Risk for Price: Using Generalized Demand System for Asset Pricing *

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Abstract

I construct a non-parametric pricing kernel with consumption prices and expenditure by decomposing consumer's indirect utility function. This pricing kernel establishes the fundamental connection between the intertemporal financial asset-holding and the intra-temporal consumption portfolio. I examine the pricing kernel in explaining variation of expected returns across diverse equity portfolios. This pricing kernel is more successful than traded-factor models and simple consumption-based models. Allowing the generalized non-homothetic preference, I find the price of good requires relatively greater risk compensation in the good-service two-sector economy. Dissecting the risk premium of expenditure and the risk premium of relative good price, I show that shrinking expenditure share in the good sector helps explain the structural transformation of risk premium.

Keywords: Revealed Preference, Relative Price, Consumption-based Asset Pricing

JEL Classification: D11, E31, G12

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1 Introduction

Question

Consumption decision and wealth-management decision are routine decisions of consumers. Food price in the grocery store, meal price in the restaurant induce the spending decision of consumers across consumption categories. On the other sides, the return of financial assets also induces the spending decision of consumers across time and space. The intrinsic connection of consumption commodity market and the financial asset market invokes the natural research question for the financial economist: Can commodity price help the financial economist make precise prediction for financial asset return? This paper aims to investigate the innate linkage of spot commodity market and financial asset market under the general economy environment.

Consumer's shopping cart covers the chicken wing, the toothpaste, and the visit to dental clinics. Consumer's price for purchasing a commodity reflects the commodity's marginal contribution to consumer welfare. For example, corn price in 1900 and the Subway sandwich price in 2020 allows economists to reasonably compare the corn in 1900 and the Subway sandwich in 2020.

Correlation with consumer's welfare determines the discount rate of financial asset. In the equity market, dividend flow of company stocks have different correlation with commodity prices. Taking into account the uncertain fluctuation of consumption price, when consumers hold shares in companies that move simultaneously with consumer prices, consumers require compensation for taking additional consumption price risks.

How to quantify the risk price for bearing the fluctuation of consumer's price? Do the cereal price and the restaurant dining price perfectly elicit their impacts over consumer welfare? Should we anticipate equalized amounts of risk compensation across commodities?

Methodology

To answer these questions, I investigate consumer's intra-temporal preference and the inter-temporal preference simultaneously. I describe the consumer preference with indirect utility function, to straightforwardly evaluate the welfare impact of high-dimensional consumption prices. I conduct a non-parameteric decomposition for equation that decides the spending share, and the Euler equation that determines the expected excess return of equity assets. I further conduct the non-parametric analysis in the heterogeneous-agent economy. I construct the artificial consumer that are consistent with the aggregate consumption spending and financial market Stochastic Discount Factor. The non-parameteric analysis of single consumer fully holds for the artifical consumer in the heterogeneous-agent economy.

For the consumption commodity with smaller price-elasticity, consumer is less able to evade the impact of price increase. Consumer spends a smaller fraction of expenditure for such necessity commodity when she allocate a larger expenditure. Simultaneously, the consumer charges higher risk-compensation when the stock return is correlated with the necessity, compared with the company whose stock return is correlated with the luxury.

In the economy with income growth, asymmetric price elasticity implies the the structural transformation of risk premium. When evaluating the risk premium of consumption price, I use expenditure share to amend the linear Stochastic Discount Factor. In the economy where correlation between the economy fundamental and the stock return are stationary, the long-run shift of expenditure share gives direct explanation for the long-run evolution of risk-premium.

Literature

The literature of Consumption-based Asset Pricing incorporate multiple consumption sectors to analyze the consumer welfare in the more granular view. (Yogo, 2006; Belo, 2010; Yang, 2011; Eraker et al., 2016) considers the durable consumption sector and the non-durable consumption sector. (Piazzesi et al., 2007) include the housing sector. (Dittmar et al., 2020) include the energy sector. Above researches considers the CES functional form, hence tractable analysis is available. (Ait-Sahalia et al., 2004) considers the nondurable consumption sector and the luxury sector, consumer's utility is separable in the quantities of each consumption sector. (Lochstoer, 2009) use Stone-Geary preference for the necessity-luxury two-sector economy. (Pakoš, 2011) allows the heterogeneous elasticity of the durable sector and non-durable sector in the CES functional form. These three articles allows the non-homotheticity in consumer preference.

The literature of structural transformation quantitatively evaluates whether the non-homothetic preference is capable to explain the long-run shift of consumption composition. (Boppart, 2014) uses PIGL (Price-independent Generalized Linear) preference, quantifies the income effect in the two-sector economy with unbalanced productivity growth; (Comin et al., 2021) use non-homothetic CES preference, separate the income effect and the price effect in a three-sector model. There have been a long tradition in identifying the micro consumption decision with non-homothetic preference. (Deaton and Muellbauer, 1980) discuss the convenience in estimating the PIGLOG (Price-independent Generalized LOG) preference. (Blundell et al., 1994; Parodi et al., 2020) exploits PIGLOG prefernce to simultaneously estimate the consumption portfolio and inter-temporal saving decision using the granular consumption data.

A strand of Macroeconomic literature investigates the parametrized aggregation conditional on thick-tail distribution: (Houthakker, 1955; Levhari, 1968; Lagos, 2006) explores the production function, (Wang and Wen, 2012; Ai et al., 2013) studies the investment adjustment cost function. As the parallel literature, non-parametric aggregation answers whether it is possible to understand the aggregate outcome without detailed knowledge of underlying distribution: (Muellbauer, 1976) discuss the existence of static consumer under price-independent generalized linear preference; (Jackson and Yariv, 2019) question the existence of dynamic investor when Gorman-preference is not available; (Hulten, 1973; Gabaix, 2011; Baqaee and Farhi, 2019) discuss the aggregate production function of the production network; (Baqaee and Burstein, 2021b) discuss a particular scenario for the existence of aggregate consumer where individual consumers have non-homothetic preference. Empirically, the non-parametric aggregation helps to construct the sufficient statistics for the economy with heterogeneous decision-makers and distribution of decision outcomes.

Contribution

This paper primarily contributes to the asset pricing literature that studies the endogenous determination of asset price and commodity price. Price elicits the primitive aggregate shock in (Papanikolaou, 2011) and (Johnson, 2011). Price reveals the producer's marginal utility in (Belo, 2010), consumer's marginal utility in (Lochstoer, 2009). Sticky wage increases correlation between the aggregate dividend flow and the consumer welfare in (Favilukis and Lin, 2016). (Roussanov et al., 2021) estimates the risk premium of core-CPI across multiple financial assets. I contribute to this literature by using indirect utility function to describe the consumer welfare. This allows me to deliver the innate connection between the consumer welfare and the spot consumer's commodity price. In particular, my estimation framework admits generalized non-homothetic preference. Compared with the consumption-based asset pricing literature of heterogeneous goods (Ait-Sahalia et al., 2004; Yogo, 2006; Piazzesi et al., 2007; Lochstoer, 2009; Belo, 2010; Pakoš, 2011; Yang, 2011; Eraker et al., 2016; Dittmar et al., 2020), this paper differs in the use of non-parametric analysis. The theoretical prediction applies for a broad class of parameterized preference.

Last but not least, this paper contributes to the emerging macro-finance literature of structural transformation. (Hou and Van Dijk, 2019) explains the disappearance of size premium. (Belo et al., 2021) documents the rising share of intangible capital in the market valuation of listed firm. (Crouzet and Eberly, 2021) incorporate the intangible capital to ease the quantitative tension between the physical capital investment rate and the firm valuation. I contribute to this literature in explaining the gradual-shift of Stochastic Discount Factor in the economy with income growth. (Smith and Timmermann, 2021) investigates the diminishing risk premia across equity portfolios.

Layout

The paper is organized as follows. Section 2 provides the approximation of Euler Equation in financial asset holding. Section 3 estimates the Euler Equation, and tests the consumption preference in the pricing kernel. In Section 4, I discuss how the price-decomposition of SDF helps find the risk-premium, and how it helps explain the long-run evolution of SDF. Section 5 constructs the consistent artificial representative consumer in the economy with heterogeneous consumers under generalized consumption preference. In Section 6, I discuss the existing conflicts in the estimation of aggregate consumption data. Section 7 concludes.

2 Decomposition of Pricing Kernel

2.1 Description of Consumer Decision

I consider the discrete-time infinite-horizon consumption bundle allocation problem of a representative consumer. The set of commodity category is fixed set $\mathcal{J}$. The state of the world is described by the $\{\{z_t\}_{t=0}^{\infty}\}$. Motion of history path is $z^{t+1} = (z^t, z_{t+1})$. The consumer is endowed with the stream of labor earning $\tilde{L}$: at path z, at time point t, the labor endowment is ℓ_t.

The consumer participates in the competitive financial market. The price of financial security[1] is $P^s_{k,t}$, the payout is $D_{k,t}$. The consumer participates in the Walras commodity market. The spot price of commodity is $P_{j,t}$ per unit. The consumer earns wage w_t per unit of labor.

Now I describe the consumer's life-time consumption problem (P.0),

$$
\begin{aligned}
\overline{U}_0(\vec{\theta}_0) \quad &= \quad \sup_{\tilde{C},\tilde{\theta}} \quad U(\tilde{c}) \\
s.t. \qquad & \sum_k \theta_{k,t} \cdot (P_{k,t} + D_{k,t}) + w_t \cdot \ell_t = \sum_j P^c_{j,t} \cdot C_{j,t} + \sum_k \theta_{k,t+1} \cdot P_{k,t}, \\
& c_{j,t} \geq 0; \quad \sum_k \theta_{k,t+1} \cdot P_{k,t} \geq \underline{a}.
\end{aligned}
\tag{P.0}
$$

The utility function $U(\tilde{C})$ describes the consumer's preference over life-time consumption bundle, I specify the preference in details later. The vector $\vec{\theta}_t$ describes the shares of financial security held by the consumer. The financial constraint is constructed to avoid the Ponzi-game, and $\underline{a}$ is constructed to never bind, in the same argument of Chapter 8 of (Ljungqvist and Sargent, 2012).

I skip the exact specification for the producer in the economy, because above description applies for arbitrary economy where the aggregate consumption allocation is consistent with the rational dynamic consumption decision. This identification methodology is similar with (Yogo, 2006), where the details of production is irrelevant for the consumption asset pricing model.

2.1.1 Intra-temporal Preference

The indirect utility function $V(P, E) : \mathcal{P} \times \mathbb{R}_{++} \to \mathbb{R}$ is defined as

$$
\begin{aligned}
V(P, E) = \max_{\vec{C} \in \mathcal{X}} \quad & g(C_1, C_2, \ldots, C_J) \\
s.t. \quad & \sum_{j \in \mathcal{J}} P_j \cdot C_j \leq E.
\end{aligned}
\tag{S.1}
$$

Here, the direct utility function $g(\cdot)$ describes the consumer's preference over the consumption bundle $(C_1, C_2, \ldots, C_J)$.

[1] I use the subscript s for the financial security.

Definition 1. *Define the **absolute share** of j-th sector as* ω_j*,*

$$\omega_j \equiv \frac{P_j \cdot C_j}{E}.$$

*Define the **relative expenditure share** between the k-th sector and the j-th sector as* $\mathcal{S}_{k,j}$*,*

$$\mathcal{S}_{k,j} \equiv \frac{\omega_k}{\omega_j}.$$

*Define the **core-IDU** as the value of IDU given price vector* P *and 1 unit consumption spending* E*,*

$$V^*(P) \equiv V(P, 1).$$

Define the (k, j)*-pair **price elasticity** as* $\eta_{k,j}$*,*

$$\eta_{k,j} \equiv -\frac{\mathcal{D}_{k,j} V^*(E^{-1} \cdot P)}{\mathcal{D}_k V^*(E^{-1} \cdot P)} \cdot (E^{-1} \cdot P_j).$$

The matrix of price-elasticity η is well-defined as long as the absolute expenditure share is strictly positive [2].

2.1.2 Inter-temporal Preference

I assume the preference of representative consumer over the consumption stream $\tilde{C}$ can be represented by the utility function $\mathcal{U} : \mathbb{R}_+^{\mathcal{J} \times \infty \times Z} \to \mathbb{R}$. In particular, I consider the simple inter-temporal preference where the consumer has CRRA coefficient γ over the utility flow $u_t = g(\vec{C}_t)$,

$$\mathcal{U}(\tilde{C}) = \lim_{T \to \infty} \mathbb{E}[\sum_{t=1}^{T} \beta^t \cdot \frac{g(\vec{C}_t)^{1-\gamma}}{1-\gamma}]. \tag{1}$$

I use utility function $u(\vec{C}) = \frac{g(\vec{C})^{1-\gamma}}{1-\gamma}$ to represent the life-time equivalent utility of consumption bundle $\vec{C}$. I use indirect utility function $v(\vec{P}, E) = \frac{V(\vec{P},E)^{1-\gamma}}{1-\gamma}$ to represent the life-time equivalent utility from consumption spending E and commodity price $\vec{P}$. Lemma 1 ensures the legitimate conversion from dynamic optimization problem with consumption bundle to the dynamic optimization problem with total consumption expenditure [3].

[2]For example the Cobb-Douglas direct utility function $u(C_1, C_2) = C_1^{\Omega_1} \cdot C_2^{\Omega_2}$ implies the constant expenditure share, but the matrix of price-elasticity η is still well-defined as

$$\eta = \begin{bmatrix} \Omega_1 + 1 & \Omega_2 \\ \Omega_1 & \Omega_2 + 1 \end{bmatrix}.$$

[3]This conversion is directly used in the (Parodi et al., 2020), here I formally verify the legitimate conversion following (Stokey, 1989).

Lemma 1. *Define the problem (P.1) as*

$$\begin{aligned}
\overline{U}_0(\vec{\theta}_0) \quad &= \quad \sup_{\tilde{C},\tilde{\theta}} \quad \lim_{T\to\infty} \mathbb{E}[\sum_{t=1}^{T} \beta^t \cdot u(\vec{C}_t)] \\
s.t. \qquad & \sum_k \theta_{k,t} \cdot (P^s_{k,t} + D_{k,t}) + w_t \cdot \ell_t = \sum_j P_{j,t} \cdot C_{j,t} + \sum_k \theta_{k,t+1} \cdot P^s_{k,t}, \\
& C_{j,t} \geq 0; \quad \sum_k \theta_{k,t+1} \cdot P^s_{k,t} \geq \underline{a}.
\end{aligned} \tag{P.1}$$

Define the problem (P.2) as

$$\begin{aligned}
\overline{V}_0^{\text{New}}(\vec{\theta}_0) \quad &= \quad \sup_{\tilde{E},\tilde{\theta}} \quad \lim_{T\to\infty} \mathbb{E}[\sum_{t=1}^{T} \beta^t \cdot v(\vec{P}_t, E_t)] \\
s.t. \qquad & \sum_k \theta_{k,t} \cdot (P^s_{k,t} + D_{k,t}) + w_t \cdot \ell_t = E_t + \sum_k \theta_{k,t+1} \cdot P^s_{k,t}, \\
& E_t \geq 0; \quad \sum_k \theta_{k,t+1} \cdot P^s_{k,t} \geq \underline{a}.
\end{aligned} \tag{P.2}$$

Optimization problems (P.1) yields equivalent value as the optimization problem (P.2). For each optimal policy C^* in problem (P.1), E^* such that

$$E^*_t = \sum_{j\in\mathcal{J}} P_{j,t} \cdot C^*_{j,t}, \quad \forall t, z^t \tag{2}$$

is an optimal policy in the optimization problem (P.2).

In the appendix, I directly show the linkage between the optimal consumption plan and the optimal expenditure plan. [4]

For the representative consumer, the wealth-constraint never binds, so the shadow price wealth-constraint is zero along the optimal path of expenditure. Optimal expenditure decision implies

$$\lambda_t = V(\vec{P}_t, E_t)^{-\gamma} \cdot \mathcal{D}_E V(\vec{P}_t, E_t). \tag{3}$$

where λ_t is the shadow price of (t, z) budget constraint in the optimization problem (P.2), after the correction of subjective discount rates and natural probability.

Definition 2. *Define the* ***real Stochastic Discount Factor*** $\tilde{M}$ *as*

$$\tilde{M}(\vec{P}_t, E_t) \equiv V(\vec{P}_t, E_t)^{-\gamma} \cdot \mathcal{D}_E V(\vec{P}_t, E_t) \cdot P_{J,t}. \tag{4}$$

The real pricing kernel augments the natural distribution of economic states and subjective discount rate when determining the financial asset price. The formula $V(\vec{P}_t, E_t)^{-\gamma}$ is H.D.0, while the formula $\mathcal{D}_E V(\vec{P}_t, E_t)$ depends on the choice of numeiraire in the economy environment. I include $P_{J,t}$ to focus on the relative term $\frac{P_{J,t}}{E_t}$, and to avoid the unnecessary discussion of numeiraire choice.

[4] Alternatively, one can show the equivalence with shadow price, assuming the transversality condition holds.

2.2 Decomposition of SDF

The real Stochastic Discount Factor $\tilde{M}$ helps evaluate the price formation in the financial markets. Hereafter, I use notation $\tilde{m} = \log(\tilde{M})$, and $\mathrm{d}\tilde{m}$ for the change of $\tilde{m}$.

Theorem 1. *First-Order Approximated SDF is*

$$\mathrm{d}\tilde{m} = -\sum_{j=1}^{J} b_j(\vec{P}, E) \cdot \omega_j \cdot (\mathrm{d}p_j - \mathrm{d}p_J) - b_e(\vec{P}, E) \cdot (\mathrm{d}e - \mathrm{d}p_J) + o(h). \tag{5}$$

with high-order term $o(h)$ for $h = \max\{\{\mathrm{d}p_j\}_j, \mathrm{d}e\}$. The risk price vector b is

$$\begin{aligned} b_j(\vec{P}, E) &= -\,[\gamma \cdot \frac{\mathcal{D}_E V(\vec{P}, E) \cdot E}{V(\vec{P}, E)} + 1] + \sum_{i=1}^{J} \eta_{j,i}(\vec{P}, E), \\ b_e(\vec{P}, E) &= -\sum_{j=1}^{J} [b_j(\vec{P}, E) - 1] \cdot \omega_j. \end{aligned} \tag{6}$$

The marginal utility from consumption expenditure are decomposed into terms related with the vector of absolute expenditure share ω, and the matrix of price-elasticity η. The risk-aversion parameter for utility flow γ and the term $\frac{\mathcal{D}_E V(\vec{P}, E) \cdot E}{V(\vec{P}, E)}$ determines the absolute level of risk price $\vec{b}$. The vector of absolute expenditure share ω, and the matrix of price-elasticity η decides the relative level of risk price $\vec{b}$. Here, I directly decompose the marginal utility of spending $V[\vec{P}_t, E_t]^{-\gamma} \cdot \mathcal{D}_E V(\vec{P}, E)$, assuming non-trivial relative risk-aversion parameter γ over utility flow [5].

The real Stochastic Discount Factor contains the term $\mathcal{D}_E V(\vec{P}, E)$, so the decomposition involves the matrix of price-elasticity η. Further, the approximation of fluctuation in the term $\mathcal{D}_E V(\vec{P}, E)$ is mainly determined by the Hessian Matrix of core-IDU function V^*. Now, I briefly explain the intuition behind the algebra. The indirect utility function is Homogeneous Degree of Zero, so the marginal utility of expenditure is the summation of marginal utility contributed by the consumption price in each consumption sector,

$$\mathcal{D}_E V(\vec{P}, E) \cdot E = -\sum_{j=1}^{J} \mathcal{D}_j V(\vec{P}, E) \cdot P_j.$$

Importantly, the absolute expenditure share ω_j tells us the contribution of consumption sector j toward the marginal utility of expenditure,

$$\omega_j = \frac{P_j \cdot \mathcal{D}_j V(\vec{P}, E)}{\sum_i P_i \cdot \mathcal{D}_i V(\vec{P}, E)}.$$

[5] The author finished the preliminary decomposition in late August, 2021, so this work is independent of the (Baqaee and Burstein, 2021a). (Baqaee and Burstein, 2021a) implements the non-parametric decomposition for the consumer welfare $V(\vec{P}, E)$, by measuring the equivalent spending $\hat{E}$ conditional on achieving the same level of utility $V(\vec{P}, \hat{E}) = V(\vec{P}_B, E_B)$. If $\hat{E} > E$, consumer utility is higher given $(\vec{P}_B, E_B)$.

