An Auto-Residual Factor Model

Malek Alkshaik*

First Version: June 6, 2025
This Version: November 7, 2025

Abstract

An Auto-Residual Factor Model that comprises of the market factor, the size factor, residual short term reversal, residual momentum, and residual long term reversal proves robust in 5 out of 5 samples tested and is the top performer of its peers on a Sh^2 basis in our main US sample. Additionally, the factor model uses no fundamental data due to the use of PCA when constructing the residual factors. This leads to an extremely useful tool when benchmarking in samples where fundamental data is unavailable or unreliable.

Keywords: Factor Modelling; Asset Pricing Anomalies; and Residual Factors;
JEL Classification: G12; G14; G15

---

*Correspondence: ma386@st-andrews.ac.uk; University of St Andrews, Fife, Scotland. The author would like to thank Ruslen Tuneshev, Maria Chiara Iannino, and Joao Cunha for their dedicated supervision, and Tianyi Xie for his aid in data collection.

1 Introduction

In a highly dimensional and noisy world, parsimonious factor models are how we, as financial economists, structure our views of how securities prices are formed. Fama and French (2015) interpret the market through the dividend-discount framework: by rearranging the present-value equation, they show that firms with higher book-to-market ratios, lower reinvestment rates, and stronger expected earnings should all earn higher expected returns.[1] Hou et al. (2015) approach the problem from a different theoretical angle but arrive at a similar conclusion: they posit a world in which a firm's cost of capital, and thus its expected return is determined by q theory. In their model, firms continue to invest until all positive NPV projects are exhausted, meaning that the firms' reinvestment rates and profitability levels jointly set their cost of capital. Although their motivations differ, both papers ultimately analyse the issue through an efficient markets lens.

Contrastingly, Stambaugh and Yuan (2017) motivate their factor model from an inefficient perspective. Termed the 'mispricing factor model', the authors attempt to take an empirical approach by combining a string of previously published asset pricing anomalies and creating two factors that are linear combinations of separate clusters. Daniel et al. (2019) maintain the idea of mispricing driving returns, but avoid using the combined anomalies approach and settle on an individual short horizon behavioural factor and a long horizon behaviour factor, combined with the market factor, to attempt to explain markets in that way.[2]

Barillas and Shanken (2018) introduce a novel factor model that uses a Bayesian framework to select from a set of candidate factors. Similarly, other papers have used regularisation techniques to select a number of candidate factors for different applications (DeMiguel et al., 2020; Kozak et al., 2020; and Freyberger et al., 2020). Finally, Swade et al. (2023) apply a technique of sequentially applying the factor with the largest alpha to the previous benchmark until no more candidate factors show any mean-variance

---

[1] All models mentioned include the market factor as their central piece, based on the work of Sharpe (1964) and Lintner (1965).

[2] All models, with the exception of Daniel et al.'s (2019), also include a size factor.

efficiency (MVE) contribution. From a set of 200 candidate factors, they find that the use of a 15 factor model possesses no statistically significant pricing errors from the unselected candidate factors.

Our contribution presents three key advancements in the study of the cross section of stock returns. Firstly, we extend the existing literature on residual factors, which previously identifies residual momentum and residual short term reversal (Blitz et al., 2011, 2013), by introducing a third residual factor: residual long term reversal. Residual long term reversal not only shows persistence across samples and markets, but it also shows significant outperformance (sharpe ratio) of its standard counterpart in 4/5 samples tested. Furthermore, in our main US sample between 1972-2022, residual long term reversal subsumes its standard counterpart in spanning regressions.

Secondly, we propose a unified theoretical framework to explain the existence and outperformance of these residual factors. Building on Jegadeesh et al.'s (2022) finding that standard autocorrelative factors[3] arise from the interaction between noise traders and informed investors, our framework posits that if informed traders not only posses superior information about individual securities, but are also aware and aim to exploit risk premia, then by orthogonalising to these systematic components, residual factors increase exposure to noise traders. This direct positioning against noise traders explains the significant outperformance of residual factors compared to their standard counterparts.

Finally, given the observed pervasiveness of these residual factors across samples and markets, the dominance of autocorrelative factors when machine learning techniques are applied to explain and predict markets,[4] and the theoretical grounding provided by our framework based on Jegadeesh et al. (2022), we develop an Auto-Residual Factor Model.[5] This model includes the market factor, the size factor, residual short term reversal, residual momentum, and our residual long term reversal factor. A key feature is that we deviate from the literature in the choice of risk factors. Standard papers on idiosyncratic

---

[3] We use the term 'autocorrelative factors' to represent factors based on past returns. Notable ones being short term reversal, momentum, and long term reversal.

[4] see Kelly et al. (2019), Gu et al. (2020), and Freyberger et al. (2020) among others.

[5] The term "Auto-Residual" derives from the factors' reliance on return autocorrelation and the use of residual returns in constructing the portfolios.

or residual factors orthogonalise securities against the FF3 before sorting. We propose the use of Principal Component Analysis (PCA) to extract latent factors to use as the right-hand-side (RHS) variables to obtain residuals, before using those residual returns to construct our residual factors.

Auto-Residual Factor Model:

$$R_{i,t} = \alpha_i + \beta_i Mkt_t + \beta_i ME_t + \beta_i rSTR_t + \beta_i rMOM_t + \beta_i rLTR_t + \varepsilon_{i,t} \tag{1}$$

This leads to a factor model which not only outperforms its peers on a maximum squared sharpe ratio basis in our main US sample (2.62 vs 2.34; which is the next highest), but is also constructed entirely without fundamental data, making it particularly valuable for pre-sample periods or international markets where such data is often unavailable or unreliable. We emphasise the use of the maximum squared sharpe ratio metric on the back of a series of papers by Barillas and Shanken (2017, 2018, and 2020). Previously, the literature has used a standard set of test assets (typically $5x5$ size and value sorts) to assess an asset pricing model's pricing ability; however, as the authors emphasise, the choice of which test assets are chosen greatly influences which pricing model is deemed as the best. They also show analytically that when all plausible test assets are used to assess a model, the one with the highest maximum squared sharpe ratio leads to the smallest amount of pricing error.

Furthermore, the residual factors used in our model not only subsume their non-residual autocorrelative counterparts, but are also unexplained (showing significant alpha) by the competing models tested in our analysis.

1.1 Related Literature on Standard and Residual Autocorrelative Factors

Evidence in early finance suggested that stock prices followed a random walk, thus implying that stock returns exhibit no autocorrelation (Fama, 1965). And while the use of technical analysis was ubiquitously applied in futures markets early on, the academic literature saw its first use of past returns in the application to stocks with De Bondt and Thaler (1985, 1987). In their seminal papers, the authors showed that stocks which had recently performed well, in multiple horizons, often would reverse in their performance in a relative measure. Chopra et al. (1992) further provide evidence of the existence of the long term reversal effect and find that the effect is particularly strong in January but is distinct from any tax loss selling effects. In a similar vein, Jegadeesh (1990) and Lehmann (1990) find reversal patterns in stock returns, but in much shorter horizons. The former author focuses on monthly reversals, whereas the latter finds evidence of reversals at a weekly horizon and attributes the effect to 'bid ask bounce'. In contrast to the concept of reversal, when examining medium term horizons—specifically, the stock performance over the past year—stock returns actually demonstrate a momentum effect where past winners tend to continue winning and past losers continue to lag behind. Building on the foundational work of Ball and Brown (1968), Jegadeesh and Titman (1993, 2001) substantiate this observation and highlight how traditional benchmarks fail to account for this phenomenon. Additionally, Rouwenhurst (1998) underscores the pervasiveness of momentum across international markets. Interestingly, however, Fama and French (2012) present evidence suggesting the absence of momentum in Japanese markets.

Residual/idiosyncratic factors can trace their roots back to the work of Grundy and Martin (2001), who find that standard momentum portfolios have time varying exposures to Fama French factors. Furthermore, the authors find that momentum portfolios that ex post hedge those exposures provide significantly better risk adjusted returns. Later, Ang et al. (2006) extend the literature on residual factors where they analyse the relationship between idiosyncratic volatility and the expected return on a stock. To extract idiosyncratic risk the authors run time series regressions for each security against the FF3 and calculate the volatility of the residuals of the regression. Gutierrez and Prinsky (2007) were the first to incorporate past residual cumulative returns to predict future returns. They find that using 'firm specific abnormal returns' to sort portfolios yields a significantly higher sharpe ratio than standard returns when constructing momentum portfolios. Not only that, but the continuation of performance is substantially longer. While standard momentum portfolios require monthly rebalancing to show positive results, the authors' residual momentum continues to perform at much longer horizons. Blitz et al. (2013) extend the literature once more to look at residual short term reversal, where the authors look at the most recent month's residual return, and bet on a reversal of performance over the next month. They find again that orthogonalising to systematic risk factors significantly increases the sharpe ratio of the sorted portfolio.

1.2 Auto-Residual Theoretical Framework

Jegadeesh et al. (2022) propose that short term reversals, momentum, and – with a minor addition – long term reversals occur due to the interplay between noise traders and informed investors, whose activities disrupt accurate price discovery. They argue that short term reversals are especially pronounced during periods of heightened noise trading, which they empirically validate by analysing retail investor order flow as a proxy for noise trading. Their dynamic theoretical model distinguishes clearly between short horizon noise traders, who trade without regard to fundamental information, and uninformed investors (who we group in with noise traders), who underreact to signals they do not generate themselves. The persistence of positions by noise traders rather than immediate liquidation contributes to price underreaction to new information, thereby facilitating momentum. Our minor addition of this framework to explain the effects of long term reversal is simply that uninformed investors have a propensity to herd, and when these investors continue to hold their position after changes in the securities fundamentals, it takes time for them to liquidate their positions, thus leading to long term reversals. Evidence of investors' propensity to herd can be found in Teh and De Bondt (1997).

To motivate and explain the performance of our residual factors we employ this same framework, but with the addition of a simple yet profound inclusion. If informed investors not only possess superior information to their noise-trading counterparts at the individual security level, but are also aware of sources of risk premia, leading them to tilt their portfolios in the direction of those premia, then by orthogonalising our historical return measure against these known factors we dilute the informed component, resulting in trades that are increasingly positioned against those held by noise traders. This positioning opposite to noise traders delivers markedly superior risk-adjusted returns for residual factors and supports our evidence from spanning regressions that traditional autocorrelative factors are merely noisy stand-ins for their residual equivalents. This "trade-against-noise-traders" perspective is also essential for reconciling our factors with Sharpe's (1991) arithmetic of active management: in a zero-sum world every gain outside the market portfolio must be financed by someone else's loss. Our residual factor framework fits that arithmetic neatly, attributing abnormal performance to consistently bearing the other side of uninformed investors' trades—a story that is both empirically supported and theoretically consistent. Such 'hedging' or orthogonalising is done on ex ante basis, meaning exposures are not perfectly reduced, but evidence of the efficacy of orthogonalising can be seen in Table 4, where t-statistics for the standard long-term reversal FF5 betas shrink or change sign once we switch to residual long term reversal.

