FUNDING LIQUIDITY AND ARBITRAGE EFFICACY ☆,☆☆

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Abstract

Funding liquidity, the ability to raise funds, affects arbitrage activity in correcting mispricing. We propose a new measure of funding liquidity through the corresponding arbitrage activity, namely the arbitrage efficacy. In a model where arbitrageurs exploit mispricing subject to endogenous leverage constraint, we capture funding liquidity (arbitrage efficacy) as the marginal leverage raised (the marginal percentage of error correction achieved) against additional mispricing. Ample funding liquidity leads to positive arbitrage efficacy, such that sufficient leverage is financed and higher correction is achieved against additional mispricing. If an external shock makes the constraint binds, funding liquidity dives and drives arbitrage efficacy negative. To test our model predictions, we estimate the arbitrage efficacy implied by the arbitrage activity in correcting the S&P 500 future-cash basis. We find that 1. the periods of negative arbitrage efficacy coincide with the ex-post financial market turmoils. 2. Arbitrage efficacy is correlated with other measures of funding liquidity. 3. The basis is more sensitive to arbitrage efficacy and other funding liquidity measures during the periods of negative arbitrage efficacy, which evidences the liquidity-induced amplification effect. (*JEL* G01 G10 G14)

1. Introduction

Arbitrageurs (called arbs in short hereafter), such as hedge funds, tend to heavily utilize leverage debts to absorb external demand and supply shocks. The high use of leverage can enhance

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their ability to capitalize on mispricing opportunities and reduce pricing anomalies, which is referred to as "smart money" by Akbas et al. (2015). However, it also exposes them to potential funding liquidity risk, as the arbs may hit the funding constraint and amplify external shock, rather than absorbing them. Shleifer and Vishny (1997) demonstrate a potential loss spiral, such that arbs with poor past performance face funding withdrawal from the lenders and have to liquidate their position, which put further pressure on asset price. Brunnermeier and Pedersen (2009) illustrate the liquidity spiral, such that adverse shocks to funding liquidity condition reduce the arbs' ability to provide market liquidity, which will raise margin requirement and further jeopardize the funding liquidity condition.

As external shocks to the financial market are inevitable, understanding and identifying the liquidity-induced amplification effect are vital for intermediaries and policy makers (Brunnermeier and Oehmke 2013, Shin 2016)[4]. Extant literature tends to study funding liquidity directly through the available arbitrage capital (Comerton-Forde et al. 2008, Adrian and Shin 2010, Akbas et al. 2016) and indirectly through the asset pricing impacts (Fontaine and Garcia 2011, Garleanu and Pedersen 2011, Nagel 2012, Frazzini and Pedersen 2014, Golez, Jackwerth and Slavutskaya 2017). However, one cannot tell when funding illiquidity becomes so severe that the amplification will be triggered. Rather, the amplifications are empirically documented by setting arbitrary thresholds on the measures of funding liquidity or by identifying the ex-post market turmoils (Comerton-Forde et al. 2008, Drehmann and Nikolaou 2013, Schuster and Uhrig-Homburg 2015). Our paper attempts to fill this gap by a complementary study on the funding liquidity and the nonlinear consequences under the binding funding constraints.

We start from the market structure of Shleifer and Vishny (1997), and extend for endogenous leverage constraint. Arbs capitalize on a risky mispricing opportunity by the use of equity and leverage; Leverage debt is financed from outside financiers, who set a constraint to protect their own capital from the arbs' potential insolvency. We derive the competitive equilibrium of

[4]Shin (2016) points out that dealer banks with over-stretched leverage not only transmit external shocks, but also amplify these external shocks through the self-reinforcing downward spiral in leverage. Financial markets will always be subject to external shocks, the task for policymakers is to mitigate the endogenous, second-round effects by helping intermediaries to be more resilient.

the model where arbs choose the optimal leverage position to capitalize on mispricing subject to the leverage constraint. Due to the endogenous leverage constraint, an external shock to equity or arbitrage risk will be amplified when the leverage constraint is hit. It is consistent with the liquidity-induced (leverage-induced) amplification effect (Brunnermeier and Pedersen 2009, Garleanu and Pedersen 2011).

Next, we investigate the model implication on the arbs' funding liquidity. Unlike the extant literature that investigate funding liquidity through the shadow cost of capital (Brunnermeier and Pedersen 2009, Garleanu and Pedersen 2011) and the size of arbitrage violation (Fontaine and Garcia 2011, Akbas et al. 2016), we define funding liquidity through the arbitrage activity in response to mispricing. More specifically, funding liquidity ℓ is defined as the marginal leverage debt raised by the arbs in order to bear against additional mispricing error, which reflects the ability to raise leverage. When the arbs are far from the leverage constraint, ℓ is close to unity, such that the arbs can raise ample leverage to explore a better mispricing opportunity. A shock to equity or arbitrage risk will slightly reduce funding liquidity. However if an extreme shock makes the leverage constraint binds, ℓ dives closely to zero. It implies that arbs can barely raise any extra leverage.

While the leverage position is difficult to observe in practise, it reflects on the arbitrage activity in correcting mispricing, i.e. the percentage of mispricing correction achieved by the arbs. It is defined as the ratio of the arbs' investment (equity plus leverage) over the size of mispricing error. In this regard, we introduce the arbitrage efficacy θ, i.e. the marginal mispricing correction achieved by the arbs in response to additional mispricing error. Arbitrage is (in)effective when the marginal correction is positive (negative), such that arbs are able to achieve higher (lower) mispricing correction with larger error. We find that ample funding liquidity leads to effective arbitrage ($\theta > 0$) since sufficient leverage debts can be financed. However when the leverage constraint is hit, funding liquidity is dampened, which renders arbitrage ineffective ($\theta < 0$). It generates an important prediction of our model: the sign of arbitrage efficacy θ can be viewed as a signal of the binding leverage constraint and a warning about the liquidity-induced amplification effect.

To examine our model predictions, we design an empirical strategy to capture the arbitrage efficacy. We choose to look into the arbitrage activity of the future-cash basis implied by the S&P500 index and E-mini future from September 1997 (the earliest possible time for E-mini future) to June 2015. We first estimate the fair value of the future contract implied by the cost of carry model, and capture the future-cash basis as the difference between the spot and fair price of the future contract. Setting an initial window of 500 days, we next estimate the mispricing correction using the Generalized Error Correction Model with a recursive setup. The implied arbitrage efficacy is computed as the OLS estimator of regressing the daily mispricing corrections (in absolute value) on the daily mispricing errors (in absolute value).

To begin, I depicts the moving arbitrage efficacy on a 250-day (the effective trading days in one year) window along with the VIX index from Chicago Board of Options Exchange (CBOE) in Figure 1. The trajectory of the implied arbitrage efficacy is rather informative about the liquidity condition of the financial market. We note that the periods of negative arbitrage efficacy tend to coincide with spikes in the VIX index, except for the flash crash event in 2010. In particular, the implied arbitrage efficacy drops sharply below zero from June 2007, which matches the build-up of the 2007 global financial crisis. It can be a good tool for policy makers to identify the funding liquidity condition among the financial markets as well as the potential amplifications due to the binding funding constraints.

Motivated by the rudimentary plot against VIX index, we formally test the model predictions using the monthly implied arbitrage efficacy, i.e. the OLS estimator of regressing the daily mispricing corrections (absolute value) on the daily mispricing errors (absolute value) over each month. 184 observations are obtained from March 2000 to June 2015. First, we document a significant correlation between the implied arbitrage efficacy and other broad measures of funding liquidity over time, e.g. TED spread, Libor-Repo spread, default spread, term spread, VIX index, funding liquidity measure of Fontaine and Garcia (2012), and the broker-dealer leverage factor from Adrian, Etula, and Muir (2014). In particular, the implied arbitrage efficacy closely links to TED and Libor-repo spread with a correlation over 50 percent. Other funding illiquidity measures also reach a correlation of more than 20 percents.

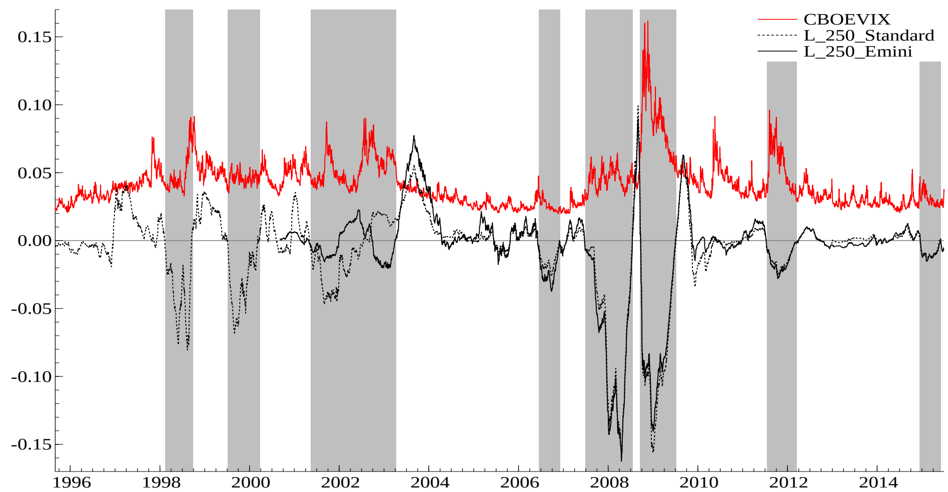

Figure 1: The implied arbitrage efficacy and the CBOE VIX index over time
The figure plots the arbitrage efficacy on a 250-day rolling window, implied by the S&P500 index and E-mini future (black solid line), and the S&P500 index and the standard future (black dotted line). Each point bases on the previous 250 days of data. Details of the measure construction are provided in Section 3. The periods of negative arbitrage efficacy is shaded. The CBOE VIX index is depicted over the same period of time (red solid line).

It indicates that the arbitrage efficacy implied by a single S&P 500 future-cash relationship captures the information of market-level funding liquidity condition. Second, we find that variations in the implied arbitrage efficacy is mainly explained by funding illiquidity measures, e.g. TED and term spread, rather than measures of market illiquidity and arbitrage risk, e.g. the VIX index, the illiquidity measure of Amihud (2002) and Pastor and Stambaugh (2003) and the idiosyncratic volatility of Ang et al. (2006). These measures together explain around 40 percent of the variations in the monthly implied arbitrage efficacy.

Our main objective is to establish the nonlienarity arose under the binding leverage constraint. Specifically, we investigate whether the future-cash basis is more sensitive to the changes in arbitrage efficacy when arbitrage efficacy becomes negative. On aggregate, we find strong negative relationships between changes in the basis and the implied arbitrage efficacy, before and after controlling for other measures of arbitrage frictions. It indicates that more effective arbitrage reduces the future-cash basis. Conditional on the positive arbitrage efficacy, the relationship becomes insignificant and indifferent from zero. However, given the negative arbitrage efficacy where leverage constraints become binding, the estimated coefficients on the implied arbitrage efficacy become sizable and highly significant, as well as other funding illiquidity measures, such as TED spread, default spread and term spread. Now a small shock to funding liquidity condition is associated with large increment in the basis, which confirms the amplification effect attributed to the binding leverage constraints. We find similar results using the noise measure of Hu, Pan and Wang (2013) that captures mispricing in the U.S. treasury bond market. It further suggests that funding illiquidity spillovers to the U.S. treasury bond market during the periods of the binding leverage constraint. A robustness check employs the future-cash bases that obtained from other major index market around the world, e.g. Germany and Japan. Overall, the arbitrage efficacy can be a great tool for policy makers to understand the funding liquidity condition and identify the binding leverage constraint that leads to amplification.

Our theoretical analysis contributes to the growing literature on the link between funding liquidity, market liquidity and mispricing. Funding liquidity is often captured by the shadow cost of capital (Brunnermeier and Pedersen, 2009; Garleanu and Pedersen, 2011) and the size of uncorrected mispricings (Vayanos and Weill, 2006; Garleanu and Pedersen, 2011; Fontaine and Garcia, 2012). Rather, we offer an alternative way to understand funding liquidity through the corresponding arbitrage activity: the marginal mispricing correction achieved by the arbs in response to additional mispricing. Our analyses offer a great tool to identify the binding leverage constraint and the amplification with rigorous theoretical backing, without setting ambiguous threshold and ex-post event dates.

Our paper also makes contributions to the empirical literature on funding liquidity. First, we introduce a distinct measurement for funding liquidity as the arbitrage efficacy, which can be captured from any verified arbitrage relationship. Extant literature has developed a number of approaches to measure funding liquidity. Some base on stated interest rates that reflects the cost of raising funds (Garleanu and Pedersen, 2011; Drehmann and Nikolaou, 2013). Others take funding illiquidity as a friction of arbitrage, liquidity provision or market making. Comerton-Forde et al. (2008) and Adrian and Shin (2010) assess the intermediaries' funding liquidity by directly investigating their balance sheet. Akbas et al. (2015, 2016) investigate the capital flow to hedge funds who conduct arbitrage[5]. While these papers tend to focus on the availability of arbitrage capital, others, like Garleanu and Pedersen (2011), Fontaine and Garcia (2012), Nagel (2012) and Frazzini and Pedersen (2014), pay attention to the size of arbitrage violation due to funding illiquidity.

