe ratio	0.971	1.134
p-value(SR_{CMV}-SR_{UMV})		0.000
α		8.447
$t(\alpha)$		4.912
TC	0.149	0.175

portfolio.[19] Table IA.3 shows that after imposing a leverage constraint on the conditional mean-variance multifactor the performance is not only robust but it also delivers a higher Sharpe ratio and abnormal return (alpha) relative to the unconstrained case.

IA.4 Exploiting each factor's own volatility

We now study the robustness of the performance of the conditional mean-variance multifactor portfolios to timing each factor using its own volatility. Table IA.4 shows that the performance of the conditional mean-variance portfolio is similar to that when using market volatility to time the factors. In particular, the out-of-sample and net-of-costs Sharpe ratio of the conditional mean-variance multifactor portfolio that uses each factor's own volatility as conditioning variable is 1.168, only slightly higher than that of the portfolio that uses market volatility as conditioning variable, which is 1.126.

[19] Moreira and Muir (2017) consider a 50% limit on the extra leverage that the volatility-managed portfolios can have over that of the unconditional factors and find that their results are robust to this leverage constraint. In unreported results, we confirm that our results are virtually unchanged when we consider a 50% constraint on the leverage that the conditional mean-variance portfolio can have over that of the unconditional multifactor portfolio. However, in this section we present the results for the more restrictive case with a 20% leverage constraint.

Table IA.4: Performance exploiting each factor's own volatility

This table reports the performance of the multifactor portfolios constructed timing each factor using its own volatility. Each of the four panels of this table reports the performance of a different method for *choosing* multifactor portfolio weights: Panel A for the conditional fixed-weight multifactor portfolio that ignores transaction costs as considered by Moreira and Muir (2017); Panel B for the conditional mean-variance multifactor portfolio that ignores transaction costs, which is obtained by solving problem (6) ignoring the transaction-cost term TC(η), but timing each factor using its own volatility; Panel C for the conditional fixed-weight multifactor portfolio considered by Moreira and Muir (2017) extended to account for transaction costs; Panel D for the conditional mean-variance multifactor portfolio obtained by solving problem (6) accounting for transaction costs, but timing each factor using its own volatility. For each panel, each of the four columns reports the performance of the chosen weights *evaluated* in a different way: (1) in sample ignoring transaction costs, (2) out of sample ignoring transaction costs, (3) out of sample net of transaction costs but ignoring trading diversification, and (4) out of sample with transaction costs with trading diversification. The sample period and quantities reported for each portfolio are the same as in Table 2.

	(1) In-sample without TC		(3) Out-of-sample without TC		(4) Out-of-sample with TC without trad. div.		(5) Out-of-sample with TC with trad. div.	
	UMV	Cond.	UMV	Cond.	UMV	Cond.	UMV	Cond.
Panel A: Conditional fixed-weight multifactor portfolio ignoring transaction costs								
Mean	0.415	0.602	0.753	0.783	0.413	0.316	0.531	0.530
Standard deviation	0.288	0.347	0.580	0.520	0.580	0.520	0.580	0.520
Sharpe ratio	1.441	1.735	1.299	1.506	0.713	0.608	0.916	1.019
p-value(SR_{CMV}-SR_{UMV})		0.000		0.000		0.986		0.012
α		18.724		13.986		−3.338		7.806
$t(\alpha)$		5.738		5.012		−1.529		3.322
TC					0.340	0.467	0.222	0.253
Panel B: Conditional mean-variance multifactor portfolio ignoring transaction costs								
Mean	0.415	0.807	0.753	0.957	0.413	0.361	0.531	0.629
Standard deviation	0.288	0.402	0.580	0.542	0.580	0.542	0.580	0.542
Sharpe ratio	1.441	2.008	1.299	1.768	0.713	0.667	0.916	1.161
p-value(SR_{CMV}-SR_{UMV})		0.000		0.000		0.718		0.012
α		33.003		35.058		2.693		20.137
$t(\alpha)$		8.135		5.830		0.551		3.893
TC					0.340	0.596	0.222	0.329
Panel C: Conditional fixed-weight multifactor portfolio optimizing transaction costs								
Mean	0.301	0.315	0.595	0.628	0.349	0.345	0.446	0.473
Standard deviation	0.219	0.223	0.459	0.445	0.459	0.445	0.459	0.445
Sharpe ratio	1.379	1.416	1.296	1.410	0.761	0.775	0.971	1.061
p-value(SR_{CMV}-SR_{UMV})		0.006		0.001		0.329		0.002
α		0.957		5.938		1.240		4.782
$t(\alpha)$		3.075		3.831		0.970		3.507
TC					0.246	0.283	0.149	0.155
Panel D: Conditional mean-variance multifactor portfolio optimizing transaction costs								
Mean	0.301	0.505	0.595	0.698	0.349	0.348	0.446	0.510
Standard deviation	0.219	0.267	0.459	0.437	0.459	0.437	0.459	0.437
Sharpe ratio	1.379	1.892	1.296	1.598	0.761	0.796	0.971	1.168
p-value(SR_{CMV}-SR_{UMV})		0.000		0.000		0.306		0.000
α		17.567		15.501		2.690		10.216
$t(\alpha)$		8.687		6.167		1.262		4.547
TC					0.246	0.350	0.149	0.188