The rest of the paper is organised in the following manner. Section 2 will outline the data and methodology of the empirical analysis. Section 3 will look specifically at residual long term reversal and the use of different time horizons. Section 4 looks at the details of the 3 residual factors more broadly. Section 5 looks at machine learning in residual factors. Section 6 looks at out of sample residual factors through the use of pre and international samples. Section 7 looks at the landscape of competing asset pricing models and how they compare to our Auto-Residual Factor Model. And finally, section 8 investigates the inevitable transaction cost considerations of our residual factors.

2 Data and Methodology

For our main US sample, we use the CRSP monthly dataset from 1972-2022 with stocks that are listed on either the NYSE, AMEX, or Nasdaq. We only use those securities with a share code of 10 or 11. We also make use of a pre-sample for out of sample testing from 1932-1971. For our international samples, we use the Compustat global files from

1992-2022 and follow Fama and French (2012) in our selection of markets. Splitting up markets into 4 sub-regions which are the US, Europe, APAC ex Japan, and Japan. For our tests which include other factor models, we source the data from a combination of our own replication and the authors' website. The list of models is found in Table 1. We also use the 12 US industry portfolios taken from Ken French's website for our industry neutral analysis.

[Insert Table 1 Here]

Residual factors are constructed similarly to traditional factors—with one key distinction: instead of using raw returns, we use residual returns obtained from regressions that control for systematic risk. For each security, residuals are estimated using 24 months of data with a chosen regressor. The specific residual return used as the sorting signal depends on the factor: Portfolios include stocks that have both the necessary prior return history covering $t-24$ to t and a valid market equity measure at the end of month t.

Residual short term Reversal: Uses the most recent month's residual return.

Residual Momentum: Utilizes residual returns from the period $t-12$ to $t-2$.

Residual long term Reversal: Draws on residual returns from $t-24$ to $t-13$.

In the US, six value weighted portfolios are formed by intersecting two size groups (Small and Big) and the historical residual return over the relevant period. The size groups are determined using the median NYSE market equity at month end, while the residual signal groups are defined by the 30th and 70th NYSE percentiles. Our residual short term reversal and residual momentum are rebalanced monthly, whereas our residual long term reversal is rebalanced semi annually.

For international markets where NYSE breakpoints are unavailable, we use 'pseudo NYSE' breakpoints which are breakpoints calculated from firms above the 80th percentile of market cap in a given sample. Fama and French (2012) estimate that the top 10% of securities in US samples are equivalent to securities in the large portion of US sorts (stocks above the NYSE median), therefore if the relationship between NYSE ranking and security size is linear, then the whole selection of NYSE stocks is proxied by the top 20% of securities by size. Therefore, we use the top 10% breakpoint for the size split as Fama and French (2012) but we use the top 20% of firms by size for the 'pseudo NYSE' signal breakpoints. This is a minor but logical amendment for constructing factors in an international setting.

Avid readers of the methodology may notice 2 more notable deviations from the previous literature. The first is the use of 24 months of historical data rather than the standard 36 months. We do this due to the difference it makes in the number of selectable securities in the international samples. At any one month for our earlier international markets, this change can increase the number of selectable securities by 15%-20% where the number of securities is particularly low. We emphasise that in the data rich samples such as the US; the results are unimpacted by the change from the change of 36 months of returns to 24 months of returns.

The 2nd notable deviation is the standard definition of long term reversal. While there is no unified definition of long term reversal Fama and French (2012) use the time frame of $t-60$ to $t-13$ rather than our $t-24$ to $t-13$. Our reasoning is 2 fold. The first is to line up the regressions for residual long term reversal, residual momentum and residual short term reversal. This means that all 3 factors use 24 months of data in the residual process and thus follow a consistent methodology. We show in Table 2 that the differences in the results in the US are minor between $t-60$ to $t-13$ and $t-24$ to $t-13$. The 2nd reason is again related to selectable securities in our pre-sample and international samples. The difference between the number of securities with 5 years of non missing prior returns and 2 years is significant. And to study patterns in security prices, we simply want to maximise the number of securities before conducting our sorts. That way, we don't run the risk of certain pockets of the portfolio being too concentrated in a few securities.

The residual factors that we use for our Auto-Residual Factor Model use PCA constructed regressors rather than traditional risk factors such as those from FF5 for example. To extract our principal components, each month we take our entire cross section of stocks with the previous 24 months of returns data and extract the 5 principal components from the K x N matrix, where K is the number of securities with valid data (around 5000 in the US in December 2001, for example), and N is the number periods which is 24 months. Importantly, unlike Grundy and Martin (2001), this method of extracting latent factors is done on an ex ante basis. Rather than standard PCA, we also test the RP-PCA of Lettau and Pelger (2020) but find no material difference in the results. For robustness, we also test principal components extracted from 24 months of daily data from the entire cross section; again, results are very similar, but we omit the results in the paper in the interest of brevity.

3 Residual Long Term Reversal

[Insert Table 2 Here]

We can see from Table 2 that long term reversal is relatively stable across look back periods as well as rebalancing frequency, showing that the effect is robust to construction methodology. Furthermore we see a similar pattern in the construction of the residual factors. The data also illustrates that for almost all look back horizons, semi annual rebalancing is what maximises the sharpe ratio of the strategy. We also see consistent improvement in the sharpe ratio across from standard to residual factors when holding the look back and holding constant. This is best highlighted by the mean sharpe ratios of all horizons and holding periods, with the mean of standard long term reversal at 0.27, and the mean of residual long term reversal at 0.39. We also see from Table 3 that in this particular sample, our chosen factor subsumes its standard counterpart.

[Insert Table 3 Here]

In Table 4, we also employ the FF5 model to examine the dynamics of residual long term reversal and its conventional counterpart. Notably, the standard version shows a statistically insignificant alpha, with positive exposures to the market, value, and reinvestment factors, and a negative exposure to profitability. The positive exposure to value and negative exposure to profitability are particularly intuitive, as firms that have previously underperformed often exhibit financial distress, leading to lower profitability and higher book to market ratios, which in turn justify a higher discount rate. Conversely, residual long term reversal exhibits a positive alpha and loads negatively on the value factor. This suggests that the conventional long term reversal factor is embedded within the value premium. However, once it is isolated, we observe a significantly different set of stocks entering the long and short positions of the factors.

[Insert Table 4 Here]

Table 5 also shows that the standard long term reversal and the value factor both show insignificant alphas when spanned against each other, whereas residual long term reversal not only shows significant alphas on both the 5 factor model of Fama and French, but also the value factor in isolation, again showing that the methodology of constructing residual factors, does successfully 'hedge' away the influence the regressors have on the final portfolios.

[Insert Table 5 Here]

4 Residual Factors More Broadly

This section looks at the details of standard and residual autocorrelative factors in our main US sample between 1972-2022.

4.1 Choice of Regressor in Factor Construction

When constructing residual factors, we've seen that the standard procedure in the literature is to use the FF3 as the RHS variable to extract residual returns. While this is a sensible suggestion, it's clear that the use of that model vs the q4 factor model is somewhat arbitrary. In Table 6 we show summary statistics of the 3 residual factors using different RHS regressors. The data demonstrates (across the residual factors) that there is a reasonable step in the sharpe ratio when going from the standard factor to the residual factor with the CAPM as the RHS variable, and then another further step in performance when using multi-factor models as the regressors.

[Insert Table 6 Here]

The final row of each panel in the table uses PCA to extract latent factors, rather than the use of a previously proposed asset pricing model. Where PCA is defined as:

$$W_k = \underset{W^\top W = I_k}{\arg\max}\ \operatorname{tr}\Big(W^\top \Sigma W\Big) \tag{2}$$

where $X \in \mathbb{R}^{n \times p}$ is the mean-centred data matrix, $\Sigma = \frac{1}{n} X^\top X$ its sample covariance, k the desired number of principal components, $W \in \mathbb{R}^{p \times k}$ an orthonormal loading matrix ($W^\top W = I_k$), and $\operatorname{tr}(W^\top \Sigma W)$ the total variance captured by those k directions.

This has several advantages. The first is that it lets the data speak for itself directly, rather than making assumptions about which factors we conjecture are relevant. Related to the first point, different markets and markets through time are dynamic and can often shift regime leading to a different set of factors becoming prevalent, therefore the use of rolling PCAs has the distinct advantage of being able to dynamically allocate to different drivers of returns when they become influential. Additionally, the use of PCA vs conventional multi-factor models does not require the use of any fundamental data. This means that the factors are still easily constructed in samples where fundamental data is unavailable or unreliable. Furthermore, the use of PCA can often be simpler and far less time consuming than generating multi-factor models. A few lines of code should suffice when using PCA, whereas replicating many factor models requires tedious data wrangling.

Finally, the use of PCA has no inherent look ahead bias, nor is it exposed to any corrected or restated data. Therefore, for the reasons outlined above, the remainder of the empirical analysis will use the residual factor versions which use PCR (principal component regression) to extract residuals rather than the conventional FF3 or FF5. We conduct no formal analysis of the optimal number of principal components to extract when constructing residual factors. We do this in the interest of robustness. Rather, we pick the number 5 as this is a common number of factors in popular asset pricing models.

4.2 Spanning Regressions

In Table 3, we report spanning regressions of our PCA derived residual factors on their conventional counterparts. In our U.S. main sample, each residual factor exhibits a positive, statistically significant alpha when regressed on its standard version. In contrast, when we flip the regression—testing standard factors against their residual equivalents—none of the alphas remain significant, and those for short-term reversal and momentum are actually negative. In spanning regressions, such asymmetry implies that the dominant factor "fully subsumes" the other: it alone captures all cross-sectional information, and a MVE portfolio constrained against short selling would allocate entirely to the subsuming factor. These findings indicate that the traditional autocorrelative factors largely serve as noisy proxies for their residual counterparts.

[Insert Figure 1 Here]

4.3 Industry Neutrality in Residual Factors

Industry neutrality in standard autocorrelative factors is fairly well understood. Hameed et al. (2015) see that when the standard short term reversal factor is constructed to be industry neutral, we see a significant increase in the sharpe ratio. Unlike standard short term reversal, the authors provide evidence that the industry neutral short term reversal is prevalent across subgroups of stocks, including larger more liquid stocks. Typical industry neutrality is done by categorising stocks into industry groups when generating sorts using cut off points for a specific industry. Interestingly this can be done differently when constructing residual factors. Rather than generating cut off points for each industry cluster, simply including industry portfolios in the regression leads to industry neutral residual factors. This approach is loosely used in Blitz et al. (2013), but it is later used more directly in Blitz et al. (2020). It should be noted that a potential pitfall with this approach is that it limits the number of sector classifications one can use. This is because when the number of regressors K (the number of industry portfolios) approaches

N (the number of observations) it leads to perfect fitting in the regression, where the error terms (residual returns) become $= 0$. While this pitfall is suboptimal, the use of this methodology can be useful where industry portfolio returns are available, but the specific industry of each security is unavailable. In this scenario, the regression based technique to industry neutrality will allow the portfolio to be constructed, whereas if the data is unavailable for which industry groups belong to which securities, the portfolios are unable to be formed. The regression based approach is also often how practitioners generate industry neutral and factor neutral portfolios (Paleologo, 2021).

