Liquid Factor Models

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Abstract

I propose factor models which use low-cost liquid hedging instruments to yield directly-actionable hedges. I combine the liquid instruments to create less-correlated liquid factors and introduce an estimation method to further handle multicollinearity and stabilize estimates. Analyzing the universe of US-listed exchange-traded funds shows that liquid factor models have explanatory power similar to or exceeding other factor models, yield more stable coefficients out-of-sample which may reduce unnecessary re-hedging and costs, and that these benefits are relatively stronger for significant coefficients and out to at least one quarter. Furthermore, these benefits hold for diverse asset classes and even some foreign assets. This suggests liquid factor models give us cheap ways to alter factor exposures, which is crucial for large portfolios that are not easily altered, work across asset classes, and may help better estimate fund alphas.
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1 Introduction

Factor models help us estimate what risks a portfolio is exposed to and how much returns come from factor exposures. Ideally, we even get information on which exposures are significant/reliable. The factors and exposures yield a composite benchmark which can help us assess managers and their portfolios. In industry, factor models which work across asset classes allow us to look at risk for entire portfolios or across a firm. We may then hedge or alter the estimated risk exposures.

We consider these motivations separately because not all factor models address them well. Some factors are not risk-related: they merely correlate with performance. Some models, particularly some commonly-used in industry, decompose returns and do not indicate statistical significance or admit an unexplained part of returns. Some factors are expensive to trade, complicating their use as a benchmark since trading costs are material. Many factor models are single-asset models which do not allow combining risks across asset classes much less finding if portfolios have cross-asset risks. Some factors may even be unhedgeable risks like illiquidity.

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A common distinction between factor models is what is observed versus estimated. Holdings-based models observe exposures from characteristics of holdings and use cross-sectional analysis to estimate latent (unobserved) factors. Returns-based models observe factor changes (often returns) and use time series analysis to estimate factor exposures.

Returns-based models have some advantages over holdings-based models: they are not fooled by "window dressing" (when reported holdings differ from typical holdings); returns include realized trading costs; and, the estimation lets us narrow our focus to significant factors. Returns also help assess sensitivity to factors we might not easily estimate: we can see if an equity portfolio has more sensitivity to the yield curve or credit conditions than similar equity portfolios; and, we can see if a bond portfolio might have equity market or size exposure. A returns-based approach may also be the only hope for estimating the factor exposures of an opaque investment fund.

Holdings-based models can assess returns for exposures to factors which might not be traded or even observed but are still of interest. This is a powerful feature. However, as a portfolio's turnover increases, holdings at any one time are decreasingly informative about historic performance. We also cannot easily hedge exposures and a holdings-based approach is limited if hedging would require altering a portfolio which is large or illiquid.[1]

A subset of observable factors are those which are tradeable. Factor tradeability has important implications. Tradeability lets us easily change a portfolio's risk exposures. Furthermore, when assessing managers we often ask if they have *alpha*: an expected abnormal return which can be isolated from systematic returns. If factors are not tradeable, we cannot isolate alpha from beta and we lose that dimension of manager assessment.[2]

I propose a liquid factor model: a returns-based model using cheaply-tradeable, liquid factors which market activity (and thus revealed preference) suggests are important factors. This model uses liquid instruments which are transparent, visible by all market participants, and not subject to estimation changes or uncertainty. The model also adds two innovations: (1) it combines these liquid instruments into intuitive, less-correlated factors to focus on risks with durable economic meaning across time; and, (2) it uses a shrinkage estimation technique which preferences explaining returns with liquid factors we expect to be most-relevant *a priori*.

The goal is not necessarily to have higher explanatory power than other factor models. Rather, the goal is to create a model that allows for easily hedging estimated risk exposures while reducing unnecessary hedging and still explaining enough of returns to be useful. To "easily hedge" factors, we should be able to build the factors from highly-liquid and capital-efficient instruments like Treasury bills, notes, and bonds or futures.[3]

This is especially important for large portfolios: a pension fund, for example, may be unable to alter most of its portfolio or deploy much capital for hedging. In that case, an *overlay* solution using derivatives and other liquid instruments is crucial. The capital-efficient nature of these instruments means they are often more liquid (since intermediation requires less capital) and thus incur lower trading costs than altering the portfolio. Using overlay may also be more tax-efficient from preferential accounting to not forcing the portfolio to incur taxable gains.

I find that liquid factor models have in-sample explanatory power similar to or better than accepted academic models. The estimated exposures explain more and are more stable than these other models out-of-sample, even out to one quarter after estimation. These results are stronger

[1]A holdings-based model may be better for a low-turnover portfolio which has changed strategy or manager. In that case, past returns may be expected to be materially different from future returns.

[2]We do not propose this model as a path to portable alpha. If the liquid factor model shows significant alpha, that alpha might be explained by an expensive omitted factor or could be due to unhedgeable risks like illiquidity. Thus we are also not proposing models for fund replication.

[3]Government debt in repo greatly reduces capital usage; futures require posting minimal margin.

still if we focus on exposures whose estimates are statistically significant. The results also hold for diverse asset classes and even some groups of foreign assets. This suggests liquid factor models offer methods to hedge single- and multi-asset portfolios which are cheap, more stable over time, and reduce the cost of hedge rebalancing. Finally, we find that an often-ignored factor, changes in the level of the yield curve, are often significant and not subsumed by analyzing excess returns.

We will first discuss why a model using liquid factors can be beneficial but has not been the focus of most research. Then, we can consider what makes a good liquid factor; analyze a couple funds to build intuition and see the problems of collinearity. We conclude by analyzing all US-listed exchange-traded funds (ETFs) as of 30 November 2021 both in- and out-of-sample.

2 Why Liquid Factors?

Most academic factor models look at cross-sectional regressions (which imply long-short portfolios) or macroeconomic variables. However, liquid factors can be seen as a logical progression of factor models shifting toward observability and tradeability.

While factor models began with the CAPM (Lintner 1965; Mossin 1966; Sharpe 1963, 1964; Treynor 1961), researchers later realized we might need multiple factors (Merton 1973; Rosenberg 1974; Ross 1976) or time-varying factors (Harvey 1989; Rothschild 1985).

2.1 Types of Factor Models and Accuracy

Connor (1995) breaks factor models into three types. Fundamental models are holdings-based: they estimate latent factor returns using cross-sections of security/issuer characteristics and then use holdings to aggregate latent factor returns. Macroeconomic and statistical models are returns-based: they estimate exposures to observed changes in time series of macroeconomic variables and bond yields or (tradeable) instrument returns. I break statistical models into style models, which create factors from portfolios using sorts on characteristics, and *liquid factor models*, which create factors from liquid instruments. While these differences seem minor, some practitioners have strong feelings about the primacy of model types. These factor types also define the spectrum of factor observability, from latent (unobservable), observable, and tradeable, to liquidly-tradeable.

Fundamental models like Rosenberg (1974) and the Barra models estimate latent returns for firm characteristics like size and earnings yield. Macroeconomic models like Rosenberg and Guy (1976) and Chen, Roll, and Ross (1986) use changes in variables like inflation, industrial production, yield curve slope, bond credit spreads, and energy prices to model portfolio returns.

Statistical models emerged later with style factor models like Fama and French (1992, 1993, 2016), Hou, Xue, and Zhang (2015), and Hou et al. (2019, 2021) using factors from returns of long-short portfolios created by sorting on firms' characteristics. Stivers (2018) shows, using PLS (which we also use), that the Fama and French (1992) factors are ICAPM factors. Sharpe (1988, 1992) use factor-related indices to assess a fund's asset allocation mix.

These model types also differ in accuracy. Connor finds that fundamental and statistical models outperform macroeconomic models, with fundamental models slightly outperforming statistical models. However, Dai, Lu, and Xiu (2019) find that statistical models outperform fundamental models. Zhang et al. (2021) show that statistical models using tradeable factors are closer to the Hansen and Jagannathan (1991) bound than models using latent or non-tradeable factors. Kan and Robotti (2008) find macroeconomic factor models perform worse than the style models of Fama and French (1992, 2016), CCAPM, ICAPM, and even CAPM since models with (forward-looking) tradeable factors are more robust to misspecification.

2.2 Cost of Hedging Factor Exposures

The cost of hedging also matters. Patton and Weller (2020) estimate the HML (value) and WML (momentum) factors cost a few percent to trade. Novy-Marx and Velikov (2016) find the trading costs of many factors erase or exceed the net factor return and that even lower-turnover factors like HML (value) and SMB (size) have non-trivial trading costs (4–5 bp/month). Chen and Velikov (2023) suggest the Fama-French factors (and others) have no net return after transactions costs.

Novy-Marx and Velikov (2016) discuss how to reduce trading costs for those factors; however, no approach makes transactions costs trivial and all add tracking error. Bašić et al. (2024) note that prior studies did not consider the cost of shorting stocks or try to create factor-mimicking portfolios that reduce trading costs. While their factor portfolios outperform Novy-Marx and Velikov (2016), they still find that the value factor loses about 1%/year to transactions costs and the size factor 1%–2%/year. These results become even more uncertain given the frequent, material, and unpublicized changes in Fama-French factors documented by Akey, Robertson, and Simutin (2023).

While the preceding works suggest trading less to reduce transactions costs (but increasing tracking error), they do not consider trading something other than long-short sort portfolios.

2.3 Why Are Liquid Factors Missing From Other Models?

If liquid factors are so good, why are they not used in the Rosenberg (1974)/Barra model or in Fama and French (1992, 2016)? Why did Chen, Roll, and Ross (1986) not use an equity index? We can get some answers if we (briefly) review the history of financial futures.

FIA (2022) notes how new financial futures are. The CME introduced FX futures in 1972 and the CBOT introduced 30-year Treasury bond futures in 1977. The CME then debuted Eurodollar futures in 1981 and S&P 500 futures in 1982; the CBOT debuted 10-, 5-, and 2-year note futures in 1982, 1988, and 1991; and, the CME debuted Russell 2000 (small cap) equity index futures in 2000. Globally, exchanges like LIFFE and SIMEX introduced financial futures in the 1980s. Most of these debuts took years to become liquid. Liquid factors are not in most well-known factor models because liquid financial futures did not exist when many of those models were published.

However, it is not surprising that Cremers, Petajisto, and Zitzewitz (2012) find replacing the Fama and French (1992) factors with liquidly-traded analogues yields a superior model with lower unexplained excess returns. Berk and van Binsbergen (2015) find that a factor model using simpler, more-liquid factors outperforms style models using large cross-sectional portfolios. Elkamhi, Lee, and Salerno (2022, 2023) also note the power (and ease) of using liquid factors from different asset classes for explaining returns.[4] Finally, Fama and French (2023) note that "details of factor construction are arguable" and "parameter instability and statistical estimation error" can make models "unreliable." This too would seem to recommend the use of simpler, liquidly-traded factors.

2.4 The Need for Fixed Income Factors

Liquid factors have occasionally appeared in older factor models. The macroeconomic model of Chen, Roll, and Ross (1986) and the style factor model of Fama and French (1993) use a liquid factor: the slope of the yield curve (long-term government bonds minus short-term bills).

Almost all of the preceding studies were focused on modeling equity returns. For fixed income factors, Litterman and Scheinkman (1991) and Litterman, Scheinkman, and Weiss (1991) suggest changes in the level, slope, and curvature of the yield curve. Knez, Litterman, and Scheinkman

[4]Among the prior literature, our work is most closely aligned with Cremers, Petajisto, and Zitzewitz (2012) and Elkamhi, Lee, and Salerno (2022), though we add a few more factors and use a more robust method of estimation.

(1994) suggests a short term (monetary policy) factor for short-term instruments. Knez, Litterman, and Scheinkman (1994) and Boudt, Paulus, and Rosenthal (2017) suggest short-term credit spreads may be useful factors — with Boudt, Paulus, and Rosenthal (2017) finding a TED spread threshold that suggests the spread could be an economic state variable as in the ICAPM. Chen, Roll, and Ross (1986), Fama and French (1993), Fung and Hsieh (2004), and Hasanhodzic and Lo (2007) all suggest a longer-term bond credit spread — though that was not liquidly-traded when those were written. As for real rates, Chen, Roll, and Ross (1986) and Brière and Signore (2012) suggest inflation factors; however, neither consider a liquid version of such a factor.

Finally, what should we make of almost no models using changes in the level of the yield curve? From conversations with many researchers, one consistent counterclaim has been made: that subtracting off the (short-term) risk-free rate to get excess returns renders changes in the yield curve level to be superfluous. This confusion between long- and short-term interest rates has been documented by Shue, Townsend, and Wang (2025) and our findings agree with their work.

2.5 An Equity Factor Zoo?

These studies are part of a proliferation of published factors that Cochrane (2011) proclaimed a "zoo." Many papers suggested that few factors are likely to be reliable out-of-sample. Harvey, Liu, and Zhu (2015), Feng, Giglio, and Xiu (2020), Hou, Xue, and Zhang (2020), and Chordia, Goyal, and Saretto (2020) tested large lists of factors for which are useful, redundant, or not useful.

Both Bessembinder, Burt, and Hrdlicka (2024) and Swade et al. (2024) note that factors need to be considered jointly (though they ignore collinearity concerns). Bessembinder, Burt, and Hrdlicka (2024) suggests there may be as many as 40 relevant factors while Swade et al. (2024) state 10–20 factors is more likely, since some statistically significant factors are economically immaterial. Harvey, Liu, and Zhu (2015) notes the difference between common factors, which includes liquid factors, and characteristics factors, based on metrics unique to a given company. They also suggest that theoretically-based factors should face a lower hurdle than empirically-found factors.

Almost all of these papers only assess equity portfolios using equity factors, mostly based on firm characteristics. Harvey, Liu, and Zhu (2015) is the only study to also assess yield curve, credit, and inflation factors. He, Kelly, and Manela (2017) develops factors based on intermediary funding liquidity – and is the only paper testing any factors on multiple asset classes. Their work, with Du, Hébert, and Li (2023), implies the relevance of yield curve and credit factors for equity portfolios.

Finally, we should clarify that we are not seeking to predict a risk premium; rather, we are only seeking to explain risk. Thus we take no position on the criticism of factor models in Welch and Goyal (2008). However, we are pleased to see that Pesaran and Timmerman (1995) and Goyal, Welch, and Zafirov (2024) find fixed income factors to be informative for equities.

3 Finding Liquid Factors

To make our liquid factors specific, it helps to think about the rise of liquid hedging instruments and the characteristics of an ideal factor. These ideas can then guide us to pick specific factors now and to modify or append factors in the future. Satisfying these guidelines greatly enhances the utility of our model. The advantage of having liquid tradeable factors is that our factor model becomes an alpha model, as in the APT of Ross (1976) and the portable alpha of Miller and Meckel (1999), Arnott (2002), and Kung and Pohlman (2004).

An alpha model is a powerful tool: If the model does not show significant alpha, the portfolio is unlikely to have alpha; and, we might even believe that our model captures most of the portfolio's dynamics. If the portfolio being analyzed has alpha, the model should show significant alpha. The converse, however, is not certain: if the model shows significant alpha, we cannot conclude the portfolio has alpha: we might have omitted factors which would eliminate that alpha. Models using non-liquid factors have more potential problems: what those models show as alpha could be due to untradeable or illiquid factors which are poorly priced or cannot be traded without paying a wide bid-ask spread which would erase the alpha (aka an "arbitrage channel").

3.1 Characteristics of Better Liquid Factors

Given the precedingdiscussion, promising characteristics of liquid factors include: liquidity, ease of trading, risk shifting, capital efficiency, and reduced rebalancing.

We want the instruments in a factor to be liquid: narrower bid-ask spreads make it cheaper to hedge a risk factor. We also want instruments which can be traded quickly; requiring a long negotiation or optimization process adds cost and uncertainty to the hedging price. We prefer risk-shifting contracts since these are the only way to hedge systemative risks and revealed preference (that the contracts exist) suggests speculators want to provide liquidity. We prefer instruments like derivatives and government bonds because they are capital-efficient: they require less capital to hold them (through margin or repo). We want factors which do not need to be rebalanced regularly since that increases transactions costs over time. We do not require factors to be compensated risks (offering a risk premium). As noted by Cochrane (2011), a risk premium is not a necessary condition for a factor and factors which merely reduce risk can be helpful in the short-run.

Given all of these criteria, the factors most usable for pricing are a few liquid instruments with actively-traded, capital-efficient instruments (like listed derivatives) that we can trade directly and quickly with little-to-no adjustment or rebalancing over time. If our analysis is looking at a long time horizon, we might prefer these factors to be compensated risks.

3.2 Suggested Liquid Factors

The above discussion suggests certain types of factors we should use. To be clear: none of these are US-specific.[5] This caveat is important since we may be dealing with a global portfolio.

For equity risks, we should first consider a large-cap equity index: Cremers, Petajisto, and Zitzewitz (2012) suggest it be market capitalization-weighted. Their analysis also suggests liquid analogs of some Fama-French factors: an SMB/size factor (Russell 2000 minus S&P 500 index returns) and an HML/value factor (Russell 3000 Value minus Russell 3000 Growth index returns).[6] We can generalize these to the outperformance of a small- versus large-cap index and of a value versus growth split of an index. If we lack a small-cap equity index, we may need to create a size factor from a broad-based and a large-cap index.

For interest rate risks, Litterman and Scheinkman (1991) suggests our first factors should be changes in the level, slope, and curvature of the yield curve. Litterman, Scheinkman, and Weiss (1991) note that the curvature (bump) factor relates to interest rate volatility.[7] Factor construction is again important: we want mutually orthogonal factors to avoid multicollinearity issues.

For credit risks, we should consider changes in credit spreads of corporate bonds, as in Chen, Roll, and Ross (1986), Fama and French (1993), Fung and Hsieh (2004), and Hasanhodzic and Lo (2007). Some markets have strong market segmentation: insurers may be restricted to holding only "investment-grade" bonds. Thus we consider both investment-grade (IG) and high yield (HY) credit

[5] There are versions of these factors for other large markets which are also liquid. Therefore, while US-focused instruments may be mentioned in what follows, that should not be read as constraining these ideas to a US context.

[6] The use of the Russell 3000 indices is odd since the CME has active futures on the Russell 1000 and 2000 Value and Growth indices, but not on any Russell 3000 indices.

[7] The term "bump" comes from a class of yield curve models in the Two-Plus family.

indices.[8] We might also consider a short-term credit spread as in Boudt, Paulus, and Rosenthal (2017) though the elimination of LIBOR complicates creating this factor.

While prolonged high inflation is uncommon in developed economies, we should still consider an inflation-surprise factor as in Chen, Roll, and Ross (1986). Thus we consider the yield spread between similar-maturity non- and inflation-indexed government bonds (aka "breakeven inflation").

We also should consider an equity volatility factor since implied-volatility-related instruments are now liquidly traded. Thus we consider a volatility index factor (like VIX or VSTOXX) as in Hasanhodzic and Lo (2007). Since the yield curve bump relates to interest rate volatility, an equity volatility factor makes for a symmetry of volatility factors.

Almost all of these factors are available with liquid instruments (futures or government bonds) which allow us to go short or long. Credit has long been an exception; however, in the US there are now credit indices (iBoxx IG and HY) that underlie liquid ETFs and less-liquid futures.[9] In 2018, the CBOE introduced futures on these ETFs and the ETFs are now the most liquid US corporate bond ETFs and the third- and first-largest in their categories.

4 Estimating More Stable Model Coefficients

A major concern with estimating factor exposures is multicollinearity. Multicollinearity often manifests as coefficients which vary across time while the aggregate risk changes much less. That means we could change the coefficients (a little more of one beta and a little less of another beta) and get almost the same residual risk.

Multicollinearity is concerning if the estimation results guide hedging: unstable coefficient estimates could lead hedgers to pay unnecessary transactions costs for an adjustment to their hedge which does not materially alter their risk. Multicollinearity also affects inferences about the statistical significance of coefficient estimates by inflating coefficient standard errors, reducing t-stats. We show this by briefly discussing variance inflation factors in Appendix A.

We handle multicollinearity in two ways: we use correlation-reducing variable combinations; and, we use an estimation approach that eliminates almost all multicollinearity by preferencing certain variables. We discuss some of the methodology for doing this because we strongly suspect there are other liquid factors: some similar to those we use here for investments in Europe, Japan, and elsewhere; and, some factors even for the US which are not yet liquid and well-understood. Thus the methodology here can help guide us to other liquid factors in the future.

Finally, while we are using liquid factors, illiquidity in the instruments or parts of portfolios we assess can cause issues such as falsely-significant alpha and distorted betas. In the data analysis section, we mention a few ways to handle these issues.

4.1 A Way Forward: Partial Least Squares

How should we proceed when the data are beset by multicollinearity? The partial least squares (PLS) approach of Wold (1966, 1982) is promising.[10] PLS involves finding orthogonal regressors

[8] In-process work by van Binsbergen, Nozawa, and Schwert (2025) and Dickerson and Nozawa (2024) supports the utility of these particular factors.

[9] The iBoxx investment grade (IG) and high yield (HY) bond indices debuted at the end of 1998. The indices became more accessible with the listing of an IG ETF (LQD) in 2002 and a high yield ETF (HYG) in 2007. Index details are in Goldman Sachs (2006a,b) and Goldman Sachs and IIC (2006). More discussion of the ETFs and their benefits can be found in Dannhauser (2017).

[10] Sharpe (1988, 1992) handled multicollinearity with quadratic programming which we do not explore.

from groups of covariates and reinforcing orthogonality by then iteratively estimating coefficients using the residuals from each step as the response for the next step.

PLS can be seen in the continuum regression framework of Stone and Brooks (1990) and lies on the spectrum between ordinary least squares and principle components regression. While the regressors are latent variables in the original PLS approach, de Jong (1993) shows an approach which preserves interpretability and does not require latent variables. Our approach is like a simplified version of de Jong (1993) or Type I ANOVA.[11] One could also view our approach as a version of the Schapire (1990) boosting machine learning algorithm.

PLS is typically used when we are "data-rich but theory-poor." In our case, we have basic theory from *a priori* knowledge of the liquidity of hedging instruments and portfolio metadata (*i.e.* the dominant asset class). Thus equity portfolios should probably first hedge equity factors and fixed income portfolios should probably first hedge yield curve factors. This approach also makes clear that we may want to construct regressors from combinations of variables based on our *a priori* knowledge of risk factors. To distinguish between standard PLS and our *theory-informed* PLS, we will hereforward refer to our procedure specifically as TIPLS.

In preferring some blocks of variables over others, TIPLS falls into the large class of biased or shrinkage estimators which includes ridge regression and the LASSO. These estimators trade unbiasedness for lower variance out-of-sample. In our case, this is good: we want our betas to be less volatile so we spend less money adjusting hedges unnecessarily.

We also posit that, in our case, the TIPLS bias is acceptable and even desirable. While ridge regression and the LASSO have a bias that is not immediately obvious, the TIPLS bias pushes us toward choosing factors which are cheaper to hedge and more likely to be related to the portfolio.

Equity index factors often have non-zero covariance with yield curve factors. We may not have a view on whether that covariance comes mostly from equities or the yield curve. (This is an area where we are theory-poor.) For an equity portfolio, we would preference equity factors and sweep that covariance into the equity index factor exposure; an equity index factor is probably a better fit and more robust choice versus sharing the covariance with the fixed income factor, as in OLS. If the equity index factor is more liquid, we should also prefer using it to hedge more of the covariance. Thus the TIPLS bias pushes our estimated exposures to hedge using more liquid instruments which are more aligned with the portfolio's holdings.

4.2 Correlation-Reducing Variable Combinations

Within the asset class groupings mentioned above, we can combine factors to create new, less-correlated factors which will have more stable coefficients over time.

Suppose we have two variables, A and B, with correlation ρ and identical mean and variance: $A, B \sim (\mu, \sigma^2)$. Based on economic reasoning or just the correlation of A and B, we might suspect A and B share some common randomness. We may want exposure to that common randomness in our model; however, we also want exposure to differences from the common randomness.

To make this concrete, these variables could be returns to a large- and small-cap index. We could then take A and B and create economically meaningful factors such as $A + B$ and $B - A$. These combinations give us a broader large-plus-small-cap market index ($A + B$) and a small-cap outperformance aka size factor ($B - A$).

The correlation of the $A + B$ and $A - B$ combinations requires some arithmetic. if A and B

[11] Weisberg (2014) notes that Type I (sequential) ANOVA can be appropriate for some problems.

have the same variance, we can simplify further:

$$\operatorname{corr}(A+B, A-B) = \frac{\operatorname{cov}(A+B, A-B)}{\sqrt{\operatorname{var}(A+B)}\sqrt{\operatorname{var}(A-B)}} = \frac{\operatorname{cov}(A^2) - \operatorname{cov}(B^2)}{\sqrt{\operatorname{var}(A+B)}\sqrt{\operatorname{var}(A-B)}} \tag{1}$$

$$= \frac{\sigma_A^2 - \sigma_B^2}{\sqrt{\sigma_A^2 + \sigma_B^2 + 2\rho\sigma_A\sigma_B}\sqrt{\sigma_A^2 + \sigma_B^2 - 2\rho\sigma_A\sigma_B}} = 0 \quad \text{if } \sigma_A = \sigma_B. \tag{2}$$

This is helpful: we can combine the variables to get two new factors which are uncorrelated. If the variables have differing variances, these new factors will not be perfectly uncorrelated; however, they are likely to have much lower correlation than A and B.

4.2.1 The Tension with Interpretability

While we could make one of our factors $A + B$, there may be some mild discomfort with this. For example, if we are considering equity index factors, most other factor models use a large-cap market factor like A. Because of that, we might want to instead use A and $B - A$ so we can more easily compare our model with other models having an A factor.

In this case, using A and $B - A$ allows some collinearity to preserve factors that are preferred by an analyst or client. This may still lower the magnitude of the correlation between these new factors, A and $B - A$, if the correlation ρ of A and B is sufficiently high.

The correlation of the A and $B - A$ combinations is:

$$\operatorname{corr}(A, B-A) = \frac{\operatorname{cov}(A, B-A)}{\sqrt{\operatorname{var}(A)}\sqrt{\operatorname{var}(B-A)}} = \frac{\rho\sigma_A\sigma_B - \sigma_A^2}{\sigma_A\sqrt{\sigma_B^2 + \sigma_A^2 - 2\rho\sigma_A\sigma_B}} \tag{3}$$

$$= \frac{\rho\sigma_B - \sigma_A}{\sqrt{(\sigma_B - \sigma_A)^2 - 2(\rho - 1)\sigma_B\sigma_A}} = -\sqrt{\frac{1-\rho}{2}} \quad \text{if } \sigma_A = \sigma_B. \tag{4}$$

Assuming equal variances, for which ρ does this variable combination help us? The dividing line is a correlation between A and $B - A$ that is equal to the correlation of A and B. If we set the correlation of A and $B - A$ equal to ρ and square both sides, we get equality in magnitude at:

$$\rho^2 = \frac{1-\rho}{2} \quad \Longrightarrow \rho = \frac{-1 \pm 3}{4}. \tag{5}$$

For $\rho = -1$ the pair A and $B - A$ are perfectly anti-correlated; and, for $\rho \in (-1, 0.5)$ we have that $|\rho| < |\operatorname{corr}(A, B-A)|$ — which does not make for a more statistically stable combination. However, when $\rho > 0.5$ we get a correlation lower in magnitude for the $(A, B - A)$ pair:

$$(\operatorname{corr}(A, B-A))^2 = \frac{1-\rho}{2} < \rho^2 \quad \forall \rho \in (0.5, 1]. \tag{6}$$

We see that for any correlation $\rho > 0.5$ between A and B, the correlation of A and $B - A$ will be smaller in magnitude — which should reduce the effects of multicollinearity. For example, if the correlation ρ of A and B is 0.7, then $\operatorname{corr}(A, B-A) = -0.39$; and, for $\rho = 0.9$ (a typical correlation among equity indices), $\operatorname{corr}(A, B-A) = -0.22$.

4.2.2 Creating Less-Collinear Equity Factors

The above reasoning gives statistical justification for the creation of our equity factors. For a size factor, we will indeed look at the Russell 2000 index returns minus the S&P 500 index returns. For a value factor, the Russell 1000 Value and Growth index futures are more liquid than the Russell 2000 Value and Growth index futures. Thus for a value factor we will look at the Russell 1000 Value index returns minus the Russell 1000 Growth index returns.

4.2.3 Creating Less-Collinear Yield Curve Factors

For yield curve exposure, we need to create level, slope, and curvature (aka "bump" or "belly") factors. To avoid making the factors mechanically collinear, slope and curvature factors should have 0 duration and the curvature should have 0 slope. We scale positions in each tenor so that all instruments have the same duration.

The level factor averages changes of the most liquid tenors: 3-months, 2-, 5-, 10-, and 30-years. Since these instruments have different durations, we average yield changes rather than returns.[12] A nice feature of this is that the level coefficient gives an (opposite-signed) effective duration.

The slope factor is created from changes in yield differences between the 30-year and 3-month tenors and the 10- and 2-year tenors. Since these tenor ranges differ, we create slopes by dividing each yield difference by the duration difference. We considered differences in tenors (29.75 and 8), durations (about 18.95 and 6.7), and log-durations. The means of slope estimates were closer and their differences were less variable when we divided by differences in durations or log-durations. Since dividing by durations seems intuitive, we have done that.