IA.5 Exploiting value spreads

In the main body of the manuscript, we use market volatility as the only conditioning variable for the conditional mean-variance multifactor portfolios. In this section, we show that exploiting each factor's value spread as a conditioning variable in addition to market volatility does not help to significantly improve the out-of-sample and net-of-costs performance of the conditional multifactor portfolios.

The value spread can be computed as the difference between the (lagged) aggregate book-to-market ratio of the stocks in the factor's long leg minus that of the stocks in the factor's short leg.[20] We then define the weight assigned to the kth factor in our parametric portfolio as

$$\theta_{k,t} = a_k + b_k \frac{1}{\sigma_t} + c_k \text{BM}_{k,t},$$

where σ_t is the market volatility in month t and $\text{BM}_{k,t}$ is the kth factor's value spread in month t. Parameters a_k, b_k and c_k are jointly estimated for the nine factors we consider as the solution of the mean-variance optimization problem in (6).

Table IA.5 reports the performance of the conditional mean-variance multifactor portfolios that exploit as conditioning variable each factor's value spread in addition to market volatility. The table shows that the conditional mean-variance multifactor portfolio that exploits each factor's value spread in addition to market volatility attains an out-of-sample and net-of-costs Sharpe ratio of 1.145, which is only 1.7% larger than that of the conditional mean-variance multifactor portfolio that exploits only market volatility (1.126). Therefore, our results show that using each factor's value spread as a conditioning variable does not improve substantially the performance of the portfolios that use only market volatility.

IA.6 Exploiting business-cycle variables

This section assesses whether the performance of our conditional mean-variance multifactor portfolio can be improved by exploiting factors related to the business cycle. We entertain this possibility by accounting for the four business cycle macroeconomic

[20] Like Fama and French (1992), the book-to-market for June of year t is the book equity for the last fiscal year end in $t-1$ divided by market capitalization for December of year $t-1$.

Table IA.5: Conditioning on value spread and market volatility

This table reports the performance of the conditional mean-variance multifactor portfolios that exploit as conditioning variable each factor's value spread in addition to market volatility. Each of the four panels of this table reports the performance of a different method for *choosing* multifactor portfolio weights: Panel A for the conditional fixed-weight multifactor portfolio that ignores transaction costs as considered by Moreira and Muir (2017); Panel B for the conditional mean-variance multifactor portfolio that ignores transaction costs, but exploits both each factor's value spread and market volatility as conditioning variables; Panel C for the conditional fixed-weight multifactor portfolio considered by Moreira and Muir (2017) extended to account for transaction costs; Panel D for the conditional mean-variance multifactor portfolio that accounts for transaction costs and exploits both each factor's value spread and market volatility as conditioning variables. For each panel, each of the four columns reports the performance of the chosen weights *evaluated* in a different way: (1) in sample ignoring transaction costs, (2) out of sample ignoring transaction costs, (3) out of sample net of transaction costs but ignoring trading diversification, and (4) out of sample with transaction costs with trading diversification. The sample period and quantities reported for each portfolio are the same as in Table 2.

