Factor and stock-specific disagreement and trading flows*

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Abstract

We study how disagreement on both factor and stock-specific risk exposures across many investors and securities impacts asset prices. Our theoretical analyses predict that disagreement about factor dynamics drives larger flows into portfolios that are more exposed to the factors. These concentrated bets on the factor lead to higher volatility and reduced diversification benefits. We then test these predictions using a novel empirical setting – exchange-traded funds (ETFs). We find that when factor disagreement rises, ETFs that mimic the factor see increased flows, higher forward-looking volatility risk, and a higher forward-looking correlation among the stocks in the ETF.

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The proliferation of Exchange-Traded Funds (ETFs) has made trading portfolios of assets easier. The perception is that these vehicles provide investors with cheap access to well-diversified portfolios and improve risk sharing. From an asset-pricing perspective, a greater distribution of risks across agents should result in lower aggregate risk. However, many studies find that the introduction of index-linked assets, such as ETFs, has had the opposite effect of increasing excess volatility (see, inter alia Ben-David et al., 2018). While prior studies attribute their findings to liquidity shocks from uninformed investors propagating between assets via an arbitrage channel, this study shows that an alternative economic mechanism — heterogeneous beliefs — is also at play. Namely, we provide evidence that speculative trading due to disagreement in subjective expectations about the stock-specific and factor components of returns can induce (i) crowded trades and (ii) higher volatility and correlation risk for the index and its underlying assets. This contrasts a risk-sharing mechanism in that ETFs facilitate speculative trades leading to exorbitant shifts in wealth.

On the theoretical front, we answer *why* concentrated positions are created (flow generated from time-varying factor disagreement) and *how* the risks are embedded in asset prices (higher factor volatility and stock correlations). On the empirical front, we quantify the effects of disagreement-induced trading within a unique setting: ETFs. We thus use the term "factor" to refer to any common component of returns that investors could be concerned about, e.g., exposure to technology or healthcare. Additionally, our analysis exploits information in ETF-linked options, which have grown in recent prominence, to examine how disagreement impacts ex-ante and long-run perceptions, rather than just realizations, of risk.

The canonical asset pricing framework assumes that variations in the returns of individual securities are a function of factor risk exposures and stock-specific shocks. This sets up a dichotomy: with many investors and stocks, there are a near infinite number of dimensions along which investors can disagree on the stock-specific portion of returns. However, this form of disagreement (henceforth, *stock-specific disagreement*) can wash out in the aggregate. In contrast, investors can disagree on far fewer dimensions regarding the portion of returns attributed to factor exposure (henceforth, *factor disagreement*). This generates the possibility of group-think and concentrated exposure (both long and short) across investors. One would think that each form of disagreement differs in its impact on asset prices and thus has different consequences for financial risk.

To demonstrate why, consider investors trading on their beliefs about the prospects of the U.S. technology sector. On the one hand, if investors' beliefs are dispersed, meaning that investors believe that stock-specific shocks to firms are more likely to drive next period's returns than common factors, then investors will choose to take disparate positions in individual technology stocks, such as Microsoft Corporation and Apple Corporation. On the other hand, if investors believe that the returns of all technology firms will be driven by these firms' exposures to a common systematic "technology" factor, then investors will likely choose a portfolio of technology stocks. For instance, investors may trade the Invesco QQQ Trust Series 1 ETF (NASDAQ: QQQ), an ETF that tracks the returns of the Nasdaq 100 index. Consequently, a large flow into (out of) QQQ suggests that investors are predominately trading on the systematic (firm-specific) component of technology stock returns.

We begin by constructing an economic model that codifies this intuition. Specifically, we consider a pure exchange economy with multiple Lucas trees that are exposed to both factor risk and stock-specific shocks. The model's primary innovation is that investors can disagree about both dimensions of returns. Our model features both periods when there is strong disagreement about the factor and periods when there is strong disagreement about stock-specific returns. Although there is still considerable disagreement in the periods when agents disagree about the stock-specific shocks, different agents take uncorrelated bets on different stocks, and hence the aggregate effect of disagreement on the factor-mimicking portfolios is muted. In contrast, factor disagreement drives all investors to take correlated bets on the systematic component of returns, inducing a large impact on the portfolios that mimic the factors, which are represented as ETFs in the data.

The model's key predictions are threefold: first, higher factor disagreement increases the exposure of investors to the factor. In the context of our empirical analysis, this translates into greater flow into securities that are primarily exposed to factor risk (i.e., ETFs). Second, higher factor disagreement increases the return volatility of ETFs, as these instruments closely align with the systematic risk factor. Third, this increase in factor volatility causes the correlations between pairs of securities that have large loadings on the common factor to increase. This reduces the diversification benefits of holding an ETF. Importantly, these predictions are a product of a frictionless economy in which time-varying subjective beliefs lead to these results. This contrasts with the extant literature on ETFs, in which similar predictions derive from either micro-structure frictions (e.g., heterogeneity in liquidity needs) or limited participation (e,g., the notion that ETFs are pure retail products).

We then use the return and flow dynamics of ETFs and their underlying assets to test the model's predictions. ETFs provide an ideal crucible for our tests for three primary reasons. First, it is difficult to empirically analyze how disagreement directly impacts asset prices without first assessing disagreement's relationship with trading flows. In any theoretical framework, changes in disagreement induce trading between agents due to shifting exposures. This coincides with changes in the risks and returns of assets thus linking disagreement to asset prices. Most papers in this area of research study the latter association without providing any evidence on the former association between disagreement and flows (see, e.g., Buraschi et al. (2014) and Daniel et al. (2021)).

ETFs allow us to circumvent the aforementioned shortcomings and examine how differences in disagreement across ETFs and time directly impact trading activity, and consequently affect asset prices. As ETFs track specific indices, and investment "themes" and "styles," market participants are incentivized to maintain the connection between the underlying asset value of the ETF and that of its component securities (see, e.g., Ben-David et al. (2018) for details). If there is even an infinitesimally higher cost of trading a basket of securities than directly trading an index itself, then an increase (decrease) in aggregate demand for diversified factor exposure should lead to the creation (destruction) of ETFs.

Our first piece of analysis exploits this connection to show how changes in factor or stock-specific disagreement drive changes in the creation or destruction of ETFs. In particular, we find that a one-standard-deviation higher factor disagreement (relative to stock-specific disagreement) leads to a 0.11-standard-deviation higher flow into the ETF. This positive association between the relative amount of factor vis-á-vis stock-specific disagreement and trade flows is robust to controlling for both ETF and time fixed effects, and a variety of confounding variables that could drive ETF flows, e.g., lagged ETF returns. Altogether, the positive relation between factor disagreement and trading flows is in line with our model's theoretical predictions.

Second, we are the first to our knowledge to exploit the richness of the ETF options market, which today composes more than 40% of all option volume, for our analysis.[1] Most papers linking ETF trading activity to risk use realized returns for their analyses. By using ETF option prices, we can link disagreement directly to changes in investor perceptions, rather than noisy realizations, of risk. Examining risk-neutral moments allows us to more accurately measure forward-looking and long-dated shifts in the volatility of, and correlation between, securities exposed to a given factor. Finally, ETFs provide us with a large cross section of investment factors and styles in which to analyze our model's novel predictions. We document a significant amount of heterogeneity in disagreement, volatility, and correlations across ETFs.

In keeping with our theoretical analysis, we find that risk-neutral or forward-looking ETF

[1] See, e.g., the Wall Street Journal article "Just as Hot as ETFs: Options on ETFs" from December 9, 2019.

volatility is strongly related to the relative amount of disagreement about the common (i.e., factor or systematic) component of an ETF's returns. A one-standard-deviation increase in our relative factor disagreement measure leads to volatility rising by about 0.10 standard deviations, even when accounting for ETF and time fixed effects and a battery of ETF-level controls.

Finally, as disagreement increases the volatility of the common component of returns, the correlation between pairs of securities in the ETF also increases. To show this, we compute the average risk-neutral correlation between all pairs of securities within an ETF and find that increased disagreement about systematic risk exposure of an ETF is indeed strongly and positively related to the average intra-ETF correlation. When the relative amount of factor disagreement rises by one standard deviation, then the average correlation between all pairs of securities with an ETF rises by about 0.05 standard deviations. This indicates that higher factor disagreement about the drivers of an ETF's returns is associated with a decline in the diversification benefits of holding the ETF, as a result of the anticipated increase in flows into the ETF. Moreover, we find that this loss of diversification benefits is concentrated among one- to six-month horizons. This contrasts with other papers that focus on losses in diversification benefits over one month or less (see, e.g., Da and Shive (2018)).

Contributions to the literature. Our paper is related to the literature on general equilibrium models with heterogeneous beliefs such as Harrison and Kreps (1978), Detemple and Murthy (1994), Zapatero (1998), and Basak (2000), among others.[2] This literature has mainly focused on economies with a single stock or Lucas tree.[3] In contrast, we consider an

[2]Models with disagreement include Shalen (1993), Scheinkman and Xiong (2003), Basak (2005), Berrada (2006), Buraschi and Jiltsov (2006), Jouini and Napp (2007), David (2008), Dumas et al. (2009), Xiong and Yan (2010), Cvitanic and Malamud (2011), Cvitanic et al. (2012), Bhamra and Uppal (2014), Buraschi et al. (2014), Cujean and Hasler (2017), Ehling et al. (2018), and Atmaz and Basak (2018).

[3]For a few exceptions see Fedyk et al. (2013) who study the survival of agents with biased beliefs in an economy with multiple assets, Buraschi et al. (2014) who consider an economy with two different consumption goods, and Hansen (2015) who study asset prices in an economy with heterogeneous beliefs and preferences with multiple Lucas trees.

economy featuring multiple Lucas trees with a factor structure to study the implications of disagreement on factor versus stock-specific risk.

Our paper also adds to the literature on the empirical relations between disagreement and asset prices (e.g., Berkman et al. (2009); Chen et al. (2002); Diether et al. (2002); Park (2005)) and disagreement and trading activity (e.g., Ajinkya et al. (1991); Bessembinder et al. (1996); Goetzmann and Massa (2005)). In the spirit of this literature, we construct measures of disagreement using the dispersion of analysts' earnings estimates. However, unlike many of the aforementioned studies, we use these analysts' estimates to construct measures of disagreement that are specific to each ETF in our sample. Our ETF-specific measures capture both the extent to which brokers disagree about the common factor underlying an ETF and the extent to which analysts disagree about the idiosyncratic component of an ETF's earnings. These two proxies map to our notions of factor and stock-specific disagreement, respectively.

A major contribution of our study is to elucidate a distinct and novel driver of index and ETF activity — i.e., time-varying heterogeneous beliefs and its relationship to the demand for factor exposure. Many have used the arbitrage relationship between ETFs and their underlying securities for analysis. Ben-David et al. (2018) analyzes how the level of ETF ownership predicts single security volatility and mispricing, and Da and Shive (2018) analyze how ETF trading activity induces changes in physical correlation. Baltussen et al. (2019) provide evidence that the availability of easy to access index products has altered the structure of serial dependence in index returns (see also Agarwal et al., 2018).

This literature argues that shocks transmitted from ETFs to their constituent stocks via an arbitrage channel lead to excess volatility and correlation effects, focusing on the incentives of arbitrageurs to trade on the difference in net asset values. Critically, the shocks to demand for indexation in these frameworks are *non-fundamental* in nature (e.g., Brown et al., 2021, for a parsimonious example of the mechanism). In contrast, our paper provides evidence of a *fundamental* origin of trades. ETFs complete the market only if it is cheaper to trade them (due to the services provided by authorized participants) than a basket of individual assets. From this perspective, our work develops an understanding of the upstream drivers of index-oriented demand, elucidating the reasons that arbitrage trading may be happening in the first place.

A concurrent literature has shown that ETFs increase informational efficiency. For example, Huang et al. (2021) show that hedge funds strategically deploy a long-short strategy using single stocks versus their related industry ETFs. A higher prevalence of this strategy predicts higher unexpected earnings and lower post-earnings-announcement drift. Antoniou et al. (2022) provide follow-on evidence that higher price-efficiency induced by ETF ownership leads to higher sensitivity of real investment decisions to a firms own stock price. Similar to our work, this strand of the literature suggests a fundamental role for ETF trading activity.

Finally, another important innovation of our paper versus previous work is in the use of ETFs and ETF options to analyze the dynamic effects of *disagreement* on ETF and individual security flows, volatility, and correlations. Option-based measures are uniquely suited to speak to the forward-looking and long-dated implications of this activity. The excess volatility and correlation literature finds primarily short-run and mean-reverting effects, which suggests that mispricing drives the results. In contrast, our results are driven by a fundamental shock that affects risk premia.[4]

1 Model and theoretical predictions

To study how factor and stock-specific disagreement impact investors' decisions to trade either an ETF or individual securities, we consider a pure exchange economy with incomplete information about cash flow dynamics. Specifically, the investors in the economy have different

[4]Our analysis is therefore also closely aligned with the work on indexation (see, e.g. Barberis et al., 2005; Bond and Garcia, 2022; Brogaard et al., 2019).

beliefs about the contribution of factor versus stock-specific shocks to observable dynamics of dividends. For the purpose of simplicity, we assume that there is only one common factor and we interpret the value-weighted portfolio of all the individual stocks as a passive ETF, such as SPY or QQQ. Extending the model to multiple factors and "ETFs" would be simple, but more numerically intensive to solve. Moreover, as we want to focus on the main economic intuition and the theoretical predictions we test in Section 2, we relegate many of the details of the model to the Online Appendix A.2.

1.1 Preferences and cash flows

Preferences. Our setting is a standard continuous time pure exchange economy populated by J investors. We assume that each agent has power utility so that the lifetime expected utility of investor j is

$$\mathbb{E}^j \left[\int_0^\infty e^{-\rho t} \frac{c_{j,t}^{1-\gamma}}{1-\gamma} dt \right]. \tag{1}$$

Here, ρ represents the time discount factor, γ is the coefficient of relative risk aversion, and the expectation is taken with respect to the *subjective* belief of investor j. This heterogeneity in beliefs, which we describe in section 1.2, is the only type of investor heterogeneity we consider.

Cash flows. The economy features N stocks, with stock n paying a flow of dividends $D_{n,t}$ at time t. We interpret a claim on aggregated dividends, i.e., $D_t = \sum_{n=1}^N D_{n,t}$, as representing the ETF in our setting. For example, D_t represents the claim to the total dividends of all NASDAQ stocks at time t, while $D_{n,t}$ represents the claim to the dividends of a single stock in the index. Since NASDAQ dividends are a small fraction of total consumption and our model abstracts from multiple factors, we also assume that there is an additional endowment stream paying E_t at time t. This additional endowment can be interpreted as including labor income and the dividends of stocks that we do not analyze or that are not part of the specific ETF of interest. The reason for including the additional endowment is to break the strong link between dividends and aggregate consumption.[5]

Aggregate consumption in the economy is therefore $C_t = E_t + D_t$. The dynamics of the endowment stream, E_t, is

$$dE_t = \mu_E E_t dt + \sigma_E E_t dw_{E,t}. \tag{2}$$

The dividend of stock n at time t is

$$D_{n,t} = D_{n,0} E_t e^{z_t + \epsilon_{n,t}}. \tag{3}$$

Here, $D_{n,0}$ is the initial dividend of stock n at time zero, E_t reflects how the average level of dividends changes over time (e.g., the fact that the level of dividends is often higher in good times than bad), z_t reflects the component of dividend growth that is common across all stocks (i.e., the common factor), and $\epsilon_{n,t}$ represents a stock-specific component of dividend growth. This common factor z_t and the stock-specific components are assumed to evolve according to

$$dz_t = \mu_z dt + \sigma_z dw_{z,t} \tag{4}$$

and

$$d\epsilon_{n,t} = \mu_n dt + \sigma_n dw_{n,t}, \tag{5}$$

for $n = 1, \ldots, N$, where $w_{z,t}$ and $w_{n,t}$ are mutually independent standard Brownian motions.

[5] Our model and its predictions are related to the composition of disagreement and its effects on a single ETF. Our empirical analysis utilizes the cross-section of ETFs. We thus verify that our empirical results are robust to the inclusion of both ETF and time fixed effects.

1.2 Disagreement

As noted in equation (1), investors in this economy have heterogeneous beliefs and disagree about the dynamics of the common factor, z_t, and the firm-specific components of dividends, $\epsilon_{n,t}$ for $n \in N$. That is, we assume that while agents can perfectly observe the *realized* values of z_t and $\epsilon_{n,t}$, they *disagree* about the dynamics of these shocks. To focus on disagreement related to the stocks in this economy and not the underlying fundamentals of the economy itself, we do not allow for any disagreement about the dynamics of the additional endowment process E_t.[6]

Factor disagreement. Specifically, we assume that each agent $j = 1, \ldots, J$ believes that the factor evolves according to

$$dz_t = \mu^j_{z,t} dt + \sigma_z dw^j_{z,t}, \quad \text{where} \quad \mu^j_{z,t} = \mu_z + \sigma_z \Delta^j_z s_t. \tag{6}$$

Here, Δ^j_z is a scalar that reflects whether agent j is optimistic ($\Delta^j_z > 0$) or pessimistic ($\Delta^j_z < 0$) about the factor. Moreover, and as we elaborate below, $s_t \in (0, 1)$ is a time-varying process that captures the fraction of total disagreement that is attributed to the factor at time t.