Contrastingly to short term reversal, Moskowitz and Grinblatt (1999) actually show that momentum relies on rotating sectors to generate its profitability. Termed 'industry momentum' they show that most of the momentum's predictive signal is in its ability to bet on well performing sectors, and when an industry neutral momentum factor is constructed, its performance is significantly diminished. This phenomenon of industry rotation is often associated with the tendency of momentum strategies to have particularly large crashes. Bets in concentrated positions such as ones loaded in particular sectors can often go particularly wrong, hence the idea of momentum exhibiting 'large tail risk'. Asness et al. (2000) look directly at industry neutrality in standard long term reversal portfolios. And while the author finds strong evidence to suggest that value portfolios benefit from industry neutrality, he finds weak evidence to suggest that industry neutrality actually inhibits the performance of long term reversal portfolios. This is a surprising result as the value factor is fairly highly correlated with the long term reversal factor.

[Insert Table 7 Here]

When it comes to industry neutrality in residual factors, we find varying results. Industry neutral residual short term reversal sees a significant sharpe ratio increase vs residual short term reversal as seen in the standard factor. The jump from a sharpe ratio of 1.14 to 1.71 is fairly staggering for simply including the 12 industry portfolios of Fama French alongside the 5 principal components. Conversely, residual momentum diverges from standard momentum regarding industry neutrality: its industry-neutral and standard versions deliver virtually identical performance (Sharpe ratios of 0.96 vs.

0.95). Analysing industry neutrality in the context of residual long term reversal we see the same relationship for the standard factor vs the residual factor. Like standard long term reversal, when industry neutrality is introduced into residual long term reversal, we see a decline in performance, with a significant fall in sharpe ratio from 0.49 to 0.16.

5 Machine Learning in Residual Factors

5.1 Latent Factors in the PCA

In our method of constructing residual factors, we use PCA each month to extract the 5 principal components of the previous 24 months of data from the entire cross section of firms with valid data. We then use this data as the regressor for each security to extract residuals. While the factors in this paper were constructed from the PCA use only ex ante data, observing latent factors in general is an ex post exercise. And therefore a useful exercise to understand markets is to see which and what factors are driving returns, and how they are related to the asset pricing models we have in financial economics. In Table 8 we see large correlation matrices comparing latent factors to a number of factors taken from our competing models from Table 1, and a number of macro factors taken from Goyal and Welch (2008). The table consists of 3 panels. The first uses standard PCA to extract 5 latent factors. The second and third use Lettau and Pelger (2020) RPPCA, with the former applying a gamma of 0, where the latter applies a gamma of ∞, and an orthogonalization condition.[6]

[Insert Table 8 Here]

Table 8 delivers a striking critique of the empirical asset pricing literature. Across all three PCA specifications, only the first principal component exhibits any substantial correlation with established factors—subsequent components barely register. Importantly, Figure 2 shows each specification's first component loads heavily on both the market and size factors. Moreover, in two of the specifications our residual short term reversal factor

---

[6] Readers can find more about the parameters of RPPCA in Lettau and Pelger (2020).

emerges as the third strongest correlate (0.33 and 0.23), while in the remaining specification, our residual long term reversal factor occupies that slot (0.22). This evidence underscores that our residual factors are integral building blocks of the covariance matrix governing the cross section of stock returns.

[Insert Figure 2 Here]

Other noteworthy findings include: Hou et al.'s (2015) ROE factor correlates strongly with our inflation series,[7] and Stambaugh and Yuan's (2017) MGMT factor shows a pronounced relationship with returns on AAA rated bonds.

5.2 Beyond OLS to Extract Residuals

When extracting residuals from regressions in finance, we generally see authors use the simplest and most commonly known regression technique; OLS. And while OLS is a deep yet simple and intuitive tool, other methods of regression exist. Specifically, ones which can capture interactions and non linearity. In our exercise, we apply 7 different specifications of regression - ones which we find to be the most intuitive - to extract residuals for the purpose of generating residual factor portfolios.[8] The first is basic OLS as done in all our previous analysis and is used in the further analysis later in the paper. The second specification will be similar to the one used by Kelly et al. (2024), where we will apply a ridge regression. In their model, the authors set the lambda parameter to 0 as they have a scenario where $K > N$, however when $N > K$, required to extract non zero residuals, setting lambda to 0 collapses the model into a standard OLS, therefore we set lambda to 0.1 to achieve a modest amount of regularization. Our third will differ from the rest as it will include a different number of principal components. 10 principal components will be extracted from the cross section, as opposed to 5 for all other models, however a lasso regression, similar to Kozak et al. (2020), will be used with the hyperparameter chosen for each unique regression to keep exactly 5 non zero betas. Our

---

[7] Some macro factors are observed ex post and may not have been available in real time.

[8] For more on the regression techniques used in the exercise, see Hastie et al. (2008), James et al. (2013), and Geron (2022).

fourth specification uses standard OLS but includes interaction terms. The fifth includes not only interaction terms, but also power terms to attempt to capture non linearitys.[9] This type of technique is often known as a polynomial regression. Our sixth specification looks to capture non linearities directly, and employs a local regression, where each part of the data is fitted separately but on a rolling basis.[10] Finally, our last specification uses the popularly used random forest, which creates multiple simulated datasets via bootstrapping, and then uses decision trees for the regression, before averaging the results to generate a prediction. For all specifications, residuals are simply the real returns of a security minus the model prediction.

[Insert Figure 3 Here]

Surprisingly, the portfolios built using OLS perform the best. Our specifications that apply Lasso and random forest come closest, but the simplicity of OLS, in this context, is clearly king. While local regression and ridge regression lie on opposite sides of the bias variance trade off spectrum, its interesting to note they both perform poorly. While the reasoning isn't entirely clear, what is encouraging is that we see all 3 portfolios' ($rMOM$, $rSTR$, and $rLTR$) performance move in tandem depending on the model.

6 Out of Sample Residual Factors

Analysing the performance of our 3 residual factors in our main US samples, we see fairly exceptional performances on a sharpe ratio basis. Given these high sharpe ratios, it is important to test if these factors exist in other datasets to ensure the genuine existence of our factors and is not simply due to the data dredging process. Robustness being the key word in empirical research, the analysis of international markets and the use of pre-samples is often how we as financial economists ensure the phenomena do genuinely exist, and will continue to exist, likely due to structural reasons. We use 5 distinct data sets for our analysis. Our main US sample uses data from 1972-2022 and is the sample with which

---

[9] We use first and second order terms in our polynomial regression.

[10] We use a span of 20%, i.e an OLS is fit on 20% of the data and roles 1% until a curve has been fitted on the entire dataset, similar to a spline.

all of our prior analyses have been conducted. Our second sample is our 'US pre-sample'. This is the use of any data available on the CRSP tapes before 1972: the actual years include 1932-1971. Our 3rd, 4th, and 5th samples equate to the international regions of Fama and French (2012). All 3 international samples only use developed markets and are split up into a European sample, an APAC ex Japan sample, and a Japanese sample.

We note while papers such as Huij and Lansdorp (2017), Chang et al. (2018), Lin (2019), and Blitz et al. (2020), investigate residual momentum and residual short term reversal in international markets, we emphasise that their use of decile or quintile sorts rather than traditional Fama French style construction can have a large impact on performance. Furthermore, while decile sorts are often used for the purpose of studying trading strategies, Fama French style construction is often used for constructing factors for explanatory applications as in all popular factor models. This distinction is important for our Auto-Residual Factor Model. Not only that, but we also highlight that none of the papers apply the use of PCA to extract residuals. Rather than using sample specific factors (as implicit in PCA), they choose 1 factor model, often either FF3 or FF5 which have varying success in explanatory power depending on the specific market. Finally, as mentioned earlier, to our knowledge, residual long term reversal has not been studied in any market.

Analysing Table 9 and Figure 4, we see encouraging results to believe in the genuine existence of our residual factors. In all 5 distinct samples, we see significant sharpe ratio increases in the case of residual short term reversal vs its standard counterpart. Not only that, but in Europe, we had a 3 fold increase in sharpe ratio, and in APAC ex Japan, the strategy progresses from a negative sharpe ratio to a positive one.

[Insert Table 9 Here]

[Insert Figure 4 Here]

In a similar vein, we see the same effect for momentum. Again, in all 5 distinct samples, we see significant sharpe ratio increases. European markets see the smallest increase in sharpe ratio, just an increase of 1.58 times. Other samples show closer to double the sharpe ratio, where the US pre-sample and APAC ex Japan is closer to 3 times bigger, with Japan seeing a 5 fold increase. Finally, residual long term reversal sees significant increases in performance in 4 out of the 5 samples. With around 5 fold increases found in Europe and APAC ex Japan.

Japan is the one 1 sample that sees a fall in sharpe ratio between long term reversal and its residual counterpart. This is unsurprising as Japan has often held the title as distinct from other financial markets. Asness (2011) documents how conventional momentum does not exist in Japan, in line with our results. Furthermore, Hanauer (2014) concludes that the lack of standard momentum in Japan is unsurprising due to 'different market dynamics'. Additionally, Fama and French (2017) also document that both their investment and profitability factors explain no variation in average stock returns in Japan. Given Japanese markets' objection to standard anomalies, 2 out of 3 residual anomalies seeing positive results is encouraging and thus our Auto-Residual Factor Model should still provide a useful tool in the Japanese market.

7 Competing Models

Our Auto-Residual Factor Model:

$$R_{i,t} = \alpha_i + \beta_i Mkt_t + \beta_i ME_t + \beta_i rSTR_t + \beta_i rMOM_t + \beta_i rLTR_t + \varepsilon_{i,t} \tag{3}$$

Sharpe (1964), Fama and French (1993,2015), Hue et al. (2015), Stambaugh and Yuan (2017), Barillas and Shanken (2018), and Daniel et al. (2020) all propose asset pricing models, in addition to our own. All but Barillas and Shanken's (2018) and Stambaugh and Yuan's (2017) models are proposed with a unified theme.

This section looks at these competing models' size factors, our residual factors' alphas regressed against the competing models, and a direct comparison of our model vs others using the maximum squared sharpe ratio.

7.1 Size Factors

From our selection of asset pricing models, we have 3 distinct size factors coming from different models. Fama and French (1993) construct their size factor with value neutrality in mind. They use 2x3 sorts of value and size, and then take the average return of the small value and small growth, in the same vein, they take the average return of the large value and large growth portfolios, and then long the small return and short the large return. Hou et al. (2015), use the same approach but wish to neutralise against 2 factors rather than just value when constructing their size portfolios. They neutralise against their reinvestment and return on equity factors, thereby using 2x3x3 sorts to achieve this. Finally, Stambaugh and Yuan (2017) take a completely different approach to their size factor, doubling the sharpe ratio of their size factor vs the 2 aforementioned size factors. They construct their size factor by using stocks least likely to be mispriced, as identified by the measures used to construct their mispricing factors.

An astute reader will notice that all 3 factors require fundamental data to construct. For our size factor however, we take by far the simplest approach. Annually we split our universe of stocks into 2 groups based on the NYSE median of size, the factor goes long a value weighted portfolio of the stocks below the NYSE median, and goes short a value weighted portfolio of the stocks above the NYSE median. This portfolio is then rebalanced annually. Our size factor has a very similar sharpe ratio to Fama and French's (1993) SMB with a correlation to it in our sample of 91%, but most notably, does not require any fundamental data.

7.2 Factor Alphas

[Insert Table 10 Here]

In Table 10, using our main US sample, we test the ability of these different factor models to explain our residual factors. For all models, both residual momentum and residual short term reversal have statistically significant alphas. Not only are all alphas significant at the 1% level, but in many of the tests we see t tests that are upwards of

5, with a t statistic of residual short term reversal against the mpf above 8. In contrast, residual long term reversal does have positive alphas against all models, but loses its 5% statistical significance against 2 of the models.

