Evaluating the Impact of Portfolio Mandates*

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Abstract

This paper evaluates the effectiveness of portfolio mandates on capital allocation. We argue that a firm's cost of capital is not a good measure of mandate effectiveness. Instead, evaluating the real impact of mandates requires examining their effect on sectoral capital allocation. Contrary to the prediction of endowment-based models, we show that mandates may have a negligible impact on the cost of capital and yet significantly influence the allocation of capital across sectors. Using a production-based model calibrated to match key asset-pricing and macroeconomic moments, we estimate that a significant portion of the mandate remains effective in shaping equilibrium capital allocation, even when there is little disparity in the cost of capital across sectors.

Keywords: ESG, cost of capital, capital allocation.
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1 Introduction

Impact investing, a strategy aimed at generating social and environmental impact alongside financial returns, has grown tremendously over the last decade. Portfolio "screens" or "mandates" are common implementations of ESG-investing strategies. Such policies aim to restrict capital allocation to specific firms to increase target firms' cost of capital and make it more costly for them to fund their operations. PricewaterhouseCoopers (2022) forecasts that ESG-related assets under management are expected to increase from \$18.4tn in 2021 to \$33.9tn by 2026, with ESG assets on pace to constitute 21.5% of total global assets under management. Bloomberg Intelligence (2021) expects global ESG assets to exceed \$53 trillion by 2025, representing more than a third of total assets under management. On the other hand, partly on the grounds that it reduces investment returns, several states in the US have introduced proposals *against* impact investing (Donefer, 2023), and twenty-five US states have sued the Biden Administration to halt a Department of Labor rule that prioritizes ESG concepts into retirement-fund regulations (Mayer, 2023).

Despite the large sums of assets being allocated to impact investing and the controversy about its costs and benefits, the academic literature to date provides a skeptical view of its effectiveness. In their pioneering work, Heinkel, Kraus, and Zechner (2001) and, more recently, Berk and van Binsbergen (2021) argue that impact-investing policies have a negligible impact on targeted firms' cost of capital and are, therefore, ineffective in influencing capital allocation. Along the same lines, there is a large literature that uses the change in the cost of capital to measure the effectiveness of various ESG-motivated policies.[1]

In this paper, we argue that the change in the cost of capital is generally not a good measure of the change in capital allocation. We also demonstrate in a quantitative model that portfolio mandates can lead to significant differences in capital allocation despite minimal differences in the cost of capital. Our analysis has wider implications than just ESG considerations and also

[1]See, for instance, the article from McKinsey "Why ESG is Here to Stay," which discusses how ESG scores are related to the cost of capital. The article states "...there have been more than 2,000 academic studies, and around 70 percent of them find a positive relationship between ESG scores on the one hand and financial returns on the other, whether measured by equity returns or profitability or valuation multiples. Increasingly, another element is the cost of capital. Evidence is emerging that a better ESG score translates to about a 10 percent lower cost of capital." For a further discussion of the effect of ESG on the cost of capital, see Edmans (2023).

encompasses situations in which portfolio constraints are imposed to impact the activity of investors because of other considerations, such as economic sanctions.[2]

Heinkel, Kraus, and Zechner (2001), Berk and van Binsbergen (2021), and the literature using the cost of capital to measure the effectiveness of ESG-related policies reach their conclusions based on the analysis of an endowment economy. In such an economy, a firm's dividends are *exogenous*, and only its asset returns depend on market-clearing prices. In this paper, we revisit the conclusion that impact-investing policies have a negligible impact on targeted firms' cost of capital and are, therefore, ineffective in influencing capital allocations by studying the effect of portfolio mandates in a model of a production economy. In contrast to an endowment economy, in a production economy *both* dividends (payoffs or output) and asset returns are determined endogenously in equilibrium. We show that this has important implications for understanding the real effects of portfolio mandates: in particular, portfolio mandates can lead to large differences in the equilibrium allocation of physical capital across firms despite negligible differences in the cost of capital across these firms.

To understand the intuition driving our result that portfolio mandates can lead to significant changes in capital allocation despite a negligible effect on the cost of capital, we study three versions of a production economy that has two sectors, which consist of green firms and brown firms. First, we consider the case of a single representative investor in an economy with no friction. Second, we consider the case of two groups of investors, of which only one group is constrained by the portfolio mandate while the other is unconstrained, and there are still no frictions in the economy. Finally, we extend the one-period, two-investor, two-sector model to a multiperiod setting with a variety of frictions and other features that allow it to match the macroeconomic and asset-pricing moments in the data

In an economy with a single representative investor, the allocation of capital across sectors depends only on the representative investor's risk aversion and the properties of the production function, which determine asset returns. The production function we consider is of the standard type, $Y = A\,K^{\alpha}$, where Y is output, A is an exogenous productivity shock, K is the capital that is invested, and $\alpha \in [0, 1]$ is the returns-to-scale parameter. The case of $\alpha = 0$ represents

[2]For example, Article 5 of Regulation (EU) No 833/2014, enacted after the onset of the war between Russia and Ukraine, states that "It shall be prohibited to directly or indirectly purchase, sell, provide investment services for or assistance in the issuance of, or otherwise deal with transferable securities" `https://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=CELEX:32022R0328`.

an *endowment* economy, in which output $Y = A\,K^{\alpha} = A$; thus, output depends only on the exogenously specified productivity shocks. The case of $\alpha = 1$, where output is given by $Y = A\,K$, represents the typical "AK" model of a production economy with constant returns to scale. The realized return or cost of capital (we use the terms "return" and "cost of capital" interchangeably depending on the context) is the ratio of output to investment, $R = Y/K = AK^{\alpha}/K = AK^{\alpha-1}$. In particular, for the case of an endowment economy ($\alpha = 0$), the realized return is inversely proportional to the capital invested, $R = AK^{-1}$. In contrast, for a production economy with constant returns to scale ($\alpha = 1$), the realized return is entirely *unrelated* to the capital invested, $R = A$. Thus, in a constant-returns-to-scale production economy, the cost of capital is not affected at all by changes in physical capital. Conversely, even a large change in physical capital allocation may be associated with no change in the costs of capital across sectors.

This raises the question about the appropriate value for the returns-to-scale parameter, α. Empirical estimates from the macroeconomic literature indicate that returns to scale are nearly constant in the US economy, i.e., $\alpha \approx 1$. In a series of influential papers, Hall (1988, 1990) argues that market power and increasing return to scale can explain procyclical productivity in the US. In more recent work, Ahmad, Fernald, and Khan (2019) argue that returns to scale are constant or slightly decreasing; however, they also suggest that there may be increasing returns to scale in specific industries or regions or the presence of factors such as technological progress and network effects. Therefore, increasing returns to scale might be particularly relevant for "green technologies" where learning-by-doing and increased scale have dramatically decreased costs over the past twenty-five years.[3]

Next, we consider an economy with two types of investors, "unconstrained" and "constrained," where the constrained investors face a portfolio mandate that forces them to invest a certain fraction of their wealth in the green sector. To measure the effectiveness of a portfolio mandate on the sectoral allocation of physical capital in equilibrium, we introduce the concept of "*mandate pass-through.*" To illustrate this main idea, consider an economy where both investors have equal wealth and both sectors have identical risk-return tradeoffs so that the optimal unconstrained allocation for both investors is to hold 50% of their portfolio in each sector. Suppose a

[3] For example, Way, Ives, Mealy, and Farmer (2022) argue that, unlike traditional technologies such as oil and gas, clean-energy technologies are on learning curves, where costs drop as a power law of cumulative production. In Section 3.3, we explain in greater detail that estimates from the macroeconomic literature suggest that $\alpha \approx 1$ is the more empirically relevant case.

mandate requires constrained investors to have 75% of their portfolio in the green sector. If we were to ignore the mandate's effect on equilibrium asset prices, the capital allocated to the green sector is $(50\% + 75\%)/2 = 62.5\%$, instead of the 50% in the absence of a mandate. We refer to this difference, 12.5%, as the *maximum* mandate pass-through.

In equilibrium, the imposition of a mandate in favor of the green sector raises the price of green assets and lowers that of brown, making brown assets more attractive on a risk-return basis. As a result of the higher return on brown assets, the unconstrained investor will invest more than 50% in the brown sector, undoing part of the effect of the portfolio mandate. If, after accounting for general equilibrium effects, the overall allocation of capital to green assets is, say, only 56.25%, then the equilibrium mandate pass-through is only 6.25%. Thus, the *effective* mandate pass-through ratio, defined as the ratio of the equilibrium to maximum mandate pass-through, is $6.25\%/12.5\% = 50\%$; that is, 50% of the mandate survives the equilibrium effects.

We analyze the equilibrium effects of portfolio mandates in two steps. First, to develop the intuition for our main result, we study the equilibrium effects of heterogeneous investors in a simple single-period (two-date) economy with two sectors, "green" (G) and "brown" (B), and two types of investors, "constrained" and "unconstrained." For this simple model, we derive the quantities of interest in closed form. This model shows that for the case of an endowment economy ($\alpha = 0$), because $R_G/R_B = (A_G/A_B) \times (K_G/K_B)^{-1}$, a portfolio mandate designed to increase K_G relative to K_B, leads to a corresponding decrease in R_G relative to R_B. Thus, unconstrained investors are strongly incentivized to shift their portfolio toward the B sector. Therefore, in equilibrium, the response of the unconstrained investors to the change in relative return across sectors undoes a large part of the effect of the portfolio mandate; consequently, the effective mandate pass-through ratio is small. On the other hand, for the case of a constant-returns-to-scale production economy ($\alpha = 1$), $R_G/R_B = (A_G/A_B)$; thus, a portfolio mandate designed to increase K_G relative to K_B does not affect at all R_G relative to R_B. As a result, unconstrained investors have no incentive to shift their portfolio toward the B sector. Therefore, in equilibrium, the unconstrained investors' response does not offset at all the effect of the portfolio mandate. Thus, the effective mandate pass-through ratio is a full hundred percent, even though the change in the cost of capital is zero.