	(1) In-sample without TC		(3) Out-of-sample without TC		(4) Out-of-sample with TC without trad. div.		(5) Out-of-sample with TC with trad. div.	
	UMV	Cond.	UMV	Cond.	UMV	Cond.	UMV	Cond.
Panel A: Conditional fixed-weight multifactor portfolio ignoring transaction costs								
Mean	0.415	0.602	0.753	0.783	0.413	0.316	0.531	0.530
Standard deviation	0.288	0.347	0.580	0.520	0.580	0.520	0.580	0.520
Sharpe ratio	1.441	1.735	1.299	1.506	0.713	0.608	0.916	1.019
p-value(SR_{CMV}-SR_{UMV})		0.000		0.000		0.983		0.011
α		18.724		13.986		−3.338		7.806
$t(\alpha)$		5.738		5.012		−1.529		3.322
TC					0.340	0.467	0.222	0.253
Panel B: Conditional mean-variance multifactor portfolio ignoring transaction costs								
Mean	0.415	0.753	0.753	1.024	0.413	0.265	0.531	0.682
Standard deviation	0.288	0.388	0.580	0.634	0.580	0.634	0.580	0.634
Sharpe ratio	1.441	1.940	1.299	1.614	0.713	0.417	0.916	1.075
p-value(SR_{CMV}-SR_{UMV})		0.000		0.000		1.000		0.036
α		28.189		29.582		−12.890		17.076
$t(\alpha)$		7.593		5.388		−2.555		3.326
TC					0.340	0.759	0.222	0.342
Panel C: Conditional fixed-weight multifactor portfolio optimizing transaction costs								
Mean	0.301	0.315	0.595	0.628	0.349	0.345	0.446	0.473
Standard deviation	0.219	0.223	0.459	0.445	0.459	0.445	0.459	0.445
Sharpe ratio	1.379	1.416	1.296	1.410	0.761	0.775	0.971	1.061
p-value(SR_{CMV}-SR_{UMV})		0.005		0.001		0.332		0.002
α		0.957		5.938		1.240		4.782
$t(\alpha)$		3.075		3.831		0.970		3.507
TC					0.246	0.283	0.149	0.155
Panel D: Conditional mean-variance multifactor portfolio optimizing transaction costs								
Mean	0.301	0.506	0.595	0.758	0.349	0.339	0.446	0.568
Standard deviation	0.219	0.277	0.459	0.496	0.459	0.496	0.459	0.496
Sharpe ratio	1.379	1.827	1.296	1.529	0.761	0.684	0.971	1.145
p-value(SR_{CMV}-SR_{UMV})		0.000		0.000		0.929		0.000
α		16.408		15.239		−1.502		11.498
$t(\alpha)$		7.612		4.910		−0.566		4.033
TC					0.246	0.419	0.149	0.190

Table IA.6: Performance exploiting market volatility and macro variables

This table reports the performance of the conditional mean-variance multifactor portfolio that only exploits inverse market volatility (CMV) and the performance of the conditional mean-variance multifactor portfolios that exploit inverse market volatility and one macroeconomic variable as defined in equation (IA1). For each multifactor portfolio, we report the annualized mean, standard deviation, Sharpe ratio of out-of-sample net returns, and p-value for the difference between the Sharpe ratios of the conditional multifactor portfolio exploiting inverse market volatility and one macroeconomic variable ($SR_{CMV_{Macro}}$) and the conditional multifactor portfolio exploiting only inverse market volatility (SR_{CMV}). The table also reports the alpha of the time-series regression of the out-of-sample net returns of the conditional portfolio exploiting inverse market volatility and a macroeconomic variable on those of the conditional portfolio exploiting inverse market volatility, alpha Newey-West t-statistic, and out-of-sample transaction costs of the conditional portfolios. The portfolios are constructed exploiting all nine factors in our dataset. We use an expanding-window approach and the out-of-sample period spans from January 1977–December 2020.