Second, our empirical study sheds new light on the nonlinearity between funding liquidity and the size of mispricing that arises under the binding funding constraint. To distinguish the stress periods where constraints tend to be binding, Comerton-Forde et al. (2008) select a threshold at the 25th percentile of their measurement exogenously; Schuster and Uhrig-Homburg (2015) determine the nonlinear relationship endogenously within a regime-switching model; Drehmann and Nikolaou (2013) allocate the stress regime by the ex-post market turmoils. Our paper, however, are able to identify the binding funding constraint by the sign of the arbitrage efficacy and find statistically significant empirical evidence of the nonlinearity, which strongly supports the liquidity-induced amplification (Shleifer and Vishny, 1997; Brunnermeier and Pedersen, 2009).

Last but not least, our paper contributes to the literature of the future-cash pricing system. Kumar and Seppi (1994) and Roll, Schwartz and Subrahmanyam (2007) link the basis to market illiquidity. Our results suggest that the ineffective arbitrage due to funding illiquidity deters the movements of future and cash market prices toward the ideal of zero basis especially when the leverage constraint is binding.

The paper proceeds as follows. In section 2, we extend the theoretical framework of Stein (2009) and explore funding liquidity through the arbitrage activity. In section 3, the empirical design is introduced to best capture the arbitrage efficacy from the theoretical work, as well as the application to the S&P500 index and E-mini future arbitrage. Section 4 summarizes the empirical results, including the time-series plot of the implied arbitrage efficacy, the linkages with other measures of funding liquidity and the asymmetric effects that arises under the binding leverage constraints. Finally section 5 concludes.

[5]Chordia et al. (2005) and Fleckensitein et al. (2014) use the flows into bond, equity funds and hedge funds as the measure for funding condition in financial intermediaries.

2. The Model

2.1. Market structure

We consider the market structure similar to Shleifer and Vishny (1997), where an asset with a fundamental value, V, trades for three periods, $t = 1, 2, 3$. At period 1, noise traders arrive with a pessimistic shock of size, s_1, that pushes the asset price away from fundamental. Then the arbs attempt to correct the mispricing and prevent the asset from trading at a distressed price. Denoting the arbs' total (arbitrage) funds as f_1, we derive the market clearing price at period 1 by[6]:

$$P_1 = V - s_1 + f_1. \tag{1}$$

There exist two different market states at period 2. Under a bad state, noise deepens such that $s_{2,b} > s_1$ with a probability, $q > 0$, in which case the price becomes:

$$P_{2,b} = V - s_{2,b} + f_{2,b}, \tag{2}$$

where $f_{2,b}$ is the total funds available in a bad state. Under a good state with a probability, $1 - q$, noise disappears (i.e. $s_{2,g} = 0$), and the asset price converges towards fundamental, $P_{2,g} = V$. Finally, at period 3, price is assumed to revert to fundamental, $P_3 = V$.

Following Stein (2009), we allow the arbs to employ leverage to exploit the mispricing opportunity. Specifically, the arbs hold an equity, f_1^e and choose to raise leverage, f_1^d. Without loss of generality, equity and leverage are raised at zero interest rate. Thus, the total arbitrage fund available at period 1 becomes the sum of equity and leverage:

$$f_1 = f_1^e + f_1^d. \tag{3}$$

If the arbs can access to arbitrage funds without any friction, they are able to eliminate any mispricing and guarantee the law of one price. In practice, however, they are faced with several

[6]The demand of noise trader is $(V - s_t)/P_t$, and that of arbitrageurs is f_t/P_t. The market clearing price is then determined as demand equals to unit asset supply.

constraints on equity and leverage. To accommodate the financial constraints observed in the real world, we introduce Assumptions (i) to (iv) on the equity and leverage.

(i) The equity providers do not withdraw their funds early (at period 2).

(ii) The equity is constrained by

$$f_1^e < s_{2,b}, \tag{4}$$

These assumptions imply that equity funds cannot be guaranteed to bet against the potentially deepening noise shock at period 2.[7] For a sufficient equity supply, i.e. $f_1^e \geq s_{2,b}$, the arbs are able to fully correct the mispricing without leverage and enforce the law of one price by investing, $f_1 = s_1$ at period 1, and $f_2 = s_{2,b}$ at period 2. The cap on equity allows us to focus on the leverage that is adopted to conduct arbitrage.

(iii) The leverage debt f_1^d must be repaid in full at period 2.

It indicates that leverage is available in a short term. If bad state occurs, arbs will have to conduct arbitrage with the remaining equity only, such that $f_{2,b}^d = 0$. The available equity at period 2 can be expressed as:

$$f_{2,g} = f_1^e + f_1 \left(\frac{V}{P_1} - 1 \right) \text{ and } f_{2,b} = f_1^e + f_1 \left(\frac{P_{2,b}}{P_1} - 1 \right), \tag{5}$$

where the second terms on the right hand side of the equations are the returns of investment at period 1. Notice that in the bad state, return of the initial investment is negative, which indicates that arbs might lose all their equity funds and fail to repay the leverage debt if they lever up too excessively. As a result, financiers, who provide the leverage debts, would set a leverage constraint to guarantee solvency of the arbs.

[7] Our setting of equity is different from that in Stein (2009). The equity supply in Stein's model can be infinite but costly, which is designed to study the longer-run question on capital structure. Our model setting is closer to Gromb and Vayanos (2002; 2010), and imposes a constraint on the size of equity. It allows us to focus on the shorter-run arbitrage activity, such that arbs are induced to use leverage to exploit various mispricing. Both the equity constraint or a positive cost of equity will prevent arbs from making full mispricing correction.

(iv) The maximum leverage that arbs are allowed to borrowed is bounded, such that[8]

$$f_1^d \leq D_U,$$

where we call D_U the leverage upper bound.[9]

The leverage upper bound is imposed exogenously in the model of Stein (2009). However, financiers in our model are informative about the arbs and the market, e.g. equity, the fundamental value and the asset price, and thus tend to set the leverage upper bound accordingly. We introduce the setting of leverage upper bound under the assumption that informed financiers set the rate of return as the riskless rate (zero in our model). In other words, financiers must ensure that the potential loss under a bad state must be covered by the arbs' equity. This no-default condition at period 2 can be expressed as: $f_{2,b} \geq 0$, such that the remaining equity under bad state is non-negative. Following (5), the leverage upper bound D_U is derived as:[10]

$$D_U = \frac{1}{2}\left(\sqrt{\left(f_1^e + s_{2,b} - s_1\right)^2 + 4f_1^e\left(V - s_{2,b}\right)} - \left(f_1^e + s_{2,b} - s_1\right)\right) \tag{6}$$

We formally summarize a basic property about the leverage upper bound as follows.

Proposition 1. *Consider the model with the market structure in Section 2.1 and hold the Assumption (i) to (iv). The leverage constraint, D_U is positively related to the arbitrageurs' equity, f_1^e, and is negatively related to the future noise shock, $s_{2,b}$.*

The proposition states that the leverage upper bound is endogenously determined by the equity and the expected future noise shock. A negative equity shock stems from negative economic or financial market condition, which tightens the leverage upper bound. The future

[8] Leverage ratio is often defined as the ratio of total funding (equity and leverage) over equity funding. Since equity is exogenous in our model, a cap on the leverage ratio is identical to a cap on the leverage debt, f_1^d.

[9] The lower bound is naturally given as $D_L = -f_1^e$, indicating that the arbs can lend their equity in full to other arbs.

[10] Our endogenous leverage upper bound is set similar in spirit to Gromb and Vayanos (2002; 2010), which also require the margin loans to be riskless, such that default is not possible. An alternative way of setting the leverage upper bound is to control the value-at-risk. In Brunnermeier and Pedersen (2009), financiers allows the arbs to default but impose an upper bound on the default probability. It yields a margin constraint which can be interpreted as a value-at-risk constraint.

noise, $s_{2,b}$ determines the magnitude of the crash (rare event) in the bad state, which reflects the risk of crisis. The leverage upper bound is reduced with higher crisis risk, as financiers notice that arbs are more likely to default in the bad state. As we will see next, this leverage upper bound has important implications for the equilibrium and the amplification effect—the property that a moderate external shock will trigger significant pricing impact.

2.2. The equilibrium

We now consider the equilibrium. The arbs face a simple trade-off: they are induced to raise as much short-term debt as they can, to invest at period 1 and to exploit the positive return at good state. On the other hand, the arbs may take a cautious leverage position, in order to capitalize on a better opportunity at period 2 if bad state occurs. Subject to the leverage constraint, $f_1^d \leq D_U$, the risk-neutral arbs choose the optimal f_1^d to maximize their expected total wealth at period 3 under perfect competition, which is given by

$$E\left(f_3^e\right) = (1-q)f_{2,g} + q\frac{V}{P_{2,b}}f_{2,b}, \tag{7}$$

The first order condition with respect to f_1^d is derived as

$$R^1 \geq R^2, \tag{8}$$

where $R^1 = \frac{V}{P_1} - 1$ is the return of investing at period 1 and holding to price convergence, and $R^2 = q\left(\frac{V}{P_{2,b}} - 1\right)$ represents the expected return of investing at period 2. For $R^1 > R^2$, arbs opt to borrow as much as they can to exploit the return of investing at period 1, but subject to the binding leverage constraint. Therefore the max-leverage strategy is adopted, such that

$$f_1^{d*} = D_U.$$

Only when the two returns are indifferent, $R^1 = R^2$, the partial leverage strategy becomes optimal with $f_1^{d*} < D_U$. After rearranging, the partial leverage strategy is expressed as

$$f_1^{d*} = \frac{V - (1-q)\left(f_1^e + s_{2,b} - s_1\right) - \sqrt{\left(V - (1-q)\left(s_1 + s_{2,b} - f_1^e\right)\right)^2 + 4Vq\left(1-q\right)\left(s_{2,b} - f_1^e\right)}}{2\left(1-q\right)} \tag{9}$$

We now demonstrate the impact of a shock to arbitrage equity f_1^e on the arbs' leverage position under loose/binding constraint. Let $j \in (p, m)$, where the superscripts p and m indicate the partial-leverage strategy and the max-leverage strategy, respectively.

Proposition 2. *Consider the model with the market structure in Section 2.1 and hold the Assumption (i) to (iv). A shock to arbitrageurs equity f_1^e affects leverage raised by the arbs, f_1^{d*}, such that $\left(\frac{\partial f_1^{d*}}{\partial f_1^e}\right)^p < 0$ and $\left(\frac{\partial f_1^{d*}}{\partial f_1^e}\right)^m > 0$.*

Propositions 1 and 2 together imply that a negative shock to equity (a positive shock to crisis risk) increases the risk of hitting the leverage upper bound. Under the partial-leverage equilibrium, arbs choose to raise more leverage after equity drops, while financiers tend to reduce the leverage upper bound. Therefore a large reduction in equity or a significant rise in crisis risk is likely to force the arbs to adopt the max-leverage strategy.

More importantly, Proposition 2 shows how a shock to equity can be amplified with the endogenous leverage upper bound. An equity shock under the partial- and max-leverage equilibrium has opposite impact on the leverage position. Arbs under the partial-leverage strategy will raise more leverage debt after a negative equity shock. Recall that $P_1 = V - s_1 + f_1^e + f_1^d$. Therefore leverage serves as a cushion to smooth the price fluctuation at period 1. Under the max-leverage equilibrium, however, the negative equity shock will force financiers to tighten the leverage constraint (Proposition 1), which results in less available leverage for the arbs. Now the equity shock spillovers to the leverage side through the endogenous leverage upper bound, which pushes the price further away from fundamental. Therefore, the arbs, instead of absorbing the shock with leverage, become the amplifier of the shock and induce significant price impact at period 1. Similarly, a shock to the crisis risk $s_{2,b}$ will also trigger amplification. Leverage drops with the crisis risk since arbs are induced to choose more cautious leverage position to avoid a greater crash at period 2. It drops faster under the max-leverage equilibrium, which leads to larger pricing impact at period 1.

Amplification in our model is induced by the endogenous leverage upper bound. Similarly, an equity shock is amplified in Gromb and Vayanos (2002; 2010) due to margin constraints and in Brunnermeier and Pedersen (2009) due to the margin spiral. However, the crisis risk shock is not amplified by the performance-based arbitrage mechanism in Shleifer and Vishny (1997). Due to the exogenous-given funding constraint, the crisis risk does not affect initial price when the funding constraint is hit. Rather, our model extension for endogenous leverage upper bound gives rise to the amplification of a future crisis risk shock. Furthermore, we not only demonstrate the cause of amplification, but in the next section we also attempt to identify when amplification occurs with a thorough study of the arbs' funding liquidity.

2.3. Funding liquidity and arbitrage efficacy

We now turn our attention to the arbs' funding liquidity condition. Given that the level of equity funds and arbitrage risk that does not change constantly over a short horizon, we posit that the arbs' funding liquidity determines how they adjust the leverage position in response to mispricing, s_1. We capture the arbs' funding liquidity[11] from this arbitrage activity:

$$\ell = \frac{\partial f_1^{d*}}{\partial s_1}. \tag{10}$$

Specifically, ℓ measures the marginal leverage financed by the arbs to bear against one more unit of mispricing error. Highly positive ℓ indicates that arbs can easily lever up to exploit the mispricing opportunity. However, arbs may find it difficult to raise leverage when ℓ is low or even negative. Let ℓ^j be the funding liquidity associated with different leverage strategies, $j \in (p, m)$, we characterize its properties as follows.