Finally, to assess the *quantitative* effects of portfolio mandates on the financial and real sectors, we use a dynamic general-equilibrium model of a production economy that is calibrated to match asset-pricing and macroeconomic moments in the US. For the case of constant returns to scale ($\alpha = 1$) and no portfolio constraints, our model is a canonical real-business-cycle model, similar to that in King, Plosser, and Rebelo (1988) and Jermann (1998), among many others. Just as in the simple single-period model, we consider an economy characterized by two sectors with different technologies, "green" and "brown," and two types of investors, "constrained" and "unconstrained." However, we relax many of the simplifying assumptions made in the single-period model. In particular, we consider an infinite-horizon economy in discrete time where investors have Epstein-Zin recursive preferences, consume in each period, and are endowed with one unit of labor that they supply to firms inelastically. Firms are all-equity financed, incur convex capital-adjustment costs (e.g., Hayashi, 1982), and choose labor and investment to maximize shareholder value subject to a capital-accumulation constraint. We solve for the equilibrium in this economy and then study the effect of a portfolio mandate on the equilibrium stock returns (cost of capital) and capital allocations in the two sectors.

The quantitative multiperiod model confirms the intuition of the simple one-period model. In equilibrium, the optimal portfolio decisions of the unconstrained investor "undo" some of the effects of the portfolio mandate. This occurs because unconstrained investors face a trade-off. On the one hand, the desire to diversify pushes the portfolio towards a 50/50 allocation. On the other hand, by making the brown sector more attractive from a risk-reward perspective, the mandate induces unconstrained investors to tilt their portfolios toward it. We find, however, that portfolio mandates retain a quantitatively significant impact in equilibrium under a realistic calibration that matches asset-pricing and macroeconomic moments of the US economy. For example, under our baseline calibration with a mandate forcing the constrained investor to hold 75% of the portfolio in the green sector, the *effective* mandate pass-through ratio is about 22%. Higher levels of risk aversion, leading to higher and more realistic risk premia, increases the unconstrained investor's desire to hold a diversified portfolio and strengthens the equilibrium real effect of portfolio mandates. Higher values of return to scale also strengthen the real impact of portfolio mandates. In contrast, the effect on the equilibrium cost of capital or Sharpe ratio of the two types of firms remains negligible, consistent with existing evidence.

In summary, our analysis suggests that in a dynamic general equilibrium production economy designed to match the macroeconomic and asset-pricing moments of the US economy, portfolio mandates can have a quantitatively significant impact on aggregate capital allocation, even if their effect on the cost of capital is negligible. This result sharply contrasts the conclusion drawn from studying endowment economies, where, because dividends are exogenous, there is a direct relation between firms' cost of capital and equilibrium capital allocations.

The main contribution of our paper is to study how much of the intended effect of portfolio mandates is undone in equilibrium. Our paper makes two key points. First, we highlight that studying the effects of portfolio mandates in an endowment economy, as most of the finance literature on portfolio mandates has done, is likely to lead to misleading conclusions. In particular, to measure the effectiveness of portfolio mandates, it is essential to focus on the *quantity* of capital flowing to the mandated sectors instead of the effect on the *cost* of capital. This insight is similar to that of Berk and Green (2004), who, in the context of the mutual-fund-performance literature, have emphasized the importance of measuring fund flows instead of risk-adjusted returns. Second, we quantify the impact of portfolio mandates on capital allocation. Specifically, we show, in a general-equilibrium production-economy model calibrated to match key macroeconomic and asset-pricing moments, that the real effect of portfolio mandates can be substantial, even if their impact on the cost of capital is negligible.

Our paper relates to the growing literature on socially responsible investing. This literature consists of two main strands: exclusion (exit) and engagement (voice). The first strand of this literature focuses on a "discount-rate channel" in that it studies the effects of limiting (or excluding entirely) investment in certain firms from an investor's portfolio on the cost of capital of targeted firms. The key mechanism in this literature is reduced risk-sharing that affects the cost of capital in an endowment economy (e.g., Heinkel, Kraus, and Zechner, 2001; Zerbib, 2019, 2022; Berk and van Binsbergen, 2021; Pastor, Stambaugh, and Taylor, 2021, 2022; Pedersen, Fitzgibbons, and Pomorski, 2021; Broccardo, Hart, and Zingales, 2022; De Angelis, Tankov, and Zerbib, 2022). Notably, Heinkel, Kraus, and Zechner (2001) and Berk and van Binsbergen (2021) focus on the result that the effect on risk premia is small if profit-seeking investors can substitute for the capital they are restricted from holding. Our paper revisits this evidence by considering a production economy and studies the quantitative effects of portfolio mandates in a calibrated model designed to match key asset-pricing and macroeconomic moments.

In a recent paper, Dangl, Halling, Yu, and Zechner (2023a) studied how different types of investor *preferences* affect equilibrium capital allocation. They find that if investments are endogenous, the effect of social preferences on corporate decisions may be sizable even if the difference in the cost of capital between the green and brown sectors is negligible. Dangl, Halling, Yu, and

Zechner (2023b) extend this analysis to the case of time-varying social preferences. Unlike them, we show that portfolio mandates can affect capital allocations across sectors—despite small differences in the cost of capital across these sectors—in a standard macroeconomic framework with the portfolio mandate imposed on only a fraction of investors. We also illustrate that the degree of the returns to scale has a crucial impact on the ability of portfolio mandates to influence equilibrium capital allocation.

Finally, Hong, Wang, and Yang (2023) introduce decarbonization capital in a representative-agent dynamic stochastic general-equilibrium model and investigate the effectiveness of sustainable finance mandates in mitigating externalities within the economy. In their economy, the mandate affects all investors and is, therefore, by definition, effective. In contrast, we study an economy where only a fraction of investors is constrained. Because unconstrained investors can trade against constrained investors, in equilibrium, they can potentially undo the effect of mandates. Our finding that mandates can substantially impact equilibrium capital allocation aligns with their conclusion that mandates can effectively address externalities.

The second strand of literature focuses instead on the "cash-flow channel." Broccardo, Hart, and Zingales (2022), following Hart and Zingales (2017), conclude that "voice" is more effective than "exit." Oehmke and Opp (2022) focus on activist investors who care about the social cost of investing in brown firms and provide a corporate perspective on the economics of motivated investors: socially responsible activists subsidize firms to adopt clean technologies. Chowdhry, Davies, and Waters (2019) show that if a firm cannot credibly commit to social goals, such subsidies take the form of investment by socially-minded activists. Our paper does not contribute directly to this strand of literature; however, our focus on production economies allows us to consider jointly the cash-flow and discount-rate channels emphasized separately by the engagement and exclusion literature, respectively.

The rest of the paper proceeds as follows. In Section 2, we develop intuition in a simple one-period (two-date) general equilibrium model that we can solve analytically. In Section 3, we assess the real impact of portfolio mandates in a multiperiod general-equilibrium model with heterogeneous investors that is calibrated to match asset-pricing and macroeconomic moments in the US economy. Section 4 concludes.

2 A single-period equilibrium model with portfolio mandates

To understand the economic intuition driving our key results, in this section, we consider a single-period general-equilibrium economy with several simplifying assumptions that make transparent the economic forces at work. Then, to establish the quantitative implications of portfolio mandates, in the next section, we consider a multiperiod model without these simplifying assumptions.

2.1 The simple single-period model

2.1.1 Firms

We assume that there are two sectors in the economy, green and brown, and we refer to them using the subscripts G and B, respectively. Each of these sectors consists of a large number of atomistic, identical, all-equity-financed firms. Output Y_j in each sector $j = \{G, B\}$ is given by the production function

$$Y_j = A_j K_j^{\alpha}, \quad j = \{G, B\}, \tag{1}$$

where $\alpha \geq 0$ is the returns-to-scale parameter, A_j denotes a random productivity shock, and K_j is the aggregate capital invested in sector j. The (gross) return on equity, or cost of capital, of a firm in sector j is defined as

$$R_j = \frac{Y_j}{K_j} = A_j K_j^{\alpha-1}, \quad j = \{G, B\}. \tag{2}$$

The case of $\alpha = 0$ corresponds to an *endowment* economy in which the output of sector j reduces to $Y_j = A_j$, and, therefore, is entirely exogenous depending only on the specification of the productivity shock A_j, while the cost of capital, $R_j = Y_j/K_j = A_j/K_j$, is inversely related to capital allocation: a high cost of capital R_j is associated with a low capital allocation K_j. For $\alpha = 1$, which corresponds to a constant-returns-to-scale production economy, the output of sector j is $Y_j = A_j K_j$, while the cost of capital, $R_j = Y_j/K_j = A_j$, is completely independent of the capital allocation K_j. Thus, the relation between the cost of capital (returns) and capital allocation becomes weaker as the returns-to-scale parameter α deviates from zero and is entirely absent when α equals one.