In the consumer theory, this property is known as "Roy Identity" or the "Shepard's Lemma". The price-elasticity $\eta_{j,i}$ is constructed to describe the sensitivity of $\mathcal{D}_j V(\vec{P}, E)$ (marginal utility of consumption sector j) toward the consumption price i. Overall, the H.D.0 property and the "Roy Identity" produces the term of price-elasticity η in the Equation (6),

$$\begin{aligned} \mathrm{d}\log \mathcal{D}_E V(\vec{P}, E) = & - \sum_{j=1}^{J} \omega_j \cdot (\mathrm{d}p_j - \mathrm{d}e) \\ & - \sum_{j=1}^{J} \omega_j \cdot [\sum_{i=1}^{J} \eta_{j,i}(\vec{P}, E)] \cdot (\mathrm{d}p_i - \mathrm{d}e) + o(h). \end{aligned} \tag{7}$$

Intuitively, the fluctuation of consumption price $\mathrm{d}p_i$ impacts the marginal utility of expenditure, through each consumption sector j. It looks as if the consumer's marginal utility of expenditure is a complicated production function of consumption prices, as in the decomposition of production function in (Gabaix, 2011) and (Baqaee and Farhi, 2019).

We can also decompose the fluctuation of $V(\vec{P}_t, E_t)^{-\gamma}$ in the similar way. Further, the fluctuation of utility is simpler, because we only need the information of first-order derivatives. Fluctuation of SDF is now the summation of the fluctuation of utility, and the fluctuation of marginal utility of expenditure. The decomposition analysis of SDF is Independent of the Consumption Hierarchy [6], as long as we have full information for the matrix of price-elasticity η.

Proposition 1. *Given the security k and the security f, real total return $\tilde{R}_{k,t\to t'}$ and $\tilde{R}_{f,t\to t'}$ satisfy*

$$\mathbb{E}[\frac{\tilde{M}_{t'}}{\tilde{M}_t} \cdot (\tilde{R}_{k,t\to t'} - \tilde{R}_{f,t\to t'})|\mathcal{I}_t] = 0. \tag{8}$$

Corollary 1. *Given the security k and the security f, total return $R_{k,t\to t'}$ and $R_{f,t\to t'}$ satisfy*

$$\mathbb{E}[\frac{\tilde{M}_{t'}}{\tilde{M}_t} \cdot (R_{k,t\to t'} - R_{f,t\to t'})|\mathcal{I}_t] \approx 0. \tag{9}$$

Here, the essential identification assumption is the path-independent preference and the interior expenditure decision. Because the INADA condition is assumed for $\lim_{E\to 0} \mathcal{D}_E V(\vec{P}, E) = \infty$, so the constraints of strictly positive expenditure are never binding. I exploit the Euler equation to identify the pricing kernel decomposed from the Indirect Utility Function. Here, the Euler equation describes the rational decision across different financial assets, so the choice of

[6] This is similar with the Network-Independence property in (Baqaee and Farhi, 2019).

deflator has neutral role in identifying the real pricing kernel [7].

Similar with (Cochrane, 1996), expected return is decomposed as

$$\mathbb{E}_t[R^e_{k,t+1}] = b_e \cdot \mathbb{E}_t\left[\mathrm{d}e_{t+1} \cdot R^e_{k,t+1}\right] + \sum_j b_j \cdot \omega_{j,t} \cdot \mathbb{E}_t\left[\mathrm{d}p_{j,t+1} \cdot R^e_{k,t+1}\right]. \tag{10}$$

The absolute expenditure share is directly observed in the aggregate data of personal consumption expenditure and micro-data of consumption panel. The matrix of price-elasticity is deep parameters in the economy. The fluctuation of price-elasticity is negligible in the economy with well-defined Representative consumer, and the in economy with Heterogeneous consumers and perfect risk-sharing. Therefore, above proposed decomposition helps the financial economists from heavy parametric assumptions in the economy with high-dimensional consumption portfolio. Compared to the literature of the consumption-habit, researchers no longer need to carefully assume the structure of habit and the dynamic of habit.

One might ask where is the "primitive macroeconomic shock" in this decomposition exercise. In the true world, the "primitive macroeconomic shock" enters into the expenditure-scaled price vector $E^{-1} \cdot \vec{P} = (E^{-1} \cdot P_1, E^{-1} \cdot P_2, \ldots, E^{-1} \cdot P_J)$. Because the expenditure-scaled price vector $E^{-1} \cdot \vec{P}$ is the sufficient statistic for the marginal utility of consumer, Theorem (1) of decomposing SDF actually constructs **the sufficient statistics** for the consumer welfare in a generalized economy environment with the "primitive macroeconomic shock". The prices works as this sufficient statistics of shocks in the consumption-based asset pricing model, so the financial economists no longer explicitly mine the "primitive macroeconomic shock". The absolute expenditure share $\vec{\omega}$ works as the sufficient statistic for the magnitude of impact, so the estimation is less dependent on the model parameters in describing the economy environment.

Suppose we use the format of traditional literature of asset pricing: identify the "primitive macroeconomic shock" empirically, and then simulate the shock-propagation of "primitive macroeconomic shock" in a quantitative model. Both the expenditure-scaled price vector $E^{-1} \cdot \vec{P}$ and the stochastic discount factor $\widetilde{M}$ can be represented in the vector of "primitive macroeconomic shock". This exercise would work for a long list of asset pricing models. In the true world, the major shock in the economy is different in different periods of economy. For example, the China-U.S. trade war had been the focus of financial media during 2015-2018, while the energy price was the focus during War-time in 2022. It is more difficult to explicitly tracks a particular source of "primitive macroeconomic shock" than tracing the sufficient statistics, let alone quantify the welfare-impact of a particular "primitive macroeconomic shock".

[7] The choice of deflator is non-trivial if we consider the Euler equation of longing a particular type of financial assets. Though treasury bonds provides almost risk-free fixed monetary coupon, the welfare-amount of risk-free coupon can still fluctuate. In addition, in the economy where there exists no well-defined composite consumption good, it is difficult to pin down the "effective numeraire", eg. gold, water, leisure, etc. From this perspective, it requires further efforts to understand the nominal risk-free rate.

The decomposition of SDF in the Theorem (1) is a legitimate approximation using the Taylor Theorem. The fluctuation of $E^{-1} \cdot \vec{P}$ might contain both the expected drift term, the cyclical term, and the unexpected shock term. The Taylor expansion is legitimate as long as the change is incremental and tiny. If the expected drift term, the cyclical term, and the unexpected shock term are all sufficiently tiny, the equation of approximation holds. However, the econometric models for identifying the vector of risk price $\vec{b}$ have particular requirement for the process of expenditure-scaled prices $E^{-1} \cdot \vec{P}$. In the online appendix, I discuss the identification assumptions involved in Theorem (1), for econometric methods commonly used by the researcher of asset pricing literature.

The decomposition of SDF in the Theorem (1) is particularly useful in the economy with **multiple commodities**. If we ignore the evolution of consumption basket along the economic history, combining the consumption expenditure, the CPI (consumption-price index) provided by the statistic authority is the sufficient statistic for gauging the consumer welfare. Unfortunately, the evolution of consumption basket is non-trivial when we look at the long economic history. The proportion of consumption expenditure toward the food sector is high in the early decades when the economy is poor, while it is low in the modern period when the economy is richer. Also, we observe non-trivial fluctuation of consumption basket along the business cycle. When the financial economists take the consumption basket seriously, the relative price across different consumption commodities is informative in gauging the consumer welfare. As the result, it is difficult to infer the financial market pricing kernel accurately without knowing the detailed consumption prices with respect to the consumption commodities.

One might ask why the composite price index provided by the bureau of statistic can't work as the ideal deflator for the financial market. In the economy with multiple commodities, if the consumer has symmetric preference across consumption commodities, the price elasticity η will also be symmetric,

$$\sum_{i=1}^{J} \eta_{j,i}(\vec{P}, E) \equiv \sum_{i=1}^{J} \eta_{1,i}(\vec{P}, E).$$

Under this scenario, the Tornqvist index $\mathbf{P}_{\text{Tornqvist}}$ provided by NIPA indeed works as the sufficient statistic in Equation (7),

$$\begin{aligned} \mathrm{d}\log \mathcal{D}_E V(\vec{P}, E) &= -\,[1 + \sum_{i=1}^{J} \eta_{1,i}(\vec{P}, E)] \cdot \sum_{j=1}^{J} \omega_j \cdot (\mathrm{d}p_i - \mathrm{d}e) + o(h) \\ &\approx -\,[1 + \sum_{i=1}^{J} \eta_{1,i}(\vec{P}, E)] \cdot \mathrm{d}\log \mathbf{P}_{\text{Tornqvist}} \end{aligned} \tag{11}$$

Unfortunately, symmetric price elasticity η imposes an overly strong simplification for the true world. As the result, the financial market pricing kernel favors knowing the detailed consumption prices across consumption sectors.

3 Estimation

In this section, I estimate the pricing kernel of financial assets in a two-sector economy $\mathcal{J} = \{g, s\}$. I choose the service price as the deflator P_J. I identify the parameters (b_g, b_e) in the pricing kernel with relative expenditure and relative good price [8]

$$\mathrm{d}\tilde{m} \approx -b_e \cdot (\mathrm{d}e - \mathrm{d}p_s) - b_g \cdot \omega_g \cdot (\mathrm{d}p_g - \mathrm{d}p_s). \tag{13}$$

Moment $g_{k,\mathcal{T}}(\theta)$ is sample mean of the Euler equation in holding the risky asset k,

$$\begin{aligned} &g_{k,\mathcal{T}}(\theta) \\ \equiv &\frac{1}{T} \cdot \sum_{t=1}^{T} [1 - b_e \cdot (\mathrm{d}e_{t+1} - \mathrm{d}p_{s,t+1}) - b_g \cdot \omega_{g,t+1} \cdot (\mathrm{d}p_{g,t+1} - \mathrm{d}p_{s,t+1})] \cdot R^e_{k,t+1} \end{aligned} \tag{14}$$

I denote $g_{\mathcal{T}}(\theta)$ as the $K-$dimension vector of $\{g_{k,\mathcal{T}}(\theta)\}_{k=1}^{K}$. The GMM estimator is

$$\hat{\theta} \equiv \min_{\theta \in \Theta} \quad g_{\mathcal{T}}(\theta)' \cdot W \cdot g_{\mathcal{T}}(\theta) \tag{15}$$

In the appendix, I specify the identification assumption for the GMM estimator as the consistent Extremum Estimator.

3.1 Data Description

3.1.1 Data Construction

I use the Annual data during 1964-2019 in Table 2.3.4, Table 2.3.5 from the NIPA website to construct the sector-level price and total non-durable consumption expenditure. I consider the good sector and the service sector:

- good: food grocery, apparel, other non-durable goods. I remove energy from the commodity sector referring (Nakamura, 2008).
- service: health care, food-away, recreation, financial service, and other service. I remove public transportation, housing from the service sector, following (Hazell et al., 2020).

I construct the Fisher index [9] as the sector-level price index. For all nominal time series, I use the price of service sector as the deflator to construct the

[8] The pricing kernel in the two-sector economy is

$$\mathrm{d}\tilde{m} \approx -b_g \cdot \omega_g \cdot \mathrm{d}p_g - b_s \cdot \omega_s \cdot \mathrm{d}p_s - b_e \cdot \mathrm{d}e. \tag{12}$$

I deflate the nominal good price and nominal expenditure with the nominal service price, so $\mathrm{d}p_s^s$ is constant zero.

[9] It is also known as price index implied by the chained real quantity. Fisher index is widely used by the Statistic Department in countries using the Kuznets' National Accounting system.

relative expenditure, the relative price of good. Figure (1) visually shows the peak of nominal prices at around 1980. I deflate the nominal time series using the nominal service price, to ensure the stationary relative price and the stationary relative expenditure.

Figure 1: **Plot of Price Indice**

[—See the Figure appendix—]

I use the equity portfolios from the DataLibrary of Kenneth French's website. I choose the the diverse equity portfolios based on the prominent anomalies in equity return.

- 5-Quintile Size portfolios, size is measured as the market value of outstanding common stock;
- 5-Quintile Book-to-Market portfolios, book asset is measured as the total asset in the balanced sheet;
- 5-Quintile Operating Profitability portfolios, operating profitability is measured as the EBITDA over the total asset;
- 5-Quintile Investment portfolios, investment rate is measured as the change rate of total asset;
- 10-Tercile Momentum portfolios, momentum is measured as the accumulate return of nearby 11-months one month ago.

These equity portfolios covers the representative cross-section anomalies in the equity market of United States. The portfolio design of passive funds refers the cross-section return anomalies, so the artificial equity portfolios reasonably mimick investors' market practice. I estimate the pricing kernel during the time interval of 1964-2019, when the 5 sets of equity portfolios are available.

3.1.2 Descriptive Statistic

Panel (a) of Table (1) provides the descriptive statistic for the two time series: the relative expenditure $\mathrm{d}e - \mathrm{d}p_s$, and the relative good price $\mathrm{d}p_g - \mathrm{d}p_s$. The first column illustrates the mean annual growth rate of relative expenditure is 1.27%, mean annual growth rate of relative good price is -1.33%. The second and the third columns show the auto-correlation coefficient in each time series, the correlation of relative expenditure and the relative price. The AR(1) coefficient for relative expenditure is 0.36, and 0.47 for the relative good price. Panel (b) of Table 1 provides the correlation coefficient between the relative good price with business cycle indices. Correlation between the relative price of good and the aggregate labor input is insignificant. Correlation between the relative good price and the market excess return is significantly negative. There exists no mechanical correlation between the time-series factors and the main indicators or equity market. Pairwise correlation between the relative good price and the sub-category price are provided in Table (A.2). I require stationary time-series factors when constructing the pricing kernel, this allows the intuitive interpretation of risk price for covariance. Dicker-Fuller test of relative good price and other main-sector price are provided in Table (A.3) in the table appendix.

Table 1: **Descriptive Statistic**

[—See the Table appendix—]

3.2 Cross-section Expected Return

3.2.1 IDU Pricing Kernel

Table (2) provides evaluation of estimation. For objective evaluation of model fit, I listed the traditional Asset Pricing models in column (1)-(4). Column "CAPM" considers the excess return of market portfolio exposes the fluctuation of SDF, the pricing kernel is assumed to be

$$\mathrm{d}\tilde{m} \approx -b_m \cdot r^e_{mkt}. \tag{16}$$

Column "FF-5" considers the Fama-French 5-factor model exposes the fluctuation of SDF that contingent on high-dimensional state variables, the pricing kernel is assumed to be

$$\mathrm{d}\tilde{m} \approx -b_m \cdot r^e_{mkt} - \sum_{k=2}^{5} b_k \cdot f_k. \tag{17}$$

Column "C-ND" considers using direct utility function to describe the marginal utility. In particular, I assume there exists well-defined non-durable composite good C_{nd} and the chained-quantity index describes the quantity of composite good C_{nd}. The utility flow is $u(C_{nd}) = \frac{C_{nd}^{1-\gamma}}{1-\gamma}$. The pricing kernel is decomposed as

$$\mathrm{d}\tilde{m} \approx -\gamma \cdot \mathrm{d}c_{nd}. \tag{18}$$

Column "C-D" considers the durable stock affects the utility flow of representative consumer. Construction of durable stock C_d follows (Yogo, 2006). The utility flow is $u(C_{nd}, C_d) = \frac{g(C_{nd},C_d)^{1-\gamma}}{1-\gamma}$. The pricing kernel is decomposed as

$$\mathrm{d}\tilde{m} \approx -\gamma \cdot \mathcal{D}_{nd}g \cdot \mathrm{d}c_{nd} - \gamma \cdot \mathcal{D}_{d}g \cdot \mathrm{d}c_{d}. \tag{19}$$

For comparable analysis, column "P-D" considers the durable stock affects the utility flow of representative consumer. This assumption is similar with the two-stage budget system in (Parodi et al., 2020). The durable stock acts as the parameter for the indirect utility function from budget allocation decision in non-durable consumption bundle. The utility flow is $u(\vec{C}, C_d) = \frac{V(\vec{P},E_{nd};C_d)^{1-\gamma}}{1-\gamma}$.

When inferring the fluctuation of marginal utility from non-durable expenditure, I consider the effect from change of durable stock,

$$\mathrm{d}\tilde{m} \approx \underbrace{-b_e \cdot (\mathrm{d}e - \mathrm{d}p_s) - b_g \cdot \omega_g \cdot (\mathrm{d}p_g - \mathrm{d}p_s)}_{\text{Durable Stock is fixed}} \qquad \underbrace{-b_d \cdot \mathrm{d}c_d}_{\text{Quantity Change of Durable}} \quad . \tag{20}$$

Though it is an unfinished research agenda to construct the representative consumer with aggregate durable stock, (Yogo, 2006) shows the empirical importance in inferring the SDF based on business cycle of durable stock. Here, I include the fluctuation of aggregate durable stock to overcome the omitted component of SDF.

I calculate the sample mean of absolute moments as the MAPE (Mean Absolute Pricing Error) of portfolio's annual excess return. If the SDF $\mathrm{d}\tilde{m}$ is mis-specified, the financial economist would observe the high MAPE.

$$\text{MAPE} = \frac{1}{K} \sum_k \left| \frac{1}{T} \cdot \sum_{t=1}^{T} (1 + \mathrm{d}\tilde{m}_{t+1}) \cdot R^e_{k,t+1} \right|. \tag{21}$$

If an investor use the mis-specified SDF $\mathrm{d}\tilde{m}$ to hedge the aggregate risk and construct risk-neutral investment strategy (more accurately, "perceived" risk neutral investment strategy, if the investor believes the accuracy of this SDF), high MAPE implies that the investor can still go a long way to construct high α based on this rmis-specified risk-neutral investment strategy. When reading the statistics of MAPE, the magnitude of MAPE reflects the magnitude of potential α in the corresponding investment strategy.

The IDU-pricing kernel larges compress the potential of mining α in the equity market, because it well describes the aggregate risk. MAPE decreases from the 1.93% to 0.39% when I use the IDU-pricing kernel, in comparison with the pricing kernel of market portfolio excess return. This comparison shows the equity market of United States is consistent with the consumer's rational choice of wealth management and consumption portfolio. MAPE is 0.84% for the pricing kernel of non-durable composite good. This comparison shows that approximated non-durable composite good using the chained-quantity generate larger model error than the general decomposition of indirect utility function.

For straight-forward comparison of model fitness, I report the RMSE (Root Mean Square Error) defined as

$$\text{RMSE} = \sqrt{\frac{1}{K} \sum_k \left| \frac{1}{T} \cdot \sum_{t=1}^{T} (1 + \mathrm{d}\tilde{m}_{t+1}) \cdot R^e_{k,t+1} \right|^2}. \tag{22}$$

Statistic of RMSE provides similar information with MAPE. Here is the subtle difference between the statistic RMSE and the statistic MAPE: if we observe the low MAPE and the high RMSE simultaneously, it tells us the pricing error $\left| \frac{1}{T} \cdot \sum_{t=1}^{T} (1 + \mathrm{d}\tilde{m}_{t+1}) \cdot R^e_{k,t+1} \right|$ is dispersed across assets. In other words, the proposed SDF $\mathrm{d}\tilde{m}$ works extremely bad for certain asset k.