Proposition 3. *Consider the model with the market structure in Section 2.1 and hold the Assumption (i) to (iv).*

[11] Notice that this definition of funding liquidity is far from similar to the extent literature. Brunnermeier and Pedersen (2009) and Garleanu and Pedersen (2011) measure it by the marginal value of an extra dollar used, i.e. the shadow cost of capital. In Brunnermeier and Pedersen (2009), the shadow cost of capital is nil when leverage constraint is loose, since there is no arbitrage return as price recovers to fundamental. But, it becomes highly positive when the leverage constraint becomes binding.

(i) For any equity f_1^e and arbitrage risk $s_{2,b}$, $0 < \ell^m < 0.5 < \ell^p < 1$.

(ii) A shock to equity f_1^e or arbitrage risk $s_{2,b}$ reduces funding liquidity: $\frac{\partial \ell^j}{\partial f_1^e} > 0$ *and* $\frac{\partial \ell^j}{\partial s_{2,b}} < 0$ *for* $j = p, m$.

(iii) If a shock to equity f_1^e or arbitrage risk $s_{2,b}$ makes leverage constraint binds, ℓ dives, such that at the corner solution we have:

$$\lim_{f_1^{d*} \to D_U} \ell^p > \ell^m.$$

Proposition 3 describes the level of the arbs' funding liquidity. Funding liquidity is always positive, such that arbs are able to lever up against larger mispricing. Even when the max-leverage strategy is adopted, financiers are willing to loose the upper bound since larger s_1 indicates higher arbitrage return when price converges. Funding liquidity stays below unity due to Assumption (i) to (iv) that limits the use of equity and leverage. A key determinant of the arbs' funding liquidity is the leverage upper bound they are faced with. When their leverage position is far from the upper bound, i.e. the partial-leverage strategy is adopted, funding liquidity is significantly higher than that under the max-leverage strategy, where the upper bound is hit.

Furthermore, under the partial-leverage strategy, a negative equity shock (or a positive crisis risk shock) slightly reduces the arb's funding liquidity ℓ, since the shock increases the risk of hitting the leverage upper bound (Proposition 1). If the leverage upper bound is hit after an extremely large equity shock, funding liquidity is dampened significantly, i.e. $\ell^m \ll \ell^p$. Overall, we note that our definition of funding liquidity ℓ does capture important information about the arbs' ability to raise leverage under the partial- and the max-leverage strategies. More surprisingly, funding liquidity tend to dive below 0.5 when the arbs are forced to adopted the max-leverage strategy. Recall that Proposition 2 demonstrate the amplification arises under the max-leverage strategy, where the leverage upper bound is hit. Thus we could identify when the arbs enters the max-leverage strategy by their funding liquidity.

In practise, we do not directly observe the leverage position f_1^{d*}. Following Cai et al. (2018, 2019), we note that the arbs' leverage position will reflects on the (observable) percentage of mispricing correction:

$$\kappa = \frac{f_1}{s_1} = \frac{f_1^e + f_1^{d*}}{s_1}. \tag{11}$$

κ captures the percentage of mispricing correction achieved by the arbs at period 1, which depends upon the leverage f_1^{d*} that arbs are able to finance. In one extreme, $\kappa = 0$, suggesting that arbs decide to wait, $f_1^d = -f_1^e$ and $f_1 = 0$. In another extreme $\kappa = 1$, implying that arbs can raise sufficient leverage to achieve full correction, such that $f_1^e + f_1^d = s_1$. As shown in Proposition 3, funding liquidity ℓ is less than unity, which indicates that full correction is not possible under our model[12].

To capture the information about funding liquidity ℓ, we now define the arbitrage efficacy, i.e. the arb's ability to eliminate additional mispricing errors, as

$$\theta = \frac{\partial \kappa}{\partial s_1} = \frac{\ell - \kappa}{s_1}. \tag{12}$$

θ measures the marginal mispricing correction achieved by the arbs in response to one more unit of mispricing. Positive (negative) θ means that arbs can achieve higher (less) percentage of correction in response to larger mispricing, which is denoted as (in)effective arbitrage. Arbitrage efficacy is closely related to funding liquidity ℓ, such that the more marginal leverage raised by the arbs, the higher marginal correction can be achieved. More importantly, we find that arbitrage becomes ineffective, i.e. $\theta < 0$, when the leverage constraint binds.

Proposition 4. *Consider the model with the market structure in Section 2.1 and hold the Assumption (i) to (iv).*

*(i) (**Effective arbitrage**) Under the loose leverage constraint, we have:* $\theta^p > 0$.

*(ii) (**Ineffective arbitrage**) Under the binding leverage constraint, we have:* $\theta^m < 0$.

Proposition 4 is intuitive. Under the partial-leverage strategy, arbs retain a strong ability to raise leverage in order to exploit higher mispricing ($0.5 < \ell^p < 1$). As a result, higher

[12]In practice, mispricing correction can be captured by the Generalized Error Correction model as suggested in Cai et al. (2015). However from the international data of 20 countries index-future arbitrage, it is almost rare to observe the full error correction in empirical applications due to the existence of arbitrage frictions.

percentage of mispricing correction, $\theta^p > 0$, is achieved. However, when funding liquidity is deteriorated under the max-leverage strategy, leverage is limited. Thus arbs fail to make higher corrections against mispricing, such that arbitrage becomes ineffective, $\theta^m < 0$. Given a level of equity f_1^e and crisis risk, $s_{2,b}$, if we observe a positive arbitrage efficacy, it indicates that the partial-leverage strategy is adopted. Any future changes in f_1^e and $s_{2,b}$ will not be amplified. However, if we observe a negative arbitrage efficacy, it signals the max-leverage strategy being adopted by the arbs. Shocks to f_1^e and $s_{2,b}$ are amplified by the leverage upper bound as shown in Proposition 2. Therefore, arbitrage efficacy not only reflects the arbs' funding liquidity, the negative sign also signals that amplification is triggered.

Literature has documented the market instability that arises under the binding leverage constraints due to amplification (Shleifer and Vishny 1997, Brunnermeier and Pedersen 2009, Duffie 2010, Mitchell and Pulvino 2012). However, it is difficult to quantitatively identify whether amplification is at work or not. Studies like Comerton-Forde et al. (2008), Schuster and Uhrig-Homburg (2015) and Drehmann and Nikolaou (2013) provide evidence of the amplification when the leverage constraints are likely to be binding. However, the identification is rather arbitrary, such as a exogenous threshold, endogenous regime-switching and ex-post event study. By the analysis of funding liquidity through arbitrage activity, our paper provides the theoretical support to signal whether the leverage constraint is hit or not, and thus the amplification. Knowing the tightness of the leverage constraint and especially when the constraint is hit is key to understand amplification during the financial crisis.

2.4. Numerical examples

We use an numerical example to illustrate the amplification of an external shock, the arb's funding liquidity and arbitrage efficacy. Let the fundamental value of the asset be, $V = 1$, probability of bad state be, $q = 0.1$, and the initial noise shock be $s_1 = 0.3$, the equity holding be $f_1^e = 0.1$ and the future noise be $s_2^b = 0.4$. We consider two cases, the impacts of an equity shock and an crisis risk shock.

To see the impact of an equity shock, we allow the size of equity shock to vary from -0.1 to

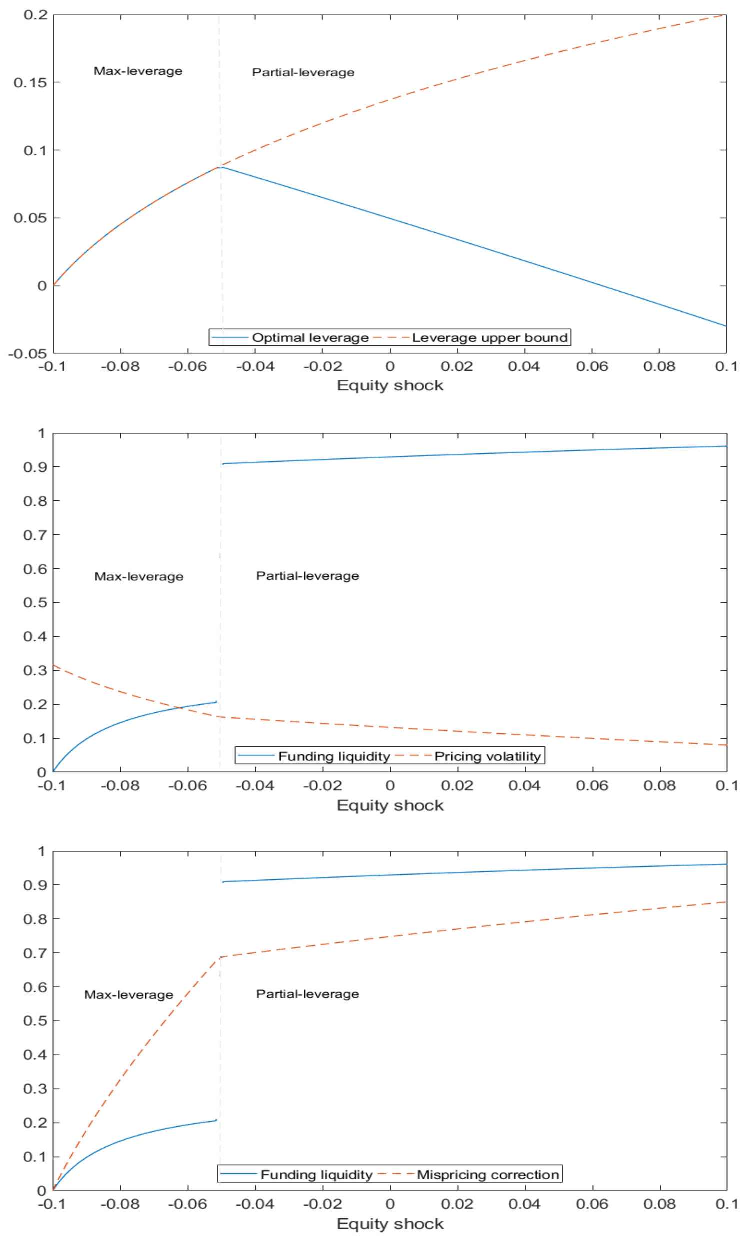

Figure 2: Plots of leverage position, leverage constraint, funding liquidity and pricing volatility
These figures plot the impacts of f_1^e on the optimal leverage position, leverage upper bound, funding liquidity and pricing volatility. We let the fundamental value of the asset be, $V = 1$, probability of bad state be, $q = 0.1$, the initial noise shock s_1 be $s_1 = 0.3$, the equity holding be $f_1^e = 0.1$ and the future noise be $s_2^b = 0.4$. These figures consider the shock to f_1^e, varying from -0.1 to $+0.1$. The left hand side of the vertical dashed line implies that the max-leverage strategy is adopted, while the partial-leverage strategy is adopted in the right hand side.

+0.1. The top left panel of Figure 2 plots the shock to equity f_1^e against the optimal leverage position f_1^{d*} and the leverage constraint, D_U. The leverage constraint becomes binding when the negative equity shock is too large, i.e. left hand side of the vertical dashed line, such that arbs are forced to adopt the leverage upper bound set by financiers. When the leverage constraint is hit, the equity shock leads to decline in leverage which induces significant pricing impacts. As shown in the top right panel, pricing volatility[13] becomes more sensitive to equity shocks under the max-leverage strategy (the liquidity-induced amplification). Funding liquidity defined as the marginal leverage keeps dropping with lower equity funding. It remains at high level, close to one, when the upper bound is not hit. However, it dives to a lower level, around 0.3, after the large negative equity shock makes the constraint binds, and it declines sharply hereafter. Finally, the bottom panel of Figure 2 plots the funding liquidity and the mispricing correction. When the leverage upper bound is not hit, $\ell - \kappa > 0$. (12) implies that arbitrage is effective. However, when the upper bound is hit, funding liquidity dives and makes $\ell - \kappa < 0$, such that arbitrage become ineffective.

For the impact of a crisis risk shock, we allow the crisis risk shock to vary from -0.1 to $+0.1$. The top right panels of Figure 3 display the result of an crisis risk shock on the optimal leverage position and the leverage upper bound. Arbs tend to deleverage after a positive shock to s_2^b, while financiers reduce the leverage upper bound at a faster rate. If the shock is too large, the leverage upper bound is hit despite the fact that the arbs are deleveraging. Funding liquidity again dives to lower level and pricing volatility sharply rises. Arbitrage tend to be effective under the partial leverage strategy, but ineffective once the leverage upper bound is hit.

3. The Empirical Design

In this section, we introduce an empirical design to capture arbitrage efficacy in practice. We first introduce the strategy to capture the arbs' mispricing correction in response to the

[13]Following Hombert and Thesmar (2014), price volatility is defined as $\sigma = E_1\left(\mid \frac{P_2 - P_1}{P_1} \mid + \mid \frac{P_3 - P_2}{P_2} \mid\right)$.