We assume that the productivity shocks A_j are normally distributed, that is, $A_j \sim \mathcal{N}(\mu_{A_j}, \sigma_{A_j})$, $j = \{G, B\}$, and their correlation is ρ, so that the covariance between them is given by $\text{Cov}[A_G, A_B] =$

$\rho\,\sigma_{A_G}\sigma_{A_B}$. Thus, from (2), we see that the moments of asset returns are

$$\mathbb{E}[R_j] = \mu_{A_j} K_j^{\alpha-1}, \quad j = \{G, B\}, \tag{3}$$

$$\mathrm{Var}[R_j] = \sigma_{A_j}^2 K_j^{2(\alpha-1)}, \quad j = \{G, B\}, \tag{4}$$

$$\mathrm{Cov}[R_G, R_B] = K_G^{\alpha-1} K_B^{\alpha-1}\,\mathrm{Cov}[A_G, A_B] = K_G^{\alpha-1} K_B^{\alpha-1} \rho\,\sigma_{A_G}\sigma_{A_B}. \tag{5}$$

Denoting the risk-free interest rate by R_f, the Sharpe ratio of investing in sector j is

$$\mathrm{SR}_j = \frac{\mathbb{E}[R_j] - R_f}{\sqrt{\mathrm{Var}[R_j]}}, \quad \text{and}$$

the price of risk in sector j is defined as

$$[\text{Price of risk}]_j = \frac{\mathbb{E}[R_j] - R_f}{\mathrm{Var}[R_j]}.$$

2.1.2 Investors

We consider an economy with a continuum of identical investors who live for one period (two dates), and each investor has initial wealth K_0. A fraction x of investors is constrained to follow a portfolio mandate, and we refer to them using the subscript c. The remaining fraction $1 - x$ is unconstrained, and we refer to them using the subscript u. For tractability, we assume that both types of investors have constant absolute risk aversion (CARA) preferences with an identical coefficient of risk aversion γ.[4]

At $t = 0$, each investor $i = \{u, c\}$ needs to choose what fraction of her wealth to invest in G and B firms, $w_{G,i}$ and $w_{B,i}$, where the firms have random gross returns R_G and R_B. The remaining fraction $1 - w_{G,i} - w_{B,i}$ is invested in the risk-free asset with a gross risk-free return R_f. These portfolio choices at $t = 0$ result in the following consumption at time $t = 1$:

$$C_{1,i} = K_0\big(R_f + w_{G,i}(R_G - R_f) + w_{B,i}(R_B - R_f)\big), \quad i = \{u, c\}, \tag{6}$$

with the expected value of consumption at time $t = 1$ being

$$\mathbb{E}[C_{1,i}] = K_0\big(R_f + w_{G,i}(\mathbb{E}[R_G] - R_f) + w_{B,i}(\mathbb{E}[R_B] - R_f)\big), \quad i = \{u, c\}, \tag{7}$$

[4] In the multiperiod model studied in the next section, we will allow for more general preferences.

and the variance of consumption being

$$\text{Var}[C_{1,i}] = K_0^2\big(w_{G,i}^2\sigma_{R_G}^2 + w_{B,i}^2\sigma_{R_B}^2 + 2w_{G,i}w_{B,i}\,\text{Cov}[R_G, R_B]\big), \quad i = \{u, c\}, \tag{8}$$

where $\mathbb{E}[R_j]$, for $j = \{G, B\}$, is the expected return (expected cost of capital) in sector j, $\sigma_{R_j}^2$ is the variance of return in sector j, and $\text{Cov}[R_G, R_B]$ is the covariance between the returns in the two sectors.[5]

Unconstrained investors. Unconstrained investors chose their portfolios $w_{j,u}$, $j = \{G, B\}$, by maximizing the expected utility of terminal consumption. Specifically, they solve the following problem,

$$\max_{w_{G,u}, w_{B,u}} \mathbb{E}\left[-\frac{1}{\gamma}e^{-\gamma\, C_{1,u}}\right], \tag{9}$$

with $C_{1,u}$ given in equation (6). Because we have assumed that the productivity shocks are normally distributed, and preferences are of the CARA type, the problem in equation (9) is equivalent to

$$\max_{w_{G,u}, w_{B,u}} \mathbb{E}\left[C_{1,u}\right] - \frac{\gamma}{2}\,\text{Var}\left[C_{1,u}\right]. \tag{10}$$

The optimal portfolio for unconstrained investors, using the moments given in (7) and (8), is the familiar mean-variance portfolio

$$w_{G,u} = \frac{\left(\mathbb{E}[R_G] - R_f\right)\sigma_{R_B}^2 - \left(\mathbb{E}[R_B] - R_f\right)\text{Cov}[R_G, R_B]}{K_0\gamma\left(\sigma_{R_G}^2\sigma_{R_B}^2 - \text{Cov}^2[R_G, R_B]\right)}, \tag{11}$$

$$w_{B,u} = \frac{\left(\mathbb{E}[R_B] - R_f\right)\sigma_{R_G}^2 - \left(\mathbb{E}[R_G] - R_f\right)\text{Cov}[R_G, R_B]}{K_0\gamma\left(\sigma_{R_G}^2\sigma_{R_B}^2 - \text{Cov}^2[R_G, R_B]\right)}. \tag{12}$$

Note that the optimal portfolio weights $w_{j,u}$ depend on the risk-free rate, R_f, and the moments of stock returns, R_G and R_B, which, we see from equations (3)–(5) will depend on the equilibrium capital allocations to the two sectors, K_G and K_B.

Constrained investors. The restriction on portfolio investment takes the form of a portfolio "*mandate*" to hold a given fraction of wealth in the two sectors.[6] Thus, for constrained investors,

[5] It is straightforward to show that the correlation between the returns R_G and R_B is equal to the correlation between the technology shocks, A_G and A_B, which we denote by ρ.

[6] An alternative way to model portfolio restrictions is by using portfolio "screens" on the B asset that limits investment in the B sector. For instance, if the constrained investor is not allowed to invest in B at all, then

the portfolio weights $w_{G,c}$ and $w_{B,c}$ are not a choice variable but are dictated by the mandate,

$$w_{G,c} = \overline{w}_G \quad \text{and} \quad w_{B,c} = \overline{w}_B. \tag{13}$$

2.1.3 Equilibrium

While each investor is atomistic and takes the return on the risk-free asset, R_f, and the two risky assets, R_G and R_B, as given, in equilibrium, these returns are determined *endogenously* by the aggregate capital that flows to the G and B sectors and the production technologies in these two sectors. The equilibrium allocation of capital K_G and K_B in the economy and the risk-free rate R_f are obtained by imposing the condition that the capital market clears in the G and B sectors and that the aggregate quantity of risk-free borrowing/lending is zero:

$$K_G = K_0\big(x\, w_{G,c} + (1-x)\, w_{G,u}\big), \tag{14}$$

$$K_B = K_0\big(x\, w_{B,c} + (1-x)\, w_{B,u}\big), \tag{15}$$

$$0 = x(1 - w_{G,c} - w_{B,c}) + (1-x)(1 - w_{G,u} - w_{B,u}), \tag{16}$$

where $w_{j,i}$ for $j = \{G, B\}$ and $i = \{u, c\}$ are given in equations (11), (12), and (13).

The system of equations characterizing the optimal choices of investors and equilibrium in the production model described above (i.e., equations (11), (12), (14), (15), and (16)) does not admit a closed-form solution for generic values of the returns-to-scale parameter α. But, we can obtain an analytical solution for the two special cases of interest: an endowment economy ($\alpha = 0$) and a constant-returns-to-scale production economy ($\alpha = 1$). Comparing these two cases then allows us to explain the intuition for why small differences in the cost of capital in the G and B sectors can be associated with large differences in the capital allocated to these sectors. Finally, to show that the insights for these special cases extend also to other values of the returns-to-scale parameter, we solve the model numerically for $\alpha > 0$.

In our analysis, we will be interested in two quantities in particular. One, the relation between the *cost* of capital across the two sectors, R_G/R_B, to the *quantity* of capital across these sectors, K_G/K_B. Two, the *effective mandate pass-through*. Formally, we measure the effective

$w_{B,c} = 0$. In what follows, we analyze the case of portfolio mandates. The results for a model with portfolio screens are qualitatively similar. The results are also qualitatively similar if one were to mandate only the investment in G, while leaving the investment in B to be a choice variable.

mandate pass-through in an economy with a returns-to-scale parameter α as the following ratio

$$\text{Effective mandate pass-through} = \frac{K_G\big|_x^\alpha - K_G\big|_{x=0}^\alpha}{xK_0\left(\overline{w}_G - w_{G,u}\big|_{x=0}^\alpha\right)}, \tag{17}$$

where $K_G\big|_x^\alpha$ is the capital allocated to the G sector in equilibrium when a fraction x of investor is constrained by the mandate, $K_G|_{x=0}^\alpha$ is the capital that would be allocated to the G sector in an economy without a mandate, while $w_{G,u}|_{x=0}^\alpha$ is the agents' portfolio weight in the G sector in an economy where no investor faces a mandate. The numerator in equation (17) represents the actual reallocation of capital to the G sector as a result of the mandate, while the denominator represents the maximum effect of a mandate on the capital allocation to G, ignoring equilibrium pricing effects. Thus, the effective mandate pass-through ratio tells us what percentage of the maximum effect of the mandate is actually achieved in equilibrium.

2.2 Solution for an endowment economy ($\alpha = 0$)

We start by studying an endowment economy ($\alpha = 0$), which is the case that has been studied in the literature (e.g., Heinkel, Kraus, and Zechner, 2001; Berk and van Binsbergen, 2021).

2.2.1 Without a portfolio mandate

To establish the benchmark, we start by examining the equilibrium for the case of an endowment economy ($\alpha = 0$) when there are no portfolio mandates and so all investors are unconstrained. In terms of the model described above, this is equivalent to setting $x = 0$.

Proposition 1. *In an endowment economy ($\alpha = 0$) in which no investor is constrained by a portfolio mandate ($x = 0$), the equilibrium optimal portfolio weights, using (11) and (12), are*

$$w_{G,u}\big|_{x=0}^{\alpha=0} = \frac{\mu_{A_G} - \gamma\big(\sigma_{A_G}^2 + \text{Cov}[A_G, A_B]\big)}{(\mu_{A_G} + \mu_{A_B}) - \gamma\,\text{Var}[A_G + A_B]}, \tag{18}$$

$$w_{B,u}\big|_{x=0}^{\alpha=0} = \frac{\mu_{A_B} - \gamma\big(\sigma_{A_B}^2 + \text{Cov}[A_G, A_B]\big)}{(\mu_{A_G} + \mu_{A_B}) - \gamma\,\text{Var}[A_G + A_B]},$$

implying that the aggregate capital allocation to the two sectors is

$$K_G\big|_{x=0}^{\alpha=0} = K_0 \times w_{G,u}\big|_{x=0}^{\alpha=0},$$

$$K_B\big|_{x=0}^{\alpha=0} = K_0 \times w_{B,u}\big|_{x=0}^{\alpha=0},$$

and that the risk-free interest rate is

$$R_f\big|_{x=0}^{\alpha=0} = \frac{1}{K_0}\Big((\mu_{A_G} + \mu_{A_B}) - \gamma \operatorname{Var}[A_G + A_B]\Big),$$

where $\operatorname{Var}[A_G + A_B] = \sigma_{A_G}^2 + 2\operatorname{Cov}[A_G, A_B] + \sigma_{A_B}^2$. *Thus, the ratio of the returns (cost of capital) in the two sectors is*

$$\frac{R_G\big|_{x=0}^{\alpha=0}}{R_B\big|_{x=0}^{\alpha=0}} = \frac{A_G}{A_B}\left(\frac{K_G\big|_{x=0}^{\alpha=0}}{K_B\big|_{x=0}^{\alpha=0}}\right)^{-1} = \frac{A_G}{A_B}\left(\frac{w_{G,u}\big|_{x=0}^{\alpha=0}}{w_{B,u}\big|_{x=0}^{\alpha=0}}\right)^{-1}. \tag{19}$$

To obtain a solution that is even more transparent, one can consider the special case where the productivity shocks in the G and B sectors are the same, i.e., $A_G = A_B = A$, $\mu_{A_G} = \mu_{A_B} = \mu_A$, and $\sigma_{A_G} = \sigma_{A_B} = \sigma_A$.

