

Factor Investing Funds: Replicability of Academic Factors and After-Cost Performance

Abstract

Do factor investing funds successfully capture the premiums associated with academic factors? We explore this question using the growing number of factor investing funds that seek to capture those premiums. While, on average, such funds do not outperform, we find that the factor investing funds with the portfolios that most closely match their academic factors—determined using our novel, holding-based ‘active characteristic share’ measure—significantly outperform those that less closely match. Furthermore, adjusting for stock size, we conclude that the answer to our question is “yes” for closely-matching factor investing funds, which net of costs duplicate the paper performance of the long side of academic factors.

Factor Investing Funds: Replicability of Academic Factors and After-Cost Performance

1. Introduction

Factor investing is a style of investment management which seeks to identify outperforming investments using particular characteristics, or factors, that academic research has identified as associated with positive abnormal returns. Such research has a long history. Early work (e.g., Black, Jensen, and Scholes, 1972) tended to focus on instances in which the empirical relation between beta and returns was inconsistent with the CAPM's predictions, while later work (e.g., Fama and French, 1993) has tended to focus on instances in which the empirical relations between other variables—such as size and book-to-market—and returns could not be explained by the CAPM. Today, academic research has identified hundreds of potentially anomalous return patterns (Harvey, Liu, and Zhu, 2016), generating a “factor zoo” (Cochrane, 2011, pg. 1063).

This research has had a significant influence on the fund industry, with fund companies now offering investment products linked to specific anomalies. These products—which go by many names (e.g., ‘smart beta’ or ‘strategic beta’) and which we call ‘factor investing’—aim to capitalize on a given anomaly by systematically buying stocks with certain quantifiable characteristics. Consider the volatility anomaly (first shown in Haugen and Heins, 1975), which suggests that low volatility stocks outperform high volatility stocks. A factor investing fund built on that anomaly would focus on systematically buying low volatility stocks (as factor investing funds tend to not have short positions).

While the goal of factor investing funds is, in general, like that of traditional actively managed funds—i.e., provide investors with a portfolio that outperforms the market—how they accomplish that goal differs. Factor investing funds do not attempt, as typical active funds would, to perform conventional market timing or individual stock selection. Factor investing funds also

do not attempt, as typical passive funds would, to track standard indexes such as the S&P 500. Accordingly, factor investing funds can be considered a hybrid of passive and active management, using systematic, rules-based approaches with the expectation that those approaches will, in the long run, result in outperformance.¹

Based on the growth of factor investing funds, investors appear to believe that those long-run outperformance expectations will be realized. At the start of our time period in 2006, we identify only 16 factor investing mutual funds managing a combined \$10 billion in assets, whereas at the end of time period in 2020, we identify 207 factor investing mutual funds managing a combined \$359 billion in assets. Put another way, the number of funds and combined assets have, over our time period, experienced compound annual growth rates of about 19% and 27%, respectively. That substantial growth has occurred in contrast with the decreasing popularity of traditional actively managed equity mutual funds, which, as a group, had negative net cash flows every year from 2006 to 2020.²

In this study, we consider whether factor investing funds can, in practice, deliver outperformance. We group factor investing funds into four broad styles: dividend, volatility, momentum, and q-factor (i.e., profitability and investment).^{3,4} Other broad styles or more finely-tuned styles would be of interest, but we are constrained by what styles fund companies have actually deployed and by the need for tractable groupings.

¹ Factor investing funds could also provide investors a variety of alternative factor exposures that could help portfolio diversification; however, this provision is not the primary thrust of fund companies' advertisements.

² See Table 43 in the Investment Company Institute's (ICI's) *2022 Investment Company Fact Book*, specifically the column for active domestic equity funds.
www.icifactbook.org/pdf/2022_factbook.pdf

³ The term q-factor comes from the Hou, Xue, and Zhang (2015) q-factor model, which includes factors related to profitability and investment. They chose that name for their model because it is built, to some extent, on the q-theory of investment. We define each of the styles in greater detail later in the paper.

⁴ In the context of our analysis, we do not consider funds focused on small cap stocks or value stocks to be factor investing funds, as those are common fund styles that have been thoroughly studied for decades (see, e.g., Jensen, 1968).

We document that the performance of factor investing funds has, on average, not justified their growth. An equal-weighted portfolio of factor investing funds during our time period has a net CAPM alpha of -1.17% per year (t -stat = -1.71). If we further account for size and value exposures—using the multifactor CPZ6 model suggested by Cremers, Petajisto, and Zitzewitz (2013)—the net alpha does increase to 0.30% per year, but it is not statistically distinguishable from zero (t -stat = 0.74).⁵ Viewed in the context of the broader academic literature, this result is, perhaps, not surprising. McClean and Pontiff (2016) show that anomalies' return premiums tend to decrease substantially out of sample, which suggests that even a factor investing fund executed perfectly and without cost or constraint may have trouble delivering for investors. During our time period, for example, the CPZ6 alpha of the traditional momentum factor, *umd* (up minus down), is -1.16% per year (t -stat = -0.36).

These average results, however, conceal significant variation across factor investing funds, particularly related to the extent to which the funds actually match the theoretical academic factors that they claim to mimic. We introduce a new measure, 'active characteristic share' (ACS), that measures, at the holdings (portfolio weight) level, how similar a factor investing fund's portfolio is to the long-side portfolio of the fund's theoretical factor. To construct ACS, we adapt the Cremers and Petajisto (2009) active share measure, which compares the portfolio weights of an actively managed fund with the portfolio weights of a standard benchmark (e.g., the S&P 500 or Russell 2000). Our adaption follows the same procedure, but instead of comparing with the

⁵ We tend, in most cases, to report results using both the CAPM and the CPZ6 model because, as we discuss later, how investors regard size and value exposures is uncertain. As demonstrated by the results here though, the CAPM and the CPZ6 model tend to reach similar conclusions, albeit with different economic magnitudes. Note that we, as explained later, deliberately do not account for exposures related to each factor investing fund style (e.g., the Fama and French (2015) profitability and investment factors).

portfolio weights of a standard benchmark, ACS compares a factor investing fund's portfolio weights with those of the fund's theoretical academic factor.

Active share and active characteristic share are designed for different purposes and need to be interpreted in different ways. Active share is designed to determine how actively a traditional actively managed fund is picking individual stocks. Consequently, a low active share is an indication of little stock picking, or closet indexing, which is likely not what investors in traditional active funds expect (see, for example, Cremers and Curtis, 2016). Active characteristic share, conversely, is designed to determine how similar a factor investing fund is to its theoretical academic counterpart. A low ACS is thus indicative of high similarity to that counterpart, which is likely what investors in factor investing funds expect.

The average ACS for factor investing funds equals about 79%, suggesting that their holdings tend to be substantially different from those of their theoretical counterparts. That average is also similar to the average active share for traditional actively managed equity funds (see, e.g., Cremers, Fulkerson, and Riley, 2022b). In other words, factor investing funds are typically as different from their theoretical counterparts as traditional actively managed equity funds are from their benchmarks. There is, however, substantial variation in the ACS of factor investing funds. The factor investing funds in the low ACS tercile have an average ACS of about 65%, while those in the high ACS tercile have an average of about 90%. This level of variation allows us to test the impact of ACS on factor investing fund performance.

We form equal-weighted portfolios of factor investing funds in the low and the high ACS terciles and compare their respective net performances. We find that factor investing funds with low ACS significantly outperform those with high ACS. Using the CAPM, the difference is 3.82% per year (t -stat = 3.89), and using the CPZ6 model, the difference is 1.08% per year (t -stat = 2.01).

That relative performance holds if we instead execute the test using fixed-effects panel regressions that control for other fund characteristics (such as size and active share).⁶ Therefore, we conclude that the more similar a factor investing fund's portfolio is to that of the anomaly it claims to mimic, the better the fund subsequently performs.

In addition, factor investing funds with low ACS are, net of costs, able to match the performance of their theoretical counterparts, which are executed at no cost. The net CPZ6 alpha of the equal-weighted portfolio of low ACS funds is only 0.36% per year less than that of a portfolio of those funds' costless theoretical counterparts—a difference that is economically small, just 3 basis points per month, and statistically insignificant (t -stat = 0.57). Switching to CAPM alpha does show a greater difference in performance, 1.38% per year, but the difference remains statistically insignificant at conventional levels (t -stat = 1.60). Accordingly, while the strength of the evidence varies depending on how size and value exposures are treated, our evidence consistently suggests that low ACS factor investing funds, on average and after costs, match the performance of the theoretical academic factors that they claim to mimic.

Our conclusion comes with important caveats. First, our procedure for matching factor investing funds with their theoretical counterparts adjusts for stock size. When comparing, say, the performance of a factor investing fund with a volatility style to that of its theoretical counterpart, we are not comparing the fund with the volatility anomaly as originally conceived. We are instead comparing the fund to the volatility anomaly within the size range in which the fund tends to invest.⁷ Second, we focus solely on the long side of anomalies. When we consider a factor

⁶ In this setting, we also find a significant negative relation between net CPZ6 alpha and turnover, which is consistent with there being a trade-off between portfolio similarity and trading costs. Similarly consistent with that trade-off, we find, in a separate analysis, that the portfolios of factor investing funds tend to change more slowly than those of their theoretical counterparts.

