Factor Based Pension Portfolio Strategies for Sustainable Withdrawals

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Abstract

This research contributes by exploring the use of factor investment strategies to create sustainable drawdown (withdrawal) strategies. Stock portfolios constructed from FTSE 350 were sorted by four factors, (size, book to market, profitability and volatility) and were then analysed. The results showed that most of our constructed portfolios will sustain the 4% fixed real rate of withdrawal. Furthermore, 4 portfolios (H,Lv (value and low volatility stocks), S,H,Lv (small, value and low volatility stocks), S,Lp,Lv (small, low profitability and low volatility stocks) and H,Lp,Lv (value, low profitability and low volatility stocks)) were identified to sustain withdrawals up to 10%. This result suggests that the low volatility factor seems to be a driver of withdrawal sustainability. This research also studied conditional expected failure time to further examine withdrawal sustainability and our results showed that the four mentioned portfolios have very similar conditional expected failure time. Our relative stability measure proposed showed inconsistent results with the withdrawal success rate but a much stronger consistency with the result of the failure time point.

Section 1

Background and Introduction

Retirement savings are part of a complex financial planning behaviour and require a multidimensional model that involves individuals to take certain decisions (Husin and Rahman, 2013). In fact, they require individuals to act and make decisions on something that they find hard to visualise, since savings for retirement require prompt action for something that will be used in the future. Global demographic changes and retirement savings adequacy have shown that voluntary pension is a new paradigm for the retirement system. The topics of retirement, saving and pensions continue to be the focus of interest among both policymakers and the media. This interest is stimulated by press stories concerning the adequacy of personal saving, the state of pension funds, the looming 'demographic crisis' and so on.

In the UK, there are broadly 2 pension vehicles: the defined contribution plan (DC) and the defined benefit plan (DB). While the DB is usually an occupational plan, the DC can be a personal or occupational plan. The defined benefit plan has certain peculiarities but the defined contribution plan (often known as money purchase plans) are effectively a savings holding which is invested for capital growth and/or income with the objective to fund the retirement phase of financial planning.

A defined-contribution (DC) pension scheme provides an income for a pensioner after retirement from a fund built-up from investing a series of contributions during their period of employment. The financial risk is taken by the member of the scheme and there is no guarantee of a fixed benefit level at retirement. The savings journey is split into two phases. During the accumulation (or pre-retirement) phase, the scheme member and/or their employer contribute to the pension fund, which is invested in a portfolio of assets with a particular risk profile. In the distribution (or post-retirement) phase the pensioner receives periodic income from the fund to provide support in old age.

Prior to 2015, in the UK, a pensioner had to purchase an annuity (guaranteed income from an insurance company) with whatever value of his pension fund at age 75. The introduction of the pension's freedom legislation in April 2015 significantly changed the pension investment landscape especially with the abolition of the need to annuitize at age 75. The abolition of this need effectively created a number of flexibilities, however, a consequence of this is that pension advisers now need to offer solutions providing sustainable income through retirement entirely with the use of the DC plan.

Following this, the investment journey of the investor usually poses two main considerations for advisers designing retirement strategies.

a) How much should be saved or accumulated to create a sufficient pot for decumulation? (Decumulation is used interchangeably as pension withdrawal henceforth). This encompasses the two fundamental objectives of a traditional portfolio investment namely, capital growth and/or income generation.
b) What withdrawal strategy should be adopted during retirement to ensure a sustainable rate of withdrawal? It is this consideration that makes a third investment objective apparent; stability/sustainability of the portfolio.

As Merton (2014) stated, the construction of investment portfolios for both the pre-retirement and retirement phases has been relatively neglected in the study of retirement planning, even more obvious is the lack of insight into the construction of retirement income strategies to provide appropriate solutions in line with the changing (more flexible) retirement savings environment. Hence, the motivation for this research is to explore how an investment strategy (factor investing) may provide a better "fixed-real" withdrawal experience relative to the market and to propose and examine the use of a relative stability measure as an indicator of withdrawal stability/sustainability of portfolios.

We identify 4 factors of excess return and then created portfolios based on these factors. We also proposed a measure of relative stability using the proportionality factor of reversion from the Augmented Dickey Fuller model (β – speed of reversion), and σ which is one unit deviation of the shock (error term) hence, $\frac{|\beta|}{\sigma}$. This was then estimated for the portfolios formed. We then establish the performance of these portfolios during withdrawals in comparison to their market and then assessed if the proposed measure of stability can be used as an indicator of stability/sustainability.

In doing this, we have carried out a simulation analysis to establish the success rates of the portfolios at different rates of withdrawal. The success rate is defined as the number of paths out of the total paths created when the portfolio balance is equal to or greater than £0. In addition to this, we have also assessed the correlation of the success rates with the relative stability measure to test whether it can be used as a proxy for portfolio withdrawal stability/sustainability.

We find that diversified portfolios constructed based on individual factors such as size, book to market, profitability and volatility offer the potential to be more successful in sustaining withdrawals of up to 8% compared to the FTSE350 market. When these factors are combined, more portfolios are created with some of which sustained withdrawals at 10% with high levels of success. Four particular portfolios were identified as the most successful; *H,Lv* (value and low volatility stocks), *S,H,Lv* (small, value and low volatility stocks), *S,Lp,Lv* (small, low profitability and low volatility stocks) and *H,Lp,Lv* (value, low profitability and low volatility stocks)). The result indicates that the low volatility factor *Lv* as well as the value factor *H* appears to be drivers of sustainable withdrawals.

We also find that these successful portfolios produced bequest funds (residual portfolio fund usually left as death benefit for beneficiaries) of up to 72% of the largest bequest fund obtained from the entire portfolios formed. In addition to this, when failure occurs, these portfolios sustained withdrawals for a considerable amount of time before failing. Furthermore, whilst the implication of the relative stability measure of the portfolios was not consistent with the results of the success rates, it appears to be more reliable when comparing portfolios with similar returns.

The rest of this paper is organized as follows. Section 2 reviews the literature and Section 3 describes the data and methodology. Section 4 presents the results and Section 5 concludes.

Section 2

Literature Review

Withdrawals at the drawdown phase of the retirement journey are generally either fixed or variable, nominal or real, and the most attention has generally been given to fixed, real withdrawals since Bengen (1994) showed that an initial withdrawal rate of 4%, with annual withdrawals subsequently adjusted by inflation, was 'safe' in the sense that, historically, this strategy never depleted a portfolio in the US in less than 30 years. Pfau, (2010), Blanchett et al, (2016) however showed in subsequent research that over the 115 years between 1900 and 2014, a 60% equity-40% bond portfolio of U.S. stocks and bonds had a failure rate[1] of 4.7%, and portfolios with at least 70% in U.S. stocks had an even lower (3.5%) failure rate. In other markets, much higher failure rates are experienced with the 4% rule.

The 4% fixed-real strategy (Bengen, 1994) is where 4% of the initial portfolio value is set at the initial withdrawal value and this value is subsequently adjusted for inflation in subsequent years. Other variations of the 'fixed' strategy include 'floor and ceiling' strategy (Bengen, 2001), where the 4% from the fixed-real approach is subject to an upper and lower bound; 'modified 4%' strategy where Clyatt (2005) specified that the withdrawal amount in any year should be 4% of the portfolio value in that year rather than the year of drawdown inception; 'constant probability of failure' strategy (see Frank, Mitchell and Blanchett 2011) where the goal is to determine the percentage that can be withdrawn each year based on the idea of maintaining a constant probability of

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[1] *Failure Rate is the number of periods relative to the total number of simulated periods (or periods considered) when the specified withdrawal rate is not sustained through the period.*

failure through time; ‘decision rule’ strategy (see Guyton and Klinger 2006) where dynamic rules such as capital preservation and prosperity rules are used to guide withdrawals; ‘safe reset’ strategy (see Stein and DeMuth 2005), where the withdrawal rate is a function of the retiree's age and adjusted only for inflation for five years before being reset to a new withdrawal rate determined by the expected number of years remaining in the person's retirement.

More recently the perfect withdrawal rate (PWR) was introduced by Suarez et al, (2015), the methodology involves using Monte Carlo methods to create a distribution of the PWR with a view to considering the likelihood of any particular withdrawal rate leading to success/failure over the ensuing decumulation period. Some fixed strategies have also taken advantage of incorporating the use of annuities for example, ‘half annuity’ strategy (see Updegrave 2007) and ‘delayed annuity’ strategy (see Clements 2007). Fixed withdrawals are generally easier to understand hence why it is attractive to advisers as it is simple and practical to convey to clients. Also, in the case of fixed real withdrawals, they preserve purchasing power. However, they do not adjust to changing market conditions or life expectancy: this may lead to depletion of a retirement portfolio earlier than desired, with calamitous results.

Meanwhile, variable withdrawals do adjust to changing conditions and hence reduce or eliminate the risk of failure. They encompass a broad set of strategies in which withdrawals are adjusted based on changing life expectancy (Dus et al, 2005), changing market conditions (Estrada, 2016), or both (Stout and Mitchell, 2006). However, they typically are more difficult to understand and implement (see Stout, 2008) and may require a retiree to reduce their real consumption at some point. An optimising retiree might choose to do this depending on their view of longevity risk, as discussed by Milevsky and Huang (2011). In fact, Lui et al (2009) concluded that dynamic and sophisticated decision rules are not viable strategies for the investing public.

By far, the most attention (both in practice and literature) has been given to the 4% fixed-real strategy by Bengen (1994) however, it is not uncommon to find a blend of both fixed and variable withdrawal strategies in practice and in fact, some of these have been prominent in existing literature. Following the examination of available literature by Spitzer, Jeffrey and Sandeep (2007), if attention is confined to real withdrawals from a portfolio over 30 years, the literature appears to contain conflicting results. Withdrawal rates considered safe or sustainable vary from 3 percent to more than 6 percent, while optimal asset allocations range from 50 percent to 100 percent stock. The results are all plausible because the outcome depends on the subjective definition of sustainable and safe.

Whilst the decision of how much is a sustainable amount to drawdown (withdraw) is an important one, an even more important question is what investment strategy should be adopted to potentially maximize the initial issue of sustainable amount. As Merton, (2014) put it, the construction of investment portfolios for the retirement journey is the ‘known unknowns’. One of the objectives of this study is to explore how an investment strategy (factor investing) may provide a better “fixed-real” withdrawal experience relative to the market.

There are a number of portfolio investment strategies targeting either the contents (asset) of the portfolio or the mechanism of investment, and whilst there has been some contributions to the latter such as Clare et al (2020) who found that smoothing the returns on individual assets by simple trend following techniques is a potent tool to enhance withdrawal rates, not much has been done on designing content based strategies with the objective of enhancing the decumulation/withdrawal phase of the retirement journey.

In terms of the content/asset-based strategy, the literature focusses almost exclusively on bond and equity combinations (i.e., various percentage compositions are used to form the portfolios for analysis) however, diversification to other asset classes such as commodities has been shown to dramatically improve the risk-return possibilities for investors for example, Clare et al, (2016) introduced commodities, real estate, and credit and compared the decumulation possibilities with equity/bond portfolios; they found that the former offers a better withdrawal rate experience in general.

The inclusion of bonds in drawdown portfolios are generally to control risk whilst the equity content is the growth/income element of these portfolios. This research attempts to explore asset-based strategies that may enhance the drawdown experience by focusing on equities only portfolios to see if certain peculiarities of stock selection (factor equity strategy) will offer this enhanced experience. This equity only portfolio offers the opportunity to focus on the growth element of portfolios as it is an apparent driver of sustainable rates of withdrawal. Furthermore, this approach strips the analysis of any influence (positive or negative as it may be) of any other asset class and focuses on the exclusive features of the asset class to highlight the strategy itself. More so, Estrada (2016) concluded that an all-equity portfolio is a simple and very effective strategy for retirees to implement and Lui et all (2009) finds that, for real withdrawal rates of 4% or less and expected time horizons of 20 years or less, asset allocation is not an important factor in determining the probability of success, since all asset allocation models tested from 100% bonds to 100% stocks succeeded nearly 100% of the time.

When designing portfolios for the purpose of decumulation, certain parameters for consideration are important. These include Identifying sources of excess return, risk minimization and portfolio durability against shocks. There is a vibrant literature showing that equity strategies involving selection of stocks based on factors such as size, book to market, volatility and profitability have been established to offer returns in excess of the market. This stemmed from the realization that the CAPM (capital asset pricing model) beta was not the only parameter explaining excess return. Researchers such as Banz (1981), Bhandari (1988), Stattman (1980), Barr et al (1985), Chan, Hamao, and Lakonishok (1991), Basu (1983), Fama French (1992 - 2015), Haugen and Heins (1975), Blitz and Vliet (2007), Ang, Hodrick, Xing, and Zhang (2006), Fu (2009) and Huang, Liu, Rhee, and Zhang (2010) are some of the literature all showing that these 4 factors (size, book to market, profitability and volatility) explain excess return to varying degrees. However, as stated above, returns are only one consideration for the withdrawal (drawdown) phase of retirement.

