Web-Appendix to

Volatility estimation and forecasts based on price durations*

Seok Young Hong† Ingmar Nolte‡ Stephen J. Taylor§ Xiaolu Zhao¶

This version: 26 January 2021

Abstract

We investigate price duration variance estimators that have long been neglected in the literature. In particular, we consider simple-to-construct non-parametric duration estimators, and parametric price duration estimators using autoregressive conditional duration specifications. This paper shows i) how price duration estimators can be used for the estimation and forecasting of the integrated variance of an underlying semi-martingale price process and ii) how they are affected by discrete and irregular spacing of observations, market microstructure noise and finite price jumps. Specifically, we contribute to the literature by constructing the asymptotic theory for the non-parametric estimator with and without the presence of bid/ask spread and time discreteness. Further, we provide guidance about how our estimators can best be implemented in practice by appropriately selecting a threshold parameter that defines a price duration event, or by averaging over a range of non-parametric duration estimators. We also provide simulation and forecasting evidence that price duration estimators can extract relevant information from high-frequency data better and produce more accurate forecasts than competing realized volatility and option-implied variance estimators, when considered in isolation or as part of a forecasting combination setting.

Keywords: Price durations; Volatility estimation; High-frequency data; Market microstructure noise; Forecasting.

*We are grateful to Federico M. Bandi (the Editor), the Associate Editor and two anonymous referees, whose valuable comments have greatly improved our paper. We would also like to thank Torben Andersen, Dobrislav Dobrev and Oliver Linton, as well as participants of the faculty seminars at Aarhus, Cambridge, Kent, Reading and the Bank of England, the ESEM 2016, Konstanz-Lancaster 2015 and ESRC-NWDTC 2015 workshops, and the SoFiE 2016, IAAE 2016 and Financial Econometrics Lancaster 2016 conferences for helpful comments. This paper was previously circulated under the title "More Accurate Volatility Estimation and Forecasts Using Price Durations". Support from the ESRC-FWF grant 'Bilateral Austria: Order Book Foundations of Price Risks and Liquidity: An Integrated Equity and Derivatives Markets Perspective' (ES/N014588/1) is gratefully acknowledged. The usual disclaimer applies.

†Lancaster University Management School, Lancaster LA1 4YX, UK. email: s.y.hong@lancaster.ac.uk

‡Lancaster University Management School, Lancaster LA1 4YX, UK. email: i.nolte@lancaster.ac.uk.

§Lancaster University Management School, Lancaster LA1 4YX, UK. email: s.taylor@lancaster.ac.uk.

¶Contact author. International Business College, Dongbei University of Finance and Economics, Dalian 116025, China. Phone +86 411847 10248, email: zhaoxiaolu@ibc.dufe-edu.cn.

Web-Appendix A: Comparison of density functions

For the choice of a suitable density function for the scaled price durations we first consider LR tests for the four reference stocks: HD, MCD, AXP and IBM. The results in Tables 10, 13, 16 and 19 show that the Burr density is preferred over the Weibull and Exponential densities most of the time over a wide range of price change threshold values δ.

Corresponding LB test results for LB statistics with lags 30 and 50 are presented in Tables 11, 14, 17 and 20. For the majority of the months the null hypothesis of i.i.d. distributed generalized residuals cannot be rejected at the 1% and 5% significance levels, which indicates that the price duration dynamics are well captured by the HACD specification.

The associated density forecast (DF) test results in Tables 12, 15, 18 and 21 show that the Burr density clearly outperforms the other two distributional assumptions, by giving the highest percentages of months in which the null is not rejected at either the 1% or 5% significance level. From the three densities considered the Burr density provides the best fit for the scaled price durations.

Overall, the test results for the four reference stocks indicate that the HACD-Burr combination fits the price duration data best.

Finally, we present in Table 22 the LB and DF tests results for all 19 stocks, when the price change threshold δ is selected using the "3-times-spread" rule. We observe that the HACD-Burr model fits the price durations data well.

Table 10: LR test results, HD

δ(ticks)	2	3	4	5	6	7	8	9	10
Wei. vs. Burr	505.77	260.88	155.93	100.55	67.86	45.56	35.09	24.14	21.30
Exp. vs. Burr	574.24	307.80	189.89	127.16	87.73	63.65	51.02	38.30	34.75
Exp. vs. Wei.	68.47	46.92	33.96	26.32	19.72	18.34	16.15	13.91	13.85
Wei. vs. Burr	1.00	1.00	1.00	1.00	1.00	0.95	0.93	0.74	0.68
Exp. vs. Burr	1.00	1.00	1.00	1.00	1.00	0.99	0.96	0.89	0.84
Exp. vs. Wei.	0.78	0.75	0.69	0.69	0.67	0.69	0.63	0.61	0.61

Notes: The first three rows are the LR test statistics (averaged over 132 months), and the last three rows are LR test results presented as proportions of the months in which the null is rejected at the 1% significance level. The assumed density under the null is stated first in the 1st column.

Table 11: LB test results for 30 and 50 lags, HD

δ(ticks)	2	3	4	5	6	7	8	9	10
30 lags 1% significance level									
Exp.	0.98	0.95	0.97	0.98	0.97	0.98	0.94	0.92	0.89
Weibull	0.97	0.94	0.95	0.98	0.95	0.98	0.93	0.92	0.92
Burr	0.87	0.86	0.90	0.92	0.95	0.95	0.94	0.86	0.89
30 lags 5% significance level									
Exp.	0.86	0.89	0.91	0.90	0.93	0.93	0.89	0.87	0.85
Weibull	0.82	0.85	0.88	0.86	0.89	0.92	0.87	0.87	0.86
Burr	0.66	0.70	0.78	0.76	0.80	0.83	0.81	0.77	0.80
50 lags 1% significance level									
Exp.	0.94	0.96	0.96	0.96	0.98	0.99	0.96	0.92	0.89
Weibull	0.93	0.95	0.96	0.96	0.96	0.98	0.95	0.93	0.92
Burr	0.87	0.91	0.93	0.90	0.96	0.99	0.96	0.87	0.90
50 lags 5% significance level									
Exp.	0.82	0.86	0.91	0.89	0.92	0.95	0.90	0.86	0.86
Weibull	0.79	0.83	0.90	0.86	0.90	0.95	0.88	0.86	0.88
Burr	0.67	0.73	0.81	0.80	0.88	0.89	0.83	0.80	0.86

Notes: The upper part of the table are LB test results for 30 lags, and the lower part are the results for 50 lags. Significance levels of 1% and 5% are considered. Each figure is the proportion of months in which the null is not rejected.

Table 12: DF test results, HD

δ(ticks)	2	3	4	5	6	7	8	9	10
1% significance level									
Exp.	0.00	0.00	0.01	0.03	0.11	0.34	0.31	0.52	0.53
Weibull	0.00	0.02	0.02	0.08	0.21	0.36	0.49	0.60	0.67
Burr	0.21	0.57	0.69	0.80	0.86	0.95	0.92	0.88	0.89
5% significance level									
Exp.	0.00	0.00	0.00	0.01	0.03	0.20	0.23	0.32	0.44
Weibull	0.00	0.00	0.01	0.04	0.11	0.25	0.30	0.45	0.53
Burr	0.14	0.43	0.56	0.67	0.76	0.85	0.80	0.81	0.83

Notes: DF test results for significance levels of 1% and 5% are presented. Each figure is the proportion of months in which the null that the assumed density is the true density is not rejected.

Table 13: LR test results, MCD

δ(ticks)	2	3	4	5	6	7	8	9	10
Wei. vs. Burr	460.17	268.57	181.59	129.51	91.43	68.76	52.41	40.05	32.77
Exp. vs. Burr	577.22	328.81	219.52	156.81	113.24	86.20	62.74	53.14	46.29
Exp. vs. Wei.	117.05	60.24	37.93	27.09	21.33	17.38	10.87	12.03	12.24
Wei. vs. Burr	1.00	1.00	0.99	0.98	0.95	0.88	0.84	0.78	0.73
Exp. vs. Burr	1.00	1.00	0.99	0.99	0.97	0.93	0.84	0.88	0.84
Exp. vs. Wei.	0.87	0.64	0.57	0.55	0.50	0.52	0.45	0.46	0.45

Notes: The first three rows are the LR test statistics (averaged over 132 months), and the last three rows are LR test results presented as proportions of the months in which the null is rejected at the 1% significance level. The assumed density under the null is stated first in the 1st column.

Table 14: LB test results for 30 and 50 lags, MCD

δ(ticks)	2	3	4	5	6	7	8	9	10
30 lags 1% significance level									
Exp.	0.93	0.96	0.98	0.95	0.98	0.99	0.93	0.93	0.89
Weibull	0.92	0.96	0.96	0.95	0.98	0.98	0.93	0.94	0.92
Burr	0.90	0.87	0.89	0.86	0.94	0.96	0.91	0.90	0.86
30 lags 5% significance level									
Exp.	0.83	0.86	0.88	0.86	0.87	0.96	0.88	0.90	0.83
Weibull	0.82	0.83	0.86	0.83	0.86	0.95	0.84	0.89	0.85
Burr	0.73	0.67	0.74	0.76	0.82	0.89	0.77	0.80	0.77
50 lags 1% significance level									
Exp.	0.90	0.92	0.97	0.95	0.98	0.99	0.90	0.92	0.88
Weibull	0.90	0.92	0.97	0.95	0.98	0.98	0.89	0.93	0.90
Burr	0.89	0.89	0.92	0.92	0.96	0.98	0.89	0.89	0.86
50 lags 5% significance level									
Exp.	0.85	0.87	0.88	0.89	0.96	0.98	0.86	0.86	0.86
Weibull	0.84	0.86	0.87	0.88	0.92	0.97	0.85	0.86	0.88
Burr	0.76	0.73	0.77	0.80	0.86	0.91	0.79	0.81	0.80

Notes: The upper part of the table are LB test results for 30 lags, and the lower part are the results for 50 lags. Significance levels of 1% and 5% are considered. Each figure is the proportion of months in which the null is not rejected.

Table 15: DF test results, MCD

δ(ticks)	2	3	4	5	6	7	8	9	10
					1% significance level				
Exp.	0.00	0.04	0.11	0.15	0.27	0.39	0.51	0.51	0.48
Weibull	0.01	0.07	0.18	0.21	0.34	0.43	0.57	0.63	0.61
Burr	0.24	0.55	0.75	0.80	0.83	0.92	0.88	0.85	0.84
					5% significance level				
Exp.	0.00	0.00	0.07	0.08	0.13	0.27	0.43	0.38	0.36
Weibull	0.01	0.03	0.10	0.13	0.23	0.32	0.40	0.47	0.48
Burr	0.14	0.45	0.61	0.70	0.72	0.83	0.83	0.80	0.76

Notes: DF test results for significance levels of 1% and 5% are presented. Each figure is the proportion of months in which the null that the assumed density is the true density is not rejected.

Table 16: LR test results, AXP

δ(ticks)	2	3	4	5	6	7	8	9	10	11	12
Wei. vs. Burr	678.13	382.69	253.40	172.94	128.72	98.31	74.79	59.54	44.10	35.64	28.78
Exp. vs. Burr	759.60	435.54	292.96	206.03	155.16	121.43	94.94	75.91	59.25	52.71	42.91
Exp. vs. Wei.	81.47	52.85	39.56	29.77	26.70	22.26	19.39	18.16	15.46	15.49	14.66
Wei. vs. Burr	1.00	1.00	1.00	1.00	0.99	0.99	0.96	0.95	0.89	0.77	0.65
Exp. vs. Burr	1.00	1.00	1.00	1.00	0.99	0.99	0.98	0.95	0.95	0.89	0.84
Exp. vs. Wei.	0.64	0.71	0.72	0.72	0.66	0.63	0.63	0.64	0.65	0.60	0.63

Notes: The first three rows are the LR test statistics (averaged over 132 months), and the last three rows are LR test results presented as proportions of the months in which the null is rejected at the 1% significance level. The assumed density under the null is stated first in the 1st column.

Table 17: LB test results for 30 and 50 lags, AXP

δ(ticks)	2	3	4	5	6	7	8	9	10	11	12
					30 lags 1% significance level						
Exp.	0.93	0.93	0.95	0.98	0.98	0.97	0.96	0.97	0.95	0.87	0.92
Weibull	0.91	0.93	0.95	0.97	0.95	0.96	0.96	0.96	0.95	0.88	0.92
Burr	0.86	0.86	0.82	0.92	0.90	0.92	0.89	0.89	0.92	0.89	0.90
					30 lags 5% significance level						
Exp.	0.79	0.89	0.86	0.90	0.92	0.91	0.83	0.91	0.92	0.83	0.90
Weibull	0.73	0.88	0.85	0.88	0.89	0.90	0.83	0.92	0.91	0.82	0.90
Burr	0.60	0.69	0.67	0.75	0.73	0.82	0.77	0.81	0.83	0.77	0.82
					50 lags 1% significance level						
Exp.	0.89	0.95	0.98	0.97	0.98	0.96	0.94	0.98	0.95	0.87	0.91
Weibull	0.89	0.95	0.97	0.95	0.96	0.97	0.95	0.98	0.95	0.89	0.92
Burr	0.85	0.92	0.88	0.89	0.92	0.94	0.93	0.96	0.92	0.90	0.90
					50 lags 5% significance level						
Exp.	0.74	0.89	0.86	0.90	0.92	0.95	0.89	0.96	0.92	0.83	0.89
Weibull	0.73	0.88	0.83	0.88	0.89	0.95	0.89	0.94	0.91	0.85	0.89
Burr	0.65	0.75	0.77	0.79	0.80	0.88	0.85	0.83	0.86	0.80	0.85

Notes: The upper part of the table are LB test results for 30 lags, and the lower part are the results for 50 lags. Significance levels of 1% and 5% are considered. Each figure is the proportion of months in which the null is not rejected.

Table 18: DF test results, AXP

δ(ticks)	2	3	4	5	6	7	8	9	10	11	12
					1% significance level						
Exp.	0.00	0.00	0.00	0.02	0.08	0.13	0.27	0.35	0.45	0.46	0.52
Weibull	0.00	0.00	0.00	0.04	0.12	0.16	0.34	0.45	0.54	0.55	0.64
Burr	0.14	0.45	0.57	0.70	0.74	0.82	0.83	0.86	0.86	0.85	0.86
					5% significance level						
Exp.	0.00	0.00	0.00	0.01	0.02	0.06	0.16	0.20	0.30	0.30	0.40
Weibull	0.00	0.00	0.00	0.02	0.05	0.08	0.22	0.27	0.36	0.48	0.51
Burr	0.11	0.35	0.42	0.51	0.66	0.64	0.74	0.76	0.78	0.74	0.80

Notes: DF test results for significance levels of 1% and 5% are presented. Each figure is the proportion of months in which the null that the assumed density is the true density is not rejected.

Table 19: LR test results, IBM

δ(ticks)	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Wei. vs. Burr	1226.36	756.87	519.93	386.78	289.61	237.13	192.94	162.79	138.47	113.99	98.94	86.18	77.35	68.19	62.71	52.73	47.47	40.66	35.94
Exp. vs. Burr	1456.15	904.80	613.80	457.87	340.02	280.01	234.20	195.56	165.85	142.20	126.07	108.08	101.79	89.13	79.64	69.18	66.39	60.73	52.39
Exp. vs. Wei.	286.91	147.92	93.87	71.09	52.99	44.79	40.66	34.32	31.24	30.33	27.14	24.45	24.41	22.38	21.90	19.86	17.90	20.12	15.73
Wei. vs. Burr	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.97	0.99	0.98	0.93	0.92	0.84	0.85	0.75
Exp. vs. Burr	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.98	1.00	0.98	0.94	0.89	0.92	0.93	0.88
Exp. vs. Wei.	0.86	0.83	0.77	0.71	0.69	0.69	0.66	0.70	0.70	0.69	0.66	0.63	0.65	0.68	0.65	0.64	0.64	0.65	0.63

Notes: The first three rows are the LR test statistics (averaged over 132 months), and the last three rows are LR test results presented as proportions of the months in which the null is rejected at the 1% significance level. The assumed density under the null is stated first in the 1st column.