⁷ Factor investing funds tend to invest in relatively large stocks. For example, about 52% of fund-quarter holdings snapshots for q-factor funds are matched to the theoretical counterpart q-factor portfolio linked to the largest NYSE size quintile.

investing fund with a dividend style, we are not comparing its performance with that of a long-short dividend factor, but only with that factor's long side. Third, our comparison does not evaluate whether it is a prudent choice to attempt to replicate the performance of the long side of a particular anomaly within a particular size group. We only evaluate whether, given that the choice was made, the fund successfully replicated the performance.⁸

The above caveats, put together, may set the bar for the factor investing funds in our analysis lower than first impressions could suggest. They also require us to draw more conservative conclusions—namely, we cannot (and do not) claim that any group of factor investing funds is able to capture the premiums associated with the factors as originally conceived in the academic literature. The caveats, however, do allow to meet the factor investing funds on their own terms, rather than imposing a comparison against a hypothetical that the funds are not attempting to match.

2. Contribution to the Literature

Our paper contributes to several strands of the literature. First, we show that, despite their costs and constraints, some factor investing funds can successfully mimic the characteristic-based anomalous return patterns, albeit not necessarily the premiums, documented in the academic literature. That is, subject to caveats, we find that the academic factors we study can be replicated under real-world conditions. The past literature provides multiple reasons to expect such replication to be challenging.

Trading costs are an especially large hurdle in the successful implementation of a factor investing strategy. Hanna and Ready (2005) and Novy-Marx and Velikov (2016) demonstrate that,

⁸ We likewise do not take a position here on the validity or persistence of the anomalies themselves (e.g., Novy-Marx, 2012, Chordia, Subrahmanyam, and Tong, 2014, Harvey, Liu, and Zhu, 2016, McClean and Pontiff, 2016, Barillas and Shanken, 2018, Arnott, Harvey, Kalesnik, and Linnainmaa, 2019, Gu, Kelly, and Xiu, 2020, and Harvey and Liu, 2021) or on how the anomalies should have performed during our time period due to predictable time variation (e.g., Baker and Wurgler, 2006, Stambaugh, Yu, and Yuan, 2012, Frazzini and Pedersen, 2014, Akbas, Armstrong, Sorescu, and Subrahmanyam, 2015, and Ben-David, Li, Rossi, and Song, 2022).

in general, the trading costs associated with anomalies are substantial—particularly for anomalies with high turnover—and Edelen, Evans, and Kadlec (2013) find that mutual funds have substantial trading costs (along with their disclosed fees). In addition, the existence of direct trading costs leads to indirect costs. Hou, Xue, and Zhang (2020) show that anomalies weaken considerably after eliminating microcap stocks and when switching portfolios from equal to value weighting. Liquidity requirements and associated trading costs necessitate that all factor investing funds, at least to some extent, lean towards larger stocks, which could be detrimental to their performance. Furthermore, Asness and Frazzini (2013) and Han, Huang, and Zhou (2021) both find that the strength of anomalies is greatly reduced if the underlying portfolios are not continually rebalanced based on new information. Factor investing funds must, therefore, trade off any attempt to capitalize on the latest information against trading costs, which likewise could be detrimental to their performance.⁹

While most prior work on the practical replicability of academic anomalies tends to estimate costs, we follow Jordan and Riley (2015) and Patton and Weller (2020) and use the returns of mutual funds, which allows us to identify the practical replicability that can be produced after actual costs. While the use of actual costs is not entirely absent, the existing literature on the operationalization of factor investing that does consider them has representativeness concerns and still, ultimately, relies on estimates. For example, Frazzini, Israel, and Moskowitz (2014) use actual costs in their analysis, but those costs are from a single, unnamed money manager and are used, in the end, to estimate the costs of unexecuted trading strategies. As far as we are aware, we are the

⁹ Along similar lines, there are tax implications for factor investing strategies (see, e.g., Vadlamudi and Bouchev, 2014, Santodomingo, Nemtchinov, and Li, 2016, and Goldberg, Hand, and Cai, 2019). Taxes are a general concern in investment management, but the concern is heightened for strategies with higher turnover. Addressing the direct impact of taxes on factor investing fund performance is, however, outside of the scope of our analysis.

first paper to demonstrate that, with some caveats, actual investors have realized the theoretical performance of academic factors.

Other large hurdles in the successful implementation of a factor investing strategy are the constraints related to short positions and scalability. With respect to the former, Avramov, Chordia, Jostova, and Philipov (2013) and Israel and Moskowitz (2013) both show evidence that many anomalies derive a substantial portion of their premiums from their short positions. An anomalous premium driven in significant part by its short side would be, seemingly, impossible to capture successfully through long-only investment. With respect to the latter, the scalability of strategies designed to capture anomalies is uncertain (e.g., Korajczyk and Sadka, 2004, Frazzini, Israel, and Moskowitz, 2014, Ratcliffe, Miranda, and Ang, 2017, and Li, Chow, Pickard, and Garg, 2019). An anomaly available at a limited capacity may not be available at a capacity meaningful for investment companies. Factor investing funds tend to be long only and, due to competition, cannot limit the total assets pursuing a given anomaly, so our analysis identifies the replicability that is being delivered under the constraints that exist in practice.¹⁰

Second, we contribute to the literature concerning the understanding of factor investing funds as a tool for investors. Prior work on this topic has focused on how well factor investing funds perform as a group and has found little evidence of group outperformance (e.g., Glushkov, 2016, Mateus, Mateus, and Soggiu, 2020, and Ben-David, Franzoni, Kim, and Moussawi, 2022). Our results are generally consistent with those findings, but our concentration is not on overall group performance. Instead, we novelly focus on the cross section of factor investing funds. Prior work has considered different broad styles in the cross section, but we are the first to analyze the

¹⁰ Kahn and Lemmon (2016, pg. 17) note that a key differentiator between other recent novel mutual fund styles, such as “130/30 strategies or portable alpha,” and factor investing funds is that those other strategies “typically encouraged investors to drop long-held constraints (e.g., on short selling),” whereas factor investing funds retain such constraints.

cross section using a holdings-level portfolio analysis. With that analysis, we demonstrate that there are substantial differences between factor investing funds with respect to how closely their portfolios match their equivalent theoretical factor portfolios. Those differences, in turn, have significant performance implications, with low active characteristic share funds not only outperforming high active characteristic share funds but also showing performance comparable to their equivalent theoretical factor portfolios. Thus, we demonstrate that, from the perspective of investors, there is a readily identifiable subset of factor investing funds that offer meaningfully greater value than the typical factor investing fund.

Finally, our results provide a unique angle within the more general literature on the value of active mutual fund management. Within the body of research covering traditional actively managed equity funds, there is a decades-long debate about whether the managers of such funds tend to create value for investors. Some of that work indicates little value (e.g., Carhart, 1997, and Fama and French, 2010), and some indicates substantial value (e.g., Glode, 2011, and Berk and van Binsbergen, 2015). Within that debate, there is also a middle ground in which the focus is not on whether such funds in general or whether such funds on average create value but on whether there is a particular segment of such funds creating value (e.g., Kosowski, Timmermann, Wermers, and White, 2006, Cremers and Petajisto, 2009, Amihud and Goyenko, 2013, and Doshi, Elkamhi, and Simutin, 2015).

Our results using active characteristic share suggest that, among factor investing funds, the segment framework is an accurate descriptor, although our results must be aligned with the prior work with subtlety, since factor investing funds are not active in a traditional sense. Factor investing funds do use a non-passive approach and, unlike traditional actively managed equity funds, they have not already been repeatedly and painstakingly analyzed, which gives our results

validity and distinctive insight with respect to the overall question of value creation. Our results, however, cannot speak to issues like individual stock selection (e.g., Wermers, 2000) or market timing (e.g., Bollen and Busse, 2001) because the managers of factor investing funds do not, in a conventional manner, select individual stocks or time the market.

3. Key Measures

3.1. Active Share

We calculate the active share measure introduced by Cremers and Petajisto (2009) using the implementation of Cremers (2017):

$$\text{Active Share} = 100\% - \sum_{i=1}^N \min(w_{\text{fund},i}, w_{\text{benchmark},i}), \quad (1)$$

where $w_{\text{fund},i}$ is the percentage weight on stock i in the fund's portfolio and $w_{\text{benchmark},i}$ is the percentage weight on stock i in the benchmark portfolio. The calculation is made over the set of all stocks N that are in both the fund and the benchmark, ignoring all non-equity positions. Active share is bound between 0% and 100%, with an active share of 0% indicating that the fund and benchmark have identical holdings (i.e., fully passively invested) and an active share of 100% indicating no overlapping holdings (i.e., highly actively invested).

We identify the benchmark for each fund using Cremers and Petajisto's (2009) minimum active share approach. That is, the benchmark that results in the lowest active share for a given fund is considered that fund's benchmark. Our set of benchmarks is similar to that used by Cremers and Petajisto (2009) and matches that used by Cremers, Fulkerson, and Riley (2022a and 2022b). It includes the S&P 500, S&P 400, S&P 600, Russell 1000, Russell 2000, Russell 3000, Russell Midcap, and the value and growth components of each of those benchmarks.

