On the Relevance of Variances and Correlations for Multi-Factor Investors

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Abstract. Investors believing in diversification through multi-factor investing should prioritize factor variances over correlations when evaluating conditional portfolio risk. I decompose the variance of an equally-weighted multi-factor portfolio into two components, average variance (AV) across and average correlation (AC) between factors, and show that both components are needed to explain current portfolio risk. However, the subsequent forecasting exercise reveals that only AV predicts future multi-factor risk and return over the short run, therefore supporting the idea of short-term persistence in risk and a variance-in-mean relationship uncovered only when focusing on AV. The results show robustness in various instances and caution investors to bet on a dysfunctional variance-in-mean relationship by forming a volatility-managed multi-factor portfolio.

Keywords. *multi-factor investing, risk-return tradeoff / forecasting, variances, correlations, multi-asset, state variables, predictors, volatility-managed portfolios*

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1 Introduction

Diversification, integral to both investing and life more generally, has a historical foundation dating back centuries.[1] Today, the idea is typically linked to the influential work of Markowitz (1952), who formally establishes the concept within the famous mean-variance framework and describes the role of correlations among assets in achieving an optimal risk-return tradeoff. Estimating these co-measures using financial market data is done countless times a day, as they are necessary inputs for models in various fields of finance. With growing awareness of diversification's relevance, new ideas have refined and expanded the concept, while making it more accessible for investors. Prominent examples are the capital asset pricing model (CAPM), the idea of (mostly) efficient markets, and the birth of the passive fund management industry. Specifically, product developments such as index mutual and exchange-traded funds (ETFs) have marked important milestones, providing low-cost diversification to the masses.

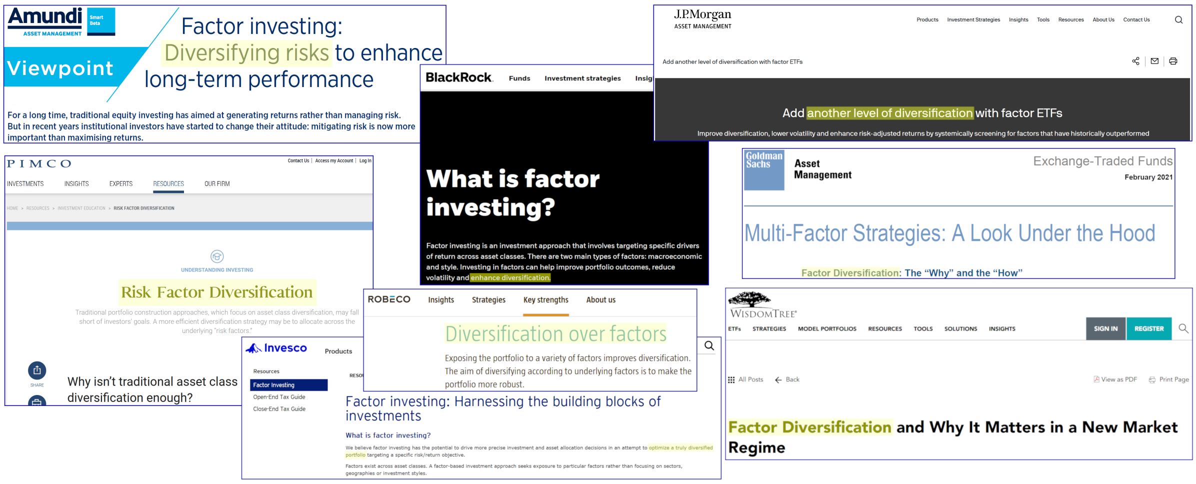

Figure 1: Examples of Asset Managers Advertising Diversification Through Factor Investing.
The figure combines screenshots from websites of various asset managers, which propose to enhance diversification by engaging in factor investing. Relevant text portions on diversification are highlighted in yellow.
Screenshots were compiled on November 11, 2022 from the following sources:
Amundi (https://www.amundi.com/institutional/files/nuxeo/dl/9102f718-3202-4362-b41d-056aaf0024e7),
BlackRock (https://www.blackrock.com/us/individual/investment-ideas/what-is-factor-investing),
Goldman Sachs Asset Management (https://www.gsam.com/content/dam/gsam/pdfs/us/en/fund-resources/investment-education/look-under-the-hood-multi-factor-strategies.pdf?sa=n&rd=n),
Invesco (https://www.invesco.com/us/resources/factor-investing?audienceType=Investor), J.P. Morgan Asset Management (https://am.jpmorgan.com/us/en/asset-management/adv/investment-strategies/etf-investing/capabilities/factor-etfs/),
Robeco (https://www.robeco.com/me/key-strengths/quant-investing/glossary/diversification-over-factors.html),
Pimco (https://europe.pimco.com/en-eu/resources/education/understanding-risk-factor-diversification),
WisdomTree Investment (https://www.wisdomtree.com/investments/blog/2020/05/13/factor-diversification-and-why-it-matters-in-a-new-market-regime)

Since models are typically not only assessed for theoretical elegance but also for their predictive performance, various researchers have empirically tested the implications of the CAPM. The results indicate that, while the market factor helps in assessing stock (portfolio) risk, other cross-sectional risk sources unrelated to the market also impact equity returns (Fama and French (1993)). Those insights have given rise to factor investing, enabling investors to harvest risk premia from these alternative sources of risk, alongside the traditional market risk premium. Consequently, the asset management industry has developed new products, often called 'smart beta', to make the approach more accessible to investors. Following this narrative, factor investing, particularly in a broad sense

[1] References to 'naive' diversification (dividing exposure equally among risky opportunities) trace back to the Talmud's origins (Duchin and Levy (2009)).

across multiple styles and asset classes (ASCLs), offers significant potential for diversification by combining these uncorrelated sources of compensated risk.[2] This has prompted asset managers to actively market the benefits of diversification through factor investing (see Figure 1). For long-term factor investors, it is, thus, crucial to assess future factor correlations when forming a strategic allocation.[3]

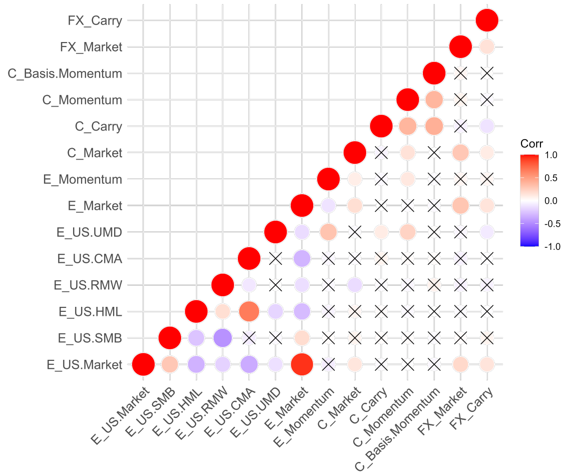

Figure 2: Correlations of Factor Excess Returns.
This figure shows Pearson correlation coefficients for 14 monthly factor excess return TS and results of t-tests with the null hypothesis of no correlation. Data covers the full investigation period from July 1971 to December 2018. Coefficients are represented by circles, of which size and color indicate correlation magnitude and sign. Insignificant coefficients are marked with crosses. The factor labels' first letters (at the left and lower margin) indicate the associated ASCL of a given factor, where *C, E*, and *FX* are abbreviations for *commodity, equity*, and *foreign exchange*, respectively. Consult Table A4 and Section 3 for an overview of all factor TS analyzed. Detailed data set information is available in Appendix A.1 and Vincenz and Zeissler (2024). An overview of the abbreviations used in the course of this paper is provided in Table A3.

Various academic studies, highlighting benefits of diversification across factors, have shaped this assessment.[4] Using a broad set of factors constructed over three ASCLs with data ranging back to July 1971,[5] I can corroborate an overall uncorrelated behavior (Figure 2). Since numerous studies support time-varying correlations in asset return series generally[6] and in factor/style returns specifically (e.g., Ilmanen (2022)), I also observe temporal dynamics, revealing that the broadly uncorrelated pattern remains stable over time.[7] Moreover, when comparing traditional diversification (across asset classes) with a broad multi-factor strategy, the latter is notably more effective in enhancing the risk-return tradeoff – especially during periods of turbulences, such as the great financial crisis.[8]

[2] The appendix of Vincenz and Zeissler (2024) provides a more detailed literature review on factor investing.
[3] Selecting factors for the investment universe with positive expected future excess returns from various proposed phenomena is also crucial, but beyond my study's scope. Refer to Ilmanen (2022) for a summary of common criteria from the literature to evaluate a factor's long-term outlook.
[4] As examples, refer to Bender et al. (2010), Asness et al. (2013), Houweling and Van Zundert (2017), or Ilmanen et al. (2021).
[5] The data set is introduced in Section 3.
[6] See Longin and Solnik (1995), Longin and Solnik (2001), Ang and Chen (2002), Cappiello et al. (2006), or Preis et al. (2012).
[7] Figure A1 presents supporting evidence (rolling factor correlations). Correlations broadly appear time-varying, with some factor pairs experiencing periods of spiking correlation. However, these spikes usually do not occur simultaneously across all factor pairs. For other pairs, a stable uncorrelated pattern persists over time.
[8] Figure A2 illustrates the diversification benefits of uncorrelated factors by showing cumulative log excess returns of four factor strategies, capturing broad market exposure across U.S. and international equity, commodity, and FX. Moreover, the figure illustrates the cumulative performance of two multi-factor portfolios. The first naively allocates across the four market factors (resembling traditional diversification), while the second defines diversification more broadly and allocates naively across the entire factor set, thus achieving a better risk-adjusted return by volatility reduction (see also Figure 3).

The empirical support for diversification benefits through multi-factor investing prompts a key question: Given the narrative of close to zero (or even negative) correlations, should multi-factor investors actually care about conditional correlations between the factors in their portfolio? Or should they focus on factor variances, since these are the main driver of portfolio risk under the assumption of zero correlations? To the best knowledge of the author, no existing study covers these questions.

I contend that long-short multi-factor investors, guided by academic research and asset manager advertisements, expect to hold a portfolio of uncorrelated return streams. Consequently, they are primarily concerned with simultaneous increases in variances across factors as drivers of their aggregate portfolio risk. Conversely, correlations are deemed fixed and without time-varying impact on overall risk. Put simply, despite their confidence in multi-factor diversification (and zero correlations), these investors remain vulnerable to synchronous spikes in variances. After a shock, this vulnerability should translate into higher expected future multi-factor returns. Specifically, since asset return variances tend to show short-term persistence, investors should expect variances to stay elevated after a broad spike, resulting in higher expected future multi-factor risk and potentially increased returns in the short run, assuming a variance-in-mean relationship.

The empirical investigation covers 14 long-short factor strategies across three ASCLs, equity, commodity, and foreign exchange (FX), from July 1971 to December 2018. Using a simple multi-factor benchmark, i.e. an equally-weighted monthly-rebalanced portfolio of all factors, I analyze the roles of average variance (AV), average correlation (AC), and the product of both (as proxy for portfolio variance) in explaining current and future benchmark risk and return. As the in-sample (IS) analysis shows, all three variables help to span contemporaneous variance of the equally-weighted multi-factor portfolio, indicating that both components, variances and correlations, are needed to explain current benchmark risk. However, AV emerges as the best-performing predictor of future multi-factor variance over a one-year horizon. Importantly, only AV also demonstrates explanatory power for future one-year average returns. Specifically, my results indicate a variance-in-mean relationship which is revealed solely after untangling the opposing effects of AV and AC on future returns.

This finding is robust across various subperiods, short forecast horizons (e.g., six months or two years), and multivariate specifications incorporating macroeconomic and market predictors or alternative approximations of multi-factor risk as control variables. Extending the forecast window to 60 and 120 months reveals that other variables gain relevance for longer-term forecasts, for example AC as well as external macro and market variables. I observe the strongest predictability of future benchmark variance at the longest window (ten years), while for returns, the best-working horizon is five years. Notably, return predictability is most pronounced when multi-factor returns exhibit mean-reversion, both AC and AV contribute to the forecast, and additional variables provide significant excess information about future returns. Using all regressors tested, the model explains around 74% of the IS variation in future mean returns over the five-year horizon. An analysis of alternate equally-weighted multi-factor portfolios, linked to ASCLs or styles, confirms AV's importance over AC as a driver of future multi-factor risk and returns in the short run.[9] Moreover, employing AV, AC, and a simple proxy for overall portfolio variance as predictive variables in an out-of-sample (OS) exercise affirms that primarily factor variances drive

[9] Only for portfolios combining either solely market-, carry-, or FX-associated factors, the IS evidence of a variance-in-mean relationship is comparably weak or (for the latter) even non-existent.

future returns. At the one-year horizon, the associated $OS\text{-}R^2$ of AV (observed in isolation) ranges from 10.36% to 13.48%, depending on the starting period of the test.

The important insight of these results is the investment implication for multi-factor investors: Be cautious when betting on a dysfunctional variance-in-mean relationship (as suggested for instance in Moreira and Muir (2017)). As an important secondary finding, I report a declining trend in multi-factor returns over the course of the investigation period, whereas lasting trends in AV or AC are absent. Put differently, while both components of multi-factor risk have remained stationary over time, multi-factor investing has lost some of its attractiveness (in terms of risk compensation).

The paper broadly contributes to the factor investing literature, mainly by empirically exploring the impact of the two components of multi-factor portfolio risk – factor variances and correlations – on the future portfolio risk-return tradeoff. Moreover, it connects to broader areas such as forecasting aggregate assets' risk-return tradeoff, examining variance and correlation risk premia, and assessing the effectiveness of macroeconomic and market data in describing and predicting asset returns. The findings bear importance for academics and practitioners. Researchers, from an asset-pricing perspective, should emphasize characterizing periods of synchronously spiking variances across cross-asset factor strategies, which are often linked to elevated global uncertainty and precede times of higher-than-average risk and returns for multi-factor investors . During such times, staying invested might require increased risk appetite and risk-bearing capabilities. For practitioners, identifying AV as forecaster of multi-factor variance and returns can enhance their ability to assess and manage conditional risk-return tradeoffs.

The paper is organized as follows: Section 2 discusses the importance of variances and correlations for multi-factor investors, motivating the hypothesis that investor focus on variances when assessing short-term portfolio risk. Section 3 introduces the data set of long-short factor returns, which is analyzed in the subsequent Section 4, covering the empirical application. In the last Section 5, I conclude with a short summary of the findings. The paper includes appendices with additional remarks (Appendix A) and exhibits (Appendix B), which are referenced throughout the paper. Abbreviations used in this paper are listed in Table A3.

2 The Importance of Variances and Correlations for Multi-Factor Investors

Consider an investor employing N long-short factor strategies to gain (naive) diversification by holding and maintaining (through frequent rebalancing) an equally-weighted factor portfolio. The conditional variance of portfolio returns is given by

$$\sigma^2_{p,t} = \sum_{i=1}^{N}\sum_{k=1}^{N} w_{i,t} w_{k,t} \sigma_{ik,t} = \frac{1}{N^2}\sum_{i=1}^{N}\sum_{k=1}^{N} \sigma_{ik,t}, \tag{1}$$

where $\sigma_{ik,t}$ represents the conditional covariance of factor i with factor k at time t and $w_{i,t}$ $[w_{k,t}]$ defines the portfolio weights of factors i $[k]$, which are simply set to $w_{i,t} = w_{k,t} = \frac{1}{N}$. An attractive feature of multi-factor investing is the documented uncorrelated behavior among long-short factor strategies (see Footnote 4 in Section 1). Therefore, an investor who diversifies across factor styles and ASCLs may expect to hold a portfolio of risky, yet uncorrelated (on average, positive) return streams, or put differently, a set of truly orthogonal risk premia (with

$\sigma_{ik,t} = \sigma_{ik} = 0, \forall i \neq k$). This investor has a simpler perception of portfolio risk:

$$\sigma^2_{p,t} = \frac{1}{N^2} \sum_{i=1}^{N} \sigma^2_{i,t}. \tag{2}$$

In other words, an investor assured of uncorrelated factor returns is only concerned with factor variances as driver of (time-varying) portfolio risk; correlations are deemed irrelevant.

This perspective differs from that of a typical diversified long-only equity investor, who knowingly holds a portfolio of moderately (or even highly) correlated assets. For such an investor, changes in both variances and covariances can have a considerable impact on overall portfolio variance. Accordingly, studies identify variance and correlation risk premia, compensating those who enable investors to hedge against stochastic variance and correlation fluctuations.[10] In this context, return co-movements become particularly crucial during market stress, that is, when diversification is needed most and often breaks down, as empirical studies have observed (e.g., Ang and Chen (2002) or Preis et al. (2012)).

Pollet and Wilson (2010) describe a model linking stock return correlations to the market risk premium, while showing simultaneously that AV may be un- or even negatively related to the risk premium. Empirically, they find higher AC to predict higher future U.S. equity market excess returns. Conversely, when controlling for AC, AV exhibits a negative yet insignificant link, supporting the model's hypothesis that changes in aggregate risk can translate into shifting correlations between stocks. However, Chen and Petkova (2012) report divergent results for the U.S. equity market. They find AV as only significant predictor of future market excess returns and highlight differences in sample period, data frequency, and stock universe as potential reasons for deviations to Pollet and Wilson (2010). More recently, Jondeau et al. (2019) assess AV and AC in comparison to average skewness as predictors for future U.S. stock excess returns and report favorable results for the latter variable. [11]

In prior studies on equity, AV (or average volatility) is typically understood as a proxy for aggregate idiosyncratic risk (e.g., Campbell et al. (2001), Goyal and Santa-Clara (2003), Bali et al. (2005), or Jondeau et al. (2019)), while AC is often regarded as measure of aggregate market risk (e.g., Pollet and Wilson (2010), Jondeau et al. (2019)).[12] For a multi-factor investor, AV across all factors in the portfolio quantifies the conditional risk of investing in a single arbitrary factor (idiosyncratic component), while AC proxies the conditional benefits of diversifying across factors (systematic component).[13] The total conditional risk of the multi-factor portfolio depends on both components and two key considerations: (1) Is it generally a risky time to invest in any arbitrary factor strategy? (2) Does broad diversification across factors offer risk reduction benefits for multi-factor investors, as suggested in the literature (see Section 1)? Starting from these considerations, we can think further about what a spike in either of the two components implies.

To emphasize the implications of surging AC, we can consider an extreme scenario, that is, AC climbs from near 0 to 1, while factor variances and unconditional mean returns remain stable. In this case, instead of an

[10] See for instance Carr and Wu (2009), Bollerslev et al. (2009), and Buss et al. (2017) on variance risk premia and Driessen et al. (2009), Buraschi et al. (2014), Mueller et al. (2017), and Buss et al. (2017) on correlation risk premia.

[11] Following Jondeau et al. (2019), I observe average skewness as control variable in my empirical analysis, see Section A.2.1.

[12] AC is also sometimes associated with investor disagreement (see e.g. Buraschi et al. (2014) or Jondeau et al. (2019)).

[13] Instead of measuring diversification benefits (i.e. low [high] AC, high [low] benefits), one may view the systematic component as proxy for systematic multi-factor risk (low [high] AC, low [high] systematic risk), akin to Pollet and Wilson (2010). The interpretation as diversification benefits follows authors such as Campbell et al. (2001) or Buss et al. (2017).

investment universe with N orthogonal risky strategies, the investor now can only access one sort of compensated risk exposure mimicked by all N strategies. In other words, the formerly distinguishable risk factors have become a single integrated risk factor, preventing the investor from realizing any further diversification benefits through multi-factor investing.[14] Essentially, this would imply that we have circled back to a single factor affecting all asset returns, reminiscent of the CAPM – despite the empirical evidence suggesting diversification through multi-factor investing due to low factor correlations (see Section 1).

Examining the idiosyncratic component, a surge in AV signals higher risk for an investment in any arbitrary factor, indicating cross-sectional variance clustering. This refers to periods with concurrently elevated return variation across the factor universe. What could be an economic interpretation of such shocks?

Considering factor premia as compensation for distinct risks, a rise in a factor's (stochastic) variance increases the risk for investors harvesting the premium. For instance, after an economic shock, investors might be less willing, capable, and likely to bear a factor's risk, leading to disinvestment and reduced risk-sharing.[15] Alternatively, more individuals may be willing to compensate others for risk-bearing, supplying more risk to share in aggregate. In both cases, a risk-based explanation might further suggest that the increased risk implies a higher conditional risk compensation.

Another prominent reasoning for outperforming factor strategies is widespread irrational investor behavior, which is exploitable by sophisticated market players with the necessary funds and risk appetite.[16] From this perspective, increasing factor variance implies that exploiting the market anomaly caused by irrational investors has become more risky. Assuming again an economic shock, investors might be less willing, capable, and likely to exploit the anomaly, causing broader market distortions and more noise trader risk for the investors remaining to exploiting the anomaly. Alternatively, the shock may fuel the anomaly by prompting more irrational investor behavior, increasing stakes for investors betting against the irrationality.[17] Both cases again highlight scenarios of greater risk, yet also greater profit opportunities for factor investors.[18]

Importantly, both risk-based and behavioral perspectives on factors can align with the idea of exogenous economic shocks affecting factor strategies' conditional risk and return. After such shocks, factor investors will likely need more risk appetite and capabilities to bear risk to stay invested. Therefore, at the aggregate multi-factor level, events triggering AV spikes should introduce significant uncertainty about any risky endeavor's future and have economic implications on a global scale, given a broad set of factors, covering assets of many countries across the world, various ASCLs, and different factor styles. For multi-factor investors, even those diversified across

[14] The term "*no-place-to-hide* state variable", introduced by Buss et al. (2017), also suits AC. Following the notion of Buraschi et al. (2014), it underscores correlation risk as a non-diversifiable risk within (stock) portfolios.

[15] Investors' funding liquidity may suffer from shocks due to destabilizing margins (which increase in illiquidity), leading to excessive deleveraging and liquidity spirals. As Brunnermeier and Pedersen (2009) outline, destabilizing margins are particularly evident when fundamentally evaluating investments becomes challenging for financiers, which could be the case following a major sudden shock. The authors also highlight the correlation between market liquidity and volatility and emphasize cross-sectional co-movements in liquidity, since typically investors facing capital constraints sell multiple assets.

[16] Exploiting market anomalies is usually not without risks for investors (e.g., De Long et al. (1990)).

[17] An example might be a shock severely stressing a substantial portion of investors. Porcelli and Delgado (2009) show experimentally that stress leads individuals to rely on established reactions to risk, potentially intensifying pre-existing behavioral biases when making financial decision (see also Starcke and Brand (2012) for a literature summary on decision making under stress). For instance, collective stress during a recession due to fears of job loss illustrates this phenomenon.

[18] I abstract from cases where factors cease to produce excess returns, for instance due to investor correcting their behavioral biases, causing anomalies to disappear. Moreover, I do not delve into data mining as explanation for factor performance; refer to Footnote 3 for more on this topic.

several strategies with supposedly zero correlations, it poses a serious challenge to mitigate such broad-based shocks.

Why is that? By defining $AV_t = \frac{1}{N}\sum_{i=1}^{N}\sigma_{i,t}^2$ and re-arranging Equation 2, the portfolio risk when all assets are assumed to be uncorrelated, it becomes clearer:

$$\sigma_{p,t}^2 = \frac{1}{N^2}\sum_{i=1}^{N}\sigma_{i,t}^2 = \frac{AV_t}{N}. \tag{3}$$

For a multi-factor investor assuming zero correlations, the impact of the systematic component (diversification benefit) is deemed fixed. Therefore, $\sigma_{p,t}^2$ depends solely on AV, representing the idiosyncratic component (risk of investing in any arbitrary factor), and on the number of factor strategies (N) available for naive diversification. To further simplify Equation 3, we can assume, as in Pollet and Wilson (2010), that all factors exhibit the same variance, i.e. $\sigma_{i,t}^2 = \sigma_{k,t}^2 = \sigma_t^2$. In this case, $AV_t = \frac{1}{N}\sum_{i=1}^{N}\sigma_{i,t}^2 = \sigma_t^2$, and total portfolio risk is given by:

$$\sigma_{p,t}^2 = \frac{AV_t}{N} = \frac{\sigma_t^2}{N}. \tag{4}$$

When AV (aka σ_t^2) spikes, the only risk management tool available to lower portfolio risk to pre-spike levels, aside from reducing overall risk exposure through disinvestment, is to include additional sensible factor strategies in the portfolio, thereby enhancing diversification benefits and reducing risk.[19]

However, identifying new, reliable and uncorrelated strategies is time-consuming and costly. While today's factor zoo offers myriad candidate strategies, this was not the case in the 1970s at the beginning of my data sample, when gathering data was more inconvenient and there was less data available to conduct meaningful backtests. Today, however, identifying candidate strategies is no longer a major challenge, but rather evaluating their sensibility and reliability. In sum, the number of strategies available to investors, N, is likely to be costly to increase and, therefore, rather persistent, especially in the short run. [20] Consequently, the only feasible way to decrease risk is to disinvest, i.e. to reduce exposure to the risky multi-factor portfolio.

As asset return variance exhibits short-term persistence (e.g., Cardinale et al. (2021)), investors may anticipate a cross-sectional variance shock to persist in the short run. For example, investors constrained by portfolio variance limits may hesitate to invest or remain invested after a spike in variances across factors that is likely to show some persistence. Therefore, less-risk appetite of (multi-)factor investors following a spike might lead to higher expected risk and required return compensation in the upcoming period, that is, a variance-in-mean relationship.[21] As a result, conditional AV should assist in short-term forecasting of multi-factor risk and returns. Conversely,

[19] This idea follows Campbell et al. (2001) and Connor et al. (2006), who find increased idiosyncratic stock volatility over time. In consequence, they argue, more randomly-chosen stocks have become necessary to reduce portfolio risk to any given level.

[20] While I focus on the short run, it is also unrealistic to assume investors in a competitive environment can endlessly discover new, reliable, and uncorrelated factor strategies, even with ample time, means, and willpower to conduct research. Ultimately, the existence of an infinite number of profitable systematic trading strategies, which all can be rationalized by a compelling (risk-based or behavioral) narrative, seems implausible.

[21] The concept implies a positive association between expected risk and investor compensation. Various studies (e.g., Campbell (1987), Glosten et al. (1993), Harvey (2001)) have tested the validity of this idea using equity (market) data, however, with only limited success, as Pollet and Wilson (2010) summarizes. More recently, Moreira and Muir (2017) have presented conflicting evidence by building volatility-managed portfolios, which deliver risk-adjusted outperformance by exploiting a weakening in the risk-return tradeoff when volatility spikes. The authors highlight that variance is stochastic and highly-persistent, while having no explanatory power for future returns; their results and investment implications remain highly debated in the literature. Liu et al. (2019), Cederburg et al. (2020), Barroso and Detzel (2021), and Angelidis and Tessaromatis

as investors expect factors to proxy orthogonal risks (at least in the short run[22]), AC between factors should lack predictive power. In other words, acknowledging stochastic factor variances but assuming fixed near-zero correlations may lead investors to compensate only for variance, but not for correlation risk.

The setup has another testable implication. The difference between Equations 1 and 2 lies in multi-factor investors' *perception* of uncorrelated factors. Yet, this is not entirely accurate – some factors are not perfectly orthogonal and correlations fluctuate to some extent over time (see Section 1). Thus, I would expect both components, AV and AC, to be necessary to decompose contemporaneous portfolio variance.

A final implication emerges from the above line of thought. In the short run, multi-factor investors focus on variances, *perceiving* factors as totally uncorrelated. However, some time of higher-than-excepted correlations between factors may lead investors to reassess, potentially abandoning some strategies with diminishing diversification benefits. Accordingly, correlations may show more relevance in the medium/long run. This expectation is supported by the findings of Buss et al. (2017), who find the variance risk premium most relevant at relatively short horizons, while the correlation risk premium shows more predictive power at longer horizons when compared to the former premium.

3 Factor Data

This study employs a subset of monthly end-of-month return TS for diversified long-short factor portfolios, initially introduced in Vincenz and Zeissler (2024). The set spans 14 factor strategies within the ASCLs equity (comprising U.S. single stocks and international equity indices), commodity, and FX.[23]

In detail, the equity section includes the five well-known Fama-French U.S. factors and two international equity country index factors. Commodity factors are derived from exchange-traded commodity futures contracts for 31 commodities. Lastly, FX factors are based on monthly spot and forward U.S. dollar exchange rates of 69 currencies. Factors are constructed by cross-sectional ranking assets based on factor characteristics and forming long-short portfolios with the top/bottom 16.67% of assets. Granular transaction costs are applied for more realistic returns (see Table A4 in the Appendix). For details on the construction and methodological differences to other papers, see Appendix A.1 and Vincenz and Zeissler (2024). Factor returns are ex-ante volatility-scaled (10%) with an expanding window of all previous returns.[24] Unless stated otherwise, returns discussed are log excess returns, in line with Pollet and Wilson (2010).

As a simple multi-factor strategy, I use the equally-weighted 'naive' benchmark, akin to Vincenz and Zeissler (2024). It includes all available factors at a given time, representing the broad market of factor premia. Additionally,

(2023) critique OS implementability or after-cost profitability (i.e. limits to arbitrage). Conversely, DeMiguel et al. (2021) present a volatility-managed multi-factor strategy outperforming OS and after transaction costs.

[22] Investors perceiving factor returns as uncorrelated might be especially convinced in the short run, given that correlations typically exhibit a slower pace of change compared to volatilities (see Frazzini and Pedersen (2014)).

[23] Table A4, taken from Vincenz and Zeissler (2024), provides a detailed overview of the factors. Compared to Vincenz and Zeissler (2024), the dataset here differs in two ways: I exclude factors with negative average monthly returns, except for E_US.SMB, leading to the exclusion of C_Value, FI_Carry_Slope, FI_Momentum, FX_Momentum and FX_Value. I keep the Fama-French U.S. Size factor, since it is quite common in the literature. Additionally, I exclude fixed income factors due to their limited number of observations and convert the unbalanced set into a balanced one.

[24] This approach sizes each factor with equal ex-ante volatility, enhancing diversification by reducing the risk contribution of highly volatile factors (e.g., Blin et al. (2021), Vincenz and Zeissler (2024)). It fits along the lines of Equation 4, which assumes all factors have the same variance.

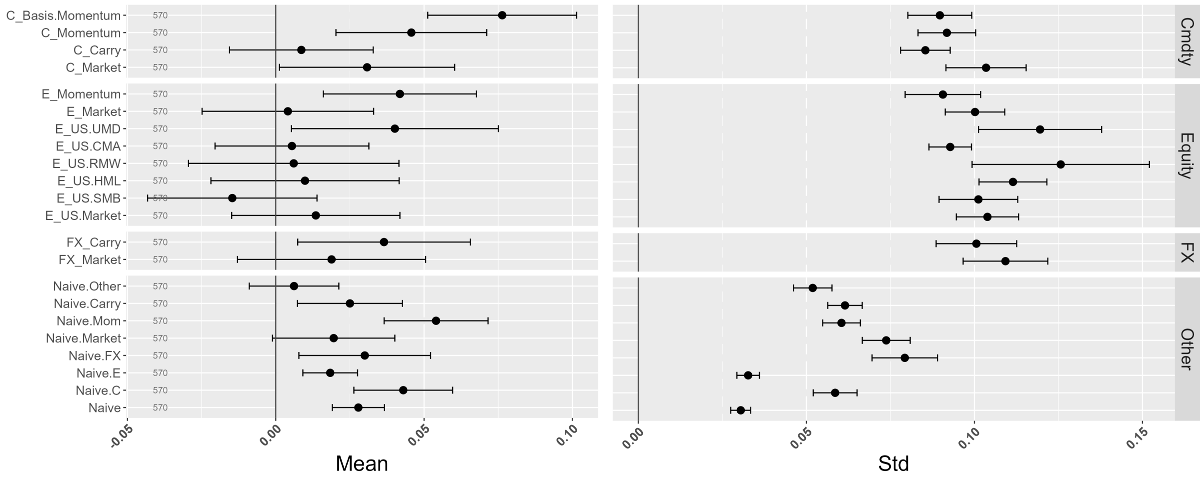

Figure 3: Mean and Standard Deviation of Factor Excess Returns.
This figure presents summary statistics for the set of 14 monthly factor excess return TS and the various multi-factor portfolios. In detail, the left panel reports the annualized arithmetic mean with associated two-sided 95% confidence intervals as box plots, while the right panel analogously reports the annualized standard deviation. Data covers the full investigation period from July 1971 to December 2018. The factor labels' first letters (at the left margin) indicate the associated ASCL of a given factor, where *C, E*, and *FX* are abbreviations for *commodity, equity*, and *foreign exchange*, respectively. The grey numbers on the right beside the factor labels report the available number of monthly observations per factor during the investigation period starting in July 1971 and ending in December 2018. Standard errors for normal confidence intervals are obtained by bootstrapping (see Appendix A.6 for more information). Monthly mean returns and their standard errors are annualized by multiplying with 12, while standard deviations and their standard errors are multiplied with $\sqrt{12}$. Consult Table A4 and Section 3 for an overview of all factor TS analyzed. Detailed data set information is available in Appendix A.1 and Vincenz and Zeissler (2024). An overview of the abbreviations used in the course of this paper is provided in Table A3.

I consider two sorts of equally-weighted substrategies, defined by factor association to either (1) a respective ASCL or (2) a specific factor style.[25] Monthly rebalancing ensures consistent exposure for all equally-weighted strategies.

For a concise overview of return series in the balanced set, Figure 3 presents means and standard deviations over the investigation period from July 1971 to December 2018, with associated two-sided 95% confidence intervals as box plots. As shown, mean returns and volatilities vary notably within and across ASCLs. In sum, the balanced set offers a global perspective on risk factors, making it ideal for studying the relevance of variances and correlations for multi-factor investors.[26]

Figure 4 reports cumulative log excess returns of all factors and the naïve benchmark over time. In line with the low correlations depicted in Figure 2, combining factors – even in a naive manner – substantially reduces portfolio volatility (see also Figure 3). Even during the great financial crisis (or the associated NBER recession), in which several factors crashed at some point, the drawdown of the naive benchmark is comparably small. This is because the worst declines in individual factor strategies during this period did not occur entirely synchronously, preserving some diversification benefits. The overall smooth performance of the naive benchmark is reassuring for multi-factor investors and asset managers advocating diversification across factors. In sum, these descriptive

[25] Styles include carry, momentum, and market. Non-attributable strategies are categorized as 'Other'. Most factors' style is evident from their name. Commodity basis-momentum falls under momentum, while U.S. single-stock value, size, quality, and investment factors are non-attributable.

[26] A significant caveat of using this extensive historical dataset is that the main hypothesis - multi-factor investors' focus on variances - presumes investor awareness of these systematic investment strategies and their low cross-correlations in the first place. Both was likely not broadly given at the beginning of the sample period. Thus, for robustness, I re-estimate the main regression specification using subsamples.

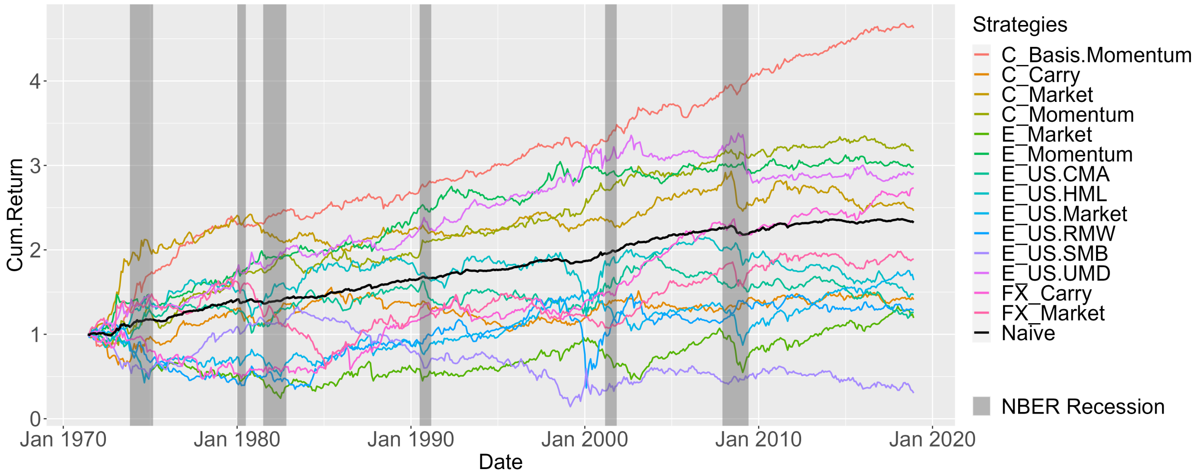

Figure 4: Cumulative Log Excess Returns of Factors and Naïve Benchmark.
This figure plots cumulative log excess returns of the constructed set of 14 monthly factor TS and of the naïve benchmark, which equally weighs all available factors in the investment universe at a given point in time, over the full investigation period from July 1971 to December 2018. Moreover, gray shading is highlighting periods of recessions in the U.S., as defined by the NBER. The factor labels' first letters (plotted at the right margin) indicate the associated ASCL of a given factor, where *C, E*, and *FX* are abbreviations for *commodity, equity*, and *foreign exchange*, respectively. Consult Table A4 and Section 3 for an overview of all factor TS analyzed. Detailed data set information is available in Appendix A.1 and Vincenz and Zeissler (2024). An overview of the abbreviations used in the course of this paper is provided in Table A3.

statistics contribute to the perception of factor strategies as largely uncorrelated return streams, raising questions about the relevance of factor correlations for multi-factor investors.

4 Empirical Application

In Section 4.1, I outline the methodology for my key measures, which proxy the conditional variance of the multi-factor portfolio. Subsequently, Section 4.2 presents the main IS results, followed by a summary of robustness tests in Section 4.3.

4.1 Methodology - Approximations for Benchmark Variance

As suggested by Pollet and Wilson (2010), I approximate the variance of the equally-weighted multi-factor benchmark by the product of two components. Specifically, at time t, I observe the product of the weighted AV of individual factors (AV_t) and the weighted AC between all factor pairs (AC_t):

$$\hat{\sigma}^2_{\text{Naive},t} \approx AV_t \times AC_t \tag{5}$$

Pollet and Wilson (2010) analytically show that this product equals the portfolio variance when all portfolio assets share the same variance. Empirically, they confirm its viability as a proxy for U.S. stock market portfolio variance.

While Pollet and Wilson (2010) use daily stock data over a quarter to estimate the two components (for predictions over the subsequent quarter), I use a different approach for my set of $N = 14$ monthly factor TS. At time t, AV is the equally-weighted average across sample variances of all factors, estimated over the previous twelve months:

$$AV_t = \frac{1}{N}\sum_{i=1}^{N}\hat{\sigma}^2_{i,t_{-12}:t} \tag{6}$$

Similarly, AC is the equally-weighted average of all [27] elements in the sample correlation matrix. The sample correlation between factor i and k, $\hat{\rho}^2_{ik,t-12:t}$, is again estimated using the last twelve observations[28]:

$$AC_t = \frac{1}{N^2} \sum_{i=1}^{N} \sum_{k=1}^{N} \hat{\rho}^2_{ik,t-12:t} \tag{7}$$

Following Pollet and Wilson (2010), I construct two competing conditional variance measures based solely on the equally-weighted portfolio's return data. The first, VAR_t, is the sample variance of the multi-factor benchmark over the preceding 12 months. The second, $VAR.G_t$, is derived by estimating a GARCH(1,1) model with an expanding window and extracting the one-step ahead forecast.

Name	Symbol	Obs	Min	Mean	Max	Std	Skewness	Kurtosis	Autocorr (1)
Average Variance	AV_t (12M)	559	0.018	0.086	0.433	0.067	2.43	6.83	0.98
Average Correlation	AC_t (12M)	559	1.484	9.671	24.203	4.788	0.79	0.06	0.92
Naive Variance	VAR_t (12M)	559	0.001	0.008	0.043	0.008	2.21	4.83	0.95
Naive Variance GARCH	$VAR.G_t$	559	0.004	0.010	0.119	0.008	6.73	65.67	0.72

Table 1: Approximations for Benchmark Variance.
This table lists the approximations for the variance of the multi-factor benchmark. Additionally, column 'Obs' reports the available number of monthly observations per variable over the full investigation period, starting in July 1971 and ending in December 2018. Moreover, in the last seven columns the table provides summary statistics of the predictor TS, specifically the minimum and maximum monthly observation as well as monthly arithmetic mean, standard deviation (all stated as percentage), skewness, excess kurtosis, and the first-order autocorrelation. The TS of the variables were tested to rule out the possibility of containing unit roots with sufficient confidence (see Table A5). Refer to Section 4.1 for information on all variables that approximate (components of) multi-factor variance (i.e. AV_t, AC_t, VAR_t, and $VAR.G_t$). An overview of the abbreviations used in the course of this paper is provided in Table A3.

Table 1 presents summary statistics for the constructed measures, and Figure 5 illustrates their dynamics over the investigation period. Consistent with previous research highlighting diversification benefits across factors, AC remains low throughout the sample period, averaging 0.1 with a standard deviation of 0.05. Furthermore, AC_t is positively skewed, indicating periods of correlation clustering. However, AC_t never surpasses 0.24 over the entire investigation period, a relatively low level compared to stock correlations.[29] Thus, even during times of synchronously rising correlations, multi-factor investors still retain some diversification benefits.

When comparing summary statistics for AV_t and the sample variance estimated from multi-factor returns (VAR_t), the latter's mean (0.01%) is much lower than the former's (0.09%). This aligns with the findings of Pollet and Wilson (2010) regarding their respective measures for the stock market. Moreover, AV_t exhibits notably higher volatility than the benchmark variance measure (0.07% vs. 0.01%), also consistent with Pollet and Wilson (2010). Additionally, both series show positive skewness and excess kurtosis to a much greater extent than AC_t, indicating periods of substantial simultaneous volatility spikes across multiple factors.