Table 20: LB test results for 30 and 50 lags, IBM

δ(ticks)	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
30 lags 1% significance level																			
Exp.	0.76	0.88	0.92	0.94	0.98	0.98	0.96	0.97	0.96	0.98	0.97	0.91	0.98	0.99	0.95	0.93	0.91	0.95	0.91
Wei.	0.72	0.87	0.89	0.90	0.97	0.98	0.94	0.95	0.98	0.99	0.97	0.93	0.97	0.95	0.87	0.89	0.91	0.89	0.86
Burr	0.68	0.80	0.83	0.81	0.89	0.92	0.89	0.89	0.92	0.92	0.92	0.94	0.94	0.91	0.95	0.92	0.91	0.91	0.88
30 lags 5% significance level																			
Exp.	0.60	0.77	0.81	0.83	0.90	0.92	0.89	0.87	0.90	0.92	0.87	0.87	0.92	0.92	0.90	0.90	0.83	0.89	0.89
Wei.	0.55	0.76	0.78	0.81	0.89	0.91	0.89	0.86	0.92	0.91	0.84	0.88	0.91	0.89	0.83	0.86	0.83	0.82	0.83
Burr	0.43	0.59	0.65	0.67	0.72	0.75	0.75	0.75	0.75	0.82	0.72	0.82	0.82	0.83	0.83	0.80	0.80	0.78	0.80
50 lags 1% significance level																			
Exp.	0.72	0.89	0.89	0.96	0.95	0.98	0.98	0.98	0.98	0.99	0.97	0.89	0.98	0.99	0.95	0.92	0.92	0.93	0.90
Wei.	0.70	0.86	0.88	0.94	0.96	0.98	0.98	0.98	0.99	1.00	0.98	0.91	0.96	0.96	0.89	0.88	0.92	0.88	0.86
Burr	0.69	0.83	0.84	0.91	0.92	0.95	0.94	0.93	0.96	0.97	0.92	0.92	0.94	0.93	0.95	0.91	0.95	0.90	0.87
50 lags 5% significance level																			
Exp.	0.58	0.71	0.80	0.86	0.91	0.92	0.91	0.90	0.93	0.96	0.86	0.87	0.95	0.95	0.90	0.90	0.89	0.89	0.87
Wei.	0.53	0.69	0.77	0.83	0.88	0.90	0.89	0.89	0.95	0.95	0.86	0.89	0.93	0.89	0.82	0.85	0.87	0.83	0.83
Burr	0.49	0.60	0.65	0.67	0.81	0.80	0.79	0.80	0.82	0.86	0.79	0.83	0.84	0.85	0.81	0.82	0.85	0.83	0.83

Notes: The upper part of the table are LB test results for 30 lags, and the lower part are the results for 50 lags. Significance levels of 1% and 5% are considered. Each figure is the proportion of months in which the null is not rejected.

Table 21: DF test results, IBM

δ(ticks)	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1% significance level																			
Exp.	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.02	0.06	0.08	0.18	0.17	0.23	0.27	0.30	0.35	0.37	0.45	0.47
Wei.	0.00	0.00	0.00	0.01	0.01	0.01	0.02	0.02	0.06	0.10	0.18	0.22	0.25	0.35	0.33	0.41	0.41	0.48	0.54
Burr	0.00	0.00	0.11	0.31	0.45	0.54	0.56	0.64	0.75	0.78	0.83	0.83	0.84	0.87	0.88	0.85	0.87	0.87	0.84
5% significance level																			
Exp.	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.02	0.02	0.03	0.07	0.11	0.13	0.17	0.23	0.25	0.25	0.36	0.33
Wei.	0.00	0.00	0.00	0.01	0.00	0.01	0.00	0.02	0.02	0.04	0.12	0.16	0.15	0.20	0.20	0.27	0.33	0.39	0.37
Burr	0.00	0.00	0.07	0.20	0.30	0.40	0.39	0.48	0.57	0.67	0.70	0.76	0.78	0.75	0.77	0.77	0.80	0.77	0.75

Notes: DF test results for significance levels of 1% and 5% are presented. Each figure is the proportion of months in which the null that the assumed density is the true density is not rejected.

Table 22: Diagnostic test results for 22 DJIA stocks

	LB30(1%)	LB30(5%)	LB50(1%)	LB50(5%)	DF(1%)	DF(5%)
HD	0.93	0.84	0.95	0.83	0.80	0.68
MCD	0.91	0.75	0.94	0.80	0.82	0.73
AXP	0.88	0.69	0.91	0.77	0.73	0.55
IBM	0.94	0.80	0.95	0.83	0.73	0.57
AA	0.90	0.75	0.92	0.80	0.80	0.70
BA	0.87	0.73	0.92	0.82	0.87	0.79
CAT	0.95	0.84	0.95	0.86	0.67	0.51
DD	0.91	0.82	0.96	0.86	0.82	0.67
DIS	0.92	0.78	0.98	0.84	0.92	0.78
GE	0.96	0.80	0.93	0.85	0.82	0.62
JNJ	0.91	0.72	0.91	0.77	0.80	0.68
JPM	0.89	0.70	0.89	0.77	0.58	0.42
KO	0.90	0.73	0.94	0.81	0.83	0.73
MMM	0.96	0.83	0.97	0.89	0.79	0.69
MRK	0.90	0.77	0.92	0.86	0.77	0.61
PG	0.92	0.73	0.94	0.80	0.77	0.63
T	0.92	0.81	0.93	0.84	0.81	0.70
UTX	0.92	0.81	0.96	0.88	0.86	0.69
WMT	0.95	0.78	0.92	0.80	0.79	0.61
XOM	0.91	0.77	0.94	0.83	0.44	0.28
INTC	0.81	0.64	0.85	0.70	0.73	0.67
MSFT	0.85	0.67	0.90	0.77	0.73	0.58
Avg.	0.91	0.76	0.93	0.81	0.77	0.63

Notes: LB and DF test results from the MLE of the HACD-Burr model in equations (13), (14) and (15). The price durations are obtained with δ^* given by the "3-times-spread" rule. Each figure in the table is the proportion of months in which the null is not rejected at the stated significance level.

Web-Appendix B: Forecasting results

Table 23: Volatility estimator means and standard deviations

		AA	AXP	BA	CAT	DD	DIS	GE	HD	IBM	JNJ	JPM	KO	MCD	MMM	MRK	PG	T	UTX	WMT	XOM	avg.
											Mean											
1	PDV	.313	.248	.219	.231	.218	.240	.241	.238	.174	.149	.277	.161	.189	.168	.204	.150	.221	.190	.174	.195	.210
2	ANP_1	.281	.230	.206	.218	.201	.216	.214	.216	.167	.136	.254	.146	.173	.159	.186	.138	.198	.178	.159	.179	.193
3	ANP_2	.292	.241	.215	.231	.209	.220	.219	.225	.172	.141	.265	.151	.180	.167	.195	.143	.205	.187	.166	.185	.200
4	NP	.279	.231	.207	.219	.202	.218	.213	.217	.167	.137	.256	.147	.174	.160	.187	.139	.198	.179	.160	.180	.193
5	PAV_1	.318	.270	.240	.259	.232	.240	.236	.246	.191	.157	.294	.168	.197	.190	.218	.159	.219	.208	.184	.207	.222
6	PAV_2	.318	.263	.235	.258	.227	.232	.234	.244	.185	.153	.289	.162	.196	.185	.216	.155	.220	.202	.182	.200	.218
7	$PABV_1$	.304	.261	.232	.253	.225	.231	.227	.238	.185	.151	.285	.161	.189	.184	.208	.153	.209	.201	.177	.201	.214
8	$PABV_2$	.312	.257	.230	.254	.223	.228	.230	.239	.182	.148	.284	.159	.190	.180	.210	.152	.214	.198	.177	.197	.213
9	RK	.321	.321	.270	.268	.240	.239	.262	.261	.232	.231	.239	.237	.237	.237	.248	.248	.191	.189	.158	.156	.239
10	$RKNP$	.322	.265	.236	.260	.228	.237	.236	.245	.185	.154	.289	.163	.198	.184	.218	.155	.223	.203	.183	.199	.219
11	$TSRV$	.292	.278	.244	.234	.215	.203	.239	.230	.209	.199	.214	.202	.219	.212	.227	.218	.172	.164	.141	.133	.212
12	SBV	.300	.248	.222	.245	.216	.219	.220	.230	.175	.142	.271	.152	.184	.175	.201	.145	.206	.191	.170	.190	.205
13	RV_5	.312	.257	.229	.252	.224	.227	.228	.238	.180	.149	.281	.158	.192	.181	.210	.151	.217	.198	.177	.195	.213
14	SRV_5	.317	.259	.233	.256	.226	.231	.231	.241	.182	.151	.284	.160	.194	.184	.213	.153	.219	.201	.179	.197	.216
15	ATM	.402	.272	.240	.267	.215	.243	.237	.244	.202	.153	.293	.159	.194	.189	.212	.150	.207	.209	.181	.193	.223
											Standard Deviation											
1	PDV	.176	.182	.108	.120	.107	.122	.161	.123	.092	.074	.188	.072	.090	.083	.095	.067	.115	.092	.081	.102	.113
2	ANP_1	.165	.178	.109	.121	.107	.119	.150	.121	.096	.075	.186	.073	.092	.085	.097	.069	.113	.094	.082	.104	.112
3	ANP_2	.173	.187	.111	.126	.112	.123	.157	.127	.100	.080	.198	.078	.096	.089	.104	.073	.121	.099	.087	.107	.117
4	NP	.166	.180	.110	.122	.108	.121	.151	.122	.097	.076	.188	.074	.093	.085	.098	.069	.115	.095	.083	.104	.113
5	PAV_1	.189	.206	.122	.139	.123	.134	.175	.139	.112	.090	.226	.088	.107	.105	.125	.085	.132	.109	.096	.120	.131
6	PAV_2	.188	.206	.120	.138	.125	.136	.176	.140	.110	.095	.226	.091	.111	.105	.126	.085	.138	.112	.099	.117	.132
7	$PABV_1$	.187	.202	.119	.139	.121	.130	.173	.136	.108	.086	.221	.085	.101	.101	.117	.081	.125	.107	.093	.118	.128
8	$PABV_2$	.189	.204	.120	.138	.125	.133	.174	.139	.109	.092	.223	.089	.107	.102	.122	.083	.134	.112	.097	.116	.130
9	RK	.189	.189	.207	.208	.122	.122	.139	.139	.125	.125	.135	.135	.176	.176	.141	.142	.113	.112	.093	.094	.144
10	$RKNP$	.188	.206	.121	.139	.125	.137	.174	.138	.108	.095	.222	.089	.110	.098	.125	.084	.137	.111	.099	.115	.131
11	$TSRV$	.178	.176	.194	.189	.113	.111	.130	.129	.118	.115	.126	.123	.166	.164	.132	.128	.105	.102	.088	.086	.134
12	SBV	.180	.193	.117	.134	.120	.127	.164	.132	.105	.088	.213	.085	.104	.100	.115	.079	.130	.106	.093	.114	.125
13	RV_5	.185	.201	.118	.136	.122	.132	.167	.135	.108	.092	.221	.087	.108	.101	.126	.082	.136	.106	.097	.119	.129
14	SRV_5	.185	.201	.120	.137	.123	.134	.170	.137	.107	.093	.222	.088	.108	.103	.125	.086	.135	.109	.097	.117	.130
15	ATM	.192	.175	.089	.101	.089	.100	.140	.104	.088	.064	.172	.065	.073	.070	.080	.059	.102	.086	.069	.075	.100

Notes: Mean and standard deviation statistics for 15 daily volatility estimators for 20 stocks using data from January 2002 to December 2012. The means and standard deviations are for annualized volatilities.

Table 24: Volatility estimator autocorrelations

		AA	AXP	BA	CAT	DD	DIS	GE	HD	IBM	JNJ	JPM	KO	MCD	MMM	MRK	PG	T	UTX	WMT	XOM	avg.
												First order										
1	PDV	.850	.888	.834	.825	.786	.828	.845	.831	.845	.812	.878	.775	.801	.748	.773	.768	.840	.789	.777	.722	.811
2	ANP_1	.791	.817	.763	.758	.705	.755	.776	.728	.760	.711	.824	.712	.660	.687	.672	.639	.751	.706	.710	.636	.728
3	ANP_2	.788	.790	.778	.772	.693	.721	.754	.701	.741	.673	.810	.665	.586	.675	.641	.604	.726	.703	.674	.598	.705
4	NP	.793	.815	.763	.741	.693	.744	.775	.724	.768	.706	.823	.711	.632	.694	.680	.639	.756	.708	.728	.645	.727
5	PAV_1	.772	.774	.754	.743	.695	.646	.734	.649	.704	.622	.760	.613	.500	.266	.470	.434	.672	.657	.633	.557	.633
6	PAV_2	.759	.684	.769	.783	.666	.564	.693	.608	.720	.511	.726	.583	.428	.333	.448	.453	.613	.626	.493	.527	.599
7	$PABV_1$	.774	.794	.746	.740	.680	.650	.726	.650	.720	.613	.756	.596	.486	.377	.511	.501	.684	.661	.585	.545	.640
8	$PABV_2$	.780	.677	.776	.777	.655	.584	.689	.550	.734	.472	.721	.560	.415	.482	.444	.417	.616	.616	.517	.520	.600
9	RK	.764	.760	.761	.746	.751	.752	.749	.760	.685	.678	.634	.624	.733	.729	.641	.630	.708	.714	.623	.601	.702
10	$RKNP$	.711	.640	.760	.747	.661	.633	.681	.617	.709	.583	.713	.671	.440	.637	.522	.564	.650	.614	.517	.628	.635
11	$TSRV$	.760	.729	.695	.636	.748	.721	.777	.738	.661	.611	.567	.484	.703	.669	.604	.556	.705	.673	.511	.433	.649
12	SBV	.783	.683	.776	.785	.732	.760	.708	.696	.726	.627	.741	.609	.438	.307	.578	.504	.656	.647	.636	.524	.646
13	RV_5	.738	.612	.764	.782	.737	.700	.713	.687	.666	.661	.721	.690	.514	.451	.343	.628	.664	.709	.674	.439	.645
14	SRV_5	.742	.666	.768	.763	.687	.609	.681	.588	.697	.553	.727	.613	.435	.279	.470	.333	.628	.612	.535	.460	.592
15	ATM	.941	.936	.926	.947	.933	.943	.936	.955	.948	.891	.925	.943	.939	.936	.908	.932	.904	.943	.942	.855	.929
												Fifth order										
1	PDV	.723	.823	.725	.730	.704	.741	.715	.747	.763	.707	.694	.666	.717	.670	.659	.667	.734	.688	.679	.694	.712
2	ANP_1	.651	.749	.652	.650	.613	.649	.648	.636	.696	.597	.585	.597	.564	.604	.544	.545	.640	.594	.595	.602	.621
3	ANP_2	.635	.711	.660	.664	.596	.602	.620	.609	.682	.545	.534	.551	.508	.590	.507	.503	.601	.585	.547	.561	.591
4	NP	.640	.750	.657	.634	.605	.640	.651	.619	.697	.587	.582	.586	.545	.601	.550	.549	.643	.604	.601	.608	.617
5	PAV_1	.623	.695	.642	.640	.588	.530	.604	.558	.651	.496	.495	.504	.430	.211	.355	.358	.548	.555	.520	.523	.526
6	PAV_2	.590	.591	.637	.664	.542	.451	.545	.530	.649	.393	.425	.464	.356	.268	.312	.360	.492	.520	.384	.469	.482
7	$PABV_1$	.625	.719	.633	.635	.584	.527	.597	.554	.664	.496	.501	.495	.425	.303	.395	.411	.575	.549	.480	.518	.534
8	$PABV_2$	.595	.590	.641	.651	.524	.467	.539	.485	.653	.372	.425	.456	.351	.404	.321	.336	.495	.507	.402	.465	.484
9	RK	.608	.601	.669	.648	.633	.629	.646	.653	.573	.562	.516	.501	.599	.590	.557	.550	.654	.656	.492	.468	.590
10	$RKNP$	.553	.546	.634	.628	.542	.514	.527	.533	.645	.462	.424	.515	.360	.553	.362	.445	.486	.511	.389	.540	.508
11	$TSRV$	.585	.549	.600	.544	.617	.583	.660	.628	.535	.504	.449	.393	.559	.518	.527	.483	.641	.608	.392	.334	.535
12	SBV	.616	.595	.649	.659	.610	.651	.544	.603	.652	.505	.441	.497	.367	.245	.415	.424	.501	.554	.500	.476	.525
13	RV_5	.545	.543	.619	.666	.588	.591	.537	.603	.622	.527	.417	.570	.405	.388	.232	.477	.491	.596	.512	.355	.514
14	SRV_5	.566	.584	.627	.638	.554	.492	.525	.509	.629	.440	.430	.488	.345	.218	.319	.256	.476	.511	.395	.391	.470
15	ATM	.836	.852	.838	.860	.833	.872	.820	.871	.814	.758	.801	.846	.845	.812	.822	.841	.809	.816	.843	.749	.827

Notes: First-order and fifth-order autocorrelation statistics for 15 daily volatility estimators for 20 stocks using data from January 2002 to December 2012.

