The Taxonomy of Tail Risk

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Abstract

We use tail events at different levels of severity to define an asset's tail risk and to decompose the latter into a systematic and an idiosyncratic component. The systematic component captures an asset's tendency to experience joint tail losses with the market and generalizes a classic tail dependence coefficient. The idiosyncratic component, on the other hand, consists of two parts: idiosyncratic tail risk that leads to asset-specific tail losses and tail risk cushioning that dampens the tail losses emanating from the market. Tail risk cushioning is a novel concept that arises naturally in our framework, is consistent with the previous two and completes the taxonomy of tail risk. We examine the performance of our tail risk decomposition on a large dataset, confirming some previous results on tail risk and uncovering new theoretical and empirical findings.

Keywords: Tail Dependence, Tail Risk Decomposition, Tail Risk Classification, Systematic Tail Risk, Idiosyncratic Tail Risk, Tail Risk Cushioning

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1. Introduction

Classic finance theory argues that diversification can eliminate idiosyncratic but not systematic risk and therefore, only exposure to the latter earns a risk premium. Exposure to the former is not rewarded with a premium since it can and should be diversified away (see, for example, Chen and Sears, 1984; Statman, 1987). Indeed, diversification is often referred to as the only free lunch in finance. However, many examples e.g., the financial crisis 2007-2009, the Eurozone crisis 2010-2011 or the crash due to the recent pandemic, illustrate that in practice diversification can abjectly fail to protect investors from tail events. According to an old saying in investment circles, the only things that go up in a crisis are correlations. Assets which before a crisis had low or even negative correlations leading to a well-diversified portfolio, become highly interdependent during the crisis, therefore compounding losses instead of offsetting them. These considerations have led to a different approach to investing that can be summarized in two words: *concentrated portfolio*.[1]

By construction, concentrated portfolios carry significant idiosyncratic risk. Assuming under-diversification, various theories predict positive relationship between the idiosyncratic risk and the expected stock returns in the cross section (see, for example, Levy, 1978; Merton, 1987; Malkiel and Xu, 2003). Under-diversified investors will demand a return compensation for bearing idiosyncratic risk. However, in stark contrast, Ang et al. (2006) find that in the cross-section high idiosyncratic volatility in one month predicts abysmally low average returns in the next month, which they

[1]For evidence on concentrated portfolios see Polkovnichenko (2005) who shows that the median number of stocks in household portfolios is two at several points between 1989 and 1998 and increases to three in 2001. Similarly, Goetzmann and Kumar (2008) find that during 1991–1996, the median number of stocks in a portfolio of individual investors is three. These findings however, do not necessarily imply irrationality - under-diversification can be entirely consistent with rational investors (see, for example, Roche, Tompaidis, and Yang, 2013 and Van Nieuwerburgh and Veldkamp, 2010. See also the interview by the manager of Henderson European Focus Fund explaining that this approach to portfolio construction is in direct response to the demand by clients which he defends as making economic sense in the following link https://www.newstatesman.com/politics/2019/05/a-truer-active-more-idiosyncratic-portfolio-2.

call ‘a substantive puzzle’.[2]

Volatility measured as standard deviation or variance of portfolio returns may not be an adequate risk measure, especially for undiversified portfolios. For example, ample empirical evidence shows that individuals value losses and gains differently, usually assigning greater weight to losses (Tversky and Kahneman, 1979; Barberis, 2013). Downside risk is particularly important when asset returns are asymmetrically distributed and investors are averse to disasters. Menezes, Geiss, and Tressler (1980) argue that investors tend to avoid positions that may lead to large losses even though they may have low probability. Rietz (1988) and Barro (2006) show that tail risks are important in explaining some of the asset pricing puzzles.

The literature lacks a consensus on unique definition for the concept of ”systematic tail risk” associated with an asset. For instance, some studies base their analysis on statistical moments (e.g., Levy and Arditti, 1975; Ang et al., 2006; Conrad, Dittmar, and Ghysels, 2013), while others employ co-moments (e.g., Harvey and Siddique, 2000; Dittmar, 2002 and François et al., 2022). Nevertheless, research relying on moment- and co-moment-based risk metrics offers only indirect insights into how tail risk influences asset pricing. Direct evidence regarding the role of tail risk remains inconclusive.

A number of studies that investigate the impact of Value-at-Risk (VaR) on expected returns find a positive correlation (e.g., Bali, Demirtas, and Levy, 2008; Bali and Cakici, 2004). However, these studies do not differentiate between the systematic and idiosyncratic components of VaR. More recently, Atilgan et al. (2020) find a robust negative effect of tail risk, proxied by VaR, on expected returns. They attribute this effect to behavioral biases which suggests the significance of idiosyncratic tail risk in asset pricing. However, other studies find insignificant or negative results

[2]For a recent discussion of the idiosyncratic volatility puzzle and the related empirical studies see Stambaugh, Yu, and Yuan (2015); Hou and Loh (2016) and Chichernea and Slezak (2013).

when examining both idiosyncratic and systematic tail risk, while they do find some supporting evidence for a hybrid tail risk measure (e.g., Bali, Cakici, and Whitelaw, 2014).

Chabi-Yo, Ruenzi, and Weigert (2018) utilize the classic tail dependence coefficient of Sibuya (1960) to proxy systematic tail risk and discover that the corresponding risk premium for this measure is considerable. On the other hand, van Oordt and Zhou (2016) rely on Arzac and Bawa (1977) asset pricing model and introduce the concept of "tail beta" which is calculated as the product of a tail dependence coefficient and the relative tail risk. They find that systematic tail risk, proxied by tail beta, is linked to future stock returns but does not earn a significantly positive risk premium. Stoja et al. (2023) suggest that the presence of "common features" in systematic tail risk measures may be the underlying reason for the contradictory results observed in these two studies. Interestingly, van Oordt and Zhou (2016) find that stocks with high (low) tail betas have high (low) tail dependence with the market, as intuition would suggest, but also high (low) idiosyncratic risk (see their Table 1). This implies positive correlation between tail dependence and idiosyncratic risk. However, Chabi-Yo, Ruenzi, and Weigert (2018) find that idiosyncratic risk correlates negatively with tail dependence (see their Table 2). Because in these models, idiosyncratic and systematic (tail) risk are not necessarily mutually consistent, it is not clear then what drives these sharply conflicting results or how to reconcile them.

Chabi-Yo, Ruenzi, and Weigert (2018) apply the classic tail dependence coefficient of Sibuya (1960) as a measure of systematic tail risk and find that the risk premium corresponding to this measure is substantial. Relying on the asset pricing model of Arzac and Bawa (1977), van Oordt and Zhou (2016) propose the tail beta - computed as the product of a tail dependence coefficient and the relative tail risk - as a new systematic tail risk measure. They find that this measure is associated with future stock returns but not with a significantly positive tail risk premium. Stoja et al. (2023) suggest that *common features* in systematic tail risk measures may be the reason behind the contradictory findings in these two studies. Interestingly, van Oordt and Zhou (2016) find that stocks with high (low) tail betas have high (low) tail dependence with the market, as intuition would suggest, but also high (low) idiosyncratic risk (see their Table 1). This implies positive correlation between tail dependence and idiosyncratic risk. However, Chabi-Yo, Ruenzi, and Weigert (2018) find that idiosyncratic risk correlates negatively with tail dependence (see their Table 2). Because in these models, idiosyncratic and systematic (tail) risk are not necessarily mutually consistent, it is not clear then what drives these sharply conflicting results or how to reconcile them.

Unlike systematic tail risk, *idiosyncratic tail risk* has attracted much less attention. Huang et al. (2012) model idiosyncratic tail risk with a two-step procedure. In the first step, stock returns are regressed on systematic risk factors. Then, in the second step idiosyncratic tail risk is estimated as the tail index of the regression residuals. This approach, standard in the literature in the context of idiosyncratic risk, warrants careful consideration in the context of idiosyncratic *tail* risk. We discuss this issue in detail in Section 2.

In this paper, we make the following contributions to the literature on tail risk and asset prices. We propose novel measures of *systematic* and *idiosyncratic* tail risks. These measures arise organically from a decomposition of the tail risk of asset returns and are mutually consistent. In addition to these types of tail risk, we propose a novel concept, *tail risk cushioning* – the tendency of an asset to dampen tail risk that emanates from the systematic factor – and propose a measure that encapsulates it. Tail risk cushioning arises naturally in our framework, is fully consistent with the previous two types and completes the *taxonomy of tail risk*. We relate the systematic, idiosyncratic and cushioning component to other measures of tail risk and examine their impact on asset returns. Specifically, we measure how much of the (expected) return of an asset can be attributed to each of these components. Following the literature (see, for example, Bali, Cakici, and Whitelaw, 2014), we apply the Fama and MacBeth (1973) methodology and use a large cross section of stock returns and the Fama-French systematic factors to estimate the significance and magnitude of the premia earned by exposure to the systematic, idiosyncratic and cushioning components of tail risk.

We define tail risk as the probability of a (joint) exceedance of certain thresholds. The thresholds are defined by VaR which shows how much an investor is likely to lose with a given probability over a given horizon. VaR has been extensively embraced by regulators and practitioners in financial markets under the Basel II and III frameworks as the basis of risk measurement for the purpose of ensuring regulatory capital adequacy, risk management and strategic planning. In extensive empirical exercises, we find a significant positive risk premium associated with the systematic component of tail risk as well as a significant negative premium for tail risk cushioning. However, we find that exposure of a portfolio to idiosyncratic tail risk earns a negative risk premium, contradicting the theory but extending the findings of Ang et al. (2006), among others, on the negative relation between expected stock returns and idiosyncratic volatility to idiosyncratic *tail* risk. Our findings are qualitatively similar, although statistically more significant, if instead of tail risk, one examines the impact of (the components of) downside risk - defined as the tendency of an asset to generate losses - on expected returns.

The paper is structured as follows. In Section 2, we lay out the theoretical framework and discuss an array of properties of these new measures of tail risk. In Section 3, we present our empirical results, while Section 4 summarizes the paper. The Appendix contains the proofs of our theoretical results.

2. Theoretical Framework

2.1. Systematic Tail Risk Defines its Idiosyncratic Counterpart

Two prominent papers that study systematic tail risk are Chabi-Yo, Ruenzi, and Weigert (2018) and van Oordt and Zhou (2016). Both their systematic tail risk measures rely, to different degrees, on the classic coefficient of tail dependence proposed by Sibuya (1960) (see their equations (1) and (7)). Essentially, they measure the systematic tail risk of an asset by joint occurrences of extreme events - occurrences when both the market and the asset exceed some thresholds, in both cases their VaRs. Following this approach, *idiosyncratic* tail risk can then be defined by the outcomes in which the asset is in distress (i.e., exceeds its VaR) while the market is not.

An approach that relies on a threshold such as VaR to estimate idiosyncratic tail risk contrasts sharply with other measures. For example, Huang et al. (2012) use a two-step procedure to estimate idiosyncratic tail risk. In the first step, stock returns are regressed on systematic risk factors. Then, in the second step idiosyncratic tail risk is estimated as the tail index of the residuals of the regression. In the context of the Sibuya-based tail risk approach, the two-step procedure can lead to misclassification of tail events. Some events may be "double counted" as both systematic and idiosyncratic and other events may be included in the calculation of idiosyncratic tail risk when in fact they should not. A careful consideration of these issues and a systematic categorization of tail events is important given their paramount importance for stock returns and since most tail events are idiosyncratic (see, for example, Bali, Cakici, and Whitelaw, 2014.

To illustrate, suppose that at some severity level α, the market (systematic risk factor) has a VaR of -5 percent but on a particular day generates a return of 10 percent. Suppose further that an asset has the same VaR of -5 percent, a (tail) beta of one but on the same day generates a return of 2 percent. Thus, the residual term is -8 percent which is large but does not result in a tail event for the asset because its VaR has not been breached. However, the two–stage procedure would classify this as an idiosyncratic tail event. Suppose on another day, the market generates a return of -6 percent and the asset generates a return of -15 percent. While the residual term of -9 percent is large, it would appear incorrect to classify this as an idiosyncratic tail event. One could argue that since the market has already breached its VaR, this event should count as a systematic tail event. Indeed, this observation is the essence of the classic tail dependence coefficient of Sibuya (1960) (see also Joe, 1997) which forms the basis of many systematic tail risk measures, including those of van Oordt and Zhou (2016) and Chabi-Yo, Ruenzi, and Weigert (2018) as well as *systemic risk* measures such as CoVaR of Adrian and Brunnermeier (2016).

In our framework, for any given level of an asset's total tail risk, systematic tail risk accounts for some part of tail risk, while the idiosyncratic component (composed of idiosyncratic tail risk and tail risk cushioning) accounts for the remaining part. Thus, for a given level of tail risk, a stock with high systematic tail risk will tend to have a low idiosyncratic component and vice versa. Therefore, this approach is the direct analogue of the total volatility decomposition of a stock into systematic and idiosyncratic volatility in the Single-Index Model (SIM - see, for example, Sharpe, 1963). This is an important feature of the model and an advantage relative to other frameworks in which it is not clear how systematic and idiosyncratic tail risk relate to each other. Of course, this approach is not without its own issues. Going back to the example, suppose the market on a particular day generates a return of -4 percent while the asset has a return of -6 percent. Since the asset's VaR has been breached, this would contribute to idiosyncratic tail risk even though the residual term is -2 percent, many times smaller in absolute value than the -8 percent term which previously did not contribute to idiosyncratic tail risk. However, this issue ensues from the decision of an investor as to what level of return constitutes a severe tail event, i.e. the rather arbitrary but unavoidable decision as to where exactly is the threshold - the point demarcating moderate losses from tail event losses - that the VaR condition imposes. Any model, including that of van Oordt and Zhou (2016) or Chabi-Yo, Ruenzi, and Weigert (2018), that relies on cut-off points for the definition of tails would be subject to this issue.[3]

2.2. A Tree-Model of Asset Returns

This section presents a simple model of asset returns and shows how it leads directly to our decomposition of tail risk. Assume that a SIM holds and the excess return r_i of stock i is approximately equal to (tail) beta β_i times the market's excess return r_m, where the latter exceeds its threshold with the time-independent probability f.

We assume $\beta_i \geq 0$ for the sake of consistency with the literature and consider two regimes. In the first regime, which occurs with probability p_i, $\beta_i > 0$ and the error term is distributed with a "moderate" dispersion (denoted ϵ_i). Hence, stock i's excess return does not deviate significantly from the prediction of the SIM. More specifically, stock i does exceed its threshold whenever the market does (denoted r_i and r_m). By analogy, stock i does not exceed its threshold whenever the market does not (denoted R_i and R_m). In the second regime, which materializes with the complementary probability $1 - p_i$, $\beta_i = 0$ and the error is distributed with a "large" dispersion (E_i). In this regime, asset i exceeds its threshold independently of the market with probability q_i due to a large negative error term materializing (E_i^-) or, with probability $1 - q_i$, does not exceed it due to a moderate or large positive error term (E_i^+).

For simplicity, all probabilities are assumed to be time-independent. However,

[3] In a wider context, Supper, Irresberger, and Weiß (2020) explicitly caution "... that several key results from the literature (e.g., Chabi-Yo, Ruenzi, and Weigert, 2018 [...]) need to be treated with care" as the dependent structure could be misestimated. As a way to alleviate the impact that this choice may have on the results, we employ a wide range of cut-off points.

this assumption is not essential - the setting can be generalized easily to allow for time-dependent probabilities.

Figure 1 depicts the event tree which illustrates the different paths that lead to the mutually-exclusive and collectively-exhaustive outcomes i.e., joint tail events represented by the final nodes.

[Figure 1]

The outcomes in Figure 1 correspond then to the following four tails. In tail $T_{\emptyset}$, no threshold exceedance has occurred; in tail $T_{\{m\}}$, the market has exceeded its threshold but not the asset; in tail $T_{\{i\}}$ the asset has exceeded its threshold but not the market. Finally, in tail $T_{\{i,m\}}$, both have exceeded their respective thresholds. Figure 2 depicts these outcomes.