[27] In defining AC, I deviate from Pollet and Wilson (2010), who exclude diagonal elements in the sample correlation matrix when calculating their measure (i.e. $AC_t = \frac{1}{N(N-1)} \sum_{i=1}^{N} \sum_{k \neq i} \hat{\rho}^2_{ik,t-12:t}$). Following their approach for my set of long-short factors results in periods with AC falling below zero (see Figure A5). In consequence, the variance approximation $\hat{\sigma}^2_{\text{Naive},t} \approx AV_t \times AC_t$ takes on insensible negative values, leading me to prefer the definition in Equation 7.

[28] I adopt a rolling 12-month horizon to align with the estimation window for variances when calculating AV (similar to Pollet and Wilson (2010)). However, this is relatively short for estimating correlations. Frazzini and Pedersen (2014), dealing with monthly returns, use one-year horizons for standard deviations and five-year windows for correlations, requiring at least twelve non-missing data points for the former and 36 observations for the latter. Their argument for this approach is evidence that correlations change more slowly than volatilities.

[29] For instance, the AC measure constructed by Pollet and Wilson (2010) (based on U.S. equity data) averages 0.237 and reaches a maximum of 0.646 over their sample period.

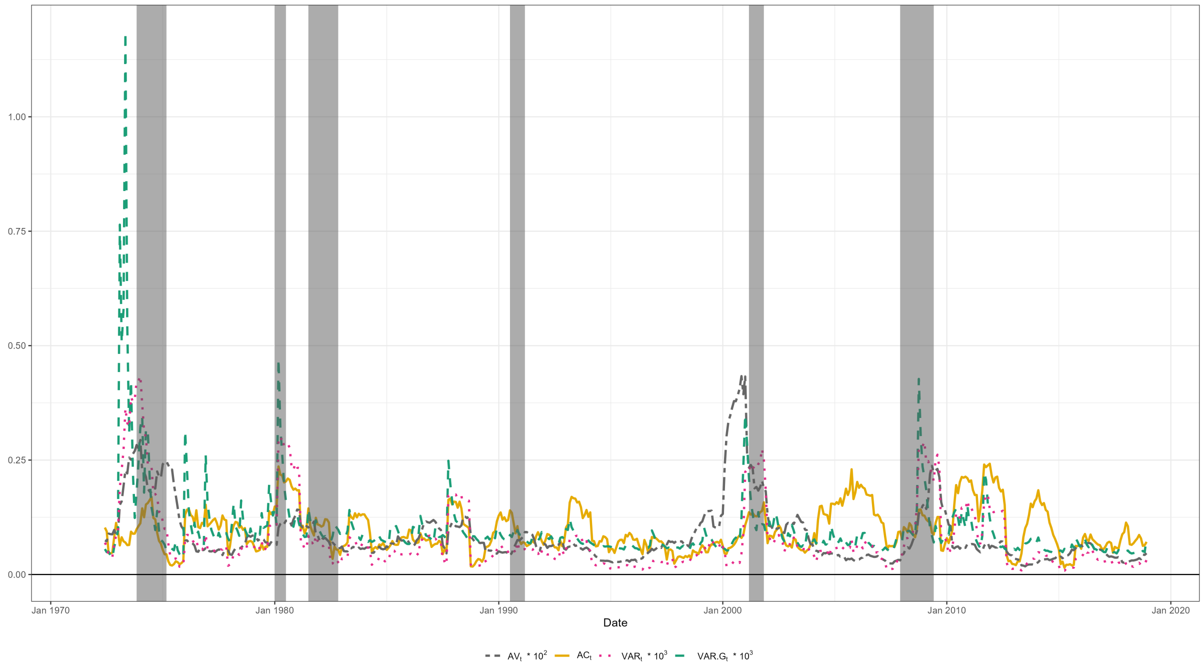

Figure 5: Regressors over Time. Approximations of Benchmark Variance.
This figure presents the dynamics of the approximations of multi-factor variance used as regressors over the full investigation period from July 1971 to December 2018. Moreover, gray shading is highlighting periods of recessions in the U.S., as defined by the NBER. Refer to Section 4.1 for information on all variables that approximate (components of) multi-factor variance (i.e. AV_t, AC_t, VAR_t, and $VAR.G_t$). An overview of the abbreviations used in the course of this paper is provided in Table A3.

Table 1 also provides insights into the series' persistence by presenting first autocorrelation coefficients. All series exhibit substantial, some extreme, serial correlation, with AV_t showing the highest coefficient (0.98), and $VAR.G_t$ the lowest (0.72). While observing high persistence is generally in line with results by Campbell et al. (2001) and Pollet and Wilson (2010) on equity market variance, it is more pronounced here due to the use of rolling overlapping windows $VAR.G_t$, using an expanding window for fitting the GARCH model, displays the lowest persistence (which is still considerable in absolute terms).

Given the high serial correlation, I test whether the series contain unit roots by conducting Augmented Dickey-Fuller tests (see Table A5 in the Appendix). However, I reject the null hypothesis of existing unit roots at the five percent level and proceed to analyze the TS in levels. Furthermore, a visual analysis of the series in Figure 5 reveals temporary spikes, but no lasting trend. These findings align with prior research on U.S. equity data. When analyzing their AV measure, Pollet and Wilson (2010) identify multiple peaks and troughs but no discernible trend. Moreover, studies by Schwert (1989), Campbell et al. (2001), and Connor et al. (2006) find no sustained rise in U.S. equity market volatility over time. Similarly, my data indicates highly volatile periods for multi-factor investors but no clear overall increase in risk (variance) throughout the study period.

Supplementing these results, I pinpoint periods with major spikes in the measures (see Table A6 in the Appendix). AV_t surged in the early to mid 1970s, concurrent with the OPEC oil embargo, rising global inflation, and the ensuing U.S. recession. Afterwards, AV_t peaked only twice to a similar extent – during the Dotcom Bubble burst and subsequent global economic slowdown in the early 2000s, and in 2009, coinciding with the conclusion of the Global Financial Crisis and the associated U.S. recession. The spikes in AV_t during these times of increased economic and financial uncertainty are not entirely surprising and match those detected by Jondeau et al. (2019)

for U.S. stock data. Importantly, the peaks in AC_t do not coincide with these renowned periods, providing further evidence of factor diversification benefits, even in such uncertain times. Thus, in accordance with the narrative in Section 2, I observe occasional surges in the risk of an arbitrary factor investment, particularly during periods of heightened global uncertainty; yet, broad diversification has consistently reduced risk for multi-factor investors, even during these challenging times.

Finally, an important observation concerns spikes in VAR_t, the sample variance of the multi-factor return series. Three out of four peaks in VAR_t coincide (partially) with AV_t spikes, indicating that AV_t has a relatively greater impact on total variance dynamics of the multi-factor benchmark compared to AC_t.[30] Furthermore, examining the TS behavior reveals that both total multi-factor variance and the idiosyncratic component (AV) tend to be higher during economic downturns, roughly aligning with NBER recessions. This pattern mirrors prior findings on the counter-cyclical behavior of equity variance measures relative to the business cycle[31] and supports the narrative of major global economic shocks triggering increased AV, as discussed in Section 2.

As there are various studies examining other variables to predict aggregate risk and returns, I consider numerous control variables to provide a comprehensive comparison. Details are provided in the Appendix. Specifically, I observe other approximations of multi-factor risk, covering higher moments (average skewness/kurtosis), return dispersion (cross-sectional variance/skewness/kurtosis), and the two TS phenomena momentum and value (Section A.2.1). Additionally, I further control for the influence of external macroeconomic and market data, such as inflation, fiscal balance, money supply, yield curve steepness, business cycle information, and market-implied volatility (Section A.2.2).

4.2 IS Results

Following Pollet and Wilson (2010), Section 4.2.1 tests how the regressors relate contemporaneously to multi-factor variance.[32] Hereafter, Section 4.2.2 assesses their power to forecast future benchmark variance over different horizons. Accordingly, Section 4.2.3 reports results for (mean) return forecasts. To be concise, recurring tables covering regression results for the various horizons, referenced frequently in the following, are placed in Appendix B (see Table A7 to A19). An aggregate perspective, summarizing results across all horizons, is provided for the variance forecasting in Figures 6 and 7 and for the return forecasting in Figures 8 and Figure 9.

4.2.1 Variance Decomposition

Table A7a shows the first set of OLS regression results, concerning the constructed approximations for benchmark risk. In the first two columns, using either solely AV or AC to explain contemporaneous benchmark variance, both estimated coefficients show notable t-statistics (2.26 and 5.07, respectively) and exhibit a positive sign. Similar to

[30] This is in line with the results of Jondeau et al. (2019), who find in their study of U.S. equity data that market variance and AV show similar dynamics over time.

[31] An early study by Officer (1973) explores the link between U.S. equity market variability and industrial production as proxy for business fluctuations. Campbell et al. (2001) find higher market-, industry-, and firm-level volatility for U.S. equity during recessions; Connor et al. (2006) broadly confirm these results. Jondeau et al. (2019) report similarities in the dynamics of U.S. stock market variance and a measure of AV across stocks, also noting that market variance spikes typically during NBER recessions. Moreira and Muir (2017) observe co-movement in volatility across their factor set (mostly US-equity-based) and report widespread spikes in recessions. In contrast, Pollet and Wilson (2010) find no evident link between their AV and AC measures and NBER recessions.

[32] Here and in the following regressions, the rolling measures estimated from the monthly data use overlapping windows.

the findings of Pollet and Wilson (2010), AV accounts for a larger portion of variation (31%) compared to AC (23%). However, the pattern is less pronounced than in their study covering the U.S. stock market.[33]

The third column combines both measures linearly and reports even larger t-statistics for both AV (2.32) and AC (6.55), but also for the estimate of the intercept term (-3.15), which in this specification establishes significance for the first time. Moreover, the specification is able to explain slightly more variation (57%) than the sum of the individual approaches. Subsequently, Column four shows results for the product of AV and AC. The term appears highly significant, with a t-statistic of 4.44, and the model accounts for a higher portion of variation (69%) as the specification combining both terms linearly. In addition, the estimate of the intercept term appears - again - insignificant in this scenario. However, the equation term is not reaching the magnitude of statistical significance (t-statistic of 39.1) and explanatory power (R^2 of 98%) that is reported by Pollet and Wilson (2010) in their study using daily and quarterly stock market data.

The fifth column provides results for a model combining all previous equation terms. In this setup, only the coefficient associated with the product of AV and AC is still significantly different from zero (t-statistic of 6.52), while R^2_{adj} gets only a minor boost by incorporating the individual terms and rises from 69% to 72% (comparing with the previous column).

The following columns (six to twelve) report findings for the additional risk approximations, observed in isolation, while the last column shows results for a specification combining all previous variables. Only CSK_t (column ten) achieves a significant estimate standalone (t-statistic of -4.05), but the R^2_{adj} is small (3%) compared to AV_t, AC_t, and their product. When observing all variables simultaneously, the coefficients of almost all variables (with VAL_t as exception) are accompanied by absolute t-statistics higher than 2, with those of AV_t and the product of AV_t and AC_t being exceptionally high (-6.87 and 13.83, respectively). In terms of variation explained, this model is slightly better than the specification solely based on AV_t, AC_t, and their product (72% vs. 87%, compare column five), indicating that the additional predictors lead to some modest improvements. However, the estimate of the intercept shows significance for the more complex model in the last column, contrasting the simpler model in column five.

Analogously to Table A7a, Table A7b shows the second set of OLS regression results, concerning the external candidate predictors. The first seven columns report results for each predictor individually, the following six for successively combining the variables, and the last three columns for specifications combining all analyzed regressors. Considered standalone, only inflation (column one; t-statistic of 2.44), the business cycle indicator ADS (columns six; t-statistic of -3.06), as well as the VIX (columns seven; t-statistic of 4) exhibit statistically significant estimates, where an increase in INFLTN or VIX leads to increased contemporaneous benchmark variance, while an increase in ADS has the opposite effect.[34] This indicates that in times of rising (falling) inflation or market-implied equity volatility, investors in a naively diversified multi-factor strategy face more (less) return variation, while an improving (worsening) business environment typically comes with lower (higher) variance in returns. The observation of negative signs estimated for the coefficients of the business cycle variables is consistent with the notion of a counter-cyclical dynamic between variance measures and the business cycle, as established

[33] In their study, Pollet and Wilson (2010) report a R^2 of around 70% (37%) for AV (AC) when decomposing contemporaneous variance.

[34] The effect of the second business cycle indicator, CFNAI, is similar in terms of its direction compared to ADS, but the coefficient estimate and t-statistic associated are notably smaller in absolute magnitude.

by various other authors (see Section 4.1). When comparing the standalone explanatory power of the external variables with those of the variance approximations, only the VIX shows similar strong performance (R^2_{adj} of 30%). This does not come as a surprise given the VIX's interpretation as market-implied measure of (equity) risk.

Turning to the specifications that successively combine the predictors (columns eight to 13), only INFLTN stays relevant (in terms of t-statistic) in all of the model versions except for the last iteration, in which the VIX is added and, thereafter, the only variable with a significant coefficient estimate. When also considering AV, AC the product of both, and the other risk approximations in parallel to all external predictors (last three columns of Table A7b), several variables jointly show significant estimates. Most importantly, the product of AV and AC appears - as before - highly relevant for explaining contemporaneous variance.

In sum, the analysis shows sufficient support of the product of AV and AC being an adequate approximation of benchmark variance, even after controlling for various other candidate predictors. This is in line with the results of Pollet and Wilson (2010) in the context of the stock market and the framework of multi-factor investing outlined in Section 2.

4.2.2 Variance Forecasting

Similar to Pollet and Wilson (2010), I analyze in the next step whether the contemporaneous regressors have power to forecast future benchmark variance, calculated over the subsequent 12-month period. Table A8a shows the first set of OLS regression results, concerning the constructed approximations for benchmark risk. The first two columns explore models solely based on either AV or AC as explanatory variable. Here, only AV exhibits an estimated coefficient with a notable t-statistic (4.68); using the associated model allows to explain roughly 12% of variation in future variance. This finding is underlined by column three, which shows a specification combining both variables with only marginally higher R^2_{adj}. In comparison: Pollet and Wilson (2010) find both variables predicting future stock market variance standalone, but with AV appearing as the more powerful predictor.

Column four shows that the product of AV and AC, which has shown strong explanatory power for contemporaneous variance, is (to a lesser extent) also useful in the forecasting exercise (t-statistic of 2.24; R^2_{adj} of 3%), similarly to the results of Pollet and Wilson (2010). However, column five reveals that after controlling for both AV and AC, the estimated coefficient of the product term appears no longer significant, while the coefficient of AV (AC) exhibits a t-statistic of 2.96 (0.75). This suggests that the information relevant for forming predictions about future multi-factor portfolio variance is inherent to the variances across underlying factors (i.e. AV), in contrast to the correlations between those factors (i.e. AC).

Column six to nine provide insights into how the competing measures of benchmark variance, based on the multi-factor return series, performs standalone, as well as combined with AV. Regardless of whether comparing the variables standalone or in a combined setting, AV emerges as the better performing predictor and explains a notably higher portion (12%, standalone) of variation in future benchmark variance than the variance measure based on benchmark returns (3%) and the GARCH approach (6%).[35] However, the specification jointly applying AV_t and $VAR.G_t$ (column nine) indicates that predictions based on AV can be slightly improved by incorporating GARCH: Both coefficient estimates show significance and R^2_{adj} is slightly higher compared to observing AV standalone.

[35] As an interesting side note: For this rather short forecast horizon, the GARCH estimate is a better predictor of future variance than simply use contemporaneous (short-term) variance. This is, indeed, no surprise given that GARCH models are favorably applied in short-term volatility forecasting.

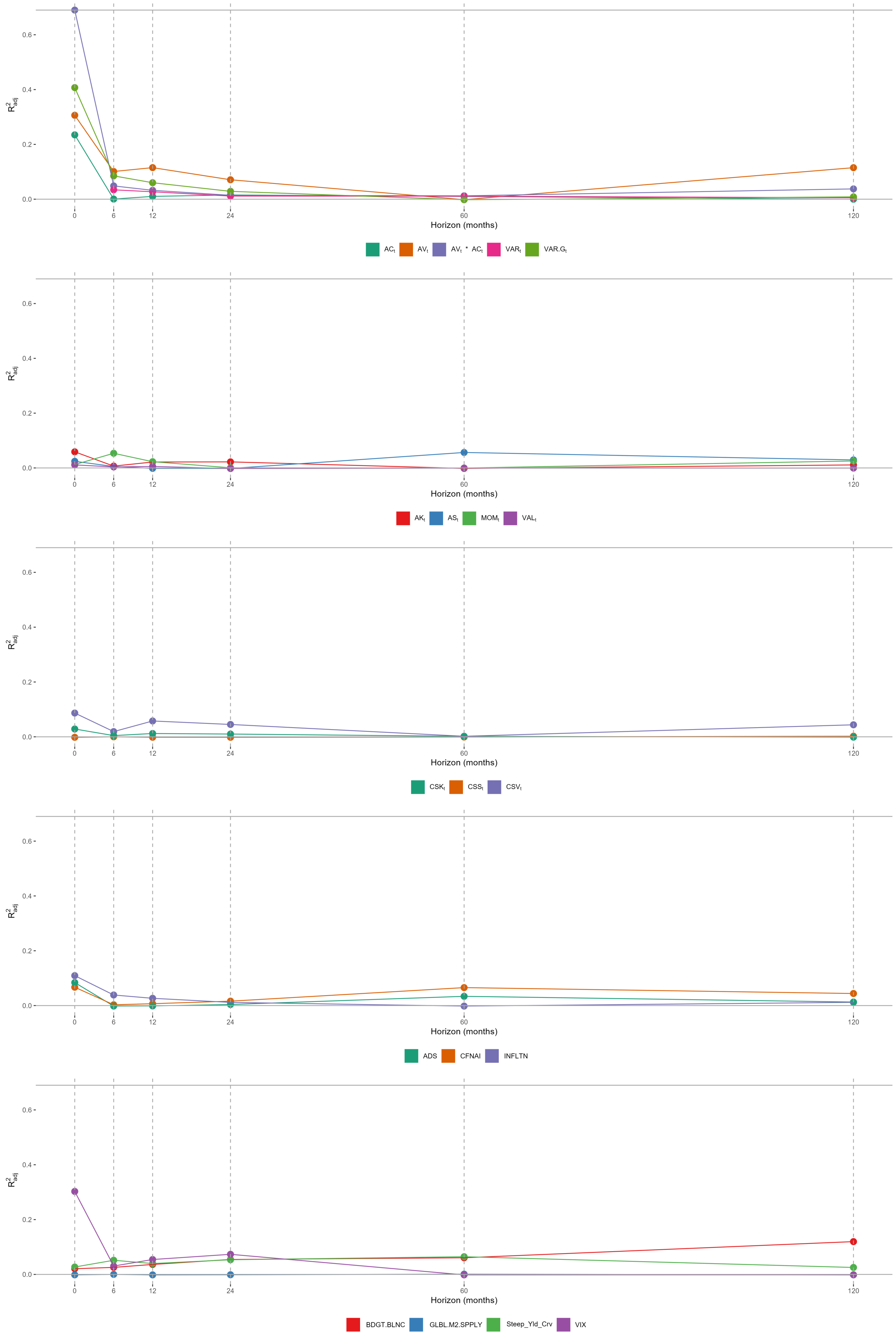

Figure 6: IS Explanatory Power of One-Regressor Specifications per Forecast Horizon. Variance Decomposition/Forecasting. (Caption on the next page.)

Figure 6: IS Explanatory Power of One-Regressor Specifications per Forecast Horizon. Variance Decomposition/Forecasting.
This figure illustrates the adjusted R^2 (R^2_{adj}) of the various measures used as explanatory variable for decomposing/predicting the conditional variance of the naive multi-factor portfolio, when each variable is applied in isolation over the various tested forecast horizons (the results of the decomposition of contemporaneous variance are indicated by the horizon labeled '0'). More information on the associated regression results (for instance the R^2_{adj}, expressed as number) are provided in Tables A7 to A12. Refer to Sections 4.1 (multi-factor variance), A.2.1 (multi-factor risk), and A.2.2 (external predictors) for an overview of explanatory variables in the regressions. An overview of the abbreviations used in the course of this paper is provided in Table A3.

The subsequent columns (ten to 16) report findings for the various other approximations of benchmark risk, observed in isolation, while the last column provides a combined model of all previous predictors. While CSV_t, CSK_t, and MOM_t - taken in isolation - exhibit significant estimates, the explanatory power (in terms of R^2_{adj}) of both is well below that established for AV. When considering all risk approximations jointly, several variables besides AV show significant estimates, but the coefficient of AV exhibits the highest t-statistic, highlighting AV as the main driver of predictability. Notably, the estimates of AV so far show significance in all tested regression specifications and the signs of the estimates are consistently positive across all model variations in Table A8a.

To summarize, investors can expect a rise (fall) in benchmark variance over the upcoming year after a rise (fall) in current AV, while no statistically significant effect between AC and subsequent variance can be detected. Importantly, this is in line with the predictions following from the view on multi-factor investing outlined in Section 2.

Table A8b shows the second set of OLS regression results, concerning the external candidate predictors. No external predictor - except for the VIX - exhibits a significant coefficient estimate when tested standalone. When combining the various variables, also other measures show more notable t-statistics in some cases, as for instance the business cycle indicator ADS (columns 12-13) or INFLTN (columns 15-16). However, even when controlling for these other variables (columns 14-16), the coefficients of AV still shows the highest observable t-statistics.

Besides the base scenario with a forecast window of 12 months, I also examine specifications over other forecast horizons to check for robustness of the results. Figure 6 provides a visual summary of the explanatory power per predictor (applied in a one-regressor specification) over the different horizons, while Figure 7 summarizes the statistical significance for each regressor and horizon. More details can be found in the appendix. Specifically, Table A9 shows results when forecasting future variance over the subsequent six-month period. Importantly, the findings qualitatively stay the same; while the forecast performance of AV is slightly less pronounced compared to the one-year period, it is still notable compared to the other variables.[36] The same holds true for a forecast horizon of 24-months (see Table A10). However, the performance is even less pronounced than observed for the six-month window.[37]

When moving to even longer forecast windows, an interesting pattern can be observed. Table A11 provides regression results when forecasting benchmark variance over the subsequent 60-month period. Five findings are

[36] A small difference in the findings, compared to the 12-month horizon, is that at this short horizon, using $VAR.G_t$ as predictor (t-statistic of 3.94 standalone) seems more promising in terms of significance than AV_t, despite the still strong performance of AV_t as predictor standalone (t-statistic of 3.59). Additionally, also MOM_t appears even better suited for the shorter window (t-statistic of 3.06) when compared to the one-year period.

[37] One notable aspect of the 24-month period is that it is the only among the various forecast horizons in which the coefficient of AC, which is consistently negative across different horizons, exhibits a significant estimate when observed on its own. However, when using both AV and AC, AC's t-statistic drops below 2, while AV stays significant. As a side note: Also the VIX shows relevance as predictor for this forecast horizon (standalone and combined with other variables, such as AV). In fact, it appears to be even more promising than for the six- and 12-month windows.

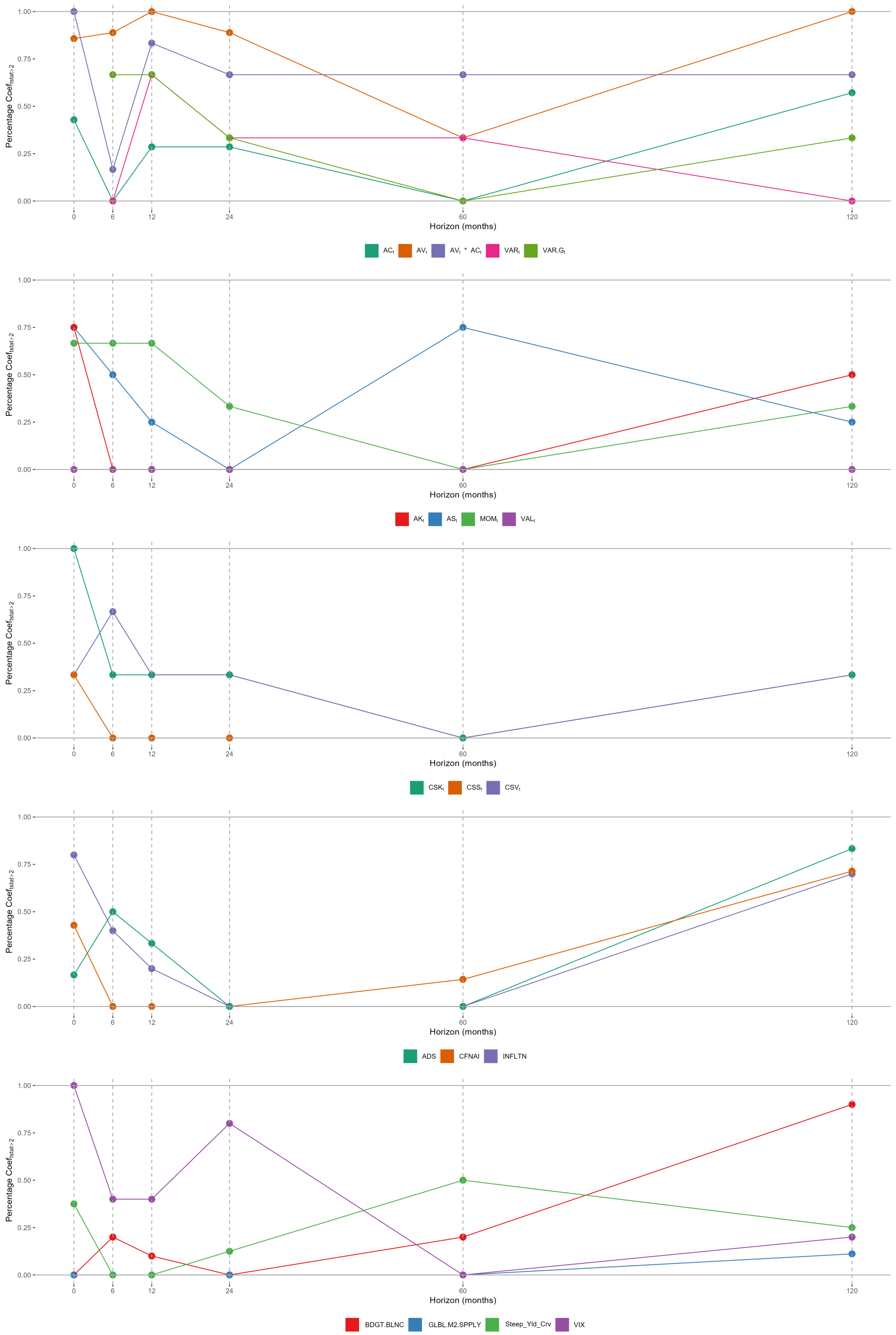

Figure 7: IS Statistical Significance per Regressor and Forecast Horizon. Variance Decomposition/Forecasting. (Caption on the next page.)

Figure 7: IS Statistical Significance per Regressor and Forecast Horizon. Variance Decomposition/Forecasting. This figure summarizes, for each of the explanatory variables used in decomposing/predicting the conditional variance of the naive multi-factor portfolio and per forecast horizons (the results of the decomposition of contemporaneous variance are indicated by the horizon labeled '0'), the percentage of t-statistics (over all tested regression specifications which include the given regressor) that exhibit an absolute value higher than 2. More information on the associated regression results (for instance the t-statistics, expressed as number) are provided in Tables A7 to A12. Refer to Sections 4.1 (multi-factor variance), A.2.1 (multi-factor risk), and A.2.2 (external predictors) for an overview of explanatory variables in the regressions. An overview of the abbreviations used in the course of this paper is provided in Table A3.

striking. First, the dominant role of AV (and its sidekick, $VAR.G_t$) as explanatory variable vanishes, while AC stays irrelevant; observed standalone or as linear combination (columns one to three), no coefficient associated with both variables exhibits significance at this horizon. Second, multi-factor variance shows (however, insignificant) mean-reverting behavior over the five-year period (compare the negative coefficient estimate in column six in Table A11a to Table A10a or A8a), broadly in line with the findings of Cardinale et al. (2021). Third, the product of AV and AC in this setting appears only to be helpful when used in combination with other predictors (especially AV); almost all coefficients associated exhibit significance across the specifications. The coefficient estimated is in all cases negative, i.e. the product – a proxy for the portfolio variance – also captures the mean-reverting dynamic that is observed for the sample variance measure. Fourth, while the variables approximating benchmark variance seem more important to forecast short-term benchmark variance, the external predictors gain importance for longer periods (comparing R^2_{adj} in column 12 of Table A8b, Table A10b, and Table A11b). The VIX is the only external predictor which behaves notably different, meaning equity-implied volatility proves relatively more helpful for shorter horizons and, therefore, resembles – not surprisingly – the other benchmark variance approximations.[38] Fifth, while none of the other risk approximations is of major help to improve the forecasts at this horizon, AS_t represents an exception and shows significant coefficient estimates in isolation as well as combined with the other predictors.

In the final step, I test the relationships applying a 120-month forecast horizon (see Table A12). For this extensively-long time period, mean-reversion of benchmark variance is - at a first glance - not evident anymore and AV is again the dominant predictor of the set of variance approximations reported in Table A12a, while the coefficients of AC and of the product of both variables are insignificant (observed in isolation).

In addition, Table A12b (together with the previous tables) reveals that the set of external predictors, while showing low explanatory power compared to the set of variance approximations at short horizon, is strongly (and almost monotonically) gaining importance for future variance forecasts when the forecast window is extended. For the 10-year period, the specification combining all external predictors (see column 13 in Table A12b) explains around 52% of variation in future benchmark variance, while the model combining the variance [risk] approximation terms (see column five [17] in Table A12a) only establishes a R^2_{adj} of roughly 15% [31%].

The dominant of the external explanatory variables at this horizon is the fiscal balance indicator, of which the coefficient estimate exhibits significance in nearly all associated specifications (t-statistic of 8.9 standalone), and which manages to explain around 12% in variation standalone (column 2). The positive coefficient is indicating that a fiscal balance surplus (deficit) typically leads to higher (lower) benchmark variance in the future. While yield curve steepness shows a significant estimate standalone, the explanatory power seems to get subsumed by the other

[38] The higher and more robust informativeness of the VIX for shorter variance forecast horizons seems also sensible when considering the construction of the index using 30-days option contracts on the S&P 500.

predictors in most of the specifications with multiple regressors. The inflation indicator, with a coefficient estimate insignificantly different from zero standalone, becomes relevant when applied together with BDGT.BLNC, but shows again insignificant estimates when also the business cycle series are considered in parallel. However, for all specifications based on even more variables (in columns 13-16 in Table A12b), the t-statistics associated with the inflation coefficient are comparably high (for instance -14.65 when considering all variables in column 16). The estimated sign of the coefficient is in all tested specifications consistently negative, indicating that a rise in global inflation predicts a fall in future multi-factor variance over long horizons. Interestingly, CFNAI and ADS show no notable forecasting performance when observed in isolation, but in almost all other specifications together with the remaining variables. Lastly, for the VIX no relevant predictive power is observed at the ten-year horizon, similarly to the five-year window.

While the insignificant results of AC and the product of AV and AC in a standalone model are somewhat contrary to the idea formulated in Section 2 of relevance assumed at longer horizons, both coefficients (together with AV), as well as those of other risk approximations show significant estimates for the last set of models that combines all regressors (see columns 14-16 in Table A12b). In other words, while the macro and market backdrop plays the dominant role when forecasting multi-factor risk at long horizons, variance (and broader risk) approximations help to further sharpen the prediction. The positive signs of the estimates indicate that - when controlling for the various external predictors that explain notable variation in future variance - higher AC, as well as AV, today leads to higher variance for the multi-factor portfolio in the far distant future. The mostly negative estimates for the product of both variables, which are observed in combination with other predictors, reveal that there is some more nuanced mean-reverting behavior at work after having controlled for the strong effect of AV. Lastly, it is notable that the variance forecasting results appear most accurate for the longest, followed by the two shortest, and trailed by the intermediate horizons (compare R^2_{adj} of last column's specification in the lower part of the previously referenced tables).

In sum, changing the forecast horizon essentially does not change the main finding observed at the 12-month window, as comparing the $R^2_{\text{adj}}$of AV, AC, and the other measures in Figure 6 (or the robustness of the statistical significance in Figure 7) highlights: AV is a comparably strong predictor of future multi-factor risk at short horizons (i.e. from six up to 24 months), while AC between factors is not of major importance . The pattern breaks at the 60-month horizon, where mean-reversion in benchmark variance comes into play and AV is not the sole dominating predictor anymore to forecast benchmark variance, but only shows some predictive power in combination with the product of AV and AC. For the longest forecast window of ten years, even though macro and market variables seem more relevant for the long-term outlook on future multi-factor risk, all three predictors - AV, AC, and the product thereof - show significant t-statistics when controlling for external variables. This fits the narrative of Section 2, suggesting that correlations between factors (in contrast to their variances) are not relevant for future benchmark variance and returns in the short run, but should ultimately be relevant in the long run.

Finally, it is worth to emphasize that the relationships observed at the 12-month horizon, when focusing either on AV or AC standalone, are consistent across all windows tested (meaning coefficients never change their sign, as captured by Table A20), i.e. higher AV [AC] predicts higher [lower] future multi-factor risk. This indicates that both components of overall portfolio variance work in opposite directions and that AV is the driving force behind the persistent in overall variance. Going further, the findings also explain the tendency of overall variance to show stronger, more significant persistence at shorter horizons and mean-reverting behavior in the medium run: For shorter windows, only the (positive) influence of AV is significant and, therefore, outweighs the (negative) influence of AC. For the 60-month period, the coefficients of both variables are insignificant, but the (negative) impact of AC is stronger (see Table A11a), leaving this forecast horizons to be the only one documenting a mean-reverting behavior in portfolio variance.

4.2.3 (Mean) Return Forecasting

In an intermediate step, before exploring how the regressors relate to future returns, I decompose contemporaneous monthly benchmark returns. Section A.3.1 in the Appendix reports and shortly describes the results (see also Table A13). In sum, no variable of the set of variance approximations (suggested in Section 4.1) is well able to explain contemporaneous multi-factor returns on a standalone basis, but combined with a broader set of other predictors, AV_t exhibits significant estimates in two specifications. Overall, the dynamics of contemporaneous returns are best explained by momentum as well as reversal patterns, (equity-)market-implied volatility, and the global money supply.

Next, I explore how the regressors relate to future returns at short horizons, i.e. one month into the future. The results are reported in Table A14. Notably, while AV has been previously of no use to decompose contemporaneous one-month benchmark returns, I report a t-statistic of 2.14 for the standalone model in column one of Table A14a, with R^2_{adj} of around 1%. The only other variable showing a coefficient estimate with a t-statistic surpassing 2 in Table A14 is observed for the product of AV and AC, also in the standalone specification (column four). The bottom line: While at least AV (and also the product term) shows some weak explanatory power standalone (in line with the predictions following from the view on multi-factor investing outlined in Section 2), predicting multi-factor returns over the upcoming month is quite challenging. This is particularly true since variables that were previously useful in explaining contemporaneous one-month returns (e.g. value signal) do not enhance one-month-ahead return forecasts.

Therefore, I also examine the predictability of future mean returns over the upcoming year. Table A15 reports the associated results. Most regressors still exhibit insignificant coefficient estimates in the various specifications, rendering them again as not helpful predictors in the short run.[39] However, assuming a holding period of more than one month greatly enhances the statistical significance and explanatory power of the AV term. When considered in isolation (Table A15a, column one), the t-statistic is about 4.15 (vs. 2.14 for the one-month forecasting window) and the model is able to explain roughly 10% of the variation in future mean returns (vs. previously 1%). Moreover, the model based on AV shows more predictive power than the competing variance measures based on the benchmark return series (Table A15a, column six to nine). Overall, these observations are in line with the predictions following from the view on multi-factor investing outlined in Section 2, implying that AV (in contrast to AC) predicts future return at relatively short horizons.[40]

As before in the context of variance forecasts, I also examine specifications over other forecast horizons to check for robustness of the results. The results are reported extensively in the appendix. Figure 8 and Figure 9 roughly

[39] As a side note: Some of the other risk proxies, such as CSV_t, also exhibits significant estimates with an absolute t-statistics higher than 2 in some specifications, but the explanatory power observed in isolation is marginal compared to AV_t.

[40] It should be noted, however, that when controlling for the set of external variables (Table A15b, last column), the relevance of AV is overshadowed by other variables such as CSV_t or the momentum signal.

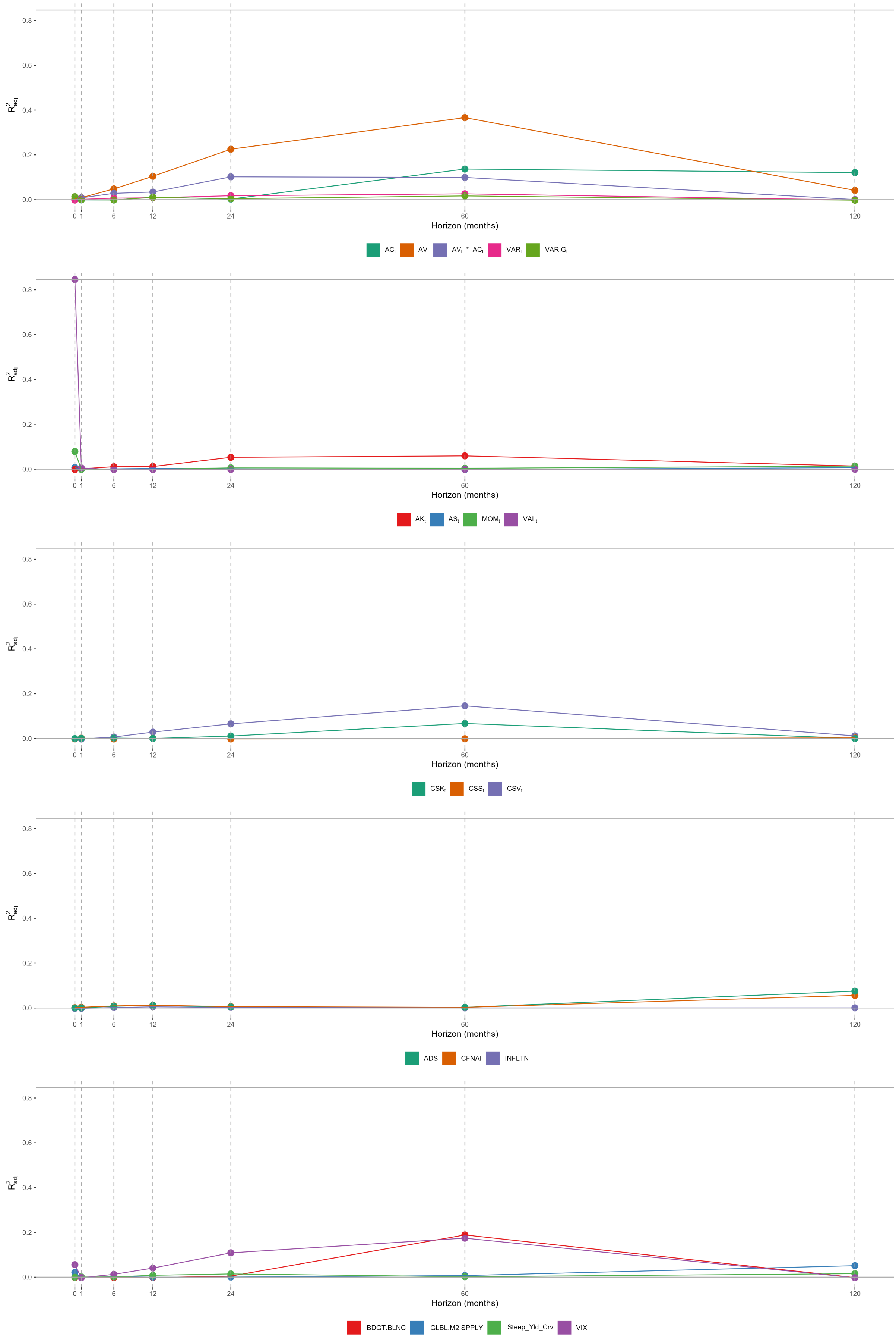

Figure 8: IS Explanatory Power of One-Regressor Specifications per Forecast Horizon. Return Decomposition/Forecasting. (Caption on the next page.)

Figure 8: IS Explanatory Power of One-Regressor Specifications per Forecast Horizon. Return Decomposition/Forecasting.
This figure illustrates the adjusted R^2 (R^2_{adj}) of the various measures used as explanatory variable for decomposing/predicting the conditional return of the naive multi-factor portfolio, when each variable is applied in isolation over the various tested forecast horizons (the results of the decomposition of contemporaneous returns are indicated by the horizon labeled '0'). More information on the associated regression results (for instance the R^2_{adj}, expressed as number) are provided in Tables A13 to A19. Refer to Sections 4.1 (multi-factor variance), A.2.1 (multi-factor risk), and A.2.2 (external predictors) for an overview of explanatory variables in the regressions. An overview of the abbreviations used in the course of this paper is provided in Table A3.

summarize the findings of the various predictors over the different horizons in terms of explanatory power and statistical significance. Specifically, Table A16 shows results when forecasting future returns over the subsequent six-month period. Importantly, the findings qualitatively stay the same; while the forecast performance of AV is less pronounced compared to the one-year period, it is still notable relative to the other variables, i.e. the model only applying AV achieves the highest R^2_{adj} (roughly 5%) across the one-regressor models.

When extending to a two-year horizon (see Table A17), the impact of AV becomes even more pronounced (with a R^2_{adj} of around 23% when observing the variable individually, see for instance Figure 8), while for instance benchmark variance (VAR_t) still shows insignificant coefficient estimates.[41] In other words, while we can not observe the typically assumed variance-in-mean relationship (see Pollet and Wilson (2010)) when using only benchmark return data, it is present when instead applying AV as predictor for risk. Moreover, the variation explained by external predictors roughly doubles compared to the 12-month period (while R^2_{adj} and t-statistics are still comparably low vs. the model solely based on AV).[42] This is a first indication that the influence of the macro and market variables on multi-factor returns is getting more important for longer holding periods, similar to the findings in the context of variance forecasting.