Table 25: RMSE values, 15 individual volatility estimators, three horizons, trade data

		AA	AXP	BA	CAT	DD	DIS	GE	HD	IBM	JNJ	JPM	KO	MCD	MMM	MRK	PG	T	UTX	WMT	XOM
										one day ahead											
1	PDV	.0048	.0031	.0028	.0039	.0030	.0029	.0049	.0021	.0021	.0013	.0041	.0013	.0012	.0067	.0023	.0021	.0014	.0028	.0010	.0026
2	ANP_1	.0049	.0032	.0029	.0041	.0031	.0029	.0051	.0021	.0021	.0013	.0042	.0013	.0012	.0068	.0023	.0021	.0014	.0028	.0011	.0027
3	ANP_2	.0049	.0031	.0029	.0041	.0031	.0028	.0051	.0021	.0021	.0013	.0041	.0012	.0012	.0068	.0023	.0021	.0014	.0027	.0011	.0027
4	NP	.0050	.0033	.0029	.0041	.0032	.0029	.0053	.0021	.0021	.0013	.0043	.0013	.0012	.0069	.0023	.0021	.0015	.0028	.0011	.0028
5	PAV_1	.0051	.0031	.0029	.0042	.0030	.0027	.0051	.0021	.0021	.0013	.0041	.0012	.0012	.0075	.0024	.0022	.0015	.0028	.0011	.0027
6	PAV_2	.0050	.0031	.0030	.0042	.0030	.0027	.0052	.0021	.0021	.0014	.0041	.0012	.0012	.0074	.0024	.0022	.0015	.0028	.0011	.0028
7	$PABV_1$	.0051	.0031	.0029	.0042	.0031	.0027	.0051	.0022	.0021	.0013	.0041	.0012	.0012	.0073	.0024	.0022	.0015	.0028	.0011	.0027
8	$PABV_2$	.0049	.0031	.0030	.0041	.0030	.0027	.0052	.0021	.0021	.0014	.0041	.0012	.0012	.0071	.0025	.0021	.0015	.0028	.0011	.0028
9	RK	.0050	.0045	.0036	.0059	.0035	.0029	.0065	.0035	.0025	.0017	.0069	.0017	.0038	.0073	.0032	.0024	.0024	.0029	.0015	.0034
10	$RKNP$	.0050	.0031	.0030	.0042	.0030	.0027	.0052	.0022	.0021	.0014	.0042	.0012	.0012	.0069	.0024	.0022	.0015	.0028	.0011	.0029
11	$TSRV$	.0050	.0043	.0036	.0060	.0035	.0031	.0064	.0036	.0026	.0018	.0070	.0018	.0039	.0073	.0032	.0024	.0025	.0031	.0016	.0036
12	SBV	.0049	.0031	.0029	.0041	.0030	.0026	.0052	.0021	.0021	.0013	.0041	.0012	.0012	.0074	.0024	.0022	.0015	.0027	.0011	.0028
13	RV_5	.0051	.0032	.0030	.0042	.0030	.0027	.0052	.0022	.0021	.0014	.0042	.0012	.0012	.0072	.0024	.0022	.0015	.0028	.0011	.0029
14	SRV_5	.0050	.0031	.0029	.0041	.0030	.0027	.0052	.0022	.0021	.0014	.0041	.0012	.0012	.0075	.0024	.0025	.0015	.0028	.0011	.0029
15	ATM	.0060	.0031	.0028	.0042	.0032	.0024	.0054	.0021	.0026	.0016	.0049	.0014	.0013	.0068	.0024	.0023	.0017	.0028	.0013	.0027
										one week ahead											
1	PDV	.0188	.0121	.0105	.0148	.0117	.0109	.0201	.0075	.0090	.0051	.0152	.0051	.0041	.0285	.0089	.0083	.0051	.0113	.0037	.0110
2	ANP_1	.0195	.0130	.0110	.0157	.0127	.0113	.0214	.0079	.0094	.0055	.0163	.0053	.0043	.0290	.0094	.0086	.0055	.0119	.0041	.0115
3	ANP_2	.0194	.0130	.0110	.0159	.0124	.0109	.0213	.0080	.0093	.0055	.0164	.0052	.0043	.0290	.0095	.0086	.0056	.0117	.0041	.0117
4	NP	.0196	.0132	.0110	.0158	.0129	.0113	.0222	.0079	.0095	.0056	.0165	.0053	.0043	.0291	.0095	.0086	.0057	.0121	.0041	.0118
5	PAV_1	.0196	.0132	.0112	.0163	.0128	.0111	.0218	.0083	.0095	.0056	.0164	.0052	.0045	.0337	.0101	.0097	.0061	.0120	.0045	.0122
6	PAV_2	.0191	.0134	.0113	.0161	.0125	.0117	.0227	.0082	.0095	.0060	.0170	.0052	.0044	.0330	.0102	.0089	.0059	.0118	.0042	.0127
7	$PABV_1$	.0194	.0133	.0112	.0164	.0128	.0110	.0218	.0085	.0096	.0056	.0165	.0053	.0047	.0322	.0105	.0095	.0062	.0120	.0046	.0122
8	$PABV_2$	.0191	.0135	.0113	.0159	.0125	.0115	.0228	.0084	.0098	.0061	.0172	.0053	.0046	.0310	.0107	.0088	.0063	.0120	.0043	.0127
9	RK	.0194	.0191	.0132	.0224	.0130	.0105	.0284	.0152	.0110	.0072	.0290	.0073	.0191	.0301	.0138	.0096	.0120	.0119	.0067	.0142
10	$RKNP$	.0193	.0136	.0113	.0166	.0122	.0115	.0228	.0084	.0095	.0060	.0177	.0053	.0044	.0295	.0098	.0085	.0059	.0118	.0043	.0126
11	$TSRV$	.0191	.0181	.0129	.0222	.0130	.0120	.0279	.0160	.0111	.0074	.0299	.0082	.0192	.0303	.0137	.0097	.0123	.0126	.0072	.0147
12	SBV	.0192	.0133	.0112	.0161	.0121	.0108	.0227	.0083	.0095	.0058	.0173	.0051	.0044	.0334	.0098	.0095	.0057	.0116	.0042	.0125
13	RV_5	.0197	.0135	.0111	.0166	.0117	.0114	.0221	.0081	.0093	.0060	.0175	.0053	.0044	.0316	.0097	.0092	.0059	.0114	.0042	.0127
14	SRV_5	.0193	.0132	.0112	.0163	.0123	.0115	.0227	.0082	.0097	.0059	.0173	.0052	.0044	.0338	.0099	.0109	.0058	.0120	.0042	.0130
15	ATM	.0228	.0141	.0104	.0167	.0134	.0093	.0225	.0068	.0110	.0063	.0197	.0054	.0046	.0291	.0092	.0088	.0063	.0116	.0043	.0134
										one month ahead											
1	PDV	.0841	.0547	.0486	.0713	.0520	.0472	.0861	.0356	.0428	.0259	.0696	.0231	.0172	.1144	.0434	.0358	.0254	.0521	.0180	.0557
2	ANP_1	.0831	.0566	.0493	.0741	.0552	.0489	.0893	.0371	.0437	.0267	.0720	.0234	.0180	.1157	.0448	.0369	.0265	.0535	.0189	.0562
3	ANP_2	.0837	.0571	.0490	.0742	.0547	.0478	.0886	.0380	.0434	.0268	.0722	.0234	.0187	.1160	.0452	.0375	.0269	.0528	.0194	.0574
4	NP	.0824	.0570	.0499	.0744	.0543	.0494	.0928	.0376	.0438	.0267	.0725	.0229	.0180	.1158	.0455	.0369	.0271	.0540	.0193	.0569
5	PAV_1	.0820	.0587	.0504	.0750	.0576	.0497	.0904	.0392	.0445	.0272	.0713	.0237	.0201	.1310	.0480	.0412	.0308	.0541	.0208	.0593
6	PAV_2	.0813	.0599	.0491	.0724	.0549	.0538	.0937	.0387	.0445	.0285	.0739	.0233	.0200	.1267	.0471	.0387	.0294	.0524	.0201	.0603
7	$PABV_1$	.0806	.0584	.0506	.0753	.0579	.0493	.0912	.0414	.0449	.0273	.0720	.0238	.0216	.1263	.0499	.0407	.0315	.0537	.0218	.0597
8	$PABV_2$	.0808	.0603	.0495	.0708	.0548	.0530	.0943	.0403	.0454	.0293	.0750	.0239	.0211	.1202	.0492	.0385	.0316	.0529	.0207	.0606
9	RK	.0813	.0901	.0503	.0868	.0546	.0439	.1253	.0666	.0442	.0299	.1223	.0345	.0858	.1090	.0653	.0384	.0626	.0526	.0320	.0601
10	$RKNP$	.0817	.0589	.0488	.0739	.0526	.0531	.0947	.0386	.0435	.0282	.0775	.0230	.0204	.1164	.0439	.0369	.0285	.0524	.0197	.0611
11	$TSRV$	.0788	.0849	.0484	.0833	.0528	.0484	.1217	.0694	.0438	.0300	.1286	.0383	.0857	.1080	.0651	.0394	.0633	.0531	.0340	.0616
12	SBV	.0815	.0590	.0492	.0726	.0535	.0502	.0943	.0390	.0439	.0273	.0740	.0230	.0202	.1271	.0461	.0412	.0279	.0521	.0198	.0603
13	RV_5	.0838	.0609	.0489	.0757	.0521	.0521	.0914	.0377	.0435	.0281	.0753	.0231	.0204	.1232	.0454	.0397	.0275	.0507	.0195	.0619
14	SRV_5	.0826	.0587	.0489	.0731	.0539	.0530	.0933	.0382	.0448	.0278	.0749	.0229	.0199	.1288	.0455	.0458	.0275	.0532	.0197	.0615
15	ATM	.0916	.0693	.0507	.0799	.0603	.0473	.0945	.0408	.0459	.0299	.0931	.0253	.0201	.1154	.0418	.0347	.0263	.0542	.0219	.0617

Table 26: MCS p-values, RMSE loss function, 15 individual forecasts, three horizons, trade data