Finally, the curvature factor is created from yield differences between the 30- and 5-year tenors minus the yield difference between 5-year and 3-month tenors. This results in subtracting the 5-year yield twice and is the same as the numeric formula for a centered divided-difference to approximate the second derivative. We account for the difference in tenor ranges (*e.g.* 30Y−5Y, 5Y−3M) by dividing by them by their duration differences (about 14.5 and 4.5).

The volatilities of government bond *yields* tend to increase with maturity. In statistical inference, we assume homoskedasticity and do not want any observation to have more influence than another. We did not create spreads with equal-volatility contributions from each tenor since eliminating the mechanical presence of DV01 or slope seemed more important than eliminating the effect of greater economic uncertainty in later tenors. However, that approach merits further research.

The data often show a correlation between the level and curvature of the yield curve. While we have eliminated any mechanical correlations between yield curve factors, that does not eliminate their correlation due to mean-variance utility, since the curvature is related to interest rate volatility. This is like the correlation between the VIX and excess returns on the S&P 500. This is yet another reason for the TIPLS estimation approach.

4.2.4 Creating Less-Collinear Credit Factors

For credit factors, we would like to eliminate their exposure to the yield curve. While we rarely know the slope exposure of a credit instrument, we can hedge the duration component of returns by subtracting returns from a similar-duration government bond. Using returns also yields factors similar to the duration-times-spread credit factors of Ben Dor et al. (2007).[13]

[12] Changes in yields have the same effect in the model as using returns since yield changes time negative durations are returns to a first-order (linear) approximation.

[13] We compute Treasury returns for this by multiplying yield changes by −1 times the government bond duration.

To handle market segmentation, we use IG and HY indices. The iBoxx IG and HY indices have duration similar to 10- and 5-year Treasury notes. Thus we can create roughly zero-duration factors by looking at changes in the yield spread (or returns) between the iBoxx IG and 10-year Treasuries (IG10) and the iBoxx HY and 5-year Treasuries (HY5).

However, these two zero-duration indices, IG10 and HY5, might still be correlated. For this reason, we choose to do as with equity indices and create factors using IG10 and HY5−IG10. This creates IG credit and pure HY (above-IG) credit factors.

One caveat is in order here: the correlation between IG10 and HY5 is 0.67. While this is above the $\rho = 0.5$ threshold mentioned earlier, we should still expect some mild collinearity between IG10 and HY5−IG10. It may make sense to monitor the correlation between these factors or, in some cases, to instead look at IG10 and HY5 (versus IG10 and HY5IG10).

The definitions of all the preceding factors are summarized in Table 1.

[Table 1 about here.]

4.3 Iterative/Sequential Estimation

With less-correlated versions of our factors, we can discuss TIPLS estimation.[14] We use basic prior information about a portfolio, *e.g.* the portfolio holds little or a lot of fixed income, to pre-specify which factors to preference in our model. Pre-specifying the estimation order avoids data snooping while still allowing us to make sharper inferences about exposures more likely to matter.

However, preferencing does not mean we should eliminate any factors from estimation. For example, while equity portfolios are most likely to be exposed to equity index factors, some equities (like construction, durables, and REITs) may have significant exposure to level/DV01 and slope factors. Similarly, some fixed income (like investment grade and high yield corporates) may have significant exposure to equity index or sector factors.

One relevant question this does not resolve is handling the intercepts from the various regressions. Since TIPLS estimation is equivalent to Type I ANOVA, we know it merely changes how much uncertainty we apportion to each factor; the intercept term is unaffected. We could cumulate the intercept estimates to get the aggregate intercept estimate; however, this would be subject to cumulated roundoff error. Therefore, we also run an OLS model with all of the factors present to recover the intercept and its standard error.

Thus our estimation of factor exposures for portfolio excess returns y using less-correlated factors $\tilde{x}$ is done as follows:

1. Estimate a model with all factors to get the aggregate intercept and its standard error.
2. Number the factors $\tilde{x}_b$ in order of preference: $b \in \{1, \ldots, k\}$.
3. Iterate through the factors:
   (a) Remember the beta estimates and standard errors; and,
   (b) Pass residuals along to the next iteration of the loop.
4. Determine the R^2 from the fraction of variance left in the ending residuals versus starting (*i.e.* the variance of portfolio excess returns).

[14] For those familiar with PLS or structural equation modeling, note that we lack latent variables and thus any outer model where we use measurements to infer the latent variables.

Algorithm 1 Simplified TIPLS Block-by-Block Estimation

procedure ESTIMATE TIPLS MODEL($y, \tilde{x}_{b\in\{1,\dots,k\}}$)
 $\tilde{y}_I \leftarrow y$
 $\hat{\alpha}, \hat{\sigma}_{\hat{\alpha}} \leftarrow \text{OLS}(\tilde{y}_I \sim \alpha + \tilde{x}_1\beta_1 + \cdots + \tilde{x}_k\beta_k + \epsilon_I)$
 $\tilde{y}_1 \leftarrow y$
 for $b = 1$ to k **do**
  $\hat{\beta}_b, \hat{\sigma}_{\hat{\beta}_b}, \hat{\epsilon}_b \leftarrow \text{OLS}(\tilde{y}_b \sim \alpha + \tilde{x}_b\beta_b + \epsilon_b)$
  $\tilde{y}_{b+1} \leftarrow \hat{\epsilon}_b$
 end for
 return $\hat{\alpha}, \hat{\sigma}_{\hat{\alpha}}, \{\hat{\beta}_b, \hat{\sigma}_{\hat{\beta}_b}\}_{b\in\{1,\dots,k\}}, \hat{\epsilon}_k$
end procedure

5 Data Analysis: Comparison With Other Factor Models

Having discussed the possible benefits of liquid factors, creating less-correlated factors, and preference-ordered estimation, we move on to examine the performance of these potential improvements and compare their effects with other factor models.

We will first briefly examine an equity-dominant value fund (Berkshire Hathaway, BRK-B) and a large high yield fund (Blackrock High Yield, BHYIX) to see how the model and its innovations perform at an individual fund level. We will then take a more general view and analyze the universe of US-listed ETFs (using some of the insights from looking at individual funds).

5.1 Assessing the Value of Model Innovations

The model we have proposed has two innovations: feature engineering, the combination of liquid instruments into more meaningful factors; and, TIPLS estimation, a form of shrinkage that preferences explaining returns with more liquid/relevant factors. To assess the value of these two innovations, most of the analysis here will look at four models: two incomplete liquid factor models with one or both innovations turned off, the full liquid factor model with both innovations, and a popular academic model for comparison.

The most basic liquid factor model is to use all of the liquid instruments discussed, but neither combining them into meaningful factors nor using TIPLS estimation. This is a "kitchen sink" joint OLS regression as discussed in § 8.2.4 of Greene (2003). We then add the innovation of feature engineering by combining liquid instruments to reduce collinearity. This model is still estimated jointly using OLS but uses the corrected covariates. Finally, we add the innovation of sequential TIPLS (shrinkage) estimation, as discussed in Section 4. That is the full liquid factor model.[15]

We also compare the preceding with two Fama-French models: the Fama and French (2016) 5-factor (equity) model plus Carhart momentum factor (FFC6); and, the Fama and French (1993) 5-factor (bond) model.

Fama and French (1993) has some of the same factors as Chen, Roll, and Ross (1986), except they are forward-looking.[16] Unfortunately, data for Fama and French (1993) is not available on Ken French's (nor DFA's) website so we cannot make direct comparisons to their model. To proceed, we endow the Fama-French bond model with liquid factors mimicking the original factors: YCSlope, IG10, R2KSPX, R1KVMG, and SP500.xs. Since Fama and French (1993) created a residual market

[15]In the code, these models are referred to as the raw (and raw.xformed), mid, and full models.

[16]We cannot use Bai, Bali, and Wen (2019) in light of its retraction due to the issues noted by Dickerson, Mueller, and Robotti (2023).

factor (RMO) after regressing on the other factors, this suggests we estimate the first four factors jointly and then estimate the SP500.xs variable second in a sequenced estimation. We call this replication FF93L, the "L" denoting that it uses liquid factors (unlike the original model).

Since we use this set of models throughout the paper, they are summarized in Table 2.

[Table 2 about here.]

Comparing the performance of these models lets us see the benefit of each aspect of the liquid factor model and how these compare with other models commonly used in academia and industry.

5.2 Summary Statistics of Factors

We begin by looking at data for US factors from the start of 2014 to 30 May 2025. While we could look at a longer time frame, we use a shorter time frame for a few reasons. First, we use daily data so this period has much more data than studies using monthly data. Second, investment managers (and mandates) change over time: long-run studies encompassing multiple managers/objectives are unlikely to be informative, while studies on shorter time periods might have offered insight on those individual mandates. Third, some investment funds we examine have not been around for multiple decades. Requiring a long history to analyze a fund imposes a form of survivorship bias. Finally, we use this time frame (and shorter time frames in some cases) to demonstrate that useful insights need not require large numbers of observations.

For reproducibility (since the code will be made public), the data are for indices from Yahoo Finance and FRED.[17] We have done analyses using ETFs and front-contract futures (with rolling) and the results are almost identical. We can see the summary statistics for the liquid model raw variables in Table 3 and for the correlation-reduced factors (with definitions) in Table 4.

[Table 3 about here.]

[Table 4 about here.]

The raw variables (liquid instruments) are often more variable than the correlation-reduced factors. However, the more important question is whether our factors have lower correlations than the original raw variables.

Figure 1 (left) shows the correlations among liquid instruments used to create the liquid factors. We can see strong correlations among the equity indices (S&P 500, Russell 2000, Russell 1000 Value and Growth) as well as the VIX (equity volatility index). We also see strong correlations among the Treasuries (especially notes and bonds), and also between the IG and HY credit indices. Furthermore, the IG index is correlated with the notes and bonds while the HY index is correlated with the equity indices.

[Figure 1 about here.]

Figure 1 (right) shows the correlations among the correlation-reduced factors created from liquid instruments. For these factors, we can see that almost all correlations are small with the largest (negative) correlation being between the level and bump (curvature) factor for the yield curve and smaller positive correlations between the credit indices and the S&P 500.

Summary statistics for the Fama and French (1992, 2016) models are shown in Table 5. Figure 2 shows the correlations among those factors and their liquid analogues. We see high correlations

[17] The results are materially the same if using Reuters or Bloomberg data.

among the market factors (Mkt.RF and SP500), size factors (SMB3, SMB5, and R2KSPX), and value factors (HML3, HML5, and R1VMG). That further confirms that the liquid analogues are valid substitutes for the Fame-French factors.

However, given the care which the FF3 model takes to avoid collinearity, it is surprising to see the moderately high (0.6) correlation between the Value (HML) factors and the FF5 CMA factor. This should lead to some instability of HML and CMA beta estimates over time.

[Table 5 about here.]

[Figure 2 about here.]

5.3 Initial Exploration: Two Funds

To explore the performance of the liquid factor model, we will briefly examine the returns for two investment funds. We will compare a liquid-instrument version of the model with model innovations added incrementally to show the benefits of those innovations, as discussed in Section 4. We will also compare these with the FFC6 and FF93L models. This will show how the models perform at an individual fund level.

For our two funds, we will examine an equity-dominant value fund (Berkshire Hathaway, BRK-B) and a large high yield fund (Blackrock High Yield, BHYIX) to see how the model and its innovations perform at an individual fund level. We will then take a more general view and analyze the universe of US ETFs (using some of the insights from looking at individual funds). The summary statistics for the first two funds' excess returns are shown in Table 6; the summary statistics for the set of funds appear later.

[Table 6 about here.]

The equity value fund has averaged an excess return of around 10%/year since the start of 2015 while the high yield fund has averaged an excess return of about 2.5%/year over the same period. Plots of the funds' performance are shown in Figure 3.

[Figure 3 about here.]

We can begin to quantify the improvement in model precision from feature engineering by computing variance inflation factors (VIFs) for the covariates. For the Kitchen Sink model, VIFs range from incredibly high values like 64, 35, and 29 for the S&P 500, 10Y UST, and Russell 1000 Growth factors to tolerable lows of 1.2 and 2.4 for the 3M UST bill and 5Y TIPS. For the Corrected Covariates model, VIFs top out at 5.3 for the S&P 500 with all other VIFs at 2.7 or lower and four factors having VIFs of 1.1 or 1.2.

5.3.1 Equity Value Fund

Estimates for the three versions of the liquid factor model, with increasing innovations, plus the FFC6 model are shown for the equity value fund in Table 7.

[Table 7 about here.]

We can see that the significance of a few intuitive factors improved due to feature engineering and use of a sequential estimation methodology. We can also note that the Sequential Shrinkage model has t-stats that are higher for a handful of factors and moderately smaller for a few factors.

This is especially true for the few factors which still had higher correlations (SP500 vs VIX and HY, YCLevel vs YCBump and IG). This is the preferencing bias at work.

The reduction in VIFs and the model being able to more cleanly estimate factor exposures suggests that creating lower-correlation factors and using sequential TIPLS estimation methods both help control for collinearity.

We can also see how the full liquid factor model compares to a well-known model like the Fama-French 5-factor model with Carhart momentum added. The full liquid factor model, Sequential Shrinkage, has almost the same beta for the market factor and a similar beta for the value factor. However, the liquid model does not estimate the size factor as being significant. Furthermore, we can see that the Fama-French model misses that this equity fund has some exposures to the level of interest rates and equity volatility — although the volatility exposure is so economically small as to be ignorable. All models show that the fund does not seem to have significant alpha.

It should come as little surprise that Berkshire Hathaway looks like a value equity fund with exposure to interest rates and possible overweights to financials and durables. Furthermore, it seems like Warren Buffet has been honest with us all along: his suggestion that people invest in index funds seems to agree with Berkshire Hathaway having no significant alpha.

These results beg the question: Is the Fama-French model better? It has a slightly higher R^2 than the liquid model; however, we cannot cheaply hedge with the SMB and HML baskets. In contrast, the liquid factor model tells us the size factor is likely not significant while value and the level of interest rates are significant. Furthermore, we can easily and cheaply trade Russell 1000 Value and Growth futures and Treasury bonds or futures. For a small loss of R^2, we learn about exposures beyond equity factors and that the fund may not be exposed to a size factor.

5.3.2 High Yield Bond Fund

Table 8 shows the three liquid models and the liquid substitute for the Fama and French (1993) model. All of the models have a harder time modeling fixed income fund returns versus equity fund returns: R^2's are in the high 50% range, at best. This is probably, in part, due to the lower liquidity of fixed income instruments: price changes may take more than a day to be realized.

[Table 8 about here.]

However, there is also likely an omitted variable. The Fama and French (1993) model lacks the most important factor found by Litterman and Scheinkman (1991): YCLevel, the change in the level of interest rates (isomorphic with duration). Their model also lacks HY5IG10, a high yield (versus investment grade) credit factor. For evaluating portfolios containing any bonds or overweight large capital-expenditure and durable goods firms, the omission of yield curve level changes would seem to be an issue. For evaluating portfolios containing corporate or other risky bonds, the omission of a high-yield credit factor could also be an issue.

We can see that the FF93L model does a much worse job than the other models at describing the fund's returns in terms of factors. The FF93 model has no credit spread of high-yield instruments above the investment-grade credit spread. That the high-yield factor adds explanatory power confirms that it is not just a scaling of the investment-grade factor; more than one credit spread is needed to capture the corporate bond market. The FF93 model also misses the level of the yield curve (minor, in this case) and the yield curve bump (associated with interest rate volatility) which is the third-most important yield curve factor. The long bump exposure suggests a short interest-rate volatility exposure, as per Litterman, Scheinkman, and Weiss (1991), and might arise from interest rate calls on the bonds held by this fund.

To check if the low explainability is due to liquidity concerns, we rerun the preceding models on monthly data. Table 9 shows the results of doing so and the much higher R^2's would suggest that the problem was indeed due to illiquidity.

[Table 9 about here.]

The monthly models exhibit a few differences from the daily models. First, the relative significance of the IG10 and HY5IG10 factors shift when moving from daily to monthly data. The daily returns loaded similarly on the high-yield credit factor HY5IG10 and the investment-grade credit factor IG10. Since the pure HY credit factor is HY5−IG10, that could be interpreted as loading on only the HY5 factor. The monthly returns, however, load more heavily on IG10: 1.19 vs 0.55 on HY5IG10. Interpreting those in terms of IG10 and HY5 would suggest the monthly data shift to a weighting of 0.64 IG10 and 0.55 HY5. This could be due to lags in changes to credit indices or to risk first manifesting in pure HY spreads and (if persistent) then in IG spreads.

In both daily and monthly returns, the models suggest a loading on interest rates (the YCLevel factor). While unsurprising for a bond fund, the YCLevel coefficient can be interpreted as an effective duration. An effective duration of about 0.67 in both full liquid models seems low for a fixed income fund and suggests the fund does some interest rate hedging.

5.4 Stability of Alpha and Beta Estimates

Having estimated these models, we should ask how stable the alpha and beta estimates are out of sample. Since it is difficult to visualize this across multiple funds, we return to explaining returns for Berkshire Hathaway, an equity value fund, using the three liquid factor models with increasing innovations and the FFC6 model.

To compare the Kitchen Sink model to the other models we must transform the Kitchen Sink betas and estimation variance-covariance matrix to get comparable betas and standard errors. How to do this is discussed in Appendix C.

Having done this, we create rolling windows of 250 days (roughly one year) and fit each of the three liquid factor models as well as the FFC6 model.[18] We can see these in Figure 4. Note that we only show the FFC6 coefficients for factors with analogues in the liquid factor model.

[Figure 4 about here.]

The most obvious result from these plots is that the Kitchen Sink model with all of the liquid instruments is greatly beset by collinearity issues. Comparable estimates (in red) are the most volatile. Collinearity makes interpreting the model difficult.

The Corrected Covariates model creates less-correlated factors from those instruments, yielding less volatile estimates. However, the estimates (in light blue) are still often more variable than those of the Sequential Shrinkage model which also uses TIPLS sequential estimation.

This lower variability is not just of econometric importance. When investors hedge their portfolios, more unstable beta estimates means those investors who choose to hedge will incur higher and possibly unnecessary transactions costs. They might also incur greater anxiety or trust a model less from having to rehedge more.

[18] We did not use longer windows since that would yield less variable coefficient estimates and be uninformative. We did not use shorter time windows because we worried about potential seasonality effects. In practice, we have used longer time windows and, on rare occasions, shorter time windows. Unsurprisingly, we found that doing so yields less-noisy and noisier estimates; however, we also found that the stability-enhancing aspects of the liquid factor model were materially helpful with both longer and shorter estimation windows.

While these plots allow us to see the differences in variability across models, they do not allow objective comparison. It might seem like the Sequential Shrinkage (*i.e.* full liquid factor model) is superior, but is that objectively so? Table 10 shows means and standard deviations of coefficient estimates (and R^2) across the three liquid models and the FFC6 model.

[Table 10 about here.]

We can see clearly how the Kitchen Sink model has the noisiest estimates across almost all coefficients and the highest R^2: merely recombining coefficient estimates into more meaningful metrics is not sufficient to overcome problems introduced by collinearity — though having fewer constraints does yield a higher in-sample R^2. The Corrected Covariates model has less-noisy estimates in general than the Kitchen Sink model; and, we can also see how the full liquid factor model (Sequential Shrinkage) has parameters which tend to be more stable, especially for the preferenced factors.

Compared to the FFC6 model, the full liquid model has similarly stable estimates for the two main equity factors, S&P 500 versus Mkt.RF (market) and R1KVMG versus HML (value). However, the FFC6 model seems to have a spurious loading on the SMB (size) factor, misses exposure to the level of interest rates, and, has elevated variability in the HML (value) and CMA (investment) factors. This is likely due to their correlation as noted earlier.

The lower variability of most factor betas suggests that the Kitchen Sink model could lead to more unnecessary rehedging than either the Corrected Covariates or Sequential Shrinkage models. Higher transactions costs would make it a more expensive model to use for hedging. While the variability of coefficients is similar for the Sequential Shrinkage and Fama-French+Carhart 6-factor model, the much higher cost to trade the FFC6 model documented by Novy-Marx and Velikov (2016) and Bašić et al. (2024) would make the FFC6 model also more expensive to use for hedging.

This example is helpful with visualizing the differences in coefficient stability: the plots in Figure 4 show the instability caused by collinearity cannot be fixed by combining coefficients. In the next section, we will be more thorough by analyzing all US-listed ETFs. That will allow us to conduct a more objective out-of-sample analysis.

5.5 US Exchange-Traded Funds: In-Sample

We can take a wider look at the performance of the liquid factor model by applying it to a large universe of investment funds: 2,310 US exchange-traded funds (ETFs) which were alive as of 30 November 2021. Two of these funds had no returns we could find; and, four funds had data issues which prevented any analysis. We determine the main asset class from the fund category or asset class holdings data, if available. If these are not available or the fund category implies no clear answer (*e.g.* a fund category of "Multiasset"), the main asset is set to "Unclear." The ETFs comprise a range of categories (84, including an "Unknown" category) and asset classes, with the main fund asset being one of fixed income, equities, commodities, real estate, or a foreign currency.

We fit the liquid factor model in partial and full forms, as in the prior examples, over the period of 2014–2025Q1. We use the main asset class to determine the order of factor estimation for the Sequential Shrinkage (full) liquid factor model. For equity-dominant funds, we use a preference ordering of equity factors, yield curve factors, credit factors, then volatility-related and inflation factors. For fixed-income-dominant funds, we use a preference ordering of fixed-income factors, credit factors, equity factors, and then volatility and inflation factors. Specifically:

Equity Ordering: SP500.xs, R2KSPX, R1KVMG, YCLevel, YCSlope, IG10, HY5IG10, YCBump, InflSurp5Y, and VIX.

Fixed-Income Ordering: YCLevel, YCSlope, IG10, HY5IG10, YCBump, SP500.xs, R2KSPX, R1KVMG, InflSurp5Y, and VIX.

We do not explore changing the order of factor estimation since this would likely lead to overfitting.

Since the Kitchen Sink model uses the liquid *instruments* but not the factors, we back-transform the coefficients and coefficient covariance matrix for the liquid instruments to give coefficients and standard errors to those for liquid factors.

Table 11 shows average coefficients and t-stats as well as the standard deviation of coefficients for ETFs holding different asset classes. We note first that the partial and full models do better at explaining the returns for bond, equity, and real estate funds than for commodity funds or funds with an unclear dominant asset class.

[Table 11 about here.]

[Table 12 about here.]

If we compare coefficients among these tables, we can see that the back-transformed Kitchen Sink (Panel A) model often has exaggerated coefficients, larger standard deviations, and smaller t-stats compared to the Corrected Covariates (Panel B) and Sequential Shrinkage (Panel C) models. The larger standard deviations of coefficients agrees with the higher variability of Kitchen Sink coefficients in the prior fund examples. We will investigate that more thoroughly shortly.

The Corrected Covariates model has coefficients much closer to those of the Sequential Shrinkage (full) model; however, the full model often has a smaller standard deviation of the estimated coefficients. Finally, the full model often has more significant t-statistics for factors expect to be significant *a priori* (*e.g.* equity factors for equities, yield curve factors for fixed income).

There are a number of prior expectations we would have for these coefficients: that fixed income funds would be the most sensitive to yield curve factors and would also be sensitive to credit spreads; that equities would be the most sensitive to equity factors; that commodities would be the most sensitive to inflation; and, that real estate would be sensitive to equity and yield curve factors since real estate funds can include mortgage and (real estate) equity investments. We also might expect equity funds to have an average S&P 500 beta of 1.

Looking at Panel C, we can see that these prior expectations largely hold: bond funds are the most sensitive to changes in the level and slope of the yield curve. However, the bond funds are, on average, not sensitive to the yield curve bump (which is related to interest rate volatility) and are not the most sensitive to credit spreads. Equity funds are almost the most sensitive to the equity factors (though commodity funds are slightly more sensitive) and among the least sensitive to changes in the level and slope of the yield curve. Commodity funds are the most sensitive to changes in breakeven inflation as well as being sensitive to equity markets (a surprise), barely sensitive to yield curve factors, but sensitive to credit factors. Real estate funds are indeed sensitive to equity and yield curve factors — as well as credit factors.

Panel C also has some surprises. The significant factor sensitivities of different asset classes almost give signatures to identify the main asset class. Bond ETFs are sensitive to yield curve changes, but also to equity markets and the equity value factor, perhaps because corporate bonds have some equity beta and equity value relates to the book value of assets creditors could seize in a default. The lack of sensitivity to the yield curve bump (related to rate volatility) but sensitivity to credit among real estate funds is also surprising (since real estate investments have collateral). The funds with an unclear main asset were typically sensitive to equity markets, equity size, changes in the yield curve level, and changes in breakeven inflation.

Finally, the average equity fund S&P 500 beta is not 1; it is 0.76. However, if we add the R2KSPX beta of 0.245, we do get close to 1. While we could note that these averages are not weighted by assets under management, the average S&P 500 beta suggests the S&P 500 is not fully representative of all equities held by these funds. Furthermore, we should note that the presence of some foreign equity ETFs as well as some leveraged and inverse-exposure ETFs could lead to divergences from an average beta of 1.

[Table 13 about here.]

[Table 13 about here.]

[Table 13 about here.]

[Table 13 about here.]

Since we have fund categories, we can also examine how the liquid factor model works for individual fund categories. This will also allow us to examine a number of more nuanced expectations. Since we have shown the differences between the partial and full liquid factor models, here we only look at results for the full (Sequential Shrinkage) model.

Table 13 shows the coefficients for each fund category. Since the table is long, the fund categories have been grouped by related asset classes. This is to facilitate comparison between potentially similar fund categories. These groupings may differ from the main asset class in Table 11 since some fund categories are grouped apart from their main asset class held. Convertibles and preferred shares are in a hybrid bond/equity grouping; energy LP, equity energy, equity precious metals, and natural resources are in the commodities grouping despite holding equities as their main asset class.

With this finer grouping of funds, we can check a number of additional expectations we might have. For example, we would expect leveraged funds (Trading-Lev.) to have larger coefficients than most funds in that asset class and inverse funds (Trading-Inv.) to have opposite-sign coefficients. We find that to be true for all of those (bond, equity, and commodity) funds.

The YCLevel coefficient, a negative effective duration, implies greater duration for longer-term bond funds, less for intermediate-term bonds funds, less still for short-term bond funds, and the least for ultra-short-term bond funds. This holds across both general bond funds (Long-Term Bond, Intermediate Bond, Short-Term Bond, Ultrashort Bond), government bond funds (Long-Term Govt, Intermediate Govt, Short-Term Govt), and muni funds (Muni National Long-Term, Muni National Intermediate, Muni National Short-Term). The general bond and government bond funds tend to have similar exposures while the muni funds have much lower exposure.

Corporate bond funds to be have more significant exposure to credit spreads than other bond funds. That is true for both IG and HY credit spreads. For IG credit spreads, the exposure is greater for Corporate Bond and Long-Term Bond funds than for Intermediate Bond and Short-Term Bond funds, which have about the same exposure. Similarly, high yield bond funds have more exposure to the HY spread above the IG spread (HY5IG10) with HY Bond, HY Muni, Long-Term Bond, Corporate Bond, Intermediate Bond, and Short-Term Bond having exposures from most to least. While we might expect government bonds to not be significantly exposed to credit spreads, that is not the case. Their exposure tends to be less than similar-duration general/corporate/HY bond funds, but it is on average significant. EM Bond funds have, on average, significant exposure to IG and HY credit spreads with greater exposure to HY5IG10.

Turning to more unusual bond funds, Bank Loan funds have, on average, a positive YCLevel exposure (*i.e.* negative effective duration) likely due to their exposure to floating rates or short-term borrowing to lend out longer term (maturity transformation); and, the funds are significantly exposed to credit spreads, especially the HY5IG10 credit spread.

Some of the most unusual average coefficients are for Nontraditional Bond funds with many lacking significant exposure to the level, slope, or bump of the yield curve but having significant exposure to credit spreads and equity markets.

We should also note that municipal bond funds have some of the lowest R^2's among all bond funds. This is likely due to liquidity and uncertainty about tax policy affecting those funds much more than for other bond funds.

Convertible bond funds have significant equity market, equity size, and equity volatility exposure as well as some credit exposure. This is not surprising since issuers of convertibles tend to be lower credit/early growth firms. Convertibles funds also have significant negative equity value exposure and significant IG and HY credit exposures.

Preferred Stock funds have, on average, significant negative betas to YCLevel (*i.e.* positive effective duration) only slightly higher than the average Short-Term Bond fund and have a significant positive equity market beta. Since preferreds tend to be issued by firms in more mature industries, we might also expect a positive value exposure and a negative size exposure. While Preferred Stock funds have a significant positive average equity value beta, they also have a significant positive average equity size beta. Furthermore, the funds have an average significant exposure to HY credit spreads (more so than IG spreads) and significant positive exposure to equity volatility. This may be due to preferred stocks being often issued by financial firms.