Since the investors observe z_t we must have

$$\underbrace{\mu^j_{z,t} dt + \sigma_z dw^j_{z,t}}_{\text{Agent } j\text{'s perception}} = \underbrace{\mu_z dt + \sigma_z dw_{z,t}}_{\text{Realization}}, \tag{7}$$

In other words, Equation (7) implies that each agent j attributes the observed variation in the factor to either the mean ($\mu^j_{z,t} dt$) or innovation ($\sigma_z dw^j_{z,t}$) components. This decomposition is determined by the degree to which the agent is either optimistic or pessimistic about the factor. Specifically, we can use the definition of $\mu^j_{z,t}$ from Equation (7) to relate the shock perceived

[6]In order to focus on the role of disagreement related to the securities market, we do not allow for any correlation between E_t and the dividends beyond the explicit dependence of the dividends on E_t. Introducing an additional correlation would not change our theoretical predictions.

by investor j to the true shock under the objective measure as

$$dw_{z,t}^{j} = dw_{z,t} - \Delta_{z}^{j} s_t dt. \tag{8}$$

There is a straightforward economic interpretation underlying Equation (8): if investor j is *optimistic* about the factor z_t, such that $\Delta_z^j > 0$, then the investor is likely to perceive a positive change in the factor as representing an increase in the expected component (since $\mu_{z,t}^j > \mu_z$) and a small (or even negative) innovation (since $dw_{z,t}^j < dw_{z,t}$). The converse holds true if the investor is *pessimistic* about the factor, i.e., when $\Delta_z^j < 0$.

Stock-specific disagreement. We also assume that agent j believes that the dynamics of the idiosyncratic component of stock n's dividends are

$$d\epsilon_{n,t} = \mu_{n,t}^{j} dt + \sigma_n dw_{n,t}^{j}, \qquad \text{where} \quad \mu_{n,t}^{j} = \mu_n + \sigma_n \Delta_n^{j} \left(1 - s_t\right). \tag{9}$$

Here, investor j is optimistic (pessimistic) about the dynamics of stock n if $\Delta_n^j > 0$ ($\Delta_n^j < 0$). Using the fact that all agents can observe $\epsilon_{n,t}$ and following the logic underlying Equation (8) allows us to express the degree to which agent j's perception of the idiosyncratic shock to stock n differs from the objective realization as

$$dw_{n,t}^{j} = dw_{n,t} - \Delta_n^{j}(1 - s_t)dt. \tag{10}$$

Intuitively, this type of disagreement allows agents to have different beliefs about the *relative* performance of the individual constituents of an index or ETF.

Composition of disagreement. In the above, s_t is a process that governs the composition of the disagreement in the economy. Specifically, we assume that

$$s_t = \frac{1}{1 + e^{-\delta_t}}, \quad \text{where} \;\; d\delta_t = \kappa_\delta \left(\bar{\delta} - \delta_t\right) dt + \sigma_\delta dw_{\delta,t}, \tag{11}$$

and $w_{\delta,t}$ is a standard Brownian motion that is independent of all other shocks. Since s_t is bounded between zero and and one, it can be interpreted as the amount of systematic versus idiosyncratic disagreement in the economy. As s_t approaches one, there is less disagreement about idiosyncratic processes $\epsilon_{n,t}$ and more disagreement about the factor process z_t.

To focus on the mix between factor and stock-specific disagreement, we have assumed a simple structural form for the beliefs of the investors in the economy. First, we keep the total disagreement constant. One could include an additional factor that changes the total level of disagreement in addition to the composition of the disagreement. Second, we assume that the composition effect is driven by an independent source of uncertainty. One could potentially generalize this by allowing the composition of disagreement to be correlated with the fundamental shocks in the economy.[7] One way to endogenize the belief structure is by assuming that agents learn or follow a specific updating rule. The outcome would in most cases lead to total disagreement varying over time and a composition effect, s_t, that correlates with the fundamental shocks to the dividends.

1.3 Model Intuition

To build intuition about the building blocks of the model and their link to our empirical results, in this section we present a concrete example of the mechanism. Our starting point is an economy with a single stock and two agents (a pessimist and optimist). For the purpose of

[7] An untabulated extension of the model shows that separating the factor and idiosyncratic aspects of disagreement and correlating disagreement with fundamentals shocks lead to similar results.

illustration and without loss of generality, we assume that the stock is expected to earn 6% per period under the *objective* probability measure. Investor O is an optimist who subjectively believes that the stock will yield 8% per period, whereas Investor P is a pessimist who subjectively believes that the stock will only yield 4% per period, i.e., the two agents are symmetric in their disagreement about the mean return. Both agents have equal wealth at time 0, but due to their disagreement, Investor O allocates a larger proportion of their wealth to the stock than Investor P.

If the stock returns a positive 10% in the next period, then the stock's price will increase for two reasons. First, the positive shock will increase the amount of dividends the firm pays, thereby increasing the firm's price. Second, and more importantly, the fact that the investors disagree about the value of the firm will amplify the price increase. The positive shock will cause Investor O to become wealthier than investor P due to their higher initial position in the stock. Thus, from a wealth-weighted perspective, the stock will become priced closer to Investor O's 8% expected return than Investor P's 4% expected return. The reverse, however, is also true. If the stock were to fall by 10%, then the stock's price would fall to reflect the (relatively wealthier) pessimist's view more than the (relatively poorer) optimist's view. This amplification effect on the stock's price from disagreement (i.e., higher highs after positive shocks; lower lows after negative) will lead to higher stock return volatility.

For the purpose of this example, we will refer to the situation described above as "scenario one," and sequentially incorporate the additional features of our model starting with (i) $N \to \infty$ stocks, (ii) then the common factor z_t, and finally (iii) the composition of disagreement s_t.

(i) $N \to \infty$ stocks and agents. Suppose the the economy is now populated by the same two investors but features $N \to \infty$ stocks with idiosyncratically varying dividends. If the investors' beliefs are randomly distributed across the stocks (i.e., each investor is optimistic about one subset of stocks, pessimistic about another subset, and neutral in regards to a third subset), then each agent's wealth is distributed across many stocks, and each agent is asymmetrically exposed to many of these stocks' orthogonal shocks. As a result, their exposure to any single shock will be extremely small. Thus, if the same 10% shock hits one stock in the economy, this will have a muted effect on an agent's wealth. This is because (i) agents have economically small exposures to each stock, and (ii) any positive shock to one stock is likely to be offset by a negative shock to another. This type of dispersed stock-specific disagreement effectively decreases the amplification effect described in the previous example with one stock.

If this economy were to also feature multiple agents, then we would obtain the same conclusion as the two-agent case above: each individual agent's wealth would be exposed to a large number of uncorrelated shocks, and any amplification effect of disagreement on return volatility would be low by the virtue of the agents' diversified holdings across the large number of securities. We refer to this situation as "scenario two."

(ii) Common factor z_t. The second feature we add is a common factor that drives part of the return variation across all individual stocks. When stock returns depend on a common factor, and agents are either optimistic or pessimistic only about the factor's prospects, then there are, in reality, far fewer dimensions along which agents disagree for this subset of stocks. This is because, in contrast to scenario two above, speculating on the common factor may have large wealth effects, since factor innovations induce groups of stocks to move in unison. Thus, as disagreement about the common factor increases, the economy is closer to the single-stock case described under scenario one above. The "single stock," however, is now a well-diversified portfolio of N stocks and moves in response to fluctuations in the factor (i.e., z_t). Thus, inter-stock correlations also rise when agents wish to speculate on factor exposures.

(iii) Composition of disagreement s_t. The final, and perhaps most novel, feature of our economy is that the share of disagreement can vary continuously between the two scenarios described above. This quantity, s_t, not only allocates the fixed amount of disagreement across agents between the idiosyncratic and factor components of returns but also elicits trading activity in our model.

When s_t is close to zero, then agents primarily disagree about stocks' idiosyncratic shocks, and the economy is similar to that described under scenario two. In contrast, when s_t is close to one, then disagreement across agents is occurring largely along the factor dimension and agents act in almost unison as either optimists or pessimists in regards to the factor. This mimics the single-stock economy represented by scenario one, where disagreement amplifies volatility.

The key is that in order to transition between the two extremes as s_t moves, the agents must reblance their portfolios. For example, when s_t is near one, agents want to hold the same well-diversified portfolio (diversifying away idiosyncratic risks), but to varying degrees depending on their level of optimism or pessimism. However, as s_t transitions to zero, then agents desire very disparate positions in individual stocks, reflecting their randomly distributed disagreement on idiosyncratic portions of returns. This rebalancing shows up as trading activity.

1.4 Asset-pricing moments

We are primarily interested in how disagreement s_t affects (i) the return dynamics of the ETF and (ii) the return correlations between pairs of individual stocks in the ETF. To analyze how disagreement affects these asset-pricing moments, we first need to define the relevant state variables. With these in hand, we can write the returns associated with each individual security and the ETF, which we interpret as a "passive" value-weighted index of the N individual stocks. Finally, given these asset returns, we can compute the volatilities and correlations of interest.

To start, the state variables in the economy are (i) the N dividend shares, (ii) the $J-1$ consumption shares, and (iii) the fraction of factor disagreement, s_t. We collectively define state variables (i) and (ii) as X_t. Since our focus is on the share of factor disagreement, we will generally examine how key equilibrium quantities, such as asset-pricing moments, evolve as a function of s_t. Next, we assume that the equilibrium price of stock n represents the claim to the stream of the firm's dividends. This allows us to express the price of stock n at time t as

$$P_{n,t} = P_n\left(X_t, s_t\right) = \mathbb{E}_t\left[\int_t^\infty \frac{M_u}{M_t} D_{n,u} du\right], \tag{12}$$

where M_t is the equilibrium stochastic discount factor under the objective belief (i.e., the probability measure of the true data generating process). An application of Ito's lemma to Equation (12) yields the return process for each stock.

$$dR_{n,t} = \frac{dP_{n,t} + D_{n,t}dt}{P_{n,t}} = \mu_{R_n,t}dt + \sigma'_{R_n,t}dw_t, \tag{13}$$

Here, $w_t = (w_{E,t}, w_{\delta,t}, w_{z,t}, w_{1,t}, \ldots, w_{N,t}) \in \mathbb{R}^{N+3}$ is a vector of all of the Brownian shocks in the economy. Hence, the equilibrium loading of stock n onto each of the $N+3$ shocks in the economy is given by the vector of diffusion coefficients $\sigma_{R_n,t} = \sigma_{R_n}\left(X_t, s_t\right) \in \mathbb{R}^{N+3}$. We collect these loadings in the matrix Σ, the n^{th} column of which is $\sigma_{R_n,t}$. Finally, we define $\omega_p \in \mathbb{R}^N$, where $\sum_{n=1}^N \omega_{p,n} = 1$ are the portfolio weights of an arbitrary portfolio p. In the special case that $\omega_{p,n} = P_{n,t}/\sum_{k=1}^N P_{k,t}$ for all n stocks, then ω_p corresponds to the ETF. This is because we define the ETF to represent the value-weighted portfolio of the N individual securities.

With this notation in hand, the instantaneous standard deviation of portfolio p is then

$$std\left(dR_{p,t}\right) = \sqrt{\omega_p'\Sigma'\Sigma\omega_p}. \tag{14}$$

Similarly, the instantaneous correlation of the returns of portfolios p and q is

$$corr(dR_{p,t}, dR_{q,t}) = \frac{\omega_p'\Sigma'\Sigma\omega_q}{std\left(dR_{p,t}\right) std\left(dR_{q,t}\right)}. \tag{15}$$

1.5 Factor exposure

Another goal is to understand how factor versus individual-stock disagreement impacts the trading of the investors in the economy. In a frictionless model such as ours, there are two key features that complicates the comparison to the data, and therefore require additional assumptions. First, the ETF is a redundant security in our setting.[8] Hence, there is no intrinsic demand for the ETF in the model since investors could, in principle, trade the individual stocks underlying the ETF. Therefore we assume that agents prefer to trade the ETF instead of the underlying stocks if the agents' goal is to take on factor exposure. The economic intuition underlying this argument is that in a model with even a small transaction cost for trading individual stocks, the investors would prefer buying the relatively cheap-to-trade ETF over the individual stocks to gain exposure to the common factor shock.

Second, trading volume is difficult to define and generally depends on the asset structure. For instance, if investors can trade claims that replicate their optimal consumption path, then trading volume is trivially zero. In contrast, if investors can only trade individual stocks, then trading volume will typically be non-zero. Instead of focusing on trading *volume*, we therefore focus on the total *exposure* of each agent to the factor. It is natural to assume that changes in an agent's exposure to the factor arise from the agent trading securities related to the factor. Thus, if we define the equilibrium wealth of investor j as $W_{j,t} = W_j\left(X_t, s_t\right)$, then Ito's lemma lets us express the investor's wealth dynamics as

$$dW_{j,t} = \ldots dt + \sigma'_{W_j,t} dw_t. \tag{16}$$

From equation (16), the exposure of agent j to each of the underlying shocks in the economy can be represented by the diffusion coefficients, $\sigma_{W_j,t} \in \mathbb{R}^{N+3}$. These exposures can either be

[8] The ETF is a value weighted portfolio of the individual stocks, and hence it can be replicated.

positive or negative. Since we are interested in the total amount of exposure (either long or short) to the factor, we define the total amount of factor exposure in this economy as

$$TE_{ETF,t} = \sum_{j=1}^{J} |\sigma_{W_j,z,t}|, \tag{17}$$

where $\sigma_{W_j,z,t}$ is the loading on the shock to the factor. While we employ the aforementioned equation as our primary measure of exposure to the factor shock, Section A.4 in the Online Appendix shows that the key model predictions defined below are robust to alternative measures of exposure and trading activity also.

1.6 Model predictions for volatility, correlation, and factor exposure

We obtain testable predictions from the model by conducting Monte Carlo simulations aimed at capturing the equilibrium relations between the degree of factor disagreement (s_t) and key quantities such as (i) total factor exposure, (ii) the risk of the ETF, and (iii) the average correlation (i.e., the diversification benefits) within the ETF.

Simulation details. Our main simulation of the model considers an economy in which there are $N = 10$ individual stocks and $J = 2N = 20$ agents. Our baseline analysis considers a "symmetric" economy in which each stock is followed by an equal number of pessimists and optimists. Moreover, we assume that N agents are optimistic (pessimistic) about the factor. While not necessary, this assumption ensures that the model's predictions are not simply driven by an ex ante imbalance between the proportions of optimists and pessimists in the economy. The key feature we need in regards to beliefs is that there is sufficient "dispersion" in the beliefs about the individual stocks. This ensures that stock-specific disagreement cannot be distilled into disagreement between two blocks of investors, thereby mimicking factor disagreement.

Overall, as half of the agents in the economy are optimistic about the factor and half are pessimistic about the factor, we refer to high s_t times as periods of high *factor disagreement*. Similarly, when s_t is low, most disagreement surrounds the idiosyncratic component of dividends. As such, we label these times as periods of high *stock-specific disagreement*. Moreover, we assume that each agent starts with the same initial consumption shares, and the initial dividend shares of the stocks are the same.

Simulation results. Figure 2 shows the exposures of the agents in the economy to the stock-specific and factor shocks (as defined in equation (17)). The left (right) plot shows the exposures to the shocks when $s_t = 0.05$ ($s_t = 0.95$) and represents the exposures in a state of low (high) factor disagreement. As the economy transitions to a state of higher factor exposure, the share of factor disagreement increases. In the middle plot, we show the factor exposure as a continuous function of the share of factor disagreement (s_t). The figure shows that as the economy moves from low to high factor disagreement, the positions of the agents become more concentrated on the factor shock and less concentrated on the individual shocks (stocks). Put differently, agents take large speculative bets on the aggregate stock portfolio (ETF) instead of the individual stocks as we have more factor disagreement.

In Figure 2, we plot the standard deviation of the ETF and the average stock market correlation of the individual stocks in the ETF as we move from stock-specific (low s_t) to factor disagreement (high s_t). As one can see, both the volatility of the ETF and the average correlation increases in the share of factor disagreement. The reason for this is that, as we move from stock-specific to factor disagreement, the economy looks more and more like an economy with bets between two large groups of investors who have opposing views on the factor. Such "correlated" bets have larger impacts on the aggregate economy than stock-specific disagreement, as bets on individual stocks tend to diversify in the cross-section.[9]

[9] One way to look at this is to consider a limiting version of our economy as the number of assets approaches infinity ($N \to \infty$). If agents are either optimists or pessimists for each individual stock, i.e., the perturbation of the belief is $+/-\,\Delta\,(1 - s_t)$ on each individual stock with equal probability, then the consumption shares only

Based on the figures and the intuition developed in Section 1.3 we have the following three model predictions:

Testable Predictions. *Increased factor disagreement (higher s_t) leads to:*

(a) Larger flows into the ETF (i.e., more common factor exposure);

(b) Higher ETF-level return volatility;

(c) Higher average correlations between the stock returns of securities in the ETF (i.e., lower diversification benefits).

2 Empirical evidence

This section describes the data and empirical measures used to evaluate the three predictions of the model outlined in Section 1. Section 2.1 motivates the economic connection between ETFs and our theory. Section 2.2 provides an overview of the set of ETFs we use to test the relations among disagreement, fund flows, and risk-neutral volatility and correlation, while Section 2.3 describes our empirical measures of factor and stock-specific disagreement regarding an ETF. Sections 2.4 through 2.6 then use these measures to test our predictions.

2.1 Institutional details

The previous section introduced the key mechanism underlying our model: agents have the ability to trade on a combination of their stock-specific beliefs (i.e., the idiosyncratic component of returns) and their factor beliefs (i.e., the common component of returns). Suppose that J investors are interested in technology stocks. On the one hand, if these J investors have very

depend on factor disagreement.

different beliefs about individual stocks, then they will hold disparate positions in individual firms, e.g., Apple, Microsoft, and Tesla. On the other hand, if the agents have a strong desire to speculate on the technology sector as a whole by purchasing a portfolio of technology stocks, e.g., the Nasdaq 100, then these J agents will act as if there are only effectively two agents in the economy: one that is optimistic about the technology "factor" and another that is pessimistic about it.

In reality, the transition from a state in which agents are only trading the individual technology stocks to one in which they are trading the Nasdaq 100 index can be executed in multiple ways. The first is the approach presented above, i.e., trading individual firms such that agents eventually hold a value-weighted portfolio of all of the stocks underlying the Nasdaq 100 index. A second approach is to purchase Nasdaq futures, and a third approach is to purchase a Nasdaq ETF, such as QQQ. The mere existence of these alternatives for gaining factor exposure reflects the fact that agents not only desire this kind of exposure, but are also subject to frictions that these alternatives likely mitigate (see, e.g., Ross (2015, 1976) on the issue of non-redundancy).