I report the CV-R^2 (Campbell-Vuolteenaho R^2) defined as

$$R^2 = 1 - \frac{\sum_k \left[\frac{1}{T} \cdot \sum_{t=1}^{T} (1 + \mathrm{d}\tilde{m}_{t+1}) \cdot R^e_{k,t+1}\right]^2}{\sum_k \left[\frac{1}{T} \cdot \sum_{t=1}^{T} R^e_{k,t+1}\right]^2}. \tag{23}$$

The time-series average expected excess return $\frac{1}{T} \cdot \sum_{t=1}^{T} R^e_{k,t+1}$ reads as the Y-variable. Assume the SDF factor f_{t+1}, the time-series average covariance term $\frac{1}{T} \cdot \sum_{t=1}^{T} \mathrm{d}f_{t+1} \cdot R^e_{k,t+1}$ reads as the X-variable. This statistic evaluates the model-fitness cross-assets, $R^2 = 1 - \frac{\sum_k (y_k - \vec{b} \cdot \vec{X})^2}{\sum_k y_k^2}$. Compared with RMSE, the statistic CV-R^2 only focus on the model-fit, where the pricing error is scaled by the time-series average expected excess return. It is interesting to see that the quantity-based consumption model "C-ND" (Non-durable consumption quantity) and "C-D" (Non-durable consumption quantity and durable consumption quantity) already out-performs the traded-factor model "CAPM" (MKT factor) and "FF-5" (Fama-French five-factor model). Further, the price-based consumption model "P-ND" (relative good price, relative non-durable expenditure) and "P-D" (relative good price, relative non-durable expenditure, durable consumption quantity) has higher CV-R^2.

I report the p-value for the J-stat of GMM estimation. The J-stat is defined as the objective function value at the optimal parameter in the GMM estimation,

$$\mathcal{J} \equiv T \cdot g_{\mathcal{T}}(\theta^*)' \cdot W^* \cdot g_{\mathcal{T}}(\theta^*). \tag{24}$$

If we observe the J-stat to be tiny, that means we don't observe variation in the moment of Euler Equation $(1 + \mathrm{d}\tilde{m}_{t+1}) \cdot R^e_{k,t+1}$. This would leads to over-identification of parameter $\vec{b}$ [10]. Because the construction of testing assets use the diverse equity portfolios, p-value of J-stat is high under all specifications of SDF. Therefore, there exists no concern of over-identification.

Table 2: **Model Fitness**

[—See the Table appendix—]

Figure (2) illustrates the improvement of model fit for the six asset pricing models. The testing assets are colored for straight-forward interpretation. As in the figure, the improvement from the model "C-ND" to the model "P-ND" mainly occurs in the Size-BM testing assets and the Momentum assets. The improvement occurs because Size-BM testing assets has considerable dispersion of factor-loading toward the relative good price, while the Momentum assets has considerable dispersion of factor-loading toward the relative expenditure.

[10] An extreme case would be that the asset returns are highly correlated, $R^e_{k,t+1} \equiv k \cdot R^e_{1,t+1}$, then we in fact use the single moment for asset-type No.1. When using the single moment to identify two parameters b_g and b_e, we have the issue of over-identification.

For intuitive interpretation of approximated Euler equation, I consider the "quasi-equivalent" Fama-Macbeth two-step regression[11], constructed in the approach of (Cochrane, 1996). The diverse equity portfolio encompasses five famous cross-section asset-pricing anomalies. In particular, the operating-profitability related testing assets has smaller correlation with the value-related testing assets. Therefore, construction of diverse equity portfolio already addresses the critique of linear reformation in testing assets. I also consider the GLS regression for each model, using the weight-matrix suggested by (Lewellen et al., 2010), to mitigate the concern of strong correlation across testing assets. Because the expenditure share is involved, I consider two setups of identification to evaluate how the IDU decomposition reduce the pricing error. The statistic "OLS-R^2" and "GLS-R^2" takes the original time series of relative price $\mathrm{d}p_g - \mathrm{d}p_s$ as the time-series factor, then evaluates the explanatory power of covariance over sample-average excess return assuming the remained pricing error as zero. As in Table (2), the "OLS-R^2" of IDU pricing kernel is the high across the six asset pricing models. The "GLS-R^2" is comparable with the pricing kernel constructed with the direct utility function. The statistic "COLS-R^2" and "CGLS-R^2" considers the case where the remained pricing error can be nonzero. Under the lens of "COLS-R^2" and "CGLS-R^2", cross-section explanatory power in column "P-ND" is comparable with column "C-ND" and "C-D". Both setups of Fama-Macbeth two-step regression consider time-series factors and the equity excess return has stationary correlation coefficient.

Figure 2: **Fitness of Asset Pricing Models**

[—See the Figure appendix—]

Table (3) investigates whether the choice of testing assets affects the point estimate of b_e and b_g. When using the Size-BM 25 portfolios point estimate of b_e and b_g slightly changes to 30.05 and -68.26. When using the industry portfolios point estimate of b_e and b_g slightly changes to 33.27 and -69.95. In these two setups, over-identification hypothesis are both rejected. In addition, The IDU-pricing kernel generates smaller MAPE than both the traded-factor pricing kernel and real-quantity pricing kernel. The point estimate of "Anomaly Factors Pricing Kernel" qualitatively changes in different setup of testing assets [12]. The point estimates of the "Real-quantity Pricing Kernel" are quantitatively

[11] Fama-Macbeth two-step regression requires more assumptions than the GMM estimation of SDF, see the explanation in (Cochrane, 1996). In particular, when the testing assets is not sufficiently diversified, the sampler-average exces return of testing asset has non-trivial idiosyncratic noise. For example, for major asset pricing models, the industry-30 portfolios generally has poor model-fitness when gauging the model fit-ness using the R^2 in the 2nd stage regression. Nonetheless, Fama-Macbeth two-step regression delivers straightforward interpretation of risk-premium and risk-exposure, so I provide the comparison.

[12] In the online appendix, Table A.5 reports the point estimate of $\vec{b}$ in the "Anomaly Factors Pricing Kernel" constructed with Fama-French five factors. The risk price for the operating profitability factor b_{Profit} has large standard error in the "Mix-30" portfolio, using the diverse equity portfolios. However, the point estimate of b_{Profit} is 5.80, and is empirically significant. The risk price for the value-growth factor b_{BM} is -2.33, with non-trivial standard error in the

similar. This further verifies the equity market operates consistently with the consumer's rational choice.

Table 3: **Parameters**

[—See the Table appendix—]

The time-series average risk-premium is calculated in the 2nd step regression, without the intercept term or with the intercept term. In Panel (D) of Table (2), the time-series average risk-premium for the relative expenditure $\mathrm{d}e - \mathrm{d}p_s$ is significantly positive, the risk-premium for the relative good price $\mathrm{d}p_g - \mathrm{d}p_s$ is significantly negative, when the testing assets is the diverse equity portfolios.

Figure 3: **Factor-Loading of Benchmark Model**

[—See the Figure appendix—]

Figure (3) illustrates the distribution of factor-loading. As in second plot of Figure (3), there are significant correlation between the factor-loading to relative price and the sample-average excess return. Estimated risk-premium for the relative good price $\mathrm{d}p_g - \mathrm{d}p_s$ is significantly negative, when using other testing assets such as the Size-BM 25 portfolios and the Industry-30 portfolios. However, the estimated risk premium for the relative expenditure $\mathrm{d}e - \mathrm{d}p_s$ is less stable. The unstable estimation partly attributed to the distribution of factor-loading in the Size-BM 25 portfolios and the Industry-30 portfolios: the factor-loading (portfolio beta) toward the time-series factor $\mathrm{d}e - \mathrm{d}p_s$ is empirically noisy, located around the zero, while the the factor-loading (portfolio beta) toward the time-series factor $\mathrm{d}p_g - \mathrm{d}p_s$ is deeply negative locating in the left-side of origin.

The "Anomaly Factors Pricing Kernel" is constructed with the equity portfolios, the correlation between the traded factors and the testing assets is more stable, compared with the correlation between the consumption-based macro factors. For the model-fitness in the Fama-Macbeth two-step regression, "Anomaly Factors Pricing Kernel" is supposed to out-perform the consumption-based macro factors. Using the testing assets "Mix-30", we see the exception: the IDU-pricing kernel is comparable with the "Anomaly Factors Pricing Kernel". This occurs because the correlation between the testing assets and the change of relative consumption prices are generally non-zero. Further, the dispersion of correlation is sufficient, so we have sufficient instruments to identify the risk-premium and the latent parameters of risk-price. On the opposite side, if we use the traditional testing assets such as the Size-BM 25 portfolios, the dispersion of correlation is highly linear, so the identification is weaker. The Industry-30 portfolios provide another extreme situation for identification. Although we have

"Size-BM 25" portfolio. However, the point estimate of b_{BM} is -5.80 using the "Industry-30" portfolio, with small standard error. Admittedly, the short time-interval of annual observations restrict the estimation power for the high-dimension SDF.

sufficient variation for identifying the parameters, we have weak estimation for the risk-loading in industry portfolios due to the non-trivial idiosyncratic noise. Therefore, when calculating the statistics of model-fitness such as the "OLS-R^2" and "GLS-R^2", these statistics are often negative numbers [13].

3.2.2 Robustness Check

Table (4) further investigates where is the improvement of model fit, by estimating the pricing kernel in separate groups of testing assets. For the 5 portfolios of Operating Profitability plus the 5 portfolios of Investment, the "Anomaly Factors Pricing Kernel" and the "Real-quantity Pricing Kernel" generates the smaller model error. For the Size-BM testing assets and the Momentum assets, the model fit is qualitatively improved. This is consistent with the observation in Figure (3).

Table 4: **Subgroup of Testing Assets**

[—See the Table appendix—]

Because the traded-factors helps identifying the portfolio of high risk-exposure and the portfolio of low risk-exposure, overall the traded-factors out-performs the consumption-based asset pricing models, in the Fama-Macbeth regression. However, traded factors doesn't help eliciting the SDF. As the result, it is almost impossible to obtain robust estimate with the traded factors. The point estimates of risk price of Fama-French 5-factor model is qualitatively different in three subsets of testing assets: (a) 5 Size + 5 BM; (b) 5 Operating Profitability + 5 Investment; (c) 10 Momentum. As the opposite side, the IDU-pricing kernel is quantitatively similar in the 10 portfolios of momentum, and in the 5 portfolios of Operating Profitability plus the 5 portfolios of Investment.

If we admit the intercept term when estimating the factor-loading, asset pricing model with traded factors performs worse than the consumption-based asset pricing model. Non-negligible noise are produced when estimating the factor-loading to true SDF indirectly via the traded-factors. If a financial economist attempts to explain the growing cross-section anomaly by incorporating more anomaly factors in equity assets, he or she will over-fit a particular cross-section anomaly because of the volatile noise term[14]. In the opposite direction, the IDU-pricing kernel is motivated by multiple consumption sectors, so it provides the economically-reasonable explanation of high-dimensions in pricing kernel [15].

[13] It is possible that an over-fitting wrong asset pricing model generates the large "OLS-R^2" and "GLS-R^2", under the wrong guidance of idiosyncratic noise.

[14] In the online appendix, Table A.6 reports the point estimate of risk price $\vec{b}$ and estimation of risk premium for the Fama-French 5-factor model. The estimation overfit the testing assets in each subgroup of testing assets.

[15] So far I hasn't observed the advantage of IDU pricing kernel in high-frequency environment. We are in lack of the high-frequency macroeconomic data, while the financial markets provides more timely information, so it is more practical to mimick the successful trading strategy for identifying the risk-premium.

Table (5) investigates whether the construction procedure of price index affects the point estimate of b_e and b_g. When using the first-order difference of Tornqvist Index, the point estimate is quantitatively similar. If I use the average change of category-level price index, the point estimate of relative expenditure is no longer significant, but the mis-measured change of relative price is still significant. Construction of Fisher-Index takes consideration of a broad class of consumption preference. Construction of Tornqvist Index caters to the homothetic preference. However, using simply the cross-category average price produces non-negligible approximation error. Because construction of Fisher-Index and Tornqvist Index takes the consumption hierarchy into consideration, approximation errors are smaller when I use these price index for the simplified two-sector economy.

Table 5: **Price Index**

[—See the Table appendix—]

Table (6) investigates whether the construction procedure of consumption sector affects the point estimate of b_e and b_g. When using the classification of Non-durable good and service of NIPA, the point estimate of b_g is quantitatively similar. When using the classification of good and service of NIPA as in (Boppart, 2014), the point estimate of b_e is quantitatively similar.

Table 6: **Consumption Sector**

[—See the Table appendix—]

Price-independent budget constraint ensures the Indirect Utility function $V(\vec{P}, E)$ as the Homogeneity Degree of Zero. As the result, we have the identity as

$$b_e = -\sum_{j=1}^{J}(b_j - 1) \cdot \omega_j.$$

under the estimation assumption of constant risk price $\vec{b}$.

Table (7) exploits the identity of Price-Independent Budget Constraint, estimate the risk price $\vec{b}$ given different setups. I consider two alternative setups as below

- I set the deflator as $p_J = p_g$ to identify parameter (b_e, b_s). The variation of real pricing kernel is decomposed into the variation of relative expenditure and relative price of service (with respect to the good price).

$$\mathrm{d}\tilde{m} \approx -b_e \cdot (\mathrm{de} - \mathrm{d}p_g) - b_s \cdot \omega_s \cdot (\mathrm{d}p_s - \mathrm{d}p_g). \quad (25)$$

- I identify parameter (b_g, b_s). The variation of real pricing kernel is decomposed into the variation of the relative expenditure, relative price of good, relative price of service (with respect to wage).

$$\mathrm{d}\tilde{m} \approx -b_g \cdot \omega_g \cdot (\mathrm{d}p_g - \mathrm{de}) - b_s \cdot \omega_s \cdot (\mathrm{d}p_s - \mathrm{de}) - (\mathrm{de} - \mathrm{d}p_s). \quad (26)$$

I find the model fit is qualitatively similar with the main table. In addition, point estimate of risk price is quantitatively reasonable, compared with the Table (3). Due to small magnitude of b_s, the first-stage estimate is inaccurate for b_s, but estimated b_g is quantitatively close.

Table 7: **Alternal Estimation for Parameters**

[—See the Table appendix—]

4 Application

4.1 Necessity Premium

On the one side, the consumer preference determines how the consumer allocate the financial wealth across different states of economy and different periods. On the other side, the consumer preference determines the intra-period consumption basket. Theoretically, fundamental connection is expected to exist between the financial market pricing kernel and the intra-period consumption portfolio. The price elasticity η plays the role is the vector of risk price $\vec{b}$ in Theorem 1. Now I use the Proposition 2 to interpret the matrix of price elasticity η using the consumption portfolio.

Proposition 2. *Given $\mathcal{S}_{k,j}$ is non-trivial constant, the log-change of relative share is decomposed into the price effect and the income effect,*

$$\begin{aligned} \mathrm{d}s_{k,j} =&(1-\eta_{k,k}+\eta_{j,k})\cdot \mathrm{d}p_k - (1-\eta_{j,j}+\eta_{k,j})\cdot \mathrm{d}p_j - \sum_{i\neq k,j}(\eta_{k,i}-\eta_{j,i})\cdot \mathrm{d}p_i \\ &+\sum_i(\eta_{k,i}-\eta_{j,i})\cdot \mathrm{d}e + o(h). \end{aligned} \tag{27}$$

with $s_{k,j}=\log(\mathcal{S}_{k,j})$, $h=(\mathrm{d}p,\mathrm{d}e)$ and $\mathrm{d}s_{k,j}=s_{k,j}(p+\mathrm{d}p,e+\mathrm{d}e)-s_{k,j}(p,e)$. The term $o(h)$ is higher-order in h in the sense that $\lim_{||h||\to 0}\frac{o(h)}{||h||}=0$ under sup-norm $||h||\equiv \sup_j|h_j|$.

All else equal, 1% increase of sector-k price P_k contributes to $1-\eta_{k,k}+\eta_{j,k}$ increase of log-relative share $S_{k,j}$ [16]. Price-elasticity is required to be greater than 0, so consumer's utility is deteriorated after the increase of price.

If the price-elasticity $\eta_{k,k}+\eta_{k,j}<\eta_{j,j}+\eta_{j,k}$, all else equal, consumption portfolio is more rigid toward the impact of sector-k price $\mathrm{d}p_k$. Substitution across commodities wouldn't largely alleviate the consumer's utility in response

[16] For consistent notation, I use the upper-case character for the nominal price and the nominal expenditure. I use the lower-case character for the log nominal price and the log expenditure. I use the affix d for small changes, eg. $\mathrm{d}p_k=\log P_{k,t+1}-\log P_{k,t}$.

to the increase of sector-k price. In particular, if $1 - \eta_{k,k} < 1$, relative share in the pair of sectors (k, j) increases in the sector-k price [17].

The equation of approximation in Proposition 2 departs from the demand system in (Deaton and Muellbauer, 1980), in the sense that the demand system is identified using the pairwise equations of relative share, while (Deaton and Muellbauer, 1980; Parodi et al., 2020) identify the PIGLOG demand system using equations of absolute share. Adoption of relative share develops the identification strategy in (Comin et al., 2021). Here, I relax the assumption of constant price elasticity across sectors, so the relative share is exploited to directly understand the dynamic of consumption composition. (Deaton and Muellbauer, 1980; Parodi et al., 2020; Comin et al., 2021) implement the identification based on parametric assumption of PIGLOG preference or Generalized Non-homothetic CES preference. Here, the equation of non-parametric decomposition guides to a more straight-forward interpretation of fluctuating price, spending and the consumption portfolio.

The approximation of relative expenditure share develops the non-parametric approach in (Baqaee and Farhi, 2019). (Baqaee and Farhi, 2019) defines the "pseudo elasticity" for the production function referring (Morishima, 1967). Here, I decompose the relative share $S_{k,j}$ implied by the core-IDU function V^* using the "micro-fluctuation" as the spending-scaled price vector $(E^{-1} \cdot P_1, E^{-1} \cdot P_2, \ldots, E^{-1} \cdot P_J)$. In other words, I represent the relative expenditure change using the fluctuation of consumer's price P and total expenditure E.

Proposition 3. *Define the* ***Engel Slope*** *for the sector pair (k, j) as*

$$\mathrm{ES}_{k,j}(\vec{P}, E) = \lim_{\mathrm{d}e \to 0} \frac{s_{k,j}(p, e + \mathrm{d}e) - s_{k,j}(p, e)}{\mathrm{d}e}, \tag{28}$$

the risk price vector satisfies

$$b_k - b_j \quad = \quad \mathrm{ES}_{k,j}(\vec{P}, E). \tag{29}$$

This identity in Proposition 3 has straight-forward implication for seeking the risk premium. All else equal, if the consumer allocates a smaller fraction of spending toward the necessity sector, correspondingly, we shall witness the consumer charges higher risk compensation for the equity portfolio correlated with the necessity spot price. We have long-lasting curiosity toward the consumption portfolio. I use name "Engel Slope" for the partial derivatives of relative expenditure share with respect to the expenditure change, because the statistician Ernst Engel proposed this observation early in the 19th century. Functional form of indirect utility function affects both the consumption portfolio and the marginal utility, so we have the mirrored relationship between the Engel Slope and the risk-price.

[17] In (Comin et al., 2021), Sato-style non-homothetic CES implies constant price elasticity across sectors $\eta_{k,i} \equiv \eta$. (Comin et al., 2021) estimate η in $(0, 1)$. The parameter restriction affects the estimate of price-elasticity. Generally, for the heterogeneous-firm economy with monopolistic competition in the derived literature of (Melitz, 2003), $\eta_{k,k} - \eta_{j,k} > 1$ delivers the reasonable equilibrium outcome with producer's strategic pricing.

In particular, the gap between the risk price coefficient $\{b_j\}_j$ is exactly the marginal effect of expenditure effect (income effect) in the consumption portfolio. There exists the fundamental relationship between the risk-price of consumption sector and the position of consumption sector along the Engel curve. Proposition 3 formally state this result.

Table (8) investigates more detailed specification of non-durable consumption. Inside the non-durable good sector, I consider the food category and the non-food category. Inside the service sector, I also separate the food category and the non-food category. I implement the estimation of detailed consumption sectors gradually.