Turning to equity funds, the fund categorization of large- versus small-cap or value versus growth concurs with the constructed liquid factors. Across large-, mid-, and small-cap funds (foreign and domestic) we see that large-cap funds load the least on the R2KSPX size factor, mid-cap funds load more, and small-cap funds load the most on the factor. Across all these (domestic and foreign), value funds load positively on the R1KVMG value factor while growth funds load negatively on the factor. These loadings are, on average, significant including for foreign funds. Interestingly, blend funds tend to load positively (and significantly) on the R1KVMG value factor, albeit to a lesser degree than value funds. Also, foreign funds load more heavily on the R2KSPX size factor, perhaps because they tend to hold smaller firms than US funds.

Financial and Utilities funds are more exposed to the yield curve level: they have an average t-stat that is higher in magnitude than most other equity funds. While the Utilities funds have a negative exposure (positive effective duration) on the order of an Intermediate Bond fund, the Financial funds have a positive exposure (negative effective duration) of magnitude similar to a Short-Term Bond fund. This is likely due to banks being more exposed to floating interest rates through direct exposure and because they are constantly rolling shorter-term funding while lending out over longer terms (maturity transformation).

Communications and Technology funds typically load significantly on equity size (positively) and equity value (negatively), perhaps because those sectors include more startup (growth) firms and firms holding fewer hard assets versus other sectors. In contrast, Consumer Defensive (*i.e.* staples) and Industrials tend to load significantly and positively on equity value. Surprisingly, Consumer Cyclical funds tend to have a significant negative loading on equity value. Health funds are not typically significantly exposed to equity value.

Also interesting among the equity funds is that almost all foreign equity funds (Diversified EM, Diversified Pacific/Asia, Europe Stock, Foreign Large or S/M Value/Blend/Growth, India Equity, Latin American Stock, and Pacific/Asia ex-Japan Stock) have a significant exposure to the US equity market with betas around 0.7 or higher. Only China Region and Japan Stock funds have a US equity market beta that is below 0.65 (both around 0.6 and significant).

For commodity-related funds, the most consistent significant exposure is to breakeven inflation (positive), bolstering the idea that commodities can protect against inflation shocks.[19] Only the Natural Resources funds are not significantly exposed to changes in breakeven inflation.

Most, but not all, of the commodities funds are significantly and positively exposed to equity markets; Managed Futures funds are not significantly exposed to equity markets and Trading-Inverse Commodity funds are significantly but negatively exposed to equity markets. Similarly, most commodity funds are significantly and positively exposed to IG and HY (above IG) credit spreads with the exception of Managed Futures funds (not significantly exposed to the IG spread) and Trading-Inverse Commodities (significantly but negatively exposed to IG and HY spreads). Some commodity funds have significant positive exposure to changes in the yield curve level (negative effective duration) while only Equity Precious Metals and Trading-Leveraged Commodities have significant positive effective duration. Few commodity fund categories have a significant mean exposure to the yield curve bump or equity volatility.

Global Real Estate and (US) Real Estate funds are significantly and positive exposed to the equity market, size, and value factors (likely due to ownership of some property and that implying the assets being backed by hard assets). They are also typically significantly exposed to changes in the level of the yield curve (with effective duration similar to an Intermediate Bond fund) as well as IG and HY credit spreads. Interestingly, (US) Real Estate funds are not significantly exposed to changes in breakeven inflation while Global RE funds are significantly and positively exposed to breakeven inflation. Furthermore, neither fund grouping is on average significantly exposed to the yield curve bump (which is associated with interest rate volatility). This may be due to commercial real estate not having the same refinancing options as residential real estate. Unsurprisingly, no real estate funds are typically exposed to equity volatility.

The last asset class grouping is foreign exchange, specifically Single Currency funds. These typically have exposure to equity markets and size as well as credit spreads that are statistically significant but economically minor. More relevant is the exposure to changes in the level of the yield curve (of similar effective duration to LT Bond funds) as well as the slope and bump of the yield curve and changes in breakeven inflation.

In general, across all of the asset groupings and fund categories, the model behaves as we would expect: we see value and growth equity funds have sensible R1KVMG coefficients; large- and small-cap equity funds have sensible R2KSPX coefficients; longer-term bond portfolios tend to have more negative YCLevel coefficients with the magnitude corresponding to durations we would expect; corporate bonds have small-but-significant equity market exposure; corporate bonds have exposure to IG and HY credit as appropriate; convertibles and preferred stock funds are significantly exposed to equity market volatility; real estate funds have significant exposure to equity and yield curve factors; and, commodities have significant exposure to inflation surprises.

Finally, we should examine the mean alpha of the fund groupings. In general, almost all of the alphas are not significantly different from zero. This provides more reassurance that our model is well-specified as in Merton (1973) and captures a lot of the important variation in fund returns.

Interestingly, the only groupings with significant alpha are the Trading-Inverse Debt and Trading-Inverse Equity funds, which have significant *negative* alpha (-5.5 bp and-23.9 bp). The Trading-Inverse Commodity funds have a mean alpha that is -18.5 bp but not quite statistically significant. Given the many concerns about these funds (see, for example, SEC (2023)), we could argue that even these funds have coefficient estimates we would expect.

[19]While commodities are sometimes proposed as protection against inflation, that is only true before the shocks to inflation are realized.

6 US Exchange-Traded Funds: Out-of-Sample

The prior analysis was in-sample. Since the universe of ETFs analyzed was funds with reserved tickers as-of 30 November 2021, we can also conduct a true out-of-sample analysis. Given that the full liquid factors model uses shrinkage estimators, we might expect it to show show smaller out-of-sample variance than the other models.

A proper out-of-sample analysis must consider survivorship issues: funds can be closed and delisted. We can detect this by watching for a string of days without data. While tickers may be reused by another fund, SEC (2016) guidelines (and historical information) on ticker reuse suggest reuse does not happen within at least 90 days.[20]

An ETF could close near the start of the out-of-sample period (20211130). We eliminate any funds with at least 65 consecutive trading days (about 90 calendar days) of no returns that includes 20211130. This prevents us including later funds using that ticker — which could add a survivorship bias. If a delisting occurs during the out-of-sample period, we use only the data up to the delisting and not after (to avoid ticker reuse). Handling delistings at the start of the out-of-sample period reduces our universe of 2,308 funds by 20 to 2,288.[21]

We begin by revisiting the idea of coefficient stability. Changes in model coefficients may cause a need to rehedge. To assess how big these changes are, we look at how coefficients change when we re-estimate the model as time moves forward. Since either a positive or negative change could result in a rehedge, we look at absolute changes. Table 14 shows the mean absolute 1-day-forward coefficient changes for the three versions of the liquid factor model as well as the Fama-French (FFC6 and FF93L) models. This lets us see if the innovations of the liquid factor model are beneficial out-of-sample and compare that with the performance of the Fama-French models.

[Table 14 about here.]

We can see that the 1-day changes are generally small, apart from the YCSlope coefficient for harder-to-model asset classes (commodities, FX). Comparing liquid factor models, we can see the value of feature engineering: the Corrected Covariates model has smaller beta changes than the Kitchen Sink model for all but equities alpha and VIX beta; and, those differences are minimal. Similarly, we can see the value of TIPLS estimation: the Sequential Shrinkage model has smaller beta changes for all but the bonds YCSlope beta and a few alphas; and, those differences are also small. Putting the innovations together, we can see that the changes for the Sequential Shrinkage model are often an order of magnitude smaller than for the Kitchen Sink model.

Comparing the Sequential Shrinkage liquid factor model to the Fama-French (FFC6, FF93L) models, we see that for comparable factors, the full liquid factor model has smaller coefficient changes than the Fama-French models, often by a factor of 2 or more. The only estimate with larger average changes is the equity alpha, and the difference between the models for that is minimal. Using liquid instruments, creating less-correlated liquid factors, and estimating coefficients using TIPLS all help yield a more stable model out-of-sample across multiple asset classes.

We might wonder if these benefits hold up as we look further ahead in time: do coefficient estimates hold up well a month or quarter later? To assess this, we look at changes versus models estimate 21 and 65 trading days (one month and one quarter) later.

[Table 15 about here.]

[20] SEC (2022) amended this to allow a small number of tickers to be reserved for 2 years or in perpetuity.

[21] The eliminated tickers are BNKD, BNKU, CAPE, CHGX, DSTX, FDIV, GBUG, GERM, IBTA, INFR, IVES, JDIV, JHMU, JUSA, KNGS, MJUS, NRGD, NRGU, RCD, SHLD. CAPE is an interesting case since the ticker reuse involved the fund shifting to a new investment management firm.

Looking at the average monthly beta changes in Table 15, the Kitchen Sink model has some large changes. For example, an average equity market beta change of 0.4 in one month is not immaterial. As with the 1-day changes though, the changes for the Corrected Covariates model are smaller in almost all cases except the commodities and equities alphas and the equities credit spreads (IG10 and HY5IG10). The Sequential Shrinkage model has smaller beta changes than the Corrected Covariates model for almost all cases, apart from bonds, commodities, and FX alphas, real estate R2KSPX, and bonds YCSlope. Yet again, we see that out-of-sample feature engineering and TIPLS estimation have yielded models which are more stable.

Comparing with the Fama-French models, the liquid factor model has smaller beta changes for comparable coefficients for all except the equities alpha and commodities equity market beta.

Looking at the average quarterly beta changes in Table 16, the Kitchen Sink model has some large changes. For example, an average equity market beta change of 0.8 in one month is troubling. As with the 1-day and monthly changes, the Corrected Covariates model has smaller changes in almost all cases except some alphas, the credit spreads for bonds, equities, real estate, and the unclear asset class grouping, and the commodities VIX beta. In those cases the betas are generally almost identical. The Sequential Shrinkage model has smaller beta changes than the Corrected Covariates model for almost all cases, apart from equities and real estate R2KSPX, bonds YCLevel, YCSlope, IG10, and HY5IG10, FX YCSlope, and commodities IG10. In the cases where the Sequential Shrinkage model does not have the smallest average coefficient changes, the difference is usually very small. Some of the Sequential Shrinkage alpha changes are slightly larger than those in the Corrected Covariates model.

Comparing with the Fama-French models, almost all changes for the Sequential Shrinkage liquid factor model are smaller than for the Fama-French models except for the equities equity market and real estate equity size betas and commodities, equities, and unclear-asset alphas.

[Table 16 about here.]

Therefore, it seems that the full liquid factor model is the most stable model analyzed out-of-sample. Having consistently smaller beta changes across 1-day, 1-month, and 1-quarter timelines suggests that the model is probably closer to properly explaining fund returns across multiple asset classes and investment strategies.

However, this analysis does not reflect how we would probably use the model: most analysts would not pay attention to statistically insignificant coefficients. Therefore, we repeat the preceding analysis but only for coefficients with a t-stat greater than 2 in magnitude.

To handle insignificant coefficients, we set all insignificant coefficients to 0; and, we only compare coefficients when at least one of them is significant. This could be a harsh practice because a coefficient flipping in and out of significance would result in averaging multiple changes between a typical coefficient and 0. However, coefficients flipping in and out of significance do make us question the model, so we will retain this possibly-harsh approach.

In Table 17, we compare beta changes for significant coefficients. For 1-day significant changes, the Corrected Covariates model has smaller changes than the Kitchen Sink model for all but the bonds, commodity, equity, and real estate alphas; commodity and unclear asset equity value; commodity, equity, and real estate HY5IG10; commodity, equity, FX, and real estate inflation surprise, and all VIX betas except for real estate. Furthermore, changes involving a significant coefficient were 13%–28% for Kitchen Sink versus 27%–36% for Corrected Covariates.

The Sequential Shrinkage model has smaller significant beta changes than the Corrected Covariates model for all but the FX equity size, bond and commodity equity value, commodity and FX YCSlope, and inflation surprise for real estate and unclear assets. Sequential Shrinkage also has more significant coefficients: 26%–43% of comparisons involve a significant coefficient. Finally, the Sequential Shrinkage model has smaller changes than the Fama-French models for all but the equity and FX alphas, S&P 500 betas for bonds, commodities, equities, and FX, EqSize for bonds, EqValue for commodities and FX, and YCSlope for commodities. When the more innovative model has larger significant beta changes, the difference between the two models is minimal.

While the performance of the full liquid factor model is generally superior for significant coefficients as well, we do see more cases where it is close to other models. This is likely due to the harshness of this test as discussed above. It is therefore impressive that the models which reduce standard errors are still largely dominant.

[Table 17 about here.]

Tables 18 and 19 show even stronger relative results for comparisons 21 and 65 trading days (about one month and one quarter) in the future.

[Table 18 about here.]

For 21-day comparisons, the Corrected Covariates model had smaller significant beta changes than the Kitchen Sink model for all except commodities, bonds, equities and real estate alphas, equity size for unclear assets, IG10 for commodities, equities, and unclear assets, HY5IG10 for all asset classes except unclear assets, and VIX for commodities, equities, FX, and unclear assets. The Sequential Shrinkage model performed even better having smaller significant beta changes for all except commodities alpha and the YCSlope for bonds and FX. The Sequential Shrinkage model also had smaller significant beta changes than the Fama-French models for commodities and equities alphas, commodities and FX EqMkt, and FX YCSlope. Furthermore, in cases where the model with more innovations had larger significant beta changes, the two were often very close.

[Table 19 about here.]

For 65-day comparisons, the Corrected Covariates model had smaller significant beta changes than the Kitchen Sink model for all coefficients except bond, commodity, and equity alphas; unclear asset equity size; commodity and FX equity value; commodity, FX, and unclear asset IG10; commodity, equity, and real estate HY5IG10; inflation surprise for commodities, equities, and real estate, and VIX for commodities, equities, FX, and real estate. The Sequential Shrinkage model again beats the Corrected Covariates model with smaller significant beta changes for all coefficients except commodity alpha, real estate equity size, bonds YCLevel, and YCSlope for bonds and FX. The Sequential Shrinkage model also has smaller significant beta changes than the Fama-French models for all coefficients except commodity and equity alpha, EqMkt for commodities, equities, and FX, and YCSlope for commodities and FX. Again, when the more innovative model has larger significant beta changes, the difference between the models is often small.

The fraction of changes involving significant coefficients in these models varies from 16%–31% for the Kitchen Sink model to 28%-47% for the Sequential Shrinkage model. Thus the full liquid factor model, Sequential Shrinkage, performs best at out-of-sample stability and is even more stable when we look at significant coefficients. Furthermore, the model is able to reduce collinearity such that there are more significant coefficients: the model better focuses on what is relevant.

Another metric of interest is out-of-sample R^2. This can be illuminating because it typically shows diminished explanatory power since there is an element of forecasting future betas. Furthermore, out-of-sample R^2 can even be negative if a model adds noise to an out-of-sample explanation. Table 20 shows the out-of-sample R^2s summarized by asset class and model.

[Table 20 about here.]

Out-of-sample R^2s yield two stark observations. First, the transformed Kitchen Sink coefficients are useful for comparison but add noise out-of-sample. That is surprising but yet another negative to doing a Kitchen Sink regression and combining coefficients afterward. Second, the Fama-French models perform comparatively worse out-of-sample for all asset classes except equities.

Otherwise, the three versions of the liquid factor model have similar out-of-sample R^2s. The Sequential Shrinkage model performs a little better for bond funds, a little worse for commodity, FX, and real estate funds, and about the same for equity and unclear/multi-asset funds. Furthermore, even these out-of-sample R^2 differences are small among the liquid factor models.

The models also age differently: the drop in out-of-sample R^2 as we attempt to explain fund returns further out is not the same. The fall-off in out-of-sample R^2 from 1-day forward to 65-days forward is, in general, the least for the Sequential Shrinkage model, greater for the Corrected Covariates model, greater still for the Kitchen Sink model, and greatest for the Fama-French models.

This suggests that (1) using liquid instruments improves out-of-sample R^2; (2) the Sequential Shrinkage (full) model improves out-of-sample model stability (which likely reduces hedging costs); and, (3) the full model ages more gracefully in that its coefficients degrade most slowly.

Given the similarity of out-of-sample R^2 among the various liquid factor models, why not use any of them? Why go to the trouble of forming factors and estimating sequentially? The answer lies in the preceding tables: model stability. With other models, we must accept more variable model coefficients and, if we use those to hedge, will pay more in transactions costs.

However, as noted previously, we would not likely use insignificant betas for hedging or replication. How do out-of-sample R^2s compare when we use only significant betas? Table 21 shows the out-of-sample R^2s when using only significant betas, summarized by asset class and model.

[Table 21 about here.]

Here we see greater differentiation between the models. Using only significant betas saves on trading costs, but it also greatly reduces the out-of-sample R^2s for *most* models. The Kitchen Sink model has a negative out-of-sample R^2 (*i.e.* it *adds* noise) for commodities and unclear/multi-asset funds; for FX, R^2s are in the single digits (versus in the twenties with all coefficients); and, R^2s for bonds, equities, and real estate are 1/3 to 1/2 lower versus for all coefficients. The Kitchen Sink with Transformed coefficients adds less noise than with using all coefficients. The Corrected Covariates R^2s are typically 1/10 to 1/5 lower (versus for all coefficients) for bond, commodity, and FX funds while they are almost unchanged for equity-dominant funds (equities, real estate, and unclear/multi-asset). The Fama-French models have much lower out-of-sample R^2s than the Corrected Covariates or Sequential Shrinkage models for all but equity and real estate funds.

The Sequential Shrinkage model, on the other hand, has out-of-sample R^2s which are almost unchanged from those when looking at all coefficients. The model's preferencing makes it more stable even when we use coefficients which are not statistically significant. The Sequential Shrinkage liquid factor model also has the highest R^2s in almost all cases.

Potential users of these models must balance multiple tensions. The full liquid factor (Sequential Shrinkage) and Fama-French models for some asset classes have the most stable coefficients out-of-sample; The full liquid factor model is even more stable when we only consider significant coefficients. The out-of-sample R^2s for many of the incomplete liquid factor models are similar if we use all coefficients; however, versions of the model with fewer innovations have more variable coefficients, which would reduce trust in the model and greatly increase hedging costs.

This all suggests we use the full liquid factor model with only significant coefficients. In that case, coefficient changes are much smaller while the out-of-sample R^2s are almost unchanged from using all coefficients. Other models, however, see reduced out-of-sample R^2s. The full liquid factor model also degrades slower in terms of out-of-sample R^2s as we look farther ahead in time. These results highlight that comparisons of factor models which ignore model stability or factor significance when measuring model may be unhelpful.

7 Conclusion

Our goal was not to create a factor model with higher explanatory power than other models but to create a model that allows for easily hedging factor exposures and increases out-of-sample stability to reduce unnecessary hedging (and costs) — while still explaining enough of returns to be useful.

As we have noted, these demands are especially important for large portfolios, such as those held by pensions and endowments. However, these demands are also important for private capital investments like hedge funds, private equity, and private credit as well as illiquid publicly-traded portfolios. Such portfolios may be difficult or impossible to alter. In these cases, an overlay solution using derivatives and other liquid instruments is crucial to adjusting a portfolio's risk; a solution which requires trading the portfolio's assets is often infeasible.

Hedging with liquid factors may be more capital-efficient since futures, swaps, and government bonds with repo require less capital to hold than adding factor portfolios of securities. This hedging may also incur lower trading costs than adjusting portfolio holdings, and may even be more tax-efficient due to preferential tax treatment of some derivatives, not forcing the portfolio to incur taxable gains, and allowing short-term gains to be held until they become long-term gains.

We began with theoretical justifications for why using factors built from liquidly-tradeable instruments should be beneficial. We then made these ideas concrete with specific instruments which the market, through revealed preference, has supported: their very liquidity suggests that we should consider them. That gave us base instruments which are consistent, transparent, and cheap. We then created low-correlation factors from small combinations of these instruments to isolate particular risks (feature engineering); and, we estimated these exposures with theory-informed partial least squares (TIPLS), a machine learning/shrinkage method that preferences using more relevant and liquid factors. Together, these two innovations yielded the full liquid factor model.

We found that the full liquid factor model offers explanatory power very similar to models currently preferred in academia and often has higher beta t-stats and alphas closer to 0. Out-of-sample, the full liquid factor model performs superior to the other models: it has the lowest variability of beta estimates and the highest explanatory power using all or only significant coefficients. It does this out to at least one quarter and performs at this high level across asset classes.

These benefits have value to academics, who will have less spurious changes in estimates to explain when comparing models and can compare asset classes beyond equities (and even across asset classes). These benefits also have value to litigation consultants who want stable estimates of factor effects that account for macroeconomic risks, as discussed in Robertson, Akey, and Simutin (2025). Finally, it has value to practitioners who want confidence in their risk models and prefer less unnecessary trading to lower transactions costs if they choose to hedge some factors.

We also found evidence that the nearly-universal omission of changes in the level of the yield curve from factor models is not justified: that fixed income, FX, and real estate funds are often significantly exposed to this factor along with some multi-asset, equity, and commodity funds. This corroborates the long-short interest rate confusion studied in Shue, Townsend, and Wang (2025).

The economics behind liquid factors mean the ideas are general: liquid factor models can be ported to other developed markets. We could easily use the EUR, GBP, JPY, or AUD yield curve; the Stoxx Europe 600 and MSCI Europe Small Cap; FTSE 100 and 250; Nikkei 225 and TOPIX; or

ASX 200 and ASX Small Ordinaries; VSTOXX, IVI, Nikkei 225 Volatility, and A-VIX indices; and, iBoxx IG and HY indices for Europe and the UK.[22] One could also study how well the American versions of those factors correlate with less-liquid analogues in those countries: Panel D of Table 13 suggests foreign value and growth funds may be well-modeled with the US value factor.

Liquid factor models also offer benefits over the latent factor models common in industry. The factors are transparent and easy to compute. An investor exposed to these factors can easily and cheaply hedge their exposure: they merely trade the instruments used to build the liquid factors. Liquid factor models are also more reproducible: changes in the estimation universe do not affect past factor returns. Furthermore, the factors here are not subject to quiet revision as documented by Akey, Robertson, and Simutin (2023) with Fama-French factors.

Most useful is that the full liquid factor model performs best where we want it to: out-of-sample for factors significant enough to consider hedging. Trading these factors costs a few basis points versus research suggesting trading the factors in current academic models costs a few hundred basis points per year. This is also our best indication of relative hedging costs. Finally, liquid factors can be hedged using derivatives which requires less capital versus factors in academic models which cannot be traded using derivatives and thus use more capital.

The liquid factor model offers a more stable, cheaper, actionable version of the Fama and French (1992, 1993) models while also adding other cross-asset factors. We believe this demonstrates clearly the value of using liquid factors.

Declaration of Interest

We have received no grants, payments, honoraria, patents, nor anything else of value for this work. We are also not involved with any advocacy groups related to the work. We are employed by an investment management firm which uses models like those discussed above.

Appendices

A Variance Inflation Factors

In linear modeling, a variance inflation factor (VIF) shows how much the variance of a coefficient estimate has been increased due to multicollinearity (correlation across model variables).

If we regress each factor i on the other model variables, we get residuals $\hat{\epsilon}_{it}$ and an $\hat{R}_i^2$ that estimates a squared joint correlation of variable i with other model variables. This affects the variance of a coefficient estimate $\hat{\beta}_i$ directly:

$$\text{var}(\hat{\beta}_i) = \frac{\text{var}(\hat{\epsilon}_{it})}{(n-1)\,\text{var}(X_i)} \times \underbrace{\frac{1}{1-\hat{R}_i^2}}_{VIF_i} . \tag{7}$$

Since the model variable R_i^2 yields a multiplier, we call that multiplier the variance inflation factor.

From the VIF_i formula, we see that as model variables become more multicollinear ($\hat{R}_i^2$ rises), the VIF_i also rises. What this means practically is that we should prefer to estimate our models with variables that are not jointly correlated — or with combinations of variables that are less correlated — because we will then be more certain of our coefficient estimates. Furthermore, this

[22] The ASX Small Ordinaries and A-VIX indices are not yet traded as futures; however, they are accessible via swaps and the Small Ordinaries is also accessible via ETFs.

makes the coefficients less likely to change (spuriously) across time. Put simply: models with less correlated/more orthogonal factors are better in terms of estimation noise and coefficient variability.

B Issues with Illiquid and Lagging Returns

While we think liquid factors may be superior for assessing instrument and portfolio returns, there are potential issues which can arise if these returns are for illiquid or lagging instruments.

Suppose we analyze daily returns for a portfolio, but some instruments' prices are published with a lag. This can happen for illiquid instruments which may only trade every few days or which trade even less frequently but are occasionally marked to an estimated price. This can also happen for foreign instruments whose market close does not align with the US market close. In these cases, we will likely see estimated factor coefficients biased toward 0.

Lagging price updates can induce autocorrelations in the returns we model and in how those returns align with liquid factors. There are a few ways we can handle these lagging prices.

A common way to handle lagging prices is to aggregate returns to a coarser time scale such as weekly, monthly, or quarterly. This however, loses all the variation between those date points which results in a loss of estimation efficiency. If some prices are lagging, we should see factors explaining those returns changing as we move to a (coarser) time scale less affected by lagging prices. We might also see loadings shift from one factor to a related factor if, on the coarser time scale, cross-correlations start to spread a shock in one factor to other factors. Finally, we might see some factors become less significant if they do not have risk premia but mostly hedge volatility.

Alternately, we can add lagged terms for some factors to our model. Specifically, we would add lags for factors we suspect might be relevant to explaining lagging prices. The lags should only cover a short time period, sufficient to catch lagging price updates: overnight for foreign instruments, maybe a few days for less-liquid instruments. Since this keeps the more frequent data at a cost of a handful more of parameters, this does not cause much of a loss in estimation frequency. The total hedge is then the sum of the coefficients (contemporaneous and lagged) for a factor. However, this approach means we might have to live with what seems to be a noisy hedge — a hedge that has more tracking error on a daily basis but less on a coarser time scale and which accurately reflects what we would see if prices did not lag.[23]

Finally, if we care about volatility, we should remember that bid-ask bounce is a greater proportion of daily returns. As documented in the high-frequency realized variance literature, we might see that volatility estimation is more efficient but biased higher when estimated using daily versus monthly returns. Lagging prices also tend to smooth out volatility. Thus we might want to fit a time series model to recover the lag structure, consider the part of volatility due to bid-ask bounce, and use this with the lag structure to infer the volatility of the driving data generating process. Together, these steps can help us better see the risk-return tradeoff of an investment.

C Transforming Kitchen Sink Coefficients

Some Kitchen Sink coefficients are easy to transform for comparison with other model forms. The intercept and VIX betas are clearly identical. Isolating groups of coefficients makes a few identities clear upon inspection. First, we disentangle the equity market and size factor betas: $\beta_{SP500.xs} = \beta_{SP500} + \beta_{R2000}$ and $\beta_{R2KSPX} = \beta_{R2000}$. Next, we disentangle the credit and inflation betas: $\beta_{IG10} = \beta_{IBOXIG} + \beta_{IBOXHY}$, $\beta_{HY5IG10} = \beta_{IBOXHY}$, and $\beta_{InflSurp5Y} = -\beta_{TIPS5Y}$.

[23]This suggests a test for lagging price updates: if the tracking error does not scale with the square root of time.

The equity value and remaining fixed income betas require a little more work or assumptions. Sometimes, there is no clear solution and so we must approximate.

For the equity value factor, we are trying to equate

$$\underbrace{\beta_{R1KVMG}(R1VAL - R1GRO)}_{\text{Corrected Covariates form}} \stackrel{?}{=} \underbrace{\beta_{R1VAL}R1VAL + \beta_{R1GRO}R1GRO}_{\text{Kitchen Sink form}}. \tag{8}$$

If $\beta_{R1VAL} = -\beta_{R1GRO}$, this is trivial: $\beta_{R1KVMG} = \beta_{R1VAL} = -\beta_{R1GRO}$. However, if these coefficients are not equal and of opposite sign, we hit an impasse.

Will an average work? Since value and growth are often defined as opposite ends on a spectrum of valuation metrics, we might expect that $\beta_{R1VAL} \approx -\beta_{R1GRO}$. In that case, if we equate $\beta_{R1KVMG} = \frac{\beta_{R1VAL}-\beta_{R1GRO}}{2}$, we then have that

$$\begin{aligned}
&\overbrace{\frac{\beta_{R1VAL} - \beta_{R1GRO}}{2}}^{\beta_{R1KVMG}}\overbrace{(R1VAL - R1GRO)}^{R1KVMG} && (9)\\
&\quad =\frac{\beta_{R1VAL}R1VAL + \beta_{R1GRO}R1GRO}{2} - \frac{\beta_{R1VAL}R1GRO + \beta_{R1GRO}R1VAL}{2} \\
&\quad \approx\frac{\beta_{R1VAL} - \beta_{R1GRO}}{2}R1VAL - \frac{\beta_{R1VAL} - \beta_{R1GRO}}{2}R1GRO && (10)\\
&\quad =\beta_{R1VAL}R1VAL + \beta_{R1GRO}R1GRO. && (11)
\end{aligned}$$

Since this is the most tractable assumption, we will make this. However, this is another source of possible differences between the Kitchen Sink and Corrected Covariates models.