In the withdrawal phase of retirement, the stability of the retirement portfolio is also an apparent consideration. In creating our proposed measure of stability, the concept of mean reversion is employed. Generally, the theory that financial asset markets are efficient is a staple of contemporary finance. While the stylized version of this theory maintains that financial information is disseminated efficiently and, consequently, stock prices are not predictable, there has been a growing body of literature that questions the universality of such a theory. The growth of discontentment with this theory prompted work on other theories that could help to explain the market phenomenon. The proposition that was suggested in financial economics (by keeping in view the idea that "what goes up must come. down"), is the theory of mean reversion. It is effectively the tendency of the market returns to come back to historic values pulled by a gravitational force. In 1985 DeBondt and Thaler effectively established weak forms of market inefficiencies when they documented that stock returns tend to be mean reverting. Other researchers have found evidence of negative autocorrelation, or "mean reversion," in stock returns over long intervals (such as Poterba and Summers (1988), and more recently, Bessembinder et al. (1995), Balvers et al. (2000), and Gropp (2004 (a)).

Using the Augmented Dickey Fuller model to obtain the speed of reversion (regressing coefficient), we propose that this measure in isolation may not give all the information required for a rational decision (on stability) as it only tells on how fast but not how far. To do this we suggest an *ad hock* measure that is relative to volatility which is effectively how far away on average the returns are from the mean. Hence, we propose the measure $\frac{|\beta|}{\sigma}$ which is the speed of reversion per unit deviation of shock. The higher this measure, the more the stability expected.

There are generally 2 approaches adopted to investigate sustainable withdrawal rates. One set of studies often use rolling historical periods, such as 30-year periods from 1951 to 1980, 1952 to 1981, and so on. Therefore, for example, given a 50-year data, sub data of 30-year blocks are obtained in a rolling (overlapping) period fashion. If the results indicate that all portfolios survived at least 30 years for withdrawal rates of 4 percent for example, these studies conclude that a 4 percent withdrawal rate is "sustainable." This approach clearly requires a sizeable data in order to obtain substantial amounts of sub data. Pioneer studies such as Bengen (1994 and subsequent updates), Blanchett (2007 and 2008), Pfau (2010), Cooley, Hubbard, and Walz (1998) are some examples of many employing this approach in their study.

The second approach employs the use of simulations (Monte-Carlo and Bootstrap are the common techniques) where for example, various simulations of a 30 year returns are obtained from an original 30-year period data; this approach is usually employed where there are data constraints (size). Guyton and Klinger (2006), Milevsky, Ho and Robinson (1997), Spitzer, Jeffrey and Sandeep (2007), Pye (2000) and Tezel (2004) are also some examples of this method. Clearly, these 2 approaches in their structure attempt to consider the effect of sequential risk (sequence of returns) and presenting the results in the form of probability estimates of success or failures.

An important question therefore is whether the choice of method used to represent the future affects estimates of the sustainability of a retirement portfolio. Cooley, Hubbard, and Walz (2003) used both methods to calculate portfolio success rates (the percentage of retirement experiences during which the retirement portfolio provided planned withdrawals and finished the period with a positive value) for a range of withdrawal rates, portfolio compositions, and payout periods. Their results showed that both models were generally consistent; a 4 percent withdrawal rate could be sustainable over a 30-year period with a balanced portfolio, but to ensure a 90 percent success rate, a larger allocation to equity and withdrawal rates of less than 4 percent were necessary.

Other less common methods have also been introduced. Ragsdale, Seila, and Little (1994) provide a mathematical algorithm that uses discounted cash flows to determine the optimal withdrawal rate from tax-deferred retirement portfolios and in a new approach, Milevsky and Robinson (2005) introduce the concept of a stochastic present value, which addresses the withdrawal issue from an actuarial perspective.

Spitzer, Jeffrey and Sandeep (2007) argue that the use of the simulation method provides more extensive examination as it creates thousands of different combinations of sub-periods. They pointed that this is more suited for the probability-based conclusion of sustainability.

Section 3

Data and Methodology.

Data has been sourced entirely from DataStream and the sample period covered was from 1969 to 2019. A universe of FTSE 350 and not the FTSE all share index was considered mainly because according to Alan, Rajesh and Angela (2009), London Stock Exchange exhibits a large "tail" of small and illiquid stocks, which are almost certainly not part of the tradable universe of the major institutional investors that make up a large part of the UK market.

Accounting data with year ending in December is used and the portfolios are reconstructed at the beginning of January every year based on the mentioned accounting data. The following data is gathered for sorting the portfolio:

- ME is basically the market capitalisation which is commonly defined as the share price multiplied by the number of shares outstanding.
- Book equity (BE) is the common shareholders' equity in a company and is calculated as the difference between Total Assets and Total Liabilities. The data for BE is between 1996 to 2019, as this data only commenced from 1996 in Datastream[2].

[2] *The data (particularly for BE) had the issue of missing data. Therefore, each year the missing data is excluded, and the available data forms the universe from which the factor portfolios are formed. Furthermore, data for BE only commenced from 1996 on Datastream hence, the analysis in this research have all been harmonised to commence from 1996.*

- The total return was estimated from the total return index of the FTSE All Share with the base date of 1964 and the estimated returns for the FTSE 350 constituents was then extracted from this.
- EBIT (earnings before interest and tax) data was used as a measure of profitability and the profitability factor is obtained as EBIT/BE (as in Fama and French (1992)).
- The measure of volatility was represented by the standard deviation of the monthly return of stocks.

In the following, we explain how FTSE350 companies are sorted according to different criteria. For sorting of market capitalization, the median of the ranked market capitalization for each year was obtained and stocks ranked above this value constituted the large stock portfolio while those ranked below form the small stock portfolio. The '*S'* (small) and '*B*' (big) portfolios are formed here.

For sorting of the book to market factor, which was estimated by the ratio of book value over market capitalisation (BE/ME), the 30^{th} and 70^{th} percentiles break points were established (after ranking in descending value) where stocks within the 30^{th} percentile form the value portfolio and stocks within the 70th percentile formed the growth portfolio. Negative BE firms were excluded in the portfolio formation. The '*H'* (value) and '*L*' (Growth) portfolios are formed here.

For sorting according to profitability, the $EBIT_{(t)}/BE_{(t-1)}$ ratio was used to define the profitability factor and using the 30^{th} and 70^{th} percentile approach above; the '*Hp*' (high profitability) and *'Lp'* (low profitability) portfolios are formed here.

For sorting of volatility, stock volatility at the beginning of each year was estimated as the standard deviation for the previous 12months and using the 30th and 70th percentile approach above, the '*Hv*' (high volatility) and '*Lv*' (low volatility) portfolios are formed here.

Following this, we constructed the portfolios which are studied in this research and the total monthly return of each of these portfolios for each year is estimated. Every January, portfolios were constructed based on 1, 2 and 3 combination of factors (1, 2 and 3 portfolio sorts) to examine the individual and combined effects of using these factors to create strategies and the market cap weighted annual return was then obtained for each portfolio. Assessment of the portfolios were done relative to the market.

Table 1

Summary Statistics Table:

Data has been harmonised to commence from 1996. 24 periods of data (1996 – 2019) has been considered with a monthly frequency. The market equity (ME) of the constituent stock was ranked in descending order and the median was obtained. Stocks above the median are classed as large stock and that bellow are small stocks. The book equity (BE) of the index constituents was also obtained and then the ratio BE/ME was ranked. The top 30th percentile is classed as value stock while the bottom 30th percentile are the growth stock. The ratio EBIT/BE (earnings before interest and tax (EBIT)) was used to obtain high profit and low profit portfolios using the same top and bottom 30th percentile approach. The standard deviation of the monthly total returns of the stocks was also obtained and with the percentile approach, high and low volatility portfolios were obtained. Portfolios were first created based on the stocks that fit in the criteria of the 4 main factors (size, book to price, profitability, and volatility). Intersects (stocks qualifying for more than one factor) were then created based on 2 and 3 factor intersects. The colour coded portfolios do not have active portfolios in all the periods considered hence, a zero-portfolio return has been entered for such periods. This may however appear to skew their results.

S	*Small Stock Portfolios*
B	*Big Stock Portfolios*
L	*Low Book to Market Ratio (Growth Stock Portfolios)*
H	*High Book to Market (Value Stock Portfolios)*
Hp	*High Profitability Stock Portfolios*
Lp	*Low Profitability Stock Portfolios*
Hv	*High Volatility Stock Portfolios*
Lv	*Low Volatility Stock Portfolios*

1 Factor Portfolios	Avg. Number of Stocks Per Year	No. of Periods with an Active Portfolio (out of 24)	Mean (Dependent Variable)	Standard Deviation (Dependant Variable)	Standard Error of Coefficient	R^2	Durbin Watson Stat.	No. of Observation
Small Portfolio	77	24	1.29E-04	0.0566	0.0435	0.4105	1.96	287
Big Portfolio	77	24	9.78E-05	0.0563	0.0404	0.4862	1.99	287
Value Portfolio	59	24	1.22E-04	0.0710	0.0500	0.4497	1.979	287
Growth Portfolio	59	24	-1.50E-05	0.0570	0.0400	0.4910	2.00	287
High Profitability Portfolio	60	24	1.15E-04	0.0567	0.0405	0.4927	1.99	287
Low Profitability Portfolio	60	24	5.18E-05	0.0736	0.0590	0.4570	1.98	287
High Volatility Portfolio	77	24	-1.43E-05	0.1100	0.0811	0.4578	1.98	287
Low Volatility Portfolio	77	24	9.36E-05	0.0383	0.0283	0.4555	2.00	287

2 Factor Portfolios	Avg. Number of Stocks Per Year	No. of Periods with an Active Portfolio (out of 24)	Mean (Dependent Variable)	Standard Deviation (Dependant Variable)	Standard Error of Coefficient	R^2	Durbin Watson Stat.	No. of Observation
S, L	6	24	2.04E-04	0.0940	0.0722	0.4106	2.00	287
S, H	42	24	7.50E-05	0.0643	0.0501	0.3921	1.98	287
S, Hp	9	24	3.94E-05	0.0678	0.0539	0.3683	2.00	287
S, Lp	37	24	6.62E-05	0.0674	0.0525	0.3928	1.98	287
S, Hv	41	24	2.31E-04	0.0877	0.0655	0.4440	1.97	287
S, Lv	41	24	1.51E-05	0.0379	0.0289	0.4214	1.99	287
B, L	53	24	-1.53E-05	0.0571	0.0407	0.4941	1.99	287
B, H	16	24	1.14E-04	0.0793	0.0564	0.5004	2.00	284
B, Hp	50	24	-2.91E-05	0.0585	0.0415	0.4981	1.99	287
B, Lp	22	24	5.70E-05	0.0773	0.0572	0.4516	1.99	287
B, Hv	37	24	5.87E-05	0.1053	0.0747	0.4990	2.00	287
B , Lv	35	24	9.57E-05	0.0399	0.0295	0.4569	1.99	287
L, Hp	44	24	-2.31E-05	0.0574	0.0409	0.4956	1.99	287
L, Lp	6	24	-4.08E-06	0.0882	0.0655	0.4494	1.99	287
L, Hv	17	24	1.99E-04	0.1207	0.0891	0.4580	1.98	287
L, Lv	18	24	5.30E-05	0.0403	0.0298	0.4544	2.00	287
H, Hp	2	24	3.99E-04	0.9548	0.0698	0.4655	1.99	287
H, Lp	39	24	9.44E-05	0.0783	0.0586	0.4393	1.98	287
H, Hv	23	24	1.17E-04	0.1126	0.0836	0.4512	1.98	287
H, Lv	9	24	2.51E-06	0.0479	0.0351	0.4652	2.00	287
Hp, Hv	15	24	2.28E-04	0.1196	0.0875	0.4667	1.99	287
Hp, Lv	16	24	8.35E-05	0.0468	0.0335	0.4903	2.00	287
Lp, Hv	25	24	-9.90E-06	0.1218	0.0894	0.4629	1.99	287
Lp, Lv	10	24	3.31E-05	0.0529	0.0392	0.4539	1.99	287

3 Factor Portfolios	Avg. Number of Stocks Per Year	No. of Periods with an Active Portfolio (out of 24)	Mean (Dependent Variable)	Standard Deviation (Dependant Variable)	Standard Error of Coefficient	R^2	Durbin Watson Stat.	No. of Observation
B, L, Hp	41	24	-3.62E-05	0.0570	0.0410	0.4930	1.99	287
B, L, Lp	5	24	-4.08E-06	0.0884	0.0650	0.4603	1.99	287
B, H, Lp	9	24	4.37E-05	0.0967	0.0667	0.5306	1.97	284
B, Lp, Hv	8	24	-8.99E-05	0.1262	0.0925	0.4653	1.99	287
B, Hp, Hv	11	24	4.09E-05	0.0471	0.0338	0.4877	1.99	287
B, L, Hv	13	24	2.08E-04	0.1199	0.0887	0.4540	1.99	287
B, H, Hv	6	24	8.17E-05	0.1384	0.0994	0.4864	1.99	287
B, Lp, Lv	4	24	3.29E-05	0.0566	0.0415	0.4648	2.00	287
B, Hp, Lv	15	24	4.09E-04	0.0471	0.0338	0.4877	1.99	287
B, L, Lv	15	24	4.73E-05	0.1262	0.0925	0.4653	1.99	287
L, Hp, Hv	10	24	2.55E-04	0.1208	0.0890	0.4594	1.98	287
L, Hp, Lv	13	24	4.46E-05	0.0445	0.0323	0.4727	2.00	287
S, L, Hp	3	24	4.80E-05	0.0975	0.0719	0.4588	2.01	287
S, H, Lp	29	24	2.52E-05	0.0692	0.0525	0.4270	1.97	287
H, Lp, Hv	16	24	-9.20E-06	0.1222	0.0884	0.4789	1.99	287
H, Lp, Lv	6	24	-2.47E-05	0.0479	0.0351	0.4652	2.00	287
S, Lp, Hv	16	24	1.72E-04	0.1031	0.0780	0.4299	1.97	287
S, H, Hv	18	24	1.54E-04	0.1002	0.0752	0.4379	1.98	287
S, Lp, Lv	6	24	-8.12E-05	0.0429	0.0336	0.3871	2.02	287
S, H, Lv	6	24	-1.20E-05	0.0398	0.0307	0.4092	2.01	287
All Market	265	24	8.02E-05	0.0561	0.0401	0.4900	1.99	287
B, H, Lv	3	23	8.95E-05	0.0599	0.0421	0.5068	2.00	287
S, Hp, Lv	2	14	1.52E-04	0.0462	0.0343	0.4500	2.00	287
S, L, Hv	3	21	4.07E-04	0.1176	0.0876	0.4474	1.99	287
S, Hp, Hv	4	21	3.55E-04	0.1231	0.0909	0.4560	1.99	287
S, L, Lv	0.417	8	5.11E-20	0.3573	0.0270	0.4325	1.97	287
H, Hp, Lv	0.417	9	0.00E+00	0.0435	0.0322	0.4551	2.00	287
H, Hp, Hv	0.417	10	6.56E-04	0.1273	0.0918	0.4912	1.94	282
S, H, Hp	1	14	3.60E-05	0.1150	0.0850	0.4650	1.99	282
S, L, Lp	1	13	5.65E-19	0.1420	0.1020	0.4880	1.99	287
L, Lp, Lv	1	21	6.55E-05	0.0558	0.0411	0.4577	1.99	287
L, Lp, Hv	2.7	16	-6.55E-05	0.0117	0.0852	0.4738	2.00	287
B, H, Hp	0.625	7	3.45E-05	0.0780	0.0550	0.5100	1.99	287

Mean Reversion and Proposed Relative Stability Measure.