Table 8: **Detailed Consumption Sectors**

[—See the Table appendix—]

As the first step, I introduce the detailed good sector as below

- I estimate parameters $(b_{g,f}, b_{g,n}, b_s)$ in the three-sector economy: Food Good, Non-food Good, and the service

$$\begin{aligned} \mathrm{d}\tilde{m} \approx & - b_{g,f} \cdot \omega_{g,f} \cdot (\mathrm{d}p_{g,f} - \mathrm{d}e) - b_{g,n} \cdot \omega_{g,n} \cdot (\mathrm{d}p_{g,n} - \mathrm{d}e) \\ & - b_s \cdot \omega_s \cdot (\mathrm{d}p_s - \mathrm{d}e) - (\mathrm{d}e - \mathrm{d}p_s). \end{aligned} \tag{30}$$

- I estimate parameters $(b_{g,f}, b_{g,n}, b_e)$ using the service price as the deflator,

$$\begin{aligned} \mathrm{d}\tilde{m} \approx & - b_{g,f} \cdot \omega_{g,f} \cdot (\mathrm{d}p_{g,f} - \mathrm{d}p_s) - b_{g,n} \cdot \omega_{g,n} \cdot (\mathrm{d}p_{g,n} - \mathrm{d}p_s) \\ & - b_e \cdot (\mathrm{d}e - \mathrm{d}p_s). \end{aligned} \tag{31}$$

Estimation of $b_{g,f}$ is quantitatively close with b_g in the estimation of good-service two-sector economy. Due to the inaccurate estimate of $b_{g,n}$, it is difficult to conclude whether the `Non-food` good is more superior than the `Food` good. When using the service price as the deflator, estimation of b_e is quantitatively close with b_e in the estimation of good-service two-sector economy.

As the second step, I introduced the detailed service sector.

- I estimate parameters $(b_{g,f}, b_{g,n}, b_s)$ in the three-sector economy: Good, Food Service, Non-Food Service

$$\begin{aligned} \mathrm{d}\tilde{m} \approx & - b_g \cdot \omega_g \cdot (\mathrm{d}p_g - \mathrm{d}e) \\ & - b_{s,f} \cdot \omega_{s,f} \cdot (\mathrm{d}p_{s,f} - \mathrm{d}e) - b_{s,n} \cdot \omega_{s,n} \cdot (\mathrm{d}p_{s,n} - \mathrm{d}e) - (\mathrm{d}e - \mathrm{d}p_s). \end{aligned} \tag{32}$$

- I estimate parameters $(b_{g,f}, b_{g,n}, b_e)$ using the Non-Food Service price as the deflator,

$$\begin{aligned} \mathrm{d}\tilde{m} \approx & - b_g \cdot \omega_g \cdot (\mathrm{d}p_g - \mathrm{d}p_{s,n}) - b_{s,f} \cdot \omega_{s,f} \cdot (\mathrm{d}p_{s,f} - \mathrm{d}p_{s,n}) \\ & - b_e \cdot (\mathrm{d}e - \mathrm{d}p_{s,n}). \end{aligned} \tag{33}$$

Estimation of (b_g, b_e) is quantitatively close with (b_g, b_e) in the estimation of good-service two-sector economy. However, I don't find accurate estimate of $(b_{s,f}, b_{s,n})$.

As the final step, I consider the completely-specified economy.

- I estimate parameters $(b_{g,f}, b_{g,n}, b_{s,f}, b_{s,n})$ in the four-sector economy: Food Good, Non-Food Good and Food Service, Non-Food Service.

$$\begin{aligned} \mathrm{d}\tilde{m} \approx & - b_{g,f} \cdot \omega_{g,f} \cdot (\mathrm{d}p_{g,f} - \mathrm{de}) - b_{g,n} \cdot \omega_{g,n} \cdot (\mathrm{d}p_{g,n} - \mathrm{de}) \\ & - b_{s,f} \cdot \omega_{s,f} \cdot (\mathrm{d}p_{s,f} - \mathrm{de}) - b_{s,n} \cdot \omega_{s,n} \cdot (\mathrm{d}p_{s,n} - \mathrm{de}) - (\mathrm{de} - \mathrm{d}p_s). \end{aligned} \tag{34}$$

- I estimate parameters $(b_{g,f}, b_{g,n}, b_{s,f}, b_e)$ using the Non-Food Service price as the deflator,

$$\begin{aligned} \mathrm{d}\tilde{m} \approx & - b_{g,f} \cdot \omega_{g,f} \cdot (\mathrm{d}p_{g,f} - \mathrm{d}p_{s,n}) - b_{g,n} \cdot \omega_{g,n} \cdot (\mathrm{d}p_{g,n} - \mathrm{d}p_{s,n}) \\ & - b_{s,f} \cdot \omega_{s,f} \cdot (\mathrm{d}p_{s,f} - \mathrm{d}p_{s,n}) - b_e \cdot (\mathrm{de} - \mathrm{d}p_{s,n}). \end{aligned} \tag{35}$$

Because the non-food good sector and the food-service sector have small expenditure share in the consumption portfolio, the evolution of expenditure share is non-trivial. I use the sample-average expenditure share, multiplied by the cyclical term in the time-series of expenditure share, for the estimation of Table (8). I use the size, profitability, investment and momentum portfolios as testing assets, for the estimation of Table (8). As illustrated in Figure (A.1), this group of testing assets has avoids the weak identification from the weak covariance (risk-loading).

When dissecting the non-durable Good sector into the Food-Good category and the Non-Food Good category, I observe the risk-price for the Food-Good $b_{g,f} = -78.77$ and the risk-price for the Non-Food Good $b_{g,n} = -93.24$ are quantitatively similar. When dissecting the non-durable Service sector into the Food-Service category and the Non-Food Service category, I observe the risk-price for the Food-Good $b_{s,f} = 290.57$, while the risk-price for the Non-Food Service $b_{s,n} = -34.09$ with non-trivial inaccuracy. Therefore, separating the Food-Service category and the Non-Food Service category improves the asset pricing model, by distinguishing the consumption sectors with different risk-price. When considering the Food-Good category, the Non-Food Good category, Food-Service category and the Non-Food Service category together, we still observe the similar risk price between the Food-Good category and the Non-Food Good category, and the largely positive risk price for the Food-Service category.

This is qualitatively consistent with the empirical evidence in (Comin et al., 2021), where the service sector is more superior (luxury) than the manufacturing sector (here, it is the non-Food Good category). Further, (Comin et al., 2021) concludes the manufacturing sector is more superior (luxury) than the agricultural sector (here, it is the Food Good category), while I arrive to this conclusion when admitting the 4 consumption categories together.

In the online appendix Table (A.9), I provide the point estimate for the risk price using the Industry-30 portfolios. Positions of the Food-Good category, the Non-Food Good category, Food-Service category and the Non-Food Service category are qualitatively similar with the estimation of Table (8). In the online appendix Table (A.10), I provide the point estimate for the risk price using the diverse equity portfolios and the original time series of expenditure share. Positions of the Food-Good category, the Non-Food Good category, Food-Service category and the Non-Food Service category are qualitatively similar with the estimation of Table (8), when the deflator is chosen as the bottom-service category. In the online appendix Table (A.11), I provide the point estimate for the risk price using the Size-BM 25 portfolios. For the estimation when separating the Food-Service category and the Non-Food Service category. it is quantitative similar with the estimation of Table (8) [18].

As we allow the more detailed consumption sectors, the point estimate for the risk price of relative expenditure b_e stays in the range $(20, 30)$, quantitatively close with the simple economy of two-sectors. Moreover, the identity of H.D.0 indirect utility function $b_e = -\sum_{j=1}^{J}(b_j - 1)\cdot\omega_j$ loosely hold when we consider the more detailed consumption sectors [19].

4.2 Long-run Shift of Risk Premium

When the excess return are correlated with the change of relative expenditure growth and the relative good price,

$$\begin{aligned} &R^e_{k,t+1} - \mathbb{E}_t[R^e_{k,t+1}] \\ =&\beta_{k,e}\cdot\left(\mathrm{d}e_{t+1} - \mathbb{E}_t\left[\mathrm{d}e_{t+1}\right]\right) + \sum_j \beta_{k,j}\cdot\left(\mathrm{d}p_{j,t+1} - \mathbb{E}_t\left[\mathrm{d}p_{j,t+1}\right]\right) + \nu_{k,t+1}, \end{aligned} \tag{36}$$

Cross-section time-series average return is decomposed as

$$\frac{1}{T}\cdot\sum_{t=1}^{T} R^e_{k,t+1} = \lambda(\tilde{\omega}_{\mathcal{T}})\cdot\vec{\beta} + \frac{1}{T}\cdot\sum_{t=1}^{T}\epsilon_{k,t+1}. \tag{37}$$

where the risk-premium contingent on the share path $\tilde{\omega}_{\mathcal{T}}$ is

$$\lambda(\tilde{\omega}_{\mathcal{T}}) = \frac{1}{T}\cdot\sum_{t=1}^{T}\begin{bmatrix}\overline{b}_e(\omega_t) & \overline{b}_1(\omega_t) & \cdots & \overline{b}_{J-1}(\omega_t)\end{bmatrix}\cdot\Sigma. \tag{38}$$

I use notation $\overline{b}_e(\omega) = \frac{b_e}{1-b_e\cdot\mu_e-\sum_j b_j\cdot\omega_{j,t}\cdot\mu_j}$ and $\overline{b}_j(\omega) = \frac{b_j\cdot\omega_{j,t}}{1-b_e\cdot\mu_e-\sum_j b_j\cdot\omega_{j,t}\cdot\mu_j}$. This is the time-series average risk-premium as in the Fama-Macbeth two-step regression.

[18] However, when separating the Food-Good category and the Non-Food Good category, the testing assets of Size-BM 25 portfolios provides insufficient instruments for the Non-Food Good category.

[19] The expenditure share in the Food-Good category $\omega_{g,f} \approx 0.15$, the Non-Food Good category $\omega_{g,n} \approx 0.2$, Food-Service category $\omega_{s,f} \approx 0.15$ and the Non-Food Service category $\omega_{s,n} \approx 0.50$.

Figure (4) shows that the change of conditional expected return is positively correlated with the factor-loading of relative good price. The positive correlation also exists in size-sorted portfolios among value firms and the Fama-French industry portfolios. I illustrate the distribution of factor-loading and the change of risk premium, in Figure (A.3) and Figure (A.2) in the Figure appendix.

Figure 4: **Conditional Expected Return: Factor-loading**

[—See the Figure appendix—]

Table (9) lists the estimation of risk price and risk premium during the time-interval: (a) 1935-1989; (b) 1950-2005; (c) 1965-2019. I construct the early sample with equal length of the benchmark sample. For the consistent testing assets during the three time blocks, I use the Size-BM 25 portfolios. In Panel (A), we can't observe the decline of risk price b_g for the relative good price. On the other side, we observe the rough decline of risk premium λ_g from the time-interval 1935-1989 toward the time-interval 1950-2005, the empirically significant decline of risk premium λ_g from the time-interval 1950-2005 toward the time-interval 1965-2019. Identifying the long-run shift of risk-premium is empirically challenging given the short history of NIPA statistics. Although Table (9) is not rigorous identification, it provides corroborative evidence for the theoretical prediction of structural transformation induced by the economic growth in the asset pricing market. In the online appendix, Table A.8 reports

Table 9: **Long-run Shift of Risk Premium**

[—See the Table appendix—]

the similar estimation with Table (9) using the good, Food-service and non-Food service sector. The observation for the good sector is qualitatively similar with Table (9). However, the early time-series of Food-service with respect to the non-Food service is qualitatively different with the recent time-series, especially in the volatility of time-series. As such, I didn't observe comparable estimation for the Food-service sector, among the three time-intervals.

Figure (5) shows the distribution of factor loading in the diverse equity portfolios. In particular, the size premium is mainly contributed by the dispersion in factor loading of relative price. As the share of good declines, risk premium contributed by the relative price no longer dominates the risk premium contributed by the relative expenditure. This provides an alternative explanation for the diminished size premium.

Figure 5: **Distribution of Covariance: Cross-section Anomalies**

[—See the Figure appendix—]

Theoretically, if the risk premium contributed by the relative good price decays, lowering the risk-exposure toward the relative good price, increasing the risk-exposure toward the relative expenditure will maintains the high Sharpe-ratio. In other words, the frontier of Sharpe-ratio has been shifting, and this incremental shift is predictable as the economy technology growth keeps forward.

Figure (6) shows the distribution of factor loading in the industry portfolios. The equity portfolio constructed based on the cross-section anomalies manually discovered the stable exposure toward the pricing kernel. Portfolios have positive risk-exposure toward the positive risk-premium in relative expenditure, and the negative risk-exposure toward the negative risk-premium in relative good price. Compared to the anomaly portfolios, heuristic industry portfolios performs poor because they don't simultaneously exploit the positive risk-premium in relative expenditure and the negative risk-premium in relative good price.

Figure 6: **Distribution of Covariance: Fama-French 30 Industries**

[—See the Figure appendix—]

5 Aggregation

In this part, I discuss the price-decomposition of pricing kernel in the heterogeneous-agent economy. In this economy, the consumer n has indirect utility function $V^{(n)}(\vec{P}, E^{(n)})$ for intra-temporal consumption decision. Hereafter, I don't explicitly specify the CRRA functional form because it is incorporated by the general functional form $V^{(n)}(\vec{P}, E^{(n)})$. Consumer (n) has the labor endowment $\{\ell_t^{(n)}\}$, initial endowment of financial security $\vec{\theta}_0^{(n)}$.

Definition 3. *The price system (P, P^s) and the consumption allocation $\tilde{C}$ constitutes the (Heterogeneous Consumer) Competitive Equilibrium if*

1. *$\tilde{C}^{(n)}$ solves problem*

$$\begin{aligned} \overline{U}_0(\vec{\theta}_0^{(n)}) \quad &= \quad \sup_{\tilde{C}, \tilde{\theta}} \quad \lim_{T\to\infty} \mathbb{E}[\sum_{t=1}^{T} \beta^t \cdot u^{(n)}(\vec{C}_t)] \\ s.t. \qquad & \sum_k \theta_{k,t} \cdot (P^s_{k,t} + D_{k,t}) + w_t \cdot \ell_t^{(n)} = \sum_j P_{j,t} \cdot C_{j,t} + \sum_k \theta_{k,t+1} \cdot P^s_{k,t}, \\ & C_{j,t} \geq 0; \quad \sum_k \theta_{k,t+1} \cdot P^s_{k,t} \geq \underline{a}. \end{aligned} \tag{P.1-HA}$$

2. *commodity market (j,t) clears in the demand side $\sum_{n\in\mathcal{N}} C_{j,t}^{(n)} = \mathbf{C}_{j,t}$.*

3. *commodity market clears in the supply side, labor market clear, given the model specification of producers;*

4. *financial security market clears, given the model specification of foreign borrowing and lending.*

At the equilibrium path $\{\vec{P}^*, E^*, \lambda^*\}$, $\mathcal{D}_E V^{(n)}(\vec{P}^*, E^{(n),*})$ is the marginal utility of consumption expenditure $E^{(n)}$. I choose the consumer (1) as the benchmark consumer for the aggregation analysis. Construct the Negishi-weight $\alpha(1) = 1$, and

$$\alpha^*(n) = \frac{\mathcal{D}_E V^{(1)}(\vec{P}^*, E^{(1),*})}{\mathcal{D}_E V^{(n)}(\vec{P}^*, E^{(n),*})}. \tag{39}$$

The distribution of expenditure $\{E^{(n),*}\}_{(n)}$ solves the auxiliary-optimization problem,

$$\begin{aligned} V(\vec{P}, \mathbf{E}; \alpha) \equiv \max_{E} \quad & \frac{1}{N} \cdot \sum_{n \in \mathcal{N}} \alpha(n) \cdot V^{(n)}(\vec{P}, E(n)) \\ s.t. \quad & \frac{1}{N} \cdot \sum_{n \in \mathcal{N}} E(n) \leq \mathbf{E}. \end{aligned} \tag{SP.1}$$

I denote the aggregate consumption spending on the equilibrium path as $\mathbf{E}^* = \frac{1}{N} \cdot \sum_{n \in \mathcal{N}} E^{(n),*}$.

Theorem 2. *In the economy where price system (P, M) and quantity system $(\{\tilde{c}^{(n)}\}_{n \in \mathcal{N}}, \{\tilde{\ell}^{(n)}\}_{n \in \mathcal{N}})$ constitute a Competitive Equilibrium for N heterogeneous consumers with preference $\{\succeq^{(n)}\}$, there exists a* ***Representative Consumer*** *with preference $\succeq^{\mathcal{N}}$ such that*

- *price system (P, M) and quantity system $(\sum_{n \in \mathcal{N}} \tilde{c}^{(n)}, \sum_{n \in \mathcal{N}} \tilde{\ell}^{(n)})$ constitute a Competitive Equilibrium for N homogeneous consumers with preference $\succeq^{\mathcal{N}}$.*

The indirect utility function of the Representative Consumer is $V(\vec{P}, \mathbf{E}; \alpha)$ with the Negishi weight constructed in equation (39). Along the equilibrium path, the Representative Consumer has identical marginal utility of expenditure with the Benchmark Consumer

$$\mathcal{D}_E V(\vec{P}^*, \mathbf{E}^*; \alpha) = \mathcal{D}_E V^{(1)}(\vec{P}^*, E^{(1),*}), \tag{40}$$

and the absolute expenditure share of artificial consumer $V(\vec{P}^*, \mathbf{E}^*; \alpha)$ is identical with observed aggregate expenditure share,

$$\vec{\omega}(\vec{P}^*, \mathbf{E}^*; \alpha) = \sum_{n \in \mathcal{N}} \frac{E^{(n),*}}{\sum_{m \in \mathcal{N}} E^{(m),*}} \cdot \vec{\omega}^{(n)}(\vec{P}^*, E^{(n),*}) \tag{41}$$

The weight α_n reflects the shadow price of consumer (n)'s budget constraint in the Competitive Equilibrium. If we take the aggregation consumption bundle as if the Representative Consumer's choice, the Negishi weight works as if it is the "Taste" of representative consumer over individual consumers. Recall we use the expenditure share over commodities to reveal the single-consumer's preference over consumption bundle. Here, we use the expenditure allocation across consumers to reveal the Representative Consumer's social preference over individual consumers.

By constructing the representative consumers consistent with the aggregate consumption expenditure and the fluctuation of SDF, the representative consumer's indirect utility function also reveals the financial market SDF $\{M_t\}$. Decomposition of indirect utility function in Section (2) is non-parametric, so previous analysis holds in the heterogeneous-consumer economy. This allows the economist to track the marginal utility of investor even if we fail to explicitly identify who is the unconstrained financial market investor.

In the economy with additive utility flow, where the financial market is complete for the consumer, $\{\alpha^*(n)\}_n$ is invariant along the equilibrium path [20]. In other words, in the economy with perfect risk-sharing, we have fixed Negishi weight [21]. The numerator $\mathcal{D}_E V^{(1)}(\vec{P}^*, E^{(1),*})$ in the Negishi weight can't be removed, because we require the Representative Consumer's welfare comparable with the benchmark consumer (1). Only in this way, fluctuation of Representative Consumer's welfare is meaningful for tracking the financial market SDF.