The yield curve coefficient transformations are tougher to divine since they form a system of simultaneous equations.

$$\begin{aligned}
&\beta_{3M}T3M + \beta_2 T2Y + \beta_5 T5Y + \beta_{10}T10Y + \beta_{30}T30Y \\
&\quad =\beta_{Lvl}YCLevel + \beta_{Slp}YCSlope + \beta_{Bmp}YCBump - \beta_{IG10}(-8.72)T10Y + && (12)\\
&\qquad - \beta_{HY5IG10}((-4.73)T5Y - (-8.72)T10Y) - \beta_{InflSurp5Y}T5Y \\
&\quad =\beta_{Lvl}\frac{T3M + T2Y + T5Y + T10Y + T30Y}{5} + \frac{\beta_{Slp}}{2}\left(\frac{T30Y - T3M}{19.2 - 0.25} + \frac{T10Y - T2Y}{8.72 - 1.98}\right) \\
&\qquad + \beta_{Bmp}\left(\frac{T30Y - T5Y}{19.2 - 4.73} - \frac{T5Y - T3M}{4.73 - 0.25}\right) + \beta_{IBOXIG}8.72T10Y && (13)\\
&\qquad + \beta_{IBOXHY}(4.73T5Y - 8.72T10Y) - \beta_{TIPS5Y}T5Y.
\end{aligned}$$

We can then move known coefficients from the Kitchen Sink model to the left hand side:

$$\begin{aligned}
&\beta_{3M}T3M + \beta_2 T2Y + (\beta_5 - 4.73\beta_{IBOXHY} - \beta_{TIPS5Y})T5Y \\
&\qquad + (\beta_{10} + 8.72(\beta_{IBOXIG} - \beta_{IBOXHY}))T10Y + \beta_{30}T30Y \\
&\quad =\beta_{Lvl}\frac{T3M + T2Y + T5Y + T10Y + T30Y}{5} + \frac{\beta_{Slp}}{2}\left(\frac{T30Y - T3M}{19.2 - 0.25} + \frac{T10Y - T2Y}{8.72 - 1.98}\right) && (14)\\
&\qquad + \beta_{Bmp}\left(\frac{T30Y - T5Y}{19.2 - 4.73} - \frac{T5Y - T3M}{4.73 - 0.25}\right).
\end{aligned}$$

Consolidating by instrument implies the following set of equations:

$$\begin{aligned}
T3M: & \frac{\beta_{Lvl}}{5} & -\frac{\beta_{Slp}}{2(19.2-0.25)} & +\frac{\beta_{Bmp}}{4.73-0.25} & = \beta_{3M} && (15)\\
T2Y: & \frac{\beta_{Lvl}}{5} & -\frac{\beta_{Slp}}{2(8.72-1.98)} & & = \beta_{2} && (16)\\
T5Y: & \frac{\beta_{Lvl}}{5} & & -\frac{\beta_{Bmp}(19.2-0.25)}{(19.2-4.73)(4.73-0.25)} & = \beta_5 - 4.73\beta_{IBOXIG} &&\\
& & & & \quad - \beta_{TIPS5Y} && (17)\\
T10Y: & \frac{\beta_{Lvl}}{5} & +\frac{\beta_{Slp}}{2(8.72-1.98)} & & = \beta_{10} + 8.72\beta_{IBOXIG} &&\\
& & & & \quad - 8.72\beta_{IBOXHY} && (18)\\
T30Y: & \frac{\beta_{Lvl}}{5} & +\frac{\beta_{Slp}}{2(19.2-0.25)} & +\frac{\beta_{Bmp}}{19.2-4.73} & = \beta_{30}. && (19)
\end{aligned}$$

Thankfully, we can recover a solution for the coefficients through combinations of equalities like those used to construct the yield curve factors.

If we add all five equalities, we get:

$$\begin{aligned}
\beta_{Lvl} &= \beta_{3M} + \beta_2 + \beta_5 - 4.73\beta_{IBOXIG} - \beta_{TIPS5Y} + \beta_{10} + 8.72(\beta_{IBOXIG} - \beta_{IBOXHY}) + \beta_{30} && (20)\\
&= \beta_{3M} + \beta_2 + \beta_5 + \beta_{10} + \beta_{30} + 3.99\beta_{IBOXIG} - 8.72\beta_{IBOXHY} - \beta_{TIPS5Y}. && (21)
\end{aligned}$$

Adding the T10Y and T30Y equalities and subtracting the T2Y and T3M equalities, we get:

$$\frac{\beta_{Slp}(19.2+8.72-1.98-0.25)}{(19.2-0.25)(8.72-1.98)} = -\beta_{3M} - \beta_2 + \beta_{10} + 8.72(\beta_{IBOXIG} - \beta_{IBOXHY}) + \beta_{30}, \quad (22)$$

$$\implies \beta_{Slp} = \frac{(19.2-0.25)(8.72-1.98)}{19.2+8.72-1.98-0.25}\left(-\beta_{3M} - \beta_2 + \beta_{10} + 8.72(\beta_{IBOXIG} - \beta_{IBOXHY}) + \beta_{30}\right). \quad (23)$$

Adding the T3M and T30Y and subtracting $2\times$ the T5Y equalities, we get:

$$\frac{\beta_{Bmp}3(19.2-0.25)}{(19.2-4.73)(4.73-0.25)} = \beta_{3M} - 2(\beta_5 - 4.73\beta_{IBOXIG} - \beta_{TIPS5Y}) + \beta_{30}, \quad (24)$$

$$\implies \beta_{Bmp} = \frac{(19.2-4.73)(4.73-0.25)}{3(19.2-0.25)}\left(\beta_{3M} - 2(\beta_5 - 4.73\beta_{IBOXIG} - \beta_{TIPS5Y}) + \beta_{30}\right). \quad (25)$$

D Kitchen Sink Liquid Instrument Estimations by Asset Class

[Table 22 about here.]

References

Akey, Pat, Adriana Z. Robertson, and Mikhail Simutin. *Noisy Factors*. Working Paper 3930228. SSRN, 2023. DOI: 10.2139/ssrn.3930228.

Arnott, Robert D. "Risk Budgeting and Portable Alpha". *Journal of Investing* 11.2 (2002), pp. 15–22. DOI: 10.3905/joi.2002.319501.

Bai, Jennie, Turan G. Bali, and Quan Wen. "RETRACTED: Common risk factors in the cross-section of corporate bond returns". *Journal of Financial Economics* 131.3 (2019), pp. 619–642. DOI: 10.1016/j.jfineco.2018.08.002.

Bailey, Natalia, George Kapetanios, and M. Hashem Pesaran. "Measurement of factor strength: Theory and practice". *Journal of Applied Econometrics* 36.5 (2021), pp. 587–613. DOI: 10.1002/jae.2830.2021.

Bašić, Filip et al. "Transaction Cost-Optimized Equity Factors around the World". *Journal of Portfolio Management* 50.6 (2024), pp. 40–73. DOI: 10.3905/jpm.2024.1.599.

Ben Dor, Arik et al. "DTS^{SM} (Duration Times Spread)". *Journal of Portfolio Management* 33.2 (2007), pp. 77–100. DOI: 10.3905/jpm.2007.674795.

Berk, Johnathan B. and Jules H. van Binsbergen. "Measuring Skill in the Mutual Fund Industry". *Journal of Financial Economics* 118.1 (2015), pp. 1–20. DOI: 10.1016/j.jfineco.2015.05.002.

Bessembinder, Hendrik, Aaron Burt, and Christopher M. Hrdlicka. *More factors matter and factors matter more than you might think: The role of time variation in factor premia.* Working Paper 3992041. SSRN, 2024. DOI: 10.2139/ssrn.3992041.

Boudt, Kris, Ellen C.S. Paulus, and Dale W.R. Rosenthal. "Funding liquidity, market liquidity and TED spread: A two-regime model". *Journal of Empirical Finance* 43 (2017), pp. 143–158. DOI: 10.1016/j.jempfin.2017.06.002.

Brière, Marie and Ombretta Signore. "Inflation-Hedging Portfolios: Economic Regimes Matter". *Journal of Portfolio Management* 38.4 (2012), pp. 43–58. DOI: 10.3905/jpm.2012.38.4.043. URL: https://ssrn.com/abstract=1945818.

Carhart, Mark M. "On Persistence in Mutual Fund Performance". *Journal of Finance* 52.1 (1997), pp. 57–82. DOI: 10.1111/j.1540-6261.1997.tb03808.x.

Cboe Global Markets. *Cboe Global Markets Launches Cboe® iBoxx® iShares® $ High Yield Corporate Bond Index (IBHY) Futures.* Sept. 2018. URL: https://ir.cboe.com/sites/cboe-ir-v1/files/cboe/news-and-events/press-releases/2018/cboe-launches-ibhy-futures-plans-for-ibig-9-10-18-final.pdf.

Chen, Andrew Y. and Mihail Velikov. "Zeroing In on the Expected Returns of Anomalies". *Journal of Financial and Quantitative Analysis* 58.3 (2023), pp. 968–1004. DOI: 10.1017/S0022109022000874.

Chen, Nai-Fu, Richard Roll, and Stephen A. Ross. "Economic Forces and the Stock Market". *Journal of Business* 59.3 (1986), pp. 383–403. DOI: 10.1086/296344.

Chordia, Tarun, Amit Goyal, and Alessio Saretto. "Anomalies and False Rejections". *Review of Financial Studies* 33.5 (2020), pp. 2134–2179. DOI: 10.1093/rfs/hhaa018.

Cochrane, John H. "Presidential Address: Discount Rates". *Journal of Finance* 66.4 (2011), pp. 1047–1108. DOI: 10.1111/j.1540-6261.2011.01671.x.

Connor, Gregory. "The Three Types of Factor Models: A Comparison of Their Explanatory Power". *Financial Analysts Journal* 3 (1995), pp. 42–46. DOI: 10.2469/faj.v51.n3.1904.

Cremers, Martijn, Antti Petajisto, and Eric Zitzewitz. "Should Benchmark Indices Have Alpha? Revisiting Performance Evaluation". *Critical Finance Review* 2.1 (2012), pp. 1–48. DOI: 10.1561/104.00000007.

Dai, Chaoxing, Kun Lu, and Dacheng Xiu. "Knowing Factors or Factor Loadings, or Neither? Evaluating Estimators of Large Covariance Matrices with Noisy and Asynchronous Data". *Journal of Econometrics* 208.1 (2019), pp. 43–79. DOI: 10.1016/j.jeconom.2018.09.005.

Dannhauser, Caitlin D. "The impact of innovation: Evidence from corporate bond exchange-traded funds (ETFs)". *Journal of Financial Economics* 125.3 (2017), pp. 537–560. DOI: 10.1016/j.jfineco.2017.06.002.

de Jong, Sijmen. "SIMPLS: An alternative approach to partial least squares regression". *Chemometrics and Intelligent Laboratory Systems* 18.3 (1993), pp. 251–263. DOI: 10.1016/0169-7439(93)85002-X.

Dickerson, Alexander, Philippe Mueller, and Cesare Robotti. "Priced risk in corporate bonds". *Journal of Financial Economics* 150.2 (2023), p. 103707. DOI: 10.1016/j.jfineco.2023.103707.

Dickerson, Alexander and Yoshio Nozawa. *Pricing Corporate Bonds with Credit Risk Primitives*. Working Paper 4822379. SSRN, 2024. DOI: 10.2139/ssrn.4822379.

Du, Wenxin, Benjamin Hébert, and Wenhao Li. "Intermediary balance sheets and the treasury yield curve". *Journal of Financial Economics* 150.3 (2023), p. 103722. DOI: 10.1016/j.jfineco.2023.103722.

Elkamhi, Redouane, Jacky S. H. Lee, and Marco Salerno. "Factor Investing Using Capital Market Assumptions". *Journal of Portfolio Management* 48.2 (2022), pp. 119–143. DOI: 10.3905/jpm.2021.1.291.

— "Portfolio Tilts Using Views on Macroeconomic Regimes". *Journal of Portfolio Management* 49.3 (2023), pp. 7–24. DOI: 10.3905/jpm.2022.1.438.

Fama, Eugene F. and Kenneth R. French. "The Cross-Section of Expected Stock Returns". *Journal of Finance* 47.2 (1992), pp. 427–465. DOI: 10.1111/j.1540-6261.1992.tb04398.x.

— "Common Risk Factors in the Returns on Stocks and Bonds". *Journal of Financial Economics* 33.1 (1993), pp. 3–56. DOI: 10.1016/0304-405X(93)90023-5.

— "Dissecting Anomalies with a Five-Factor Model". *Review of Financial Studies* 29.1 (2016), pp. 69–103. DOI: 10.1093/rfs/hhv043.

— *Production of U.S. $Rm - Rf$, SMB, and HML in the Fama-French Data Library*. Working Paper 4629613. SSRN, 2023. DOI: 10.2139/ssrn.4629613.

Feng, Guanhao, Stefano Giglio, and Dacheng Xiu. "Taming the Factor Zoo: A Test of New Factors". *Journal of Finance* 75.3 (2020), pp. 1327–1370. DOI: 10.1111/jofi.12883.

Fung, William and David A. Hsieh. "Hedge Fund Benchmarks: A Risk-Based Approach". *Financial Analysts Journal* 60.5 (2004), pp. 65–80. DOI: 10.2469/faj.v60.n5.2657.

Futures Industry Association. *Financial Futures 50th Anniversary*. 2022. URL: https://vimeo.com/767157667.

Goldman Sachs & Co. *The GS $ High Yield Indices: Rules & Methodology*. Mar. 2006. URL: https://www.goldmansachs.com/investop/High_Yield_Final_2-27-06.pdf.

— *The GS $ Investment Grade Indices: Rules & Methodology*. Mar. 2006. URL: https://www.goldmansachs.com/investop/Inv_Grade_Final_2-27-06.pdf.

Goldman Sachs & Co. and International Index Company. *International Index Company to Purchase Goldman Sachs' Credit Indices*. Sept. 2006. URL: https://www.goldmansachs.com/investop/IICGS-final-release-FINAL.pdf.

Goyal, Amit, Ivo Welch, and Athanasse Zafirov. "A Comprehensive 2022 Look at the Empirical Performance of Equity Premium Prediction". *Review of Financial Studies* 37.11 (2024), pp. 3490–3557. DOI: 10.1093/rfs/hhae044.

Greene, William H. *Econometric Analysis*. 5th ed. Prentice Hall, 2003.

Hansen, Lars Peter and Ravi Jagannathan. "Implications of Security Market Data for Models of Dynamic Economies". *Journal of Political Economy* 99.2 (1991), pp. 225–262. DOI: 10.1086/261749.

Harvey, Campbell R. "Time-Varying Conditional Covariances in Tests of Asset Pricing Models". *Journal of Financial Economics* 24.2 (1989), pp. 289–317. DOI: 10.1016/0304-405X(89)90049-4.

Harvey, Campbell R., Yan Liu, and Heqing Zhu. "...and the Cross-Section of Expected Returns". *Review of Financial Studies* 29.1 (2015), pp. 5–68. DOI: 10.1093/rfs/hhv059.

Hasanhodzic, Jasmina and Andrew W. Lo. "Can Hedge-Fund Returns Be Replicated?: The Linear Case". *Journal of Investment Management* 5.2 (2007), pp. 5–45.

He, Zhiguo, Bryan Kelly, and Asaf Manela. "Intermediary asset pricing: New evidence from many asset classes". *Journal of Financial Economics* 126.1 (2017), pp. 1–35. DOI: 10.1016/j.jfineco.2017.08.002.

Hou, Kewei, Chen Xue, and Lu Zhang. "Digesting Anomalies: An Investment Approach". *Review of Financial Studies* 28.3 (2015), pp. 650–705. DOI: 10.1093/rfs/hhu068.

— "Replicating Anomalies". *Review of Financial Studies* 33.5 (2020), pp. 2019–2133. DOI: 10.1093/rfs/hhy131.

Hou, Kewei et al. "Which Factors?" *Review of Finance* 23.1 (2019), pp. 1–35. DOI: 10.1093/rof/rfy032.

— "An Augmented q-factor Model with Expected Growth". *Review of Finance* 25.1 (2021), pp. 1–41. DOI: 10.1093/rof/rfaa004.

Kan, Raymond and Cesare Robotti. "Model Comparison Using the Hansen-Jagannathan Distance". *Review of Financial Studies* 22.9 (2008), pp. 3449–3490. DOI: 10.1093/rfs/hhn094.

Knez, Peter J., Robert Litterman, and José Scheinkman. "Explorations Into Factors Explaining Money Market Returns". *Journal of Finance* 49.5 (1994), pp. 1861–1882. DOI: 10.1111/j.1540-6261.1994.tb04784.x.

Kung, Edward and Lawrence Pohlman. "Portable Alpha". *Journal of Portfolio Management* 30.3 (2004), pp. 78–87. DOI: 10.3905/jpm.2004.412322.

Ladekarl, Jeppe and Edgar E. Peters. "Emerging Market Currency: The Common Risk Factor in Emerging Markets". *Journal of Investing* 22.3 (2013), pp. 135–143. DOI: 10.3905/joi.2013.22.3.135.

Lintner, John. "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets". *The Review of Economics and Statistics* 47.1 (1965), pp. 13–37. DOI: 10.2307/1924119.

Litterman, Robert, José Scheinkman, and Laurence Weiss. "Volatility and the Yield Curve". *Journal of Fixed Income* 1.1 (1991), pp. 49–53. DOI: 10.3905/jfi.1991.692346.

Litterman, Robert B. and José Scheinkman. "Common Factors Affecting Bond Returns". *Journal of Fixed Income* 1.1 (1991), pp. 54–61. DOI: 10.3905/jfi.1991.692347.

Merton, Robert C. "An Intertemporal Capital Asset Pricing Model". *Econometrica* 41.5 (1973), pp. 867–887. DOI: 10.2307/1913811.

Miller, Todd and Timothy S. Meckel. "Beating Index Funds With Derivatives". *Journal of Portfolio Management* 25.5 (1999), pp. 75–87. DOI: 10.3905/jpm.1999.319708.

Mossin, Jan. "Equilibrium in a Capital Asset Market". *Econometrica* 34.4 (1966), pp. 768–783. DOI: 10.2307/1910098.

Novy-Marx, Robert and Mihail Velikov. "A Taxonomy of Anomalies and Their Trading Costs". *Review of Financial Studies* 29.1 (2016), pp. 104–147. DOI: 10.1093/rfs/hhv063.

Patton, Andrew J. and Brian M. Weller. "What You See is Not What You Get: The Costs of Trading Market Anomalies". *Journal of Financial Economics* 137.2 (2020), pp. 515–549. DOI: 10.1016/j.jfineco.2020.02.012.

Pesaran, Mohammad Hashem and Allan Timmerman. "Predictability of Stock Returns: Robustness and Economic Significance". *Journal of Finance* 50.4 (1995), pp. 1201–1228. DOI: 10.2307/2329349.

Robertson, Adriana Z., Pat Akey, and Mikhail Simutin. "Noisy Factors in Law". *The University of Chicago Law Review* 92.3 (2025), pp. 769–832.

Rosenberg, Barr. "Extra-Market Components of Covariance in Security Returns". *Journal of Financial and Quantitative Analysis* 9.2 (1974), pp. 263–274. DOI: 10.2307/2330104.

Rosenberg, Barr and James Guy. "Prediction of Beta from Investment Fundamentals: Part One, Prediction Criteria". *Financial Analysts Journal* 32.3 (1976), pp. 60–72. DOI: 10.2469/faj.v32.n3.60.

Ross, Stephen A. "The arbitrage theory of capital asset pricing". *Journal of Economic Theory* 13.3 (1976), pp. 341–360. DOI: 10.1016/0022-0531(76)90046-6.

Rothschild, Michael. *Asset Pricing Theories*. Technical Working Paper 44. NBER, 1985. DOI: 10.3386/t0044.

Schapire, Robert E. "The Strength of Weak Learnability". *Machine Learning* 5.2 (1990), pp. 197–227. DOI: 10.1007/bf00116037.

Securities and Exchange Commission. *Release 34-77395: Joint Industry Plan; Order Approving Amendment No. 3 to the National Market System Plan for the Selection and Reservation of Securities Symbols*. Mar. 2016. URL: https://www.sec.gov/files/rules/sro/nms/2016/34-77395.pdf.

— *Release 34-95040: Joint Industry Plan; Order Approving Amendment No. 4 to the National Market System Plan for the Selection and Reservation of Securities Symbols*. June 2022. URL: https://www.sec.gov/files/rules/sro/nms/2022/34-95040.pdf.

— *Updated Investor Bulletin: Leveraged and Inverse ETFs*. Aug. 2023. URL: https://www.investor.gov/introduction-investing/general-resources/news-alerts/alerts-bulletins/investor-alerts/sec.

Sharpe, William F. "A Simplified Model for Portfolio Analysis". *Management Science* 9.2 (1963), pp. 277–293. DOI: 10.1287/mnsc.9.2.277.

— "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk". *Journal of Finance* 19.3 (1964), pp. 425–442. DOI: 10.1111/j.1540-6261.1964.tb02865.x.

— "Determining a Fund's Effective Asset Mix". *Investment Management Review* 2.6 (1988), pp. 59–69.

— "Asset Allocation: Management Style and Performance Measurement". *Journal of Portfolio Management* 18.2 (1992), pp. 7–19. DOI: 10.3905/jpm.1992.409394.

Shue, Kelly, Richard Townsend, and Chen Wang. *Long-Short Interest Rate Confusion*. Working Paper. 2025.

Stivers, Adam. "Equity Premium Predictions with Many Predictors: A Risk-based Explanation of the Size and Value Factors". *Journal of Empirical Finance* 45.1 (2018), pp. 126–140. DOI: 10.1016/j.jempfin.2017.10.004.

Stone, Mervyn and Rodney J. Brooks. "Continuum Regression: Cross-Validated Sequentially Constructed Prediction Embracing Ordinary Least Squares, Partial Least Squares and Principal Components Regression". *Journal of the Royal Statistical Society, Series B* 52.2 (1990), pp. 237–269. DOI: 10.1111/j.2517-6161.1990.tb01786.x.

Swade, Alexander et al. "Factor Zoo (.zip)". *Journal of Portfolio Management* 50.3 (2024), pp. 11–31. DOI: 10.3905/jpm.2023.1.561.

Treynor, Jack L. *Market Value, Time, and Risk*. Working Paper 2600356. SSRN, 1961. DOI: 10.2139/ssrn.2600356.

van Binsbergen, Jules H., Yoshio Nozawa, and Michael Schwert. "Duration-Based Valuation of Corporate Bonds". *Review of Financial Studies* 38.1 (2025), pp. 158–191. DOI: 10.1093/rfs/hhae054.

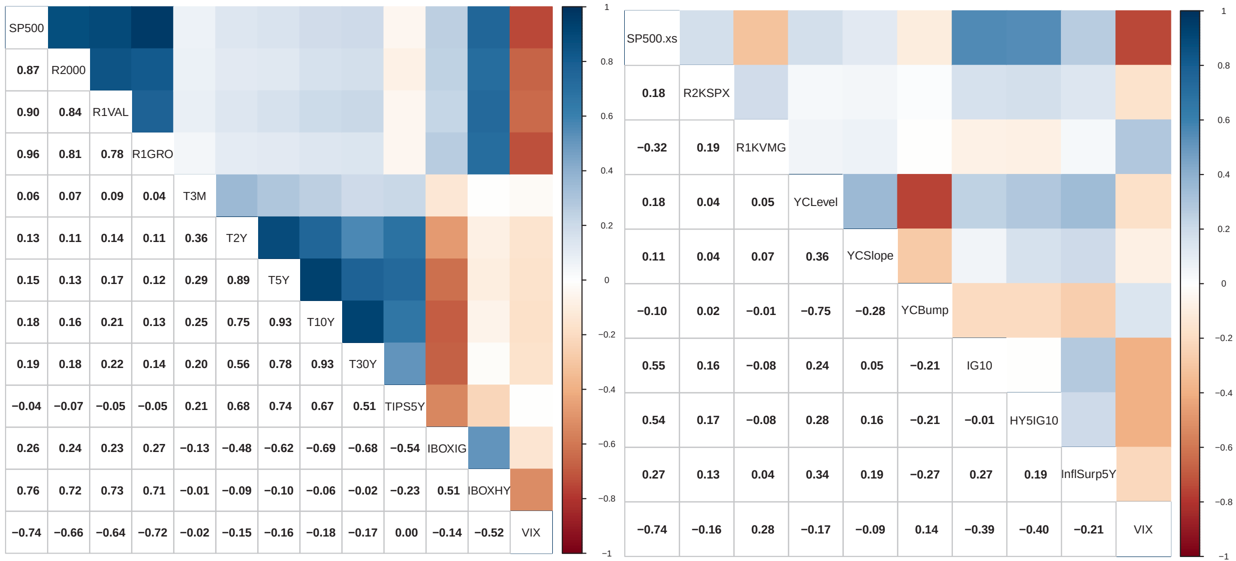
Weisberg, Sanford. *Applied Linear Regression*. 4th ed. John Wiley & Sons, Inc., 2014. URL: http://users.stat.umn.edu/~sandy/alr4ed/index.html.

Welch, Ivo and Amit Goyal. "A Comprehensive Look at The Empirical Performance of Equity Premium Prediction". *Review of Financial Studies* 21.4 (2008), pp. 1455–1508. DOI: 10.1093/rfs/hhm014.

Wold, Herman. "Estimation of principal components and related models by iterative least squares". *Multivariate Analysis. Proceedings of the International Symposium on Multivariate Analysis*. Ed. by Paruchuri Krishnaiah. 1966, pp. 391–420.

— "Soft modeling: the basic design and some extensions". *Systems under Indirect Observation: Causality, Structure, Prediction, Part II*. Ed. by Karl G. Jōreskog and Herman Wold. 1982, pp. 1–54.

Zhang, Xiang et al. "Tradable or nontradable factors—what does the Hansen–Jagannathan distance tell us?" *International Review of Economics & Finance* 71 (2021), pp. 853–879. DOI: 10.1016/j.iref.2020.10.013.



Liquid Instruments | Liquid Factors

Figure 1: Correlation plots from daily data over 2014–20250530 for raw liquid instruments (left) and liquid factors (right). The liquid instruments show strong correlations among equity indices, among US Treasury bonds (but not bills), between the iBoxx IG index and Treasury bonds, and among the iBoxx HY, VIX, and equity indices. Credit correlations are opposite-sign from typical since credit factors are in returns, not yield changes. For the liquid factors, average and maximum correlations are lower.The negative correlations between yield curve level vs curvature and VIX vs S&P 500 are sensible since the yield curve curvature and VIX are both volatility-related factors.

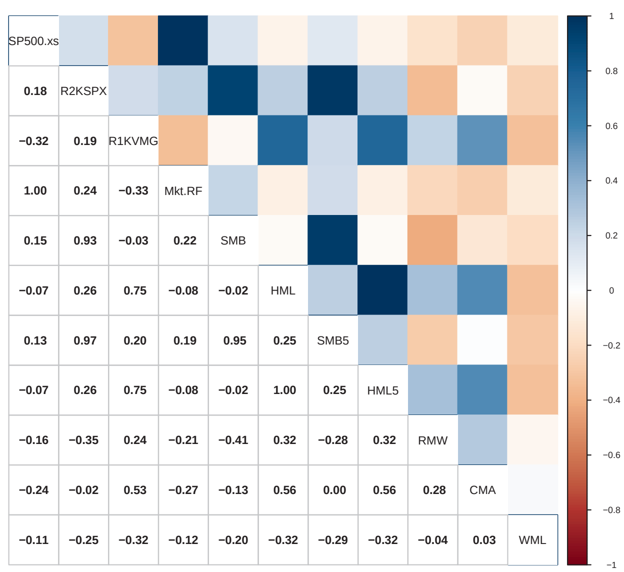
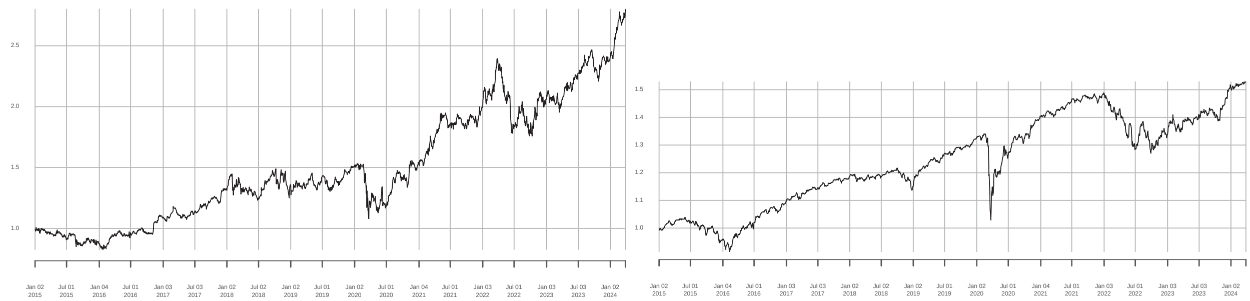


Figure 2: Correlation plots for the Fama-French 3- and 5-factor model factors, the liquid analogues of the FF3 factors, and the Carhart momentum factor. We expect high correlations among Mkt.RF and SP500; SMB3, SMB5, and R2KSPX; and, HML3, HML5, and R1VMG. Surprising, however, is the moderately high correlation between the Value (HML) factors and the FF5 CMA factor.