3.2. Active Characteristic Share

We calculate our adaption of active share, active characteristic share (ACS), using the same basic calculation:

$$\text{ACS} = 100\% - \sum_{i=1}^N \min(w_{\text{fund},i}, w_{\text{c-benchmark},i}), \quad (2)$$

where $w_{\text{fund},i}$ is the percentage weight on stock i in the fund's portfolio and $w_{\text{c-benchmark},i}$ is the percentage weight on stock i in the characteristic-benchmark portfolio, which is a long-only theoretical portfolio that tracks the same academic factor as the factor investing fund. The calculation is made over the set of all stocks N that are in both portfolios. As with active share, we employ a minimum approach to benchmark identification for ACS, but rather than search over a set of standard benchmark portfolios, we search over a set of size-based matching-factor benchmark portfolios. For example, for a factor investing fund with a volatility style, we compare the fund portfolio against a series of long-only portfolios investing in low volatility stocks within different NYSE size quintiles.¹¹

We discuss those portfolios in greater detail later in the paper and in the appendix, but here we note a more general point on the measurement of ACS. As with the benchmarks used to calculate active share, those used to calculate ACS require the drawing of somewhat arbitrary lines. Active share is calculated using a set of benchmarks defined on a size-value axis, but traditional active funds do not hold strictly to those benchmarks' boundaries. Cremers, Fulkerson, and Riley (2022b), by way of illustration, show that a substantial number of active small cap funds that

¹¹ Within a given size quintile of a factor investing style, there are still several choices that can meaningfully alter a factor investing fund's portfolio composition—for example, what the exact group of stocks evaluated for inclusion is, what the key measure is and how it is calculated, whether sector weights are considered, how often and when the portfolio is rebalanced, and how the included stocks are weighted. The SPDR SSGA US Large Cap Low Volatility Index ETF and the Invesco S&P 500 Low Volatility ETF are both volatility-style factor investing funds focused on large cap stocks, but as of September 22, 2022, only two of their top ten holdings were the same.

self-declare their benchmark to be the Russell 2000 invest a significant portion of their assets in stocks with market caps too large for Russell 2000 inclusion.¹² Likewise, ACS is calculated using a set of benchmarks we build on a size-factor axis, but factor investing funds will not hold strictly to those benchmarks' boundaries. Both traditional active funds and factor investing funds are too numerous and bespoke to allow perfect binning. Importantly though, the resultant imperfections in the calculation of those measures should bias against finding any relation between them and performance. The implication of the somewhat arbitrary lines we draw to calculate ACS is, therefore, not to raise suspicions about the validity of our estimates but to indicate that our estimates are closer to lower bounds.

In terms of application, ACS is designed for a different type of fund and must be interpreted in a different manner than active share. Active share measures how actively a traditional active fund invests relative to its benchmark, while ACS measures how closely a factor investing fund is tracking the theoretical factor portfolio that motivated the fund's creation. A low ACS for a factor investing fund means that it is closely replicating the long side of its motivating factor, which would likely be consistent with investor expectations. A low active share for a traditional active fund, conversely, means that it is not very different from its standard benchmark, which would likely be inconsistent with investor expectations.

3.3. Fund Performance

Our primary measure of fund performance is the alpha calculated from the following model:

$$r_{\text{fund},t} - r_{f,t} = \alpha + \sum_{j=1}^N \beta_j * \text{Factor}_{j,t} + \varepsilon_t, \quad (3)$$

¹² Consider further a theoretical fund that invests one half of its capital passively in the S&P 500 and the other half passively in the Russell 2000. That fund would have a meaningfully positive active share despite being fully passively managed.

where $r_{\text{fund},t}$ is the return on the fund in period t , $r_{f,t}$ is the risk-free return in period t , and α is the performance of the fund after adjusting for a set of factors. We consider two sets of factors in our analysis. The first set is a single market factor (i.e., the CAPM of Sharpe, 1964, and Lintner, 1965), and the second set is six factors adopted from Cremers, Petajisto, and Zitzewitz (2013).¹³ We use the market factor, the two size factors, and the three value factors from their seven-factor model, but exclude the momentum factor. Within our framework, we cannot properly evaluate the performance of a factor investing fund with a momentum style if our factor set includes the momentum factor (as it would remove any performance benefits while leaving any implementation costs, ensuring underperformance).¹⁴ We estimate Eq. (3) using monthly returns in most instances—using daily returns only in occasional, noted cases. We focus on net returns, as those returns indicate the after-cost performance experienced by actual investors.¹⁵

We tend to report results using both the CAPM and the CPZ6 model throughout the paper because both models are potentially informative. Berk and van Binsbergen (2016) and Barber, Huang, and Odean (2016) both argue that the CAPM best explains investors' capital allocations, suggesting that the CAPM is the appropriate model choice. Evans and Sun (2021), however, show that, during our time period, size and value exposures are less likely than in the past to be considered alpha by investors, suggesting that the CPZ6 model is the appropriate model choice. Given this grey area, viewing results using both models allows us to determine whether our conclusions are dependent on the treatment of funds' size and value exposures. While our broad

¹³ Cremers, Petajisto, and Zitzewitz's (2013) models are similar to the Carhart (1997) model, but they correct for several problems that arise when using the Carhart (1997) model to evaluate mutual funds (e.g., the Carhart (1997) model's bias against small cap funds and bias towards large cap funds).

¹⁴ For the same reason, we also do not include factors such as the Frazzini and Pedersen (2014) betting-against-beta factor or the Fama and French (2015) profitability and investment factors.

¹⁵ Both of our models assume that passive investing is costless, which means, to the extent that passive investing is not costless, our estimates of net alpha will understate true net alpha (see, e.g., Berk and van Binsbergen, 2015). However, during our time period, the benchmarks underlying the factors in the CPZ6 model could all have been invested in using very low-cost passive funds.

conclusions are typically similar using the CAPM and the CPZ6 model, the economic magnitudes often vary substantially.

4. Data

We construct our sample of factor investing mutual funds using multiple sources. To build our initial pool, we use any fund identified by Morningstar Direct as ‘strategic beta’ or by ETF.com as ‘smart beta’. We further add any funds from several investment management firms well known for factor investing (e.g., DFA, AQR, Janus, Eaton Vance/Atlanta Capital, Intech, and Perkins). From that sample, we drop any funds domiciled outside the U.S., not focused on U.S. equities, or focused on ESG (Environmental, Social, and Governance) or sector investing.¹⁶ We also drop any funds that focus on exposure solely along the size-value axis (e.g., large-cap growth or small-cap value).

Finally, we manually filter our sample by reading the investment objective in each fund’s prospectus. Those prospectus readings are also used to identify the broad style (dividend, volatility, momentum, or q-factor) of each factor investing fund. As a general guideline, dividend funds focus on stocks paying relatively high dividends; volatility funds focus on stocks with relatively low volatility; momentum funds focus on stocks with relatively high past returns; and q-factor funds focus on stocks with relatively high profitability and/or stocks making relatively few investments. The exact investment procedures used, however, vary from fund to fund within a given style.¹⁷ Placing all factor investing into four broad styles causes a loss in precision—each fund cannot be

¹⁶ ESG and sector funds are dropped because they are not associated with an academic factor and because their management style, which often involves conventional individual stock selection and market timing, tends to be incongruent with that of factor investing funds.

¹⁷ The q-factor style has the greatest within-style variance with respect to investment procedures. Funds with that style often consider a broad range of inputs that can be generally summarized, either rhetorically or empirically, as associated with the investment and profitability factors in the Hou, Xue, and Zhang (2015) q-factor model.

placed perfectly—but some level of imprecision is necessary for tractability. Importantly, to the extent placement is imperfect, we should be biased against finding our key results.¹⁸

For most of our data on our final sample of factor investing funds, we use the CRSP Survivor-Bias-Free Mutual Fund Database. The exception is fund holdings, which are primarily from Thomson Reuters, although we supplement them using the CRSP holdings database starting in 2009.¹⁹ The holdings from Thomson Reuters are joined with our CRSP data through the WFICN variable provided by MFLINKS. We include both traditional open-end mutual funds and exchange traded funds (ETFs), and we collapse all share classes of a fund into a single fund (again through the WFICN variable). Fund characteristics and returns are an asset-weighted average of the share-class values, except for assets, which are summed across all share classes.

Our sample period begins in January 2006, as there were only a small number of factor investing funds prior to that point in time, and ends in September 2020. Over that period, we identify 233 unique factor investing funds through the procedure explained above. Figure 1 shows the time series of the number of factor investing funds and their total assets under management. In January 2006, our sample contains only 16 factor investing funds. From that point forward, the number of factor investing funds increases steadily until stabilizing in recent years, with 207 funds in September 2020. Total assets under management show a similar trend, with \$10 billion in assets in January 2006 and \$359 billion in September 2020.

[Insert Figure 1]

¹⁸ For example, we find that factor investing funds with low ACS outperform those with high ACS. A fund should have, in expectation, an average ACS if placed perfectly but an above average ACS if placed imperfectly. Consequently, when we perform our tercile sort, imperfect placement should negatively impact the dependability of both the high and low ACS groups, pushing us towards a null result.

¹⁹ We do not supplement the Thomson Reuters holdings using the CRSP holdings until after 2008 because Schwarz and Potter (2016) find that the CRSP holdings have inaccurate data prior to that point.

Table 1, Panel A provides descriptive statistics on our full sample of factor investing funds. Most of the observations are at the fund-quarter level, but returns are at the fund-month level. The average active share for this sample of funds is 65%, which is near Cremers and Petajisto's 60% threshold for being a closet-index fund. Hence, from a stock-picking perspective, factor investing funds tend not to be very active, which is not unexpected given that factor investing funds do not engage in conventional individual stock selection. The average adjusted R^2 of 93.3%, calculated here using the CPZ6 model and daily returns, suggests that most of the returns on factor investing funds can be explained by traditional exposures (i.e., market, large cap vs. small cap, and growth vs. value). The expense ratio for factor investing funds is, on average, 0.45% per year, which is (i) less than the average for traditional active equity funds but (ii) greater than the average for traditional passive equity funds.²⁰ The average turnover ratio for factor investing funds is 68.2% per year, which again is less than that of traditional active equity funds but greater than that of traditional passive equity funds.²¹

[Insert Table 1]

Panel B of Table 1 covers the same characteristics, but by fund style. Looking across the styles, active share tends to be relatively high (71.7%) and adjusted R^2 tends to be relatively low (90.6%) for dividend funds, while the reverse is true of q-factor funds (58.2% and 95.9%). Expense ratios are similar across the styles, but momentum funds have notably higher turnover at 107.2% per year.