Corollary 1. *In an endowment economy ($\alpha = 0$) in which no investor is constrained by a portfolio mandate ($x = 0$), and the productivity shocks in the G and B sectors are the same, in equilibrium*

$$w_{G,u}\big|_{x=0}^{\alpha=0} = w_{B,u}\big|_{x=0}^{\alpha=0} = \frac{1}{2},$$

$$K_G\big|_{x=0}^{\alpha=0} = K_B\big|_{x=0}^{\alpha=0} = \frac{1}{2}K_0,$$

$$R_f\big|_{x=0}^{\alpha=0} = \frac{1}{K_0}\big(2\mu_A - 2\gamma(\sigma_A^2 + \operatorname{Cov}[A_G, A_B])\big),$$

so that, in the absence of portfolio mandates, the ratio of the returns in the two sectors is

$$\frac{R_G\big|_{x=0}^{\alpha=0}}{R_B\big|_{x=0}^{\alpha=0}} = \frac{A}{A}\left(\frac{K_G\big|_{x=0}^{\alpha=0}}{K_B\big|_{x=0}^{\alpha=0}}\right)^{-1} = 1. \tag{20}$$

2.2.2 With a portfolio mandate for all investors

We now study the effect of introducing in an endowment economy ($\alpha = 0$) a portfolio mandate for all investors, which, in terms of the general model described above, is equivalent to setting $x = 1$.

Proposition 2. *In an endowment economy ($\alpha = 0$) in which all investors face a portfolio mandate ($x = 1$), the optimal portfolio weights for all investors are*

$$w_{G,c}\big|_{x=1}^{\alpha=0} = \overline{w}_G,$$

$$w_{B,c}\Big|_{x=1}^{\alpha=0} = \overline{w}_B = 1 - \overline{w}_G,$$

implying that the aggregate capital allocation to the two sectors is

$$K_G\Big|_{x=1}^{\alpha=0} = K_0 \times \overline{w}_G,$$
$$K_B\Big|_{x=1}^{\alpha=0} = K_0 \times \overline{w}_B,$$

so that the ratio of the returns in the two sectors is

$$\frac{R_G\big|_{x=1}^{\alpha=0}}{R_B\big|_{x=1}^{\alpha=0}} = \frac{A_G}{A_B}\left(\frac{\overline{w}_G}{\overline{w}_B}\right)^{-1}, \tag{21}$$

which, for the special case in which the productivity shocks are the same across the two sectors, simplifies to

$$\frac{R_G\big|_{x=1}^{\alpha=0}}{R_B\big|_{x=1}^{\alpha=0}} = \left(\frac{\overline{w}_G}{\overline{w}_B}\right)^{-1}. \tag{22}$$

Comparing (19) with (21), or the more transparent (20) with (22), we see that a mandate that imposes that $\overline{w}_G > w_{G,u}$ and/or $\overline{w}_B < w_{B,u}$ will lead to a corresponding change in the relative cost of capital in the two sectors. That is, the cost of capital is tightly linked to portfolio mandates, which is the finding in the literature that relies on endowment models (e.g., Heinkel, Kraus, and Zechner, 2001; Berk and van Binsbergen, 2021).[7]

2.3 Solution for a constant-returns-to-scale production economy ($\alpha = 1$)

We now study the effect of portfolio mandates in a constant-returns-to-scale production economy ($\alpha = 1$) and contrast it to that in an endowment economy, which we studied above. Just as before, to establish the benchmark, we start by looking at the equilibrium when no investor faces a portfolio mandate, which is equivalent to setting $x = 0$ in the general single-period model.

[7] Moreover, because in this section we are looking at the setting where all investors are constrained by the mandate (i.e., $x = 1$), not surprisingly, we find that the

$$\text{Effective mandate pass-through} = \frac{K_G\big|_{x=1}^{\alpha=0} - K_G\big|_{x=0}^{\alpha=0}}{K_0 \times x \times \left(\overline{w}_G - w_{G,u}\big|_{x=0}^{\alpha=0}\right)} = \frac{K_0\overline{w}_G - K_0 w_{G,u}\big|_{x=0}^{\alpha=0}}{K_0 \times \left(\overline{w}_G - w_{G,u}\big|_{x=0}^{\alpha=0}\right)} = 1.$$

2.3.1 Without a portfolio mandate

Proposition 3. *In a constant-returns-to-scale production economy ($\alpha = 1$) in which no investor is constrained by a portfolio mandate ($x = 0$), the optimal portfolio weights in equilibrium are*

$$w_{G,u}\big|_{x=0}^{\alpha=1} = \frac{\mu_{A_G} - \mu_{A_B} + \gamma K_0\big(\sigma^2_{A_B} - \mathrm{Cov}[A_G, A_B]\big)}{\gamma K_0 \,\mathrm{Var}[A_G - A_B]}, \tag{23}$$

$$w_{B,u}\big|_{x=0}^{\alpha=1} = \frac{\mu_{A_B} - \mu_{A_G} + \gamma K_0\big(\sigma^2_{A_G} - \mathrm{Cov}[A_G, A_B]\big)}{\gamma K_0 \,\mathrm{Var}[A_G - A_B]}, \tag{24}$$

implying that the aggregate capital allocation to the two sectors is

$$K_G\big|_{x=0}^{\alpha=1} = K_0 \times w_{G,u}\big|_{x=0}^{\alpha=1},$$
$$K_B\big|_{x=0}^{\alpha=1} = K_0 \times w_{B,u}\big|_{x=0}^{\alpha=1},$$

and that the risk-free interest rate is

$$R_f\big|_{x=0}^{\alpha=1} = \frac{\mu_{A_G}\sigma^2_{A_B} + \mu_{A_B}\sigma^2_{A_G} - \mathrm{Cov}[A_G, A_B]\,(\mu_{A_G} + \mu_{A_B}) - \gamma K_0\left(\sigma^2_{A_G}\sigma^2_{A_B} - \mathrm{Cov}^2[A_G, A_B]\right)}{\mathrm{Var}[A_G - A_B]}, \tag{25}$$

where

$$\mathrm{Var}[A_G - A_B] = \sigma^2_{A_G} - 2\,\mathrm{Cov}[A_G, A_B] + \sigma^2_{A_B}. \tag{26}$$

The ratio of the returns (cost of capital) in the two sectors is

$$\frac{R_G\big|_{x=0}^{\alpha=1}}{R_B\big|_{x=0}^{\alpha=1}} = \frac{A_G}{A_B}, \tag{27}$$

which is independent of the capital allocated to these two sectors. For the special case in which the productivity shocks are the same across the two sectors, this ratio reduces to

$$\frac{R_G\big|_{x=0}^{\alpha=1}}{R_B\big|_{x=0}^{\alpha=1}} = 1. \tag{28}$$

2.3.2 With a portfolio mandate for all investors

We now study the effect of introducing in a constant-returns-to-scale production economy ($\alpha = 1$) a portfolio mandate for all investors, which is equivalent to setting $x = 1$ in the general model.

Proposition 4. *In a constant-returns-to-scale production economy ($\alpha = 1$) in which all investors are constrained by a portfolio mandate ($x = 1$), in equilibrium, the optimal portfolio weights for all investors are the mandated weights,*

$$w_{G,c}\big|_{x=1}^{\alpha=1} = \overline{w}_G, \tag{29}$$

$$w_{B,c}\big|_{x=1}^{\alpha=1} = \overline{w}_B = 1 - \overline{w}_G, \tag{30}$$

implying that the aggregate capital allocation to the two sectors is

$$K_G\big|_{x=1}^{\alpha=1} = \overline{w}_G K_0,$$

$$K_B\big|_{x=1}^{\alpha=1} = \overline{w}_B K_0.$$

The ratio of the returns in the two sectors, then, is

$$\frac{R_G\big|_{x=1}^{\alpha=1}}{R_B\big|_{x=1}^{\alpha=1}} = \frac{A_G}{A_B}, \tag{31}$$

which, for the special case in which the productivity shocks are the same across the two sectors, again reduces to

$$\frac{R_G\big|_{x=1}^{\alpha=1}}{R_B\big|_{x=1}^{\alpha=1}} = 1. \tag{32}$$

Comparing (27) with (31), or (28) with (32), we see that a mandate that imposes that $\overline{w}_G > w_{G,u}|_{x=0}^{\alpha=1}$ and/or $\overline{w}_B < w_{B,u}|_{x=0}^{\alpha=1}$ will *not* lead to any change in the relative cost of capital across the two sectors. That is, the cost of capital is unrelated to portfolio mandates, contrary to the finding in the literature that relies on endowment models.

Furthermore, we see that a mandate is fully effective in shifting capital from the B to the G sector. That is, a mandated $\overline{w}_G > w_{G,u}|_{x=0}^{\alpha=1}$ results in a capital allocation $K_0\overline{w}_G > K_0 w_{G,u}|_{x=0}^{\alpha=1}$. This, of course, is not surprising because the mandate is imposed on all investors. In the next section, we study the effectiveness of a portfolio mandate imposed on only a fraction of the investors and show that, if $\alpha = 1$, then the effective mandate pass-through is still 100 percent.

2.3.3 With a portfolio mandate for only some investors

In this section, we study a constant-returns-to-scale production economy in which a fraction $0 < x < 1$ of investors are constrained in their portfolio choice. Because portfolio mandates do not have any effect on the returns in the two sectors when $\alpha = 1$, the equilibrium in this more general case can be characterized by exploiting the results we have for the two special cases where no investors face the mandate ($x = 0$) and where all investors face the mandate ($x = 1$).