Corollary 2. *Given invariant distribution of Negishi-weight $\{\alpha^*(n)\}_n$ along the equilibrium path, the log-change in real marginal utility of expenditure for the representative consumer approximately equals*

$$\mathrm{d}\tilde{m} = -\sum_{j=1}^{J} b_j(\alpha) \cdot \omega_j \cdot (\mathrm{d}p_j - \mathrm{d}p_J) - b_e(\alpha) \cdot (\mathrm{d}\boldsymbol{e} - \mathrm{d}p_J) + o(h). \tag{42}$$

where α is the artificial Negishi-weight, $\vec{\omega}$ is the aggregate expenditure share, $\boldsymbol{e}$ is the (log) aggregate total consumption expenditure, and the vector $b(\alpha)$ is written with aggregate expenditure share $\vec{\omega}$ and representative consumer's price-elasticity η

$$\begin{aligned} b_j(\alpha) =& -[\frac{\mathcal{D}_E V(\alpha) \cdot E}{V(\alpha)} + 1] + \sum_{i=1}^{J} \eta_{j,i}(\alpha), \\ b_e(\alpha) =& [\frac{\mathcal{D}_E V(\alpha) \cdot E}{V(\alpha)} + 1] + 1 - \sum_{j=1}^{J} \omega_j \cdot \sum_{i=1}^{J} \eta_{j,i}(\alpha). \end{aligned} \tag{43}$$

In the economy with generalized consumption preference, the representative consumer implied by the competitive equilibrium outcome might depart from the individual consumer, in the sense that the functional form of indirect utility function is different. This result departs from (Jackson and Yariv, 2019),

[20] Along the equilibrium path, $\{\alpha^*(n)\}_n$ might vary, if the economy has idiosyncratic labor endownment or non-trivial wealth-constraint.

[21] The international finance literature often consider the integrated economy with fixed Negishi weight. I depart from the Constant Social Planner's problem because the consumption allocation is implemented by the financial market and the commodity market. On the equilibrium path, the aggregate wealth allocation might be inconsistent with the Representative Consumer constructed from the intra-temporal consumption allocation.

because I don't pursue the similarity in the functional form of utility function. This result departs from (Baqaee and Burstein, 2021b), because I recover the Representative Consumer using the approach of "revealed social preference". I only require the consistent marginal utility of aggregate spending and the consistent aggregate consumption portfolio. Because I impose less restriction for the Representative Consumer, the aggregate wealth (positions of financial security) might be inconsistent with the Representative Consumer constructed from intra-temporal consumption allocation.

The construction of Negishi weight departs from (Saez and Stantcheva, 2016) and (Bhandari et al., 2021). Both (Saez and Stantcheva, 2016) and (Bhandari et al., 2021) use the marginal utility of consumption as the denominator of Negishi weight. In (Saez and Stantcheva, 2016), the numerator of the Negishi weight is chosen to be consistent with the optimal fiscal tax transfer. In (Bhandari et al., 2021), the numerator is the average marginal utility of consumption across consumers. Compared with (Saez and Stantcheva, 2016) and (Bhandari et al., 2021), I use benchmark consumer's marginal utility as the numerator, this ensures the reverse-engineered representative consumer has identical marginal utility with the financial market investor. To be clear, my construction of Negishi weight is the result implied by consumption allocation in a competitive equilibrium. I don't rely on the government or other legal authority assigning the consumption across consumers directly.

The generalized decomposition of SDF in the Corollary (2) is helpful for connecting this paper with previous empirical results of consumption-based asset pricing models. Typical consumption-based asset pricing models directly assume the representative consumer is well-defined. Some of these models work well to some extent, in the sense that we established the consensus that aggregate consumption quantity helps explaining the expected returns. Corollary (2) attempts to establish the equivalence between the economy with heterogeneous-consumers, and an artificial economy with representative consumer. Therefore, the empirical analysis in this article serves under the assumption of the Corollary (2). Although Corollary (2) puts strong assumptions over the distribution of Negishi weight, it serves as the benchmark to compare the true data and hypothetically ideal economy.

6 Discussion

6.1 Price versus Quantity

In the economy with Heterogeneous Consumers, decomposition of Indirect Utility Function allows reverse-engineering the representative consumer. In particular, in the economy with perfect risk-sharing, price-elasticity of consistent representative consumer is approximately weighted price-elasticity, given general consumption preference. On the other hand, when using the direct utility function to describe the consumption preference, the direct utility function of effective Representative Consumer also takes the weighted formula, using the effective Negishi-weight constructed in Equation (39) . Under the assumption of time-invariant effective Representative Consumer, estimating the SDF using the consumption quantities is identical with estimating the SDF using the consumption prices. In Table 2, we observe the improvement of model-fit from the model of nondurable consumption quantity "C-ND" toward the model of relative good price and relative expenditure "P-ND". This can be purely driven by a more detailed specification of consumption sectors.

Table (10) investigates whether the decomposition of Indirect Utility Function has different empirical implication when we allow more detailed specification of consumption sectors in the quantity-based consumption asset pricing models. As shown in the estimation named "DU" (direct utility function), when using the "chained-quantity" of good and service as the time-series factors for the linear SDF, the point estimate of first-stage is inaccurate. Although non-parametric decomposition with Direct Utility function and the decomposition with Indirect Utility function are theoretically equivalent, the approximation error of model are empirically different. When using the non-parametric decomposition with Indirect Utility function, consumption expenditure share provides the sufficient statistic of impact from the change of consumption price, while this is infeasible when decomposing the Direct Utility function.

When using the aggregate consumption data to approximate the SDF, the equivalence between non-parametric decomposition with Direct Utility function and the decomposition with Indirect Utility function deeply relies on the well-defined Representative Consumer. Accuracy for the SDF with Direct Utility function is fatally reduced when we fail to track the marginal investor in the financial market. If we write down the SDF in the fluctuation of quantities and shares

$$\begin{aligned}
\mathrm{d}\tilde{m} &= -\sum_{j=1}^{J} b_j(\vec{P}, E) \cdot \omega_j \cdot (\mathrm{d}p_j - \mathrm{d}p_J) - b_e(\vec{P}, E) \cdot (\mathrm{d}e - \mathrm{d}p_J) + o(h) \\
&= -\sum_{j=1}^{J} b_j(\vec{P}, E) \cdot \omega_j \cdot [(\mathrm{d}\log\omega_j - \mathrm{d}\log\omega_J) - (\mathrm{d}C_j - \mathrm{d}C_J)] \\
&\quad - b_e(\vec{P}, E) \cdot (\mathrm{d}e - \mathrm{d}p_J) + o(h).
\end{aligned}$$

We need to accurately track the time-series of $\{\mathrm{d}\log\omega_j\}_j$ for the marginal investor in the financial market [22]. This is difficult in the practice of financial

[22]One would doubt the construction of price index in the NIPA consumption data: effectively we are using the change of aggregate consumption quantities and aggregate consumption shares. Here, we want to be careful with the so-called "Quantity" for consumption sectors in the NIPA statistic. The bureau of statistics first collect the nominal revenues, and nominal expenditures when accounting for the output and consumption. In their sample of economy activities, the staff in bureau of statistics the infer the change of price. Next they construct the price index using the classic formula of sufficient statistics. As the final step, the quantity is derived by the nominal time-series and the price indice. In short words, the nominal time-series and the price indice are the **direct observation** of economy status, while the implied quantities are the **derived statistics**. The "Quantity" of "Food-at-Home" in NIPA table is not comparable with the "Tons" of apple consumed, or the "Calories" of food contained.

institutions.

I further investigate whether the Stone-Geary preference provides a good description, as specified in (Lochstoer, 2009). In the Column "Good", I assume zero habit in the good sector $u(C_g, C_s) = C_g^{\alpha} \cdot (C_s - X_s)^{1-\alpha}$, decompose the pricing kernel as

$$\mathrm{d}\tilde{m} \approx -a_g \cdot \mathrm{d}q_g - a_s \cdot (\mathrm{d}p_s - \mathrm{d}p_g). \tag{44}$$

The point estimate is in-significant in the first-stage estimate. In the Column "Service", I assume zero habit in the service sector $u(C_g, C_s) = (C_g - X_g)^{\alpha} \cdot C_s^{1-\alpha}$, decompose the pricing kernel as

$$\mathrm{d}\tilde{m} \approx -a_g \cdot (\mathrm{d}p_g - \mathrm{d}p_s) - a_s \cdot \mathrm{d}q_s. \tag{45}$$

The point estimate for a_s is 22.31 with considerable accuracy in the first-stage estimate, consistent with (Lochstoer, 2009). In addition, the model fitness is comparable with the previous exercise of pricing kernel decomposed from the Indirect Utility Function. Applying Stone-Geary Preference requires the researchers to identify which is the sector with near-zero consumption habit. Therefore, given more consumption data in the future, if researchers want to extend the pricing kernel for more granular consumption structure, they need to carefully explore the structure of habits for the consumption hierarchy is more subtle.

As in Table 6, I extend the estimation for three-sector economy: Good,Food-Service,Non-Food Service. The Stone-Geary preference is extended correspondingly

- The Food-Service sector has zero consumption-habit,

$$u(C_g, C_{s,f}, C_{s,n}) = (C_g - X_g)^{\alpha_g} \cdot C_{s,f}^{1-\alpha} \cdot (C_{s,n} - X_{s,n})^{\alpha_{s,n}}.$$

- The Non-Food Service sector has zero consumption-habit,

$$u(C_g, C_{s,f}, C_{s,n}) = (C_g - X_g)^{\alpha_g} \cdot (C_{s,f} - X_{s,f})^{\alpha_{s,f}} \cdot C_{s,n}^{1-\alpha}.$$

In Column "Food", the estimation assumes the food-service with zero-habit, for the relative price of non-food service with respect to food service $p_{n,f}$, the risk price is significantly positive, while the typical Stone-Geary preference predicts the negative risk price. In Column "Non-Food", I consider the non-food service category with zero-habit, for the relative price of good with respect to non-food service $p_{g,n}$, the absolute value of risk price is larger than that of quantity growth in non-food service q_n. However, the typical Stone-Geary preference predicts the risk price $p_{g,n}$ in smaller magnitude.

Table 10: **Quantity Index**

[—See the Table appendix—]

In summary, although the Stone-Geary preference qualitatively captures the essence of non-homothetic preference. it also raises quantitative doubts in the aggregate consumption data. Because Stone-Geary preference is a special case for non-homothetic preference, if the financial economist meets difficulty in identifying the structure of consumption-habit, she can skip identifying the structure of consumption-habit, by using the sufficient-statistic of expenditure share in decomposing the Indirect Utility Function. Further, because we simply use the level of absolute expenditure share as the sufficient statistics, the requirement is loose for accurately measuring the expenditure. This explains why the superior empirical performance of SDF decomposed from the indirect utility function, using the fluctuation of relative consumption prices and aggregate expenditure. Overall, the general IDU decomposition would be more flexible than fully-parameterized Stone-Geary preference.

6.2 Excessive Necessity Premium

The exact aggregation of price-elasticity relies on the stylized economy environment, here I simply documents the quantitative gap between the aggregate-elasticity measured from the financial asset returns and the micro-elasticity.

In the estimation with aggregate consumption data, for the pair good-to-service, the Engel-slope implied by the risk price is $\frac{b_e+b_g-1}{\omega_s}$, roughly in the parameter interval $(-65, -70)$. In the parallel research, I estimate the Engel slope of good-to-service share for synthetic households constructed from Consumer Expenditure Survey. Estimated Engel slope $\mathrm{ES}_{g,s} \approx -0.49$. This estimate is consistent with the estimate of (Boppart, 2014) and (Comin et al., 2021), where the quantitative magnitude of expenditure effect (income effect) is small. Though the price-elasticity identified from financial market pricing kernel indeed quantitatively departs from the micro price-elasticity, the sign of Engel slope is consistent with the risk price in my previous estimate. The alignment of Engel slope is also observed in the pair of Food-Service and Good. Estimated Engel slope for the pair of FoodService-Good is $\mathrm{ES}_{fs,g} \approx 0.8$.

Several important assumptions worth further investigation to explain this quantitative tension.

The distribution of effective Negishi weights are assumed to be invariant, for a well-defined time-invariant effective Representative Consumer. If the macro primitive shocks leads the both the fluctuation of consumption price, and the distribution of consumption expenditure, we would witness significant distortion of aggregate price elasticity.

Commodity price is assumed to be identical across consumers. In reality, geographical price dispersion of service sector can be non-trivial [23]. Location-specific service price can introduce another layer of consumer-level idiosyncratic risk. In particular, individual consumer is unlikely to separate the local price shock and the aggregate price shock. Given the financial market plays non-

[23] Though I aggregate the restaurant dining, entertainment and other sub-industries into the main service sector, this effort might have limited role.

trivial role in information aggregation, it is reasonable to see the over-reaction of SDF toward the consumption price fluctuation.

Analysis of aggregating price-elasticity is much more complicated for the economy environment with recursive utility function. It is almost impossible to deliver analytical aggregation result with recursive utility function. Exploring the channel of long-run risk helps address the quantitative puzzles. Since direct inference is almost impossible, I expect the quantitative investigation to be done in the future research.

7 Conclusion

Pricing kernel decomposed with indirect utility function allows the financial economist to evaluate the risk price of consumption price in equity market. The approximated SDF with relative prices improves the model-fit of financial-asset Euler Equation, compared with traded-factor asset pricing model and simple consumption-based asset pricing model.

Due to the non-parametric connection between intra-temporal consumption choice and the inter-temporal financial wealth management, there exists exact identity between the price-elasticity of relative expenditure share and the risk price of consumption price. In particular, the risk price of necessity commodity price is in larger magnitude. Qualitatively, this prediction holds in the estimated aggregate pricing kernel.

From estimation of aggregate pricing kernel and micro-elasticity, I document the quantitative gap between the aggregate price-elasticity implied by the financial pricing kernel and the micro elasticity in consumers' expenditure share. Under stylized economy environment, aggregate price-elasticity is non-trivial aggregation of consumers' price-elasticity. It requests further research examine the satisfactory quantitative explanation in realistic economy environment, under the moment restriction of cross-section consumption distribution.

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A Figure

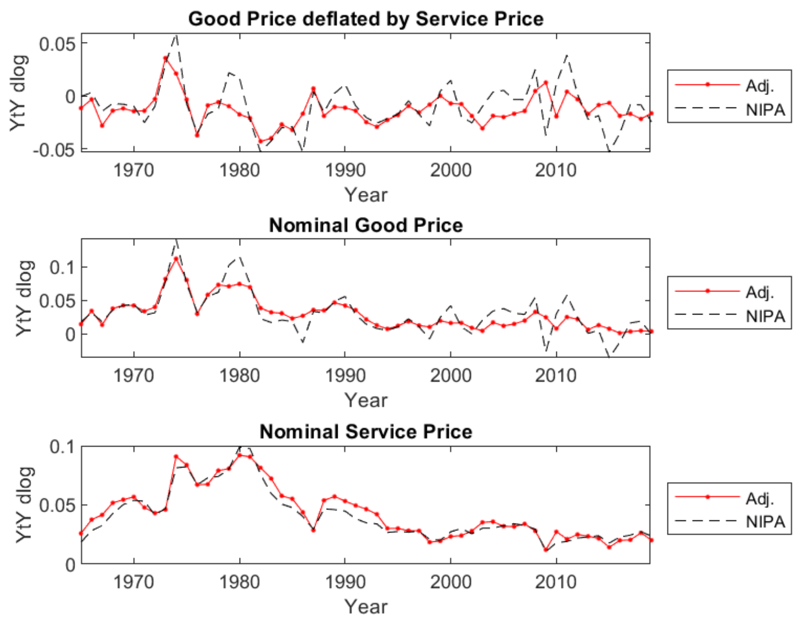

Figure 1: Plot of Price Indice

Description: The X-axis is time-axis, the first row of figure plots the annual log change of relative good price with respect to the service price $\mathrm{d}p_g - \mathrm{d}p_s$. The second row of figure plots the annual log change of nominal good price $\mathrm{d}p_g$. The third row of figure plots the annual log change of nominal service price $\mathrm{d}p_s$. The red thick line plots the price index using the definition of sectors described in sector 3. The dark dashed line plots the price index using the original definition of sectors in NIPA.

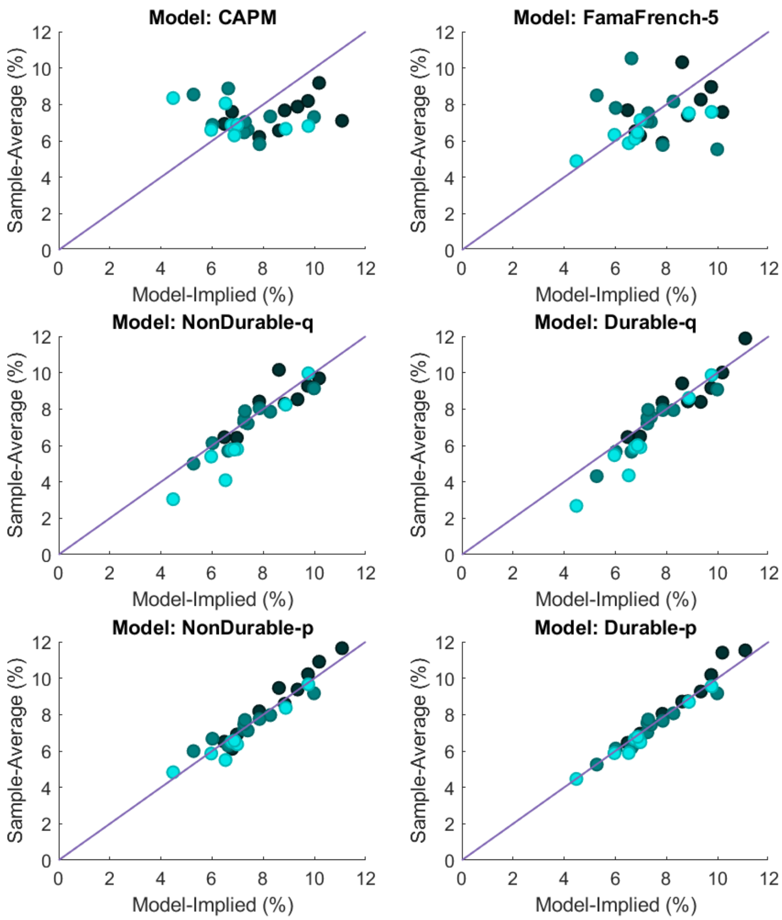

Figure 2: Fitness of Asset Pricing Models

Description: The X-axis is the model-predicted excess return, $\mathbb{E}_{\mathcal{T}}[-\mathrm{d}m \cdot R^e_{k,t+1}]$. The Y-axis is the average excess return in sample, $\mathbb{E}_{\mathcal{T}}[R^e_{k,t+1}]$. The dark-green dots are Size-BM portfolios. The green dots are Profitability-Investment portfolios. The light-green dots are Momentum portfolios.

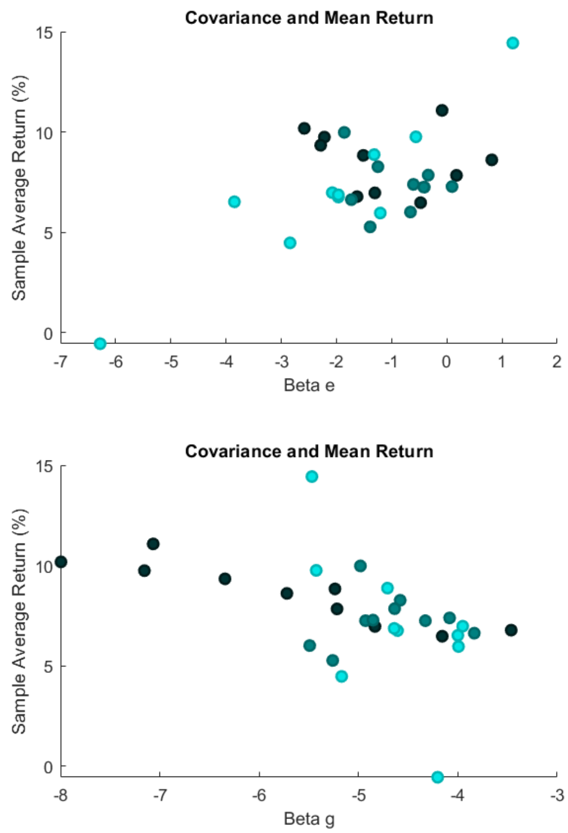

Figure 3: Factor-Loading of Benchmark Model

Description: The X-axis is risk-exposure from the Fama-Macbeth 2-step regression, $\vec{\beta}$. The Y-axis is the average excess return in sample, $\mathbb{E}_{\mathcal{T}}[R^e_{k,t+1}]$. The dark-green dots are Size-BM portfolios. The green dots are Profitability-Investment portfolios. The light-green dots are Momentum portfolios.