Moving to a 60-month forecast window, Table A18 reports similarities, but also some interesting differences to the previous findings. As for the shorter horizons, the estimate of the AV coefficient is consistently positive and highly significant in nearly all associated specifications, achieving a t-statistic (R^2_{adj}) of 7.9 (37%) standalone - the best performance for AV across the tested horizons. In addition, AC now also shows significance in many specifications, exhibiting a t-statistic (R^2_{adj}) of -4.32 (14%) standalone. Using both variables, the regression is able to explain almost 47% of the variation in future mean returns. The product of both variables, while showing significance standalone (t-statistic of 3.02), however, seems to be of no additional explanatory help when applied together with the single terms (column four to five). In general, the gain in importance of AC as predictor fits the idea that correlations should also at some point in time come into consideration for multi-factor investors, as outlined in Section 2. The consistently negative sign of the estimates suggests that a spike in AC predicts lower future returns, while the relationship is - as before - positive for AV.

In addition, some of the additional approximations of multi-factor risk also show more explanatory power. For instance, CSV_t and CSK_t standalone exhibit t-statistics (R^2_{adj}) of 4.64 (15%) and -3.26 (7%), respectively. Besides these strong results for the variance and some of the risk approximations, also the set of external predictors now appears more useful. This is especially true for the fiscal balance indicator and the VIX. For both, the coefficient

[41] Additionally, I report significant estimates for the VIX, the product of AV and AV, as well as CSV_t in the one-regressor model, but the explanatory power (R^2_{adj} of around 11%, 10%, and 7%, respectively) is small compared to AV (23%).

[42] Specifically, the model in column 12 of Table A17b (A15b) [A16b], based on all external predictors except for the VIX (which as a measure of implied volatility behaves intuitively more similar to the variance approximations), establishes a R^2_{adj} of around 6% (3%) [1%] for the 24-month (12-month) [6-month] horizon.

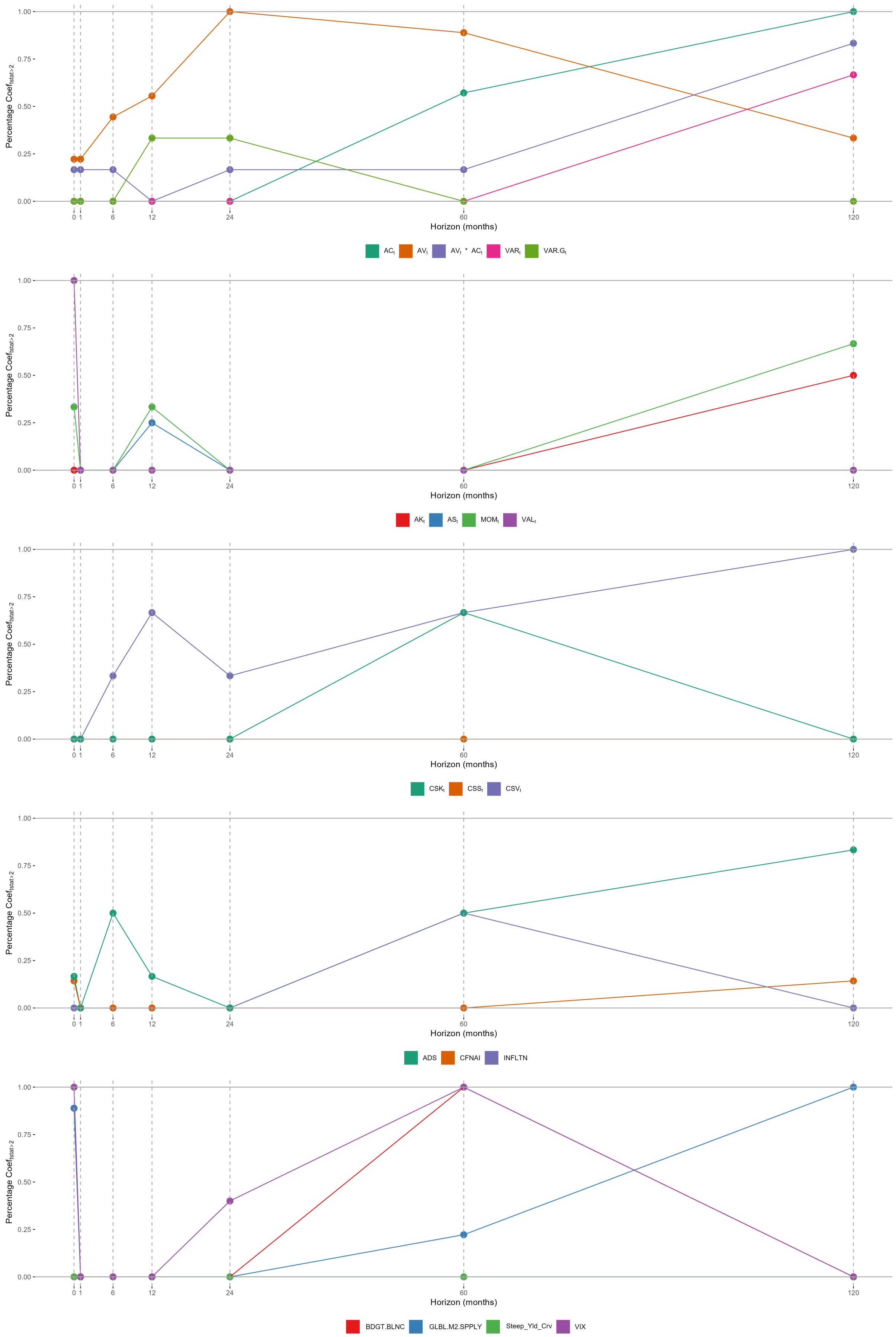

Figure 9: IS Statistical Significance per Regressor and Forecast Horizon. Return Decomposition/Forecasting. (Caption on the next page.)

Figure 9: IS Statistical Significance per Regressor and Forecast Horizon. Return Decomposition/Forecasting. This figure summarizes, for each of the explanatory variables used in decomposing/predicting the conditional return of the naive multi-factor portfolio and per forecast horizons (the results of the decomposition of contemporaneous returns are indicated by the horizon labeled '0'), the percentage of t-statistics (over all tested regression specifications which include the given regressor) that exhibit an absolute value higher than 2. More information on the associated regression results (for instance the t-statistics, expressed as number) are provided in Tables A13 to A19. Refer to Sections 4.1 (multi-factor variance), A.2.1 (multi-factor risk), and A.2.2 (external predictors) for an overview of explanatory variables in the regressions. An overview of the abbreviations used in the course of this paper is provided in Table A3.

estimate is statistically different from zero in all models associated, and when considered individually, they are able to explain roughly 19% and 17% of variation in future returns, respectively (as illustrated in Figure 8). Overall, applying all predictors jointly leads to a R^2_{adj} of roughly 74% at the 60-month forecast horizon, which is impressive even considered for an IS estimate.

In the last step, I apply a ten-year forecast window (see Table A19). AV is overall not a relevant predictor anymore at this long horizon (R^2_{adj} of 4% when considered standalone) and only shows a significant estimate when combined with benchmark variance, all other risk approximations, or all available predictors. Instead, AC establishes a t-statistic (R^2_{adj}) of -2.23 (12%) when observed individually and proves significant across all associated specifications, while the coefficient is consistently estimated with a negative sign. In addition, the product of both terms now appears useful in all models when observed with other variables, while the forecast performance is still weak standalone. Of the other approximations for benchmark risk, I find some predictive power for variables such as CSV_t (showing significance in all specifications with standalone R^2_{adj} of 1%), but also the results in this category are generally weaker compared to the 60-month period.

Furthermore, Table A19b, reporting the results for the external predictors, provides evidence that the macro and market variables also are less helpful for the 120-month window. Only the global money supply and business cycle variables show partially interesting results, but the broader explanatory power is far from what is evident for the 60-month window.[43] In detail, all external variables together explain about 24% of variation in future mean return over the ten-year horizon, while the same model explains roughly 58% over the five-year horizon (compare column 13 in Table A19 to Table A18). Indeed, this marks a difference between the variance and (mean) return forecasting exercises. While the predictability (taken R^2_{adj} as measure) for variance forecasts is strongest for the longest forecast window tested (ten years), it peaks at the 60-month window for return forecasts. [44]

To summarize, the results for shorter horizons (ranging from one month to two years) validate the observations made at the one-year horizon. Specifically, AV emerges as the most promising predictor when compared to the second component of portfolio variance (AC) or any other measures used as control variables, as nicely outlined in Figure 8 and Figure 9. However, the most explanatory power to predict future returns is achieved for the five-year forecast horizon, at which the variance of multi-factor returns exhibits mean-reversion, AC - in addition to the peak performance of AV - starts to matter for and contributes to risk/return forecasts, and macro and market variable (as well as other risk approximations) are able to deliver noteworthy excess information about future

[43] More in detail, for the coefficient of the global money supply variable, I establish a t-statistic of -2.39 for the one-regressor model, while the coefficient is consistently significant and negative across all specifications estimated. For the business cycle indicator ADS, the associated t-statistic is 2.23 for the one-regressor model and the coefficient is significantly different from zero in all tested models, except for one. The signs of the coefficients are in all cases positive.

[44] To be more specific in this context: For variance forecasts, the model applying all predictors shows a R^2_{adj} of 39% (6-month), 42% (12-month), 26% (24-month), 35% (60-month), and 63% (120-month). For mean return forecasts, the same model shows an R^2_{adj} of 16% (6-month), 26% (12-month), 40% (24-month), 74% (60-month), and 50% (120-month).

returns. Again, it is worth to highlight that the positive relationship observed for AV (applied standalone) at the 12-month horizon shows consistency across all windows tested (captured by Table A20), i.e. higher AV predicts higher future multi-factor returns. The negative sign of AC's coefficient is similarly consistent, with the exception of the one-month horizon.

These results are again highly insightful. As I reported before in the context of variance forecasting, they indicate that both components of overall portfolio variance also work in opposite directions when predicting future returns. This is important, since it explains why VAR_t, the contemporaneous overall portfolio variance, shows no significant relationship with future returns across any of the horizons tested: By combining both components, the opposing effects are diluted and the net effect is less informative. Therefore, when authors such as Moreira and Muir (2017) state that variance/volatility does not predict returns, they are correct on the surface (meaning that overall variance of the multi-factor portfolio does not predict returns), but overlook the more nuanced patterns concerning the idiosyncratic and systematic components of the overall measure. These patterns suggest that the often assumed variance-in-mean relationship is indeed present in the data, however, only when focusing on the component of multi-factor variance that is relevant to investors (following the argument in Section 2), namely AV.

4.3 Robustness and Further Analysis

For the main part of this paper, I have measured AV and AC in levels, following the idea that multi-factor investors care about their portfolio's variance also in terms of the level and (as assumed) perceive factor correlations as persistently (close to) zero, which is a fixed absolute threshold. In an additional analysis in Section A.3.2, I further examine this topic by exploring an alternate approach of measuring AV and AC in relative terms instead of levels, specifically by calculating relative changes over time. Overall, the findings suggest that contemporaneous levels of the measures are the primary drivers of short- and long-term future multi-factor risk and return, whereas relative changes between months do not exhibit significant explanatory power.

Similar to Pollet and Wilson (2010), I check for further robustness of the main finding - that is, the dominance of AV as predictor of future short-term multi-factor returns compared to AC - by estimating the regression models that contain each predictor in isolation for different subperiods of the total investigation period starting in July 1971 and ending in December 2018. The results are reported and outlined more detailed in Section A.3.3 in the Appendix (see also Table A22) and deliver additional support for the hypothesis that multi-factor investors should mainly care about variances, at least in the short run. For instance, with one exception, all coefficients associated with AV over the different subperiods show notable t-statistics higher than 3 and the same positive relationship that is also observed over the full sample. Moreover, the relationship is especially evident in the most recent subsample.

Another possible way to check for further robustness is to test the impact of AV and AC on short-term risk and returns when observing other multi-factor portfolios than the naive factor portfolio over all available factor strategies, as done so far. Therefore, I use the two different sets of equally-weighted strategies, which are described in Section 3 and formed based on a factor's association to either an ASCL or factor style, to forecast variances and returns for each multi-factor portfolio.

I start with forecasts of portfolio variances over the upcoming 12-month period; detailed results can be found in the Appendix (Section A.3.4). The findings of this robustness test broadly support the main results presented in Section 4.2.2. Another interesting observation is that the measure of variance based on the returns of the respective portfolio return series comes with stronger predictive power for future risk - compared to AV - of multi-factor portfolios with a lower number of considered factors (N). This is in line with the idea that total portfolio variance is a better proxy for risk of undiversified factor portfolios with low N, for which the variance better reflects the information of the single factor variances, that is lost due to diversification for high N.

In addition to variances, I similarly check for robustness concerning the findings on multi-factor return forecasting, when observing other multi-factor portfolios than the naive factor portfolio over all available factor strategies. The results are outlined in Section A.3.5 of the Appendix. In sum, these findings mostly deliver further support for the relevance of variances (compared to correlations) for future returns of multi-factor returns. Only for portfolios containing either the market-, carry-, or FX-associated factors, the IS evidence of a variance-in-mean relationship is comparably weak or even non-existent (for the latter), which is puzzling.

The IS results presented so far suggest that average factor variance is a viable predictor of multi-factor (mean) returns over the short-term future, for instance the upcoming one-month or one-year period, while AC carries no notable predictive information for these horizons. I further evaluate this claim OS by predicting returns of the next month (or over the next year, alternatively), while using only data available up to the current month to conduct the forecast. The findings are reported more extensively in Section A.4.2. Overall, the main pattern remains robust: AV appears as the more useful predictor to forecast multi-factor returns at the one- and twelve-month period when compared to AC or other variance approximations. Specifically, when observed individually, AV establishes a $OS\text{-}R^2$ between 10.36% and 13.48% for return forecasts over the upcoming year. In addition, observing the recursively-estimated coefficients over time indicates that while AV - with a consistently positive relationship since the end of the 1980s - has reached its peak impact on the return forecast (i.e. the highest absolute coefficient estimate) only a few years ago, AC has at the same time reached its lowest impact so far. Moreover, the tendency of lasting increases in AV's estimated coefficient during NBER recessions fits the idea of major exogenous economic shocks as potential trigger for a rise in variances across factors and its impact on future multi-factor returns, as discussed in Section 2.

While the statistical insights so far are interesting, their economic implications for investment decisions, potentially in real-time, are still left to explore. Therefore, I also construct simple timing strategies, similar to Moreira and Muir (2017), to test the viability of varying exposure to the equally-weighted multi-factor portfolio according to different approximations for portfolio variance, specifically AV, AC and the sample variance. However, my ex-ante expectations are somewhat different to those stated in Moreira and Muir (2017), since the authors build their volatility-managed portfolios based on empirical evidence supporting persistence in variances and opposing a variance-in-mean relationship. In contrast, my findings so far suggest a more nuanced picture, in which the variance-in-mean relationship is only uncovered when focusing on the variable that should be of main interest for multi-factor investors believing in uncorrelated factors, as outlined in Section 2, namely AV. The results of these trading strategies are reported in Section A.5 and overall caution multi-factor investors to bet on a dysfunctional variance-in-mean relationship.[45]

[45] An additional robustness test shows that this overall picture is unchanged when refraining from ex-ante volatility scaling the factor return series before forming the naive multi-factor portfolio and conducting the timing exercise.

5 Conclusion

Should conditional correlations matter to multi-factor investors?

The factor investing literature emphasizes diversification benefits across factors. Accordingly, asset managers actively promote these benefits of multi-factor investing. In this context, I hypothesize that multi-factor investors perceived factor correlations as close to zero and, thus, focus on factor variances in their short-term risk assessment. Using 14 factor return series across various ASCLs, factor styles, and an extensive historical data period, this paper empirically finds support for the hypothesis.

While both variance components, AV and AC, explain current multi-factor risk, only AV predicts future portfolio risk and return at short horizons (one month to two years), supporting the hypothesis. Specifically, the findings confirm short-term persistence in risk and a variance-in-mean relationship: Higher AV unlike AC, leads to significantly increased future risk and returns for multi-factor investors. Notably, the variance-in-mean relationship is discernible only by disentangling the opposing effects of AV and AC, and not by observing the overall 'diluted' variance of the multi-factor portfolio.

For medium- and long-term multi-factor risk and return forecasts (60 and 120 months), other predictors, such as ACalong with external macro and market variables, gain relevance. The strongest predictability for future benchmark variance is observed at the longest window of 120 months, while 60 months works best for return forecasts. Interestingly, return predictability peaks when multi-factor return variance exhibits mean-reversion, AC (besides AV) starts contributing to risk/return forecasts, and additional control variables provide further excess information about future returns. The model, using all tested regressors, explains around 72% of the IS variation in future mean returns over this horizon.

Besides other robustness tests, such as exploring different control variables and analyzing IS performance across subperiods and for alternative equally-weighted multi-factor strategies, an OS analysis validates AV as the key component of benchmark risk, enhancing forecasts of short-term multi-factor returns. The important implication of these results for multi-factor investors is to be cautious when relying on a dysfunctional variance-in-mean relationship, as suggested by Moreira and Muir (2017). In addition to the main findings, the paper presents further observations on multi-factor investing. For instance, while the risk of an arbitrary factor investment (proxied by AC) and the diversification benefits of multi-factor investing (proxied by AC) have remained stable, multi-factor investing has become less attractive over time in terms of risk compensation.

The paper contributes to various strands in the literature, such as factor investing, forecasting risk-return tradeoffs, variance and correlation risk premia, and the effectiveness of macroeconomic and market data in predicting asset returns. It also highlights potential areas for future research, such as better characterizing periods of synchronously spiking variances across cross-asset factor strategies. These periods coincide with times of elevated global economic uncertainty and are typically followed by higher-than-average risk and returns for diversified multi-factor investors. Additionally, the study only incidentally touches on the strengthening explanatory power of external macro and market predictors for longer forecast horizons, peaking at the five-year horizon. Therefore, further exploring the relevance of macro and market data for multi-factor returns, particularly at longer horizons, promises a deeper understanding of long-term multi-factor premia.

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A Appendix: Supplementary Text

A.1 Details to Factor Series

A.1.1 Factor Construction

The following description concerning the factor construction is taken from the paper by Vincenz and Zeissler (2024):[46]

> For the set of global **currencies** we use monthly spot and forward exchange rates in U.S. dollars and build end-of-month series starting from January 1971 to December 2018. The data are primarily sourced from Bloomberg and missing data was filled with data from GFD[47]. Using bid-ask data, we can account for transaction costs during trading. The total sample contains 69 currencies (...). We note that the effective sample size of available currencies can vary over time, given that some emerging country currencies become available only at later time periods and other currencies cease to exist i.e. due to the adoption of the Euro. We exclude countries within the euro area after they adapted the euro starting in January 1999. (...) Our data cleansing process includes data removals in case of large deviation of the covered interest rate parity. We remove negative ask and mid implied yields in non-developed markets, ask and mid implied yields in developed markets below -5%, and ask and mid implied yields above 150% in all markets. Additionally we remove data where bid implied yields are above 150%.
>
> To proxy for a currency market factor, we follow the approach of Lustig et al. (2011) and construct the dollar factor. The dollar factor describes a long position in the set of all available currencies against the U.S. dollar and the performance is measured over a 1-month holding period.
>
> Our measure of carry is the implied yield in line with literature. The implied yield for long positions is calculated using the current 1-month forward bid rate (f^b) and the current spot ask rate (s^a), given that an investor would purchase the forward at the bid price and sell after the 1 month holding period at the spot ask price
>
> $$IY_t^l = \frac{f_t^b}{s_t^a}(1 + r_t^d) - 1, \tag{8}$$
>
> where r_t^d is the (domestic) U.S. dollar interest rate. Given that covered interest rate parity holds, the implied yield should approximate the interest rate differential between the USD and the respective foreign currency (Riddiough and Sarno (2018)). Furthermore, the implied yield for short positions is calculated in an analogous way, using the current 1-month forward ask rate (f^a) and the current spot bid rate (s^b) instead
>
> $$IY_t^s = \frac{f_t^a}{s_t^b}(1 + r_t^d) - 1 \tag{9}$$
>
> For currency momentum, we sort according to the historical 12-month currency excess return and leave out the most recent month

[46] Note that in a more recent version of the paper by Vincenz and Zeissler (2024), the authors have adopted some changes to their factor universe. The paper at hand, however, is based on the factors as described in the following, which is an excerpt of an older version of Vincenz and Zeissler (2024).

[47] Methodology inherited from Dockner et al. (2018).

$$M_t = \prod_{s=t-11}^{t-1} (1 + r_s) - 1, \tag{10}$$

where r_t is the currency spot return. For the value measure, we refer to the concept of purchasing power parity in order to determine whether a currency is under- or overvalued. Our measure is the five-year change in the real exchange rate, which can be formulated as the 5-year currency spot rate adjusted by the consumer price index of the foreign country relative to the U.S. over the same period. We therefore use the same methodology as Asness et al. (2013), who restrict their analysis to the G10 countries, and extend it to a broader currency sample.

For our global **fixed income** universe we construct zero curves for 45 international local currency government bond markets (21 developed and 24 emerging countries) starting in December 1994 and ending in December 2018 using data from Bloomberg. (...) Moreover, we use monthly data, i.e. end-of-month data of local bond yields and, analogously to currencies, build end-of-month time series. In order to have meaningful duration-representative returns, we aggregate returns into a bucket where bonds with a time to maturity of five to ten years are grouped. This maturity-bucketing is analog to the methodology used by JP Morgan[48], who also form several maturity buckets for instance within their JPM GBI index. Sovereign bond returns will be presented from the perspective of a U.S. investor. Similar to currencies, we note that the effective sample size of available countries can vary over time, given limited data availability for particular countries. (...)

We introduce a number of risk premia evident in the fixed income universe. The market factor constitutes of a GDP-weighted long position in the seven largest countries[49] in terms of real GDP and a short position in the risk-free rate[50].

Our measure of carry is defined similar to Koijen et al. (2018) as the term spread within the maturity bucket:

$$C_t = y_t^{10y} - y_t^{5y}. \tag{11}$$

For momentum we take again the common measure of the 12-1 month historical US dollar return, i.e.

$$M_t = \prod_{s=t-11}^{t-1} (1 + r_s^{m_t}) - 1, \tag{12}$$

where $r_t^{m_t}$ is the return of a bond at time t with remaining maturity m_t. Our measure of value is defined as the nominal yield (y) on the bond minus current inflation[51] (Inf^{yoy}) to derive a real bond yield level:

$$V_t = y_t - Inf_t^{yoy}. \tag{13}$$

As stated in Brooks and Moskowitz (2017), our measures of carry, momentum and value can be interpreted in a natural economic way, namely that carry provides information about expected future

[48] Brooks and Moskowitz (2017) follow the same approach.
[49] United States, Great Britain, Japan, China, Germany, India, France
[50] ICE LIBOR USD 1-month rate is taken from Bloomberg.
[51] Opposed to expected inflation as is used e.g. in Brooks and Moskowitz (2017) or Asness et al. (2013).

yields without changes to the yield curve, momentum signals trends in yield changes and value indicates the level of yields with respect to a fundamental anchor namely inflation.

Our **commodity** data collection includes liquid and exchange-traded commodity futures contracts for 31 commodities sourced via Bloomberg. The majority of contracts is identical to those used by Szymanowska et al. (2014) and Boons and Prado (2019). Our sample of futures data starts in July 1959 and ends in December 2018. (...) The returns of individual commodities are calculated using a roll-over strategy as done by the authors above. In line with their reasoning, we calculate first and second nearby contract returns given these are usually more liquid. In order to avoid contract positions close to expiration and the resulting notice days or erratic volume and price behavior, we restrict expiration of each commodity contract to be after $t + 2$. To account for transaction costs, we will apply a relative half spread of 4.4 basis points suggested by Marshall et al. (2012).

For commodities the market factor is, analog to currencies, a long position in the set of all available commodity futures at any specific timepoint. Our carry signal is derived from the basis (B_t), which indicates whether a commodity futures curve is in contango (positive basis) or in backwardation (negativ basis):

$$B_t = \frac{F_t^{T_2}}{F_t^{T_1}} - 1, \tag{14}$$

where $F_t^{T_n}$ is the n-th nearby futures contract at time t. We take long (short) positions in commodities in backwardation (contango), i.e. purchase (sell) relatively cheap (expensive) first-nearby futures contracts given the term-structure of the futures curve.

For momentum we use in accordance with previous asset classes the 12-1 month historical return of the first-nearby futures contract as signal

$$M_t = \prod_{s=t-11}^{t-1} (1 + r_s^{T_1}) - 1, \tag{15}$$

where $r_s^{T_1}$ is the return of the first nearby (T_1) futures contract at time s[52].

The commodity value measure is based on the negative five year cumulative return[53] commonly used by other authors such as Asness et al. (2013):

$$V_t = -\left(\prod_{s=t-59}^{t} (1 + r_s^{T_1}) - 1 \right) \tag{16}$$

The last and most recent commodity factor is basis-momentum, which is defined as a combination of (B_t) and momentum (M_t):

$$BM_t = \prod_{s=t-11}^{t} (1 + r_s^{T_1}) - \prod_{s=t-11}^{t} (1 + r_s^{T_2}). \tag{17}$$

[52] We require at least one return $r_s^{T_1}$ for the calculation of the momentum signal above, i.e. in case of missing data we will still be able to generate a momentum signal and therefore expand the momentum signal availability in the cross-section.

[53] Analog to momentum, we will require at least one return $r_s^{T_1}$ for the calculation of the value signal in order to broaden the cross-sectionally available assets.

The motivation of the signal according to Boons and Prado (2019) is that it contains relevant slope and curvature information, determined by market participants seeking positions on the futures curve at different locations.

For our analysis on international **equity indices** we include a total of 49 Morgan Stanley Capital International (MSCI) country total return indices, all sourced via Bloomberg, to our equity cross-section and construct end-of-month series starting in January 1970. The respective indices are all quoted in USD. (...) As outlined by Zaremba (2019), MSCI indices, followed by Datastream Global Equity Indices, are the most popular choice on a country-level equity perspective, given the calculation transparency, consistency in index calculation and result comparability across a broad number of countries. Bhojraj and Swaminathan (2006) investigate factor momentum on equity index level from 1970 to 1999 using MSCI data and find momentum during the first year and reversals during the following year. We acknowledge the heterogenity among the selected countries and therefore are cautious with the final results of the factor construction.

In our analysis, we proxy transaction costs with 10 basis points for each country index in each month. Analog to the other asset classes, we assume this proxy will be again more conservative given full transaction costs are incurred monthly, even when the position remains unchanged.

We construct a market-capitalization weighted equity index benchmark with the seven largest countries in terms of market capitalization to represent the equity market factor financed with the risk-free rate. Given market capitalization data from the country indices becomes available not until August 1995, we proxy the equity market returns solely with the US equity index returns, starting in 1970. Our equity index momentum measure is the 12-1 month cumulative return as in Equation 12. [54] Finally, we replicate the five classical U.S. **equity single stock** factors (from Fama and French (1993) and Fama and French (2015)) as well as the momentum factor from Carhart (1997) and add them to our factor universe.

At the end of month t we rank assets according to the above described signals and form six portfolios. In the case of an available total sample size below six, at least one asset will be selected for the top and bottom sixth portfolio. This selection approach has the advantage that it can account for a varying sample size. We take long (short) positions in assets based on the top (bottom) sextile in accordance with the standard methodology i.e. done by Lustig et al. (2011) for currencies. At the end of each month, the portfolios are rebalanced. Each asset in the long (short) portfolio is then weighted equally.

A.1.2 Differentiation to Literature

The following description of methodological differences of the considered factors to the relevant academic literature is taken from the paper by Vincenz and Zeissler (2024):[55]

For **currencies**, in contrast to most other literature (i.e. Menkhoff et al. (2012) or Lustig et al. (2011) we include the United States, i.e. USD, to the currency sample. All currencies above are generally quoted

[54] Opposed to the other presented asset classes, we refrain from constructing a **carry** and **value** factor within the equity index space due to the heterogeneity of countries.For carry the dividend yield could be considered as underlying characteristic measurement and for value the cyclically-adjusted price earnings ratio. However, perceived discrepancies in terms of e.g. shareholder value and accounting methodologies, among other differences, across the presented countries led us to refrain from constructing such factors.

[55] As noted before, in a more recent version of the paper by Vincenz and Zeissler (2024) the authors have adopted some changes to their factor universe. The following excerpt is taken from an older version of Vincenz and Zeissler (2024).

against the USD, however with the inclusion of the USD as a separate investable currency, this has the effect that the pair USD/USD constitutes a neutral portfolio position. From a practical perspective, this gives an investor the chance to stay in USD (i.e. invest into the currency pair USD/USD) in case the other investment opportunities are less attractive, i.e. due to negative carry across all currencies. In addition, we use inflation and GDP data from local sources (including several emerging market countries) and rely on the credibility of local authorities supplementing sound data. As mentioned in the Appendix A.1.1, we apply a data cleansing procedure when covered interest parity is violated.

For **fixed income**, opposed to currency data, we do not have bid-ask data available for the zero-coupon yield curves. In order to account for transaction costs, we therefore approximate the zero-coupon bond spreads using currency-related spreads for each country[56]. Our heuristics includes a spread multiple of 1.5 of the country-representative FX-spread. Given that we calculate returns from the perspective of a US investor, total transaction costs (including currency conversion costs) for a fixed income investor consist therefore of a 2.5 multiple of currency transaction costs. Given that total transaction costs are incurred at each month, even if the bond is not entirely sold but only rebalanced, we assume this approach will be more conservative than in practice. Total fixed income transaction costs will be represented as follows:

$$s_t^{spread} = \left| s_t^a - s_t^b \right|, \tag{18}$$

where s_t^a (s_t^b) is the ask (bid) spot exchange rate at time t. Equation 18 represents the currency spot spread.

$$rx_{t+1} = \frac{s_{t+1}^b - 1.5 s_{t+1}^{spread}}{s_t^a} - 1 \tag{19}$$

where rx_t is the currency spot excess return at time t.

Following the construction of fixed income returns including transaction costs, we construct a long-term maturity bucket averaging returns between tenors of five to ten years. We consider this duration bucket to proxy for returns with highest loadings on duration risk.

We use a broader cross-section of 31 **commodities** to construct factors for this asset class (cp. Boons and Prado (2019) and Szymanowska et al. (2014) who use 21 commodities respectivley).

Given the global representativeness of the asset-classes currencies, fixed income and commodities, we follow suit with **equities** and resort to MSCI equity indices. From a practical perspective and given real-world investment constraints, country indices provide investors with a simple, mostly feasible, diversified and cost-efficient way to implement an equity factor strategy. However, we acknowledge that there is great heterogeneity across the different countries and the results of global factor portfolios based on heterogeneous country specific indicators shall be treated cautiously. This great heterogeneity among countries leads us to construct only a market and momentum factor, which can be constructed most consistently in our view, for equity indices.

[56] González-Rozada and Yeyati (2008) show that time variation in bond spreads is explained by global factors which we assume are also implicit in currency spreads.

A.2 Additional Methodology

A.2.1 Other Approximations for Benchmark Risk

There is a long list of papers that consider return dispersion as explanatory variable for aggregated asset returns (for instance Stivers and Sun (2010), Maio (2016), or Stöckl and Kaiser (2021)) and risk (see e.g. Bekaert and Harvey (2000) or Stivers (2003)).[57] Return dispersion describes how closely the returns of a cross-section of assets move in lockstep over a observed time period and is typically measured as the cross-sectional variance (volatility) of returns of the set of assets (see e.g. Maio (2016)). Garcia et al. (2014) formally show it's usefulness as consistent and asymptotically efficient estimator for aggregate idiosyncratic volatility, with the key advantages of being model-free and observable at any frequency. Cooper, Ma, and Maio (Cooper et al.) develop an asset-pricing model that accompanies the common market factor by two additional sources of risk, of which one is defined by the cross-sectional variance of various common long-short factors.

Empirically, Maio (2016) for instance reports return dispersion being negatively related to future stock market excess returns over various horizons in his IS test and also finds statistically and economically evidence for it's viability as OS predictor, while the recent results of Stöckl and Kaiser (2021) deliver further support for it's predictive power IS and OS. Following this literature, I define cross-sectional variance as

$$CSV_t = \frac{1}{N}\sum_{i=1}^{N}(r_{i,t} - \bar{r}_{i_1:i_N,t})^2, \tag{20}$$

where $r_{i,t}$ defines the monthly log return of the factor at time t and $\bar{r}_{i_1:i_N,t}$ the average return over the cross-section of N factors.

Authors such as Jondeau et al. (2019) or Stöckl and Kaiser (2021) explore the viability of higher return moments for predicting fluctuations in future aggregated stock excess returns, following studies that point to possible theoretical reasons for these metrics to show relevance. For example, a negative skewness, associated with occasionally large negative returns, is often interpret as sign of tail/crash risk (see e.g. Brunnermeier and Pedersen (2009), Kozhan et al. (2013), or Bollerslev et al. (2015)). Moreover, early work by authors such as Kraus and Litzenberger (1976) has already explored skewness preferences of investors in the context of asset-pricing and established - similar to the traditional CAPM - that only the non-diversifiable part of an asset's skewness, i.e. the co-skewness of the asset with the market portfolio, should demand a risk compensation. In a more recent study, Schneider et al. (2020) – building on authors such as Kraus and Litzenberger (1976) – show that accounting for (co)skewness helps to explain empirically well-documented low-risk anomalies based on beta and volatility risk measures. Jondeau et al. (2019) provide theoretical insights as well as empirical evidence that support the relevance of (rising) average skewness as predictor of (falling) future stock market returns. Additionally, Jondeau et al. (2020) show that this relationships also holds when forecasting the returns of index futures.

[57] In addition to predicting TS dynamics of aggregated asset returns, other papers such as Jiang (2010) also explore whether return dispersion is priced in the cross-section of stock returns. For an overview and discussion of cross-sectional measures of dispersion and their relevance in a broader context, see for instance Stöckl and Kaiser (2021).

Similar to Jondeau et al. (2019),[58] I first calculate sample skewness for factor i at time t using return data of the previous 12 months, denoted $\hat{Sk}_{i,t_{-12}:t}$.[59] At time t, average skewness is then defined as (equally-weighted) average of the sample skewness of all N individual factors:

$$AS_t = \frac{1}{N}\sum_{i=1}^{N} \hat{Sk}_{i,t_{-12}:t} \tag{22}$$

Moreover, I define average kurtosis analogously based on the sample excess kurtosis of the individual factor TS. For factor i, I estimate the sample excess kurtosis using the last 12 observations and denote the variable as $\hat{K}_{i,t_{-12}:t}$.[60] Average kurtosis at time t is then constructed as (equally-weighted) average of the sample excess kurtosis of all N individual factors:

$$AK_t = \frac{1}{N}\sum_{i=1}^{N} \hat{K}_{i,t_{-12}:t} \tag{24}$$

Name	Symbol	Obs	Min	Mean	Max	Std	Skewness	Kurtosis	Autocorr (1)
Average Skewness	AS_t (12M)	559	-1.133	-0.17	0.752	0.296	-0.14	0.79	0.88
Average Kurtosis	AK_t (12M)	559	-0.932	-0.164	1.249	0.379	0.83	0.57	0.86
Cross-Sectional Variance	CSV_t	559	0	0.001	0.018	0.001	7.51	90.47	0.35
Cross-Sectional Skewness	CSS_t	559	-0.891	-0.004	1.201	0.152	1.71	22.56	0.04
Cross-Sectional Kurtosis	CSK_t	559	0.018	31.136	913.059	67.141	6.88	66.85	0.10
Momentum	MOM_t (12M)	559	-0.008	0.002	0.01	0.003	-0.44	1.21	0.93
Value	VAL_t (60M)	505	-0.02	0	0.03	0.005	0.95	4.28	0.18

Table A1: Other Approximations for Benchmark Risk.
This table lists the alternative approximations for the risk of the multi-factor benchmark. Additionally, column 'Obs' reports the available number of monthly observations per variable over the full investigation period, starting in July 1971 and ending in December 2018. Moreover, in the last seven columns the table provides summary statistics of the predictor TS, specifically the minimum and maximum monthly observation as well as monthly arithmetic mean, standard deviation (for CSS_t and CSK_t, these metrics are reported in thousands), skewness, excess kurtosis, and the first-order autocorrelation. The TS of the variables were tested to rule out the possibility of containing unit roots with sufficient confidence (see Table A5). If necessary (only in the case of VAL_t (60M)), the TS were transformed (by calculating differences) and re-tested (see Table A5). For information on the variables that approximate multi-factor risk (i.e. AS_t, AK_t, CSV_t, CSS_t, CSK_t, MOM_t, and VAL_t), refer to Section A.2.1. An overview of the abbreviations used in the course of this paper is provided in Table A3.

In addition to these two variables based on higher moments, I additionally calculate cross-sectional measures of dispersion in higher moments, as done by Stöckl and Kaiser (2021), who also provide a more detailed overview

[58] While Jondeau et al. (2019) use daily stock data to estimate their monthly standardized measure of skewness, this is infeasible given that only monthly factor return data is available. Therefore, instead of relying on daily data, I construct both measures using the $N = 14$ monthly factor series and a one-year lookback window, as done before in the context of AV and AC. Additionally, Jondeau et al. (2019) standardize their measure of skewness for each asset using a historic measure of the asset's volatility. Since the factor TS are already ex-ante volatility scaled, I refrain from further adjustments when calculating $Sk_{i,t_{-12}:t}$.

[59] For transparency, I use the following formula to calculate sample skewness:

$$\hat{Sk}_{i,t_{-12}:t} = D_1 * \sum_{t=t_{-12}}^{t} \left(\frac{r_{i,t} - \bar{r}_{i,t_{-12}:t}}{\hat{\sigma}_{i,t_{-12}:t}}\right)^3, \tag{21}$$

where $\bar{r}_{i,t_{-12}:t}$ defines the average return over the previous 12 months, and $D_1 = \frac{12}{(12-1)*(12-2)} = \frac{6}{55}$.

[60] For transparency, I use the following formula to calculate sample excess kurtosis:

$$\hat{K}_{i,t_{-12}:t} = D_4 * \sum_{t=t_{-12}}^{t} \left(\frac{r_{i,t} - \bar{r}_{i,t_{-12}:t}}{\hat{\sigma}_{i,t_{-12}:t}}\right)^4 - D_5, \tag{23}$$

with $D_4 = \frac{12*(12+1)}{(12-1)*(12-2)*(12-3)} = \frac{26}{165}$ and $D_5 = \frac{3*(12-1)^2}{(12-2)*(12-3)} = \frac{121}{30}$.

of the relevant literature motivating these metrics. Cross-sectional skweness, for instance, acts as an approximation of aggregated idiosyncratic skweness, as shown formally by Garcia et al. (2014) (see Stöckl and Kaiser (2021)). Moreover, in the asset-pricing model of Cooper, Ma, and Maio (Cooper et al.), the third source of risk (besides the market factor and cross-sectional variance of several long-short portfolios) is defined as the cross-sectional skewness over their set of factor returns.

Following Stöckl and Kaiser (2021), I define cross-sectional skewness as

$$CSS_t = \frac{1}{N}\sum_{i=1}^{N}\frac{(r_{i,t} - \bar{r}_{i_1:i_N,t})^3}{CSV_t^3} \tag{25}$$

and cross-sectional kurtosis as

$$CSK_t = \frac{1}{N}\sum_{i=1}^{N}\frac{(r_{i,t} - \bar{r}_{i_1:i_N,t})^4}{CSV_t^4}. \tag{26}$$

Finally, I include simple proxies for two common signals often used for predicting returns (cross-sectionally[61] as well as over time), namely momentum and value (see for instance Moskowitz et al. (2012), Gupta and Kelly (2019), Ilmanen et al. (2021), and Vincenz and Zeissler (2024)).[62] Both measures are derived using past return data of the equally-weighted multi-factor portfolio and should control for two distinct features often observed for return TS: persistence (i.e. positive autocorrelation) over the short and reversals over the long run. Specifically, the momentum signal at time t, MOM_t, is the arithmetic mean return estimated over the previous twelve months. Conversely, the value signal VAL_t compares cumulative returns over the last five years.[63]

As before, Table A1 reports summary statistics for the constructed TS and Figure A3 shows the dynamics of the variables over the investigation period. Measures constructed using rolling overlapping windows are fairly persistent, while dispersion measures, conversely, display less autocorrelation. Since the results of conducted Augmented Dickey-Fuller tests lead to a rejection of the null hypothesis considering the existence of unit roots (see Table A5) for almost all series, I proceed to analyze the TS of all variables in levels, with the exception of VAL_t. For VAL_t, I transform the TS into first differences.

The evidence pointing to a stochastic trend in the TS of the value signal underlines an observation that could already be inferred from previous charts showing the cumulative performance of the equally-weighted benchmark,

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[61] For this reason, both concepts are also underlying some factors in the investment universe of this study, presented in Section 3.

[62] The viability of predicting future asset returns by relying on past return data has been discussed in the literature for quite some time (see for instance Fama (1965) or Lo and MacKinlay (1988)). In the last decade, Moskowitz et al. (2012) have revisited the matter of TS momentum in their influential study and found supporting evidence for return persistence in futures on various ASCLs, namely equity indices, currencies, commodities, and bonds. Since then, the discussion is far from over. Various other articles have either delivered further support for the profitability of TS momentum (e.g. Georgopoulou and Wang (2017) or Hurst et al. (2017)) or raised new doubts (e.g. Goyal and Jegadeesh (2018) or Huang et al. (2020)). While not the main focus of the paper at hand, investigating TS momentum in the context of multi-factor investing adds a new perspective to this discourse.