Proposition 5. *In a constant-returns-to-scale production economy $\alpha = 1$ where only a fraction $0 < x < 1$ of investors face the portfolio mandate, in equilibrium, the optimal portfolio weights of unconstrained investors, $w_{G,u}|_{0<x<1}^{\alpha=1}$ and $w_{B,u}|_{0<x<1}^{\alpha=1}$, are given by (23) and (24), respectively. The portfolio weights of the group of investors facing portfolio mandates, $w_{G,c}|_{0<x<1}^{\alpha=1}$ and $w_{B,c}|_{0<x<1}^{\alpha=1}$, are given by (29) and (30), respectively. The aggregate capital allocations $K_G|_{0<x<1}^{\alpha=1}$ and $K_B|_{0<x<1}^{\alpha=1}$ are obtained by substituting into equations (14) and (15) the expressions for the portfolio weights $\{w_{G,u}, w_{B,u}, w_{G,c}, w_{B,c}\}$ for the case ($\alpha = 1, 0 < x < 1$). The risk-free interest rate is given by the expression in (25).*

The ratio of the returns (cost of capital) in the two sectors is still independent of the capital allocated to these two sectors: $R_G/R_B = A_G/A_B$, and, noting that $w_{G,c}|_{x=1}^{\alpha=1} = \overline{w}_G$, the effectiveness of the mandate is 100 percent, that is,

$$\textit{Effective mandate pass-through} = \frac{K_G|_{0<x<1}^{\alpha=1} - K_G|_{x=0}^{\alpha=1}}{x \times K_0 \times \left(\overline{w}_G - w_{G,u}|_{x=0}^{\alpha=1}\right)}$$

$$= \frac{K_0 \left(x\, w_{G,c}|_{x=1}^{\alpha=1} + (1-x)\, w_{G,u}|_{x=0}^{\alpha=1}\right) - K_0 \times w_{G,u}|_{x=0}^{\alpha=1}}{x \times K_0 \times \left(\overline{w}_G - w_{G,u}|_{x=0}^{\alpha=1}\right)} = 1.$$

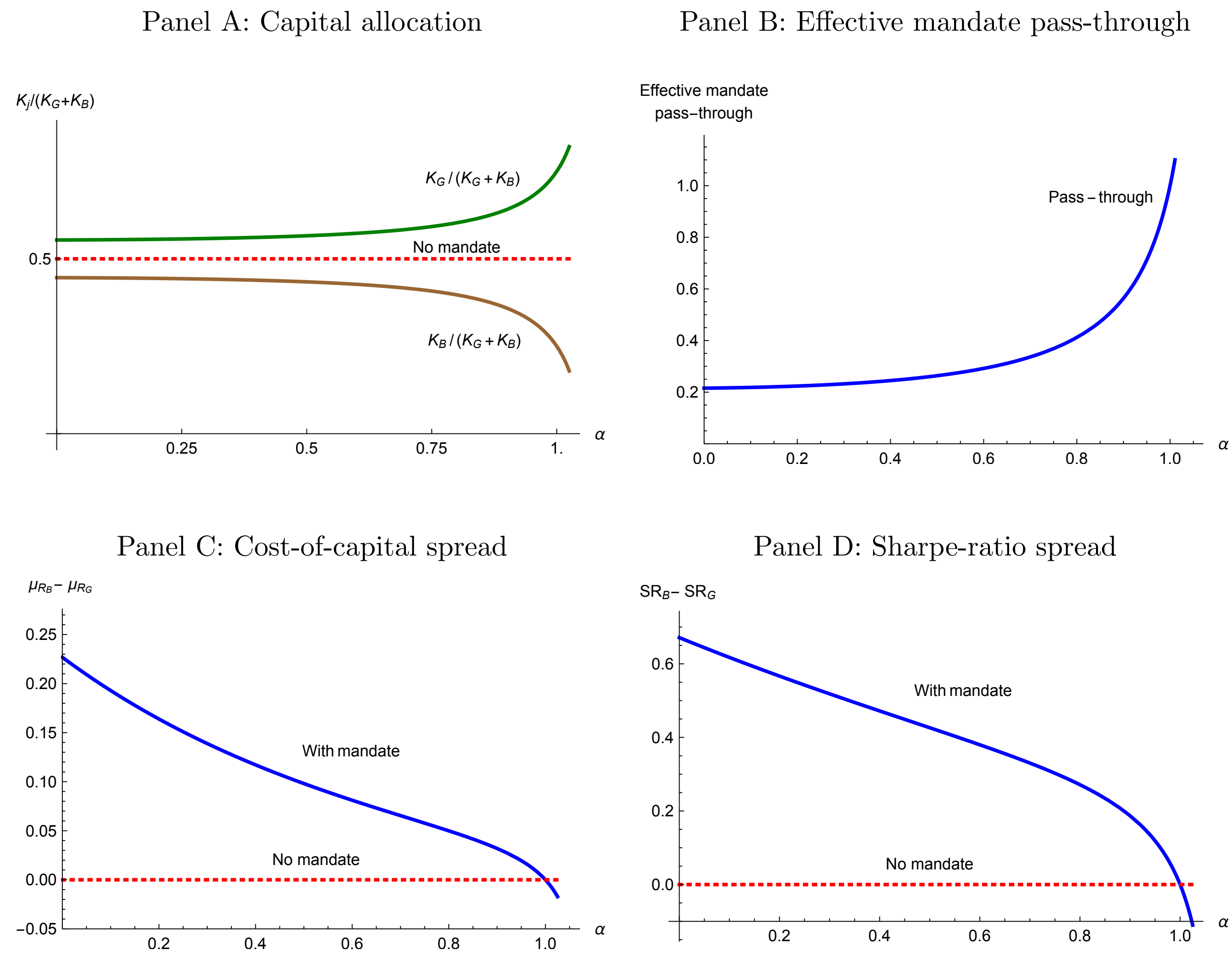
2.4 Solution for a general production economy ($\alpha \geq 0$)

In the sections above, we have studied the equilibrium for two special values of the returns-to-scale parameter for which the model can be solved analytically: the case of $\alpha = 0$, which corresponds to an exchange economy, and the case of $\alpha = 1$, which corresponds to a constant-returns-to-scale exchange economy. In this section, to show that the results we have obtained for the case of $\alpha = 1$ generalize to other values of α, we solve the model numerically for different values of $\alpha \geq 0$.

The model we consider is of an economy in which $x = 50\%$ of the investors face a mandate to invest $w_{G,c}|_{x=0.5}^{\alpha} = \overline{w}_G = 75\%$ of their portfolio in sector G and $w_{B,c}|_{x=0.5}^{\alpha} = \overline{w}_B = 25\%$ in sector B. The quantities of interest obtained from the numerical solution are illustrated in Figure 1, where the parameter values used are: $K_0 = 1$, $\mu_{A_G} = \mu_{A_B} = 1.05$, $\sigma_{A_G} = \sigma_{A_B} = 0.15$, $\rho = 0$, $\gamma = 5$.

Figure 1: Equilibrium capital allocation and cost of capital

Panel A shows the equilibrium capital allocation across the G sector (green line) and B sector (brown line) as a function of the returns-to-scale parameter, α. The dashed red line is the capital allocation without a portfolio mandate. Panel B shows the equilibrium mandate pass-through, defined in equation (17). Panel C shows $\mathbb{E}[R_B] - \mathbb{E}[R_G]$, the spread between the expected return in the B and G sectors, as a function of α. Panel D shows the spread between the Sharpe ratios of the returns in the B and G sectors, $\text{SR}_B - \text{SR}_G$, as a function of α. The parameter values used to generate these plots are: $K_0 = 1$, $\mu_{A_G} = \mu_{A_B} = 1.05$, $\sigma_{A_G} = \sigma_{A_B} = 0.15$, $\rho = 0$, $\gamma = 5$, $x = 0.5$, $\overline{w}_G = 0.75$, $\overline{w}_B = 0.25$.



Panel A of Figure 1 shows how the equilibrium allocation of physical capital varies with the returns-to-scale parameter, α. The panel shows that the mandate increases the allocation of capital in the G sector relative to the benchmark no-mandate case in which the allocation is 50 percent. Thus, the effectiveness of the mandate increases with the returns-to-scale parameter, α.

Panel B of Figure 1 shows the equilibrium mandate pass-through, defined as the capital allocated to the G sector in equilibrium as a fraction of the maximum allocation that would result if we ignored the equilibrium effects on asset prices. Formally, we construct the effective mandate pass-through as defined in equation (17). In the calibration used in Figure 1, $w_{G,u}|_{x=0}^{\alpha} = 50\%$ and $\overline{w}_G = 75\%$, therefore, with $K_0 = 1$ and $x = 0.5$, the denominator equals 12.5%. As Panel B shows, the mandate's effectiveness is small (about 25%) for low values of the returns-to-scale parameter α but can be substantial as α approaches 1, reaching a value of 100 percent when $\alpha = 1$.

This result contrasts sharply with the effect of α on the cost of capital. Panel C of Figure 1 shows that the mandate to invest in the G sector creates a positive spread between the cost of capital in the B and G sectors, $\mathbb{E}[R_B] - \mathbb{E}[R_G]$. The mandate, by creating excess demand for G capital, increases its price and lowers its required return (cost of capital) relative to the B sector. However, the panel also shows that, in equilibrium, the spread $\mathbb{E}[R_B] - \mathbb{E}[R_G]$ decreases with α. In fact, for the case where the returns-to-scale parameter is $\alpha = 1$, the difference in the cost of capital between the two sectors is zero (Panel C), while the difference in the capital allocation is extremely large (Panel A). The results in Panels A and C illustrate that one does not need a higher cost of capital for the B sector relative to the G sector to reduce the capital flowing to the B sector.

Panel D shows that the mandate to invest in the G sector also increases the Sharpe ratio of the brown asset relative to the green asset, making it more attractive for the unconstrained investor to invest in the brown sector. Just as for the cost of capital, the difference in the Sharpe ratio of the B asset relative to the G asset is zero when $\alpha = 1$.