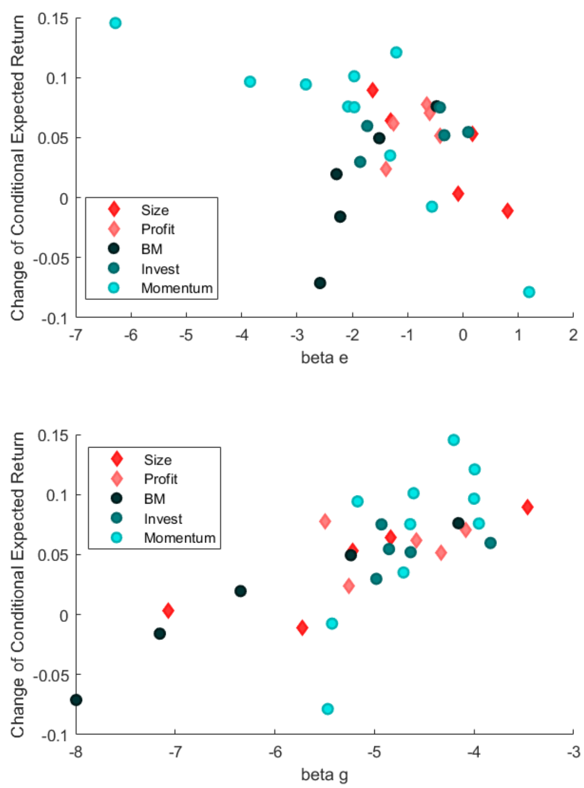

Figure 4: Conditional Expected Return: Factor-loading

Description: The X-axis is risk-exposure from the Fama-Macbeth 2-step regression, $\vec{\beta}$. The Y-axis is the change of 15-year average excess return between the sample start and the sample end, $\mathbb{E}_T[R^e_{k,t+1}] - \mathbb{E}_0[R^e_{k,t+1}]$. The dark-red dots are Size portfolios. The light-red dots are Profitability portfolios. The dark-green dots are BM portfolios. The green dots are Investment portfolios. The light-green dots are Momentum portfolios.

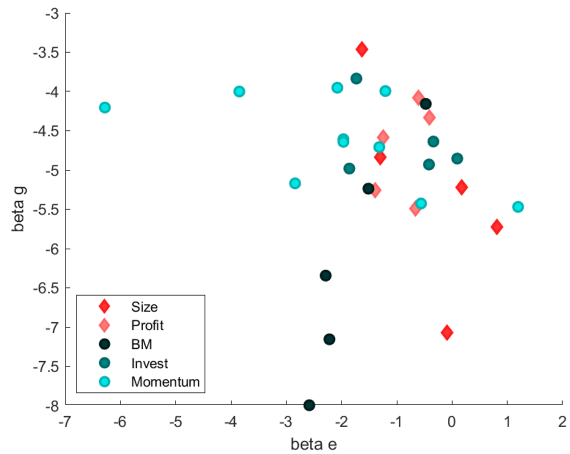

Figure 5: Distribution of Covariance: Cross-section Anomalies

Description: The X-axis is risk-exposure from the Fama-Macbeth 2-step regression, $\vec{\beta}$. The Y-axis is the average excess return in sample, $\mathbb{E}_{\mathcal{T}}[R^e_{k,t+1}]$. The dark-green dots are Size-BM portfolios. The green dots are Profitability-Investment portfolios. The light-green dots are Momentum portfolios.

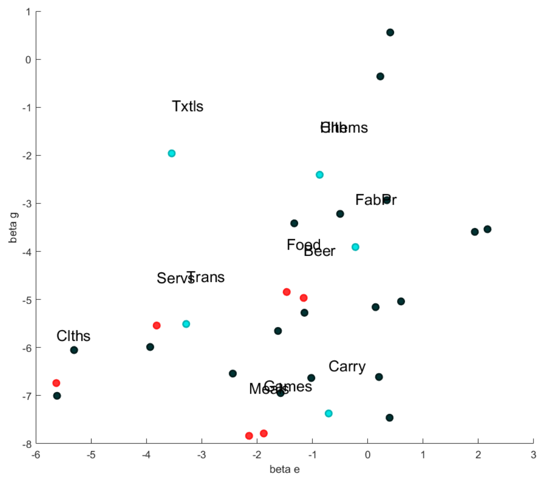

Figure 6: Distribution of Covariance: Fama-French 30 Industries

Description: The X-axis is risk-exposure from the Fama-Macbeth 2-step regression, $\vec{\beta}$. The Y-axis is the average excess return in sample, $\mathbb{E}_{\mathcal{T}}[R^e_{k,t+1}]$. The dark-green dots are Size-BM portfolios. The green dots are Profitability-Investment portfolios. The light-green dots are Momentum portfolios.

B Table

Table 1: Descriptive Statistic

Description: Time span of sample is during 1965-2019. All the standard error are Newey-West adjusted with two-year lag. The VIX time series is available during 1990-2019. Standard errors are in parenthesis. T-stat is in brackets.

Panel (a): Time Series - Statistic			
	Mean(pct)	SE(pct)	AR(1)
de	1.27	1.28	0.36
$(s.e.)[t]$	(0.21)	(0.13)	[3.06]
dp	-1.33	1.38	0.47
$(s.e.)[t]$	(0.24)	(0.23)	[3.52]
Panel (b): Business Cycle - Correlation			
	MKT	Hour	Output
$\mathrm{Corr}(z, \mathrm{d}p_{g/s})$	-0.35	0.01	0.01
$[t]$	[-2.35]	[0.06]	[0.03]
	VIX	EP-Y10	CAY
$\mathrm{Corr}(z, \mathrm{d}p_{g/s})$	0.13	0.22	-0.21
$[t]$	[0.73]	[2.08]	[-1.05]

Table 2: Model Fitness

Description: The model-fit of asset pricing models with traded factors are reported in the 2nd column and 3rd column. The 2nd column uses the single MKT factor. The 3rd column uses the Fama-French 5-factors. The model-fit of models with change of consumption quantities are reported in the 4th column and the 5th column. The 4th column uses the change of quantity index in the nondurable consumption sector. The 5th column augments the 6th column with the quantity change in the durable stock. The model-fit of asset pricing models with change of relative prices are reported in the 6th column and the 7th column. The 6th column uses the change of relative good price, the change of relative expenditure in the nondurable consumption sector, with deflator as the service price. The 7th column augments the 6th column with the quantity change in the durable stock. Construction of durable stock, quantity index and price index are decribed in Section (3). In Panel (A), statistics of model-fit are reported for the GMM estimation outcome. Construction of MAPE (Mean Absolute Pricing Error), RMSE (Root Mean Square Error), CV-R^2 (Campbell-Vuolteenaho R^2) and J-pval (p-value for the J-stat) are decribed in Section (3). In Panel (B), statistics of model-fit are reported for the Fama-Macbeth two-step regression. Fama-Macbeth regression estimate the time-series average risk premium in the 1st step. Definition of time-series average risk premium are described in the online estimation appendix. OLS-R^2 calculate the Fama-Macbeth two-step regression without intercept term in the 2nd step, and similarly for GLS-R^2. COLS-R^2 calculate the Fama-Macbeth two-step regression with intercept term in the 2nd step, and similarly for CGLS-R^2.

	Model Fitness					
	Traded Factors		Quantities		Relative Prices	
	CAPM	FF-5	C-ND	C-D	P-ND	P-D
	Panel (A): GMM					
MAPE	1.93	1.68	0.84	0.80	0.39	0.26
RMSE	2.94	2.68	1.19	1.16	0.47	0.37
CV-R^2	-0.43	-0.19	0.77	0.78	0.96	0.98
J-pval	93.71	81.78	96.17	92.35	91.97	90.43
	Panel (B): Fama-Macbeth Two-step Regression					
OLS-R^2	-0.58	-0.38	-9.64	0.14	0.51	0.52
GLS-R^2	-0.02	0.01	0.02	0.02	0.01	0.03
COLS-R^2	0.14	0.66	0.53	0.56	0.58	0.58
CGLS-R^2	0.01	0.03	0.08	0.09	0.06	0.09

Table 3: Parameters

Description: This table reports the point estimate of risk price in GMM estimation, the estimate of risk premium in Fama-Macbeth two-step regression. Sample is during 1965-2019. Panel (A-C) reports the estimate of GMM estimation. **Risk Price** reports the vector $\vec{b}$. Estimation uses the asset pricing model with relative prices "P-ND". Estimation are reported for different setups of testing assets. In the 2nd column and the 3rd column, teststing assets **Mix 30** uses the size, BM ratio, profitability, investment and momentum portfoliors. In the 4th column and the 5th column, teststing assets **Size-BM 25** uses the 25 portfolios double sorted based on size and BM ratio. In the 6th column and the 7th column, teststing assets **Industry 30** uses the 30-industry portfolios. All testing assets are from the Data Library of Kenneth French. In constructiong the weight matrix for GMM, "1st-Stage" uses the Identity Matrix, "2nd-Stage" uses the asymptotical variance of "1st-Stage" estimatin. Stationary covariance matrix of moments are assumed. T-stat is reported in brackets. Description of statistics in Table (2) applies.

	Specification of Testing Assets					
	Mix 30		Size-BM 25		Industry 30	
	Panel (A): Risk Price					
	1st-Stage	2nd-Stage	1st-Stage	2nd-Stage	1st-Stage	2nd-Stage
b_e	28.87	29.77	30.05	33.72	33.27	33.88
$[t]$	[1.69]	[24.18]	[2.61]	[13.06]	[4.38]	[24.98]
b_g	-72.10	-72.61	-68.26	-63.83	-69.95	-67.92
$[t]$	[-2.23]	[-22.26]	[-2.90]	[-11.68]	[-3.04]	[-17.21]
	Panel (B): Stats of GMM					
MAPE	0.39		0.38		0.84	
RMSE	0.47		0.51		0.99	
CV-R^2	0.96		0.96		0.78	
J-pval		91.97		81.48		94.03
	Panel (C): Test Statistic					
Test-t		[-12.23]		[-4.24]		[-7.34]

Table 3: Parameters, Continued

Description: Panel (D-E) reports the estimate of Fama-Macbeth two-step regression. **Risk Premium** reports the time-series average risk premium in sample. In 2nd Column, 4th Column, and 6th Column, risk premium are estimated under the assumption of zero pricing error. Estimation of risk premium is **without** the intercept term in the 2nd step of regression. In 3rd Column, 5th Column, and 7th Column, risk premium are estimated **with** the intercept term in the 2nd step of regression. T-stat is reported in brackets. Calculation of t-stat the simple standard error. Description of statistics in Table (2) applies.

	Specification of Testing Assets					
	Mix 30		Size-BM 25		Industry 30	
	Panel (D): Risk Premium					
	without	with	without	with	without	with
λ_e	0.84	0.98	0.38	0.43	-0.06	-0.19
$[t]$	[1.92]	[2.26]	[0.55]	[0.64]	[-0.17]	[-0.59]
λ_g	-1.73	-1.09	-1.56	-1.28	-1.43	-0.20
$[t]$	[-4.15]	[-2.02]	[-4.19]	[-2.50]	[-3.34]	[-0.50]
α	-	3.51	-	1.99	-	6.96
$[t]$	-	[1.13]	-	[0.63]	-	[2.62]
	Panel (E): Stats of Two-step Regression					
OLS-R^2	0.51		0.63		-1.49	
GLS-R^2	0.01		-0.38		-0.10	
COLS-R^2		0.58		0.67		0.10
CGLS-R^2		0.06		0.01		0.06

Table 4: Subgroup of Testing Assets

Description: This table reports the point estimate for the vector $\vec{b}$ using different setups of testing assets. "Size-BM" uses the 5 size, 5 BM ratio portfoliors. "Profit-IK" uses the 5 profitability, and 5 investment portfoliors. "Momentum 10" uses the 10 momentum portfoliors. All testing assets are from the Data Library of Kenneth French. Other description in Table (3) applies.

	Specification of Testing Assets					
	Size-BM		Profit-IK		Momentum	
	Panel (A): Risk Price					
	1st-Stage	2nd-Stage	1st-Stage	2nd-Stage	1st-Stage	2nd-Stage
b_e	25.15	28.65	40.79	42.74	28.93	22.55
$[t]$	[2.05]	[4.57]	[2.74]	[5.41]	[1.50]	[4.24]
b_g	-71.94	-62.63	-62.93	-72.75	-75.93	-87.51
$[t]$	[-3.11]	[-5.97]	[-1.90]	[-5.00]	[-2.07]	[-5.89]
	Panel (B): Stats of GMM					
MAPE	0.33		0.36		0.24	
RMSE	0.41		0.42		0.34	
CV-R^2	0.92		0.88		0.99	
J-pval		25.15		45.58		15.08
	Panel (C): Test Statistic					
Test-t		[-2.43]		[-1.52]		[-3.51]

Table 4: Subgroup of Testing Assets, Continued

Description: This table reports the point estimate for the vector $\vec{b}$ using different setups of testing assets. "Size-BM" uses the 5 size, 5 BM ratio portfoliors. "Profit-IK" uses the 5 profitability, and 5 investment portfoliors. "Momentum 10" uses the 10 momentum portfoliors. All testing assets are from the Data Library of Kenneth French. Other description in Table (3) applies.

	Specification of Testing Assets					
	Size-BM		Profit-IK		Momentum	
	Panel (D): Risk Premium					
	without	with	without	with	without	with
λ_e	0.16	0.12	-0.66	-0.33	1.38	1.71
$[t]$	[0.29]	[0.22]	[-0.95]	[-0.50]	[2.62]	[2.94]
λ_g	-1.51	-1.03	-1.42	0.39	-2.13	-0.41
$[t]$	[-3.66]	[-1.75]	[-3.27]	[0.53]	[-4.64]	[-0.40]
α	-	2.85	-	8.87	-	8.69
$[t]$	-	[0.87]	-	[2.18]	-	[1.85]
	Panel (E): Stats of Two-step Regression					
OLS-R^2	0.65		-0.48		0.81	
GLS-R^2	0.18		-0.57		0.18	
COLS-R^2		0.85		0.06		0.86
CGLS-R^2		0.25		0.28		0.44

Table 5: Price Index

Description: This table reports the point estimate for the vector $\vec{b}$ using different setups of testing assets. "Fisher" constructs the Fisher-index as the sector-level price, the implied price deflator from chained quantity index. "Tornqvist" constructs the Tornqvist Index as sector-level price, the implied price deflator from expenditure-share weighted index. "Simple" uses the average change of sub-category price index in each sector. All estimation use the "Mix 30" portfolios as the testing assets. Other description in Table (3) applies.

	Risk Price					
	Fisher		Tornqvist		Simple	
	1st-Stage	2nd-Stage	1st-Stage	2nd-Stage	1st-Stage	2nd-Stage
b_e	28.87	29.77	26.05	27.12	16.92	17.47
$[t]$	[1.69]	[24.18]	[1.38]	[19.30]	[0.86]	[10.95]
b_g	-72.10	-72.61	-71.63	-70.05	-87.67	-85.88
$[t]$	[-2.23]	[-22.26]	[-4.86]	[-51.77]	[-2.14]	[-24.50]

Table 6: Consumption Sector

Description: This table reports the point estimate for the vector $\vec{b}$ using different setups of testing assets. "Benchmark" constructs the price in non-durable good-service sector, where Energy and Housing are excluded. "NIPA-Nondurable" constructs the price in nondurable good sector and service sector, using NIPA definition. "NIPA-Good" constructs the price in good sector and service sector, using NIPA definition. All estimation use the "Mix 30" portfolios as the testing assets. Other description in Table (3) applies.

	Risk Price					
	Benchmark		NIPA-Nondurable		NIPA-Good	
	1st-Stage	2nd-Stage	1st-Stage	2nd-Stage	1st-Stage	2nd-Stage
b_e	28.87	29.77	36.54	37.41	25.81	25.28
$[t]$	[1.69]	[24.18]	[4.75]	[29.66]	[2.69]	[14.61]
b_g	-72.10	-72.61	-72.01	-72.20	-61.28	-63.29
$[t]$	[-2.23]	[-22.26]	[-2.36]	[-14.13]	[-2.46]	[-18.15]

Table 7: Alternal Estimation for Parameters

Description: This table reports the point estimate for the vector $\vec{b}$ using different pricing kernels with different deflators. "Service" uses the pricing kernel of relative expendture and relative good price, with respect to the service price. "Good" uses the pricing kernel of relative expendture and relative service price, with respect to the good price. "Expenditure" uses the pricing kernel of relative good price and relative service price, with respect to the expenditure. All estimation use the "Mix 30" portfolios as the testing assets. Other description in Table (3) applies.

	Risk Price					
	Service		Good		Expenditure	
	1st-Stage	2nd-Stage	1st-Stage	2nd-Stage	1st-Stage	2nd-Stage
b_e	28.87	29.77	27.03	26.64		
$[t]$	[1.69]	[24.18]	[1.40]	[23.41]		
b_g	-72.10	-72.61			-72.97	-71.72
$[t]$	[-2.23]	[-22.26]			[-2.33]	[-17.36]
b_s			6.12	6.71	-0.66	-4.27
$[t]$			[0.11]	[2.12]	[-0.02]	[-1.03]

Table 8: Detailed Consumption Sectors

Description: This table reports the estimation using multiple consumption sectors. Sample is during 1965-2019. Panel (A) reports the point estimate for the vector $\vec{b}$ using GMM estimation. "Good" considers the Food-good category and non-Food good category. "Service" considers the Food-service category and non-Food service category. "All" considers the four categories: Food-good, non-Food good, Food-service and the non-Food service. de use the nominal expenditure as the deflator, dp use the bottom-category price as the deflator. Estimation use the 5-size, 5-profitability, 5-investment and 10-momentum portfolios. The expenditure in each sector is detrended. Other description in Table (2) and Table (3) applies.

	Risk Price							
	Good			Service			All	
$b_{g,f}$	-78.77	-79.06	b_g	-109.81	-109.76	$b_{g,f}$	-133.35	-133.16
$[t]$	[-1.57]	[-1.58]	$[t]$	[-4.01]	[-4.00]	$[t]$	[-3.05]	[-3.04]
$b_{g,n}$	-93.24	-92.69				$b_{g,n}$	-82.39	-81.43
$[t]$	[-2.05]	[-2.04]				$[t]$	[-1.48]	[-1.48]
b_s	19.78		$b_{s,f}$	290.57	290.16	$b_{s,f}$	324.37	321.90
$[t]$	[1.14]		$[t]$	[2.14]	[2.11]	$[t]$	[2.25]	[2.23]
			$b_{s,n}$	-34.09		$b_{s,n}$	-50.74	
			$[t]$	[-1.22]		$[t]$	[-1.38]	
b_e		22.37	b_e		24.33	b_e		26.91
$[t]$		[2.60]	$[t]$		[2.84]	$[t]$		[2.61]
	Deflator							
	de	dp_s		de	$dp_{s,n}$		de	$dp_{s,n}$
	GMM statistic							
MAPE	0.32	0.32		0.22	0.22		0.23	0.24
RMSE	0.40	0.40		0.31	0.31		0.28	0.29
Cohen-R^2	0.98	0.98		0.99	0.99		0.99	0.99
J-pval	76.44	76.33		78.57	78.72		72.36	72.70

Table 8: Detailed Consumption Sectors, Continued

Description: Panel (B) reports the estimation of risk premium (without intercept term) using multiple consumption sectors. Other description in Table (2) and Table (3) applies.