[Figure 2]

The four areas in Figure 2 correspond to the four possible outcomes i.e., joint tail events in Figure 1 that materialize due to three binary events: the realization of the market (systematic factor) return - whether it is above or below a given threshold, the occurrence of the first or the second regime and, in the latter case, the realization of the idiosyncratic shock - whether it is above or below a given threshold. Below we show that the parameter values of f, p_i and q_i in the event tree can be uniquely calculated from the observed data on the tail events.

2.3. The Taxonomy of Tail Risk

When the thresholds in the model above delineate extreme (or tail) events, we can interpret the areas in Figure 2 as follows: the region $T_{\emptyset}$ corresponds then to the day-to-day moderate losses as well as gains. There is an extensive literature that examines various asset pricing predictions in this region. In fact, the majority of asset pricing studies relate to this area. More recently and, in particular, since the financial crisis of 2007-2009, there has been rather intense interest in the asset pricing implications of the joint tail $T_{\{i,m\}}$ (e.g., Baruník and Nevrla, 2022, Bollerslev, Patton, and Quaedvlieg, 2022, Chabi-Yo, Ruenzi, and Weigert, 2018, van Oordt and Zhou, 2016). This tail proxies the *systematic tail risk* which, theoretically, should have important implications for asset pricing although the empirical findings are mixed as discussed above.

Similarly important but to date overlooked, are the remaining two tails. In tail $T_{\{i\}}$ the stock i return exceeds its respective threshold whenever the market return *does not*. Therefore, this tail captures *idiosyncratic tail risk* of stock i. The tendency of an asset to exceed its threshold when the market does not is an undesirable property and therefore, investors can only be induced to hold this asset if they are compensated with an adequate risk premium. The corresponding theoretical result is rigorously stated in Subsection 2.5. below and proved in the Appendix.

By analogy, tail $T_{\{m\}}$ corresponds to outcomes where the market exceeds its threshold but stock i *does not*. Therefore, this tail captures an important property of stock i: *tail risk cushioning* – i.e. the tendency of an asset to dampen the losses emanating from the market. Assets that have this property would be in high demand, especially during periods of market turbulence and would therefore, be compensated with lower expected returns. This claim is also rigorously stated in Subsection 2.5. and the proof is given in the Appendix.

Intuitively, stock i's exposure to systematic tail risk can be defined as its tendency (not) to exceed its threshold when the market does (not). Figure 2 illustrates that this situation occurs in the joint tail $T_{\{i,m\}}$ ($T_{\emptyset}$) where both returns are simultaneously below (above) their respective VaR thresholds. Similarly, idiosyncratic behaviour is displayed whenever stock i diverges strongly from the market, i.e. either stock i or the market exceeds its VaR but not *both* at the same time. This occurs in joint tail $T_{\{m\}}$ when the market exceeds its threshold but not the asset and in the joint tail $T_{\{i\}}$ when asset i does exceeds its threshold but not the market.

If the prediction of classical finance theory on the reward to risk exposure extends to tail risk, then only exposure to systematic tail risk should earn a risk premium. Idiosyncratic tail risks are supposed to be diversified away and investors would not be compensated with any premia for exposure to such risks. For example, Hwang, Xu, and In (2018) find that for portfolios with a small number of stocks, naïve diversification not only outperforms more sophisticated diversification techniques but is also *less* exposed to tail risk. However, for large portfolios, naïve diversification maintains its superior performance but *increases* tail risk. Without a clear demarcation and classification of tail risk into systematic and idiosyncratic it is challenging to understand and interpret these results.

It is important to emphasize that in the foregoing discussion, the meaning of tails can be "expanded" to all outcomes below the median return which effectively modifies tail risk to downside risk (see also Bali, Cakici, and Whitelaw, 2014). The three components of risk are still valid and mutually-consistent although they would now represent systematic downside risk, idiosyncratic downside risk and downside risk cushioning.

2.4. The Definition of Tail Risk Measures

In this subsection, we formally derive our measures of systematic tail risk, idiosyncratic tail risk and tail risk cushioning. Define x_0, x_m, x_i and x_{im} as the respective probabilities of the outcomes $T_\emptyset$, $T_{\{m\}}$, $T_{\{i\}}$ and $T_{\{i,m\}}$. In this case, the event tree in

Figure 1 leads to a system of linear equations as follows:

$$\begin{cases} Pr(T_{\emptyset}) = x_0 = (1-f)\cdot p_i + (1-f)\cdot(1-p_i)\cdot(1-q_i) \\ Pr(T_{\{i\}}) = x_i = (1-f)\cdot(1-p_i)\cdot q_i \\ Pr(T_{\{m\}}) = x_m = f\cdot(1-p_i)\cdot(1-q_i) \\ Pr(T_{\{i,m\}}) = x_{im} = f\cdot p_i + f\cdot(1-p_i)\cdot q_i \end{cases}$$

The last probability $Pr(T_{\{i,m\}})$, for example, is the sum of the probability $f \cdot p_i$ encapsulating the market exceeding its threshold followed by the asset and the probability $f \cdot (1-p_i) \cdot q_i$ encapsulating the market and the asset exceeding their respective thresholds independently.

In the following discussion, we define the threshold for the asset i equal to $VaR_i^{\alpha_i}$ and for the market equal to $VaR_m^{\alpha_m}$ at the corresponding severity levels α_i and α_m. Then, the probabilities of the tails $Pr(T_{\{i\}})$ and $Pr(T_{\{m\}})$ are equal to $x_i = \alpha_i - x_{im}$ and $x_m = \alpha_m - x_{im}$. As the sum of the probabilities of the four collectively-exhaustive and mutually-exclusive outcomes must be one, the following unique solutions for f, q_i and p_i obtain:

$$f = \alpha_m, \tag{1}$$

$$p_i = \frac{x_{im} - \alpha_i\alpha_m}{\alpha_m - \alpha_m^2}, \tag{2}$$

$$q_i = \frac{\alpha_m(\alpha_i - x_{im})}{\alpha_m\,(1+\alpha_i-\alpha_m) - x_{im}}. \tag{3}$$

The probabilities p_i and q_i are well-defined only if $\alpha_i\alpha_m \leq x_{im} < \alpha_m(1+\alpha_i-\alpha_m)$. Note that by construction, $x_{im} \leq \alpha_m$ and $x_{im} \leq \alpha_i$.

As stock i's excess return r_i closely follows the prediction $\beta_i r_m$ with probability

p_i, this probability captures the systematic part of the tail risk of asset i. With the complementary probability $1 - p_i$, asset i, independently of the market, either exceeds its threshold or it does not. The former event occurs with probability q_i and captures the idiosyncratic tail risk, while the latter occurs with the complementary probability $1-q_i$ and captures the tail risk cushioning of asset i. Formally, we define:

Systematic Tail Risk (STR):

$$STR_i \equiv STR_i(\alpha_i, \alpha_m) \equiv p_i = \frac{x_{im} - \alpha_i \alpha_m}{\alpha_m - \alpha_m^2} \tag{4}$$

Idiosyncratic Tail Risk (ITR):

$$ITR_i \equiv ITR_i(\alpha_i, \alpha_m) \equiv (1 - p_i) q_i = \frac{x_i}{1 - \alpha_m} = \Pr\big(T_{\{i\}} | T_{\{i\}} \cup T_{\emptyset}\big) \tag{5}$$

Tail Risk Cushioning (TRC):

$$TRC_i \equiv TRC_i(\alpha_i, \alpha_m) \equiv (1 - p_i)(1 - q_i) = \frac{x_m}{\alpha_m} = \Pr\big(T_{\{m\}} | T_{\{m\}} \cup T_{\{i,m\}}\big), \tag{6}$$

where

$$x_{im} \equiv x_{im}(\alpha_i, \alpha_m) = \Pr\big(r_i < F_i^{-1}(\alpha_i), r_m < F_m^{-1}(\alpha_m)\big). \tag{7}$$

Corollary 1 and Proposition 4 below, provide details on α_i and α_m which allow for a relatively straightforward estimation of the impact of the different components of tail risk in (4)-(6) on stock returns.

In the context of our model, the Systematic Tail Risk p_i can be interpreted as a coefficient of tail dependence, with values bounded between 0 and 1, that captures joint VaR exceedances by asset i's returns and the market returns. In particular, when $\alpha_m = \alpha_i$ and $p_i = 1$, then the market exceeding its VaR leads always to stock i exceeding its VaR. However, if $p_i = 0$ then VaR exceedances by the market and asset i are independent.

On the other hand, the complementary probability $1-p_i$ can be decomposed into two parts. The first part, Idiosyncratic Tail Risk, corresponds to asset i exceeding its threshold independently of the market and the second part, Tail Risk Cushioning, corresponds to asset i not exceeding its threshold independently. We show below that the systematic tail risk measure p_i is similar to that of Chabi-Yo, Ruenzi, and Weigert (2018). However, the other two measures are novel in the literature.

It is straightforward to see that measures (4)-(6) are valid and mutually-consistent for any level of alpha below the median ($\alpha_m, \alpha_i \leq 50$ percent) although their meaning now generalizes to *systematic*, *idiosyncratic* and downside risk *cushioning*.[4] We return to this important point in the empirical exercises in Section 3.

2.5. Properties of the Measures of Tail Risk

The tail risk measures that emerge from the framework laid out above have a surprisingly rich array of properties which we elaborate on in this section. Importantly, our tail risk measures are closely related to long-established coefficients of tail dependence. Specifically, the next result shows that the lower (upper) tail dependence coefficient of Sibuya (1960), usually denoted λ_L (λ_U), is a limit case of $STR_i(\alpha_i, \alpha_m)$ when $\alpha_m = \alpha_i = \alpha$. These classic coefficients are of crucial importance in the Extreme Value Theory (EVT) literature (see, e.g., Joe, 1997).

[4]Note that if $\alpha_i = \alpha_m = 0.5$, then these measures would bear some resemblance to Bollerslev, Patton, and Quaedvlieg (2022) semibetas.

Proposition 1.

$$\begin{aligned}\lim_{\alpha\to 0} STR_i(\alpha,\alpha) &= \lambda_L \equiv \lim_{\alpha\to 0} \Pr\big(r_i < F_i^{-1}(\alpha) | r_m < F_m^{-1}(\alpha)\big),\\ \lim_{\alpha\to 1} STR_i(\alpha,\alpha) &= \lambda_U \equiv \lim_{\alpha\to 1} \Pr\big(r_i > F_i^{-1}(\alpha) | r_m > F_m^{-1}(\alpha)\big),\end{aligned}$$

Proof. See Appendix.

By allowing for any value of the severity level α, STR generalizes the classic coefficients of tail dependence to *arbitrary* severity levels of extreme events. This feature is paramount in empirical studies which rely on multivariate extreme tails because the limited number of observations in these tails make such studies practically infeasible. Moreover, by allowing for cases where $\alpha_m \neq \alpha_i$, $STR(\alpha_i, \alpha_m)$ provides another flexible feature useful in empirical studies.

We note here that naïvely generalizing λ_L by computing the conditional probability

$$\lambda_L(\alpha) = Pr\left\{X_i \leq F_i^{-1}(\alpha) | X_m \leq F_m^{-1}(\alpha)\right\}, \tag{8}$$

may result in misleading inferences. In particular, when asset i is independent of the market, $\lambda_L(\alpha) = \alpha$ implies that their dependence increases in the severity level α, while our measure yields $STR_i = 0$ for any severity level α.

In our next proposition, we show that $STR_i(\alpha_i, \alpha_m)$ can be interpreted as "quantile beta":

Proposition 2.

$$STR_i(\alpha_i, \alpha_m) = \frac{Cov(I(r_i < F_i^{-1}(\alpha_i)), I(r_m < F_m^{-1}(\alpha_m)))}{Var(I(r_m < F_m^{-1}(\alpha_m)))}, \tag{9}$$

where $I(.)$ is the indicator function.

Proof. See Appendix.

Unlike the traditional CAPM β and its numerous offspring, the ratio of the covariance and variance in (9) is computed not for asset and market returns but for their respective indicator functions. The "quantile beta" is related to the tail risk measures in Baruník and Nevrla (2022) and in Schreindorfer (2020) and also to the negative semibeta of Bollerslev, Patton, and Quaedvlieg (2022). The latter authors estimate the dependence between market return and asset return conditional on the joint occurrence of negative events for both the market and asset i. STR generalizes their negative semibeta by allowing for any value of the severity levels α_i and α_m.

Furthermore, STR is also closely related to another classic measure of tail dependence. Huang (1992) proposes the measure $E[\kappa|\kappa \geq 1]$, which is the expected number of tail events given that at least one has occurred (see also Hartmann, Straetmans, and Vries, 2004). It is straightforward to show that in the bivariate case $E[\kappa|\kappa \geq 1] = \frac{2}{2-p_i}$. Finally, our STR is closely connected to χ-measure proposed by Coles, Heffernan, and Tawn (1999) and used by Poon, Rockinger, and Tawn (2004) to study stock returns. It is again straightforward to verify that in the bivariate lower tails case

$$\lim_{\alpha \to 0} p_i = \chi \tag{10}$$

where $\chi = \lim_{s \to -\infty} \frac{Pr(S<s,T<s)}{Pr(S<s)}$. An equivalent result holds in the upper tails.

Another essential aspect of this framework is that our Idiosyncratic Tail Risk and Tail Risk Cushioning measures generalize, respectively, the mixed lower-upper tail dependence coefficient (λ_{LU}) and the mixed upper-lower tail dependence coefficient (λ_{UL}) - see, for example, Joe (1997).

Proposition 3.

$$\begin{aligned}
\lim_{\alpha \to 0} ITR_i(\alpha, 1-\alpha) &= \lambda_{LU} \equiv \lim_{\alpha \to 0} \Pr\big(r_i < F_i^{-1}(\alpha) | r_m > F_m^{-1}(1-\alpha)\big), \\
\lim_{\alpha \to 0} TRC_i(1-\alpha, \alpha) &= \lambda_{UL} \equiv \lim_{\alpha \to 0} \Pr\big(r_i > F_i^{-1}(1-\alpha) | r_m < F_m^{-1}(\alpha)\big).
\end{aligned}$$

Proof. See Appendix.

As in the case of STR, the Idiosyncratic Tail Risk and Tail Risk Cushioning generalize the mixed tail dependence coefficients to *any* severity level of tail events. This is an important feature of these measures, especially in the context of asset returns with positive dependence in the systematic factor which is typically the overwhelming majority of assets. In that case, it may be practically impossible to estimate the tail dependence coefficients in the mixed tails due to the lack of or exceptionally low number of observations in these tails.

The connection between our tail risk measures and the classical tail dependence coefficients is important and it can be shown theoretically that they impact the expected excess returns. Chabi-Yo, Ruenzi, and Weigert (2018) prove in their Theorem 3 that, under weak assumptions, the expected excess return of a risky asset i is an increasing (decreasing) function of its λ_L (λ_U) with the systematic factor, i.e. the market return. Our Proposition 1 and their Proposition 3 imply then the following corollary.

Corollary 1. *The expected excess return of risky asset i, $E[R_i] - R_f$, increases in $\lim_{\alpha \to 0} STR_i(\alpha, \alpha)$ and decreases in $\lim_{\alpha \to 1} STR_i(\alpha, \alpha)$.*

We can show that, under the same weak assumptions, ITR and TRC have in the limit a similarly unambiguous impact on expected excess returns of risky assets.

Proposition 4. *The expected excess return of risky asset i, $E[R_i] - R_f$, increases in $\lim_{\alpha \to 0} ITR_i(\alpha, 1 - \alpha)$ and decreases in $\lim_{\alpha \to 0} TRC_i(1 - \alpha, \alpha)$.*

Proof. See Appendix.

These results suggest that our measures of tail dependence will impact excess returns not only in the limit as the joint tail probability vanishes, but also for moderate values of α, in particular, when these measures of tail risk become measures of downside risk.[5] In Section 3, we rely on measures (4)-(6) with α_i and α_m as specified in Corollary 1 and Proposition 4 to estimate the impact of the different components of tail and downside risk on stock returns.