The standard Dickey-Fuller Test for on an AR (1) process is based on the regression form (where R_t is portfolio returns):

$$R_t = \varphi(R_{t-1}) + \varepsilon_t$$

$$-1 < \varphi < 1 \text{ and } |\varphi| < 1 \text{ for a reverting process}$$

$$E[R_t] = \mu = constant$$

Where, $R_t = R_t - E(R_t)$ may be interpreted as the distance to the stationary mean.

The test for stationarity (unit root) here is whether $\varphi = 1$

The unit root tests described above is valid if the time series R_t is well characterized by an AR (1) process with white noise errors. Many financial time series, however, have a more complicated dynamic structure than is captured by a simple AR (1) model. Said and Dickey (1984) augment the basic autoregressive unit root test to accommodate general ARMA (Auto Regressive Moving Average) models with unknown orders and their test is referred to as the augmented Dickey Fuller (ADF) model.

Subtracting R_{t-1} on both sides gives:

$$R_t - R_{t-1} = \varphi R_{t-1} - R_{t-1} + \varepsilon_t$$

giving:

$$R_t - R_{t-1} = (\varphi - 1)\, R_{t-1} + \varepsilon_t$$

thus, can be written as:

$$\Delta R_t = \beta R_{t-1} + \varepsilon_t$$

where,

$$\beta = \varphi - 1$$

R_t is returns at time t

$$\Delta R_t = R_t - R_{t-1} \; and \; \varepsilon_t \sim N(0, \sigma^2)$$

Therefore, from the standard Dickey Fuller test for AR (1) above, since $\beta = \varphi - 1$, the null hypothesis test becomes 1+β = 1. The role of the ADF hypothesis test is to consider the null hypothesis that $\beta = 0$, which would indicate that the process is a random walk and thus non mean reverting. If the hypothesis that $\beta = 0$ can be rejected, then the following movement of the return series is proportional to the current return and thus it is unlikely to be a random walk hence, mean reverting.

The test statistic (ADF_τ), is given as: $ADF_\tau = \frac{\beta}{SE(\beta)}$

$SE(\beta)$ is the standard error of the regressing coefficient β

Andrews and Chen (1994) showed that the cumulative response of a series of AR (1) order is given as *CIR* = 1/ (1- φ) hence, strengthening the view of using φ (or β) as an indicator of stability. But, while β can be used to measure the speed of reversion following shocks, it does not give any information on the dispersion of the return series around its stable mean state, hence, does not tell how far from it reverts. If the effect of withdrawing from a portfolio can be considered an 'off system' shock, and if β shows the speed of reversion following dispersions from the mean, then a measure that will reflect the magnitude of deviation from the stable state as well as the speed of reversion may be a useful tool.

If ε_t is a measure of what is not explained by the modelled series (in its stable state), then deviations from ε_t should reflect exogeneous strains such as the one imposed by drawing down on a portfolio. Therefore, σ, which is 1 standard deviation of shock ε_t, would give a unit measure of deviation from the 'stable' system.

Looking at this in another way, from the ADF equation above,

Variance of $R_t = \frac{\sigma^2}{1-\varphi^2}$

Hence, the deviation / dispersion of returns around its mean is proportionate to σ. The *ad hock* measure $\frac{|\beta|}{\sigma}$ was proposed as a measure of relative stability where σ is 1 standard deviation of shock[3]. The intention is that this measure is expected to give an indication of how quickly a portfolio will revert to its mean following 1 unit deviation from its 'system' shock (such as a withdrawal).

Sustainability Analysis.

A success rate analysis was carried out on the portfolios using the Monte-Carlo simulation method. Monte Carlo simulations are often used when the problem at hand has a probabilistic component. An expected value of that probabilistic component can be studied using Monte Carlo due to the law of large numbers (as the number of identically distributed, randomly generated variables increases, their sample mean (average) approaches their theoretical mean).

3. 1 standard deviation of shock is obtained by standardising the sum of squared residuals by the degree of freedom (n-1) and the obtaining the square root of the result.

The traditional Monte Carlo methodology assumes the returns an investor is able to achieve are equally likely over the entire retirement period. The generally adopted normal distribution for stock prices was stipulated for the simulation. The stipulated mean was the average monthly return of the portfolios, and the stipulated standard deviation was the standard deviation of the monthly returns of the portfolios.

For each portfolio, the methodology created 10,000 paths, which each path containing a time-series of 288 monthly returns, randomly generated from the mentioned estimated mean and standard deviation. An accumulated pot of £100,000 was then used to calculate the value of portfolio at each time point in the path, considering both the simulated monthly rate of return and the withdrawals, which are made at the end of every 12 monthly period (end of each year).

The rates of withdrawal assessed were 4%, 6%, 8% and 10%. The first withdrawal is taken at the end of the first month of investment and then subsequently at the end of each year (so there will be 2 withdrawals in the 1st year). This is because generally, for a retail client, the demand for a drawdown portfolio is usually triggered following his request to start releasing an income. This withdrawal sum is then increased yearly by a rate of inflation. Annual CPI inflation rates for the corresponding periods of the data was obtained from the Bank of England Library (available online[4]) to create a pool of 24 annual inflation rates. The rate used to annually increase the withdrawal sum was then selected randomly from this pool (using a simple excel random select function).

So, for example, for a 4% withdrawal, if a portfolio made 2% return in the 1st month, then there will be £98,000 to invest in the 2nd month {(£100,000 x 1.02) - £4,000} and if 5% return was made in month 2, then there will be £102,900 to invest in month

[4] *Available at https://www.ons.gov.uk/economy/inflationandpriceindices/timeseries/d7g7/mm23*

3 (£98,000 x 1.05) and if 3% is made in month 3 then there will be £105,987 to invest in month 4 (£102,900 x 1.03). Assuming there was £120,000 at the end of the first 12 months (year 1) after the withdrawal at the end of year, and the inflation rate has been 2%, then in month 13 (start of the 2nd year) the client will have £115,920 to invest {(£120,000-(£4,000 x 1.02)}.

For each of the withdrawal rates, the final portfolio balance at the end of the period for each portfolio for 10,000 simulations are then obtained. So, for example, portfolio 'A' will have 10,000 portfolio balances for 4% withdrawals and another 10,000-portfolio balance for 6% withdrawals and the same at 8% and 10% withdrawals.

At each withdrawal rate, the success rate is defined as the number of times (out of the 10,000) a portfolio having a ending balance greater than or equal to £0.

To determine the reliability of the relative stability measure, a correlation analysis of the success rates and relative stability measure is also analysed since a more stable portfolio would be expected to have a better success rate.

Section 4

Results

In this section we examine the returns of the 52 portfolios considered. We also examine the results obtained from the Monte-Carlo simulation to identify how successful the portfolios are at the various withdrawal rates. Also, we show the result of the failure point assessment which identifies how long on average the portfolios sustained withdrawals before failing. Furthermore, the average portfolio residual value for the portfolios is also presented. In addition to these, the proposed relative stability measure for the portfolios is shown and the correlation result of this measure with the success rate and failure point is also presented.

Section 4.1

Portfolio Returns

Tables 4.1

Summary of Portfolio Returns

Portfolio	*Avg. Number of Stocks Per Year*	*No. of Periods with an Active Portfolio (out of 24)*	*Average Monthly Returns*
All Market	*268*	*24*	*1.17%*

1 Factor Portfolios	Avg. Number of Stocks Per Year	No. of Periods with an Active Portfolio (out of 24)	Average Monthly Returns	Average Monthly Excess Returns
Small Portfolio	134	24	1.26%	0.09%
Big Portfolio	134	24	1.17%	0.00%
Value Portfolio	59	24	1.23%	0.06%
Growth Portfolio	59	24	1.22%	0.05%
High Profitability Portfolio	60	24	1.12%	-0.05%
Low Profitability Portfolio	60	24	1.00%	-0.17%
High Volatility Portfolio	77	24	1.40%	0.23%
Low Volatility Portfolio	77	24	1.03%	-0.14%

	High Volatility Portfolio	Low Volatility Portfolio
Sharpe Ratio	0.13	0.26

2 Factor Portfolios	Avg. Number of Stocks Per Year	No. of Periods with an Active Portfolio (out of 24)	Average Monthly Returns	Average Monthly Excess Returns
S, L	6	24	1.14%	-0.03%
S, H	42	24	1.23%	0.06%
S, Hp	9	24	0.92%	-0.25%
S, Lp	37	24	1.41%	0.24%
S, Hv	41	24	1.22%	0.05%
S, Lv	41	24	1.08%	-0.09%
B, L	53	24	1.22%	0.05%
B, H	16	24	1.11%	-0.06%
B, Hp	50	24	1.16%	-0.01%
B, Lp	22	24	0.97%	-0.20%
B, Hv	37	24	1.23%	0.06%
B , Lv	35	24	1.05%	-0.12%
L, Hp	44	24	1.18%	0.01%
L, Lp	6	24	1.00%	-0.17%
L, Hv	17	24	1.25%	0.08%
L, Lv	18	24	0.95%	-0.22%
H, Hp	2	24	0.55%	-0.62%
H, Lp	39	24	1.34%	0.17%
H, Hv	23	24	1.05%	-0.12%
H, Lv	9	24	1.41%	0.24%
Hp, Hv	15	24	1.42%	0.25%
Hp, Lv	16	24	0.84%	-0.33%
Lp, Hv	25	24	1.04%	-0.13%
Lp, Lv	10	24	1.18%	0.01%

3 Factor Portfolios	Avg. Number of Stocks Per Year	No. of Periods with an Active Portfolio (out of 24)	Average Monthly Returns	Average Monthly Excess Returns
B, L, Hp	41	24	1.18%	0.01%
B, L, Lp	5	24	1.00%	-0.17%
B, H, Lp	9	24	1.12%	-0.05%
B, Lp, Hv	8	24	0.97%	-0.20%
B, Hp, Hv	11	24	1.47%	0.30%
B, L, Hv	13	24	1.25%	0.08%
B, H, Hv	6	24	0.92%	-0.25%
B, Lp, Lv	4	24	1.14%	-0.03%
B, Hp, Lv	15	24	0.83%	-0.34%
B, L, Lv	15	24	0.95%	-0.22%
L, Hp, Hv	10	24	1.36%	0.19%
L, Hp, Lv	13	24	0.75%	-0.42%
S, L, Hp	3	24	1.19%	0.02%
S, H, Lp	29	24	1.26%	0.09%
H, Lp, Hv	16	24	1.25%	0.08%
H, Lp, Lv	6	24	1.34%	0.17%
S, Lp, Hv	16	24	1.46%	0.29%
S, H, Hv	18	24	1.21%	0.04%
S, Lp, Lv	6	24	1.28%	0.11%
S, H, Lv	6	24	1.25%	0.08%

The FTSE 350 universe was considered. The market equity (ME) of the index constituent stock was ranked in descending order and the median was obtained. Stocks above the median are classed as large stock and that bellow are small stocks. The book equity (BE) of the index constituents was also obtained and then the ratio BE/ME was ranked. The top 30th percentile is classed as value stock while the bottom 30th percentile are the growth stock. The ratio EBIT/BE (earnings before interest and tax (EBIT)) was used to obtain high profit and low profit portfolios using the same top and bottom 30th percentile approach. The standard deviation of the monthly total returns of the stocks was also obtained and with the percentile approach, high and low volatility portfolios were obtained. Portfolios were first created based on the stocks that fit in the criteria of the 4 main factors (size, book to price, profitability, and volatility). Intersects (stocks qualifying for more than one factor) were then created based on 2 and 3 factor intersects. After sorting into the individual portfolios based on the various factors, the weighted monthly total returns (total returns times weighted market cap of the portfolio) of the constituent stocks for each year was obtained. Then the average monthly returns through the data period was calculated. The Sharpe Ratio is estimated as the average portfolio return through the entire period less the risk-free rate then divided by the portfolio standard deviation. The risk-free rate is the UK 1month Treasury Bill

The returns result presented in tables 4.1 above shows that the more factors are combined to form a strategy, the rarer it is to find valid stocks. That said, for the portfolios that have active stock through the entire period, most of them have a reasonable number of stocks per year.