Most papers in the ETF literature assume that the primary friction is a participation cost for uninformed traders (see, e.g., Bond and Garcia, 2022). This ignores evidence, however, that informed traders also use ETFs (and, of course, futures) in their trading activity (see, e.g., Huang et al., 2021). Based on this evidence we propose an alternative friction that affects both retail and sophisticated traders alike: the cost of purchasing or shorting a basket of many securities versus purchasing or shorting an ETF or future. Take, for example, a large fundamental hedge fund that would like to purchase \$100m of the Nasdaq 100. Given that their value add to investors comes from understanding stocks' fundamental values, not from superior trade execution, trading via an index product (e.g., an ETF) would be considerably more efficient than routing multiple stock orders through a relatively expensive program trading platform, such as one run by Goldman Sachs.

From this perspective, the creation and redemption activity of ETFs is a natural measure of changing factor demand that we link to changes in disagreement. We focus on ETF trading activity as our main measure of the demand for the underlying factors, since ETFs cover a wide variety of investment themes, styles, and factors, and span a relatively long time period. As Box et al. (2021) highlight, a large proportion of ETF trading volume stems from fundamental demands of investors. We therefore also test our main hypothesis using other measures of exposure and trading activity. For instance, and building upon the similarities between ETFs and futures, we analyze the relationship between disagreement and futures open interest.[10] This analysis lends additional credence to our proposed mechanism.

Finally, a natural question is why we are focused on ETFs and not passive, indexed mutual funds. First, and perhaps most importantly, mutual funds do not have timely pricing data, specifically on options. This inhibits our ability to test the predicted relations between disagreement, and forward-looking measures of volatility and correlations. Second, mutual fund holdings data are reported quarterly. In contrast, data on ETF constituents, ETF flows, and ETF-options prices are available at a much higher frequency, i.e., daily. This timely availability of data is important as many of the relationships we find regarding diversification benefits last approximately six months.

2.2 Data and summary statistics

Our sample begins in January 2012, which is the first month in which ETF Global began providing granular and comprehensive data on ETF flows and constituents, and ends in December

[10]Unlike forwards, futures are standardized and have well known and transparent no-arbitrage relationships to their underlying cash securities. Program trading operations at investment banks have for decades imposed this future-cash relationship through their trading operations. ETF issuers have similarly clear rules about how to maintain the ETF versus underlying asset relationship. It should thus come as no surprise that it is these same investment banks that are usually those authorized to execute these rules, i.e., as authorized participants. See Evans et al. (2022) for a more detailed explanation of the role arbitrage and market makers play in maintaining this relationship.

2020. As the top panel of Figure 3 shows, ETFs are a relatively nascent security that only began trading in 1993 with the introduction of the SPDR S&P 500 Trust ETF (NYSE: SPY). While ETF trading volumes represented less than 5% of total dollar trading volume in the 1990s, the ETF market has come to represent approximately 25% to 30% of total dollar trading volume since 2010. This rapid increase in popularity reflects, in large part, the fact that ETFs provide investors with relatively cheap access to a wide variety of investment factors and styles. Beyond the fact that granular data on ETF holdings are only available beginning in 2012, there is an additional benefit of starting our sample period at this point in time: our theoretical analyses assumes that the dynamics of disagreement, and consequently flows into and out of an ETF, are stationary. It would appear that ETF trading activity has achieved a stable equilibrium in the time period underlying our analyses.

With these benefits of ETFs in mind, our analysis focuses on a small set of highly liquid US-focused equity ETFs for which options on both the ETF and its constituent stocks are actively traded. This focus allows us to elicit accurate measures of the forward-looking volatility and correlation risk associated with the investment factor, theme, or style that the ETF tracks. The 13 ETFs in our sample are SPY (SPDR S&P 500 ETF Trust), DIA (SPDR Dow Jones Industrial Average ETF Trust), QQQ (Invesco QQQ Trust Series 1), XLK (Technology Select Sector SPDR Fund), XLB (Materials Select Sector SPDR), XLE (Energy Select Sector SPDR), XLI (Industrial Select Sector SPDR), XLP (Consumer Staples Select Sector SPDR), XLV (Health Care Select Sector SPDR), XLY (Consumer Discretionary Select Sector SPDR), XOP (SPDR S&P Oil & Gas Exploration & Production ETF), XBI (SPDR S&P Biotech ETF), and IBB (iShares Biotechnology ETF).

Although these 13 ETFs represent only a small number of the approximately 2,200 distinct ETFs that are now trading in U.S. markets, the bottom panel of Figure 3 demonstrates that these ETFs represent just under half of all dollar trading volume in US ETFs in the recent decade. Moreover, these ETFs represent a variety of investment styles. Three ETFs track broad market indicies, while 10 track many of the various sectors underlying the U.S. economy. Thus, our sample represents an economically sizable portion of the US equity market.[11]

Table 1 reports a number of summary statistics related to the ETFs that comprise our sample. For instance, the table shows that the largest ETF in our sample is SPY, which has a net asset value (NAV) of \$227.23b. The smallest ETF in our sample is XOP, the Oil & Gas Exploration & Production ETF, with a net asset value of \$44.90b. Beyond showing relatively large differences in NAVs across ETFs, the table also shows large differences in the market capitalizations of the equities underlying these ETFs. For instance, the biotech (healthcare) firms underlying XBI (XLV) have a combined market value of \$710.31b (\$2795.04b).

ETF trading activity. We measure the *relative* trading activity associated with each ETF in one of two ways. First, we define the net flow into ETF m in month t as

$$\mathrm{NFlow}_{m,t} = \sum_{\tau=1}^{T_t} \mathrm{NetFlow}_{m,t,\tau} \Bigg/ \sum_{j=1}^{J} \mathrm{ME}_{m,j,t}. \tag{18}$$

Here, $\mathrm{NetFlow}_{m,t,\tau}$ represents the net flow into ETF m on trading day τ of month t (expressed in dollars and from ETF Global), and T_t captures the total number of trading days in month t. To gain a sense of the economic magnitude of these monthly net flows, and to make flows comparable across ETFs with different market capitalizations, we scale the net flows by the aggregate market capitalization of the J stocks underlying ETF m at the end of month t. The economic intuition behind this scaling is that flows into and out of an ETF should only affect

[11]While a number of other economically large ETFs exist, they are excluded from our sample because they feature very little option-trading activity at the index level. For instance, while both VOO (Vanguard S&P 500 ETF) and IVV (iShares Core S&P 500 ETF) are two ETFs that have net asset values in excess of \$200b there are typically fewer than 1,000 options linked to either VOO or IVV traded each day. In contrast, hundreds of thousands of options linked to SPY are typically traded each day. These differences in options-trading volume at the index level imply that we can elicit risk-neutral moments for SPY, but we are unable to elicit the same moments for VOO and IVV.

the risks of the ETF and its constituent securities if these flows are large relative to the size of the underlying stocks.

Second, we complement the previous measure with the dollar-trading volume in ETF m in month t relative to the contemporaneous dollar-trading volume in the ETF's constituent stocks:

$$\text{DRVol}_{m,t} = \sum_{\tau=1}^{T_t} \text{DVol}_{m,t,\tau} \Bigg/ \left(\sum_{j=1}^{J} \sum_{\tau=1}^{T_t} \text{DVol}_{m,j,t,\tau} - \sum_{\tau=1}^{T_t} \text{NetFlow}_{m,t,\tau} \right). \tag{19}$$

Here, $\text{DVol}_{m,t,\tau}$ represents the dollar trading volume associated with ETF m on trading day τ of month t, and $\text{DVol}_{m,j,t,\tau}$ denotes the dollar trading volume associated with constituent j of ETF m on trading day τ of month t. All other variables follow the same definition as those in equation (18). We subtract daily net flows from the dollar-trading volume of the constituent stocks so as to avoid double counting any trading activity in the individual stocks that arises due to investors also trading the ETF.[12] The economic intuition underlying equation (19) is that the value of $\text{DRVol}_{m,t}$ will be higher when market participants have a greater demand for exposure to the common factor provided by the ETF rather than the idiosyncratic exposures of its constituent stocks.

Table 1 reports the average value of the absolute net flows into each ETF over the average month of the sample period, as well as the average dollar-trading volume in each ETF and its constituent stocks. The table shows that approximately \$30b flows into and out of SPY each month, and around \$1b moves into and out of XBI and XLB, the biotech and materials ETFs. While the nominal value of these latter flows is an order of magnitude smaller than the flow for SPY, they represent similar magnitudes relative to the aggregate values of the underlying equities held by each of these three ETFs. Similarly, the table shows that there is, on average, 10 to 20 times as much dollar-trading volume in individual stocks relative to an ETF.

[12]In untabulated results we show that our findings are robust to not making this small adjustment to the denominator.

Other summary statistics. Beyond the summary statistics outlined above, Table 1 also reports the average number of analysts following the average firm underlying each ETF, or $\mathbb{E}\left[\text{Analysts}\right]$. The table shows that the ETFs are well balanced in terms of their analyst coverage, as most underlying stocks are followed by an average of 15 analysts. This fact is useful, as Section 2.3 uses data related to analyst forecasts to construct measures of ETF-level disagreement, which maps to the notion of disagreement in the model. Unreported summary statistics also indicate that almost all of the individual stocks underlying these ETFs are optioned – a fact that we exploit later in this section when we use the options market to estimate forward-looking measures of volatility and correlation risk for each ETF in the sample.

2.3 Measuring systematic and idiosyncratic disagreement

Our empirical analysis requires measures of factor and stock-specific disagreement for each ETF. This allows us to empirically determine the extent to which s_t in equation (11) is closer to zero (more stock-specific disagreement) or one (more factor disagreement). Following Buraschi et al. (2014), we estimate these measures using IBES data. IBES captures quarterly earnings estimates from *informed* Wall Street analysts; measures generated from these data are therefore particularly useful in highlighting how our results are driven by broader fundamentals (i.e., heterogenous beliefs) rather than dynamics from largely *uninformed* retail beliefs or flows.

For the stock-specific disagreement measure, we first calculate the mean absolute value of next period's earnings estimates (Est) for each stock across all analysts. These estimates are subject to the standard filters applied to the unadjusted IBES data file. For instance, we remove all analyst revisions reported after a firm's earnings announcement date. Moreover, we choose each analyst's most recent estimate, removing observations where we know analysts coverage has stopped or where IBES has recommended removal through their own proprietary analysis.

Finally, we remove stale information by deleting forecasts that are outstanding for more than 180 days. The measure of stock-specific disagreement surrounding the stocks underlying ETF m at time t, denoted by $\text{StockDisagree}_{m,t}$, is then the weighted sum of the individual security disagreement measures across all stocks in the ETF, or

$$\text{StockDisagree}_{m,t} = \sum_{j=1}^{J} w_{j,m,t} \cdot \left[\frac{\frac{1}{A}\sum_{a=1}^{A} |\text{Est}_{a,j,t} - \overline{\text{Est}}_{j,t}|}{|\overline{\text{Est}}_{j,t}|} \right]. \tag{20}$$

Here, $\text{Est}_{a,j,t}$ is the earnings estimate of analyst a for stock j at time t, $\overline{\text{Est}}_{j,t}$ is the average earnings estimate across all analysts for a given security at time t, and $w_{j,m,t}$ is the weight of stock j in ETF m at time t, drawn from ETF Global. Since these weights come from ETF Global, they typically represent the relative market capitalization of the security in the ETF.

The measure of factor disagreement for ETF m at time t, denoted by $\text{FactorDisagree}_{m,t}$, is constructed by first summing the forecasts of all component securities of an index across all analysts employed by a broker. This "bottom up" approach mimics the methodology used by macroeconomic groups at brokerage firms when estimating earnings for the S&P 500 and other indexes (e.g., Darrough and Russell (2002)). We then compute the disagreement across brokers rather than analysts in the sample,

$$\text{FactorDisagree}_{m,t} = \frac{\frac{1}{B}\sum_{b=1}^{B} |\sum_{j=1}^{J} \text{Est}_{b,j,t} - \overline{\text{Est}}_{m,t}|}{|\overline{\text{Est}}_{m,t}|}, \tag{21}$$

where B is the total number of brokers in the sample, $\text{Est}_{b,j,t}$ is the earnings estimate of broker b for security j, and $\overline{\text{Est}}_{m,t}$ is the equal-weighted average earnings estimate for ETF m across all brokers. Intuitively, this measure captures the extent to brokerage firms disagree about the valuation of an index rather than the degree to which analysts disagree about the valuation of an individual constituent of the index. Since the weights in our sample are based on the market capitalization of security j versus total market capitalization of all stocks in ETF m, our estimate of $\text{Est}_{b,j,t}$ is simply broker b's earnings-per-share forecast from IBES multiplied by the shares outstanding from Compustat.

In constructing this measure of factor disagreement, we find that individuals brokers do not necessarily cover all securities in a given ETF. For instance, while almost all brokers cover stocks with large market capitalizations, only some brokers cover stocks with smaller market capitalizations. The dropoff in coverage is not linear, and falls precipitously from over 80% of market capitalization for the top 10 brokers to less than 30% for those outside of the top 10. With these limitations in mind, we apply two filters to the data. First, we restrict our attention to the disagreement among the top 10 brokers in the sample. Second, if a given broker b does not cover a particular stock j at time t, we assume that the broker's estimate for the earnings-per-share of that stock corresponds to the consensus estimate for that security's expected earnings. Since smaller stocks are less covered by analysts, but also have smaller weights in a given ETF, this imputation has a minimal effect on our results.

To gauge the *relative* importance of factor disagreement for a given ETF m in a given month t, we define the share of factor disagreement $s_{m,t}$ as

$$s_{m,t} = \frac{\text{FactorDisagree}_{m,t}}{\text{FactorDisagree}_{m,t} + \text{StockDisagree}_{m,t}}. \tag{22}$$

When this ratio $s_{m,t}$ approaches one, then the majority of the disagreement regarding the prospects of an ETF is related to the prospects of the underlying factor that the ETF tracks. In contrast, when $s_{m,t}$ approaches zero, then the majority of the disagreement surrounding an ETF is driven by the (idiosyncratic) prospects of the stocks that constitute the ETF. As such, $s_{m,t}$ provides us with an empirical measure of s_t from equation (11), which governs the amount of disagreement about the dynamics of a stock's idiosyncratic shocks (when $s_t \to 0$) and the amount of disagreement about the common factor (when $s_t \to 1$).[13] We plot the time-series dynamics of $s_{m,t}$ for SPY in Figure 4.

Figure A.1.1 in the Online Appendix plots the values of stock-specific (or idiosyncratic) and factor disagreement for SPY, defined following equations (20) and (21). The majority of disagreement related to SPY is driven by the idiosyncratic prospects of the constituent stocks of that ETF. However, factor disagreement exceeds stock-specific disagreement in the period surrounding the 2013 debt-ceiling crisis, before the 2016 presidential election, and upon the onset of the economic effects of the COVID-19 pandemic that began in early 2020.[14]

2.4 Disagreement and flows

This section shows that increases in disagreement about the common factor underlying each ETF (i) predict an increase in ETF inflows and (ii) are related to increased ETF trading activity. That is, when investors face more disagreement about the common component of an investment style rather than stock-specific disagreement about the constituent stocks, then they would rather trade the portfolio than trade the constituent stocks. This supports the first key prediction of the model in Section 1.6 that posits that higher levels of factor disagreement (i.e., higher values of s_t) predict an increased demand for factor exposure, as shown in Figure 1. Empirically this demand will manifest itself in both greater assets dedicated to an ETF, implying flows into (i.e., the creation of) the ETF, and greater ETF versus underlying trading activity.

We thus examine the relation between the relative amount of factor disagreement and the

[13] Section A.5 in the Online Appendix shows that this empirical method for measuring the relative amount of factor disagreement maps to the notion of the share of disagreement in our theoretical model.

[14] We provide summary statistics for this relative disagreement measure in Table A.1.1 of the Online Appendix, and report the correlation between these measures of relative disagreement across each pair of ETFs in Table A.1.2 of the Online Appendix.

relative amount of trading in the ETF by estimating the following panel regression:

$$\text{TActivity}_{m,\tau} = \alpha_m + \delta_t + \beta_2 \text{s}_{m,t} + \boldsymbol{\beta}\boldsymbol{X}_{m,t-1}^T + \varepsilon_{m,t}. \tag{23}$$

Here, $\text{TActivity}_{m,\tau}$ represents either the net flow (see Equation (18)) into ETF m at month $t+1$ or dollar trading volume (see Equation (19)) in ETF m in month t. $\text{s}_{m,t}$ corresponds to the relative amount of factor disagreement regarding ETF m from equation (22), while $\boldsymbol{X}_{m,t-1}$ represents a vector of control variables, and includes one-month ETF returns, absolute returns, and lagged flows, as flows in month t may simply arise from investors chasing high returns in month t (Dannhauser and Pontiff, 2019). Similarly, this vector includes the average bid-ask spread of the ETF, as investors are less likely to invest in ETFs with larger transactions costs.

To be consistent with our theory, we also include ETF and time fixed effects, denoted by δ_t and α_m, respectively. Time fixed effects absorb common shocks that affect all ETFs simultaneously (e.g., the effect of the Tax Cuts and Jobs Act of 2017, which triggered an inflow of funds into the equity market), while the ETF fixed effects absorb fixed differences in the level of flows and disagreement across ETFs, e.g., the fact that flows are unconditionally higher for SPY compared to XBI, the biotechnology sector ETF. Standard errors are clustered by time, all regressions are estimated at the monthly frequency (see, e.g., Buraschi et al. (2014)), and both our independent and dependent variables are standardized.