	CMV	Sentiment	Baa-Aaa	Slope	Claims	Production
Mean	0.507	0.615	0.536	0.581	0.509	0.505
Standard deviation	0.450	0.559	0.469	0.489	0.458	0.454
Sharpe ratio	1.126	1.099	1.141	1.189	1.110	1.112
p-value(SR_{CMV}-SR_{UMV})		0.630	0.308	0.115	0.864	0.886
α		2.445	1.410	5.400	−0.463	−0.420
$t(\alpha)$		0.683	1.091	2.064	−0.832	−0.763
TC	0.188	0.199	0.194	0.192	0.196	0.196

variables considered by Herskovic, Moreira, and Muir (2020): 1) the Moody's Baa-Aaa spread, 2) the slope of the term structure, 3) initial claims, and 4) industrial production. In addition, given the outperformance of the conditional mean-variance multifactor portfolio on high-sentiment regimes (see Section 3.3), we also consider a sentiment dummy variable based on the Baker and Wurgler (2006) index. More precisely, the weight on each factor k is defined as

$$\theta_{k,t} = a_k + b_k \frac{1}{\sigma_t} + c_k \text{Macro}_{lt}, \tag{IA1}$$

where Macro_{lt} is the value of the macroeconomic variable l at time t. Table IA.6 shows the Sharpe ratios of the conditional multifactor portfolio that exploits inverse market volatility and one macroeconomic variable is not statistically different from that of the conditional multifactor portfolio exploiting only inverse market volatility. Accordingly, including business cycle variables does not enhance the performance of the conditional

Table IA.7: Performance of alternative conditional multifactor portfolio

This table reports the performance of the unconditional multifactor portfolio (UMV), our conditional multifactor portfolio that exploits inverse market volatility (CMV), and the alternative conditional multifactor portfolio (ALT) obtained by solving problem (IA4). For each multifactor portfolio, we report the annualized mean, standard deviation, Sharpe ratio of out-of-sample net returns, and p-value for the difference between the Sharpe ratios of the unconditional multifactor portfolio and the conditional multifactor portfolios. The table also reports the alpha of the time-series regression of the conditional multifactor portfolio out-of-sample net returns on those of the unconditional portfolio, alpha Newey-West t-statistic, and out-of-sample transaction costs of the conditional multifactor portfolios. The portfolios are constructed exploiting all nine factors in our dataset. The out-of-sample period spans from January 1977–December 2020.

	UMV	CMV	ALT
Mean	0.446	0.507	1.122
Standard deviation	0.459	0.450	1.129
Sharpe ratio	0.971	1.126	0.994
p-value(SR_{CMV}-SR_{UMV})		0.001	0.448
α		8.195	38.301
$t(\alpha)$		4.416	2.308
TC	0.149	0.188	0.303

mean-variance portfolio that already exploits inverse market volatility as a conditioning variable.[21]

IA.7 Alternative conditional multifactor portfolio

Our conditional multifactor portfolios rely on the assumption that the conditional weight on the kth factor is an affine function of inverse market volatility:

$$\theta_{k,t} = a_k + b_k \frac{1}{\sigma_t}. \tag{IA2}$$

One may wonder whether a less parsimonious conditional multifactor portfolio would perform better. To address this concern, in this section we evaluate the performance of an alternative conditional multifactor portfolio obtained by using the estimator of the conditional covariance matrix considered by Chernov, Lochstoer, and Lundeby (2021):

$$\Sigma_t = (1 - \lambda)V_t + \lambda\Sigma_{t-1}, \tag{IA3}$$

[21] In unreported results, we confirm that the results presented in this section are robust to considering interactions between inverse market volatility and macroeconomic variables.

where V_t is the nonlinear shrinkage covariance matrix of Ledoit and Wolf (2020) using the daily returns in month t and $\lambda = 0.94$, that is, the λ used by the RiskMetrics model. We then compute the alternative conditional multifactor portfolio by solving the following problem:

$$\max_{\theta_t \geq 0} \quad \theta_t^\top \mu - \frac{\gamma}{2} \theta_t^\top \Sigma_t \theta_t - TC(\theta_t), \tag{IA4}$$

where μ is the vector of means estimated from an expanding window of factor returns.[22]

Table IA.7 reports the performance of the alternative conditional multifactor portfolio. We observe that our proposed conditional multifactor portfolio outperforms the alternative portfolio in terms of Sharpe ratio.

