Informed Trading in the Index Option Market

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Abstract. We estimate a structural model of informed trading in option markets. We decompose option order flow into exposures to the underlying asset (through the option delta) and its volatility (through the option vega). We then use these order flow exposures to predict changes in the underlying asset and volatility in a vector auto regressive (VAR) model. The model measures informed trading in the aggregate option market, as option order flows can be meaningfully combined across options with different strike prices and maturities. Further, the order flow aggregation increases statistical power, which is necessary to identify informed trading on the two components. The model also yields a novel price impact parameter of volatility speculation. Estimates using options on the S&P500 confirm that option trades are indeed informed about changes in both the underlying and volatility, although the magnitude of the former is substantially larger.

Key words: Options, informed trading, price impact

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1. Introduction

It is well known that investors may use option markets to exploit private information on future movements of both the underlying asset and its volatility. Showing this empirically, however, is challenging. For example, volatility is not directly observable, and extracting the process from option prices requires a structural model. Also, a large cross-section of sometimes hundreds of options are traded simultaneously, which differ in strike price, expiration date, and type (put or call). An appropriate model should account for the high correlations in option returns, order flows, and liquidity. Further, option prices generally have substantial microstructure noise. The main aim of this paper is to address these challenges, in a model that jointly estimates informed option trading on both the direction and the volatility of the underlying asset.

We develop a methodological framework that recognizes that an option trade provides exposure to the underlying asset (through the option delta) and to its volatility (through the option vega). Accordingly, we construct two order flow exposures by multiplying option net order flow (defined as buyer-originated minus seller-originated volume) with the option delta or vega, which we coin "delta order flow" and "vega order flow". The advantage is that these order flows can be meaningfully aggregated over all trades in the cross-section of options. We then relate the two option order flows to changes in the underlying price and its volatility in a vector autoregressive (VAR) model. The model builds on the seminal work of Hasbrouck (1991), who estimates a two-equation VAR model relating order flows to price changes. The main contribution of our paper is to relate the delta and vega order flows to changes in the underlying asset and its volatility.

Our model offers three key advantages in modeling informed trading in options markets. First, we estimate a single VAR model that uses trades in the cross-section of options across strike prices and maturities. Our results are thus representative of informed trading and liquidity of the aggregate option market. Effectively, we impose economic structure from a theoretical option pricing model to summarize the information content in a large number of order flows in just two variables – delta and vega order flow. This structure allows us to disentangle volatility and price movements, and increase the statistical power to detect informed trading on both components.

Second, the model yields a novel price impact parameter of vega order flow on the level of volatility. It represents the (il)liquidity of volatility speculation. The economic mechanism is that traders with private information on the future evolution of volatility use options to exploit this information. In equilibrium, market makers observe the vega order flow, and respond by adjusting option prices (and implied volatilities). Our estimated price impact of vega order flow on volatility captures this adjustment, and can be interpreted as a measure of the degree of informed volatility trading.

Third, the model accounts for the strong return and order flow correlations across options. In the model, a trade in one option affects returns of others through its impact on the underlying price and volatility. The delta and vega order flows capture cross-option trading strategies, such as straddles, strangles or butterfly spreads.[1] Indeed, we aggregate the *net* delta and vega order flows across options, such that for example a delta-neutral straddle trade has no contribution to delta order flow.[2]

We estimate the VAR model using trade and quote data from 2014 of SPX options written on the S&P 500 index. We construct the delta and vega order flow variables at an hourly frequency, and use the VIX as volatility proxy. In the VAR model we further account for informed trading directly in the underlying asset, by adding a separate fifth equation with the net order flow of the ETF on the S&P500 (SPY).

The main results are as follows. In a given hour, a one standard deviation shock to the vega order flow variable increases the index volatility by 0.07 percentage points (e.g., increasing the average volatility of 11.50% to 11.57%). This effect is 13% of the standard deviation of volatility changes, and reflects the private information of investors speculating on volatility changes. Together, the order flow variables explain 27% of the variation in volatility, leaving 76% to public information arrival.

Further, the estimates show that a one standard deviation delta order flow shock increases the underlying by $0.93 (or 4.8 basis points relative to an average price of $1,930). This effect is

[1] Such option strategies are indeed widely used, see Fahlenbrach and Sandås (2010).

[2] An option straddle involves buying a put and a call, where the negative and positive delta exposures cancel out to leave only volatility exposure.

smaller than a one standard deviation shock in the ETF order flow, which moves the underlying by \$1,50. In fact, the *per dollar* price impact of ETF order flow is 2.9 times larger than the option order flow, after adjusting for the fact that delta order flow shocks have a larger standard deviation than ETF shocks. The higher price impact of ETF flow indicates a higher degree of informed trading in this market, which is puzzling as the SPY ETF is a very liquid market compared to the SPX options. A potential explanation is that the order sizes of option trades are generally an order of magnitude larger than ETF trades, which suggests that the ETF attracts relatively small but informed traders, whereas option markets attract investors wanting to trade in size.

Other results of the VAR model are mostly as expected. The order flow in the ETF is highly correlated with the delta order flow, which suggests investors use both options and the ETF when looking for directional exposure. All three order flow variables are highly autocorrelated at the hourly frequency. The model also confirms the well known leverage effect, which is the negative correlation between the underlying price and volatility. This effect emphasizes the importance of modeling both processes jointly.

Our model imposes a strong economic structure on the data through two crucial assumptions. We validate these assumptions with two tests. First, using a Wald test, we verify that the delta and vega price impacts do not vary across options with different expiration dates or strike prices. This means we can indeed pool delta and vega order flows across options. Second, we test the information loss of our model, which uses a two-factor option pricing model (underlying and volatility) to summarize the price changes across options with different strike prices and maturities. Our analysis, following Bakshi, Cao, and Chen (2000), indicates that our model explains 99.2% of the variation in individual option price changes. Together, the two tests validate our imposed structure, and provide statistical support for a five-equation system to capture the information content of hundreds of option prices and order flows. In addition, our model uses the standard identifying assumptions from the ordering of the equations in VAR models.[3]

Our paper is most closely related to Pan and Poteshman (2006), who show that option order flow is informative about the directionality of the underlying, and Ni, Pan, and Poteshman (2008),

[3]In the robustness section we consider several different orderings of the equations. All results hold.

who focus on the impact of a variable similar to vega order flow on future realized volatility. Our work complements these papers, and offers several methodological contributions and novel empirical findings. First, and most importantly, our framework jointly models the informational content of both option order flow components.[4] Second, Ni, Pan, and Poteshman (2008) focus on an the at-the-money option, whereas our VAR model aggregates volatility information from the whole cross-section of option prices. The imposed structure yields additional statistical power. Third, our empirical findings are based on order flows that are constructed from publicly available trade and quote data, whereas Pan and Poteshman (2006) and Ni, Pan, and Poteshman (2008) use non-public data of non-market maker position-opening order flow disaggregated by investor type. Fourth, our methodology measures permanent price changes due to informed trading, and is not affected by temporary demand pressures. And fifth, the focal point of our empirical study is informed trading on market-wide volatility rather than the volatility of individual stocks.

The aim of our paper is also closely related to Chordia, Kurov, Muravyev, and Subrahmanyam (2017) who study the impact of weekly index option order flow on subsequent S&P 500 index returns. Their study differs from ours both in terms of methodology and empirical findings. Chordia, Kurov, Muravyev, and Subrahmanyam (2017) find that SPX order flow is uninformative about future S&P 500 index returns, whereas the order flow of other index options, namely options on ISE 250 Index, Russell 2000 Index, Nasdaq-100 Index and the S&P Mid Cap Index 400 (all traded on the International Securities Exchange (ISE)), provide significant predictive power. In contrast, our results provide strong support for a direct connection between order flow in SPX and S&P 500 index returns and their volatility. In addition, Chordia, Kurov, Muravyev, and Subrahmanyam (2017) find that high net put order flow of ISE index options predicts negative index returns, and attribute this to informed market makers. We find strong support for the opposite effect which is in line with Chen, Joslin, and Ni (2016), who show that monthly net order flow in deep-out-of-the-money SPX put options predicts low index returns. We generalize this result as our order flow variables are constructed from all option trades. In addition, Chen, Joslin, and Ni (2016) and Chordia, Kurov, Muravyev, and Subrahmanyam (2017) focus on position opening

[4] This approach provides new insights regarding the relative information content of option and spot markets by comparing the price impact of various order flow variables (see Easley, O'Hara, and Srinivas (1998)).

trades, whereas we sign option trades and weigh them by their price and volatility exposure and hence our order flow is directly observable by market participants.

The literature on analyzing the impact of option trades on the volatility of the underlying is still relatively scarce. Bollen and Whaley (2004) show that option-implied volatilities are positively related to the net buying pressure for both S&P 500 index option and options on individual stocks (see also Garleanu, Pedersen, and Poteshman, 2009).[5] Puhan (2014) combines daily changes in open interest of different option categories to identify straddle positions (which provide exposure to volatility). She relates open interest changes to future realized volatility and finds significant effects, especially around macroeconomic news announcements. Figlewski and Frommherz (2016) extract the volatility processes for groups of options sorted on moneyness and put-call characteristics from high frequency option quote data on the S&P500, the E-mini and the SPDR ETF. They show that out-of-the-money options lead innovations in volatility.

A substantial literature focusses on the contribution of option markets to price discovery. In extensions of the Hasbrouck (1995) information shares analysis to option markets, several papers find modest price discovery through option quotes (Chakravarty, Gulen, and Mayhew (2004), Holowczak, Simaan, and Wu (2006) and Rourke (2013)). More recently, Patel, Putnis, and Michayluk (2016) propose an improved version of the Hasbrouck (1995) information share measure that accounts better for the typical noise in option quotes, and accordingly find that options contribute 33% to price discovery. Using a rather different approach, Muravyev, Pearson, and Broussard (2013) find that violations of put-call parity are resolved by adjustments of option prices, which they interpret as evidence of no price discovery in the option markets.

2. A Price Impact Model for the Aggregate Option Market

This section presents a novel methodology to measure the price impact of option order flow. We extend the Vector Auto Regressive (VAR) model of Hasbrouck (1991) to option markets and explicitly take into account (i) the cross-option correlations in order flows, (ii) the cross-option

[5]They also find that approximately 20% of the price impact is reversed the next day, suggesting that a transitory price pressure is an important component of the positive price impact.

price impacts, and (iii) the fact that options can be used to speculate on both the underlying as well as its volatility.