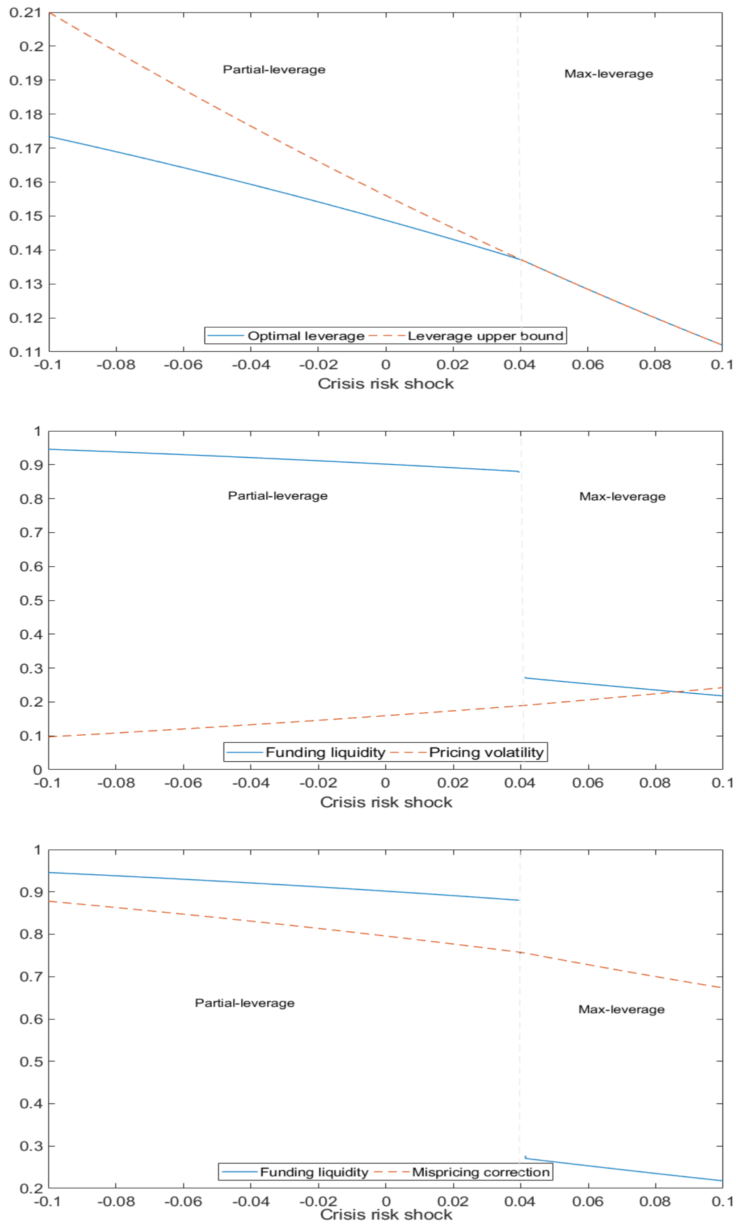

Figure 3: Plots of leverage position, leverage constraint, funding liquidity and pricing volatility

These figures plot the impacts of s_2^b on optimal leverage position, leverage upper bound, funding liquidity and pricing volatility. We let the fundamental value of the asset be, $V = 1$, probability of bad state be, $q = 0.1$, and the initial noise shock s_1 be $s_1 = 0.3$, the equity holding be $f_1^e = 0.1$ and the future noise be $s_2^b = 0.4$. These figures display the impact of a shock to s_2^b, varying from -0.1 to $+0.1$. The left hand side of the vertical dashed line implies that the partial-leverage strategy is adopted, while the max-leverage strategy is adopted in the right hand side.

mispricing opportunity they face. Second, we demonstrate the design to capture the arbitrage efficacy as a measure of funding liquidity. Finally, we describe the underlying data that are applied to the strategy for our empirical application.

3.1. Measuring and mispricing correction

Suppose p_t and p_t^* is the natural log of the asset's spot and fundamental price of an asset at date t, then z_t, the unarbitraged mispricing error at date t, can be easily obtained by:

$$p_t - p_t^* = z_t. \tag{13}$$

Under the assumption of frictionless arbitrage, arbs will take place and correct mispricing in no time, $z_t = 0$. In practice, however, one tends to observe persistent price deviation from the fundamental value, which indicates that arbitrage is far from frictionless. To capture the essence of mispricing correction in our theoretical model, let z_t be stationary but serially correlated, such that

$$z_t = \phi z_{t-1} + \varepsilon_t,\, \varepsilon_t \sim iid\left(0,\, \sigma_\varepsilon^2\right) \tag{14}$$

where ε is the mispricing innovations. Subtracting z_{t-1} on both side, we obtain

$$z_t - z_{t-1} = \kappa z_{t-1} + \varepsilon_t$$

where $\kappa = \phi - 1$. κ captures the mispricing correction achieved by arbs in correcting the past mispricing error z_{t-1}, and lies between -1 to 0. In one extreme, $k = -1$ or $\kappa = -100\%$ implies full correction of the past mispricing, z_{t-1}. In another extreme, $\kappa = 0$ indicates that no correction has been made by the arbs. Taking the first difference of eq. (13), we obtain the standard error correction model (ECM) that is used to estimate κ:

$$\Delta p_t = \kappa z_{t-1} + \Delta p_t^* + \varepsilon_t.$$

More generally, we follow Cai et al. (2017), who introduce a two-period generalised ECM to capture the mispricing correction achieved by arbs, such that:

$$\Delta p_t = \kappa z_{t-1} + \lambda^* z_{t-2} + \delta \Delta p_t^* + \gamma \Delta p_{t-1} + \mu_t, \;\; \mu_t \sim iid(0,\, \sigma_\mu^2) \tag{15}$$

where Δ is the difference operator, the lagged one error correction term, κ, captures the initial mispricing correction. The interpretation of the error correction term as the impact of arbitrage has been widely documented in the literature (Dwyer et al., 1996; Martens, Kofman and Vorst, 1998; Tse, 2001).

Notice that the mispricing correction κ estimated in Eq. (15) is a static measure, which represents the average correction to mispricing within a period of time. In order to capture the changes in κ in response to the corresponding error, we introduce the following recursive approach. First, we set a starting window of N days to run the regression in Eq. (15), and assign the estimated κ_t to the ending date of the window, date t. κ_t represents the mispricing correction achieved by the arbs attempting to eliminate a pool of N past mispricing, z_{t-1}, $z_{t-2} \ldots$, z_{t-N}. Next, we continue to obtain κ_{t+1} by adding the next mispricing error observed, z_t, into the starting regression, and now κ_{t+1} indicate the average error correction in response to a pool of $N+1$ past errors, z_t, z_{t-1}, $z_{t-2} \ldots$, z_{t-N}. Therefore the difference $\Delta\kappa_{t+1} = \kappa_{t+1} - \kappa_t$ is the changes in mispricing correction with respect to the additional error z_t. Finally, we adjust the value of each κ_t due to the changing pool size. Note that $\Delta\kappa_{t+1}$ represents the changes in correction by adding one error z_t into a pool of N past errors. Similarly, $\Delta\kappa_{t+i}$ thus displays the changes in correction by adding one error z_{t+i-1} into a pool of $N+i-1$ past errors. However due to the different size of the pool, directly comparing between $\Delta\kappa_{t+1}$ and $\Delta\kappa_{t+i}$ can be misleading and subject to underestimation. We thus adjust the value of $\Delta\kappa_{t+i}$ in order to control for the pool size by $\Delta\hat{\kappa}_{t+i} = \Delta\kappa_{t+i} \times \frac{N+i-1}{N}$. Accordingly, we adjust the value of κ_{t+i} as $\hat{\kappa}_{t+i} = \hat{\kappa}_{t+i-1} + \Delta\hat{\kappa}_{t+i}$. The loop strategy is displayed in Table 1. Employing this modification, we will end up with a daily series of adjusted $\hat{\kappa}_t$ and the corresponding error z_{t-1}.

Estimated κ	Pool size	Modification on $\Delta\kappa$	Adjusted $\hat{\kappa}$
κ_t	N	N/A	$\hat{\kappa}_t = \kappa_t$
κ_{t+1}	$N+1$	$\Delta\hat{\kappa}_{t+1} = \Delta\kappa_{t+1} \times \frac{N}{N}$	$\hat{\kappa}_{t+1} = \hat{\kappa}_t + \Delta\hat{\kappa}_{t+1}$
κ_{t+2}	$N+2$	$\Delta\hat{\kappa}_{t+2} = \Delta\kappa_{t+2} \times \frac{N+1}{N}$	$\hat{\kappa}_{t+2} = \hat{\kappa}_{t+1} + \Delta\hat{\kappa}_{t+2}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
κ_{t+i}	$N+i$	$\Delta\hat{\kappa}_{t+i} = \Delta\kappa_{t+i} \times \frac{N+i-1}{N}$	$\hat{\kappa}_{t+i} = \hat{\kappa}_{t+i-1} + \Delta\hat{\kappa}_{t+i}$

Table 1: Modification of κ to control for the pool size

3.2. Measuring arbitrage efficacy

Given that there are T observations of data, $\{\hat{\kappa}_t, z_{t-1}\}_{t=1}^{T}$. By regressing the absolute value of z_{t-1} on the absolute value of $\hat{\kappa}_t$, we obtain the estimate of arbitrage efficacy, L, as the OLS estimator for the slope coefficient:

$$\mid \hat{\kappa}_t \mid = \hat{\kappa}_0 + \theta \mid z_{t-1} \mid + \epsilon_t, \; \epsilon_t \sim iid(0, \sigma_\epsilon^2) \tag{16}$$

or more specifically,

$$\theta = \frac{\text{Cov}\left(\mid \hat{\kappa}_t \mid, \mid z_{t-1} \mid\right)}{\text{Var}\left(z_{t-1}\right)}, \tag{17}$$

where L represents how correction respond to the size of mispricing, i.e. arbitrage efficacy. Note that we take the absolute value of $\hat{\kappa}_t$ since the estimated $\hat{\kappa}_t$ are negative from the GECM, and also take the absolute value of z_{t-1} in order to represent the size of error. L is positive when $\text{Cov}\left(\mid \hat{\kappa}_t \mid, \mid z_{t-1} \mid\right) > 0$, such that mispricing correction $\hat{\kappa}_t$ increases with mispricing error, z_{t-1}. L becomes negative when $\text{Cov}\left(\mid \hat{\kappa}_t \mid, \mid z_{t-1} \mid\right) < 0$, such that mispricing correction $\hat{\kappa}_t$ is rather decreasing with mispricing error, z_{t-1}. It implies that arbitrage becomes ineffective.

The methodology can be applied to various assets classes, markets and countries with a recognizable arbitrage relationship in order to evaluate funding liquidity in broad scope. In this paper we will apply this methodology to the future-cash basis implied by the S&P 500 cash index and E-mini future.

3.3. Data

We choose to estimate the implied arbitrage efficacy in the S&P 500 index future-cash basis because of several advantages. First, the cost-of-carry model can be applied to determine the

	Mean	Median	Minimum	Maximum	Std Dev
ΔI	0.016	0.056	-9.469	10.957	1.263
Δp	0.017	0.065	-10.399	13.197	1.288
$p - I$	0.115	0.001	-1.689	2.030	0.516
$p - p^*$	0.074	0.058	-2.201	1.792	0.213
$\mid p - p^* \mid$	0.156	0.113	0.000	2.201	0.164
r	2.01	1.42	-0.02	6.24	2.077
q	1.83	1.86	1.07	3.37	0.387

Table 2: Basic descriptive statistic

The table reports the the descriptive statistics for all variables. The sample used is the daily series of the S&P 500 index and its E-mini futures contract covering the period September 16, 1997 to June 30, 2015. ΔI (Δp) is the first difference of log spot (futures) price. The log fundamental value is computed as $p^*_{t,T} = I_t + (r_t - q_t)\,\tau_t$ where I_t is the log of cash index, r_t is the annualized risk-free (3 month T-bill) interest rate on an investment for the period , and q_t is the annualized dividend yield on the index. All numbers are recorded in percentage point terms.

fundamental value of the E-mini future contract, which is indicated in the following relationship to hold in equilibrium:

$$p^*_{t,T} = I_t + (r_t - q_t)\,\tau_t, \tag{18}$$

where $p^*_{t,T}$ is the natural log of the fundamental price of E-mini future contract with a maturity date T implied from cost of carry model; I_t is the log spot price of the S&P 500 index; r_t and q_t is the risk-free interest rate and dividend yield of the asset, respectively; $\tau_t = T - t$ is the time to maturity. Then, the future-cash basis at date t can be estimated as the difference between the E-mini future price $p_{t,T}$ and the fundamental price $p^*_{t,T}$. For completeness, we collect our proxies for risk-free interest rate: the US three-month T-bill rate, and dividend yields on the S&P 500 index. We focus on the most actively-traded future contracts that have 3-month to maturity and roll over every quarter (March, June, September, and December) into successive contracts that have 3-month to maturity.[14] All data are sourced from Datastream. We provide the summary statistics of the daily S&P 500 index and E-mini S&P 500 future data in Table 2.