Berkshire Hathaway (Equity Value Fund) Blackrock High Yield Fund

Figure 3: Cumulated return for the equity-dominant investment fund, Berkshire Hathaway (left) and the bond-dominant investment fund, Blackrock High Yield Institutional (right).

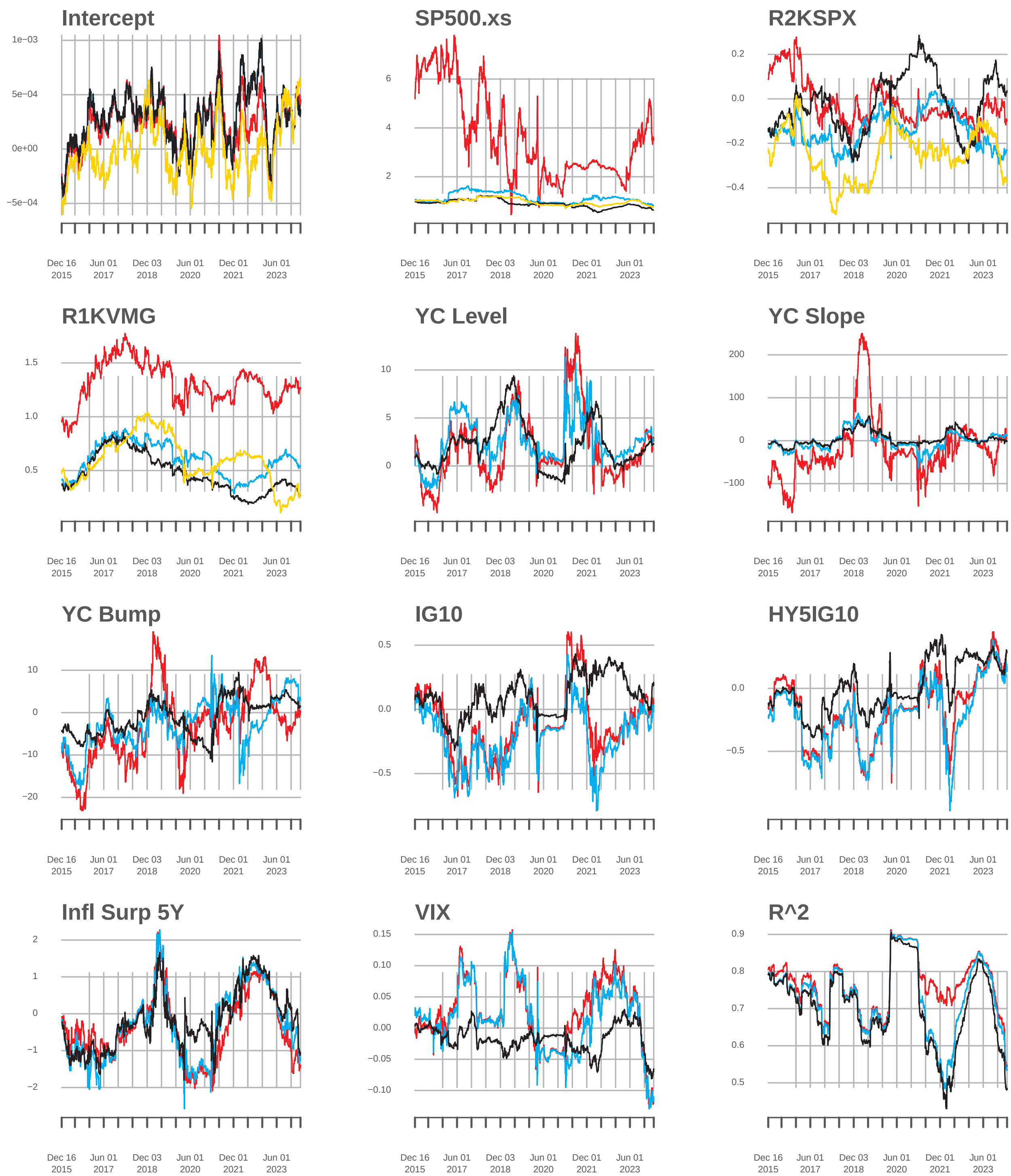


Figure 4: Rolling beta estimates for the three versions of the liquid factor model using 1-year windows for data from 2015–2024Q1. The beta estimates from the various models are denoted by line colors: Kitchen Sink in red; Corrected Covariates in light blue; the full liquid model, Sequential Shrinkage, in **black**; and, comparable factors from FFC6 in gold. The Kitchen Sink estimates are especially noisy due to the high collinearity between the numerous equity and fixed income instruments.

Dictionary of Liquid Factor Definitions		
Description	Short Name	Formula
Large Cap Equity Index (excess)	SP500.xs	S&P 500 log return − 3M UST yield/250
Equity Size Factor	R2KSPX	Russell 2000 − S&P 500 log returns
Equity Value Factor	R1KVMG	Russell 1000: Value − Growth log returns
USD Yield Curve Level	YCLevel	$\Delta \frac{3M+2Y+5Y+10Y+30Y}{5}$ UST yields
USD Yield Curve Slope	YCSlope	$\Delta \left[\frac{30Y-3M}{18.95} + \frac{10Y-2Y}{6.7}\right]/2$ UST yields
USD Yield Curve Bump	YCBump	$\Delta \left[\frac{30Y-5Y}{14.5} - \frac{5Y-3M}{4.5}\right]$ UST yields
IG Credit (zero-duration)	IG10	iBoxx IG log return − UST 10Y return
HY Credit (zero duration)	HY5IG10	HY5 − IG10
HY+IG Credit (zero-duration)	*HY5*	*iBoxx HY log return − UST 5Y return*
Breakeven Inflation	InflSurp5Y	Δ(5Y US TIPS - 5Y UST) yields
Equity Index Volatility	VIX	ΔVIX/100

Table 1: Summary of correlation-reduced liquid factors. Note that the iBoxx indices use returns for the index and US Treasuries instead of yield changes since returns are readily available. The italicized HY5 entry is used to calculate the HY Credit liquid factor but is not a factor itself.

Model Name	Summary
Kitchen Sink	OLS regression using all liquid instruments used to create liquid factors; no feature engineering or TIPLS estimation.
Corrected Covariates	OLS regression using liquid factors created from combining certain liquid instruments; no TIPLS estimation.
Sequential Shrinkage	Theory-implied PLS regression using liquid factors created from combining certain liquid instruments.
FFC6	Fama and French (2016) 5-factor model plus Carhart (1997) momentum factor yielding a 6-factor equity model.
FF93L	Fama and French (1993) 5-factor bond model using liquid factors in place of the original (unavailable) factors.

Table 2: Model definitions used to compare model performance and assess benefits of liquid factor model innovations.

Daily Log Returns, Yield Changes (USTs, TIPS), and Changes (VIX): 2014–20250530						
Instrument	Min	Mean	Median	Max	Std. Dev.	Ann. Vol.
S&P 500	-12.77%	3.9 bp	3.2 bp	9.09%	110 bp	17.4%
Russell 2000	-15.40%	1.9 bp	2.2 bp	8.98%	141 bp	22.2%
Russell 1000 Value	-12.80%	2.3 bp	1.8 bp	9.46%	106 bp	16.8%
Russell 1000 Growth	-13.18%	5.2 bp	6.6 bp	11.16%	126 bp	19.9%
3M UST ret.	-0.09%	0.0 bp	0.0 bp	0.06%	0.7 bp	0.1%
2Y UST ret.	-0.67%	-0.2 bp	0.0 bp	1.13%	10.0 bp	1.6%
5Y UST ret.	-1.47%	-0.4 bp	0.0 bp	1.51%	26.0 bp	4.1%
10Y UST ret.	-2.53%	-0.5 bp	0.0 bp	2.62%	45.9 bp	7.3%
30Y UST ret.	-5.57%	-0.8 bp	0.0 bp	5.95%	96.7 bp	15.3%
5Y TIPS ret.	-1.56%	-0.2 bp	0.0 bp	1.66%	26.6 bp	4.2%
3M UST Δyld	-23 bp	0.1 bp	0.0 bp	34 bp	2.8 bp	45 bp
2Y UST Δyld	-57 bp	0.1 bp	0.0 bp	34 bp	5.1 bp	80 bp
5Y UST Δyld	-32 bp	0.1 bp	0.0 bp	31 bp	5.5 bp	87 bp
10Y UST Δyld	-30 bp	0.1 bp	0.0 bp	29 bp	5.3 bp	83 bp
30Y UST Δyld	-31 bp	0.0 bp	0.0 bp	29 bp	5.0 bp	80 bp
5Y TIPS Δyld	-35 bp	0.1 bp	0.0 bp	33 bp	5.6 bp	89 bp
iBoxx IG	-5.13%	-0.2 bp	2.6 bp	7.13%	52.6 bp	8.3%
iBoxx HY	-5.65%	-0.5 bp	1.1 bp	6.34%	52.8 bp	8.4%
VIX	-44.24%	1.0 bp	-43.7 bp	76.82%	7.91%	125.1%

Table 3: Summary statistics for daily returns, yield changes (for USTs and TIPS), and changes (for VIX) of the raw liquid model variables over 20140101–20250530. Note that for the VIX, a 1% change above would be like a change from an 18% volatility to a 18.18% volatility. We can see the much greater variability of the equity, credit, and volatility variables — as well as longer-term bond returns.

Daily Liquid Factor Data: 2014–20250530						
Instrument	Min	Mean	Median	Max	Std. Dev.	Ann. Vol.
SP500.xs	-12.77%	3.4 bp	5.9 bp	9.07%	112 bp	17.7%
R2KSPX	-5.69%	-2.0 bp	-0.5 bp	6.13%	70 bp	11.1%
R1KVMG	-7.00%	-2.8 bp	-2.7 bp	5.98%	79 bp	12.5%
YCLevel	-0.24%	0.1 bp	0.2 bp	0.30%	4.1 bp	64 bp
YCSlope	-1.7 bp	0.0 bp	0.0 bp	3.5 bp	0.4 bp	6 bp
YCBump	-6.6 bp	0.0 bp	0.0 bp	8.4 bp	1.3 bp	21 bp
IG10	-5.59%	0.6 bp	1.1 bp	5.74%	39 bp	6.2%
HY5IG10	-8.02%	-0.1 bp	0.5 bp	4.59%	41 bp	6.5%
InflSurp5Y	-34 bp	0.0 bp	0.0 bp	22 bp	4.0 bp	64 bp
VIX	-44.24%	1.0 bp	-43.7 bp	76.82%	7.91%	125.1%

Table 4: Summary statistics for daily observations of the correlation-reduced liquid factors over 20140101–20250530. While the equity factors still show high variability, the credit factors are less variable and the yield curve measures are far less variable than the constituent US Treasuries.

Daily Fama French Factor Returns: 2014–20250530						
Instrument	Min	Mean	Median	Max	Std. Dev.	Ann. Vol.
Mkt.RF	-12.01%	4.5 bp	6.0 bp	9.65%	114.6 bp	18.1%
SMB3	-3.54%	-1.1 bp	-2.0 bp	5.45%	64.8 bp	10.2%
HML3	-5.02%	-0.9 bp	-4.0 bp	6.70%	86.9 bp	13.7%
SMB5	-4.58%	-1.3 bp	-3.0 bp	5.71%	67.4 bp	10.7%
HML5	-5.02%	-0.7 bp	-4.0 bp	6.70%	86.9 bp	13.7%
RMW	-2.23%	1.5 bp	1.0 bp	4.24%	51.2 bp	8.1%
CMA	-2.91%	-0.6 bp	-2.0 bp	2.48%	46.5 bp	7.4%
WML	-14.36%	1.3 bp	6.0 bp	5.90%	106.3 bp	16.8%

Table 5: Summary statistics for daily observations of the Fama and French (1992, 2016) and Carhart (1997) factors over 20140101–20250530. The SMB factor differs materially between the 3-factor and 5-factor model while the HML factor does not. The market factor also differs slightly from the S&P 500 liquid factor; and, the SMB factor is slightly less variable than the R2KSPX liquid factor.

Daily Excess Log Returns for Investment Funds: 2015–2024Q1						
Investment Fund	Min	Mean	Median	Max	Std. Dev.	Ann. Vol.
Berkshire Hathaway (BRK-B)	-10.09%	3.8 bp	4.0 bp	10.98%	123.8 bp	19.6%
Blackrock High Yield (BHYIX)	-4.28%	1.0 bp	0.0 bp	3.69%	46.5 bp	7.4%

Table 6: Summary statistics from daily observations of excess returns for two profiled investment funds, equity value and high yield bond, over 20150101 to 20240329.

Models: Liquid Factor, Increasing in Model Innovations + FFC6: Equity Fund, 2015–2024Q1 Daily Data												
	Kitchen Sink			Corr. Covariates		Sequential Shrinkage					FFC6	
Instrument	Est.	t	Factor	Est.	t	Est.	t	Order	ΔR^2	Factor	Est.	t
Intercept	1.9 bp	1.3	Intercept	2.2 bp	1.6	2.2 bp	1.6	–		Intercept	-0.0 bp	-0.1
SP500	2.35	26.7	SP500.xs	1.02	38.0	0.87	63.6	1	65.8%	Mkt.Rf	0.89	75.5
R2000	-0.08	-4.3	R2KSPX	-0.09	-4.7	0.02	0.9	2	0.0%	SMB	-0.29	-13.0
R1K Value	-0.00	-0.0	R1KVMG	0.48	26.2	0.42	24.6	3	7.6%	HML	0.51	24.1
R1K Growth	-1.20	-23.0										
										RMW	-0.06	-2.0
										CMA	0.03	0.7
										WML	0.01	1.1
T3M	0.50	1.1	YCLevel	1.89	3.7	1.97	5.9	4	0.5%			
T2Y	-0.44	-0.7	YCSlope	1.16	0.3	3.47	0.9	5	0.0%			
T5Y	0.43	0.4	YCBump	-1.15	-0.8	0.40	0.4	8	0.0%			
T10Y	0.21	0.1										
T30Y	-0.36	-0.4										
IBOXIG	0.05	1.2	IG10	-0.18	-3.7	0.05	1.7	6	0.0%			
IBOXHY	-0.20	-4.4	HY5IG10	-0.20	-4.4	0.004	0.2	7	0.0%			
TIPS5Y	0.90	2.6	InflSurp5Y	-0.13	-0.5	-0.03	-0.1	9	0.0%			
VIX	0.01	0.6	VIX	-0.01	-0.9	-0.03	-4.3	10	0.2%			
R^2 (in-sample)		77.1%			75.1%				74.1%			76%

Table 7: Estimation results for three versions of the liquid factor model and the Fama-French+Carhart 6-factor model for the equity-dominant investment fund over 20150101–20240329. The Kitchen Sink model uses liquidly-traded variables; the Corrected Covariates model creates factors which reduce collinearity; and, the Sequential Shrinkage model estimates the coefficients sequentially to further reduce collinearity and prefer more relevant and liquid factors. Note that for the Sequential Shrinkage model, the rows are not exactly in order of estimation. This out-of-order presentation was chosen to group yield curve variables for comparison with the Kitchen Sink model.

Liquid Factor Models, Increasing in Model Innovations: Fixed Income Fund, 2015–2024Q1 Daily Data												
	Kitchen Sink			Corr. Covariates		Sequential Shrinkage				FF93L		
Instrument	Est.	t	Factor	Est.	t	Est.	t	Order	ΔR^2	Est.	t	Order
Intercept	1.0 bp	1.8	Intercept	0.4 bp	0.7	0.4 bp	0.7	–		0.5 bp	0.7	–
T3M	-0.36	-1.8	YCLevel	-3.12	-14.4	-0.66	3.1	1	0.5%			
T2Y	-0.07	-0.3	YCSlope	-1.59	-1.0	1.88	0.8	2	0.0%	-3.66	-1.9	1
T5Y	1.32	2.7	YCBump	1.39	2.2	6.33	13.8	5	4.3%			
T10Y	0.10	0.2										
T30Y	-0.63	-1.7										
IBOXIG	0.04	2.2	IG10	0.58	28.1	0.48	26.5	3	24.9%	0.44	24.3	1
IBOXHY	0.53	26.9	HY5IG10	0.54	26.9	0.44	30.8	4	23.2%			
SP500	-0.18	-4.6	SP500.xs	0.00	0.2	0.01	2.4	6	0.1%	0.09	15.2	2
R2000	0.02	2.0	R2KSPX	0.03	3.0	0.03	4.0	7	0.4%	0.09	8.4	1
R1K Value	0.09	5.6	R1KVMG	0.03	3.9	0.02	3.1	8	0.2%	-0.02	-1.7	1
R1K Growth	0.07	2.9										
TIPS5Y	-0.73	-4.8	InflSurp5Y	-0.15	-1.4	-0.32	-3.0	9	0.2%			
VIX	0.01	2.1	VIX	0.01	2.1	0.00	1.0	10	0.0%			
R^2 (in-sample)		57.5%			57.2%				53.8%			32.7%

Table 8: Estimation results for three versions of the liquid factor model and the Fama-French bond model for the fixed income-dominant investment fund over 20150101–20240329. The Kitchen Sink model uses liquidly-traded instruments; the Corrected Covariates model creates factors which reduce collinearity; and, the Sequential Shrinkage model estimates the coefficients sequentially to further reduce collinearity and prefer more relevant and liquid factors. Variance inflation factors are very high for betas in the Kitchen Sink model (many more than 10) and much lower for the Corrected Covariates and Sequential Shrinkage. For the Sequential Shrinkage model, the rows are not fully in order of estimation; YCBump was estimated after the IG10 and HY5IG10 factors but grouped with yield curve variables for comparison to the Kitchen Sink model.

Liquid Factor Models, Increasing in Model Innovations: Fixed Income Fund, 2015–2024Q1 Monthly Data												
	Kitchen Sink			Corr. Covariates		Sequential Shrinkage				FF93L		
Instrument	Est.	t	Factor	Est.	t	Est.	t	Order	ΔR^2	Est.	t	Order
Intercept	5.2 bp	1.0	Intercept	-18.5 bp	-2.6	-18.5 bp	-2.6	–		-19.8 bp	-1.7	–
T3M	1.06	2.9	YCLevel	-2.95	-6.0	-0.67	3.1	1	0.3%			
T2Y	-1.38	-1.8	YCSlope	1.97	0.4	-20.50	-1.4	2	1.7%	-8.83	-1.0	1
T5Y	2.20	1.6	YCBump	6.58	4.3	7.94	5.1	5	3.9%			
T10Y	1.84	1.8										
T30Y	-0.76	-0.7										
IBOXIG	0.18	2.9	IG10	0.99	14.0	1.19	16.0	3	68.6%	1.16	13.8	1
IBOXHY	0.81	15.3	HY5IG10	0.74	11.1	0.55	7.0	4	9.1%			
SP500	-0.57	-3.7	SP500.xs	0.08	2.5	0.00	0.1	6	0.0%	0.08	-0.5	2
R2000	-0.01	-0.5	R2KSPX	0.05	1.9	-0.02	-0.6	7	0.1%	0.06	1.3	1
R1K Value	0.27	3.3	R1KVMG	-0.03	-1.1	-0.07	-2.2	8	0.7%	-0.12	-2.6	1
R1K Growth	0.32	3.7										
TIPS5Y	-0.80	-2.0	InflSurp5Y	-2.55	-2.6	-3.09	-2.4	9	0.8%			
VIX	0.00	0.2	VIX	0.05	2.8	0.03	2.0	10	0.5%			
R^2 (in-sample)		96.1%			92.5%				85.7%			73.1%

Table 9: Estimation results from three versions of the liquid factor model and the Fama-French bond model for monthly returns of the fixed income-dominant investment fund over 20150101–20240329. The Kitchen Sink model uses liquidly-traded instruments; the Corrected Covariates model creates factors which reduce collinearity; and, the Sequential Shrinkage model estimates the coefficients sequentially to further reduce collinearity and prefer more relevant and liquid factors. For the Sequential Shrinkage model, the rows are not fully in order of estimation; YCBump was estimated after the IG10 and HY5IG10 factors but grouped with yield curve variables for comparison to the Kitchen Sink model.

	Kitchen Sink		Corrected Covariates		Sequential Shrinkage			FFC6	
Coefficient	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\mu}$	$\hat{\sigma}$	Coefficient	$\hat{\mu}$	$\hat{\sigma}$
Intercept	2.3 bp	2.3 bp	2.8 bp	2.5 bp	2.8 bp	2.5 bp	Intercept	0- bp	2.5 bp
SP500.xs	3.67	1.77	1.13	0.21	0.90	0.16	Mkt.RF	0.97	0.13
R2KSPX	-0.02	0.10	-0.14	0.08	-0.02	0.13	SMB	-0.25	0.11
R1KVMG	1.32	0.19	0.60	0.16	0.47	0.18	HML	0.60	0.22
YCLevel	1.82	3.57	2.62	2.43	2.17	2.48			
YCSlope	-19.1	71.6	-0.18	21.5	4.71	16.2			
YCBump	-3.21	8.46	-2.41	5.76	-0.99	4.27			
IG10	-0.12	0.25	-0.21	0.22	0.10	0.16			
HY5IG10	-0.16	0.26	-0.21	0.26	0.01	0.20			
InflSurp5Y	-0.38	0.89	-0.32	0.97	-0.17	0.79			
VIX	0.03	0.05	0.02	0.05	-0.02	0.02			
							RMW	-0.07	0.12
							CMA	0.07	0.30
							WML	0.05	0.13
R^2 (in-sample)	77%	7.2%	73%	9.6%	71%	10.4%	R^2	76%	7.9%

Table 10: Mean and standard deviation of rolling coefficient estimates for the three liquid factor models and the Fama-French+Carhart 6-factor model using 1-year windows for data from 2015–2024Q1. The Corrected Covariates model has less noisy estimates than the Kitchen Sink model; and, the full liquid factor model (Sequential Shrinkage) has parameters which tend to be more stable, especially for preferenced factors. Compared to the FFC6 model, the full liquid model has similarly stable estimates for the two main equity factors (market and value/HML). However, the FFC6 model seems to spuriously load on the size/SMB factor while missing exposure to the level of interest rates. The FFC6 model also may have elevated variability in the HML and CMA factors due to their correlation.

	Int.	SP500	R2KSPX	R1KVMG	YCLvl	YCSlp	IG10	HY5IG10	YCBmp	InfSur5Y	VIX	N/R^2
A. Aggregated Kitchen Sink Back-Transformed Liquid Factor Models, by Main Asset												
	Int.	SP500	R2KSPX	R1KVMG	YCLvl	YCSlp	IG10	HY5IG10	YCBmp	InfSur5Y	VIX	N/R^2
Bonds	-0.1 bp	0.027	0.016	0.013	-0.863	1.361	0.323	0.129	1.098	-0.326	0.001	443
$\bar{t}$-stat	*-0.00*	*0.46*	*1.05*	*1.27*	*-2.38*	*1.17*	*16.44*	*5.68*	*1.69*	*-2.09*	*1.25*	50.6%
$sd(\hat{\beta})$	0.000	0.420	0.122	0.062	5.837	46.096	0.288	0.284	9.225	1.096	0.006	
Commodities	-3.8 bp	0.095	0.150	0.173	5.483	-0.600	0.173	0.146	-6.798	-4.001	-0.006	73
$\bar{t}$-stat	*-0.23*	*1.87*	*5.15*	*6.94*	*2.70*	*-0.42*	*1.83*	*1.82*	*-1.67*	*-5.68*	*-1.57*	32.2%
$sd(\hat{\beta})$	0.001	0.869	0.232	0.296	11.880	61.965	0.390	0.543	26.330	8.214	0.021	
Equities	-2.0 bp	0.060	0.220	0.028	0.780	-3.988	0.179	0.140	-0.170	-0.653	-0.004	1602
$\bar{t}$-stat	*-0.44*	*4.13*	*9.93*	*2.97*	*0.46*	*-0.50*	*2.70*	*2.09*	*-0.19*	*-1.21*	*-0.71*	62.5%
$sd(\hat{\beta})$	0.001	2.053	0.441	0.488	6.851	39.967	0.368	0.417	16.188	2.641	0.019	
FX	-0.8 bp	-0.031	0.007	0.021	-1.853	4.033	0.141	0.117	1.950	-0.949	0.001	6
$\bar{t}$-stat	*-0.90*	*-0.41*	*0.41*	*1.59*	*-2.31*	*0.73*	*4.08*	*3.57*	*0.81*	*-3.73*	*0.26*	19.2%
$sd(\hat{\beta})$	0.000	0.038	0.026	0.030	1.941	7.607	0.045	0.078	4.389	0.585	0.006	
Real Estate	-1.3 bp	0.646	0.257	0.364	-4.288	-21.826	0.430	0.271	1.346	-0.003	0.007	36
$\bar{t}$-stat	*-0.65*	*5.21*	*9.26*	*13.85*	*-2.90*	*-1.86*	*6.40*	*4.17*	*0.23*	*-0.08*	*1.51*	59.5%
$sd(\hat{\beta})$	0.000	0.449	0.174	0.152	3.759	22.473	0.239	0.147	5.828	0.964	0.012	
Unclear	-3.0 bp	-0.055	0.050	0.082	4.332	4.156	0.230	0.112	-8.523	-2.770	0.011	148
$\bar{t}$-stat	*-0.41*	*0.87*	*1.64*	*1.39*	*0.97*	*-0.08*	*3.62*	*1.57*	*-0.99*	*-3.22*	*0.29*	30.3%
$sd(\hat{\beta})$	0.001	2.240	0.390	0.314	10.609	98.663	0.462	0.538	21.084	4.189	0.089	
B. Aggregated Corrected Covariates Liquid Factor Models, by Main Asset												
	Int.	SP500	R2KSPX	R1KVMG	YCLvl	YCSlp	IG10	HY5IG10	YCBmp	InfSur5Y	VIX	N/R^2
Bonds	-0.1 bp	0.057	0.014	0.014	-3.519	-6.487	0.321	0.134	2.515	0.363	0.001	443
$\bar{t}$-stat	*0.06*	*4.10*	*0.90*	*1.50*	*-19.98*	*-4.34*	*16.24*	*5.99*	*5.09*	*2.38*	*1.30*	50.2%
$sd(\hat{\beta})$	0.000	0.131	0.103	0.067	6.458	40.658	0.288	0.280	4.540	1.088	0.006	
Commodities	-3.4 bp	0.425	0.152	0.175	-2.144	8.424	0.161	0.148	7.303	3.989	-0.006	73
$\bar{t}$-stat	*-0.06*	*12.04*	*5.08*	*7.14*	*-1.81*	*0.96*	*1.65*	*1.84*	*2.17*	*5.75*	*-1.65*	31.9%
$sd(\hat{\beta})$	0.001	0.537	0.228	0.274	10.092	28.082	0.376	0.507	16.513	8.373	0.021	
Equities	-1.4 bp	0.659	0.220	0.044	-0.478	0.743	0.183	0.148	2.665	0.657	-0.003	1602
$\bar{t}$-stat	*0.09*	*30.64*	*9.66*	*3.58*	*-0.90*	*-0.11*	*2.70*	*2.13*	*1.21*	*1.22*	*-0.69*	61.7%
$sd(\hat{\beta})$	0.001	0.687	0.421	0.388	3.313	14.997	0.386	0.380	7.757	2.552	0.019	
FX	-0.8 bp	0.044	0.009	0.026	-3.541	10.434	0.134	0.122	4.391	0.967	0.001	6
$\bar{t}$-stat	*-0.85*	*2.48*	*0.50*	*2.13*	*-9.36*	*3.78*	*3.88*	*3.75*	*3.88*	*3.84*	*0.21*	18.9%
$sd(\hat{\beta})$	0.000	0.074	0.026	0.030	1.698	3.047	0.047	0.078	2.561	0.590	0.006	
Real Estate	-0.6 bp	0.768	0.252	0.324	-5.234	-12.338	0.404	0.271	4.368	0.091	0.006	36
$\bar{t}$-stat	*-0.27*	*22.51*	*9.04*	*13.55*	*-7.81*	*-2.10*	*6.10*	*4.22*	*2.08*	*0.26*	*1.37*	59.0%
$sd(\hat{\beta})$	0.000	0.172	0.171	0.121	1.148	12.217	0.247	0.152	3.635	0.940	0.011	
Unclear	-2.6 bp	0.219	0.061	0.087	-2.168	1.731	0.216	0.125	3.025	2.858	0.010	148
$\bar{t}$-stat	*-0.25*	*6.37*	*1.57*	*1.82*	*-5.13*	*-0.85*	*3.53*	*1.71*	*1.47*	*3.37*	*0.28*	29.7%
$sd(\hat{\beta})$	0.001	0.431	0.321	0.252	5.912	39.433	0.439	0.475	12.166	4.247	0.089	
C. Aggregated Sequential Shrinkage (Full) Liquid Factor Models, by Main Asset												
	Int.	SP500	R2KSPX	R1KVMG	YCLvl	YCSlp	IG10	HY5IG10	YCBmp	InfSur5Y	VIX	N/R^2
Bonds	-0.0 bp	0.026	0.013	0.010	-3.011	-6.885	0.392	0.193	3.940	0.215	0.001	443
$\bar{t}$-stat	*0.06*	*3.85*	*0.86*	*1.39*	*-20.27*	*-3.25*	*21.60*	*8.79*	*8.53*	*1.81*	*0.99*	48.4%
$sd(\hat{\beta})$	0.000	0.053	0.071	0.047	7.138	35.985	0.384	0.298	5.609	0.999	0.003	
Commodities	-3.2 bp	0.195	0.154	0.106	0.902	19.072	0.886	0.783	16.430	2.391	-0.002	73
$\bar{t}$-stat	*-0.06*	*9.25*	*4.67*	*4.15*	*1.98*	*2.10*	*11.73*	*12.01*	*8.02*	*3.37*	*-1.15*	27.0%
$sd(\hat{\beta})$	0.001	0.228	0.209	0.200	11.339	30.101	1.144	1.000	20.724	6.883	0.012	
Equities	-1.5 bp	0.760	0.245	0.062	-0.403	-0.701	0.039	0.009	0.966	0.412	0.002	1602
$\bar{t}$-stat	*0.09*	*60.38*	*10.66*	*4.43*	*-1.17*	*-0.37*	*1.23*	*0.32*	*0.68*	*0.85*	*0.76*	61.0%
$sd(\hat{\beta})$	0.001	0.756	0.429	0.322	3.363	15.517	0.216	0.186	4.432	2.150	0.009	
FX	-0.9 bp	0.073	0.033	0.019	-3.433	9.069	0.069	0.056	3.207	0.849	0.002	6
$\bar{t}$-stat	*-0.85*	*8.74*	*2.34*	*1.79*	*-14.29*	*3.51*	*2.92*	*2.44*	*4.35*	*3.69*	*1.24*	18.3%
$sd(\hat{\beta})$	0.000	0.130	0.041	0.037	2.144	2.596	0.019	0.031	1.456	0.471	0.002	
Real Estate	-0.5 bp	0.776	0.368	0.275	-5.246	-12.721	0.275	0.138	3.063	0.035	0.005	36
$\bar{t}$-stat	*-0.27*	*42.58*	*12.95*	*12.32*	*-12.00*	*-2.28*	*5.93*	*3.03*	*2.30*	*0.12*	*2.00*	58.3%
$sd(\hat{\beta})$	0.000	0.099	0.191	0.109	1.376	10.950	0.129	0.087	1.666	0.744	0.006	
Unclear	-2.6 bp	0.235	0.115	0.094	-1.500	1.933	0.128	0.044	2.353	2.224	0.009	148
$\bar{t}$-stat	*-0.25*	*15.50*	*2.86*	*2.08*	*-6.06*	*-0.68*	*2.90*	*0.28*	*1.08*	*2.89*	*1.61*	28.7%
$sd(\hat{\beta})$	0.001	0.815	0.348	0.214	5.501	36.373	0.249	0.225	4.996	3.339	0.039	

Table 11: Partial and Full Liquid Factor Models, Aggregated by Main Asset. Data are for 2,304 US ETFs alive as of 30 November 2021 over the period 2014–2025Q1. The main asset is determined from fund category and asset class holding data. Bond and commodity funds are fit with a fixed income factor ordering; equity, real estate, and unknown funds are fit with an equity factor ordering.