Consider an asset with N options traded on it, where individual options are indexed by $n = 1, \ldots, N$ according to the characteristics strike price, expiration date and put/call identifier. We first recognize that the main determinants of option price changes are the changes in the value of the underlying and its volatility (e.g., Bakshi, Cao, and Chen, 1997).[6] The option price change can be written as

$$d^o_{n,t+1} \quad = \quad \Delta_{n,t}\, d^u_{t+1} + \nu_{n,t}\, d^v_{t+1} + \epsilon_{n,t+1}, \tag{1}$$

where $d^o_{n,t+1} = P_{n,t+1} - P_{n,t}$ is the price change of option n at time $t+1$, $\Delta_{n,t}$ and $\nu_{n,t}$ are the delta and vega of the option, and d^u and d^v denote the price changes in underlying and its volatility, i.e., $d^u_{t+1} = P^u_{t+1} - P^u_t$ and $d^v_{t+1} = v_{t+1} - v_t$. The error $\epsilon_{n,t+1}$ contains terms of order dt of the option pricing model. Throughout the paper, we define the vega as the first partial derivative of the option price with respect to volatility. One of the key challenges is that volatility is not directly observable, and therefore needs to be estimated from market data.

We propose to disaggregate an option trade into its exposure to the underlying asset (through the options delta) and to its volatility (through the options vega). The key advantage of this linear transformation is that option order flows are now expressed in the same units, and can be meaningfully aggregated across options. We define the aggregate net dollar exposure to the underlying by x^Δ_t and to the volatility by x^ν_t, in time period t, by

$$x^\Delta_t \quad = \quad \sum_i Q_i \times \Delta_{i,t} \times BuySell_i \times P^u_{t-1}, \tag{2}$$

$$x^\nu_t \quad = \quad \sum_i Q_i \times \nu_{i,t} \times BuySell_i \times v_{t-1}. \tag{3}$$

We sum over all option trades, indexed by i, in the fixed time interval between $t-1$ and t. We abuse notation slightly, and denote by respectively $\Delta_{i,t}$ and $\nu_{i,t}$ the delta and vega of the

[6] In the Heston model, the impact of other input variables on option price changes are negligible at high frequencies (e.g., changes in the time to maturity and interest rates).

particular option trade i. Further, Q_i is the trade volume in number of options, and $BuySell_i$ a binary variable that equals 1 for a buyer originated trade and -1 for a seller originated trade. In the remainder of the paper, x_t^{Δ} and x_t^{ν} are called *delta order flow* and *vega order flow*, respectively.

The term v_{t-1} refers to the level of volatility, but is expressed in dollars. That is, for a volatility of 11% we use a price of \$0.11. Effectively, we treat volatility as if it were a tradeable asset. Then, an option gives an exposure to this hypothetical asset, proportional to the options vega.[7] Accordingly, both the delta and vega order flow variables are denoted in dollars.

We next relate the delta and vega order flows to the price changes of the underlying (d_t^u) and volatility (d_t^v) in a Vector Auto Regressive (VAR) framework. We include the dollar order flow traded in the underlying asset directly (x_t^u) as an additional equation. This accounts for trading strategies that involve simultaneous trading in options and the underlying. Note that the order flow x_t^u is measured in the same unit as x_t^{Δ}, as both represent dollar order flow exposure to the underlying. The difference however is that the latter is constructed from option order flows.

The empirical model can be written as follows:

$$
\begin{aligned}
d_t^u = &\quad c_1 + \sum_{l=1} A_{1,l} d_{t-l}^u + \sum_{l=0} B_{1,l} d_{t-l}^v + \sum_{l=0} C_{1,l} x_{t-l}^u + \sum_{l=0} D_{1,l} x_{t-l}^{\Delta} + \sum_{l=0} E_{1,l} x_{t-l}^{\nu} + \varepsilon_{1,t} \\
d_t^v = &\quad c_2 + \sum_{l=1} A_{2,l} d_{t-l}^u + \sum_{l=1} B_{2,l} d_{t-l}^v + \sum_{l=0} C_{2,l} x_{t-l}^u + \sum_{l=0} D_{2,l} x_{t-l}^{\Delta} + \sum_{l=0} E_{2,l} x_{t-l}^{\nu} + \varepsilon_{2,t} \\
x_t^u = &\quad c_3 + \sum_{l=1} A_{3,l} d_{t-l}^u + \sum_{l=1} B_{3,l} d_{t-l}^v + \sum_{l=1} C_{3,l} x_{t-l}^u + \sum_{l=0} D_{3,l} x_{t-l}^{\Delta} + \sum_{l=0} E_{3,l} x_{t-l}^{\nu} + \varepsilon_{3,t} \\
x_t^{\Delta} = &\quad c_4 + \sum_{l=1} A_{4,l} d_{t-l}^u + \sum_{l=1} B_{4,l} d_{t-l}^v + \sum_{l=1} C_{4,l} x_{t-l}^u + \sum_{l=1} D_{4,l} x_{t-l}^{\Delta} + \sum_{l=0} E_{4,l} x_{t-l}^{\nu} + \varepsilon_{4,t} \\
x_t^{\nu} = &\quad c_5 + \sum_{l=1} A_{5,l} d_{t-l}^u + \sum_{l=1} B_{5,l} d_{t-l}^v + \sum_{l=1} C_{5,l} x_{t-l}^u + \sum_{l=1} D_{5,l} x_{t-l}^{\Delta} + \sum_{l=1} E_{5,l} x_{t-l}^{\nu} + \varepsilon_{5,t},
\end{aligned}
\tag{4}
$$

where $A_{i,l}, ..., E_{i,l}$ are constant coefficients. Equation (4) emphasizes the ordering of the equation in the structural VAR. This identifies the orthogonal structural innovations, as it restricts the contemporaneous innovation in one variable to affect that of another, but not the reverse.[8] In

[7] Consider the following example. An price of an option with a vega of 10 increases by \$0.10 after a 1% point increase in volatility. The equivalent vega flow is $\nu \times v_t = 10 \times v_t$, which also increases by \$0.10 after v_t increases from \$0.10 to \$0.11.

[8] Our approach is equivalent to estimating the reduced form VAR, and applying a Cholesky decomposition on

particular, the first equation of the system, d_t^u, is affected by all other variables contemporaneously but it does not affect others. In the last equation, the vega order flow x_t^ν contemporaneously affects all other variables but x_t^ν is not affected by others.

While in general the ordering is arbitrary, we believe that the ordering in Equation (4) is the most natural. First, we follow Hasbrouck (1991) by placing the price changes first and the order flows second. This is motivated by sequential trade models, where order flows cause returns because of asymmetric information and informed trading, whereas returns do not directly cause order flows. In the choice between the ordering of the price changes d^u and d^v, we note the former is a traded asset whereas the latter (the volatility) is not. Being a traded asset, d^u should respond fast and efficiently to new information, such that it is affected by contemporaneous innovations in volatility. The reverse does not hold, since volatility moves slower as it is only indirectly traded through a wide range of options.

With respect to the order flows, we also believe our suggested ordering is the most conservative approach. The focus of the paper is on the information content of the delta and vega order flow on the underlying and the volatility process. We want to measure the information content in these series after controlling for any information captured by trading in the underlying itself (x_t^u). Accordingly, our proposed ordering attributes any contemporaneous information captured in for example both x_t^u and x_t^Δ to the former when predicting d_t^u. While this ordering may create a bias by reducing the predictability of x^Δ on d^u, it is the most conservative approach for our purposes.

The proposed model yields several practical advantages. First, the lags in the VAR system naturally account for any GARCH like feature of the volatility process. Second, the volatility process is extracted from the cross section of options. This allows for a simple separation of informed trading in the two components. Third, the analysis of option markets typically requires a data reduction, as the number of actively traded options is typically large (considering put and call options, and strike prices and various expiration dates).[9] To reduce the data, some papers propose to model the option prices as a function of time to maturity and moneyness (e.g., Aït-

the residuals (see chapter 9.4 in Hasbrouck (2007)).

[9] Chan, Chung, and Fong (2002) also use an extension of the Hasbrouck (1991) model to option markets, but use only data of one representative put and call option.

Sahalia and Lo, 2000). In this paper we apply a data reduction motivated by a two-factor option pricing model.

3. Empirical Setup

3.1 Data

Our trade and quote data are collected from Reuters Tick Historical and cover the time period from January 2, 2014 until December 31, 2014. Our option data set consists of SPX options which are European-style contracts written on the S&P 500 index. Options differ in strike price, put/call identifier and expiration dates. Options in our dataset are part of the third-Friday expiration cylce and constitute the most liquid option contracts written on the S&P 500 index. For each option trade, we observe the exact time of the trade (to the millisecond) as well as the trade price and volume. Quote data consist of the best bid and ask prices and associated quantities, which are used to classify each trade as a market buy or sell order. In addition to option data, we collect corresponding trade and quote data for the SPY ETF which we use as a tradeable proxy for the underlying asset.

We apply a range of standard data filters. First, we discard options with less than 7 or more than 180 calendar days to maturity. Long-term options are very infrequently traded, and short-term options are adversely affected by changes in the expiration cycle. Second, we remove individual trades for which the dollar volume exceeds 10 million USD, as these are likely negotiated over the counter (OTC).[10] And third, we avoid open and close rotations by keeping trades between 8:45am to 2:45pm local time, leaving a sample of six full trading hours.

3.2 Model Design Choices and Implementation

The empirical implementation of the proposed model requires several design choices such as the sample frequency and the construction of the order flow variables x_t^{Δ} and x_t^{ν} as well as the factor changes d_t^u and d_t^v. We describe our approach to construct these below.

[10] This filter drops less than 0.09% of the trades.