Second, the E-mini future is one of the most traded future contracts. It contains large numbers of financial intermediaries, such as hedge fund and investment banks, that will exploit

[14] In the appendix, we test the robustness by investigating the implied arbitrage efficacy from other S&P 500 future contracts (e.g. standard contracts that have 3-, 6-, 9-month to maturity) and other major index-future market (e.g. Germany and Japan).

any arbitrage opportunity in the market. It also offers liquidity at lower cost (Domowitz and Steil, 1999), attracts traders even with modest capital and provides higher pricing efficiency (Hasbrouck, 2003; Kurov and Lasser, 2004). Thus the arbitrage efficacy implied by the S&P 500 future-cash basis is more likely to reflect the ability of a large number of future-cash arbitrageurs and to infer the market-level funding liquidity.

Third, the E-mini future has been trading since September 16, 1997, which covers the periods of rapid growth in hedge fund industry[15] and some noticeable market events, like the burst of dot-com bubble, the recent financial crisis 2007-2008, the Flash Crash in 2010 and the European sovereign debt crisis. The comprehensive time-period helps to verify the validity of the arbitrage efficacy as a measure of funding liquidity in different market circumstances, especially the extreme ones.

4. Main results

4.1. Arbitrage efficacy over time

We first present some aggregate level results of the implied arbitrage efficacy of S&P 500 index-future arbitrage over the period from September 16, 1997 to June 30, 2015 in Table 3. The aggregate level of L is positive at 0.172 over full sample, showing that arbitrage activity in general tends to actively react to larger mispricing errors. The sub-sample results, however, vary but consistent with the ex post market condition. The first sub-sample covers the periods of turmoil from September 1997 to October 2002, which marks the collapse of Dot-com bubble, the 911 attacks and the market downturn in 2002. The implied arbitrage efficacy in this period is recorded at -0.006 with t-statistic of -1.89, which implies that one percent changes in mispricing error leads to 0.6% drop in the mispricing error. It suggests a low funding liquidity and the possibility of binding constraints. The second sub-sample from November 2002 to May 2007 captures the bull market with ample liquidity, where our measure

[15]According to the data of Fung and Hsieh (2013) gathering from BarclayHedge, HFR, Lipper-Tass and Hedgefund.net, the hedge fund industry starts to grow dramatically after 2000. The number of funds and the asset under management (AUM) are more than five times larger in 2010 than that in 2000.

Sample period	Implied arbitrage efficacy	t-stat
09/1997 - 06/2015	0.172***	(4.98)
09/1997 - 10/2002	-0.006*	(-1.89)
11/2002 - 05/2007	0.109***	(6.28)
06/2007 - 04/2009	-0.197***	(-9.77)
05/2009 - 12/2011	-0.019***	(-2.57)
01/2012 - 06/2015	-0.004	(-0.94)

Table 3: The implied arbitrage efficacy of S&P 500 index-future arbitrage in full and sub-sample
The table reports the implied arbitrage efficacy estimated from (17) over full sample and sub-samples based on ex post market events. The implied arbitrage efficacy is estimated by Eq. (17), where mispricing correction and mispricing error are estimated from the S&P 500 index and E-mini future arbitrage relationship. All t-statistic are computed with Newey-West standard errors. ***, ** and * indicate significance at 1%, 5% and 10% levels, respectively.

captures the same story, i.e. positive at 0.109 with statistical significance. The third sub-sample consists the financial crisis of 2007 - 09 as the market was subject to severe uncertainty and liquidity dry-out. The implied arbitrage efficacy L is significantly negative (-0.197 with t-statistic of -9.77), which suggests that funding liquidity is severely deteriorated and funding constraints are binding. The period from May 2009 to the end of 2011 marks the flash crash and sovereign debt crisis in US and EU. The implied arbitrage efficacy improved to -0.019 but still significantly negative. The final sub- sample captures another bull market along with numerous liquidity events along with injection schemes to improve liquidity condition. Our funding liquidity measure is recorded at -0.004, which is indifferent to zero.

For the purpose of tracking the innovation of funding liquidity, the time-series variation of θ is in favor, while the aggregate level of θ is of little importance. We thus capture the moving arbitrage efficacy in Eq. (17) on a rolling window basis for a rudimentary test. In particular, we take a window of $T = 250$, and the 250-day moving arbitrage efficacy is assigned to the ending date τ of the window as θ_τ, representing the average arbitrage efficacy over the previous 250 days. In doing so, a daily series of θ with 3721 observations from September 6, 2000 to June 30, 2015 are obtained, which allow us to examine the dynamics of funding liquidity. Figure 4 depicts the daily series of arbitrage efficacy θ implied by the S&P 500 index-future arbitrage relationship over a 250-day rolling window. At first glance, we see that the implied arbitrage efficacy θ varies through time; the daily series of θ fluctuates gently in the early stage of the sample period from 2000 to 2007; θ remains positive, except for the liquidity concern

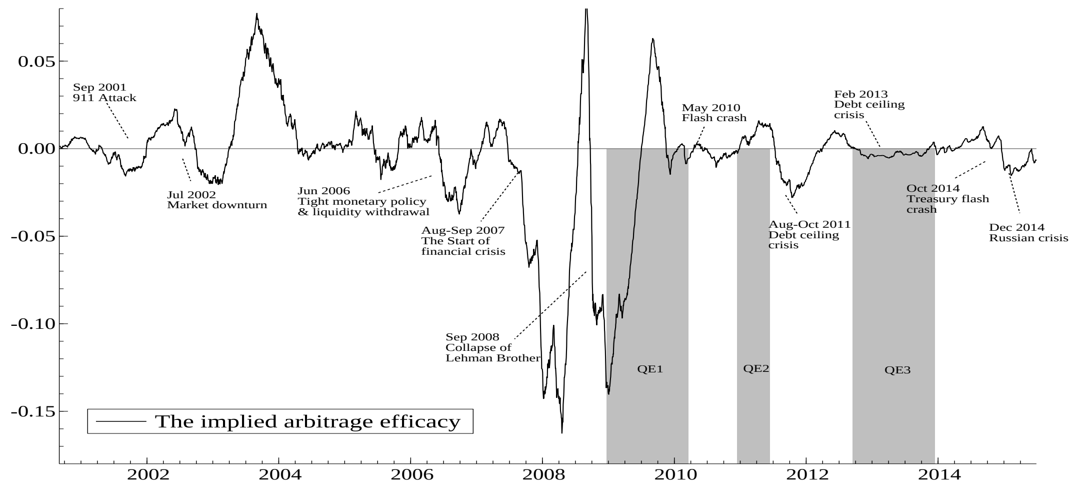

Figure 4: The implied arbitrage efficacy of S&P 500 index-future arbitrage over time, March 2, 2000 to June 30, 2015
The figure shows the arbitrage efficacy implied by the arbitrage relationship between S&P 500 index and E-mini future, computed through a 250-days rolling window. The periods of the three round of QE are shaded, along with some major financial market events. The data is sampled from March 2, 2000 to June 30, 2015 at daily frequency.

during mid 2006. It is after 2007 that the variation in θ began to be remarkably volatile. It becomes negative from June 2007, and even drop to its bottom, at around -0.16, in early 2008 and the collapse of Lehman Brother in September 2008. After the Global financial crisis in 2007-2009, θ retains its stability till the end of our sample period. The sign of θ is negative, i.e. arbitrage is ineffective, with most of the market crashes and turmoils during our sample periods. The major liquidity-related events include: liquidity withdrawal in 2006, the Global Financial Crisis in 2007-2008, the Debt Ceiling (dollar shortage) crisis in 2011 and the Debt Ceiling crisis in 2013.

4.2. Arbitrage efficacy and other measures of funding liquidity

The rolling-window plot of the implied arbitrage efficacy provides some rudimentary results that consist with our model predictions. Started from this subsection, we document some formal evidence to verify our model predictions. We construct the arbitrage efficacy on a monthly basis as the slope estimator in (16) over each month, denoted as θ^m. We obtain 184 monthly observations from March 2000 to June 2015.

We first investigate whether the implied arbitrage efficacy θ^m is associated with other funding liquidity measure. Typical measures for funding illiquidity and credit conditions includes: the TED spread, e.g. the spread between the three-month risky LIBOR rate and the three-month risk free T-Bill yield, which measures the funding illiquidity among the financial intermediaries; Repo spread is the spread between 3-month LIBOR and the repo rate; Default spread is the spread between BAA and AAA rated corporate bonds; Term spread is the spread between the yield on 10-year Treasury bonds and the 3-month T-bill rate. We also have the VIX index as the ex ante risk-neutral expectation of the future market volatility (referred to as the "fear" index)[16], the treasury market funding illiquidity measure ($FGilliq$) from Fontaine and Garcia (2012) and the broker-dealer leverage factor ($BDlev$) from Adrian, Etula, and Muir (2014). Details on data sources are provided in the Appendix.

Table 4 reports the summary of pairwise correlation matrix between levels and changes in these funding illiquidity measures. Our measure of funding liquidity is negatively related to all the other funding illiquidity variables in monthly frequency. The pairwise correlations in levels are highest for TED spread (-56.3%) and lowest for term spread (-2.7%), which is the only measure that is insignificant at 10% levels. Although our funding liquidity measure is constructed using only the S&P 500 index-future arbitrage relationship, which reflects the funding liquidity condition among the participating arbitrageurs in the index-future arbitrage, it contains some information of market-wide funding illiquidity and credit condition.

To further investigate the linkage between the arbitrage efficacy and other measures of funding liquidity and arbitrage risk, we report the OLS regression of the implied arbitrage efficacy on several important measures. Other than the funding illiquidity measures introduced in the last section, we also include the following variables of arbitrage frictions that are available on monthly bases from March 2000 to June 2015. We have the Amihud (2002) illiquidity measure ($Illiq$) that indicates the level of stock market illiquidity, the aggregate liquidity measure ($PSilliq$) from Pastor and Stambaugh (2003) that poses a illiquidity cost to conduct arbitrage, and the idiosyncratic risk ($Idio$) from Ang et al. (2006) that captures the arbitrage

[16]VIX index tends to reflects the market condition. Literature uses VIX index to proxy for funding liquidity condition, as Brunnermeier and Pedersen (2009) finds that funding illiquidity is closely related to market volatility.

Correlation matrix in levels								
	θ^m	TED	$Repo$	Def	$Term$	VIX	$FGilliq$	$BDLev$
θ^m	1.000							
TED	-0.563***	1.000						
$Repo$	-0.551***	0.935***	1.000					
Def	-0.249***	0.507***	0.677***	1.000				
$Term$	-0.027	-0.250***	0.018	0.284***	1.000			
VIX	-0.245***	0.522***	0.639***	0.755***	0.279***	1.000		
$FGilliq$	-0.124*	0.416***	0.399***	0.327***	-0.299***	0.123***	1.000	
$BDlev$	-0.283***	0.532***	0.450***	0.165***	-0.222***	0.136***	0.065*	1.000

Table 4: Pairwise correlation between the implied arbitrage efficacy and other measures of funding illiquidity, March 2000 to June 2015
The table reports the correlation matrix among the implied arbitrage efficacy θ^m and other funding liquidity measures in levels. TED is TED spread between the three-month risky LIBOR rate and the three-month risk free T-Bill yield; $Repo$ is Libor-repo spread between 3-month LIBOR and the repo rate; Def is the default spread between BAA-AAA rated corporate bonds; $Term$ is the term spread between the yield on 10-year Treasury bonds and the 3-month T-bill rate; VIX is the CBOE VIX index; $FGilliq$ is the the funding liquidity measure of Fontaine and Garcia; $BDlev$ is the Broker-dealer leverage factor of Adrian et al. (2014). ***, ** and * indicate significance at 1%, 5% and 10% levels, respectively.

risk exposed to arbitrageurs in the stock market. Details of data sources are provided in the Appendix.

The results in Table 5 show that these explanatory variables of funding liquidity and arbitrage frictions do indeed capture some variation in the implied arbitrage efficacy with the expected sign. When consider individually, higher market risk and credit risk are associated with worse funding liquidity, and thus lower arbitrage efficacy. In particular, column (2) shows that TED spread explains more than 30% of the variation in the implied arbitrage efficacy, while VIX index have a much smaller explanation power as seen in column (1). Column (3) reports the joint regression with funding illiquidity measures. Coefficients of TED and term spread are negative and statistically significant at 1% level, while that of VIX index becomes positive and close to zero. When consider the variables of market illiquidity and arbitrage risk jointly in column (4), we find that measures of market illiquidity and idiosyncratic risk have limited explanatory power, while funding liquidity measures, e.g. TED and term spread, still serve as the dominant explanatory variables. It again shows that our measure is closely related to the existing measure of funding liquidity. The adjusted R^2 is 37%, which implies that more than 60% of the variation in the implied arbitrage efficacy is unexplained.