[63] The value signal is inspired by Asness et al. (2013) and is given for factor i at time t by

$$VAL_t = ln(\frac{\bar{P}_{t-60}}{P_t}), \tag{27}$$

where P_t is the cumulative return index of the naive multi-factor portfolio at time t and $\bar{P}_{t-60}$ is the average cumulative return index five years ago, estimated from $t-65$ to $t-54$. Since returns are additive, calculating momentum or value signals per individual factor and averaging over those yields the same aggregate measure, which therefore could also be labeled 'average' momentum/value, similar to some of the other variables in the set. Intuitively, a [negative] positive value of VAL_t is interpreted as [over-] undervaluation.

such as Figure 4: The long-lasting attractiveness of engaging in multi-factor investing, supported for instance by a significantly positive unconditional mean return (see Figure 3), has been on a decline over the investigation period for the given factor universe.[64] This phenomenon is highlighted in Figure A6, by contrasting the behavior of the value signal - before taking first differences - with the performance dynamics of the naive portfolio. As the figure reveals, VAL_t is almost constantly negative, implying persistent 'overvaluation' of the naive strategy (when following the narrative of the value concept) or more generally that the cumulative return over the last five years has been persistently positive most of the time. This finding is in line with significantly positive unconditional mean returns of the naive portfolio and generally supporting the attractiveness of multi-factor investing. However, as Figure A6 further illustrates, VAL_t clearly shows an upward trajectory over time and finally crosses zero late at the end of the investigation period, indicating that five year cumulative returns have declined over time and lately been negative for the first time. Therefore, it seems sensible for investors to have this trend in mind when projecting past unconditional (multi-)factor performance into the future.

A.2.2 External Predictive Variables

I resort to a subset of the variables described and applied in Vincenz and Zeissler (2024) to allocate a universe of factors across different ACs based on OS return predictions. The set is motivated by the relevant asset-pricing literature (see Vincenz and Zeissler (2024)). With one exception[65], I focus on those variables providing a large number of observations (i.e. covering my full investigation period); the detailed set and summary statistics are displayed in Table A2.

Name	Symbol	Obs	Min	Mean	Max	Std	Skewness	Kurtosis	Autocorr (1)
Inflation Regime	INFLTN	570	-0.010	0.038	0.148	0.030	1.68	2.24	0.99
Global Fiscal Balance	BDGT.BLNC	570	-0.078	-0.029	-0.002	0.015	-0.90	0.81	0.99
Money Supply	GLBL.M2.SPPLY	570	-1.558	0.096	2.398	0.379	0.64	6.17	0.03
Steepness of the Yield Curve	Steep_Yld_Crv	570	-0.027	0.017	0.044	0.012	-0.62	0.10	0.95
Chicago Fed National Activity Index	CFNAI	570	-4.298	-0.095	1.956	0.827	-1.50	4.51	0.93
Aruoba Diebold Scotti Index	ADS	570	-4.672	-0.083	2.720	0.802	-1.19	4.69	0.84
VIX Index	VIX	397	0.095	0.202	0.614	0.079	1.71	4.44	0.83

Table A2: Candidate Predictors.
This table lists the external variables by their names and associated shortcut symbols used in the rest of the paper. Additionally, column 'Obs' reports the available number of monthly observations per variable over the full investigation period, starting in July 1971 and ending in December 2018. Moreover, in the last seven columns the table provides summary statistics of the predictor TS, specifically the minimum and maximum monthly observation as well as monthly arithmetic mean, standard deviation, skewness, excess kurtosis, and the first-order autocorrelation. The TS of the external predictors were tested to rule out the possibility of containing unit roots with sufficient confidence. If necessary (only in the case of GLBL.M2.SPPLY), the TS were transformed (by calculating differences) and re-tested (see Table A5). For information on the external predictors refer to Section A.2.2, while a detailed description of the data set is provided in Vincenz and Zeissler (2024). An overview of the abbreviations used in the course of this paper is provided in Table A3.

Specifically, variables associated with inflation, fiscal balance, money supply, steepness of the yield curve, two U.S. business cycle indicators (Chicago Fed National Activity Index and Aruoba Diebold Scotti Index), as well as

[64] This finding is broadly in line with the results of authors such as Chordia et al. (2014) or Green et al. (2017), who report falling average premia for factors (anomaly portfolios) over time.

[65] Specifically, I include the VIX despite the shorter-than-desired TS, since controlling with a measure of market-implied volatility seems important in the context of volatility/variance approximations, as were discussed in Section 4.1.

market-implied volatility enter the analysis as predictors. All predictors represent proxies for global economic and financial conditions to match the mostly globally-oriented factor universe.[66]

As was the case for the variables previously introduced as regressors, Table A2 also reports the first autocorrelation coefficient of the external predictors. Similar for instance to the proxies of multi-factor variance discussed in Section 4.1, the series exhibit generally a very high degree of persistence (especially INFLTN and BDGT.BLNC). After testing the TS to rule out the possibility of containing unit roots (see Table A5), I reject the null hypothesis for all external predictors except for GLBL.M2.SPPLY, which shows a persistent level increase over most of the investigation period. In consequence, I transform the TS of GLBL.M2.SPPLY by forming first differences, re-test the series, and subsequently reject the null hypothesis of existing unit roots. Finally, FigureA4 shows the dynamics of the variables over the investigation period.

A.3 IS Results

A.3.1 Monthly Return Decomposition

In an intermediate step, before exploring how the regressors relate to future returns, I decompose contemporaneous monthly benchmark returns; Table A13 shows the results. In sum, only a few variables exhibit a coefficient estimate significantly and consistently differing from zero in the various tested model specifications.

Naturally, two of these variables are those calculated directly from past and contemporaneous return data, namely MOM_t and VAL_t. In detail, the momentum signal achieves an absolute t-statistic higher than 5 and a R^2_{adj} of roughly 8% standalone, whereas VAL_t shows absolute t-statistics higher than 20 in all associated models and explains around 85% standalone. These results indicate that contemporaneous monthly multi-factor returns are partly explained by patterns of short-term persistence in returns (here captured over the last year) and - to a large extent - by patterns of mean reversion over longer horizons (captured over five years).[67] Other variance and risk approximations are generally not useful for the decomposition when observed individually, but combined with a broader set of other predictors, AV_t exhibits significant estimates in two specifications.

The second exception is the global money supply indicator, which shows t-statistics larger than 3 standalone as well as in all of the other models (with one exception), when combined with the remaining variables. The sign of the estimated coefficients for GLBL.M2.SPPLY indicates that lose (tight) monetary conditions are associated with higher (lower) contemporaneous monthly multi-factor returns. The third and last variable showing importance over several specifications is equity-implied volatility (VIX), with absolute t-statistics higher than 2 and negative signs of the coefficient observed consistently across all models. The latter observation implies that an increase in current implied volatility typically comes with lower contemporaneous monthly returns.

To sum up, the dynamics of contemporaneous returns are best explained by momentum as well as reversal patterns, (equity-)market-implied volatility, and the global money supply. The model using all variables together achieves a R^2_{adj} of around 91% (see last column in Table A13b).

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[66] For more details on (the construction of) each predictor, refer to Vincenz and Zeissler (2024).

[67] Specifically, the consistently positive sign of the estimated momentum coefficients suggests that positive current momentum typically comes with positive current returns, while the negative sign of the value coefficient indicates that an increase in contemporaneous undervaluation is typically observed together with negative current returns.

A.3.2 Relative Measures of AV and AC

For the main part of this paper, I have measured AV and AC in levels, following the idea that multi-factor investors care about their portfolio's variance also in terms of the level. This view further fits the assumed investor perception of factor correlations being persistently (close to) zero and mostly irrelevant for the TS dynamics of overall portfolio risk. For example, an investor with this strong belief will presumably worry less about a recent increase in AC from 0.03 to 0.06, even though it is a spike of 100%, because the level is still close to zero.

In an additional analysis, I further examine this topic by exploring an alternate approach of measuring AV and AC in relative terms instead of levels, specifically by calculating relative changes over time.[68] The results are presented in Table A20, covering the decomposing and forecasting of multi-factor variances, and in Table A21, showing similar results for returns.

Overall, the findings suggest that contemporaneous levels of the measures are the primary drivers of short- and long-term future multi-factor risk and return, whereas relative changes between months do not exhibit significant explanatory power. In detail, none of the relative measures is useful in forecasting future variances, independent of the chosen forecast window,[69] and only $(\Delta AC_{t-1:t})/AC_{t-1}$ achieves a significant coefficient when forecasting returns over the next six-month period.

When decomposing current variances and one-month returns, the significant coefficient of $(\Delta AV_{t-1:t})/AV_{t-1}$ indicates that an increase in relatively-measured AV comes with higher contemporaneous multi-factor variance (in line with the results for the level of AV) and negative contemporaneous returns (contrary to the consistently positive coefficient estimated for the level of AV). The latter finding suggests that a relative increase in AV today is typically accompanied by a negative shock to returns and has no predictive power for future returns, while a high level of AV today has no significant relationship with current returns, but seems to predict higher future returns.

A.3.3 (Mean) Return Forecasting Using Subsamples

Similar to Pollet and Wilson (2010), I check for further robustness of the main finding - that is, the dominance of AV as predictor of future short-term multi-factor returns compared to AC - by estimating the regression models that contain each predictor in isolation for different subperiods of the total investigation period starting in July 1971 and ending in December 2018. The results are reported in Table A22.

With one exception (the first subperiod from June 1972 to October 1983), the coefficients of AV show all notable t-statistics higher than 3 and the same positive relationship that is also observed over the full sample. For the last subperiod from August 2006 to December 2017), the t-statistic even surpasses 6. In contrast, all estimates for AC over the different subperiods appear insignificant, with the previously mentioned exception of the first subsample, in which the pattern reverses and AC plays the dominant role (however, with a negative sign and a seemingly counter-intuitive relationship), while AV shows no significance.

Moreover, Table A22 also reports results when considering conditional market variance, estimated over the last 12 monthly observations, as explanatory variable. While the estimate of the associated coefficient appears insignificant over the full sample and the first two subsamples, the variable establishes significant estimates in the

[68] For transparency, $(\Delta AC_{t-1:t})/AC_{t-1}$ is defined as $(AC_t - AC_{t-1})/AC_{t-1}$.

[69] However, the relationship for $(\Delta AV_{t-1:t})/AV_{t-1}$ is consistently positive across all horizons (as it is for the level of AV), supporting the narrative of (relatively) higher risk today implying higher risk in the future.

latter two subperiods, in which also AV shows the strongest performance. However, AV stills outperforms market variance in both subperiods in terms of R^2_{adj} and RMSE.

Overall, the results - especially in the more recent period - deliver additional support for the hypothesis that multi-factor investors mainly should care about variances, at least in the short run.

A.3.4 Variance Forecasting of Other Multi-Factor Portfolios

Another possible way to check for further robustness is to test the impact of AV and AC on short-term risk and returns when observing other multi-factor portfolios than the naive factor portfolio over all available factor strategies, as done so far. Therefore, I use the two different sets of equally-weighted strategies, which are described in Section 3 and formed based on a factor's association to either an ASCL or factor style, to forecast variances and returns for each multi-factor portfolio. I look at two different versions of the introduced measures of AV and AC. First, as before, I use AV and AC calculated over all available factors in the data set. Second, I also calculate both measures solely using the factors underlying the respective multi-factor portfolio considered for the forecast.

I start with forecasts of multi-factor variance over the upcoming 12-month period. The results for the three naive strategies formed on ASCLs are reported in Table A23. With the exception of the strategy based only on equity factors (Naive.E), AV and AC - calculated using all factors - do not seem useful for variance forecasts for other multi-factor portfolios.[70] However, when measuring AV and AC only over the factors constituting the respective mulit-factor portfolio, the results support the idea of variances as the dominant predictor of future multi-factor risk. Taken in isolation, the estimated coefficient of AV shows significance for all three portfolio, with t-statistics between 2.14 and 7.62, while AC only shows relevance for the equity-only strategy (-2.07).

Interestingly, the latter findings is - in some sense - in line with the arguments provided in Section 2. The equity factor segment comprises two market factors, based on equity country indices and U.S. single stock data, respectively. Notably, these are highly correlated (see Figure 2) and this high correlation is - naturally - quite persistent (see Figure A1), since it is 'by design'. Given that the equity-only multi-factor portfolio contains only a fraction of the strategies that constitute the naive portfolio over all factors, the risk of the former portfolio is expected to be strongly influenced by the two highly-correlated factors, due to less diversification. Therefore, fitting this observation into the narrative provided in Section 2, an investor engaging in the equity-only multi-factor strategy probably does *not* perceive his portfolio is to be constructed from totally uncorrelated streams of returns, given the two obviously highly-correlated strategies. The findings support this idea, since AC only shows relevance for variance forecasts of Naive.E, but not the other two portfolios.

In addition to the portfolios formed on ASCLs, Table A24 also shows the findings for the multi-factor strategies based on factor styles. The results are generally in line with those for the set of ASCL-based strategies, i.e. AV - calculated using only the strategies contained in the respective multi-factor portfolio - seems to show the strongest predictive power, when compared to AC. More in detail, the estimated coefficient of AV (AC) shows t-statistics between 2.26 (-1.96) and 9.58 (1.56), when evaluated in isolation. Moreover, while AV - calculated using all available factors - overall appeared as not helpful for the ASCL-based portfolios (except for the equity-only strategy), the measure now shows significant coefficient estimates for every except the carry-only portfolio, when

[70] The explanatory power for the equity-only strategy is likely due to the equity bias in the overall factor universe, i.e. mostly equity factors are used to calculate AV and AC over the whole factor set.

observed standalone. This is - as before - in line with the equity-bias in the overall sample of factors and the fact that carry is the only style-segment which includes no equity factor strategy, therefore being only marginally diversified (across two ASCLs) and having the least in common with the overall naive strategy across all factors, when compared with the other style-based portfolios.

Considering both tables and all alternative naive strategies analyzed, the coefficient of AV (AC) - when evaluated in isolation - exhibits t-statistics between 2.14 (-2.07) and 9.58 (1.56), with an average t-statistic of 4.66 (0.12) and a median of 3.63 (1.09). Therefore, the findings of this robustness test broadly support the main results presented in Section 4.2.2. Finally, another interesting observation is that the measure of variance based on the returns of the respective portfolio return series comes with stronger predictive power for future risk - compared to AV - of multi-factor portfolios with a lower number of considered factors (N). This is in line with the idea that total portfolio variance is a better proxy for risk of undiversified factor portfolios with low N, for which the variance better reflects the information of the single factor variances, that is lost due to diversification for high N.

A.3.5 (Mean) Return Forecasting of Other Multi-Factor Portfolios

In addition to variances, I similarly check for robustness concerning the findings on multi-factor return forecasting, when observing other multi-factor portfolios than the naive factor portfolio over all available factor strategies, as done before. The results for the three naive strategies formed on ASCLs are reported in Table A25.

With the exception of the strategy based only on FX factors (Naive.FX), AV (compared to AC) appears as the main driver of future multi-factor returns, when observed in isolation. Specifically, the coefficient estimated for AV shows a t-statistic of 6.32 for Naive.E and 3.14 for Naive.C, compared to 0.32 for Naive.FX, while no estimate of AC establishes notable significance. Moreover, and again with the exception of Naive.FX, multi-factor variance (and its approximation, the product of AV and AC) also shows significant coefficient estimates. In sum, these findings deliver further support for the relevance of variances for future returns of multi-factor investors, but also highlight the difficulty to forecast returns for FX-associated multi-factor strategies, which do not seem related to contemporaneous proxies of risk.

I also conduct the analysis for the naive strategies formed on factor styles; the results are shown in Table A26. For this setup, the results are more mixed, indicating that portfolios based on styles (across ASCLs) behave differently than those formed on ASCLs (across styles). For the market and carry portfolios, no coefficient estimate across the several regressors and models establishes significance, with the exception of the coefficient of AV, calculated across all available factors and observed together with the measure of portfolio variance (last column).[71] While this is only weak evidence, it is supportive of the role of factor variances as main driver of multi-factor returns. Still, it is notable that the IS evidence of a variance-in-mean relationship is so weak for these two portfolios (and even non-existent for the FX-based strategy mentioned before), compared to the findings of the various other analyses conducted.

For Naive.Mom, I find both AV and correlation as important drivers of future returns. This is not surprising, since the portfolio contains two pairs of factors that are highly-correlated, namely the commodity momentum and commodity basis-momentum strategies, as well as the two equity momentum factors based on country indices and

[71] This findings is especially striking, since the portfolio based on market factors contains the two highly-correlated equity market factors, as outlined in Section A.3.4.

U.S. single stocks (see Figure 2). Finally, the findings for the portfolio with the three remaining factors (Naive.Other) are - again - in line with the previous results and highlight the relevance of factor variances for future multi-factor returns.

Finally, I again consider both tables: Aggregated over all alternative naive strategies analyzed, the coefficient of AV (AC) - when evaluated in isolation - exhibits t-statistics between 0.12 (-2.5) and 15.75 (0.67), with an average t-statistic of 3.96 (-0.38) and a median of 1.92 (-0.37). This last comparison highlights that the main finding reported in Section 4.2.3 broadly persist when considering less diversified multi-factor portfolios (i.e. based on fewer factors), while they are less emphasized in the latter case.

A.4 OS Test

A.4.1 Methodology

To check for further robustness, I also consider OS forecasting to avoid look-ahead bias when evaluating the relationships. Specifically, I recursively estimate the regression models - while using (overlapping) data only available up to a certain point in time - to subsequently forecast the mean return over the upcoming period. These forecasts are then compared to those of a historical mean model, using only the constant term as explanatory variable, where e_N and e_A define the vector of OS errors from the historical mean model and the OLS model, respectively. As measures of accuracy, I observe $OS\text{-}R^2$ and $\Delta RMSE$ as outlined in Welch and Goyal (2007) or Pollet and Wilson (2010):

$$OS\text{-}R^2 = 1 - \frac{MSE_A}{MSE_N} \tag{28}$$

$$\Delta RMSE = \sqrt{MSE_N} - \sqrt{MSE_A} \tag{29}$$

Moreover, I calcualte $MSE\text{-}F$, which is the F-statistic suggested by McCracken (2007):

$$MSE\text{-}F = P * \frac{MSE_N - MSE_A}{MSE_A}, \tag{30}$$

where P is the number of OS observations.

A.4.2 Results

The IS results presented so far suggest that average factor variance is a viable predictor of multi-factor (mean) returns over the short-term future, for instance the upcoming one-month or one-year period, while AC carries no notable predictive information for these horizons. I start the OS evaluation of this claim by predicting returns of the next month, while using only data available up to the current month to conduct the forecast. Table A27 presents the OS results for the main regression models, using a varying number of observations ($R = 91, 131, 171$) to fit the regression for conducting the first forecast.[72]

As can be inferred when comparing Panel a with b and c, the results are to some extent sensitive to the chosen start of the OS exercise (R). Nevertheless, some broader patterns emerge across the three Panels: First, the model

[72] The methodology of the OS exercise is described in Section A.4.1. The following discussion of the results focuses on $OS\text{-}R^2$ as metric to evaluate the forecast performance. However, choosing $\Delta RMSE$ instead leads to qualitatively similar conclusions.

solely based on AV (AC) outperforms (underperforms) the historical mean model in terms of $OS\text{-}R^2$ in all three applications.[73] Second, the competing variance proxy based on the return series of the overall factor portfolio, VAR_t, performs worse than the historical mean model in one of the three applications, and worse than AV in all of them. Last, using the product of AV and AC yields the best results in each analysis, always followed by AV observed standalone on the second rank. Therefore, the main pattern generally remains robust across all tested R: AV appears as the more useful predictor to forecast multi-factor returns at the one-month period when compared to AC.

In the next step, I test the relationship over the one-year horizon, as done before in the IS analysis. Table A28 presents the associated OS results for the main regression models, using a varying number of observations ($R = 80, 120, 160$) to fit the regression for conducting the first forecast of the mean return over the upcoming 12-month period. All three panels essentially show the same patterns. In line with the IS findings, AV is the dominant predictor with an $OS\text{-}R^2$ for the one-regressor model between 10.36% ($R = 80$) and 13.48% ($R = 160$). None of the predictive models tested achieves similar forecast performance, whether based on one or multiple regressors. One the contrary, AC is performing notably worse than the historical mean model in the OS exercise, with a standalone $OS\text{-}R^2$ between -10.18% ($R = 80$) and -7.12% ($R = 160$). In sum, AV clearly seems more of an important driver for future returns.

Table A28 also indicates that forecasting multi-factor returns using multi-factor variance (VAR_t) or the product of AV and AC, which is expected to proxy for multi-factor variance, performs considerably worse than simply using AV. This suggests that important information gets lost when only considering the aggregated return TS of the equally-weighted multi-factor benchmark or when blending the two components of benchmark variance into one measure. Moreover, applying both components AV and AC jointly in a multivariate model helps to improve the forecast performance compared to observing AC, the product of both components, or VAR_t standalone, with $OS\text{-}R^2$ between 0.90% ($R = 80$) and 7.14% ($R = 160$). Still, this model, influenced by the uninformative noise of AC, performs only as the second-best option in all three scenarios, always following the model merely based on AV.

Finally, to evaluate the persistence of the relationship between future returns and the proxies for multi-factor variance, I report in Figure A7 the recursively estimated coefficients when forecasting returns over the next 12 months based on an univariate regression framework. In general, the figure confirms findings already established in the course of conducting IS tests across subsamples (see Section A.3.3 and Table A22). Considering AV, the estimate is negative at the start of the investigation period, successively reverses, and finally turns (and consistently stays) positive at the end of the 1980s. Afterwards, the relationship becomes even more pronounced until reaching a peak at the end of the sample (in line with the results in Table A22). For AC, the estimate is generally more negative in the beginning of the sample, before gradually moving closer to zero over time (compare also to Table A22). So while AV has reached its peak impact on the return forecast (i.e. the highest absolute coefficient estimate) only a few years ago, AC has at the same time reached its lowest impact so far. Moreover, the bottom panel of Figure A7 shows that the estimated coefficients of VAR_t and the product of AV and ACexhibit similar dynamics over time, in line with the idea that both variables are considered being multi-factor variance approximations. The

[73] AV (AC) establishes $OS\text{-}R^2$ of 0.20%, 0.91%, and 1.11% (-1.32%, -0.28% and -0.23%) for $R = 91$, $R = 131$, and $R = 171$, respectively.

relationship between either of the two variables and future returns is negative at the beginning and turns positive in the 2000s, showing a visible upwards trend over time.

Another interesting observation can be drawn from the upper panel of Figure A7. During the recessions in the early 2000s (associated with the dotcom bubble) and the late 2000s (great financial crisis), the figure reveals a notable spike in the coefficient of AV, indicating that a spike in variances across factors is typically followed by higher future returns.[74] Both times, the coefficient subsequently reaches a new all-time high. This observation fits the idea of major exogenous economic shocks as potential trigger for a rise in variances across factors and its impact on future multi-factor returns, as discussed in Section 2.

A.5 Trading Strategies - Timing

A.5.1 Methodology

While the statistical insights so far are interesting, their economic implications for investment decisions, potentially in real-time, are still left to explore. In the following, I therefore generate simple timing strategies, similar to authors such as Moreira and Muir (2017), Cederburg et al. (2020), Barroso and Detzel (2021), or DeMiguel et al. (2021). The goal is to test the viability of varying exposure to the equally-weighted multi-factor portfolio according to different approximations for portfolio variance, specifically AV, AC and the sample variance.[75]

However, my ex-ante expectations are somewhat different to those stated in Moreira and Muir (2017). The authors build their volatility-managed portfolios based on empirical evidence against a variance-in-mean relationship. This, together with documented persistence in variances, makes it attractive to reduce the exposure in volatile times, thereby lowering (managing) the risk and enhancing the risk-return tradeoff.

In contrast, the IS findings established so far suggest a more nuanced picture. Specifically, the overall variance of the multi-factor portfolio is actually less persistent as one would assume based on prior studies' results. This is mainly due to the opposing influences of both variance components, i.e. higher AV [AC] relates to higher [lower] future variance. Moreover, while I indeed find no evidence of a significant variance-in-mean relationship when observing overall, "diluted" portfolio variance, it can be uncovered when considering both variance components (and their - again- opposing effects) separately: Higher AV predicts higher future multi-factor returns. Given these insights, inverse volatility/variance timing, as proposed by Moreira and Muir (2017), should not lead to broad risk-adjusted outperformance.

Therefore, I start by defining two overarching forms of trading strategies constructed from the signal s, based on contrary investment philosophies. The first form, in line with the ideas in Moreira and Muir (2017), follows a "risk-managing" approach to timing, meaning that the exposure to the multi-factor portfolio in the upcoming month is calculated as the inverse of the signal s at the end of month t ($w_{s,MM,t_{+1}} = \frac{1}{s_t}$). The second form instead does the exact opposite, yielding a "risk-embracing" strategy which increases exposure as the signal increases. In more detail, I observe a linear scaling ($w_{s,LIN,t_{+1}} = s_t$), matching the linear regression framework of the IS analysis, as well as a transformation that should help to reduce extreme leverage ($w_{s,SQRT,t_{+1}} = \sqrt{s_t}$).

[74] The same behavior is, to a much smaller extent, also visible in the 1990s recession. While the coefficient also increases notably during the first recession in the 1980s, it afterwards still exhibits a negative sign. The only recession in the sample without a lasting increase in the coefficient of AV is the second in the 1980s.

[75] As a side note: The signal TS used here are identical to those used in the IS analysis, meaning in case of variables such as AV or AC that they are calculated from log returns. However, when calculating the performance of the timing strategies, I consider arithmetic returns.

The monthly returns of the timing strategy based on signal s are then derived as

$$\mathrm{r}_{s,t_{+1}} = w_{s,t_{+1}} \times r_{t_{+1}}, \tag{31}$$

where $r_{t_{+1}}$ is the return of the static naive multi-factor portfolio.

To be aligned with the IS analysis, I additionally observe timing strategies with longer holding periods. For these strategies, given a holding period of H months, the weight for the upcoming period, $w_{s,t_{+1}}$, is carried forward for the next $(H - 1)$ months, assuming that during this period, monthly trading activity only involves rebalancing to maintain the target weight. For each combination of signal s and holding period $\forall H > 1$, I form H parallel strategies. The first begins the timing efforts at the earliest period possible (given data availability), while each of the remaining strategies subsequently delays the start of the timing exercise by a month compared to is predecessor. Then, I average at time t over the monthly weights and returns of all parallel strategies available to arrive at an aggregate strategy for signal s. This procedure's goal is to mitigate the results being driven by a specific starting point.

To measure the benefits of the resulting timing strategies, I calculate various measures of (risk-adjusted) performance, for instance the annualized information ratio as in Ilmanen et al. (2021), using the return TS of the timed strategy and the associated static portfolio (with a constant weight of 1 over the same period) as benchmark.[76] Similar to Moreira and Muir (2017), I scale all timing strategies ex-post to a volatility equal to the unconditional volatility of the static multi-factor portfolio before calculating performance measures, thereby enhancing comparability. For establishing statistical inference, I perform bootstraps with 1000 replications to estimate standard errors and p-values (see Appendix A.6 for more information).

Finally, I briefly want to stress some differences between my methodology, the one applied by Moreira and Muir (2017), and that of other papers in this area. First and foremost, the main focus of Moreira and Muir (2017) (and other authors such as Liu et al. (2019), Cederburg et al. (2020), Barroso and Detzel (2021), and Angelidis and Tessaromatis (2023)) is on the volatility management of different, individually observed factors. In contrast, the analysis provided is concerned with multi-factor investing, aligning with the focus of recent research by DeMiguel et al. (2021). Additionally, Moreira and Muir (2017) (and also DeMiguel et al. (2021) as well as Angelidis and Tessaromatis (2023)) build their multi-factor portfolios from different sets of U.S. equity factors, while the multi-factor portfolio observed in the paper at hand is more diverse in terms of covering more ASCLs and a wider geographic breadth (see Section 3).[77] Moreover, in their analysis Moreira and Muir (2017) focus on timing portfolios which combine the factors in their set so that the portfolio is unconditionally mean-variance efficient (Angelidis and Tessaromatis (2023) follow this procedure). In other words, they chose the static weight of a respective multi-factor portfolio so that the IS Sharpe Ratio is maximized given the underlying set of factors. In

[76] Note that the "static" equally-weighted portfolio is only static in the sense that the factor exposure is the same in each period. However, as mentioned before in Section 3, the portfolio has still to be rebalanced every month to maintain this constant exposure over time. All timing efforts described here are conducted on top of this monthly rebelancing of the equally-weighted portfolio.

[77] While Moreira and Muir (2017) include a FX carry factor in their analysis of individual factors and also cover credit-risk factors (based on corporate bond return data) as well as international stock market indices in their Internet Appendix, these strategies are not included in their multi-factor portfolios. Other authors dealing with volatility management also often solely cover factors and anomaly portfolios constructed from U.S. equity data (see for instance Liu et al. (2019), Cederburg et al. (2020), or Barroso and Detzel (2021)).

contrast, my analysis centers on an approach to multi-factor investing that is agnostic to any expectations of future returns and refrains from further optimization, using instead a simple equally-weighted portfolio (after having conducted an ex-ante volatility scaling, see Section 3).[78]

A.5.2 Results

The results of the timing exercise are reported in Table A29. Once more, the evidence broadly points rather towards the existence of a variance-in-mean relationship than against it. Starting with the strategies constructed in the spirit of Moreira and Muir (2017) (MM), I find actually none of the 18 risk-managing approaches with significant outperformance at the 5% level. The same holds true for the bulk of risk-embracing strategies, indicating that neither reducing nor increasing risk exposure based on contemporaneous conditional variance proxies helps to enhance the risk-return tradeoff, since the former [latter] is not only reducing [increasing] future risk, but also future returns.

Specifically, only one of the total of 54 constructed strategies shows significance at the 5% level, namely the approach scaling weights according to the square root (SQRT) of AV and holding on to the exposure for the next five years.[79] This finding is generally in line with the IS results and can be explained with the distinct feature of this horizon, at which the predictive power of AV for future returns is the strongest across all windows observed, while it is simultaneously the only horizon for which AV does not significantly forecast future risk (see for instance Table A20 and Table A21). While this indeed marks a weakening in the risk-return tradeoff, its dynamic is opposite to the ideas in Moreira and Muir (2017), who build their volatility-managed portfolios based on the evidence of risk persistence and a missing link between conditional risk and future returns.

In any case, the big picture established from the constructed trading strategies caution multi-factor investors to bet on a dysfunctional variance-in-mean relationship. This picture is unchanged when refraining from ex-ante volatility scaling the factor return series before forming the naive multi-factor portfolio (to be more aligned with Moreira and Muir (2017) or Barroso and Detzel (2021)), as Table A30 reveals.

A.6 Details to Bootstrap Methods and Inference

To calculate standard errors for different summary statistics (e.g. mean or standard deviation) of a return TS, I rely on non-parametric bootstrapping (see for instance Davison and Hinkley (1997)). In detail, I generate 1000[80] bootstrap samples, each of which has the same size as the original TS, by randomly drawing monthly observations with replacement from the original sample. Subsequently, I compute the different summary statistics under consideration for each bootstrap sample and derive the standard errors of the statistics.

[78] Moreira and Muir (2017) include, when examining TS alphas of their multi-factor portfolios (presented in Table VI), a risk parity portfolio as control variable, which follows a conceptually similar idea as the multi-factor portfolios observed here. In Barroso and Detzel (2021), the twelfth footnote contains results on an equally-weighted multi-factor portfolio, implying they use a similar weighting method as the provided paper (except for the ex-ante volatility-scaling, which I omit in a robustness test, see Table A30). By contrast, DeMiguel et al. (2021) use a conceptually different approach. They refrain from using fixed relative weights for the components of theirs multi-factor portfolio and instead allow the weights to change dependent on market volatility.

[79] In the face of such weak results, I do not adjust the statistical inference for multiple comparisons, which overall would be sensible given the set numerous strategies tested and reduce the established significance even further. Similarly, I do not consider additional transaction costs that would accrue due to changing exposure to the (monthly-rebalanced) multi-factor portfolio based on the respective predictive signal.

[80] Concerning the number of samples, I follow authors such as Brandt et al. (2009) or Barroso and Santa-Clara (2015) (as the former state in their study and the latter in the Online Appendix of their work).

Using the standard errors from the non-parametric bootstrap, I construct (non-bias-adjusted, i.e. centered around the original estimate of the statistic) normal confidence intervals, one- or two-sided depending on application.

B Appendix: Additional Tables and Figures

B.1 Figures

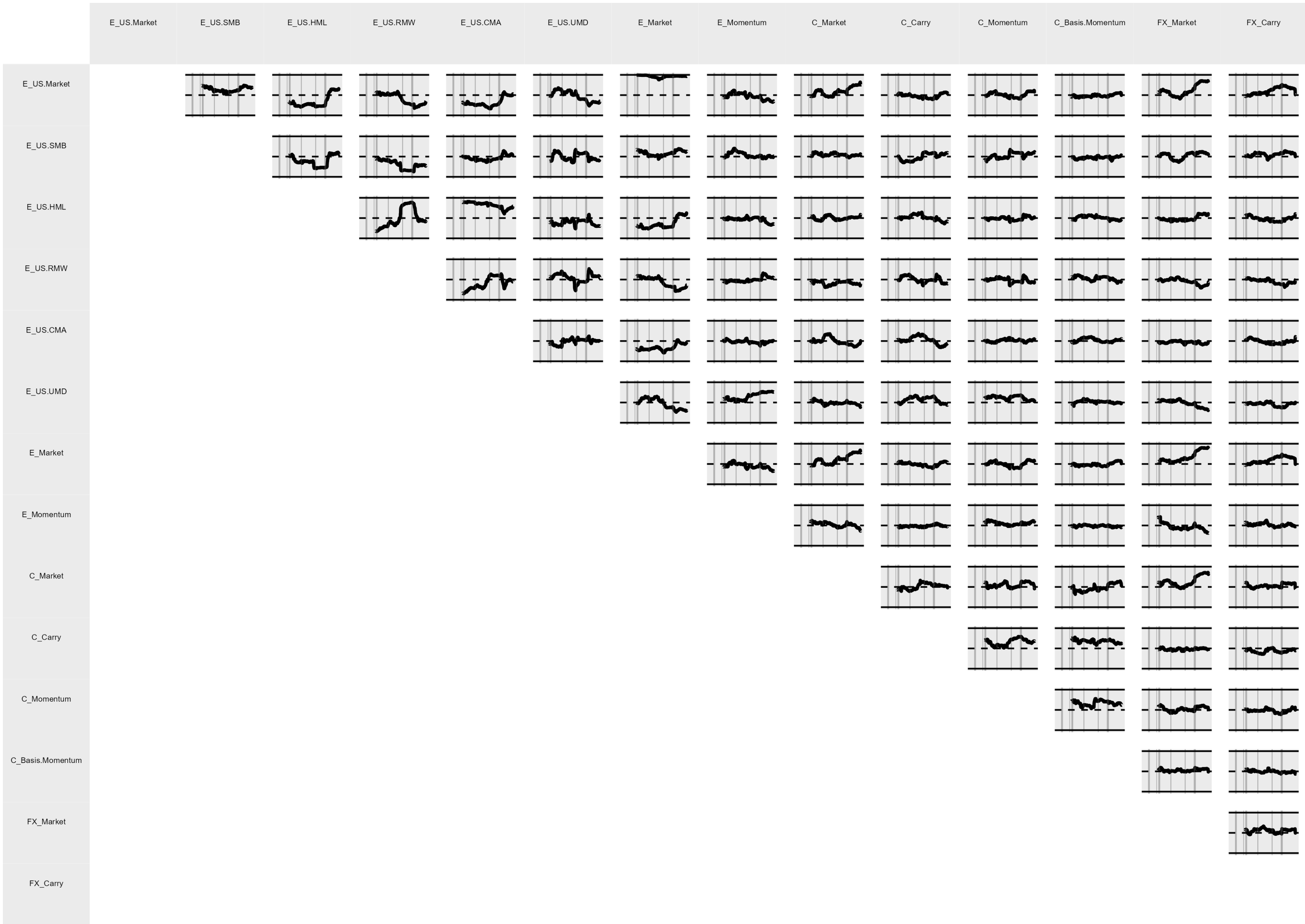

Figure A1: Ten-Year Rolling Correlations of Factor Excess Returns. (Caption on the next page.)

Figure A1: Ten-Year Rolling Correlations of Factor Excess Returns.
This figure presents Pearson correlation coefficients for the set of 14 monthly factor excess return TS over the full investigation period from July 1971 to December 2018, estimated over a rolling window of ten years (120 months) and arranged similar to a common correlation matrix. Correlation pairs are either depicted as solid black block (perfectly correlated elements along the diagonal) or as sparkline (off-diagonal elements). Each sparkline comes with three surrounding horizontal lines: The two solid lines mark correlations of -1 and 1, respectively, while the dashed line identifies a correlation of 0. Moreover, for each sparkline gray shading is highlighting periods of recessions in the U.S., as defined by the NBER. The factor labels' first letters (at the left and upper margin) indicate the associated ASCL of a given factor, where *C, E*, and *FX* are abbreviations for *commodity, equity*, and *foreign exchange*, respectively. Consult Table A4 and Section 3 for an overview of all factor TS analyzed. Detailed data set information is available in Appendix A.1 and Vincenz and Zeissler (2024). An overview of the abbreviations used in the course of this paper is provided in Table A3.

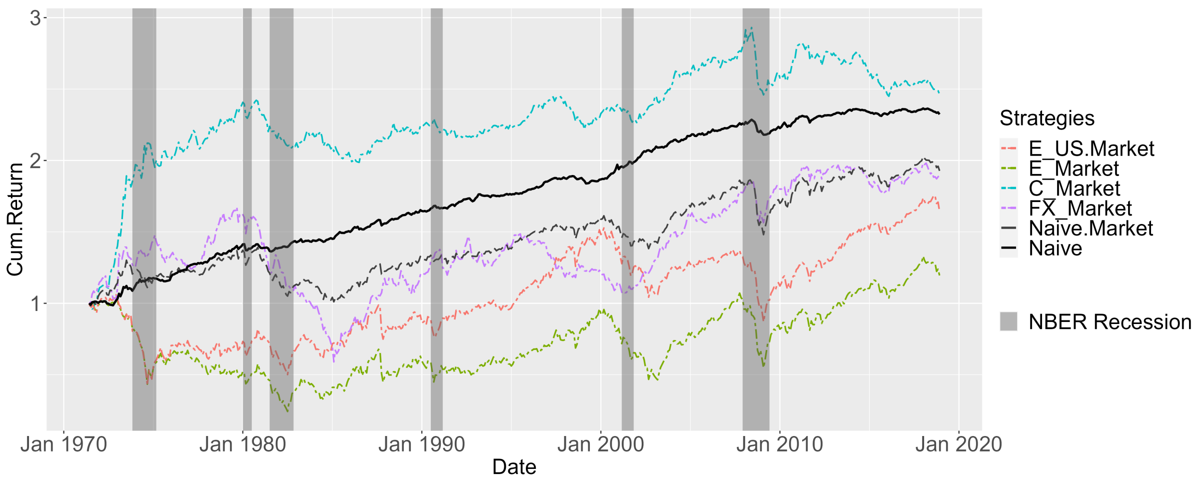

Figure A2: Cumulative Log Excess Returns of Market Factors and Naïve Benchmark.
This figure plots cumulative log excess returns of the four monthly market factor TS, the equally-weighted strategy combining all those market factors, and of the naïve benchmark, which equally weighs all available factors in the investment universe at a given point in time, over the full investigation period from July 1971 to December 2018. Moreover, gray shading is highlighting periods of recessions in the U.S., as defined by the NBER. The factor labels' first letters (plotted at the right margin) indicate the associated ASCL of a given factor, where *C, E*, and *FX* are abbreviations for *commodity, equity*, and *foreign exchange*, respectively. Consult Table A4 and Section 3 for an overview of all factor TS analyzed. Detailed data set information is available in Appendix A.1 and Vincenz and Zeissler (2024). An overview of the abbreviations used in the course of this paper is provided in Table A3.