Sampling frequency: We sample data at an hourly frequency, due to the high degree of microstructure noise in option prices. Options are characterized by relatively large bid ask spreads and tick sizes (relative to their prices). Therefore, option prices typically adjust infrequently and slowly, only after sufficiently large changes in the underlying price or volatility. The volatility process is particularly noisy, as it is not traded directly and contains measurement error.[11]

Delta and vega: A second important design choice regards the delta and vega calculation for the order flow variables in Equations (2) and (3). We use the smile-consistent option pricing model of Heston (1993) to calculate deltas and vegas for each individual option trade. Our methodology for calculating these option sensitivities can be summarized as follows. First, we utilize the theoretical, functional relationship between the (observable) VIX index and the (unobservable) spot volatility to simultaneously calibrate the Heston model to intra-daily option data and the VIX index on the first day of our sample. Our approach avoids filtering techniques and high-dimensional optimization (Broadie, Chernov, and Johannes (2007), Christoffersen, Jacobs, and Mimouni (2010)) and is easy to extend to other pricing models or option markets. Second, we use the calibrated parameters from the first day of the sample and VIX index values on the next trading day to obtain the out-of-sample S&P 500 spot volatility. Third, equipped with estimates of the spot volatility and the structural parameters of the model, we then calculate deltas and vegas for all option trades on the second day using standard Fourier inversion techniques. The procedure is then repeated for all days in our sample. Our methodology allows for a conceptually easy construction of the trading exposure to the underlying S&P 500 (x^{Δ}) and the S&P 500 volatility (x^{v}) using the definitions in Equations (2) and (3). We provide a more detailed description of our methodology in Appendix A.

Volatility process: We use the VIX index values to estimate the volatility process of the S&P 500 at a one minute frequency. We briefly describe our construction approach. We follow a

[11]For more advanced statistical methods to account for (microstructure) noise, see e.g., Mykland and Zhang (2017).

standard procedure, and define the time-t VIX index with maturity τ by

$$VIX_t(\tau) = \left[\frac{2\, e^{r\tau}}{\tau} \int_{\mathbb{R}_0^+} \frac{O(t, t+\tau, k)}{k^2} dk \right]^{1/2} \tag{5}$$

where $O(t, K, T)$ denotes the mid price of an out-of-the-money (OTM) option (with strike K and maturity T) where moneyness is defined relative to the forward price of the underlying. To approximate the integral in Equation (5), we first construct an option pricing function that is continuous in the strike K. We follow the literature and interpolate the implied volatilities of OTM options by a cubic polynomial and extrapolate the curve by fixing the implied volatilities of options beyond the traded strike range to the nearest available market implied volatility (for these procedures, see Broadie, Chernov, and Johannes (2007) and Carr and Wu (2009)). We then use a simple, adaptive Gauss-Kronrod quadrature method to calculate the integral in Equation (5). We sample our option data at a minute frequency to construct these data intra-daily, Appendix A describes the link between the VIX and spot volatility in more detail.

Price of the underlying: We back out the underlying price process from option prices. By estimating implied futures prices constructed from put-call parity we follow the convention in the option pricing literature in order to avoid estimating a time-varying dividend yield and its term structure. To this end, for each t on a one-minute grid, we collect every put/call pair with identical strike price and identical maturity date and use market mid quotes to solve put-call parity for unobservable futures prices. At a given time t and for a fixed option maturity T, we average the implied futures prices over all available strikes for which a put and a call price is available to obtain the option implied futures price $F_{t,T}$.[12] We average the changes over all short-term futures in our sample to calculate d_t^u. Our empirical results are robust to using alternative approaches.

[12]We use linearly-interpolated rates from the OptionMetrics zerocurve file as a proxy for the risk-free rate of interest.

4. Empirical Results

4.1 Summary Statistics

Individual option trades: Table 1 reports summary statistics of individual option trades. Column (1) shows that trades are generally large with an average size of $95,000 which is about 20 times larger than a typical stock trade (of approximately $5,000). Note that the dollar risk that changes hands in these trades is even larger, considering the embedded leverage options offer. The average SPX option trade is 56.5 contracts, where each contract consists of 100 options. Column (1) further reveals that 41% of the option trades are calls (leaving 59% puts); and that 61.5% of the trades are buyer originated (leaving 38.5% seller originated).[13] The relatively strong buying demand in put options suggests that investors use options to hedge downside risk in the S&P 500 index. The overall sample consists of more than 1.7 million trades.

The standard deviation of the deltas is 0.34, which suggests there is considerable variation in the exposure options offer with respect to the underlying. The average vega is 73.2, with a standard deviation of 50.3 suggesting options also vary substantially in their volatility exposure. To interpret these numbers, after an increase in (annualized) volatility from e.g. 15% to 16%, the price of a typical option increase by approximately $73.2 \times 0.01 = \$0.73$. Combined, these numbers indicate that investors actively trade options with a wide variety of characteristics, presumably to get varying exposures to the underlying and volatility.

Column (1) further reveals that out-of-the-money options are traded relatively frequently, as the average moneyness (0.945) is less than one.[14] The average time to maturity is 0.11 years, or about 40 calendar days.

The remaining columns report summary statistics for nine subsets of the data, sorted by time to maturity (ttm) and moneyness. Specifically, we make three buckets by ttm according to the cutoff values of 30 and 60 days. We define moneyness as $F_{t,T}/K$ for calls and $K/F_{t,T}$ for puts and use cutoff values of 0.95 and 1.0 to group options into either deep out-of-the money,

[13]The average trade direction (1 for a buyer originated trade and -1 for a seller originated trade) is 0.23, which implies that $0.5 + 0.5 \times 0.23 = 0.615$ or 61.5% is buyer originated.

[14]We define moneyness for calls by Strike/Forward price, and for puts by Forward price/Strike. This way, in-the-money options have a large absolute delta.

out-of-the-money or in-the money. While these cutoff values are arbitrary, we choose them such that they have a reasonable number of trades in each subset. These subsamples will be used throughout the paper. Each particular subset can be identified in the table by the corresponding average moneyness and ttm. As expected, the subgroups which contain near-the-money options yield the highest exposure to volatility (those in columns (3),(6) and (9)). Similarly, the subsets in columns (4), (7) and (10) yield the highest exposure to the underlying.[15]

Aggregated order flow variables: Table 2 provides summary statistics of all variables used in the estimation of Equation (4), including hourly order flow variables (in exposures of 100 millions USD),[16] as well as the price changes in the underlying asset and its volatility. Our sample period is characterized by selling pressure in the SPY ETF, as its average order flow (x^u) is –39 million USD per hour (with a standard deviation of 457 million USD). The delta order flow constructed for option trades, measured in the same unit, is positive with 181 million USD per hour, and is more than twice as volatile with a standard deviation of 1,024 million USD. This finding is in line with the summary statistics for individual trades, and confirms that investors in the SPX market trade in size.

The standard deviation of vega order flow is about 60 times smaller than the standard deviation of delta flow, but as a pricing factor the volatility is 26 times riskier than the underlying. Accordingly, the magnitude of the risk that changes hands through a one-standard deviation delta order flow trade is about twice as large as that of the vega flow. To see this, the annualized volatility level is on average 11.5%, with an hourly standard deviation of 0.6 percentage points. The standard deviation of percentage changes in volatility is $5.2\% = 0.006/0.115 \times 100$. The spot price is \$1,930 on average, and its difference d^u has a standard deviation of \$3.98 per hour. The standard deviation of the underlying return is $0.2\% = 3.98/1,913 \times 100$, which is about 26 times smaller.

[15]These subsets contain deep in the money calls (with a delta close to one) and deep in the money puts (with a delta just below zero). This explains why the average delta of trades in these subsets ranges between 0.3 and 0.5.

[16]This unit is most convenient when estimating the structural VAR.

4.2 Price Impact Model

We estimate the VAR model in Equation (4) with two lags, where the lag structure is determined using the Akaike Information Criterion. Following Hasbrouck (1991), we assume the trading process restarts at the beginning of each trading day, and therefore we set all initial lagged values to zero.

Table 3 provides the long run impulse response functions (IRFs) of the model. The table contains the five-by-five matrix of IRFs, which show the impact of an impulse in one variable (in columns) on the cumulative impact on all other variables (in rows). We allow each shock to iterate through the system for four hours. The table also shows the size of each shock in the first column. For convenience, see Figure 1 for a plot of the most relevant IRFs over time.

The table offers two main results. First, column (6) shows that a one-standard deviation shock in x^{ν} increases d^{v} by 0.07. This implies a volatility increase from 11.5% to 11.57% if current volatility is at the sample average, and corresponds to 0.14 standard deviations of d^{v}. This magnitude is economically sizeable because it is caused by vega order flow. This leaves the remaining variation in volatility to be driven by public information arrival (and the other order flows).

The second main result of Table 3 is shown in Column (5), where the long-run impact of a shock in x^{Δ} on d^{u} is 93.14 cents (\$0.93). This can be interpreted as a change in the average value of the underlying from \$1,913 to \$1,913.93; an increase of 4.9 basis points. The effect is economically large as it corresponds to 0.5 standard deviation of d^{u}, which is \$1.89 at the hourly frequency. Interestingly, the impact of an order flow shock in the ETF (x^{u}) is even larger still with \$1.50 per standard deviation (column (4)). Further, note that the size of ETF shocks are much smaller than the size of delta flow shocks: \$459 million vs \$821 million. Accordingly, the *per-dollar* price impact of an ETF shock is 2.9 times larger than when the same exposure is obtained through options.[17]

Table 3 offers several additional results. The coefficients on the diagonal of the table reveal the

[17] The dollar impact of an ETF order flow shock is \$1.50/4.59, which is 2.9 times larger than that for options: \$0.93/8.21.

long-term impact of an impulse on the same variable. All variables are positively autocorrelated, because the long-term impacts are greater than the size of the structural shocks (shown in the first column). Consistent with the leverage effect, we see that x^Δ negatively affects d^v and x^ν negatively affects d^u.[18] Further, x^Δ and x^ν are uncorrelated in the long-run: a shock in one does not affect the other. Indeed, x^ν and x^Δ mechanically have a positive correlation for call option trades and a negative correlation for put option trades — on average, the two opposing effects seem to cancel out. Lastly, delta flow significantly causes ETF flow x^u, but not the other way around. This is likely a result of the ordering of the equations, where we allow a contemporaneous shock in x^Δ to affect x^u, but not the other way around.

The plots in Figure 1 reveal the cumulative impacts over time. For brevity, we only show the shocks of the order flow variables on the changes in the price and volatility. The figure reveals that the majority of the effects occur in the first hour.

Table 4 contains the regression coefficients of the VAR estimated by OLS. The omitted coefficients in the columns reveal the ordering of the equations in the structural model (which identify the structural residuals). The results of the IFRs are clearly visible in the coefficients as well. Perhaps noteworthy is the strong negative effect of d^v on d^u (t-stat of 35.1), which is the well known leverage effect.[19] Further, the positive contemporaneous coefficient of x^Δ on x^u (0.10), suggests that traders obtain exposure to the underlying by trading simultaneously options and the ETF. The coefficient would have been negative if traders were to hedge positions in options by trading the ETF. The table also clearly reveals the aforementioned positive autocorrelations in the order flow variables.