Aggregated Fama-French (FFC6, FF93L) Factor Models, by Main Asset

	Int.	EqMktXs	EqSize	EqValue	RMW	CMA	WML	YCSlp	IG10	N/R^2
Bonds	-0.1 bp	0.055	0.048	-0.019	—	—	—	9.516	0.363	443
$\bar{t}$-stat	*-0.08*	*6.26*	*2.78*	*-0.87*	—	—	—	*4.41*	*29.79*	40.5%
$sd(\hat{\beta})$	2.5 bp	0.129	0.137	0.096	—	—	—	41.474	0.376	
Commodities	-3.3 bp	0.382	0.347	-0.055	—	—	—	56.627	0.493	73
$\bar{t}$-stat	*-0.00*	*19.26*	*8.31*	*-1.23*	—	—	—	*6.09*	*7.67*	24.1%
$sd(\hat{\beta})$	12.4 bp	0.445	0.396	0.166	—	—	—	80.399	1.032	
Equities	-2.7 bp	0.728	0.122	0.081	-0.062	-0.012	-0.012	—	—	1602
$\bar{t}$-stat	*-1.04*	*65.81*	*4.48*	*3.71*	*-0.23*	*0.23*	*-0.95*	—	—	61.3%
$sd(\hat{\beta})$	7.4 bp	0.729	0.362	0.376	0.322	0.295	0.136	—	—	
Real Estate	-2.9 bp	0.745	0.184	0.152	0.065	0.093	-0.122	—	—	36
$\bar{t}$-stat	*-1.37*	*41.05*	*4.98*	*4.49*	*1.40*	*1.96*	*-5.85*	—	—	51.5%
$sd(\hat{\beta})$	2.0 bp	0.091	0.152	0.152	0.111	0.130	0.061	—	—	
Unclear	-3.2 bp	0.225	0.012	0.081	-0.108	0.107	-0.026	—	—	154
$\bar{t}$-stat	*-0.63*	*15.29*	*0.40*	*0.87*	*-1.23*	*1.56*	*-0.75*	–	—	19.9%
$sd(\hat{\beta})$	10.9 bp	0.769	0.255	0.289	0.283	0.297	0.143	—	—	

Table 12: Fama-French (1992) plus Carhart (1997) Six-Factor and Fama-French (1993) Models, Aggregated by Main Asset. Data are for 2,304 US ETFs alive as of 30 November 2021 over the period 2014–2025Q1. The main asset is determined from fund category and asset class holding data. The Fama-French (1993) model has liquid factors substituted in for its original covariates since the model data are not available on Ken French's website.

A. Multi-Asset/Unknown												
	Int.	SP500	R2KSPX	R1KVMG	YCLvl	YCSlp	IG10	HY5IG10	YCBmp	InfSur5Y	VIX	N/R^2
Unknown	-0.9 bp	0.514	0.147	0.036	-1.203	-1.328	0.117	0.035	1.504	0.702	0.002	623
$\bar{t}$-stat	*-0.07*	*43.38*	*5.59*	*2.46*	*-6.33*	*-1.20*	*5.38*	*1.17*	*1.89*	*1.40*	*0.99*	55.9%
$sd(\hat{\beta})$	0.001	0.468	0.314	0.200	3.193	22.909	0.311	0.197	4.074	2.272	0.009	
Alloc: 15%–30% Eq	1.3 bp	0.119	0.024	0.010	-1.267	-5.412	0.768	0.614	9.680	0.107	0.002	4
$\bar{t}$-stat	*0.11*	*13.58*	*1.79*	*0.42*	*-4.03*	*-1.37*	*24.32*	*21.56*	*11.49*	*0.45*	*1.31*	46.1%
$sd(\hat{\beta})$	0.000	0.062	0.035	0.048	1.979	9.848	0.253	0.218	4.074	0.162	0.003	
Alloc: 30%–50% Eq	0.2 bp	0.116	0.068	0.062	-2.806	-16.007	0.875	0.604	8.788	0.363	-0.001	8
$\bar{t}$-stat	*0.18*	*19.72*	*4.36*	*4.20*	*-8.96*	*-3.53*	*30.20*	*24.53*	*15.01*	*0.50*	*-1.38*	65.9%
$sd(\hat{\beta})$	0.000	0.046	0.103	0.104	2.615	19.067	0.519	0.398	4.908	1.341	0.003	
Alloc: 50%–70% Eq	-0.9 bp	0.431	0.124	0.055	-1.634	-5.484	0.090	-0.004	1.047	0.478	0.001	8
$\bar{t}$-stat	*-0.42*	*60.56*	*9.60*	*4.96*	*-9.73*	*-2.40*	*5.67*	*0.53*	*1.93*	*2.59*	*0.46*	61.9%
$sd(\hat{\beta})$	0.000	0.187	0.136	0.123	0.937	6.600	0.095	0.133	2.278	0.487	0.008	
Alloc: 70%–85% Eq	-2.0 bp	0.636	0.188	0.093	-0.678	-3.114	0.068	0.023	2.042	0.467	-0.001	7
$\bar{t}$-stat	*0.02*	*70.48*	*11.99*	*8.38*	*-3.75*	*-1.79*	*4.09*	*0.95*	*2.09*	*2.78*	*0.02*	64.3%
$sd(\hat{\beta})$	0.000	0.217	0.195	0.176	1.607	11.470	0.174	0.131	2.272	1.195	0.010	
Alloc: 85%+ Eq	-0.5 bp	0.690	0.412	0.333	-1.388	-13.999	0.347	0.224	4.642	1.946	0.011	1
$\bar{t}$-stat	*-0.18*	*42.77*	*17.00*	*16.86*	*-3.47*	*-3.16*	*8.62*	*5.77*	*3.81*	*4.86*	*5.75*	57.6%
$sd(\hat{\beta})$	—	—	—	—	—	—	—	—	—	—	—	
Infrastructure	-0.1 bp	0.735	0.236	0.259	-3.145	-2.210	0.190	0.165	4.364	0.730	0.002	7
$\bar{t}$-stat	*0.14*	*57.03*	*11.02*	*16.98*	*-9.69*	*-0.23*	*5.83*	*5.56*	*4.83*	*2.54*	*1.41*	70.7%
$sd(\hat{\beta})$	0.000	0.181	0.227	0.112	1.578	7.244	0.128	0.101	2.367	0.712	0.008	
Tactical Alloc	-0.6 bp	0.358	0.111	-0.009	-0.242	-5.443	0.341	0.284	4.520	1.240	-0.001	11
$\bar{t}$-stat	*-0.27*	*24.74*	*5.06*	*-0.53*	*-1.13*	*-0.91*	*10.64*	*10.03*	*5.36*	*3.40*	*-0.24*	47.0%
$sd(\hat{\beta})$	0.000	0.285	0.088	0.065	1.163	8.022	0.481	0.455	6.484	1.018	0.005	
Trading–Misc.	-17.1 bp	-0.941	-0.066	0.075	-2.218	-2.317	-0.000	0.190	0.424	0.169	0.060	14
$\bar{t}$-stat	*-1.28*	*-7.98*	*-1.06*	*0.21*	*3.19*	*-1.09*	*-1.05*	*0.36*	*-1.16*	*-0.79*	*3.13*	15.6%
$sd(\hat{\beta})$	0.003	1.952	0.461	0.243	8.233	46.427	0.255	0.281	6.786	1.622	0.110	
World Alloc	-0.6 bp	0.356	0.172	0.082	-0.506	0.596	0.318	0.212	5.077	1.640	0.002	10
$\bar{t}$-stat	*-0.45*	*26.70*	*8.73*	*5.12*	*-1.37*	*0.17*	*11.60*	*8.28*	*6.31*	*4.75*	*0.81*	47.7%
$sd(\hat{\beta})$	0.000	0.233	0.089	0.121	1.286	12.225	0.375	0.289	5.377	1.620	0.009	
B. Fixed Income												
	Int.	SP500	R2KSPX	R1KVMG	YCLvl	YCSlp	IG10	HY5IG10	YCBmp	InfSur5Y	VIX	N/R^2
Bank Loan	0.1 bp	0.024	-0.009	0.027	0.515	-2.523	0.337	0.233	4.552	0.153	0.002	5
$\bar{t}$-stat	*-0.14*	*5.10*	*-0.91*	*4.19*	*3.09*	*-1.25*	*22.05*	*16.36*	*10.89*	*0.92*	*3.58*	32.6%
$sd(\hat{\beta})$	0.000	0.020	0.013	0.013	0.514	2.372	0.147	0.080	1.858	0.331	0.002	
Corporate Bond	0.6 bp	0.015	0.007	0.004	-5.646	-19.331	0.634	0.033	3.990	0.158	0.001	26
$\bar{t}$-stat	*0.70*	*4.09*	*0.67*	*0.82*	*-29.69*	*-9.18*	*54.32*	*2.40*	*12.18*	*1.55*	*1.79*	69.3%
$sd(\hat{\beta})$	0.000	0.008	0.024	0.010	2.205	9.287	0.168	0.060	1.097	0.511	0.001	
EM Bond	-0.4 bp	0.025	0.008	0.013	-2.399	-6.820	0.466	0.302	4.955	0.395	0.000	17
$\bar{t}$-stat	*-0.71*	*3.32*	*0.68*	*1.42*	*-9.05*	*-2.19*	*18.79*	*12.57*	*8.08*	*1.74*	*0.25*	34.7%
$sd(\hat{\beta})$	0.000	0.025	0.020	0.010	2.826	9.807	0.287	0.178	3.173	0.338	0.002	
EM Local-FX Bond	-0.6 bp	0.042	0.016	0.032	-2.046	4.260	0.380	0.303	6.645	0.391	-0.003	8
$\bar{t}$-stat	*-0.16*	*4.48*	*1.19*	*1.88*	*-6.12*	*1.71*	*12.09*	*10.77*	*7.37*	*1.15*	*-2.43*	16.5%
$sd(\hat{\beta})$	0.000	0.049	0.027	0.031	1.100	8.286	0.236	0.195	3.943	0.587	0.003	
HY Bond	0.4 bp	0.040	0.003	0.013	-1.125	1.766	0.667	0.624	9.236	-0.100	0.000	46
$\bar{t}$-stat	*1.36*	*8.54*	*0.39*	*1.81*	*-4.35*	*0.54*	*28.89*	*37.34*	*22.23*	*-0.75*	*0.57*	67.6%
$sd(\hat{\beta})$	0.000	0.021	0.022	0.014	1.477	5.704	0.177	0.160	2.453	0.390	0.002	
HY Muni	0.8 bp	0.027	0.009	0.067	-3.743	-12.225	0.587	0.303	5.157	-0.094	0.006	4
$\bar{t}$-stat	*0.18*	*1.16*	*0.66*	*2.92*	*-9.72*	*-2.83*	*12.72*	*6.06*	*3.81*	*0.25*	*2.72*	19.0%
$sd(\hat{\beta})$	0.000	0.035	0.014	0.042	1.078	3.017	0.285	0.147	3.415	0.585	0.005	
Infl. Prot. Bond	1.0 bp	0.016	0.023	0.020	-5.306	-9.395	0.181	0.020	1.934	3.098	0.003	12
$\bar{t}$-stat	*1.54*	*4.54*	*2.92*	*3.48*	*-33.99*	*-0.95*	*13.74*	*1.96*	*8.05*	*30.57*	*5.59*	57.0%
$sd(\hat{\beta})$	0.000	0.006	0.022	0.010	4.026	25.987	0.118	0.019	2.686	1.509	0.001	
Int. Bond	0.3 bp	0.009	0.007	0.009	-6.140	-13.899	0.239	0.037	1.874	0.124	0.000	24
$\bar{t}$-stat	*0.25*	*2.66*	*0.66*	*1.88*	*-56.34*	*-11.00*	*25.60*	*2.74*	*7.93*	*2.46*	*0.22*	69.6%
$sd(\hat{\beta})$	0.000	0.010	0.020	0.010	0.977	6.205	0.080	0.051	0.767	0.280	0.001	
Int. Govt	0.7 bp	0.012	0.009	0.014	-5.955	-3.968	0.089	0.027	2.328	0.145	-0.000	12
$\bar{t}$-stat	*0.75*	*2.81*	*0.96*	*2.18*	*-69.75*	*-5.13*	*8.76*	*1.59*	*8.18*	*2.58*	*-0.29*	59.7%
$sd(\hat{\beta})$	0.000	0.013	0.023	0.014	0.802	5.715	0.117	0.047	1.283	0.260	0.001	
LT Bond	1.1 bp	0.010	0.010	-0.000	-13.071	-67.414	1.056	0.009	0.446	-0.180	0.001	5
$\bar{t}$-stat	*1.65*	*2.35*	*1.53*	*-0.10*	*-49.86*	*-27.02*	*85.48*	*1.25*	*0.67*	*-1.05*	*0.91*	83.2%
$sd(\hat{\beta})$	0.000	0.010	0.004	0.004	1.083	4.063	0.291	0.024	2.196	0.226	0.001	
LT Govt	0.3 bp	-0.033	0.003	-0.005	-16.617	-90.287	0.227	-0.122	-7.830	0.021	0.000	11
$\bar{t}$-stat	*0.31*	*-4.61*	*0.49*	*-0.58*	*-65.01*	*-38.24*	*9.47*	*-4.93*	*-12.25*	*1.31*	*0.17*	77.6%
$sd(\hat{\beta})$	0.000	0.024	0.010	0.009	6.934	54.187	0.181	0.129	6.288	0.352	0.001	
Multisector Bond	-0.4 bp	0.019	0.022	0.016	-3.004	-8.734	0.334	0.196	3.658	0.241	0.000	14
$\bar{t}$-stat	*-0.50*	*2.84*	*1.54*	*1.76*	*-18.86*	*-4.04*	*17.66*	*9.89*	*7.12*	*1.49*	*-0.25*	44.7%
$sd(\hat{\beta})$	0.000	0.022	0.053	0.030	2.483	6.497	0.200	0.162	2.981	0.257	0.003	
Muni CA LT	1.2 bp	-0.006	-0.017	0.020	-2.591	-7.994	0.309	0.039	1.303	-0.144	0.002	2
$\bar{t}$-stat	*0.34*	*-0.73*	*-1.28*	*1.62*	*-9.06*	*-2.21*	*11.45*	*1.96*	*1.53*	*-0.89*	*1.50*	13.3%
$sd(\hat{\beta})$	0.000	0.001	0.005	0.002	0.158	2.746	0.036	0.033	0.132	0.229	0.001	
Muni MN	-1.8 bp	0.004	-0.020	0.003	-2.777	-8.580	0.140	0.092	-0.139	-0.004	-0.001	1
$\bar{t}$-stat	*-1.78*	*0.51*	*-1.82*	*0.36*	*-15.84*	*-4.19*	*4.68*	*3.34*	*-0.26*	*-0.02*	*-0.67*	24.6%
$sd(\hat{\beta})$	—	—	—	—	—	—	—	—	—	—	—	

Table 13: Full Models, by Asset Class, Fund Category. Data are for 2,304 US ETFs alive as of 30 November 2021 over the period 20140101–20250530. The asset class groupings are based on fund categories. Bond, commodity, and foreign exchange funds are fit with a fixed income factor ordering; equity, real estate, and unknown funds are fit with an equity factor ordering. Multiasset funds were fit with an equity ordering if a main asset class could not be determined.

B. Fixed Income (cont.)												
	Int.	SP500	R2KSPX	R1KVMG	YCLvl	YCSlp	IG10	HY5IG10	YCBmp	InfSur5Y	VIX	N/R^2
Muni Natl Int.	-0.1 bp	0.005	-0.005	0.020	-2.930	-8.313	0.261	0.100	1.415	-0.161	0.001	17
$\bar{t}$-stat	*-0.25*	*0.55*	*-0.81*	*3.21*	*-19.21*	*-4.83*	*17.69*	*5.92*	*3.30*	*-1.31*	*2.02*	37.3%
$sd(\hat{\beta})$	0.000	0.015	0.013	0.010	0.892	4.011	0.137	0.084	0.745	0.281	0.001	
Muni Natl LT	-0.4 bp	0.011	0.002	0.037	-3.926	-12.733	0.430	0.197	3.480	-0.205	0.004	7
$\bar{t}$-stat	*-0.10*	*1.35*	*-0.26*	*3.01*	*-13.67*	*-3.75*	*14.72*	*7.39*	*3.87*	*-0.97*	*2.68*	21.0%
$sd(\hat{\beta})$	0.000	0.025	0.035	0.022	0.778	4.078	0.111	0.162	2.290	0.353	0.001	
Muni Natl ST	-0.6 bp	-0.001	0.119	-0.011	-0.832	-0.750	0.135	0.054	1.380	-0.124	0.002	7
$\bar{t}$-stat	*-0.71*	*-0.13*	*14.45*	*-1.11*	*-7.80*	*-0.79*	*12.89*	*1.59*	*2.70*	*-1.65*	*3.60*	25.5%
$sd(\hat{\beta})$	0.000	0.011	0.299	0.055	0.463	2.001	0.061	0.093	1.453	0.548	0.003	
Muni NY Int.	3.4 bp	-0.002	0.021	0.025	-2.473	-7.737	0.246	0.093	2.757	-0.174	0.003	1
$\bar{t}$-stat	*1.25*	*-0.07*	*0.59*	*0.79*	*-3.85*	*-1.10*	*3.73*	*1.48*	*1.39*	*-0.27*	*0.81*	1.3%
$sd(\hat{\beta})$	—	—	—	—	—	—	—	—	—	—	—	
Nontrad. Bond	0.8 bp	0.065	0.067	0.048	-0.006	-5.056	0.714	0.549	6.984	1.838	-0.001	9
$\bar{t}$-stat	*0.58*	*4.75*	*2.08*	*2.01*	*1.99*	*-0.75*	*18.58*	*13.04*	*7.32*	*4.32*	*-0.13*	40.7%
$sd(\hat{\beta})$	0.000	0.063	0.126	0.077	5.126	16.862	0.513	0.602	5.583	2.790	0.004	
ST Bond	0.5 bp	0.008	0.005	0.007	-2.333	3.075	0.190	0.028	2.513	0.138	0.000	24
$\bar{t}$-stat	*0.34*	*2.41*	*0.31*	*1.79*	*-27.50*	*3.26*	*21.39*	*2.25*	*11.10*	*2.37*	*0.94*	49.4%
$sd(\hat{\beta})$	0.000	0.011	0.020	0.008	0.662	3.079	0.113	0.062	1.110	0.175	0.001	
ST Govt	0.6 bp	0.003	0.008	0.007	-2.189	5.811	0.002	0.000	1.231	0.030	0.000	7
$\bar{t}$-stat	*0.97*	*2.09*	*-0.31*	*1.12*	*-48.71*	*14.01*	*0.45*	*-1.33*	*10.58*	*2.33*	*0.63*	46.9%
$sd(\hat{\beta})$	0.000	0.003	0.028	0.012	0.793	4.777	0.016	0.016	0.411	0.155	0.000	
Trading–Inv. Debt	-5.5 bp	0.057	-0.041	0.015	30.340	141.273	-0.387	0.081	12.910	-0.553	0.002	7
$\bar{t}$-stat	*-4.11*	*1.01*	*0.07*	*0.25*	*65.52*	*27.46*	*-12.10*	*-6.71*	*1.06*	*-1.59*	*0.46*	73.4%
$sd(\hat{\beta})$	0.000	0.081	0.095	0.031	22.539	134.357	0.292	0.465	20.456	0.709	0.003	
Trading–Lev. Debt	-0.2 bp	-0.023	-0.043	-0.019	-29.406	-112.939	0.554	0.147	-4.448	0.714	0.001	5
$\bar{t}$-stat	*0.42*	*-0.55*	*-0.13*	*-0.14*	*-59.65*	*-15.93*	*8.70*	*2.92*	*1.27*	*2.16*	*0.57*	55.7%
$sd(\hat{\beta})$	0.000	0.077	0.086	0.041	21.444	124.150	0.573	0.586	19.237	0.751	0.004	
UltraST Bond	-0.7 bp	0.000	0.004	0.006	-0.227	-0.298	0.076	0.008	0.718	0.078	0.000	28
$\bar{t}$-stat	*-0.72*	*0.49*	*-0.76*	*1.92*	*-5.21*	*0.93*	*9.27*	*0.98*	*3.03*	*-0.62*	*0.96*	12.4%
$sd(\hat{\beta})$	0.000	0.014	0.024	0.018	0.320	4.477	0.121	0.038	0.795	0.665	0.002	
World Bond	0.3 bp	0.027	0.028	0.014	-5.152	3.556	0.227	0.184	5.605	0.478	0.002	9
$\bar{t}$-stat	*-0.13*	*3.33*	*2.00*	*1.30*	*-18.81*	*1.57*	*9.27*	*7.86*	*7.76*	*1.89*	*1.24*	23.4%
$sd(\hat{\beta})$	0.000	0.016	0.016	0.014	2.353	7.884	0.133	0.126	2.515	0.452	0.001	
C. Fixed Income-like/Equity-Linked												
	Int.	SP500	R2KSPX	R1KVMG	YCLvl	YCSlp	IG10	HY5IG10	YCBmp	InfSur5Y	VIX	N/R^2
Convertibles	0.7 bp	0.248	0.143	-0.232	1.694	-0.275	1.082	0.860	14.010	-0.993	0.004	3
$\bar{t}$-stat	*0.62*	*22.23*	*8.18*	*-16.05*	*3.75*	*-0.01*	*26.75*	*24.57*	*13.23*	*-3.31*	*2.30*	57.4%
$sd(\hat{\beta})$	0.000	0.012	0.014	0.016	0.385	3.285	0.093	0.115	2.130	0.010	0.002	
Preferred Stock	0.2 bp	0.368	0.226	0.105	-3.251	-27.683	0.196	0.222	2.815	0.297	0.013	13
$\bar{t}$-stat	*0.18*	*28.51*	*10.89*	*5.93*	*-10.46*	*-8.13*	*6.48*	*6.42*	*2.45*	*0.75*	*7.24*	42.7%
$sd(\hat{\beta})$	0.000	0.160	0.146	0.079	1.342	7.671	0.068	0.128	2.873	0.383	0.007	
D. Equity												
	Int.	SP500	R2KSPX	R1KVMG	YCLvl	YCSlp	IG10	HY5IG10	YCBmp	InfSur5Y	VIX	N/R^2
China Region	-3.6 bp	0.592	0.173	-0.034	-0.010	5.698	0.021	0.069	-0.371	-0.185	-0.001	35
$\bar{t}$-stat	*-0.79*	*20.20*	*3.54*	*-0.66*	*0.14*	*0.74*	*0.49*	*0.65*	*-0.12*	*-0.13*	*-0.39*	17.3%
$sd(\hat{\beta})$	0.000	0.159	0.110	0.121	1.060	7.998	0.112	0.112	1.505	1.213	0.005	
Communications	-3.3 bp	0.840	0.235	-0.140	-0.690	3.553	-0.120	-0.108	-0.160	0.013	0.006	9
$\bar{t}$-stat	*-1.24*	*54.33*	*9.48*	*-6.20*	*-1.55*	*0.71*	*-1.80*	*-1.87*	*-0.09*	*0.11*	*2.04*	63.7%
$sd(\hat{\beta})$	0.000	0.229	0.193	0.145	1.317	9.689	0.156	0.135	1.961	0.471	0.009	
Cons. Cyclical	-1.0 bp	0.991	0.503	-0.100	-1.361	-6.412	-0.015	-0.053	-0.732	0.151	0.007	20
$\bar{t}$-stat	*-0.24*	*53.61*	*17.54*	*-4.58*	*-2.90*	*-1.29*	*-0.03*	*-1.44*	*-0.45*	*0.38*	*2.72*	65.0%
$sd(\hat{\beta})$	0.000	0.233	0.258	0.186	2.146	9.007	0.132	0.136	1.811	0.757	0.005	
Cons. Defensive	1.9 bp	0.588	0.103	0.227	-1.248	-0.186	0.026	0.084	1.846	-0.824	-0.002	8
$\bar{t}$-stat	*0.58*	*42.46*	*2.52*	*13.24*	*-3.30*	*-0.13*	*1.16*	*2.50*	*1.57*	*-2.29*	*-1.44*	46.6%
$sd(\hat{\beta})$	0.000	0.088	0.245	0.086	0.683	6.689	0.093	0.079	1.108	0.270	0.004	
Divers. EM	-1.6 bp	0.695	0.159	0.066	-0.843	3.349	0.154	0.041	1.942	0.304	-0.002	64
$\bar{t}$-stat	*-0.47*	*42.94*	*6.08*	*3.37*	*-1.83*	*0.84*	*3.46*	*0.93*	*1.49*	*0.86*	*-1.34*	47.4%
$sd(\hat{\beta})$	0.000	0.126	0.052	0.088	0.684	6.442	0.071	0.073	1.461	0.885	0.005	
Divers. Pac/Asia	-0.8 bp	0.750	0.129	0.099	-1.295	2.519	0.093	0.044	1.806	0.323	-0.005	1
$\bar{t}$-stat	*-0.41*	*71.28*	*7.82*	*6.96*	*-4.53*	*0.80*	*3.18*	*1.58*	*2.03*	*1.11*	*-3.65*	68.2%
$sd(\hat{\beta})$	—	—	—	—	—	—	—	—	—	—	—	
Europe Stock	-0.4 bp	0.800	0.165	0.168	0.133	-2.201	0.246	0.011	2.778	0.693	-0.003	20
$\bar{t}$-stat	*0.11*	*55.66*	*7.22*	*8.84*	*0.30*	*-0.57*	*5.93*	*0.87*	*2.38*	*1.72*	*-1.59*	57.4%
$sd(\hat{\beta})$	0.000	0.084	0.049	0.098	1.963	7.677	0.119	0.122	2.230	0.524	0.002	
Financial	1.3 bp	0.968	0.597	0.462	3.243	0.201	0.058	0.076	0.636	-0.127	0.003	20
$\bar{t}$-stat	*0.87*	*57.76*	*24.51*	*25.16*	*8.78*	*-0.06*	*1.71*	*1.80*	*0.46*	*-0.43*	*1.07*	72.4%
$sd(\hat{\beta})$	0.000	0.157	0.300	0.151	2.651	15.724	0.136	0.105	2.355	0.836	0.007	
Fgn Lg Blend	-0.3 bp	0.690	0.150	0.126	-0.855	0.151	0.144	0.067	2.514	0.477	-0.001	58
$\bar{t}$-stat	*-0.12*	*57.78*	*7.97*	*8.45*	*-2.50*	*0.13*	*4.33*	*1.85*	*2.63*	*1.49*	*-1.10*	63.5%
$sd(\hat{\beta})$	0.000	0.154	0.052	0.043	1.221	5.917	0.062	0.066	1.255	0.361	0.004	
Fgn Lg Growth	-0.7 bp	0.768	0.123	-0.013	-1.481	-5.527	0.134	0.025	1.739	0.635	-0.000	14
$\bar{t}$-stat	*-0.25*	*55.28*	*5.04*	*0.53*	*-3.52*	*-1.16*	*3.41*	*0.82*	*1.71*	*1.56*	*-0.39*	56.3%
$sd(\hat{\beta})$	0.000	0.074	0.064	0.140	1.279	6.323	0.095	0.079	2.229	0.570	0.003	
Fgn Lg Value	-0.2 bp	0.709	0.206	0.241	-0.388	6.421	0.208	0.082	3.895	1.086	0.000	30
$\bar{t}$-stat	*0.01*	*56.76*	*10.47*	*15.29*	*-1.15*	*1.73*	*6.40*	*2.59*	*3.92*	*3.26*	*-0.19*	64.8%
$sd(\hat{\beta})$	0.000	0.092	0.055	0.067	0.812	5.713	0.044	0.048	1.299	0.437	0.003	
Fgn S/M Blend	-0.2 bp	0.768	0.211	0.110	-0.921	-2.085	0.186	0.026	2.708	0.984	-0.000	4
$\bar{t}$-stat	*0.15*	*70.56*	*12.67*	*7.75*	*-3.37*	*-0.54*	*6.35*	*1.03*	*3.13*	*3.44*	*-0.20*	69.8%
$sd(\hat{\beta})$	0.000	0.056	0.026	0.018	1.965	5.416	0.029	0.058	1.974	0.508	0.002	

Table 13: Full Models, by Asset Class, Fund Category (cont.). Data are for 2,304 US ETFs alive as of 30 November 2021 over the period 20140101–20250530. The asset class groupings are based on fund categories. Bond funds are fit with a fixed income factor ordering. Convertibles funds were fit with a fixed income factor ordering; preferred funds were fit with an equity factor ordering.