The 1 sort portfolio returns show the that the size, value and profitability premium is present with the profitability premium being the largest at 12 basis points and the value premium being the smallest at 1 basis point every month. Although, the absolute total monthly returns presented does not confirm the volatility anomaly, however, the anomaly is confirmed in risk adjusted terms using the Sharpe ratio. Furthermore, the result shows that investing in either small, value, growth and high volatility stocks would have individually provided better returns in comparison to holding the market. High volatility stock investment strategy provided the best return during this period.

The 2 sort portfolios show that 11 of the 24 portfolios formed outperformed the market (*S,H; S,Lp; S,Hv; B,L; B,Hv; L,Hp; L,Hv; H,Lp; H,Lv; Hp,Hv; and Lp,Lv*) with the high profitability – high volatility portfolio returning the highest return (25 basis point over the market). The 3 sort portfolios produced 20 active portfolios throughout the entire sample period. 12 of these portfolios outperformed the market (*B,L,Hp; B,Hp,Hv; B,L,Hv; L,Hp,Hv; S,L,Hp; S,H,Lp; H,Lp,Hv; H,Lp,Lv; S,Lp,Hv; SLpLv* *and S,H,Lv; and S,H,Hv) with the B,Hp,Hv* producing the best return within this sort and the entire sorts (30 basis points over the market returns).

Section 4.2

Success Rate

Table 4.2

Summary of Success Rate Analysis for 1 Sort Portfolios

Success Rates						
Portfolio	Average Monthly Return	Observed Standard Deviation	Success @ Rate 4% Withdrawal	Success Rate @ 6% Withdrawal	Success Rate @ 8% Withdrawal	Success Rate @ 10% Withdrawal
FTSE 350	1.17%	0.0400	99.94%	97.85%	87.66%	63.38%
Small Portfolio	1.26%	0.0441	99.94%	98.43%	88.70%	68.80%
Big Portfolios	1.17%	0.0403	99.96%	97.92%	86.61%	62.49%
Value Portfolio	1.23%	0.0528	99.42%	93.63%	79.80%	58.68%
Growth	1.22%	0.0406	99.97%	98.75%	90.13%	68.33%
High Profitability	1.12%	0.0403	99.88%	96.97%	83.79%	57.85%
Low Profitability	1.00%	0.0543	96.25%	81.21%	59.70%	36.48%
High Volaility	1.40%	0.0812	92.87%	81.35%	67.35%	50.55%
Low Volatility	1.03%	0.0283	100.00%	99.42%	89.47%	54.20%

The FTSE 350 universe was considered. Portfolios based on the factors of size, book to market ratio, profitability and volatility were formed. Hence, primary portfolios of 'S' (small stocks), 'B' (large stocks), 'H' (value stocks), 'L' (Growth stocks), 'Hp' (high profitability), 'Lp' (low profitability), 'Hv' (high volatility) and 'Lv' (low volatility) were formed. After sorting into the individual portfolios based on the various factors, the weighted monthly total returns (total returns times weighted market cap of the portfolio) of the constituent stocks for each year was obtained. Then the average monthly returns through the data period was calculated. Withdrawal rates of 4%, 6%, 8% and 10%of the initial portfolio value was considered. A 10,000 Monte-Carlo simulation was carried out on each portfolio for each withdrawal rate. This assumed a normal distribution with replacements. For each simulation, a final portfolio value was established. This was done by assuming a starting portfolio value of £100,000 and this was compounded with the monthly simulated returns. 2 withdrawals are made in the first year (January and December) and subsequently every December. The sum withdrawn was increased by the inflation rate randomly selected from a pool of annual inflation rate for the period under consideration (1996 – 2019). The inflation rate was obtained from the Bank of England online library. Success is defined as an ending portfolio balance greater than or equal to £0.

The table above (table 4.2) shows the result of investing in a diversified portfolio based on the individual factors. In addition to the FTSE350 market, all the individual factor portfolios had success rates at the 4% withdrawal rate ranging from 92%[5] (high volatility portfolio) to 100% (low volatility portfolio), most of which (including the

[5] *A 92% success rate implies that 1800 simulated 24yr periods out of the total 10000 simulations, the portfolio value at the end of the period was less than £0*

FTSE350 market) achieved a 99% success rate. The portfolios were less successful at the 6% withdrawal rate but still considerably successful. With the exception of the low profitability and high volatility portfolios, the portfolios returned success rates ranging from 93% (value portfolios) to 99% (low volatility portfolios); the market had 97% success.

At the 8% withdrawal rate, more significant levels of failure (1-success rate) were observed. At this withdrawal, only growth portfolios returned a 90% success rate; the market returned 87% while low profitability returned 59% (the lowest). Others had success rates ranging from 67% (high volatility) to 89% (low volatility). At 10% withdrawal rate, it is fair to say all the portfolios failed as only success rates ranging from 36% (low profitability portfolios) to 68% (small and growth portfolios) was observed

Table 4.3

Summary of Success Rate Analysis for 2 Sort Portfolios

			Success Rates			
Portfolio	Average Monthly Return	Observed Standard Deviation	Success @ Rate 4% Withdrawal	Success Rate @ 6% Withdrawal	Success Rate @ 8% Withdrawal	Success Rate @ 10% Withdrawal
S, L	1.14%	0.0724	90.97%	75.22%	56.77%	38.77%
S, H	1.23%	0.0498	99.68%	95.32%	82.79%	60.31%
S, Hp	0.92%	0.0551	93.50%	75.36%	51.29%	29.61%
S, Lp	1.41%	0.0518	99.93%	97.98%	90.77%	74.90%
S, Hv	1.22%	0.0656	96.01%	84.28%	68.04%	49.27%
S, Lv	1.08%	0.0292	100.00%	99.62%	92.00%	60.01%
B, L	1.22%	0.0406	99.97%	98.66%	89.72%	68.56%
B, H	1.11%	0.0560	97.51%	86.94%	67.55%	45.60%
B, Hp	1.16%	0.0414	99.90%	97.52%	85.09%	60.43%
B, Lp	0.97%	0.0566	94.72%	77.50%	54.73%	32.45%
B, Hv	1.23%	0.0745	91.70%	78.51%	61.10%	43.44%
B , Lv	1.05%	0.0295	100.00%	99.37%	89.39%	55.90%
L, Hp	1.18%	0.0408	99.91%	98.13%	87.79%	63.80%
L, Lp	1.00%	0.0649	90.44%	71.71%	51.30%	32.64%
L, Hv	1.25%	0.0891	84.22%	67.49%	52.27%	37.94%
L, Lv	0.95%	0.0298	99.98%	97.83%	77.34%	37.61%
H, Hp	0.55%	0.0783	52.24%	30.62%	16.09%	8.95%
H, Lp	1.34%	0.0574	99.36%	94.28%	82.31%	64.21%
H, Hv	1.05%	0.0837	79.60%	60.40%	43.45%	28.95%
H, Lv	1.41%	0.0351	100.00%	99.95%	98.84%	90.86%
Hp, Hv	1.42%	0.0873	91.05%	78.19%	64.25%	49.10%
Hp, Lv	0.84%	0.0334	99.65%	89.93%	57.44%	21.92%
Lp, Hv	1.04%	0.0893	74.38%	54.83%	38.81%	26.28%
Lp, Lv	1.18%	0.0392	99.95%	98.59%	88.79%	64.61%

The FTSE 350 universe was considered. Portfolios based on the factors of size, book to market ratio, profitability and volatility were formed. Hence, primary portfolios of 'S' (small stocks), 'B' (large stocks), 'H' (value stocks), 'L' (Growth stocks), 'Hp' (high profitability), 'Lp' (low profitability), 'Hv' (high volatility) and 'Lv' (low volatility) were formed. Intersects (stocks qualifying for more than one factor) were then created based on 2 factor intersects. After sorting into the individual portfolios based on the various factors, the weighted monthly total returns (total returns times weighted market cap of the portfolio) of the constituent stocks for each year was obtained. Then the average monthly returns through the data period was calculated. Withdrawal rates of 4%, 6%, 8% and 10%of the initial portfolio value was considered. A 10,000 Monte-Carlo simulation was carried out on each portfolio for each withdrawal rate. This assumed a normal distribution with replacements. For each simulation, a final portfolio value was established. This was done by assuming a starting portfolio value of £100,000 and this was compounded with the monthly simulated returns. 2 withdrawals are made in the first year (January and December) and subsequently every December. The sum withdrawn was increased by the inflation rate randomly selected from a pool of annual inflation rate for the period under consideration (1996 – 2019). The inflation rate was obtained from the Bank of England online library. Success is defined as an ending portfolio balance greater than or equal to £0.

Table 4.3 above shows the success rate results for the 2 sort portfolios. There are a total of 24 portfolios created. At the 4%, *S,Lv; B,Lv* and *H,Lv* portfolios had a 100% success rate. Nine other portfolios had a success rate of 99% and another eight had a success rate of over 90%. The *H,Hp* portfolio had the worst success rate of 54%.

At 6% withdrawal rate, *S,Lv; B,Lv* and *H,Lv* portfolios had the highest success rate (99%) however, only 7 portfolios had success rates in excess of 90% (ranging from 94.28% to 98.66%. The remaining portfolios had success rates ranging from 30.62% to 89.93%. At the 8% rate of withdrawal, the *H,Lv* portfolio was the most successful with a success rate of 98%. The *S,Lv* portfolio had a success rate of 92% whilst the *S,Lp* portfolio had a success rate of 90% (this portfolio had success rates of 99% and 97% at the 4% and 6% withdrawal rates respectively. Only the *H,Lv* portfolio had a 90% success rate at the 10% rate of withdrawal whilst the remaining portfolios had a success rate of between 8% and 74%.

Table 4.4

Summary of Success Rate Analysis for 3 Sort Portfolios

			Success Rates			
Portfolio	Average Monthly Return	Observed Standard Deviation	Success @ Rate 4% Withdrawal	Success Rate @ 6% Withdrawal	Success Rate @ 8% Withdrawal	Success Rate @ 10% Withdrawal
B, L, Hp	1.18%	0.0400	99.94%	98.10%	88.54%	64.95%
B, L, Lp	1.00%	0.0650	90.46%	72.40%	50.01%	31.94%
B, H, Lp	1.12%	0.0680	92.38%	76.39%	57.99%	40.42%
B, Lp, Hv	0.97%	0.0924	68.13%	48.63%	33.83%	22.71%
B, Hp, Hv	1.47%	0.0875	91.59%	79.85%	65.32%	51.42%
B, L, Hv	1.25%	0.0888	84.87%	66.86%	51.72%	37.27%
B, H, Hv	0.92%	0.0991	59.39%	41.10%	28.53%	19.23%
B, Lp, Lv	1.14%	0.0410	99.88%	97.22%	84.23%	58.34%
B, Hp, Lv	0.83%	0.0340	99.47%	88.63%	55.24%	21.59%
B, L, Lv	0.95%	0.0300	100.00%	97.63%	78.05%	38.59%
L, Hp, Hv	1.36%	0.0890	88.03%	73.15%	58.65%	44.42%
L, Hp, Lv	0.75%	0.0323	99,24%	82.81%	43.77%	12.97%
S, L, Hp	1.19%	0.0720	92.47%	78.22%	59.85%	43.11%
S, H, Lp	1.26%	0.0530	99.48%	94.32%	81.88%	61.36%
H, Lp, Hv	1.25%	0.0882	85.15%	68.15%	52.13%	37.93%
H, Lp, Lv	1.34%	0.0392	99.99%	99.67%	96.00%	81.83%
S, Lp, Hv	1.46%	0.0785	95.84%	85.89%	72.82%	57.75%
S, H, Hv	1.21%	0.0756	91.39%	76.88%	58.86%	42.01%
S, Lp, Lv	1.28%	0.0346	100.00%	99.82%	96.44%	81.13%
S, H, Lv	1.25%	0.0312	100.00%	99.95%	97.75%	81.98%

The FTSE 350 universe was considered. Portfolios based on the factors of size, book to market ratio, profitability and volatility were formed. Hence, primary portfolios of 'S' (small stocks), 'B' (large stocks), 'H' (value stocks), 'L' (Growth stocks), 'Hp' (high profitability), 'Lp' (low profitability), 'Hv' (high volatility) and 'Lv' (low volatility) were formed. Intersects (stocks qualifying for more than one factor) were then created based on 3 factor intersects. After sorting into the individual portfolios based on the various factors, the weighted monthly total returns (total returns times weighted market cap of the portfolio) of the constituent stocks for each year was obtained. Then the average monthly returns through the data period was calculated. Withdrawal rates of 4%, 6%, 8% and 10%of the initial portfolio value was considered. A 10,000 Monte-Carlo simulation was carried out on each portfolio for each withdrawal rate. This assumed a normal distribution with replacements. For each simulation, a final portfolio value was established. This was done by assuming a starting portfolio value of £100,000 and this was compounded with the monthly simulated returns. 2 withdrawals are made in the first year (January and December) and subsequently every December. The sum withdrawn was increased by the inflation rate randomly selected from a pool of annual inflation rate for the period under consideration (1996 – 2019). The inflation rate was obtained from the Bank of England online library. Success is defined as an ending portfolio balance greater than or equal to £0.