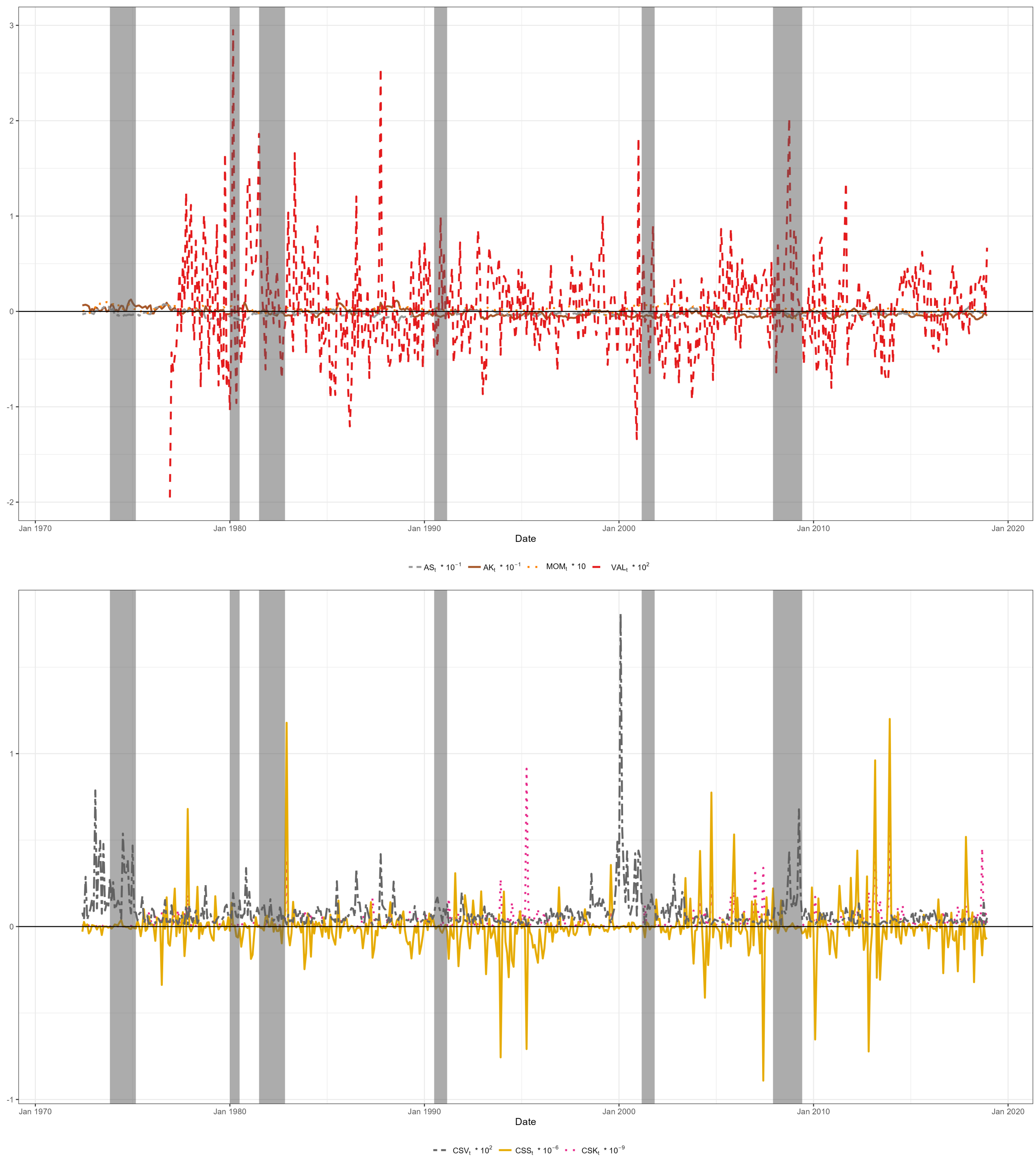

Figure A3: Regressors over Time. Approximations of Benchmark Risk.
This figure presents the dynamics of the approximations of multi-factor risk used as regressors over the full investigation period from July 1971 to December 2018. Moreover, gray shading is highlighting periods of recessions in the U.S., as defined by the NBER. For information on the variables that approximate multi-factor risk (i.e. AS_t, AK_t, CSV_t, CSS_t, CSK_t, MOM_t, and VAL_t), refer to Section A.2.1. An overview of the abbreviations used in the course of this paper is provided in Table A3.

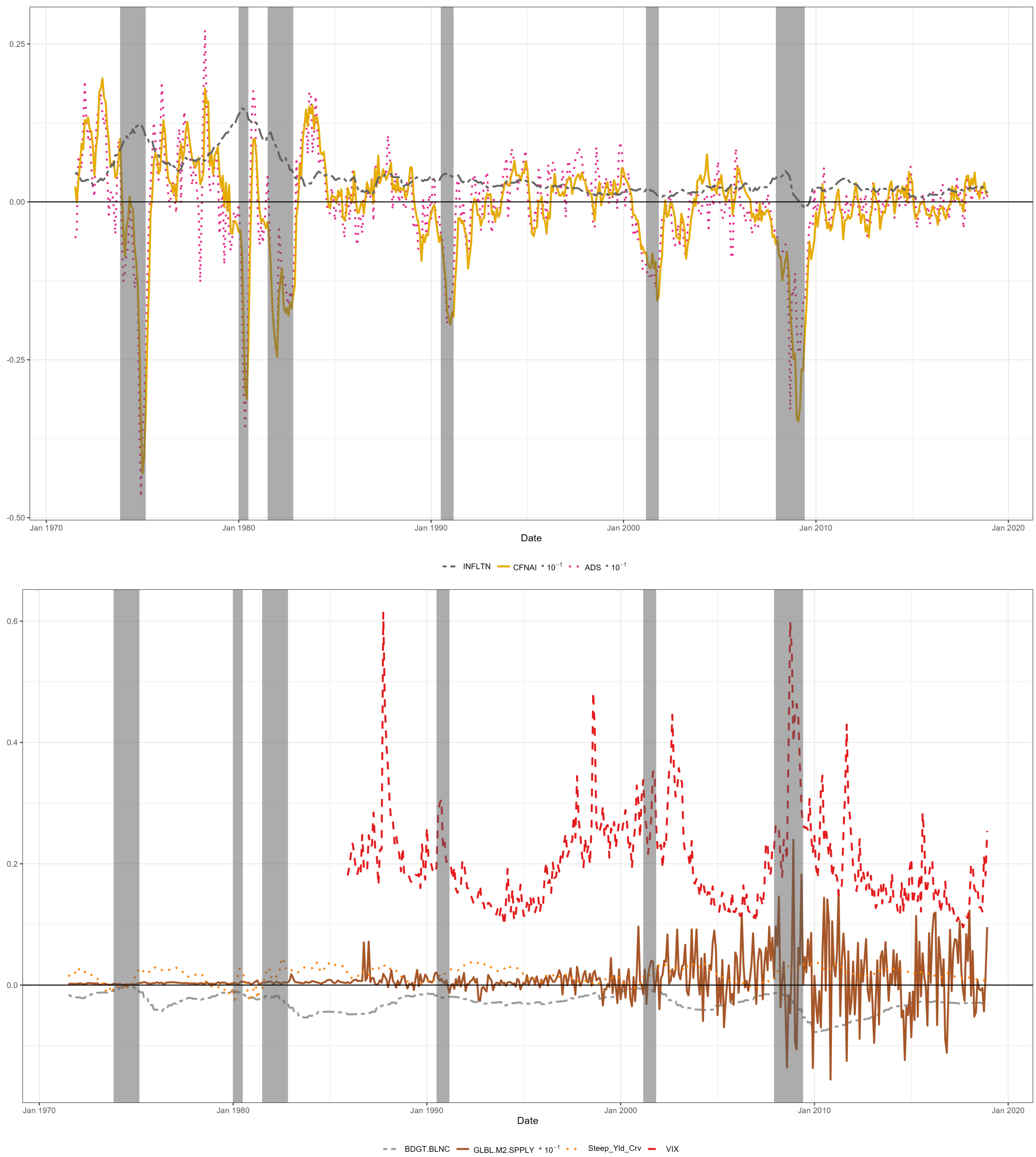

Figure A4: Regressors over Time. External Variables.
This figure presents the dynamics of the external variables used as regressors over the full investigation period from July 1971 to December 2018. Moreover, gray shading is highlighting periods of recessions in the U.S., as defined by the NBER. For information on the external predictors refer to Section A.2.2. An overview of the abbreviations used in the course of this paper is provided in Table A3.

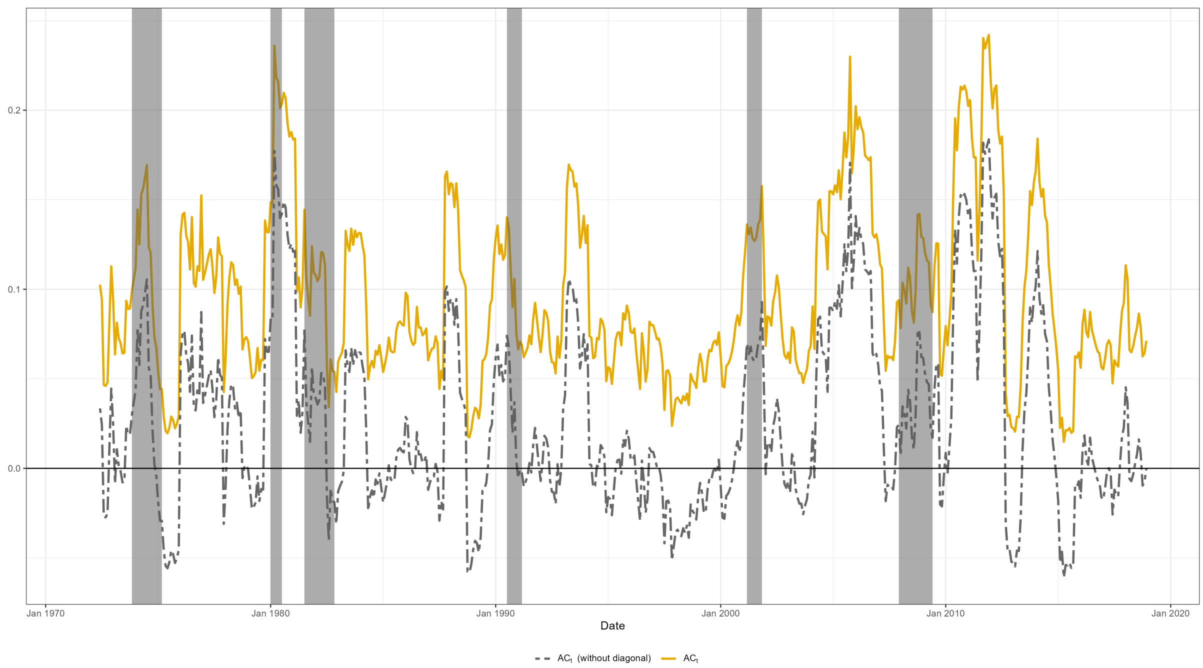

Figure A5: Two Definitions of AC over Time.
This figure presents the dynamics of two different definitions for AC over the full investigation period from July 1971 to December 2018. Specifically, AC_t is defined as $AC_t = \frac{1}{N^2} \sum_{i=1}^{N} \sum_{k=1}^{N} \hat{\rho}^2_{ik,t-12:t}$, while AC_t(without diagonal) follows $AC_t = \frac{1}{N(N-1)} \sum_{i=1}^{N} \sum_{k \neq i} \hat{\rho}^2_{ik,t-12:t}$. Moreover, gray shading is highlighting periods of recessions in the U.S., as defined by the NBER. An overview of the abbreviations used in the course of this paper is provided in Table A3.

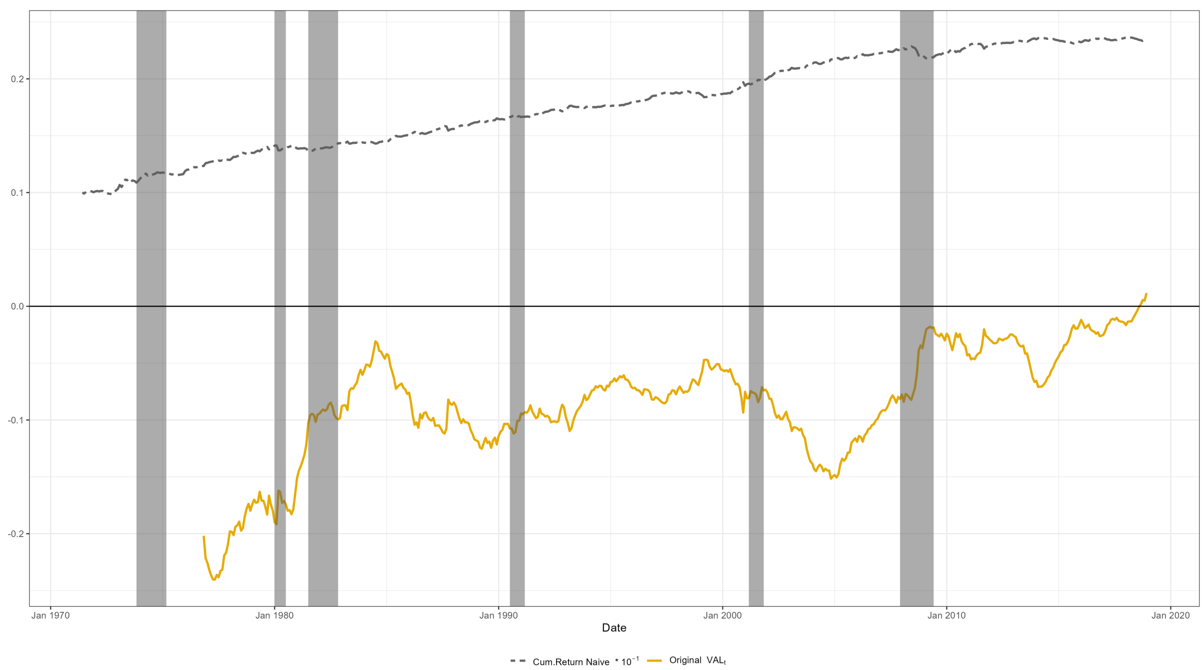

Figure A6: Original Value Signal and Cumulative Log Excess Returns of Naïve Benchmark over Time.
This figure presents the 'original' value signal, i.e. before taking first differences of the TS due to evidence of a unit root (see Table A5), and cumulative log excess returns of the naïve benchmark, which equally weighs all available factors in the investment universe at a given point in time, over the full investigation period from July 1971 to December 2018. Moreover, gray shading is highlighting periods of recessions in the U.S., as defined by the NBER. For more information on the value signal VAL_t, refer to Section A.2.1. Consult Table A4 and Section 3 for an overview of all factor TS analyzed. Detailed data set information is available in Appendix A.1 and Vincenz and Zeissler (2024). An overview of the abbreviations used in the course of this paper is provided in Table A3.

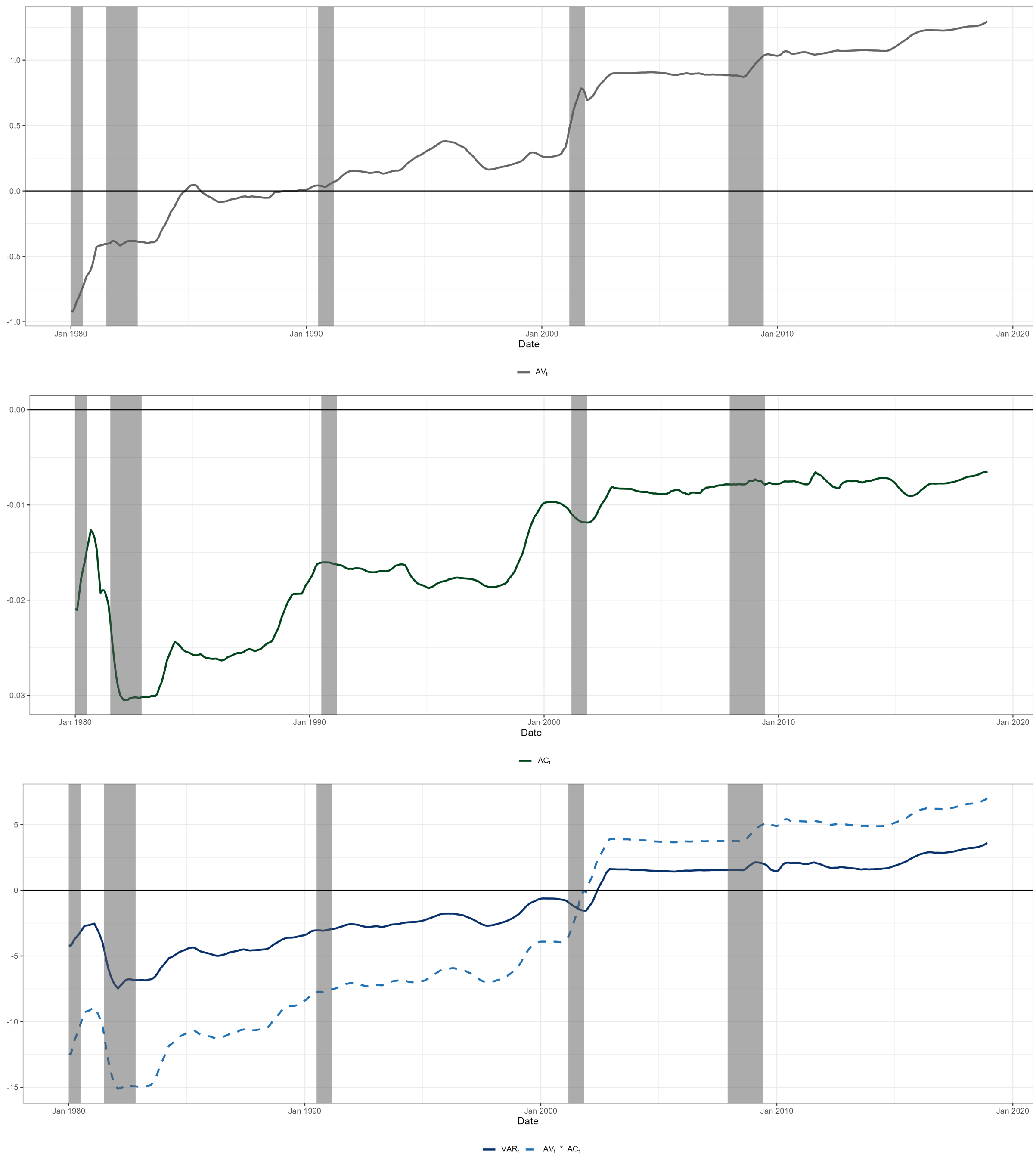

Figure A7: Expanding Coefficient Estimates. Mean Return Forecasting (12M). R = 80.
This figure reports the TS of coefficients estimated recursively in the course of forecasting (rolling) mean returns of the equally-weighted benchmark over the next 12-month period in an OS setting (see Table A28, $R = 80$, i.e. 80 observations are used for the initial estimation). Specifically, each line represents the estimated coefficient of an univariate linear model using the respective predictive variable as well as an intercept to subsequently predict future returns. Consult Table A4 and Section 3 for an overview of factors constituting the equally-weighted multi-factor portfolio. Detailed data set information is available in Appendix A.1 and Vincenz and Zeissler (2024). Refer to Section 4.1 for information on all variables that approximate (components of) multi-factor variance (i.e. AV_t, AC_t, VAR_t, and $VAR.G_t$). An overview of the abbreviations used in the course of this paper is provided in Table A3.

B.2 Tables

Abbreviation	Full Expression
AC	average correlation
ASCL	asset class
AV	average variance
CAPM	capital asset pricing model
ETF	exchange-traded fund
FX	foreign exchange
IS	in-sample
OS	out-of-sample
TS	time(-)series

Table A3: Abbreviations.
The table lists all abbreviations introduced and used in the course of this paper to provide a quick overview.

Symbol	Test statistic	Lag order	No. Diff.
AV_t (12M)	-4.29	4	0
AC_t (12M)	-6.25	5	0
VAR_t (12M)	-6.27	9	0
$VAR.G_t$	-5.84	10	0
AS_t (12M)	-6.01	1	0
AK_t (12M)	-7.64	1	0
CSV_t	-4.62	10	0
CSS_t	-6.67	8	0
CSK_t	-5.36	8	0
MOM_t (12M)	-6.24	8	0
VAL_t (60M)	-8.00	4	1

(a) Risk Approximations

Symbol	Test statistic	Lag order	No. Diff.
INFLTN	-6.05	4	0
BDGT.BLNC	-4.69	6	0
GLBL.M2.SPPLY	-8.28	9	1
Steep_Yld_Crv	-4.42	3	0
CFNAI	-5.15	9	0
ADS	-5.49	8	0
VIX	-7.20	2	0

(b) External Predictors

Table A5: Unit Root Tests.
This table reports results of the Augmented Dickey-Fuller tests (see Dickey and Fuller (1979)) conducted for the monthly TS that are used as regressors, i.e. the risk approximations (Panel a) and external candidate predictors (Panel b). The unit root tests are based on regressions that include both a constant and time trend. Specifically, the table lists for each TS the resulting test statistic, the number of lags included in the test, as well as the number of transformations (i.e. differencing) that were necessary before rejecting the null hypothesis of a unit root at the five percent level. The five percent critical value associated is -3.41. The lag order is selected according to the Akaike (AIC) information criterion, with a maximum number of ten lags considered. Refer to Sections 4.1 (multi-factor variance), A.2.1 (multi-factor risk), and A.2.2 (external predictors) for an overview of explanatory variables in the regressions. An overview of the abbreviations used in the course of this paper is provided in Table A3.

Asset class	Factor	Characteristics	Measurement	Source	Transaction costs	Instrument	No. assets
FX	Market	Equally-weighted	$\frac{1}{N}$	Lustig et al. (2011)	Bid-ask spread	Forwards	69
	Carry	Implied-Yield	$\frac{f_t}{s_t}(1+r_t^d)-1$	Lustig et al. (2011)			
Commodity	Market	Equally-weighted	$\frac{1}{N}$	Szymanowska et al. (2014)	4.4 bps rel. half-spread	Futures	31
	Carry	Basis (First-to-Second)	$\frac{F_t^{T_2}}{F_t^{T_1}}-1$	Szymanowska et al. (2014)			
	Momentum	12-1 month return	$\prod_{s=t-11}^{t-1}(1+r_s^{T_1})-1$	Bakshi et al. (2019)			
	Basis-Momentum	12 month return difference	$\prod_{s=t-11}^{t}(1+r_s^{T_1})-\prod_{s=t-11}^{t}(1+r_s^{T_2})$	Boons and Prado (2019)			
Equity Index	Market	Market-cap weighted	$\frac{MV_{t,i}}{\sum_i MV_{t,i}}$	Fama and French (1993)	10 bps p.m.	MSCI Indices	49
	Momentum	12-1 month return	$\prod_{s=t-11}^{t-1}(1+r_s)-1$	Asness et al. (2013)			
Equity Single Stocks	Market	Market-cap weighted	$\frac{MV_{t,i}}{\sum_i MV_{t,i}}$	Fama and French (1993)	20 bps p.m.	Stocks	24781
	Size	Market capitalization	MV_t	Fama and French (1993)			
	Momentum	12-1 month return	$\prod_{s=t-11}^{t-1}(1+r_s)-1$	Carhart (1997)			
	Value	Book-to-market	BtM_t	Fama and French (1993)			
	Quality	Operating Profitability	OP_t	Fama and French (2015)			
	Investment	Investment	ΔTA_t	Fama and French (2015)			

Table A4: Factor Replication Overview.
This table borrowed from Vincenz and Zeissler (2024) describes the construction of factors across different ASCLs and links to the relevant literature. Each month, Vincenz and Zeissler (2024) cross-sectionally rank the characteristics for each factor and go long (short) the top (bottom) 16.67% quantile of assets. f_t is the currency forward rate at time t, s_t is the spot exchange rate at time t, r_t^d is the domestic interest rate (here referred to as the 1-month U.S. interest rate), $CPI_t^{f,d}$ is the consumer price index of the foreign (domestic) country f (d) at time t. $F_t^{T_i}$ is the value of the i-th nearby futures contract at time t expiring at time T, y_t^{i-y} is the i year interest rate at time t, BtM_t is the book-to-market ratio, OP the operating profitability ratio and TA total assets. FX, commodity, fixed income and equity index data are from Bloomberg and Global Financial Data. Equity single stock factors are constructed using the CRSP/Compustat database. For more information on the factor construction, see Vincenz and Zeissler (2024). An overview of the abbreviations used in the course of this paper is provided in Table A3.

No.	Period	Length	Mean
1	Apr 1973 - May 1974	14	0.242
2	Jul 1974 - Jul 1975	13	0.218
3	Feb 2000 - Dec 2001	23	0.291
4	Apr 2009 - Sep 2009	6	0.225

(a) AV_t

No.	Period	Length	Mean
1	Mar 1980 - Feb 1981	12	20.207
2	Apr 1993 - May 1993	2	16.814
3	Jun 2005 - Oct 2005	5	18.862
4	Dec 2005 - Sep 2006	10	18.390
5	Jun 2010 - May 2011	12	19.697
6	Aug 2011 - Jul 2012	12	21.147

(b) AC_t

No.	Period	Length	Mean
1	Feb 1973 - Sep 1974	20	0.031
2	Mar 1980 - Feb 1981	12	0.027
3	Jan 2001 - Dec 2001	12	0.023
4	Oct 2008 - Sep 2009	12	0.025

(c) VAR_t

No.	Period	Length	Mean
1	Feb 1973 - Sep 1973	8	0.057
2	Nov 1973 - Jul 1974	9	0.025
3	Jan 1976 - Mar 1976	3	0.023
4	Dec 1976 - Jan 1977	2	0.021
5	Oct 1979 - Nov 1979	2	0.018
6	Mar 1980 - Aug 1980	6	0.026
7	Oct 1987 - Dec 1987	3	0.020
8	Dec 2000 - Apr 2001	5	0.025
9	Sep 2008 - Feb 2009	6	0.025
10	Sep 2011 - Nov 2011	3	0.019

(d) $VAR.G_t$

Table A6: Periods with Peaks in Approximations for Benchmark Variance.
This table lists - per panel - periods showing extreme increases in one of the different measures analyzed as approximations for the variance of the multi-factor benchmark. Specifically, each panel reports per period the beginning and ending date, the length of the period (in months), as well as the mean estimated over all observations within the period (stated as percentage). A period with an extreme increase in the underlying variable is defined as a time frame of at least two consecutive months, of which all values associated fall into the upper 10% quantile of all observations available. For a convenient comparison, the mean estimates over all observations available are 0.09% (AV_t), 0.1 (AC_t), 0.01% (VAR_t), and 0.01% ($VAR.G_t$). Refer to Section 4.1 for information on all variables that approximate (components of) multi-factor variance (i.e. AV_t, AC_t, VAR_t, and $VAR.G_t$). An overview of the abbreviations used in the course of this paper is provided in Table A3.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
(Intercept)	0 [1.322]	0 [0.121]	**0 [-3.15]**	0 [0.574]	0 [0.981]	**0 [4.601]**	**0 [6.488]**	**0 [7.324]**	**0 [16.085]**	**0 [16.817]**	**0 [3.389]**	**0 [14.01]**	**0 [6.24]**
AV_t (12M)	**0.063 [2.258]**		**0.066 [2.316]**		-0.042 [-1.049]								**-0.09 [-6.87]**
AC_t (12M)		**0.001 [5.066]**	**0.001 [6.55]**		0 [-1.114]								**0 [-3.518]**
AV_t (12M) * AC_t (12M)				**0.87 [4.438]**	**1.221 [6.521]**								**1.444 [13.833]**
AS_t (12M)						0 [-1.077]							**0 [-3.095]**
AK_t (12M)							0 [1.86]						**0 [2.483]**
CSV_t								0.019 [1.823]					**0.004 [2.525]**
CSS_t									0 [0.218]				**0 [2.414]**
CSK_t										**0 [-4.053]**			**0 [-3.634]**
MOM_t (12M)											0.003 [0.432]		**-0.003 [-2.472]**
VAL_t (60M)												0.001 [1.754]	-0.001 [-1.897]
RMSE (‰)	0.063	0.066	0.049	0.042	0.040	0.075	0.073	0.072	0.076	0.075	0.075	0.061	0.022
R^2_{adj} (%)	30.62	23.48	57.30	69.00	72.06	2.42	5.89	8.68	-0.18	2.85	1.22	1.11	86.81
No. Obs.	559	559	559	559	559	559	559	559	559	559	559	505	505

(a) Risk Approximations

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
(Intercept)	**0 [2.612]**	**0 [2.884]**	**0 [16.211]**	**0 [3.185]**	**0 [6.178]**	**0 [7.02]**	0 [-1.049]	0 [1.77]	0 [1.649]	0 [1.596]	0 [1.526]	0 [1.586]	0 [-1.026]	0 [0.895]	0 [1.858]	0 [1.892]
AV_t (12M)														**-0.07 [-7.289]**	**-0.064 [-6.563]**	**-0.068 [-7.004]**
AC_t (12M)														0 [-0.981]	0 [0.262]	0 [-0.147]
AV_t (12M) * AC_t (12M)														**1.115 [10.445]**	**1.022 [9.103]**	**1.07 [11.345]**
AS_t (12M)															**0 [-2.19]**	**0 [-2.475]**
AK_t (12M)															**0 [3.518]**	**0 [3.171]**
CSV_t																0.002 [0.993]
CSS_t																0 [0.964]
CSK_t																**0 [-2.47]**
MOM_t (12M)																**-0.002 [-2.774]**
VAL_t (60M)																0 [-1.186]
INFLTN	**0.001 [2.44]**							**0.001 [2.794]**	**0.001 [2.45]**	**0.001 [2.748]**	**0.001 [2.567]**	**0.001 [2.58]**	0 [-0.593]	0 [-1.639]	**-0.001 [-3.212]**	**0 [-2.271]**
BDGT.BLNC		0.001 [0.693]						0 [0.213]	0 [0.218]	0 [0.111]	0 [-0.313]	0 [-0.34]	-0.001 [-1.08]	0 [-0.509]	0 [0.587]	0 [0.43]
GLBL.M2.SPPLY			0 [0.352]						0 [1.902]	0 [1.726]	0 [1.512]	0 [1.624]	0 [1.389]	0 [1.338]	0 [1.392]	0 [1.59]
Steep_Yld_Crv				-0.001 [-0.797]						0 [-0.233]	-0.001 [-0.731]	-0.001 [-0.716]	0.001 [1.767]	**0.001 [2.641]**	**0.001 [2.604]**	**0.001 [3.357]**
CFNAI					0 [-1.905]						0 [-1.298]	0 [-0.322]	0 [-1.437]	**0 [-2.517]**	**0 [-4.196]**	**0 [-4.369]**
ADS						**0 [-3.06]**						0 [-0.903]	0 [-1.355]	0 [0.279]	0 [0.013]	0 [-0.506]
VIX							**0 [4.003]**						**0 [4.027]**	**0 [4.451]**	**0 [3.44]**	**0 [4.169]**
RMSE (‰)	0.071	0.075	0.076	0.075	0.073	0.073	0.049	0.071	0.071	0.071	0.069	0.069	0.042	0.020	0.018	0.017
R^2_{adj} (%)	10.92	2.08	-0.15	2.70	6.70	8.40	30.27	10.89	11.00	10.97	16.19	16.62	49.32	88.43	90.87	91.52
No. Obs.	559	559	559	559	559	559	397	559	559	559	559	559	397	397	397	397

(b) External Predictors

Table A7: Variance Decomposition of the Naive Portfolio. (Caption on the next page.)

Table A7: Variance Decomposition of the Naive Portfolio.
This table reports OLS results of regressing contemporaneous (12-month rolling) multi-factor return variance (dependent variable) on contemporaneous risk proxies (Panel a) and external candidate predictors (Panel b). Data covers the full investigation period from July 1971 to December 2018. The regressor names are shown at the left margin. For return-based measures, the estimation period (in months) is disclosed in brackets alongside the regressor's name. Below each estimated coefficient, associated t-statistics (adjusted based on the methods proposed in Newey and West (1987) and Newey and West (1994)) are reported in squared brackets. Coefficients with absolute t-statistics above 2 are highlighted in bold. The last three rows show per model root mean squared error (RMSE), adjusted R^2 (R^2_{adj}), and number of observations used for estimation. Consult Table A4 and Section 3 for an overview of factors constituting the equally-weighted multi-factor portfolio. Detailed data set information is available in Appendix A.1 and Vincenz and Zeissler (2024). Refer to Sections 4.1 (multi-factor variance), A.2.1 (multi-factor risk), and A.2.2 (external predictors) for an overview of explanatory variables in the regressions. The TS of the regressors were tested to rule out the possibility of containing unit roots with sufficient confidence. If necessary (only in the case of VAL_t (60M) and GLBL.M2.SPPLY), the TS were transformed (by calculating differences) and re-tested (see Table A5). An overview of the abbreviations used in the course of this paper is provided in Table A3.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
(Intercept)	**0**	**0**	**0**	**0**	0	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	0
	[3.238]	**[4.454]**	**[2.595]**	**[4.004]**	[1.149]	**[4.648]**	**[5.368]**	**[3.32]**	**[2.864]**	**[5.47]**	**[5.901]**	**[9.936]**	**[15.951]**	**[12.876]**	**[4.665]**	**[16.94]**	[-1.3]
AV_t (12M)	**0.038**		**0.037**		**0.07**			**0.039**	**0.032**								**0.132**
	[4.678]		**[4.747]**		**[2.964]**			**[4.154]**	**[3.206]**								**[4.633]**
AC_t (12M)		0	0		0												**0**
		[-1.571]	[-1.271]		[0.753]												**[2.495]**
AV_t (12M) * AC_t (12M)				**0.19**	-0.369												**-1.293**
				[2.239]	[-1.746]												**[-3.178]**
VAR_t (12M)						**0.166**		-0.025									**0.51**
						[2.035]		[-0.307]									**[2.174]**
$VAR.G_t$							**0.223**		**0.131**								0.056
							[4.087]		**[2.185]**								[0.438]
AS_t (12M)										0							0
										[0.127]							[-0.184]
AK_t (12M)											0						0
											[1.037]						[-0.214]
CSV_t												**0.015**					-0.001
												[3.547]					[-0.312]
CSS_t													0				0
													[0.514]				[0.195]
CSK_t														**0**			0
														[-3.432]			[-0.252]
MOM_t (12M)															**0.004**		**0.006**
															[2.158]		**[3.107]**
VAL_t (60M)																-0.001	0
																[-1.798]	[0.524]
RMSE (‰)	0.070	0.074	0.070	0.073	0.069	0.074	0.072	0.070	0.069	0.075	0.074	0.072	0.075	0.074	0.074	0.062	0.053
R^2_{adj} (%)	11.52	1.03	12.15	3.24	13.42	2.71	6.01	11.40	13.23	-0.15	2.17	5.78	-0.16	1.21	2.28	0.55	25.14
No. Obs.	547	547	547	547	547	547	547	547	547	547	547	547	547	547	547	493	493

(a) Risk Approximations

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
(Intercept)	**0**	**0**	**0**	**0**	**0**	**0**	0	**0**	**0**	**0**	**0**	**0**	0	**0**	**0**	**0**
	[3.552]	**[4.503]**	**[11.226]**	**[5.648]**	**[5.651]**	**[6.155]**	[1.745]	**[2.559]**	**[2.382]**	**[2.486]**	**[2.532]**	**[2.551]**	[0.192]	**[-3.172]**	**[-2.839]**	**[-2.903]**
AV_t (12M)														**0.11**	**0.098**	**0.095**
														[5.667]	**[5.53]**	**[6.082]**
AC_t (12M)														**0**	0	0
														[2.071]	[1.109]	[0.997]
AV_t (12M) * AC_t (12M)														**-0.757**	**-0.592**	**-0.586**
														[-3.404]	**[-3.114]**	**[-3.001]**
AS_t (12M)															**0**	0
															[2.033]	[1.808]
AK_t (12M)															0	0
															[-0.598]	[-0.658]
CSV_t																0.002
																[0.563]
CSS_t																0
																[0.074]
CSK_t																0
																[0.087]
MOM_t (12M)																0.001
																[0.757]
VAL_t (60M)																0
																[-0.512]
INFLTN	0							0	0	0	0	0	0.001	0.002	**0.002**	**0.002**
	[1.893]							[1.001]	[0.942]	[0.728]	[1.03]	[0.969]	[1.064]	[1.951]	**[2.207]**	**[2.757]**
BDGT.BLNC		0.001						0.001	0.001	0.001	0.001	0.001	0	-0.001	-0.001	**-0.001**
		[1.379]						[0.994]	[0.889]	[0.604]	[0.674]	[0.647]	[-0.262]	[-1.489]	[-1.757]	**[-2.498]**
GLBL.M2.SPPLY			0						0	0	0	0	0	0	0	0
			[0.23]						[0.894]	[0.758]	[1.338]	[1.335]	[1.069]	[1.359]	[1.369]	[1.209]
Steep_Yld_Crv				-0.001						-0.001	0	0	0	0	0	0
				[-1.785]						[-1.007]	[-0.662]	[-0.58]	[-0.439]	[0.036]	[-0.119]	[-0.211]
CFNAI					0						0	0	0	0	0	0
					[0.513]						[0.589]	[1.142]	[0.773]	[0.161]	[0.901]	[0.521]
ADS						0						**0**	**0**	0	0	0
						[0.166]						**[-2.065]**	**[-2.243]**	[-1.886]	[-1.805]	[-1.622]
VIX							**0**						0	0	**0**	0
							[2.385]						[1.893]	[0.941]	**[2.035]**	[1.699]
RMSE (‰)	0.075	0.074	0.076	0.074	0.076	0.076	0.058	0.074	0.074	0.074	0.073	0.073	0.056	0.046	0.044	0.044
R^2_{adj} (%)	2.65	3.64	-0.17	3.97	0.69	-0.12	5.44	4.61	4.54	5.25	6.51	6.89	11.14	38.92	42.78	42.35
No. Obs.	558	558	558	558	558	558	385	558	558	558	558	558	385	385	385	385

(b) External Predictors

Table A8: Variance Forecasting (12M) of the Naive Portfolio..
This table reports OLS results of regressing (rolling) multi-factor return variance over the next 12-month period (dependent variable) on contemporaneous risk proxies (Panel a) and external candidate predictors (Panel b). To prevent repetition, refer to Table A7 or Table A13 for a detailed table description.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
(Intercept)	**0**	**0**	**0**	**0**	0	**0**	**0**	**0**	0	**0**	**0**	**0**	**0**	**0**	**0**	**0**	0
	[2.792]	**[4.228]**	**[2.294]**	**[3.642]**	[1.168]	**[4.423]**	**[4.246]**	**[2.826]**	[1.816]	**[5.731]**	**[5.605]**	**[7.413]**	**[11.238]**	**[8.913]**	**[4.599]**	**[14.449]**	[-0.691]
AV_t (12M)	**0.046**		**0.045**		0.062			**0.044**	**0.035**								**0.099**
	[3.591]		**[3.852]**		[1.851]			**[2.74]**	**[2.401]**								**[2.324]**
AC_t (12M)		0	0		0												0
		[-0.866]	[-0.556]		[0.397]												[1.04]
AV_t (12M) * AC_t (12M)				**0.296**	-0.181												-0.586
				[2.445]	[-0.63]												[-0.981]
VAR_t (12M)						0.24		0.023									0.053
						[1.87]		[0.162]									[0.135]
$VAR.G_t$							**0.341**		**0.239**								0.201
							[3.938]		**[3.043]**								[0.996]
AS_t (12M)										0							0
										[0.863]							[0.091]
AK_t (12M)											0						0
											[0.784]						[-0.396]
CSV_t												0.012					**-0.011**
												[1.207]					**[-2.689]**
CSS_t													0				0
													[1.515]				[1.495]
CSK_t														**0**			0
														[-2.229]			[-0.369]
MOM_t (12M)															**0.008**		**0.008**
															[3.064]		**[3.146]**
VAL_t (60M)																-0.001	0
																[-1.345]	[0.492]
RMSE (‰)	0.091	0.096	0.091	0.094	0.091	0.094	0.092	0.091	0.089	0.096	0.096	0.095	0.096	0.096	0.093	0.083	0.075
R^2_{adj} (%)	10.11	0.05	10.03	4.85	10.07	3.42	8.51	9.97	13.67	0.54	0.67	1.91	0.00	0.45	5.35	0.38	17.63
No. Obs.	553	553	553	553	553	553	553	553	553	553	553	553	553	553	553	499	499

(a) Risk Approximations

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
(Intercept)	**0**	**0**	**0**	**0**	**0**	**0**	0	0	0	**0**	**0**	**0**	0	**0**	**0**	**0**
	[3.509]	**[3.428]**	**[11.528]**	**[4.263]**	**[5.788]**	**[6.25]**	[1.744]	[1.892]	[1.896]	**[2.086]**	**[2.026]**	**[2.055]**	[-0.68]	**[-2.723]**	**[-2.747]**	**[-2.524]**
AV_t (12M)														**0.1**	**0.084**	**0.102**
														[3.148]	**[2.618]**	**[2.261]**
AC_t (12M)														0	0	0
														[0.16]	[-0.617]	[-0.265]
AV_t (12M) * AC_t (12M)														-0.476	-0.252	-0.414
														[-1.301]	[-0.689]	[-1.087]
AS_t (12M)															**0**	**0**
															[2.779]	**[2.587]**
AK_t (12M)															0	0
															[-1.592]	[-1.502]
CSV_t																**-0.011**
																[-2.457]
CSS_t																0
																[1.571]
CSK_t																0
																[0.594]
MOM_t (12M)																0.003
																[1.415]
VAL_t (60M)																0
																[0.151]
INFLTN	**0.001**							0.001	0.001	0	0	0	0.002	**0.003**	**0.003**	**0.003**
	[2.034]							[1.504]	[1.311]	[1.022]	[1.313]	[1.26]	[1.943]	**[2.709]**	**[2.842]**	**[3.058]**
BDGT.BLNC		0.001						0.001	0.001	0	0	0	0	-0.001	**-0.001**	**-0.001**
		[1.115]						[0.683]	[0.701]	[0.237]	[0.346]	[0.329]	[-0.482]	[-1.904]	**[-2.565]**	**[-2.648]**
GLBL.M2.SPPLY			0						0	0	0	0	0	0	0	0
			[-1.098]						[-0.527]	[-0.641]	[-0.567]	[-0.542]	[-0.33]	[-0.055]	[-0.246]	[-0.339]
Steep_Yld_Crv				-0.002						-0.001	-0.001	-0.001	0	0	0	0
				[-1.772]						[-1.374]	[-1.289]	[-1.203]	[-0.262]	[-0.19]	[-0.207]	[-0.31]
CFNAI					0						0	0	0	0	0	0
					[0.485]						[0.497]	[0.966]	[1.243]	[0.743]	[1.656]	[1.019]
ADS						0						0	**0**	**0**	**0**	0
						[0.042]						[-1.32]	**[-2.553]**	**[-2.7]**	**[-2.569]**	[-1.873]
VIX							**0**						**0**	0	0	0
							[2.519]						**[2.189]**	[0.226]	[1.72]	[1.897]
RMSE (‰)	0.093	0.094	0.095	0.093	0.095	0.095	0.078	0.093	0.093	0.092	0.092	0.091	0.074	0.064	0.062	0.060
R^2_{adj} (%)	3.86	2.56	0.01	5.16	0.29	-0.18	3.05	4.73	4.61	6.32	6.77	6.99	11.46	32.14	36.92	39.36
No. Obs.	564	564	564	564	564	564	391	564	564	564	564	564	391	391	391	391