IA.8 Alternative risk measures

In Section 3.2, we show that the conditional mean-variance multifactor portfolio delivers an out-of-sample Sharpe ratio net of transaction costs larger than that of the unconditional multifactor portfolio and the volatility-managed portfolios of individual factors. We now confirm that this insight also holds when we compute return-to-risk ratios using the maximum drawdown and the Value-at-Risk instead of the volatility of portfolio returns that is used to construct the Sharpe ratio. Figure IA.1 shows that regardless of the risk measure we consider, the conditional multifactor portfolio offers a better risk-return tradeoff than that of the unconditional multifactor portfolio and the volatility-managed portfolios of individual factors.

IA.9 Out-of-sample multifactor portfolio weights

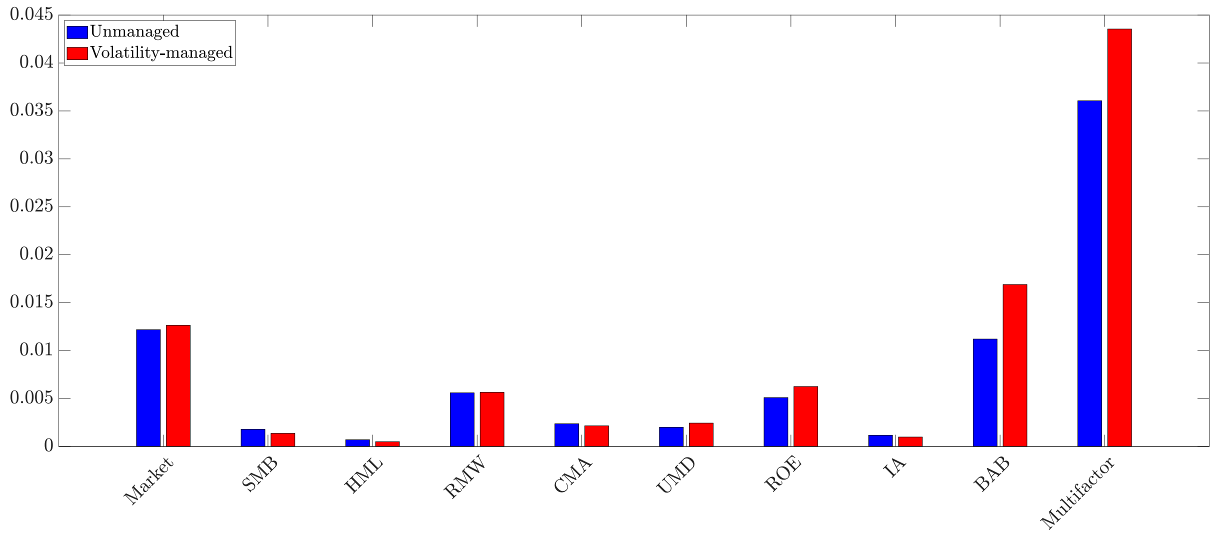
In Section 4.3, we explain that the conditional mean-variance portfolio allows us to adjust the *relative* weight of each factor with market volatility. The analysis in Section 4.3 is in sample, that is, we estimate the optimal conditional multifactor portfolio using the entire sample from January 1977 to December 2020. We now confirm that this insight is true also for the out-of-sample factor weights computed using an expanding window. Figure IA.2 shows that the relative weight of the factors varies with market

[22] It is challenging to estimate conditional mean returns, and thus, we use an unconditional estimate of means for simplicity.