Regression of θ^m in levels on:								
	(1)		(2)		(3)		(4)	
VIX	-0.021**	(-2.01)			0.016*	(1.77)	0.008	(0.06)
TED			-0.972***	(-4.19)	-1.374***	(-4.10)	-1.364***	(-3.95)
Def					0.094	(0.48)	0.117	(0.63)
$Term$					-0.154***	(-3.92)	-0.115***	(-2.75)
$FGilliq$					0.058	(1.26)	0.091*	(1.75)
$Illiq$							2803.	(0.09)
$PSilliq$							0.541	(0.52)
$Idio$							15.98	(0.97)
R-square	0.054		0.313		0.361		0.373	
Obs	184		184		184		184	

Table 5: Determining the implied arbitrage efficacy, March 2000 to June 2015
This table reports the OLS regressions that estimate the relation between the implied arbitrage efficacy and factors for funding liquidity and other arbitrage frictions. TED is TED spread between the three-month risky LIBOR rate and the three-month risk free T-Bill yield; *Repo* is Libor-repo spread between 3-month LIBOR and the repo rate; Def is the default spread between BAA-AAA rated corporate bonds; $Term$ is the term spread between the yield on 10-year Treasury bonds and the 3-month T-bill rate; VIX is the CBOE VIX index; $FGilliq$ is the the funding liquidity measure of Fontaine and Garcia; *Illiq* is the Amihud (2002) illiquidity measure; $PSilliq$ is the aggregate liquidity measure from Pastor and Stambaugh (2003); $Idio$ is the idiosyncratic risk captured from Ang et al. (2006). The Unit root test (ADF) rejects the null in levels for the monthly implied arbitrage efficacy θ^m. Thus it is regressed in levels. All coefficient are reported at 10^2 level for better view. The constant term is not reported. All t-statistic are computed with Newey-West standard errors. ***, ** and * indicate significance at 1%, 5% and 10% levels, respectively, and reported in the bracket.

4.3. Nonlinearity

Our model predicts that amplification arises due to the binding leverage constraint, such that an external shock, e.g. an equity shock, can spillover to the leverage debt and cause more mispricing. In other words, mispricing tend to be more sensitive to an external shock when the leverage constraint binds. In this section, we examine the relation between the size of mispricing and the implied arbitrage efficacy. To identify the periods of the binding leverage constraint, we refer to the sign of the implied arbitrage efficacy, rather than setting an arbitrary threshold or ex-post event dates. We capture the future-cash basis in absolute value from the S&P 500 index and E-mini future using Eq. (13) and Eq. (18), denoted as $\mid z_t \mid$. Since other arbitrage frictions, such as market condition, arbitrage risk and illiquidity also affect the size of mispricing error, we add the following explanatory variables. The controls include the lagged mispricing itself, the VIX index to control for market conditions, TED spread, Default spread and term spread to control for credit market conditions, the idiosyncratic risk ($Idio$) to control for arbitrage risk in the stock market, the Amihud (2002) illiquidity measure ($Illiq$)

to control for the level of stock market illiquidity.[17]

Results of the OLS regressions in changes are reported in Table 4.3. Column (2) reports the relation between mispricing error and the control variables over the full sample from 2000 to 2015. Idiosyncratic risk and TED spread are significantly and positively related to the size of mispricing error, such that higher arbitrage risk and credit cost is associated with larger mispricing. In column (3) where the implied arbitrage efficacy is included, the coefficient on L^m is statistically significant at 1% levels with a negative sign (-2.537). The adjusted R^2 increases from 0.579 to 0.623. Columns (4) and (5) display the conditional results on the positive and negative implied arbitrage efficacy, respectively. When $\theta^m > 0$, i.e. leverage constraint tends to be slack, loading on the implied arbitrage efficacy drops (in absolute term) to -0.792 and becomes insignificant. Other funding liquidity measures, such as TED spread and Default spread, also witness a decline in the coefficients and the significance. However idiosyncratic risk obtains a higher and statistically significant loading, 6.246, which is the dominant driver of the size of mispricing in these periods of time. However, when $\theta^m < 0$, i.e. leverage constraints become binding, loading on θ^m sharply increases (in absolute term) to -3.286, and it is statistically significant at 1% level. Coefficients on TED spread and Default spread also are more than doubled and become significant, while that on the idiosyncratic risk becomes indifferent from zero. These evidence implies that a moderate external shock to the funding liquidity condition has a large impact on the size of mispricing when $\theta^m < 0$, which is consistent with the liquidity-induced amplification effect.

In addition, we also repeat the regression on the noise measure from Hu, Pan and Wang (2013), which captures the observed market-wide mispricing in the US treasury bond market. This exercise may also answer a question: whether and when funding liquidity in stock market will spillover to the treasury bond market?[18] Existing literature shows that commonality in liquidity appears when funding constraints tend to be binding (Brunnermeier and Pedersen, 2009; Fontaine, Garcia and Gungor, 2015). Therefore we expect to see a closer relationship

[17]Results are not dissimilar if we control for market-wide illquidity measure of Pastor and Stambaugh (2003).

[18]Chordia, Sarkar, and Subrahmanyam (2005) and Goyenko and Ukhov (2009) provide evidence of liquidity spillovers between equity and bond markets in the US.

Regression of future-cash basis $\mid z \mid$ in changes on:										
	(1)		(2)		(3)		(4)		(5)	
lag $\Delta \mid z \mid$	-0.205	(-1.61)	-0.418***	(-5.52)	-0.340***	(-4.56)	-0.294***	(-4.06)	-0.517***	(-4.26)
$\Delta\theta^m$	-4.728***	(-6.14)			-2.537***	(-3.67)	-0.792	(-1.16)	-3.286***	(-3.30)
ΔVIX			-0.007	(-0.30)	0.008	(0.36)	0.034	(0.89)	0.038	(1.07)
$\Delta Idio$			4.868***	(2.75)	5.525***	(3.23)	6.426***	(-3.17)	-0.415	(-0.22)
ΔTED			0.200***	(5.14)	0.138***	(3.86)	0.096*	(1.95)	0.181***	(2.93)
ΔDef			0.087*	(1.72)	0.068*	(1.72)	0.055	(1.23)	0.127**	(2.02)
$\Delta Term$			-0.021	(-1.11)	-0.023	(-1.28)	-0.019	(-1.04)	-0.002	(-0.06)
$\Delta Illiq$			-0.341	(-0.77)	-0.012	(-0.02)	-0.603	(-1.24)	-0.475	(-0.56)
R-square	0.349		0.579		0.623		0.444		0.769	
Obs	183		183		183		104		78	

Table 6: Determine the size of mispricing error in S&P 500 index-future arbitrage, March 2000 to June 2015

This table reports the OLS regression of mispricing error captured in the index-future arbitrage relation on the implied arbitrage efficacy controlling for various variables of funding liquidity, market liquidity and arbitrage risk in changes. Columns (1) to (3) regress on full sample. Column (4) reports the conditional regression on positive arbitrage efficacy ($L^m > 0$), while column (5) on negative arbitrage efficacy ($L^m < 0$). All t-statistic are computed with Newey-West standard errors. ***, ** and * indicate significance at 1%, 5% and 10% levels, respectively, and reported in the bracket.

between the noise measure and the implied arbitrage efficacy during the periods of binding leverage constraints.

We replace the stock market controls, i.e. idiosyncratic risk $Idio$ and market liquidity $Illiq$, to the aggregate market-wide illiquidity ($PSilliq$) of Pastor and Stambaugh (2003) and the treasury market funding illiquidity measure ($FGilliq$) of Fontaine and Garcia (2012). Details on data sources are provided in the Appendix. Results of regressions in changes are drew on Table 4.3. Although θ^m, on its own, has a significant negative relation to the noise measure (Column 1), the coefficient drops to -9.052 after the controls are introduced in column 3 with a t-statistic of 1.57. All controls produce the correct sign in relating to the noise measure with mix results in significance. VIX index and term spread are the dominating drivers with significant coefficient at 1% levels. We are more interested in the conditional regressions on the sign of the implied arbitrage efficacy. In column (4) where $\theta^m > 0$, VIX index and Term spread remain the dominating drivers of the noise measure but with a smaller loading, comparing to the aggregate results. Coefficient on L^m has even larger decline (in absolute term) to only 0.322. However column (5) illustrate the arise of nonlinearity under $\theta^m < 0$. Comparing to the

Regression of the noise measure ($noise$) in changes on:										
	(1)		(2)		(3)		(4)		(5)	
lag $\Delta noise$	0.522***	(-3.35)	0.473***	(4.01)	0.472***	(4.05)	0.576***	(3.40)	0.323***	(2.86)
$\Delta\theta^m$	-12.473**	(-1.98)			-9.052	(-1.57)	0.322	(0.03)	-11.89**	(-2.00)
ΔVIX			1.087***	(4.61)	1.178***	(4.59)	0.865***	(3.63)	1.121***	(3.44)
ΔTED			0.345	(0.75)	0.090	(0.18)	0.981*	(1.97)	-0.674	(-1.34)
ΔDef			0.943	(1.58)	0.908	(1.49)	-0.601	(-0.70)	2.214**	(2.60)
$\Delta Term$			0.758***	(3.53)	0.758***	(3.65)	0.475**	(2.38)	0.997***	(3.15)
$\Delta FGilliq$			0.182	(1.35)	0.254*	(1.68)	-0.059	(-0.38)	0.339	(1.47)
$\Delta PSilliq$			0.287	(0.49)	0.301	(0.55)	1.325**	(2.22)	-0.591	(-0.83)
R-square	0.269		0.503		0.611		0.463		0.734	
Obs	183		183		183		104		78	

Table 7: Determine the size of mispricing error in U.S. treasury bond market, March 2000 to June 2015

This table reports the OLS regression of the noise measure on the implied arbitrage efficacy controlling for various variables of funding liquidity, market liquidity and arbitrage risk in changes. Columns (1) to (3) regress on full sample. Column (4) reports the conditional regression on positive arbitrage efficacy, while column (5) on negative arbitrage efficacy. Column (6) runs the conditional regression on the negative arbitrage efficacy that avoids the period of the global financial crises (Jan 2007 to May 2009). All t-statistic are computed with Newey-West standard errors. ***, ** and * indicate significance at 1%, 5% and 10% levels, respectively, and reported in the bracket.

results in column (4), loading on VIX index increases from 0.865 to 1.121; loading on default spread sharply rises from -0.601 (indifferent from zero) to 2.214; loading on term spread also doubles from 0.475 to 0.997. It implies that noise measure is more sensitive to the funding liquidity shock when the implied arbitrage efficacy becomes negative. More importantly, the estimate coefficient on θ^m is sizable (-11.89) and highly significant at a 5% significance level, which represents the spillover effect of funding liquidity shock from stock market to treasury bond market.

5. Conclusion

In this paper we propose an alternative way to study the arbitrageurs' funding liquidity through the corresponding arbitrage activity. Under the limit to arbitrage framework with endogenous leverage constraint, we allow arbitrageurs to choose the optimal leverage position subject to the equity and leverage constraints. The arbitrageurs' funding liquidity is defined as the marginal leverage financed to bear against additional mispricing error. We find that arbitrageurs tend to raise sufficient leverage against higher mispricing when they are far from the leverage constraint. Such activity is reflected on the arbitrage efficacy, which is defined as the marginal percentage of mispricing correction achieved by the arbitrageurs. When funding liquidity is ample, higher marginal mispricing correction can be achieved since more leverage funds enter the market to correct mispricing. However when the leverage constraint is hit, funding liquidity is deteriorated, such that insufficient leverage can be financed to bet against higher mispricing due to the leverage constraint. It leads lower marginal mispricing correction, i.e. arbitrage efficacy becomes negative. We thus use arbitrage efficacy to infer for funding liquidity, and more importantly the negative sign of arbitrage efficacy signals the binding leverage constraint, where amplifications occur.

We empirically estimate the implied arbitrage efficacy from the S&P 500 future-cash basis, and find statistically significant evidence that the implied arbitrage efficacy is related to other broad measure of funding liquidity. More importantly, the sign of the implied arbitrage efficacy identifies the binding leverage constraint, such that the periods of negative arbitrage efficacy coincide with the episodes of liquidity crises within the sample period and exhibit strong amplification effects. The measure of implied arbitrage efficacy thus provides vital and helpful tool for policy maker and regulators to evaluate the funding condition among the financial intermediaries and the potential existence of amplification due to the binding funding constraint. Moreover the measure could be used to investigate the effects of the implementation of the liquidity injection schemes, such as quantitative easing, and help to evaluate their efficacy.

Our work provides a number of direction that future researches might address. First, it would be interesting to extend the empirical methodology to several other stock market arbitrage relation, or even difference asset classes, to capture the more information about market-wide funding liquidity condition. In doing so, one might be able to properly address the spillover and contagion effect in funding liquidity especially during the crisis period. Second, Brunnermeier and Pedersen (2009) suggest that funding illiquidity among arbitrageurs leads to the phenomenon of flight to liquidity, flight to quality, and commonality in liquidity. Hence, the implied arbitrage efficacy can be a good tool to distinguish the sample into two regimes: the period of loose and binding funding constraint, and empirically examine these hypotheses. Third, our paper only focuses on the initial mispricing correction, while the pattern of subsequent price recovery also reflects the impediments faced by arbs, as suggested by Duffie (2010). Combining both immediate and subsequent pricing dynamics might generate more fruitful results. Fourth, the amplification effect under binding funding constraint can lead to long-lasting consequences, which has been explored in the macroeconomics literature. It would be interesting to empirically verify the impact of funding liquidity in the financial sector on overall economics, especially during the period of binding leverage constraints.