AA		AXP		BA		CAT		DD		DIS		GE		HD		IBM		JNJ		JPM		KO		MCD		MMM		MRK		PG		T		UTX		WMT		XOM	
one day ahead																																							
.00	15	.00	11	.02	11	.00	9	.00	9	.00	11	.02	11	.00	11	.00	11	.00	11	.00	11	.00	11	.00	9	.00	5	.00	9	.00	11	.00	11	.00	11	.00	11	.00	11
.01	5	.00	9	.02	9	.00	11	.00	11	.00	1	.02	9	.00	9	.00	15	.00	9	.00	9	.00	9	.00	11	.00	14	.00	11	.00	9	.00	9	.01	10	.00	9	.00	9
.01	7	.00	13	.02	6	.00	5	.00	4	.00	9	.02	4	.01	10	.00	9	.00	15	.00	15	.00	15	.00	15	.00	11	.01	6	.02	15	.00	15	.01	4	.00	15	.00	13
.02	13	.04	4	.05	8	.00	7	.00	2	.01	2	.03	15	.03	14	.02	4	.00	13	.00	4	.00	1	.00	7	.00	13	.01	8	.02	12	.00	8	.04	9	.00	14	.00	10
.02	9	.04	2	.06	10	.02	13	.02	15	.01	4	.03	14	.03	7	.04	10	.00	12	.00	2	.01	4	.00	1	.00	6	.01	10	.02	14	.01	13	.04	8	.00	7	.00	14
.09	11	.04	3	.06	5	.02	10	.02	3	.01	6	.03	6	.03	4	.04	1	.00	6	.02	12	.01	10	.00	4	.00	7	.01	15	.02	13	.01	4	.04	6	.00	13	.00	8
.16	4	.04	7	.06	12	.02	6	.02	5	.01	3	.03	10	.14	13	.04	14	.00	14	.02	13	.02	13	.00	8	.00	9	.01	13	.02	6	.13	6	.05	14	.00	6	.00	6
.22	6	.04	8	.06	14	.02	14	.02	7	.01	13	.03	12	.14	5	.04	2	.00	8	.02	10	.02	2	.34	13	.00	8	.01	14	.02	10	.13	12	.06	5	.00	10	.00	4
.34	10	.04	10	.06	7	.02	15	.06	6	.02	8	.03	3	.21	8	.09	13	.00	10	.56	8	.06	6	.34	2	.00	12	.01	5	.02	5	.13	10	.06	1	.00	5	.02	7
.47	8	.10	6	.06	13	.02	8	.06	8	.02	14	.03	13	.21	12	.09	6	.00	4	.56	3	.06	3	.57	3	.00	10	.01	7	.02	8	.13	14	.06	2	.00	12	.02	12
.47	14	.36	5	.06	4	.02	4	.13	10	.06	5	.06	2	.21	6	.09	8	.01	2	.70	6	.06	7	.61	5	.00	4	.01	4	.02	7	.13	7	.29	15	.00	8	.02	5
.58	2	.64	1	.09	2	.02	3	.13	14	.11	10	.06	8	.21	15	.39	5	.13	5	.71	14	.06	8	.84	10	.00	2	.06	3	.02	2	.13	5	.29	7	.01	4	.02	15
.66	12	.87	15	.17	3	.02	12	.37	1	.11	7	.06	5	.21	2	.61	7	.13	3	.71	5	.42	5	.84	14	.01	3	.06	12	.02	4	.13	2	.56	13	.85	2	.02	2
.66	3	.88	12	.45	1	.02	2	.60	12	.13	12	.06	7	.22	3	.68	12	.13	7	.71	7	.42	14	.84	6	.35	15	.08	2	.02	3	.13	3	.56	3	.85	3	.02	3
1	1	1	14	1	15	1	1	1	13	1	15	1	1	1	1	1	3	1	1	1	1	1	12	1	12	1	1	1	1	1	1	1	1	1	12	1	1	1	1
one week ahead																																							
.03	15	.00	9	.01	9	.00	9	.00	5	.02	6	.00	9	.00	11	.00	11	.00	11	.00	9	.00	9	.00	9	.02	5	.00	9	.02	11	.00	11	.00	5	.00	11	.00	11
.13	5	.00	11	.01	11	.00	11	.00	7	.02	10	.00	11	.00	9	.00	15	.00	9	.00	11	.00	11	.00	11	.02	7	.00	11	.02	7	.00	9	.00	4	.00	9	.00	15
.15	4	.00	8	.01	8	.00	5	.01	4	.02	14	.00	10	.00	8	.00	9	.00	10	.00	13	.02	10	.00	7	.02	14	.00	7	.02	9	.00	5	.00	7	.00	5	.00	9
.35	2	.00	10	.01	6	.00	13	.01	9	.02	8	.00	14	.00	7	.00	7	.00	6	.00	15	.02	2	.00	8	.02	6	.00	8	.02	12	.00	7	.00	11	.00	7	.00	8
.35	13	.02	13	.02	12	.00	7	.01	15	.02	11	.00	8	.00	10	.01	14	.00	12	.00	8	.02	8	.00	15	.02	12	.00	6	.02	13	.00	8	.03	2	.00	8	.00	13
.35	3	.02	6	.02	5	.00	10	.01	11	.02	4	.00	13	.00	5	.01	8	.00	14	.00	14	.02	4	.00	5	.02	13	.00	10	.03	5	.00	13	.14	8	.00	6	.00	6
.35	7	.02	12	.02	14	.00	14	.01	2	.02	7	.00	15	.02	6	.01	5	.00	15	.00	6	.02	15	.00	13	.02	8	.00	12	.03	14	.00	15	.14	14	.00	14	.00	12
.35	14	.02	15	.02	7	.00	6	.01	14	.02	13	.00	12	.02	12	.01	6	.00	13	.00	12	.02	13	.00	14	.02	11	.00	5	.03	6	.00	6	.14	6	.00	10	.00	14
.35	9	.02	5	.05	13	.00	12	.01	8	.02	5	.00	4	.03	14	.01	10	.00	8	.00	10	.02	5	.00	10	.02	9	.00	14	.07	8	.00	10	.14	3	.00	13	.00	10
.35	10	.02	7	.05	10	.00	15	.01	6	.02	2	.00	5	.03	3	.04	4	.00	7	.00	5	.02	6	.00	12	.02	10	.00	4	.30	15	.00	12	.32	9	.00	15	.00	5
.35	12	.02	4	.05	3	.00	3	.01	3	.52	1	.00	6	.13	4	.12	2	.00	5	.00	4	.02	7	.00	3	.05	4	.00	3	.30	4	.00	4	.32	10	.00	3	.00	7
.73	8	.02	2	.05	2	.00	2	.01	12	.52	12	.00	2	.27	13	.12	12	.05	2	.00	7	.11	3	.00	4	.05	3	.01	13	.30	2	.00	14	.35	12	.00	4	.00	4
.73	6	.02	3	.05	4	.00	4	.14	10	.52	3	.00	7	.27	2	.12	3	.05	3	.01	2	.53	14	.00	6	.18	2	.01	2	.30	3	.00	3	.80	15	.00	2	.00	3
.73	11	.02	14	.77	1	.02	8	.91	13	.52	9	.00	3	.33	1	.13	13	.05	4	.02	3	.83	12	.00	2	.53	15	.57	15	.56	10	.00	2	.80	13	.01	12	.01	2
1	1	1	1	1	15	1	1	1	1	1	15	1	1	1	15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
one month ahead																																							
.11	3	.01	9	.03	7	.00	9	.00	5	.01	10	.02	9	.00	11	.35	7	.00	15	.00	9	.00	9	.00	9	.00	7	.00	11	.01	7	.00	11	.03	5	.00	11	.00	13
.11	15	.01	11	.03	5	.00	15	.00	7	.01	6	.02	11	.00	9	.39	5	.00	8	.01	11	.00	11	.00	11	.00	5	.00	9	.01	5	.00	9	.04	7	.00	9	.00	14
.11	13	.01	15	.11	15	.00	13	.00	15	.01	8	.02	10	.00	8	.39	8	.00	11	.01	15	.01	5	.00	12	.00	6	.03	7	.01	14	.00	7	.04	4	.00	7	.00	10
.11	14	.01	8	.11	4	.01	11	.00	2	.01	12	.02	8	.00	7	.39	15	.00	6	.01	8	.01	8	.00	8	.00	14	.04	8	.01	13	.00	5	.04	2	.00	8	.00	8
.11	2	.01	13	.11	8	.01	5	.00	6	.01	14	.02	12	.00	5	.39	6	.02	9	.03	14	.01	15	.00	13	.00	12	.04	5	.01	12	.00	8	.63	15	.00	5	.00	7
.11	5	.01	6	.11	9	.03	3	.00	3	.01	5	.02	6	.00	12	.39	14	.02	10	.03	10	.01	7	.00	10	.00	13	.04	12	.01	8	.01	6	.63	3	.00	6	.00	15
.11	1	.03	12	.69	6	.03	7	.00	8	.01	13	.02	14	.00	15	.39	12	.02	13	.03	6	.01	3	.00	7	.00	8	.04	4	.01	6	.01	10	.66	14	.00	15	.00	6
.11	4	.03	14	.69	12	.03	2	.00	9	.01	7	.02	15	.00	3	.39	9	.02	5	.03	13	.01	2	.00	14	.00	10	.09	6	.01	11	.01	12	.66	8	.00	12	.00	12
.11	12	.07	5	.78	2	.03	10	.03	14	.01	4	.02	4	.00	6	.39	4	.02	7	.03	12	.01	6	.00	6	.02	3	.10	3	.01	3	.01	14	.68	11	.00	3	.00	5
.13	6	.07	10	.85	3	.03	4	.03	4	.01	2	.02	13	.00	10	.52	2	.02	14	.03	4	.40	13	.00	5	.05	15	.17	13	.01	9	.01	13	.68	6	.00	14	.00	11
.13	10	.19	3	.86	14	.06	12	.03	12	.22	11	.02	7	.00	14	.72	13	.02	3	.06	3	.73	1	.00	15	.05	4	.29	2	.01	10	.01	3	.68	10	.00	13	.00	9
.37	9	.19	7	.86	13	.07	14	.81	11	.56	3	.03	5	.00	4	.72	10	.05	12	.15	7	.73	10	.00	3	.08	2	.29	14	.01	4	.01	4	.68	9	.00	4	.00	3
.37	8	.19	2	.88	10	.11	6	.81	10	.60	1	.07	2	.03	13	.72	11	.05	4	.15	2	.73	12	.04	2	.54	1	.67	10	.01	2	.01	2	.68	12	.00	10	.00	4
.37	7	.19	4	.88	1	.68	1	.82	13	.60	15	.07	3	.03	2	.72	3	.05	2	.20	5	.82	4	.06	4	.54	9	.67	1	.57	1	.49	15	.68	1	.00	2	.31	2
1	11	1	1	1	11	1	8	1	1	1	9	1	1	1	1	1	1	1	1	1	1	1	14	1	1	1	11	1	15	1	15	1	1	1	13	1	1	1	1

Notes: MCS p-values for the RMSE loss function. In each stock sub-panel we report the corresponding p-values on the left and on the right we list the estimator numbers ranked in the ascending orders of the p-values. Estimators 1 to 15 are: PDV, ANP_1, ANP_2, NP, PAV_1, PAV_2, $PABV_1$, $PABV_2$, RK, RKNP, TSRV, SBV, RV_5, SRV_5 and ATM.

Table 27: QLIKE values, 15 individual volatility estimators, three horizons, trade data

		AA	AXP	BA	CAT	DD	DIS	GE	HD	IBM	JNJ	JPM	KO	MCD	MMM	MRK	PG	T	UTX	WMT	XOM
											one day ahead										
1	PDV	.0275	.0278	.0288	.0279	.0293	.0345	.0317	.0257	.0326	.0295	.0306	.0321	.0323	.0367	.0269	.0304	.0267	.0336	.0273	.0283
2	ANP_1	.0278	.0288	.0297	.0287	.0308	.0344	.0337	.0262	.0325	.0307	.0316	.0321	.0318	.0386	.0279	.0315	.0276	.0341	.0278	.0293
3	ANP_2	.0280	.0281	.0298	.0287	.0298	.0319	.0337	.0258	.0316	.0297	.0309	.0306	.0315	.0385	.0281	.0311	.0276	.0334	.0278	.0289
4	NP	.0284	.0294	.0297	.0289	.0315	.0346	.0359	.0265	.0331	.0309	.0319	.0326	.0320	.0394	.0285	.0316	.0291	.0348	.0283	.0305
5	PAV_1	.0286	.0283	.0302	.0295	.0295	.0310	.0329	.0263	.0321	.0300	.0304	.0302	.0317	.0414	.0284	.0322	.0284	.0340	.0289	.0299
6	PAV_2	.0282	.0282	.0309	.0294	.0290	.0311	.0336	.0260	.0313	.0311	.0306	.0301	.0313	.0410	.0286	.0318	.0283	.0334	.0286	.0297
7	$PABV_1$	.0289	.0286	.0301	.0294	.0298	.0309	.0331	.0268	.0319	.0297	.0304	.0305	.0326	.0410	.0289	.0325	.0284	.0336	.0295	.0304
8	$PABV_2$	.0279	.0282	.0308	.0290	.0290	.0308	.0334	.0261	.0312	.0309	.0307	.0300	.0317	.0405	.0290	.0318	.0288	.0333	.0289	.0297
9	RK	.0285	.0462	.0388	.0425	.0360	.0342	.0601	.0438	.0417	.0411	.0761	.0487	.0917	.0486	.0446	.0400	.0492	.0350	.0412	.0442
10	$RKNP$	.0282	.0290	.0315	.0297	.0295	.0309	.0334	.0269	.0323	.0315	.0315	.0308	.0316	.0398	.0292	.0329	.0286	.0335	.0287	.0305
11	$TSRV$	.0284	.0451	.0385	.0431	.0363	.0374	.0593	.0451	.0428	.0435	.0756	.0512	.0912	.0485	.0440	.0405	.0491	.0383	.0429	.0467
12	SBV	.0277	.0281	.0304	.0286	.0289	.0300	.0336	.0262	.0309	.0308	.0311	.0294	.0311	.0415	.0282	.0321	.0281	.0327	.0284	.0294
13	RV_5	.0288	.0291	.0307	.0294	.0289	.0310	.0337	.0263	.0316	.0316	.0313	.0306	.0314	.0411	.0289	.0331	.0289	.0327	.0287	.0304
14	SRV_5	.0280	.0279	.0304	.0292	.0291	.0306	.0338	.0264	.0314	.0311	.0309	.0297	.0313	.0415	.0285	.0333	.0285	.0331	.0286	.0298
15	ATM	.0337	.0277	.0286	.0307	.0334	.0276	.0371	.0278	.0417	.0358	.0360	.0360	.0337	.0380	.0305	.0369	.0335	.0343	.0333	.0325
											one week ahead										
1	PDV	.0199	.0194	.0197	.0192	.0223	.0257	.0240	.0173	.0286	.0235	.0217	.0248	.0206	.0379	.0199	.0276	.0194	.0273	.0186	.0234
2	ANP_1	.0206	.0211	.0207	.0204	.0246	.0268	.0269	.0184	.0293	.0254	.0236	.0256	.0210	.0392	.0216	.0291	.0210	.0286	.0204	.0244
3	ANP_2	.0207	.0208	.0206	.0206	.0235	.0248	.0266	.0186	.0285	.0246	.0233	.0247	.0210	.0393	.0221	.0286	.0210	.0276	.0204	.0245
4	NP	.0209	.0214	.0207	.0204	.0251	.0269	.0293	.0185	.0297	.0256	.0236	.0254	.0214	.0395	.0219	.0292	.0223	.0293	.0209	.0255
5	PAV_1	.0206	.0211	.0211	.0213	.0240	.0248	.0267	.0193	.0291	.0251	.0227	.0242	.0219	.0433	.0228	.0303	.0227	.0285	.0220	.0258
6	PAV_2	.0201	.0214	.0210	.0210	.0229	.0253	.0270	.0187	.0280	.0261	.0235	.0239	.0212	.0426	.0226	.0284	.0216	.0264	.0208	.0257
7	$PABV_1$	.0204	.0212	.0212	.0213	.0241	.0246	.0268	.0201	.0291	.0249	.0225	.0244	.0230	.0427	.0237	.0307	.0229	.0281	.0228	.0262
8	$PABV_2$	.0198	.0216	.0209	.0204	.0227	.0248	.0267	.0191	.0284	.0261	.0237	.0242	.0218	.0413	.0232	.0284	.0224	.0263	.0211	.0257
9	RK	.0204	.0366	.0271	.0310	.0264	.0227	.0532	.0355	.0348	.0346	.0629	.0395	.0819	.0454	.0392	.0360	.0427	.0274	.0315	.0373
10	$RKNP$	.0204	.0218	.0208	.0216	.0228	.0249	.0270	.0191	.0287	.0260	.0248	.0246	.0213	.0402	.0226	.0283	.0215	.0257	.0209	.0263
11	$TSRV$	.0201	.0350	.0263	.0309	.0262	.0257	.0518	.0369	.0357	.0363	.0625	.0420	.0814	.0450	.0384	.0364	.0421	.0299	.0332	.0388
12	SBV	.0200	.0214	.0207	.0208	.0228	.0238	.0271	.0189	.0279	.0256	.0243	.0235	.0211	.0426	.0221	.0292	.0211	.0260	.0205	.0255
13	RV_5	.0209	.0221	.0206	.0217	.0222	.0253	.0273	.0185	.0279	.0260	.0247	.0241	.0214	.0421	.0224	.0303	.0219	.0255	.0204	.0265
14	SRV_5	.0202	.0210	.0206	.0211	.0230	.0248	.0270	.0188	.0281	.0256	.0241	.0235	.0210	.0422	.0221	.0301	.0212	.0262	.0207	.0259
15	ATM	.0239	.0202	.0183	.0218	.0257	.0187	.0302	.0163	.0354	.0285	.0247	.0261	.0227	.0382	.0215	.0310	.0238	.0258	.0209	.0298
											one month ahead										
1	PDV	.0212	.0212	.0230	.0237	.0257	.0292	.0298	.0202	.0380	.0330	.0235	.0275	.0203	.0544	.0271	.0378	.0235	.0335	.0205	.0366
2	ANP_1	.0210	.0225	.0234	.0246	.0276	.0303	.0317	.0214	.0378	.0344	.0250	.0276	.0209	.0558	.0283	.0397	.0247	.0344	.0215	.0373
3	ANP_2	.0212	.0224	.0230	.0245	.0269	.0287	.0310	.0218	.0367	.0342	.0245	.0269	.0214	.0559	.0285	.0400	.0246	.0335	.0219	.0380
4	NP	.0211	.0226	.0238	.0247	.0271	.0308	.0341	.0217	.0380	.0341	.0250	.0268	.0208	.0558	.0287	.0395	.0259	.0350	.0220	.0378
5	PAV_1	.0207	.0228	.0238	.0249	.0278	.0287	.0313	.0219	.0374	.0343	.0232	.0268	.0226	.0611	.0299	.0410	.0274	.0341	.0226	.0389
6	PAV_2	.0206	.0232	.0228	.0241	.0262	.0302	.0316	.0218	.0361	.0349	.0243	.0256	.0226	.0578	.0290	.0398	.0254	.0315	.0224	.0392
7	$PABV_1$	.0198	.0224	.0236	.0246	.0277	.0281	.0315	.0229	.0373	.0340	.0228	.0266	.0237	.0593	.0309	.0408	.0278	.0331	.0232	.0389
8	$PABV_2$	.0201	.0229	.0226	.0231	.0258	.0294	.0311	.0223	.0361	.0350	.0242	.0259	.0234	.0559	.0298	.0400	.0267	.0312	.0228	.0389
9	RK	.0206	.0408	.0247	.0280	.0274	.0236	.0616	.0334	.0342	.0359	.0616	.0424	.0774	.0509	.0447	.0409	.0481	.0328	.0314	.0432
10	$RKNP$	.0208	.0226	.0224	.0244	.0251	.0301	.0320	.0216	.0359	.0344	.0256	.0255	.0232	.0558	.0278	.0394	.0255	.0308	.0222	.0411
11	$TSRV$	.0201	.0387	.0236	.0268	.0259	.0247	.0594	.0345	.0335	.0357	.0618	.0442	.0764	.0493	.0446	.0421	.0470	.0328	.0326	.0441
12	SBV	.0205	.0222	.0225	.0239	.0256	.0288	.0315	.0217	.0354	.0337	.0245	.0251	.0226	.0573	.0287	.0415	.0249	.0317	.0220	.0393
13	RV_5	.0215	.0238	.0226	.0253	.0251	.0298	.0315	.0210	.0360	.0343	.0250	.0254	.0231	.0578	.0292	.0413	.0244	.0305	.0218	.0410
14	SRV_5	.0210	.0223	.0224	.0243	.0259	.0298	.0313	.0215	.0361	.0340	.0247	.0250	.0224	.0581	.0284	.0423	.0246	.0314	.0220	.0401
15	ATM	.0221	.0248	.0226	.0246	.0272	.0254	.0336	.0227	.0379	.0393	.0261	.0288	.0234	.0529	.0276	.0360	.0239	.0303	.0250	.0386