3. Empirical Analysis of Tail Risk Measures

3.1. Summary Statistics

In our extensive empirical exercises, as is standard in the literature, we use daily and monthly data for all common stocks in the American Stock Exchange (AMEX), National Association of Securities Dealers Automated Quotations (NASDAQ) and New York Stock Exchange (NYSE) markets. Our data is obtained from the Center for Research in Security Prices (CRSP) and covers the period from January 1968 to December 2021. In the empirical exercises, we abide by standard practice and include only stocks with share codes 10 or 11 and with the minimum of two years of data available in every five years. To calculate the Book-to-Market ratios, we obtain the firm accounting data from the CRSP-Compustat Merge database. This results in a sample of 3,278,028 stock-month observations with the average of 5,059 stocks per month although this number varies between 2,149 and 7,932 stocks in each month during the period we examine. Data on the risk-free rate and on the market excess return for the same period are obtained from the Kenneth French's online data library.

The tail risk measures that we propose are computed as follows. At the end of each month, we calculate STR, ITR and TRC for a stock using the previous 5 years of return observations of the market and the stock. We use the lower tail of the actual empirical distribution of excess returns to calculate a non-parametric measure of VaR following the literature (see, for example, Atilgan et al., 2020). Specifically, VaR is

[5] Because we focus on the lower part of the asset returns distribution (i.e. when $\alpha \leq 50$ percent), we do not investigate the case when $\lim_{\alpha\to 1} STR_i(\alpha, \alpha)$ and leave it instead for future research.

calculated as the α percentile (where $\alpha \in \{0.05, 0.1, 0.2, 0.3, 0.4, 0.5\}$) of the daily excess returns over the past five year as of the end of month t with the restriction that at least 500 non-missing return observations should exist. Having determined the thresholds defined by VaR_i and VaR_m, we then compute the probabilities x_m, x_i and x_{im} as the respective probabilities of the outcomes $T_{\{m\}}$, $T_{\{i\}}$ and $T_{\{i,m\}}$ as the number of observations that fall into each tail divided by the total number of observations over that period. With the probabilities x_m, x_i and x_{im}, it is then straightforward to obtain the tail risk measures in (4)-(6) with α_i and $\alpha_i = m$ as specified in Corollary 1 and Proposition 4.

Table 1 presents the summary statistics of the distribution of different risk and return measures of the cross-section of U.S. stocks. We average these measures for a stock over the period in which it shows up in the sample. For each measure, the table reports the mean and standard deviation as well as the skewness and different quantile levels. The cross-sectional skewness of excess returns is highly negative as is the coskewness with the market. This is indicative of a considerable number of stocks in the sample with extremely poor performance. Moreover, in line with extensive and well-established evidence, the excess returns have fat tails.

Table 2 presents the correlation matrix for the variables in Table 1. These correlations are obtained for tail risk measures estimated at $\alpha = 10$ percent severity level threshold and are in line with previous studies (see, e.g., van Oordt and Zhou, 2016). We obtain similar results for other severity levels. In line with the literature, the table highlights the tendency of large and liquid stocks to have high beta. Interestingly, these stocks also tend to have high systematic tail risk. We also observe that idiosyncratic tail risk and tail risk cushioning are negatively correlated with systematic tail risk. As we argued above, this is a feature of the model that ensues from the fact that in our framework a stock whose dynamics in the tails are governed by the systematic factor must display weak idiosyncratic dynamics in those tails.

[Tables 1 - 2]

While the correlations in Table 2 shed light on the interactions across variables, they cannot offer any insights into the non-monotonicities or other dynamics of the data that may influence the results. To that end, and following the literature (see, for example, Bali, Cakici, and Whitelaw, 2014), we examine the dynamics of the data in detail and present the results in Table 3. Panels A through C in this table report the mean of the monthly median of the various characteristics for the stocks in each quintile sorted by STR, ITR and TRC, respectively.

Specifically, Panel A in Table 3 reports the characteristics for the portfolios sorted on STR. The systematic tail risk measure is negatively and monotonically related to idiosyncratic tail risk and tail risk cushioning. This result mirrors the classic total volatility decomposition in the SIM (see Sharpe, 1963).

Interestingly, stocks with high STR are more liquid, larger in size and higher priced. The intuition behind these findings is that large stocks, representing a bigger share of the market, make a larger contribution to systematic tail risk while the tail events of the smaller stocks are more likely to be idiosyncratic. Large stocks and liquid stocks, on average, have low returns. On the other hand, stocks with low STR are small, illiquid stocks that, all else equal should have high returns.

Both market beta and cokurtosis also increase in STR, implying that stocks with high systematic tail risk are more exposed to market risk as well as downside risk. This is intuitive and in line with the findings of Bali, Cakici, and Whitelaw (2014). Cokurtosis is estimated as in Ang et al. (2006) and measures the strength of the relation between the market cubic returns and individual stock returns. If the asset and the market have a large positive cokurtosis, they tend to experience positive and negative extreme events simultaneously. It is therefore, unsurprising that stocks with high STR have higher cokurtosis.

Moreover, momentum is positively and monotonically related to systematic tail risk. Stocks with high (low) STR tend to be past winners (losers). Therefore, STR-sorted portfolios should display the well-documented momentum effect (see also Bali, Cakici, and Whitelaw, 2014). However, book-to-market (B/M) ratios decrease with STR which suggests a negative relation between STR and the value premium.

Panel A also reports two properties of the stock return distribution - realized volatility and coskewness. The latter measures the direction and strength of the relation between individual stock returns and squared market returns. A preference for positive skewness implies a negative price for coskewness risk. Stocks with high STR have low realised volatility as well as coskewness, indicating the importance of these effects for the risk premium of STR.

[Table 3]

Panel B in Table 3 reports the same characteristics as panel A for portfolios sorted on ITR rather than on STR. The stock characteristics move in the opposite direction to their dynamics when sorted on STR across all the quintiles. Stocks with high ITR have lower betas, are smaller, lower priced and less liquid. They also tend to be recent losers but, as expected, tend to not experience extreme events simultaneously with the market. They also have a larger value premium, higher volatility and coskewness. These relations are all monotonic.

In panel C of Table 3, we report the characteristics of portfolios sorted on our third measure, Tail Risk Cushioning. Our results show that TRC is negatively related to beta, size, price, momentum, cokurtosis and positively related to book-to-market, illiquidity, realised volatility and coskewness. Consistent with the previous two sets of results, these relations are all monotonic. We examine the robustness of these results with sorting into deciles (results not reported). The conclusions based on sorting into deciles are the same as those obtained from the sorting into quantiles.

3.2. Persistence Analysis of Tail Risk Measures

Following the literature (e.g., van Oordt and Zhou, 2016 and Bali, Cakici, and Whitelaw, 2014), we inspect the transition probability with which a stock belonging to tail risk quintile j over a specific period jumps to quintile i in the subsequent period. The intuition is that if the categorization of a stock in a particular tail risk quintile is informative about its future (relative) tail risk, then persistence of such categorization is a necessary condition. If not, then these tail risk measures would serve only as summary statistics without any information about the future tail risk of the stock. Therefore, if the transition probabilities of a tail risk measure are equal across quantiles (i.e., around 20 percent), then a stock in a particular tail risk quintile over a period has an equal chance of jumping into any of the other quintiles in the following period. It follows that such categorization cannot inform about its future tail risk. By analogy, if the categorization of a stock into a particular tail risk quintile is informative about its future exposure to tail risk, then these measures are persistent and the elements in the main diagonal of the transition matrix would be considerably larger than the off-diagonal elements.

Figure 3 depicts the persistence transition matrices for the tail risk measures proposed. Each panel of this figure corresponds to one of the tail risk components and clearly highlights the tendency of a stock belonging to a particular quintile over a period (in this case one year) to remain in that quintile in the subsequent period (year). This can be seen in the diagonal elements of the transition matrices which represent the frequencies of remaining in the same quintile. For all three transition matrices, the values of the diagonal elements are always higher than 20 percent and, in the case of STR and ITR, can be over 70 percent for the lowest and highest exposure quintiles. The corresponding values for TRC are never lower than 60 percent. Importantly, for all three risk measures, if a stock does change categorization, it is most likely to jump into an adjacent category. The probability of jumping two or more categories is negligibly small.

[Figure 3]

To correct for overlapping in the estimation samples of the measures in the previous exercise, in Figure 4, we report the transition probabilities for a horizon of five years.[6] Each panel of this figure shows the relative transition probabilities with which a stock belonging to quintile j in year t moves to quintile i in year $t + 5$. Even though the transition probabilities of a stock staying in the same quintile are somewhat lower than in the previous exercise, in all cases they are still considerably larger than 20 percent, confirming the findings of Bali, Cakici, and Whitelaw (2014) and van Oordt and Zhou (2016). Moreover, the probabilities of staying in the lowest and highest exposure quintiles are always considerably larger than those of transitioning to other quintiles. Consequently, we conclude that past tail risk measures carry information about future tail risk.

[Figure 4]

3.3. Portfolio Sorting Analysis

First, we examine the impact of the tail risk measures on expected returns regardless of other canonical determinants of expected returns via portfolio sorting. At the beginning of every month from 1968 to 2021, we estimate the tail risk measures for all stocks in the NYSE, AMEX and NASDAQ markets using daily data over the previous five years.

We observe that stocks with high systematic tail risk exposure are generally large. For example, if we sort stocks into five quintiles based on their STR, the average size of stocks in quintile 5 of STR is 42 times larger than that of quintile 1

[6] We use five years of historical data for the estimation of the measures to be consistent with our empirical investigation of tail risk premium in the later sections.

stocks. The opposite is true for ITR and TRC, where the average size of stocks in quintile 1 is more than 20 times higher than that of quintile 5. This is intuitive as small stocks are more prone to tail events even during calm periods of the market. The significant impact of size on expected stock returns is well established in the literature. Therefore, to account for the size effect we resort to bivariate sorting. We sort the stocks in our sample into 25 portfolios, first on size and then on one of the tail risk measures. Following Fama and French (1993), at the beginning of every month, we first sort stocks into size quintiles based on their market capitalization at the end of the previous month using the quintile breakpoints of all NYSE stocks. Then, within each size quintile, stocks are sorted further into five quintiles based on their tail risk measures obtained at that time. For each sorted portfolio, we calculate value-weighted excess returns over the next one month. In Table 4, we report the average excess returns and the corresponding Newey and West (1987) t-statistics of the sorted portfolios over the 1968-2021 period. The return of a long-short strategy which, within each size quintile, buys the portfolio of stocks with the highest tail risk exposure (the fifth quintile) and sells the portfolio of stocks with the lowest tail risk exposure (the first quintile) along with its alpha from the Carhart (1997) four-factor model, are reported in the last two columns. We present the results for measures calculated using the ten percent tail threshold. The results for other tail thresholds are similar.

[Table 4]

In Table 4, the size effect can be clearly seen as the average excess returns reduce almost monotonically going from small to large size quintiles. Interestingly, we observe mixed results for the relationship between tail risk and expected returns. Although portfolios with high STR exposure generally earn higher returns in small-size quintiles with positive and not significant Carhart (1997) four-factor model alphas, they earn lower returns in large-size quintiles. We also observe that the risk premia for ITR and TRC are not statistically different from zero and their signs vary across quintile portfolios. We observe similar results for alternative settings of the sorting, including using the next two to six month returns after portfolio creation, and using equally weighted returns instead of value-weighted returns. These results are similar to those obtained by Bali, Cakici, and Whitelaw (2014) and van Oordt and Zhou (2016) but contradict the findings of Chabi-Yo, Ruenzi, and Weigert (2018). A possible explanation for these results is that expected stock returns are influenced by several other factors which the portfolio sorting exercise does not account for. Indeed, the results in Table 3 above which highlight clearly the monotonic relationship between the tail risk measures and the canonical measures such as beta, momentum, book-to-market and others support this conclusion. This issue can be addressed with the Fama and MacBeth (1973) method which is a two-step cross sectional regression to examine the relation between expected return and factor betas. Therefore, in the next section, we investigate the tail risk premia using this framework.

3.4. Fama and MacBeth (1973) Cross-Sectional Regression

In the Fama and MacBeth (1973) cross-sectional regression analysis, we estimate the factor betas and other risk measures using time series data in the first step, and then, the relation between returns and these variables is estimated in a second step with a cross sectional regression.

Subrahmanyam (2010) highlights that the number of variables shown to predict stock returns in the cross-section is in excess of fifty (see also Chib and Zeng, 2020 for a recent study on the extensive number of risk factors in asset pricing). Controlling for all of these variables is clearly infeasible and thus, we focus on the most widely used ones in the literature and those that, intuitively, are most likely to be correlated with our tail risk measures (see also Bali, Cakici, and Whitelaw, 2014).

In Table 5, we report the results of the Fama and MacBeth (1973) cross-sectional regression of monthly excess returns of all listed US stocks on our tail risk measures and also on other canonical measures. Specifically, the excess returns of each stock relative to the T-bill rate over the following month is regressed on the explanatory variables estimated from historical data over the previous five years. We report the time series average of the coefficients estimated monthly for each variable in six different models. These coefficients capture the premia per unit of risk and are reported with the respective Newey and West (1987) t-statistics (in parentheses).

[Table 5]

The regressors in the Models I to III contain varying sets of canonical risk measures including CAPM beta, book-to-market, size, momentum, volatility, illiquidity, coskewness and cokurtosis (see, e.g., van Oordt and Zhou, 2016; Bali, Cakici, and Whitelaw, 2014 and references therein). Book-to-market is measured as the ratio of the book value from the previous fiscal year adjusted for investment tax credits, deferred taxes and preferred shares divided by the market capitalization at the end of the previous calendar year (see, for example, Fama and French, 1993). Size is calculated as the natural logarithm of market capitalization at the end of the previous month. Momentum is the average of previous year returns excluding the last month (see, e.g., Huang et al., 2012). Volatility is the standard deviation of daily returns. Illiquidity is proxied by average daily illiquidity in the last year, where the latter is calculated as the ratio of the absolute daily return over daily dollar volume (see Amihud, 2002). Coskewness and cokurtosis are computed as in Ang et al. (2006).

Estimates in Models I-III are consistent with results reported in the literature. At the level of individual stocks, the CAPM beta earns a negative or insignificant risk premium when it is calculated from past daily returns (see, e.g., Bali, Engle, and Murray, 2016 for an extensive discussion of this finding). Book-to-market is associated with higher expected return and it is highly significant when we include only CAPM beta and size in the regression. However, with additional risk factors included (Model III), book-to-market becomes only marginally significant. Size affects expected returns negatively and is significant. Momentum, illiquidity and cokurtosis are all statistically significant with the signs of the premia consistent with theoretical predictions. Volatility is significantly associated with lower expected returns, reflecting the volatility feedback and leverage effects (see Black, 1976; Campbell and Hentschel, 1992 among others). Finally, coskewness is not significant, which is probably due to the high level of measurement noise (see, e.g., Bali, Engle, and Murray, 2016).

Models IV to VI include our proposed measures of tail risk (4)-(6) with α_i and α_m as specified in Corollary 1 and Proposition 3. Table 5 shows the results calculated at ten percent VaR. We note that STR exhibits the expected positive sign and is highly significant. This suggests that investors are rewarded for bearing the systematic tail risk, which is in line with the theoretical predictions laid out in Corollary 1 above. Importantly, the inclusion of systematic tail risk in the regression does not substantially alter the significance or the magnitude of other coefficients. We conclude, therefore, that systematic tail risk captures a distinct risk not present in the other canonical factors.

Similarly, the risk premium associated with TRC is also of the expected negative sign as suggested by Proposition 3 and is statistically significant. Therefore, this finding supports the theoretical prediction that investors are willing to pay a higher price (i.e. accept lower expected returns) to hold stocks with the ability to cushion large losses generated by the market.

However, we find an unexpected result regarding the idiosyncratic tail risk. The risk premium of ITR is negative although only marginally significant. This suggests that investors are not compensated with higher expected returns when investing in stocks with higher idiosyncratic tail risk. This finding seems at odds with the theoretical prediction of Proposition 3 but mirrors the findings on idiosyncratic volatility of Ang et al. (2006) among others.

3.5. Time-varying Crash Fears

Chen, Joslin, and Tran (2012) argue that the risk premium for disaster risk increases substantially after a disaster (see also Gennaioli, Shleifer, and Vishny, 2015 who propose a theoretical model where investors overstate the fear of a future market crash following the occurrence of a tail event). Therefore, we examine the impact of the realization of a market tail event on the three components of tail risk. To that end, and following the literature (see, for example, Chabi-Yo, Ruenzi, and Weigert, 2018), we divide our data set into two subsamples centered around large tail events: the "Post–market crash" subsample containing 5 years after a market tail event and the "Remaining years" subsample. However, because of the pandemic, our sample of market crashes contains three more days which occurred in March 2020 and has a total of 13 worst days.[7] As these three days occurred towards the end of our sample, we only have just one more year of data after the crash. The results of this analysis are presented in Table 6.