D. Equity (cont.)	Int.	SP500	R2KSPX	R1KVMG	YCLvl	YCSlp	IG10	HY5IG10	YCBmp	InfSur5Y	VIX	N/R^2
Fgn S/M Growth	-5.0 bp	0.716	0.241	-0.077	-2.407	-6.319	0.117	0.031	1.637	0.503	0.010	2
$\bar{t}$-stat	*-1.66*	*31.12*	*6.93*	*-2.46*	*-3.60*	*-0.70*	*1.65*	*0.57*	*0.94*	*0.77*	*2.84*	36.8%
$sd(\hat{\beta})$	0.000	0.085	0.073	0.020	1.217	13.295	0.106	0.056	2.013	0.084	0.007	
Fgn S/M Value	-0.0 bp	0.729	0.210	0.171	-0.958	-0.375	0.185	0.077	3.012	1.295	-0.000	7
$\bar{t}$-stat	*0.17*	*56.08*	*10.50*	*10.46*	*-2.62*	*0.29*	*5.02*	*1.89*	*3.19*	*3.30*	*-0.65*	60.9%
$sd(\hat{\beta})$	0.000	0.073	0.038	0.071	1.261	10.266	0.024	0.073	2.007	0.877	0.005	
Health	-0.1 bp	0.858	0.467	-0.101	-2.268	2.707	-0.147	-0.084	-1.953	-1.184	0.000	31
$\bar{t}$-stat	*-0.14*	*38.71*	*11.73*	*-1.98*	*-3.66*	*0.56*	*-1.92*	*-0.81*	*-0.81*	*-1.90*	*-0.17*	48.9%
$sd(\hat{\beta})$	0.000	0.230	0.386	0.221	1.866	10.688	0.156	0.124	3.136	0.797	0.006	
India Equity	1.2 bp	0.716	0.125	0.150	-0.199	6.584	0.182	0.102	3.688	-1.131	-0.003	9
$\bar{t}$-stat	*0.76*	*36.94*	*4.11*	*5.68*	*-0.38*	*1.14*	*3.35*	*1.98*	*2.24*	*-2.08*	*-1.29*	37.1%
$sd(\hat{\beta})$	0.000	0.085	0.017	0.022	0.248	2.558	0.031	0.031	0.324	0.655	0.007	
Industrials	1.8 bp	1.000	0.537	0.163	0.528	-0.068	0.037	-0.069	-0.153	0.610	0.003	16
$\bar{t}$-stat	*1.08*	*56.83*	*22.48*	*10.00*	*1.32*	*-0.14*	*0.88*	*-1.60*	*-0.04*	*1.37*	*0.62*	58.4%
$sd(\hat{\beta})$	0.000	0.095	0.216	0.207	0.734	8.608	0.111	0.094	2.057	1.072	0.005	
Japan Stock	-0.6 bp	0.632	0.117	0.104	1.347	2.396	0.004	0.005	-0.184	-0.398	-0.007	12
$\bar{t}$-stat	*-0.24*	*38.62*	*4.48*	*4.81*	*2.72*	*0.50*	*0.26*	*0.06*	*-0.10*	*-0.87*	*-3.29*	41.0%
$sd(\hat{\beta})$	0.000	0.158	0.045	0.037	3.276	7.832	0.050	0.045	1.703	0.447	0.005	
Large Blend	1.2 bp	0.780	0.057	0.044	-0.104	-1.114	0.033	0.013	0.383	0.098	0.002	167
$\bar{t}$-stat	*0.95*	*130.46*	*4.92*	*3.61*	*-0.53*	*-0.54*	*0.94*	*0.77*	*0.40*	*0.37*	*1.14*	75.2%
$sd(\hat{\beta})$	0.001	0.242	0.147	0.093	1.516	10.324	0.134	0.143	2.084	0.886	0.007	
Large Growth	-0.1 bp	1.036	0.053	-0.231	-0.634	-3.605	-0.123	-0.088	-1.181	0.049	0.004	62
$\bar{t}$-stat	*0.04*	*83.78*	*2.31*	*-17.78*	*-2.12*	*-1.31*	*-3.21*	*-2.63*	*-1.40*	*0.15*	*2.50*	76.8%
$sd(\hat{\beta})$	0.000	0.222	0.158	0.127	1.423	10.171	0.112	0.087	1.895	0.559	0.007	
Large Value	1.6 bp	0.795	0.196	0.276	0.391	4.174	0.092	0.076	1.923	0.141	-0.001	78
$\bar{t}$-stat	*1.16*	*80.80*	*11.86*	*26.49*	*1.48*	*1.14*	*4.07*	*2.69*	*2.23*	*0.68*	*-0.58*	75.2%
$sd(\hat{\beta})$	0.000	0.181	0.132	0.128	1.070	8.826	0.074	0.110	3.789	0.914	0.007	
LatAm Stock	-2.3 bp	0.993	0.394	0.321	-3.219	0.700	0.341	0.233	7.573	1.983	0.001	5
$\bar{t}$-stat	*-0.47*	*32.08*	*8.22*	*7.69*	*-3.83*	*0.01*	*3.98*	*2.90*	*2.92*	*2.40*	*0.32*	32.3%
$sd(\hat{\beta})$	0.000	0.106	0.057	0.063	1.199	7.257	0.052	0.035	1.132	0.143	0.004	
L/S Equity	-0.9 bp	0.374	0.135	-0.046	0.022	-3.251	-0.058	-0.062	-0.463	0.624	-0.002	12
$\bar{t}$-stat	*-0.43*	*25.04*	*5.88*	*-0.80*	*0.30*	*-0.35*	*-1.07*	*-1.25*	*-0.05*	*1.60*	*-1.17*	32.5%
$sd(\hat{\beta})$	0.000	0.215	0.201	0.225	1.452	10.310	0.126	0.117	2.081	0.572	0.008	
Mid Blend	1.4 bp	0.902	0.453	0.160	0.045	-2.183	0.013	-0.000	0.526	0.106	0.004	39
$\bar{t}$-stat	*0.96*	*70.86*	*29.32*	*13.60*	*-0.03*	*-1.26*	*0.64*	*0.04*	*0.55*	*0.57*	*2.46*	75.4%
$sd(\hat{\beta})$	0.000	0.185	0.180	0.093	0.684	11.083	0.081	0.060	1.567	0.588	0.004	
Mid Growth	0.2 bp	1.046	0.478	-0.185	-0.976	-8.835	-0.132	-0.109	-1.565	0.055	0.008	23
$\bar{t}$-stat	*0.32*	*62.73*	*22.03*	*-8.40*	*-2.39*	*-2.35*	*-2.77*	*-2.41*	*-1.44*	*0.42*	*3.03*	67.6%
$sd(\hat{\beta})$	0.000	0.168	0.162	0.226	1.305	9.833	0.126	0.090	1.671	0.629	0.008	
Mid Value	2.0 bp	0.905	0.506	0.353	0.224	-3.235	0.091	0.065	1.721	0.439	0.004	25
$\bar{t}$-stat	*1.23*	*62.30*	*26.31*	*25.63*	*0.61*	*-0.74*	*2.91*	*1.97*	*1.85*	*1.14*	*2.58*	76.6%
$sd(\hat{\beta})$	0.000	0.123	0.201	0.108	1.244	8.460	0.085	0.066	1.443	0.799	0.003	
Misc. Region	-2.1 bp	0.720	0.170	0.172	0.059	6.124	0.191	0.058	3.862	1.170	-0.002	50
$\bar{t}$-stat	*-0.50*	*38.24*	*5.94*	*6.33*	*-0.30*	*1.06*	*3.31*	*1.50*	*2.38*	*2.30*	*-0.46*	38.4%
$sd(\hat{\beta})$	0.000	0.217	0.099	0.108	2.066	14.053	0.139	0.225	2.732	1.506	0.006	
Misc. Sector	-5.1 bp	1.000	0.563	-0.045	-1.886	-9.456	0.041	-0.087	-0.093	0.946	0.003	22
$\bar{t}$-stat	*-0.78*	*46.65*	*15.67*	*-1.47*	*-3.85*	*-1.27*	*1.07*	*-0.81*	*0.25*	*1.63*	*1.57*	53.6%
$sd(\hat{\beta})$	0.001	0.210	0.284	0.152	2.100	11.543	0.176	0.142	2.617	0.941	0.008	
Pac/Asia ex-JP Eq	-0.9 bp	0.752	0.136	0.073	-1.173	2.191	0.189	0.048	1.726	0.326	-0.002	10
$\bar{t}$-stat	*-0.24*	*50.03*	*5.80*	*3.97*	*-2.93*	*0.55*	*4.58*	*1.24*	*1.49*	*0.90*	*-0.87*	52.0%
$sd(\hat{\beta})$	0.000	0.095	0.041	0.116	0.790	4.415	0.099	0.045	1.913	0.799	0.004	
Small Blend	1.5 bp	0.893	0.772	0.169	0.136	0.625	-0.057	-0.048	-0.150	0.125	0.005	50
$\bar{t}$-stat	*0.93*	*50.21*	*46.64*	*12.30*	*0.07*	*-0.28*	*-2.01*	*-2.31*	*-0.52*	*0.28*	*2.92*	74.2%
$sd(\hat{\beta})$	0.000	0.189	0.196	0.106	0.717	10.276	0.068	0.083	1.816	0.664	0.006	
Small Growth	1.1 bp	0.982	0.791	-0.062	-0.010	-4.959	-0.169	-0.137	-2.240	0.347	0.006	16
$\bar{t}$-stat	*0.54*	*51.35*	*40.56*	*-5.38*	*-0.90*	*-1.43*	*-4.90*	*-4.68*	*-2.57*	*0.71*	*2.75*	64.6%
$sd(\hat{\beta})$	0.000	0.217	0.165	0.127	0.859	10.648	0.081	0.117	1.814	0.842	0.003	
Small Value	2.3 bp	0.994	0.941	0.312	0.484	-1.331	-0.008	-0.033	0.028	0.311	0.004	18
$\bar{t}$-stat	*1.26*	*50.05*	*48.03*	*21.52*	*1.54*	*-0.24*	*-0.45*	*-1.26*	*0.01*	*0.83*	*2.59*	76.5%
$sd(\hat{\beta})$	0.000	0.077	0.138	0.097	1.072	5.336	0.048	0.084	0.979	0.274	0.004	
Technology	-1.4 bp	1.162	0.321	-0.388	-1.398	-5.897	-0.179	-0.171	-2.334	0.062	0.010	57
$\bar{t}$-stat	*-0.11*	*56.37*	*9.25*	*-14.63*	*-2.44*	*-1.15*	*-2.43*	*-2.56*	*-1.25*	*0.32*	*2.92*	64.9%
$sd(\hat{\beta})$	0.001	0.272	0.316	0.217	1.552	12.903	0.170	0.144	2.858	0.919	0.009	
Trading–Inv. Eq	-23.8 bp	-1.862	-0.552	-0.071	2.121	2.646	-0.063	0.042	-0.806	-1.296	-0.004	65
$\bar{t}$-stat	*-2.23*	*-25.06*	*-4.87*	*-2.19*	*0.82*	*0.01*	*-0.62*	*-0.19*	*-0.32*	*-0.46*	*-0.32*	26.9%
$sd(\hat{\beta})$	0.002	1.142	0.796	0.777	8.887	26.442	0.387	0.364	7.741	5.100	0.015	
Trading–Lev. Eq	-4.8 bp	2.365	0.721	0.167	-2.193	-2.054	0.136	0.023	4.483	2.208	0.002	78
$\bar{t}$-stat	*-0.29*	*41.46*	*8.83*	*2.31*	*-1.30*	*-0.30*	*0.82*	*-0.02*	*0.48*	*0.85*	*0.35*	47.3%
$sd(\hat{\beta})$	0.001	0.744	0.879	0.771	9.732	32.958	0.517	0.456	12.546	4.958	0.021	
Utilities	1.8 bp	0.645	0.107	0.324	-4.906	-6.455	0.182	0.229	3.975	-0.362	0.003	8
$\bar{t}$-stat	*0.79*	*39.18*	*4.74*	*15.48*	*-11.63*	*-1.36*	*4.18*	*5.63*	*3.13*	*-0.98*	*1.28*	50.4%
$sd(\hat{\beta})$	0.000	0.039	0.224	0.120	1.170	3.197	0.148	0.047	1.687	0.687	0.002	

Table 13: Full Models, by Asset Class, Fund Category (cont.). Data are for 2,304 US ETFs alive as of 30 November 2021 over the period 20140101–20250530. The asset class groupings are based on fund categories. Equity funds are fit with an equity factor ordering.

	Int.	SP500	R2KSPX	R1KVMG	YCLvl	YCSlp	IG10	HY5IG10	YCBmp	InfSur5Y	VIX	N/R^2
E. Commodity-Linked												
Commodities Broad	1.0 bp	0.081	0.066	0.116	2.055	17.336	0.481	0.435	9.032	5.418	-0.002	17
$\bar{t}$-stat	*0.52*	*4.48*	*2.34*	*4.67*	*3.61*	*2.81*	*8.72*	*8.27*	*5.72*	*10.67*	*-0.70*	16.0%
$sd(\hat{\beta})$	0.000	0.024	0.030	0.065	1.667	7.440	0.200	0.186	4.529	1.967	0.002	
Energy LP	-2.0 bp	0.803	0.500	0.452	0.867	0.066	0.265	0.141	8.698	5.573	0.008	18
$\bar{t}$-stat	*0.01*	*25.58*	*10.57*	*11.70*	*0.57*	*-0.07*	*3.20*	*1.88*	*3.26*	*6.51*	*1.62*	33.5%
$sd(\hat{\beta})$	0.001	0.211	0.170	0.144	1.189	9.258	0.139	0.207	2.779	2.969	0.007	
Equity Energy	-1.7 bp	1.102	0.817	0.670	4.283	10.094	0.180	0.099	5.534	7.488	0.005	13
$\bar{t}$-stat	*-0.20*	*35.01*	*16.55*	*17.29*	*5.37*	*1.09*	*2.40*	*1.26*	*2.26*	*9.58*	*1.05*	49.4%
$sd(\hat{\beta})$	0.000	0.159	0.264	0.278	2.998	13.248	0.121	0.136	3.032	2.676	0.004	
Eq Prec. Metals	-0.3 bp	0.265	0.265	0.060	-10.494	27.881	1.146	0.869	26.355	5.787	0.002	11
$\bar{t}$-stat	*-0.09*	*6.08*	*3.85*	*1.09*	*-8.50*	*2.00*	*8.87*	*7.02*	*6.94*	*4.89*	*0.28*	14.0%
$sd(\hat{\beta})$	0.000	0.096	0.121	0.046	7.074	9.570	0.290	0.384	8.223	1.697	0.004	
Managed Futures	-0.6 bp	0.028	0.023	-0.011	2.943	-3.108	-0.081	-0.119	-2.573	1.391	-0.004	5
$\bar{t}$-stat	*-0.35*	*1.78*	*1.23*	*-1.28*	*6.12*	*-1.01*	*-1.28*	*-2.66*	*-2.27*	*3.80*	*-2.08*	10.5%
$sd(\hat{\beta})$	0.000	0.053	0.017	0.028	3.818	12.010	0.174	0.104	1.733	0.900	0.004	
Nat. Resources	0.8 bp	0.348	0.270	0.167	4.542	26.669	1.498	1.393	25.910	0.612	-0.006	28
$\bar{t}$-stat	*0.32*	*18.96*	*9.20*	*7.51*	*6.00*	*3.14*	*21.63*	*23.48*	*14.70*	*0.36*	*-2.63*	49.2%
$sd(\hat{\beta})$	0.000	0.068	0.161	0.159	2.816	17.578	0.217	0.210	5.010	2.544	0.004	
Trading–Inv. Com.	-18.3 bp	-0.227	-0.141	-0.158	11.376	-28.346	-1.276	-0.944	-23.746	-9.716	-0.004	9
$\bar{t}$-stat	*-1.88*	*-2.29*	*-0.89*	*-1.18*	*8.84*	*-1.18*	*-4.59*	*-2.39*	*-3.30*	*-4.62*	*-1.34*	8.8%
$sd(\hat{\beta})$	0.002	0.245	0.246	0.288	16.727	37.776	1.400	1.272	20.699	7.514	0.021	
Trading–Lev. Com.	-13.1 bp	0.279	0.118	0.228	-10.404	37.404	1.678	1.216	30.518	11.141	0.008	8
$\bar{t}$-stat	*-0.53*	*2.76*	*1.11*	*1.48*	*-8.88*	*0.79*	*5.82*	*2.88*	*3.75*	*5.30*	*1.34*	11.3%
$sd(\hat{\beta})$	0.002	0.251	0.227	0.287	18.624	51.034	1.448	1.458	27.172	7.471	0.026	
F. Real Estate												
	Int.	SP500	R2KSPX	R1KVMG	YCLvl	YCSlp	IG10	HY5IG10	YCBmp	InfSur5Y	VIX	N/R^2
Global RE	-1.4 bp	0.681	0.234	0.204	-4.594	-6.012	0.302	0.106	3.382	0.560	0.001	8
$\bar{t}$-stat	*-0.86*	*53.99*	*12.09*	*12.28*	*-14.36*	*-1.74*	*9.29*	*3.45*	*3.48*	*1.73*	*0.46*	61.3%
$sd(\hat{\beta})$	0.000	0.069	0.041	0.044	0.891	4.463	0.051	0.045	0.661	0.354	0.003	
Real Estate	-0.3 bp	0.819	0.427	0.324	-5.709	-15.878	0.300	0.149	3.484	-0.118	0.008	23
$\bar{t}$-stat	*-0.11*	*41.01*	*13.64*	*13.17*	*-11.71*	*-2.60*	*5.67*	*3.06*	*2.26*	*-0.43*	*3.10*	56.2%
$sd(\hat{\beta})$	0.000	0.080	0.208	0.088	1.371	12.173	0.127	0.088	1.548	0.700	0.005	
G. Foreign Exchange												
	Int.	SP500	R2KSPX	R1KVMG	YCLvl	YCSlp	IG10	HY5IG10	YCBmp	InfSur5Y	VIX	N/R^2
Single Currency	-0.8 bp	0.028	0.016	0.022	-3.030	10.684	0.180	0.157	4.996	0.715	0.001	6
$\bar{t}$-stat	*-0.85*	*3.25*	*1.16*	*1.96*	*-12.21*	*3.91*	*7.13*	*6.67*	*6.75*	*3.11*	*0.40*	17.7%
$sd(\hat{\beta})$	0.000	0.045	0.025	0.016	2.657	4.116	0.185	0.199	3.322	0.313	0.003	

Table 13: Full Models, by Asset Class, Fund Category (cont.). Data are for 2,304 US ETFs alive as of 30 November 2021 over the period 2014–2025Q1. The asset class groupings are based on fund categories. Commodity and FX funds are fit with a fixed income factor ordering; real estate funds are fit with an equity factor ordering.

	Average Absolute 1-Day Coefficient Change, $\lvert\hat{\beta}_{\tau} - \hat{\beta}_{\tau-1}\rvert$										
Asset Class	Intercept	SP500.xs	R2KSPX	R1KVMG	YCLevel	YCSlope	IG10	HY5IG10	YCBump	InfSur5Y	VIX
Kitchen Sink Model with Transformed Coefficients for Comparison											
Bonds	0.09 bp	0.0163	0.0017	0.0015	0.1028	0.7011	0.0050	0.0046	0.1952	0.0241	0.00021
Commodities	0.79 bp	0.1411	0.0150	0.0133	0.8745	6.0414	0.0426	0.0396	1.6647	0.2031	0.00185
Equities	0.33 bp	0.0599	0.0064	0.0057	0.3672	2.5153	0.0179	0.0164	0.7029	0.0858	0.00078
FX	0.21 bp	0.0375	0.0039	0.0035	0.2223	1.5529	0.0109	0.0102	0.4192	0.0534	0.00049
Real Estate	0.35 bp	0.0662	0.0068	0.0060	0.3925	2.6157	0.0185	0.0177	0.7509	0.0964	0.00083
Unclear	0.51 bp	0.0879	0.0097	0.0084	0.5438	3.7796	0.0272	0.0248	1.0530	0.1250	0.00120
Corrected Covariates Model											
Bonds	0.09 bp	0.0021	0.0013	0.0013	0.0394	0.2385	0.0050	0.0045	0.1100	0.0239	0.00021
Commodities	0.78 bp	0.0175	0.0110	0.0109	0.3295	1.9751	0.0414	0.0380	0.9227	0.1988	0.00182
Equities	0.34 bp	0.0076	0.0048	0.0048	0.1433	0.8628	0.0179	0.0163	0.4015	0.0855	0.00079
FX	0.21 bp	0.0046	0.0029	0.0029	0.0844	0.5170	0.0106	0.0099	0.2426	0.0523	0.00048
Real Estate	0.35 bp	0.0079	0.0051	0.0051	0.1475	0.9580	0.0184	0.0175	0.4139	0.0949	0.00083
Unclear	0.50 bp	0.0109	0.0071	0.0070	0.2061	1.2673	0.0263	0.0237	0.5758	0.1209	0.00117
Sequential Shrinkage Model											
Bonds	0.10 bp	0.0008	0.0011	0.0007	0.0280	0.2719	0.0039	0.0036	0.0607	0.0199	0.00010
Commodities	0.81 bp	0.0060	0.0092	0.0062	0.1752	1.8079	0.0294	0.0280	0.3977	0.1626	0.00085
Equities	0.34 bp	0.0039	0.0042	0.0030	0.0658	0.6668	0.0104	0.0101	0.1386	0.0643	0.00037
FX	0.21 bp	0.0015	0.0023	0.0016	0.0426	0.4638	0.0074	0.0069	0.0995	0.0398	0.00021
Real Estate	0.35 bp	0.0043	0.0045	0.0032	0.0691	0.7578	0.0109	0.0111	0.1471	0.0701	0.00039
Unclear	0.51 bp	0.0052	0.0060	0.0040	0.0978	0.9892	0.0151	0.0144	0.2044	0.0942	0.00057
Fama-French (FFC6, FF93L) Models											
Asset Class	Intercept	EqMkt	EqSize	EqValue	YCLevel	YCSlope	IG10	RMW	CMA	WML	
Bonds	0.16 bp	0.0012	0.0022	0.0019	—	0.3627	0.0065	—	—	—	—
Commodities	0.85 bp	0.0062	0.0115	0.0096	—	1.8669	0.0332	—	—	—	—
Equities	0.32 bp	0.0041	0.0059	0.0056	—	—	—	0.0063	0.0089	0.0051	—
FX	0.23 bp	0.0017	0.0032	0.0027	—	0.5154	0.0090	—	—	—	—
Real Estate	0.37 bp	0.0046	0.0066	0.0065	—	—	—	0.0074	0.0103	0.0058	—
Unclear	0.52 bp	0.0062	0.0094	0.0089	—	—	—	0.0096	0.0145	0.0081	—

Table 14: Out-of-Sample Mean Absolute 1-Day Beta Changes Across Model Forms, by Asset Class. Data are for 2,288 US ETFs alive as of 30 November 2021 over the period 20211130–20250530. Beta changes are determined from rolling regressions where the prior year of data is used. Asset class groupings are based on fund categories. Commodity and FX funds are fit with a fixed income factor ordering; real estate funds are fit with an equity factor ordering. Note that the average coefficient change 1-day in the future is smallest in almost all cases for the full liquid factor model (Sequential Shrinkage). This suggests that feature engineering and shrinkage estimation does lower out-of-sample variance.

	Average Absolute 21-day Coefficient Change, $\lvert\hat{\beta}_{\tau} - \hat{\beta}_{\tau-21}\rvert$										
Asset Class	Intercept	SP500.xs	R2KSPX	R1KVMG	YCLevel	YCSlope	IG10	HY5IG10	YCBump	InfSur5Y	VIX
Kitchen Sink Model with Transformed Coefficients for Comparison											
Bonds	0.42 bp	0.1020	0.0111	0.0095	0.6477	4.3841	0.0329	0.0288	1.1978	0.1601	0.0014
Commodities	3.78 bp	0.9258	0.0897	0.0888	5.1811	35.7020	0.2530	0.2337	10.0250	1.3003	0.0124
Equities	1.65 bp	0.3994	0.0436	0.0405	2.2854	15.2688	0.1161	0.1008	4.3653	0.5640	0.0051
FX	0.87 bp	0.2175	0.0225	0.0206	1.1320	8.6677	0.0659	0.0569	2.3900	0.2833	0.0030
Real Estate	1.54 bp	0.4316	0.0426	0.0369	2.3960	16.5276	0.1122	0.1051	4.4555	0.5932	0.0049
Unclear	2.53 bp	0.6104	0.0656	0.0576	3.5122	24.0600	0.1798	0.1562	6.7022	0.8424	0.0082
Corrected Covariates Model											
Bonds	0.42 bp	0.0133	0.0083	0.0083	0.2738	1.5290	0.0325	0.0285	0.7224	0.1579	0.0014
Commodities	3.79 bp	0.1021	0.0642	0.0712	1.8672	12.1432	0.2502	0.2210	5.5311	1.2650	0.0123
Equities	1.66 bp	0.0487	0.0326	0.0352	0.9025	5.2427	0.1187	0.1012	2.4873	0.5514	0.0051
FX	0.86 bp	0.0245	0.0162	0.0171	0.5594	3.2386	0.0652	0.0564	1.5713	0.2686	0.0029
Real Estate	1.48 bp	0.0474	0.0306	0.0314	0.8819	5.9388	0.1122	0.1029	2.4378	0.5854	0.0049
Unclear	2.50 bp	0.0676	0.0474	0.0488	1.3089	8.1968	0.1785	0.1481	3.5931	0.8131	0.0080
Sequential Shrinkage Model											
Bonds	0.48 bp	0.0058	0.0073	0.0046	0.2644	2.1564	0.0296	0.0269	0.4884	0.1299	0.0007
Commodities	3.96 bp	0.0391	0.0535	0.0405	1.2406	11.3531	0.2102	0.1933	2.8710	1.0783	0.0058
Equities	1.64 bp	0.0312	0.0307	0.0215	0.4619	4.2645	0.0662	0.0636	0.8793	0.4247	0.0025
FX	0.87 bp	0.0098	0.0128	0.0091	0.3269	3.0374	0.0478	0.0428	0.7342	0.2088	0.0014
Real Estate	1.48 bp	0.0305	0.0308	0.0198	0.4805	5.0827	0.0686	0.0667	0.9877	0.4362	0.0023
Unclear	2.47 bp	0.0434	0.0416	0.0294	0.6713	6.7513	0.1055	0.0996	1.3223	0.6269	0.0039
Fama-French (FFC6, FF93L) Models											
Asset Class	Intercept	EqMkt	EqSize	EqValue	YCLevel	YCSlope	IG10	RMW	CMA	WML	
Bonds	0.82 bp	0.0078	0.0177	0.0147	—	2.8681	0.0478	—	—	—	
Commodities	4.04 bp	0.0377	0.0744	0.0649	—	11.5781	0.2267	—	—	—	
Equities	1.56 bp	0.0313	0.0381	0.0404	—	—	—	0.0417	0.0649	0.0371	
FX	0.99 bp	0.0104	0.0203	0.0167	—	3.3368	0.0543	—	—	—	
Real Estate	1.67 bp	0.0326	0.0371	0.0449	—	—	—	0.0450	0.0655	0.0350	
Unclear	2.55 bp	0.0468	0.0595	0.0636	—	—	—	0.0595	0.1065	0.0571	

Table 15: Out-of-Sample Mean Absolute 21-day (Monthly) Beta Changes Across Model Forms, by Asset Class. Data are for 2,288 US ETFs alive as of 30 November 2021 over the period 20211130–20250530. Beta changes are determined from rolling regressions where the prior year of data is used. Asset class groupings are based on fund categories. Commodity and FX funds are fit with a fixed income factor ordering; real estate funds are fit with an equity factor ordering. Note that the average coefficient change 21 days in the future is smallest in almost all cases for the full liquid factor model (Sequential Shrinkage). This suggests that feature engineering and shrinkage estimation does lower out-of-sample variance.