Table 4.4 above shows the result of success rates for the 3 sort portfolios (20 portfolios were considered). At the 4% withdrawal rate, 3 portfolios (*H,Lp,Lv; S,Lp,Lv* and *S,H,Lv*) had 100% success rate whilst 6 other portfolios had a 99% success rate including the *H,Lp,Lv* portfolio with 99.99% success. Six other portfolios had over 90% success rate whilst the remaining portfolios produced success rates ranging from 59% to 88%. At 6% rate of withdrawal, *H,Lp,Lv; S,Lp,Lv* and *S,H,Lv* were the only portfolios with 99% success rates and only 4 other portfolios had a success rate of over 90% (between 94% and 98%).

At the 8% rate of withdrawal, *H,Lp,Lv; S,Lp,Lv* and *S,H,Lv* had respective success rates of 96%, 96% and 97%. These were the only ones above 90% success rates. The remaining portfolios produced success rates ranging from 33% to 88%. The *H,Lp,Lv*; *S,Lp,Lv* and *S,H,Lv* all had 81% success rate at the 10% withdrawal rate whilst the remaining portfolios had success rates ranging from 19% to 64%.

In summary, 52 portfolios were considered and at the 4% withdrawal rate, the market and 27 other portfolios sustained this rate of withdrawal with a success rate of between 99%-100%. Another 16 portfolios were able to sustain this rate of withdrawal with at least a 90% success. The worst performing portfolio was the *H,Hp* (value and high profitability stocks) with a success rate of 52%.

None of the portfolios (including the market) had a 100% success rate at 6% withdrawal but 25 portfolios had a success rate of at least 90% with portfolios *Lv* (Low Volatility stocks), *S,Lv* (Small and Low Volatility Stocks), *B,Lv* (Big and Low Volatility stocks), *S,H,Lv* (Small, value, low volatility stocks), *S,Lp,Lv* (Small, Low Profitability and Low Volatility stocks), *H,Lp,Lv* (Value, Low profitability and Low Volatility stocks) and *H,Lv* (Value, low volatility stocks) having the highest success rate of 99%.

At 8% withdrawal rate, the *H,Lv* portfolio was the most successful with 98% success. The *S,H,Lv* portfolio had a 97% success rate. Two other portfolios were also as successful; the *H,Lp,Lv* portfolio had a success rate of 96% (this portfolio also had a success rate of 99% at the 4% and 6% withdrawal rates) and the *S,Lp,Lv* portfolio had a success rate of 96% (this portfolio also had a success rate of 100% and 99% at the 4% and 6% withdrawal rates respectively). The worst performing portfolio (*H,Hp*) had a success rate of 16.09%.

At the 10% withdrawal rate, only the *H,Lv* portfolio had a decent success rate (90%). The *S,H,Lv*, *H,Lp,Lv* and *S,Lp,Lv* had a success rate of 81% respectively. These were the best performing portfolios. The remaining portfolios had success rates ranging from 9% to 69%.

These results tend to align with the observations from Athavale and Goebel (2011), who noted that scenarios in which average returns exceed the withdrawal rate do not necessarily lead to portfolio success (as can be seen at the 10% withdrawal rate where most of the portfolios had very low success rates but annual returns in excess of 12%). Second, scenarios in which average returns are lower than the withdrawal rate do not necessarily result in portfolio failure. Third, low standard deviations do not necessarily result in portfolio success. Fourth, while some retirement experiences will result in failure, the nature of equity returns causes others to be immensely successful. Taken together, they concluded that although larger returns and smaller standard deviations contribute to portfolio success, these are not sufficient conditions to ensure success, and other factors including the timing of returns and the occurrence of negative or positive runs are also important.

Notably also is that the 4 most successful portfolios (*H,Lv*; *S,H,Lv*; *H,Lp,Lv* and *S,Lp,Lv)* had the low volatility factor in their construction and 3 of these had the value factor in addition. As it seems, the combined effect of these factors tends to enhance sustainability.

Section 4.2

Failure Point

The possibility of failure (used interchangeably with failure rate) has often been referred to as more important to an investor as it shows an indication of the possibility of a withdrawal rate being unsustainable. In addition to this (and within the context of this research), it indicates the reliability of a portfolio strategy for withdrawal purposes. In fact, a retiree ultimately has to balance the trade-off between higher withdrawals and a higher possibility of failure and choose a feasible combination of these variables that he/she finds acceptable.

However, the failure rate is silent about when a strategy failed during the failure period. As it seems, a strategy that sustained a retiree's withdrawal plan halfway through his retirement is very different from another that carried him 90% of the way. In the work by Estrada (2017), 2 variables were proposed; one measured how long before the end of the retirement period a strategy failed and the other measured what proportion of the retirement period a strategy sustained a retiree's withdrawals. The work concluded that these variables, together with the failure rate provided a better picture of the main risk retirees have to bear during retirement. It is important to note that the failure rate is implicitly defined by the success rate (Failure rate = 1 – Success rate) as used by Cooley, Hubbard and Walz (1998).

Estrada's work pointed out that although 2 strategies may be exposed to the same level of risk using the failure/success rate assessment, the use of shortfall years (one of the measures proposed) which implicitly gave an indication of the strategy's stability could tell a different and more informative story.

Table 4.5

Summary of Failure Point (1 sort portfolios)

	Default Period (Years)											
	4% Withdrawal Rate			6% Withdrawal Rate			8% Withdrawal Rate			10% Withdrawal Rate		
	Min Period	Average Period	Max Period	Min Period	Average Period	Max Period	Min Period	Average Period	Max Period	Min Period	Average Period	Max Period
FTSE 350	18	21	24	11	19	24	8	18	24	7	16	24
Small Portfolio	16	21	23	11	19	24	7	17	24	6	15	24
Big Portfolios	15	21	24	12	20	24	7	18	24	6	16	24
Value Portfolio	11	19	24	8	18	24	6	16	24	5	14	24
Growth	18	20	21	11	19	24	8	18	24	6	16	24
High Profitability	14	20	24	11	19	24	8	18	24	6	16	24
Low Profitability	9	19	24	7	17	24	5	15	24	4	14	24
High Volaility	6	17	24	4	15	24	4	14	24	3	12	24
Low Volatility	-	-	-	16	21	24	10	19	24	8	17	24

Following on from the details of table one, the default periods (failure point) were obtained. This is the month (the particular month during the 288 months of each simulation) when the failure occurred. The minimum, maximum and average failure periods are presented in the table. The empty cells indicate a 100% success rate (no failure).

Table 4.5 above shows the summary of the failure points for the 1 sort set of portfolios. The FTSE350 and the small and big portfolios all sustained the 4% rate of withdrawal up to the 21st (out of 24) year on average during failed periods. These was the furthest failure point in this category at this withdrawal rate. Although the growth portfolios for example had marginally better success rates compared to these 3 portfolios (99.97% Vs 99.94% and 99.96%), the growth portfolio only sustained withdrawals during failed periods up to the 20th year on average. The low volatility portfolio had no failed periods at this withdrawal rate.

At the 6% withdrawal rate, the low volatility portfolio sustained withdrawals up to the 21st year on average during failed periods making it the most durable at this withdrawal rate during failed periods. Although the small portfolio had a better success rate compared to big portfolios at this rate of withdrawal, the big portfolios were able to sustain withdrawals for an extra year on average (year 20 Vs year 19). This is also observed between the low profitability and high volatility portfolios.

At the 8% withdrawal rate, the low volatility portfolio sustained withdrawals up to the 19th year on average during failed periods making it the most durable at this withdrawal rate during failed periods. The remaining portfolios produced failure points ranging on average from the 14th to the 18th year. The low volatility portfolio also appeared to be the most stable at the 10% withdrawal rate based on the failure point as it sustained withdrawals on average up till the 17th year.

Table 4.6

Summary of Failure Point (2 sort portfolios)

	Default Period *(Years)*											
	4% Withdrawal Rate			6% Withdrawal Rate			8% Withdrawal Rate			10% Withdrawal Rate		
	Min Period	Average Period	Max Period	Min Period	Average Period	Max Period	Min Period	Average Period	Max Period	Min Period	Average Period	Max Period
S, L	6	18	24	4	16	24	4	14	24	3	12	24
S, H	13	20	24	8	18	24	6	16	24	5	15	24
S, Hp	8	19	24	7	17	24	5	15	24	4	13	24
S, Lp	15	20	24	9	18	24	6	16	24	5	15	24
S, Hv	7	18	24	6	16	24	4	15	24	3	13	24
S, Lv	-	-	-	16	21	24	11	19	24	7	17	24
B, L	16	18	21	11	19	24	8	18	24	6	16	24
B, H	10	19	24	7	17	24	6	16	24	5	14	24
B, Hp	16	20	23	10	19	24	8	18	24	6	16	24
B, Lp	10	19	24	7	17	24	5	15	24	4	13	24
B, Hv	6	17	24	5	16	24	4	14	24	3	12	24
B , Lv	-	-	-	13	21	24	9	19	24	7	17	24
L, Hp	15	19	24	11	19	24	7	18	24	6	16	24
L, Lp	7	18	24	6	16	24	4	14	24	3	13	24
L, Hv	5	16	24	4	15	24	3	13	24	3	12	24
L, Lv	20	21	23	12	21	24	9	19	24	7	16	24
H, Hp	5	16	24	4	14	24	3	12	24	3	10	24
H, Lp	11	19	23	7	17	24	6	16	24	5	14	24
H, Hv	5	16	24	4	15	24	4	13	24	3	11	24
H, Lv	-	-	-	16	20	23	11	19	24	8	17	24
Hp, Hv	6	16	24	4	15	24	4	13	24	3	12	24
Hp, Lv	14	22	24	11	20	24	7	18	24	7	15	24
Lp, Hv	4	16	24	4	14	24	3	12	24	2	11	24
Lp, Lv	19	19	19	12	20	24	8	18	24	7	16	24

Following on from the details of table one, the default periods (failure point) were obtained. This is the month (the particular month during the 288 months of each simulation) when the failure occurred. The minimum, maximum and average failure periods are presented in the table. The empty cells indicate a 100% success rate (no failure).

Table 4.6 above shows the summary of the failure point for the 2 sort portfolios. At 4% withdrawal rate, on average, most of the portfolios did not sustain this withdrawal as long as the FTSE350 did (year 21) however, whilst the *S,Lv; B,Lv* and *H,Lv* portfolios did not have any failed period, only the *Hp,Lv* portfolio was able to sustain withdrawals for an additional year on average compared to the FTSE350.

At 6% withdrawal rate, only 6 portfolios had a further failure point on average compared to the market (year 19). The average failure point (year 21) was achieved by portfolios *S,Lv; B,Lv* and *L,Lv* and the *H,Lv* portfolio had an average failure point at year 20. At the 8% withdrawal rate, fewer portfolios (4 in number) had an average failure point better than that of the market (year 18). These 4 portfolios including the *S,Lv; B,Lv* and *H,Lv* sustained withdrawals for an additional year on average and this was the furthest failure point at this withdrawal rate. At the 10% withdrawal rate, only 3 portfolios had an average failure point better than that of the market (year 16). These 3 portfolios including the *S,Lv; B,Lv* and *H,Lv* sustained withdrawals for an additional year on average and this was the best failure point at this withdrawal rate.

Table 4.7

Summary of Failure Point (3 sort portfolios)

	Default Period *(Years)*											
	4% Withdrawal Rate			6% Withdrawal Rate			8% Withdrawal Rate			10% Withdrawal Rate		
	Min Period	Average Period	Max Period	Min Period	Average Period	Max Period	Min Period	Average Period	Max Period	Min Period	Average Period	Max Period
B, L, Hp	19	21	22	12	19	24	7	18	24	6	16	24
B, L, Lp	8	18	24	6	16	24	5	14	24	4	13	24
B, H, Lp	8	18	24	5	16	24	5	14	24	4	13	24
B, Lp, Hv	5	16	24	4	14	24	3	12	24	2	11	24
B, Hp, Hv	5	16	24	4	15	24	4	13	24	3	12	24
B, L, Hv	5	16	24	4	14	24	3	13	24	3	11	24
B, H, Hv	4	15	24	3	13	24	3	11	24	2	10	24
B, Lp, Lv	17	21	24	9	19	24	7	18	24	6	16	24
B, Hp, Lv	8	22	24	8	20	24	8	17	24	6	15	24
B, L, Lv	-	-	-	12	20	24	8	19	24	7	16	24
L, Hp, Hv	6	16	24	4	14	24	4	13	24	3	12	24
L, Hp, Lv	13	21	24	10	20	24	7	17	24	6	14	24
S, L, Hp	7	18	24	5	16	24	4	14	24	4	13	24
S, H, Lp	12	19	24	6	18	24	6	16	24	5	14	24
H, Lp, Hv	5	16	24	4	14	24	3	13	24	3	12	24
H, Lp, Lv	-	-	-	12	19	23	9	18	24	6	16	24
S, Lp, Hv	6	17	24	4	15	24	4	14	24	3	12	24
S, H, Hv	6	17	24	5	15	24	4	14	24	4	12	24
S, Lp, Lv	-	-	-	14	19	24	9	18	24	7	17	24
S, H, Lv	-	-	-	17	21	24	11	20	24	8	17	24

Following on from the details of table one, the default periods (failure point) were obtained. This is the month (the particular month during the 288 months of each simulation) when the failure occurred. The minimum, maximum and average failure periods are presented in the table. The empty cells indicate a 100% success rate (no failure).