7.3 Direct Comparison

When comparing asset pricing models directly, the classic approach is the use of spanning regressions or the ability to minimise the alpha of a set of test assets as used in Hue et al. (2015) and Fama and French (2015). Barillas and Shanken (2017) however show that this approach is sensitive to the choice of test assets and is not the optimal method for comparing the power of models. They show analytically that the true relevant metric for comparison of models is the maximum squared sharpe ratio. This is the sharpe ratio squared of the MVE portfolio of the factors in the factor model.

[Insert Figure 5 Here]

Figure 5 shows the maximum squared sharpe ratio of the competing models as well as the Auto-Residual Factor Model. The figure illustrates a key finding in our paper; that our model – in this sample – comes closest to spanning the ex post mean variance frontier. We again emphasise that although the Auto-Residual Factor Model does score highest in the key criteria, the real contribution of the model as a tool for financial economists is its fundamental data free construction which, outside of the CAPM, is not the case for any of the competing models.

[Insert Table 11 Here]

In Table 11 we also see the maximum squared sharpe ratio of our model across our different samples and also show the allocation made to each factor when creating the mean variance efficient portfolio from the factors of the model. In the US, the allocation is clearly dominated by residual short term reversal and residual momentum as they amount to around 90% of the allocation. We see the same pattern when it comes to our pre and international samples.

7.4 Complex Models and Factor Mimicking Portfolios

Our analysis in the previous section compares models which are built via the principle of parsimony. Complex models exist however, and their authors claim these models while losing their interpretability because of their use of a large number of variables and parameters, provide better performance and therefore should be used as the yard stick. While the parsimonious models used are created via the typical Fama French methodology which uses NYSE sorts and splits factors into their small and large versions, complex models are usually constructed via factor mimicking portfolios (FMPs), which rather than using sorts, use regressions. FMPs mechanically provide portfolios which have 1 unit exposure to the underlying characteristic which is used as the x variable in the regression which allows the researcher to construct portfolios based on a firms exposure to a specific phenomenon such as inflation or a latent factor extracted via PCA. These FMPs, unlike their traditional counterparts, weight firms equally and do not use NYSE breakpoints and therefore are generally even more susceptible to the criticism that they only exist due to the absence of incorporating frictions. Examples of complex models are found in Kelly et al (2019) and Kozak et al. (2020) where the authors show the superiority of complex models vs their parsimonious counterparts on an sharpe ratio basis.

[Insert Figure 6 Here]

While we acknowledge the evidence of the superior performance of complex models, Figure 6 demonstrates the improved performance is only partially due to complexity. When we apply the FMP methodology, which is necessarily used in complex models, to our factor model we see a great increase in Sh^2 while still only using the same information used to construct our traditional Auto-Residual factor model. Hellum et al. (2025) in their analysis attempt to tame the influence of smaller firms on the pricing models by removing micro caps. But Figure 6 shows how the FMP methodology inflates the sharpe ratio even after this is done, and therefore comparing Fama French style factors to complex model derived factors is an unequal comparison.

8 Trading Cost Considerations

A common critique of empirical research done in asset pricing is the exclusion of transaction costs when generating factor portfolios. Critics argue that if a factor is not actually investible, since its existence is due to the absence of frictions in paper portfolios, then it should not be deemed as an explanatory variable.

Novy Marx and Velikov (2016) investigate the effects of transaction costs on a range of asset pricing anomalies including conventional momentum and short term reversal. They conclude while the profits of momentum survive transaction costs, short term reversals do not. Additionally, Blitz et al. (2013) find that the standard short term reversal factor does not survive transaction costs, however its residual counterpart does.

[Insert Table 12 Here]

In Table 12 we see the numbers for turnover and sharpe ratio of our factors vs their standard counterparts. Furthermore, we also include a version of our factors which we term ‘turnover aware residual factors’. For these factors, we simply reduce the frequency in which they rebalance and show the subsequent sharpe ratios. For our residual short term reversal factor, we rebalance bimonthly which reduces the turnover considerably but still keeps a respectable sharpe ratio. For our residual momentum portfolio, we rebalance semi annually, which again maintains a respectable sharpe ratio, and finally for our residual long term reversal, we rebalance annually, which again sees turnover fall significantly yet sees the sharpe ratio unchanged.

Looking at models and transaction costs directly, Detzel et al. (2022) write a paper in which they analyse asset pricing models on the maximum squared sharpe ratio basis after transaction costs. The authors show – somewhat expectedly – that the superiority of models differs before and after transaction costs. While the monthly rebalanced versions of our residual factors would struggle in this exercise, our turnover aware factors would fair much better, with evidence from Detzel et al.’s (2022) paper showing cost mitigating strategies, such as banding, greatly benefit higher turnover strategies such as conventional momentum.

9 Conclusion

Fama and French (2018) call for discipline in the factor model space by invoking 2 simple steps when proposing models to explain the cross section of stock returns. They express that models should be motivated by ‘theory, even an umbrella theory’ and provide evidence of robustness through out of sample testing. Our Auto-Residual Factor Model not only employs a unified theory, which is an extension of the work of Jegadeesh et al. (2022), and is pervasive across a variety of different samples, but also scores highest on the key criteria of comparing asset pricing models, all while only requiring returns and market capitalisation for its construction.

References

[1] Ang, A., Hodrick, R.J., Xing, Y. and Zhang, X., 2006. The cross-section of volatility and expected returns. The Journal of Finance, 61(1), pp.259-299.

[2] Asness, C., 2011. Momentum in Japan: The exception that proves the rule. The Journal of Portfolio Management, 37(4), pp.67-75.

[3] Asness, Cliff S., R. Burt Porter, and Ross L. Stevens. 2000. "Predicting Stock Returns Using Industry-Relative Firm Characteristics." Available at SSRN: https://ssrn.com/abstract=213872.

[4] Ball, R. and Brown, P., 1968. An empirical evaluation of accounting income numbers. Journal of Accounting Research, 6(2), pp.159-178.

[5] Barillas, F. and Shanken, J., 2017. Which alpha? The Review of Financial Studies, 30(4), pp.1316-1338.

[6] Barillas, F. and Shanken, J., 2018. Comparing asset pricing models. The Journal of Finance, 73(2), pp.715-754.

[7] Barillas, F., Kan, R., Robotti, C. and Shanken, J., 2020. Model comparison with Sharpe ratios. Journal of Financial and Quantitative Analysis, 55(6), pp.1840-1874.

[8] Blitz, D., Hanauer, M.X. and Vidojevic, M., 2020. The idiosyncratic momentum anomaly. International Review of Economics & Finance, 69, pp.932-957.

[9] Blitz, D., Huij, J. and Martens, M., 2011. Residual momentum. Journal of Empirical Finance, 18(3), pp.506-521.

[10] Blitz, D., Huij, J., Lansdorp, S. and Verbeek, M., 2013. short term residual reversal. Journal of Financial Markets, 16(3), pp.477-504.

[11] Chang, R.P., Ko, K.C., Nakano, S. and Rhee, S.G., 2018. Residual momentum in Japan. Journal of Empirical Finance, 45, pp.283-299.

[12] Chopra, N., Lakonishok, J. and Ritter, J.R., 1992. Measuring abnormal performance: Do stocks overreact? Journal of Financial Economics, 31(2), pp.235-268.

[13] Daniel, K., Hirshleifer, D. and Sun, L., 2020. Short- and long-horizon behavioral factors. The Review of Financial Studies, 33(4), pp.1673-1736.

[14] De Bondt, W.F. and Thaler, R., 1985. Does the stock market overreact? The Journal of Finance, 40(3), pp.793-805.

[15] De Bondt, W.F. and Thaler, R.H., 1987. Further evidence on investor overreaction and stock market seasonality. The Journal of Finance, 42(3), pp.557-581.

[16] DeMiguel, V., Martin-Utrera, A., Nogales, F.J. and Uppal, R., 2020. A transaction-cost perspective on the multitude of firm characteristics. The Review of Financial Studies, 33(5), pp.2180-2222.

[17] Detzel, A., Novy-Marx, R. and Velikov, M., 2023. Model comparison with transaction costs. The Journal of Finance, 78(3), pp.1743-1775.

[18] Ehsani, S. and Linnainmaa, J.T., 2022. Factor momentum and the momentum factor. The Journal of Finance, 77(3), pp.1877-1919.

[19] Fama, E.F., 1965. The behavior of stock-market prices. The Journal of Business, 38(1), pp.34-105.

[20] Fama, E.F. and French, K.R., 1993. Common risk factors in the returns on stocks and bonds. Journal of Financial Economics, 33(1), pp.3-56.

[21] Fama, E.F. and French, K.R., 2012. Size, value, and momentum in international stock returns. Journal of Financial Economics, 105(3), pp.457-472.

[22] Fama, E.F. and French, K.R., 2015. A five-factor asset pricing model. Journal of Financial Economics, 116(1), pp.1-22.

[23] Fama, E.F. and French, K.R., 2017. International tests of a five-factor asset pricing model. Journal of Financial Economics, 123(3), pp.441-463.

[24] Fama, E.F. and French, K.R., 2018. Choosing factors. Journal of Financial Economics, 128(2), pp.234-252.

[25] Freyberger, J., Neuhierl, A. and Weber, M., 2020. Dissecting characteristics nonparametrically. The Review of Financial Studies, 33(5), pp.2326-2377.

[26] Géron, A., 2022. Hands-on machine learning with Scikit-Learn, Keras, and TensorFlow: Concepts, tools, and techniques to build intelligent systems. " O'Reilly Media, Inc.".

[27] Grundy, B.D. and Martin, J.S.M., 2001. Understanding the nature of the risks and the source of the rewards to momentum investing. The Review of Financial Studies, 14(1), pp.29-78.

[28] Gu, S., Kelly, B. and Xiu, D., 2020. Empirical asset pricing via machine learning. The Review of Financial Studies, 33(5), pp.2223-2273.

[29] Gutierrez Jr, R.C. and Prinsky, C.A., 2007. Momentum, reversal, and the trading behaviors of institutions. Journal of Financial Markets, 10(1), pp.48-75.

[30] Hameed, A. and Mian, G.M., 2015. Industries and stock return reversals. Journal of Financial and Quantitative Analysis, 50(1-2), pp.89-117.

[31] Hanauer, M., 2014. Is Japan different? Evidence on momentum and market dynamics. International Review of Finance, 14(1), pp.141-160.

[32] Hastie, T., 2009. The elements of statistical learning: data mining, inference, and prediction.

[33] Hou, K., Xue, C. and Zhang, L., 2015. Digesting anomalies: An investment approach. The Review of Financial Studies, 28(3), pp.650-705.

[34] Huij, J. and Lansdorp, S., 2017. Residual momentum and reversal strategies revisited. Available at SSRN 2929306.

[35] James, G., Witten, D., Hastie, T. and Tibshirani, R., 2013. An introduction to statistical learning (Vol. 112, No. 1). New York: springer.

[36] Jegadeesh, N., 1990. Evidence of predictable behavior of security returns. The Journal of Finance, 45(3), pp.881-899.