[Table 6]

We find that the impact of STR on returns is much stronger in the years subsequent to a market crash. The impact of STR on returns is almost three times as high in the "Post–market crash" subsample with a coefficient for the impact of STR of 0.034 in contrast to a coefficient of 0.016 for the "Remaining years". This finding is similar to that of Chabi-Yo, Ruenzi, and Weigert (2018) and supports the theoretical model of Gennaioli, Shleifer, and Vishny (2015). However, whereas TRC is not significant in periods following market crashes, it appears to be relevant in the remaining years with a statistically significant coefficient of -0.023. This suggests that investors are willing to pay a premium for stocks that cushion potential blows

[7]The market crash dates are: 19 October 1987, 26 October 1987, 31 August 1998, 14 April 2000, 29 September 2008, 09 October 2008, 15 October 2008, 20 November 2008, 1 December 2008, 08 August 2011, 09 March 2020, 12 March 2020 and 16 March 2020.

emanating from the market during relatively calmer periods but rather surprisingly this effect does not appear to be priced following a market crash. While at face value this looks like a paradox, an explanation for it may be that in highly turbulent periods, risk aversion increases sufficiently to deter under-diversified investors from market participation (see, for example, Zhou, 2020 and Meister and Schulze, 2022 for evidence that household market participation decreases substantially following a market crash). This, in turn, reduces any impact these investors may have on the pricing of stocks. Finally, ITR has a negative but insignificant impact on expected returns in both, the post-market crash and the remaining periods. This result is consistent with the findings of Bali, Cakici, and Whitelaw (2014) so not entirely surprising.

3.6. The Impact of Downside Risk on Expected Returns

In Section 2, we argued that the three components of risk are valid and mutually consistent if the meaning of tails is "expanded" to all outcomes below the median return which effectively modifies tail risk to downside risk (see, for example, Bali, Cakici, and Whitelaw, 2014). Thus, we now turn to examining the premia of the downside (tail) risk measures for α ranging from 5 to 50 percent and report the results of the Fama and MacBeth (1973) cross-sectional regression in Table 7.

The findings are consistent with those in Table 5. Specifically, the risk premium of systematic downside (tail) risk is positive and highly significant at every threshold level α defining the downside (tail) risk. Similarly the risk premium of downside (tail) risk cushioning is negative and significant with the only exception at five percent severity level. The puzzling negative risk premium associated with idiosyncratic tail risk can also be observed at almost all levels of idiosyncratic downside risk. Indeed, for α ranging from 50 to 20 percent there seems to be strongly significant evidence of a negative impact of ITR on expected returns. At these high levels of α, ITR is a proxy for idiosyncratic (semi-) volatility rather than tail risk. In this context, this result is not surprising and entirely in line with the findings of Ang et al. (2006), among others, that stocks with high idiosyncratic volatility have very low average returns even after controlling for exposure to aggregate volatility. Interestingly, the ITR risk premium flips back to the positive sign indicated by Proposition 3 at the five percent severity level of tail risk.[8]

Interestingly, we observe that the impact of tail risk components on expected returns becomes slightly weaker when the tail threshold reduces below 10 percent. This is largely true for all three measures but especially for ITR and TRC, suggesting that the impact of downside risk on expected returns becomes weaker when the overall downside risk becomes tail risk. At face value, this finding suggests that investors care more about downside risk rather than tail risk.

[Table 7]

In the above analysis, the tail risk measures are estimated with a window of five years of daily data. To check the robustness of the results obtained with this window size, we repeat the investigation and examine the premia of the tail risk measures at different tail thresholds but now with the tail risk measures estimated with a window of two years of daily data. The results of the Fama and MacBeth (1973) cross-sectional regressions are reported in Table 8. Although the significance of the results has reduced somewhat as illustrated by the slightly lower t-statistics, the results are qualitatively similar to those of Table 7. An exception is the risk premium for TRC which is now significant for *all* levels of α (recall that in the previous exercise, TRC was significant only for α from 50 to 10 percent). Similar to previous results, STR is significantly and positively related to expected returns for all levels of α examined whereas ITR is significantly but negatively related to expected returns for levels of α between 50 and 20 percent. Interestingly and in line

[8]We did not carry out the investigation for one percent tail threshold since the exceptionally low number of observations made the estimation of the measures infeasible.

with the finding in the previous table, although negative and significant for most levels of α, the ITR risk premium switches to the positive sign (though statistically insignificant) at the five percent severity level of tail risk.

[Table 8]

In a similar exercise but with a different focus, we modify the above investigation and examine the premia of the tail risk measures at different threshold levels α for three months ahead returns. In each Fama and MacBeth (1973) cross-sectional regression, the three months-ahead, i.e. the cumulative excess return of a stock from $t+1$ to $t+3$ is regressed on its time-t tail risk measures as well as other canonical risk measures. The results, presented in Table 9, are largely similar to those in Table 7. The difference is that, while the coefficients seem to have increased and in some cases doubled across all three risk measures, the statistical significance has reduced slightly though generally still well above the usual significance levels. For example, at $\alpha = 50$ percent, the impact of STR increases from 0.034 on the one-month ahead expected returns to 0.069 on the three-month ahead expected returns (with the respective Newey and West (1987) t-statistics 6.997 and 5.772). Similarly, these coefficients are 0.011 and 0.024 at $\alpha = 5$ percent (with Newey and West (1987) t-statistics of 3.938 and 3.043 respectively). Similar observations can be made for ITR and TRC. This suggests that the impact of these downside risk measures may take more than one month to "show up" in expected returns although the reduced precision makes it harder to detect it deep in the tails.

[Table 9]

3.7. Robustness Analysis

3.7.1. Examining the Risk Premia after Further Screening

Next, we examine the risk premia of the tail risk measures after screening the sample of stocks for factors that may have biased our previous results. More specifically and following the literature (see Bali, Cakici, and Whitelaw, 2014), we conduct the Fama and MacBeth (1973) cross-section regressions after further screening as follows. We screen for *Size* by removing stocks with prices less than $5 and stocks smaller than the smallest size decile of NYSE stocks; screen for *Illiquidity* by removing stocks with illiquidity higher than that of the top decile of NYSE stocks; we remove stocks with idiosyncratic volatility higher than that of the top decile of NYSE stocks (Volatility 1); we remove the top 20 percent realized volatility stocks within the NYSE stock only sample (Volatility 2); we screen for *Winners* (*Losers*) by removing stocks with momentum higher (lower) than that of the top (bottom) decile of NYSE stocks; we screen for *Momentum* by removing both winners and losers. Moreover, we screen further by removing stocks with returns during the previous month higher than that of the top decile of NYSE stocks (1-month Winners); and finally by removing stocks with previous month returns lower than that of the bottom decile of NYSE stocks (1-month Losers). In each cross-sectional regression, the one month-ahead excess return of a stock is regressed against its risk measures of CAPM beta, size, book-to-market, momentum, illiquidity, volatility, coskewness, cokurtosis, and the proposed tail risk measures. Table 10 presents the results which show that regardless of the screening factor, STR always has a positive and significant impact on stock returns. ITR on the other hand continues to have a negative impact on returns although in many cases it is insignificant. Finally, TRC remains negatively and, in the majority of cases, significantly related to returns.

[Table 10]

3.7.2. Including All Three Tail Risk Components

As a further robustness check, we examine the simultaneous impact of all three tail risk measures on expected returns in the Fama and MacBeth (1973) cross-sectional regression. The results of this investigation are reported in Table 11 for different levels of the tail threshold and are entirely in line with our findings discussed above. Specifically, we find evidence of a positive and significant systematic downside (tail) risk premium, and a significantly negative downside (tail) risk cushioning premium. The risk premium associated with ITR continues to be negative and is significant when it proxies idiosyncratic downside risk (i.e. at high levels of the threshold levels α). It becomes insignificant or positive at a risk severity level $\alpha = 20$ percent or below.

[Table 11]

3.7.3. Long-term Predictive Power of the Three Tail Risk Measures

In a recent contribution, Atilgan et al. (2020) find that tail risk (VaR) of a stock has a significantly negative impact on its expected returns which, they argue, is driven by retail investors' underreaction to tail risk. Stocks with higher tail risk should have lower prices and hence, higher expected returns to compensate investors for the considerable chance of significant drops in the value of their investments. Moreover, it is also a stylized fact that (tail) risk measures are highly persistent. However, if this persistence is overlooked due to e.g., behavioral bias or slow diffusion of information, then stocks will be overpriced which in turn leads to the negative relationship between a stock's VaR and its expected returns.

An important implication of not accounting for the persistence in tail risk and the ensuing mispricing is that its current value should predict future returns. Figure 4 illustrates the cross-sectional persistence - stocks in high (low) STR, ITR and TRC categories tend to remain in those categories even after 5 years (for time-series persistence of tail risk see, for example, Polanski, Stoja, and Windmeijer, 2019). With a small modification to the exercise in Atilgan et al. (2020), we now examine whether the current values of the tail risk measures estimated at a range of severity levels α have predictive power for returns in the long term[9]. Figure 5 depicts the values of the risk premia associated with the STR, ITR and TRC measures in the Fama and MacBeth (1973) cross-sectional regression explaining the next one- to twelve-month returns. A common pattern can be seen in all three measures.

The risk premium is largest in magnitude for high alpha in all three cases. Moreover, although the magnitudes decrease precipitously from the second month, they remain considerable for four to six months, beyond which they flatten out. Taken together, these results imply a consistent underreaction of investors to tail risk, supporting the findings of Atilgan et al. (2020) which they argue may be due to behavioral biases.

[Figure 5]

3.7.4. Sub-sample and Other Robustness Analyses

We observe in our data that the vast majority of market crashes are in the latter part of the sample. Indeed, ten out of 13 market crash dates occured after the year 1999. To examine whether and how this clustering of market crashes impacts our results, we conduct sub-sample analysis where we split the sample into prior- and post-2000. Specifically, the first sub-sample covers the period January 1986 – December 1999, whereas the second covers the period January 2000 – December 2021. We then conduct Fama and MacBeth (1973) cross sectional regressions on both these sub-samples separately. The results (not reported but available upon request) convey a similar message in both sub-samples. STR has a positive and statistically significant impact on returns. TRC and ITR have a negative and mostly statistically significant impact on returns. In all three cases, the economic impact was larger in the second sub-sample. However, the results suggest that while STR is significant

[9]Atilgan et al. (2020) on the other hand look at the predictive power of the VaR-sorted decile portfolios for the one- to twelve-months ahead expected returns.

at all severity levels of α examined, ITR and TRC measures are significant only for $\alpha \geq 20$ percent, i.e. when they serve as proxies of downside rather than tail risk.

Another sub-sample analysis we conducted was to omit the impact of the U.S. – China Trade Tensions ensuing the mutual imposition of tariffs and other trade barriers at the beginning of 2018 which had a large impact on the economies of both countries. However, the evidence suggests that it affected more larger U.S. companies with multinational operations relative to their smaller counterparts (see, for example, Amiti, Redding, and Weinstein, 2019). In this sub-sample analysis, we omit from the sample the period January 2018 – December 2021 and conduct Fama and MacBeth (1973) cross sectional regressions on sub-sample January 1968 – December 2017. The results (not reported but available upon request) support the previous conclusions: STR has a positive and statistically significant impact on returns, while TRC and ITR have a negative and mostly statistically significant impact on expected returns.

4. Conclusion

There are several studies that examine the relationship between (different measures of) systematic tail risk and expected returns. The impact of idiosyncratic tail risk on stock returns, on the other hand, has attracted much less attention.

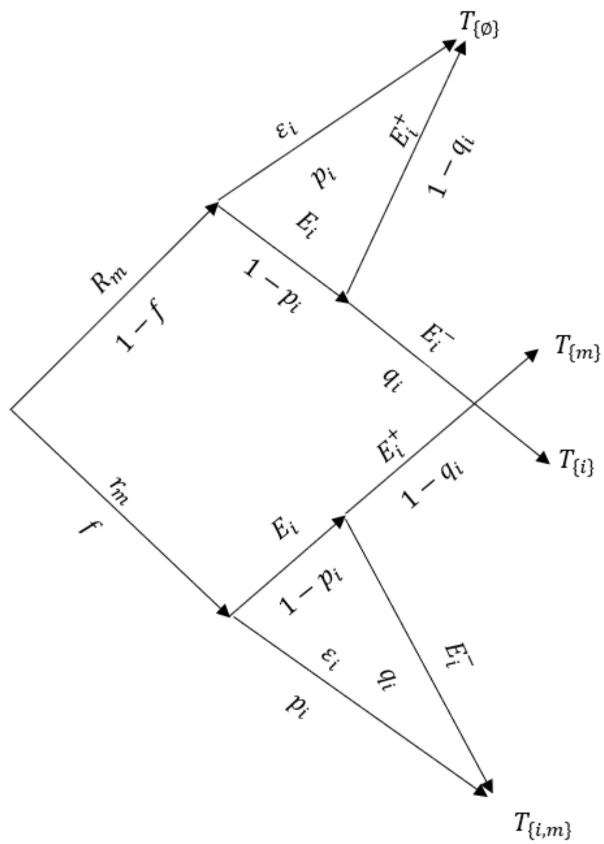
In this paper, we decompose the tail risk of stock returns into systematic and idiosyncratic parts, with the latter being further decomposed into the tendency of a stock to contribute to or dampen tail risk. These three components of tail risk correspond, respectively, to systematic tail risk, idiosyncratic tail risk and tail risk cushioning of a stock.

In the theoretical part, we propose a simple model of asset returns and show how it leads directly to our decomposition of tail risk. From this model, we derive closed-form measures for the three aforementioned tail risk components that can be estimated empirically. The explicit formulae for the derived measures allow for their detailed studies, and we prove a number of their properties. In particular, we show that STR generalizes the classic lower (upper) tail dependence coefficient of Sibuya (1960) to any level of severity of extreme events. Moreover, we prove that all our measures have in the limit an unambiguous impact on expected excess returns.

In the empirical part, we extensively investigate the impact of systematic and idiosyncratic components on asset returns. We find, in particular, that our measure of systematic tail risk has a considerable impact on stock returns which confirms the findings reported by Chabi-Yo, Ruenzi, and Weigert (2018). Moreover, we find evidence that exposure of a stock portfolio to tail risk cushioning earns a significant negative risk premium as suggested by our theoretical results. However, evidence suggests that exposure of a portfolio to idiosyncratic tail risk earns a negative risk premium extending the findings of Ang et al. (2006), among others, on the negative impact of idiosyncratic volatility on expected stock returns to idiosyncratic tail risk. The components of downside risk have an economically similar although statistically stronger impact on expected returns relative to their tail risk counterparts. Our findings on idiosyncratic tail and downside risk add to the existing wealth of results on idiosyncratic risk that contradicts theoretical predictions, an issue which clearly deserves further study.