²⁰ See Figure 6.7 in the ICI's *2022 Investment Company Fact Book* for information on expense ratios for traditional active and passive equity funds.
www.icifactbook.org/pdf/2022_factbook.pdf

²¹ See, for example, Table 1 in Crane and Crotty (2018) for average turnover ratios for traditional active and passive equity funds.

5. Performance of Factor Investing Funds as a Group

We first examine the general performance of factor investing funds. In Panel A of Table 2, we regress each fund's net monthly returns against either the CPZ6 model or the CAPM and report descriptive statistics for the resulting alphas and alpha t -statistics. Using the CPZ6 model, the average alpha is -0.19% per year. The associated t -statistics have 25th and 75th percentiles of -1.104 and 0.746 , respectively, suggesting that most of the CPZ6 alphas for factor investing funds are statistically indistinguishable from zero. Using the CAPM, the average alpha for factor investing funds is -2.35% per year. The associated t -statistics have a 25th percentile of -1.80 , suggesting that, with respect to the CAPM, a substantial number of factor investing funds have a statistically significant negative alpha.

We repeat that analysis by style in Panel B. The dividend and volatility styles have positive average CPZ6 alphas (0.88% and 1.35% per year), although the average associated t -statistics are statistically insignificant (0.56 and 0.58). In a likewise manner, the momentum and q-factor styles have negative average CPZ6 alphas (-1.93% and -0.99% per year) with statistically insignificant average associated t -statistics (-1.17 and -0.66). Using the CAPM, the average alpha for each style is negative, but again, the average associated t -statistic for each style is statistically insignificant.

[Insert Table 2]

If we switch from this fund-weighted approach to a time-weighted approach, by forming equal-weighted portfolios of factor investing funds, we find some notable differences in our results. As shown in Panel C, the portfolio of dividend funds has an economically and statistically significant positive alpha of 1.54% per year (t -stat = 1.92), while the portfolio of momentum funds has an economically and statistically significant negative alpha of -2.36% per year (t -stat = -2.75).

The CAPM alphas of those portfolios are both statistically indistinguishable from zero. Therefore, dividend (momentum) funds have meaningfully outperformed (underperformed) using a time-weighted approach only for investors who account for size and value exposures in addition to market exposure.

Figure 2 shows the net cumulative abnormal returns, based on the CPZ6 model, on one dollar invested in each of those equal-weighted portfolios. Consistent with Table 2, the portfolio of dividend funds has the highest cumulative abnormal return (\$1.23), and the portfolio of momentum funds has the lowest (\$0.73); however, they arrive at those ends in different manners. The portfolio of dividend funds gains a small, but steady, amount of alpha over our time period, whereas the portfolio of momentum funds loses a significant amount of alpha early. In April 2006, the momentum fund portfolio is worth \$1.01, but by November 2009, it has decreased in value to \$0.80. The performance of the momentum fund portfolio is not particularly strong from that point forward, but it also does not experience another value decrease of that magnitude.

[Insert Figure 2]

To summarize, we find that, with few exceptions, factor investing funds, as a group, do not deliver positive alpha on average, which is consistent with the prior literature (e.g., Malkiel, 2014).

6. Comparison with Theoretical Factor Investing Portfolios

In this section, we first create and evaluate the academically motivated theoretical portfolios that the factor investing funds claim to mimic. We then compare those theoretical portfolios with factor investing funds' actual portfolios.

6.1. Performance of Theoretical Factor Investing Portfolios

We use CRSP and Compustat to construct theoretical portfolios for each of our styles: dividend, volatility, momentum, and q-factor. While the details are available in the appendix, we

construct the theoretical portfolios such that (i) like the factor investing funds themselves, the theoretical portfolios are long only, (ii) there are distinct portfolios for each of the four styles, and (iii) within each style, there is a separate portfolio for each NYSE size quintile.

Table 3 shows the annualized CPZ6 alphas of our theoretical portfolios. To ensure our portfolios are properly constructed, we compare their performance with that of their corresponding publicly available parallels—specifically those available from the Hou-Xue-Zhang q-factors data library.²² We execute the analysis, for both our portfolios and their public parallels, using the returns averaged across each style’s set of size-based portfolios.²³ As shown, there tends to be a close match with respect to performance between our portfolios and their public parallels. For example, our dividend portfolio has a CPZ6 alpha of 1.98% per year (t -stat = 1.64), while the publicly available parallel portfolio has a CPZ6 alpha of 2.34% per year (t -stat = 1.62). Furthermore, the correlation between those two series of returns is 96%. The largest difference in alpha, occurring for volatility, is only 0.61% per year, or about 5 basis points per month—which seems a small amount for a long-only equity portfolio.

[Insert Table 3]

Apart from momentum, for which performance is poor, there is some evidence that the theoretical portfolios outperformed during our time period (January 2006 to September 2020). Consequently, given the alphas for the factor investing funds in the prior section, an initial basic comparison suggests that, on average, factor investing funds are unable to successfully capture the performance of their theoretical counterparts.

²² The library is available at www.global-q.org. The library makes returns available, but not underlying holdings.

²³ While we form five size-based portfolios for each style, the public parallels have only, depending on the style, two or three sized-based portfolios.

6.2. Comparing Factor Investing Funds and Their Theoretical Counterparts

We next compare these theoretical portfolios to their matching factor investing funds. Here, we specifically consider how each's holdings change over time and how effective the factor investing funds are at obtaining exposure to their nominal factor.

In Table 4, Panel A, we present the average annual changeover for each factor investing fund style and the average annual changeover for those funds' minimum ACS benchmarks (i.e., the portfolio that most closely matches the fund's portfolio from the set of size-based theoretical portfolios tracking the fund's nominal factor). We define changeover as the active share of a portfolio relative to itself one year ago (in particular, June to June). Changeover is indicative of the net change in a portfolio over time, with 0% indicating no net change and 100% indicating a completely new set of stocks. Prior work (e.g., Beck, Hsu, Kalesnik, and Kostka, 2016) has shown that theoretical factor portfolios often require frequent trading—leading to trade-offs in practical implementation—and thus we want to consider whether factor investing funds change their portfolios at the same rate as their theoretical counterparts.²⁴

With the exception of the volatility style, the factor investing funds have lower changeover than their theoretical counterparts. For example, the factor investing funds with a momentum style have an average annual changeover of 63.0%, compared to 88.0% for the theoretical momentum portfolios. Such gaps indicate that, compared to their theoretical counterparts, factor investing funds shift their portfolios at a slower pace. Given the trade-off in practice between trading costs and tracking error, it is possible that factor investing funds are making an optimal choice; however,

²⁴ We prefer here to use changeover instead of turnover because changeover captures the net impact on the portfolio. A fund could make a large number of mostly offsetting trades, which would result in a high turnover, but a low changeover. Or, a fund could make no trades, not even to rebalance, which would result in zero turnover, but a positive changeover. Therefore, if a factor investing fund's changeover is less than that of its minimum ACS benchmark, that tends to suggest the fund chose not to make trades necessary to fully track their benchmark.

the necessity of having to make that trade-off could explain the limited evidence of factor investing funds delivering positive alpha to investors.

[Insert Table 4]

In Panel B, we consider the extent to which factor investing funds' portfolios generate exposure to the long sides of the factors captured by their counterpart theoretical portfolios. A factor investing fund could have zero alpha, but still add value to an investor's portfolio, if the fund provides significant exposure to a desired factor. Even after accounting for the CPZ6 model factors, the factor investing funds still tend to generate significant exposures to their factors. While the average changeover of factor investing funds with a momentum style is lower than that of their theoretical counterparts, those funds still generate a momentum factor exposure of 0.39 (t -stat = 5.91), such that an investor interested in momentum factor exposure could obtain it through those funds.^{25,26}

7. Impact of Active Characteristic Share on Factor Investing Fund Performance

In this section, we explore the relation between active characteristic share and the future performance of factor investing funds. We consider the relation in both a univariate context—using portfolios of factor investing funds with low and high ACS—and in a multivariate context—using a panel regression featuring ACS as a predictive variable.

²⁵ If we instead measure exposure using publicly available versions of the equivalent long-short factors, we find results similar to those reported here, with economically large and statistically significant exposures within each style, except for the q-factor style.

²⁶ In untabulated results, we also follow a procedure analogous to Daniel, Grinblatt, Titman, and Wermers (1997) to calculate the value-weighted characteristic rank for both the factor investing funds and their minimum active share benchmarks. We find that the factor investing funds tilt towards their targeted characteristics more than their minimum active share benchmarks.

7.1. Active Characteristic Share of Factor Investing Funds

We begin by assessing the active characteristic share of factor investing funds. Table 5, Panel A shows the distribution of active characteristic share for our full sample and for each style. In the full sample, the average ACS is 78.7%. That value is similar to the average active share shown for traditional actively managed equity funds in Cremers, Fulkerson, and Riley (2022b). Thus, the average factor investing fund is as different from the theoretical factor portfolio it claims to mimic as the average traditional actively managed equity fund is from its benchmark.