Our results have several important implications. First, traders who are privately informed about the "fundamental" volatility level indeed trade options to exploit this information. Option market makers observe the order flows, and accordingly update the prices of *all* options to reflect this volatility information. It is important to emphasize that the volatility price process is extracted from all options in the cross-section of strike prices. This suggests that we indeed

[18] The intuition is that x^Δ positively predicts d^u, and therefore negatively d^v because of the negative correlation between d^u and d^v. Similarly, x^ν positively predicts d^v, and therefore negatively d^u.

[19] Indeed, the underlying and volatility process are strongly negatively correlated, with a correlation of -0.86 at the hourly frequency.

measure fundamental volatility information, and not a transitory impact that could be caused by a temporary trading demand. We find it unlikely that a transitory demand pressure in a single option would affect prices of all options and therefore the volatility process.

A second implication is that we confirm that delta order flow exposure indeed affects the price of the S&P 500. While this may seem obvious at first, the empirical literature in fact has many contradicting findings. Several papers find no meaningful impact of option order flow on the underlying (e.g., Stephan and Whaley (1990), Chan, Chung, and Fong (2002), Chordia, Kurov, Muravyev, and Subrahmanyam (2017)), whereas others do (e.g., Pan and Poteshman (2006), Muravyev (2016)).

A third implication of our findings is that cross-option price impacts could be very significant for large investors, who wish to trade multiple options. Indeed, a trade in one option affects the prices of others through its impact on the underlying and volatility. Accordingly, we should not analyze the liquidity in the order books of individual options, but rather estimate it through a joint system.

Fourth, our finding that ETF order flow has a 2.9 times higher per dollar price impact than delta flow hints at a certain form of market segmentation between SPX option markets and the SPY ETF. Option trades are much larger in terms of dollar volume and risk, which suggests that large investors prefer to trade in size using options. In contrast, the ETF market attracts relatively small and informed traders. The low price impact of delta flow is consistent with Muravyev and Pearson (2015), who find that option bid-ask spreads are relatively small after they control for a particular execution timing effect.

4.3 Testing the Imposed Structure

In this section we evaluate two crucial assumptions the structural model imposes on the data. We first test whether we are allowed to aggregate the delta and vega order flows across different options. Specifically, we test whether the price impact of delta flow and vega flow is the same when the variables are constructed from trades in different types of options. Second, we test how much information we lose when summarizing the cross section of option prices with just a two-dimensional process.

Price impacts for different types of options: We construct the delta and vega order flows separately for trades in the nine subsets of options introduced in Section (4.1), which are sorted by time to maturity and moneyness. We estimate the equations in the VAR using the original delta and vega flows as in Table (4), but add the flows of the individual subgroups. For each regression we do a single Wald test, testing whether all coefficients on the subgroups are jointly equal to zero.[20]

The results of the Wald tests are as follows. For the d^v equation, the p-value of the joint test for the vega flows is 0.15, so we fail to reject equality of the vega flows across subgroups. However, the p-value for the delta flows is 0.0145, suggesting that the impact of delta flow on volatility differs significantly across the subgroups. Further inspection reveals that the coefficient of delta flow on volatility is higher for options with a longer time to maturity. We are not too worried about this rejection, because it is not a coefficient of interest. For the d^u equation, the p-value of the Wald test is 0.12 for the delta flows, and 0.37 for the vega flows. For both, we fail to reject equality of the coefficients for the different subgroups of the data. These results are reassuring, and suggest that the price impact of delta flow and vega flow does not differ across options with different time to maturity or moneyness.

Information loss using a two-dimensional process: The five equation VAR model is designed to summarize the information captured in the large state space of options prices and order flows across strike prices, expiration dates and puts and calls. To evaluate the performance of this data reduction technique we follow Bakshi, Cao, and Chen (2000), and test the model in Equation (1) by regressing the price change of individual options, with characteristics n={*Strike, Expiration date,Put-call identifier*}, on the predicted price change according to the option pricing model:

$$d^o_{n,t} = \beta_0 + \sum_{l=0}^{2} \beta_{l,1} \Delta_{n,t-l-1} d^s_{t-l} + \sum_{l=0}^{2} \beta_{l,2} \nu_{n,t-l-1} d^v_{t-l} + \varepsilon_{n,t}, \qquad (6)$$

where $d^o_{n,t}$ is the price change of the option, and the terms $\Delta_{n,t} d^s_t$ and $\nu_{n,t} d^v_t$ represent the options

[20]We have nine subgroups, but by adding the aggregate delta and vega flows we loose one subgroup that becomes the base case. With two lags in the system, we jointly test whether the 8*3=24 coefficients on the delta flows (or vega flows) are equal to zero. We repeat the test for each equation in the VAR.

exposure (delta or vega) multiplied by the factor (change in underlying price or change in volatility). The regression contains two lags to be consistent with the VAR model, and is estimated once for the full sample of options. If the structure we impose from a two-factor option pricing model is correct, the regression should yield coefficients $\beta_0 = 0$, $\sum_{l=0}^{2} \beta_{l,1} = 1$, and $\sum_{l=0}^{2} \beta_{l,2} = 1$. Indeed, for these parameter values the actual option price changes are fully captured by the predicted values of the two factor model. Further, the R-Squared of the regression equals one in theory.

Table 5 provides estimation results. Column (1) shows the full sample regression. We estimate an R-Squared of 99.2%, which suggests that our model does an excellent job at summarizing the information content in option price changes. Further, we see that $\sum_{l=0}^{2} \beta_{l,1} = 0.997$ and $\sum_{l=0}^{2} \beta_{l,2} = 0.899$, which both are close to one from an economical point of view. This also confirms that option returns are strongly affected by changes in volatility, and that options are useful tools to speculate on changes in volatility. The t-statistic on coefficient $\Delta \times d^s$ is extremely large (5,551) due to the high R-Squared and the sheer size of the data (851,051 observations).[21] For this reason, we do reject equality to one for both $\sum_{l=0}^{2} \beta_{l,1}$ and $\sum_{l=0}^{2} \beta_{l,2}$ (the t-statistics are 14.6 and 37.3, respectively). Column (1) further shows that the lagged coefficients are smaller, but still statistically significant. This motivates the use of the lags in the system. Compared to Bakshi, Cao, and Chen (2000), the two factor model works much better with more recent data, mainly because markets have become more efficient. Using SPX option data from 1994, they find coefficients of $\beta_1 = 0.80$, $\beta_2 = 0.41$, and an R-Squared of 59%.

We repeat the exercise for the nine subsets of options sorted by time to maturity and moneyness. In general, the model works very well. We see it performs slightly weaker for options with a low moneyness and short time to maturity (column 2). In this case, the $\sum_{l=0}^{2} \beta_{l,1} = 0.612$ and $\sum_{l=0}^{2} \beta_{l,2} = 1.35$. These coefficients are likely affected by the model misspecification of the Heston model, where we impose a fixed mean reversion parameter in the volatility process (see Eraker, 2004). Further, this subset contains options with very low prices, and we know that the microstructure noise is more severe here as tick sizes are relatively larger. The model works better for the remaining columns, which all have an R-Squared exceeding 92%.

[21] This analysis uses data of all individual options (one observation per option-day-hour), whereas the VAR used one observation per day-hour).

From an economical standpoint our model seems to work well. However, the rejection of equality to one for $\sum_{l=0}^{2} \beta_{l,1}$ and $\sum_{l=0}^{2} \beta_{l,2}$ means that actual option price changes differ from what our model predicts. This implies that either the deltas and vegas contain errors; or that the true data generating process for option prices contains additional factors (see e.g., Christoffersen, Heston, and Jacobs (2009) or Bardgett, Gourier, and Leippold (2015)). As a consequence, there is some bias in the delta and vega order flows used in the VAR model. This issue could be tackled by using a more advanced option pricing model. However, we see no economic channel how such misspecification could alter our findings. In fact, if the misspecification were random, the bias would make it more difficult to find evidence of informed trading.

5. Robustness Analysis

We have executed the following robustness analyses. All the results can be found in the Web Appendix.

Ordering of the equations. An important but somewhat arbitrary choice is the ordering of the equations in the structural VAR. The ordering determines in which direction a contemporaneous correlation between two variables goes: only one is allowed to affect the other, but not the reverse. Otherwise, the system is under identified. The ordering in turn determines the structural innovations, the regression coefficients, and the impulse response functions. While with five equations we have 120 potential combinations, there are two alternative orderings that deserve most attention. We estimated one version where we reorder d^u and d^v; which switches the direction of the leverage effect. In this case, d^v strongly causes d^u, and the same holds for contemporaneous order flows. The long-run impulse response functions however, are very similar to the original estimates. We estimated a second version where we put x^u before x^{Δ} and x^{ν} (instead of after). In this case, the information contemporaneously captured in x^u and x^{Δ} is attributed to the latter. Accordingly, the coefficient of x^{Δ} on d^u is estimated more precisely (a higher t-stat). The long-run coefficients in this version are very similar to the main specification, both in terms of economic magnitudes and significance levels.

Sampling Frequency. We have estimated the VAR model using data sampled at the half-hour frequency. We double the number of lags in the system to be consistent with the main specification. An important result is that the coefficient of vega order flow on volatility is about 40% smaller than in the main specification (but still statistically significant). This suggests that at higher frequencies, the variance in the volatility process starts to get dominated by microstructure noise. Adding lags in the system does not seem to capture this problem, which hint at a certain low-frequency components in the data generating process. Interestingly, the coefficients on delta order flow and ETF flow are virtually unaffected. This result suggests that the order flow information on the direction of the underlying is incorporated very quickly into prices, whereas volatility information is much more slowly reflected. A likely explanation is that spot volatility is not directly tradeable, such that volatility information indeed only gets incorporated into the volatility price after a sufficient number option prices have updated.

Price of the underlying. In the main specification we use the forward price (extracted from option data) to proxy for the value of the underlying asset. The advantage is that this is not confounded by dividend yields paid before option expiration. In a robustness check, we estimated the VAR using the S&P500 ETF price as the underlying instead. Results are virtually identical in this specification. This result is reassuring, as evidence suggests that the ETF market typically leads in price discovery (Hasbrouck, 2003). At the one-hour frequency however, this channel is of no concern.