AppendixA. The arbitrage efficacy implied by the other markets

In this appendix, we will investigate the arbitrage efficacy estimated by other future contracts of S&P 500 index, and other major index-future markets.

First we capture the implied arbitrage efficacy using the standard S&P 500 future contract. The standard contract has the same underlying asset: S&P 500 index, as the E-mini future. Comparing to the standard contract, the E-mini contract offers a smaller size of contract and lower margin, which attracts more high frequency traders and market makers. However, the standard contract was first introduced in 1982, which offers a larger samples. However we choose to start the sample in 11 June 1990 due to considerations with respect to data quality. In particular, hedge funds industry, regarded as real world arbitrageurs, prospers in the early 1990s. We then compute the 250-day moving arbitrage efficacy using the same methodology introduced in Section 3.

The top panel in Figure A.5 shows the daily series of the 250-day moving arbitrage efficacy from June 1993 to June 2015, implied by the standard future contracts, while the bottom one captures the arbitrage efficacy implied by the E-mini future contracts from September 2000 to June 2015. The plots are similar in terms of the periods of negativity and the magnitudes of negativity, except for early 2000s. The arbitrage efficacy implied by standard contract displays more significant and persistent negativity in value from mid 2001 to mid 2002, when the market was under uncertainty after 911 attack. Due to the larger sample period, the

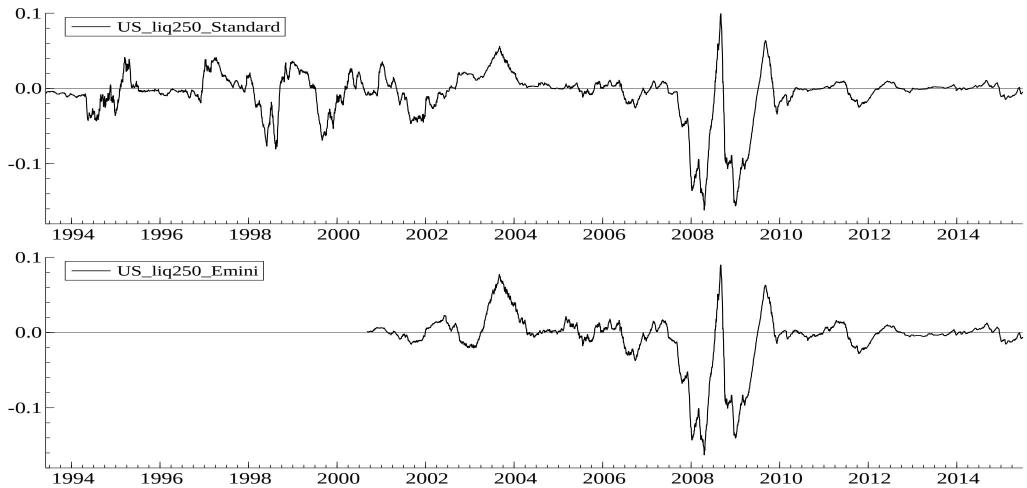

Figure A.5: The arbitrage efficacy implied by the S&P 500 index and standard future contract
The figures show the arbitrage efficacy computed through a 250-days rolling window. The top figure is implied by the arbitrage relationship between S&P 500 index and the standard future contract from June 1993 to June 2015, while the bottom figure is implied by the S&P 500 E-mini future from September 2000 to June 2015.

arbitrage efficacy implied by standard contract is also informative about the market turmoil in 1994, the global turmoil in mid 1998 and the bust of dot-com bubble starting from late 1999. It drops below zero in these periods and persists for at least six months.

Next, we seek for international evidence by investigating two major index-future markets: the DAX index (Germany) and the Nikkei 225 index (Japan). We have two future contracts for the Nikkei 225 index, one traded in the Osaka Exchange and one in the Chicago Mercantile Exchange (CME), and one future contract for DAX index traded in EUREX Deutschland. Take the Osaka-traded contract for example, it starts from 1988, but we choose to start the analysis in 1995. The reason being that the 3-month money rate in Japan, used as the risk free rate, is available from 1995. Similarly, although the future contract for DAX index is available from 1990, we choose to start from 1995 due to data quality. Also as a result of the introduction of the euro in 1998, we use the 3-month Frankfurt Interbank Offer Rate as the risk free rate from the start but replace it with the Euro Interbank Offered Rate from December 1998.

Figure A.6 depicts the arbitrage efficacy implied by the DAX index and future along with the 10-year government bond yield for some European countries (Greece, Italy, Portugal and

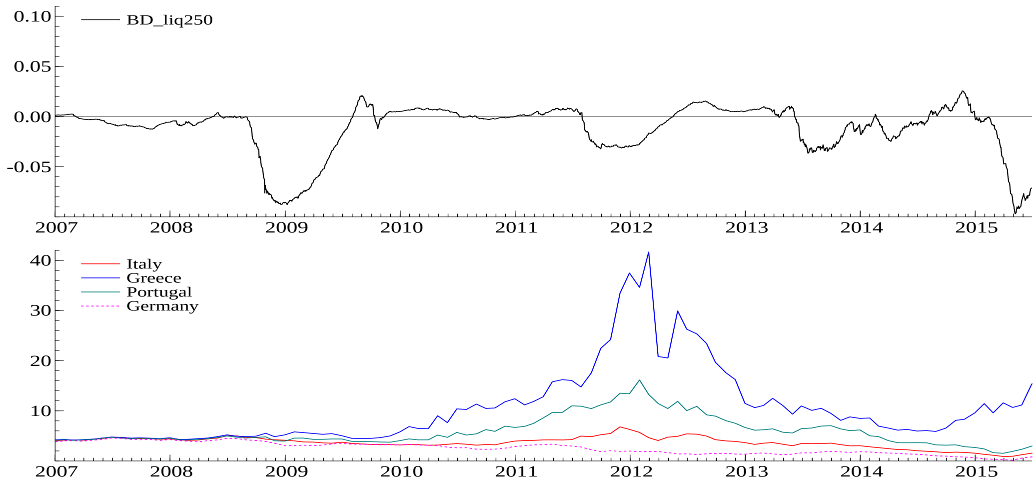

Figure A.6: The arbitrage efficacy implied by the DAX index and the standard future contract
The top figure depict the arbitrage efficacy implied by the DAX index from January 2007 to June 2015, computed through a 250-days rolling window. The bottom figure shows the monthly 10-year government bond yield for Greece, Italy, Portugal and Germany at the same period.

Germany) after 2007. Although the implied arbitrage efficacy is available from 1998, we focus on what happen after 2007 to investigate the impact of the 2007 global financial crises and the 2010 European debt crisis. The implied arbitrage efficacy is close to zero in 2007 and early 2008. It is the collapse of Lehman Brothers in September 2008 that drives it below zero significantly. It reaches the lowest point at -8.7% in early 2009 and starts to improve. The build-up of the European debt crisis since 2010 does not seem to affect the implied arbitrage efficacy. It drops below zero from mid 2011 to mid 2012, when the government bond yields are at their peak (See Greece for example). The implied arbitrage efficacy grow above zero in late 2012, as the ECB calmed the markets by announcing free unlimited support for all eurozone countries. 2015 marks another rises in the long-term bond yields due to uncertainties in Greece. The implied arbitrage efficacy responds and declines to the -10%. Notice also that the arbitrage efficacy also drop below zero in mid 2013. This period coincides with the dollar appreciation, which leads to tightening global liquidity (Avdjiev et al. 2017, 2018). Evidence of funding illiquidity is also documented in the violation of covered interest parity after mid 2013.

Figure A.7 plots the arbitrage efficacy implied by the Nikkei 225 index and the two future

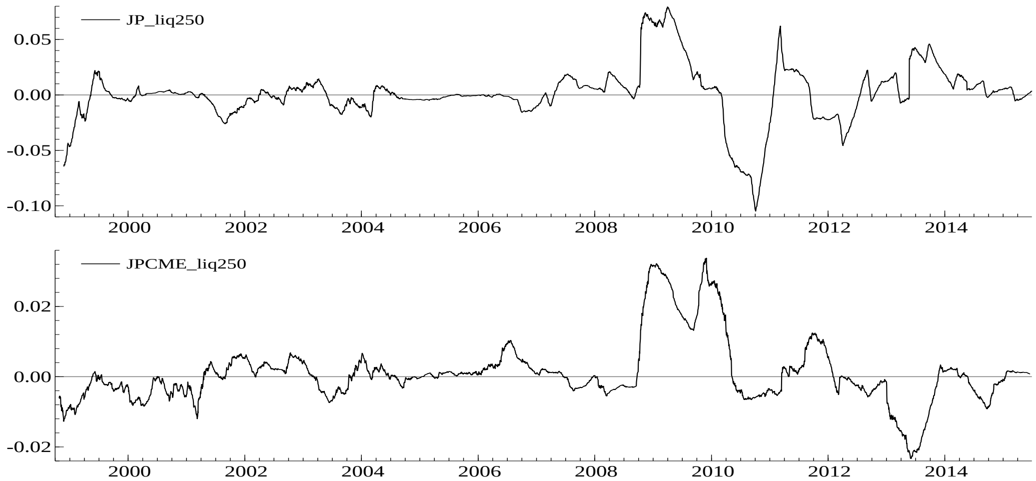

Figure A.7: The arbitrage efficacy implied by the Nikkei 225 index and the future contracts
The top figure is implied by the Nikkei 225 index and the standard future contract traded in Osaka, and the bottom figure is implied by the Nikkei 225 index and the standard future contract traded in CME. The implied arbitrage efficacy is computed through a 250-days rolling window from October 1998 to June 2015.

contracts. The top figure is implied by the future contracts traded in Osaka, while the bottom one is implied by those traded in CME. The two trajectories are both negative in 1998, but remain close to zero in early 2000s. Unlike the arbitrage efficacy implied by S&P 500 and DAX indices, the collapse of Lehman Brothers in September 2008 rather pushes the trajectories up above zero significantly (7% and 3%, respectively).

For a more clear view of the implied arbitrage efficacy in US, Germany and Japan, we plot them in Figure A.8 covering the period of June 2006 and June 2015. Both the S&P 500 index-implied arbitrage efficacy capture the build-up of the global financial crisis in 2007, while those implied by DAX and Nikkei index are rather unaffected. The collapse of Lehman Brothers in September 2008 drives the arbitrage efficacy implied by S&P 500 and DAX indices below zero significantly, while pushes that implied by Nikkei index up above zero. It implies that the Japanese market is served as a safe haven in this crisis period. Similar evidence is found in late 2011 where dollar shortage and sovereign debt crises hit the U.S. and the Europe. Starting from 2013, arbitrage efficacy captured in the Germany and Japan market tend to be more negative than those in the U.S. market, consistent with the period of dollar appreciation that reduces global liquidity.

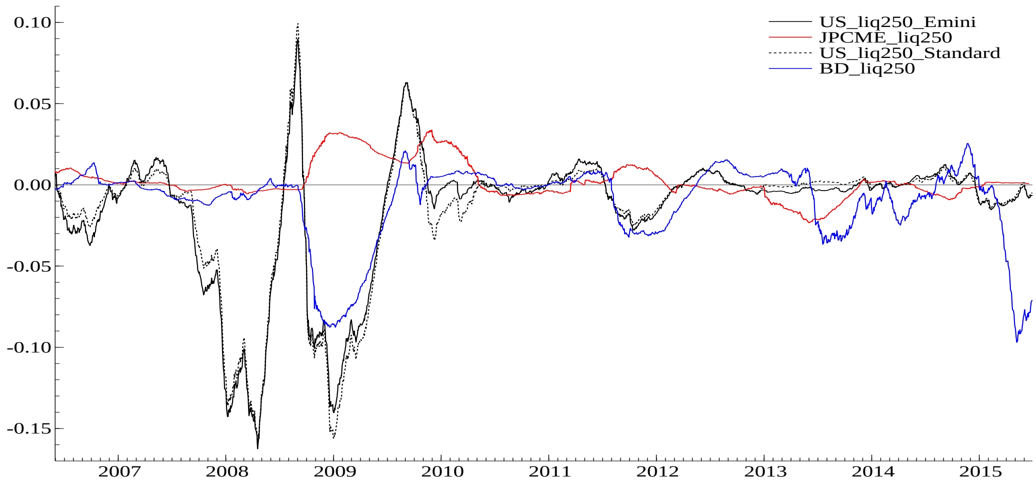

Figure A.8: The arbitrage efficacy implied by the S&P500 index, DAX index, and Nikkei index
The figures depict the arbitrage efficacy computed through a 250-days rolling window from October 1998 to June 2015. The top figure is implied by the arbitrage relationship between DAX index and the standard future contract, the middle figure is implied by the Nikkei 225 index and the standard future contract traded in Osaka, and the bottom figure is implied by the Nikkei 225 index and the standard future contract traded in CME.

AppendixB. Proofs

In this appendix, we provide the proofs for the propositions derived from our model.