[Insert Table 5]

Panel B of Table 5 divides the full sample and each style into ACS terciles and reports the distributions for each's low and high terciles. This division shows a substantial variation in ACS among factor investing funds. Within the full sample, the low ACS tercile has an average ACS of 64.7%, nearing the level at which Cremers and Petajisto (2009) would call a traditional actively managed equity fund a closet indexer. The high ACS tercile, conversely, has an average ACS of 90.3%, which suggests little overlap between the fund and its matching theoretical factor portfolio.

7.2. Performance of Portfolios of Funds with Low and High Active Characteristic Share

We first examine the relation between ACS and performance using a portfolio approach. Specifically, at the start of each quarter, we sort factor investing funds, within either the full sample or within each style, into terciles based on their most recently available measure of ACS (as of that time).²⁷ Then, we form equal-weighted portfolios using the low and high ACS terciles. Here, we begin our analysis in July 2010, instead of January 2006, because having few funds available early in our sample leads to sparsely populated ACS terciles early in our sample.²⁸

²⁷ If a fund's most recently available measure of ACS is more than 12 months old, then we consider ACS missing for that fund. Like active share, ACS has a high autocorrelation, so staleness is not a primary concern.

²⁸ Our earlier performance results still obtain in this reduced time period.

Table 6 shows the annualized alphas of the resulting portfolios. Starting with the CPZ6 model, we find that, in the full sample, the low ACS portfolio significantly outperforms the high ACS portfolio. The difference in alpha is 1.08% per year (t -stat = 2.01).²⁹ In isolation though, the low ACS portfolio does not outperform to a statistically significant degree—the alpha is 0.71% per year, but the t -statistic is only 1.34. Hence, the full sample results do not indicate a long-only positive-alpha opportunity for investors. Those results do, however, indicate an economically meaningful opportunity cost to choosing a high ACS fund over a low ACS fund. Moreover, the results are consistent with low ACS funds, which more closely match their equivalent theoretical factor portfolios, providing greater value to investors.

[Insert Table 6]

The results using the CPZ6 model to compare the low and high ACS portfolios within each style are similar to the full sample results, albeit not statistically significant. For each of the four styles, the low ACS portfolio has a greater alpha than the high ACS portfolio. The primary notable deviation is that the low ACS portfolio among volatility-style factor investing funds does have both an economically and statistically significant positive alpha (2.09% per year, t -stat = 1.99). Of further note, we find, in untabulated results, that if we add the publicly available version of the appropriate long-short factor for each style to the CPZ6 model—e.g., adding the umd factor to the CPZ6 model when evaluating the portfolios of factor investing funds with a momentum style—that the low ACS portfolios tend to have statistically and economically greater exposures to those factors. The low ACS portfolio among momentum-style factor investing funds, for instance, has a umd exposure of 0.22, while the high ACS portfolio among the same funds has a umd exposure of

²⁹ High ACS funds, on average, have higher expense ratios than low ACS funds, but that difference cannot explain our results. On a gross basis, the low ACS portfolio still outperforms the high ACS portfolio based on the CPZ6 model by 0.98% per year (t -stat = 1.81).

only 0.14. The t -statistic associated with the difference in those values is 1.88. Thus, it is the case that, on average, a factor investing fund's relative level of ACS corresponds to its relative level of exposure to its claimed factor.³⁰

If we switch to the CAPM, the full sample low ACS portfolio continues to outperform the full sample high ACS portfolio, but to a larger degree. The difference in alpha is 3.82% per year (t -stat = 3.89). That result is, however, driven by the underperformance of the high ACS portfolio, as the low ACS portfolio, like before, does not outperform in isolation, with an alpha of -0.13% per year (t -stat = -0.12). Similarly, within the styles, the low ACS portfolios outperform the high ACS portfolios to a larger economic and statistical degree using the CAPM. The only difference within a style that is not statistically significant at conventional levels occurs among dividend-style factor investing funds (1.78% per year, t -stat = 1.32).

7.3. Performance of Active Characteristic Share Portfolios Relative to Theoretical Portfolios

The result that the portfolios of factor investing funds with low ACS outperform those with high ACS leaves a key question unanswered: do funds with low ACS perform as well as the theoretical factor portfolios they claim to mimic? We consider that question here by comparing the performance of equal-weighted portfolios of factor investing funds with low ACS against usage-weighted portfolios of those funds' comparable-size theoretical factor portfolios.

We present results from that comparison in Table 7. In particular, we report the difference between the annualized after-cost alphas of the low ACS fund portfolios and the annualized cost-free alphas of their theoretical counterparts. Using the CPZ6 model and the full sample, there is an insignificant difference in performance between the low ACS portfolio and its theoretical

³⁰ Accounting for those differences in factor exposures in our calculations of portfolio alphas in Table 6 would, in effect, remove the benefits of a successful implementation of a factor investing strategy. We want our test to have the potential to differentiate between the performances of two volatility-style factor investing funds that are identical, except that one has twice the volatility factor exposure as the other.

counterpart. The low ACS portfolio underperforms, but by only 0.36% per year (t -stat = -0.57), or about 3 basis points per month. If the CAPM is used instead, the difference in alpha is larger at 1.38% per year, but still statistically insignificant (t -stat = 1.60). Within the styles, the differences in alpha between the low ACS portfolios and their theoretical counterparts are, in a few cases, large economically, but with one exception (the q-factor style using the CAPM), the differences are not significant statistically. Thus, whatever the performance of their theoretical counterparts is, the evidence suggests that the factor investing funds with low ACS tend to match it—with the strength of the evidence varying, to some degree, depending on how size and value exposures are treated.³¹ Importantly though, as discussed earlier, this result should not be taken as evidence that low ACS factor investing funds are capturing academic factor premiums (i) of the size originally indicated in the literature or (ii) of the size available during our sample period.

[Insert Table 7]

Perhaps unsurprisingly, repeating the same test using high ACS funds shows they tend to significantly underperform their theoretical counterparts.³² In the full sample, the annualized difference in CPZ6 alpha is 1.55% (t -stat = -2.06). The outcome is similar using the CAPM (1.92%

³¹ On the surface, there may appear to be a discrepancy between the results here and the results in Tables 3 and 6. Illustrating by example, the low ACS portfolio of volatility funds in Table 6 has a CPZ6 alpha of about 2.7% per year less than that of the average theoretical volatility portfolio reported in Table 3, but in Table 7 we see a difference in CPZ6 alpha between the low ACS portfolio of volatility funds and its theoretical counterparts of just 0.5% per year. The discrepancy can, however, be attributed to the combined effect of (i) anomaly strength tending to decrease as stock size increases and (ii) factor investing funds tending to buy relatively large stocks. The theoretical volatility portfolio linked to the largest NYSE size quintile underperforms the one linked to the smallest by 3.9% per year, but about 73% of the fund-quarter holdings snapshots for factor investing funds with a volatility style are matched to the theoretical volatility portfolio linked to the largest NYSE size quintile.

³² The low and high ACS portfolios do not have the same theoretical counterparts because the funds within each of the portfolios are not matched to an identical set of comparable-size theoretical factor portfolios. For example, even within a style, such as volatility, the group of funds with low ACS can be matched to a combination of size-based theoretical volatility portfolios different from the high ACS group's combination. In untabulated tests though, we find no significant difference in performance between the theoretical counterparts of the low and high ACS portfolios.

per year, $t\text{-stat} = -2.53$).³³ Within each style, the high ACS funds also consistently underperform, albeit to economic and statistical degrees ranging from clearly insignificant to clearly significant.

Viewed broadly, not only do factor investing funds with low ACS outperform those with high ACS, but there is also meaningful evidence that those with low ACS perform as well as their equivalent theoretical factor portfolios, even though the latter is executed without cost. Factor investing funds with high ACS, conversely, show little evidence of matching performance.

7.4. Multivariate Impact of Active Characteristic Share

We conclude our analysis of ACS by testing whether its predictive power with respect to factor investing fund performance holds after accounting for other fund characteristics and alternative measures of activeness. To perform this test, we use a panel regression with annualized quarterly fund alphas (estimated from daily returns) as the left-hand-side variable. The most recently available measure of ACS as of the start of the quarter is the key right-hand-side variable. We also include on the right, as fund characteristics, lagged measures of size, expense ratio, turnover ratio, and performance; as alternative measures of activeness, lagged active share and lagged R^2 —measured here relative to the appropriate comparable-size theoretical factor portfolio; and, as additional controls, style and year-quarter fixed effects.

The results from this analysis are shown in Table 8. In columns (1) and (2), the impact of ACS is evaluated after controlling for fund characteristics, but before controlling for alternative measures of activeness. A one standard deviation decrease in ACS is associated with an increase in annualized CPZ6 alpha of 0.31% ($=2.58 \times 0.122$) and with an increase in annualized CAPM alpha of 1.17% ($=9.61 \times 0.122$). Using either performance model, the impact is statistically

³³ Factor investing funds as an entire group also, on average, underperform their theoretical counterparts. The annualized difference in CPZ6 alpha is 1.08% per year ($t\text{-stat} = -1.71$), and the annualized difference in CAPM alpha is 1.75% per year ($t\text{-stat} = -2.37$).

significant, with t -statistics of -2.30 and -5.35 , respectively. Thus, the impact of ACS is robust to controlling for fund characteristics, although, like in earlier tests, the manner in which size and value exposures are accounted for (or not accounted for) does impact the strength of the results.