Table 28: MCS p-values, QLIKE loss function, 15 individual forecasts, three horizons, trade data

AA		AXP		BA		CAT		DD		DIS		GE		HD		IBM		JNJ		JPM		KO		MCD		MMM		MRK		PG		T		UTX		WMT		XOM	
one day ahead																																								
1	11	1	1	1	11	1	8	1	1	1	9	1	1	1	1	1	1	1	1	1																			
.00	15	.00	11	.00	11	.00	11	.00	11	.00	1	.00	11	.00	11	.00	11	.00	11	.00	11	.00	11	.00	9	.00	11	.00	9	.00	9	.00	11	.00	11	.00	11	.00	11
.00	7	.00	9	.00	9	.00	9	.00	9	.00	11	.00	9	.00	9	.00	9	.00	9	.00	9	.00	9	.00	11	.00	9	.00	11	.00	11	.00	15	.01	4	.00	9	.00	9
.00	13	.00	4	.00	6	.00	10	.00	4	.00	2	.00	4	.01	10	.00	15	.00	15	.00	15	.00	15	.04	15	.00	7	.00	10	.00	15	.00	9	.01	5	.00	15	.00	15
.00	5	.00	10	.00	10	.00	5	.00	15	.00	4	.00	15	.02	15	.00	4	.00	13	.00	2	.00	4	.04	7	.00	5	.00	8	.00	13	.00	8	.01	9	.00	7	.00	4
.00	9	.00	13	.00	13	.00	6	.00	2	.00	9	.00	14	.02	7	.00	2	.00	10	.00	4	.00	1	.04	1	.00	13	.00	15	.00	10	.01	13	.01	2	.00	14	.00	7
.06	4	.00	2	.00	8	.00	7	.04	5	.00	3	.00	3	.02	4	.00	10	.00	14	.02	12	.00	2	.04	4	.00	8	.00	13	.00	14	.02	4	.01	15	.00	6	.00	13
.15	11	.25	7	.00	14	.00	13	.04	7	.00	6	.00	2	.03	14	.22	1	.00	4	.02	10	.00	10	.04	2	.00	10	.00	7	.00	7	.02	6	.01	7	.00	8	.00	10
.15	10	.52	8	.00	12	.00	15	.31	3	.01	13	.00	6	.03	5	.22	5	.00	6	.18	13	.00	13	.04	8	.00	6	.00	6	.00	5	.02	10	.01	6	.00	5	.00	5
.15	6	.52	5	.00	5	.00	14	.42	10	.01	5	.00	12	.03	2	.22	3	.00	2	.18	14	.03	7	.60	5	.00	14	.00	4	.00	6	.02	14	.01	10	.01	10	.00	8
.29	14	.52	12	.00	7	.02	8	.71	14	.01	8	.00	8	.32	13	.36	7	.08	8	.18	3	.12	3	.60	10	.00	4	.00	14	.00	12	.02	7	.01	1	.01	13	.00	14
.75	8	.52	3	.00	3	.07	4	.85	1	.07	7	.00	13	.32	12	.36	14	.08	12	.50	8	.12	6	.75	3	.01	12	.00	5	.00	2	.02	12	.01	8	.01	4	.00	2
.77	2	.52	6	.00	2	.07	3	.85	6	.07	10	.01	10	.37	6	.36	13	.43	5	.50	6	.12	5	.77	13	.01	2	.01	3	.00	8	.03	5	.02	3	.03	12	.02	6
.77	3	.83	14	.00	4	.08	2	.85	8	.07	14	.06	7	.40	8	.36	6	.57	7	.70	1	.12	8	.77	14	.03	3	.02	12	.00	4	.03	2	.02	14	.13	2	.03	12
.77	12	.83	1	.74	1	.08	12	.85	12	.17	12	.06	5	.69	3	.36	8	.57	3	.70	7	.33	14	.77	6	.44	15	.02	2	.00	3	.03	3	.83	13	.13	3	.10	3
1	1	1	15	1	15	1	1	1	13	1	15	1	1	1	1	1	12	1	1	1	5	1	12	1	12	1	1	1	1	1	1	1	1	1	12	1	1	1	1
one week ahead																																								
.01	15	.00	11	.00	9	.00	9	.00	9	.00	4	.00	9	.00	11	.00	11	.00	11	.00	11	.00	11	.00	9	.00	7	.00	9	.00	9	.00	11	.00	4	.00	11	.00	9
.06	4	.00	9	.00	11	.00	11	.00	4	.00	2	.00	11	.00	9	.00	15	.00	9	.00	9	.00	9	.00	11	.00	5	.00	11	.00	11	.00	9	.00	5	.00	9	.00	11
.06	5	.00	13	.00	6	.00	5	.00	5	.00	6	.00	4	.00	7	.01	9	.00	6	.00	13	.01	4	.07	7	.00	11	.00	8	.00	13	.00	7	.00	2	.00	5	.00	15
.15	13	.00	10	.07	8	.00	10	.00	7	.00	1	.00	15	.00	8	.01	4	.00	15	.00	14	.01	2	.08	15	.00	9	.00	7	.00	7	.00	5	.00	11	.00	7	.00	6
.15	3	.00	8	.07	5	.00	13	.00	11	.00	13	.00	13	.00	5	.01	2	.00	10	.00	12	.01	15	.08	8	.00	13	.00	6	.00	5	.00	8	.00	7	.00	8	.00	13
.20	2	.00	6	.07	14	.00	7	.00	2	.00	8	.00	14	.06	10	.01	10	.00	14	.00	10	.01	10	.08	5	.00	8	.00	5	.02	14	.00	15	.05	3	.00	6	.00	8
.20	7	.00	4	.07	7	.00	6	.14	15	.00	11	.00	10	.06	6	.05	7	.00	13	.00	8	.01	3	.08	4	.00	6	.00	10	.10	15	.00	13	.10	1	.00	4	.00	14
.20	9	.00	12	.07	12	.00	14	.14	14	.02	7	.00	12	.06	12	.05	5	.00	12	.00	2	.01	7	.59	10	.00	12	.00	13	.10	4	.00	4	.39	9	.00	14	.00	5
.44	10	.00	5	.07	13	.00	12	.14	3	.02	5	.00	2	.10	4	.12	3	.00	8	.01	15	.03	1	.59	13	.00	14	.00	3	.10	2	.00	6	.39	8	.00	10	.00	7
.60	14	.00	2	.07	3	.00	15	.14	12	.07	10	.00	8	.10	3	.12	8	.00	4	.01	4	.03	5	.59	6	.00	10	.00	12	.12	12	.00	10	.39	6	.00	2	.00	12
.60	11	.01	7	.07	2	.00	3	.24	6	.07	14	.00	6	.10	14	.77	1	.00	5	.01	3	.03	8	.59	12	.00	4	.00	4	.48	3	.00	2	.55	14	.00	3	.00	4
.60	6	.01	14	.07	10	.00	2	.60	10	.16	3	.00	7	.19	2	.77	14	.02	2	.01	6	.14	13	.59	3	.00	2	.00	14	.62	6	.00	12	.55	12	.09	15	.00	10
.76	12	.01	3	.07	4	.00	4	.60	8	.16	12	.00	3	.28	13	.77	6	.02	7	.01	5	.14	6	.59	2	.00	3	.00	2	.67	8	.00	14	.55	15	.09	13	.00	2
.86	1	.55	15	.33	1	.01	8	.80	1	.16	9	.00	5	.51	1	.85	12	.06	3	.01	7	.83	14	.59	14	.82	15	.30	15	.67	10	.00	3	.55	10	.09	12	.00	3
1	8	1	1	1	15	1	1	1	13	1	15	1	1	1	15	1	13	1	1	1	1	1	12	1	1	1	1	1	1	1	1	1	1	1	13	1	1	1	1
one month ahead																																							
.11	13	.01	9	.00	7	.01	9	.00	5	.01	6	.01	9	.01	11	.05	5	.00	15	.00	11	.00	9	.00	9	.00	7	.00	11	.03	14	.00	11	.03	5	.00	11	.00	13
.11	3	.01	11	.00	5	.01	13	.00	7	.01	10	.01	11	.01	9	.05	7	.00	8	.00	9	.00	11	.00	11	.00	5	.00	9	.03	5	.00	9	.03	7	.00	9	.00	10
.11	2	.02	13	.05	4	.02	5	.00	2	.01	8	.01	4	.01	8	.10	4	.00	6	.01	10	.00	5	.03	8	.00	14	.01	7	.06	13	.00	7	.03	4	.00	8	.01	11
.11	14	.02	6	.05	9	.02	11	.00	3	.01	5	.01	10	.01	7	.10	15	.03	9	.01	14	.02	15	.03	10	.00	6	.01	8	.06	12	.00	5	.29	2	.00	15	.01	14
.37	4	.02	15	.05	2	.02	4	.00	9	.01	13	.01	15	.01	15	.10	2	.03	11	.01	13	.02	3	.03	12	.00	13	.01	6	.06	7	.00	8	.59	3	.00	7	.01	9
.37	15	.02	8	.05	3	.02	7	.01	4	.01	12	.01	7	.01	3	.10	1	.03	5	.01	4	.02	2	.03	13	.00	12	.02	5	.06	11	.00	6	.69	1	.00	6	.01	6
.37	5	.02	5	.05	11	.04	3	.01	6	.01	14	.07	12	.01	6	.10	6	.03	10	.02	2	.02	7	.03	15	.00	10	.02	13	.06	8	.00	10	.69	9	.00	5	.01	12
.37	10	.02	4	.05	6	.04	10	.11	15	.01	4	.07	6	.01	5	.10	3	.03	3	.02	15	.06	1	.03	6	.00	3	.02	4	.06	6	.06	4	.69	11	.00	12	.01	5
.53	1	.02	10	.66	1	.04	2	.11	14	.01	7	.07	13	.01	12	.10	14	.03	2	.02	6	.06	4	.03	7	.01	8	.27	3	.06	3	.06	12	.69	12	.00	3	.01	8
.53	6	.02	3	.66	8	.04	14	.38	8	.01	2	.07	2	.01	4	.10	13	.03	13	.02	8	.06	8	.03	14	.01	2	.27	12	.08	9	.06	14	.69	6	.00	4	.01	7
.54	12	.02	12	.74	13	.04	6	.38	11	.32	3	.10	5	.01	10	.10	8	.03	4	.02	3	.06	6	.03	3	.01	4	.30	14	.08	10	.06	2	.69	14	.03	14	.01	15
.54	9	.02	2	.75	15	.04	15	.38	12	.36	1	.10	14	.01	2	.10	10	.61	7	.35	12	.37	13	.24	5	.14	15	.33	2	.08	4	.18	3	.69	8	.03	13	.01	3
.77	11	.39	14	.75	12	.04	12	.71	1	.36	15	.13	3	.23	14	.10	12	.61	14	.52	1	.37	10	.33	2	.14	1	.49	10	.08	2	.63	13	.69	10	.03	10	.01	4
.77	8	.41	7	.87	10	.47	1	.86	10	.36	11	.13	8	.23	13	.10	9	.61	12	.52	5	.63	12	.33	4	.14	9	.66	15	.55	1	.72	15	.84	13	.03	2	.32	2
1	7	1	1	1	14	1	8	1	13	1	9	1	1	1	1	1	11	1	1	1	7	1	14	1	1	1	11	1	1	1	15	1	1	1	15	1	1	1	1

Notes: MCS p-values for the QLIKE loss function. In each stock sub-panel we report the corresponding p-values on the left and on the right we list the estimator numbers ranked in the ascending orders of the p-values. Estimators 1 to 15 are: PDV, ANP_1, ANP_2, NP, PAV_1, PAV_2, $PABV_1$, $PABV_2$, RK, RKNP, TSRV, SBV, RV_5, SRV_5 and ATM.