In summary, Figure 1 shows that to fully understand the channels through which portfolio mandates can have an effect, it is essential to consider production models. Models without production, such as the endowment models of Heinkel, Kraus, and Zechner (2001) and Berk and van Binsbergen (2021), where output is exogenous, can lead to the inference that a low cost-of-capital spread also implies a negligible effect on the allocation of real capital across the B and G sectors, which is not true in general. As the case of constant returns to scale shows, the difference in returns can be zero, yet the mandate's real effect can be substantial. Thus, studying the difference in cost of capital for firms in the B and G sectors is generally not the best way to evaluate whether portfolio mandates are effective; instead, one should directly measure the physical capital in each sector.

The results of this section, obtained from an analytically tractable model, illustrated the *qualitative* impact of portfolio mandates in a general-equilibrium production economy. To assess these claims *quantitatively*, we now turn to a state-of-the-art dynamic general-equilibrium production economy model.

3 A multiperiod equilibrium model with portfolio mandates

In this section, we embed portfolio mandates in a canonical neoclassical general equilibrium model with production that is then calibrated to match empirical macroeconomic and asset-pricing moments. Our model, when returns to scale are constant ($\alpha = 1$) and there are no portfolio constraints, is a canonical real-business-cycle model, similar to King, Plosser, and Rebelo (1988) and Jermann (1998), among many others.[8] We use this model to assess quantitatively the impact of portfolio mandates in equilibrium.

In the baseline version of the model, we assume that the technologies for the firms in the green and brown sectors are identical. In the absence of mandates, the equilibrium in this economy implies that each investor allocates an equal fraction of its risky portfolios to the two sectors. As a result, in equilibrium, capital is equally distributed between the green and brown sectors. Portfolio mandates distort this allocation directly, through the portfolio constraint, and indirectly through the equilibrium effect on prices. Solving for the equilibrium in this economy allows us to assess the magnitudes of these distortions quantitatively. In particular, the analysis in this section highlights that the qualitative effects identified in the simple model of Section 2 are also quantitatively substantial. In particular, portfolio mandates can significantly impact the allocation of real capital even when the difference in the cost of capital in the two sectors is negligible.

3.1 The multiperiod model with frictions

3.1.1 Investors

We consider an infinite-horizon economy in discrete time $t = \{0, 1, \ldots\}$. Just as in the previous section, the economy is populated by a continuum of measure-one investors who are infinitely lived and supply labor and invest in firms with one of two production technologies: G and B. A fraction

[8] For example, our model is identical to King, Plosser, and Rebelo (1988) if we shut down capital adjustment costs and set the utility of leisure to zero, and is similar to Jermann (1998) with the only difference being the adjustment cost specification—we use quadratic adjustment costs as in Hayashi (1982).

x of the investors is constrained (c) in that it is subject to a portfolio mandate to hold the risky assets in a given fixed proportion. The remaining fraction $1-x$ of investors is unconstrained (u).

Let $W_{i,t}$, $C_{i,t}$, and $L_{i,t}$ represent, respectively, the net worth, consumption, and labor supply of investor $i=\{u,c\}$. Investors are endowed with one unit of labor that they supply inelastically, that is, $L_{i,t}=1$ for all i and t for the wage ω_t. Let $B_{u,t+1}$ denote the face value at time $t+1$ of the one-period risk-free bond held by the unconstrained investors and by $R_{f,t}$ the risk-free rate; hence, $B_{u,t+1}/R_{f,t}$ represents the time t value of the holdings of the risk-free bond. We denote by $w_{G,i,t}$ and $w_{B,i,t}$ the share of the investible wealth of investor i that is invested in the G and B sectors, respectively. The cum-dividend time-t values of the green and brown firms are, respectively, $V_{G,t}$ and $V_{B,t}$, with dividends $D_{G,t}$ and $D_{B,t}$.

We assume investors have Epstein-Zin recursive preferences with risk aversion γ, elasticity of intertemporal substitution ψ and time-discount parameter β. The unconstrained investor solves

$$U_u(W_{u,t}) = \max_{\{C_{u,t},w_{G,u,t},w_{B,u,t}\}} \left\{(1-\beta)C_{u,t}^{1-1/\psi} + \beta\left(\mathbb{E}_t[U_u(W_{u,t+1})^{1-\gamma}\right)^{\frac{1-1/\psi}{1-\gamma}}\right\}^{\frac{1}{1-1/\psi}}, \tag{33}$$

subject to the intertemporal budget constraint

$$W_{u,t+1} = (W_{u,t}+\omega_t L_{u,t} - C_{u,t})\left(R_{f,t} + w_{G,u,t}(R_{G,t+1}-R_{f,t}) + w_{B,u,t}(R_{B,t+1}-R_{f,t})\right) + \Upsilon_{u,t+1}, \tag{34}$$

where the return $R_{j,t+1}=V_{j,t+1}/V_{j,t}$, $j=\{G,B\}$, with $V_{j,t}$ denoting firm j's value, defined later in equation (36). The term $\Upsilon_{u,t+1}$ in equation (34), represents net lump-sum transfers received by unconstrained investors. Allowing for such transfers helps with the stability of the numerical solution. Without transfers, the constrained investors' wealth share can drift toward zero or one for long periods.[9] In aggregate, the lump-sum transfer is zero-sum, that is, $x\Upsilon_{c,t}+(1-x)\Upsilon_{u,t}=0$. The optimality conditions for the problem (33)–(34) results in three standard Euler equations, one for each of the three financial assets, that is, the bond and the stocks for G and B firms.

The constrained investors' problem is identical to that of the unconstrained investor, with the only difference being that constrained investors cannot choose their equity shares; instead, they face a mandate to invest in the G and B sectors in given proportions, $\overline{w}_{j,c}\in(0,1)$, $j=\{G,B\}$.

[9] We assume that the net transfer to unconstrained investors is a small fraction ξ of the difference in the wealth of the constrained and unconstrained investors. Hence the transfer $\Upsilon_{u,t+1}$ to an unconstrained investor is positive if the constrained investor's wealth is larger than that of the unconstrained and negative otherwise. Since the wealth of each type of investor is close to 50% in equilibrium, the size of the net transfer is very small.

As a result, the optimality conditions for constrained investors consist of a single Euler equation, characterizing the optimal consumption decision.

3.1.2 Firms

There are two types of firms, G and B, which make optimal hiring and investment decisions to maximize shareholders' value. As in a standard neoclassical model, we assume that firms incur convex capital-adjustment costs when making investment decisions (e.g., Hayashi, 1982). We assume that firms are all-equity financed, with investors being the shareholders. Investors' consumption and portfolio decisions result in a flow of capital $K_{j,t}$, $j = \{G, B\}$ into the two sectors of the economy. Firms operate in a perfectly competitive market and produce identical goods but are subject to different productivity shocks.

Firms produce output $Y_{j,t}$ according to a Cobb-Douglas production function

$$Y_{j,t} = (K_{j,t})^{\alpha\theta}(A_{j,t}L_{j,t})^{(1-\theta)}, \tag{35}$$

where $\theta \in [0,1]$ controls the relative importance of capital in the production and $\alpha \in [0,1]$ is a returns-to-scale parameter. The production function exhibits constant returns to scale if $\alpha = 1$ and declining returns to scale if $\alpha < 1$. The quantity $A_{j,t}$ in equation (35) denotes a stationary process representing neutral (TFP) productivity shocks. This shock may contain aggregate or firm-specific components; the aggregate component may have stationary and non-stationary components.

Firms choose labor $L_{j,t}$ and investment $I_{j,}$ to maximize shareholder value. Formally, firm j's value $V_{j,t}$ results from the solution of the following problem

$$V_{j,t}(K_{j,t}) = \max_{L_{j,t}, I_{j,t}} D_{j,t}(K_{j,t}) + \mathbb{E}_t\left[\mathbb{M}_{u,t+1}V_{j,t+1}(K_{j,t+1})\right], \tag{36}$$

where $\mathbb{M}_{u,t+1}$ is the stochastic discount factor (SDF) of the unconstrained investors, the marginal investors in this economy. When maximizing shareholder value, firms take $\mathbb{M}_{u,t+1}$ as given. The optimization in (36) is subject to the capital accumulation equation, which, using $\delta > 0$ to denote capital depreciation, is

$$K_{j,t+1} = (1-\delta)K_{j,t} + I_{j,t}. \tag{37}$$

As is well known, the firm value $V_{j,t}(K_{j,t})$ can be written as $V_{j,t}(K_{j,t}) = K_{j,t} \times Q_{j,t}$, with Qj, t denoting Tobin's Q, or the market-to-book ratio. In the absence of adjustment costs, Tobin's Q is equal to 1, in which case, $V_{j,t}(K_{j,t}) = K_{j,t}$ because capital can be instantaneously transferred to and from consumption. In the presence of adjustment costs, there is a wedge between the price of installed capital (firm value) and uninstalled capital (consumption), and therefore, Tobin's Q will, in general, be different from one.

3.1.3 Labor

In equation (36), $D_{j,t}(K_{j,t})$ represents the dividends firm j distributes to its shareholders. To define this quantity, we need to describe how wages are set in the model. If labor markets were perfectly flexible, the aggregate wage would be far too volatile, having the same properties as output; this would also counterfactually imply that profits and dividends are counter-cyclical and that equity volatility is too low. As shown by Favilukis and Lin (2016), introducing wage rigidity into a production-economy model makes wages, profits, and dividends behave more like in the data and improves the model's asset-pricing performance. Because asset prices are crucial for our mechanism, we introduce wage rigidity in a reduced-form manner.