Table 4.7 above shows the summary of the failure point for the 3 sort portfolios. At 4% withdrawal rate, on average, most of the portfolios did not sustain this withdrawal as long as the FTSE350 did (year 21) however, the *B,L,Lv; H,Lp,Lv; S,Lp,Lv* and *S,H,Lv* portfolios did not have any failed period. Whilst the *B,Hp,Lv* and the *S,H,Lp* portfolios have similar success rates at this withdrawal rate, the *B,Hp,Lv* portfolio sustained withdrawals for an extra 3 years during failed periods on average hence further agreeing with Estrada's work.

Only 4 portfolios had a further failure point on average compared to the market (year 19) at the 6% withdrawal rate. The furthest average failure point (year 21) was achieved by portfolio *S,H,Lv*. *B,Hp,Lv*; *B,L,Lv* and the *L,Hp,Lv* portfolio had an average failure point at year 20. The *H,Lp,Lv* portfolio and 3 others had the same average failure point as the market.

At the 8% withdrawal rate, the *S,H,Lv* portfolio sustained withdrawals for the longest period (year 20) and this is 2years more than that of the market (year 18). The *H,Lp,Lv and S,Lp,Lv* portfolios had the same failure point on average with the market whilst the *B,L,Lv* portfolio had a 19year average failure point. Again the difference in the success rate and failure point implications can be spotted; whilst the *B,L,Lv* has a success rate of 78% at this 8% withdrawal rate compared to the *B,L,Hp* portfolio with 88% success rate; it sustained withdrawals for an additional 1 year on average during failed periods. Also, the *L,Hp,Hv* and the *S,H,Hv* both have similar success rate of 58% however, the *S,H,Hv* portfolio sustained withdrawals for an additional year.

At the 10% withdrawal rate, only 2 portfolios had an average failure point further than that of the market (year 16). These were the *S,Lp,Lv and S,H,Lv*. The *H,Lp,Lv* and 3 other portfolios had the same failure point an average as the FTSE350 market.

In summary the results of this section show that the 4 most successful portfolios (*H,Lv*, *H,Lp,Lv*, *S,Lp,Lv* and *H,Lp,Lv*) identified earlier did not fail through all the simulated periods at the 4% withdrawal rate. At 6% withdrawal rate, *H,Lv* and *S,H,Lv* had similar failure rates (1-success rate) but *S,H,Lv* sustained this withdrawal rate one year more year on average during failure periods. At the 6% and 8% withdrawal rate, *H,Lp,Lv* and *S,Lp,Lv* had similar failure rates (*S,Lp,Lv* had a marginally smaller failure rate in both cases) and on average, they both had the same failure point. At 8% withdrawal rate *H,Lv* had a lower failure rate than the *S,H,Lv* portfolios but the *S,H,Lv* portfolio sustained withdrawals for and additional year on average during failure periods. *H,Lv* had a much lower failure rate than the *S,H,Lv* portfolio at the 10% withdrawal rate however they both had the same failure point on average during failure periods.

This result generally agrees with Estrada (2017) as it indicates that the failure point which sometimes gives a different but deeper information compared to that of the failure/success rate can be used as an added layer for assessing the sustainability of portfolios. In addition to this, the result also shows that the 4 most successful portfolios provided competitive average failure points as they produced either the furthest average failure point or very close to the furthest achieved average failure point through all the withdrawal rates.

Section 4.3

Residual Value

Table 4.8

Summary of Residual Value (1 sort portfolios)

	Residual Value											
	4%			6% Withdrawal Rate			8% Withdrawal Rate			10% Withdrawal Rate		
	Min Value	Average Value	Max Value	Min Value	Average Value	Max Value	Min Value	Average Value	Max Value	Min Value	Average Value	Max Value
FTSE 350	£ 2,284.34	£ 1,878,268.00	£ 25,200,000.00	£ 736.62	£1,436,302.00	£ 28,800,000.00	£ 44.46	£1,080,572.00	£ 13,900,000.00	£ 20.55	£ 867,066.10	£ 22,100,000.00
Small Portfolio	£ 6,860.54	£ 2,507,579.00	£ 58,800,000.00	£ 781.50	£1,999,972.00	£ 37,900,000.00	£ 212.99	£1,548,990.00	£ 23,000,000.00	£ 267.06	£1,233,076.00	£ 19,800,000.00
Big Portfolios	£ 11,311.58	£ 1,866,595.00	£ 23,600,000.00	£ 552.18	£1,428,593.00	£ 18,000,000.00	£ 64.33	£1,079,156.00	£ 21,100,000.00	£ 92.91	£ 873,421.00	£ 12,900,000.00
Value Portfolio	£ 1,204.74	£ 2,342,001.00	£ 59,800,000.00	£ 30.39	£1,874,107.00	£ 53,100,000.00	£ 34.71	£1,586,401.00	£ 49,900,000.00	£ 461.12	£1,408,782.00	£ 35,100,000.00
Growth	£ 25,566.65	£ 2,216,887.00	£ 24,000,000.00	£ 458.70	£1,716,590.00	£ 24,000,000.00	£ 273.68	£1,300,293.00	£ 15,800,000.00	£ 16.34	£1,035,851.00	£ 13,300,000.00
High Profitability	£ 3,071.67	£ 1,623,580.00	£ 15,600,000.00	£ 228.16	£1,197,416.00	£ 27,500,000.00	£ 189.55	£ 913,957.40	£ 12,200,000.00	£ 66.01	£ 755,293.60	£ 10,500,000.00
Low Profitability	£ 103.54	£ 1,135,687.00	£ 33,200,000.00	£ 216.18	£ 920,116.10	£ 19,900,000.00	£ 79.24	£ 835,261.80	£ 16,300,000.00	£ 134.53	£ 777,517.90	£ 21,900,000.00
High Volaility	£ 111.09	£ 4,229,864.00	£474,000,000.00	£ 33.02	£3,816,678.00	£ 269,000,000.00	£ 196.21	£3,827,451.00	£ 259,000,000.00	£ 56.51	£3,772,973.00	£ 226,000,000.00
Low Volatility	£ 15,745.06	£ 1,201,234.00	£ 8,596,260.00	£2,910.00	£ 830,172.80	£ 7,445,220.00	£ 365.10	£ 548,858.40	£ 4,981,550.00	£ 77.04	£ 382,484.10	£ 3,466,004.00

Following on from the details of table one, the residual value presented shows the minimum, average and maximum residual portfolio values during successful periods

Table 4.8 shows the summary of the residual values for the 1 sort portfolios. At the 4% withdrawal rate the high volatility stocks produced on average the largest residual value (£4.2m). This is considerably higher than what the market produced on average (£1.8m). Small, value and growth portfolios were the other portfolios that produced higher than market average residual value. Although, it is tempting to assume that the scale of the residual value is entirely informed by the volatility of portfolios especially when one considers that the high volatility portfolios have the highest level of deviation (8%); however, this result tends to indicate that the size of the residual value is also informed by other factors, potentially the sequence of returns. This is evident by considering that although the value portfolio has a larger deviation compared to the small portfolio (5% Vs 4%), the small portfolio produced a larger average residual value (£2.5m Vs £2.3m). The low profitability portfolio produced the lowest average residual value of £1.1m.

At the 6% and 8% withdrawal rate, the same comparative performance was observed where the high volatility portfolio produced the highest average residual value and the small value and growth being the other portfolios that produced higher average residual values compared to the market. Also, the low profitability portfolio produced the lowest average residual value. At the 10% withdrawal rate, in addition to the same comparative performance repeating itself, the big sort portfolios also produced a better average residual value compared to the market.

Table 4.9

Summary of Residual Value (2 sort portfolios)

	Residual Value											
	4%			6% Withdrawal Rate			8% Withdrawal Rate			10% Withdrawal Rate		
	Min Value	Average Value	Max Value	Min Value	Average Value	Max Value	Min Value	Average Value	Max Value	Min Value	Average Value	Max Value
S, L	£ 122.68	£ 1,872,170.00	£ 76,800,000.00	£ 76.94	£1,755,446.00	£ 105,000,000.00	£ 27.99	£1,654,220.00	£ 50,100,000.00	£ 77.76	£1,698,103.00	£ 143,000,000.00
S, H	£ 769.80	£ 2,285,108.00	£ 76,100,000.00	£ 623.68	£1,836,283.00	£ 32,700,000.00	£ 539.65	£1,451,374.00	£ 23,800,000.00	£ 109.89	£1,334,180.00	£ 41,400,000.00
S, Hp	£ 48.06	£ 873,752.60	£ 30,800,000.00	£ 164.38	£ 732,709.20	£ 21,700,000.00	£ 91.53	£ 674,065.40	£ 22,000,000.00	£ 48.97	£ 632,424.30	£ 20,100,000.00
S, Lp	£ 1,508.46	£ 4,059,528.00	£ 88,400,000.00	£ 517.95	£3,298,507.00	£ 64,600,000.00	£ 264.84	£2,713,717.00	£ 45,500,000.00	£ 12.07	£2,359,656.00	£ 45,400,000.00
S, Hv	£ 57.63	£ 2,282,783.00	£ 91,000,000.00	£ 249.04	£1,993,630.00	£ 101,000,000.00	£ 79.10	£1,802,033.00	£ 52,500,000.00	£ 48.36	£1,779,211.00	£ 77,700,000.00
S, Lv	£ 72,167.12	£ 1,420,278.00	£ 10,300,000.00	£ 281.18	£1,012,135.00	£ 10,800,000.00	£ 46.58	£ 670,788.60	£ 9,671,709.00	£ 174.37	£ 476,462.90	£ 4,406,521.00
B, L	£ 8,317.72	£ 2,185,777.00	£ 19,200,000.00	£ 75.69	£1,711,850.00	£ 29,900,000.00	£ 105.53	£1,301,171.00	£ 25,400,000.00	£ 36.07	£1,032,983.00	£ 15,800,000.00
B, H	£ 1,064.15	£ 1,611,829.00	£ 49,900,000.00	£ 1.17	£1,339,014.00	£ 45,400,000.00	£ 20.85	£1,168,295.00	£ 33,900,000.00	£ 115.55	£1,066,127.00	£ 24,400,000.00
B, Hp	£ 1,793.68	£ 1,837,152.00	£ 18,500,000.00	£ 852.42	£1,406,642.00	£ 17,400,000.00	£ 169.84	£1,086,009.00	£ 14,600,000.00	£ 82.77	£ 855,543.40	£ 12,200,000.00
B, Lp	£ 233.01	£ 1,027,020.00	£ 25,000,000.00	£ 11.01	£ 867,850.00	£ 29,700,000.00	£ 4.92	£ 799,844.30	£ 20,400,000.00	£ 25.60	£ 778,976.50	£ 22,500,000.00
B, Hv	£ 8.84	£ 2,435,016.00	£127,000,000.00	£ 12.17	£2,300,040.00	£ 129,000,000.00	£ 2.31	£2,215,851.00	£ 112,000,000.00	£ 329.67	£2,147,736.00	£ 130,000,000.00
B , Lv	£ 35,260.89	£ 1,284,120.00	£ 7,575,887.00	£ 513.69	£ 920,612.10	£ 6,113,209.00	£1,196.93	£ 609,911.10	£ 6,299,184.00	£ 56.88	£ 438,873.60	£ 7,561,920.00
L, Hp	£ 20,207.35	£ 1,968,435.00	£ 25,200,000.00	£ 6.38	£1,497,568.00	£ 24,500,000.00	£ 13.92	£1,159,013.00	£ 20,000,000.00	£ 313.68	£ 911,587.90	£ 12,900,000.00
L, Lp	£ 89.22	£ 1,179,342.00	£ 46,400,000.00	£ 159.88	£1,060,886.00	£ 51,700,000.00	£ 177.52	£ 989,526.40	£ 26,000,000.00	£ 327.02	£1,019,743.00	£ 38,300,000.00
L, Hv	£ 195.14	£ 2,909,136.00	£360,000,000.00	£ 37.77	£2,963,997.00	£ 521,000,000.00	£ 230.76	£2,876,092.00	£ 569,000,000.00	£ 7.95	£2,997,257.00	£ 129,000,000.00
L, Lv	£ 6,760.93	£ 898,660.60	£ 5,106,878.00	£ 205.20	£ 613,116.40	£ 6,686,467.00	£ 68.00	£ 408,870.10	£ 5,242,709.00	£ 7.73	£ 304,381.50	£ 3,037,873.00
H, Hp	£ 52.29	£ 434,776.40	£ 28,100,000.00	£ 0.86	£ 454,590.90	£ 25,900,000.00	£ 107.78	£ 545,747.70	£ 16,900,000.00	£ 51.08	£ 516,342.30	£ 11,600,000.00
H, Lp	£ 1,083.60	£ 3,269,919.00	£ 65,900,000.00	£ 323.45	£2,743,621.00	£ 65,200,000.00	£ 17.80	£2,360,434.00	£ 52,200,000.00	£ 63.43	£2,031,171.00	£ 47,800,000.00
H, Hv	£ 32.49	£ 1,578,216.00	£ 96,200,000.00	£ 92.03	£1,576,041.00	£ 140,000,000.00	£ 23.25	£1,624,559.00	£ 85,100,000.00	£ 619.52	£1,766,399.00	£ 116,000,000.00
H, Lv	£130,876.10	£ 4,021,342.00	£ 32,800,000.00	£3,929.16	£3,261,406.00	£ 30,500,000.00	£ 667.31	£2,484,818.00	£ 26,800,000.00	£ 151.86	£1,814,932.00	£ 26,400,000.00
Hp, Hv	£ 382.50	£ 4,611,907.00	£252,000,000.00	£ 26.39	£4,376,845.00	£ 205,000,000.00	£ 225.04	£4,447,160.00	£ 504,000,000.00	£ 493.76	£4,788,017.00	£ 837,000,000.00
Hp, Lv	£ 20.72	£ 621,573.40	£ 5,761,747.00	£ 260.87	£ 415,287.50	£ 5,365,097.00	£ 10.91	£ 310,507.80	£ 4,395,045.00	£ 4.55	£ 251,648.60	£ 2,626,347.00
Lp, Hv	£ 17.92	£ 1,609,764.00	£163,000,000.00	£ 30.85	£1,770,947.00	£ 272,000,000.00	£ 66.90	£1,915,204.00	£ 252,000,000.00	£ 386.62	£1,858,378.00	£ 87,800,000.00
Lp, Lv	£ 11,885.51	£ 1,954,306.00	£ 24,600,000.00	£ 45.66	£1,495,049.00	£ 27,700,000.00	£ 5.64	£1,125,482.00	£ 14,300,000.00	£ 87.93	£ 893,934.40	£ 21,900,000.00