Table 29: RMSE values, 14 combination volatility estimators, three horizons, trade data

		AA	AXP	BA	CAT	DD	DIS	GE	HD	IBM	JNJ	JPM	KO	MCD	MMM	MRK	PG	T	UTX	WMT	XOM
	one day ahead																				
1	PDV	.0048	.0031	.0028	.0039	.0030	.0029	.0049	.0021	.0021	.0013	.0041	.0013	.0012	.0067	.0023	.0021	.0014	.0028	.0010	.0026
2	SBV	.0049	.0031	.0029	.0041	.0030	.0026	.0052	.0021	.0021	.0013	.0041	.0012	.0012	.0074	.0024	.0022	.0015	.0027	.0011	.0028
3	ATM	.0060	.0031	.0028	.0042	.0032	.0024	.0054	.0021	.0026	.0016	.0049	.0014	.0013	.0068	.0024	.0023	.0017	.0028	.0013	.0027
4	$PDV + SBV$	.0047	.0030	.0028	.0039	.0029	.0027	.0050	.0021	.0020	.0013	.0040	.0012	.0012	.0069	.0023	.0021	.0014	.0027	.0010	.0026
5	$PDV + ATM$	.0046	.0027	.0025	.0036	.0027	.0022	.0047	.0018	.0021	.0013	.0038	.0011	.0011	.0065	.0021	.0020	.0014	.0024	.0010	.0024
6	$SBV + ATM$	.0046	.0028	.0026	.0037	.0027	.0022	.0048	.0018	.0020	.0013	.0039	.0011	.0011	.0068	.0022	.0020	.0014	.0025	.0010	.0025
7	$Duration4$	.0048	.0031	.0028	.0040	.0031	.0028	.0051	.0021	.0021	.0013	.0041	.0012	.0012	.0068	.0023	.0021	.0014	.0028	.0010	.0027
8	$SBV + Duration4$	.0048	.0031	.0028	.0040	.0030	.0027	.0051	.0021	.0021	.0013	.0041	.0012	.0012	.0068	.0023	.0021	.0014	.0027	.0010	.0027
9	$ATM + Duration4$	.0046	.0030	.0027	.0039	.0029	.0026	.0049	.0020	.0020	.0013	.0039	.0012	.0012	.0067	.0022	.0020	.0014	.0026	.0010	.0026
10	$RV10$	.0049	.0030	.0028	.0041	.0029	.0026	.0051	.0021	.0021	.0013	.0041	.0012	.0013	.0070	.0023	.0021	.0015	.0027	.0011	.0028
11	$PDV + RV10$	.0049	.0030	.0028	.0041	.0029	.0026	.0050	.0021	.0020	.0013	.0041	.0012	.0013	.0069	.0023	.0021	.0014	.0027	.0010	.0028
12	$ATM + RV10$	.0048	.0029	.0028	.0041	.0029	.0025	.0050	.0020	.0020	.0013	.0040	.0012	.0013	.0069	.0023	.0021	.0014	.0026	.0010	.0028
13	$Duration4 + RV10$	.0048	.0030	.0028	.0041	.0029	.0026	.0050	.0021	.0020	.0013	.0040	.0012	.0012	.0069	.0023	.0021	.0014	.0027	.0010	.0027
14	All	.0048	.0029	.0028	.0040	.0029	.0025	.0050	.0020	.0020	.0013	.0040	.0011	.0012	.0068	.0023	.0021	.0014	.0026	.0010	.0027
	one week ahead																				
1	PDV	.0188	.0121	.0105	.0148	.0117	.0109	.0201	.0075	.0090	.0051	.0152	.0051	.0041	.0285	.0089	.0083	.0051	.0113	.0037	.0110
2	SBV	.0192	.0133	.0112	.0161	.0121	.0108	.0227	.0083	.0095	.0058	.0173	.0051	.0044	.0334	.0098	.0095	.0057	.0116	.0042	.0125
3	ATM	.0228	.0141	.0104	.0167	.0134	.0093	.0225	.0068	.0110	.0063	.0197	.0054	.0046	.0291	.0092	.0088	.0063	.0116	.0043	.0134
4	$PDV + SBV$	.0186	.0126	.0107	.0154	.0117	.0103	.0211	.0078	.0090	.0054	.0162	.0050	.0041	.0297	.0093	.0087	.0053	.0112	.0039	.0116
5	$PDV + ATM$	.0186	.0121	.0095	.0147	.0110	.0084	.0198	.0060	.0090	.0053	.0154	.0044	.0040	.0279	.0082	.0079	.0052	.0103	.0033	.0115
6	$SBV + ATM$	.0185	.0130	.0103	.0155	.0115	.0092	.0208	.0069	.0090	.0056	.0167	.0048	.0041	.0297	.0088	.0083	.0054	.0109	.0037	.0119
7	$Duration4$	.0190	.0128	.0108	.0155	.0123	.0110	.0211	.0077	.0092	.0054	.0160	.0052	.0042	.0289	.0093	.0085	.0054	.0117	.0040	.0115
8	$SBV + Duration4$	.0189	.0128	.0108	.0156	.0122	.0108	.0213	.0078	.0092	.0054	.0162	.0051	.0042	.0291	.0094	.0086	.0055	.0115	.0040	.0116
9	$ATM + Duration4$	.0185	.0125	.0104	.0153	.0117	.0100	.0206	.0071	.0091	.0054	.0154	.0049	.0041	.0285	.0089	.0083	.0054	.0112	.0038	.0114
10	$RV10$	.0190	.0132	.0107	.0161	.0120	.0108	.0224	.0084	.0095	.0059	.0174	.0053	.0054	.0306	.0099	.0089	.0062	.0115	.0043	.0128
11	$PDV + RV10$	.0189	.0131	.0107	.0159	.0119	.0106	.0221	.0082	.0093	.0058	.0172	.0052	.0051	.0302	.0098	.0088	.0060	.0114	.0042	.0125
12	$ATM + RV10$	.0187	.0131	.0106	.0158	.0118	.0105	.0220	.0081	.0093	.0058	.0171	.0052	.0051	.0302	.0097	.0087	.0060	.0113	.0042	.0126
13	$Duration4 + RV10$	.0189	.0130	.0106	.0158	.0120	.0105	.0219	.0080	.0092	.0056	.0169	.0051	.0047	.0296	.0097	.0087	.0058	.0114	.0041	.0122
14	All	.0187	.0129	.0105	.0156	.0118	.0103	.0216	.0079	.0091	.0056	.0166	.0051	.0046	.0294	.0095	.0086	.0058	.0113	.0040	.0121
	one month ahead																				
1	PDV	.0841	.0547	.0486	.0713	.0520	.0472	.0861	.0356	.0428	.0259	.0696	.0231	.0172	.1144	.0434	.0358	.0254	.0521	.0180	.0557
2	SBV	.0815	.0590	.0492	.0726	.0535	.0502	.0943	.0390	.0439	.0273	.0740	.0230	.0202	.1271	.0461	.0412	.0279	.0521	.0198	.0603
3	ATM	.0916	.0693	.0507	.0799	.0603	.0473	.0945	.0408	.0459	.0299	.0931	.0253	.0201	.1154	.0418	.0347	.0263	.0542	.0219	.0617
4	$PDV + SBV$	.0812	.0561	.0483	.0718	.0520	.0469	.0887	.0370	.0424	.0264	.0713	.0225	.0180	.1169	.0445	.0378	.0263	.0511	.0187	.0575
5	$PDV + ATM$	.0851	.0586	.0473	.0722	.0517	.0421	.0878	.0355	.0420	.0272	.0735	.0219	.0174	.1130	.0407	.0337	.0247	.0508	.0184	.0577
6	$SBV + ATM$	.0803	.0616	.0485	.0732	.0537	.0469	.0894	.0387	.0418	.0275	.0765	.0232	.0189	.1162	.0420	.0356	.0256	.0516	.0197	.0587
7	$Duration4$	.0823	.0562	.0491	.0733	.0539	.0481	.0887	.0369	.0433	.0265	.0714	.0231	.0178	.1154	.0446	.0367	.0263	.0530	.0188	.0565
8	$SBV + Duration4$	.0818	.0564	.0489	.0731	.0535	.0477	.0891	.0372	.0431	.0265	.0716	.0229	.0181	.1158	.0447	.0372	.0265	.0525	.0189	.0570
9	$ATM + Duration4$	.0816	.0569	.0483	.0729	.0528	.0460	.0881	.0367	.0427	.0268	.0708	.0226	.0177	.1146	.0432	.0357	.0258	.0521	.0188	.0567
10	$RV10$	.0808	.0615	.0467	.0713	.0536	.0496	.0956	.0408	.0437	.0281	.0772	.0247	.0263	.1175	.0476	.0386	.0322	.0520	.0217	.0606
11	$PDV + RV10$	.0806	.0606	.0466	.0711	.0533	.0486	.0943	.0400	.0433	.0277	.0763	.0243	.0246	.1165	.0472	.0382	.0313	.0517	.0212	.0599
12	$ATM + RV10$	.0799	.0616	.0466	.0710	.0533	.0489	.0942	.0406	.0431	.0279	.0764	.0245	.0250	.1164	.0467	.0379	.0312	.0516	.0214	.0602
13	$Duration4 + RV10$	.0810	.0595	.0469	.0714	.0533	.0478	.0929	.0390	.0431	.0273	.0750	.0238	.0223	.1155	.0466	.0378	.0299	.0519	.0206	.0590
14	All	.0804	.0596	.0468	.0712	.0531	.0474	.0922	.0390	.0428	.0272	.0744	.0237	.0218	.1150	.0460	.0374	.0294	.0516	.0205	.0589

Table 30: MCS p-values, RMSE loss function, 14 combination forecasts, three horizons, trade data

AA		AXP		BA		CAT		DD		DIS		GE		HD		IBM		JNJ		JPM		KO		MCD		MMM		MRK		PG		T		UTX		WMT		XOM	
one day ahead																																							
.00	3	.00	2	.00	2	.00	2	.00	7	.00	1	.00	3	.00	10	.00	3	.02	2	.00	3	.00	3	.00	10	.01	11	.00	3	.00	3	.00	3	.00	7	.00	3	.00	2
.00	10	.00	7	.00	8	.00	3	.00	3	.00	7	.00	13	.00	11	.00	1	.02	3	.00	2	.00	1	.00	3	.01	10	.00	2	.01	10	.00	10	.00	8	.00	2	.00	13
.00	2	.00	8	.00	7	.00	13	.01	13	.00	8	.00	2	.00	2	.00	7	.02	10	.00	13	.00	7	.00	11	.01	12	.00	10	.01	11	.00	2	.00	1	.00	10	.00	10
.00	11	.00	1	.00	4	.00	10	.01	2	.00	10	.00	11	.00	13	.00	2	.02	11	.00	10	.00	10	.00	1	.01	13	.00	7	.01	2	.00	11	.00	2	.00	1	.00	11
.00	7	.00	10	.00	10	.00	7	.01	8	.00	11	.00	7	.00	7	.00	8	.02	12	.00	11	.00	13	.00	12	.01	2	.00	8	.01	13	.00	8	.00	3	.00	11	.00	3
.00	13	.00	13	.00	13	.00	11	.01	11	.00	13	.00	10	.00	1	.00	10	.02	4	.02	7	.00	11	.00	2	.01	7	.00	11	.03	12	.00	6	.01	4	.00	4	.01	14
.00	1	.00	11	.00	11	.00	8	.01	10	.00	2	.01	8	.00	3	.00	5	.02	6	.02	8	.00	4	.00	7	.01	8	.00	4	.04	4	.00	7	.01	10	.01	13	.01	12
.06	12	.00	4	.00	1	.01	4	.02	4	.00	4	.01	14	.00	12	.00	11	.02	8	.02	4	.00	8	.00	13	.01	14	.00	13	.04	14	.00	12	.01	13	.01	8	.01	7
.06	8	.00	3	.00	12	.01	1	.02	14	.00	12	.01	12	.00	14	.00	4	.02	7	.05	14	.00	14	.00	8	.01	4	.01	12	.04	8	.00	4	.01	11	.02	12	.01	8
.07	14	.00	12	.00	3	.01	14	.02	1	.00	14	.01	4	.00	8	.00	9	.02	13	.05	1	.00	12	.00	4	.01	3	.01	14	.04	7	.00	13	.01	9	.02	7	.07	9
.07	4	.00	14	.00	14	.01	12	.02	12	.00	9	.01	9	.01	4	.00	13	.34	14	.06	12	.00	2	.02	9	.02	1	.01	1	.05	6	.78	14	.01	14	.06	6	.07	4
.85	6	.00	9	.00	9	.01	9	.02	9	.01	3	.02	1	.01	9	.20	6	.73	1	.45	9	.01	9	.05	14	.03	9	.02	9	.14	1	.78	9	.01	12	.14	14	.07	1
.85	5	.00	6	.00	6	.03	6	.15	6	.67	5	.02	6	.02	6	.20	12	.73	9	.45	6	.23	6	.24	5	.03	6	.02	6	.20	9	.78	5	.01	6	.51	9	.07	6
1	9	1	5	1	5	1	5	1	5	1	6	1	5	1	5	1	14	1	5	1	5	1	5	1	6	1	5	1	5	1	5	1	1	1	5	1	5	1	5
one week ahead																																							
.00	3	.00	10	.00	2	.01	2	.00	3	.00	2	.00	3	.00	10	.00	3	.00	2	.00	3	.00	10	.00	10	.02	10	.00	2	.00	10	.00	10	.00	7	.00	10	.00	3
.01	7	.00	11	.00	8	.01	3	.00	2	.00	1	.00	13	.00	12	.00	10	.00	10	.00	10	.00	11	.00	12	.02	11	.00	10	.00	11	.00	13	.00	8	.00	11	.04	2
.01	2	.00	2	.00	7	.01	10	.01	13	.00	7	.00	14	.00	11	.00	11	.00	3	.00	11	.00	12	.00	11	.02	2	.00	11	.00	2	.00	11	.00	2	.00	12	.04	12
.01	8	.00	12	.00	13	.01	13	.01	7	.00	10	.00	10	.00	13	.00	12	.00	13	.00	12	.00	7	.00	13	.02	13	.00	12	.00	12	.00	12	.00	13	.00	13	.04	14
.01	10	.00	13	.00	10	.01	11	.01	10	.00	13	.00	2	.00	14	.00	2	.00	14	.00	2	.00	13	.00	2	.02	12	.00	13	.00	13	.00	14	.00	10	.00	14	.04	10
.01	13	.00	14	.00	11	.01	8	.01	8	.00	8	.00	11	.00	2	.00	13	.01	12	.00	13	.00	3	.00	3	.02	6	.00	8	.00	4	.00	2	.00	1	.00	2	.04	13
.02	11	.00	3	.00	4	.01	12	.01	14	.00	11	.00	8	.00	8	.03	8	.01	11	.00	14	.00	2	.00	14	.02	14	.00	14	.00	14	.00	3	.00	11	.00	3	.04	11
.80	14	.00	8	.00	14	.01	7	.01	11	.00	12	.00	12	.00	7	.03	7	.01	6	.01	4	.00	8	.00	8	.02	8	.00	7	.00	3	.00	8	.00	14	.00	7	.04	6
.82	1	.00	7	.00	12	.01	14	.01	12	.00	4	.00	7	.00	4	.46	14	.01	8	.01	8	.00	14	.00	7	.02	4	.02	4	.02	8	.00	7	.00	3	.00	8	.04	8
.82	12	.03	6	.00	6	.01	9	.01	9	.00	14	.01	4	.00	1	.75	9	.01	4	.01	6	.00	1	.00	9	.02	7	.02	3	.02	7	.01	9	.00	9	.01	4	.04	9
.88	5	.05	4	.00	9	.01	6	.01	4	.00	9	.01	6	.00	9	.79	1	.09	7	.01	7	.00	4	.02	1	.05	3	.02	9	.11	1	.01	6	.00	12	.01	9	.04	4
.88	4	.05	9	.00	1	.01	4	.01	1	.00	6	.01	9	.00	6	.84	6	.12	9	.40	9	.00	9	.09	4	.05	9	.02	6	.11	6	.01	4	.00	4	.01	1	.04	5
.88	9	.75	1	.03	3	.69	1	.01	6	.22	3	.31	1	.06	3	.84	5	.16	5	.61	5	.00	6	.09	6	.13	1	.06	1	.20	9	.43	5	.00	6	.01	6	.04	7
1	6	1	5	1	5	1	5	1	5	1	5	1	5	1	5	1	4	1	1	1	1	1	5	1	5	1	5	1	5	1	5	1	1	1	5	1	5	1	1
one month ahead																																							
.01	3	.01	10	.02	2	.01	3	.03	3	.00	2	.11	10	.00	12	.04	3	.02	3	.00	3	.00	12	.00	10	.00	2	.07	10	.01	12	.00	13	.09	7	.00	12	.04	2
.01	7	.01	12	.09	3	.01	9	.03	2	.00	10	.11	12	.00	10	.21	2	.02	12	.00	12	.00	10	.00	12	.00	10	.08	12	.01	10	.00	10	.17	3	.00	10	.04	12
.01	1	.01	3	.09	8	.01	7	.03	10	.00	12	.11	2	.00	11	.21	10	.02	10	.00	10	.00	11	.00	11	.00	6	.08	11	.01	2	.00	11	.17	8	.00	11	.04	3
.01	8	.01	11	.09	7	.01	8	.03	7	.00	11	.11	13	.00	14	.23	13	.02	14	.00	11	.03	14	.00	2	.00	12	.08	2	.01	11	.00	14	.17	10	.00	14	.04	10
.01	5	.01	13	.09	9	.55	6	.03	13	.00	13	.11	11	.00	13	.23	7	.03	6	.00	2	.03	13	.02	13	.00	11	.17	13	.01	14	.00	12	.17	9	.00	13	.04	14
.01	2	.01	6	.09	6	.55	2	.07	8	.00	7	.11	14	.00	6	.23	8	.03	11	.00	14	.14	3	.02	14	.00	4	.19	14	.01	13	.00	2	.17	13	.00	3	.04	11
.01	4	.01	2	.60	4	.70	5	.07	12	.00	8	.11	3	.00	2	.35	11	.03	13	.01	13	.14	7	.02	3	.00	8	.22	8	.01	4	.00	8	.28	1	.00	2	.04	6
.01	9	.02	14	.69	1	.70	13	.07	14	.00	14	.11	8	.00	3	.35	12	.03	5	.08	6	.14	1	.05	6	.09	13	.22	7	.01	8	.00	4	.28	2	.00	6	.04	13
.01	13	.15	5	.69	5	.70	4	.24	11	.00	6	.11	6	.00	8	.35	9	.03	2	.08	5	.14	2	.05	8	.09	7	.22	4	.01	7	.00	7	.28	14	.00	9	.04	4
.16	10	.15	8	.69	13	.70	10	.24	6	.04	4	.11	4	.00	9	.55	14	.03	8	.08	8	.14	6	.11	4	.49	14	.22	1	.01	6	.00	3	.28	11	.00	8	.04	5
.16	11	.15	9	.69	14	.83	14	.60	9	.05	1	.13	7	.00	7	.66	1	.05	9	.08	4	.44	8	.11	9	.49	3	.22	9	.13	9	.00	9	.65	12	.00	7	.04	8
.18	14	.15	4	.69	10	.83	1	.84	4	.05	3	.13	9	.02	4	.69	4	.05	7	.08	7	.53	9	.11	7	.49	9	.22	6	.21	1	.00	6	.65	6	.33	4	.04	9
.79	6	.15	7	.85	11	.83	11	.84	1	.05	9	.26	5	.89	1	.75	5	.13	4	.30	9	.53	4	.55	5	.49	1	.40	3	.40	3	.27	1	.73	4	.48	5	.04	7
1	12	1	1	1	12	1	12	1	5	1	5	1	1	1	5	1	6	1	1	1	1	1	5	1	1	1	5	1	5	1	5	1	5	1	5	1	1	1	1