Recall from Equations (18) and (19) that we scale the trading activity of the ETF by the market capitalization and dollar-trading volume of the underlying securities, respectively, to make these measures comparable across ETFs of different sizes. As ETF flows are much smaller in magnitude than the market capitalizations and trading volumes of the underlying securities (recall the summary statistics in Table 1), our flow measures tend to take on very small values. Consequently, standardizing these variables allows us to report estimates of the impacts of disagreement on flows that are interpreted as the effects of standard deviation changes. Finally, while our model is in continuous time — i.e., all relationships between variables of interest are instantaneous — our empirical setting is in discrete time. We therefore run predictive regressions when *flow* ($\mathrm{NFlow}_{m,t}$) is the dependent variable and contemporaneous regressions when volume *level* ($\mathrm{DRVol}_{m,t}$) is the dependent variable.

Table 2 reports the results of these panel regressions. Focusing on net flows in Panel A, column (1) shows that without including any ETF and time fixed effects or controls, relatively higher amounts of factor disagreement are associated with increases in trade flows (i.e., the creation of ETF units). A one-standard-deviation higher amount of factor disagreement leads to an 0.11-standard-deviation higher flow into the ETF the following month. Moreover, column (2) shows that the same result holds true if we control for time fixed effects. Finally, column (3) shows that controlling for lagged flows, past returns, and trading costs does not alter this result. In particular, while the coefficient on lagged flows is negative (indicating that there is qualitative evidence of mean reversion in flows) and the coefficient on past returns is positive (suggesting some of the high flows in month t may be driven by return-chasing investors), neither of these coefficients is statistically significant. There is, however, a negative (positive) and significant relation between bid-ask spreads (absolute returns) and trading flows. This indicates that net flows are lower in ETFs with larger trading costs and net flows are higher in fund with more volatile returns.

Focusing on Panel B, which repeats the previous set of regressions using the monthly dollar-trading volume in the ETF relative to the dollar-trading volume in the underlying securities (i.e., $\mathrm{DRVol}_{m,t}$) as the dependent variable, yields a similar results to Panel A. Notably, the results show a positive association between the amount of factor disagreement and the degree to which investors trade the ETF versus its underlying securities in month t. Columns (4) and (5) show that a one-standard-deviation increase in factor disagreement is associated with an approximately 0.26-standard-deviation increase in the amount of trading in the ETF relative to the underlying stocks. While including both ETF fixed effects and lagged dollar-trading volume in column (6) moderates the impact of factor disagreement on dollar-trading volume due to the persistent nature of relative trading activity, the association between factor disagreement and trading activity remains positive and significant.

Combined, the tests in Panels A and B validate the intuition developed in our theoretical model. As disagreement increases in an individual firm's factor versus idiosyncratic risk, investors trade the factor exposure more aggressively. Additionally, dynamics in factor disagreement drive flow into the ETF and correspondingly out of the individual securities, consistent with our model's first prediction.

Finally, we note that the positive association between factor disagreement and flows is the opposite to that predicted by Huang et al. (2021) and Antoniou et al. (2022). In their frameworks, institutional investors use ETFs as a means to hedge the systematic risk exposures of firms, thereby isolating their investments to only the idiosyncratic components. If (i) the supply of ETFs to borrow were limited and (ii) speculative demand for shorts exogenously increased additional units of the ETF would need to be created to satisfy the hedging motive. After all, at the margin, every short position in the ETF must be accompanied by a corresponding long position in the same security. This would result in flows into the ETF as a result of higher *idiosyncratic* disagreement — the opposite of what we find. To be clear, our results are not inconsistent with the notion that institutional investors deploy ETFs in the manner suggested by these papers. Rather, our contribution is to provide a coherent economic explanation for the positive relationship between factor disagreement and ETF flows in the data.

Index Futures. While we use ETFs as a laboratory to test for a positive association between the share of factor disagreement and the demand for factor exposure, this prediction is not confined to just ETFs and should hold true in periods predating 2012. Testing the implications of our model in other asset classes is generally difficult due to a lack of available data; however, in Panel C of Table 2 we show that the same relation holds true with respect to the open interest in the index futures market (see Bessembinder et al., 1996, for a similar interpretation of futures open interest as a measure of disagreement).

Following Hong and Yogo (2012), we compute open interest in futures across all index contracts given on the last trading day for each month from the Commodity Futures Trading Commission's (CFTC) *Commitments of Traders in Commodity Futures* data and use that as our dependent variable ($\text{TActivity}_{m,t}$) in equation (23).[15] To our point of limited data, there are only three indices that have liquid futures over a long horizon – the Nasdaq, Dow Jones, and S&P 500. These correspond closely to the QQQ, DIA, and SPY ETFs, respectively.

Although we are restricted to this narrower cross-section, futures data has a much longer time series than ETF data: the S&P 500 runs from 1982, and the Dow Jones and Nasdaq samples run from 1997. Trading, however, in the first few years for each index has a clear upward time trend, similar to ETFs before 2012 (see Figure 3). We therefore maintain a balanced panel by starting our sample in January 2000. In addition, the CFTC computes separate open interest across different contract multipliers for a given index. For example, the original S&P 500 contract, with a multiplier of 250, and the E-Mini S&P 500 contract, with a multiplier of 50, are separate line items for a given month t. As the price level of the index has increased, trading has slowly shifted from the higher to lower contract multipliers. This transition to lower contract multipliers is true for all three indices. We are careful to maintain a common multiplier across contracts when summing open interest.

Table 2 present the results using futures interest in columns (7) through (9). As in columns (1) through (6), both the dependent and independent variables are standardized. A one-standard-deviation higher disagreement measure corresponds to a 0.17-standard-deviation higher

[15]See "Commitments of Traders" section at `https://www.cftc.gov/data`.

futures open interest. This is robust to the addition of date fixed effects. In column (9) we add index fixed effects and lagged futures open interest as controls. Given that our open interest measure crosses many expiry dates and contract multipliers, we do not include bid-ask spreads or returns as controls. For this specification a one-standard-deviation increase in disagreement relates to a more than 0.09-standard-deviation higher future open interest. These results further validate the notion that increases in factor disagreement translate into greater demand for exposure to the underlying factor.

Robustness. The analyses above show that there is a positive and statistically robust relation between relative disagreement and trading activity, measured in three distinct ways. We conduct two additional robustness checks to verify the economic significance of this result. First, Table A.1.3 in the Online Appendix shows that our core results are not simply a manifestation of indexation. The table includes a dummy variable that identifies the effects of disagreement for the two ETFs in our sample that track common indexes – the S&P 500 and the Nasdaq. The results show that our results are not concentrated within these commonly traded index ETFs. Second, we re-estimate equation (23) after replacing the *relative* disagreement measure from equation (22) with the level of *factor* disagreement from equation (21). The results in Table A.1.4 in the Online Appendix show that the positive and statistically significant relation between disagreement and trading activity remains, and ensures that our results are not simply a manifestation of the scaling inherent in the definition of *relative* disagreement, $s_{m,t}$.

2.5 Volatility and disagreement

Having shown that disagreement drives flows into ETFs, as predicted by Figure 1, this section documents that disagreement also drives an increase in ETF-level volatility. This confirms prediction (b) from Section 1.6, as outlined in the left panel of Figure 2. We establish this relation by showing that increases in disagreement surrounding the systematic component of ETF earnings drive increases in forward-looking ETF-level risk.

Measuring volatility risk. We first construct measures of ETF-level risk using options data, where forward-looking (risk-neutral) measures of volatility risk are readily available. We follow Bakshi et al. (2003) to construct ETF-level measures of return variation using options data on a given day t. The price of a τ-maturity security that pays the quadratic return is

$$V(t,\tau) = \int_{P(t)}^{\infty} \frac{2\left[1 - \ln(K/P(t))\right]}{K^2} \cdot \text{Call}\,(t,\tau;K)\,dK + \int_{-\infty}^{P(t)} \frac{2\left[1 + \ln(K/P(t))\right]}{K^2} \cdot \text{Put}\,(t,\tau;K)\,dK, \tag{24}$$

where K is the strike of either a call ($\text{Call}\,(t,\tau;K)$) or a put ($\text{Put}\,(t,\tau;K)$) option with maturity date $t+\tau$, and $P(t)$ is the current stock price. Next, we calculate the risk-neutral variance as

$$VAR^{Q}(t,\tau) = \exp^{r\tau} V(t,\tau) - \mu_t(\tau)^2. \tag{25}$$

Here, r represents the risk-free rate (drawn from OptionMetrics) and $\mu_t(\tau)$ following the definition from Equation (39) of Bakshi et al. (2003). Finally, we convert the daily risk-neutral *variance* into an annualized risk-neutral volatility, denoted by $\sigma^{Q}_{m,t,\tau}$, by taking the square root of equation (25) and scaling the resulting quantity by $\sqrt{252}$.

Equation (25) captures the idea that investors' exposure to a squared return contract is a function of the probability-weighted expected return squared across *all* possible share price values. One can back out these values from an infinite string of options in the positive and negative return domains by using call and put options, respectively. Following Buss and Vilkov (2012), we estimate equation (25) for a given ETF over a discretized grid of moneyness (K/S values from 0.33 to 3 by increments of 0.01), using the annual duration volatility surface files from OptionMetrics.

It is important to note that we use a variety of maturities when we compute these forward-looking measures of risk. Many of the maturities we consider are long dated (i.e., we consider options that expire 30 to 365 days in the future). This differentiates our work from other studies that examine the impact of ETF ownership on intra-ETF correlations and variance and that typically focus on higher frequency estimates of risk using realized returns (see, e.g., Ben-David et al., 2018; Da and Shive, 2018). Longer-dated options allow us to measure *changes* in expectations of the risk and diversification benefits (i.e., correlation) of an ETF, as well as changes in perceptions of risk. Existing studies characterize the relationship between ETF flow and asset prices as being short-lived and mean-reverting in nature. Consequently, any result we find would be expected to be long-dated due to the extended duration of the options we use.

Figure 5 shows the time series of risk-neutral volatility estimated using equation (25) for the most prominent ETF in our sample: SPY. In particular, the figure computes the risk-neutral volatility of SPY in two ways. First, equation (25) is estimated using options written on SPY itself (the dashed blue line in the figure). Second, we compute the risk-neutral volatility of each individual firm within SPY, and then take the weighted sum of these risk-neutral volatilities across all pairs of firms in the index, where these volatilities are weighted by the importance of a given stock in a given ETF on on a given trading day t as reported by ETF Global.

This procedure, which implicitly assumes that the returns of all pairs of stocks in a given ETF are perfectly positively correlated, results in the dashed red line shown in the figure. While these two approaches for calculating the risk-neutral volatility of an ETF are highly correlated (for instance, each measure of volatility increases surrounding the 2016 presidential election and the onset of the recession induced by the COVID-19 virus), the wedge between the two lines suggests that investors' forward-looking perceptions of the intra-ETF correlation are less than one (i.e., investors do not believe the stocks within the S&P 500 are perfectly positively correlated) and vary over time.

Following the intuition underlying our economic model in Section 1, we implement a regression analysis to examine whether disagreement regarding the relative contribution of the systematic risk of stock returns also drives (part of) the cross-sectional differences in the forward-looking volatility of an ETF in a given month. Section 2.6 then explores whether disagreement is associated with the intra-ETF correlation between stocks, as our model also predicts.

Regression analysis. We examine the relation between the relative importance of factor disagreement and ETF-level volatility risk by estimating the following panel regression:

$$\sigma^{Q}_{m,t,\tau} = \alpha_m + \delta_t + \beta_2 s_{m,t} + \boldsymbol{\beta}\boldsymbol{X}^{T}_{m,t-1} + \varepsilon_{m,t}. \tag{26}$$

As in regression (23), $\mathrm{s}_{m,t}$ corresponds to the relative amount of factor disagreement regarding ETF m, as defined in equation (22). When estimating this equation, we measure the risk-neutral volatility of ETF m at time t by focusing on options with $\tau \in \{30, 91, 182, 273, 365\}$ days to maturity. The controls we use are the exact same as in regression (23), and include net flows, ETF returns, and bid-ask spreads. We also include ETF and time fixed effects, denoted by δ_t and α_m, respectively, and we standardized variables so that each point estimate can be interpreted as the effect of a one-standard-deviation change in the variable of interest. The coefficient of interest is that on the relative disagreement measure. Similar to Table 2, we expect β_2 to be positive, as increases in factor disagreement would lead to a higher variance of the ETF overall.

Table 3 reports the results of these panel regressions. Each column shows that increases in the systematic portion of disagreement result in higher ETF-level volatility, regardless of the time-to-maturity of the underlying options. The point estimates indicate that a one- standard-deviation increase in the systematic portion of disagreement raises forward-looking volatility by between 0.10-standard-deviations (when using 91-day-to-maturity options) and 0.12 standard deviations (when using 365-day-to-maturity options). Each of these point estimates is statistically significant at the 1% level. Altogether, these tests validate the intuition developed in our theoretical model (recall Section 1.3 and Figure 2): as disagreement in the factor increases across agents, volatility increases by a statistically significant and economically large amount.

2.6 Intra-ETF correlation risk

The relationship between ETF flows and factor volatility is partially driven by changing correlations between the stocks composing the factor. In the context of our risk-neutral measure of implied volatility, one would therefore assume that correlation *risk* is related to disagreement. In this section we empirically test and confirm this prediction.

Measuring correlation risk. To construct a measure of the forward-looking diversification benefits of holding an ETF, we start by considering the definition of portfolio variance for ETF m on day t using risk-neutral measures of volatility constructed from options with τ days to maturity

$$\left(\sigma_{m,t,\tau}^{Q}\right)^{2}=\sum_{i=1}^{N}\sum_{j=1}^{N}w_{i,m,t}w_{j,m,t}\sigma_{i,t,\tau}^{Q}\sigma_{j,t,\tau}^{Q}\rho_{i,j,t,\tau}^{Q}. \tag{27}$$

Here, $\sigma_{m,t,\tau}^{Q}$ denotes the risk-neutral volatility of ETF m, $\sigma_{i,t,\tau}^{Q}$ is the risk-neutral volatility of security i, $w_{i,m,t}$ denotes the weight of security i in ETF m, obtained from ETF Global, and $\rho_{i,j,t,\tau}^{Q}$ represents the risk-neutral correlation between stocks i and j on day t with τ days to maturity. We obtain the risk-neutral volatility of both the index and the constituent stocks via equation (25).

While exchange-traded options prices provide us with readily observable estimates of $\sigma_{m,t,\tau}^{Q}$ and $\sigma_{i,t,\tau}^{Q}$, the options market does not provide us with exchange-traded claims that deliver the correlation between a pair of securities at maturity. Consequently, we obtain the *average* risk-neutral correlation between the pairs of securities that constitute ETF m on day t by "inverting" equation (27) for the average value of $\rho^Q_{i,j,t,\tau}$. That is, we define

$$\overline{\rho}^Q_{m,t,\tau} = \frac{\left(\sigma^Q_{m,t,\tau}\right)^2}{\sum_{i=1}^N \sum_{j=1}^N w_{i,m,t} w_{j,m,t} \sigma^Q_{i,t,\tau} \sigma^Q_{j,t,\tau}}. \tag{28}$$

as the risk-neutral correlation of stocks contained in ETF m at time t, calculated using options with τ days to maturity. This measure essentially reflects the time-varying wedge between the measures of index-level volatility computed using index-level and individual stock options in Figure 5.

Figure 6 displays the monthly time-series variation for the average $\tau = \{30, 91, 182, 273, 365\}$-day risk-neutral correlations ($\overline{\rho}^Q_{m,t,\tau}$) underlying two prominent ETFs in the sample: SPY and XOP, an ETF that tracks stocks involved in gas and oil expoloration and production. There are four key takeaways from this figure. First, the estimated values of $\overline{\rho}^Q_{m,t,\tau}$ clearly satisfy the requirement that $\rho^Q_{m,t,} \in [0, 1]$. Second, the figure shows that these risk-neutral correlations vary substantially over time. While the average correlation between S&P 500 stocks was between 0.50 and 0.70 in 2012 depending on the maturity of the options used to compute $\overline{\rho}^Q_{m,t,\tau}$ in equation (28), these correlations dropped as low as 0.20 during the economic expansion of the 2010s before rising back to around 0.70 during the onset of the COVID-19 crisis. Third, there is also a large degree of cross-sectional heterogeneity between the risk-neutral correlations across ETFs at a given point in time. For instance, while stocks in the S&P 500 shared an average risk-neutral correlation of about 0.70 in 2020, the firms underlying XOP had an average risk-neutral correlation of about 0.95. Thus, ETF-level correlation risk can vary both over time and between different investment styles.

The final takeaway from Figure 6 is that there are stark differences in the *term structure* of risk-neutral correlations. Notably, while short-term measures of intra-ETF correlations (estimated using 30-day options) tend to be "fast-moving" and vary substantially over time, long-term measures of intra-ETF correlations (estimated using 365-day options) are relatively "slow-moving" and consequently vary less over time. However, these differences in the levels of correlations tend to diverge during good times and compress during bad times, such as the economic and financial shock induced by COVID-19.

Regression analysis. To begin, we examine how disagreement regarding the systematic component of ETF returns may drive variation in the *level* of intra-ETF correlation risk by estimating the following panel regression:

$$\overline{\rho}^{Q}_{m,t,\tau} = \delta_t + \alpha_m + \beta_2 s_{m,t} + \boldsymbol{\beta} \boldsymbol{X}^{T}_{m,t-1} + \varepsilon_{m,t}. \tag{29}$$

Here, $\overline{\rho}^{Q}_{m,t,\tau}$ represents the average risk-neutral correlation for ETF m at time t, estimated using 30-day options; $s_{m,t}$ is the amount of factor-level disagreement related to ETF m at time t, as defined in equation (22); and the vector $X_{m,t}$ controls for ETF flows, returns, and bid-ask spreads. The regression includes both time fixed effects (δ_t) that account for common shocks that impact all ETFs at a given point in time, and ETF fixed effects (α_m) that account for unconditional differences in correlation risk across ETFs (e.g., the difference in the level of correlation between SPY and XOP in Figure 6). Finally, and similar to the tables above, both the independent and dependent variables are standardized for ease of interpretation.