	Risk Premium							
	Good			Service			All	
$\lambda_{g,f}$	-3.56	-2.67	λ_g	-2.58	-2.06	$\lambda_{g,f}$	-2.87	-2.90
$[t]$	[-4.76]	[-4.38]	$[t]$	[-4.00]	[-4.44]	$[t]$	[-3.97]	[-4.33]
$\lambda_{g,n}$	-2.06	-1.18				$\lambda_{g,n}$	-1.49	-1.52
$[t]$	[-4.17]	[-3.87]				$[t]$	[-2.84]	[-4.23]
λ_s	-0.89		$\lambda_{s,f}$	-2.21	-1.69	$\lambda_{s,f}$	-1.91	-1.94
$[t]$	[-2.34]		$[t]$	[-4.24]	[-2.88]	$[t]$	[-3.84]	[-4.27]
			$\lambda_{s,n}$	-0.52		$\lambda_{s,n}$	0.03	
			$[t]$	[-0.78]		$[t]$	[0.08]	
λ_e		0.89	λ_e		0.52	λ_e		-0.03
$[t]$		[2.34]	$[t]$		[0.78]	$[t]$		[-0.08]
	Deflator							
	de	dp_s		de	$dp_{s,n}$		de	$dp_{s,n}$
	Fama-Macbeth Two-step Statistic							
OLS-R^2	0.57	0.57		0.57	0.57		0.61	0.61
GLS-R^2	-0.01	-0.01		-0.00	-0.00		0.03	0.03

Table 8: Detailed Consumption Sectors, Continued

Description: Panel (C) reports the estimation of risk premium (with intercept term) using multiple consumption sectors. Other description in Table (2) and Table (3) applies.

	Risk Premium							
	Good			Service			All	
$\lambda_{g,f}$	-2.20	-1.52	λ_g	-2.44	-1.52	$\lambda_{g,f}$	-1.26	-1.14
$[t]$	[-2.28]	[-1.62]	$[t]$	[-3.45]	[-2.67]	$[t]$	[-1.53]	[-1.38]
$\lambda_{g,n}$	-1.73	-1.05				$\lambda_{g,n}$	-1.43	-1.32
$[t]$	[-3.48]	[-3.10]				$[t]$	[-2.72]	[-3.46]
λ_s	-0.68		$\lambda_{s,f}$	-1.91	-0.99	$\lambda_{s,f}$	-0.86	-0.75
$[t]$	[-2.06]		$[t]$	[-3.36]	[-2.35]	$[t]$	[-2.05]	[-0.08]
			$\lambda_{s,n}$	-0.92		$\lambda_{s,n}$	-0.11	
			$[t]$	[-1.71]		$[t]$	[-0.31]	
λ_e		0.68	λ_e		0.92	λ_e		0.11
$[t]$		[2.06]	$[t]$		[1.71]	$[t]$		[0.31]
α	4.38	4.38	α	2.75	2.75	α	5.96	5.96
$[t]$	[1.32]	[1.32]	$[t]$	[0.95]	[0.95]	$[t]$	[1.98]	[1.98]
	Deflator							
	de	dp_s		de	$dp_{s,n}$		de	$dp_{s,n}$
	Fama-Macbeth Two-step Statistic							
COLS-R^2	0.65	0.65		0.60	0.60		0.72	0.72
CGLS-R^2	0.06	0.06		0.05	0.05		0.09	0.09

Table 9: Long-run Shift of Risk Premium

Description: This table reports the point estimate for the vector $\vec{b}$ in different time blocks, using the asset pricing model "P-ND" with relative prices in the good-service two-sectors. All estimation use the "Size-BM 25" portfolios as the testing assets.

	Risk Price					
	1935-1989		1950-2005		1965-2019	
	Panel (A): Parameters					
	1st-Stage	2nd-Stage	1st-Stage	2nd-Stage	1st-Stage	2nd-Stage
b_e	31.77	31.76	35.00	40.13	30.05	33.72
$[t]$	[3.78]	[24.78]	[3.05]	[12.55]	[2.61]	[13.06]
b_g	-47.95	-46.15	-66.87	-63.51	-68.26	-63.83
$[t]$	[-2.69]	[-10.83]	[-2.92]	[-13.40]	[-2.90]	[-11.68]
	Panel (B): Stats of GMM					
MAPE	0.70		0.33		0.38	
RMSE	0.95		0.39		0.51	
CV-R^2	0.92		0.98		0.96	
J-pval		82.47		96.62		81.48
	Panel (C): Test Statistic					
Test-t		[-3.33]		[-3.61]		[-4.24]

Table 9: Long-run Shift of Risk Premium, Continued

Description: This table reports the estimate for the risk premium in different time blocks, using the asset pricing model "P-ND" with relative prices in the good-service two-sectors. All estimation use the "Size-BM 25" portfolios as the testing assets.

	Risk Premium					
	1935-1989		1950-2005		1965-2019	
	Panel (D): Risk Premium					
	without	with	without	with	without	with
λ_e	1.54	2.61	0.13	1.42	0.38	0.43
[t]	[2.34]	[3.81]	[0.14]	[1.52]	[0.55]	[0.64]
λ_g	-3.38	0.54	-2.98	-1.41	-1.56	-1.28
[t]	[-2.69]	[0.63]	[-4.35]	[-2.37]	[-4.19]	[-2.50]
α	-	9.92	-	6.99	-	1.99
[t]	-	[3.73]	-	[2.66]	-	[0.63]
	Panel (E): Stats of Two-step Regression					
OLS-R^2	-0.09		0.38		0.63	
GLS-R^2	-0.07		0.13		-0.38	
COLS-R^2		0.74		0.82		0.67
CGLS-R^2		0.06		0.16		0.01

Table 10: Quantity Index

Description: This table reports the point estimate for the vector $\vec{b}$ using different pricing kernels. "DU" uses the quantity growth of nondurable good and service. "Stone-Geary" uses the quantity growth of zero-habit sector, and the growth of relative prices, 1st-stage estimation outcome are reported. The sector with zero-habit is denoted with subscript h. In "SG-2", the zero-habit sector is either the good sector or the service sector. In "SG-3", the zero-habit sector is either the Food-service sector or the non-Food service sector. All estimation use the "Mix 30" portfolios as the testing assets. Other description in Table (2) and Table (3) applies.

	Risk Price							
	DU			SG-2			SG-3	
	1st-Stage	2nd-Stage		Good	Service		Food	Non-Food
q_g	50.20	41.92	q_g	34.76				
$[t]$	[0.99]	[5.48]	$[t]$	[1.27]				
q_s	1.59	6.30	q_s		22.31	q_h	19.67	21.00
$[t]$	[0.04]	[1.24]	$[t]$		[1.61]	$[t]$	[2.50]	[2.45]
$p_{g,s}$			$p_{g,s}$	-13.01	-22.37	$p_{g,h}$	-50.27	-36.29
$[t]$			$[t]$	[-0.47]	[-1.29]	$[t]$	[-3.77]	[-2.22]
						$p_{n,h}$	67.03	
						$[t]$	[3.64]	
						$p_{f,h}$		-3.43
						$[t]$		[-0.34]
	GMM statistic							
MAE	0.64			0.48	0.40		0.39	0.31
RMSE	0.81			0.61	0.48		0.49	0.44
CV-R^2	0.89			0.94	0.96		0.96	0.97
J-pval		91.69		91.48	92.58		92.53	90.69
	Fama-Macbeth Two-step Statistic							
OLS-R^2	-0.07			0.45	0.41		0.45	0.52
GLS-R^2	0.02			-0.01	0.04		0.00	0.08
COLS-R^2	0.54			0.47	0.61		0.46	0.65
CGLS-R^2	0.11			0.04	0.10		0.04	0.14

C Notation

For consistent notation, I use the upper-case character for the nominal price and the nominal expenditure. I use the lower-case character for the log nominal price $p_j = \log(P_j)$, and the log expenditure $e = \log(E)$.

I use notation $\mathcal{D}_j f$ as the derivative of function f with respect to j-th element. I use $\mathcal{D}_{j,i} f$ as the second-order derivatives of function f, where $\mathcal{D}_{j,i} f = \mathcal{D}_i \mathcal{D}_j f$. I denote the matrix of second-order derivatives as $\mathcal{H}f$ where $\mathcal{H}f_{j,i} = \mathcal{D}_{j,i} f$. I denote the first-order difference of variable x as $\mathrm{d}x = x' - x$.

Definition 1. *Define the* ***absolute share*** *of j-th sector as* ω_j*,*

$$\omega_j \equiv \frac{P_j \cdot C_j}{E}.$$

Define the ***relative expenditure share*** *between the k-th sector and the j-th sector as* $\mathcal{S}_{k,j}$*,*

$$\mathcal{S}_{k,j} \equiv \frac{\omega_k}{\omega_j}.$$

Define the ***core-IDU*** *as the value of IDU given price vector* P *and 1 unit consumption spending* E*,*

$$V^*(P) \equiv V(P, 1).$$

Define the (k, j)*-pair* ***price elasticity*** *as* $\eta_{k,j}$*,*

$$\eta_{k,j} \equiv -\frac{\mathcal{D}_{k,j} V^*(E^{-1} \cdot P)}{\mathcal{D}_k V^*(E^{-1} \cdot P)} \cdot (E^{-1} \cdot P_j).$$

I use the core-IDU function $V^*(p) = V(p, 1)$ to simplify the notation. The budget set is Homogeneous of Degree Zero,

$$\left\{ \vec{C} \in \mathcal{X} | \sum_{j \in \mathcal{J}} (k \cdot P_j) \cdot C_j \leq k \cdot E \right\} = \left\{ \vec{C} \in \mathcal{X} | \sum_{j \in \mathcal{J}} P_j \cdot C_j \leq E \right\}, \quad k > 0.$$

Because the budget set is H.D.0, when the consumption spending is positive, the the core-IDU function V^* and the indirect utility function V has relationship as $V(P, E) = V^*(E^{-1} \cdot P)$. Thorough this article, I require the core-IDU function V^* with continuous third-order derivatives.

Definition 2. *Define the* ***real Stochastic Discount Factor*** $\tilde{M}$ *as*

$$\tilde{M}(\vec{P}_t, E_t) \equiv V(\vec{P}_t, E_t)^{-\gamma} \cdot \mathcal{D}_E V(\vec{P}_t, E_t) \cdot P_{J,t}. \tag{4}$$

Definition 3. *The price system* (P, P^s) *and the consumption allocation* $\tilde{C}$ *constitutes the (Heterogeneous Consumer) Competitive Equilibrium if*

1. $\tilde{C}^{(n)}$ *solves problem*

$$\begin{aligned}
\overline{U}_0(\vec{\theta}_0^{(n)}) \quad &= \quad \sup_{\tilde{C},\tilde{\theta}} \quad \lim_{T\to\infty} \mathbb{E}[\sum_{t=1}^{T} \beta^t \cdot u^{(n)}(\vec{C}_t)] \\
s.t. \qquad & \sum_k \theta_{k,t} \cdot (P^s_{k,t} + D_{k,t}) + w_t \cdot \ell_t^{(n)} = \sum_j P_{j,t} \cdot C_{j,t} + \sum_k \theta_{k,t+1} \cdot P^s_{k,t}, \\
& C_{j,t} \geq 0; \quad \sum_k \theta_{k,t+1} \cdot P^s_{k,t} \geq \underline{a}.
\end{aligned} \tag{P.1-HA}$$

2. *commodity market* (j,t) *clears in the demand side* $\sum_{n\in\mathcal{N}} C_{j,t}^{(n)} = \mathbf{C}_{j,t}$.
3. *commodity market clears in the supply side, labor market clear, given the model specification of producers;*
4. *financial security market clears, given the model specification of foreign borrowing and lending.*

D Approximation

D.1 Relative Share

Proposition 2. *Given* $\mathcal{S}_{k,j}$ *is non-trivial constant, the log-change of relative share is decomposed into the price effect and the income effect,*

$$\begin{aligned}
\mathrm{d}s_{k,j} =&(1-\eta_{k,k}+\eta_{j,k}) \cdot \mathrm{d}p_k - (1-\eta_{j,j}+\eta_{k,j}) \cdot \mathrm{d}p_j - \sum_{i\neq k,j} (\eta_{k,i}-\eta_{j,i}) \cdot \mathrm{d}p_i \\
&+ \sum_i (\eta_{k,i}-\eta_{j,i}) \cdot \mathrm{d}e + o(h).
\end{aligned} \tag{27}$$

with $s_{k,j} = \log(\mathcal{S}_{k,j})$, $h = (\mathrm{d}p, \mathrm{d}e)$ *and* $\mathrm{d}s_{k,j} = s_{k,j}(p+\mathrm{d}p, e+\mathrm{d}e) - s_{k,j}(p,e)$. *The term* $o(h)$ *is higher-order in* h *in the sense that* $\lim_{||h||\to 0} \frac{o(h)}{||h||} = 0$ *under sup-norm* $||h|| \equiv \sup_j |h_j|$.

Proof. By Taylor's Theorem, for $s_{k,j}$ with continuous second-order derivatives in neighborhood of $a = (p,e)$, there exists $\theta \in [0,1]$ such that

$$s_{k,j}(a+h) - s_{k,j}(a) =\mathcal{D}s_{k,j}(a) \cdot h + \frac{1}{2} \cdot h^T \cdot \mathcal{H}s_{k,j}(a+\theta\cdot h) \cdot h \tag{46}$$

Denote the term $o(h;a) = \frac{1}{2} \cdot h^T \cdot s_{k,j}(a+\theta\cdot h) \cdot h$. The term $o(h;a)$ is higher-order in h in the sense that given the sup-norm $||h|| \equiv \sup_j |h_j|$, $\lim_{||h||\to 0} \frac{o(h;a)}{||h||} = 0$ for arbitrary a.

Given the optimal consumption bundle is unique, the Roy Identity tells us that **absolute share** ω can be written as

$$\omega_j = \frac{P_j \cdot C_j}{E} = \frac{P_j \cdot \mathcal{D}_j V^*}{\sum_i P_i \cdot \mathcal{D}_i V^*}. \tag{47}$$

Replacing the absolute share ω_k and ω_j, the log of **relative expenditure share** satisfies

$$\begin{aligned} s_{k,j} =& \log(\frac{\omega_k}{\omega_j}) \\ =& \log(P_k) + \log[-\mathcal{D}_k V^*] - \log(P_j) - \log[-\mathcal{D}_j V^*]. \end{aligned} \tag{48}$$

Now I explicitly decompose the term $\mathcal{D}s_{k,j}(a)\cdot h$. Recall the first-order derivative of composition satisfies $\mathcal{D}[\log \circ f(a)] = \frac{\mathcal{D}f(a)}{f(a)}$. Recall $a = (\vec{p}, e)$ and $h = (\vec{p}_B - \vec{p}, e_B - e)$, the term $\mathcal{D}s_{k,j}(a) \cdot h$ is decomposed as below,

$$\begin{aligned} \mathcal{D}s_{k,j}(a) \cdot h =& (p_{k,B} - p_k) - (p_{j,B} - p_j) \\ & + [\sum_{i=1}^{J} \mathcal{D}_{k,i} V^* \cdot (E^{-1} \cdot P_i) \cdot (p_{i,B} - p_i) \\ & \quad + \sum_{i=1}^{J} \mathcal{D}_{k,i} V^* \cdot (-E^{-1} \cdot P_i) \cdot (e_B - e)] \cdot [\mathcal{D}_k V^*]^{-1} \\ & - [\sum_{i=1}^{J} \mathcal{D}_{j,i} V^* \cdot (E^{-1} \cdot P_i) \cdot (p_{i,B} - p_i) \\ & \quad + \sum_{i=1}^{J} \mathcal{D}_{j,i} V^* \cdot (-E^{-1} \cdot P_i) \cdot (e_B - e)] \cdot [\mathcal{D}_j V^*]^{-1}. \end{aligned} \tag{49}$$

I use the matrix of **price elasticity** $\eta(\vec{P}, E)$ at $(\vec{P}, E)$. For succinct notation, I use η as the local price elasticity. Substituting $\eta_{k,i} = -\frac{\mathcal{D}_{k,i} V^*}{\mathcal{D}_k V^*} \cdot (E^{-1} \cdot P_j)$ and $\eta_{j,i} = -\frac{\mathcal{D}_{j,i} V^*}{\mathcal{D}_j V^*} \cdot (E^{-1} \cdot P_j)$, the equation (46) is written as

$$\begin{aligned} s_{k,j}(a+h) - s_{k,j}(a) =& (1 - \eta_{k,k} + \eta_{j,k}) \cdot (p_{k,B} - p_k) - (1 - \eta_{j,j} + \eta_{k,j}) \cdot (p_{j,B} - p_j) \\ & - \sum_{i \neq k,j} (\eta_{k,i} - \eta_{j,i}) \cdot (p_{i,B} - p_i) \\ & + \sum_i (\eta_{k,i} - \eta_{j,i}) \cdot (e_B - e) \\ & + \frac{1}{2} \cdot h^T \cdot \frac{\mathcal{H}s_{k,j}(a + \theta \cdot h)}{\mathcal{S}_{k,j}(a)} \cdot h \end{aligned} \tag{50}$$

For simple notation, I denote the first-order difference using $\mathrm{d}s_{k,j} = s_{k,j}(a + h) - s_{k,j}(a)$, $\mathrm{d}p_k = p_{k,B} - p_k$ and $\mathrm{d}e = e_B - e$, so the equation (46) reads as equation (27). $\square$

D.2 Dynamic Decision with IDU

Lemma 1. *Define the problem (P.1) as*

$$
\begin{aligned}
\overline{U}_0(\vec{\theta}_0) \quad &= \quad \sup_{\tilde{C},\tilde{\theta}} \quad \lim_{T\to\infty} \mathbb{E}[\sum_{t=1}^{T} \beta^t \cdot u(\vec{C}_t)] \\
s.t. \quad & \sum_k \theta_{k,t} \cdot (P^s_{k,t} + D_{k,t}) + w_t \cdot \ell_t = \sum_j P_{j,t} \cdot C_{j,t} + \sum_k \theta_{k,t+1} \cdot P^s_{k,t}, \\
& C_{j,t} \geq 0; \quad \sum_k \theta_{k,t+1} \cdot P^s_{k,t} \geq \underline{a}.
\end{aligned} \tag{P.1}
$$

Define the problem (P.2) as

$$
\begin{aligned}
\overline{V}_0^{\text{New}}(\vec{\theta}_0) \quad &= \quad \sup_{\tilde{E},\tilde{\theta}} \quad \lim_{T\to\infty} \mathbb{E}[\sum_{t=1}^{T} \beta^t \cdot v(\vec{P}_t, E_t)] \\
s.t. \quad & \sum_k \theta_{k,t} \cdot (P^s_{k,t} + D_{k,t}) + w_t \cdot \ell_t = E_t + \sum_k \theta_{k,t+1} \cdot P^s_{k,t}, \\
& E_t \geq 0; \quad \sum_k \theta_{k,t+1} \cdot P^s_{k,t} \geq \underline{a}.
\end{aligned} \tag{P.2}
$$

Optimization problems (P.1) yields equivalent value as the optimization problem (P.2). For each optimal policy C^* in problem (P.1), E^* such that

$$
E^*_t = \sum_{j\in\mathcal{J}} P_{j,t} \cdot C^*_{j,t}, \quad \forall t, z^t \tag{2}
$$

is an optimal policy in the optimization problem (P.2).

Proof. Following Theorem 7.6 and Theorem 9.2 in (Stokey, 1989), I require Assumption 9.1-9.2 in (Stokey, 1989) to ensure the proper measure space and the well-defined optimal consumption plan for the optimization problem (P.1). Similar assumptions are required to ensure the proper measure space and the well-defined optimal expenditure plan for the optimization problem (P.2). Here, I focus on comparing value and the optimal plan in problem (P.1) and problem (P.2).