[37] Jegadeesh, N. and Titman, S., 1993. Returns to buying winners and selling losers: Implications for stock market efficiency. The Journal of Finance, 48(1), pp.65-91.

[38] Jegadeesh, N. and Titman, S., 2001. Profitability of momentum strategies: An evaluation of alternative explanations. The Journal of Finance, 56(2), pp.699-720.

[39] Jegadeesh, N. and Titman, S., 2023. Momentum: Evidence and insights 30 years later. Pacific-Basin Finance Journal, 82, p.102202.

[40] Jegadeesh, N., Luo, J., Subrahmanyam, A. and Titman, S., 2022. short term reversals and longer-term momentum around the world: Theory and evidence. Nanyang Business School Research Paper (22-13).

[41] Kelly, B., Malamud, S. and Zhou, K., 2024. The virtue of complexity in return prediction. The Journal of Finance, 79(1), pp.459-503.

[42] Kelly, B.T., Pruitt, S. and Su, Y., 2019. Characteristics are covariances: A unified model of risk and return. Journal of Financial Economics, 134(3), pp.501-524.

[43] Kozak, S., Nagel, S. and Santosh, S., 2020. Shrinking the cross-section. Journal of Financial Economics, 135(2), pp.271-292.

[44] Lehmann, B.N., 1990. Fads, martingales, and market efficiency. The Quarterly Journal of Economics, 105(1), pp.1-28.

[45] Lettau, M. and Pelger, M., 2020. Estimating latent asset-pricing factors. Journal of Econometrics, 218(1), pp.1-31.

[46] Lin, Q., 2019. Residual momentum and the cross-section of stock returns: Chinese evidence. Finance Research Letters, 29, pp.206-215.

[47] Lintner, J., 1965. Security prices, risk, and maximal gains from diversification. The journal of finance, 20(4), pp.587-615.

[48] Moskowitz, T.J. and Grinblatt, M., 1999. Do industries explain momentum? The Journal of Finance, 54(4), pp.1249-1290.

[49] Novy-Marx, R. and Velikov, M., 2016. A taxonomy of anomalies and their trading costs. The Review of Financial Studies, 29(1), pp.104-147.

[50] Paleologo, G.A., 2021. Advanced Portfolio Management: A Quant's Guide for Fundamental Investors. John Wiley & Sons.

[51] Rouwenhorst, K.G., 1998. International momentum strategies. The Journal of Finance, 53(1), pp.267-284.

[52] Sharpe, W.F., 1964. Capital asset prices: A theory of market equilibrium under conditions of risk. The journal of finance, 19(3), pp.425-442.

[53] Sharpe, W.F., 1991. The arithmetic of active management. Financial Analysts Journal, 47(1), pp.7-9.

[54] Stambaugh, R.F. and Yuan, Y., 2017. Mispricing factors. The Review of Financial Studies, 30(4), pp.1270-1315.

[55] Swade, A., Hanauer, M.X., Lohre, H. and Blitz, D., 2023. Factor zoo (.zip). The Journal of Portfolio Management, Quantitative Special (2024), p.50.

[56] Teh, L.L. and De Bondt, W.F., 1997. Herding behavior and stock returns: An exploratory investigation. Revue Suisse d'Économie Politique et de Statistique, 133, pp.293-324.

[57] Welch, I. and Goyal, A., 2008. A comprehensive look at the empirical performance of equity premium prediction. The Review of Financial Studies, 21(4), pp.1455-1508.

Figure 1: Equity Curve of Short Term Reversal vs Residual Short Term Reversal

Graph of the equity curve of short term reversal and residual short term reversal with PCA construction from 1932 to 2022.

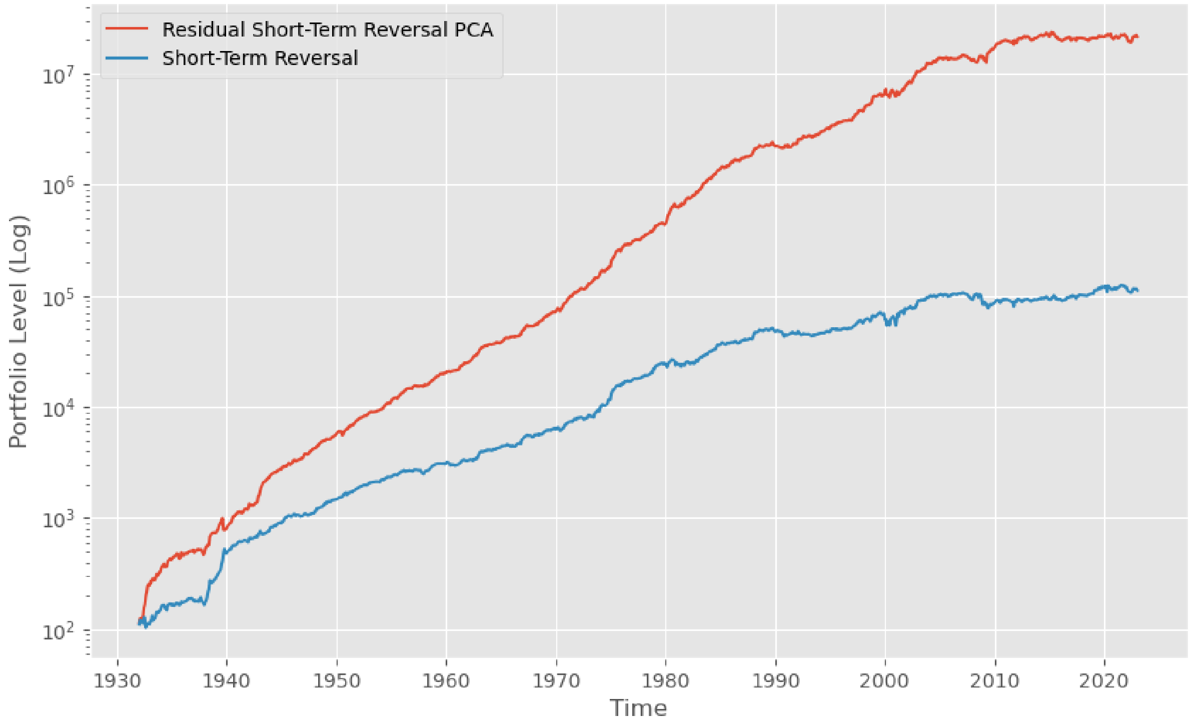


Figure 2: PC1 Loadings on Different Factors

The 3 graphs look at the correlations of factors on the 1st principle compenant of different specification of PCAs. The factors included are traded factors found across our competing models, and a number of macro factors from Goyal and Welch (2008). Graph 1 uses standard PCA, graph 2 uses RPPCA with a γ of 0, and graph 3 looks at RPPCA with a γ of ∞ with an orthogonalisation condition included in the factor extraction process.

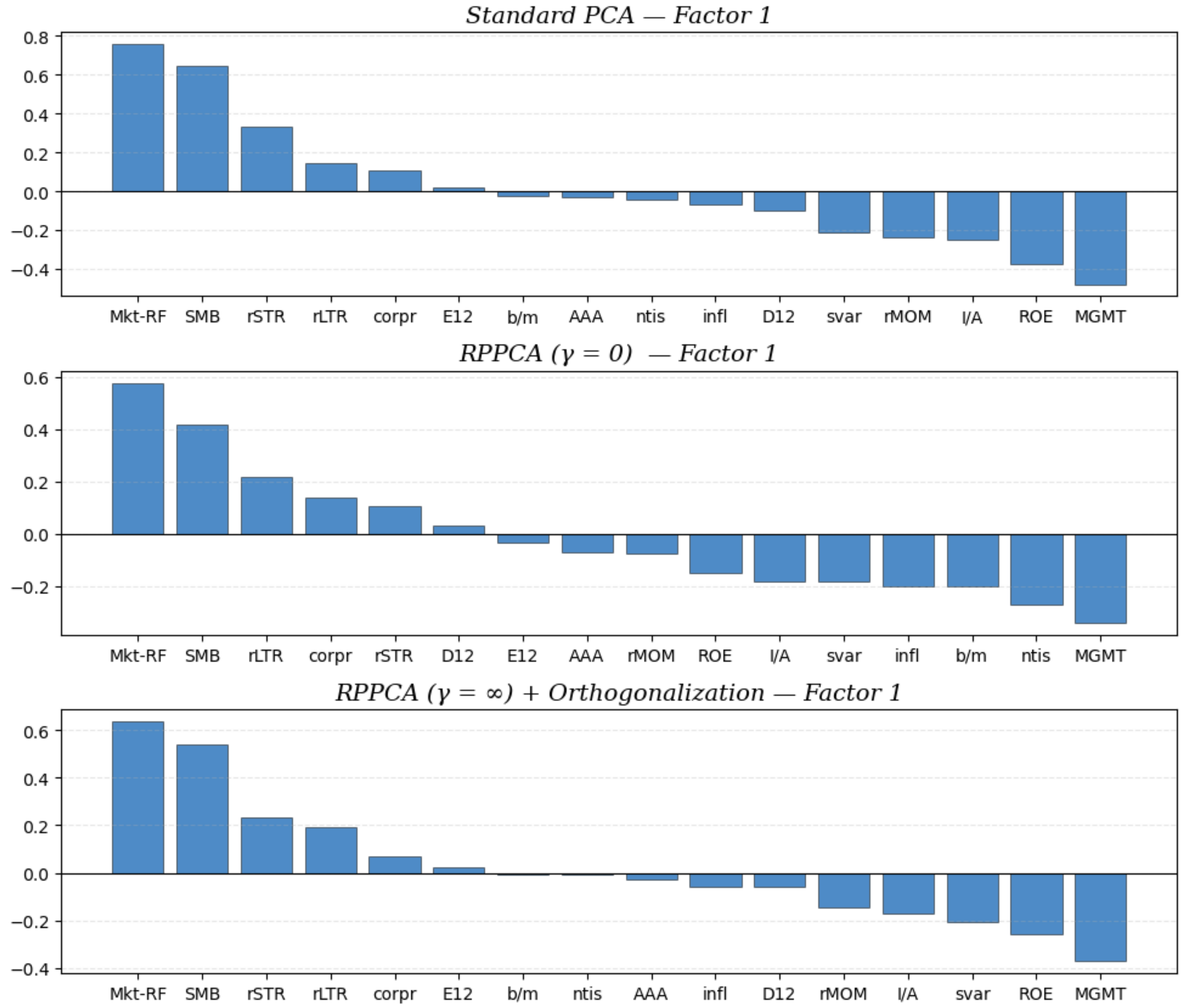
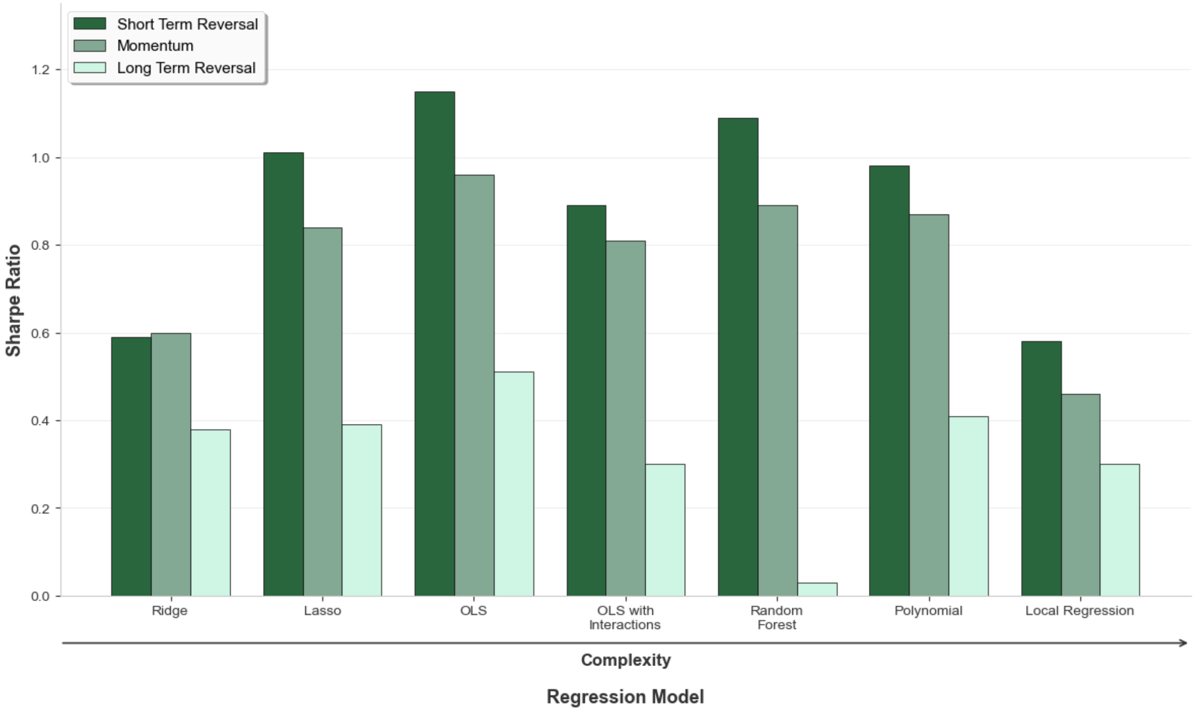