The results of estimating equation (29) are reported in Table 4. The table highlights three key takeaways. First, as shown in Column (1), there is a strong predictive relationship between correlation and disagreement. A one-standard-deviation increase in relative factor disagreement results in a 0.26-standard-deviation increase in $\overline{\rho}^{Q}_{m,t,30}$, which empirically supports the intuition underlying model prediction (c) in Section 1.6. Specifically, these results support the prediction illustrated by the right panel of Figure 2. Through the lens of the model, this association arises because more concentrated trading in the ETF (i.e., a greater exposure to the systematic risk factor) increases the covariation between related securities, and makes the market more susceptible to shocks that impact this systematic factor.

In Columns (2) and (3) we add ETF and month fixed effects, respectively, to the regression; while the coefficients are still statistically significant, the economic magnitude decreases with the addition of ETF fixed effects. Now a one-standard-deviation increase in the relative factor disagreement predicts a 0.05- to 0.09-standard-deviation increase in $\overline{\rho}^{Q}_{m,t,30}$. This result indicates that the relationship holds true both dynamically and in the cross-section. In columns (4) and (5) we add ETF flows, returns, and bid-ask spreads as controls. The statistical and economic significance of the relationship between lagged disagreement and correlation remains. Collectively, the evidence in this section further validates the economic mechanisms of our model.

Correlation term structure. One of the underlying assumptions of the model is that the relative share of factor disagreement is mean reverting. One could directly test the mean-reverting properties of $s_{m,t}$ itself. However, due to its relatively short time-series and high persistence, the analysis may be unreliable. Alternatively, one can look at how long the market anticipates the higher correlation to persist using correlations implied by options of different maturities. We therefore test how shifts in disagreement propagate through the term structure of forward-looking correlations by running the following panel regression:

$$\overline{\rho}^{Q}_{m,t,\tau_2} - \overline{\rho}^{Q}_{m,t,\tau_1} = \delta_t + \beta_2 s_{m,t} + \boldsymbol{\beta} \boldsymbol{X}^{T}_{m,t-1} + \varepsilon_{m,t}. \tag{30}$$

Here $\overline{\rho}^{Q}_{m,t,\tau_2}$ and $\overline{\rho}^{Q}_{m,t,\tau_1}$ represent average risk-neutral correlations for ETF m at time t, estimated using options of two different horizons, τ_2 and τ_1, respectively, with $\tau_2 > \tau_1$. This difference in risk-neutral correlations is regressed onto the factor-level disagreement related to ETF m at time t as defined in equation (22), and the vector $X_{m,t}$ controls for ETF flows, returns, and bid-ask spreads. Given that the dependent variable is already a measured as a difference, we add only time (monthly) fixed effects to the regression.

The results of estimating equation (30) are reported in Table 5. The columns highlight the correlation spreads that are used as independent variables for each regression. As in our other regressions, we standardize the dependent and independent variables in order to highlight the economic significance of shifts in the term structure. First, as highlighted in Columns (1), (2), and (5), most of the activity takes place in the front end of the correlation term structure, between horizons of 30 days and six months. This contrasts with the duration of correlation and volatility effects that result from the arbitrage-based mechanisms of, e.g., Ben-David et al. (2018) and Da and Shive (2018), which tend to persist less than three months.

To better understand the magnitude of these effects, we list the mean and standard deviation of each spread at the bottom of Table 5. The average correlation spread is steepest between 91 and 30 days (i.e., Column (1)), with implied correlation at 91 days being more than 0.10 correlation points higher (as per the average correlation spread denoted by "Avg Spread"), and relatively flat for longer horizons. Relative to baselines, however, the effects from an increase in relative disagreement are most pronounced on the spread between 182 and 91 days. With a one-standard-deviation increase in factor relative disagreement, the correlation spread between 30 and 91 days decreases by approximately 0.13 standard-deviations (or (0.1364*0.0881)/0.1135 = 10%), whereas the spread between 182 and 91 days decreases by more than 30% of the mean spread. Given the persistence of the spreads, both coefficients are strongly statistically significant and suggest that factor disagreement and the resulting flows into the ETF drive longer-dated anticipated correlation. Finally, relative disagreement has little predictability on the correlation term structure spread at longer horizons. Expectations hypothesis suggests that the market therefore anticipates a mean reversion of any shock to the term structure induced by a shift in relative disagreement within approximately six months. Given the relationships we have found between lagged $s_{m,t}$, and volatility and correlation risk, this implies a mean-reverting $s_{m,t}$ process.

3 Conclusion

This paper studies how factor and stock-specific disagreement affect asset prices and risk. We start by building a theoretical model to examine the interplay between heterogeneity in subjective expectations about the stock-specific and common components of expected returns. We consider a pure exchange economy with multiple Lucas trees that are exposed to both factor risk and stock-specific shocks, and we allow agents to disagree about both dimensions of returns. As such, our model features periods of strong disagreement about the factor, and periods when disagreement about stocks dominates.

Our model predicts that (i) factor disagreement increases the exposure of investors in the economy to the assets that are most closely aligned with systematic risk; (ii) factor disagreement increases the return volatility of financial instruments aligned with the common factor (e.g., ETFs); and (iii) this increase in volatility is accompanied by the increased correlations between stocks that compose the factor (i.e., reduced diversification benefits).

We then use the return and flow dynamics of ETFs and their underlying securities to test these hypotheses. In keeping with the model's predictions we find that disagreement is strongly related to ETF flows: when the proportion of factor-to-stock-specific disagreement is one-standard-deviation higher, there is a 0.11-standard-deviation higher flow into the ETF. Second, disagreement also relates closely to forward-looking and long-dated volatility: a one-standard-deviation increase in the relative factor disagreement measure leads to volatility rising by 0.06 standard deviations.

Finally, we find that increased factor disagreement is also strongly related to higher long-run correlations. Specifically, an increase in the relative magnitude of factor disagreement *reduces* the diversification benefits of holding the stocks underlying the ETF. The effects of higher factor disagreement on the term structure of risk-neutral correlations imply that agents expect these elevated levels of correlation to last approximately six months. Taken together, our theoretical and empirical results highlight the first-order effect of disagreement on trade flows, and show how flows impact cross-sectional differences in volatility and correlation risk.

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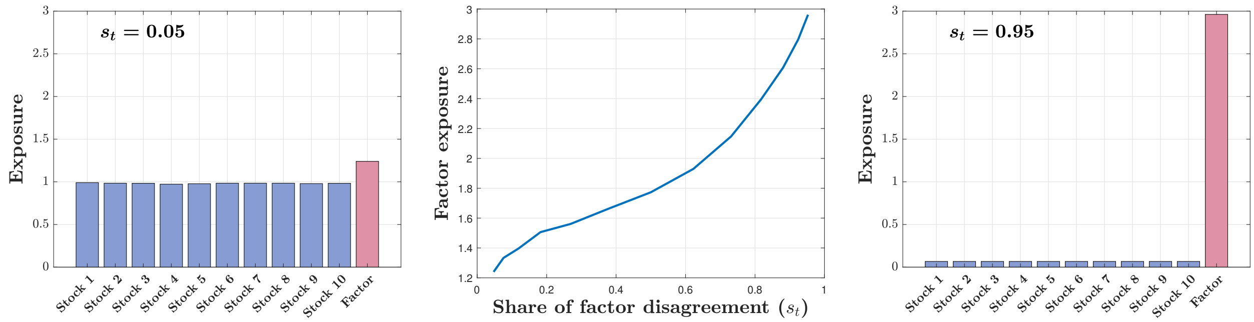

Figure 1: The figures shows exposure when $s_t = 0.05$ for each of the 10 stocks and the factor (left), the factor exposure as function of the share of factor disagreement s (middle), and the exposure when $s_t = 0.95$ for each of the 10 stocks and the factor (right). The figures are based on a 10-stock economy. All agents have the same consumption shares and each dividend share is the same for each stock. The total dividend share is 5%. The figures are based on the following parameters: $\gamma = 2$, $\rho = 0.02$, $\mu_E = 0.02$, $\sigma_E = 0.03$, $\sigma_Z = 0.03$, $\mu_Z = -0.5\sigma_Z^2$, $\sigma_n = 0.06$, $\mu_n = -0.5\sigma_n^2$ $\Delta = 0.2$, $\kappa_\delta = 0.1$, $\sigma_\delta = 0.2$, $\bar{\delta} = 0$. The results are generated by Monte-Carlo simulations based on 400,000 paths of monthly observations of length of 100 years.

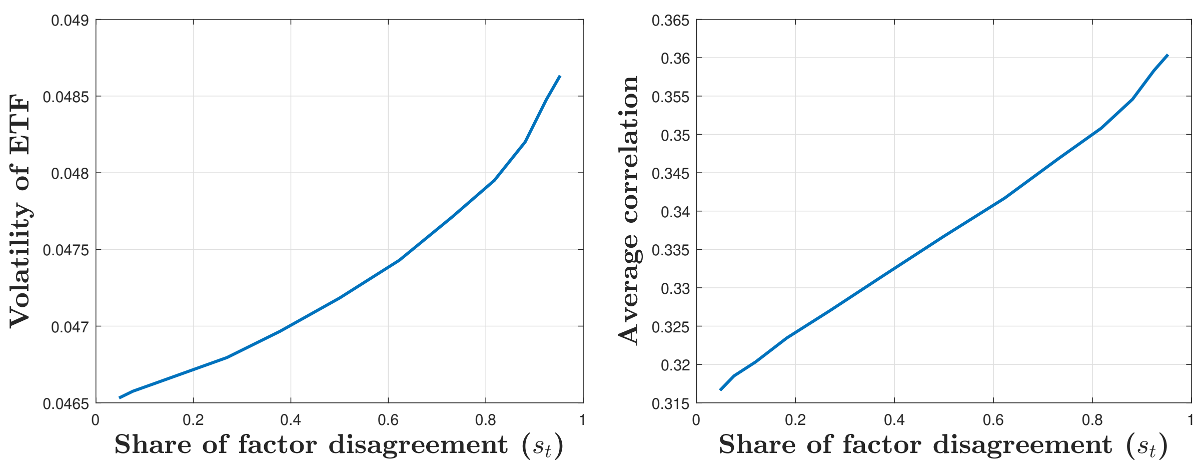

Figure 2: The figures shows the standard deviation of the ETF (left) and the average stock return correlation (right) as a function of the share of factor disagreement s. The figures are based on a 10-stock economy. All agents have the same consumption shares and each dividend share is the same for each stock. The total dividend share is 5%. The figures are based on the following parameters: $\gamma = 2$, $\rho = 0.02$, $\mu_E = 0.02$, $\sigma_E = 0.03$, $\sigma_Z = 0.03$, $\mu_Z = -0.5\sigma_Z^2$, $\sigma_n = 0.06$, $\mu_n = -0.5\sigma_n^2$ $\Delta = 0.2$, $\kappa_\delta = 0.1$, $\sigma_\delta = 0.2$, $\bar{\delta} = 0$. The results are generated by Monte-Carlo simulations based on 400,000 paths of monthly observations of length of 100 years.

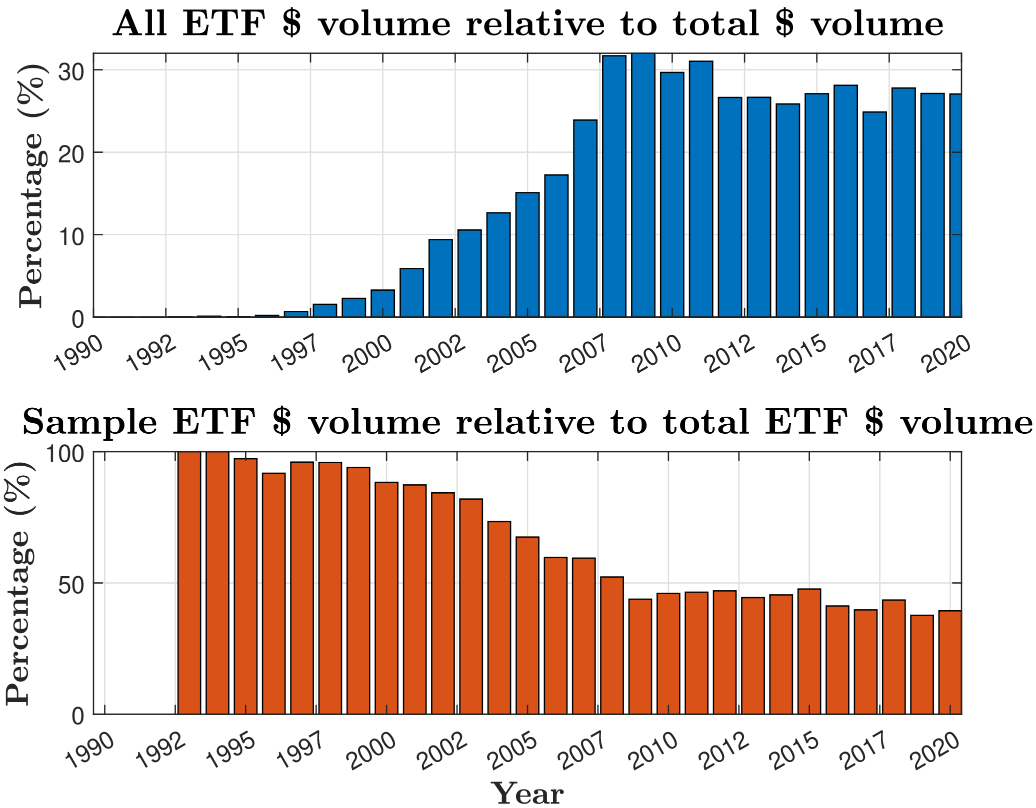

Figure 3: The figure displays the percentage of dollar-trading volume in ETFs relative to dollar-trading volume for the U.S. equity market for the period ranging from 1900 to 2020. ETFs are identified as securities in the CRSP Monthly dataset that have a share code (SHRCD) of 73. We then compute the monthly dollar trading volume of (i) all ETFs, and (ii) all securities in the CRSP Monthly universe, and aggregate these monthly trading volumes to the annual frequency within each trading year. The top panel of the figure then reports the percentage of ETF-related dollar trading volume, summed across all U.S. ETFs, relative to the aggregate amount of dollar-trading volume across all U.S. securities. The bottom panel of the figure reports the percentage of ETF-related dollar-trading volume, summed across the ETFs in our sample (see Table 1), relative to the aggregate amount of dollar-trading volume across all U.S. ETFs.

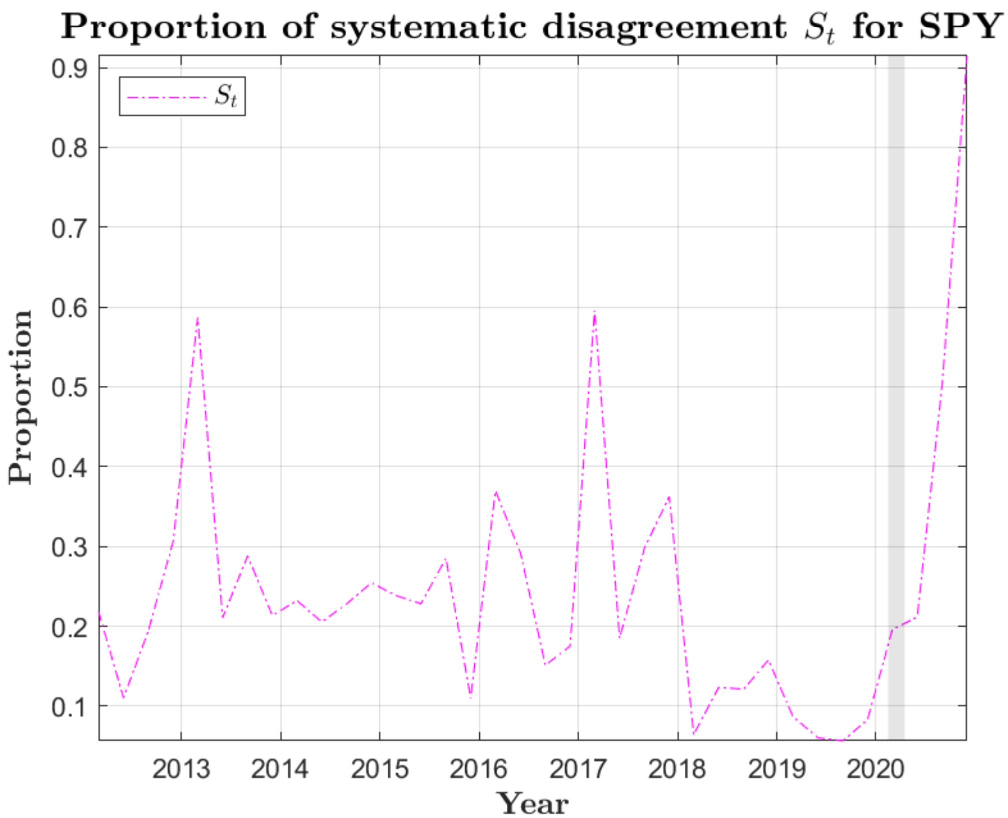

Figure 4: The figure reports the proportion of factor disagreement relative to total disagreement, as defined by equation (22) for SPY – an ETF that tracks the S&P 500. The individual components of disagreement are measured using equations (20) and (21), respectively. For the purpose of visualizing the data, we aggregate each measure of disagreement to the quarterly frequency by computing the mean value of each disagreement in a given quarter. The sample period ranges from 2012 to 2020.