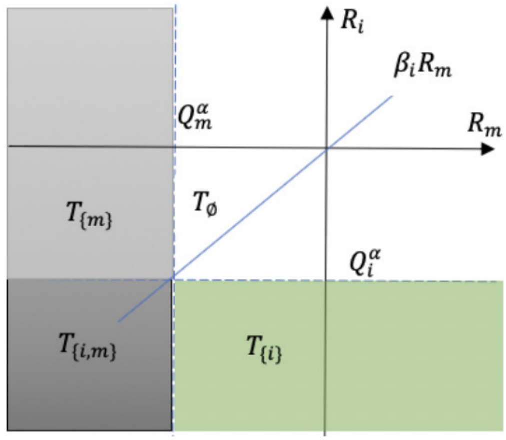
5. Figures

Figure 1: The Evolution of Stock Returns



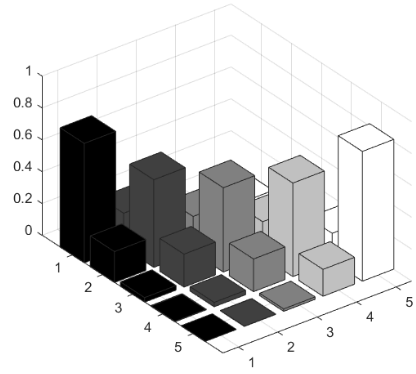
According to our tree model, market excess returns fall below (r_m) or above (R_m) a given threshold with probabilities f and $1-f$, respectively. Excess returns of asset i follow a SIM with a non-negative β_i according to one out of two possible regimes. In the first regime, $\beta_i > 0$ and the error term is distributed with a "moderate" dispersion (ϵ_i). In this regime, which occurs with probability p_i, stock i's does (not) exceed its threshold whenever the market does (not). In the second regime, which materializes with the complementary probability $1-p_i$, $\beta_i = 0$ and the error is distributed with a "large" dispersion (E_i). In this case, asset i exceeds its threshold independently of the market with probability q_i due to a large negative error term materializing (E_i^-) or, with probability $1-q_i$, does not exceed it due to a moderate or large positive error term (E_i^+).

Figure 2: The Partition of Outcome Space of Market and Stock Returns

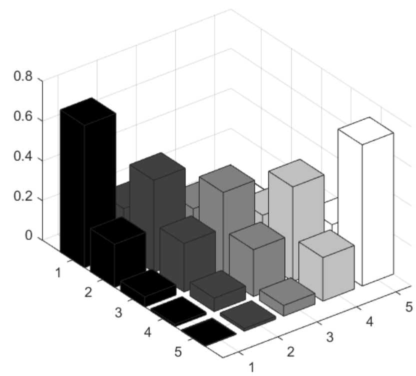


Partition of the two-dimensional outcome space into four joint tails. These joint tails correspond to the final nodes in the event tree depicted in Figure 1: in $T_\emptyset$ no exceedance has occurred (the white area), in $T_{\{m\}}$ the market but not the asset exceeds its threshold (the light grey area), in $T_{\{i\}}$ the asset but not the market exceeds its threshold (the green area), in $T_{\{i,m\}}$ both exceed their respective thresholds (the dark grey area). The dash lines depict the thresholds which in this case correspond to the quantiles $Q_m^\alpha = F_m^{-1}(\alpha)$ and $Q_i^\alpha = F_i^{-1}(\alpha)$.

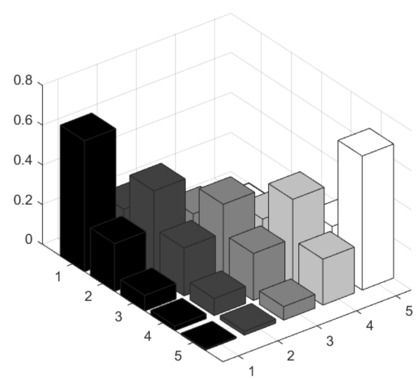
Figure 3: Persistency analysis: two consecutive years



(a) STR



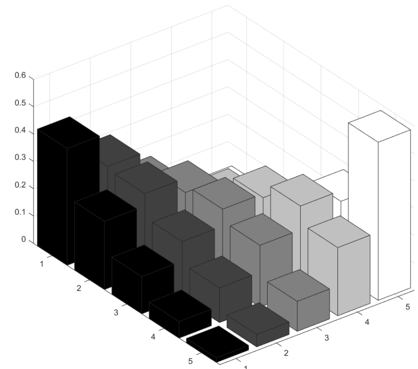
(b) ITR



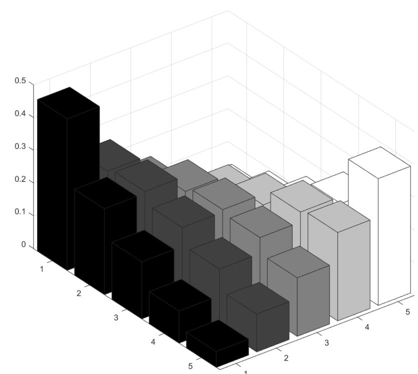
(c) TRC

These figures show the relative frequency with which a stock belonging to quintile j moves into quintile i in the next year for each tail risk measure, averaged over the whole sample period from 1968 to 2021.

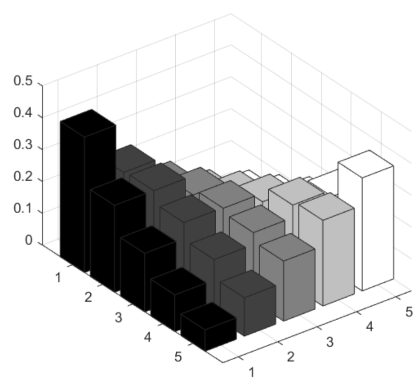
Figure 4: Persistency analysis: five years apart



(a) STR



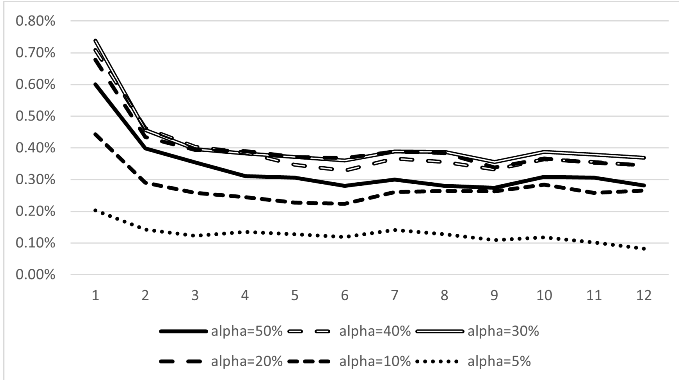
(b) ITR



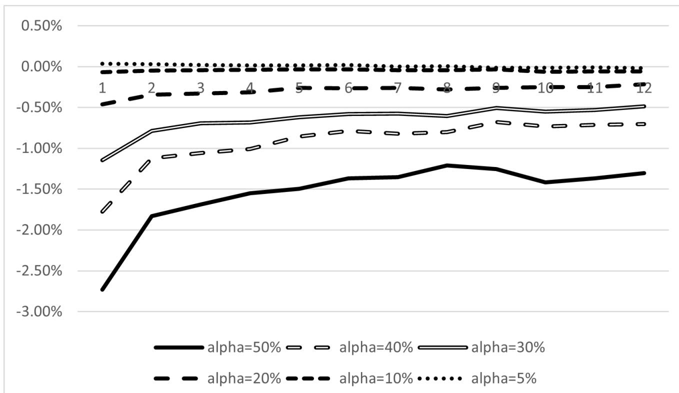
(c) TRC

These figures show the relative frequency with which a stock belonging to quintile j moves into quintile i in the next 5 years for each tail risk measure, averaged over the whole sample period from 1968 to 2021.

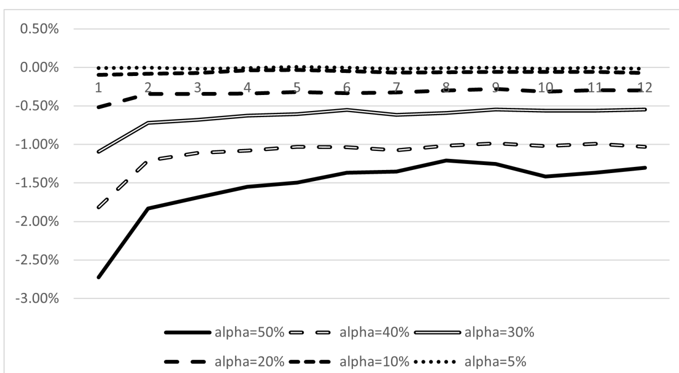
Figure 5: Tail risk premium term structure



(a) STR



(b) ITR



(c) TRC

These figures show the risk premia for the components of tail risk which investors could expect to earn for holding the risky stocks at different points ranging from one month to one year into the future. Each line is a measure estimated at a specific level of the tail threshold ranging from 5 percent to 50 percent.

6. Tables

Table 1: Descriptive statistics

Measures	mean	10% quantile	25% quantile	median	75% quantile	90% quantile	Standard deviation	skewness
Monthly excess return (%)	0.5547	-3.3400	-0.1371	0.9832	1.9995	3.7802	4.9276	-1.0227
Beta	0.7789	0.1863	0.4311	0.7596	1.0814	1.3925	0.4675	0.3729
Size	18.4573	16.0300	17.0183	18.2910	19.7652	21.1081	1.9494	0.3439
Book-to-Market	0.8671	0.2148	0.3985	0.7045	1.0779	1.6165	0.7584	2.9982
Momentum (%)	9.9489	-29.5640	-2.9612	11.9094	22.2047	39.1281	33.9412	0.8753
Illiquidity	8.9680	0.0050	0.0447	0.5205	3.9245	17.4891	33.8920	7.5401
Realized daily volatility (%)	4.2533	2.0079	2.6460	3.7373	5.2567	7.1977	2.2228	1.4457
Coskewness	-0.1602	-0.3788	-0.2234	-0.1166	-0.0419	0.0195	0.1997	-2.1864
Cokurtosis	2.9481	0.3349	0.8492	1.8117	3.9681	6.9057	3.2795	2.3968
STR	0.1487	0.0269	0.0659	0.1302	0.2162	0.2966	0.1034	0.6924
ITR	0.0648	0.0225	0.0383	0.0624	0.0874	0.1091	0.0339	0.4748
TRC	0.0594	0.0241	0.0368	0.0561	0.0790	0.0995	0.0293	0.5400

This table presents summary statistics of the cross-sectional distribution of the main variables used in this study (averaged over the whole sample period), including monthly excess return, beta, size (natural logarithm of market capitalization), book-to-market, momentum, illiquidity, realized daily volatility, coskewness, cokurtosis and the components of tail risk measures calculated at $\alpha = 10$ percent. For each variable, we show the mean, the 10% quantile, the 25% quantile, the 50% quantile (median), the 75% quantile, the 90% quantile, the standard deviation and the skewness. STR is computed as $STR(\alpha, \alpha)$; ITR is computed as $ITR(\alpha, 1-\alpha)$; TRC is computed as $TRC(1-\alpha, \alpha)$. These measures are calculated using 5 years of daily data except for monthly excess returns which is based on one month stock return and one month risk free rate. The computations of Size, Book-to-Market, Momentum and Illiquidity are discussed in detail in Section 3.4. The sample period is from January 1968 to December 2021.

Table 2: Correlation matrix between variables

	Excess return (%)	Beta	Size	Book-to-Market	Momentum (%)	Illiquidity	Realized daily volatility (%)	Co-skewness	Co-kurtosis	STR	ITR	TRC
Excess return (%)	1.0000	-	-	-	-	-	-	-	-	-	-	-
Beta	0.0340	1.0000	-	-	-	-	-	-	-	-	-	-
Size	0.1793	0.4267	1.0000	-	-	-	-	-	-	-	-	-
Book-to-Market	-0.0057	-0.1502	-0.2924	1.0000	-	-	-	-	-	-	-	-
Momentum (%)	0.4387	0.0600	0.3186	-0.0692	1.0000	-	-	-	-	-	-	-
Illiquidity	-0.0388	-0.2228	-0.3818	0.1740	-0.1397	1.0000	-	-	-	-	-	-
Realized daily volatility (%)	-0.0935	0.0080	-0.5126	0.0180	-0.1940	0.3745	1.0000	-	-	-	-	-
Coskewness	-0.0128	-0.0857	-0.1973	0.0642	-0.0691	0.0787	0.1856	1.0000	-	-	-	-
Cokurtosis	0.0305	0.2802	0.3489	-0.0540	0.0834	-0.1134	-0.2896	-0.8296	1.0000	-	-	-
STR	0.0942	0.6703	0.7515	-0.1438	0.1549	-0.2660	-0.3658	-0.1880	0.4080	1.0000	-	-
ITR	-0.0809	-0.6414	-0.6332	0.1746	-0.1422	0.2801	0.3511	0.0762	-0.3098	-0.7189	1.0000	-
TRC	-0.0710	-0.6394	-0.5846	0.1322	-0.1215	0.2355	0.3405	0.1751	-0.3363	-0.6967	0.6867	1.0000

This table presents cross-sectional correlations calculated at $\alpha = 10$ percent for the main variables used in this study (averaged over the whole sample period) including monthly excess return, beta, size (natural logarithm of market capitalization), book-to-market, momentum, illiquidity, realized daily volatility, coskewness, cokurtosis and the components of tail risk measures. The measures of each stock are averaged over the whole sample period. STR is computed as $STR(\alpha, \alpha)$; ITR is computed as $ITR(\alpha, 1-\alpha)$; TRC is computed as $TRC(1-\alpha, \alpha)$. A detailed description of the computation of these variables is given in the main text. The sample period is from January 1968 to December 2021.

Table 3: Portfolios of stocks sorted into quintiles by tail risk measures

Quintile	Beta	Size	B/M	Momentum	Illiquidity	Real Vol	Coskew	Cokurtosis	Price	STR	ITR	TRC
Panel A: Portfolios sorted on STR												
1 - Low	0.276	16.770	0.960	0.033	5.012	0.041	-0.048	0.542	6.208	0.039	0.094	0.081
2	0.548	17.506	0.843	0.036	2.011	0.038	-0.094	1.540	8.297	0.103	0.075	0.067
3	0.758	18.354	0.754	0.056	0.711	0.033	-0.123	2.618	13.400	0.164	0.058	0.052
4	0.933	19.251	0.687	0.072	0.183	0.029	-0.164	3.854	19.772	0.225	0.042	0.040
5 - High	1.184	20.585	0.588	0.088	0.032	0.025	-0.221	5.692	30.753	0.313	0.022	0.024
Panel B: Portfolios sorted on ITR												
1 - Low	1.090	20.374	0.608	0.093	0.031	0.025	-0.184	5.171	28.448	0.278	0.015	0.026
2	0.899	19.243	0.681	0.075	0.157	0.029	-0.143	3.654	19.780	0.213	0.036	0.039
3	0.770	18.530	0.736	0.061	0.539	0.032	-0.123	2.772	14.508	0.169	0.055	0.050
4	0.590	17.725	0.820	0.041	1.409	0.036	-0.107	1.820	9.487	0.115	0.078	0.064
5 - High	0.332	16.910	0.975	0.023	4.126	0.041	-0.088	0.897	6.294	0.069	0.118	0.083
Panel C: Portfolios sorted on TRC												
1 - Low	1.083	20.202	0.608	0.082	0.044	0.026	-0.195	4.961	26.827	0.270	0.027	0.016
2	0.909	19.268	0.681	0.073	0.149	0.028	-0.157	3.762	20.069	0.214	0.042	0.034
3	0.769	18.554	0.736	0.064	0.468	0.031	-0.129	2.818	14.691	0.170	0.054	0.050
4	0.589	17.765	0.818	0.049	1.277	0.036	-0.101	1.800	9.841	0.118	0.070	0.069
5 - High	0.344	17.015	0.949	0.030	3.698	0.041	-0.058	0.841	6.893	0.072	0.093	0.102

This table presents quintile portfolios formed every month from January 1968 to December 2021 by sorting stocks based on the three measures of tail risk calculated at $\alpha = 10$ percent over the past five years:(1) systematic tail risk (STR), (2) idiosyncratic tail risk (ITR), and (3) tail risk cushioning (TRC). Portfolio 1 (5) is the portfolio of stocks with the lowest (highest) tail risk. STR is computed as $STR(\alpha, \alpha)$; ITR is computed as $ITR(\alpha, 1-\alpha)$; TRC is computed as $TRC(1-\alpha, \alpha)$. Panel A reports for each STR quintile the average across the months in the sample of the median values within each month of various characteristics for the stocks – Beta, the log market capitalization (Size), the book-to-market ratio (B/M), a measure of illiquidity (Illiquidity), realised volatility (Real Vol), the coskewness (Coskew), the cokurtosis (Cokurtosis), the price in dollars (Price), and the three tail risk measures. Panels B and C present the same descriptive statistics for quintile portfolios of ITR and TRC respectively.