Figure 3: Different Regression Models

The sharpe ratio of our 3 residual autocorrelative factors using different regression techniques to extract residuals.



---

[10] The full list and definitions of marco factors can be found in Goyal and Welch (2008).

Figure 4: Standard Vs Residual Factors Across Samples

The Sharpe Ratio of standard autocorrelative factors vs their residual counterparts across different samples. The main US sample run from 1972 to 2022, the US pre-sample runs from 1932 to 1971, and the 3 international samples run from 1992 to 2022.

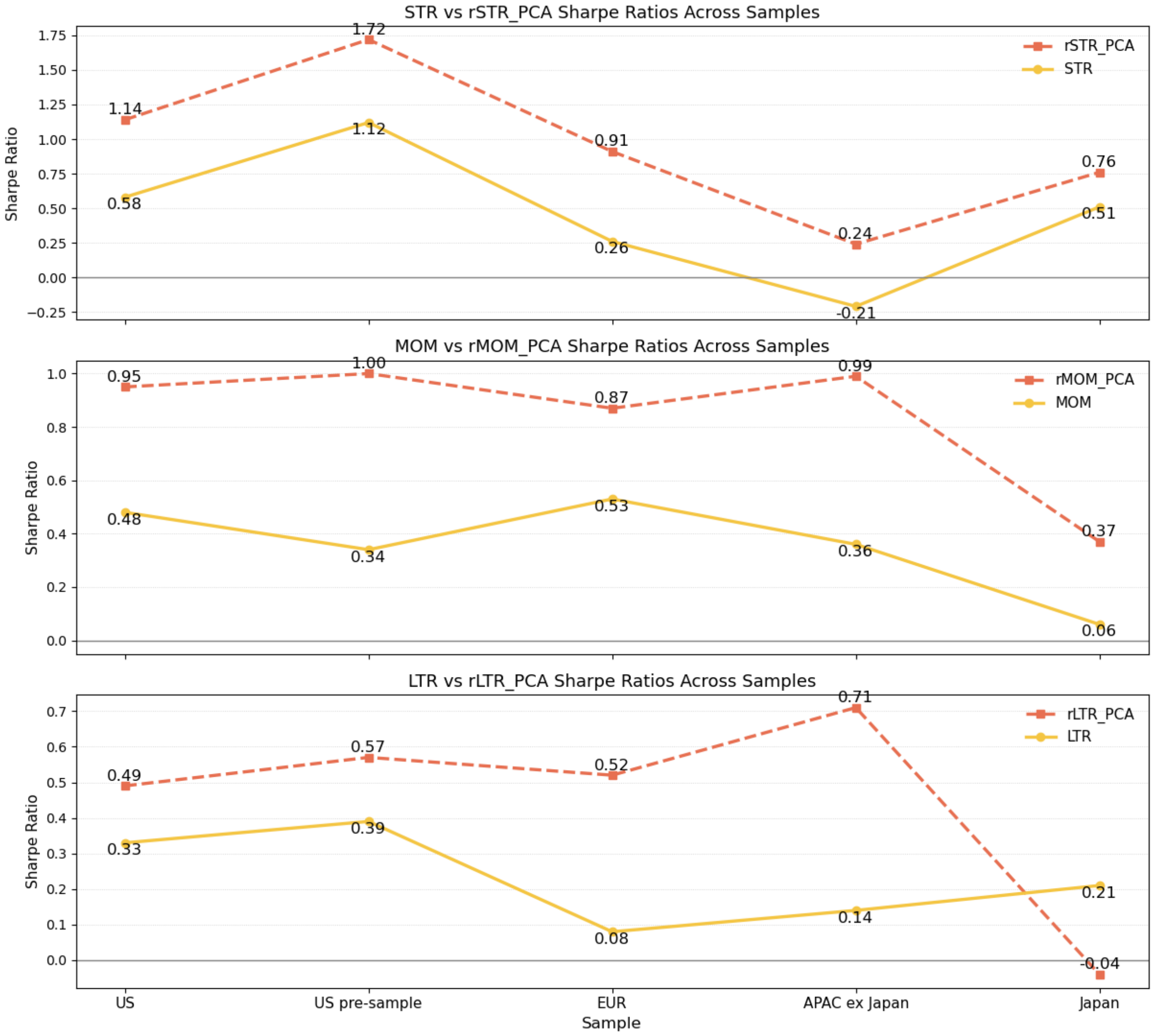


Figure 5: Maximum Squared Sharpe Ratios of Competing Models

The maximum squared sharpe ratio of competing models in our main US sample, running from 1972 to 2022.

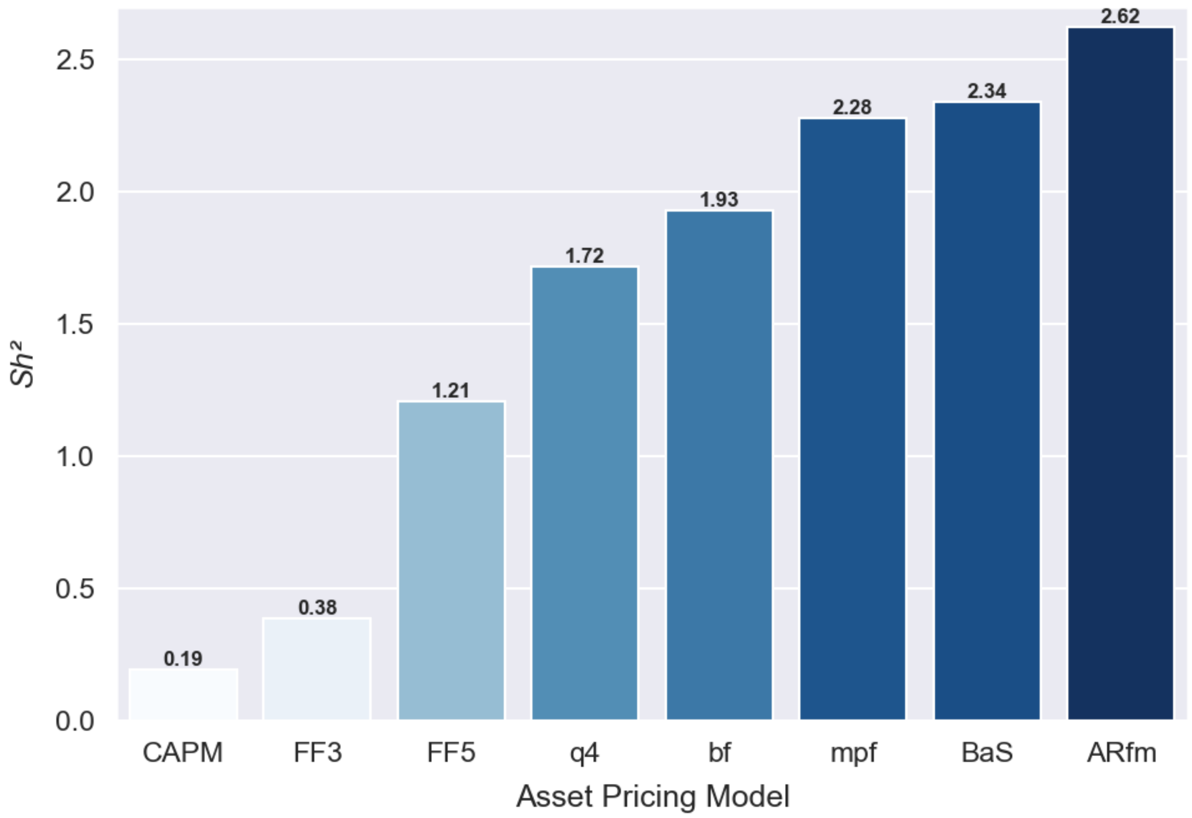


Figure 6: Sh^2 of Auto-Residual Portfolios

The maximum squared sharpe ratio of different specifications of Auto-Residual factor model. FMP uses a factor mimicking portfolio construction technique where variables are projected onto forward returns and the Betas are taken as the portfolio return. FMP Liq applies the same methodology but excludes firms below the NYSE 20th percentile of size. And FF Style is the model with the standard Fama French construction. Sample extends from 1972-2022.

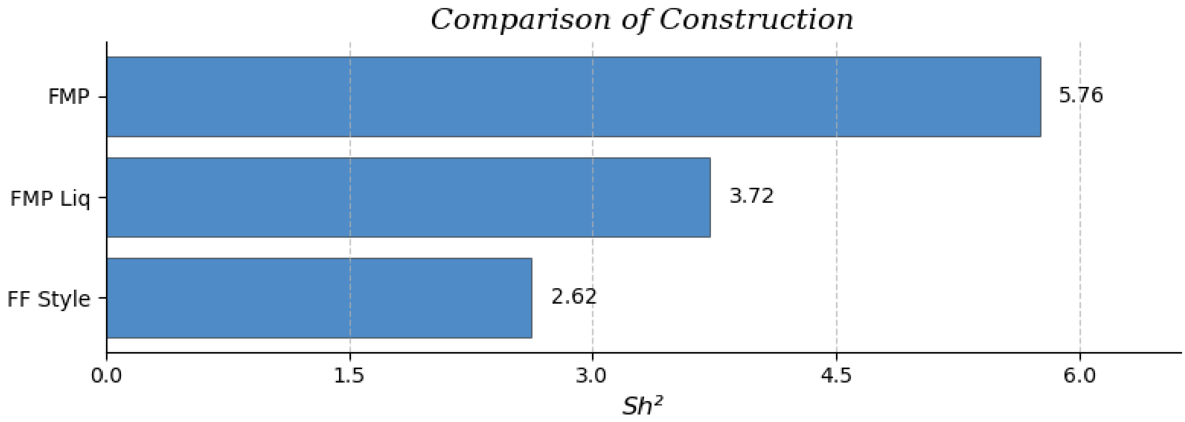


Table 1: Competing Factor Models

Contents and background information of the competing factor models used in the paper. The SMBs used in the factor models differ slightly, but are based on the same core idea. See the corresponding paper for detailed definitions of the factors.