(b) External Predictors

Table A9: Variance Forecasting (6M) of the Naive Portfolio.
This table reports OLS results of regressing (rolling) multi-factor return variance over the next 6-month period (dependent variable) on contemporaneous risk proxies (Panel a) and external candidate predictors (Panel b). To prevent repetition, refer to Table A7 or Table A13 for a detailed table description.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
(Intercept)	**0** **[3.874]**	**0** **[5.633]**	**0** **[3.341]**	**0** **[4.687]**	**0** **[2.151]**	**0** **[5.126]**	**0** **[7.544]**	**0** **[3.837]**	**0** **[3.728]**	**0** **[6.759]**	**0** **[7.44]**	**0** **[13.806]**	**0** **[26.078]**	**0** **[15.967]**	**0** **[5.291]**	**0** **[17.221]**	0 [0.287]
AV_t (12M)	**0.021** **[2.936]**		**0.02** **[2.577]**		0.036 [1.867]			**0.023** **[3.16]**	**0.018** **[2.391]**								**0.096** **[4.813]**
AC_t (12M)		**0** **[-2.333]**	0 [-1.994]		0 [0.117]												**0** **[2.283]**
AV_t (12M) * AC_t (12M)				0.091 [1.413]	-0.176 [-1.162]												**-1.025** **[-3.925]**
VAR_t (12M)						0.078 [1.352]		-0.031 [-0.67]									**0.413** **[2.777]**
$VAR.G_t$							**0.109** **[3.488]**		0.057 [1.86]								0.089 [1.153]
AS_t (12M)										0 [-0.028]							0 [0.14]
AK_t (12M)											0 [1.049]						0 [0.33]
CSV_t												**0.01** **[3.784]**					-0.001 [-0.424]
CSS_t													0 [0.269]				0 [-0.175]
CSK_t														**0** **[-2.582]**			0 [-0.063]
MOM_t (12M)															0.001 [0.49]		**0.003** **[2.16]**
VAL_t (60M)																0 [0.034]	0.001 [1.643]
RMSE (‰)	0.050	0.052	0.050	0.052	0.050	0.052	0.051	0.050	0.050	0.052	0.052	0.051	0.052	0.052	0.052	0.047	0.042
R^2_{adj} (%)	7.09	1.55	8.19	1.43	8.69	1.14	2.87	7.06	7.66	-0.19	2.21	4.51	-0.17	1.01	0.06	-0.21	17.64
No. Obs.	535	535	535	535	535	535	535	535	535	535	535	535	535	535	535	481	481

(a) Risk Approximations

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
(Intercept)	**0** **[4.567]**	**0** **[4.968]**	**0** **[14.513]**	**0** **[9.359]**	**0** **[6.437]**	**0** **[6.834]**	0 [1.823]	**0** **[3.364]**	**0** **[3.361]**	**0** **[3.966]**	**0** **[4.171]**	**0** **[4.173]**	0 [1.61]	0 [0.037]	0 [0.237]	0 [0.299]
AV_t (12M)														**0.06** **[3.665]**	**0.055** **[3.42]**	**0.055** **[3.91]**
AC_t (12M)														0 [1.303]	0 [0.93]	0 [0.875]
AV_t (12M) * AC_t (12M)														**-0.469** **[-3.264]**	**-0.403** **[-2.961]**	**-0.397** **[-2.631]**
AS_t (12M)															0 [1.3]	0 [1.147]
AK_t (12M)															0 [0.475]	0 [0.376]
CSV_t																0 [0.115]
CSS_t																0 [-0.186]
CSK_t																0 [0.258]
MOM_t (12M)																0 [-0.296]
VAL_t (60M)																0 [0.21]
INFLTN	0 [1.57]							0 [0.381]	0 [0.348]	0 [0.052]	0 [0.289]	0 [0.256]	0 [0.314]	0.001 [0.922]	0.001 [0.908]	0.001 [1.397]
BDGT.BLNC		0.001 [1.984]						0.001 [1.613]	0.001 [1.62]	0.001 [1.057]	0.001 [0.966]	0.001 [0.957]	0 [-0.298]	0 [-0.983]	0 [-1.015]	0 [-1.518]
GLBL.M2.SPPLY			0 [0.497]						0 [0.904]	0 [0.875]	0 [1.651]	0 [1.634]	0 [0.926]	0 [1.052]	0 [1.071]	0 [1.28]
Steep_Yld_Crv				**-0.001** **[-3.942]**						-0.001 [-1.735]	0 [-0.808]	0 [-0.761]	-0.001 [-1.805]	-0.001 [-1.452]	-0.001 [-1.501]	-0.001 [-1.977]
CFNAI					0 [0.712]						0 [0.717]	0 [0.914]	0 [-0.207]	0 [-0.794]	0 [-0.515]	0 [-0.292]
ADS						0 [0.483]						0 [-0.861]	0 [-1.406]	0 [-1.324]	0 [-1.304]	0 [-1.211]
VIX							**0** **[2.217]**						**0** **[2.052]**	0 [1.98]	**0** **[2.458]**	**0** **[2.063]**
RMSE (‰)	0.057	0.056	0.058	0.056	0.057	0.057	0.044	0.056	0.056	0.055	0.055	0.055	0.042	0.039	0.039	0.038
R^2_{adj} (%)	1.16	5.45	-0.12	5.35	1.61	0.40	7.34	5.40	5.38	6.73	9.06	9.03	14.63	26.72	27.42	26.50
No. Obs.	546	546	546	546	546	546	373	546	546	546	546	546	373	373	373	373

(b) External Predictors

Table A10: Variance Forecasting (24M) of the Naive Portfolio.
This table reports OLS results of regressing (rolling) multi-factor return variance over the next 24-month period (dependent variable) on contemporaneous risk proxies (Panel a) and external candidate predictors (Panel b). To prevent repetition, refer to Table A7 or Table A13 for a detailed table description.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
(Intercept)	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
	[5.085]	**[8.148]**	**[5.462]**	**[6.315]**	**[4.22]**	**[6.893]**	**[8.1]**	**[5.145]**	**[4.952]**	**[9.638]**	**[10.57]**	**[14.934]**	**[35.502]**	**[22.536]**	**[7.177]**	**[23.202]**	**[3.055]**
AV_t (12M)	0.001		0.001		**0.028**			0.005	0.001								**0.038**
	[0.146]		[0.091]		**[4.801]**			[0.725]	[0.15]								**[2.731]**
AC_t (12M)		0	0		0												0
		[-1.096]	[-1.135]		[1.787]												[1.594]
AV_t (12M) * AC_t (12M)				-0.049	**-0.303**												**-0.478**
				[-1.025]	**[-4.647]**												**[-2.7]**
VAR_t (12M)						-0.045		**-0.069**									0.135
						[-1.096]		**[-2.721]**									[1.446]
$VAR.G_t$							0.003		0								-0.003
							[0.092]		[-0.009]								[-0.073]
AS_t (12M)										**0**							0
										[2.279]							[1.233]
AK_t (12M)											0						0
											[0.093]						[-0.839]
CSV_t												0.002					0
												[1.171]					[0.321]
CSS_t													0				0
													[0.6]				[0.334]
CSK_t														0			0
														[-1.033]			[-0.533]
MOM_t (12M)															0		0.001
															[0.363]		[1.402]
VAL_t (60M)																0	0.001
																[0.77]	[1.92]
RMSE (‰)	0.031	0.030	0.030	0.030	0.030	0.030	0.031	0.030	0.031	0.030	0.031	0.031	0.031	0.031	0.031	0.030	0.028
R^2_{adj} (%)	-0.15	1.02	0.84	1.23	6.57	1.10	-0.20	1.89	-0.35	5.65	-0.18	0.20	-0.08	0.18	-0.03	-0.06	8.94
No. Obs.	499	499	499	499	499	499	499	499	499	499	499	499	499	499	499	445	445

(a) Risk Approximations

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
(Intercept)	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
	[6.833]	**[6.424]**	**[23.407]**	**[9.274]**	**[9.463]**	**[12.386]**	**[2.192]**	**[4.787]**	**[4.857]**	**[5.555]**	**[5.628]**	**[5.652]**	**[4.819]**	**[4.511]**	**[5.34]**	**[5.503]**
AV_t (12M)														**0.017**	0.008	0.01
														[2.049]	[1.421]	[1.072]
AC_t (12M)														0	0	0
														[1.842]	[1.349]	[1.22]
AV_t (12M) * AC_t (12M)														**-0.282**	**-0.17**	-0.178
														[-2.693]	**[-2.159]**	[-1.595]
AS_t (12M)															**0**	**0**
															[2.345]	**[2.483]**
AK_t (12M)															0	0
															[-0.198]	[-0.187]
CSV_t																-0.001
																[-1.126]
CSS_t																0
																[-0.382]
CSK_t																0
																[-0.097]
MOM_t (12M)																0
																[-0.371]
VAL_t (60M)																0
																[1.142]
INFLTN	0							0	0	0	0	0	-0.001	-0.001	-0.001	-0.001
	[0.103]							[-1.047]	[-0.663]	[-1.018]	[-0.594]	[-0.611]	[-1.078]	[-1.502]	[-1.559]	[-1.894]
BDGT.BLNC		**0.001**						0.001	**0.001**	0	0.001	0.001	0	0	0	0
		[2.102]						[1.969]	**[2.05]**	[0.968]	[1.526]	[1.508]	[0.565]	[0.96]	[0.805]	[1.033]
GLBL.M2.SPPLY			0						0	0	0	0	0	0	0	0
			[0.971]						[1.204]	[1.342]	[1.48]	[1.457]	[1.417]	[1.473]	[1.625]	[1.772]
Steep_Yld_Crv				-0.001						-0.001	0	0	**-0.001**	**-0.001**	**-0.001**	**-0.001**
				[-1.711]						[-1.082]	[-0.647]	[-0.631]	**[-2.365]**	**[-2.068]**	**[-2.216]**	**[-3.171]**
CFNAI					0						0	0	0	0	0	**0**
					[1.865]						[1.713]	[1.686]	[1.107]	[1.052]	[1.524]	**[2.083]**
ADS						0						0	0	0	0	0
						[1.654]						[-0.626]	[-1.264]	[-1.807]	[-1.774]	[-1.653]
VIX							0						0	0	0	0
							[0.147]						[0.395]	[1.028]	[1.899]	[1.986]
RMSE (‰)	0.032	0.031	0.032	0.031	0.031	0.032	0.032	0.031	0.031	0.031	0.029	0.029	0.028	0.027	0.026	0.026
R^2_{adj} (%)	-0.19	6.07	0.10	6.46	6.55	3.37	-0.17	6.64	6.82	9.71	16.91	16.90	26.33	30.02	35.11	34.62
No. Obs.	510	510	510	510	510	510	337	510	510	510	510	510	337	337	337	337

(b) External Predictors

Table A11: Variance Forecasting (60M) of the Naive Portfolio.
This table reports OLS results of regressing (rolling) multi-factor return variance over the next 60-month period (dependent variable) on contemporaneous risk proxies (Panel a) and external candidate predictors (Panel b). To prevent repetition, refer to Table A7 or Table A13 for a detailed table description.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
(Intercept)	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
	[8.671]	**[8.415]**	**[8.095]**	**[10.426]**	**[4.782]**	**[10.093]**	**[14.175]**	**[8.739]**	**[8.348]**	**[14.851]**	**[14.273]**	**[37.852]**	**[51.794]**	**[33.707]**	**[9.82]**	**[42.589]**	**[7.772]**
AV_t (12M)	**0.009**		**0.009**		**0.021**			**0.011**	**0.009**								**0.029**
	[2.896]		**[2.994]**		**[3.069]**			**[3.663]**	**[2.706]**								**[3.769]**
AC_t (12M)		0	0		0												**0**
		[-0.479]	[-0.198]		[1.242]												**[3.297]**
AV_t (12M) * AC_t (12M)				0.05	-0.139												**-0.244**
				[1.584]	[-1.754]												**[-2.769]**
VAR_t (12M)						0.021		-0.03									0.086
						[0.968]		[-1.713]									[1.845]
$VAR.G_t$							0.021		-0.003								**-0.065**
							[1.278]		[-0.141]								**[-2.372]**
AS_t (12M)										0							0
										[1.394]							[0.397]
AK_t (12M)											0						**0**
											[-0.584]						**[-4.211]**
CSV_t												**0.003**					0.001
												[4.049]					[0.838]
CSS_t													0				0
													[1.568]				[1.145]
CSK_t														0			0
														[-0.86]			[1.69]
MOM_t (12M)															0.001		**0.002**
															[1.5]		**[2.231]**
VAL_t (60M)																0	0
																[-0.806]	[1.05]
RMSE (‰)	0.017	0.019	0.017	0.018	0.017	0.018	0.018	0.017	0.017	0.018	0.018	0.018	0.019	0.019	0.018	0.018	0.015
R^2_{adj} (%)	11.50	-0.02	11.35	3.76	14.51	0.54	0.85	12.48	11.32	2.93	1.10	4.39	0.23	-0.13	2.54	-0.06	31.23
No. Obs.	439	439	439	439	439	439	439	439	439	439	439	439	439	439	439	385	385

(a) Risk Approximations

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
(Intercept)	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
	[6.772]	**[13.62]**	**[32.117]**	**[11.749]**	**[12.11]**	**[15.603]**	**[6.882]**	**[8.74]**	**[8.261]**	**[8.707]**	**[9.071]**	**[9.57]**	**[19.181]**	**[13.691]**	**[8.269]**	**[18.006]**
AV_t (12M)														**0.02**	**0.017**	**0.018**
														[3.807]	**[4.868]**	**[7.425]**
AC_t (12M)														**0**	**0**	**0**
														[2.205]	**[2.154]**	**[5.191]**
AV_t (12M) * AC_t (12M)														**-0.201**	**-0.157**	**-0.167**
														[-2.568]	**[-3.413]**	**[-5.923]**
AS_t (12M)															0	**0**
															[1.704]	**[3.156]**
AK_t (12M)															0	**0**
															[-1.721]	**[-3.938]**
CSV_t																0
																[1.066]
CSS_t																**0**
																[2.332]
CSK_t																**0**
																[4.601]
MOM_t (12M)																0
																[1.205]
VAL_t (60M)																0
																[1.068]
INFLTN	0							**0**	**0**	**0**	0	0	**-0.001**	**-0.001**	**-0.001**	**-0.001**
	[-0.691]							**[-2.136]**	**[-2.073]**	**[-2.218]**	[-1.394]	[-1.521]	**[-4.287]**	**[-4.84]**	**[-5.034]**	**[-14.65]**
BDGT.BLNC		**0.001**						**0.001**	**0.001**	**0.001**	**0.001**	**0.001**	**0.001**	**0.001**	0.001	**0.001**
		[8.903]						**[5.624]**	**[5.618]**	**[3.968]**	**[3.432]**	**[3.734]**	**[2.231]**	**[2.105]**	[1.87]	**[5.508]**
GLBL.M2.SPPLY			0						0	0	0	0	0	0	**0**	0
			[0.585]						[0.22]	[0.193]	[1.351]	[1.626]	[1.592]	[1.899]	**[2.128]**	[1.601]
Steep_Yld_Crv				**0**						0	0	0	0	0	0	**0**
				[-3.228]						[-0.685]	[0.779]	[0.916]	[1.092]	[1.584]	[1.444]	**[4.825]**
CFNAI					0						**0**	**0**	**0**	0	**0**	**0**
					[1.506]						**[2.665]**	**[3.353]**	**[2.488]**	[1.834]	**[2.685]**	**[4.017]**
ADS						0						**0**	**0**	**0**	**0**	**0**
						[0.887]						**[-2.97]**	**[-2.964]**	**[-2.102]**	**[-2.249]**	**[-3.528]**
VIX							0						0	**0**	0	0
							[0.221]						[-1.673]	**[-2.812]**	[-0.494]	[-0.504]
RMSE (‰)	0.020	0.019	0.020	0.020	0.020	0.020	0.020	0.018	0.018	0.018	0.017	0.017	0.013	0.013	0.012	0.011
R^2_{adj} (%)	1.18	11.96	-0.16	2.54	4.40	1.29	-0.13	17.44	17.27	17.20	29.40	31.09	52.08	57.32	61.91	63.20
No. Obs.	450	450	450	450	450	450	277	450	450	450	450	450	277	277	277	277

(b) External Predictors

Table A12: Variance Forecasting (120M) of the Naive Portfolio.
This table reports OLS results of regressing (rolling) multi-factor return variance over the next 120-month period (dependent variable) on contemporaneous risk proxies (Panel a) and external candidate predictors (Panel b). To prevent repetition, refer to Table A7 or Table A13 for a detailed table description.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
(Intercept)	**0.002**	**0.003**	0.002	**0.002**	0.001	**0.002**	0.001	**0.002**	0.001	**0.003**	**0.002**	**0.002**	**0.002**	**0.002**	0	**0.003**	**0.003**
	[2.518]	**[3.101]**	[1.775]	**[3.684]**	[0.48]	**[4.106]**	[1.16]	**[2.653]**	[0.923]	**[6.504]**	**[5.258]**	**[5.396]**	**[6.224]**	**[5.032]**	[0.22]	**[8.963]**	**[3.132]**
AV_t (12M)	0.949		0.935		2.48			1.392	0.419								-1.777
	[1.354]		[1.24]		[1.418]			[1.765]	[0.533]								[-1.463]
AC_t (12M)		-0.004	-0.004		0.01												-0.001
		[-0.454]	[-0.377]		[0.644]												[-0.084]
AV_t (12M) * AC_t (12M)				3.849	-17.452												**36.16**
				[0.529]	[-0.889]												**[2.121]**
VAR_t (12M)						-0.238		-7.072									-15.658
						[-0.028]		[-0.7]									[-1.676]
$VAR.G_t$							13.165		11.941								-11.835
							[1.208]		[1.004]								[-1.861]
AS_t (12M)										0.003							0
										[1.927]							[0.087]
AK_t (12M)											0						0
											[-0.069]						[-0.223]
CSV_t												-0.127					-0.162
												[-0.221]					[-1.045]
CSS_t													0				0
													[0.863]				[0.01]
CSK_t														0			0
														[1.595]			[-0.867]
MOM_t (12M)															**0.928**		0.118
															[5.303]		[1.35]
VAL_t (60M)																**-1.522**	**-1.503**
																[-20.58]	**[-24.292]**
RMSE (‰)	8.802	8.823	8.800	8.821	8.790	8.825	8.758	8.791	8.754	8.785	8.825	8.824	8.824	8.819	8.463	3.262	3.134
R^2_{adj} (%)	0.34	-0.12	0.20	-0.08	0.25	-0.18	1.35	0.42	1.26	0.72	-0.18	-0.15	-0.14	-0.03	7.87	84.59	85.46
No. Obs.	559	559	559	559	559	559	559	559	559	559	559	559	559	559	559	505	505

(a) Risk Approximations

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
(Intercept)	**0.002**	**0.002**	**0.002**	**0.002**	**0.002**	**0.002**	**0.007**	0.002	0.002	0.001	0.001	0.001	**0.007**	0.004	0.004	**0.003**
	[3.565]	**[2.525]**	**[4.773]**	**[2.302]**	**[6.046]**	**[5.979]**	**[4.111]**	[1.875]	[1.491]	[1.16]	[1.121]	[1.12]	**[2.667]**	[1.202]	[1.2]	**[2.538]**
AV_t (12M)														**4.677**	**4.92**	-0.464
														[2.196]	**[2.25]**	[-0.531]
AC_t (12M)														0.009	0.012	0.008
														[0.421]	[0.56]	[0.804]
AV_t (12M) * AC_t (12M)														-11.66	-15.362	12.294
														[-0.388]	[-0.494]	[1.101]
AS_t (12M)															-0.001	0
															[-0.611]	[-0.271]
AK_t (12M)															0	-0.001
															[0.325]	[-1.965]
CSV_t																-0.052
																[-0.527]
CSS_t																0
																[-1.313]
CSK_t																0
																[-1.459]
MOM_t (12M)																0.092
																[1.232]
VAL_t (60M)																**-1.759**
																[-28.115]
INFLTN	0							-0.001	0.004	0.008	0.012	0.012	-0.016	0.038	0.032	0.026
	[0.011]							[-0.035]	[0.234]	[0.529]	[0.83]	[0.783]	[-0.364]	[0.846]	[0.709]	[1.154]
BDGT.BLNC		0.004						0.004	0.005	0.023	0.036	0.035	0.023	-0.008	-0.004	0.034
		[0.14]						[0.143]	[0.171]	[0.642]	[0.954]	[0.946]	[0.585]	[-0.206]	[-0.096]	[1.675]
GLBL.M2.SPPLY			**0.004**						**0.004**	**0.004**	**0.004**	**0.004**	**0.004**	**0.004**	**0.004**	0
			[3.412]						**[3.39]**	**[3.422]**	**[3.339]**	**[3.34]**	**[3.611]**	**[4.166]**	**[4.162]**	[1.054]
Steep_Yld_Crv				0.027						0.048	0.066	0.067	0.05	0.055	0.055	-0.026
				[0.685]						[1.118]	[1.483]	[1.544]	[0.905]	[1.082]	[1.058]	[-0.992]
CFNAI					0.001						0.001	0.001	0	0	-0.001	**-0.002**
					[0.965]						[1.486]	[1.226]	[-0.094]	[-0.392]	[-0.527]	**[-2.78]**
ADS						0						0	0	0.001	0.001	**0.001**
						[0.72]						[-0.381]	[0.045]	[0.647]	[0.621]	**[2.862]**
VIX							**-0.025**						**-0.027**	**-0.038**	**-0.04**	**-0.009**
							[-2.717]						**[-2.805]**	**[-3.62]**	**[-3.767]**	**[-2.563]**
RMSE (‰)	8.793	8.793	8.691	8.787	8.780	8.785	7.942	8.793	8.690	8.675	8.645	8.643	7.743	7.467	7.461	2.336
R^2_{adj} (%)	-0.18	-0.17	2.14	-0.03	0.13	0.01	5.58	-0.35	1.83	1.98	2.50	2.36	8.88	14.59	14.28	91.48
No. Obs.	570	570	570	570	570	570	397	570	570	570	570	570	397	397	397	397

(b) External Predictors

Table A13: Return Decomposition of the Naive Portfolio. (Caption on the next page.)

Table A13: Return Decomposition of the Naive Portfolio.
This table reports OLS results of regressing contemporaneous monthly multi-factor returns (dependent variable) on contemporaneous risk proxies (Panel a) and external candidate predictors (Panel b). Data covers the full investigation period from July 1971 to December 2018. The regressor names are shown at the left margin. For return-based measures, the estimation period (in months) is disclosed in brackets alongside the regressor's name. Below each estimated coefficient, associated t-statistics (adjusted based on the methods proposed in Newey and West (1987) and Newey and West (1994)) are reported in squared brackets. Coefficients with absolute t-statistics above 2 are highlighted in bold. The last three rows show per model root mean squared error (RMSE), adjusted R^2 (R^2_{adj}), and number of observations used for estimation. Consult Table A4 and Section 3 for an overview of factors constituting the equally-weighted multi-factor portfolio. Detailed data set information is available in Appendix A.1 and Vincenz and Zeissler (2024). Refer to Sections 4.1 (multi-factor variance), A.2.1 (multi-factor risk), and A.2.2 (external predictors) for an overview of explanatory variables in the regressions. The TS of the regressors were tested to rule out the possibility of containing unit roots with sufficient confidence. If necessary (only in the case of VAL_t (60M) and GLBL.M2.SPPLY), the TS were transformed (by calculating differences) and re-tested (see Table A5). An overview of the abbreviations used in the course of this paper is provided in Table A3.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
(Intercept)	**0.001**	**0.002**	0.001	**0.001**	0.001	**0.002**	**0.002**	0.001	0.001	**0.002**	**0.003**	**0.002**	**0.002**	**0.003**	**0.002**	**0.002**	0
	[2.028]	**[2.301]**	[0.613]	**[2.524]**	[0.487]	**[3.493]**	**[3.54]**	[1.927]	[1.743]	**[5.323]**	**[5.993]**	**[5.33]**	**[6.048]**	**[6.221]**	**[4.128]**	**[5.938]**	[0.076]
AV_t (12M)	**1.349**		1.374		1.255			1.331	**1.347**								2.613
	[2.142]		[1.991]		[0.819]			[1.811]	**[2.077]**								[1.1]
AC_t (12M)		0.005	0.006		0.005												0.02
		[0.728]	[0.872]		[0.408]												[1.319]
AV_t (12M) * AC_t (12M)				**12.205**	1.34												-1.35
				[2.168]	[0.091]												[-0.036]
VAR_t (12M)						6.817		0.282									-16.244
						[1.071]		[0.037]									[-0.573]
$VAR.G_t$							3.981		0.053								-0.541
							[0.915]		[0.01]								[-0.035]
AS_t (12M)										0							0
										[0.363]							[-0.013]
AK_t (12M)											0.001						0.001
											[1.24]						[1.119]
CSV_t												0.002					-0.25
												[0.006]					[-0.78]
CSS_t													0				0
													[-1.544]				[-1.5]
CSK_t														0			0
														[-1.841]			[-1.293]
MOM_t (12M)															0.081		-0.033
															[0.5]		[-0.197]
VAL_t (60M)																-0.14	-0.121
																[-1.465]	[-1.249]
RMSE (‰)	8.756	8.799	8.751	8.758	8.751	8.787	8.796	8.756	8.756	8.802	8.788	8.803	8.787	8.790	8.800	8.236	8.119
R^2_{adj} (%)	0.88	-0.10	0.82	0.83	0.64	0.17	-0.04	0.70	0.70	-0.16	0.15	-0.18	0.18	0.10	-0.12	0.52	1.14
No. Obs.	558	558	558	558	558	558	558	558	558	558	558	558	558	558	558	504	504

(a) Risk Approximations

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
(Intercept)	**0.002**	**0.002**	**0.002**	**0.002**	**0.002**	**0.002**	**0.002**	**0.003**	**0.003**	0.002	0.002	0.002	0.004	0.001	0.001	0.002
	[3.961]	**[2.519]**	**[5.566]**	**[2.112]**	**[6.083]**	**[6.088]**	**[2.016]**	**[2.115]**	**[2.081]**	[1.699]	[1.644]	[1.631]	[1.537]	[0.41]	[0.474]	[0.651]
AV_t (12M)														2.866	3.215	2.919
														[1.925]	[1.987]	[1.387]
AC_t (12M)														0.017	0.021	0.023
														[1.172]	[1.405]	[1.434]
AV_t (12M) * AC_t (12M)														0.588	-4.82	-5.727
														[0.031]	[-0.233]	[-0.26]
AS_t (12M)															-0.001	-0.002
															[-0.64]	[-0.768]
AK_t (12M)															0.001	0.001
															[0.508]	[0.759]
CSV_t																-0.034
																[-0.098]
CSS_t																0
																[-0.679]
CSK_t																0
																[-1.06]
MOM_t (12M)																0.175
																[0.917]
VAL_t (60M)																-0.096
																[-0.776]
INFLTN	-0.003							-0.005	-0.005	0	0.004	0.003	-0.057	-0.029	-0.038	-0.058
	[-0.23]							[-0.303]	[-0.303]	[-0.005]	[0.285]	[0.21]	[-1.074]	[-0.557]	[-0.665]	[-1.043]
BDGT.BLNC		0.005						0.008	0.008	0.028	0.042	0.041	0.012	0.007	0.014	0.017
		[0.163]						[0.26]	[0.26]	[0.771]	[1.087]	[1.073]	[0.296]	[0.183]	[0.341]	[0.413]
GLBL.M2.SPPLY			0						0	0	0	0	0	0	0	0
			[-0.01]						[-0.032]	[0.009]	[0.15]	[0.173]	[0.053]	[-0.016]	[0.006]	[-0.239]
Steep_Yld_Crv				0.036						0.054	0.073	0.077	0.014	0.021	0.02	0.009
				[0.898]						[1.167]	[1.515]	[1.521]	[0.227]	[0.366]	[0.345]	[0.165]
CFNAI					0.001						0.001	0.002	-0.001	-0.001	-0.002	-0.002
					[1.4]						[1.778]	[1.25]	[-0.528]	[-0.734]	[-0.862]	[-1.157]
ADS						0.001						-0.001	0.002	0.004	0.004	0.004
						[0.755]						[-0.56]	[0.9]	[1.309]	[1.284]	[1.598]
VIX							-0.001						0	-0.011	-0.013	-0.012
							[-0.203]						[-0.076]	[-1.616]	[-1.85]	[-1.616]
RMSE (‰)	8.788	8.788	8.789	8.777	8.767	8.779	8.193	8.787	8.787	8.770	8.735	8.727	8.122	7.927	7.914	7.880
R^2_{adj} (%)	-0.16	-0.17	-0.18	0.09	0.32	0.04	-0.24	-0.32	-0.50	-0.27	0.35	0.34	-0.03	3.98	3.80	3.36
No. Obs.	569	569	569	569	569	569	396	569	569	569	569	569	396	396	396	396

(b) External Predictors

Table A14: Return Forecasting (1M) of the Naive Portfolio.
This table reports OLS results of regressing multi-factor returns of the subsequent month (dependent variable) on contemporaneous risk proxies (Panel a) and external candidate predictors (Panel b). To prevent repetition, refer to Table A7 or Table A13 for a detailed table description.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
(Intercept)	**0.001**	**0.003**	**0.002**	**0.002**	0.001	**0.002**	**0.002**	**0.001**	**0.001**	**0.003**	**0.003**	**0.002**	**0.002**	**0.002**	**0.002**	**0.002**	0.001
	[2.616]	**[4.797]**	**[2.343]**	**[3.435]**	[1.561]	**[5.088]**	**[5.461]**	**[2.783]**	**[2.666]**	**[5.543]**	**[6.446]**	**[7.606]**	**[12.212]**	**[9.67]**	**[5.896]**	**[11.362]**	[1.182]
AV_t (12M)	**1.297**		**1.274**		**1.808**			**1.538**	**1.306**								1.536
	[4.147]		**[3.832]**		**[2.465]**			**[4.509]**	**[3.968]**								[1.537]
AC_t (12M)		-0.007	-0.005		-0.001												-0.003
		[-1.425]	[-1.052]		[-0.093]												[-0.463]
AV_t (12M) * AC_t (12M)				7.002	-6.03												-0.02
				[1.351]	[-0.658]												[-0.001]
VAR_t (12M)						3.6		-3.87									-0.362
						[0.96]		[-0.977]									[-0.035]
$VAR.G_t$							**3.537**		-0.202								-1.08
							[2.319]		[-0.145]								[-0.219]
AS_t (12M)										0.001							0
										[0.66]							[0.293]
AK_t (12M)											0.001						0
											[1.01]						[-0.099]
CSV_t												**0.398**					0.127
												[3.684]					[1.221]
CSS_t													0				0
													[-1.695]				[-1.941]
CSK_t														0			0
														[-1.263]			[1.852]
MOM_t (12M)															0.037		0.092
															[0.414]		[1.235]
VAL_t (60M)																0.002	0.026
																[0.108]	[0.979]
RMSE (‰)	2.539	2.667	2.525	2.636	2.521	2.671	2.669	2.527	2.539	2.678	2.667	2.644	2.681	2.682	2.683	2.536	2.327
R^2_{adj} (%)	10.45	1.19	11.23	3.42	11.36	0.86	1.02	11.13	10.29	0.36	1.17	2.88	0.10	0.04	-0.05	-0.20	13.75
No. Obs.	547	547	547	547	547	547	547	547	547	547	547	547	547	547	547	493	493

(a) Risk Approximations

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
(Intercept)	**0.003**	**0.002**	**0.002**	**0.002**	**0.002**	**0.002**	0.001	0.003	**0.003**	0.003	0.003	0.003	0.002	0.001	0.001	0.001
	[4.416]	**[2.578]**	**[9.43]**	**[4.594]**	**[6.482]**	**[6.432]**	[0.915]	[1.992]	**[2.118]**	[1.912]	[1.865]	[1.861]	[1.043]	[0.776]	[0.442]	[0.79]
AV_t (12M)														0.594	0.973	0.805
														[0.634]	[1.044]	[0.988]
AC_t (12M)														-0.009	-0.007	-0.006
														[-0.86]	[-0.645]	[-0.704]
AV_t (12M) * AC_t (12M)														15.322	10.714	9.903
														[1.396]	[1.01]	[0.953]
AS_t (12M)															-0.002	**-0.002**
															[-1.42]	**[-2.074]**
AK_t (12M)															-0.001	0
															[-0.8]	[-0.416]
CSV_t																**0.216**
																[2.649]
CSS_t																0
																[-1.451]
CSK_t																0
																[1.516]
MOM_t (12M)																**0.174**
																[2.366]
VAL_t (60M)																0.026
																[0.908]
INFLTN	-0.007							-0.008	-0.009	-0.006	-0.004	-0.004	-0.047	-0.017	-0.012	-0.031
	[-0.704]							[-0.635]	[-0.68]	[-0.501]	[-0.343]	[-0.336]	[-0.906]	[-0.371]	[-0.287]	[-1.097]
BDGT.BLNC		-0.001						0.005	0.005	0.014	0.021	0.021	-0.001	-0.019	-0.019	-0.018
		[-0.025]						[0.18]	[0.187]	[0.507]	[0.612]	[0.622]	[-0.021]	[-0.748]	[-0.722]	[-1.07]
GLBL.M2.SPPLY			0						0	0	0	0	0	0	0	0
			[-0.631]						[-0.813]	[-0.749]	[-0.698]	[-0.688]	[-0.739]	[-0.748]	[-0.709]	[-0.721]
Steep_Yld_Crv				0.022						0.025	0.034	0.034	0.003	-0.001	0.003	0.002
				[1.381]						[0.926]	[1.263]	[1.21]	[0.098]	[-0.021]	[0.082]	[0.085]
CFNAI					0						0	0	0	0	0	-0.001
					[1.079]						[1.257]	[0.767]	[-0.28]	[-0.392]	[-0.611]	[-1.542]
ADS						0						0	0	0.001	0.001	**0.001**
						[1.065]						[-0.02]	[0.362]	[1.331]	[1.263]	**[2.033]**
VIX							0.007						0.006	-0.001	-0.002	-0.003
							[1.596]						[1.324]	[-0.154]	[-0.403]	[-0.923]
RMSE (‰)	2.688	2.697	2.695	2.684	2.678	2.680	2.574	2.687	2.684	2.671	2.646	2.646	2.525	2.290	2.258	2.213
R^2_{adj} (%)	0.51	-0.18	0.00	0.82	1.25	1.10	4.06	0.41	0.49	1.22	2.92	2.74	6.25	22.23	23.97	26.03
No. Obs.	558	558	558	558	558	558	385	558	558	558	558	558	385	385	385	385

(b) External Predictors

Table A15: Mean Return Forecasting (12M) of the Naive Portfolio.
This table reports OLS results of regressing (rolling) mean returns over the next 12-month period (dependent variable) on contemporaneous risk proxies (Panel a) and external candidate predictors (Panel b). To prevent repetition, refer to Table A7 or Table A13 for a detailed table description.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
(Intercept)	**0.001**	**0.003**	0.001	**0.002**	0.001	**0.002**	**0.002**	**0.001**	**0.001**	**0.003**	**0.003**	**0.002**	**0.002**	**0.003**	**0.002**	**0.002**	0
	[2.497]	**[3.601]**	[1.569]	**[3.323]**	[1.002]	**[4.345]**	**[4.772]**	**[2.613]**	**[2.78]**	**[5.545]**	**[5.88]**	**[6.153]**	**[9.761]**	**[7.504]**	**[4.478]**	**[6.406]**	[0.071]
AV_t (12M)	**1.289**		**1.288**		1.375			**1.427**	**1.417**								2.828
	[3.308]		**[3.288]**		[1.104]			**[3.025]**	**[3.092]**								[1.514]
AC_t (12M)		-0.001	0		0.001												0.01
		[-0.239]	[-0.039]		[0.053]												[0.948]
AV_t (12M) * AC_t (12M)				**9.284**	-0.984												-11.217
				[2.098]	[-0.084]												[-0.404]
VAR_t (12M)						4.754		-2.225									-1.923
						[1.165]		[-0.498]									[-0.107]
$VAR.G_t$							1.191		-2.912								-1.239
							[0.498]		[-1.12]								[-0.142]
AS_t (12M)										0.001							0
										[0.745]							[0.12]
AK_t (12M)											0.001						0
											[1.136]						[0.473]
CSV_t												**0.307**					-0.085
												[2.085]					[-0.626]
CSS_t													0				0
													[-0.202]				[-0.463]
CSK_t														0			0
														[-1.978]			[-0.232]
MOM_t (12M)															0.025		0.054
															[0.236]		[0.5]
VAL_t (60M)																-0.006	0.022
																[-0.134]	[0.48]
RMSE (‰)	3.785	3.882	3.785	3.824	3.785	3.866	3.882	3.782	3.778	3.877	3.858	3.866	3.883	3.875	3.882	3.611	3.452
R^2_{adj} (%)	4.82	-0.16	4.65	2.84	4.48	0.69	-0.12	4.78	4.99	0.12	1.10	0.68	-0.18	0.23	-0.15	-0.19	6.35
No. Obs.	553	553	553	553	553	553	553	553	553	553	553	553	553	553	553	499	499

(a) Risk Approximations

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
(Intercept)	**0.003**	**0.002**	**0.002**	**0.002**	**0.002**	**0.002**	0.001	0.003	**0.003**	0.003	0.003	0.003	0.003	0.001	0.001	0.001
	[4.267]	**[2.245]**	**[7.468]**	**[2.922]**	**[6.226]**	**[5.98]**	[0.995]	[1.895]	**[2.062]**	[1.974]	[1.694]	[1.702]	[1.213]	[0.394]	[0.342]	[0.527]
AV_t (12M)														1.903	2.172	2.304
														[1.519]	[1.651]	[1.809]
AC_t (12M)														0.008	0.011	0.013
														[0.758]	[0.944]	[1.275]
AV_t (12M) * AC_t (12M)														2.095	-1.664	-4.76
														[0.141]	[-0.108]	[-0.361]
AS_t (12M)															-0.001	-0.001
															[-0.88]	[-1.195]
AK_t (12M)															0	0.001
															[0.159]	[0.483]
CSV_t																0.038
																[0.377]
CSS_t																0
																[-0.447]
CSK_t																0
																[-0.175]
MOM_t (12M)																0.194
																[1.906]
VAL_t (60M)																0.058
																[1.459]
INFLTN	-0.008							-0.009	-0.008	-0.006	-0.004	-0.004	-0.066	-0.043	-0.046	-0.066
	[-0.719]							[-0.677]	[-0.628]	[-0.5]	[-0.29]	[-0.289]	[-1.204]	[-0.88]	[-0.856]	[-1.418]
BDGT.BLNC		0						0.007	0.007	0.014	0.022	0.022	0.001	-0.005	-0.002	-0.003
		[0.017]						[0.213]	[0.25]	[0.431]	[0.602]	[0.618]	[0.032]	[-0.2]	[-0.085]	[-0.09]
GLBL.M2.SPPLY			0.001						0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
			[1.114]						[1.146]	[1.198]	[1.408]	[1.435]	[1.455]	[1.406]	[1.424]	[1.38]
Steep_Yld_Crv				0.016						0.019	0.031	0.031	0.002	0.006	0.008	0.005
				[0.541]						[0.533]	[0.842]	[0.799]	[0.041]	[0.139]	[0.16]	[0.151]
CFNAI					0						0.001	0.001	-0.001	-0.001	-0.001	-0.002
					[0.957]						[1.197]	[0.803]	[-1.088]	[-1.773]	[-1.861]	[-1.998]
ADS						0						0	0.001	**0.002**	**0.002**	**0.002**
						[0.907]						[-0.016]	[1.158]	**[2.492]**	**[2.439]**	**[2.171]**
VIX							0.006						0.005	-0.002	-0.004	-0.005
							[1.066]						[0.788]	[-0.42]	[-0.615]	[-0.986]
RMSE (‰)	3.882	3.889	3.880	3.883	3.867	3.870	3.699	3.880	3.873	3.868	3.839	3.839	3.596	3.382	3.371	3.343
R^2_{adj} (%)	0.19	-0.18	0.27	0.10	0.93	0.79	1.23	0.07	0.27	0.35	1.67	1.49	5.19	15.46	15.58	15.85
No. Obs.	564	564	564	564	564	564	391	564	564	564	564	564	391	391	391	391

(b) External Predictors

Table A16: Mean Return Forecasting (6M) of the Naive Portfolio.
This table reports OLS results of regressing (rolling) mean returns over the next 6-month period (dependent variable) on contemporaneous risk proxies (Panel a) and external candidate predictors (Panel b). To prevent repetition, refer to Table A7 or Table A13 for a detailed table description.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
(Intercept)	**0.001**	**0.003**	0.001	**0.002**	0.001	**0.002**	**0.002**	**0.001**	**0.001**	**0.002**	**0.003**	**0.002**	**0.002**	**0.002**	**0.003**	**0.002**	0.001
	[2.55]	**[4.885]**	[1.941]	**[3.333]**	[1.003]	**[4.317]**	**[5.833]**	**[2.634]**	**[2.758]**	**[6.412]**	**[8.349]**	**[14.334]**	**[23.252]**	**[14.554]**	**[6.2]**	**[16.446]**	[1.052]
AV_t (12M)	**1.351**		**1.344**		**2.087**			**1.613**	**1.447**								**2.351**
	[4.102]		**[3.869]**		**[4.304]**			**[7.44]**	**[4.941]**								**[3.141]**
AC_t (12M)		-0.003	-0.001		0.005												0.006
		[-1.108]	[-0.45]		[1.046]												[1.04]
AV_t (12M) * AC_t (12M)				**8.477**	-8.361												-8.022
				[2.009]	[-1.768]												[-0.809]
VAR_t (12M)						3.497		-4.26									1.273
						[1.158]		[-1.596]									[0.206]
$VAR.G_t$							1.834		**-2.229**								-0.846
							[1.13]		**[-2.283]**								[-0.263]
AS_t (12M)										0							0.001
										[0.522]							[1.507]
AK_t (12M)											0.001						0.001
											[1.797]						[1.723]
CSV_t												**0.42**					0.068
												[4.834]					[1.286]
CSS_t													0				0
													[-0.246]				[-0.878]
CSK_t														0			0
														[-1.515]			[0.473]
MOM_t (12M)															-0.062		-0.041
															[-0.683]		[-0.721]
VAL_t (60M)																0.008	0.018
																[0.514]	[1.021]
RMSE (‰)	1.691	1.918	1.689	1.820	1.677	1.904	1.916	1.668	1.682	1.919	1.870	1.857	1.922	1.910	1.915	1.854	1.512
R^2_{adj} (%)	22.53	0.29	22.50	10.19	23.48	1.76	0.46	24.42	23.22	0.15	5.27	6.57	-0.17	1.12	0.58	-0.15	31.85
No. Obs.	535	535	535	535	535	535	535	535	535	535	535	535	535	535	535	481	481