[Insert Table 8]

We next evaluate the impact of ACS after controlling for both fund characteristics and alternative measures of activeness. First, in columns (3) through (6), we repeat columns (1) and (2) but replace ACS with either R^2 or active share. Controlling for fund characteristics, both of those alternative measures of activeness have an inconsistent impact on performance. In columns (7) and (8), we then include all fund characteristics and measures of activeness simultaneously. The economic and statistical strength of the relation between ACS and performance remains similar to what was observed before in columns (1) and (2). Using the CAPM, R^2 and active share do have some economically and statistically significant predictive power, but their impacts are statistically insignificant using the CPZ6 model.

These models also provide an opportunity to evaluate the influence of expense and turnover on performance. There is a consistent negative relation between the expense ratio and alpha. A one standard deviation increase in the expense ratio is associated with a decrease in annualized CPZ6 alpha of 0.73% ($=302.8 \times 0.0024$) and with a decrease in annualized CAPM alpha of 0.90% ($=374.0 \times 0.0024$). Notably, the impact is larger than a one-to-one ratio, which implies that factor investing funds with relatively high expense ratios do not perform worse simply because of their additional expense. To exceed a one-to-one ratio, such funds must also be underperforming on a gross basis. The relation between the turnover ratio and alpha is dependent on how alpha is measured. Using the CAPM, there is no relation, but using the CPZ6 model, there is a negative relation. A one standard deviation increase in the turnover ratio is associated with a decrease in

annualized CPZ6 alpha of 0.70% ($=1.05 \times 0.666$). That result is consistent with funds facing a trade-off between trading costs and maintaining a low active characteristic share.

Returning to our primary focus, we conclude based on these results that the ACS-performance relation among factor investing funds is robust to accounting for both fund characteristics and alternative measures of activeness, with the qualification that the strength of the relation depends on the treatment of funds' size and value exposures.

8. Conclusion

Decades of stock pricing research has resulted in the discovery of a vast and growing number of potentially anomalous return patterns. Harvey, Liu, and Zhu (2016) place their count at 316, arguing that figure is an underestimate. In recent years, these anomalies found in the academic literature have been operationalized by the mutual fund industry with the development of factor investing funds. Such funds have experienced rapid growth, with total assets having a compound annualized growth rate from 2006 to 2020 of about 27%. That growth raises an obvious, important question for investors—how well do factor investing funds perform?—but that question is also important from an academic standpoint. A substantial amount of effort has been spent (e.g., Novy-Marx and Velikov, 2016, and Hou, Xue, and Zhang, 2020) to estimate whether the premiums associated with factors formed from anomalous return patterns could be successfully obtained after costs. Factor investing funds provide a real-world test.

We find that, on average, factor investing funds do not outperform. Accounting for market risk alone, using the CAPM, the net alpha of an equal-weighted portfolio of factor investing funds is -1.17% per year (t -stat = -1.71). If we further account for size and value exposures, using the CPZ6 model, the net alpha does increase, but only to 0.30% per year (t -stat = 0.74). However, there is substantial cross-sectional variation in how closely factor investing funds' portfolios match

their theoretical counterparts' portfolios. Using active characteristic share (ACS)—an adaption of Cremers and Petajisto's (2009) original active share measure—we demonstrate that the factor investing funds with the closest matches have significantly better performance. An equal-weighted portfolio of factor investing funds in the lowest tercile of ACS outperforms an equal-weighted portfolio of funds in the highest tercile by 3.82% per year (t -stat = 3.89) using the CAPM and by 1.08% per year (t -stat = 2.01) using the CPZ6 model. Moreover, the portfolio of low ACS factor investing funds shows evidence of performing as well as a portfolio of their theoretical counterparts—despite those counterparts being executed at no cost. The difference in CAPM alpha is 1.38% per year (t -stat = 1.60), while the difference in CPZ6 alpha is just 0.36% per year (t -stat = 0.57).

We conclude, based on those results, that a significant portion of factor investing funds—i.e., those with low ACS—are able, subject to the caveats explained before, to successfully capture the anomalous return patterns identified in the academic literature. Crucially, that replication occurs after all real-world costs, suggesting that the operationalization of anomalies, perhaps momentum most notably (Korajczyk and Sadka, 2004, and Lesmond, Schill, and Zhou, 2004), may be more viable than previously thought. It is vital to note, though, that this conclusion should not be taken as evidence that there are a set of factor investing funds generating premiums of the size documented in the literature.

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Appendix: Constructing Theoretical Portfolios

We consider only common stocks with a CRSP share code of 10 or 11 and construct all portfolios using value weighting. The size sorts are carried out independently of the sorts on the other characteristics, are performed at the same frequency as a given other characteristic sort, and use the 20th, 40th, 60th, and 80th percentiles of the stocks on the NYSE.

Dividend: In the spirit of Litzenger and Ramaswamy (1979), at the end of June of each year, we sort dividend-paying stocks into quartiles based on their dividend yields over the last 12 months. Non-dividend-paying stocks are placed in their own group, resulting in five total groups. The portfolios are rebalanced annually.

Momentum: In the spirit of Jegadeesh and Titman (1993), at the end of June and December of each year, we sort stocks into quintiles based on their returns over the months $t - 12$ to $t - 2$. The portfolios are rebalanced semiannually.

Volatility: In the spirit of Ang, Hodrick, Xing, and Zhang (2006), at the beginning of each month t , we sort stocks into quintiles based on their daily total volatility in month $t - 1$. The portfolios are rebalanced monthly.

Q-Factor: Investment is measured as the annual change in total assets divided by the one-year-lagged total assets. At the end of June of each year, we sort stocks into quintiles based on that investment measure as of the end of the most recent fiscal year. Those portfolios are rebalanced annually. Profitability is measured as the income before extraordinary items divided by one-quarter-lagged book equity. At the end of each year-quarter, we sort stocks into quintiles based on the most recent available value for that profitability measure. Those portfolios are rebalanced quarterly. Both of those portfolio formation procedures are in the spirit of Hou, Xue, and Zhang (2015). The final portfolio equally invests in the valued-weighted portfolios formed using the investment and profitability measures.

Figure 1: The number of and assets of factor investing funds

This figure shows the number of factor investing funds and their combined assets under management for each month of our sample. The sample period is January 2006 to September 2020.

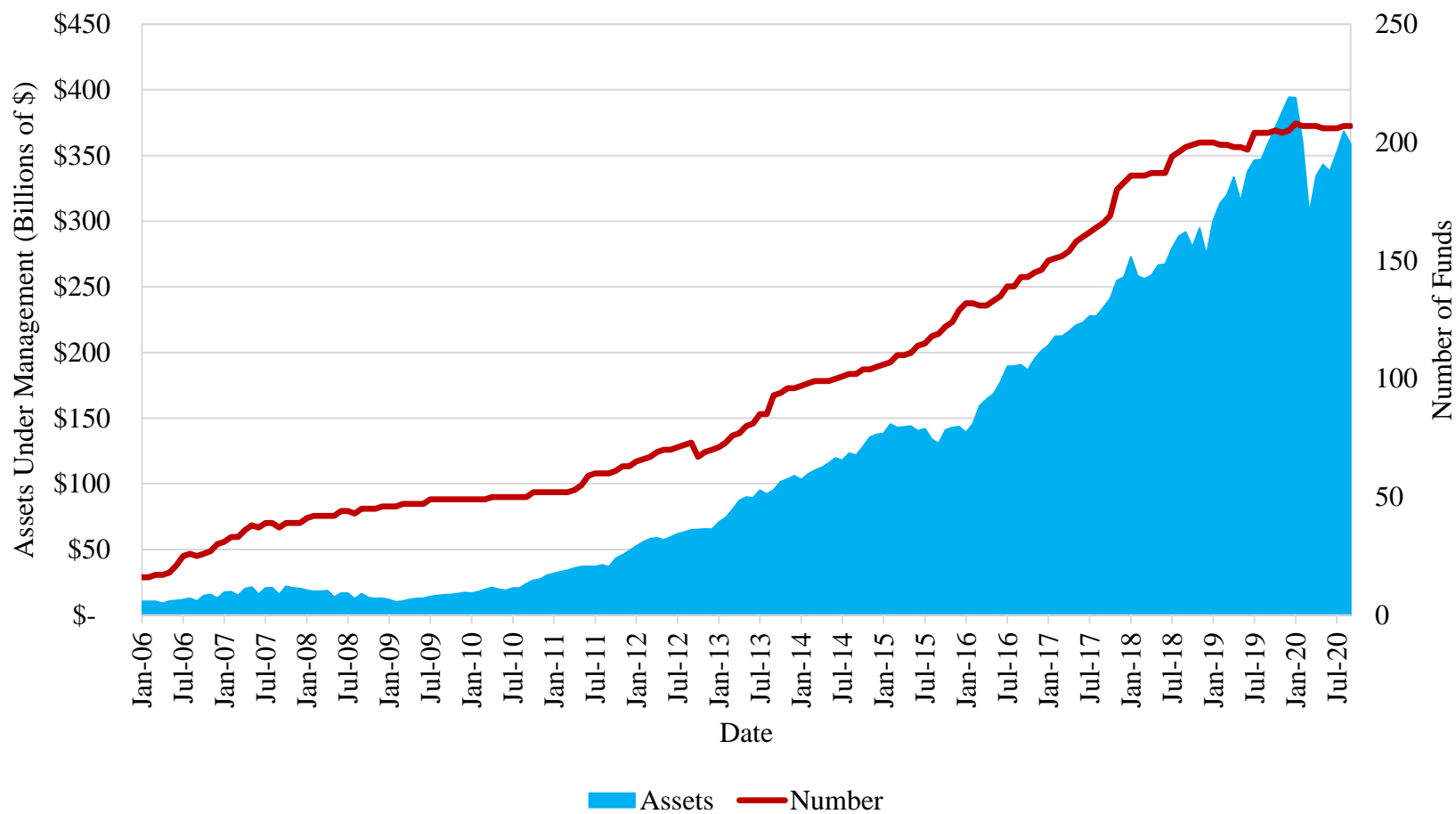


Figure 2: The net cumulative abnormal returns of factor investing funds

This figure shows the net CPZ6 model cumulative abnormal returns on one dollar invested in January 2006 in different equal-weighted portfolios of factor investing funds. Portfolios are formed using the full sample and using each individual style. The sample period ends in September 2020.