Table 4: Average excess returns of quintile portfolios sorting on size and tail risk measures

STR Quintile		1	2	3	4	5	5 - 1	Cahart alpha
Size Quintile	1	0.947	0.847	0.938	1.002	1.077	0.130	0.126
		(3.492)	(3.128)	(3.296)	(3.359)	(3.051)	(0.637)	(0.556)
	2	0.847	0.939	1.033	1.013	0.916	0.069	0.116
		(3.974)	(3.880)	(3.921)	(3.730)	(2.967)	(0.392)	(0.751)
	3	0.764	0.902	0.926	0.948	0.853	0.089	0.088
		(3.748)	(4.076)	(4.016)	(3.618)	(3.074)	(0.533)	(0.528)
	4	0.681	0.902	0.845	0.739	0.815	0.134	0.163
		(3.652)	(4.582)	(3.972)	(3.143)	(3.085)	(0.902)	(1.164)
	5	0.608	0.670	0.695	0.690	0.475	-0.133	-0.122
		(3.716)	(4.083)	(3.989)	(3.657)	(2.177)	(-0.980)	(-0.938)
ITR Quintile		1	2	3	4	5	5 - 1	Cahart alpha
Size Quintile	1	0.907	1.097	1.045	0.988	1.040	0.133	0.180
		(2.917)	(3.311)	(3.331)	(3.512)	(3.517)	(0.777)	(0.930)
	2	0.913	0.936	0.887	1.034	1.001	0.088	0.042
		(3.291)	(3.790)	(3.460)	(4.034)	(3.830)	(0.584)	(0.194)
	3	0.878	0.897	0.870	0.857	0.913	0.035	0.001
		(3.496)	(3.714)	(3.762)	(3.685)	(3.930)	(0.248)	(0.005)
	4	0.805	0.794	0.814	0.764	0.832	0.027	-0.043
		(3.357)	(3.580)	(3.751)	(3.550)	(4.040)	(0.230)	(-0.261)
	5	0.497	0.668	0.643	0.687	0.661	0.165	0.124
		(2.376)	(3.636)	(3.613)	(3.919)	(3.871)	(1.266)	(0.868)
TRC Quintile		1	2	3	4	5	5 - 1	Cahart alpha
Size Quintile	1	0.988	0.987	1.030	0.953	1.051	0.063	0.039
		(3.043)	(3.195)	(3.304)	(3.313)	(3.440)	(0.333)	(0.189)
	2	0.909	0.964	0.923	1.077	0.909	0.000	-0.032
		(3.196)	(3.608)	(3.628)	(4.213)	(3.818)	(0.000)	(-0.208)
	3	0.822	0.948	0.903	0.979	0.766	-0.056	-0.002
		(3.232)	(3.791)	(3.769)	(4.264)	(3.494)	(-0.426)	(-0.013)
	4	0.705	0.835	0.852	0.853	0.780	0.075	0.046
		(2.871)	(3.595)	(4.020)	(4.069)	(3.955)	(0.643)	(0.421)
	5	0.542	0.583	0.661	0.581	0.688	0.146	0.114
		(2.552)	(3.043)	(3.618)	(3.444)	(4.209)	(1.107)	(0.776)

This table shows the average excess returns of 25 portfolios sorted on size and a tail risk measure calculated at $\alpha = 10$ percent. Size of a stock is the natural logarithm of its market capitalization at the end of the previous month. Tail risk measures are calculated according to equations 4 - 6 (STR is computed as $STR(\alpha, \alpha)$; ITR is computed as $ITR(\alpha, 1-\alpha)$; TRC is computed as $TRC(1-\alpha, \alpha)$) using the last 5 year data. The second row in each size quintile gives the value of the Newey and West (1987) t-statistics (in brackets) for the returns on the corresponding first row. The last two columns are the average excess return of the long-short strategy which buys quintile 5 and sells quintile 1 of the tail risk within each size quintile, and their alphas in Carhart (1997) four factor models. The sample period is from January 1968 to December 2021.

Table 5: Cross-sectional analysis of tail risk and other canonical risk measures

	I	II	III	IV	V	VI
Intercept	0.011	0.033	0.047	0.053	0.049	0.050
	(5.296)	(2.983)	(6.933)	(7.704)	(7.287)	(7.341)
Beta	-0.003	0.001	0.002	0.000	0.001	0.001
	(-1.563)	(0.453)	(0.901)	(0.001)	(0.607)	(0.564)
Size		-0.001	-0.002	-0.003	-0.002	-0.002
		(-2.598)	(-6.403)	(-7.299)	(-6.570)	(-6.627)
B/M		0.002	0.001	0.001	0.001	0.001
		(3.751)	(1.360)	(1.221)	(1.372)	(1.398)
Momentum			0.007	0.007	0.007	0.007
			(4.232)	(4.481)	(4.098)	(4.132)
Illiquidity			0.001	0.001	0.001	0.001
			(4.706)	(4.571)	(4.763)	(4.743)
Real Vol			-0.195	-0.175	-0.187	-0.189
			(-3.889)	(-3.473)	(-3.765)	(-3.764)
Coskewness			-0.001	0.001	-0.002	-0.001
			(-0.503)	(0.455)	(-0.571)	(-0.233)
Cokurtosis			0.002	0.001	0.002	0.002
			(4.299)	(2.562)	(4.002)	(4.083)
STR				0.023		
				(7.393)		
ITR					-0.013	
					(-1.506)	
TRC						-0.019
						(-2.111)

Fama and MacBeth (1973) average risk premia of the proposed tail risk measures and of the canonical risk measures calculated at α= 10 percent tail threshold (with the corresponding Newey and West (1987) t-statistics in brackets). STR is computed as $STR(\alpha, \alpha)$; ITR is computed as $ITR(\alpha, 1-\alpha)$; TRC is computed as $TRC(1-\alpha, \alpha)$. In each cross-sectional regression, monthly excess return of a stock is regressed on CAPM beta, book-to-market, size, momentum, volatility, illiquidity, coskewness, cokurtosis, and the proposed tail risk measure. The sample period spans from January 1968 to December 2021.

Table 6: Cross-sectional analysis of time-varying tail risk

	Post-market crash			**Remaining years**		
	I	II	III	I	II	III
Intercept	0.061	0.053	0.053	0.047	0.046	0.047
	(5.648)	(5.018)	(5.009)	(5.496)	(5.443)	(5.503)
Beta	0.002	0.004	0.004	-0.001	0.000	-0.001
	(0.372)	(0.835)	(0.851)	(-0.612)	(-0.238)	(-0.359)
Size	-0.003	-0.002	-0.003	-0.002	-0.002	-0.002
	(-5.529)	(-4.667)	(-4.699)	(-5.147)	(-4.852)	(-4.893)
B/M	0.000	0.000	0.000	0.001	0.001	0.001
	(0.075)	(0.315)	(0.348)	(1.450)	(1.458)	(1.468)
Momentum	0.003	0.002	0.002	0.009	0.009	0.009
	(0.991)	(0.711)	(0.727)	(6.699)	(6.630)	(6.614)
Illiquidity	0.002	0.001	0.002	0.001	0.001	0.001
	(2.821)	(2.911)	(2.935)	(4.680)	(4.632)	(4.695)
Real Vol	-0.070	-0.086	-0.092	-0.244	-0.253	-0.252
	(-0.780)	(-0.968)	(-1.040)	(-4.223)	(-4.433)	(-4.321)
Coskewness	0.003	0.001	0.001	0.000	-0.003	-0.002
	(0.768)	(0.265)	(0.238)	(-0.002)	(-0.856)	(-0.471)
Cokurtosis	0.001	0.002	0.002	0.002	0.002	0.002
	(1.428)	(2.626)	(2.505)	(2.161)	(3.134)	(3.250)
STR	0.034			0.016		
	(6.163)			(4.681)		
ITR		-0.012			-0.013	
		(-0.788)			(-1.489)	
TRC			-0.014			-0.023
			(-0.857)			(-2.365)

This table shows the Fama and MacBeth (1973) average risk premia of canonical risk measures and of the proposed tail risk measures calculated at α = 10 percent tail threshold, along with their corresponding Newey and West (1987) t-statistics (in brackets). These results relate to two subsamples: the "Post–market Crash" subsample containing the 5 subsequent years after a market tail event and the "Remaining years" subsample. The market tail events are defined as the 13 worst market returns in our sample which occurred on: 19/10/1987, 26/10/1987, 31/08/1998, 14/04/2000, 29/09/2008, 09/10/2008, 15/10/2008, 20/11/2008, 1/12/2008, 08/08/2011, 09/03/2020, 12/03/2020 and 16/03/2020. In each cross-sectional regression, monthly excess return of a stock is regressed against its risk measures of CAPM beta, size, book-to-market, momentum, illiquidity, volatility, coskewness, cokurtosis, and the proposed tail risk measure. STR is computed as $STR(\alpha, \alpha)$; ITR is computed as $ITR(\alpha, 1-\alpha)$; TRC is computed as $TRC(1-\alpha, \alpha)$. The sample period is from January 1968 to December 2021.

Table 7: Cross-sectional analysis: tail risk premia at different tail thresholds with tail risk measured over a 5 year horizon

	STR						ITR						TRC					
alpha	50%	40%	30%	20%	10%	5%	50%	40%	30%	20%	10%	5%	50%	40%	30%	20%	10%	5%
Intercept	0.060	0.059	0.058	0.057	0.053	0.050	0.094	0.083	0.074	0.059	0.049	0.046	0.094	0.083	0.072	0.060	0.050	0.047
	(8.052)	(8.031)	(8.122)	(8.042)	(7.704)	(7.299)	(8.894)	(8.843)	(9.020)	(8.327)	(7.287)	(6.899)	(8.894)	(8.910)	(8.815)	(8.254)	(7.341)	(6.964)
Beta	-0.002	-0.002	-0.002	-0.002	0.000	0.001	-0.002	-0.002	-0.002	0.000	0.001	0.002	-0.002	-0.002	-0.002	-0.001	0.001	0.002
	(-0.781)	(-1.148)	(-1.104)	(-0.781)	(0.001)	(0.579)	(-0.982)	(-1.074)	(-0.821)	(-0.112)	(0.607)	(0.981)	(-0.982)	(-1.077)	(-0.879)	(-0.281)	(0.564)	(0.894)
Size	-0.003	-0.003	-0.003	-0.003	-0.003	-0.002	-0.003	-0.003	-0.003	-0.003	-0.002	-0.002	-0.003	-0.003	-0.003	-0.002	-0.002	-0.002
	(-7.592)	(-7.582)	(-7.722)	(-7.603)	(-7.299)	(-6.833)	(-7.507)	(-7.511)	(-7.566)	(-7.152)	(-6.570)	(-6.378)	(-7.507)	(-7.505)	(-7.382)	(-7.075)	(-6.627)	(-6.457)
B/M	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	(1.000)	(0.961)	(0.962)	(1.073)	(1.221)	(1.335)	(0.954)	(0.976)	(1.089)	(1.289)	(1.372)	(1.321)	(0.954)	(1.014)	(1.057)	(1.255)	(1.398)	(1.338)
Momentum	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007
	(4.609)	(4.633)	(4.636)	(4.658)	(4.481)	(4.292)	(4.642)	(4.573)	(4.419)	(4.198)	(4.098)	(4.169)	(4.642)	(4.594)	(4.508)	(4.308)	(4.132)	(4.171)
Illiquidity	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	(4.487)	(4.484)	(4.518)	(4.534)	(4.571)	(4.619)	(4.532)	(4.513)	(4.492)	(4.550)	(4.763)	(4.778)	(4.532)	(4.512)	(4.538)	(4.636)	(4.743)	(4.736)
Real Vol	-0.159	-0.150	-0.151	-0.158	-0.175	-0.189	-0.158	-0.149	-0.152	-0.168	-0.187	-0.201	-0.158	-0.149	-0.150	-0.161	-0.189	-0.197
	(-3.048)	(-2.861)	(-2.887)	(-3.036)	(-3.473)	(-3.753)	(-3.037)	(-2.845)	(-2.962)	(-3.332)	(-3.765)	(-4.043)	(-3.037)	(-2.851)	(-2.856)	(-3.116)	(-3.764)	(-3.968)
Coskewness	-0.003	-0.001	0.000	0.001	0.001	0.000	-0.002	-0.002	-0.003	-0.003	-0.002	-0.001	-0.002	-0.002	-0.001	0.000	-0.001	-0.001
	(-0.906)	(-0.483)	(-0.117)	(0.303)	(0.455)	(0.124)	(-0.734)	(-0.842)	(-1.065)	(-0.958)	(-0.571)	(-0.363)	(-0.734)	(-0.554)	(-0.281)	(-0.026)	(-0.233)	(-0.433)
Cokurtosis	0.002	0.002	0.001	0.001	0.001	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
	(2.986)	(2.846)	(2.621)	(2.381)	(2.562)	(3.359)	(2.936)	(3.029)	(2.974)	(3.500)	(4.002)	(4.270)	(2.936)	(2.850)	(3.114)	(3.461)	(4.083)	(4.205)
Tail risk	0.034	0.037	0.037	0.034	0.023	0.011	-0.066	-0.060	-0.058	-0.039	-0.013	0.013	-0.066	-0.062	-0.056	-0.046	-0.019	-0.004
	(6.997)	(7.352)	(7.602)	(7.298)	(7.393)	(3.938)	(-7.221)	(-6.877)	(-6.871)	(-4.552)	(-1.506)	(1.652)	(-7.221)	(-7.386)	(-6.582)	(-4.805)	(-2.111)	(-0.551)

This table shows the Fama and MacBeth (1973) average risk premia of canonical risk measures and of the proposed tail risk component measures calculated at 5 to 50 percent tail thresholds, along with their corresponding Newey and West (1987) t-statistics (in brackets). In each cross-sectional regression, monthly excess return of a stock is regressed against its risk measures of CAPM beta, size, book-to-market, momentum, illiquidity, volatility, coskewness, cokurtosis, and the proposed tail risk measure. STR is computed as $STR(\alpha, \alpha)$; ITR is computed as $ITR(\alpha, 1-\alpha)$; TRC is computed as $TRC(1-\alpha, \alpha)$. The sample period is from January 1968 to December 2021.