Notes: MCS p-values for the RMSE loss function. In each stock sub-panel we report the corresponding p-values on the left and on the right we list the estimator combination numbers ranked in the ascending orders of the p-values. Combinations 1 to 14 are: PDV; SBV; ATM; PDV+SBV; PDV+ATM; SBV+ATM; four duration-based estimators, estimators 1 to 4, called Duration4; SBV+Duration4; ATM+Duration4; ten RV-type estimators, estimators 5 to 14, called RV10; PDV+RV10; ATM+RV10; Duration4+RV10; All.

Table 31: QLIKE values, 14 combination volatility estimators, three horizons, trade data

		AA	AXP	BA	CAT	DD	DIS	GE	HD	IBM	JNJ	JPM	KO	MCD	MMM	MRK	PG	T	UTX	WMT	XOM
											one day ahead										
1	PDV	.0275	.0278	.0288	.0279	.0293	.0345	.0317	.0257	.0326	.0295	.0306	.0321	.0323	.0367	.0269	.0304	.0267	.0336	.0273	.0283
2	SBV	.0277	.0281	.0304	.0286	.0289	.0300	.0336	.0262	.0309	.0308	.0311	.0294	.0311	.0415	.0282	.0321	.0281	.0327	.0284	.0294
3	ATM	.0337	.0277	.0286	.0307	.0334	.0276	.0371	.0278	.0417	.0358	.0360	.0360	.0337	.0380	.0305	.0369	.0335	.0343	.0333	.0325
4	$PDV + SBV$	.0268	.0274	.0292	.0279	.0283	.0305	.0322	.0254	.0305	.0295	.0303	.0294	.0307	.0384	.0272	.0305	.0269	.0322	.0274	.0283
5	$PDV + ATM$	.0268	.0246	.0253	.0257	.0264	.0252	.0300	.0228	.0320	.0294	.0290	.0265	.0299	.0335	.0256	.0288	.0264	.0287	.0262	.0271
6	$SBV + ATM$	.0263	.0252	.0273	.0263	.0269	.0254	.0311	.0234	.0301	.0295	.0291	.0277	.0294	.0366	.0262	.0297	.0270	.0299	.0271	.0276
7	$Duration4$	.0272	.0283	.0293	.0284	.0300	.0334	.0332	.0257	.0321	.0299	.0310	.0314	.0316	.0381	.0276	.0308	.0274	.0337	.0275	.0290
8	$SBV + Duration4$	.0270	.0279	.0293	.0282	.0294	.0320	.0331	.0256	.0314	.0297	.0307	.0305	.0312	.0383	.0276	.0307	.0273	.0331	.0275	.0288
9	$ATM + Duration4$	.0262	.0267	.0277	.0272	.0281	.0295	.0315	.0241	.0310	.0294	.0294	.0288	.0307	.0364	.0266	.0295	.0267	.0314	.0266	.0282
10	$RV10$	.0277	.0277	.0295	.0290	.0284	.0296	.0337	.0259	.0306	.0298	.0316	.0302	.0343	.0387	.0282	.0309	.0281	.0319	.0281	.0298
11	$PDV + RV10$	.0275	.0275	.0292	.0287	.0283	.0295	.0333	.0256	.0304	.0295	.0314	.0298	.0330	.0383	.0279	.0307	.0276	.0318	.0278	.0295
12	$ATM + RV10$	.0271	.0271	.0290	.0284	.0279	.0286	.0328	.0253	.0299	.0294	.0307	.0297	.0330	.0380	.0276	.0304	.0276	.0312	.0277	.0294
13	$Duration4 + RV10$	.0274	.0275	.0289	.0285	.0286	.0298	.0333	.0253	.0304	.0293	.0311	.0296	.0315	.0380	.0277	.0305	.0272	.0321	.0275	.0293
14	All	.0270	.0271	.0285	.0281	.0281	.0290	.0326	.0249	.0300	.0291	.0305	.0291	.0309	.0375	.0273	.0301	.0270	.0315	.0272	.0290
											one week ahead										
1	PDV	.0199	.0194	.0197	.0192	.0223	.0257	.0240	.0173	.0286	.0235	.0217	.0248	.0206	.0379	.0199	.0276	.0194	.0273	.0186	.0234
2	SBV	.0200	.0214	.0207	.0208	.0228	.0238	.0271	.0189	.0279	.0256	.0243	.0235	.0211	.0426	.0221	.0292	.0211	.0260	.0205	.0255
3	ATM	.0239	.0202	.0183	.0218	.0257	.0187	.0302	.0163	.0354	.0285	.0247	.0261	.0227	.0382	.0215	.0310	.0238	.0258	.0209	.0298
4	$PDV + SBV$	.0193	.0201	.0199	.0198	.0219	.0234	.0252	.0178	.0273	.0241	.0229	.0232	.0200	.0398	.0208	.0277	.0199	.0259	.0192	.0240
5	$PDV + ATM$	.0194	.0177	.0166	.0176	.0198	.0172	.0236	.0133	.0278	.0231	.0196	.0191	.0192	.0350	.0178	.0251	.0187	.0223	.0162	.0237
6	$SBV + ATM$	.0190	.0194	.0183	.0190	.0209	.0187	.0252	.0153	.0269	.0241	.0215	.0215	.0196	.0382	.0194	.0258	.0196	.0234	.0181	.0245
7	$Duration4$	.0200	.0206	.0203	.0201	.0237	.0258	.0263	.0180	.0288	.0245	.0230	.0249	.0208	.0389	.0212	.0284	.0207	.0280	.0199	.0243
8	$SBV + Duration4$	.0199	.0205	.0202	.0201	.0232	.0248	.0263	.0180	.0283	.0245	.0231	.0242	.0205	.0393	.0213	.0282	.0206	.0273	.0198	.0243
9	$ATM + Duration4$	.0193	.0195	.0190	.0191	.0219	.0222	.0251	.0162	.0277	.0240	.0213	.0225	.0202	.0374	.0199	.0267	.0201	.0258	.0186	.0239
10	$RV10$	.0199	.0210	.0198	.0206	.0221	.0230	.0276	.0190	.0273	.0248	.0246	.0243	.0254	.0392	.0226	.0280	.0221	.0255	.0205	.0254
11	$PDV + RV10$	.0197	.0207	.0195	.0204	.0220	.0228	.0271	.0186	.0271	.0244	.0243	.0238	.0240	.0389	.0223	.0278	.0216	.0254	.0201	.0251
12	$ATM + RV10$	.0195	.0205	.0194	.0202	.0215	.0220	.0269	.0183	.0266	.0244	.0237	.0237	.0241	.0386	.0219	.0274	.0215	.0248	.0199	.0251
13	$Duration4 + RV10$	.0198	.0206	.0195	.0202	.0223	.0230	.0270	.0182	.0272	.0243	.0240	.0236	.0222	.0387	.0220	.0278	.0211	.0259	.0200	.0248
14	All	.0195	.0203	.0192	.0199	.0218	.0222	.0265	.0177	.0268	.0240	.0233	.0232	.0216	.0383	.0215	.0273	.0208	.0253	.0196	.0246
											one month ahead										
1	PDV	.0221	.0224	.0243	.0250	.0272	.0306	.0315	.0211	.0402	.0349	.0248	.0290	.0214	.0575	.0286	.0399	.0247	.0354	.0216	.0386
2	SBV	.0216	.0235	.0238	.0253	.0271	.0304	.0334	.0229	.0374	.0356	.0259	.0266	.0239	.0607	.0303	.0439	.0263	.0335	.0232	.0416
3	ATM	.0232	.0261	.0237	.0260	.0285	.0264	.0355	.0239	.0398	.0404	.0275	.0303	.0244	.0558	.0285	.0380	.0252	.0319	.0260	.0406
4	$PDV + SBV$	.0214	.0227	.0237	.0251	.0267	.0295	.0321	.0218	.0381	.0349	.0252	.0270	.0219	.0584	.0292	.0416	.0253	.0339	.0222	.0400
5	$PDV + ATM$	.0219	.0229	.0225	.0237	.0253	.0249	.0316	.0203	.0379	.0367	.0238	.0254	.0209	.0552	.0268	.0362	.0235	.0313	.0222	.0385
6	$SBV + ATM$	.0208	.0239	.0229	.0243	.0262	.0272	.0323	.0225	.0362	.0363	.0246	.0265	.0224	.0567	.0279	.0384	.0243	.0311	.0236	.0398
7	$Duration4$	.0219	.0233	.0246	.0257	.0283	.0312	.0332	.0222	.0396	.0357	.0258	.0286	.0219	.0586	.0296	.0414	.0259	.0360	.0225	.0395
8	$SBV + Duration4$	.0217	.0232	.0243	.0256	.0278	.0306	.0331	.0223	.0390	.0356	.0257	.0279	.0221	.0586	.0297	.0417	.0259	.0353	.0226	.0398
9	$ATM + Duration4$	.0214	.0232	.0238	.0250	.0271	.0289	.0325	.0218	.0387	.0363	.0248	.0271	.0216	.0576	.0287	.0396	.0252	.0342	.0226	.0391
10	$RV10$	.0215	.0248	.0226	.0243	.0270	.0289	.0353	.0230	.0366	.0352	.0276	.0290	.0286	.0558	.0317	.0420	.0291	.0331	.0239	.0416
11	$PDV + RV10$	.0214	.0245	.0225	.0243	.0269	.0287	.0348	.0226	.0367	.0350	.0273	.0286	.0270	.0557	.0313	.0417	.0285	.0332	.0236	.0413
12	$ATM + RV10$	.0211	.0247	.0225	.0241	.0267	.0283	.0347	.0229	.0363	.0352	.0269	.0287	.0274	.0555	.0310	.0411	.0284	.0326	.0238	.0413
13	$Duration4 + RV10$	.0215	.0242	.0228	.0245	.0272	.0289	.0345	.0224	.0372	.0350	.0270	.0282	.0251	.0562	.0310	.0416	.0277	.0337	.0233	.0408
14	All	.0213	.0241	.0227	.0243	.0269	.0285	.0341	.0223	.0369	.0351	.0265	.0280	.0245	.0560	.0305	.0410	.0273	.0333	.0233	.0406

Table 32: MCS p-values, QLIKE loss function, 14 combination forecasts, three horizons, trade data