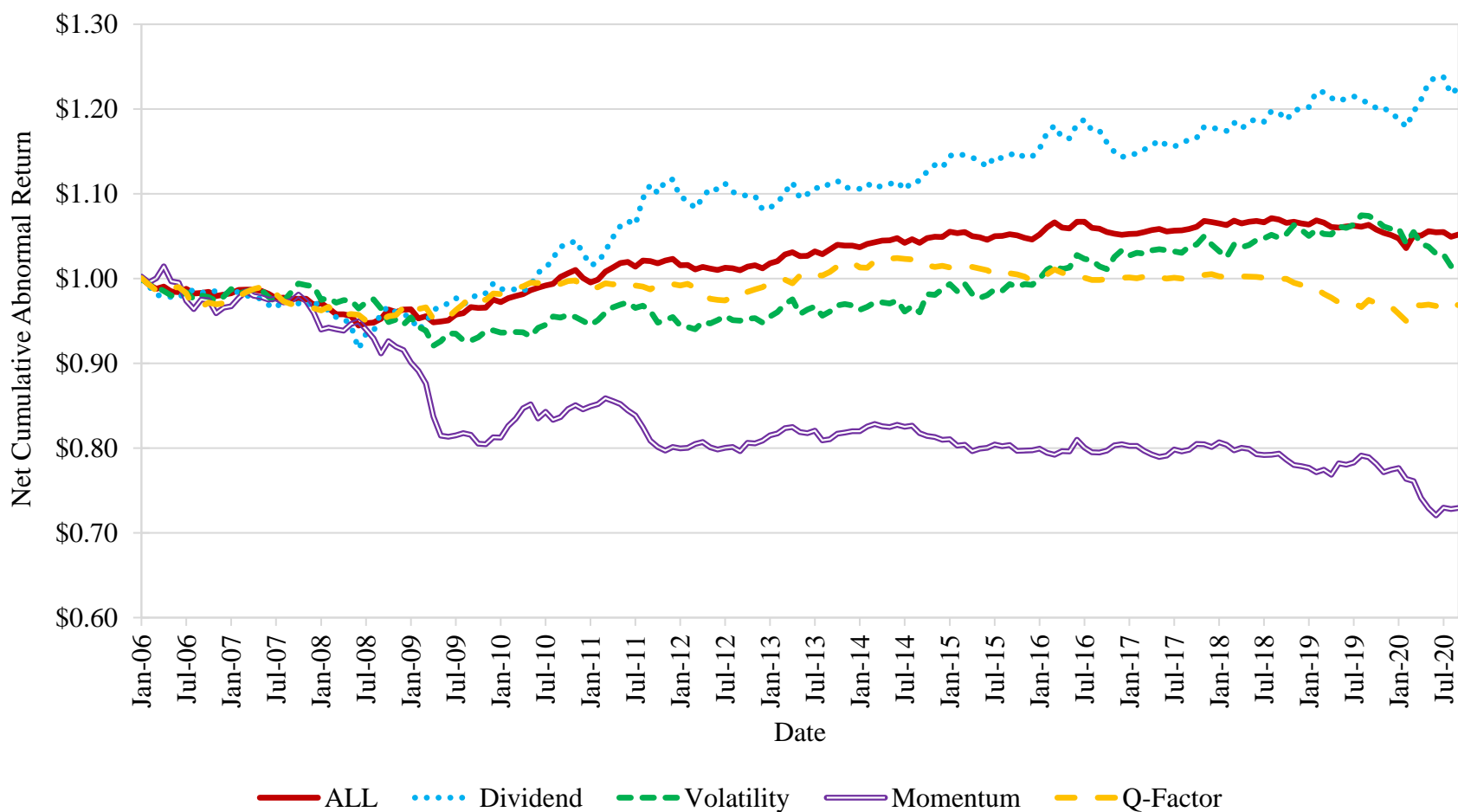


Table 1: Characteristics of factor investing funds

This table provides summary statistics for our sample of factor investing funds. Active share is calculated following Cremers and Petajisto (2009). TNA is total net assets measured in millions of dollars. Adjusted R^2 is calculated in a manner similar to Amihud and Goyenko (2013) using daily returns and the CPZ6 model. Expense is the annual expense ratio. Turnover is the annual turnover ratio. # of Stocks is the number of stocks held by the fund. Net Return is the percentage monthly net fund return. Gross Return is the percentage monthly gross fund return. Panel A provides statistics for the full sample, while Panel B provides the means for the four different styles (Dividend, Volatility, Momentum, and Q-Factor). The sample period is January 2006 to September 2020.

Panel A: Full Sample Summary Statistics

| Variable | Obs. | Mean | Median | P25 | P75 | Stdev. |
|------------------|-------|--------|--------|-------|-------|--------|
| Active Share (%) | 5393 | 64.6 | 66.7 | 53.5 | 78.4 | 17.4 |
| TNA (\$MM) | 6390 | 1263.8 | 174.4 | 46.6 | 765.6 | 3897.1 |
| Adj. R^2 (%) | 5343 | 93.3 | 96.0 | 92.0 | 98.1 | 8.6 |
| Expense (%) | 6130 | 0.45 | 0.39 | 0.29 | 0.60 | 0.24 |
| Turnover (%) | 6130 | 68.2 | 51.0 | 27.50 | 88.0 | 66.6 |
| # of Stocks | 5393 | 327 | 207 | 98 | 455 | 327 |
| Net Return (%) | 18191 | 1.04 | 1.40 | -1.06 | 3.58 | 4.55 |
| Gross Return (%) | 18191 | 1.07 | 1.43 | -1.02 | 3.61 | 4.55 |

Panel B: Mean Values by Style

| Style | # of Funds | Active Share | TNA | Adj. R^2 | Expense | Turnover | # of Stocks | Net Return | Gross Return |
|------------|------------|--------------|--------|------------|---------|----------|-------------|------------|--------------|
| Dividend | 72 | 71.7 | 2288.7 | 90.6 | 0.44 | 53.2 | 194 | 0.96 | 1.00 |
| Volatility | 38 | 65.4 | 1227.2 | 90.0 | 0.43 | 76.6 | 260 | 1.00 | 1.03 |
| Momentum | 23 | 68.0 | 508.3 | 94.6 | 0.48 | 107.2 | 315 | 1.04 | 1.08 |
| Q-Factor | 100 | 58.2 | 694.8 | 95.9 | 0.46 | 67.1 | 451 | 1.10 | 1.14 |

Table 2: The distribution of factor investing fund alpha

This table shows the annualized performance of factor investing funds using the CPZ6 model and the CAPM. Panel A shows the distribution of alphas and associated t -statistics for the individual funds. Panel B shows the mean value of alpha and the mean associated t -statistic within each style. Panel C shows, for both the full sample and style subsamples, the alphas and associated t -statistics of factor investing funds evaluated using equal-weighted portfolios. The sample period is January 2006 to September 2020.

| Panel A: Full Sample | | | | | | |
|----------------------|-----------|--------|--------|--------|--------|--------|
| Model | Variable | Mean | Median | P25 | P75 | Stdev. |
| CPZ6 | Alpha | -0.19% | -0.04% | -1.43% | 1.24% | 2.32% |
| | t -stat | -0.17 | -0.06 | -1.10 | 0.75 | 1.22 |
| CAPM | Alpha | -2.35% | -2.08% | -3.88% | -0.31% | 3.13% |
| | t -stat | -0.94 | -0.96 | -1.80 | -0.15 | 1.11 |

| Panel B: Mean Values by Style | | | | |
|-------------------------------|--------|-----------|--------|-----------|
| Model | CPZ6 | | CAPM | |
| Variable | Alpha | t -stat | Alpha | t -stat |
| Dividend | 0.88% | 0.56 | -2.56% | -0.89 |
| Volatility | 1.35% | 0.58 | -0.66% | -0.21 |
| Momentum | -1.93% | -1.17 | -0.65% | -0.24 |
| Q-Factor | -0.99% | -0.66 | -3.14% | -1.36 |

| Panel C: Equal-Weighted Portfolios | | | | |
|------------------------------------|--------|-----------|--------|-----------|
| Model | CPZ6 | | CAPM | |
| Variable | Alpha | t -stat | Alpha | t -stat |
| Full Sample | 0.30% | 0.74 | -1.17% | -1.71 |
| Dividend | 1.54% | 1.92 | -0.81% | -0.63 |
| Volatility | 0.11% | 0.17 | -0.63% | -0.82 |
| Momentum | -2.36% | -2.75 | -1.57% | -1.11 |
| Q-Factor | -0.32% | -0.71 | -1.79% | -2.28 |

Table 3: The performance of theoretical factor investing portfolios

This table compares the annualized performance of our long-only theoretical factor portfolios with those posted publicly. The factors, both ours and the public ones, are formed within different size groups, with the reported values being averages across those groups. We compare the alphas and associated t -statistics using the CPZ6 model and calculate the return correlation. The sample period is January 2006 to September 2020.