Proof for Proposition 1: We first derive the upper leverage limit D_U, by allowing for $f_{2,b} = 0$ under bad state, such that the arbs' equity in period 2 fully covers the losses generated in period 1. We have

$$f_{2,b} = f_1^e + (f_1^e + D_U)\left(\frac{P_{2,b}}{P_1} - 1\right) = 0 \tag{B.1}$$

where

$$P_1 = V - s_1 + f_1^e + D_U, \;\; P_{2,b} = V - s_{2,b}, \tag{B.2}$$

Using Eq.(B.1) and (B.2), we solve for the leverage constraint D_U:

$$D_U = \frac{1}{2}\left(m_1 - m_0\right) \tag{B.3}$$

where

$$
\begin{aligned}
m_0 &= \left(f_1^e + s_{2,b} - s_1\right) \\
m_1 &= \sqrt{\left(m_0\right)^2 + 4f_1^e\left(V - s_{2,b}\right)}.
\end{aligned}
$$

Using Eq. (B.3), we are able to write the partial derivative of D_U w.r.t f_1^e as

$$
\frac{\partial D_U}{\partial f_1^e} = \frac{1}{2}\left(\frac{2V - \left(s_1 + s_{2,b} - f_1^e\right)}{m_1} - 1\right).
$$

It is easily found that the first term in the bracket is greater than 1, since

$$
\left(2V - \left(s_1 + s_{2,b} - f_1^e\right)\right)^2 - \left(m_1\right)^2 = 4\left(V - s_1\right)\left(V - s_{2,b}\right) \geq 0.
$$

Thus we have $\frac{\partial D_U}{\partial f_1^e} > 0$.

Using Eq. (B.3), we also write the partial derivative of D_U w.r.t $s_{2,b}$ as

$$
\frac{\partial D_U}{\partial s_{2,b}} = \frac{1}{2}\left(\frac{s_{2,b} - s_1 - f_1^e}{m_1} - 1\right)
$$

It is easily found that the first term in the bracket is less than 1, since

$$
\left(s_{2,b} - s_1 - f_1^e\right)^2 - \left(m_1\right)^2 = -4f_1^e\left(V - s_1\right) < 0.
$$

Thus we have $\frac{\partial D_U}{\partial s_{2,b}} < 0$. **Q.E.D.**

Proof for Proposition 2: Consider the partial-leverage strategy first. We derive the optimal f_1^d by solving the first order condition in Eq.(8):

$$
\frac{V}{P_1} - 1 = q\left(\frac{V}{P_{2,b}} - 1\right)
$$

where

$$
P_1 = V - s_1 + f_1,\; P_{2,b} = V - s_{2,b} + f_{2,b}.
$$

Then the optimal leverage fund is given by:

$$f_1^d = \frac{n_0 - n_2}{2\,(1-q)} \tag{B.4}$$

where

$$\begin{aligned}
n_0 &= V - (1-q)\,(f_1^e + s_{2,b} - s_1) \\
n_1 &= V - (1-q)\,(s_1 + s_{2,b} - f_1^e) \\
n_2 &= \sqrt{(n_1)^2 + 4Vq\,(1-q)\,(s_{2,b} - f_1^e)}
\end{aligned}$$

We write the partial derivative of f_1^d w.r.t. f_1^e as

$$\frac{\partial f_1^d}{\partial f_1^e} = -\frac{1}{2}\left[\frac{V - (1-q)\,(s_1 + s_{2,b} - f_1^e) - 2Vq}{n_2} + 1\right]$$

It is easily found that the absolute value of the first term in the bracket is less than 1, since

$$[V - (1-q)\,(s_1 + s_{2,b} - f_1^e) - 2Vq]^2 - (n_2)^2 = -4qV\,(1-q)\,(V - s_1) < 0$$

Therefore we have $\frac{\partial f_1^d}{\partial f_1^e} < 0$.

Consider the max-leverage strategy now. We have $\frac{\partial D_U}{\partial f_1^e} > 0$ from Proposition 1. **Q.E.D.**

Proof for Proposition 3: Under the partial-leverage strategy, f_1^d is implied by Eq. (B.4). Taking the partial derivative of f_1^d in Eq. (B.4) w.r.t. s_1, we obtain the funding liquidity ℓ^p under the partial-investment strategy by

$$\ell^p = \frac{\partial f_1^d}{\partial s_1} = \frac{(1-q) + (1-q)\,n_1/n_2}{2\,(1-q)} = \frac{1}{2}\left(1 + \frac{n_1}{n_2}\right) \tag{B.5}$$

Under the assumption on equity constraint, i.e. $s_2^b > f_1^e$, we have $n_2 > n_1$. Therefore we must have $0.5 < \ell^p < 1$. If $q = 0$ or $q = 1$, ℓ^p becomes 1 as $n_2 = n_1$.

Funding liquidity under max-leverage strategy can be written as the partial derivative of

D_U in Eq. (B.3) w.r.t. s_1:

$$\ell^m = \frac{\partial D_U}{\partial s_1} = \frac{1}{2}\left(1 - \frac{m_0}{m_1}\right). \tag{B.6}$$

It is easily seen that $0 < \ell^m < 0.5$, since $0 < \frac{m_0}{m_1} < 1$. Therefore we must have $0 < \ell^m < \ell^p < 1$. Also there must be a dive in ℓ when leverage constraint becomes binding.

To see how ℓ^p and ℓ^m behave after an equity shock, we write the partial derivatives of ℓ^p w.r.t. f_1^e as:

$$\frac{\partial \ell^p}{\partial f_1^e} = \frac{1}{2} + \frac{1}{2}\frac{\partial \frac{n_1}{n_2}}{\partial f_1^e}$$

where

$$\frac{\partial \frac{n_1}{n_2}}{\partial f_1^e} = \frac{(n_2)^2 - (n_1)^2 + 2Vqn_1}{(n_2)^3} = \frac{4Vq(1-q)(s_{2,b} - f_1^e) + 2Vqn_1}{(n_2)^3} > 0.$$

Thus we have $\frac{\partial \ell^p}{\partial f_1^e} > 0$.

We also write the partial derivatives of ℓ^m w.r.t. f_1^e as

$$\frac{\partial \ell^m}{\partial f_1^e} = \frac{1}{2} - \frac{1}{2}\frac{\partial \frac{m_0}{m_1}}{\partial f_1^e}$$

where

$$\frac{\partial \frac{m_0}{m_1}}{\partial f_1^e} = \frac{(m_1)^2 - (m_0)^2 - 2(V - s_{2,b})m_0}{(m_1)^3} = \frac{2(V - s_{2,b})(f_1^e - s_{2,b} + s_1)}{(m_1)^3}.$$

We notice that the term $f_1^e - (s_{2,b} - s_1)$ tend to be negative under max-leverage strategy. Proposition 1 suggests that max-leverage strategy occurs when f_1^e is rather small. Large f_1^e will trigger the partial-leverage strategy as leverage funds is less attractive. Therefore without further mathematical derivation and loss of generality, we can assume that $f_1^e - (s_{2,b} - s_1) < 0$ for small f_1^e that triggers max-leverage strategy. Thus $\frac{\partial \frac{m_0}{m_1}}{\partial f_1^e} < 0$ and $\frac{\partial \ell^m}{\partial f_1^e} > 0$. **Q.E.D.**

Proof for Proposition 4: Under the partial-investment strategy we write the arbitrage efficacy as

$$\alpha^p = \frac{\partial \kappa}{\partial s_1} = \frac{\ell^p - \kappa}{s_1} = \frac{\ell^p}{s_1} - \frac{1}{s_1^2}\left(\frac{n_3 - n_2}{2(1-q)}\right) \tag{B.7}$$

where

$$n_3 = V - (1-q)\left(s_2^b - s_1 - f_1^e\right).$$

To show that $\alpha^p > 0$, we consider the worse case possible: the lowest α^p with the initial shock as large as s_2^b, i.e. $s_1 \to s_2^b$. Then α^p can be expressed as,

$$\begin{aligned}
[s_1 \to s_2^b]lim\alpha^p &= \frac{s_2^b\left(1-q\right)\left(\frac{n_4}{n_5}-1\right)+n_5-n_4}{2\left(1-q\right)\left(s_2^b\right)^2} \\
&= \frac{n_5-n_4}{2\left(1-q\right)\left(s_2^b\right)^2}\left(1-\frac{s_2^b\left(1-q\right)}{n_5}\right)
\end{aligned}$$

where

$$\begin{aligned}
n_4 &= V-\left(1-q\right)\left(2s_2^b-f_1^e\right) \\
n_5 &= \sqrt{\left(n_4\right)^2+4q\left(1-q\right)V\left(s_2^b-f_1^e\right)}
\end{aligned}$$

To see $[s_1 \to s_2^b]lim\alpha^p > 0$, we verify whether the following inequality holds or not:

$$n_5 > s_2^b\left(1-q\right) \tag{B.8}$$

The right hand side in Eq. (B.8) reach its largest when $q \to 0$, i.e. $RHS = s_2^b$, while the left hand side is at its lowest when $q \to 0$, i.e. $LHS = V - 2s_2^b + f_1^e$. Thus the inequality in Eq. (B.8) holds provided that

$$V > 3s_2^b - f_1^e.$$

Since the model assumes that noise shocks are much less than the fundamental value, i.e. $V \gg s_2^b$, f_1^e, the condition can be satisfied easily. Thus the inequality $[s_1 \to s_2^b]lim\alpha^p > 0$ holds, and we have $\alpha^p > 0$.

Under the max-leverage strategy the arbitrage efficacy can be expressed as

$$\alpha^m = \frac{\ell^m - \kappa}{s_1}.$$

According to Eq.(B.3) and (B.6), it becomes

$$\alpha^m = \frac{1}{2s_1^2}(m_0 + m_1) - \frac{1}{2s_1}\left(\frac{m_0}{m_1} - 1\right)$$

For α^m to be negative, it requires the following condition after rearrangement:

$$\begin{aligned} s_1 &< \frac{1}{s_2^b}\left(V\left(s_2^b + f_1^e\right) - \sqrt{V\left(V - s_2^b\right)}\left(s_2^b - f_1^e\right)\right) \\ &< \frac{2Vf_1^e}{s_2^b}. \end{aligned}$$

For simplicity, we can express it as

$$V > \frac{s_1}{2f_1^e}s_2^b.$$

Since it is reasonable to assume that $V \gg s_1,\ f_1^e,\ s_2^b$, the condition is easily satisfied. Therefore, we always have $\alpha^m < 0$. **Q.E.D.**

AppendixC. Additional data sources

This Appendix details the data sources and variable definitions for all the variables used in Section 4. Unless specified otherwise, the variable is available on a monthly basis for the whole time period, January 2000 through June 2015. For data that are available in daily series, we obtain the monthly data by taking the average over the start and the end of the month.

We obtain the VIX index from Chicago Board of Options Exchange (CBOE). TED spread is 3-month LIBOR less 3-month T-bill. Libor-repo spread is 3-month LIBOR less GC repo rate. Default spread is the spread between BAA and AAA rated corporate bonds. Term spread is the spread between the yield on 10-year Treasury bonds and the 3-month T-bill rate. These interest rate variables are available from Datastream. Other measures of funding and market liquidity is available from the authors' webpages. The monthly treasury market funding illiquidity measure of Fontaine and Garcia (2012) is collected from Fontaine's webpage. The broker-dealer leverage factor of Adrian, Etula and Muir (2014) is collected from Muir's webpage. The monthly aggregate illiquidity of Pastor and Stambaugh (2003) is collected from Pastor's webpage. The noise measure of Hu, Pan, and Wang (2013) is collected from Pan's webpage.

In addition, we construct the idiosyncratic risk measure following Ang, Hodrick, Xing and Zhang (2006), who find that stocks with high idiosyncratic risk relative to the Fama–French 3-factor model (1993) have low average return. We obtain the daily stock return and the S&P 500 Index return from the Center for Research in Security Prices (CRSP), then the daily risk-free return from the Fama–French Data Library website. We calculate the idiosyncratic risk of each stock in each month and then take the mean of the monthly aggregate idiosyncratic risk of all US stocks to run principal component analysis (PCA). The idiosyncratic risk is constructed as follows:

$$Ret_{i,t} - Ret_{riskfree,t} = \alpha_t + \beta_t \left(Ret_{spx500,t} - Ret_{riskfree}\right) + \varepsilon_{i,t}$$

$$IdioRisk_{i,t} = Std\left(\varepsilon_{i,t=1}, \varepsilon_{i,t=2}, \varepsilon_{i,t=3}, \ldots\ldots \varepsilon_{i,t=end\,day\,of\,month}\right),$$

where $Ret_{i,t}$ is the daily return of stock on day d at month t, $Ret_{riskfree,t}$ is the daily return of the 30-day T-bill from the Fama–French website, and $\varepsilon_{i,t=j}$ is the residual obtained from the regression on day j.

We also construct the Amihud (2002) illiquidity measure as follows:

$$Illiq_{i,y} = \left(\frac{1}{D_{i,y}}\right) \sum_{k=0}^{D_{i,y}} \frac{\mid R_{i,y,d} \mid}{VOLD_{i,y,d}},$$

where $D_{i,y}$ is the number of days of stock i available in year y, $R_{i,y,d}$ is the daily return of stock i on day d in year y, and $VOLD_{i,y,d}$ is the trading volume in dollars of stock i on day d in year y. The daily stock return are obtained from the Center for Research in Security Prices (CRSP).

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