Price changes versus returns. We estimated the VAR with the return on the underlying and volatility, instead of the price differences. After appropriately scaling the coefficients, the results are virtually identical to those in the main specification. We chose to report the version with price differences, as it corresponds more naturally to the analysis in Section 4.3.

The role of the SPY ETF order flow. We estimated a four equation version of the model, where we drop the SPY ETF order flow equation altogether. In this case, generally results are stronger in terms of statistical significance (although the point estimates of the coefficients are similar). Not only do we estimate fewer parameters, also note that the ETF order flow is correlated with option order flows. When adding ETF flow in the main specification of the model, it absorbs part of the predictive power.

6. Conclusion

We offer a novel framework to estimate the liquidity of the aggregate options market. To our knowledge, we are the first to propose a single model that captures trading in potentially hundreds of options, with differing strike prices, expiration dates and types (put or call). We impose economic structure on the data using theoretical option pricing models, which takes into account the high cross-option correlations in returns, order flows, and liquidity.

The empirical results using S&P 500 (SPX) options are consistent with classical models of informed trading. We find that order flow exposures to the underlying (or volatility) positively predict changes in the price (and volatility); consistent with strategic traders using options to speculate on changes in either component. As a result, order flow in one option affects the underlying and volatility, and therefore indirectly the prices of all other options. This result is key for large traders concerned about transaction costs when trading multiple options.

We believe the results are driven by asymmetric information (informed trading), and not by transitory demand pressures (inventory concerns). We extract the value of the underlying (the option forward price) and volatility from the cross-section of options. It should be unlikely that a transitory demand pressure moves all option prices simultaneously. Further, our VAR system contains several lags, such that the the price changes are permanent at least up to this lag number.

We see an important analogy with the literature on equity market fragmentation, where the same asset is traded on multiple trading venues. In the current setting, highly correlated options are traded in multiple limit order books. The economic forces of informed trading strategies across markets (or options) are similar, including the resulting correlations in order flows, returns, and liquidity.

Conceptually, our approach can easily be extended to options of individual stocks, or to other markets where groups of highly correlated assets are traded. For example, for the corporate bond market a term structure model could be used to estimate the bond pricing factors. Then, each bond trade gives exposure to these factors, which could be used in a similar structural VAR with factor returns and order flow exposures.

A. Appendix: Heston Model Details

Model definition. The Heston (1993) model has become the most important benchmark in the option pricing literature. Its main theoretical advantage stems from the mathematical tractability of its characteristic function (see Duffie, Pan, and Singleton (2000)). Under the model assumptions, the S&P 500 index S and its volatility v are described by the following stochastic differential equations under the risk-neutral measure $\mathbb{Q}$:

$$dS_t = (r-q)S_t dt + S_t v_t dW_t^1 \tag{A.7}$$

$$dv_t^2 = \kappa^{\mathbb{Q}}\left(\theta^{\mathbb{Q}} - v_t^2\right)dt + \sigma v_t\left(\rho dW_t^1 + \sqrt{1-\rho^2}dW_t^2\right) \tag{A.8}$$

where W^1 and W^2 are standard Brownian motions under $\mathbb{Q}$. Structural parameters are given by $\kappa^{\mathbb{Q}} > 0$, $\theta^{\mathbb{Q}} > 0$, $\sigma > 0$ and $\rho \in [-1,1]$. The risk-free rate is denoted r and q is the continuous dividend yield.

Due to the affine model structure, European call and put prices can be calculated by standard Fourier inversion methods (see below) and we denote $P(t, S_t, v_t, K, T, \omega)$ the time-t price of a put or call option ($\omega = 1$ for a call, $\omega = -1$ for a put) with strike K and maturity $T > t$. We collect all structural parameters and the risk-free rate in the vector Θ but drop the dependence of option prices on the parameter vector for notational simplicity. It follows from Ito's Lemma that

$$dP(t, S_t, v_t, K, T, \omega) = \frac{\partial P(t, S_t, v_t, K, T, \omega)}{\partial S_t} dS_t + \frac{\partial P(t, S_t, v_t, K, T, \omega)}{\partial v_t} dv_t + dt\text{-terms} \tag{A.9}$$

Due to the analytical tractability of the option pricing function P, the first partial derivatives with respect to S and v can also be calculated analytically and we denote these as:

$$\Delta(t, S_t, v_t, K, T, \omega) \equiv \frac{\partial P(t, S_t, v_t, K, T, \omega)}{\partial S_t} \tag{A.10}$$

$$\nu(t, S_t, v_t, K, T, \omega) \equiv \frac{\partial P(t, S_t, v_t, K, T, \omega)}{\partial v_t}. \tag{A.11}$$

To relate changes in option prices to observable variables, we exploit the theoretical relation between the spot volatility and the VIX index. It is straightforward to show that under our model assumptions, the squared VIX index is a linear function of spot variance, i.e. $VIX_t^2(\tau) = a(\Theta, \tau) + b(\Theta, \tau)v_t^2$ where a and b are functions of the structural model parameters Θ and the maturity of the options used to construct the VIX index (denoted τ, i.e. 30 days for the standard index published by CBOE). Using these theoretical results, given a parameter set Θ, option delta and vega can also be calculated by Fourier inversion.

Option pricing formulae. Duffie, Pan, and Singleton (2000) show that the generalized characteristic function for affine jump-diffusion processes is exponentially affine in the state variables $\log S_t$ and v_t^2. For the Heston model, the logarithm of the characteristic function under the risk neutral measure $\mathbb{Q}$ is given by

$$\log \Psi^{\mathbb{Q}}(u, \tau, S_t, v_t) \equiv \log E^{\mathbb{Q}} \left[e^{iu \log S_{t+\tau}} \middle| \mathcal{F}_t \right] = \mathcal{A}(u, \tau) + \mathcal{B}(u, \tau)v_t^2 + u \log S_t,$$

where $\mathcal{A}$ and $\mathcal{B}$ are complex-valued functions, $i = \sqrt{-1}$ and $E^{\mathbb{Q}}[\cdot | \mathcal{F}_t]$ denotes the risk-neutral $\mathcal{F}_t$-conditional expectation. For expositional clarity the dependence of all functions on the parameter set of the model Θ is suppressed. Following Bates (2006), the price of a European call option is given by

$$P(t, S_t, v_t, K, T, \omega = 1) = e^{-r\tau} F_{t,T} - e^{-r(T-t)} K \left(\tfrac{1}{2} + \tfrac{1}{\pi} \int_0^\infty \Re \left[\frac{e^{-iu \log K} \Psi^{\mathbb{Q}}(u, T-t, S_t, v_t)}{iu(1-iu)} \right] du \right).$$

where $F_{t,T} = e^{(r-q)(T-t)} S_t$ and $\Re$ denotes the real part of a complex number. Partial derivatives of the call price are given by

$$\frac{\partial P(t, S_t, v_t, K, T, \omega = 1)}{\partial S_t} = e^{-q(T-t)} - \frac{e^{-r(T-t)}K}{\pi} \int_0^\infty \Re\left[\frac{iu}{S_t}\frac{e^{-iu\log K}\Psi^{\mathbb{Q}}(u, T-t, S_t, v_t)}{iu(1-iu)}\right] du$$

and

$$\frac{\partial P(t, S_t, v_t, K, T, \omega = 1)}{\partial v_t} = -\frac{2v_t \times e^{-r(T-t)}K}{\pi} \int_0^\infty \Re\left[\frac{\mathcal{B}(u, T-t)\, e^{-iu\log K}\Psi^{\mathbb{Q}}(u, T-t, S_t, v_t)}{iu(1-iu)}\right] du.$$

The theoretical value of the VIX index with maturity $\tau_v = 30/365$ can be recovered from the characteristic function. One can show that for all the Heston model

$$VIX_t(\tau) = \sqrt{\theta^{\mathbb{Q}}\left(1 - \frac{1 - e^{\kappa^{\mathbb{Q}}\tau}}{\tau\kappa^{\mathbb{Q}}}\right) + \frac{1 - e^{\kappa^{\mathbb{Q}}\tau}}{\tau\kappa^{\mathbb{Q}}} v_t^2}. \tag{A.12}$$

Empirical estimation. In order to compute the delta and vega for all option transactions in our dataset, we need to calibrate the Heston model to market data. There is no standard methodology in the literature regarding the estimation of model parameters of option pricing models. We adopt a simple calibration technique and estimate model parameters for every trading in our sample using intradaily option quotes. To this end, we minimize the mean-root-squared-error of all available OTM options with maturities from 7 to 180 days as follows:

$$RMSE_t = \arg\min_{\Theta} \sqrt{\frac{1}{N_t} \sum_{i=1,\ldots,N_t} \left(P^{ma}(t_i, K_i, T_i, \omega_i) - P(t_i, S_{t_i}, v_{t_i}, K_i, T_i, \omega_i, \Theta)\right)^2}, \tag{A.13}$$

where N_t is the number of available option quotes on day t, t_i are set to minutely intervals from 9:45am to 15:45pm (on day t) and $P^{ma}(t_i, K_i, T_i, \omega_i)$ denotes the market mid-quote of option i. We use the shortest option maturity to invert the theoretical VIX formula above to obtain v_t. To limit the number of option contracts in our calibration, we restrict the calibration to the most liquid contracts, OTM contracts and contracts with short to medium maturity. For the calculation of deltas and vegas we use calibrated parameters from the previous trading day.

Our calibration procedure has two distinct features. First, our methodology circumvents the problem of filtering the latent variance process, as we calibrate the model to both option prices and the VIX index simultaneously. Alternative approaches such as those in Broadie, Chernov, and Johannes (2007) or Christoffersen, Jacobs, and Mimouni (2010) rely on filtering techniques or the calibration of v_t which leads to additional complexity in the calibration algorithm. And second, our recalibration allows us to be robust to changing market dynamics. While the Heston model may be rejected because of the imposed structure (see for instance Broadie, Chernov, and Johannes (2007) or Christoffersen, Jacobs, and Mimouni (2010)), our recalibration ensures that we are not particularly sensitive to possible model misspecification. To calculate deltas and vegas for option transactions, we use parameters estimated using option data on the previous day, so our procedure is out-of-sample. While parameters in the calibration may change over time, our assumption is that the Heston model provides a reasonable way of separating price from volatility risk using model calibrations from the previous trading day. Following the empirical set-up in Bakshi, Cao, and Chen (2000), we provide strong empirical evidence below that this approach covers more than 98% of the variation in option prices in our sample. Hence, Equation (A.9) in combination with our Heston model implementation provides a highly accurate description of option market prices and the benefit of a reduction of variables to only price and VIX risk.