Following on from the details of table one, the residual value presented shows the minimum, average and maximum residual portfolio values during successful periods

Table 4.9 above shows the summary of the residual values for the 2 sort portfolios. At 4% withdrawal rate, a total of 9 portfolios out of the 24 considered produced a better than market average residual value. The *Hp,Hv* (high profitability and high volatility sort portfolios) portfolios produced the highest average residual value of £4.6m and this is closely followed by the *H,Lv* and *S,Lp* portfolios which produced about £4m on average. The bottom 3 portfolios (*H,Hp; S,Hp* and *L,Lv)* produced average residual values of £434,000, £873,000 and £898,000. Again, the association between the average residual value, the standard deviation and potentially the sequence of return described earlier was observed as the *Hp,Hv* portfolio though had the highest average residual value, it did not produce the highest standard deviation amongst this sort.

At the 6% withdrawal rate, a total of 14 portfolios produced a better than market average residual value (£1.4m). Again the *Hp,Hv* portfolio had the highest average residual value (£4.38m) whilst the *H,Lv* portfolio produced £3.26m average residual value. The remaining better performing portfolios produced an average residual value ranging from £1.50m to £3.30m. The worst average residual values were obtained by *H,Hp* (£454,000), *L,Lv* (£613,000) and *S,Hp* (£732,000). At the 8% and 10% rates of withdrawal, the *Hp,Hv* portfolio produced the larges average residual value (£4.4m and £4.8m respectively).

There were 16 portfolios that produced a better than market average residual value at the 8% and also at the 10% rates of withdrawal. At the 8% withdrawal rate, the remaining better than market performers ranged from £1.09m to £2.88m whilst the range at the 10% rate of withdrawal was £893,000 to £2.9m. The worst residual values at the 8% rate of withdrawal were £408,000, £545,000 and £609,000 whilst

£251,000, £304,000 and £516,000 were the worst average residual values obtained at the 10%.

Table 4.10

Summary of Residual Value (3 sort portfolios)

	Residual Value											
	4%			6% Withdrawal Rate			8% Withdrawal Rate			10% Withdrawal Rate		
	Min Value	Average Value	Max Value	Min Value	Average Value	Max Value	Min Value	Average Value	Max Value	Min Value	Average Value	Max Value
B, L, Hp	£ 4,351.25	£ 1,993,430.00	£ 27,300,000.00	£ 662.00	£1,487,955.00	£ 18,900,000.00	£ 633.58	£1,118,917.00	£ 18,000,000.00	£ 30.30	£ 882,930.20	£ 16,600,000.00
B, L, Lp	£ 18.28	£ 1,203,301.00	£ 68,700,000.00	£ 263.34	£1,046,018.00	£ 49,900,000.00	£ 76.02	£1,011,382.00	£ 43,000,000.00	£ 400.41	£1,029,780.00	£ 29,500,000.00
B, H, Lp	£ 61.30	£ 1,697,900.00	£ 38,000,000.00	£ 120.19	£1,582,636.00	£ 77,200,000.00	£ 314.00	£1,459,456.00	£ 52,600,000.00	£ 130.50	£1,472,289.00	£ 52,100,000.00
B, Lp, Hv	£ 78.05	£ 1,455,601.00	£ 86,200,000.00	£ 74.66	£1,605,721.00	£ 104,000,000.00	£ 3.35	£1,707,855.00	£ 123,000,000.00	£ 331.86	£1,761,656.00	£ 155,000,000.00
B, Hp, Hv	£ 171.07	£ 5,500,905.00	£183,000,000.00	£ 345.51	£5,221,587.00	£ 596,000,000.00	£ 99.07	£4,731,887.00	£ 253,000,000.00	£ 101.49	£5,201,375.00	£ 1,390,000,000.00
B, L, Hv	£ 241.65	£ 3,009,921.00	£327,000,000.00	£ 69.45	£2,922,884.00	£ 192,000,000.00	£ 112.60	£2,892,263.00	£ 204,000,000.00	£ 154.37	£3,238,993.00	£ 326,000,000.00
B, H, Hv	£ 30.26	£ 1,416,169.00	£145,000,000.00	£ 44.74	£1,590,171.00	£ 103,000,000.00	£ 125.86	£1,729,551.00	£ 162,000,000.00	£ 22.03	£1,931,229.00	£ 112,000,000.00
B, Lp, Lv	£ 221.52	£ 1,753,158.00	£ 24,500,000.00	£ 246.78	£1,318,551.00	£ 23,400,000.00	£ 424.44	£ 982,281.50	£ 12,300,000.00	£ 44.86	£ 800,830.50	£ 18,300,000.00
B, Hp, Lv	£ 185.67	£ 592,043.50	£ 7,408,291.00	£ 67.05	£ 410,275.70	£ 6,229,462.00	£ 30.07	£ 291,366.60	£ 4,390,980.00	£ 165.16	£ 248,811.80	£ 3,074,868.00
B, L, Lv	£ 27,529.16	£ 909,845.40	£ 11,300,000.00	£ 132.72	£ 622,020.90	£ 5,513,888.00	£ 57.23	£ 409,534.00	£ 5,148,848.00	£ 86.73	£ 311,379.80	£ 4,004,559.00
L, Hp, Hv	£ 30.88	£ 4,072,424.00	£383,000,000.00	£ 189.83	£3,774,753.00	£ 447,000,000.00	£ 175.64	£3,997,758.00	£ 1,080,000,000.00	£ 382.33	£3,928,750.00	£ 451,000,000.00
L, Hp, Lv	£ 254.86	£ 451,579.80	£ 4,026,687.00	£ 77.90	£ 291,050.00	£ 5,295,370.00	£ 22.06	£ 213,941.00	£ 4,069,103.00	£ 219.31	£ 192,461.40	£ 1,466,849.00
S, L, Hp	£ 10.57	£ 2,197,735.00	£192,000,000.00	£ 93.98	£1,955,090.00	£ 97,600,000.00	£ 97.11	£1,905,505.00	£ 66,800,000.00	£ 111.62	£1,810,077.00	£ 69,300,000.00
S, H, Lp	£ 1,190.95	£ 2,537,516.00	£ 44,200,000.00	£ 97.07	£2,112,667.00	£ 48,600,000.00	£ 45.48	£1,756,374.00	£ 44,500,000.00	£ 142.68	£1,534,569.00	£ 37,600,000.00
H, Lp, Hv	£ 25.17	£ 2,709,056.00	£142,000,000.00	£ 73.85	£2,917,374.00	£ 293,000,000.00	£ 75.59	£2,918,990.00	£ 281,000,000.00	£ 125.72	£2,857,045.00	£ 150,000,000.00
H, Lp, Lv	£ 9,036.86	£ 3,254,146.00	£ 36,400,000.00	£ 632.79	£2,571,461.00	£ 38,000,000.00	£ 220.00	£1,922,782.00	£ 39,500,000.00	£ 503.22	£1,529,954.00	£ 27,800,000.00
S, Lp, Hv	£ 390.38	£ 5,036,901.00	£425,000,000.00	£ 44.51	£4,474,397.00	£ 527,000,000.00	£ 126.61	£4,255,707.00	£ 205,000,000.00	£ 205.59	£3,972,936.00	£ 183,000,000.00
S, H, Hv	£ 22.30	£ 2,377,106.00	£140,000,000.00	£ 48.93	£2,111,422.00	£ 87,200,000.00	£ 36.53	£2,180,296.00	£ 214,000,000.00	£ 37.73	£2,087,244.00	£ 108,000,000.00
S, Lp, Lv	£115,357.50	£ 2,673,551.00	£ 19,300,000.00	£2,386.38	£2,094,376.00	£ 20,800,000.00	£ 152.32	£1,546,058.00	£ 24,900,000.00	£ 29.49	£1,127,882.00	£ 16,000,000.00
S, H, Lv	£159,290.90	£ 2,465,560.00	£ 25,000,000.00	£5,909.92	£1,884,606.00	£ 17,900,000.00	£ 234.07	£1,362,404.00	£ 16,800,000.00	£ 192.76	£ 949,754.90	£ 11,900,000.00

Following on from the details of table one, the residual value presented shows the minimum, average and maximum residual portfolio values during successful periods.

Table 3.3 shows the summary of the residual values for the 3 sort portfolios and 20 portfolios were considered in this category. *B,Hp,Hv and S,Lp,Hv* produced the largest and second largest average residual value across all the considered withdrawal rates. There were 12 portfolios that produced a better than market average residual value at the 4% withdrawal rate, 15 at the 6% and 8% withdrawal rates and 16 at the 10% withdrawal rate. At the 4% rate of withdrawal, the range of the remaining better performing portfolios was £1.99m to £4.07m whilst the bottom 3 portfolios achieved £451,000, £592,000 and £909,000.

At the 6% rate of withdrawal, the range of the remaining better performing portfolios was £1.49m to £3.77m whilst the bottom 3 portfolios achieved £291,000, £410,000 and £622,000. At the 8% rate of withdrawal, the range of the remaining better performing portfolios was £1.14m to £4.00m whilst the bottom 3 portfolios achieved £213,000, £291,000 and £409,000. At the 10% rate of withdrawal, the range of the remaining better performing portfolios was £882,000 to £3.93m whilst the bottom 3 portfolios achieved £192,000, £248,000 and £311,000.

In summary, from all the 4 most successful portfolios identified earlier, the *H,Lv* portfolios consistently produced the largest average residual value during successful periods for all the rates of withdrawal (from about £4m at 4% withdrawal to about £2m at 10% withdrawal). Next to this performance was the *H,Lp,Lv* portfolio with the same consistency across all the withdrawal rates, followed by the *S,Lp,Lv* and finally the *S,H Lv* portfolio.

The *B,Hp,Hv*, *Hp,Hv* and *S,Lp,Hv* portfolios produced the largest residual value at the 4%, 6%, 8% and 10% withdrawal rates (the *B,Hp,Hv* portfolio had success rates of 91.59%, 79.85%, 65.32% and 51.42% respectively while the *S,Lp,Hv* portfolio had success rates of 95.84%, 85.89%, 72.82% and 57.75% respectively. The *Hp,Hv* portfolio had success rates of 91.05%, 78.19%, 64.25% and 49.10% respectively).

A retiree's proper management of his nest egg requires a careful balancing of two financial risks. On the one hand, the retiree may spend too much and outlive his savings; on the other hand, the retiree may unnecessarily lower his lifestyle and end up with an unintended bequest. The results of Tables 3 tend to show that increasing the volatility of a portfolio will have a positive impact on how much bequest is left (although, as explained earlier, this does not entirely explain the level of bequest). However, this comes with an increased probability of failure therefore a careful balance of bequest motive and acceptable possibility of failure will have to be carefully assessed during the design of retirement portfolios.

At the 4% withdrawal rate, the 4 most successful portfolios produced between 45% and 73% of the highest average residual value obtained and at the 6% withdrawal rate, they produced between 36% and 62% of the highest average residual value obtained. At 8% withdrawal rate, these portfolios offered between 29% and 53% of the highest average residual value and at 10% withdrawal rate it was between 18% and 35% of the highest average value obtained.