Table 8: Cross-sectional analysis: tail risk premia at different tail thresholds with tail risk measured over a 2 year horizon

	STR						ITR						TRC					
alpha	50%	40%	30%	20%	10%	5%	50%	40%	30%	20%	10%	5%	50%	40%	30%	20%	10%	5%
Intercept	0.053	0.054	0.052	0.051	0.048	0.046	0.074	0.071	0.063	0.053	0.046	0.045	0.074	0.068	0.061	0.053	0.047	0.045
	(8.264)	(8.187)	(8.085)	(7.988)	(7.738)	(7.396)	(8.946)	(8.782)	(8.936)	(8.212)	(7.200)	(7.108)	(8.946)	(8.553)	(8.622)	(8.061)	(7.413)	(7.198)
Beta	-0.002	-0.002	-0.002	-0.001	-0.001	0.000	-0.002	-0.002	-0.002	-0.001	0.000	0.000	-0.002	-0.002	-0.002	-0.001	0.000	0.000
	(-0.976)	(-1.349)	(-1.117)	(-0.799)	(-0.391)	(-0.028)	(-1.118)	(-1.300)	(-1.111)	(-0.592)	(0.048)	(0.224)	(-1.118)	(-1.196)	(-1.063)	(-0.649)	(-0.245)	(0.052)
Size	-0.003	-0.003	-0.003	-0.002	-0.002	-0.002	-0.003	-0.003	-0.003	-0.002	-0.002	-0.002	-0.003	-0.003	-0.002	-0.002	-0.002	-0.002
	(-7.717)	(-7.743)	(-7.615)	(-7.603)	(-7.370)	(-6.968)	(-7.696)	(-7.711)	(-7.715)	(-7.246)	(-6.672)	(-6.612)	(-7.696)	(-7.579)	(-7.487)	(-7.109)	(-6.744)	(-6.644)
B/M	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	(1.761)	(1.718)	(1.726)	(1.789)	(1.849)	(1.892)	(1.768)	(1.759)	(1.775)	(1.847)	(1.871)	(1.834)	(1.768)	(1.837)	(1.774)	(1.868)	(1.883)	(1.866)
Momentum	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007	0.007
	(4.656)	(4.717)	(4.728)	(4.622)	(4.495)	(4.319)	(4.676)	(4.630)	(4.490)	(4.323)	(4.257)	(4.362)	(4.676)	(4.632)	(4.574)	(4.363)	(4.286)	(4.347)
Illiquidity	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	(4.291)	(4.259)	(4.264)	(4.395)	(4.450)	(4.541)	(4.307)	(4.283)	(4.275)	(4.440)	(4.723)	(4.672)	(4.307)	(4.299)	(4.329)	(4.458)	(4.618)	(4.607)
Real Vol	-0.182	-0.170	-0.177	-0.183	-0.189	-0.194	-0.180	-0.171	-0.174	-0.182	-0.197	-0.203	-0.180	-0.172	-0.173	-0.178	-0.189	-0.197
	(-3.415)	(-3.203)	(-3.325)	(-3.473)	(-3.638)	(-3.732)	(-3.376)	(-3.205)	(-3.291)	(-3.479)	(-3.777)	(-3.862)	(-3.376)	(-3.238)	(-3.257)	(-3.347)	(-3.590)	(-3.758)
Coskewness	-0.002	-0.001	0.000	0.000	0.001	0.000	-0.002	-0.002	-0.003	-0.002	-0.002	-0.001	-0.002	-0.001	0.000	0.000	0.000	-0.001
	(-0.823)	(-0.389)	(-0.118)	(0.166)	(0.393)	(-0.042)	(-0.684)	(-0.927)	(-1.197)	(-1.092)	(-0.671)	(-0.408)	(-0.684)	(-0.414)	(-0.154)	(0.021)	(-0.067)	(-0.321)
Cokurtosis	0.002	0.002	0.002	0.002	0.002	0.003	0.002	0.002	0.002	0.003	0.003	0.003	0.002	0.002	0.002	0.003	0.003	0.003
	(3.685)	(3.645)	(3.728)	(3.400)	(3.339)	(3.976)	(3.684)	(3.815)	(3.942)	(4.373)	(4.938)	(4.986)	(3.684)	(3.705)	(4.008)	(4.659)	(4.833)	(4.913)
Tail risk	0.021	0.025	0.022	0.018	0.014	0.006	-0.040	-0.041	-0.038	-0.025	-0.003	0.009	-0.040	-0.038	-0.036	-0.028	-0.023	-0.015
	(5.417)	(6.388)	(5.729)	(5.536)	(5.457)	(2.975)	(-5.723)	(-5.818)	(-6.108)	(-4.565)	(-0.658)	(1.492)	(-5.723)	(-5.742)	(-5.590)	(-4.026)	(-3.840)	(-2.037)

This table shows the Fama and MacBeth (1973) average risk premia of canonical risk measures and of the proposed tail risk component measures calculated at 5 to 50 percent tail thresholds, along with their corresponding Newey and West (1987) t-statistics (in brackets). The tail risk measures STR, ITR and TRC are estimated over a 2-year horizon (as opposed to a 5-year horizon as in the previous results). In each cross-sectional regression, monthly excess return of a stock is regressed against its risk measures of CAPM beta, size, book-to-market, momentum, illiquidity, volatility, coskewness, cokurtosis, and the proposed tail risk measure. STR is computed as $STR(\alpha, \alpha)$; ITR is computed as $ITR(\alpha, 1-\alpha)$; TRC is computed as $TRC(1-\alpha, \alpha)$. The sample period is from January 1968 to December 2021.

Table 9: Cross-sectional analysis: tail risk premia three months ahead

	STR						ITR						TRC					
alpha	50%	40%	30%	20%	10%	5%	50%	40%	30%	20%	10%	5%	50%	40%	30%	20%	10%	5%
Intercept	0.124	0.122	0.121	0.118	0.11	0.103	0.195	0.169	0.151	0.123	0.102	0.095	0.195	0.172	0.148	0.124	0.104	0.098
	(6.083)	(6.012)	(6.080)	(5.976)	(5.603)	(5.285)	(7.113)	(6.911)	(7.076)	(6.351)	(5.372)	(4.977)	(7.113)	(7.17)	(6.795)	(6.202)	(5.378)	(5.034)
Beta	-0.001	-0.002	-0.002	-0.001	0.002	0.005	-0.002	-0.002	-0.001	0.002	0.005	0.007	-0.002	-0.002	-0.001	0.001	0.005	0.006
	(-0.171)	(-0.403)	(-0.369)	(-0.191)	(0.393)	(0.768)	(-0.334)	(-0.337)	(-0.169)	(0.263)	(0.807)	(1.123)	(-0.334)	(-0.4)	(-0.233)	(0.182)	(0.786)	(1.035)
Size	-0.006	-0.006	-0.006	-0.006	-0.005	-0.005	-0.006	-0.006	-0.006	-0.005	-0.004	-0.004	-0.006	-0.006	-0.005	-0.005	-0.004	-0.004
	(-5.544)	(-5.478)	(-5.596)	(-5.468)	(-5.106)	(-4.728)	(-5.442)	(-5.418)	(-5.47)	(-5.106)	(-4.568)	(-4.391)	(-5.442)	(-5.499)	(-5.302)	(-5.017)	(-4.603)	(-4.441)
B/M	0.003	0.003	0.003	0.004	0.004	0.004	0.003	0.003	0.004	0.004	0.004	0.004	0.003	0.003	0.004	0.004	0.004	0.004
	(2.248)	(2.232)	(2.230)	(2.301)	(2.399)	(2.484)	(2.203)	(2.251)	(2.332)	(2.445)	(2.482)	(2.446)	(2.203)	(2.251)	(2.296)	(2.414)	(2.498)	(2.467)
Momentum	0.018	0.018	0.018	0.018	0.017	0.017	0.018	0.018	0.017	0.017	0.017	0.017	0.018	0.018	0.018	0.017	0.017	0.017
	(3.850)	(3.878)	(3.903)	(3.947)	(3.798)	(3.671)	(3.881)	(3.812)	(3.695)	(3.533)	(3.489)	(3.62)	(3.881)	(3.852)	(3.806)	(3.647)	(3.537)	(3.61)
Illiquidity	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003
	(4.886)	(4.910)	(4.920)	(4.918)	(4.939)	(4.972)	(4.925)	(4.908)	(4.936)	(5.005)	(5.055)	(5.138)	(4.925)	(4.923)	(4.954)	(4.999)	(5.044)	(5.039)
Real Vol	-0.394	-0.38	-0.380	-0.386	-0.427	-0.453	-0.392	-0.378	-0.385	-0.41	-0.454	-0.486	-0.392	-0.371	-0.377	-0.399	-0.456	-0.473
	(-2.522)	(-2.408)	(-2.414)	(-2.458)	(-2.828)	(-3.027)	(-2.516)	(-2.401)	(-2.486)	(-2.686)	(-3.055)	(-3.288)	(-2.516)	(-2.361)	(-2.39)	(-2.589)	(-3.066)	(-3.205)
Coskewness	-0.005	-0.002	0.000	0.002	0.003	0.002	-0.003	-0.004	-0.006	-0.005	-0.003	-0.002	-0.003	-0.003	-0.001	0.001	-0.001	-0.002
	(-0.570)	(-0.294)	(0.050)	(0.300)	(0.430)	(0.217)	(-0.43)	(-0.518)	(-0.693)	(-0.613)	(-0.359)	(-0.187)	(-0.43)	(-0.321)	(-0.143)	(0.063)	(-0.067)	(-0.207)
Cokurtosis	0.003	0.003	0.003	0.002	0.002	0.003	0.003	0.003	0.003	0.003	0.004	0.004	0.003	0.003	0.003	0.003	0.004	0.004
	(1.850)	(1.702)	(1.640)	(1.368)	(1.504)	(2.174)	(1.837)	(1.924)	(1.853)	(2.196)	(2.543)	(2.846)	(1.837)	(1.75)	(1.974)	(2.114)	(2.633)	(2.745)
Tail risk	0.069	0.075	0.075	0.071	0.049	0.024	-0.138	-0.119	-0.115	-0.081	-0.026	0.039	-0.138	-0.127	-0.115	-0.093	-0.040	-0.006
	(5.772)	(5.750)	(5.816)	(5.439)	(5.876)	(3.043)	(-5.767)	(-5.314)	(-5.228)	(-3.377)	(-1.153)	(1.839)	(-5.767)	(-5.894)	(-5.018)	(-3.330)	(-1.543)	(-0.276)

This table shows the Fama and MacBeth (1973) average risk premia of canonical risk measures and of the proposed tail risk component measures calculated at 5 to 50 percent tail thresholds, along with their corresponding Newey and West (1987) t-statistics (in brackets) for expected returns three months ahead. STR is computed as $STR(\alpha,\alpha)$; ITR is computed as $ITR(\alpha,1-\alpha)$; TRC is computed as $TRC(1-\alpha,\alpha)$. In each cross-sectional regression, three months-ahead excess return of a stock is regressed against its risk measures of CAPM beta, size, book-to-market, momentum, illiquidity, volatility, coskewness, cokurtosis, and the proposed tail risk measure. The sample period is from January 1968 to December 2021.

Table 10: Fama and MacBeth (1973) Regressions with further screening for size, illiquidity, idiosyncratic volatility, momentum winners and losers

	Size	Illiquidity	Volatility1	Volatility2	Momentum -Winners	Momentum -Losers	Momentum	1 month Winners	1 month Losers
					Panel A: STR				
Intercept	0.042	0.040	0.034	0.038	0.051	0.043	0.040	0.046	0.040
	(5.388)	(5.009)	(4.976)	(5.432)	(7.693)	(6.301)	(6.031)	(7.017)	(5.916)
Beta	0.002	0.004	0.000	-0.001	-0.001	0.001	-0.001	-0.001	0.001
	(1.140)	(1.720)	(-0.039)	(-0.507)	(-0.678)	(0.323)	(-0.346)	(-0.251)	(0.628)
Size	-0.002	-0.002	-0.001	-0.002	-0.003	-0.002	-0.002	-0.002	-0.002
	(-4.675)	(-4.331)	(-4.824)	(-4.928)	(-7.646)	(-5.693)	(-5.721)	(-6.677)	(-5.273)
B/M	0.000	0.000	0.001	0.000	0.001	0.001	0.001	0.001	0.001
	(0.560)	(0.306)	(0.959)	(-0.379)	(1.677)	(1.753)	(2.329)	(1.538)	(2.285)
Momentum	0.007	0.007	0.008	0.007	0.010	0.006	0.010	0.004	0.010
	(4.678)	(3.875)	(4.988)	(3.404)	(3.963)	(4.566)	(5.188)	(2.519)	(5.976)
Illiquidity	-0.003	0.018	0.000	0.003	0.001	0.001	0.001	0.001	0.001
	(-0.615)	(0.546)	(1.356)	(2.604)	(4.711)	(3.861)	(3.974)	(5.259)	(3.689)
Real Vol	-0.330	-0.360	-0.135	-0.160	-0.126	-0.157	-0.115	-0.084	-0.276
	(-4.891)	(-4.741)	(-1.472)	(-1.512)	(-2.485)	(-3.084)	(-2.166)	(-1.556)	(-4.969)
Coskewness	0.000	-0.001	0.000	0.002	0.003	0.000	0.002	0.001	0.001
	(-0.121)	(-0.246)	(-0.094)	(0.515)	(0.988)	(-0.008)	(0.629)	(0.251)	(0.409)
Cokurtosis	0.000	0.000	0.001	0.000	0.002	0.001	0.001	0.002	0.001
	(0.730)	(0.264)	(1.561)	(0.394)	(2.866)	(1.946)	(2.045)	(2.670)	(1.077)
Tail risk	0.008	0.009	0.010	0.015	0.026	0.014	0.018	0.021	0.016
	(3.076)	(2.939)	(3.879)	(4.341)	(7.369)	(5.509)	(6.147)	(6.138)	(6.088)
					Panel B: ITR				
Intercept	0.042	0.039	0.033	0.039	0.047	0.041	0.037	0.042	0.038
	(5.308)	(4.835)	(4.909)	(5.534)	(7.211)	(6.132)	(5.772)	(6.488)	(5.676)
Beta	0.003	0.005	0.001	0.001	0.000	0.002	0.000	0.001	0.002
	(1.472)	(2.265)	(0.501)	(0.541)	(0.072)	(0.723)	(0.226)	(0.326)	(1.026)
Size	-0.002	-0.002	-0.001	-0.001	-0.002	-0.002	-0.002	-0.002	-0.002
	(-4.417)	(-4.092)	(-4.375)	(-4.633)	(-6.814)	(-5.245)	(-5.128)	(-5.880)	(-4.745)
B/M	0.000	0.000	0.001	0.000	0.001	0.001	0.001	0.001	0.001
	(0.524)	(0.239)	(0.970)	(-0.422)	(1.865)	(1.836)	(2.460)	(1.704)	(2.358)
Momentum	0.007	0.007	0.008	0.007	0.009	0.006	0.010	0.004	0.009
	(4.674)	(3.861)	(4.825)	(3.379)	(3.593)	(4.219)	(4.734)	(2.247)	(5.640)
Illiquidity	-0.003	0.017	0.000	0.003	0.001	0.001	0.001	0.001	0.001
	(-0.518)	(0.519)	(1.868)	(2.756)	(4.881)	(4.037)	(4.141)	(5.371)	(3.840)
Real Vol	-0.345	-0.391	-0.165	-0.229	-0.137	-0.169	-0.128	-0.095	-0.288
	(-5.273)	(-5.494)	(-1.945)	(-2.374)	(-2.750)	(-3.356)	(-2.446)	(-1.803)	(-5.234)
Coskewness	-0.002	-0.002	-0.002	-0.001	0.000	-0.002	-0.001	-0.002	-0.001
	(-0.662)	(-0.531)	(-0.692)	(-0.224)	(-0.118)	(-0.725)	(-0.205)	(-0.698)	(-0.494)
Cokurtosis	0.001	0.001	0.001	0.001	0.002	0.001	0.002	0.002	0.001
	(1.347)	(1.130)	(2.079)	(0.955)	(4.455)	(2.825)	(3.127)	(3.956)	(2.205)
Tail risk	-0.009	0.006	-0.013	-0.017	-0.017	-0.007	-0.010	-0.013	-0.009
	(-1.299)	(0.656)	(-1.792)	(-1.806)	(-1.908)	(-0.974)	(-1.245)	(-1.459)	(-1.142)

continued

Table 10: *continued*

	Size	Illiquidity	Volatility1	Volatility2	Momentum -Winners	Momentum -Losers	Momentum	1 month Winners	1 month Losers
					Panel C: TRC				
Intercept	0.042	0.040	0.034	0.039	0.048	0.042	0.038	0.043	0.039
	(5.273)	(4.925)	(4.966)	(5.481)	(7.296)	(6.192)	(5.841)	(6.579)	(5.692)
Beta	0.003	0.004	0.001	0.001	0.000	0.001	0.000	0.001	0.002
	(1.455)	(2.082)	(0.612)	(0.648)	(0.034)	(0.620)	(0.112)	(0.318)	(0.971)
Size	-0.002	-0.002	-0.001	-0.001	-0.002	-0.002	-0.002	-0.002	-0.002
	(-4.393)	(-4.103)	(-4.412)	(-4.631)	(-6.890)	(-5.316)	(-5.207)	(-5.967)	(-4.787)
B/M	0.000	0.000	0.001	0.000	0.001	0.001	0.001	0.001	0.001
	(0.541)	(0.287)	(0.978)	(-0.376)	(1.897)	(1.887)	(2.517)	(1.722)	(2.384)
Momentum	0.007	0.007	0.008	0.007	0.009	0.006	0.010	0.004	0.009
	(4.642)	(3.766)	(4.830)	(3.342)	(3.623)	(4.278)	(4.815)	(2.262)	(5.706)
Illiquidity	-0.003	0.016	0.000	0.003	0.001	0.001	0.001	0.001	0.001
	(-0.514)	(0.498)	(1.883)	(2.756)	(4.895)	(3.970)	(4.115)	(5.402)	(3.837)
Real Vol	-0.342	-0.375	-0.175	-0.235	-0.140	-0.168	-0.128	-0.098	-0.288
	(-5.236)	(-5.245)	(-2.048)	(-2.364)	(-2.802)	(-3.333)	(-2.453)	(-1.841)	(-5.219)
Coskewness	-0.001	-0.001	-0.001	0.000	0.001	-0.001	0.000	-0.001	-0.001
	(-0.416)	(-0.498)	(-0.384)	(0.000)	(0.250)	(-0.373)	(0.156)	(-0.375)	(-0.196)
Cokurtosis	0.001	0.000	0.001	0.001	0.002	0.001	0.002	0.002	0.001
	(1.196)	(0.814)	(1.943)	(0.976)	(4.523)	(2.789)	(3.112)	(4.089)	(2.264)
Tail risk	-0.017	-0.012	-0.017	-0.014	-0.023	-0.018	-0.020	-0.018	-0.014
	(-2.192)	(-1.337)	(-2.080)	(-1.616)	(-2.391)	(-2.082)	(-2.202)	(-1.950)	(-1.515)

This table shows the Fama and MacBeth (1973) average risk premia of canonical risk measures and of the proposed tail risk component measures calculated at $\alpha = 10$ percent tail thresholds, along with their corresponding Newey and West (1987) t-statistics (in brackets). The cross-section regressions are run after further screening as follows: removing stocks with price less than \$5 and stocks smaller than the smallest size decile of NYSE stocks (Size); removing stocks with illiquidity higher than that of top decile of NYSE stocks (Illiquidity); removing stocks with idiosyncratic volatility higher than that of top decile of NYSE stocks (Volatility 1); removing top 20% realized volatility stocks within the NYSE stock only sample (Volatility 2); removing stocks with momentum higher than that of top decile of NYSE stocks (Winners); removing stocks with momentum lower than that of bottom decile of NYSE stocks (Losers); removing both winners and losers (Momentum); removing stocks with previous month returns higher than that of the top decile of NYSE stocks (1-month Winners); removing stocks with previous month returns lower than that of the bottom decile of NYSE stocks (1-month Losers). In each cross-sectional regression, three months-ahead excess return of a stock is regressed against its risk measures of CAPM beta, size, book-to-market, momentum, illiquidity, volatility, coskewness, cokurtosis, and the proposed tail risk measure. STR is computed as $STR(\alpha, \alpha)$; ITR is computed as $ITR(\alpha, 1-\alpha)$; TRC is computed as $TRC(1-\alpha, \alpha)$. The sample period is from January 1968 to December 2021.