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Table 1 Summary statistics SPX option trades S&P500

The table shows the mean and standard deviation (in brackets) of option trade variables. The first column reports results for the full sample, and the remaining columns those for the nine subsets of the data. The option trades are sorted by time to maturity and moneyness. Most variables are self explanatory. The vega's are calculated with respect to the underlying volatility. The trade direction equals 1 if the trade is originated with a market buy order, and -1 if it is a market sell order. The call indicator equals 1 if the trade is in a call option, and zero if it is in a put option.

	Full Sample	Subsets								
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Volume ($1,000)	95.3	6.93	38.4	140	22.5	108	207	70.6	245	321
	(348)	(41)	(152)	(471)	(126)	(316)	(569)	(264)	(498)	(745)
Volume (#contracts)	56.49	58.47	47.93	43.66	57.23	59.40	46.99	75.40	68.54	52.11
	(187.3)	(222.3)	(162.5)	(126.1)	(234.6)	(183.2)	(113.5)	(253.9)	(149.3)	(102.2)
Price ($)	18.10	1.085	7.275	39.36	3.534	18.06	51.84	7.284	37.27	68.75
	(38.46)	(1.363)	(9.127)	(69.68)	(3.820)	(15.66)	(77.14)	(8.339)	(15.96)	(86.30)
Option Δ	0.0634	-0.0121	0.0514	0.219	-0.0195	0.0945	0.152	-0.0129	0.128	0.154
	(0.342)	(0.0198)	(0.271)	(0.639)	(0.0494)	(0.333)	(0.588)	(0.0846)	(0.410)	(0.559)
Option ν	73.20	11.63	75.61	103.8	31.19	105.0	126.0	35.87	112.5	118.1
	(50.26)	(15.50)	(39.56)	(33.97)	(27.21)	(38.30)	(31.92)	(29.99)	(31.96)	(34.41)
Trade direction (1,-1)	0.231	0.303	0.166	0.208	0.236	0.192	0.260	0.318	0.243	0.300
	(0.951)	(0.919)	(0.970)	(0.965)	(0.942)	(0.961)	(0.950)	(0.920)	(0.951)	(0.942)
Call indicator (1,0)	0.411	0.0933	0.527	0.580	0.187	0.568	0.518	0.214	0.534	0.513
	(0.492)	(0.291)	(0.499)	(0.494)	(0.390)	(0.495)	(0.500)	(0.410)	(0.499)	(0.500)
Moneyness	0.231	0.303	0.166	0.208	0.236	0.192	0.260	0.318	0.243	0.300
	(0.951)	(0.919)	(0.970)	(0.965)	(0.942)	(0.961)	(0.950)	(0.920)	(0.951)	(0.942)
Time to Maturity (years)	0.411	0.0933	0.527	0.580	0.187	0.568	0.518	0.214	0.534	0.513
	(0.492)	(0.291)	(0.499)	(0.494)	(0.390)	(0.495)	(0.500)	(0.410)	(0.499)	(0.500)
Number of trades	1,713,322	200,554	411,862	104,017	227,050	271,710	99,595	160,994	148,213	89,327

Table 2 Summary statistics returns and order flows of the mean and volatility of the S&P500

Order flow is defined as buyer originated minus seller originated trading volume, expressed in dollars. The first row shows statistics, aggregated at the hourly level, on the net order flow in the SPY ETF (in USD100s of millions). For the next two rows, we decompose each SPX option trade into exposure to the underlying S&P500 (x^{Δ}), based on the options delta, and exposure to the S&P500 volatility (x^{ν}), based on the options vega. The order flow exposures are then aggregated over all trades across options with different strike prices, expiration dates, and puts and calls. We further report statistics on the log returns of the S&P500 price (r^u) and its volatility process (r^v). Both processes, as well as the options Delta and Vega, are estimated with a Heston stochastic volatility model using the cross-section of option prices. The sample period is January to December, 2014.

	Mean	StDev	p10	p25	p50	p75	p90	N
x^u (100 million USD)	-0.39	4.72	-5.84	-2.93	-0.26	2.13	4.93	1,500
x^{Δ} (100 million USD)	1.81	8.62	-7.93	-2.80	1.46	6.24	11.86	1,500
x^{ν} (100 million USD)	0.11	0.17	-0.03	0.02	0.08	0.17	0.30	1,500
d^u (cents)	1.02	398.81	-459.62	-178.68	8.67	213.72	435.11	1,500
d^v (percentage points)	-0.01	0.60	-0.60	-0.26	-0.03	0.22	0.59	1,500

Table 3 Long-run impulse response function VAR

This table shows the long-run effects of the impulse response functions (IRFs) of the VAR model estimated using Equation (4). Long run is defined as four periods (trading hours). The first column shows the standard deviation of the structural residual of each equation, which represents the size of the shocks in the IRF. The next columns show the long-run impact of a one-standard deviation impulse to each of the variables. The superscripts ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

		Impulse:				
		d^u	d^v	x^u	x^{Δ}	x^{ν}
Response	$\sigma(\epsilon)$					
d^u (cents)	189.68	238.21***	-281.42***	149.76***	93.14***	-47.05**
		(12.25)	(13.95)	(7.01)	(4.08)	(2.22)
d^v (percentage points)	0.51	-0.03	0.52***	-0.19***	-0.10***	0.07**
		(1.03)	(19.01)	(6.50)	(2.99)	(2.27)
x^u (100 million USD)	4.59	-0.22	0.44*	5.17***	0.92***	-0.61**
		(0.92)	(1.91)	(20.21)	(3.21)	(2.27)
x^{Δ} (100 million USD)	8.21	2.31***	-1.21***	0.49	9.90***	-0.27
		(4.88)	(2.59)	(1.03)	(18.23)	(0.51)
x^{ν} (100 million USD)	0.17	0.00	0.02***	-0.01	-0.01	0.21***
		(0.41)	(2.65)	(1.24)	(1.04)	(21.18)

Table 4 Regression coefficients VAR model
This table shows the coefficients of the VAR system estimated from Equation (4). The system considers five endogenous variables reported in columns (1) to (5) respectively: the option order flow exposure to the volatility component (x^v), to the underlying component (x^Δ), and SPY ETF order flow (x^u); and the difference of the volatility process (d^v) and price (d^u) of the S&P500. The ordering of the columns and the missing coefficients correspond to the ordering of the equations in the VAR model, used for identification of the structural shocks. The letters L in the independent variable names represent the lag-operator. Inference is based on Newey-West standard errors with two lags. The superscripts ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

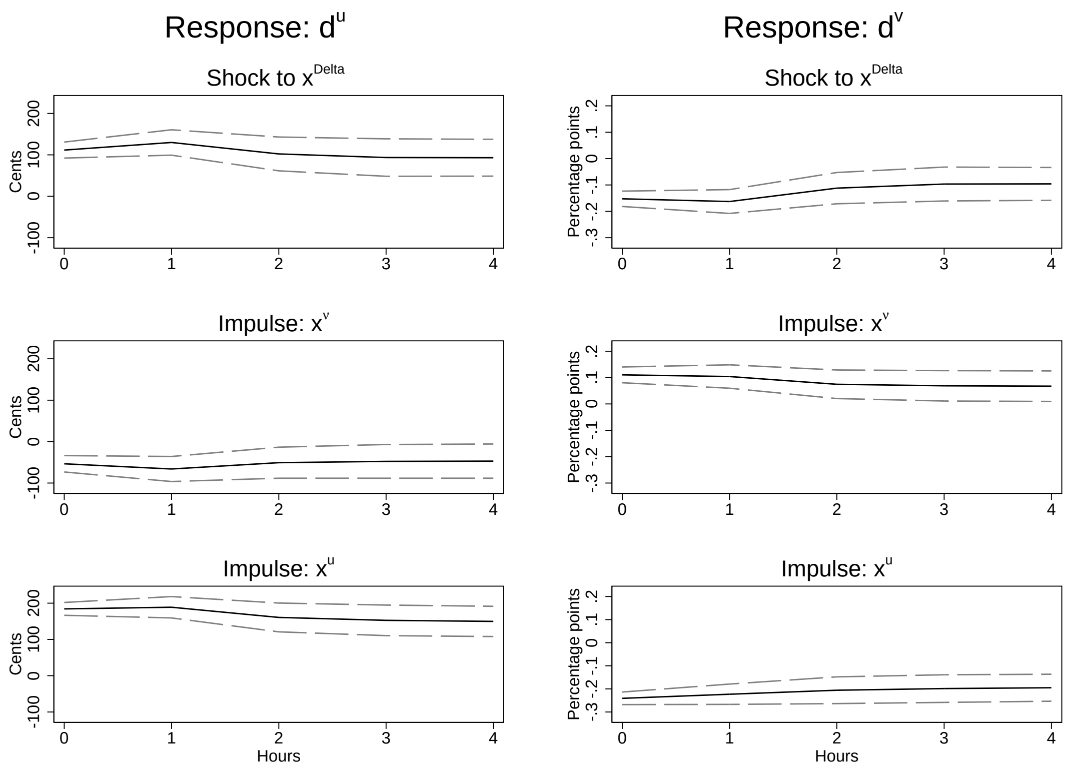
	(1) x^ν	(2) x^Δ	(3) x^u	(4) d^v	(5) d^u
x^ν	.	-1.369	-2.120*	0.529***	54.533
		(-0.6)	(-2.0)	(4.6)	(1.2)
L1 x^ν	0.142***	0.479	-0.505	-0.171	-84.239**
	(3.0)	(0.3)	(-0.6)	(-1.3)	(-2.1)
L2 x^ν	0.047	0.654	-0.206	-0.159	10.365
	(1.3)	(0.4)	(-0.3)	(-1.2)	(0.3)
x^Δ	.	.	0.100***	-0.013***	2.686***
			(6.6)	(-7.1)	(3.7)
L1 x^Δ	-0.000	0.188***	-0.017	0.002	0.243
	(-0.0)	(5.2)	(-1.0)	(0.9)	(0.3)
L2 x^Δ	-0.000	-0.066*	0.011	0.006**	0.011
	(-0.7)	(-1.8)	(0.7)	(2.3)	(0.0)
x^u	.	.	.	-0.052***	13.001***
				(-11.7)	(8.3)
L1 x^u	-0.000	-0.046	0.137***	0.014***	-0.946
	(-0.2)	(-0.8)	(3.6)	(3.0)	(-0.6)
L2 x^u	-0.000	-0.027	0.015	0.003	-4.844***
	(-0.3)	(-0.4)	(0.4)	(0.8)	(-3.8)
d^v	.	.	.	.	-517.338***
					(-35.1)
L1 d^v	0.038**	1.485*	-0.697	-0.005	51.198**
	(2.1)	(1.8)	(-1.4)	(-0.1)	(2.4)
L2 d^v	0.011	1.203	0.908*	-0.014	-0.070
	(0.5)	(1.3)	(1.8)	(-0.1)	(-0.0)
d^u					
L1 d^u	0.000	0.007***	-0.002***	-0.000	0.121***
	(0.7)	(5.4)	(-2.8)	(-0.5)	(3.6)
L2 d^u	0.000	0.002	0.001	-0.000	0.008
	(0.1)	(1.3)	(0.7)	(-0.3)	(0.2)
Constant	0.097***	1.677***	-0.185	-0.041*	-5.738
	(16.9)	(5.1)	(-1.1)	(-2.0)	(-0.8)
Observations	1,500	1,500	1,500	1,500	1,500
R-squared	0.035	0.092	0.054	0.270	0.774