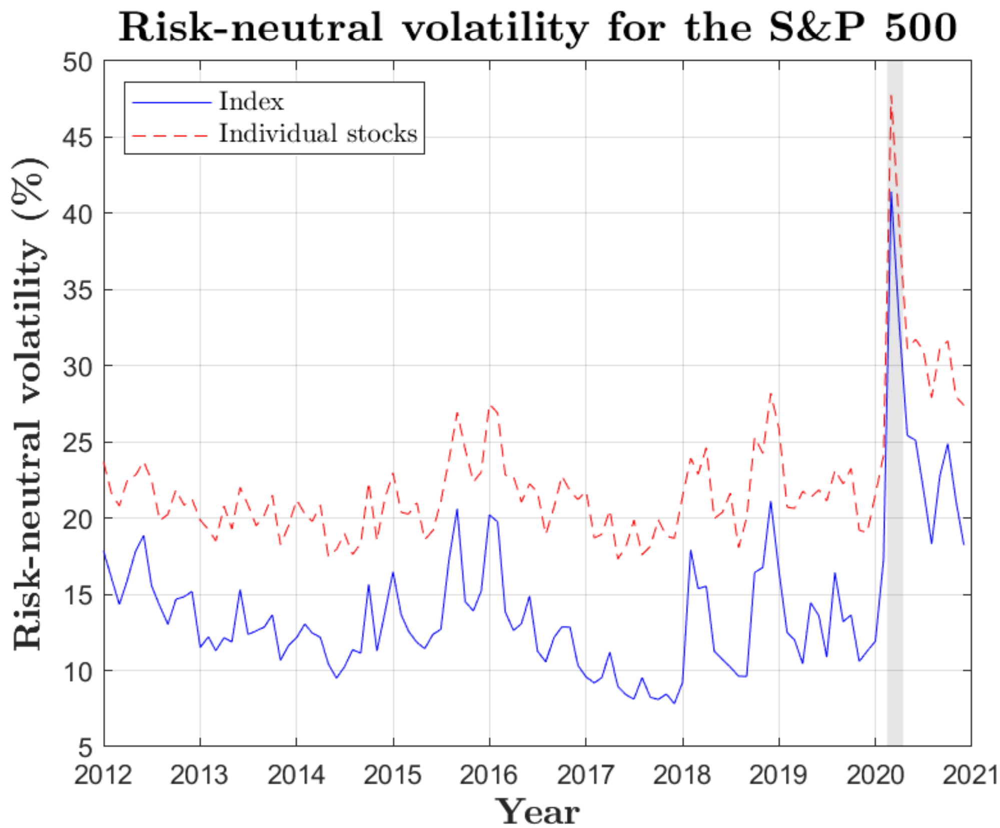

Figure 5: The figure displays the risk-neutral volatility for SPY, which tracks the S&P 500 stock market index. The risk-neutral volatility is computed in two ways. First, the solid blue line reports the volatility obtained by estimating equation (25) using ETF-linked options. Second, the dashed red line reports the volatility obtained by estimating equation (25) using individual stock options, for each stock i and j in ETF m, and then computing the index-level volatility as $\sum_{i=1}^{N}\sum_{i=1}^{N} w_{i,m,t}w_{j,m,t}\sigma^{Q}_{i,t,\tau}\sigma^{Q}_{j,t,\tau}$, where $w_{i,m,t}$ and $w_{j,m,t}$ is the weight of firm i or j in ETF m at time t, respectively, and each risk-neutral volatility (i.e., $\sigma^{Q}_{i,t,\tau}$) is obtained by taking the square root of the risk-neutral variance in equation (25). These risk-neutral volatilities are calculated on a daily basis, scaled to represent annualized volatilities, and aggregated to the monthly frequency by computing the average risk-neutral volatility within each month. In all the calculations above, we set $\tau = 30$ such that we estimate each risk-neutral volatility with 30-day options. Finally, the sample period spans January 2012 and December 2020.

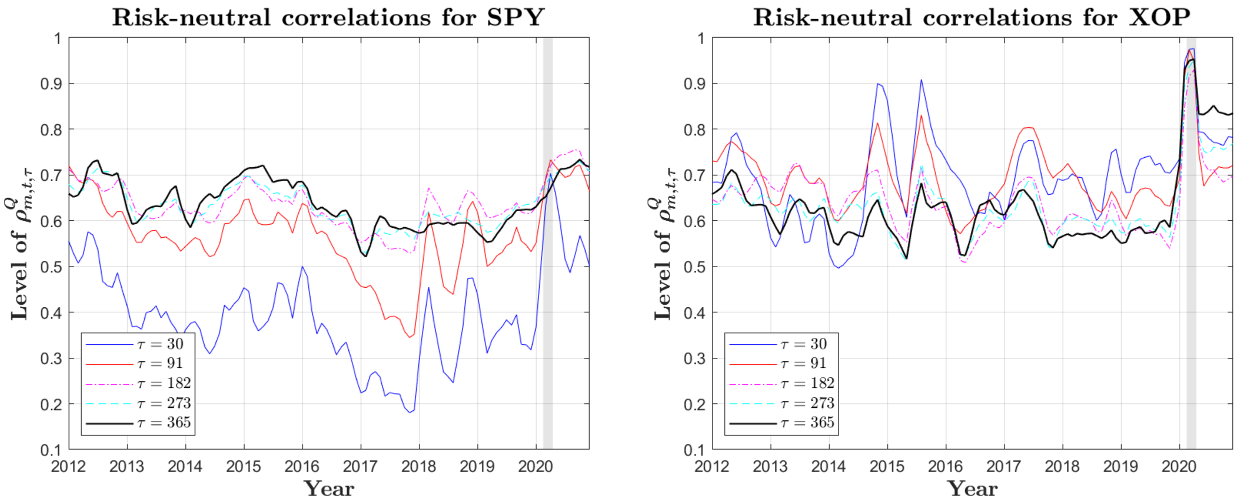

Figure 6: The figure displays the average risk-neutral correlation for the SPY ETF, which tracks the S&P 500 stock market index, and the XOP ETF, which tracks the returns of stocks involved in the exploration and production of oil and gas. The figure reports the τ = 30-, 91-, 182-, 273-, and 365-day-ahead risk-neutral correlation of each index, obtained by solving equation (28) to obtain $\overline{\rho}^{Q}_{m,t,\tau}$ for each ETF m in each month t. For the purpose of visualizing the resulting risk-neutral correlations, we apply a moving-average filter to each monthly time-series of correlations, and report the average correlation over a window of $[-1, 1]$ months around each month t. The sample period spans January 2012 to December 2020.

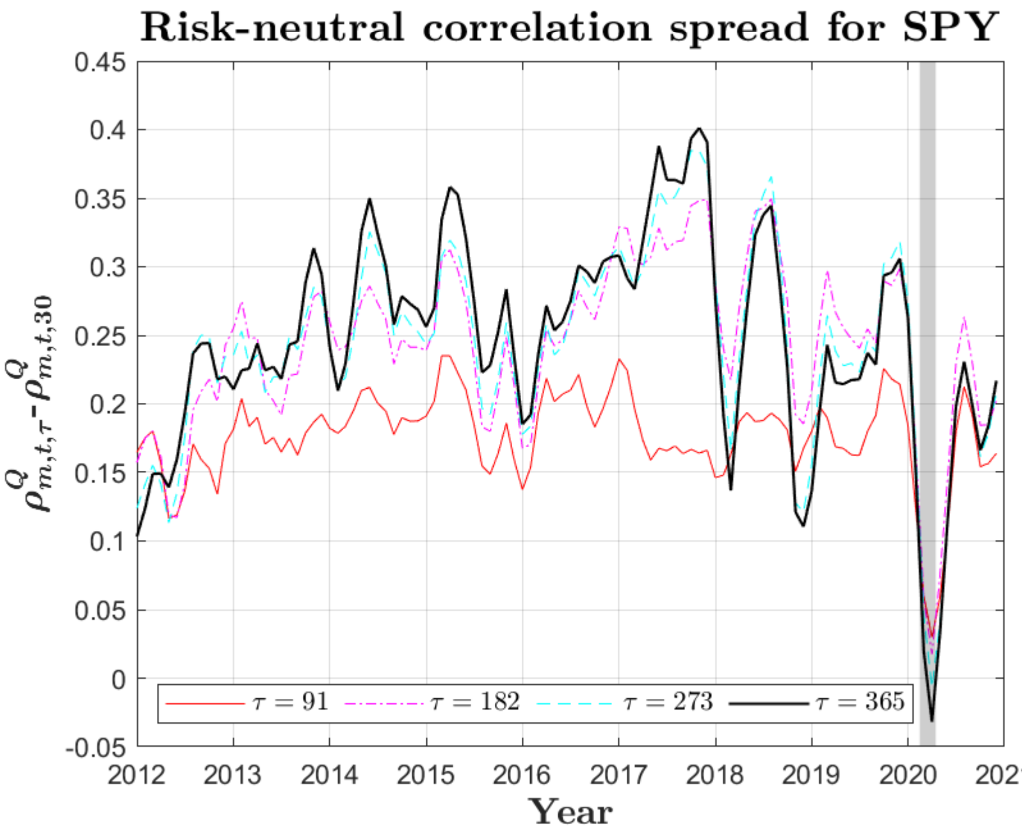

Figure 7: The figure reports the time-series of the spread in the average risk-neutral correlations among the constituents of the SPY ETF, an ETF that tracks the returns of the S&P 500 index. Specifically, the risk-neutral correlation $\overline{\rho}^Q_{m,t,\tau}$ represented by equation (28) is estimated using options with maturities of $\tau \in \{30, 91, 182, 273, 365\}$ days to maturity. With these risk-neutral correlations in hand, the figure then reports $\overline{\rho}^Q_{m,t,\tau} - \overline{\rho}^Q_{m,t,30}$ for $\tau \in \{91, 182, 273, 365\}$. As such, the figure displays the difference between various measures of the intra-ETF "long-run" correlations and the market's perception of the intra-ETF "short-run" correlation. For the purpose of visualizing the resulting risk-neutral correlation spreads, we apply a moving-average filter to each monthly time-series of correlation spreads, and report the average correlation spread over a window of $[-1, 1]$ months around each month t. The sample period spans January 2012 to December 2020.

Table 1: The table reports summary statistics for the ETFs in our sample. For each ETF, the table reports the ticker alongside the benchmark that the ETF tracks (denoted by "Style"). The net asset value of each fund is represented by "NAV ($b)", and is reported by ETF Global. Similarly, "ME ($b)" represents the average total market value represented by the stocks underlying each ETF over the sample period, and is constructed from CRSP Monthly data. "|Flow|" reports the average amount of (absolute) flow into and out of each ETF, on average, over the sample period, and is also measured in billions of dollars from ETF Global. The columns "$Vol(ETF)" and "$Vol(Stocks)" report the average amount of dollar-trading volume associated with each ETF and its underlying stocks per month, respectively. E [Analysts] reports the average number of analysts following each firm in a given ETF. The sample period ranges from 2012 to 2020.

ETF	Style	NAV ($b)	ME ($b)	|Flow| ($b)	$Vol(ETF) ($b)	$Vol(Stocks) ($b)	E [Analyst]
SPY	Market	227.23	20227.72	29.36	473.85	2867.83	16.27
DIA	Dow Jones	200.56	5840.02	2.30	19.96	585.23	21.36
QQQ	Nasdaq	133.55	6219.90	7.01	102.88	1146.36	19.58
IBB	Biotech	187.63	764.66	0.83	7.52	163.19	5.92
XLK	Technology	55.02	4760.55	1.61	12.95	738.46	21.22
XLB	Materials	49.77	577.13	0.91	6.62	93.67	15.18
XLE	Energy	69.13	1346.49	2.17	22.64	197.01	24.22
XLI	Industrials	59.98	1979.36	2.02	14.65	281.70	15.12
XLP	Cons. Staples	50.01	1942.78	1.82	11.49	187.68	14.53
XLV	Health Care	72.35	2795.04	1.90	14.14	333.28	15.58
XLY	Cons. Disc.	85.73	2408.32	1.60	9.65	468.94	19.26
XOP	Oil & Gas	44.90	1163.23	1.17	10.47	180.15	17.04
XBI	Biotech	104.43	710.31	0.96	6.06	133.18	7.33

Table 2: The table documents how changes in relative factor disagreement ($s_{m,t}$) are associated with trading activity in the ETF or index futures. These results are obtained by estimating the panel regression outlined in equation (23). Net flows into an ETF are constructed according to equation (18), relative volumes of ETF versus its components are constructed according to equation (19), open interests are computed according to the methodology of Hong and Yogo (2012), and the measure of relative disagreement is obtained via equation (22). Regressions control for one-month lagged trading activity as well as combinations of index and time (i.e., month) fixed effects. For ETF-associated regressions we also control for ETF-level returns and bid-ask spreads. The sample period underlying this regression ranges from 2012 to 2020.

	Panel A: Net Flow			Panel B: Relative Volume			Panel C: Open Interest		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
$S_{m,\tau}$	0.1139^{***}	0.1120^{***}	0.0907^{***}	0.2645^{***}	0.2743^{***}	0.0574^{***}	0.1696^{***}	0.2403^{***}	0.0903^{***}
	[3.81]	[3.48]	[3.14]	[10.23]	[11.17]	[3.04]	[4.61]	[5.81]	[3.21]
$\text{NFlow}_{m,t}$			-0.0652						
			[-1.48]						
$\text{Ret}_{m,t}$			0.0185			0.0171			
			[0.53]			[0.64]			
$\text{Bid-Ask}_{m,t}$			-0.0605^{**}			0.0006			
			[-2.37]			[0.04]			
$\lvert\text{Ret}\rvert_{m,t}$			0.0811^{**}			0.0528^{**}			
			[2.50]			[2.43]			
$\text{DRVol}_{m,t-1}$						0.7700^{***}			
						[21.85]			
$\text{OInterest}_{m,t-1}$									0.7649^{***}
									[12.60]
Date FE	No	Yes	Yes	No	Yes	Yes	No	Yes	Yes
ETF/Index FE	No	No	Yes	No	No	Yes	No	No	Yes
Observations	1,263	1,263	1,263	1,376	1,376	1,261	792	792	726
R^2	0.0131	0.0135	0.0254	0.0700	0.0753	0.5936	0.0370	0.0600	0.6067

Table 3: The table documents how changes in relative factor disagreement ($s_{m,t}$) about an ETF are associated with the *forward-looking* risk of an ETF, measured using option-implied volatility. These results are obtained by estimating the panel regression outlined in equation (26), where the forward-looking risk of an ETF is obtained via equation (25) and the measure of relative disagreement is obtained via equation (22). Each regression also controls for one-month trade flows, ETF-level returns, and bid-ask spreads, as well as combinations of ETF and time (i.e., month) fixed effects. The sample period underlying this regression ranges from 2012 to 2020.

	30 days (1)	91 days (2)	182 days (3)	273 days (4)	365 days (5)
$S_{m,t}$	0.1210***	0.0991***	0.1043***	0.1226***	0.1229***
	[4.11]	[3.43]	[3.64]	[4.40]	[4.53]
$NFlow_{m,t}$	0.1041	0.0900	0.0780	0.0705	0.0666
	[1.64]	[1.46]	[1.36]	[1.40]	[1.38]
$Ret_{m,t}$	−0.2208**	−0.2073	−0.1791	−0.1566	−0.1402
	[−2.12]	[−1.60]	[−1.20]	[−1.08]	[−1.04]
$Bid\text{-}Ask_{m,t}$	0.1563***	0.1480***	0.1280***	0.1234***	0.1237***
	[4.42]	[3.34]	[3.27]	[3.40]	[3.09]
$\lvert Ret \rvert_{m,t}$	0.2884***	0.2634***	0.2514***	0.2315***	0.2172***
	[3.80]	[3.22]	[3.39]	[3.34]	[3.34]
Date FE	Yes	Yes	Yes	Yes	Yes
ETF FE	Yes	Yes	Yes	Yes	Yes
Observations	1,263	1,263	1,263	1,263	1,263
R^2	0.2050	0.1759	0.1470	0.1323	0.1231

Table 4: The table documents how changes in relative factor disagreement ($s_{m,t}$) about an ETF are associated with the *forward-looking* risk of an ETF, measured using option-implied correlation. These results are obtained by estimating the panel regression outlined in equation (26), the forward-looking risk of an ETF is obtained via equation (25), and the measure of relative disagreement is obtained via equation (22). Each regression also controls for one-month trade flows, ETF-level returns, and bid-ask spreads, as well as combinations of ETF and time (i.e., month) fixed effects. The sample period underlying this regression ranges from 2012 to 2020.

	(1)	(2)	(3)	(4)	(5)
$S_{m,t}$	0.2612***	0.0877***	0.0649**	0.0551**	0.0469*
	[10.55]	[2.71]	[2.55]	[2.06]	[1.87]
$NFlow_{m,t}$				0.0609*	0.0416
				[1.84]	[1.51]
$Ret_{m,t}$					−0.0874**
					[−2.15]
$\lvert Ret\rvert_{m,t}$					0.1593***
					[4.46]
$Bid\text{-}Ask_{m,t}$					0.2212***
					[4.38]
Date FE	No	No	Yes	Yes	Yes
ETF FE	No	Yes	Yes	Yes	Yes
Observations	1,378	1,378	1,378	1,263	1,263
R^2	0.0682	0.0077	0.0311	0.0340	0.1222

Table 5: The table documents how changes in relative factor disagreement ($s_{m,t}$) about an ETF are associated with the term-structure of correlation of an ETF, measured by subtracting option implied *forward-looking* correlation at different horizons. The horizons used are listed in the header of the tables. The results are obtained by estimating the panel regression outlined in equation (30), and the measure of relative disagreement is obtained via equation (22). Each regression also controls for one-month trade flows, ETF-level returns, and bid-ask spreads, as well as time (i.e., month) fixed effects. The mean spreads across the sample are also listed in the table. The sample period underlying this regression ranges from 2012 to 2020.