Section 4.4

Relative Stability Measure

Table 4.11

Summary of Relative Stability Measure and Success Rate

Relative Stability Measure and Success Rate Summary						
Portfolio	Average Monthly Return	Relative Stability Measure	Success @ Rate 4% Withdrawal	Success Rate @ 6% Withdrawal	Success Rate @ 8% Withdrawal	Success Rate @ 10% Withdrawal
FTSE 350	1.17%	24.49	99.94%	97.85%	87.66%	63.38%
Small Portfolio	1.26%	18.94	99.94%	98.43%	88.70%	68.80%
Big Portfolios	1.17%	24.13	99.96%	97.92%	86.61%	62.49%
Value Portfolio	1.23%	17.09	99.42%	93.63%	79.80%	58.68%
Growth	1.22%	24.25	99.97%	98.75%	90.13%	68.33%
High Profitability	1.12%	24.4	99.88%	96.97%	83.79%	57.85%
Low Profitability	1.00%	16.88	96.25%	81.21%	59.70%	36.48%
High Volaility	1.40%	11.31	92.87%	81.35%	67.35%	50.55%
Low Volatility	1.03%	32.18	100.00%	99.42%	89.47%	54.20%
S, L	1.14%	11.79	90.97%	75.22%	56.77%	38.77%
S, H	1.23%	16.96	99.68%	95.32%	82.79%	60.31%
S, Hp	0.92%	14.15	93.50%	75.36%	51.29%	29.61%
S, Lp	1.41%	16.46	99.93%	97.98%	90.77%	74.90%
S, Hv	1.22%	13.64	96.01%	84.28%	68.04%	49.27%
S, Lv	1.08%	29.29	100.00%	99.62%	92.00%	60.01%
B, L	1.22%	24.35	99.97%	98.66%	89.72%	68.56%
B, H	1.11%	14.28	97.51%	86.94%	67.55%	45.60%
B, Hp	1.16%	24.04	99.90%	97.52%	85.09%	60.43%
B, Lp	0.97%	16.45	94.72%	77.50%	54.73%	32.45%
B, Hv	1.23%	13.39	91.70%	78.51%	61.10%	43.44%
B , Lv	1.05%	31.08	100.00%	99.37%	89.39%	55.90%
L, Hp	1.18%	24.29	99.91%	98.13%	87.79%	63.80%
L, Lp	1.00%	14.19	90.44%	71.71%	51.30%	32.64%
L, Hv	1.25%	10.31	84.22%	67.49%	52.27%	37.94%
L, Lv	0.95%	30.56	99.98%	97.83%	77.34%	37.61%
H, Hp	0.55%	13.48	52.24%	30.62%	16.09%	8.95%
H, Lp	1.34%	16.16	99.36%	94.28%	82.31%	64.21%
H, Hv	1.05%	11.04	79.60%	60.40%	43.45%	28.95%
H, Lv	1.41%	26.56	100.00%	99.95%	98.84%	90.86%
Hp, Hv	1.42%	10.70	91.05%	78.19%	64.25%	49.10%
Hp, Lv	0.84%	29.30	99.65%	89.93%	57.44%	21.92%
Lp, Hv	1.04%	10.37	74.38%	54.83%	38.81%	26.28%
Lp, Lv	1.18%	23.19	99.95%	98.59%	88.79%	64.61%
B, L, Hp	1.18%	24.30	99.94%	98.10%	88.54%	64.95%
B, L, Lp	1.00%	14.18	90.46%	72.40%	50.01%	31.94%
B, H, Lp	1.12%	12.40	92.38%	76.39%	57.99%	40.42%
B, Lp, Hv	0.97%	10.08	68.13%	48.63%	33.83%	22.71%
B, Hp, Hv	1.47%	10.57	91.59%	79.85%	65.32%	51.42%
B, L, Hv	1.25%	10.26	84.87%	66.86%	51.72%	37.27%
B, H, Hv	0.92%	9.81	59.39%	41.10%	28.53%	19.23%
B, Lp, Lv	1.14%	22.46	99.88%	97.22%	84.23%	58.34%
B, Hp, Lv	0.83%	28.92	99.47%	88.63%	55.24%	21.59%
B, L, Lv	0.95%	30.21	100.00%	97.63%	78.05%	38.59%
L, Hp, Hv	1.36%	10.35	88.03%	73.15%	58.65%	44.42%
L, Hp, Lv	0.75%	29.29	99,24%	82.81%	43.77%	12.97%
S, L, Hp	1.19%	12.80	92.47%	78.22%	59.85%	43.11%
S, H, Lp	1.26%	16.31	99.48%	94.32%	81.88%	61.36%
H, Lp, Hv	1.25%	10.86	85.15%	68.15%	52.13%	37.93%
H, Lp, Lv	1.34%	21.12	99.99%	99.67%	96.00%	81.83%
S, Lp, Hv	1.46%	11.05	95.84%	85.89%	72.82%	57.75%
S, H, Hv	1.21%	11.67	91.39%	76.88%	58.86%	42.01%
S, Lp, Lv	1.28%	22.97	100.00%	99.82%	96.44%	81.13%
S, H, Lv	1.25%	26.72	100.00%	99.95%	97.75%	81.98%

The table is broadly extracted from Table 1 (success rates) except for column 3 (Relative Stability Measure). The monthly total return (from the total return index) of the constituent stocks of the portfolios together with the market capitalisation (ME) was then used to establish the market capitalized monthly weighted return of the portfolios (from which the average monthly return *for the period under review was obtained; column 2). Using the AR(1) i.e. autoregression for each portfolio (Augmented Dickey Fuller model) the regressing coefficient β was obtained. This was then divided by 1 standard deviation of shock (to give the relative stability measure) given as 1 S.D of Shock = Square root of (sum of squared residual/n-a) where n-a = Degree of freedom (number of adjusted observations - number of regressors).*

Table 4.12

Correlation summary of Relative Stability Measure and Success Rate

	Relative Stability Measure
Relative Stability Measure	1.0000
Success Rate @ 4%	0.6094
Success Rate @6%	0.7105
Success Rate @8%	0.6076
Success Rate @10%	0.3306

The table shows the correlation between the relative stability measure (described above) with the results of the success rates.

Table 4.13

Correlation summary of Relative Stability Measure and Failure Point

	Relative Stability Measure
Relative Stability Measure	1.0000
Average Failure Point @ 4%	0.8756
Average Failure Point @6%	0.9518
Average Failure Point @8%	0.9333
Average Failure Point @10%	0.8903

The table shows the correlation between the relative stability measure (described above) with the results of the failure point.

Table 4.11 summarises the respective measure of relative stability (of which, the higher the value the better) with the various success rates. A quick glance at the table clearly shows an inconsistency between this measure and the success rates as confirmed in the correlation results in table 4.12.

The reason for this inconsistency is perhaps not farfetched. The relative stability measure is effectively a 1-dimensional measure based on standard deviation and a rate of change, but the success rate is 2 dimensional as it is a product of the combined effect of average returns and standard deviation. So, for example, the growth and high profitability portfolios (table 4.2) have similar standard deviations but potentially due to the difference in their average monthly return, they produce different levels of success rate. In the same way, portfolios *B,L,Hv* and *S,H,Lv* have the same average monthly return but different standard deviations hence, producing different levels of success.

One of the measures introduced by Estrada (2017) mentioned earlier in section 4.2 (failure point) was shown to give an even deeper information on sustainability. Again, from the reasoning of what the relative stability measure represents, one would expect that a more stable portfolio will fail at a later point of the retirement period. Table 4.13 shows that the correlation between the failure point and relative stability measure is much stronger. This result tends to indicate there may be some level of reliability of the relative stability measure after all. One potential reason for the difference in this correlation result compared to that of the success rate is that the failure rate may be providing a more rounded information about sustainability that is less about the returns but more about stability.

Furthermore, looking closely at the results in table 4.11, the relative stability measure gives a reasonably consistent indication of the level of success when 2 portfolios with similar returns are considered. Examples are portfolios Growth and *S,Hv; B,L* and *S,Hv*; *H,Lv* and *Hp,Hv*; *B,L,Hv* and *S,H Lv; B,Lv* and *H,Hv*; High profitability and *B,H,Lp; S,L* and *B,Lp,Lv; B,Hp* and *B,Hp,Hv*. This observation tends to indicate that the measure contains much of the information provided by the standard deviation as the more stable portfolio of each pairing above generally have lower standard deviation. However, it does seem also that the measure provides more indication of stability compared to the standard deviation; for example, portfolios *L,Hv* and *B,L,Hv* both have similar returns but *L,Hv* has a higher standard deviation, higher relative stability measure and is generally more successful. It also has a better failure point generally.

Section 5

Conclusion

One of the fundamental implications of the UK pension legislation in 2015 was that it placed the responsibility of ensuring the sustainability of defined contribution plans solely in the hands of holders of such plans. Constructing a retirement portfolio for the retirement phase (drawdown) of a retiree's retirement journey requires certain important considerations: generating excess return within the portfolio, deciding on a preferred withdrawal strategy including a withdrawal rate, and ensuring the portfolio is stable enough to sustain the chosen withdrawal rate. This research has focused on generating returns and withdrawal stability as they are the most factual. The choice of withdrawal rates is quite subjective, and hence we analysed a range of withdrawal rates including the current industry standard of 4% as well as 6%, 8% and 10% withdrawal rates. That way the reader can see the range of possible outcomes by varying the selected withdrawal rate.

We focused on the size, book to market ratio, profitability and volatility factors and then estimated how successful the resultant portfolios are using the Monte-Carlo simulation approach. We find that there was a size, value and profitability premium available, but the volatility premium was only captured in risk adjusted returns. We also see that investing in stocks based on their profitability measure will not have yielded returns in excess of the market. 24 two sort portfolios were considered and 11 of these outperformed the market with the Hp,Hv portfolio (high profitability-high volatility) portfolio producing the largest excess return of 25 basis points monthly. 12 of the 20 3 sort portfolios considered also outperformed the market with the B,Hp,Hv (big-high profitability-high volatility) producing the largest excess return of 30 basis points monthly.

Out of the 52 portfolios considered, the market and 27 other portfolios sustained the 4% rate of withdrawal with a success rate of between 99%-100%. Another 16 portfolios were able to sustain this rate of withdrawal with at least a 90% success. The worst performing portfolio was the *H,Hp* (value high profitability stocks) with a success rate of 52.24%. None of the portfolios (including the market) had a 100% success rate at 6% withdrawal but 25 portfolios had a success rate of at least 90% with portfolios *S,H,Lv* (Small, value, low volatility stocks) and *H,Lv* (value low volatility stocks) having the highest success rate of 99.95%. The worst success rate was 30.62%. At 8% and 10% withdrawal rates, the *H,Lv* portfolio was the most successful with 98.84% and 90% success respectively. The *S,H,Lv, H,Lp,Lv* and the *S,Lp,Lv* portfolios closely followed the *H,Lv* portfolio but only up to the 8% withdrawal rate.

In line with Estrada (2017), this research further examined the failure points of these portfolios i.e., the particular point (average) of a failure period when failure occurred. The results revealed that making conclusions based solely on the failure rates (1 – success rate) could be misleading for example, the failure (or success) rates could imply 2 portfolios have similar risk exposures, but the failure point may suggest otherwise because it gives a deeper layer of information.

However, the 4 most successful portfolios provided competitive average failure points as they produced either the furthest average failure point or very close to the furthest average failure point through all the withdrawal rates. The failure points for these portfolios compared to the market were either further or at par but the *H,Lv* portfolio was comparatively better than that of the market across all withdrawal rates.

We also assessed the average residual fund produced by the portfolios and find that increasing the volatility of a portfolio will have a positive impact on portfolio residual value although, this alone does not explain the value; the sequence of returns will also explain this. However, this comes with an increased probability of failure. Therefore, a careful balance of bequest motive and acceptable possibility of failure will have to be carefully assessed during the design of retirement portfolios. At the 4% withdrawal rate, the 4 most successful portfolios produced up to 73% of the highest average residual value obtained and up to 62% at the 6% withdrawal rate. They produced up to 53% and 35% at the 8% and 10% withdrawal rates.

We also proposed a *ad hoc* measure of relative stability which was expected to capture the same inference shown by the success rates since it is meant to infer a portfolios stability. The results showed that this measure was reasonably consistent with the failure point assessment, but low correlations were observed when compared with the success rate. An explanation for this is the difference in the dimensions making up the relative stability measure and the success rate (the relative stability measure is basically made up of measures of deviation whilst the success rate reflects the combined effect of returns and standard deviation). It was however observed that when 2 portfolios have similar returns, the measure is largely consistent in indicating the more stable one.

Key Summary of Conclusion.

- This work makes an initial contribution of considering the FTSE 350 market in the analysis of portfolio construction for drawdown purposes.

- 4 factors (size, book to market ratio, profitability, and volatility) were identified as potential sources of return in excess of the market.
- For the period considered (1996 – 2019), the profitability factor did not outperform the market and the low volatility factor (a sub factor of the volatility factor) only had a better risk adjusted return compared to the market.
- Based on this, the low volatility anomaly (presently gaining traction) was confirmed.
- Most of the portfolios formed with these factors will sustain a 4% fixed real rate of withdrawal.
- Higher average return does not necessarily imply better success rate and lower average return does not necessarily imply higher failure rate.
- Compared to the market and the entire portfolios formed, the *H,Lv*, *S,H,Lv*, *H,Lp,Lv* and *S,Lp,Lv* were identified as the most successful portfolios as they sustained up to 8% withdrawal with at least 96% success rate. The *H,Lv* sustained 10% withdrawal with 90% success. These portfolios also produced returns in excess of the market.
- Hence, this research further makes the contribution that these factor strategies above are successful equity strategies that can be used to generate returns in excess of the market and at the same time sustain higher than the generally adopted 4% fixed real rate withdrawal rate.
- Even when failure occurs, these portfolios sustained withdrawals for competitively long periods relative to the market and other portfolios.
- As a further contribution, the low volatility factor '*Lv*' (as well as the value factor '*H*') appears to be a driver of stability / sustainability. This tends to align with the low volatility anomaly as stocks offering relatively high returns at low risk; 2 factors predominantly required for a successful drawdown portfolio.

- As an additional contribution, the relative stability *ad hoc* measure proposed (which measures the speed of reversion per unit of shock) whilst it may not be consistently reflective of the success rates (because, unlike the relatively stability measure, the success rate is influenced by the level of returns), it can be used as an indicator of better stability and success when the comparative portfolios have similar return.

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