Table 5 Predictability of option price changes with S&P500 underlying on volatility.
This table shows the extent to which option price changes can be explained by the two underlying factors as proposed in Equation (1). The dependent variable is the change in an option price, which is regressed on the particular options delta times the changes in the underlying (Δd^s), and the options vega times the change in the volatility (νd^v). We add two lags to be consistent with the previous analyses. The sample uses all option data of 2014, including options with all expiration dates, strike prices and puts and calls. Column (1) shows results for the full sample, and columns (2)-(10) for various subsets of options sorted by time-to-maturity (TTM) and moneyness (see Table (2)). The first rows show the average TTM and Moneyness of the particular sample. The data is sampled at the hourly frequency, and inference is based on Newey-West standard errors with one lag. The superscripts ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

	Full Sample	Subsets								
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Avg TTM (years)	0.18	0.06	0.05	0.05	0.13	0.13	0.12	0.27	0.26	0.27
Avg Moneyness (S/K)	1.21	0.90	0.98	1.35	0.87	0.98	1.35	0.83	0.98	1.48
Δd^s	1.000***	0.622***	0.993***	0.999***	1.077***	0.997***	1.000***	1.066***	0.990***	1.000***
	(5,323.3)	(24.7)	(303.7)	(3,768.2)	(103.4)	(309.6)	(3,123.4)	(199.6)	(394.8)	(3,165.8)
L1 Δd^s	-0.003***	-0.046***	0.006***	-0.002***	0.011	0.015***	-0.004***	0.029***	0.018***	-0.005***
	(-23.6)	(-3.1)	(3.2)	(-10.0)	(1.4)	(8.1)	(-15.6)	(8.6)	(12.6)	(-22.9)
Lag2 Δd^s	-0.000	0.036**	-0.001	0.001***	0.001	0.001	-0.001***	0.001	-0.001	-0.000*
	(-1.3)	(2.4)	(-0.4)	(6.3)	(0.2)	(0.4)	(-3.6)	(0.2)	(-0.6)	(-1.7)
νd^v	0.902***	1.241***	0.917***	0.914***	0.849***	0.850***	0.862***	0.939***	0.902***	0.922***
	(324.6)	(56.1)	(155.5)	(149.9)	(97.0)	(119.2)	(134.0)	(130.6)	(130.9)	(160.5)
L1 νd^v	-0.029***	0.090***	-0.013***	-0.015***	-0.026***	-0.029***	-0.049***	-0.015***	-0.019***	-0.033***
	(-18.2)	(7.1)	(-3.5)	(-3.5)	(-3.5)	(-6.3)	(-11.8)	(-3.1)	(-4.3)	(-9.2)
L2 νd^v	0.026***	0.017	-0.001	0.003	0.039***	0.021***	0.024***	0.047***	0.023***	0.035***
	(16.3)	(1.5)	(-0.3)	(1.0)	(6.1)	(4.8)	(6.1)	(12.0)	(5.5)	(9.9)
Constant	0.012***	0.008***	0.006***	0.004***	0.013***	0.016***	0.011***	0.014***	0.021***	0.014***
	(50.0)	(13.0)	(5.5)	(7.5)	(30.7)	(13.2)	(19.2)	(36.1)	(16.4)	(25.5)
Observations	851,051	17,579	25,235	122,453	65,753	39,553	149,738	148,684	56,569	225,487
R-squared	0.992	0.863	0.963	0.995	0.935	0.962	0.995	0.925	0.956	0.993

Figure 1 Impulse response functions VAR: the impact of order flows on the underlying and volatility returns

This figure plots the IRFs of order flow shocks to the difference of the underlying (d^u) and volatility (d^v). The processes are extracted from the cross-section of option prices, and reported in basis points. The net order flow in each option is decomposed into order flow exposure to the underlying (x^u) based on its delta, and volatility (x^v) based on its vega. The order flows are aggregated across options with all strike prices, expiration dates, and puts and calls. The order flow in the SPY ETF is also added, which is in the same unit as x^u. The IRFs are based on the VAR system of equation (4), which is estimated with two lags using data sampled at the hourly frequency.



Web Appendix to

"Informed Trading in the Index Option Market"

Figure 1 Impulse response functions SVAR: Different ordering (1)
This figure plots the IRFs of the SVAR model. It is similar to Figure (1) in the paper, except that the ordering of the equations is changed. It follows: $\{x^u, x^\nu, x^\Delta, d^v, d^u\}$.

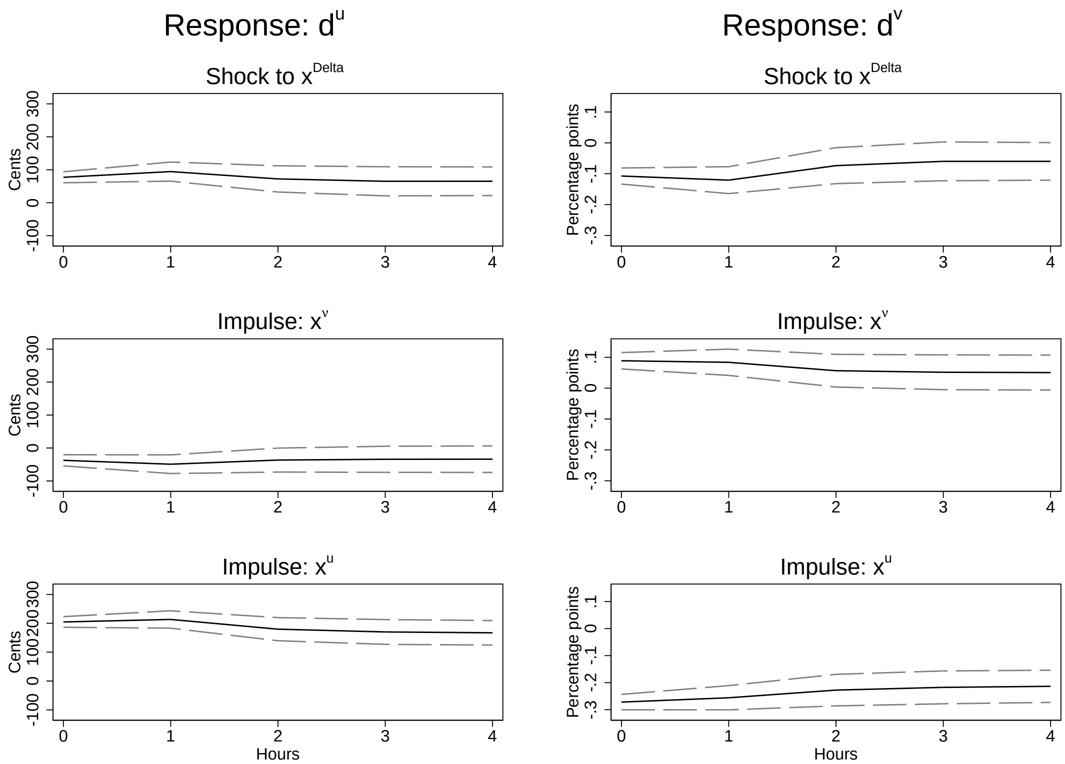


Figure 2 Impulse response functions SVAR: Different ordering (2)
This figure plots the IRFs of the SVAR model. It is similar to Figure (1) in the paper, except that the ordering of the equations is changed. It follows: $\{x^{\nu}, x^{\Delta}, x^{u}, d^{u}, d^{v}\}$.