Specifically, we assume that firms must hire at least labor $\overline{L} < L_{j,t}$ at wage $\overline{\omega}_t$, but are free to choose how much remaining labor, $L_{j,t} - \overline{L}$, to hire, and that labor is paid a competitive wage $\tilde{\omega}_t$ that clears labor markets. Because labor supply is inelastic and set to $L_{j,t} = 1$, the average wage paid is therefore $\omega_t L_{j,t} = \overline{\omega}_t \overline{L} + \tilde{\omega}_t (L_{j,t} - \overline{L})$, which, in equilibrium, is smoother than $\tilde{\omega}_t$. Note that the firm's first-order condition is independent of $\overline{L}$; therefore, this reduced-form way of modeling wage rigidity does not affect the firm's investment choice. However, it does affect dividends, wage paid, firm value, and equity return. Firm j's dividends are therefore given by

$$D_{j,t}(K_{j,t}) = Y_{j,t} - \omega_t L_{j,t} - I_{j,t} - \eta \left(\frac{I_{j,t}}{K_{j,t}} - \hat{\delta} \right)^2 K_{j,t}, \quad \eta > 0,\ \hat{\delta} > 0, \tag{38}$$

where $Y_{j,t}$ is output, defined in equation (35), $\hat{\delta} = \delta + g$ is capital depreciation δ gross of the growth rate g, and the term $\eta \left(\frac{I_{j,t}}{K_{j,t}} - \hat{\delta} \right)^2 K_{j,t}$ represents a quadratic adjustment-cost function.[10]

[10]Because we set gross depreciation to $\hat{\delta} = \delta + g$, adjustment costs are zero in the steady state.

3.1.4 Equilibrium

An equilibrium of this economy consists of the following: (i) investors' consumption and portfolio policies, $\{C_{i,t}, w_{G,i,t}, w_{B,i,t}\}$; (ii) firms' investment and hiring policies, $\{I_{j,t}, L_{j,t}\}$; (iii) wages $\tilde{\omega}_t$; (iv) prices of the two risky assets, $\{V_{G,t}, V_{B,t}\}$, and the risk-free rate, $R_{f,t}$, such that: investors maximize their lifetime utility in equation (33), firms maximize shareholder value in equation (36), and the markets for labor, the two risky assets, and the risk-free asset clear. By Walras' law, the goods market automatically clears; that is, the aggregate budget constraint holds.

3.2 Calibration

We solve numerically for an equilibrium in the economy described above using dynamic programming. We calibrate the model's parameters at an annual frequency to match key macroeconomic and asset-pricing moments. Table 1 shows the parameter values used in our baseline calibration. In our benchmark case, we consider a coefficient of relative risk aversion $\gamma = 5$. However, we also solve the model with higher risk aversion, up to $\gamma = 50$, to explore the model's implications with a more realistic value of the equity risk premium. We set the elasticity of intertemporal substitution (EIS) to $\psi = 0.2$ so that for the benchmark case of $\gamma = 5$, the investors' preferences are time-separable CRRA. We set $\beta = 0.9422$ to target a ratio of capital to output K/Y of around 2.9 in the steady state and an aggregate growth rate of $g = 1.5\%$. We assume that 50% of investors are subject to a portfolio mandate ($x = 0.5$) requiring them to hold their wealth in the ratio 75% to 25% between the G and B sectors. We set the wealth transfer parameter at the end of each period to $\xi = 0.01$.

We choose parameters for the Markov chain describing the TFP process to match the volatility and autocorrelation of Hodrick-Prescott (H-P) filtered output.[11] Specifically, we assume that the firm's productivity is separated into aggregate and industry-level components: $A_t^j = A_t Z_t^j$. The aggregate component $A_t = (1+g)^t$ captures the growth trend. The industry component $Z_t^j = 1 + z_t^j$ drives the business cycle and follows a 2-state Markov chain, with $L = 0.912$ and $H = 1.088$, and with probability $p = 0.82$ of staying in the current state.

We set the capital adjustment cost $\eta = 5$ to match investment volatility. We set the fraction of labor receiving a fixed wage $\overline{L} = 0.50$ so that the volatility of wages is about half that of output, which also implies reasonable values for the volatility and procyclicality of dividends and profits.

[11] We use a filtering parameter of 100, as proposed by Backus and Kehoe (1992).

Table 1: Parameter values

The table reports the values for the parameters used in the benchmark calibration of the multiperiod model described in this section.

Parameter	Symbol	Benchmark value
Investors		
Relative risk aversion	γ	5.0
Elasticity of intertemporal substitution	ψ	0.2
Time discount rate	β	0.9422
Fraction of constrained investors	x	0.50
Portfolio mandate in G capital	$\overline{w}_G$	0.75
Portfolio mandate in B capital	$\overline{w}_B$	0.25
Faction of labor receiving government fixed	$\overline{L}$	0.5
Investors' wealth transfer fraction	ξ	0.01
Firms		
Aggregate growth rate	g	0.015
Low TFP shock realization	L	0.912
High TFP shock realization	H	1.088
Probability of remaining in current state	p	0.82
Depreciation rate	δ	0.06
Capital adjustment cost	η	5.0
Parameter controlling the capital share	θ	0.35
Return to scale	α	1.0

We set depreciation $\delta = 0.06$, a standard value in the literature. We set capital share $\theta = 0.35$ so that 65% of output is paid to labor. In our baseline model, we set returns to scale to be constant, that is, $\alpha = 1.0$. We also solve models with decreasing returns to scale: $\alpha = 0.90$ and $\alpha = 0.80$. For these models, for the model to match the target moments (specifically, labor share and the capital-to-output ratio) β rises to 0.952 and 0.963, respectively, and $\alpha\theta$ falls to 0.32 and 0.28. Finally, to explore the implications of increasing returns to scale, we also solve the model for $\alpha = 1.02$.

In our calibration, we allow for the existence of a government sector which enables us to distinguish between total and private-sector GDP. It is well known that the latter is much more volatile than the former. To model the government in a simple way, we assume that the actual amount of labor supplied by investors is 1.35 instead of 1.0, as described in the model section above, with 1.0 working in the private sector and 0.35 in government. Unlike private-sector employees, government employees are paid a constant wage adjusted for growth. That is, the government wage rate is set to $\overline{\omega}(1+g)^t$ where $\overline{\omega}$ is the unconditional average of the detrended market-clearing

Table 2: Macroeconomics moments

The table shows macroeconomics moments from the model and compares them to corresponding quantities in the data. All variables, other than the Share of GDP, are H-P filtered. Volatility is in annual percentage units. GDP-P refers to private sector GDP. The values in the "Model" columns are obtained by solving a version of the model with a portfolio mandate that constrains 50% of investors to invest 75% of their wealth in firms in the G sector and 25% in firms in the B sector. The model is calibrated at an annual frequency. Parameter values are reported in Table 1.

	Share of GDP		Volatility (%)		Corr with GDP		Autocorr	
	Data	Model	Data	Model	Data	Model	Data	Model
GDP	1.00	1.00	2.33	2.32	1.00	1.00	0.54	0.35
GDP-P	0.80	0.81	2.74	2.85	0.91	1.00	0.48	0.35
Consumption	0.63	0.64	1.72	1.60	0.91	0.99	0.53	0.34
Investment	0.17	0.18	7.60	7.42	0.78	0.99	0.45	0.34
Wages	—	—	1.17	1.42	0.49	1.00	0.58	0.35

wage. Hence, total government expenses are equal to $0.35 \times \overline{\omega}$ and total labor income is then $(\omega_t \times 1) + (\overline{\omega} \times 0.35)$. We assume that government expenditure equals a lump-sum tax levied on total labor income. With this assumption, the problem's solution is independent of government size. The only quantity affected by government expenditure is total GDP, which is equal to the sum of private-sector GDP and government expenditure. The choice of 1.35 for total labor implies that private sector GDP is 80% of total GDP, as in the data.[12]

Table 2 compares macroeconomic moments in the data to corresponding quantities in the multiperiod model, under the assumption that 50% of investors face a mandate to invest 75% of their wealth in firms in the G sector and 25% in the B sector. The values reported in the table are obtained by simulating the model for 10,000 years and using a 100-year burn-in period. The table reports five quantities: total GDP, private-sector GDP (GDP-P), Consumption, Investment, and Wages. For each quantity, we compute the share of GDP, the volatility, the correlation with GDP, and the autocorrelation and compare them to the corresponding values in the data. The table shows that the model matches key macroeconomic moments reasonably well under the baseline parameters of Table 1. These moments remain largely unaffected by different values of risk aversion; therefore, in the table, we report results only for the benchmark case of $\gamma = 5$. The only moments significantly different from the data are the correlations of investment and wages with GDP, which, in the data,

[12] Note that labor is approximately 65% of output, so if private labor is 1.0, then private output is 1.0/0.65=1.54. Government labor, which equals government output, is 35%. Therefore private output as a share of total output is 1.54/(1.54+0.35)=81%.

Table 3: Asset-pricing moments

The table shows the annual mean and volatility of the risk-free rate, $\mathbb{E}[R_f]$ and $\sigma(R_f)$, and of the market risk premium, $\mathbb{E}[R^M - R_f]$ and $\sigma(R^M - R_f)$ obtained from a model with a portfolio mandate that constrains 50% of investors to invest 75% of their wealth in firms in the G sector and 25% in firms in the B sector. The model is calibrated at an annual frequency. The equity return is levered using a leverage ratio of 2. Parameter values are reported in Table 1. Values in the Data column are based on the sample period 1950–2021 and are from Ken French's website.

		Model				
	Data	$\gamma = 5$	$\gamma = 10$	$\gamma = 25$	$\gamma = 40$	$\gamma = 50$
$\mathbb{E}[R_f]$	0.91	5.78	5.62	5.06	4.53	4.17
$\sigma(R_f)$	2.27	3.27	3.24	3.21	3.23	3.29
$\mathbb{E}[R^M - R_f]$	8.99	1.42	1.76	2.84	3.92	4.36
$\sigma(R^M - R_f)$	17.89	16.32	16.28	16.16	16.00	16.28

are much less than in the model. This is not surprising because, with only one aggregate shock, model correlation with GDP tends to be close to 1.

Table 3 reports the asset-pricing moments: the annual mean and volatility of the risk-free rate and the equity-market risk premium. The equity return used to compute the market risk premium is levered using a factor of two, equivalent to an economy-wide 50/50 debt/equity ratio. The table shows that the model does a good job of matching the volatility of the risk-free rate and the equity risk premium in the data. However, not surprisingly, for the case of low risk aversion, $\gamma = 5$, the risk-free rate is too high, and the equity risk premium is too low. This is just a manifestation of the equity-premium puzzle. Higher values of risk aversion in the table result in values of the equity premium and risk-free rate that are closer to the data.[13]

3.3 Equilibrium effects of portfolio mandates

Below, we describe the equilibrium effects of portfolio mandates in the multiperiod model of a production economy. Table 4 contains our main quantitative results about the equilibrium effects of portfolio mandates. The fundamental goal of the analysis is to contrast capital allocation measures (Panels A and B) and cost of capital measures (Panels C and D) for different values of the returns-to-scale parameter, α.