| Construction | Factor Style | CPZ6 Alpha | CPZ6 Alpha t -stat | Correlation |
|----------------|--------------|------------------|----------------------|-------------|
| Ours Public | Dividend | 1.98% 2.34% | 1.64 1.62 | 96% |
| Ours Public | Volatility | 4.18% 4.79% | 4.84 4.94 | 97% |
| Ours Public | Momentum | -2.33% -2.21% | -2.00 -1.88 | 99% |
| Ours Public | Q-Factor | 1.23% 1.27% | 1.71 1.83 | 99% |

Table 4: Factor investing funds and their theoretical counterparts

This table compares factor investing funds with their theoretical counterparts. Panel A shows, for each style, the mean annual changeover for the factor investing funds' underlying portfolios and their size-matched theoretical counterparts' underlying long-side portfolios. Changeover compares current portfolio weights with portfolio weights one year earlier (June to June) using a calculation similar to active share. Panel B shows, for each style, the mean exposure to the long side of the size-matched theoretical factor that the factor investing funds claim to mimic, after controlling for the factors in the CPZ6 model. The sample period is January 2006 to September 2020.

| Panel A: Changeover | | |
|---------------------|----------------------------------|-------------------------------|
| Style | Long Side of Theoretical Factors | Actual Factor Investing Funds |
| Dividend | 43.2% | 34.0% |
| Volatility | 39.2% | 40.7% |
| Momentum | 88.0% | 63.0% |
| Q-Factor | 60.8% | 42.7% |

| Panel B: Factor Exposure | | |
|--------------------------|-----------------------|---------------------|
| Style | Exposure to Long Side | <i>t</i> -statistic |
| Dividend | 0.39 | 10.74 |
| Volatility | 0.37 | 6.79 |
| Momentum | 0.39 | 5.91 |
| Q-Factor | 0.07 | 1.64 |

Table 5: The active characteristic share of factor investing funds

This table shows, for our sample of factor investing funds, the distribution of active characteristic share (ACS). Panel A provides the statistics for the full sample and for each style. Panel B sorts each group into ACS terciles each quarter and provides the statistics for the low and high terciles. The sample period is January 2006 to September 2020.

| Panel A: Full Sample and By Style | | | | | | |
|-----------------------------------|------|-------|-------|-------|--------|--|
| Group | N | Mean | P25 | P75 | Stdev. | |
| Full Sample | 6006 | 78.7% | 70.7% | 87.8% | 12.2% | |
| Dividend | 1966 | 77.6% | 69.1% | 88.8% | 14.7% | |
| Volatility | 811 | 74.4% | 66.9% | 82.7% | 10.9% | |
| Momentum | 697 | 78.5% | 73.9% | 87.2% | 12.7% | |
| Q-Factor | 2731 | 80.8% | 75.5% | 88.2% | 9.8% | |

| Panel B: Low and High Terciles | | | | | | |
|--------------------------------|---------|------|-------|-------|-------|--------|
| Group | Tercile | N | Mean | P25 | P75 | Stdev. |
| Full Sample | Low | 1984 | 64.7% | 60.9% | 70.6% | 9.2% |
| | High | 2001 | 90.3% | 87.6% | 92.6% | 3.6% |
| Dividend | Low | 633 | 61.2% | 55.6% | 69.1% | 11.8% |
| | High | 656 | 91.7% | 88.3% | 95.2% | 4.6% |
| Volatility | Low | 257 | 63.5% | 60.7% | 67.6% | 6.5% |
| | High | 268 | 86.2% | 82.6% | 89.6% | 6.0% |
| Momentum | Low | 211 | 65.8% | 57.6% | 76.1% | 13.0% |
| | High | 228 | 88.7% | 84.5% | 92.8% | 5.0% |
| Q-Factor | Low | 897 | 69.1% | 63.6% | 75.5% | 7.4% |
| | High | 909 | 89.9% | 88.0% | 91.3% | 2.4% |

Table 6: The performance of low and high active characteristic share portfolios

This table shows the annualized alphas of quarterly-rebalancing equal-weighted portfolios of low and high active characteristic share factor investing funds. Alpha is measured using both the CPZ6 model and the CAPM; separate portfolios are formed using the full sample and each style; and Low and High are determined based on a within-style (except for the full sample) tercile sort. *t*-statistics calculated based on Newey and West (1986) standard errors are reported in parentheses below each alpha. The sample period is July 2010 to September 2020.

| Model | CPZ6 | | | CAPM | | |
|-------------|-------------------|-------------------|-----------------|-------------------|-------------------|-----------------|
| Tercile | Low | High | Low – High | Low | High | Low – High |
| Dividend | 1.81% (1.40) | 0.65% (0.83) | 1.17% (1.35) | 0.08% (0.03) | -1.71% (-1.25) | 1.78% (1.32) |
| Volatility | 2.09% (1.99) | 1.01% (1.00) | 1.08% (0.70) | 1.96% (1.45) | -3.07% (-1.79) | 5.02% (2.73) |
| Momentum | -0.99% (-0.72) | -2.47% (-2.44) | 1.48% (0.97) | 0.16% (0.09) | -3.64% (-2.33) | 3.80% (2.36) |
| Q-Factor | -0.06% (-0.16) | -0.88% (-1.48) | 0.82% (1.59) | -0.88% (-1.28) | -5.74% (-3.31) | 4.85% (3.60) |
| Full Sample | 0.71% (1.34) | -0.37% (-0.83) | 1.08% (2.01) | -0.13% (-0.12) | -3.95% (-2.87) | 3.82% (3.89) |

Table 7: The performance of factor investing funds relative to theoretical portfolios

This table compares the annualized CPZ6 and CAPM alphas of factor investing funds with those of the theoretical factor portfolios that the factor investing funds claim to mimic. Separate analyses are performed using the full sample and within each style. The quarterly-rebalancing equal-weighted Low and High portfolios of factor investing funds are the result of within-style (except for the full sample) tercile sorts on active characteristic share. The Theo portfolios are quarterly-rebalancing usage-weighted portfolios of the factor investing funds' comparable-size long-only theoretical factor portfolios. *t*-statistics calculated based on Newey and West (1986) standard errors are reported in parentheses below each alpha. The sample period is July 2010 to September 2020.

| Model | CPZ6 | | CAPM | |
|-------------|-------------------|-------------------|-------------------|-------------------|
| | Low – Theo | High – Theo | Low – Theo | High – Theo |
| Dividend | -0.69% (-0.68) | -2.09% (-1.55) | -0.33% (-0.30) | -0.32% (-0.15) |
| Volatility | -0.47% (-0.49) | -3.12% (-2.63) | -1.52% (-1.29) | -4.90% (-3.69) |
| Momentum | 1.72% (1.30) | -0.18% (-0.12) | -0.38% (-0.24) | -1.58% (-0.95) |
| Q-Factor | -0.59% (-0.66) | -0.76% (-0.93) | -2.26% (-1.75) | -1.96% (-1.70) |
| Full Sample | -0.36% (-0.57) | -1.55% (-2.06) | -1.38% (-1.60) | -1.92% (-2.53) |

Table 8: Multivariate relation between active characteristic share and performance

This table shows results from regressing annualized fund alphas on active characteristic share (ACS) and a number of controls. Observations are at the fund-quarter level; all explanatory variables are lagged by one quarter; and alpha is measured using daily returns and both the CPZ6 model and the CAPM. Style and year-quarter fixed effects are included in all specifications. *t*-statistics calculated from standard errors two-way clustered by fund and year-quarter are reported in parentheses below each coefficient. The sample period is January 2006 to September 2020.

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
|---------------------|---------|---------|---------|---------|---------|---------|---------|---------|
| Model | CPZ6 | CAPM | CPZ6 | CAPM | CPZ6 | CAPM | CPZ6 | CAPM |
| ACS | -2.58 | -9.61 | | | | | -3.10 | -10.6 |
| | (-2.30) | (-5.35) | | | | | (-2.40) | (-5.00) |
| R ² | | | 4.08 | 1.90 | | | 2.68 | -7.32 |
| | | | (1.46) | (0.54) | | | (0.88) | (-1.90) |
| Active Share | | | | | 0.34 | -4.83 | 1.33 | -2.62 |
| | | | | | (0.37) | (-3.77) | (1.34) | (-1.80) |
| Log TNA | -0.16 | 0.08 | -0.16 | 0.16 | -0.13 | 0.18 | -0.19 | 0.07 |
| | (-2.00) | (0.74) | (-2.02) | (1.36) | (-1.60) | (1.57) | (-2.33) | (0.64) |
| Expense | -296.5 | -361.3 | -309.3 | -450.5 | -321.2 | -391.0 | -302.8 | -374.0 |
| | (-3.83) | (-3.93) | (-4.05) | (-4.85) | (-4.22) | (-4.23) | (-3.92) | (-4.05) |
| Turnover | -0.94 | -0.21 | -0.96 | -0.42 | -1.01 | -0.01 | -1.05 | -0.29 |
| | (-2.49) | (-0.49) | (-2.51) | (-0.95) | (-2.56) | (-0.02) | (-2.63) | (-0.64) |
| Alpha | -0.06 | 0.16 | -0.06 | 0.17 | -0.06 | 0.17 | -0.07 | 0.15 |
| | (-1.67) | (6.34) | (-1.86) | (6.43) | (-1.62) | (6.64) | (-1.91) | (5.89) |
| Time FE | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
| Style FE | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
| N | 4758 | 4758 | 4672 | 4672 | 4758 | 4758 | 4672 | 4672 |
| Adj. R ² | 0.090 | 0.326 | 0.091 | 0.323 | 0.089 | 0.323 | 0.092 | 0.329 |