	Average Absolute 65-day Coefficient Change, $\lvert\hat{\beta}_\tau - \hat{\beta}_{\tau-65}\rvert$										
Asset Class	Intercept	SP500.xs	R2KSPX	R1KVMG	YCLevel	YCSlope	IG10	HY5IG10	YCBump	InfSur5Y	VIX
Kitchen Sink Model with Transformed Coefficients for Comparison											
Bonds	0.79 bp	0.1963	0.0220	0.0180	1.2215	7.9238	0.0623	0.0562	2.2348	0.3186	0.0027
Commodities	6.81 bp	1.6944	0.1695	0.1656	8.8478	65.2171	0.4510	0.4034	18.4876	2.4871	0.0234
Equities	2.99 bp	0.8150	0.0902	0.0825	4.0070	27.0975	0.2104	0.1840	8.0001	1.0998	0.0098
FX	1.49 bp	0.3727	0.0400	0.0362	1.6956	14.8343	0.1197	0.0930	4.0133	0.5406	0.0054
Real Estate	2.70 bp	0.8141	0.0847	0.0648	3.8987	26.6529	0.2090	0.1585	7.3898	1.0613	0.0084
Unclear	4.88 bp	1.2325	0.1339	0.1128	6.5741	45.9131	0.3415	0.2984	12.8839	1.6368	0.0159
Corrected Covariates Model											
Bonds	0.81 bp	0.0257	0.0164	0.0162	0.5497	2.9462	0.0624	0.0566	1.4290	0.3130	0.0027
Commodities	6.77 bp	0.1874	0.1221	0.1328	2.8937	23.7085	0.4409	0.3747	9.7176	2.3984	0.0235
Equities	3.02 bp	0.0926	0.0679	0.0733	1.5556	9.7186	0.2182	0.1850	4.5610	1.0744	0.0098
FX	1.45 bp	0.0465	0.0279	0.0306	0.9942	5.5381	0.1140	0.0924	2.9748	0.5008	0.0054
Real Estate	2.64 bp	0.0989	0.0612	0.0623	1.3766	11.6138	0.1996	0.1617	4.3279	1.0169	0.0084
Unclear	4.84 bp	0.1277	0.0985	0.0985	2.2942	15.8601	0.3431	0.2819	6.4801	1.5832	0.0156
Sequential Shrinkage Model											
Bonds	0.92 bp	0.0137	0.0150	0.0090	0.6448	4.8467	0.0689	0.0579	1.1773	0.2648	0.0014
Commodities	7.22 bp	0.0912	0.1048	0.0743	2.7258	22.5574	0.4586	0.3717	6.6234	2.2371	0.0118
Equities	3.00 bp	0.0687	0.0694	0.0467	0.9030	8.4868	0.1279	0.1232	1.7441	0.8427	0.0048
FX	1.40 bp	0.0225	0.0249	0.0160	0.6908	5.6112	0.0997	0.0849	1.7058	0.3873	0.0026
Real Estate	2.64 bp	0.0728	0.0726	0.0410	0.9706	10.1592	0.1348	0.1132	1.9886	0.8006	0.0041
Unclear	4.73 bp	0.1013	0.0923	0.0641	1.3610	13.6081	0.2015	0.1993	2.5961	1.2438	0.0081
Fama-French (FFC6, FF93L) Models											
Asset Class	Intercept	EqMkt	EqSize	EqValue	YCLevel	YCSlope	IG10	RMW	CMA	WML	
Bonds	1.61 bp	0.0170	0.0351	0.0349	—	6.2743	0.1054	—	—	—	
Commodities	6.94 bp	0.0788	0.1482	0.1380	—	23.9017	0.4780	—	—	—	
Equities	2.88 bp	0.0676	0.0760	0.0847	—	—	—	0.0824	0.1371	0.0784	
FX	1.69 bp	0.0239	0.0400	0.0387	—	6.5970	0.1124	—	—	—	
Real Estate	3.10 bp	0.0733	0.0657	0.0971	—	—	—	0.0911	0.1430	0.0643	
Unclear	4.71 bp	0.1017	0.1199	0.1420	—	—	—	0.1166	0.2386	0.1152	

Table 16: Out-of-Sample Mean Absolute 65-day (Quarterly) Beta Changes Across Model Forms, by Asset Class. Data are for 2,288 US ETFs alive as of 30 November 2021 over the period 20211130–20250530. Beta changes are determined from rolling regressions where the prior year of data is used. Asset class groupings are based on fund categories. Commodity and FX funds are fit with a fixed income factor ordering; real estate funds are fit with an equity factor ordering. Note that the average coefficient change 1-day in the future is smallest in almost all cases for the full liquid factor model (Sequential Shrinkage). This suggests that the shrinkage estimation does, in fact, lower out-of-sample variance.

	Average Absolute 1-Day Significant Coefficient Change, $\left|\hat{\beta}_\tau \mathbb{I}(\lvert t_\tau \rvert > 2) - \hat{\beta}_{\tau-1}\mathbb{I}(\lvert t_{\tau-1} \rvert > 2)\right|$											
Asset Class	Intercept	SP500.xs	R2KSPX	R1KVMG	YCLvl	YCSlp	IG10	HY5IG10	YCBmp	InfSur5Y	VIX	% Sig.
Kitchen Sink Model with Transformed Coefficients for Comparison												
Bonds	0.55 bp	0.1278	0.0109	0.0080	0.3066	3.2978	0.0141	0.0197	0.8430	0.1103	0.00120	27.6%
Commodities	4.65 bp	0.8961	0.0542	0.0292	6.5802	45.5907	0.2047	0.2182	8.5544	0.4692	0.01005	23.1%
Equities	2.51 bp	0.1437	0.0144	0.0129	2.7417	20.6697	0.1261	0.1001	5.7345	0.4397	0.00389	24.3%
FX	2.46 bp	0.2013	0.0191	0.0100	1.4458	14.5862	0.0384	0.0593	2.5423	0.3442	0.00211	13.3%
Real Estate	3.34 bp	0.2731	0.0138	0.0079	2.2735	17.8563	0.0799	0.1746	10.2510	0.5305	0.00523	22.3%
Unclear	3.02 bp	0.4602	0.0409	0.0241	3.8529	31.3671	0.2294	0.1526	6.5652	0.4332	0.00471	20.1%
Corrected Covariates Model												
Bonds	0.62 bp	0.0077	0.0078	0.0059	0.0603	0.7841	0.0134	0.0169	0.3565	0.1100	0.00121	35.2%
Commodities	6.31 bp	0.0453	0.0485	0.0343	1.5441	7.9818	0.2396	0.2276	4.1187	0.5567	0.01093	29.7%
Equities	2.72 bp	0.0117	0.0109	0.0120	0.8230	5.8904	0.1033	0.1063	2.5385	0.4767	0.00398	30.3%
FX	0.70 bp	0.0147	0.0136	0.0096	0.1442	1.5440	0.0419	0.0269	0.7030	0.4639	0.00446	26.9%
Real Estate	3.96 bp	0.0083	0.0080	0.0060	0.2136	6.6245	0.0794	0.1939	2.3838	0.6942	0.00388	35.7%
Unclear	2.65 bp	0.0404	0.0355	0.0305	0.8885	6.6726	0.1887	0.1354	2.9342	0.3987	0.00635	25.4%
Sequential Shrinkage Model												
Bonds	0.61 bp	0.0036	0.0062	0.0066	0.0349	0.5906	0.0062	0.0084	0.1458	0.0976	0.00100	37.3%
Commodities	6.25 bp	0.0226	0.0227	0.0390	0.5214	11.7465	0.0582	0.0864	2.4280	0.4459	0.00396	43.3%
Equities	2.70 bp	0.0047	0.0079	0.0083	0.2496	4.6672	0.0835	0.0648	1.2369	0.4074	0.00234	31.5%
FX	0.66 bp	0.0042	0.0157	0.0125	0.0539	1.9857	0.0114	0.0230	0.2725	0.2742	0.00239	36.7%
Real Estate	3.88 bp	0.0045	0.0057	0.0043	0.0961	3.2848	0.0471	0.1152	1.7498	0.7450	0.00184	36.2%
Unclear	2.55 bp	0.0104	0.0215	0.0214	0.5560	4.8408	0.1700	0.1199	2.9209	0.4124	0.00368	26.1%
Fama-French (FFC6, FF93L) Models												
Asset Class	Intercept	EqMkt	EqSize	EqVal	YCLvl	YCSlp	IG10	RMW	CMA	WML		% Sig.
Bonds	0.70 bp	0.0027	0.0044	0.0045	—	0.6169	0.0111	—	—	—		32.3%
Commodities	7.32 bp	0.0169	0.0291	0.0308	—	7.2157	0.0818	—	—	—		30.4%
Equities	2.59 bp	0.0047	0.0276	0.0187	—	—	—	0.0168	0.0310	0.0149		35.5%
FX	0.59 bp	0.0024	0.0079	0.0087	—	2.2784	0.0181	—	—	—		27.5%
Real Estate	4.93 bp	0.0047	0.0293	0.0152	—	—	—	0.0287	0.0228	0.0093		36.9%
Unclear	3.23 bp	0.0134	0.0561	0.0344	—	—	—	0.0280	0.0484	0.0351		23.3%

Table 17: Out-of-Sample Mean Absolute 1-Day Significant Beta Changes Across Model Forms, by Asset Class. Data are for 2,288 US ETFs alive as of 30 November 2021 over the period 20211130–20250530. Beta changes are determined from rolling regressions where the prior year of data is used. Insignificant betas are contemporaneously set to 0. Asset class groupings are based on fund categories. Commodity and FX funds are fit with a fixed income factor ordering; real estate funds are fit with an equity factor ordering. The average significant coefficient change 1-day in the future is smallest in almost all cases for the full liquid factor model (Sequential Shrinkage). This suggests that the shrinkage estimation does, in fact, lower out-of-sample variance. The change sizes also tend to be smaller here, for significant betas, than for all betas (in the prior table).

	Average Absolute 21-day Significant Coefficient Change, $\left\|\hat{\beta}_\tau \mathbb{I}(\|t_\tau\| > 2) - \hat{\beta}_{\tau-21}\mathbb{I}(\|t_{\tau-21}\| > 2)\right\|$											
Asset Class	Intercept	SP500.xs	R2KSPX	R1KVMG	YCLvl	YCSlp	IG10	HY5IG10	YCBmp	InfSur5Y	VIX	% Sig.
Kitchen Sink Model with Transformed Coefficients for Comparison												
Bonds	1.40 bp	0.3645	0.0359	0.0283	1.3257	13.0324	0.0671	0.0811	3.0981	0.4144	0.00454	29.8%
Commodities	16.22 bp	3.1302	0.2093	0.1711	17.6546	187.3328	0.9482	0.7765	30.0761	2.2959	0.03911	25.4%
Equities	6.03 bp	0.6705	0.0763	0.0714	8.9758	58.6534	0.4492	0.3694	19.1082	1.6000	0.01604	26.7%
FX	3.63 bp	0.9520	0.0562	0.0448	4.7374	37.6204	0.2140	0.2292	8.7813	1.1173	0.00853	15.8%
Real Estate	8.15 bp	0.9248	0.0823	0.0461	8.8492	65.3548	0.3774	0.5068	21.7461	2.1315	0.01702	24.3%
Unclear	7.69 bp	1.8073	0.1571	0.1248	13.6100	87.0676	0.6811	0.5669	23.7449	1.7421	0.02122	22.8%
Corrected Covariates Model												
Bonds	1.53 bp	0.0306	0.0286	0.0226	0.3775	3.3150	0.0645	0.0724	1.5746	0.4170	0.00420	37.3%
Commodities	19.38 bp	0.2065	0.1756	0.1499	6.0326	38.1392	0.9748	0.7496	17.1283	2.4021	0.04057	31.8%
Equities	6.97 bp	0.0686	0.0587	0.0646	2.6456	18.1708	0.4074	0.3575	7.6209	1.6519	0.01671	32.5%
FX	2.13 bp	0.0378	0.0497	0.0437	0.8514	7.1321	0.2305	0.1532	4.1226	1.1877	0.01091	29.9%
Real Estate	8.33 bp	0.0505	0.0461	0.0366	1.2486	19.7075	0.3678	0.5599	8.2742	2.6134	0.01535	37.2%
Unclear	6.84 bp	0.1612	0.1580	0.1203	2.7319	21.1784	0.7057	0.5540	9.7575	1.7258	0.02311	27.9%
Sequential Shrinkage Model												
Bonds	1.51 bp	0.0151	0.0230	0.0214	0.3097	3.8332	0.0403	0.0425	0.8524	0.3656	0.00310	38.8%
Commodities	19.53 bp	0.0964	0.1398	0.1124	2.3162	36.7973	0.3555	0.3417	8.4471	2.2373	0.01793	45.5%
Equities	6.89 bp	0.0368	0.0482	0.0440	1.1756	15.2047	0.2396	0.2326	4.1435	1.4020	0.00865	33.3%
FX	2.01 bp	0.0263	0.0384	0.0228	0.4048	10.1099	0.0768	0.0802	2.0461	0.8834	0.00494	38.4%
Real Estate	8.05 bp	0.0334	0.0370	0.0259	0.5727	16.2516	0.2436	0.2727	6.0285	1.9011	0.00875	36.8%
Unclear	6.58 bp	0.0743	0.1035	0.0866	1.9102	17.3636	0.5596	0.4155	8.1905	1.5890	0.01461	28.0%
Fama-French (FFC6, FF93L) Models												
Asset Class	Intercept	EqMkt	EqSize	EqVal	YCLvl	YCSlp	IG10	RMW	CMA	WML		% Sig.
Bonds	2.40 bp	0.0156	0.0306	0.0274	—	4.1220	0.0719	—	—	—		34.0%
Commodities	17.81 bp	0.0733	0.1550	0.1610	—	30.1218	0.4498	—	—	—		31.8%
Equities	5.96 bp	0.0377	0.0994	0.0811	—	—	—	0.0871	0.1440	0.0777		37.1%
FX	2.34 bp	0.0147	0.0450	0.0401	—	8.7670	0.1211	—	—	—		29.6%
Real Estate	10.15 bp	0.0348	0.1070	0.0890	—	—	—	0.1181	0.1201	0.0524		39.1%
Unclear	7.08 bp	0.0809	0.1885	0.1565	—	—	—	0.1372	0.2649	0.1528		25.6%

Table 18: Out-of-Sample Mean Absolute 21-Day (Monthly) Significant Beta Changes Across Model Forms, by Asset Class. Data are for 2,288 US ETFs alive as of 30 November 2021 over the period 20211130–20250530. Beta changes are determined from rolling regressions where the prior year of data is used. Insignificant betas are contemporaneously set to 0. Asset class groupings are based on fund categories. Commodity and FX funds are fit with a fixed income factor ordering; real estate funds are fit with an equity factor ordering. The average significant coefficient change 1-day in the future is smallest in almost all cases for the full liquid factor model (Sequential Shrinkage). This suggests that the shrinkage estimation does, in fact, lower out-of-sample variance. The change sizes also tend to be smaller here, for significant betas, than for all betas (in the prior table).

	Average Absolute 65-day Significant Coefficient Change, $\left|\hat{\beta}_{\tau}\mathbb{I}(|t_{\tau}|>2)-\hat{\beta}_{\tau-65}\mathbb{I}(|t_{\tau-65}|>2)\right|$											
Asset Class	Intercept	SP500.xs	R2KSPX	R1KVMG	YCLvl	YCSlp	IG10	HY5IG10	YCBmp	InfSur5Y	VIX	% Sig.
Kitchen Sink Model with Transformed Coefficients for Comparison												
Bonds	1.77 bp	0.4579	0.0488	0.0376	1.9181	16.7618	0.1048	0.1297	4.3538	0.5828	0.00615	30.8%
Commodities	19.77 bp	4.0511	0.2999	0.2664	22.5453	229.8208	1.3003	1.0550	44.8057	4.0064	0.05631	26.5%
Equities	7.40 bp	1.1018	0.1381	0.1246	11.3525	69.6655	0.6157	0.4915	24.3042	2.2265	0.02307	28.1%
FX	4.07 bp	1.0790	0.0801	0.0621	4.7674	43.6048	0.3935	0.2837	12.7696	1.5735	0.01261	17.4%
Real Estate	9.80 bp	1.2351	0.1443	0.0779	10.9227	80.2691	0.5778	0.5699	24.0381	2.6827	0.02001	25.2%
Unclear	9.25 bp	2.4533	0.2332	0.1929	16.3360	113.3237	0.9132	0.7727	32.0516	2.8114	0.03214	24.3%
Corrected Covariates Model												
Bonds	1.89 bp	0.0452	0.0403	0.0315	0.6673	5.0878	0.1009	0.1176	2.4171	0.5819	0.00590	38.2%
Commodities	23.25 bp	0.3135	0.2678	0.2267	6.5756	52.7878	1.3141	1.0665	21.5197	4.0120	0.05754	33.3%
Equities	8.65 bp	0.1246	0.1058	0.1121	3.3306	23.3740	0.5826	0.4823	9.8698	2.2581	0.02397	33.8%
FX	2.92 bp	0.0609	0.0576	0.0623	1.5316	10.3927	0.4181	0.2437	5.7773	1.5303	0.01621	32.2%
Real Estate	9.56 bp	0.1074	0.0829	0.0717	1.8906	25.4503	0.5728	0.6096	10.2033	3.1386	0.02001	37.3%
Unclear	8.53 bp	0.2346	0.2369	0.1699	3.7111	28.1259	0.9858	0.7456	13.6688	2.7503	0.03394	29.7%
Sequential Shrinkage Model												
Bonds	1.89 bp	0.0277	0.0336	0.0286	0.7177	7.2675	0.0855	0.0773	1.7770	0.5326	0.00404	39.5%
Commodities	23.31 bp	0.1841	0.2283	0.1627	3.9942	50.7759	0.6813	0.5641	13.8623	3.9071	0.02674	46.6%
Equities	8.52 bp	0.0814	0.0980	0.0781	1.7578	20.3940	0.3307	0.3075	5.9719	1.9555	0.01236	34.3%
FX	2.76 bp	0.0532	0.0535	0.0319	0.8297	16.0640	0.1515	0.1250	3.5490	1.1972	0.00741	40.0%
Real Estate	9.23 bp	0.0810	0.0839	0.0541	1.1273	24.6513	0.4442	0.3173	6.5294	2.4537	0.01008	36.6%
Unclear	8.17 bp	0.1526	0.1752	0.1362	2.6471	23.6628	0.7437	0.5889	10.8308	2.5796	0.02140	29.3%
Fama-French (FFC6, FF93L) Models												
Asset Class	Intercept	EqMkt	EqSize	EqVal	YCLvl	YCSlp	IG10	RMW	CMA	WML		% Sig.
Bonds	3.57 bp	0.0297	0.0547	0.0559	—	8.3151	0.1452	—	—	—		35.0%
Commodities	22.70 bp	0.1315	0.2487	0.2810	—	46.9759	0.7746	—	—	—		32.5%
Equities	7.24 bp	0.0824	0.1460	0.1385	—	—	—	0.1435	0.2331	0.1347		37.8%
FX	3.35 bp	0.0318	0.0765	0.0773	—	13.3017	0.2220	—	—	—		31.1%
Real Estate	11.33 bp	0.0795	0.1472	0.1730	—	—	—	0.1820	0.2310	0.0894		40.5%
Unclear	8.45 bp	0.1574	0.2519	0.2529	—	—	—	0.2184	0.4657	0.2348		27.3%

Table 19: Out-of-Sample Mean Absolute 65-Day (Quarterly) Significant Beta Changes Across Model Forms, by Asset Class. Data are for 2,288 US ETFs alive as of 30 November 2021 over the period 20211130–20250530. Beta changes are determined from rolling regressions where the prior year of data is used. Insignificant betas are contemporaneously set to 0. Asset class groupings are based on fund categories. Commodity and FX funds are fit with a fixed income factor ordering; real estate funds are fit with an equity factor ordering. The average significant coefficient change 1-day in the future is smallest in almost all cases for the full liquid factor model (Sequential Shrinkage). This suggests that the shrinkage estimation does, in fact, lower out-of-sample variance. The change sizes also tend to be smaller here, for significant betas, than for all betas (in the prior table).

	Out-of-Sample R^2s					
Horizon	Bonds	Commodities	Equities	FX	Real Estate	Unclear
Kitchen Sink Model						
1-day	51.7%	29.7%	65.4%	23.1%	59.7%	26.9%
21-day	48.0%	26.9%	62.3%	21.6%	58.0%	23.5%
65-day	39.8%	24.2%	56.7%	18.2%	55.4%	20.7%
Kitchen Sink Model with Transformed Coefficients for Comparison						
1-day	-94.1%	-316.1%	-216.5%	-80.6%	-178.8%	-186.0%
21-day	-103.3%	-383.8%	-218.9%	-69.6%	-144.1%	-212.6%
65-day	-112.2%	-345.5%	-203.0%	-77.6%	-86.7%	-200.2%
Corrected Covariates Model						
1-day	51.1%	30.8%	65.0%	24.1%	59.4%	27.9%
21-day	48.0%	28.1%	62.0%	22.5%	57.8%	24.6%
65-day	41.0%	25.3%	56.7%	19.1%	55.5%	21.7%
Sequential Shrinkage Model						
1-day	52.0%	25.7%	64.2%	21.9%	57.1%	28.0%
21-day	49.7%	23.5%	61.4%	20.1%	55.5%	24.8%
65-day	45.8%	21.4%	56.6%	18.3%	53.2%	22.3%
Fama-French (FFC6, FF93L) Models						
1-day	14.7%	18.3%	64.7%	8.4%	51.9%	18.7%
21-day	11.6%	16.4%	61.7%	6.7%	49.7%	15.3%
65-day	9.4%	16.3%	56.7%	5.5%	46.5%	12.5%
# Funds	437	71	1577	6	35	145

Table 20: Average Out-of-Sample R^2s Across Model Forms, by Asset Class.

	Out-of-Sample R^2s					
Horizon	Bonds	Commodities	Equities	FX	Real Estate	Unclear
Kitchen Sink Model						
1-day	29.4%	-7.9%	46.3%	2.8%	35.9%	-0.5%
21-day	23.7%	-0.4%	44.4%	3.3%	34.2%	-0.3%
65-day	21.2%	-15.5%	40.3%	6.1%	35.5%	-10.0%
Kitchen Sink Model with Transformed Coefficients for Comparison						
1-day	-35.3%	-195.3%	-177.8%	-7.4%	-94.2%	-91.1%
21-day	-47.9%	-265.7%	-177.5%	-16.3%	-56.9%	-120.9%
65-day	-53.3%	-200.1%	-163.7%	-12.5%	-16.6%	-103.4%
Corrected Covariates Model						
1-day	45.3%	30.5%	63.8%	20.4%	59.0%	27.3%
21-day	42.4%	28.1%	61.1%	19.3%	57.8%	24.4%
65-day	36.2%	26.1%	56.1%	17.9%	55.1%	22.0%
Sequential Shrinkage Model						
1-day	54.4%	25.0%	64.3%	21.3%	57.3%	28.3%
21-day	52.7%	23.2%	61.7%	19.4%	55.7%	25.3%
65-day	49.0%	21.1%	57.0%	17.2%	53.0%	23.2%
Fama-French (FFC6, FF93L) Models						
1-day	15.3%	18.0%	64.5%	8.5%	50.8%	17.8%
21-day	12.2%	16.5%	61.5%	7.3%	48.2%	14.3%
65-day	9.7%	16.7%	56.7%	5.7%	44.5%	12.0%
	437	71	1577	6	35	145

Table 21: Average Out-of-Sample R^2s Using Only Significant Betas Across Model Forms, by Asset Class.

Aggregated Kitchen Sink Liquid Instrument Models, by Main Asset

	Int.	SP500	R2K	R1Val	R1Gro	T3M	T2Y	T5Y	T10Y	T30Y	IBIG	IBHY	TIPS5Y	VIX	N/R^2
Bonds	-0.1 bp	0.044	0.016	0.012	-0.014	0.005	-0.289	0.053	-0.491	-0.123	0.194	0.129	-0.326	0.001	443
$\bar{t}$-stat	*-0.00*	*0.71*	*1.05*	*0.93*	*-0.23*	*0.16*	*-1.56*	*-0.54*	*-0.77*	*-0.47*	*11.28*	*5.68*	*-2.09*	*1.25*	50.6%
$sd(\hat{\beta})$	0.000	0.493	0.122	0.266	0.256	1.205	1.051	2.048	2.809	4.952	0.298	0.284	1.096	0.006	
Commodities	-3.8 bp	0.245	0.150	0.186	-0.159	0.169	0.750	-0.121	0.476	1.375	0.027	0.146	-4.001	-0.006	73
$\bar{t}$-stat	*-0.23*	*3.03*	*5.15*	*2.29*	*-3.24*	*0.21*	*0.46*	*0.53*	*-0.22*	*0.91*	*0.09*	*1.82*	*-5.68*	*-1.57*	32.2%
$sd(\hat{\beta})$	0.001	0.853	0.232	0.311	0.555	2.246	3.091	6.476	8.183	7.659	0.439	0.543	8.214	0.021	
Equities	-2.0 bp	0.280	0.220	0.106	0.050	0.286	0.240	-0.028	0.245	0.452	0.038	0.140	-0.653	-0.004	1602
$\bar{t}$-stat	*-0.44*	*6.55*	*9.93*	*1.94*	*-0.65*	*0.50*	*0.17*	*0.17*	*-0.02*	*0.28*	*0.74*	*2.09*	*-1.21*	*-0.71*	62.5%
$sd(\hat{\beta})$	0.001	2.065	0.441	0.847	1.409	1.640	2.339	5.276	5.262	3.331	0.255	0.417	2.641	0.019	
Real Estate	-1.3 bp	0.903	0.257	0.170	-0.559	0.182	-0.231	0.483	-3.462	0.462	0.160	0.271	-0.003	0.007	36
$\bar{t}$-stat	*-0.65*	*7.52*	*9.26*	*3.31*	*-7.66*	*0.15*	*-0.50*	*0.37*	*-1.67*	*0.37*	*2.47*	*4.17*	*-0.08*	*1.51*	59.5%
$sd(\hat{\beta})$	0.000	0.454	0.174	0.113	0.337	0.852	1.589	2.136	2.896	1.214	0.164	0.147	0.964	0.012	
Unclear	-2.9 bp	-0.005	0.049	0.163	0.005	-0.214	-0.040	1.699	-0.340	0.805	0.115	0.112	-2.699	0.010	154
$\bar{t}$-stat	*-0.43*	*1.24*	*1.59*	*1.18*	*-0.16*	*-0.14*	*-0.27*	*0.61*	*-0.58*	*0.43*	*2.08*	*1.65*	*-3.24*	*0.29*	29.9%
$sd(\hat{\beta})$	0.001	2.353	0.382	1.203	1.364	5.027	3.047	5.555	7.185	5.773	0.604	0.528	4.123	0.087	

Table 22: Partial Liquid Factor Model, Aggregated by Main Asset. Data are for 2,304 US ETFs alive as of 30 November 2021 over the period 2014–2025Q1. The main asset is determined from fund category and asset class holding data. Coefficients are for using all liquid instruments used to create factors in the model.