Acronym	**Full Name**	**Reference**	**Factors**
CAPM	Capital Asset Pricing Model	Sharpe(1964), Lintner (1965)	Mkt-Rf
FF3	Fama French 3 Factor Model	Fama and French (1993)	Mkt-Rf, SMB, HML
FF5	Fama French 5 Factor Model	Fama and French (2015)	Mkt-Rf, SMB, HML, CMA, RMW
BaS	Barillas and Shanken Factor Model	Barillas and Shanken (2018)	Mkt-Rf, Size, I/A, ROE, HMLm, MOM
q4	q4 Factor Model	Hou, Xue, and Zhang (2015)	Mkt-Rf, Size, I/A, ROE
mpf	Mispricing Factor Model	Stambaugh and Yuan (2017)	Mkt-Rf, SMB, MGMT, PERF
bf[11]	Behavioural Factor Model	Daniel, Hirshleifer, and Sun (2019)	Mkt-Rf, PEAD, FIN

---

[11] Since Daniel et al. (2019) derive their factors with a non-standard procedure, we reconstruct the same factors using the conventional approach of Fama and French (1993). This replicated short- and long-horizon behavioural factor model is used in all subsequent analyses.

Table 2: Long Term Reversal at Different Horizons

Sharpe ratios of long term reversal portfolios in the US main sample between 1972 and 2022. The rows show the period in which previous returns are measured. The columns represent the holding period of each portfolio before rebalancing. The residual long term reversal portfolios are constructed using PCA as the regressor.

	Standard Long Term Reversal				Residual Long Term Reversal			
	1	3	6	12	1	3	6	12
$t24 - t13$	0.27	0.31	0.33	0.35	0.45	0.50	0.49	0.45
$t36 - t13$	0.23	0.28	0.26	0.27	0.37	0.44	0.47	0.37
$t48 - t13$	0.26	0.26	0.22	0.24	0.29	0.36	0.37	0.39
$t60 - t13$	0.28	0.24	0.22	0.32	0.23	0.39	0.37	0.29
Mean	0.27				0.39			
T								1972-2022

Table 3: Spanning Regressions on Residual Factors vs Standard Factors

Alphas and t stats of spanning regressions of residual factors vs their standard counterparts. Panel 1 looks at where the residual factors are the left-hand-side (LHS) variables, while Panel 2 looks at the regression results where the residual factors are on the RHS, and the standard factors are on the LHS. These results span our main US sample from 1972-2022. Alphas are annual.

	(1) $y = \text{Residual}_{\text{PCA}}$, $x = \text{STD}$		**(2)** $y = \text{STD}$, $x = \text{Residual}_{\text{PCA}}$	
	α	(t)	α	(t)
STR	0.059	(5.857)***	-0.000	(-0.137)
MOM	0.048	(5.293)***	-0.006	(-0.243)
LTR	0.029	(2.854)***	0.020	(1.395)
T				1972-2022

Table 4: Standard and Residual Long Term Reversal Regressed against FF5

Standard and residual long term reversal regressed against the Fama French 5 factor model. The regressions span our main US sample from 1972-2022. The data for the Fama French 5 factor model was downloaded from Ken French's website. Alphas are annual.

		$(1)\ y = \text{LTR}, x = FF5$			$(2)\ y = \text{rLTR}_{\text{FF5}}, x = FF5$		
		α	Mkt-Rf	SMB	HML	RMW	CMA
(1)	**Estimate**	0.008	0.044	0.169	0.376	-0.181	0.257
	t-stat	(0.715)	(1.448)	(3.370)***	(5.531)***	(-2.720)***	(2.475)**
(2)	**Estimate**	0.040	-0.019	0.043	-0.148	-0.129	0.063
	t-stat	(3.789)***	(-0.081)	(1.191)	(-3.192)***	(-2.453)**	(0.837)

Table 5: Value, Long Term Reversal, and Residual Long Term Reversal

Spanning regressions of HML vs long term reversal, and then spanning regressions of HML vs residual long term reversal. These regressions use our main US sample from 1972-2022. Alphas are annual.

Regression Specification	Intercept (α)	t-statistic
Panel A: Standard long term Reversal		
$\text{HML}_t = \alpha + \beta \cdot \text{LTR}_t + \varepsilon_t$	0.022	(1.652)*
$\text{LTR}_t = \alpha + \beta \cdot \text{HML}_t + \varepsilon_t$	0.016	(1.226)
Panel B: Residual long term Reversal		
$\text{HML}_t = \alpha + \beta \cdot \text{rLTR}_{\text{PCA},t} + \varepsilon_t$	0.049	(3.335)***
$\text{rLTR}_{\text{PCA},t} = \alpha + \beta \cdot \text{HML}_t + \varepsilon_t$	0.038	(4.023)***

Table 6: Choice of Regressor in Residual Factors

Summary statistics for different portfolios of residual factors. The first rows of each panel show the standard factor, where as the rest are residual portfolios with the choice of regressor. LTR represents long term reversal, MOM represents momentum, and STR represents short term reversal. This data spans our main US sample from 1972 to 2022, however since the residual factors require 24 months of data to extract residuals, the actual returns begin in 1974.

Portfolio	**Mean Return (%)**	**Volatility (%)**	**Sharpe Ratio**	**Max Drawdown (%)**	**Market Beta**
LTR	2.69	10.05	0.27	-31.69	-0.03
rLTR$_{CAPM}$	3.30	9.18	0.36	-51.24	0.05
rLTR$_{FF3}$	3.05	7.06	0.43	-36.46	0.05
rLTR$_{FF5}$	3.23	6.29	0.50	-35.20	0.04
rLTR$_{BaS}$	2.65	6.78	0.39	-39.98	0.02
rLTR$_{q4}$	3.61	7.36	0.49	-37.64	0.02
rLTR$_{mpf}$	3.76	7.36	0.51	-30.04	0.02
rLTR$_{bf}$	3.37	7.07	0.48	-26.51	0.01
rLTR$_{PCA,(5)}$	3.13	6.87	0.46	-26.93	0.04
MOM	6.51	14.87	0.44	-59.37	-0.14
rMOM$_{CAPM}$	7.75	8.54	0.91	-25.08	-0.13
rMOM$_{FF3}$	6.83	7.51	0.91	-35.86	-0.12
rMOM$_{FF5}$	6.46	7.38	0.88	-41.92	-0.12
rMOM$_{BaS}$	6.09	7.37	0.83	-37.74	-0.13
rMOM$_{q4}$	6.77	7.73	0.88	-36.73	-0.13
rMOM$_{mpf}$	7.29	8.01	0.91	-38.89	-0.13
rMOM$_{bf}$	7.22	7.55	0.96	-28.84	-0.14
rMOM$_{PCA,(5)}$	6.51	7.37	0.88	-41.81	-0.14
STR	6.37	11.23	0.57	-30.59	0.21
rSTR$_{CAPM}$	7.65	9.43	0.81	-26.23	0.16
rSTR$_{FF3}$	9.46	7.61	1.24	-14.22	0.11
rSTR$_{FF5}$	9.17	7.08	1.29	-13.86	0.11
rSTR$_{BaS}$	8.09	6.34	1.28	-18.60	0.10
rSTR$_{q4}$	8.72	7.35	1.19	-18.62	0.11
rSTR$_{mpf}$	8.00	7.05	1.13	-22.40	0.12
rSTR$_{bf}$	9.47	7.22	1.31	-7.43	0.13
rSTR$_{PCA,(5)}$	7.97	7.09	1.12	-12.43	0.12

Table 7: Standard Residual Factors Vs Industry Neutral Residual Factors

The summary statistics of standard residual factors vs their industry neutral counterparts. The data spans our main US sample from 1972 to 2022. The industry portfolios used to obtain the industry neutrality are the 12 Fama French industry portfolios downloaded from Ken French's website.

	rSTR$_{PCA}$		rMOM$_{PCA}$		rLTR$_{PCA}$	
	Standard	Ind Neutral	Standard	Ind Neutral	Standard	Ind Neutral
Mean Return (%)	7.96	8.43	7.05	5.04	3.38	0.72
Volatility (%)	6.99	4.93	7.40	5.26	6.85	4.40
Sharpe Ratio	**1.14**	**1.71**	**0.95**	**0.96**	**0.49**	**0.16**
Max Drawdown (%)	-13.93	-7.99	-41.07	-28.50	-31.95	-18.42
Market Beta	0.12	0.08	-0.11	-0.07	0.03	0.04

Table 8: Factor Correlation Matrix

The 3 panels look at factor correlation matrix with a different type of PCA. The factors included are traded factors found across our competing models, and a number of macro factors from Goyal and Welch (2008). Panel A uses standard PCA, Panel B uses RPPCA with a γ of 0, and Panel C looks at RPPCA with a γ of ∞ with an orthogonalisation condition included in the factor extraction process.

Panel A: Standard PCA

	PC1	PC2	PC3	PC4	PC5	Mkt	SMB	rSTR	rMOM	rLTR	I/A	ROE	MGT	D12	E12	b/m	AAA	ntis	inf	crpr	svar
PC1	1.00																				
PC2	-0.02	1.00																			
PC3	0.01	0.20	1.00																		
PC4	-0.01	0.05	0.30	1.00																	
PC5	-0.01	0.05	0.08	0.29	1.00																
Mkt	**0.76**	-0.24	-0.09	-0.06	0.01	1.00															
SMB	**0.65**	-0.02	0.02	0.03	-0.02	0.29	1.00														
rSTR	0.33	-0.07	0.02	-0.02	0.01	0.26	0.20	1.00													
rMOM	-0.24	-0.04	0.09	0.02	0.02	-0.23	-0.16	-0.10	1.00												
rLTR	0.14	0.00	0.10	0.00	0.05	0.09	0.17	-0.07	**0.54**	1.00											
I/A	-0.25	0.10	0.02	0.03	0.00	-0.34	-0.23	-0.11	0.18	-0.12	1.00										
ROE	-0.38	-0.04	0.00	0.04	0.04	-0.21	-0.39	-0.24	0.38	0.11	0.05	1.00									
MGT	-0.48	0.10	-0.01	-0.03	-0.01	-0.47	-0.41	-0.25	0.41	0.01	0.36	**0.73**	1.00								
D12	-0.10	0.00	0.00	-0.03	-0.01	-0.06	-0.06	-0.03	0.06	0.03	-0.02	0.09	-0.01	1.00							
E12	0.02	0.01	-0.01	0.02	-0.02	0.07	-0.02	0.04	0.02	0.03	-0.04	0.02	-0.17	**0.58**	1.00						
b/m	-0.02	0.00	-0.01	0.00	0.01	-0.06	0.05	0.15	0.00	0.07	0.02	0.02	0.07	-0.02	0.08	1.00					
AAA	-0.03	0.00	-0.01	-0.01	0.00	-0.06	0.00	0.04	-0.01	0.10	-0.05	-0.05	**0.70**	0.40	0.04	-0.04	1.00				
ntis	-0.04	0.01	0.01	0.00	0.01	-0.04	0.01	0.12	0.07	-0.01	0.08	-0.10	0.17	0.26	0.24	0.03	0.03	1.00			
inf	-0.07	0.10	0.03	0.01	0.02	-0.11	-0.04	0.01	0.02	0.11	0.09	**0.44**	0.29	0.20	0.01	0.21	0.03	0.15	1.00		
crpr	0.10	-0.12	-0.06	-0.09	0.03	0.26	-0.05	0.11	-0.05	-0.05	-0.10	-0.02	-0.08	-0.07	-0.08	0.03	0.07	-0.05	-0.20	1.00	
svar	-0.21	0.05	0.03	0.02	0.03	-0.31	-0.17	-0.03	-0.07	-0.13	0.08	0.10	0.08	-0.13	-0.23	0.12	-0.05	-0.18	-0.19	0.03	1.00