	91-30 days (1)	182-91 days (2)	273-182 days (3)	365-273 days (4)	365-30 days (5)
$S_{m,t}$	-0.1364^{***}	-0.0960^{***}	0.0206	-0.0353	-0.1429^{***}
	[−4.61]	[−3.84]	[0.66]	[−1.27]	[−5.29]
$NFlow_{m,t}$	-0.0311	-0.0056	0.0143	0.0204	-0.0145
	[−1.09]	[−0.17]	[0.54]	[0.63]	[−0.48]
$Ret_{m,t}$	0.0724	0.0428	-0.0068	0.0078	0.0699
	[1.44]	[1.11]	[−0.17]	[0.21]	[1.47]
$\lvert Ret\rvert_{m,t}$	-0.4096^{***}	-0.2616^{***}	0.0070	0.0355	-0.3928^{***}
	[−8.96]	[−6.87]	[0.19]	[0.94]	[−8.03]
$\text{Bid-Ask}_{m,t}$	-0.1471^{**}	-0.2549^{**}	-0.0722	0.0716	-0.2270^{**}
	[−2.57]	[−2.08]	[−1.58]	[1.24]	[−2.41]
Date FE	Yes	Yes	Yes	Yes	Yes
Avg Spread	0.1135	0.0176	-0.0095	-0.0037	0.1180
Std Spread	0.0881	0.0633	0.0327	0.0338	0.1381
Observations	1,263	1,263	1,263	1,263	1,263
R^2	0.2472	0.1768	0.0061	0.0094	0.2700

A Internet Appendix

A.1 Additional tables and figures

Table A.1.1: The table presents summary statistics for the relative importance of factor disagreement ($s_{m,t}$ from equation (22)) for each ETF m in our sample. In particular, the table reports the mean, median, and standard deviation of the relative disagreement measure for each ETF, as well as the 25^{th} and 75^{th} percentile of this measure. The sample period ranges from January 2012 to December 2020.

ETF	Style	Mean	Std	$p25$	Median	$p75$
SPY	Market	0.27	0.20	0.12	0.21	0.36
DIA	Dow Jones	0.04	0.04	0.01	0.02	0.05
QQQ	Nasdaq	0.12	0.12	0.04	0.08	0.15
IBB	Biotech	0.24	0.17	0.11	0.18	0.31
XLK	Technology	0.09	0.11	0.03	0.06	0.10
XLB	Materials	0.03	0.04	0.01	0.02	0.04
XLE	Energy	0.21	0.19	0.09	0.16	0.28
XLI	Industrials	0.03	0.05	0.01	0.01	0.03
XLP	Cons. Staples	0.01	0.02	0.01	0.01	0.01
XLV	Health care	0.03	0.03	0.01	0.02	0.03
XLY	Cons. Discretionary	0.07	0.06	0.02	0.04	0.08
XOP	Oil & gas	0.31	0.19	0.19	0.25	0.40
XBI	Biotech	0.21	0.19	0.07	0.15	0.27

Table A.1.2: The table presents the correlation between the measures of the relative importance of factor disagreement ($s_{m,t}$ from equation (22)) for each pair of ETFs in our sample. The sample period ranges from January 2012 to December 2020.

	SPY	DIA	QQQ	XLK	XLB	XLE	XLI	XLP	XLV	XLY	XOP	XBI	IBB
SPY	1.00	0.23	0.09	-0.01	0.07	0.17	-0.01	0.21	0.42	0.18	0.13	0.03	0.14
DIA	-	1.00	-0.03	0.14	0.14	0.01	0.02	0.18	0.11	0.28	-0.05	-0.09	-0.03
QQQ	-	-	1.00	0.13	0.22	0.18	0.29	0.26	0.28	0.15	0.08	-0.11	0.12
XLK	-	-	-	1.00	-0.11	-0.01	0.07	-0.10	0.04	-0.01	0.04	0.12	0.01
XLB	-	-	-	-	1.00	-0.00	0.22	0.05	-0.03	0.17	-0.02	-0.08	0.15
XLE	-	-	-	-	-	1.00	0.19	0.46	0.04	0.33	0.15	0.04	0.31
XLI	-	-	-	-	-	-	1.00	0.06	0.05	0.01	0.05	0.03	0.01
XLP	-	-	-	-	-	-	-	1.00	0.30	0.48	0.10	-0.12	0.30
XLV	-	-	-	-	-	-	-	-	1.00	0.16	0.09	-0.12	0.35
XLY	-	-	-	-	-	-	-	-	-	1.00	0.05	-0.11	0.32
XOP	-	-	-	-	-	-	-	-	-	-	1.00	0.18	0.06
XBI	-	-	-	-	-	-	-	-	-	-	-	1.00	-0.11
IBB	-	-	-	-	-	-	-	-	-	-	-	-	1.00

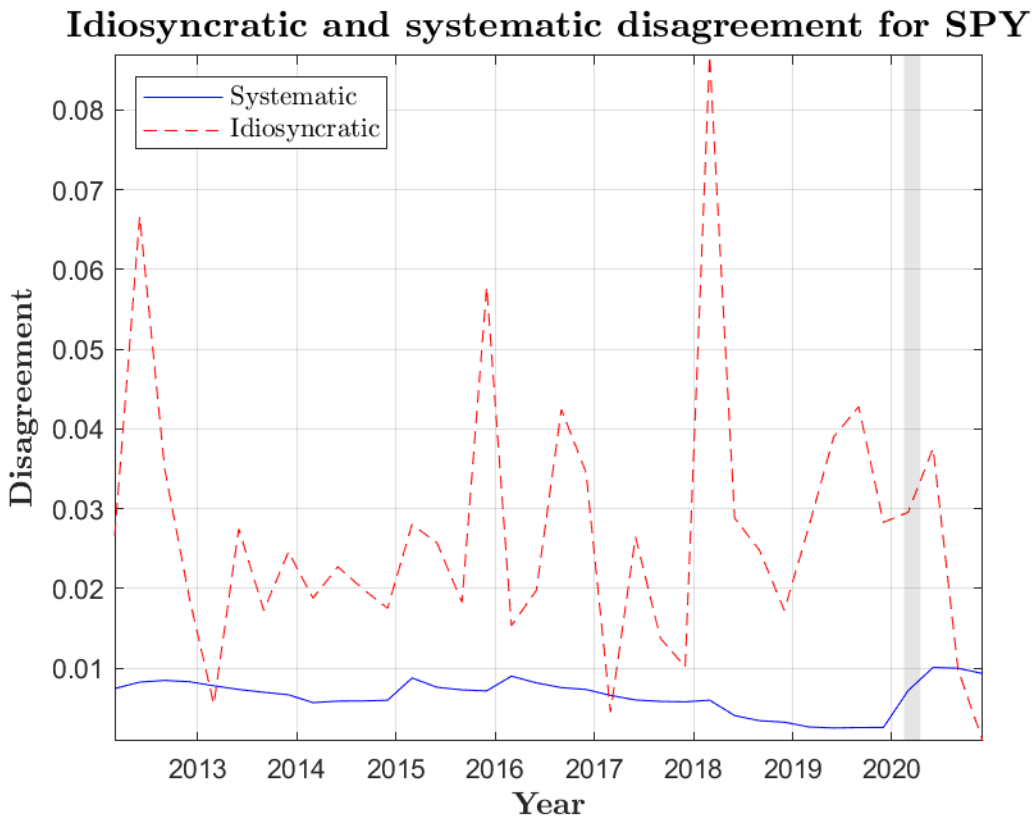

Figure A.1.1: The figure reports the levels of systematic (factor) and idiosyncratic disagreement for SPY – an ETF that tracks the S&P 500 – as measured using equations (20) and (21), respectively. For the purpose of visualizing the data, we aggregate each measure of disagreement to the quarterly frequency by computing the mean value of each disagreement in a given quarter. The sample period ranges from 2012 to 2020.

Table A.1.3: The table documents how changes in relative factor disagreement ($s_{m,\tau}$) are associated with trading activity in the ETFs. These results are obtained by estimating the panel regression outlined in equation (23) with one modification: the regression specification also includes an interaction between relative factor disagreement (denoted by $s_{m,t}$) and a dummy variable indicating where the ETF represents a broad index (denoted by $\mathbb{I}$[Index]). Here, we assume that the two indexes in our sample are the S&P 500 index and the Nasdaq-100 index. Net flows into an ETF are constructed according to equation (18), relative volumes of ETF versus its components are constructed according to equation (19), and the measure of relative disagreement is obtained via equation (22). Regressions control for one-month lagged trading activity as well as combinations of index and time (i.e., month) fixed effects. For ETF-associated regressions we also control for ETF-level returns and bid-ask spreads. The sample period underlying this regression ranges from 2012 to 2020.

	Panel A: Net Flow			Panel B: Relative Volume		
	(1)	(2)	(3)	(4)	(5)	(6)
$S_{m,\tau}$	0.1127***	0.1151***	0.0907***	0.2805***	0.2959***	0.0502***
	[3.66]	[3.40]	[3.01]	[9.05]	[11.81]	[2.70]
$S_{m,t} \times \mathbb{I}$ [Index]	0.0121	−0.0284	0.0005	−0.1788	−0.1868	0.0688
	[0.15]	[−0.36]	[0.01]	[−1.26]	[−1.53]	[1.15]
$\text{NFlow}_{m,t}$			−0.0652			
			[−1.47]			
$\text{Ret}_{m,t}$			0.0185			0.0167
			[0.53]			[0.62]
$\text{Bid-Ask}_{m,t}$			−0.0605**			0.0009
			[−2.37]			[0.05]
$\lvert\text{Ret}\rvert_{m,t}$			0.0811**			0.0537**
			[2.48]			[2.44]
$\text{DRVol}_{m,t-1}$						0.7652***
						[21.10]
Date FE	No	Yes	Yes	No	Yes	Yes
ETF/Index FE	No	No	Yes	No	No	Yes
Observations	1,263	1,263	1,263	1,261	1,261	1,261
R^2	0.0131	0.0136	0.0254	0.0728	0.0801	0.5942

Table A.1.4: The table documents how changes factor disagreement ($\text{FactorDisagree}_{m,\tau}$) are associated with trading activity in the ETFs. These results are obtained by estimating the panel regression outlined in equation (23) with one modification: the relative disagreement measure denoted by equation (22) is replaced by the factor disagreement measure denoted by equation (21). Net flows into an ETF are constructed according to equation (18) and relative volumes of ETF versus its components are constructed according to equation (19). Regressions control for one-month lagged trading activity as well as combinations of index and time (i.e., month) fixed effects. For ETF-associated regressions we also control for ETF-level returns and bid-ask spreads. The sample period underlying this regression ranges from 2012 to 2020.

	Panel A: Net Flow	Panel B: Relative Volume	Panel C: Open Interest
	(1)	(2)	(3)
$\text{FactorDisagree}_{m,\tau}$	0.1098^{***}	0.0606^{***}	0.0903^{***}
	[3.92]	[2.63]	[3.21]
$\text{NFlow}_{m,t}$	-0.0707		
	[-1.57]		
$\text{Ret}_{m,t}$	0.0243	0.0191	
	[0.68]	[0.70]	
$\text{Bid-Ask}_{m,t}$	-0.0531^{**}	0.0038	
	[-1.99]	[0.24]	
$\lvert\text{Ret}\rvert_{m,t}$	0.0770^{**}	0.0521^{**}	
	[2.40]	[2.40]	
$\text{DRVol}_{m,t-1}$		0.7696^{***}	
		[21.66]	
$\text{OInterest}_{m,t-1}$			0.7649^{***}
			[12.60]
Date FE	Yes	Yes	Yes
ETF/Index FE	Yes	Yes	Yes
Observations	1,263	1,261	726
R^2	0.0291	0.5938	0.6067

A.2 Complete Model and Simulation Details

In this section we provide the additional details of the model in Section 1. Details of the cash flow dynamics and the information structure can be found in the main body of the paper, so we do not repeat it here. To simplify some of the exposition we define the dividend diffusion coefficients for stock n as σ_{D_n}. Note that from an application of Ito's lemma on the dividend dynamics we have

$$\sigma_{D_n} = \sigma_E + \sigma_z + \sigma_n \tag{A.2.1}$$

In the reminder of this section, we stack all the Brownian shocks into a vector w_t, with $w_t = (w_{E,t}, w_{\delta,t}, w_{z,t}, w_{1,t}, \ldots, w_{N,t}) \in \mathbb{R}^{N+3}$. Similarly, define agent j's disagreement vector at time t as $\Delta_t^j = \left(0, 0, \Delta_z^j s_t, \Delta_1^j (1 - s_t), \ldots, \Delta_N^j (1 - s_t)\right)$. The disagreement vector of agent j captures how distorted the belief is about each shock in the Brownian vector w_t at time t.

A.2.1 Security markets

Agents can trade a locally risk-free asset in zero net supply, $N + 1$ stocks, and two additional zero net supply derivatives. Since the market has $N + 3$ shocks, the market is potentially complete. The locally risk-free asset follows

$$dB_t = r_t B_t dt, \tag{A.2.2}$$

where r_t is real short rate determined in equilibrium. In addition, since stock $n = 1, \ldots, N$ is a claim to $D_{n,t}$, the return dynamics of stock n is

$$dR_{n,t} = \frac{dP_{n,t} + D_{n,t}dt}{P_{n,t}} = \mu_{R_n,t}dt + \sigma'_{R_n,t}dw_t, \tag{A.2.3}$$

where $\sigma_{R_n,t} \in \mathbb{R}^{N+3}$. We solve for $\sigma_{R_n,t}$ and $\mu_{R_n,t}$ in equilibrium. We also assume that agents can trade a claim on the first Lucas tree, E_t, with return dynamics given by

$$dR_{E,t} = \frac{dP_{E,t} + E_t dt}{P_{E,t}} = \mu_{R_E,t} dt + \sigma'_{R_E,t} dw_t. \tag{A.2.4}$$

Besides the risk-free asset and the $N+1$ stocks, we assume that there are two zero net supply claims (derivatives) that agents can trade. The first derivative is linked to the shock to δ_t

$$dR_{w_\delta,t} = \mu_{w_\delta,t} dt + dw_{\delta,t}, \tag{A.2.5}$$

and the second derivative is linked to the shock to E_t,

$$dR_{w_E,t} = \mu_{w_E,t} dt + dw_{E,t}, \tag{A.2.6}$$

where $\mu_{w_\delta,t}$ and $\mu_{w_E,t}$ are determined in equilibrium. It is convenient to summarize the price system in terms of the stochastic discount factor. In our economy, agents have different beliefs, and therefore perceive different market prices of risk. Consequently, each agent perceives the stochastic discount factor differently. The dynamics of the stochastic discount factor as perceived by agent j is

$$dM_t^j = -r_t M_t^j dt - \theta_{E,t} M_t^j dw_{E,t} - \theta_{z,t}^j M_t^j dw_{z,t}^j - \sum_{n=1}^{N} \theta_{n,t}^j M_t^j dw_{n,t}^j. \tag{A.2.7}$$

Under the true measure the stochastic discount factor has the dynamics

$$dM_t = -r_t M_t^j dt - \theta_{E,t} M_t dw_{E,t} - \theta_{z,t}^j M_t dw_{z,t} - \sum_{n=1}^{N} \theta_{n,t} M_t dw_{n,t}, \tag{A.2.8}$$

where $\theta^j_{z,t} = \theta_{z,t} + \Delta^j_z s_t$ and $\theta^j_{n,t} = \theta_{n,t} + \Delta^j_n (1 - s_t)$. In equilibrium, we have $\mu_{i,t} = r_t + \theta'_t \sigma_{i,t}$ for $i = E, w_E, w_\delta, R_1, \ldots, R_N$. Hence, the expected return perceived by agent j is related to the expected return under the true measure by

$$\mu^j_{i,t} = \mu_{i,t} + \Delta^j_{z,t} s_t \sigma_{i,z,t} + \sum_{n=1}^{N} \Delta^j_{n,t} (1 - s_t) \sigma_{i,n,t}. \tag{A.2.9}$$

As noted above, if Δ^j_z (or Δ^j_n) is positive it implies that agent j is optimistic about z (or ϵ_n). In that case, we see from Equation (A.2.9) that the agent will also perceive a higher expected return provided that the loading of the asset $\sigma_{i,z,t}$ (or $\sigma_{i,n,t}$) is positive. Note that we can define the disagreement process of agent j by η^j_t that links the perceived stochastic discount factor of agent j, M^j_t, to the stochastic discount factor under the true measure, M_t, by $M^j_t = M_t / \eta^j_t$. The disagreement process is formally a Radon-Nikodym derivative with dynamics

$$d\eta^j_t = \Delta^j_z s_t \eta^j_t dw_{z,t} + \sum_{n=1}^{N} \Delta^j_n (1 - s_t) \eta^j_t dw_{n,t}. \tag{A.2.10}$$

A.2.2 Preferences

Agents maximize lifetime utility given by

$$\mathbb{E}^j \left[\int_0^\infty e^{-\rho t} \frac{c_{j,t}^{1-\gamma}}{1-\gamma} dt \right], \tag{A.2.11}$$

subject to the dynamic budget constraint

$$\begin{aligned} dW_t^j &= \left(r_t W_t^j + \pi_{E,t}^j \left(\mu_{E,t}^j - r_t\right) + \pi_{w_E,t}^j \left(\mu_{w_E,t} - r_t\right)\right) dt \\ &\quad + \left(\pi_{w_\delta,t}^j \left(\mu_{w_\delta,t} - r_t\right) + \sum_{n=1}^{N} \pi_{n,t}^j \left(\mu_{R_n,t}^j - r_t\right) - c_{j,t}\right) dt \\ &\quad + \pi_{w_E,t}^j dw_{E,t} + \pi_{w_\delta,t}^j dw_{\delta,t} \\ &\quad + \pi_{E,t}^j \sigma_{E,t}' dw_t + \sum_{n=1}^{N} \pi_{n,t}^j \sigma_{R_n,t}' dw_t, \end{aligned} \tag{A.2.12}$$

with $W_0^j = w^j$ and where $\pi_{i,t}^j$ for $i = E, w_E, w_\delta, 1, \ldots, N$ is the dollar amount invested in asset i by agent j. Note the expectation in Equation (A.2.11) and the dynamics of the wealth in Equation (A.2.12) are under the belief of agent j.

A.2.3 Equilibrium

We start by defining the equilibrium.