(a) Risk Approximations

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
(Intercept)	**0.003**	**0.003**	**0.002**	**0.002**	**0.002**	**0.002**	0	**0.003**	**0.003**	**0.003**	**0.003**	**0.003**	0.002	0	0	0
	[4.388]	**[3.315]**	**[13.626]**	**[4.113]**	**[6.606]**	**[6.932]**	[0.687]	**[2.665]**	**[2.477]**	**[2.101]**	**[2.215]**	**[2.223]**	[1.201]	[-0.061]	[-0.036]	[-0.048]
AV_t (12M)														**1.944**	**2.149**	**2.025**
														[2.393]	**[2.92]**	**[3.409]**
AC_t (12M)														0.004	0.006	0.005
														[0.457]	[0.682]	[0.83]
AV_t (12M) * AC_t (12M)														-1.827	-4.674	-2.994
														[-0.22]	[-0.588]	[-0.409]
AS_t (12M)															-0.001	-0.001
															[-0.88]	[-0.864]
AK_t (12M)															0	0
															[0.416]	[0.344]
CSV_t																0.066
																[1.48]
CSS_t																0
																[-0.659]
CSK_t																0
																[0.195]
MOM_t (12M)																-0.061
																[-0.967]
VAL_t (60M)																-0.005
																[-0.256]
INFLTN	-0.004							-0.007	-0.007	-0.004	-0.003	-0.003	-0.028	-0.002	-0.005	0.001
	[-0.437]							[-0.649]	[-0.566]	[-0.324]	[-0.214]	[-0.218]	[-0.893]	[-0.085]	[-0.188]	[0.044]
BDGT.BLNC		0.01						0.015	0.015	0.027	0.031	0.031	0.014	0.002	0.004	0.004
		[0.496]						[0.647]	[0.614]	[1.209]	[1.264]	[1.271]	[0.707]	[0.108]	[0.284]	[0.36]
GLBL.M2.SPPLY			0						0	0	0	0	0	0	0	0
			[-0.853]						[-0.865]	[-0.786]	[-0.691]	[-0.682]	[-0.823]	[-1.131]	[-1.093]	[-1.016]
Steep_Yld_Crv				0.02						0.033	0.039	0.04	0.007	0.012	0.013	0.014
				[1.106]						[1.688]	[1.838]	[1.844]	[0.31]	[0.466]	[0.534]	[0.749]
CFNAI					0						0	0	0	0	0	0
					[0.892]						[1.236]	[0.921]	[0.249]	[-0.176]	[-0.437]	[-0.094]
ADS						0						0	0	0	0	0
						[0.79]						[-0.246]	[-0.444]	[0.723]	[0.701]	[0.517]
VIX							**0.008**						**0.007**	0.002	0.001	0.001
							[2.543]						**[2.105]**	[0.597]	[0.238]	[0.347]
RMSE (‰)	1.932	1.931	1.935	1.921	1.929	1.931	1.845	1.920	1.918	1.886	1.868	1.867	1.810	1.501	1.487	1.478
R^2_{adj} (%)	0.27	0.46	0.05	1.43	0.62	0.46	10.85	1.33	1.43	4.44	6.17	6.01	12.80	39.54	40.34	40.21
No. Obs.	546	546	546	546	546	546	373	546	546	546	546	546	373	373	373	373

(b) External Predictors

Table A17: Mean Return Forecasting (24M) of the Naive Portfolio.
This table reports OLS results of regressing (rolling) mean returns over the next 24-month period (dependent variable) on contemporaneous risk proxies (Panel a) and external candidate predictors (Panel b). To prevent repetition, refer to Table A7 or Table A13 for a detailed table description.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
(Intercept)	**0.001**	**0.003**	**0.002**	**0.002**	**0.002**	**0.002**	**0.002**	**0.002**	**0.002**	**0.002**	**0.002**	**0.002**	**0.002**	**0.003**	**0.002**	**0.002**	**0.003**
	[5.009]	**[8.035]**	**[6.226]**	**[6.371]**	**[5.491]**	**[6.538]**	**[9.359]**	**[5.261]**	**[4.821]**	**[8.403]**	**[10.449]**	**[21.941]**	**[24.08]**	**[31.286]**	**[7.537]**	**[23.374]**	**[7.008]**
AV_t (12M)	**1**		**0.953**		**0.812**			**1.19**	**1.041**								0.742
	[7.904]		**[8.676]**		**[3.331]**			**[5.206]**	**[8.356]**								[1.885]
AC_t (12M)		**-0.009**	**-0.008**		**-0.009**												**-0.008**
		[-4.318]	**[-4.097]**		**[-3.531]**												**[-2.807]**
AV_t (12M) * AC_t (12M)				**4.886**	1.585												1.981
				[3.016]	[0.778]												[0.375]
VAR_t (12M)						2.44		-3.157									-1.298
						[1.491]		[-1.934]									[-0.391]
$VAR.G_t$							1.812		-0.979								-0.838
							[1.908]		[-1.061]								[-0.475]
AS_t (12M)										0							0
										[0.012]							[0.183]
AK_t (12M)											0.001						0
											[1.666]						[0.45]
CSV_t												**0.358**					**0.092**
												[4.643]					**[2.442]**
CSS_t													0				0
													[-0.534]				[-0.179]
CSK_t														**0**			**0**
														[-3.26]			**[-2.429]**
MOM_t (12M)															-0.032		-0.048
															[-1.353]		[-1.236]
VAL_t (60M)																0.007	0.011
																[0.75]	[1.187]
RMSE (‰)	0.907	1.058	0.830	1.081	0.829	1.124	1.129	0.882	0.903	1.140	1.105	1.053	1.139	1.100	1.137	1.079	0.760
R^2_{adj} (%)	36.63	13.68	46.78	9.93	46.79	2.61	1.68	39.88	36.99	-0.20	5.91	14.59	-0.10	6.73	0.38	-0.10	49.05
No. Obs.	499	499	499	499	499	499	499	499	499	499	499	499	499	499	499	445	445

(a) Risk Approximations

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
(Intercept)	**0.003**	**0.003**	**0.002**	**0.003**	**0.002**	**0.002**	**0.001**	**0.004**	**0.004**	**0.004**	**0.004**	**0.004**	**0.003**	**0.003**	**0.003**	**0.003**
	[4.852]	**[7.285]**	**[32.622]**	**[5.783]**	**[8.135]**	**[9.508]**	**[2.206]**	**[7.398]**	**[5.25]**	**[4.45]**	**[5.05]**	**[5.138]**	**[6.364]**	**[5.307]**	**[4.832]**	**[7.862]**
AV_t (12M)														**0.716**	**0.804**	**0.702**
														[2.224]	**[2.429]**	**[2.959]**
AC_t (12M)														-0.004	-0.003	-0.003
														[-1.376]	[-0.956]	[-1.406]
AV_t (12M) * AC_t (12M)														-0.106	-1.373	-0.386
														[-0.036]	[-0.434]	[-0.14]
AS_t (12M)															0	0
															[-0.842]	[-0.98]
AK_t (12M)															0	0
															[1.102]	[0.945]
CSV_t																0.033
																[1.336]
CSS_t																0
																[-1.402]
CSK_t																0
																[-1.927]
MOM_t (12M)																-0.025
																[-0.947]
VAL_t (60M)																0.001
																[0.057]
INFLTN	-0.002							**-0.009**	-0.009	-0.008	-0.007	-0.006	**-0.04**	**-0.022**	**-0.024**	**-0.023**
	[-0.25]							**[-2.444]**	[-1.402]	[-1.024]	[-0.869]	[-0.858]	**[-4.004]**	**[-2.458]**	**[-2.494]**	**[-3.843]**
BDGT.BLNC		**0.031**						**0.037**	**0.037**	**0.043**	**0.046**	**0.046**	**0.042**	**0.03**	**0.031**	**0.03**
		[2.23]						**[2.859]**	**[2.49]**	**[3.271]**	**[3.626]**	**[3.679]**	**[5.974]**	**[4.517]**	**[4.499]**	**[7.017]**
GLBL.M2.SPPLY			0						**0**	0	0	0	0	0	0	**0**
			[-1.927]						**[-2.241]**	[-1.978]	[-1.693]	[-1.701]	[-1.626]	[-1.81]	[-1.848]	**[-2.309]**
Steep_Yld_Crv				-0.006						0.016	0.02	0.019	0.007	0.005	0.005	0.005
				[-0.37]						[1.446]	[1.704]	[1.618]	[0.657]	[0.531]	[0.587]	[0.798]
CFNAI					0						0	0	0	0	0	0
					[0.64]						[1.699]	[0.474]	[0.797]	[0.816]	[0.42]	[0.764]
ADS						0						0	0	**0**	**0**	**0**
						[0.65]						[0.645]	[1.665]	**[3.076]**	**[3.105]**	**[3.18]**
VIX							**0.006**						**0.006**	**0.004**	**0.003**	**0.003**
							[2.621]						**[4.225]**	**[4.19]**	**[3.616]**	**[3.628]**
RMSE (‰)	1.152	1.038	1.148	1.151	1.150	1.150	1.070	1.007	1.001	0.988	0.971	0.969	0.753	0.610	0.602	0.584
R^2_{adj} (%)	0.03	18.85	0.70	0.20	0.30	0.29	17.42	23.45	24.16	26.10	28.48	28.51	58.35	72.43	72.93	74.17
No. Obs.	510	510	510	510	510	510	337	510	510	510	510	510	337	337	337	337

(b) External Predictors

Table A18: Mean Return Forecasting (60M) of the Naive Portfolio.
This table reports OLS results of regressing (rolling) mean returns over the next 60-month period (dependent variable) on contemporaneous risk proxies (Panel a) and external candidate predictors (Panel b). To prevent repetition, refer to Table A7 or Table A13 for a detailed table description.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
(Intercept)	**0.002**	**0.003**	**0.003**	**0.002**	**0.003**	**0.002**	**0.002**	**0.002**	**0.002**	**0.002**	**0.002**	**0.002**	**0.002**	**0.002**	**0.003**	**0.002**	**0.004**
	[5.784]	**[11.591]**	**[7.357]**	**[9.612]**	**[7.884]**	**[13.857]**	**[9.264]**	**[10.236]**	**[5.67]**	**[12.732]**	**[15.449]**	**[24.472]**	**[59.992]**	**[39.723]**	**[9.583]**	**[32.206]**	**[11.077]**
AV_t (12M)	0.194		0.173		-0.267			**0.292**	0.218								**-1.228**
	[1.069]		[1.099]		[-1.5]			**[2.447]**	[1.23]								**[-3.2]**
AC_t (12M)		**-0.005**	**-0.005**		**-0.01**												**-0.013**
		[-2.233]	**[-2.309]**		**[-3.232]**												**[-4.989]**
AV_t (12M) * AC_t (12M)				0.472	**5.015**												**19.311**
				[0.49]	**[3.064]**												**[3.907]**
VAR_t (12M)						-0.283		**-1.667**									**-8.486**
						[-0.377]		**[-2.237]**									**[-2.965]**
$VAR.G_t$							0.047		-0.535								-0.976
							[0.06]		[-0.832]								[-0.708]
AS_t (12M)										0							0
										[-0.916]							[-1.872]
AK_t (12M)											0						0
											[0.579]						[0.301]
CSV_t												**0.062**					**0.056**
												[2.244]					**[2.998]**
CSS_t													0				0
													[-1.209]				[-0.665]
CSK_t														0			0
														[-0.571]			[-0.601]
MOM_t (12M)															-0.033		**-0.067**
															[-1.671]		**[-3.9]**
VAL_t (60M)																-0.006	-0.01
																[-0.846]	[-1.643]
RMSE (‰)	0.638	0.611	0.599	0.652	0.586	0.652	0.653	0.629	0.637	0.650	0.647	0.648	0.651	0.652	0.647	0.669	0.548
R^2_{adj} (%)	4.16	12.09	15.34	0.06	18.69	-0.11	-0.22	6.81	4.43	0.68	1.39	1.18	0.31	0.04	1.35	-0.06	30.85
No. Obs.	439	439	439	439	439	439	439	439	439	439	439	439	439	439	439	385	385

(a) Risk Approximations

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
(Intercept)	**0.002**	**0.002**	**0.002**	**0.002**	**0.002**	**0.002**	**0.002**	**0.002**	**0.003**	**0.002**	**0.003**	**0.003**	**0.003**	**0.004**	**0.003**	**0.003**
	[6.486]	**[27.862]**	**[34.521]**	**[11.712]**	**[12.037]**	**[17.865]**	**[6.05]**	**[13.02]**	**[6.401]**	**[5.968]**	**[6.074]**	**[6.657]**	**[5.282]**	**[5.263]**	**[4.379]**	**[6.91]**
AV_t (12M)														-0.566	-0.47	**-0.637**
														[-1.58]	[-1.423]	**[-2.554]**
AC_t (12M)														**-0.013**	**-0.013**	**-0.015**
														[-3.639]	**[-3.98]**	**[-6.185]**
AV_t (12M) * AC_t (12M)														**10.772**	**9.907**	**11.992**
														[3.055]	**[3.202]**	**[4.919]**
AS_t (12M)															0	0
															[-1.152]	[-1.132]
AK_t (12M)															**0**	**0**
															[-2.124]	**[-2.689]**
CSV_t																**0.048**
																[2.873]
CSS_t																0
																[-1.45]
CSK_t																0
																[-0.595]
MOM_t (12M)																**-0.057**
																[-2.803]
VAL_t (60M)																-0.008
																[-0.889]
INFLTN	-0.001							-0.001	-0.002	-0.001	0	0	-0.012	-0.004	-0.001	0
	[-0.281]							[-0.761]	[-0.623]	[-0.344]	[0.083]	[0.132]	[-1.325]	[-0.516]	[-0.073]	[0.068]
BDGT.BLNC		-0.001						0	0	0.004	0.013	0.012	0.013	-0.002	-0.005	-0.006
		[-1.124]						[-0.074]	[-0.019]	[0.393]	[1.234]	[1.243]	[0.983]	[-0.257]	[-0.547]	[-0.904]
GLBL.M2.SPPLY			**-0.001**						**-0.001**	**-0.001**	**-0.001**	**-0.001**	**-0.001**	**0**	**0**	**0**
			[-2.393]						**[-2.976]**	**[-2.893]**	**[-3.076]**	**[-2.749]**	**[-2.545]**	**[-2.61]**	**[-2.629]**	**[-2.479]**
Steep_Yld_Crv				0.007						0.008	0.014	0.013	0.014	0.006	0.008	0.009
				[1.907]						[0.755]	[1.269]	[1.204]	[1.126]	[0.643]	[0.814]	[1.594]
CFNAI					0						**0**	0	0	0	0	0
					[1.793]						**[2.219]**	[0.436]	[0.146]	[0.407]	[0.22]	[0.658]
ADS						**0**						0	**0.001**	**0.001**	**0.001**	**0**
						[2.229]						[1.829]	**[2.983]**	**[4.139]**	**[4.506]**	**[3.781]**
VIX							0						0.001	-0.001	-0.001	-0.001
							[0.399]						[0.484]	[-1.219]	[-1.034]	[-1.546]
RMSE (‰)	0.653	0.654	0.636	0.648	0.634	0.628	0.777	0.653	0.633	0.628	0.601	0.595	0.671	0.561	0.548	0.530
R^2_{adj} (%)	0.05	-0.19	5.12	1.53	5.61	7.50	-0.26	-0.17	5.48	6.85	14.52	16.02	23.57	45.98	48.09	50.43
No. Obs.	450	450	450	450	450	450	277	450	450	450	450	450	277	277	277	277

(b) External Predictors

Table A19: Mean Return Forecasting (120M) of the Naive Portfolio.
This table reports OLS results of regressing (rolling) mean returns over the next 120-month period (dependent variable) on contemporaneous risk proxies (Panel a) and external candidate predictors (Panel b). To prevent repetition, refer to Table A7 or Table A13 for a detailed table description.

	(1)	(2)	(3)	(4)
(Intercept)	0	0	**0**	**0**
	[1.322]	[0.121]	**[12.273]**	**[25.12]**
AV_t (12M)	**0.063**			
	[2.258]			
AC_t (12M)		**0.001**		
		[5.066]		
$(\Delta AV_{t-1:t})/AV_{t-1}$ (12M)			**0**	
			[2.537]	
$(\Delta AC_{t-1:t})/AC_{t-1}$ (12M)				0
				[1.414]
RMSE (‰)	0.063	0.066	0.075	0.076
R^2_{adj} (%)	30.62	23.48	1.62	0.05
No. Obs.	559	559	558	558

(a) 0M

	(1)	(2)	(3)	(4)
(Intercept)	**0**	**0**	**0**	**0**
	[2.792]	**[4.228]**	**[10.154]**	**[13.992]**
AV_t (12M)	**0.046**			
	[3.591]			
AC_t (12M)		0		
		[-0.866]		
$(\Delta AV_{t-1:t})/AV_{t-1}$ (12M)			0	
			[0.34]	
$(\Delta AC_{t-1:t})/AC_{t-1}$ (12M)				0
				[1.009]
RMSE (‰)	0.091	0.096	0.096	0.096
R^2_{adj} (%)	10.11	0.05	-0.11	0.10
No. Obs.	553	553	552	552

(b) 6M

	(1)	(2)	(3)	(4)
(Intercept)	**0**	**0**	**0**	**0**
	[3.238]	**[4.454]**	**[12.025]**	**[13.229]**
AV_t (12M)	**0.038**			
	[4.678]			
AC_t (12M)		0		
		[-1.571]		
$(\Delta AV_{t-1:t})/AV_{t-1}$ (12M)			0	
			[1.592]	
$(\Delta AC_{t-1:t})/AC_{t-1}$ (12M)				0
				[0.236]
RMSE (‰)	0.070	0.074	0.073	0.074
R^2_{adj} (%)	11.52	1.03	1.22	-0.17
No. Obs.	547	547	546	546

(c) 12M

	(1)	(2)	(3)	(4)
(Intercept)	**0**	**0**	**0**	**0**
	[3.874]	**[5.633]**	**[16.529]**	**[17.343]**
AV_t (12M)	**0.021**			
	[2.936]			
AC_t (12M)		**0**		
		[-2.333]		
$(\Delta AV_{t-1:t})/AV_{t-1}$ (12M)			0	
			[1.89]	
$(\Delta AC_{t-1:t})/AC_{t-1}$ (12M)				0
				[-0.527]
RMSE (‰)	0.050	0.052	0.051	0.051
R^2_{adj} (%)	7.09	1.55	1.41	-0.14
No. Obs.	535	535	534	534

(d) 24M

	(1)	(2)	(3)	(4)
(Intercept)	**0**	**0**	**0**	**0**
	[5.085]	**[8.148]**	**[28.029]**	**[29.353]**
AV_t (12M)	0.001			
	[0.146]			
AC_t (12M)		0		
		[-1.096]		
$(\Delta AV_{t-1:t})/AV_{t-1}$ (12M)			0	
			[1.579]	
$(\Delta AC_{t-1:t})/AC_{t-1}$ (12M)				0
				[0.101]
RMSE (‰)	0.031	0.030	0.030	0.031
R^2_{adj} (%)	-0.15	1.02	0.57	-0.20
No. Obs.	499	499	498	498

(e) 60M

	(1)	(2)	(3)	(4)
(Intercept)	**0**	**0**	**0**	**0**
	[8.671]	**[8.415]**	**[46.718]**	**[51.049]**
AV_t (12M)	**0.009**			
	[2.896]			
AC_t (12M)		0		
		[-0.479]		
$(\Delta AV_{t-1:t})/AV_{t-1}$ (12M)			0	
			[0.502]	
$(\Delta AC_{t-1:t})/AC_{t-1}$ (12M)				0
				[-0.445]
RMSE (‰)	0.017	0.019	0.018	0.018
R^2_{adj} (%)	11.50	-0.02	-0.09	-0.18
No. Obs.	439	439	438	438

(f) 120M

Table A20: Variance Forecasting using Level vs. Relative Change of Variance Components.
This table lists the results of OLS regressions of the contemporaneous/future multi-factor return variance (dependent variable) on estimated contemporaneous AC and AV, both expessed in levels as well as relative differences. Each panel covers a specific forecast horizon, where the results of the decomposition of contemporaneous variance are indicated by the horizon labeled '0M'). Data covers the full investigation period from July 1971 to December 2018. The regressor names are shown at the left margin. For return-based measures, the estimation period (in months) is disclosed in brackets alongside the regressor's name. Below each estimated coefficient, the subtable reports associated t-statistics (adjusted based on the methods proposed in Newey and West (1987) and Newey and West (1994)) in squared brackets. Coefficients with absolute t-statistics above 2 are highlighted in bold. The last three rows of both subtables show - for each of the models defined in the columns above - root mean squared error (RMSE), adjusted R^2 (R^2_{adj}), and the number of observations used for the estimation. Consult Table A4 and Section 3 for an overview of factors constituting the equally-weighted multi-factor portfolio. Detailed data set information is available in Appendix A.1 and Vincenz and Zeissler (2024). Refer to Section 4.1 for information on the variables that approximate components of multi-factor variance (i.e. AV_t and AC_t). An overview of the abbreviations used in the course of this paper is provided in Table A3.

	(1)	(2)	(3)	(4)
(Intercept)	**0.002**	**0.003**	**0.002**	**0.002**
	[2.518]	**[3.101]**	**[6.9]**	**[6.847]**
AV_t (12M)	0.949			
	[1.354]			
AC_t (12M)		-0.004		
		[-0.454]		
$(\Delta AV_{t-1:t})/AV_{t-1}$ (12M)			**-0.014**	
			[-2.257]	
$(\Delta AC_{t-1:t})/AC_{t-1}$ (12M)				-0.002
				[-0.63]
RMSE (‰)	8.802	8.823	8.626	8.783
R^2_{adj} (%)	0.34	-0.12	3.80	0.27
No. Obs.	559	559	558	558

(a) 0M

	(1)	(2)	(3)	(4)
(Intercept)	**0.001**	**0.002**	**0.002**	**0.002**
	[2.028]	**[2.301]**	**[5.958]**	**[5.945]**
AV_t (12M)	**1.349**			
	[2.142]			
AC_t (12M)		0.005		
		[0.728]		
$(\Delta AV_{t-1:t})/AV_{t-1}$ (12M)			-0.001	
			[-0.204]	
$(\Delta AC_{t-1:t})/AC_{t-1}$ (12M)				0.001
				[1.193]
RMSE (‰)	8.756	8.799	8.809	8.803
R^2_{adj} (%)	0.88	-0.10	-0.17	-0.02
No. Obs.	558	558	557	557

(b) 1M

	(1)	(2)	(3)	(4)
(Intercept)	**0.001**	**0.003**	**0.002**	**0.002**
	[2.497]	**[3.601]**	**[7.049]**	**[8.982]**
AV_t (12M)	**1.289**			
	[3.308]			
AC_t (12M)		-0.001		
		[-0.239]		
$(\Delta AV_{t-1:t})/AV_{t-1}$ (12M)			-0.001	
			[-0.315]	
$(\Delta AC_{t-1:t})/AC_{t-1}$ (12M)				**0.001**
				[2.018]
RMSE (‰)	3.785	3.882	3.885	3.874
R^2_{adj} (%)	4.82	-0.16	-0.14	0.42
No. Obs.	553	553	552	552

(c) 6M

	(1)	(2)	(3)	(4)
(Intercept)	**0.001**	**0.003**	**0.002**	**0.002**
	[2.616]	**[4.797]**	**[9.243]**	**[13.518]**
AV_t (12M)	**1.297**			
	[4.147]			
AC_t (12M)		-0.007		
		[-1.425]		
$(\Delta AV_{t-1:t})/AV_{t-1}$ (12M)			0	
			[-0.274]	
$(\Delta AC_{t-1:t})/AC_{t-1}$ (12M)				0.001
				[1.652]
RMSE (‰)	2.539	2.667	2.671	2.668
R^2_{adj} (%)	10.45	1.19	-0.14	0.14
No. Obs.	547	547	546	546

(d) 12M

	(1)	(2)	(3)	(4)
(Intercept)	**0.001**	**0.003**	**0.002**	**0.002**
	[2.55]	**[4.885]**	**[12.705]**	**[16.792]**
AV_t (12M)	**1.351**			
	[4.102]			
AC_t (12M)		-0.003		
		[-1.108]		
$(\Delta AV_{t-1:t})/AV_{t-1}$ (12M)			0	
			[0.246]	
$(\Delta AC_{t-1:t})/AC_{t-1}$ (12M)				0
				[0.36]
RMSE (‰)	1.691	1.918	1.915	1.915
R^2_{adj} (%)	22.53	0.29	-0.16	-0.17
No. Obs.	535	535	534	534

(e) 24M

	(1)	(2)	(3)	(4)
(Intercept)	**0.001**	**0.003**	**0.002**	**0.002**
	[5.009]	**[8.035]**	**[19.925]**	**[24.068]**
AV_t (12M)	**1**			
	[7.904]			
AC_t (12M)		**-0.009**		
		[-4.318]		
$(\Delta AV_{t-1:t})/AV_{t-1}$ (12M)			0.001	
			[1.68]	
$(\Delta AC_{t-1:t})/AC_{t-1}$ (12M)				0
				[-0.193]
RMSE (‰)	0.907	1.058	1.130	1.137
R^2_{adj} (%)	36.63	13.68	1.09	-0.19
No. Obs.	499	499	498	498

(f) 60M

	(1)	(2)	(3)	(4)
(Intercept)	**0.002**	**0.003**	**0.002**	**0.002**
	[5.784]	**[11.591]**	**[33.426]**	**[63.544]**
AV_t (12M)	0.194			
	[1.069]			
AC_t (12M)		**-0.005**		
		[-2.233]		
$(\Delta AV_{t-1:t})/AV_{t-1}$ (12M)			0	
			[-0.048]	
$(\Delta AC_{t-1:t})/AC_{t-1}$ (12M)				0
				[0.337]
RMSE (‰)	0.638	0.611	0.652	0.652
R^2_{adj} (%)	4.16	12.09	-0.23	-0.22
No. Obs.	439	439	438	438

(g) 120M

Table A21: (Mean) Return Forecasting using Level vs. Relative Change of Variance Components.
This table lists the results of OLS regressions of the contemporaneous/future multi-factor returns (dependent variable) on estimated contemporaneous AC and AV, both expessed in levels as well as relative differences. Each panel covers a specific forecast horizon, where the results of the decomposition of contemporaneous returns are indicated by the horizon labeled '0M'). Data covers the full investigation period from July 1971 to December 2018. The regressor names are shown at the left margin. For return-based measures, the estimation period (in months) is disclosed in brackets alongside the regressor's name. Below each estimated coefficient, the subtable reports associated t-statistics (adjusted based on the methods proposed in Newey and West (1987) and Newey and West (1994)) in squared brackets. Coefficients with absolute t-statistics above 2 are highlighted in bold. The last three rows of both subtables show - for each of the models defined in the columns above - root mean squared error (RMSE), adjusted R^2 (R^2_{adj}), and the number of observations used for the estimation. Consult Table A4 and Section 3 for an overview of factors constituting the equally-weighted multi-factor portfolio. Detailed data set information is available in Appendix A.1 and Vincenz and Zeissler (2024). Refer to Section 4.1 for information on the variables that approximate components of multi-factor variance (i.e. AV_t and AC_t). An overview of the abbreviations used in the course of this paper is provided in Table A3.

Sample Period	Full Sample	6/1972 - 10/1983	11/1983 - 3/1995	4/1995 - 7/2006	8/2006 - 12/2017
(Intercept)	0.001	0.003	0.001	0.002	-0.001
	[2.616]	[2.741]	[1.674]	[2.147]	[-0.975]
AV_t (12M)	1.297	-0.013	2.096	1.172	2.341
	[4.147]	[-0.029]	[3.335]	[3.874]	[6.588]
RMSE (‰)	2.539	2.887	1.312	2.305	2.704
R^2_{adj} (%)	10.45	-0.74	11.98	18.60	13.23
No. Obs.	547	137	137	136	137

(a) AV

Sample Period	Full Sample	6/1972 - 10/1983	11/1983 - 3/1995	4/1995 - 7/2006	8/2006 - 12/2017
(Intercept)	0.003	0.006	0.003	0.002	0.001
	[4.797]	[4.37]	[3.097]	[1.388]	[1.264]
AC_t (12M)	-0.007	-0.025	-0.009	0.009	0.002
	[-1.425]	[-2.801]	[-0.683]	[0.59]	[0.409]
RMSE (‰)	2.667	2.633	1.364	2.528	2.911
R^2_{adj} (%)	1.19	16.22	4.83	2.07	-0.59
No. Obs.	547	137	137	136	137

(b) AC

Sample Period	Full Sample	6/1972 - 10/1983	11/1983 - 3/1995	4/1995 - 7/2006	8/2006 - 12/2017
(Intercept)	0.002	0.004	0.002	0.002	0
	[5.088]	[4.587]	[4.443]	[2.035]	[0.245]
VAR_t (12M)	3.6	-4.656	7.722	18.928	10.202
	[0.96]	[-0.836]	[1.286]	[2.928]	[3.576]
RMSE (‰)	2.671	2.845	1.370	2.321	2.823
R^2_{adj} (%)	0.86	2.21	3.93	17.49	5.43
No. Obs.	547	137	137	136	137

(c) Naive Variance

Table A22: Mean Return Forecasting (12M) of the Naive Portfolio Over Subsamples.
This table lists the results of OLS regressions of the (rolling) mean returns over the next 12-month period (dependent variable) on estimated contemporaneous AV_t (Panel a), AC_t (Panel b), and the VAR_t (Panel c), using the full data set (July 1971 to December 2018) as well as four subsamples with (nearly) equal number of observations. Below each estimated coefficient, the subtable reports associated t-statistics (adjusted based on the methods proposed in Newey and West (1987) and Newey and West (1994)) in squared brackets. Consult Table A4 and Section 3 for an overview of factors constituting the equally-weighted multi-factor portfolio. Detailed data set information is available in Appendix A.1 and Vincenz and Zeissler (2024). Refer to Section 4.1 for information on all variables that approximate (components of) multi-factor variance (i.e. AV_t, AC_t, VAR_t, and $VAR.G_t$). An overview of the abbreviations used in the course of this paper is provided in Table A3.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
(Intercept)	0	0	0	0	0	0	0	0	0	0	0	0	0
	[3.4]	[3.162]	[2.551]	[4.148]	[2.034]	[4.03]	[3.94]	[2.841]	[4.86]	[2.596]	[3.763]	[4.83]	[3.435]
AV_t (12M)	0.104		0.102		0.106								0.021
	[1.675]		[1.561]		[1.158]								[0.727]
AC_t (12M)		0	0		0								
		[-1.065]	[-0.911]		[-0.425]								
AV_t (12M) * AC_t (12M)				0.621	-0.049								
				[1.181]	[-0.046]								
Naive.C AV_t (12M)						0.166		0.178		0.066	0.134		
						[3.632]		[2.803]		[0.28]	[1.121]		
Naive.C AC_t (12M)							0	0		0			
							[-0.767]	[-1.366]		[-2.127]			
Naive.C AV_t (12M) * Naive.C AC_t (12M)									0.267	0.242			
									[3.33]	[0.628]			
Naive.C VAR_t (12M)											0.067	0.291	0.269
											[0.359]	[4.212]	[4.339]
RMSE (‰)	0.343	0.350	0.343	0.347	0.343	0.332	0.350	0.329	0.336	0.329	0.331	0.334	0.334
R^2_{adj} (%)	3.81	0.26	3.93	1.48	3.76	10.27	-0.06	11.29	7.79	11.56	10.21	8.91	8.85
No. Obs.	547	547	547	547	547	547	547	547	547	547	547	547	547

(a) Naive.C / $N = 4$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
(Intercept)	0	0	0	0	0	0	0	0	0	0	0	0	0
	[1.019]	[4.798]	[1.568]	[2.829]	[-0.318]	[3.006]	[4.145]	[2.081]	[2.5]	[1.289]	[3.856]	[4.752]	[1.309]
AV_t (12M)	0.078		0.078		0.139								0.081
	[2.945]		[2.894]		[3.518]								[2.656]
AC_t (12M)		0	0		0								
		[-2.102]	[-1.718]		[1.979]								
AV_t (12M) * AC_t (12M)				0.435	-0.69								
				[1.621]	[-3.358]								
Naive.E AV_t (12M)						0.06		0.058		0.086	0.064		
						[7.623]		[7.174]		[8.301]	[8.327]		
Naive.E AC_t (12M)							0	0		0			
							[-2.073]	[-0.966]		[1.712]			
Naive.E AV_t (12M) * Naive.E AC_t (12M)									0.388	-0.433			
									[1.597]	[-3.562]			
Naive.E VAR_t (12M)											-0.135	0.161	-0.068
											[-1.47]	[1.631]	[-0.635]
RMSE (‰)	0.068	0.085	0.068	0.080	0.066	0.064	0.084	0.063	0.081	0.061	0.063	0.085	0.068
R^2_{adj} (%)	37.31	1.83	38.30	13.24	41.90	45.56	6.25	45.97	11.28	49.19	46.99	2.39	37.60
No. Obs.	547	547	547	547	547	547	547	547	547	547	547	547	547

(b) Naive.E / $N = 8$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
(Intercept)	0	0.001	0.001	0.001	0.001	0	0	0	0	0	0	0	0
	[4.791]	[3.944]	[3.461]	[4.445]	[3.609]	[3.624]	[1.561]	[1.868]	[4.797]	[-0.352]	[3.047]	[4.659]	[3.862]
AV_t (12M)	-0.013		-0.019		-0.168								-0.078
	[-0.211]		[-0.293]		[-1.491]								[-1.411]
AC_t (12M)		-0.001	-0.001		-0.003								
		[-1.572]	[-1.559]		[-2.297]								
AV_t (12M) * AC_t (12M)				-0.403	1.678								
				[-0.746]	[1.886]								
Naive.FX AV_t (12M)						0.136		0.107		0.445	0.097		
						[2.141]		[1.232]		[3.648]	[0.488]		
Naive.FX AC_t (12M)							0	0		0.001			
							[1.544]	[0.926]		[1.98]			
Naive.FX AV_t (12M) * Naive.FX AC_t (12M)									0.158	-0.481			
									[2.296]	[-3.636]			
Naive.FX VAR_t (12M)											0.054	0.172	0.204
											[0.247]	[2.16]	[2.884]
RMSE (‰)	0.530	0.525	0.525	0.529	0.524	0.519	0.522	0.517	0.520	0.513	0.519	0.520	0.518
R^2_{adj} (%)	-0.16	1.52	1.40	0.12	1.80	3.66	2.69	4.57	3.33	5.74	3.52	3.51	4.19
No. Obs.	547	547	547	547	547	547	547	547	547	547	547	547	547

(c) Naive.FX / $N = 2$

Table A23: Variance Forecasting (12M) of Naive Portfolios Based on Asset Classes.
This table lists the results of OLS regressions of the (rolling) measure of multi-factor return variance over the next 12-month period (dependent variable) on variables proxying (components of) multi-factor variance, when considering the returns of different equally-weighted multi-factor strategies - based on a factor's association to an ASCL - over the full investigation period. The name of each panel shows the respective symbol of the naive strategy, as well as the number of factor premia covered by the strategy (N). Below each estimated coefficient, the subtable reports associated t-statistics (adjusted based on the methods proposed in Newey and West (1987) and Newey and West (1994)) in squared brackets. Consult Table A4 and Section 3 for an overview of factors constituting the different equally-weighted multi-factor portfolios. Detailed data set information is available in Appendix A.1 and Vincenz and Zeissler (2024). Refer to Section 4.1 for information on all variables that approximate (components of) multi-factor variance (i.e. AV_t, AC_t, VAR_t, and $VAR.G_t$). An overview of the abbreviations used in the course of this paper is provided in Table A3.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
(Intercept)	0	0	0	0	0	0	0	0	0	0	0	0	0
	[3.458]	[4.715]	[2.915]	[3.836]	[2.705]	[4.134]	[3.191]	[1.844]	[4.185]	[1.178]	[3.981]	[4.145]	[3.117]
AV_t (12M)	0.099		0.101		-0.044								0.041
	[1.973]		[2.113]		[-0.425]								[0.882]
AC_t (12M)		0	0		-0.001								
		[0.668]	[0.789]		[-0.588]								
AV_t (12M) * AC_t (12M)				1.072	1.634								
				[2.375]	[1.484]								
Naive.Market AV_t (12M)						0.138		0.129		0.126	0.07		
						[3.12]		[3.497]		[1.084]	[0.84]		
Naive.Market AC_t (12M)							0	0		0			
							[1.559]	[1.362]		[0.773]			
Naive.Market AV_t (12M) * Naive.Market AC_t (12M)									0.226	0.006			
									[4.626]	[0.031]			
Naive.Market VAR_t (12M)											0.127	0.214	0.186
											[1.107]	[4.26]	[4.67]
RMSE (‰)	0.452	0.456	0.451	0.450	0.450	0.447	0.452	0.443	0.445	0.443	0.445	0.446	0.446
R^2_{adj} (%)	1.95	0.00	2.04	2.73	2.60	4.24	1.88	5.58	5.12	5.40	4.59	4.39	4.51
No. Obs.	547	547	547	547	547	547	547	547	547	547	547	547	547

(a) Naive.Market / $N = 4$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
(Intercept)	0	0	0	0	0	0	0	0	0	0	0	0	0
	[5.368]	[6.306]	[4.894]	[7.31]	[6.009]	[3.601]	[4.291]	[2.109]	[3.473]	[3.004]	[3.06]	[3.495]	[3.02]
AV_t (12M)	0.075		0.073		-0.01								0.029
	[1.224]		[1.171]		[-0.184]								[1.038]
AC_t (12M)		0	0		-0.001								
		[-1.14]	[-1.039]		[-2.718]								
AV_t (12M) * AC_t (12M)				0.52	0.933								
				[1.093]	[2.008]								
Naive.Carry AV_t (12M)						0.17		0.156		-0.075	0.197		
						[2.262]		[1.809]		[-0.697]	[2.16]		
Naive.Carry AC_t (12M)							0	0		-0.001			
							[-1.962]	[-1.222]		[-2.35]			
Naive.Carry AV_t (12M) * Naive.Carry AC_t (12M)									0.356	0.605			
									[1.941]	[2.161]			
Naive.Carry VAR_t (12M)											-0.077	0.361	0.324
											[-0.288]	[2.024]	[1.991]
RMSE (‰)	0.213	0.218	0.212	0.216	0.211	0.197	0.212	0.194	0.204	0.190	0.197	0.204	0.203
R^2_{adj} (%)	5.10	0.75	5.61	2.81	6.50	18.64	6.04	20.95	12.91	24.00	18.63	13.36	13.88
No. Obs.	547	547	547	547	547	547	547	547	547	547	547	547	547

(b) Naive.Carry / $N = 2$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
(Intercept)	0	0	0	0	0	0	0	0	0	0	0	0	0
	[3.058]	[3.787]	[2.141]	[3.498]	[2.049]	[3.561]	[2.038]	[1.932]	[5.078]	[-0.234]	[3.586]	[4.882]	[3.108]
AV_t (12M)	0.21		0.208		0.105								0.192
	[5.964]		[6.819]		[0.564]								[3.37]
AC_t (12M)		-0.001	0		-0.001								
		[-1.219]	[-0.883]		[-1.093]								
AV_t (12M) * AC_t (12M)				1.496	1.157								
				[2.986]	[0.549]								
Naive.Mom AV_t (12M)						0.186		0.184		0.38	0.177		
						[4.235]		[3.962]		[2.769]	[0.899]		
Naive.Mom AC_t (12M)							0	0		0			
							[1.086]	[0.224]		[2.271]			
Naive.Mom AV_t (12M) * Naive.Mom AC_t (12M)									0.364	-0.516			
									[3.876]	[-1.682]			
Naive.Mom VAR_t (12M)											0.023	0.436	0.047
											[0.048]	[3.994]	[0.276]
RMSE (‰)	0.247	0.283	0.246	0.263	0.244	0.251	0.283	0.251	0.260	0.248	0.251	0.255	0.247
R^2_{adj} (%)	24.60	1.13	25.17	14.46	26.00	22.00	0.98	21.89	16.22	23.78	21.86	19.37	24.52
No. Obs.	547	547	547	547	547	547	547	547	547	547	547	547	547

(c) Naive.Mom / $N = 4$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
(Intercept)	0	0	0	0	0	0	0	0	0	0	0	0	0
	[-0.772]	[2.239]	[0.955]	[2.087]	[0.008]	[2.307]	[0.777]	[-0.893]	[3.43]	[-0.145]	[2.557]	[3.629]	[0.232]
AV_t (12M)	0.28		0.276		0.355								0.128
	[3.098]		[3.131]		[1.332]								[2.271]
AC_t (12M)		-0.001	-0.001		0								
		[-1.384]	[-1.292]		[-0.231]								
AV_t (12M) * AC_t (12M)				1.63	-0.898								
				[2.73]	[-0.427]								
Naive.Other AV_t (12M)						0.146		0.143		0.083	0.109		
						[9.576]		[21.691]		[1.121]	[3.293]		
Naive.Other AC_t (12M)							0.001	0.001		0			
							[1.43]	[1.422]		[1.124]			
Naive.Other AV_t (12M) * Naive.Other AC_t (12M)									0.641	0.273			
									[5.233]	[0.761]			
Naive.Other VAR_t (12M)											0.207	0.646	0.448
											[0.739]	[4.947]	[2.558]
RMSE (‰)	0.252	0.310	0.248	0.292	0.247	0.226	0.306	0.219	0.220	0.218	0.224	0.240	0.233
R^2_{adj} (%)	35.77	3.10	37.76	14.01	38.12	48.24	5.42	51.44	51.20	51.68	49.36	41.79	45.23
No. Obs.	547	547	547	547	547	547	547	547	547	547	547	547	547

(d) Naive.Other / $N = 4$

Table A24: Variance Forecasting (12M) of Naive Portfolios Based on Factor Styles. (Caption on the next page.)