AA		AXP		BA		CAT		DD		DIS		GE		HD		IBM		JNJ		JPM		KO		MCD		MMM		MRK		PG		T		UTX		WMT		XOM	
one day ahead																																								
.00	3	.00	7	.00	2	.00	3	.00	3	.00	1	.00	3	.00	10	.00	3	.00	3	.00	3	.00	3	.00	10	.00	10	.00	3	.00	3	.00	3	.00	7	.00	3	.00	3
.00	10	.00	1	.00	10	.00	13	.00	8	.00	7	.00	2	.00	3	.00	7	.00	2	.00	10	.00	1	.00	3	.00	11	.00	10	.00	2	.00	2	.00	1	.00	10	.00	10
.00	2	.00	2	.00	11	.00	2	.00	7	.00	13	.00	13	.00	11	.00	1	.00	8	.00	13	.00	7	.00	11	.00	12	.00	2	.00	10	.00	10	.00	8	.00	2	.00	11
.00	11	.00	8	.00	8	.00	10	.00	13	.00	8	.00	8	.00	7	.00	8	.00	7	.00	11	.00	10	.00	1	.00	13	.00	7	.00	11	.00	11	.00	4	.00	11	.00	7
.00	13	.00	10	.00	13	.00	11	.00	2	.00	4	.00	7	.00	13	.00	5	.00	10	.00	2	.00	13	.00	12	.00	7	.00	11	.00	13	.00	7	.00	3	.00	12	.00	2
.00	7	.00	13	.00	7	.00	7	.00	10	.00	11	.00	10	.00	8	.00	2	.00	4	.00	7	.00	11	.00	7	.00	2	.00	8	.00	8	.00	12	.00	2	.00	7	.00	13
.00	1	.00	11	.00	12	.00	14	.00	4	.00	2	.00	11	.00	2	.00	9	.00	11	.00	12	.00	8	.00	2	.00	8	.00	13	.00	7	.00	8	.00	13	.00	8	.00	
.00	12	.00	4	.00	4	.00	1	.00	1	.00	10	.00	14	.00	12	.00	10	.00	6	.00	1	.00	12	.00	13	.00	14	.00	12	.04	12	.00	13	.00	11	.00	13	.00	
.00	8	.00	12	.00	14	.00	8	.00	11	.00	14	.00	4	.00	1	.00	13	.00	1	.00	14	.00	14	.00	8	.00	4	.00	4	.06	4	.50	6	.00	10	.00	1	.01	14
.03	14	.00	14	.00	1	.00	12	.02	14	.00	12	.00	12	.00	14	.00	4	.00	5	.00	4	.00	4	.00	4	.00	3	.00	14	.07	14	.64	4	.00	14	.00	4	.04	1
.07	4	.00	3	.00	3	.00	4	.02	9	.00	9	.01	9	.00	4	.00	11	.00	12	.00	8	.00	2	.01	9	.05	1	.01	1	.07	1	.64	14	.00	9	.00	14	.05	9
.47	5	.00	9	.00	9	.00	9	.07	12	.05	3	.01	1	.00	9	.72	6	.00	9	.52	9	.00	9	.03	14	.05	9	.04	9	.07	6	.64	9	.00	12	.00	6	.09	4
.70	6	.00	6	.00	6	.03	6	.28	6	.66	6	.01	6	.00	6	.72	14	.00	13	.67	6	.02	6	.22	5	.05	6	.17	6	.13	9	.64	1	.00	6	.29	9	.14	6
1	9	1	5	1	5	1	5	1	5	1	5	1	5	1	5	1	12	1	14	1	5	1	5	1	6	1	5	1	5	1	5	1	5	1	5	1	5	1	5
one week ahead																																								
.00	3	.00	10	.00	2	.00	2	.00	3	.00	1	.00	3	.00	10	.00	3	.00	2	.00	10	.00	7	.00	10	.00	10	.00	10	.00	2	.00	3	.00	7	.00	10	.00	3
.00	7	.00	11	.00	7	.00	10	.00	2	.00	2	.00	2	.00	11	.00	8	.00	3	.00	11	.00	10	.00	12	.00	2	.00	11	.00	3	.00	10	.00	1	.00	11	.00	10
.00	8	.00	13	.00	8	.00	3	.00	8	.00	7	.00	13	.00	13	.00	7	.00	8	.00	13	.00	1	.00	11	.00	11	.00	12	.00	10	.00	13	.00	8	.00	7	.00	12
.00	13	.00	2	.00	13	.00	13	.00	13	.00	8	.00	10	.00	12	.00	2	.00	7	.00	2	.00	11	.00	3	.00	13	.00	2	.00	11	.00	2	.00	9	.00	13	.00	11
.00	10	.00	12	.00	10	.00	11	.00	7	.00	10	.00	11	.00	14	.00	1	.00	10	.00	12	.00	3	.00	13	.00	12	.00	13	.00	7	.00	11	.00	2	.00	8	.00	13
.00	2	.00	14	.00	4	.00	8	.00	10	.00	13	.00	14	.00	2	.00	9	.01	11	.00	14	.00	13	.00	2	.00	7	.00	14	.00	13	.00	12	.00	13	.00	12	.00	2
.00	11	.00	8	.00	11	.00	14	.03	11	.00	4	.00	8	.00	7	.00	5	.01	12	.00	3	.00	2	.01	7	.00	8	.00	8	.02	8	.00	7	.00	4	.00	2	.00	14
.48	1	.00	7	.00	14	.00	7	.04	4	.00	11	.00	7	.00	8	.00	13	.01	4	.00	8	.00	14	.01	14	.00	14	.00	7	.05	4	.00	8	.00	10	.00	14	.00	6
.48	14	.00	4	.00	12	.00	12	.04	14	.00	14	.00	12	.00	4	.00	10	.01	13	.00	4	.00	8	.01	8	.04	4	.00	4	.05	12	.02	14	.00	11	.00	3	.00	7
.56	12	.00	3	.00	1	.00	4	.04	1	.00	12	.00	4	.00	1	.00	4	.05	9	.00	7	.00	12	.04	1	.04	6	.00	3	.12	1	.02	9	.00	3	.00	9	.02	8
.72	5	.00	9	.00	9	.00	9	.04	12	.00	9	.00	9	.00	9	.00	11	.05	6	.00	1	.00	4	.04	9	.04	3	.00	9	.12	14	.02	4	.00	14	.00	4	.10	9
.72	9	.00	1	.00	6	.00	6	.04	9	.00	6	.00	6	.00	3	.60	6	.47	14	.00	6	.00	9	.39	4	.04	9	.00	1	.33	9	.02	6	.00	12	.00	1	.25	4
.72	4	.00	6	.04	3	.02	1	.04	6	.41	3	.48	1	.00	6	.60	14	.47	1	.01	9	.00	6	.44	6	.05	1	.00	6	.35	6	.23	1	.00	6	.00	6	.55	5
1	6	1	5	1	5	1	5	1	5	1	5	1	5	1	5	1	12	1	5	1	5	1	5	1	5	1	5	1	5	1	5	1	5	1	5	1	5	1	1
one month ahead																																								
.00	7	.00	10	.03	2	.08	7	.02	7	.00	2	.03	10	.00	10	.02	7	.04	3	.00	10	.00	10	.00	10	.01	2	.05	10	.04	2	.00	13	.07	7	.01	12	.19	2
.00	1	.00	12	.15	7	.08	8	.11	8	.00	7	.03	13	.00	3	.02	8	.04	6	.00	11	.00	11	.00	12	.01	8	.05	12	.04	10	.00	14	.07	8	.01	10	.37	10
.00	3	.00	11	.15	8	.55	2	.37	3	.00	8	.03	11	.00	2	.02	3	.04	5	.00	12	.03	12	.01	11	.02	7	.10	11	.04	11	.00	10	.18	1	.03	3	.37	12
.00	8	.01	14	.31	9	.55	9	.37	13	.00	1	.03	12	.00	12	.12	1	.04	9	.00	13	.03	13	.03	2	.02	9	.14	13	.04	13	.00	11	.18	9	.03	11	.37	11
.00	5	.01	13	.47	1	.55	3	.46	2	.00	13	.03	14	.00	13	.12	9	.06	7	.00	14	.04	3	.03	3	.02	6	.16	14	.04	12	.00	12	.18	13	.03	14	.37	13
.00	13	.04	3	.47	4	.55	4	.46	14	.00	10	.03	3	.00	6	.12	4	.13	8	.01	3	.04	1	.05	13	.14	4	.17	2	.04	14	.00	2	.41	4	.03	13	.37	14
.11	2	.04	6	.58	3	.55	1	.46	9	.00	4	.03	2	.00	8	.12	13	.13	2	.01	2	.04	14	.33	14	.52	1	.21	7	.04	8	.00	8	.41	14	.03	2	.37	4
.11	4	.04	2	.58	6	.55	13	.46	10	.00	11	.04	8	.00	14	.19	2	.13	14	.01	8	.08	7	.33	8	.52	13	.23	8	.04	7	.03	7	.41	11	.03	9	.37	3
.11	10	.04	9	.58	13	.55	14	.51	11	.00	14	.05	7	.00	7	.19	5	.13	12	.01	7	.24	8	.33	6	.52	14	.23	4	.04	4	.04	4	.41	10	.03	6	.37	8
.11	9	.04	8	.58	14	.55	10	.51	1	.00	12	.09	9	.01	11	.19	14	.34	10	.14	4	.32	9	.42	7	.52	10	.23	1	.04	9	.06	9	.58	2	.03	8	.37	6
.11	14	.04	7	.58	10	.55	11	.51	12	.01	9	.09	6	.04	9	.19	11	.86	13	.14	9	.49	4	.42	4	.65	11	.23	9	.04	1	.06	3	.58	12	.03	7	.37	7
.11	11	.53	5	.81	11	.55	6	.51	4	.01	6	.09	4	.16	4	.19	10	.87	11	.14	1	.53	6	.42	9	.76	3	.23	3	.04	6	.06	1	.74	3	.45	4	.55	9
.62	12	.53	4	.90	5	.55	12	.51	6	.52	3	.81	5	.29	1	.81	12	.87	4	.14	6	.53	2	.42	1	.76	12	.23	6	.37	3	.06	6	.74	5	.47	5	.78	1
1	6	1	1	1	12	1	5	1	5	1	5	1	1	1	5	1	6	1	1	1	5	1	5	1	5	1	5	1	5	1	5	1	5	1	6	1	1	1	5

Notes: MCS p-values for the QLIKE loss function. In each stock sub-panel we report the corresponding p-values on the left and on the right we list the estimator combination numbers ranked in the ascending orders of the p-values. Combinations 1 to 14 are: PDV; SBV; ATM; PDV+SBV; PDV+ATM; SBV+ATM; four duration-based estimators, estimators 1 to 4, called Duration4; SBV+Duration4; ATM+Duration4; ten RV-type estimators, estimators 5 to 14, called RV10; PDV+RV10; ATM+RV10; Duration4+RV10; All.

Web-Appendix C: Proofs of Main Results

Proof of Theorem 1. In the 'continuous case', our non-parametric estimator $NPDV_t$ can be viewed as the realized volatility estimator with respect to a particular stochastic sampling on regular grids, where the "barriers" are always equidistant and symmetric.

We shall derive the quadratic variation of the process U_n defined as

$$U_n := \delta_n^{-1}\Big(NPDV_t - [X,X]_t\Big) \equiv \delta_n^{-1}\Bigg\{\sum_{j=0}^{N_t}\Big(X_{\tau_{n,j+1}} - X_{\tau_{n,j}}\Big)^2 - [X,X]_t\Bigg\}.$$

We first note that the limit of the sample third moment, the *tricity*:

$$\delta_n{}^{-1}\sum_{j=0}^{N_{n,t}}\Big(X_{\tau_{n,j+1}} - X_{\tau_{n,j}}\Big)^3, \tag{56}$$

is crucial in the determination of the asymptotic bias in any stochastic sampling type framework, see for example Li et al. (2014, Section 1). In view of the definition of our sampling points (2) and straightforward applications of Doob's optional sampling theorem, it follows that the probability limit of (56) is zero.

Now, since by Itô's lemma we have

$$d(X_t - X_{\tau_{n,j}\wedge t})^4 = 4(X_t - X_{\tau_{n,j}\wedge t})^3 dX_t + 6(X_t - X_{\tau_{n,j}\wedge t})^2 d\left[X,X\right]_t,$$

and using standard measure change arguments through Girsanov's theorem

$$dU_t = 2\delta_n^{-1}\Bigg[\Big(X_t - X_{\tau_{n,j}\wedge t}\Big)dX_t\Bigg], \tag{57}$$

it follows that, along with weak consistency of NPDV (11) we have

$$\begin{aligned}
[U,U]_t =& \frac{4}{\delta_n^2}\sum_{j=0}^{N_{n,t}}\int_{\tau_{n,j}}^{\tau_{n,j+1}}\Big(X_s - X_{\tau_{n,j}}\Big)^2\, d[X,X]_s \\
=& \frac{2}{3}\frac{1}{\delta_n^2}\sum_{j=0}^{N_{n,t}}\Big(X_{\tau_{n,j+1}} - X_{\tau_{n,j}}\Big)^4 - \frac{8}{3}\frac{1}{\delta_n^2}\sum_{j=0}^{N_{n,t}}\int_{\tau_{n,j}}^{\tau_{n,j+1}}\Big(X_s - X_{\tau_{n,j}}\Big)^3\, dX_s \qquad (58)\\
=& \frac{2}{3}N_{n,t}\delta_n^2 + o_p(1) \xrightarrow{P} \frac{2}{3}[X,X]_t \equiv \frac{2}{3}\int_0^t \sigma_s^2\, ds. \quad \text{a.s.} \qquad (59)
\end{aligned}$$

The proof is now complete upon employing the stable limit theorem of Jacod & Shiryaev (2003, Theorem 9.7.3) applied to the stochastic sequence $\{U_n\}$. □

Proof of Lemma 1 and Theorem 2.

By Itô isometry and boundedness of σ we can see that

$$\mathrm{E}\Big(X_{\tau_{n,j+1}} - X_{\tau_{n,j}}\Big)^2 = \mathrm{E}\left(\int_{\tau_j}^{\tau_{j+1}} \sigma_s \, dW_s\right)^2 = \mathrm{E}\left(\int_{\tau_j}^{\tau_{j+1}} \sigma_s^2 \, ds\right) \leq (\sigma_*^2) \cdot \mathrm{E}(\tau_{j+1} - \tau_j),$$

from which it follows by Markov's inequality that

$$v_{\tau_{n,j}} = P\left[\left|X_{\tau_{n,j+1}} - X_{\tau_{n,j}}\right| > \delta_n\right] = O_P\left(\frac{\Delta_n}{\delta_n}\right).$$

Therefore, we have $\tau^*_{n,j+1} - \tau^*_{n,j} = O_P(\Delta_n^3/\delta_n)$ and also, since $\Delta_n = o(\delta_n^{1/3})$ it is clear that the thinned Poisson sample $\{\tau_n^*\}$ satisfies Assumption A. This proves Lemma 1.

We note that we can write

$$\Delta_n^{-1}\left(N_t^*\delta_n^2 - [X,X]_t\right) = \Delta_n^{-1}\left(\sum_{j=0}^{N_t^*}(X_{\tau^*_{n,j+1}} - X_{\tau^*_{n,j}})^2 - [X,X]_t - \mathcal{B}_t\right)$$

where $\mathcal{B}_t = \mathcal{B}_{n,t} = [N_t^*\delta_n^2 - \sum_j (X_{\tau^*_{j+1}} - X_{\tau^*_j})^2]$. Now, we see that

$$\begin{aligned}
\mathrm{E}\left(\left[X_{\tau^*_{n,j+1}} - X_{\tau^*_{n,j}}\right]^2 \Big| \mathcal{F}^*_{n,j}\right) - \delta_n^2 &= \sigma^2_{\tau_{n,j}} \mathrm{E}\left(\left[W_{\tau^*_{n,j+1}} - W_{\tau^*_{n,j}}\right]^2 \Big| \mathcal{F}^*_{n,j}\right) - \delta_n^2 \\
&= \sigma^2_{\tau_{n,j}}(2-1)!!(\tau^*_{n,j+1} - \tau^*_{n,j}) - \delta_n^2 \\
&\sim 1 \cdot \sigma^2_{\tau_{n,j}} \Delta_n^2 v_n - \delta_n^2 \qquad (60)
\end{aligned}$$

where !! means double factorial. But since $\Delta_n = O(\delta_n^{3/5})$ and $v = O(\Delta_n^3/\delta_n)$, it straightfowardly follows that the last term (60) $= O(\Delta_n^5/\delta_n - \delta_n^2) = O(\delta_n^2)$, implying that the bias term $\mathcal{B}_{n,t} = O_P(\Delta_n^5 \delta_n^{-1} \delta_n^{-1})$.

Therefore, we see that the bias contribution from the time discretization asymptotically tends to zero, since $\delta_n^1/\delta_n^{3/5} \to 0$.

Now it suffices to derive the limiting law of $\Delta_n^{-1}(\sum_j (X_{\tau^*_{n,j+1}} - X_{\tau^*_{n,j}})^2 - [X,X]_t)$. Following the same argument leading to (58) in Theorem 1 and applying Lemma 9.1. of Aït-Sahalia & Jacod (2014), we see that the desired CLT holds with asymptotic variance

$$\frac{2}{3} \cdot 3 \cdot \int_0^t \sigma_s^4 v_s \, ds$$

as required. □

Proof of Theorem 3. From (31) and the property of conditional expectation of the unobserved price change, it is straightforward to see that the leading bias term comes from

$$N'_{n,t} \cdot \frac{1}{4}\varsigma^2 \left\{ \mathcal{D}_{\tau'_{n,j+1}} - \mathcal{D}_{\tau'_{n,j}} \right\}^2 . \tag{61}$$

Now, on noting that both binary variables $\mathcal{D}_{\cdot}$ take values of 1 with probability p_a and -1 with probability $p_q (= 1 - p_a)$, we readily see that the expected value of (61) is given by $2 - 2(p_a - p_b)^2$.

Therefore, since $\varsigma_n = C_\varsigma \delta_n$, the asymptotic bias of (37) then follows from Theorem 1, completing the proof. □