Table 11: Cross-sectional analysis: including all three tail risk measures

alpha	40%	30%	20%	10%	5%
Intercept	0.085	0.074	0.065	0.055	0.049
	(7.910)	(8.315)	(8.509)	(8.073)	(7.299)
Beta	-0.003	-0.003	-0.003	-0.001	0.001
	(-1.490)	(-1.529)	(-1.129)	(-0.243)	(0.644)
Size	-0.003	-0.003	-0.003	-0.003	-0.002
	(-7.681)	(-7.882)	(-7.866)	(-7.507)	(-6.861)
B/M	0.000	0.000	0.001	0.001	0.001
	(0.908)	(0.901)	(1.045)	(1.257)	(1.264)
Momentum	0.007	0.007	0.007	0.007	0.007
	(4.693)	(4.695)	(4.606)	(4.328)	(4.218)
Illiquidity	0.001	0.001	0.001	0.001	0.001
	(4.488)	(4.475)	(4.546)	(4.706)	(4.717)
Real Vol	-0.140	-0.136	-0.143	-0.167	-0.194
	(-2.667)	(-2.599)	(-2.753)	(-3.370)	(-3.944)
Coskewness	-0.002	-0.001	0.001	0.002	0.001
	(-0.699)	(-0.526)	(0.282)	(0.646)	(0.366)
Cokurtosis	0.001	0.001	0.001	0.001	0.002
	(2.674)	(2.373)	(2.027)	(2.278)	(3.220)
STR	0.009	0.020	0.028	0.023	0.012
	(4.750)	(4.004)	(5.918)	(7.110)	(4.109)
ITR	-0.024	-0.020	-0.008	-0.002	0.015
	(-2.573)	(-2.304)	(-0.993)	(-0.315)	(1.869)
TRC	-0.034	-0.026	-0.024	-0.015	-0.009
	(-4.100)	(-3.179)	(-2.927)	(-1.837)	(-1.178)

This table shows the Fama and MacBeth (1973) average risk premia of canonical risk measures and of the proposed tail risk measures calculated at 5 to 40 percent tail thresholds, along with their corresponding Newey and West (1987) t-statistics (in brackets). In each cross-sectional regression, monthly excess return of a stock is regressed against its risk measures of CAPM beta, size, Book-to-Market, momentum, illiquidity, volatility, coskewness, cokurtosis, and all three components of tail risk. STR is computed as $STR(\alpha, \alpha)$; ITR is computed as $ITR(\alpha, 1-\alpha)$; TRC is computed as $TRC(1-\alpha, \alpha)$. The sample period is from January 1968 to December 2021.

7. Appendix

7.1. Proofs

Recall that

$$\begin{aligned} \alpha_m &= \Pr\big(r_m < F_m^{-1}(\alpha_m)\big), \quad \alpha_i = \Pr\big(r_i < F_i^{-1}(\alpha_i)\big), \\ x_{im} &\equiv x_{im}(\alpha_i, \alpha_m) = \Pr\big(r_i < F_i^{-1}(\alpha_i), r_m < F_m^{-1}(\alpha_m)\big), \end{aligned}$$

with

$$\lim_{\alpha \to 0} x_{im}(\alpha, \alpha) = 0, \quad \lim_{\alpha \to 1} x_{im}(\alpha, \alpha) = 1,$$

and that the unit probability of the outcome space can be decomposed into probabilities x_{im}, $\alpha_i - x_{im}$, $\alpha_m - x_{im}$ and $x_{im} + 1 - \alpha_i - \alpha_m$ of the tails $T_{\{im\}}$,$T_{\{i\}}$,$T_{\{m\}}$ and $T_{\emptyset}$, respectively, as illustrated in Figure 2.

Proof of Proposition 1:

By the definition (4) of $STR_i(\alpha_i, \alpha_m)$, where $\alpha = \alpha_m = \alpha_i$ and $x_{im} = x_{im}(\alpha, \alpha)$:

$$\lim_{\alpha \to 0} STR_i(\alpha, \alpha) = \lim_{\alpha \to 0} \frac{x_{im}/\alpha - \alpha}{1 - \alpha} = \frac{\lim_{\alpha \to 0}(x_{im}/\alpha - \alpha)}{\lim_{\alpha \to 0}(1 - \alpha)} = \lim_{\alpha \to 0} \frac{x_{im}}{\alpha} = \lambda_L.$$

For the upper tail limit, we compute:

$$\begin{aligned} \lim_{\alpha \to 1} STR_i(\alpha, \alpha) = \lim_{\alpha \to 1} \frac{x_{im}/\alpha - \alpha}{1 - \alpha}, \quad \textit{and}, \\ \lambda_U = \lim_{\alpha \to 1} \Pr\big(r_i > F_i^{-1}(\alpha) | r_m > F_m^{-1}(\alpha)\big) = \lim_{\alpha \to 1} \frac{x_{im} + 1 - 2\alpha}{1 - \alpha}. \end{aligned}$$

As the limit of the sum is the sum of the limits, we obtain:

$$\begin{aligned} \lim_{\alpha \to 1} STR_i(\alpha, \alpha) + (-\lambda_U) = \lim_{\alpha \to 1} (\frac{x_{im}/\alpha - x_{im} - 1 + \alpha}{1 - \alpha}) = \\ \lim_{\alpha \to 1} (\frac{x_{im}\frac{1-\alpha}{\alpha} - (1 - \alpha)}{1 - \alpha}) = \lim_{\alpha \to 1} (\frac{x_{im}}{\alpha} - 1) = 0, \end{aligned}$$

because $\lim_{\alpha\to 1} x_{im}(\alpha,\alpha) = 1$. This proves the claim that $\lim_{\alpha\to 1} STR_i(\alpha,\alpha) = \lambda_U$.

Proof of Proposition 2:

Define the random variables $Z_m = I(r_m < F_m^{-1}(\alpha_m))$ and $Z_i = I(r_i < F_i^{-1}(\alpha_i))$, where $I(.)$ is the indicator function. As $E[Z_m] = \alpha_m$ and $E[Z_i] = \alpha_i$,

$$Cov(Z_m, Z_i) = Pr(Z_m = 1, Z_i = 1)(1-\alpha_m)(1-\alpha_i) + Pr(Z_m = 0, Z_i = 0)\alpha_m\alpha_i +$$
$$Pr(Z_m = 1, Z_i = 0)(1-\alpha_m)(-\alpha_i) + Pr(Z_m = 0, Z_i = 1)(-\alpha_m)(1-\alpha_i).$$

After making the following substitutions: $x_{im} = Pr(Z_m = 1, Z_i = 1)$, $\alpha_m - x_{im} = Pr(Z_m = 1, Z_i = 0)$, $\alpha_i - x_{im} = Pr(Z_m = 0, Z_i = 1)$, $1 - x_{im} - (\alpha_m - x_{im}) - (\alpha_i - x_{im}) = Pr(Z_m = 0, Z_i = 0)$, and rearranging terms, we obtain the numerator of $STR_i(\alpha_i, \alpha_m)$. In a similar manner, we calculate $Var(Z_m) = Cov(Z_m, Z_m)$ which results in the denominator of $STR_i(\alpha_i, \alpha_m)$.

Proof of Proposition 3:

By the definition ((5)) of $ITR_i(\alpha_i, \alpha_m)$ with $\alpha = \alpha_i = 1-\alpha_m$ and $x_{im} = x_{im}(\alpha, 1-\alpha)$:

$$\lim_{\alpha\to 0} ITR_i(\alpha, 1-\alpha) = \lim_{\alpha\to 0} \frac{\alpha - x_{im}}{\alpha} = \lim_{\alpha\to 0} \Pr\big(r_i < F_i^{-1}(\alpha) | r_m > F_m^{-1}(1-\alpha)\big) = \lambda_{LU},$$

By the definition ((6)) of $TRC_i(\alpha_i, \alpha_m)$ with $\alpha = \alpha_m = 1-\alpha_i$ and $x_{im} = x_{im}(1-\alpha, \alpha)$:

$$\lim_{\alpha\to 0} TRC_i(1-\alpha, \alpha) = \lim_{\alpha\to 0} \frac{\alpha - x_{im}}{\alpha} = \lim_{\alpha\to 0} \Pr\big(r_i > F_i^{-1}(1-\alpha) | r_m < F_m^{-1}(\alpha)\big) = \lambda_{UL}.$$

Proof of Proposition 4:

In this proof, we use the notation and some results in Appendix A and Internet Appendix of Chabi-Yo, Ruenzi, and Weigert (2018). Expressions labelled (Hxx) are taken from the latter Appendix. Chabi-Yo, Ruenzi, and Weigert (2018) consider a simple theoretical model in which the representative agent with utility function

$u(.)$ maximizes her expected utility under standard regularity conditions $u'(.) > 0$, $u''(.) < 0$, $u'''(.) > 0$, $u''''(.) < 0$. In (H46) in the Internet Appendix, they express the expected excess return on any risky asset i as,

$$\begin{aligned} E[R_i] - R_f &= \frac{k_{\max} - 1}{2}(\lambda^{du}[\overline{k}, k_{\max}]\delta_i^{du}[\overline{k}, k_{\max}] + \lambda^{du}[\overline{k}, 1]\delta_i^{du}[\overline{k}, 1]) \\ &\quad + \frac{1}{2}(\lambda^{ud}[\overline{k}, 0]\delta_i^{ud}[\overline{k}, 0] + \lambda^{ud}[\overline{k}, 1]\delta_i^{ud}[\overline{k}, 1]) + (...), \quad (H46), \end{aligned}$$

where $\overline{k}$ is a specific value of gross return, $k_{\max}$ is the maximum value of gross return and,

$$\begin{aligned} \lambda^{du}[\overline{k}, k] &= \frac{u'''(k)}{u'(a)} Cov((\overline{k} - R_m)^+, (R_m - k)^+) < 0, \quad (H39), \\ \lambda^{ud}[\overline{k}, k] &= -\frac{u'''(k)}{u'(a)} Cov((R_m - \overline{k})^+, (k - R_m)^+) > 0, \quad (H40) \\ \delta_i^{du}[\overline{k}, k] &= \frac{Cov((\overline{k} - R_i)^+, (R_m - k)^+)}{Cov((\overline{k} - R_m)^+, (R_m - k)^+)}, \quad (H43), \\ \delta_i^{ud}[\overline{k}, k] &= \frac{Cov((R_i - \overline{k})^+, (k - R_m)^+)}{Cov((R_m - \overline{k})^+, (k - R_m)^+)}, \quad (H44). \end{aligned}$$

From (H46) it follows that $\delta_i^{du}[\overline{k}, k_{\max}]$ is theoretically related to the expected excess return for any $\overline{k}$. This holds, in particular, in the limit as $\overline{k} \to 0$ and $k \to k_{\max}$. In (H47), this limit is given by,

$$\lim_{\overline{k} \to 0, k \to k_{\max}} \delta_i^{du}[\overline{k}, k] =$$

$$LUTD_i \cdot \lim_{\overline{k} \to 0, k \to k_{\max}} \frac{E[(\overline{k} - R_i)(R_m - k) | R_i < \overline{k}, R_m > k] \Pr(R_m > k)}{Cov((\overline{k} - R_m)^+, (R_m - k)^+)} - (...),$$

where $LUTD_i \equiv \lambda_{LU}$. We show now that this limit decreases in λ_{LU}. First, we observe that $E[(\overline{k} - R_i)(R_m - k)|\ R_i < \overline{k},\ R_m > k] \Pr(R_m > k)$ is clearly positive. Furthermore, (H39) implies that $Cov((\overline{k} - R_m)^+,\ (R_m - k)^+) < 0$ as $\lambda^{du}[\overline{k}, k] < 0$ and $u'''(.) > 0$ and $u'(.) > 0$ by the regularity conditions imposed on the utility function u. Hence, the ratio of these two expressions is negative, which implies that

$\lim_{\overline{k}\to 0, k\to k_{\max}} \delta_i^{du}[\overline{k}, k]$ decreases in λ_{LU}. It follows then from (H46) that the expected excess return increases in λ_{LU} because $\lambda^{du}[\overline{k}, k] < 0$ by (H39). In Proposition 3, we showed that $\lambda_{LU} = \lim_{\alpha\to 0} ITR_i(\alpha, 1-\alpha)$, which completes the proof of the first claim in Proposition 4.

Furthermore, from (H46) also follows that $\delta_i^{ud}[\overline{k}, 0]$ is theoretically related to the expected excess return for any $\overline{k}$. This holds, in particular, in the limit as $\overline{k} \to k_{\max}$ and $k \to 0$. In (H48), this limit is given by,

$$\lim_{\overline{k}\to k_{\max}, k\to 0} \delta_i^{ud}[\overline{k}, k] =$$
$$ULTD_i \cdot \lim_{\overline{k}\to k_{\max}, k\to 0} \frac{E[(R_i-\overline{k})(k-R_m)|R_i>\overline{k}, R_m<k]\Pr(R_m<k)}{Cov((R_m-\overline{k})^+,(k-R_m)^+)} - (\ldots),$$

where $ULTD_i \equiv \lambda_{UL}$. We show now that this limit decreases in λ_{UL}. First, we observe that $E[(R_i - \overline{k})(k - R_m)|R_i > \overline{k}, R_m < k]\Pr(R_m < k)$ is clearly positive. Furthermore, (H40) implies that $Cov((R_m - \overline{k})^+, (k - R_m)^+) < 0$ as $\lambda^{ud}[\overline{k}, k] > 0$ and $u'''(.) > 0$ and $u'(.) > 0$ by the regularity conditions imposed on the utility function u. Hence, the ratio of these two expressions is negative, which implies that $\lim_{\overline{k}\to k_{\max}, k\to 0} \delta_i^{ud}[\overline{k}, k]$ decreases in λ_{UL}. It follows then from (H46) that the expected excess return decreases in λ_{UL} because $\lambda^{ud}[\overline{k}, k] > 0$ by (H40). In Proposition 3, we also showed that $\lambda_{UL} = \lim_{\alpha\to 0} TRC_i(1-\alpha, \alpha)$, which completes the proof of the second claim in Proposition 4.

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