Table A24: Variance Forecasting (12M) of Naive Portfolios Based on Factor Styles.
This table lists the results of OLS regressions of the (rolling) measure of multi-factor return variance over the next 12-month period (dependent variable) on variables proxying (components of) multi-factor variance, when considering the returns of different equally-weighted multi-factor strategies - based on a factor's association to a certain style - over the full investigation period. The name of each panel shows the respective symbol of the naive strategy, as well as the number of factor premia covered by the strategy (N). Below each estimated coefficient, the subtable reports associated t-statistics (adjusted based on the methods proposed in Newey and West (1987) and Newey and West (1994)) in squared brackets. Consult Table A4 and Section 3 for an overview of factors constituting the different equally-weighted multi-factor portfolios. Detailed data set information is available in Appendix A.1 and Vincenz and Zeissler (2024). Refer to Section 4.1 for information on all variables that approximate (components of) multi-factor variance (i.e. AV_t, AC_t, VAR_t, and $VAR.G_t$). An overview of the abbreviations used in the course of this paper is provided in Table A3.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
(Intercept)	0.002	0.005	0.003	0.003	0.001	0.002	0.004	0.003	0.002	0.008	0.002	0.002	0.002
	[2.069]	[2.925]	[1.914]	[3.344]	[0.861]	[1.957]	[4.536]	[3.174]	[3.417]	[4.051]	[2.026]	[3.214]	[2.195]
AV_t (12M)	2.384		2.348		3.757								0.957
	[2.201]		[2.132]		[2.286]								[2.825]
AC_t (12M)		-0.01	-0.008		0.004								
		[-1.078]	[-0.842]		[0.32]								
AV_t (12M) * AC_t (12M)				12.954	-15.892								
				[1.518]	[-1.135]								
Naive.C AV_t (12M)						2.895		3.131		-3.67	1.071		
						[3.14]		[2.639]		[-1.108]	[0.464]		
Naive.C AC_t (12M)							-0.002	-0.005		-0.016			
							[-0.57]	[-1.766]		[-4.53]			
Naive.C AV_t (12M) * Naive.C AC_t (12M)									5.307	14.673			
									[3.308]	[2.751]			
Naive.C VAR_t (12M)											3.88	5.669	4.651
											[1.031]	[3.742]	[3.193]
RMSE (‰)	5.643	5.847	5.629	5.791	5.616	5.525	5.861	5.478	5.528	5.298	5.486	5.497	5.471
R^2_{adj} (%)	7.34	0.52	7.62	2.40	7.87	11.18	0.02	12.51	11.06	18.01	12.25	12.08	12.73
No. Obs.	547	547	547	547	547	547	547	547	547	547	547	547	547

(a) Naive.C / $N = 4$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
(Intercept)	0.001	0.002	0.001	0.001	0.001	0.001	0.002	0	0.001	0	0	0.001	0
	[0.88]	[3.134]	[0.976]	[1.514]	[0.662]	[1.087]	[2.487]	[0.258]	[0.723]	[0.425]	[0.627]	[1.578]	[0.485]
AV_t (12M)	1.1		1.081		1.42								0.856
	[1.554]		[1.494]		[1.777]								[1.208]
AC_t (12M)		-0.005	-0.004		-0.001								
		[-1.087]	[-0.854]		[-0.155]								
AV_t (12M) * AC_t (12M)				6.18	-3.829								
				[0.826]	[-0.418]								
Naive.E AV_t (12M)						1.097		1.141		0.932	0.983		
						[6.325]		[5.137]		[2.661]	[6.194]		
Naive.E AC_t (12M)							-0.002	0.002		0			
							[-0.555]	[0.637]		[0.056]			
Naive.E AV_t (12M) * Naive.E AC_t (12M)									11.672	3.241			
									[2.444]	[0.517]			
Naive.E VAR_t (12M)											3.203	7.725	5.324
											[1.24]	[2.631]	[1.723]
RMSE (‰)	2.882	2.965	2.874	2.942	2.873	2.775	2.972	2.770	2.843	2.768	2.763	2.900	2.850
R^2_{adj} (%)	6.04	0.53	6.35	2.10	6.27	12.90	0.06	13.01	8.56	13.01	13.47	4.83	7.94
No. Obs.	547	547	547	547	547	547	547	547	547	547	547	547	547

(b) Naive.E / $N = 8$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
(Intercept)	0.003	0.003	0.003	0.003	0.003	0.002	0.003	0.003	0.002	0.002	0.002	0.002	0.003
	[1.827]	[1.56]	[1.449]	[2.112]	[1.105]	[1.311]	[1.247]	[1.232]	[1.708]	[0.434]	[0.841]	[1.722]	[1.715]
AV_t (12M)	-0.227		-0.242		-0.613								-0.309
	[-0.195]		[-0.207]		[-0.227]								[-0.245]
AC_t (12M)		-0.003	-0.003		-0.007								
		[-0.193]	[-0.204]		[-0.274]								
AV_t (12M) * AC_t (12M)				-2.364	4.188								
				[-0.191]	[0.14]								
Naive.FX AV_t (12M)						0.405		0.664		1.968	3.235		
						[0.318]		[0.497]		[0.392]	[0.887]		
Naive.FX AC_t (12M)							-0.002	-0.003		-0.001			
							[-0.369]	[-0.592]		[-0.142]			
Naive.FX AV_t (12M) * Naive.FX AC_t (12M)									0.201	-1.857			
									[0.147]	[-0.3]			
Naive.FX VAR_t (12M)											-3.831	0.127	0.256
											[-1.003]	[0.092]	[0.172]
RMSE (‰)	7.594	7.594	7.592	7.593	7.591	7.589	7.588	7.573	7.594	7.569	7.556	7.595	7.592
R^2_{adj} (%)	-0.14	-0.14	-0.28	-0.13	-0.45	-0.02	0.02	0.22	-0.16	0.13	0.65	-0.17	-0.29
No. Obs.	547	547	547	547	547	547	547	547	547	547	547	547	547

(c) Naive.FX / $N = 2$

Table A25: Mean Return Forecasting (12M) of Naive Portfolios Based on Asset Classes.
This table lists the results of OLS regressions of the (rolling) mean returns over the next 12-month period (dependent variable) on variables proxying (components of) multi-factor variance, when considering the returns of different equally-weighted multi-factor strategies - based on a factor's association to an ASCL - over the full investigation period. The name of each panel shows the respective symbol of the naive strategy, as well as the number of factor premia covered by the strategy (N). Below each estimated coefficient, the subtable reports associated t-statistics (adjusted based on the methods proposed in Newey and West (1987) and Newey and West (1994)) in squared brackets. Consult Table A4 and Section 3 for an overview of factors constituting the different equally-weighted multi-factor portfolios. Detailed data set information is available in Appendix A.1 and Vincenz and Zeissler (2024). Refer to Section 4.1 for information on all variables that approximate (components of) multi-factor variance (i.e. AV_t, AC_t, VAR_t, and $VAR.G_t$). An overview of the abbreviations used in the course of this paper is provided in Table A3.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
(Intercept)	0.003	0.003	0.005	0.003	0.004	0.001	0.001	0.001	0.001	0.004	0.001	0.001	0.003
	[2.604]	[2.092]	[2.339]	[2.852]	[1.643]	[0.981]	[0.665]	[0.502]	[0.97]	[2.222]	[1.097]	[0.934]	[2.104]
AV_t (12M)	-1.939		-2.01		-0.541								-2.781
	[-1.684]		[-1.641]		[-0.207]								[-4.154]
AC_t (12M)		-0.014	-0.016		-0.003								
		[-0.923]	[-1.092]		[-0.196]								
AV_t (12M) * AC_t (12M)				-21.545	-16.565								
				[-1.708]	[-0.635]								
Naive.Market AV_t (12M)						0.154		0.144		-2.228	-0.93		
						[0.119]		[0.119]		[-1.674]	[-0.638]		
Naive.Market AC_t (12M)							0	0		-0.005			
							[0.09]	[0.08]		[-1.588]			
Naive.Market AV_t (12M) * Naive.Market AC_t (12M)									0.766	4.593			
									[0.359]	[1.687]			
Naive.Market VAR_t (12M)											2.022	0.857	2.732
											[0.649]	[0.398]	[1.436]
RMSE (‰)	6.665	6.756	6.619	6.609	6.607	6.791	6.792	6.791	6.783	6.753	6.771	6.781	6.572
R^2_{adj} (%)	3.53	0.87	4.67	5.15	4.84	-0.16	-0.17	-0.33	0.09	0.60	0.27	0.15	6.03
No. Obs.	547	547	547	547	547	547	547	547	547	547	547	547	547

(a) Naive.Market / $N = 4$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
(Intercept)	0.001	0.003	0.002	0.002	0.001	0.002	0.002	0.001	0.002	0	0.002	0.002	0.002
	[1.908]	[3.341]	[2.041]	[2.876]	[0.997]	[2.248]	[0.886]	[0.556]	[2.114]	[0.08]	[2.072]	[2.115]	[1.559]
AV_t (12M)	1.082		1.055		1.856								1.244
	[1.426]		[1.365]		[1.453]								[2.014]
AC_t (12M)		-0.007	-0.006		0.001								
		[-1.034]	[-0.89]		[0.06]								
AV_t (12M) * AC_t (12M)				4.833	-9.026								
				[0.662]	[-0.661]								
Naive.Carry AV_t (12M)						0.138		0.258		1.587	-0.061		
						[0.148]		[0.269]		[0.655]	[-0.034]		
Naive.Carry AC_t (12M)							0.002	0.002		0.005			
							[0.612]	[0.638]		[0.809]			
Naive.Carry AV_t (12M) * Naive.Carry AC_t (12M)									0.549	-3.478			
									[0.232]	[-0.573]			
Naive.Carry VAR_t (12M)											0.563	0.427	-1.163
											[0.112]	[0.172]	[-0.394]
RMSE (‰)	5.004	5.046	4.995	5.045	4.991	5.057	5.050	5.048	5.056	5.043	5.056	5.056	4.999
R^2_{adj} (%)	1.90	0.28	2.07	0.30	2.08	-0.16	0.10	-0.01	-0.12	0.01	-0.33	-0.15	1.94
No. Obs.	547	547	547	547	547	547	547	547	547	547	547	547	547

(b) Naive.Carry / $N = 2$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
(Intercept)	0.004	0.006	0.005	0.004	0.005	0.003	0.008	0.007	0.004	0.006	0.003	0.004	0.004
	[4.435]	[4.07]	[3.377]	[4.712]	[3.02]	[4.041]	[5.39]	[4.814]	[4.954]	[2.6]	[4.35]	[4.829]	[4.411]
AV_t (12M)	0.782		0.74		0.287								0.406
	[1.179]		[1.095]		[0.191]								[0.254]
AC_t (12M)		-0.01	-0.01		-0.014								
		[-1.2]	[-1.152]		[-1.096]								
AV_t (12M) * AC_t (12M)				2.982	5.104								
				[0.482]	[0.353]								
Naive.Mom AV_t (12M)						1.418		1.769		3.52	5.608		
						[1.92]		[3.248]		[1.918]	[3.322]		
Naive.Mom AC_t (12M)							-0.009	-0.011		-0.007			
							[-2.503]	[-3.061]		[-1.274]			
Naive.Mom AV_t (12M) * Naive.Mom AC_t (12M)									1.461	-4.611			
									[0.871]	[-1.041]			
Naive.Mom VAR_t (12M)											-11.256	1.84	1.019
											[-2.723]	[0.937]	[0.218]
RMSE (‰)	5.364	5.366	5.343	5.385	5.342	5.292	5.275	5.123	5.370	5.111	5.160	5.363	5.362
R^2_{adj} (%)	0.78	0.68	1.35	-0.02	1.22	3.43	4.04	9.30	0.55	9.59	8.00	0.79	0.67
No. Obs.	547	547	547	547	547	547	547	547	547	547	547	547	547

(c) Naive.Mom / $N = 4$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
(Intercept)	-0.004	0	-0.005	-0.003	-0.005	-0.002	0	-0.002	-0.001	-0.002	-0.002	-0.002	-0.003
	[-2.646]	[0.036]	[-2.127]	[-1.933]	[-1.905]	[-2.243]	[-0.129]	[-1.785]	[-1.93]	[-1.288]	[-2.45]	[-1.961]	[-2.619]
AV_t (12M)	4.888		4.934		5.516								2.97
	[3.498]		[3.434]		[2.905]								[2.863]
AC_t (12M)		0.006	0.01		0.016								
		[0.583]	[0.989]		[0.883]								
AV_t (12M) * AC_t (12M)				38.677	-6.575								
				[2.547]	[-0.581]								
Naive.Other AV_t (12M)						2.446		2.442		3.275	2.171		
						[15.75]		[25.032]		[1.657]	[2.451]		
Naive.Other AC_t (12M)							0.004	0.001		0.004			
							[0.668]	[0.113]		[0.365]			
Naive.Other AV_t (12M) * Naive.Other AC_t (12M)									9.868	-3.767			
									[13.039]	[-0.422]			
Naive.Other VAR_t (12M)											1.541	10.254	5.648
											[0.348]	[8.048]	[3.78]
RMSE (‰)	5.070	6.040	5.045	5.353	5.043	4.815	6.037	4.814	4.952	4.808	4.808	5.109	4.920
R^2_{adj} (%)	29.57	0.06	30.14	21.49	30.08	36.49	0.16	36.38	32.82	36.43	36.56	28.49	33.57
No. Obs.	547	547	547	547	547	547	547	547	547	547	547	547	547

(d) Naive.Other / $N = 4$

Table A26: Mean Return Forecasting (12M) of Naive Portfolios Based on Factor Styles. (Caption on the next page.)

Table A26: Mean Return Forecasting (12M) of Naive Portfolios Based on Factor Styles.
This table lists the results of OLS regressions of the (rolling) mean returns over the next 12-month period (dependent variable) on variables proxying (components of) multi-factor variance, when considering the returns of different equally-weighted multi-factor strategies - based on a factor's association to a certain style - over the full investigation period. The name of each panel shows the respective symbol of the naive strategy, as well as the number of factor premia covered by the strategy (N). Below each estimated coefficient, the subtable reports associated t-statistics (adjusted based on the methods proposed in Newey and West (1987) and Newey and West (1994)) in squared brackets. Consult Table A4 and Section 3 for an overview of factors constituting the different equally-weighted multi-factor portfolios. Detailed data set information is available in Appendix A.1 and Vincenz and Zeissler (2024). Refer to Section 4.1 for information on all variables that approximate (components of) multi-factor variance (i.e. AV_t, AC_t, VAR_t, and $VAR.G_t$). An overview of the abbreviations used in the course of this paper is provided in Table A3.

Model	$OS\text{-}R^2$ (%)	$\Delta RMSE$ (‰)	$MSE\text{-}F$
AV_t (12M)	0.20	0.01	0.94
AC_t (12M)	-1.32	-0.06	-6.08
AV_t (12M) + AC_t (12M)	-0.55	-0.02	-2.53
AV_t (12M) * AC_t (12M)	0.74	0.03	3.46
VAR_t (12M)	-0.08	0.00	-0.38

(a) R = 91 / P = 466 / π = 5.12

Model	$OS\text{-}R^2$ (%)	$\Delta RMSE$ (‰)	$MSE\text{-}F$
AV_t (12M)	0.91	0.04	3.91
AC_t (12M)	-0.28	-0.01	-1.19
AV_t (12M) + AC_t (12M)	0.85	0.03	3.63
AV_t (12M) * AC_t (12M)	1.28	0.05	5.54
VAR_t (12M)	0.24	0.01	1.05

(b) R = 131 / P = 426 / π = 3.25

Model	$OS\text{-}R^2$ (%)	$\Delta RMSE$ (‰)	$MSE\text{-}F$
AV_t (12M)	1.11	0.05	4.34
AC_t (12M)	-0.23	-0.01	-0.87
AV_t (12M) + AC_t (12M)	1.08	0.04	4.20
AV_t (12M) * AC_t (12M)	1.45	0.06	5.69
VAR_t (12M)	0.30	0.01	1.15

(c) R = 171 / P = 386 / π = 2.26

Table A27: OS Return Forecasting (1M) of the Naive Portfolio.
This table reports the results of the OS test for forecasting (rolling) mean returns of the equally-weighted benchmark over the next month. Specifically, each panel shows the results for a different number of observations (R) used to make the first prediction; in this context, P denotes the related number of OS forecasts and π equals P/R. The first column specifies the respective model by stating the independent variables considered in excess of the intercept. The remaining columns present OS statistics that compare the respective model to the historical mean model, using only the constant term as explanatory variable. Positive $OS\text{-}R^2$ and positive difference in root mean squared forecast error (RMSE) indicate superior forecasting ability compared to the benchmark model. $MSE\text{-}F$ defines the F-statistic suggested by McCracken (2007). Consult Table A4 and Section 3 for an overview of factors constituting the equally-weighted multi-factor portfolio. Detailed data set information is available in Appendix A.1 and Vincenz and Zeissler (2024). Refer to Section 4.1 for information on all variables that approximate (components of) multi-factor variance (i.e. AV_t, AC_t, VAR_t, and $VAR.G_t$). An explaination of the OS methodology, including the measures $OS\text{-}R^2$, $\Delta RMSE$, and $MSE\text{-}F$, is provided in Section A.4.1. An overview of the abbreviations used in the course of this paper is provided in Table A3.

Model	$OS\text{-}R^2$ (%)	$\Delta RMSE$ (‰)	$MSE\text{-}F$
AV_t (12M)	10.36	0.14	51.30
AC_t (12M)	-10.18	-0.13	-41.02
AV_t (12M) + AC_t (12M)	0.90	0.01	4.04
AV_t (12M) * AC_t (12M)	-5.01	-0.07	-21.20
VAR_t (12M)	-2.12	-0.03	-9.24

(a) R = 80 / P = 444 / π = 5.55

Model	$OS\text{-}R^2$ (%)	$\Delta RMSE$ (‰)	$MSE\text{-}F$
AV_t (12M)	13.40	0.18	62.50
AC_t (12M)	-9.06	-0.12	-33.58
AV_t (12M) + AC_t (12M)	5.06	0.07	21.54
AV_t (12M) * AC_t (12M)	-1.98	-0.03	-7.83
VAR_t (12M)	0.38	0.01	1.53

(b) R = 120 / P = 404 / π = 3.37

Model	$OS\text{-}R^2$ (%)	$\Delta RMSE$ (‰)	$MSE\text{-}F$
AV_t (12M)	13.48	0.19	56.73
AC_t (12M)	-7.12	-0.10	-24.18
AV_t (12M) + AC_t (12M)	7.14	0.10	27.99
AV_t (12M) * AC_t (12M)	-1.51	-0.02	-5.42
VAR_t (12M)	0.44	0.01	1.61

(c) R = 160 / P = 364 / π = 2.28

Table A28: OS Mean Return Forecasting (12M) of the Naive Portfolio.
This table reports the results of the OS test for forecasting (rolling) mean returns of the equally-weighted benchmark over the next 12-month period. Specifically, each panel shows the results for a different number of observations (R) used to make the first prediction; in this context, P denotes the related number of OS forecasts and π equals P/R. The first column specifies the respective model by stating the independent variables considered in excess of the intercept. The remaining columns present OS statistics that compare the respective model to the historical mean model, using only the constant term as explanatory variable. Positive $OS\text{-}R^2$ and positive difference in root mean squared forecast error (RMSE) indicate superior forecasting ability compared to the benchmark model. $MSE\text{-}F$ defines the F-statistic suggested by McCracken (2007). Consult Table A4 and Section 3 for an overview of factors constituting the equally-weighted multi-factor portfolio. Detailed data set information is available in Appendix A.1 and Vincenz and Zeissler (2024). Refer to Section 4.1 for information on all variables that approximate (components of) multi-factor variance (i.e. AV_t, AC_t, VAR_t, and $VAR.G_t$). An explaination of the OS methodology, including the measures $OS\text{-}R^2$, $\Delta RMSE$, and $MSE\text{-}F$, is provided in Section A.4.1. An overview of the abbreviations used in the course of this paper is provided in Table A3.

H	Signal	Scaling	Obs	Mean	Std	SR	CE	IR	Skewness	Kurtosis	w_{P50}	w_{P75}	w_{P90}	w_{P99}
-	-	-	558	2.90	3.05	0.95	2.71	0.00	-0.15	0.31	1.00	1.00	1.00	1.00
1	VAR_t (12M)	MM	558	2.33	3.05	0.76	2.15	-25.99	-0.05	0.78	0.71	1.25	1.97	3.63
1	VAR_t (12M)	SQRT	558	2.45	3.05	0.80	2.27	-34.05	0.07	0.86	0.71	0.85	1.28	1.87
1	VAR_t (12M)	LIN	558	1.85	3.05	0.61	1.67	-46.51	0.50	1.74	0.37	0.53	1.20	2.56
1	AC_t (12M)	MM	558	2.46	3.05	0.81	2.27	-28.11	0.07	0.58	0.82	1.09	1.47	3.29
1	AC_t (12M)	SQRT	558	2.88	3.05	0.94	2.69	-3.01	-0.20	0.43	0.92	1.14	1.30	1.51
1	AC_t (12M)	LIN	558	2.73	3.05	0.90	2.55	-12.26	-0.20	0.64	0.77	1.17	1.54	2.07
1	AV_t (12M)	MM	558	2.52	3.05	0.82	2.33	-22.45	-0.14	0.33	0.97	1.30	1.89	2.82
1	AV_t (12M)	SQRT	558	2.64	3.05	0.87	2.46	-22.52	0.07	0.85	0.74	0.93	1.24	1.79
1	AV_t (12M)	LIN	558	2.09	3.05	0.69	1.91	-37.80	0.38	2.74	0.41	0.66	1.17	2.45
6	VAR_t (12M)	MM	558	2.55	3.05	0.84	2.36	-17.47	0.11	0.41	0.73	1.24	1.90	3.27
6	VAR_t (12M)	SQRT	558	2.54	3.05	0.83	2.35	-28.83	-0.01	0.60	0.76	0.90	1.34	1.96
6	VAR_t (12M)	LIN	558	1.97	3.05	0.64	1.78	-41.39	0.35	1.46	0.42	0.58	1.30	2.77
6	AC_t (12M)	MM	558	2.60	3.05	0.85	2.42	-20.65	0.15	0.69	0.85	1.06	1.40	2.81
6	AC_t (12M)	SQRT	558	2.85	3.05	0.94	2.67	-6.90	-0.19	0.38	0.94	1.14	1.31	1.51
6	AC_t (12M)	LIN	558	2.72	3.05	0.89	2.54	-14.32	-0.18	0.50	0.82	1.19	1.58	2.09
6	AV_t (12M)	MM	558	2.56	3.05	0.84	2.37	-20.83	-0.07	0.25	0.98	1.28	1.91	2.75
6	AV_t (12M)	SQRT	558	2.69	3.05	0.88	2.51	-18.92	0.01	0.75	0.75	0.94	1.25	1.78
6	AV_t (12M)	LIN	558	2.18	3.05	0.72	2.00	-34.46	0.18	2.61	0.43	0.69	1.22	2.46
12	VAR_t (12M)	MM	558	2.62	3.05	0.86	2.44	-15.28	0.07	0.32	0.79	1.19	1.91	2.82
12	VAR_t (12M)	SQRT	558	2.64	3.05	0.86	2.45	-23.00	-0.03	0.45	0.79	0.94	1.33	1.98
12	VAR_t (12M)	LIN	558	2.13	3.05	0.70	1.94	-36.39	0.31	1.20	0.46	0.70	1.42	2.90
12	AC_t (12M)	MM	558	2.74	3.05	0.90	2.56	-12.37	0.09	0.56	0.86	1.04	1.51	2.36
12	AC_t (12M)	SQRT	558	2.84	3.05	0.93	2.65	-11.14	-0.19	0.32	0.96	1.13	1.29	1.49
12	AC_t (12M)	LIN	558	2.68	3.05	0.88	2.49	-19.24	-0.23	0.46	0.88	1.19	1.56	2.04
12	AV_t (12M)	MM	558	2.42	3.05	0.79	2.23	-30.51	-0.16	0.36	0.98	1.26	1.87	2.57
12	AV_t (12M)	SQRT	558	2.79	3.05	0.91	2.60	-11.19	-0.01	0.60	0.77	0.96	1.27	1.74
12	AV_t (12M)	LIN	558	2.38	3.05	0.78	2.19	-27.08	0.11	1.82	0.47	0.74	1.30	2.45
24	VAR_t (12M)	MM	558	2.45	3.05	0.80	2.26	-31.14	-0.16	0.45	0.88	1.24	1.95	2.51
24	VAR_t (12M)	SQRT	558	2.84	3.05	0.93	2.65	-6.68	-0.03	0.33	0.82	1.09	1.36	1.80
24	VAR_t (12M)	LIN	558	2.51	3.05	0.82	2.33	-22.18	0.17	0.59	0.54	1.04	1.57	2.64
24	AC_t (12M)	MM	558	2.80	3.05	0.92	2.61	-12.11	-0.01	0.35	0.92	1.10	1.44	1.77
24	AC_t (12M)	SQRT	558	2.91	3.05	0.95	2.72	2.86	-0.18	0.30	1.00	1.08	1.25	1.46
24	AC_t (12M)	LIN	558	2.88	3.05	0.94	2.69	-2.43	-0.20	0.31	0.97	1.14	1.53	2.02
24	AV_t (12M)	MM	558	2.28	3.05	0.75	2.09	-43.17	-0.35	0.70	0.97	1.24	1.75	2.24
24	AV_t (12M)	SQRT	558	2.94	3.05	0.96	2.76	4.99	0.02	0.41	0.82	0.99	1.27	1.62
24	AV_t (12M)	LIN	558	2.74	3.05	0.90	2.56	-9.41	0.20	0.83	0.56	0.83	1.41	2.25
60	VAR_t (12M)	MM	558	2.53	3.05	0.83	2.34	-31.34	-0.19	0.46	0.88	1.04	1.73	1.98
60	VAR_t (12M)	SQRT	558	2.93	3.05	0.96	2.74	3.73	-0.00	0.33	0.93	1.08	1.27	1.68
60	VAR_t (12M)	LIN	558	2.75	3.05	0.90	2.56	-9.55	0.24	0.67	0.74	1.03	1.45	2.42
60	AC_t (12M)	MM	558	2.93	3.05	0.96	2.74	4.44	-0.07	0.40	0.94	1.12	1.26	1.41
60	AC_t (12M)	SQRT	558	2.82	3.05	0.92	2.63	-22.73	-0.21	0.33	0.98	1.12	1.17	1.22
60	AC_t (12M)	LIN	558	2.69	3.05	0.88	2.51	-28.39	-0.27	0.40	0.94	1.23	1.34	1.47
60	AV_t (12M)	MM	558	2.28	3.05	0.75	2.09	-56.89	-0.38	0.60	0.92	1.28	1.52	1.72
60	AV_t (12M)	SQRT	558	3.08 *	3.05	1.01 *	2.89 *	27.90 *	0.01	0.30	0.91	1.01	1.31	1.43
60	AV_t (12M)	LIN	558	3.09	3.05	1.01	2.91	15.33	0.17	0.39	0.75	0.95	1.61	1.81
120	VAR_t (12M)	MM	558	2.83	3.05	0.93	2.65	-6.16	-0.09	0.38	0.95	1.29	1.46	1.57
120	VAR_t (12M)	SQRT	558	2.78	3.05	0.91	2.60	-16.81	-0.09	0.39	0.91	1.02	1.18	1.62
120	VAR_t (12M)	LIN	558	2.53	3.05	0.83	2.35	-25.81	0.09	0.67	0.76	0.92	1.29	2.29
120	AC_t (12M)	MM	558	2.98 ·	3.05	0.98	2.80 ·	19.51 ·	-0.07	0.34	1.02	1.07	1.13	1.28
120	AC_t (12M)	SQRT	558	2.84	3.05	0.93	2.65	-24.84	-0.20	0.31	1.01	1.06	1.14	1.21
120	AC_t (12M)	LIN	558	2.76	3.05	0.90	2.57	-27.76	-0.24	0.32	1.01	1.12	1.29	1.45
120	AV_t (12M)	MM	558	2.70	3.05	0.89	2.51	-24.45	-0.21	0.29	1.02	1.22	1.43	1.58
120	AV_t (12M)	SQRT	558	2.89	3.05	0.95	2.70	-3.04	-0.10	0.36	0.91	1.03	1.14	1.39
120	AV_t (12M)	LIN	558	2.79	3.05	0.91	2.60	-12.17	-0.04	0.45	0.76	1.07	1.24	1.76

Table A29: Summary Statistics of Timing Strategies. (Caption on the next page.)

Table A29: Summary Statistics of Timing Strategies.
This table presents summary statistics for the set of 54 timing strategies (constructed for different holding periods (H), signals, and methods to scale the signals) as well as for the static multi-factor benchmark (in the first row). In detail, I report the number of monthly return observations, followed by annualized arithmetic mean returns (as percentages) with associated significance levels, annualized standard deviations (as percentages), annualized Sharpe Ratios with associated significance levels, annualized certainty equivalents (applying a CRRA utility with $\gamma = 4$; as percentages) with associated significance levels, annualized information ratios (vs. static benchmark, as percentages) with associated significance levels, annualized skewness, annualized excess kurtosis, and various quantiles of the distribution of weights w in the other respective columns. Monthly mean returns and certainty equivalents are annualized by multiplying with 12, while standard deviations, Sharpe Ratios and information ratios are multiplied with $\sqrt{12}$. Furthermore, I annualize monthly excess kurtosis by multiplying with $\frac{1}{12}$ and skewness by multiplying with $\frac{1}{\sqrt{12}}$. For the performance measures, i.e. mean, Sharpe Ratio, certainty equivalent and information ratio, I additionally report significance levels of tests with the null hypothesis of no outperformance compared to the relevant estimate of the static benchmark over the same period, whereby p-values are obtained via bootstrapping (see Appendix A.6 for more information). In this sense, the levels ***,**,*,$^{\cdot}$ indicate whether a given estimate is significantly greater as the relevant estimate of the static benchmark at the 0,01%, 1%, 5% or 10% level, respectively. Consult Table A4 and Section 3 for an overview of factors constituting the equally-weighted multi-factor portfolio. Detailed data set information is available in Appendix A.1 and Vincenz and Zeissler (2024). Refer to Section 4.1 for information on all variables that approximate (components of) multi-factor variance (i.e. AV_t, AC_t, VAR_t, and $VAR.G_t$). A description of the timing methodology, including the different scaling methods MM, LIN, and $SQRT$, can be found in Section A.5.1. An overview of the abbreviations used in the course of this paper is provided in Table A3.

H	Signal	Scaling	Obs	Mean	Std	SR	CE	IR	Skewness	Kurtosis	w_{P50}	w_{P75}	w_{P90}	w_{P99}
-	-	-	558	5.33	5.64	0.94	4.69	0.00	-0.08	0.24	1.00	1.00	1.00	1.00
1	VAR_t (12M)	MM	558	4.47	5.64	0.79	3.81	-22.63	-0.26	1.20	0.77	1.20	1.77	2.90
1	VAR_t (12M)	SQRT	558	4.30	5.64	0.76	3.68	-42.82	0.30	0.90	0.71	0.94	1.27	1.77
1	VAR_t (12M)	LIN	558	2.99	5.64	0.53	2.40	-56.47	0.80	2.05	0.37	0.65	1.19	2.31
1	AC_t (12M)	MM	558	4.98	5.64	0.88	4.35	-12.95	0.03	0.36	0.79	1.09	1.54	2.73
1	AC_t (12M)	SQRT	558	5.16	5.64	0.91	4.52	-13.78	-0.06	0.29	0.96	1.16	1.30	1.46
1	AC_t (12M)	LIN	558	4.83	5.64	0.86	4.19	-21.47	-0.02	0.42	0.84	1.23	1.55	1.95
1	AV_t (12M)	MM	558	4.90	5.64	0.87	4.26	-13.44	-0.09	0.24	0.94	1.28	1.91	3.22
1	AV_t (12M)	SQRT	558	4.74	5.64	0.84	4.11	-31.95	0.23	0.65	0.77	0.98	1.13	1.71
1	AV_t (12M)	LIN	558	3.72	5.64	0.66	3.12	-46.44	0.62	1.55	0.49	0.78	1.05	2.39
6	VAR_t (12M)	MM	558	4.93	5.64	0.87	4.29	-11.40	-0.06	0.50	0.83	1.19	1.79	3.01
6	VAR_t (12M)	SQRT	558	4.44	5.64	0.79	3.82	-38.92	0.24	0.73	0.74	0.94	1.28	1.79
6	VAR_t (12M)	LIN	558	3.15	5.64	0.56	2.55	-52.95	0.71	2.05	0.40	0.68	1.21	2.36
6	AC_t (12M)	MM	558	5.26	5.64	0.93	4.62	-2.91	-0.02	0.33	0.84	1.10	1.52	2.25
6	AC_t (12M)	SQRT	558	5.10	5.64	0.90	4.46	-21.17	-0.05	0.27	0.97	1.14	1.30	1.43
6	AC_t (12M)	LIN	558	4.77	5.64	0.84	4.13	-27.14	0.01	0.33	0.88	1.21	1.57	1.89
6	AV_t (12M)	MM	558	4.81	5.64	0.85	4.16	-16.87	-0.13	0.27	0.94	1.26	1.90	3.05
6	AV_t (12M)	SQRT	558	4.85	5.64	0.86	4.22	-27.54	0.15	0.50	0.80	0.99	1.14	1.73
6	AV_t (12M)	LIN	558	3.94	5.64	0.70	3.33	-41.99	0.46	1.24	0.53	0.82	1.09	2.50
12	VAR_t (12M)	MM	558	4.66	5.64	0.83	4.01	-21.50	-0.13	0.39	0.85	1.20	1.80	2.53
12	VAR_t (12M)	SQRT	558	4.78	5.64	0.85	4.15	-25.64	0.23	0.66	0.76	0.97	1.22	1.84
12	VAR_t (12M)	LIN	558	3.65	5.64	0.65	3.05	-42.23	0.67	2.09	0.45	0.73	1.16	2.49
12	AC_t (12M)	MM	558	5.39	5.64	0.95	4.75	2.89	-0.10	0.39	0.87	1.11	1.53	1.95
12	AC_t (12M)	SQRT	558	5.08	5.64	0.90	4.44	-26.70	-0.07	0.26	0.99	1.12	1.28	1.38
12	AC_t (12M)	LIN	558	4.74	5.64	0.84	4.11	-32.63	-0.05	0.30	0.93	1.20	1.55	1.78
12	AV_t (12M)	MM	558	4.43	5.64	0.78	3.78	-29.96	-0.19	0.37	0.94	1.24	1.85	2.85
12	AV_t (12M)	SQRT	558	5.07	5.64	0.90	4.44	-15.59	0.14	0.46	0.81	1.00	1.14	1.70
12	AV_t (12M)	LIN	558	4.35	5.64	0.77	3.74	-30.54	0.43	1.14	0.56	0.85	1.08	2.44
24	VAR_t (12M)	MM	558	4.29	5.64	0.76	3.63	-37.46	-0.26	0.56	0.88	1.21	1.71	2.28
24	VAR_t (12M)	SQRT	558	5.18	5.64	0.92	4.56	-8.46	0.14	0.42	0.85	1.01	1.15	1.83
24	VAR_t (12M)	LIN	558	4.50	5.64	0.80	3.88	-25.30	0.42	1.06	0.59	0.86	1.12	2.63
24	AC_t (12M)	MM	558	5.29	5.64	0.94	4.65	-2.38	-0.05	0.27	0.91	1.14	1.37	1.79
24	AC_t (12M)	SQRT	558	5.25	5.64	0.93	4.61	-10.84	-0.09	0.25	1.01	1.09	1.16	1.36
24	AC_t (12M)	LIN	558	5.14	5.64	0.91	4.50	-13.98	-0.09	0.25	0.98	1.17	1.32	1.75
24	AV_t (12M)	MM	558	4.29	5.64	0.76	3.64	-36.59	-0.26	0.49	0.93	1.31	1.77	2.44
24	AV_t (12M)	SQRT	558	5.29	5.64	0.94	4.66	-2.93	0.11	0.36	0.83	1.02	1.13	1.68
24	AV_t (12M)	LIN	558	4.87	5.64	0.86	4.26	-16.10	0.33	0.66	0.60	0.92	1.11	2.42
60	VAR_t (12M)	MM	558	4.61	5.64	0.82	3.96	-31.80	-0.22	0.30	0.97	1.20	1.71	1.86
60	VAR_t (12M)	SQRT	558	5.33	5.64	0.95	4.71	0.40	0.12	0.36	0.88	0.96	1.27	1.59
60	VAR_t (12M)	LIN	558	4.92	5.64	0.87	4.30	-13.69	0.40	0.84	0.68	0.80	1.47	2.12
60	AC_t (12M)	MM	558	5.46	5.64	0.97	4.82	11.94	-0.07	0.23	1.01	1.12	1.21	1.48
60	AC_t (12M)	SQRT	558	5.18	5.64	0.92	4.54	-26.72	-0.11	0.27	0.98	1.06	1.15	1.20
60	AC_t (12M)	LIN	558	5.00	5.64	0.89	4.35	-30.78	-0.16	0.32	0.95	1.10	1.32	1.41
60	AV_t (12M)	MM	558	4.40	5.64	0.78	3.75	-39.26	-0.26	0.35	0.93	1.32	1.71	1.96
60	AV_t (12M)	SQRT	558	5.43	5.64	0.96	4.80	7.44	0.11	0.33	0.89	0.99	1.24	1.50
60	AV_t (12M)	LIN	558	5.21	5.64	0.92	4.59	-4.51	0.32	0.57	0.70	0.87	1.45	2.01
120	VAR_t (12M)	MM	558	5.00	5.64	0.89	4.35	-16.39	-0.12	0.27	1.00	1.29	1.45	1.53
120	VAR_t (12M)	SQRT	558	5.18	5.64	0.92	4.55	-10.59	0.06	0.36	0.86	0.93	1.22	1.55
120	VAR_t (12M)	LIN	558	4.72	5.64	0.84	4.10	-21.76	0.29	0.74	0.64	0.77	1.35	2.04
120	AC_t (12M)	MM	558	5.48	5.64	0.97	4.83	20.63	-0.11	0.23	1.03	1.08	1.13	1.16
120	AC_t (12M)	SQRT	558	5.22	5.64	0.93	4.58	-29.70	-0.08	0.24	0.98	1.03	1.14	1.18
120	AC_t (12M)	LIN	558	5.10	5.64	0.90	4.46	-32.06	-0.10	0.24	0.96	1.07	1.29	1.39
120	AV_t (12M)	MM	558	4.88	5.64	0.86	4.24	-22.70	-0.14	0.25	1.01	1.29	1.54	1.62
120	AV_t (12M)	SQRT	558	5.28	5.64	0.94	4.65	-4.08	0.04	0.33	0.87	0.98	1.20	1.47
120	AV_t (12M)	LIN	558	5.01	5.64	0.89	4.39	-13.51	0.21	0.53	0.69	0.86	1.36	1.93

Table A30: Summary Statistics of Timing Strategies without Ex-Ante Volatility Scaling of Factors. (Caption on the next page.)

Table A30: Summary Statistics of Timing Strategies without Ex-Ante Volatility Scaling of Factors.
This table presents summary statistics for the set of 54 timing strategies (constructed for different holding periods (H), signals, and methods to scale the signals) as well as for the static multi-factor benchmark (in the first row), similar to Table A29. However, while the factor TS used in Table A29 are ex-ante volatility scaled (see Section 3), the given table presents the same analysis based on the unscaled series. In detail, I report the number of monthly return observations, followed by annualized arithmetic mean returns (as percentages) with associated significance levels, annualized standard deviations (as percentages), annualized Sharpe Ratios with associated significance levels, annualized certainty equivalents (applying a CRRA utility with $\gamma = 4$; as percentages) with associated significance levels, annualized information ratios (vs. static benchmark, as percentages) with associated significance levels, annualized skewness, annualized excess kurtosis, and various quantiles of the distribution of weights w in the other respective columns. Monthly mean returns and certainty equivalents are annualized by multiplying with 12, while standard deviations, Sharpe Ratios and information ratios are multiplied with $\sqrt{12}$. Furthermore, I annualize monthly excess kurtosis by multiplying with $\frac{1}{12}$ and skewness by multiplying with $\frac{1}{\sqrt{12}}$. For the performance measures, i.e. mean, Sharpe Ratio, certainty equivalent and information ratio, I additionally report significance levels of tests with the null hypothesis of no outperformance compared to the relevant estimate of the static benchmark over the same period, whereby p-values are obtained via bootstrapping (see Appendix A.6 for more information). In this sense, the levels ***,**,*,$^{\cdot}$ indicate whether a given estimate is significantly greater as the relevant estimate of the static benchmark at the 0,01%, 1%, 5% or 10% level, respectively. Consult Table A4 and Section 3 for an overview of factors constituting the equally-weighted multi-factor portfolio. Detailed data set information is available in Appendix A.1 and Vincenz and Zeissler (2024). Refer to Section 4.1 for information on all variables that approximate (components of) multi-factor variance (i.e. AV_t, AC_t, VAR_t, and $VAR.G_t$). A description of the timing methodology, including the different scaling methods MM, LIN, and $SQRT$, can be found in Section A.5.1. An overview of the abbreviations used in the course of this paper is provided in Table A3.