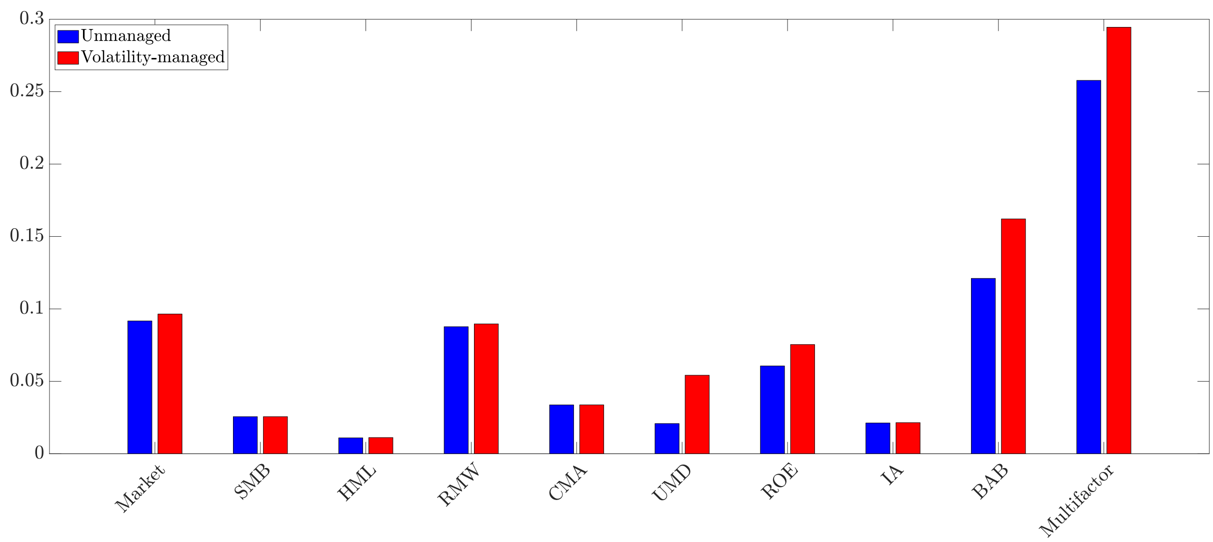
Figure IA.1: Alternative performance measures

This figure depicts in-sample and net-of-costs mean-to-risk ratios of the unmanaged and volatility-managed individual factor and multifactor portfolios. We consider the maximum drawdown and the Value-at-Risk with 95% confidence level, VaR(95), as our risk measures.

Panel A: Mean to maximum drawdown



Panel B: Mean to VaR(95)



volatility. For instance, while the weight on the market factor does not seem to change with market volatility (i.e., parameter $b_{\mathrm{MKT}} = 0$ in equation (3)), the weights on other factors, such as momentum (UMD) and betting-against-beta (BAB), change substantially with market volatility (i.e., parameter $b_k \neq 0$ in equation (3)). One can observe this from the result that the market weight is similar to that assigned by the unconditional

Figure IA.2: Out-of-sample weights of unconditional and conditional multifactor portfolios

This figure depicts the out-of-sample weights of the unconditional mean-variance multifactor portfolio (UMV, blue line), and the conditional mean-variance multifactor portfolio (CMV, solid red line), that account for transaction costs from January 1977 to December 2020. The figure also depicts the average weights of the unconditional mean-variance multifactor portfolio (E[UMV], dashed blue line) and the conditional mean-variance multifactor portfolio (E[CMV], dashed red line). Each of the nine graphs depicts the weights for a particular factor.