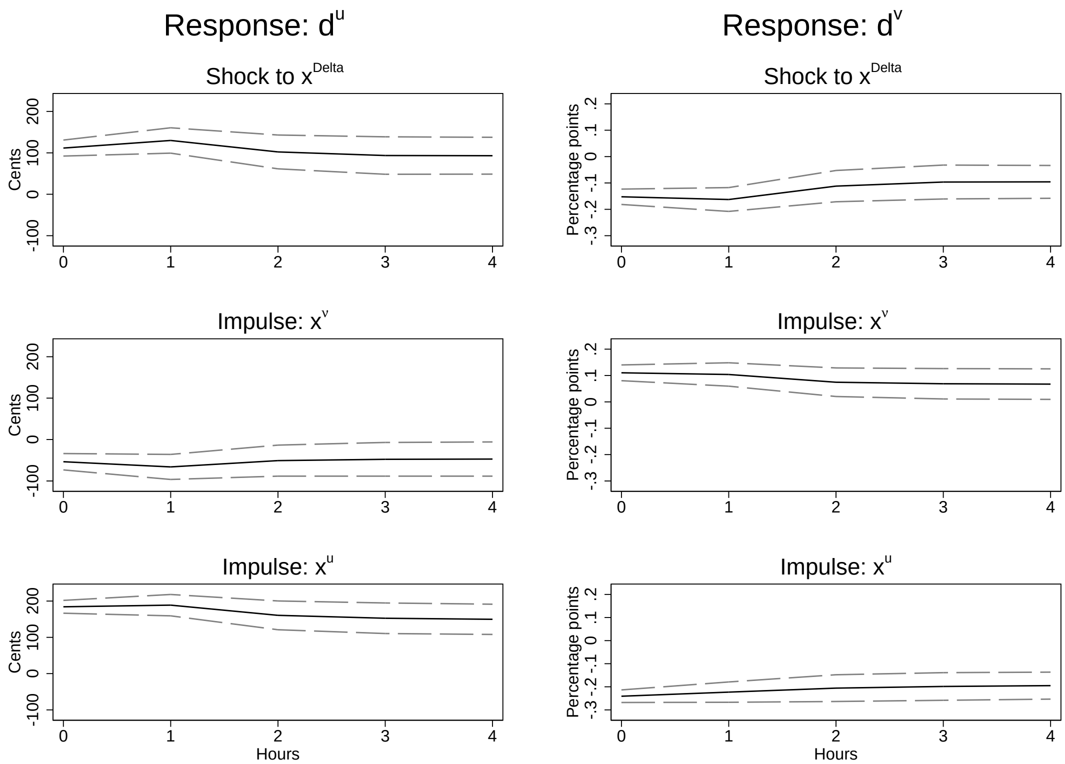


Figure 3 Impulse response functions SVAR: Half-hour sampling frequency
This figure plots the IRFs of the SVAR model. It is similar to Figure (1) in the paper, except that the sampling frequency is 30 minutes. Accordingly, we double the number of lags in the system (4), and redefine “long-run” as eight periods.

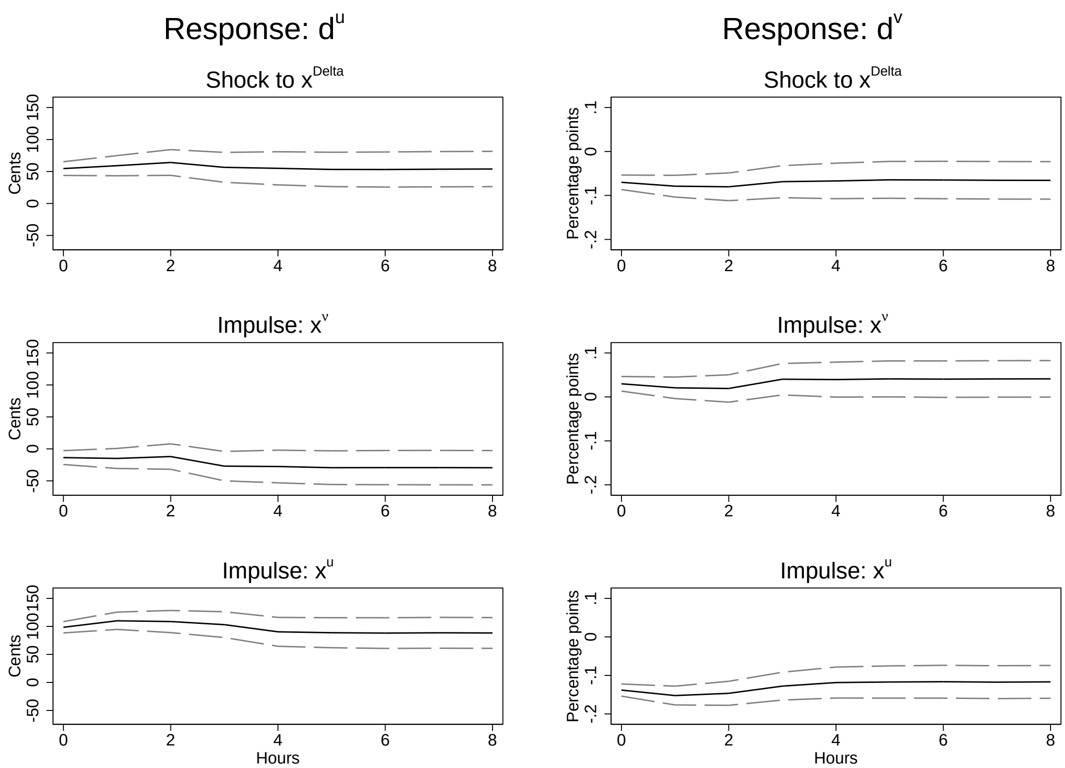


Figure 4 Impulse response functions SVAR: SPY ETF price instead of forward price
This figure plots the IRFs of SVAR model where we use the price change of the SPY ETF price, instead of the forward price obtained from the cross-section of options. Otherwise, the figure is identical to Figure (1) in the paper.

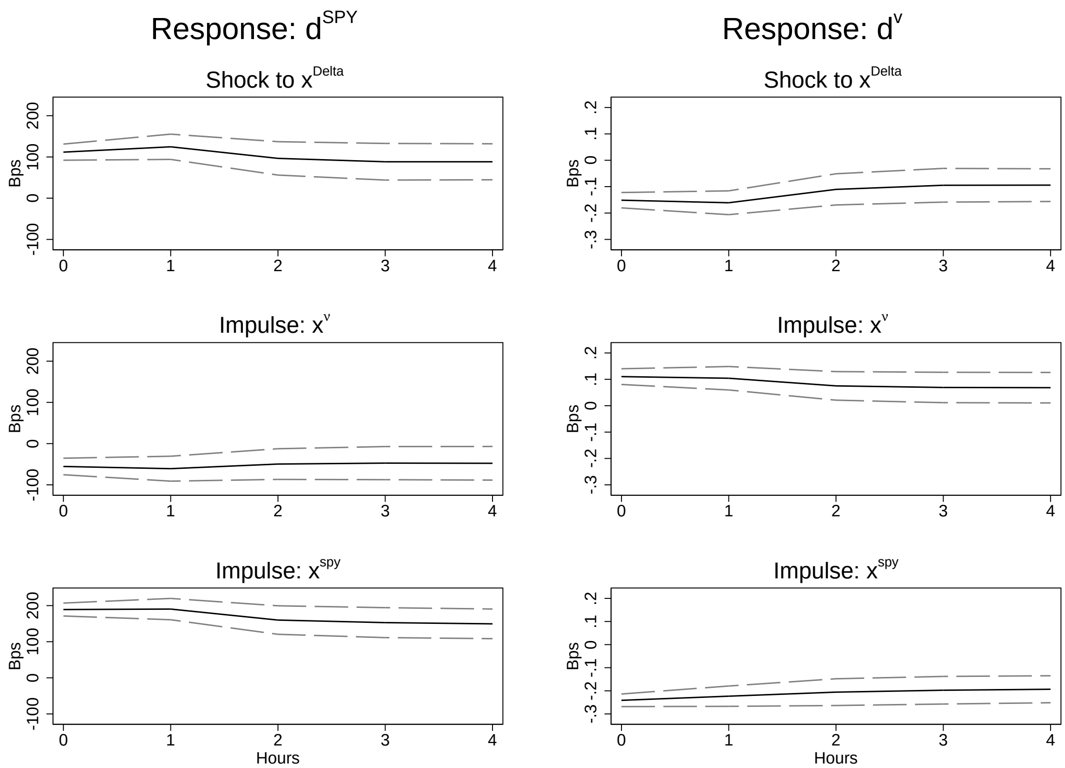


Figure 5 Impulse response functions SVAR: returns instead of price changes
This figure plots the IRFs of SVAR model where we use the *return* of the underlying (r^u) and volatility (r^v), instead of the price difference. Otherwise, the figure is identical to Figure (1) in the paper.

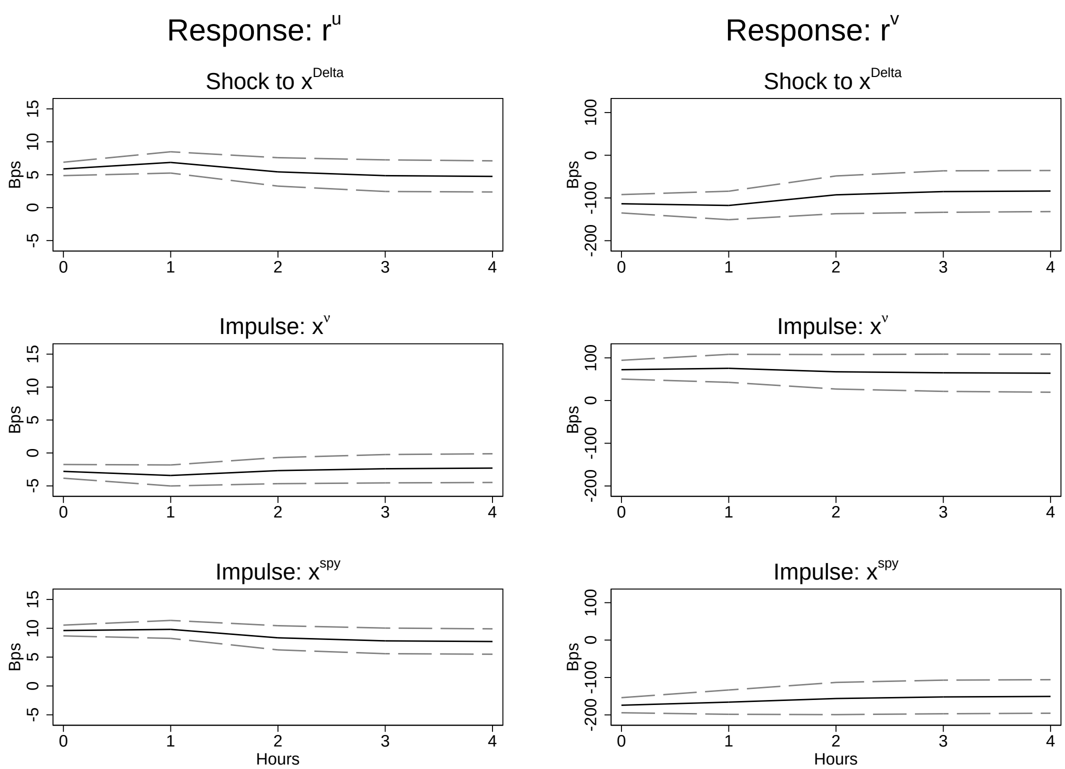


Figure 6 Impulse response functions SVAR: No SPY ETF order flow

This figure plots the IRFs of a four-equation SVAR model, where we omit the spy ETF order flow equation altogether.