Definition 1. *Given preferences, endowments, and beliefs, an equilibrium is a collection of allocations* $\left(c_{j,t}, \pi_{i,t}^j\right)$ *and a price system* $\left(r_t, \mu_{E,t}, \{\mu_{R_n,t}\}_{n=1}^N, \mu_{w_E,t}, \mu_{w_\delta,t}, \sigma_{E,t}, \{\sigma_{R_n,t}\}_{n=1}^N\right)$ *for* $j = 1, \ldots, J$ *and such that the processes* $\left(c_{j,t}, \pi_{i,t}^j\right)$ *maximize lifetime utility in Equation* (A.2.11) *subject to the dynamic budget condition in* (A.2.12) *and all market clear.*

Since the market is complete, we can solve the individual problem using Martingale methods as in Karatzas et al. (1987) and Cox and Huang (1989). The first-order conditions yield

$$c_{j,t} = \left(\kappa_j M_t / \eta_t^j e^{\rho t}\right)^{-1/\gamma}, \tag{A.2.13}$$

where κ_j is the Lagrange multiplier from the static optimization problem. The Lagrange multiplier is linked to the initial wealth of agent j. It is convenient to define $\lambda_{j,t} = \eta_t^j / \kappa_j$. By using the optimal consumption in (A.2.13) and the market clearing in the commodity market we have the following Proposition.

Proposition 1. *In equilibrium the optimal consumption of agent $j = 1, \ldots, J$ is*

$$c_{j,t} = f_{j,t} C_t, \tag{A.2.14}$$

where the consumption share, $f_{j,t}$, is

$$f_{j,t} = \frac{\lambda_{j,t}^{\frac{1}{\gamma}}}{\sum_k^J \lambda_{k,t}^{\frac{1}{\gamma}}}. \tag{A.2.15}$$

Moreover, the stochastic discount factor, M_t, is

$$M_t = e^{-\rho t} \left(\sum_j^J \lambda_{j,t}^{\frac{1}{\gamma}} \right)^{\gamma} C_t^{-\gamma}. \tag{A.2.16}$$

Proof. From the first order conditions we have

$$\lambda_{j,t} c_{j,t}^{-\gamma} = \lambda_{l,t} c_{l,t}^{-\gamma} \tag{A.2.17}$$

for any two agent j, l. Rearranging the above and using the clearing of the commodity market we get the result. □

The next proposition shows the equilibrium real short rate and the market prices of risk.

Proposition 2. *The equilibrium real rate is*

$$r_t = \rho + \gamma \left(\mu_{C,t} + \sigma_{C,t}' \mathcal{E}_t (\Delta) \right) - \frac{1}{2} \gamma (1 + \gamma) \sigma_{C,t}' \sigma_{C,t} + \frac{1}{2} \left(1 - \frac{1}{\gamma} \right) \mathcal{V}_t (\Delta) \tag{A.2.18}$$

and the market prices of risks are

$$\theta_t = \gamma\sigma_{C,t} - \mathcal{E}_t(\Delta) \tag{A.2.19}$$

where

$$\mathcal{E}_t(\Delta) = \sum_{j=1}^{J} f_{j,t}\Delta_t^j \tag{A.2.20}$$

is the consumption weighted average and

$$\mathcal{V}_t(\Delta) = \sum_{j=1}^{J} f_{j,t}\left(\Delta_t^j - \mathcal{E}_t(\Delta)\right)'\left(\Delta_t^j - \mathcal{E}_t(\Delta)\right) \tag{A.2.21}$$

is the consumption weighted average total variance of disagreement vector Δ_t^j

Proof. The results forllow from an application of Ito's lemma on the stochastic discount factor, M_t, in (A.2.16) and matching the terms with the dynamics in Equation (A.2.8). □

The risk-free rate is similar to the standard risk-free rate in a homogeneous beliefs economy with constant relative risk aversion (CRRA) preference with two exceptions. First, the true expected growth rate, $\mu_{C,t}$ is replaced with $\mu_{C,t} + \sigma'_{C,t}\mathcal{E}_t(\Delta)$. This turns out to be equivalent to the consumption share weighed average belief about the consumption growth. This is what Heyerdahl-Larsen and Illeditsch (2020b) refer to as the market view. Second, there is an additional term that depends on the consumption share weighted total variance of the disagreement vector, $\mathcal{V}_t(\Delta)$. This term is due to the speculative trade between the agents in the economy and the sign is linked to the value of γ. For $\gamma > 1$, the risk-free rate is higher when there is more disagreement in the economy. This is the channel explored in Ehling et al. (2018). We calculate stock price dynamic of stock n using Malliavin calculus. Specifically, it can be shown that the stock price loadings on the shocks are

$$\sigma_{R_n,t} = \theta_t + \frac{\mathbb{E}_t\left(\int_t^\infty \mathcal{D}_t\left(M_u D_{n,u}\right) du\right)}{\mathbb{E}_t\left(\int_t^\infty M_u D_{n,u} du\right)} \tag{A.2.22}$$

where $\mathcal{D}_t x_u = \left(\mathcal{D}_{E,t} x_u, \mathcal{D}_{\delta,t} x_u, \mathcal{D}_{z,t} x_u, \mathcal{D}_{1,t} x_u, \ldots, \mathcal{D}_{N,t} x_u\right)$ denotes the Malliavin derivative of x_u at time t. The next return dynamics is given by the next proposition.

Proposition 3. *The dynamics of stock $n = 1\ldots, N$ is*

$$dR_{n,t} = \mu_{R_n,t} dt + \sigma'_{R_n,t} dw_t \tag{A.2.23}$$

where

$$\mu_{R_n,t} = r_t + \theta'_t \sigma_{R_n,t}, \tag{A.2.24}$$

and where

$$\sigma_{R_n,t} = \sigma_{D_n} + \theta_t + \frac{\mathbb{E}_t\left(\int_t^\infty M_u\left(\mathcal{E}_u\left(\mathcal{D}_t log\left(\lambda_u\right)\right) - \gamma \mathcal{D}_t log\left(C_u\right)\right) du\right)}{\mathbb{E}_t\left(\int_t^\infty M_u D_{n,u} du\right)}, \tag{A.2.25}$$

and where $\mathcal{E}_u\left(\mathcal{D}_t log\left(\lambda_u\right)\right)$ is the consumption share weighted average of the Malliavin derivative of the log disagreement process η_u.

Proof. To derive the result note that we have

$$M_t P_{n,t} + \int_0^t M_u D_{n,u} du = \mathbb{E}_t\left[\int_0^\infty M_u D_{n,u} du\right] \tag{A.2.26}$$

From equation (A.2.26) we have that the right hand side is a local martingale. An application of Ito's lemma to the left hand side of equation (A.2.26) yields

$$d\left(M_t P_{n,t} + \int_0^t M_u D_{n,u} du\right) = \ldots dt + M_t P_{n,t} \left(\sigma_{R_n,t} - \theta_t\right)' dw_t \tag{A.2.27}$$

For the right hand side of equation (A.2.26), we apply the Clark-Ocone theorem, implying

$$d\mathbb{E}_t\left[\int_0^\infty M_u D_{n,u} du\right] = \mathbb{E}_t\left[\int_0^\infty \mathcal{D}_t \left(M_u D_{n,u}\right) du\right]' dw_t \tag{A.2.28}$$

Next we calculate the Malliavin derivative in equation (A.2.28)

$$\begin{aligned}
\mathcal{D}_t \left(M_u D_{n,u}\right) &= D_{n,u} \mathcal{D}_t M_u + M_u \mathcal{D}_t D_{n,u} \\
&= D_{n,u} \mathcal{D}_t \left(e^{-\rho u} \left(\sum_j^J \lambda_{j,u}^{\frac{1}{\gamma}}\right)^{\gamma} C_u^{-\gamma}\right) + M_u D_{n,u} \sigma_{D_n} \\
&= D_{n,u} e^{-\rho u} \gamma \left(\sum_j^J \lambda_{j,u}^{\frac{1}{\gamma}}\right)^{\gamma-1} \frac{1}{\gamma} \sum_{j=1}^J \lambda_u^{\frac{1}{\gamma}} \mathcal{D}_t log\left(\lambda_u\right) + M_u D_{n,u} \sigma_{D_n} \\
&= M_u D_{n,u} \left(\sum_{j=1}^J f_u^j \mathcal{D}_t log\left(\lambda_u\right) + \sigma_{D_n}\right) \\
&= M_u D_{n,u} \left(\mathcal{E}_u \left(\mathcal{D}_t log\left(\lambda_u\right)\right) + \sigma_{D_n}\right)
\end{aligned} \tag{A.2.29}$$

Inserting equation (A.2.29) into equation (A.2.28), equating it with the diffusion coefficients in equation (A.2.27) and solving for the stock price diffusion coefficients $\sigma_{R_n,t}$ yields the result. □

As we are interested in the exposure of the agents' wealth to the shocks in the economy, we also need to find the wealth dynamics. The next proposition characterizes the wealth dynamics of the agents in the economy.

Proposition 4. *Let W_t^j be the wealth of agent j at time t with dynamics*

$$dW_t^j = \mu_{W^j,t} W_t^j dt + \sigma'_{W^j,t} W_t^j dw_t, \tag{A.2.30}$$

Then the exposures of agent j's wealth to the shocks, $\sigma_{W^j,t}$, are

$$\sigma_{W^j,t} = \theta_t + \frac{\mathbb{E}_t \left(\int_t^\infty \mathcal{D}_t \left(M_u C_{j,u} \right) du \right)}{\mathbb{E}_t \left(\int_t^\infty M_u C_{j,u} du \right)} \tag{A.2.31}$$

Proof. Note that we have

$$M_t W_t^j + \int_0^t M_u C_{j,u} du = \mathbb{E}_t \left(\int_0^\infty M_u C_{j,u} du \right) \tag{A.2.32}$$

Following a similar approach as for the stock price diffusions coefficients, i.e., apply Ito's lemma on the left hand side and Clark-Ocone theorem on the right hand side then equating the diffusion coefficents yields the result. □

From Proposition 4 together with the wealth dynamics in Equation (A.2.12) one can calculate the optimal portfolios. As we are interested in the economy wide loading on the factor shock $w_{z,t}$, we instead directly examine the total absolute exposure to the factor shock:

$$TE_{ETF,t} = \sum_{j=1}^{J} \mid \sigma_{W^j,z,t} \mid \tag{A.2.33}$$

A.3 Simulation details

In this section we provide complete details on the simulation of the model economy from Section A.2 that underlies the testable predictions in Figures 1 and 2 in Section 1.6.

Specifying beliefs. Our main numerical illustration of the model assumes that there are $J = 2N$ agents in the economy. We set $\Delta_z^j = \Delta > 0$ for agent $j = 1, \ldots, N$. This implies that half of the agents are *optimistic* about the growth rate of the factor (recall Equation (6)). We set $\Delta_z^j = -\Delta$ for the remaining agents, implying that half of the population is *pessimistic* about the factor's growth rate. While this assumption is not necessary, and is both discussed and relaxed below, it ensures that our baseline results are not simply driven by an ex ante imbalance between optimists and pessimists. Specifically, in terms of the stock-specific beliefs underlying Equation (9) we make the following assumptions. First, we assume that all investors are optimistic about half and pessimistic about the other half of the stock-specific components. Specifically, for investor j's belief about the stock-specific component of stock n we assume that

$$\Delta_n^j = \begin{cases} \frac{\Delta}{\sqrt{N}}, & \text{if optimistic} \\ -\frac{\Delta}{\sqrt{N}}, & \text{if pessimistic} \end{cases} \tag{A.3.34}$$

We normalize the stock-specific component by $\sqrt{N}$ to keep the total disagreement about the stock-specific component equal to that of the factor disagreement for $s_t = 0.5$. Second, we assume that for each stock there are just as many optimists as pessimists. Finally, we assume that for each optimist about the factor component, there is an investor with the same beliefs about the stock-specific components that is pessimistic about the factor component. In this way, we turn off any correlation between the structure of the beliefs in the economy about the factor and stock-specific disagreement.

Overall, as half of the agents in the economy are optimistic about the factor and half of the agents in the economy are pessimistic about the factor, we refer to high s_t times as periods of high *factor disagreement*. Similarly, when s_t is low, most disagreement surrounds the idiosyncratic component of dividends. As such, we label these times as periods of high *stock-specific disagreement*.

Other parameters. Each simulation considers an economy with ten stocks ($N = 10$) and a disagreement parameter $\Delta = 0.2$. Thus, there are $J = 2N = 20$ agents in the economy. Moreover, we assume that each agent starts with the same initial consumption shares.We solve the model using a social planner problem with Pareto weight $\frac{1}{\kappa_j}$ for agent $j = 1, \ldots, N$. Starting with the same initial consumption shares is equivalent to assuming the homogeneous Pareto weights. There is a mapping between the Pareto weights and initial wealth. We also assume that the dividend shares of each stock are the same, and that the total dividends are initially 5% of total consumption.

We define the ETF as the value-weighted portfolio of all the ten stocks. As we consider a frictionless market, there is no intrinsic demand for the ETF in the model since investors could, in principal, trade the individual stocks underlying the ETF. Hence, we assume that agents prefer to trade the ETF instead of the underlying stocks in the ETF if their agent's goal is to take on factor exposure.[16] We average all model-implied results across 400,000 simulated paths of monthly data that each span 100 years.

A.4 Different trading measures

In the main body of the paper we consider the total factor exposure as our measure of flows and trading activity. In this section we examine two alternative measures. First, in the main body of the paper the exposure is the total dollar exposure. We can instead measure the exposure as the weighted average exposures of each of the investors in the economy. Specifically, we define

[16] For instance, if the ETF were an equal-weighted portfolio of the ten stocks in the economy, and agent j was holding one unit of each of stocks one to nine, and two units of stock ten, then we assume that the agent is holding one unit of the ETF and one additional unit of stock ten.

the weighted exposure as WE_t where

$$WE_t = \sum_{j=1}^{J} f^w_{j,t} \left| \frac{\sigma_{W_j,z,t}}{W_{j,t}} \right| \tag{A.4.35}$$

In addition, we consider a measure based on the excess exposure as defined in Heyerdahl-Larsen and Illeditsch (2020a). Specifically, the excess exposure is defined as

$$XE_t = WE_t - |\sigma_{ETF,z,t}|. \tag{A.4.36}$$

The excess exposure capture the the variation in wealth that is over and beyond that of a passive position. Finally, all the measure of exposures are monotonically increasing, so if we instead look at changes with respect to the factor disagreement share process, the measures also imply that the total exposure increases in the factor share process.

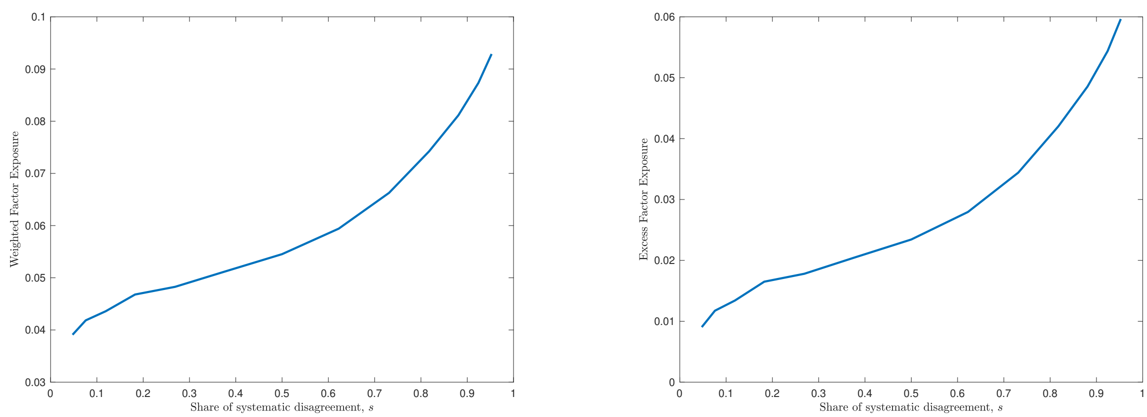

Figure A.4.2: The figure plots two measures of exposure for the baseline calibration. The right left hand plot is the weighted exposures and the right hand plot is the excess exposure. Parameters are as in the baseline case.

A.5 Empirical mapping of disagreement

In this section we discuss the relation between the empirical measure of factor disagreement in the data and the model. To do so, we compare the model implied equivalent of our empirical measure to the share process s_t. Throughout this section we keep the relative size of the stocks the same. Figure A.5.3 shows that our empirical measure of the share of factor disagreement is monotone in the factor share process s_t in our model.

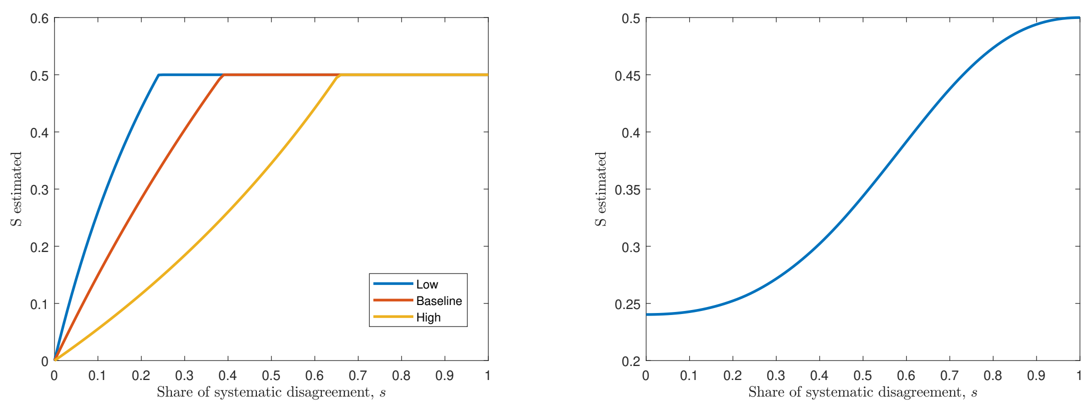

Figure A.5.3: The figures shows the relation between share process, s_t, and the empirical counterpart. The left hand plot shows the mapping for low, baseline and high idiosyncratic baseline disagreement. Idiosyncratic disagreement is $\Delta_j^i = +/-k\Delta/\sqrt{N}$, and we set $k = 0.5, 1, 3$ for low, baseline and high respectively. The right hand side is based on random disagreement where we draw Δ_j^i from normal distribution with mean zero and standard deviation of $\Delta/\sqrt{N}$