[13] Note that as we change risk aversion, EIS stays constant, which explains why the macroeconomic moments don't change much.

Table 4: Equilibrium effects of portfolio mandates

The table shows the equilibrium effect of portfolio mandates on capital allocation and cost of capital for different values of the risk aversion and returns-to-scale parameters. These values are obtained from a model with a portfolio mandate that constrains 50% of investors to invest 75% of their wealth in firms in the G sector and 25% in the B sector. Panel A reports the share of capital allocated to the green sector as a fraction of total capital, $K_G/(K_G+K_B)$; Panel B computes the effective mandate pass-through ratio defined in equation (39); that is, the fraction of the intended pass-through effect that survives general equilibrium effects; Panel C reports the difference in the equilibrium cost of capital between G and B sectors; and Panel D reports the difference in Sharpe ratios between firms in the G and B sectors. Parameter values are reported in Table 1.

	Risk aversion				
Return to scale	$\gamma = 5$	$\gamma = 10$	$\gamma = 25$	$\gamma = 40$	$\gamma = 50$
	Panel A: Capital allocation, $K_G/(K_G + K_B)$				
$\alpha = 1.02$	0.549	0.558	0.564	0.564	0.565
$\alpha = 1.00$	0.527	0.539	0.546	0.552	0.555
$\alpha = 0.90$	0.503	0.505	0.511	0.515	0.517
$\alpha = 0.80$	0.501	0.501	0.502	0.503	0.504
	Panel B: Effective mandate pass-through ratio (%)				
$\alpha = 1.02$	39.20	46.56	50.96	51.52	51.60
$\alpha = 1.00$	21.60	31.20	36.80	41.60	44.00
$\alpha = 0.90$	2.40	4.00	8.80	12.00	13.60
$\alpha = 0.80$	0.80	0.80	1.60	2.40	3.20
	Panel C: Difference in cost of capital, $R_B - R_G$ (%)				
$\alpha = 1.02$	-0.02	-0.02	0.00	0.00	0.00
$\alpha = 1.00$	0.04	0.08	0.12	0.14	0.14
$\alpha = 0.90$	0.06	0.10	0.18	0.19	0.17
$\alpha = 0.80$	0.05	0.07	0.10	0.12	0.13
	Panel D: Difference in Sharpe ratio, $SR_B - SR_G$				
$\alpha = 1.02$	-0.0004	0.0012	0.0063	0.0106	0.0120
$\alpha = 1.00$	0.0040	0.0070	0.0115	0.0148	0.0131
$\alpha = 0.90$	0.0033	0.0057	0.0104	0.0124	0.0114
$\alpha = 0.80$	0.0030	0.0037	0.0054	0.0066	0.0070

Table 4 considers values of the returns to scale parameter ranging from $\alpha = 0.80$ (decreasing returns to scale) to $\alpha = 1.02$ (increasing returns to scale). Empirical estimates from the macroeconomic literature indicate that returns to scale are nearly constant in the US economy. A series of influential papers Hall (1988, 1990) argues that market power and increasing return to scale can explain procyclical productivity in the US. In subsequent work, Basu and Fernald (1997) estimate constant or slightly decreasing returns to scale in the US economy. They note, however, that estimates of returns to scale vary at different levels of industry aggregation. While a typical industry exhibits decreasing returns, the total manufacturing and private economy show increasing returns. They suggest that there may be economies of scale at the aggregate level in that as the scale of production increases, the average cost of production decreases. Ahmad, Fernald, and Khan (2019) present new estimates of returns to scale for the US economy based on two separate industry datasets and compare them to previous estimates in the literature. They find evidence of constant or slightly decreasing returns to scale at the aggregate level in the US economy over 1989–2014, consistent with a relatively small aggregate markup in the post-1990 period. While their evidence points to constant or declining returns to scale, Ahmad, Fernald, and Khan (2019) do not rule out the possibility of increasing returns to scale in specific industries or regions or the presence of certain factors, such as technological progress or network effects. Increasing returns to scale might be particularly relevant for "green technologies" where learning by doing and increased scale have led to a dramatic decline in costs over the past 25 years; for example, Way, Ives, Mealy, and Farmer (2022) argue that, unlike traditional technologies such as oil and gas, clean-energy technologies are on learning curves, where costs drop as a power law of cumulative production. This might be particularly relevant for "green technologies" in a model with different types of capital. In light of this evidence, in our analysis in Table 4, we allow for both decreasing, constant, and (slightly) increasing returns to scale.

Panel A of Table 4 reports the equilibrium fraction of capital flowing to firms in the G sector, $\frac{K^G}{K^G+K^B}$. Because we assume that the technologies of firms in the two sectors are identical, the optimal unconstrained investor's portfolio is equally weighted between the G and B sectors. Hence, all entries in Panel A should equal 0.50 in a world without mandates. The values in the table refer, however, to the case in which constrained investors are mandated to hold 75% of wealth in firms in the G sector and 25% in the B sector, implying that the *maximum* proportion of capital allocated to the G sector as a result of the mandate is $(50\% + 75\%)/2 = 62.5\%$. Panel A shows that the *equilibrium* allocation of capital to the G sector varies between 50% to 55.5% depending on the returns-to-scale parameter α and risk aversion γ. The deviation from the unconstrained 50/50 allocation is particularly strong for the case of constant returns to scale ($\alpha = 1$) and high risk aversion. For example, the equilibrium capital allocation to the G sector is 55.5% for $\alpha = 1$ and $\gamma = 50$. Although levels of risk aversion of $\gamma = 25$ or 50 are clearly unreasonable, they are considered here as a reduced-form way of capturing high risk premia in the economy arising from, e.g., limited participation, taxes, and intermediary frictions.[14]

[14] For example, in standard habit models, (e.g., Campbell and Cochrane, 1999), while the curvature parameter in the utility function is 2, the average effective risk aversion is around 80.

To evaluate the magnitude of the equilibrium effect of mandates on capital allocation, we compute the "effective mandate pass-through," defined in (17), which we report in Panel B of Table 4. To construct the pass-through, we first compute the maximum effectiveness of a mandate, ignoring any equilibrium consideration. In our setting, because the constrained investor represents 50% of the entire mass of investors, a portfolio mandate of 75% in G and 25% in B implies that 62.5% (= $0.5 \times 75\% + 0.5 \times 50\%$) of the entire capital should be allocated to the G sector. Under this "partial equilibrium" intuition, the *maximum* deviation from the unconstrained 50/50 allocation is, therefore, $12.5\% = 62.5\% - 50\%$. Using the equilibrium allocation to G in Panel A, denoted by $K_G =|^{\alpha}_{x=50\%}$, we can then construct the effective mandate pass-through ratio of the mandate as the equilibrium pass-through expressed as a percentage of the maximum pass-through:

$$\text{Effective mandate pass-through ratio} = \frac{K_G\big|^{\alpha}_{x=50\%} - 0.50}{0.125}. \tag{39}$$

The values of the effective mandate pass-through ratio in Panel B of Table 4 show that, although general-equilibrium effects undo part of the mandate, a significant part remains effective. For example, with a risk aversion of 5 and constant returns to scale ($\alpha = 1$), about 21.60% of the mandate remains effective. Intuitively, by increasing the cost of capital of firms in the B sector, the mandate makes them more attractive to unconstrained investors who trade off higher returns for worse diversification. As risk aversion increases and risk premia increase, the equilibrium allocation further deviates from the unconstrained 50%. For relative risk aversion of $\gamma = 50$ and constant returns to scale, the pass-through is 44%. Thus, the results in Panel B show that the effect of mandates on the allocation of real capital is particularly strong when risk premia are close to their values in the data.

Panels C and D report the effect of mandates on the firms' cost of capital and Sharpe ratios, respectively. Unlike the significant impact documented in Panels A and B, the effect on the cost of capital (Panel C) and Sharpe ratios (Panel D) are minimal. For example, in our baseline calibration ($\gamma = 5, \alpha = 1.0$), the cost of capital of firms in the B sector is only four basis points higher than that of firms in the G sector (Panel C), and the Sharpe ratio is only 0.004 units higher. This negligible difference in the cost of capital contrasts with the significant mandate pass-through effect of 21.6% reported in Panel B. The contrast between the mandate's "real" and "financial" effects is even stronger when risk premia are closer to their value in the data ($\gamma = 50$). In this case, the difference in the cost of capital under constant returns-to-scale is 14 basis points, while the mandate pass-through is 44%.

In sum, the results from our quantitative model support the central intuition developed in the simple model of Section 2. Specifically, in an economy with production, the difference in the cost of capital is a poor metric to assess the real impact of portfolio mandates in equilibrium. Mandates can significantly impact capital allocation while having a negligible effect on firms' cost of capital. These findings caution against using the cost of capital to measure the effectiveness of portfolio mandates in equilibrium; instead, one should measure the flow of capital.

4 Conclusion

In this paper, we examine the impact of portfolio mandates on the allocation of physical capital in a general-equilibrium economy with production and heterogenous investors. In contrast to the existing literature that has studied impact investing in models of an endowment economy, we consider a production economy that nests the endowment economy as a special case.

To assess the quantitative importance of the effect of portfolio mandates, we study a dynamic general equilibrium production economy. Under a realistic calibration of the multiperiod model that matches asset-pricing and macroeconomic moments of the US economy, we find that the effect of portfolio mandates on the allocation of physical capital across sectors can be substantial. In contrast, the impact on the equilibrium cost of capital and Sharpe ratio of firms in the two sectors remains negligible, consistent with existing evidence.

Thus, a key takeaway of our analysis is that judging the effectiveness of portfolio mandates by studying their effect on the cost of capital of affected firms can be misleading: small differences in the cost of capital across sectors can be associated with significant differences in the allocation of physical capital across these sectors.

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