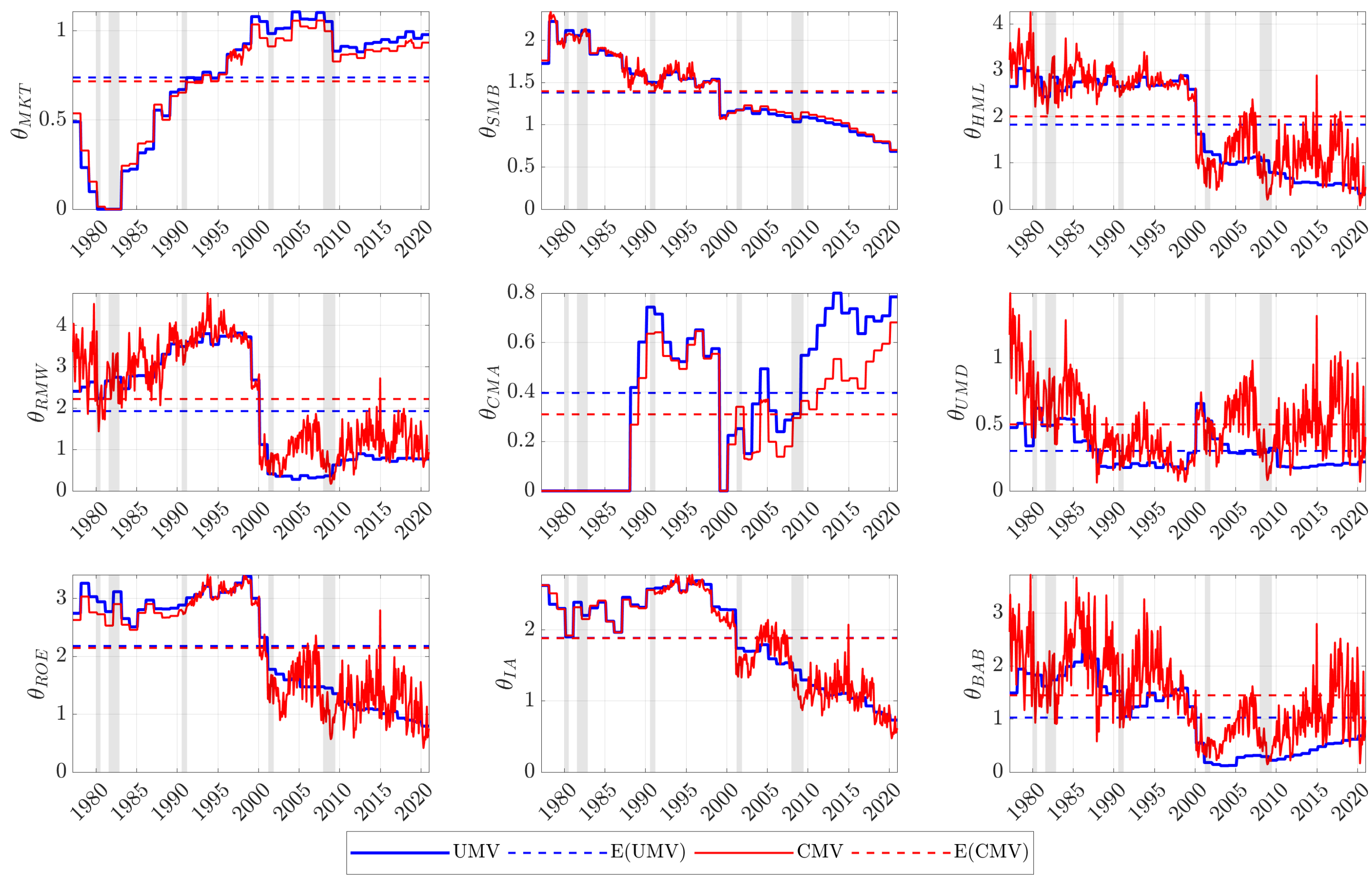


Table IA.8: Correlation of factor returns

This table reports the correlation matrix of the nine unmanaged factors over the period January 1977 to December 2020. The numbers in bold font correspond with negative correlations.

	MKT	SMB	HML	RMW	CMA	UMD	ROE	IA	BAB
MKT	1.00	0.26	**−0.22**	**−0.26**	**−0.37**	**−0.14**	**−0.25**	**−0.34**	**−0.15**
SMB		1.00	**−0.20**	**−0.46**	**−0.14**	0.03	**−0.38**	**−0.21**	**−0.07**
HML			1.00	0.20	0.67	**−0.24**	**−0.04**	0.66	0.36
RMW				1.00	0.11	0.11	0.68	0.19	0.36
CMA					1.00	**−0.02**	**−0.01**	0.91	0.33
UMD						1.00	0.52	**−0.02**	0.22
ROE							1.00	0.08	0.33
IA								1.00	0.34
BAB									1.00

multifactor portfolio, whereas the weights on UMD and BAB exhibit a much stronger time variation around the weight assigned to these factors by the unconditional multifactor portfolio. Thus, we conclude that the relative weight of each factor changes with market volatility also out of sample.

IA.10 Risk diversification of multifactor portfolios

Compared to the volatility-managed *individual-factor* portfolios, the multifactor portfolios reduce risk by diversifying across multiple factors. To illustrate this effect, Table IA.8 reports the correlation matrix for the returns of the nine unmanaged factors.[23] The correlation matrix reveals that risk diversification is an important driver of the favorable performance of the conditional *multifactor* portfolios, compared to the volatility-managed *individual-factor* portfolios. In particular, the market (MKT) and size (SMB) factors are generally negatively correlated with the other factors, and thus combining MKT and SMB with the rest of the factors helps to reduce the overall risk of the conditional multifactor portfolio.

[23]In unreported results, we find that the managed factors are highly correlated with their unmanaged counterparts (correlation in excess of 86% for every factor) and that the correlations between the managed factors are similar to those between their unmanaged counterparts.