Panel B: RPPCA $\gamma = 0$

	PC1	PC2	PC3	PC4	PC5	Mkt	SMB	rSTR	rMOM	rLTR	I/A	ROE	MGT	D12	E12	b/m	AAA	ntis	inf	crpr	svar
PC1	1.00																				
PC2	-0.26	1.00																			
PC3	-0.25	0.04	1.00																		
PC4	-0.22	-0.09	0.16	1.00																	
PC5	-0.19	-0.13	-0.05	0.22	1.00																
Mkt	**0.64**	0.07	-0.17	-0.15	0.15	1.00															
SMB	**0.54**	-0.10	0.02	-0.16	-0.10	0.29	1.00														
rSTR	0.23	-0.17	-0.10	-0.04	-0.03	0.26	0.20	1.00													
rMOM	-0.14	0.07	0.17	0.07	0.06	-0.23	-0.16	-0.10	1.00												
rLTR	0.19	0.04	0.10	-0.03	0.00	0.09	0.17	-0.07	**0.54**	1.00											
I/A	-0.17	-0.09	0.06	0.08	0.05	-0.34	-0.23	-0.11	0.18	-0.12	1.00										
ROE	-0.26	0.18	0.11	0.10	0.03	-0.21	-0.39	-0.24	0.38	0.11	0.05	1.00									
MGT	-0.37	-0.07	0.09	0.12	0.09	-0.47	-0.41	-0.25	0.41	0.01	0.36	**0.73**	1.00								
D12	-0.06	0.07	0.02	0.01	-0.04	-0.06	-0.06	-0.03	0.06	0.03	-0.02	0.09	-0.01	1.00							
E12	0.03	-0.03	0.01	0.00	0.02	0.07	-0.02	0.04	0.02	0.03	-0.04	0.02	-0.17	**0.58**	1.00						
b/m	-0.01	-0.04	-0.01	0.02	0.02	-0.06	0.05	0.15	0.00	0.07	0.02	0.02	0.07	-0.02	0.08	1.00					
AAA	-0.03	-0.02	-0.03	0.05	0.04	-0.06	0.00	0.04	-0.01	0.10	-0.05	-0.05	**0.70**	0.40	0.04	-0.04	1.00				
ntis	-0.01	-0.09	-0.02	-0.01	0.04	-0.04	0.01	0.12	0.07	-0.01	0.08	-0.10	0.17	0.26	0.24	0.03	0.03	1.00			
inf	-0.06	-0.06	-0.01	0.05	0.09	-0.11	-0.04	0.01	0.02	0.11	0.09	**0.44**	0.29	0.20	0.01	0.21	0.03	0.15	1.00		
crpr	0.07	0.14	-0.11	-0.02	0.04	0.26	-0.05	0.11	-0.05	-0.05	-0.10	-0.02	-0.08	-0.07	-0.08	0.03	0.07	-0.05	-0.20	1.00	
svar	-0.21	0.05	0.00	0.04	-0.04	-0.31	-0.17	-0.03	-0.07	-0.13	0.08	0.10	0.08	-0.13	-0.23	0.12	-0.05	-0.18	-0.19	0.03	1.00

Panel C: RPPCA $\gamma = \infty$ and Orthogonalization Condition

	PC1	PC2	PC3	PC4	PC5	Mkt	SMB	rSTR	rMOM	rLTR	I/A	ROE	MGT	D12	E12	b/m	AAA	ntis	inf	crpr	svar
PC1	1.00																				
PC2	0.05	1.00																			
PC3	-0.03	0.09	1.00																		
PC4	-0.01	0.13	0.18	1.00																	
PC5	0.08	0.16	0.05	0.26	1.00																
Mkt	**0.58**	-0.02	-0.16	-0.00	-0.04	1.00															
SMB	**0.42**	-0.13	-0.01	-0.08	-0.07	0.29	1.00														
rSTR	0.10	-0.15	-0.02	-0.05	-0.02	0.26	0.20	1.00													
rMOM	-0.08	0.14	0.17	0.03	0.06	-0.23	-0.16	-0.10	1.00												
rLTR	0.22	0.11	0.13	0.02	0.07	0.09	0.17	-0.07	**0.54**	1.00											
I/A	-0.18	0.03	0.20	0.03	0.01	-0.34	-0.23	-0.11	0.18	-0.12	1.00										
ROE	-0.15	0.19	0.05	0.05	0.00	-0.21	-0.39	-0.24	0.38	0.11	0.05	1.00									
MGT	-0.34	0.04	0.18	0.06	0.06	-0.47	-0.41	-0.25	0.41	0.01	0.36	**0.73**	1.00								
D12	0.03	0.25	-0.21	0.01	0.06	-0.06	-0.06	-0.03	0.06	0.03	-0.02	0.09	-0.01	1.00							
E12	-0.04	0.02	-0.02	-0.03	-0.00	0.07	-0.02	0.04	0.02	0.03	-0.04	0.02	-0.17	**0.58**	1.00						
b/m	-0.20	0.06	0.07	-0.03	-0.09	-0.06	0.05	0.15	0.00	0.07	0.02	0.02	0.07	-0.02	0.08	1.00					
AAA	-0.07	0.12	0.21	0.07	0.05	-0.06	0.00	0.04	-0.01	0.10	-0.05	-0.05	**0.70**	0.40	0.04	-0.04	1.00				
ntis	-0.20	-0.02	0.02	0.06	-0.08	-0.04	0.01	0.12	0.07	-0.01	0.08	-0.10	0.17	0.26	0.24	0.03	0.03	1.00			
inf	-0.20	0.04	0.05	0.05	-0.02	-0.11	-0.04	0.01	0.02	0.11	0.09	**0.44**	0.29	0.20	0.01	0.21	0.03	0.15	1.00		
crpr	0.14	0.04	-0.05	0.08	0.06	0.26	-0.05	0.11	-0.05	-0.05	-0.10	-0.02	-0.08	-0.07	-0.08	0.03	0.07	-0.05	-0.20	1.00	
svar	-0.18	-0.03	0.07	-0.02	-0.02	-0.31	-0.17	-0.03	-0.07	-0.13	0.08	0.10	0.08	-0.13	-0.23	0.12	-0.05	-0.18	-0.19	0.03	1.00

Table 9: Standard Vs Residual Factors Across Samples

The Sharpe Ratio of standard autocorrelative factors vs their residual counterparts across different samples. The main US sample run from 1972 to 2022, the US pre-sample runs from 1932 to 1971, and the 3 international samples run from 1992 to 2022.

	Sharpe Ratio					
	STR	$rSTR_{PCA}$	MOM	$rMOM_{PCA}$	LTR	$rLTR_{PCA}$
US	0.58	1.14	0.48	0.95	0.33	0.49
US pre-sample	1.12	1.72	0.34	1.00	0.39	0.57
EUR	0.26	0.91	0.53	0.87	0.08	0.52
APAC ex Japan	-0.21	0.24	0.36	0.99	0.14	0.71
Japan	0.51	0.76	0.06	0.37	0.21	-0.04

Table 10: Benchmarking our Residual Factors

Alphas and t-stats of residual factors regressed against popular asset pricing models. The sample covers the main US sample from 1972-2022. The main US sample begins in 1972 vs the standard 1963 because 1972 is the earliest date there is data for all of the factor models. Alphas are annual.

	rSTR_{PCA}		rMOM_{PCA}		rLTR_{PCA}	
	α	(t)	α	(t)	α	(t)
CAPM	0.067	(7.271)***	0.082	(7.841)***	0.030	(3.151)***
FF3	0.070	(7.340)***	0.076	(7.540)***	0.037	(3.870)***
FF5	0.078	(7.425)***	0.066	(6.060)***	0.041	(3.864)***
BaS	0.079	(7.100)***	0.033	(3.509)***	0.019	(2.158)**
q4	0.082	(7.351)***	0.048	(4.305)***	0.025	(2.097)**
mpf	0.090	(8.528)***	0.031	(3.047)***	0.011	(1.113)
bf	0.094	(7.521)***	0.032	(2.979)***	0.014	(1.284)

Table 11: Maximum Squared Sharpe Ratio of ARfm in Different Samples

Maximum Squared Sharpe Ratio and portfolio allocation for ARfm for the mean variance portfolio across the different samples. The main US sample run from 1972 to 2022, the US pre-sample runs from 1932 to 1971, and the 3 international samples run from 1992 to 2022.

	$Sh^2 Allocation$					
Sample	Mkt-Rf	Size	$rSTR_{PCA}$	$rMOM_{PCA}$	$rLTR_{PCA}$	Sh^2
US	6.9%	2.6%	45.0%	44.0%	1.4%	2.62
US pre-sample	6.1%	9.4%	45.1%	39.4%	0.0%	4.37
EUR	10.3%	8.7%	34.7%	43.2%	3.1%	1.77
APAC ex Japan	17.1%	0.0%	6.9%	66.1%	9.7%	1.21
Japan	4.6%	0.0%	61.3%	34.2%	0.0%	0.67

Table 12: Standard vs Turnover Aware Portfolios

Sharpe ratios and average turnover for our normally rebalanced standard and residual factors. The normal rebalancing for the factors are monthly for short term reversal and momentum, and semi annual for long term reversal. For our Turnover Aware Factors, short term reversal rebalances bimonthly, momentum rebalances semi annually, and long term reversal rebalances annually. The sample looks at US data running from 1972 to 2022.

	Standard Rebalancing					
	STR	$rSTR_{PCA}$	MOM	$rMOM_{PCA}$	LTR	$rLTR_{PCA}$
Sharpe Ratio	0.58	1.14	0.48	0.95	0.33	0.49
(Turnover)	(151.21)	(154.31)	(56.14)	(77.16)	(9.14)	(11.12)
	Turnover Aware Rebalancing					
	STR	$rSTR_{PCA}$	MOM	$rMOM_{PCA}$	LTR	$rLTR_{PCA}$
Sharpe Ratio	0.42	0.93	0.30	0.80	0.35	0.45
(Turnover)	(76.21)	(73.31)	(20.14)	(22.16)	(6.45)	(7.03)
T						1972-2022