Step 1: Construct problem (P.3) with psuedo constraints,

$$\begin{aligned}
\overline{V}_0(\vec{\theta}_0) &= \sup_{\tilde{E},\tilde{C},\tilde{\theta}} \quad \lim_{T\to\infty} \mathbb{E}[\sum_{t=1}^{T} \beta^t \cdot u(\vec{C}_t)] \\
s.t. \quad & \sum_j P_{j,t} \cdot C_{j,t} \leq E_t, \\
& \sum_k \theta_{k,t} \cdot (P^s_{k,t} + D_{k,t}) + w_t \cdot \ell_t = \sum_j P_{j,t} \cdot C_{j,t} + \sum_k \theta_{k,t+1} \cdot P^s_{k,t}, \\
& C_{j,t} \geq 0; \quad \sum_k \theta_{k,t+1} \cdot P^s_{k,t} \geq \underline{a}.
\end{aligned} \tag{P.3}$$

I verify Problem (P.2) generates lower value than problem (P.3),

$$\overline{V}_0^{\text{New}}(\vec{\theta}_0) \leq \overline{V}_0(\vec{\theta}_0).$$

To see this is true, I construct the auxiliary consumption bundle implied by the optimal expenditure plan E^* in problem (P.2),

$$C^{E*}_{j,t}(z^t) = \frac{E^*_t(z^t)}{P_{j,t}(z^t)} \cdot \frac{P_{j,t}(z^t) \cdot \mathcal{D}_j V^*(E^*_t(z^t)^{-1} \cdot \vec{P}_t(z^t))}{\sum_i P_{i,t}(z^t) \cdot \mathcal{D}_i V^*(E^*_t(z^t)^{-1} \cdot \vec{P}_t(z^t))}. \tag{51}$$

By construction of consumption plan C^{E*}, for all (t, z), H.D.0 core-IDU V^* ensures that

$$\sum_{j\in\mathcal{J}} P_{j,t}(z^t) \cdot C^{E*}_{j,t}(z^t) = E^*_t(z^t).$$

Notice that $V[\vec{P}_t(z^t), E^*_t(z^t)] = V^*([E^*_t(z^t)]^{-1} \cdot \vec{P}_t(z^t)) = g(\vec{C}^{E*}_t(z^t))$ at each time-state node, so the objective function values are identical. I construct the plan of financial security $\tilde{\theta}^{E*}$ exactly the same as the financial wealth plan in problem (P.2). As the consequence, for all (t, z^t),

$$\sum_{j\in\mathcal{J}} P_{j,t}(z^t) \cdot C^{E*}_{j,t}(z^t) + \sum_k \theta^{E*}_{k,t+1} \cdot P^s_{k,t} = \sum_k \theta^{E*}_{k,t} \cdot (P^s_{k,t} + D_{k,t}) + w_t \cdot \ell_t.$$

Therefore, the plan $(\tilde{C}^{E*}, \tilde{E}^*, \tilde{\theta}^{E*})$ is feasible in Problem (P.3). For arbitrary feasible expenditure plan of problem (P.2), a feasible plan can be constructed in the similar way, so I conclude problem (P.2) generates (weakly) lower value than the problem (P.3).

Recall that Problem (P.3) adds additional constraints to the Problem (P.1), so Problem (P.3) generates (weakly) lower value than Problem (P.1). Overall, I conclude problem (P.2) generates lower value than the problem (P.1).

Step 2: I verify $\overline{U}_0(\vec{\theta}_0) \leq \overline{V}_0(\vec{\theta}_0)$. Construct the implied expenditure plan E^{C*} from the optimal consumption plan C^* in problem (P.1),

$$E^{C*}_t(z^t) = \sum_{j\in\mathcal{J}} P_{j,t}(z^t) \cdot C^*_{j,t}(z^t).$$

Again, the exact inter-temporal budget constraints implies $V[\vec{P}_t(z^t), E_t^{C*}(z^t)] = V^*(E_t^{C*}(z^t)^{-1} \cdot \vec{P}_t(z^t)) = g(\vec{c}_t^*(z^t))$ at each time-state node, so objective function values are identical. A financial portfolio plan $\tilde{\theta}^{C*}$ can be constructed exactly the same as the financial wealth plan in problem (P.2). Therefore, the plan $(\tilde{E}^{C*}, \tilde{\theta}^{C*})$ is feasible in Problem (P.2). Given the enlarged feasible set of expenditure plan, I conclude Problem (P.1) generates lower value than the Problem (P.2).

Step 3: Combine step (1) and step (2), we conclude

$$\overline{U}_0(\vec{\theta}_0) = \overline{V}_0(\vec{\theta}_0).$$

Furthermore, for each optimal policy c^* in problem (P.1), E^{C*} such that

$$E_t^{C*}(z^t) = \sum_{j \in \mathcal{J}} P_{j,t}(z^t) \cdot C_{j,t}^*(z^t), \quad , \forall t, z \tag{52}$$

is also an optimal policy in the optimization problem (P.2). To see this is true, recall the optimal value $\overline{U}_0(\vec{\theta}_0)$ is attained by the consumption plan C^*, so E^{C*} attains the optimal value $\overline{V}_0(\vec{\theta}_0)$. In the symmetric argument, for each optimal policy E^* in problem (P.1), C^{E*} such that

$$C_{j,t}^{E*}(z^t) = \frac{E_t^*(z^t)}{P_{j,t}(z^t)} \cdot \frac{P_{j,t}(z^t) \cdot \mathcal{D}_j V^*(E_t^*(z^t)^{-1} \cdot \vec{P}_t(z^t))}{\sum_i P_{i,t}(z^t) \cdot \mathcal{D}_i V^*(E_t^*(z^t)^{-1} \cdot \vec{P}_t(z^t))}. \tag{53}$$

is also an optimal policy in the optimization problem (P.1). □

D.3 Stochastic Discount Factor

Lemma 2. *For the pair of sectors (i, j),*

$$\eta_{j,i} = \eta_{i,j} \cdot \frac{\omega_i}{\omega_j}. \tag{54}$$

Proof. By Theorem 9.41 in (Rudin,1964), $\mathcal{D}_{i,j}V^* = \mathcal{D}_{j,i}V^*$ as V^* has continuous second-order derivatives. Recall the definition of price-elasticity,

$$\eta_{i,j} = -\frac{\mathcal{D}_{i,j}V^*}{\mathcal{D}_i V^*} \cdot (E^{-1} \cdot P_j).$$

Switching the direction of sub-scripts yields the equation of price elasticity,

$$\eta_{j,i} = \eta_{i,j} \cdot \frac{\mathcal{D}_i V^*}{\mathcal{D}_j V^*} \cdot \frac{E^{-1} \cdot P_i}{E^{-1} \cdot P_j} = \eta_{i,j} \cdot \frac{\omega_i}{\omega_j}. \tag{55}$$

□

Theorem 1. *First-Order Approximated SDF is*

$$\mathrm{d}\tilde{m} = -\sum_{j=1}^{J} b_j(\vec{P}, E) \cdot \omega_j \cdot (\mathrm{d}p_j - \mathrm{d}p_J) - b_e(\vec{P}, E) \cdot (\mathrm{d}e - \mathrm{d}p_J) + o(h). \tag{5}$$

with high-order term $o(h)$ for $h = \max\{\{\mathrm{d}p_j\}_j, \mathrm{d}e\}$. The risk price vector b is

$$
\begin{aligned}
b_j(\vec{P}, E) =& -[\gamma \cdot \frac{\mathcal{D}_E V(\vec{P}, E) \cdot E}{V(\vec{P}, E)} + 1] + \sum_{i=1}^{J} \eta_{j,i}(\vec{P}, E), \\
b_e(\vec{P}, E) =& -\sum_{j=1}^{J} [b_j(\vec{P}, E) - 1] \cdot \omega_j.
\end{aligned}
\tag{6}
$$

Proof. By the Taylor's Theorem, in the neighborhood of $a = (\vec{p}, e)$, there exists $\theta \in [0, 1]$ such that

$$
\tilde{m}(a + h) - \tilde{m}(a) = \mathcal{D}\tilde{m}(a) \cdot h + \frac{1}{2} \cdot h^T \cdot \mathcal{H}\tilde{m}(a + \theta \cdot h) \cdot h. \tag{56}
$$

The log of **real Stochastic Discount Factor** $\tilde{M}$ satisfies

$$
\tilde{m} = -\gamma \cdot \log[V^*] + \log[\sum_{j=1}^{J} \mathcal{D}_j V^* \cdot (-E^{-2}) \cdot P_j] + \log(P_J). \tag{57}
$$

Now I explicitly decompose the term $\mathcal{D}\tilde{m}(a) \cdot h$. Recall the first-order derivative of composition satisfies $\mathcal{D}[\log \circ f(a)] = \frac{\mathcal{D}f(a)}{f(a)}$. The term $\mathcal{D}\tilde{m}(a) \cdot h - \mathrm{d}p_J$ is decomposed as below,

$$
\begin{aligned}
\mathcal{D}\tilde{m}(a) \cdot h - \mathrm{d}p_J = -\gamma \cdot [V^*]^{-1} \cdot & [\sum_{j=1}^{J} \mathcal{D}_j V^* \cdot E^{-1} \cdot P_j \cdot \mathrm{d}p_j \\
& + \sum_{j=1}^{J} \mathcal{D}_j V^* \cdot (-E^{-2}) \cdot P_j \cdot \mathrm{d}e] \\
+[\sum_{j=1}^{J} \mathcal{D}_j V^* \cdot (-E^{-2}) \cdot P_j]^{-1} \cdot & [\sum_{j=1}^{J} \mathcal{D}_j V^* \cdot (-E^{-2}) \cdot \mathrm{d}p_j \\
& + \sum_{j=1}^{J} \mathcal{D}_j V^* \cdot (2 \cdot E^{-3}) \cdot P_j \cdot \mathrm{d}e] \\
+[\sum_{j=1}^{J} \mathcal{D}_j V^* \cdot (-E^{-2}) \cdot P_j]^{-1} \cdot & [\sum_{j=1}^{J} \sum_{i=1}^{J} \mathcal{D}_{j,i} V^* \cdot (-E^{-3}) \cdot P_j \cdot \mathrm{d}p_i \\
& + \sum_{j=1}^{J} \sum_{i=1}^{J} \mathcal{D}_{j,i} V^* \cdot E^{-4} \cdot P_j \cdot P_i \cdot \mathrm{d}e].
\end{aligned}
\tag{58}
$$

Denote $A = \sum_{j=1}^{J} \mathcal{D}_j V^* \cdot P_j$. Replacing the formulas

$$\begin{aligned}
\mathcal{D}_E V(\vec{P}, E) =& \sum_{j=1}^{J} \mathcal{D}_j V^* \cdot (-E^{-2}) \cdot P_j, \\
V_j(\vec{P}, E) =& \mathcal{D}_j V^* \cdot E^{-1}, \\
\omega_k =& \frac{P_k \cdot \mathcal{D}_k V^*}{\sum_{j=1}^{J} \mathcal{D}_j V^* \cdot P_j},
\end{aligned}$$

yields the simplified $\mathcal{D}\tilde{m}(a) \cdot h - \mathrm{d}p_J$ as below,

$$\begin{aligned}
&\mathcal{D}\tilde{m}(a) \cdot h - \mathrm{d}p_J \\
&\quad = -\gamma \cdot [V^*]^{-1} \cdot [\sum_{j=1}^{J} \omega_j \cdot A \cdot E^{-1} \cdot \mathrm{d}p_j + A \cdot (-E^{-1}) \cdot \mathrm{d}e] \\
&+ [A \cdot (-E^{-2})]^{-1} \cdot [\sum_{j=1}^{J} \omega_j \cdot A \cdot (-E^{-2}) \cdot \mathrm{d}p_j + A \cdot (2 \cdot E^{-2}) \cdot \mathrm{d}e] \\
&+ [A \cdot (-E^{-2})]^{-1} \cdot [\sum_{j=1}^{J} \sum_{i=1}^{J} \mathcal{D}_{j,i} V^* \cdot (-E^{-3}) \cdot P_j \cdot P_i \cdot \mathrm{d}p_i \\
&\qquad\qquad + \sum_{j=1}^{J} \sum_{i=1}^{J} \mathcal{D}_{j,i} V^* \cdot E^{-3} \cdot P_j \cdot P_i \cdot \frac{\mathrm{d}e}{e}].
\end{aligned} \tag{59}$$

We further replace the formula with the price-elasticity η,

$$\begin{aligned}
\mathcal{D}_{j,i} V^* \cdot E^{-3} \cdot P_j \cdot P_i =& \mathcal{D}_j V^* \cdot P_j \cdot \frac{\mathcal{D}_{j,i} V^* \cdot E^{-3} \cdot P_j \cdot P_i}{\mathcal{D}_j V^* \cdot P_j} \\
=& \omega_j \cdot A \cdot (-E^{-2}) \cdot \frac{\mathcal{D}_{j,i} V^* \cdot P_i}{\mathcal{D}_j V^* \cdot E} \\
=& \omega_j \cdot A \cdot (-E^{-2}) \cdot (-\eta_{j,i}).
\end{aligned}$$

The term $\mathcal{D}\tilde{m}(a) \cdot h - \mathrm{d}p_J$ is further simplified as

$$\begin{aligned}
\mathcal{D}\tilde{m}(a) \cdot h - \mathrm{d}p_J =& -\gamma \cdot \frac{A \cdot E^{-1}}{V^*} \cdot (\sum_{j=1}^{J} \omega_j \cdot \mathrm{d}p_j - \mathrm{d}e) + (\sum_{j=1}^{J} \omega_j \cdot \mathrm{d}p_j - \mathrm{d}e) - \mathrm{d}e \\
& - \sum_{j=1}^{J} \omega_j \cdot (\sum_{i=1}^{J} \eta_{j,i} \cdot \mathrm{d}p_i - \sum_{i=1}^{J} \eta_{j,i} \cdot \mathrm{d}e).
\end{aligned} \tag{60}$$

After replacing $\frac{A \cdot E^{-1}}{V^*} = \frac{-\mathcal{D}_E V(\vec{P}, E) \cdot E}{V(\vec{P}, E)}$, the term $\mathcal{D}\tilde{m}(a) \cdot h$ is further simplified as

$$\mathcal{D}\tilde{m}(a)\cdot h = -\gamma\cdot\frac{-\mathcal{D}_E V(\vec{P},E)\cdot E}{V(\vec{P},E)}\cdot(\sum_{j=1}^{J}\omega_j\cdot \mathrm{d}p_j-\mathrm{d}e)+(\sum_{j=1}^{J}\omega_j\cdot \mathrm{d}p_j-\mathrm{d}e)$$
$$-(\mathrm{d}e-\mathrm{d}p_J)-\sum_{j=1}^{J}\omega_j\cdot(\sum_{i=1}^{J}\eta_{j,i}\cdot \mathrm{d}p_i-\sum_{i=1}^{J}\eta_{j,i}\cdot \mathrm{d}e). \tag{61}$$

Recall Lemma (2), in the pair (i,j), switching the direction of sub-scripts yields the equation of price elasticity,

$$\eta_{j,i}=\eta_{i,j}\cdot\frac{\omega_i}{\omega_j}. \tag{62}$$

This further implies the risk price of consumption category j with

$$\sum_{i=1}^{J}\omega_i\cdot\eta_{i,j}=\sum_{i=1}^{J}\omega_j\cdot\eta_{j,i}=\omega_j\cdot\sum_{i=1}^{J}\eta_{j,i}. \tag{63}$$

The term $\mathcal{D}\tilde{m}(a)\cdot h$ is further simplified as

$$\mathcal{D}\tilde{m}(a)\cdot h = -\gamma\cdot\frac{-\mathcal{D}_E V(\vec{P},E)\cdot E}{V(\vec{P},E)}\cdot(\sum_{j=1}^{J}\omega_j\cdot \mathrm{d}p_j-\mathrm{d}e)+(\sum_{j=1}^{J}\omega_j\cdot \mathrm{d}p_j-\mathrm{d}e)$$
$$-(\mathrm{d}e-\mathrm{d}p_J)-\sum_{i=1}^{J}\omega_i\cdot(\sum_{j=1}^{J}\eta_{i,j}\cdot \mathrm{d}p_i-\sum_{j=1}^{J}\eta_{i,j}\cdot \mathrm{d}e). \tag{64}$$

Hence, the First-Order Approximated Linear SDF is

$$\mathrm{d}\tilde{m}=-\sum_{j=1}^{J}b_j\cdot\omega_j\cdot \mathrm{d}p_j-b_e\cdot \mathrm{d}e+\mathrm{d}p_J+o(h).$$

has risk price $\vec{b}$ as

$$b_j = -[\gamma\cdot\frac{\mathcal{D}_E V(\vec{P},E)\cdot E}{V(\vec{P},E)}+1]+\sum_{k=1}^{J}\eta_{j,k}, \tag{65}$$

$$b_e = [\gamma\cdot\frac{\mathcal{D}_E V(\vec{P},E)\cdot E}{V(\vec{P},E)}+1]+1-\sum_{j=1}^{J}\omega_j\cdot\sum_{k=1}^{J}\eta_{j,k}. \tag{66}$$

By construction, absolute consumption shares add-up to 1,

$$\sum_{j=1}^{J}\omega_j=1.$$

Therefore, the risk price vector b satisfies

$$\sum_{j=1}^{J} \omega_j \cdot b_j + b_e = 1. \tag{67}$$

Considering the relative change using the deflator P_J, the First-Order Approximated Linear SDF is

$$\mathrm{d}\tilde{m} = -\sum_{j=1}^{J} b_j \cdot \omega_j \cdot (\mathrm{d}p_j - \mathrm{d}p_J) - b_e \cdot (\mathrm{d}e - \mathrm{d}p_J) + o(h).$$

$\square$

Proposition 3. *Define the* ***Engel Slope*** *for the sector pair* (k, j) *as*

$$\mathrm{ES}_{k,j}(\vec{P}, E) = \lim_{\mathrm{d}e \to 0} \frac{s_{k,j}(p, e + \mathrm{d}e) - s_{k,j}(p, e)}{\mathrm{d}e}, \tag{28}$$

the risk price vector satisfies

$$b_k - b_j \quad = \quad \mathrm{ES}_{k,j}(\vec{P}, E). \tag{29}$$

Proof. Recall the Proposition (2),

$$\mathrm{ES}_{k,j}(\vec{P}, E) = \sum_{i=1}^{J} \eta_{k,i}(\vec{P}, E) - \sum_{i=1}^{J} \eta_{j,i}(\vec{P}, E). \tag{68}$$

Theorem (1) tells us,

$$b_k - b_j = \sum_{i=1}^{J} \eta_{k,i}(\vec{P}, E) - \sum_{i=1}^{J} \eta_{j,i}(\vec{P}, E). \tag{69}$$

So we arrive to $b_k - b_j = \mathrm{ES}_{k,j}(\vec{P}, E)$. $\square$

Proposition 1. *Given the security k and the security f, real total return $\tilde{R}_{k,t\to t'}$ and $\tilde{R}_{f,t\to t'}$ satisfy*

$$\mathbb{E}[\frac{\tilde{M}_{t'}}{\tilde{M}_t}\cdot(\tilde{R}_{k,t\to t'}-\tilde{R}_{f,t\to t'})|\mathcal{I}_t]=0. \tag{8}$$

Proof. I refer the standard argument as in Chapter 13, (Ljungqvist and Sargent, 2012). The Lagrangian for the consumption allocation problem is

$$\begin{aligned} L_0(\vec{\theta}_0,\lambda_0,\nu_0,\nu_0^e)= \quad & \sup_{\tilde{E},\tilde{\theta},\tilde{\lambda}_t,\tilde{\nu}_t,\tilde{\nu}_t^e} \lim_{T\to\infty}\{\beta^{T+1}\cdot\mathbb{E}[\sum_k P_{k,T+1}\cdot\mathcal{D}_e v(\vec{P}_T,E_T)] \\ & +\mathbb{E}[\sum_{t=1}^{T}\beta^t\cdot[v(\vec{P}_t,E_t)+\lambda_t\cdot(\text{ budget constraint}) \\ & +\nu_t\cdot(\text{bounded total wealth}) \\ & +\nu_t^e\cdot(\text{non-negative spending})]]\}. \end{aligned} \tag{L.1}$$

Here, the *budget constraint* reads as

$$\sum_k\theta_{k,t}\cdot(P_{k,t}^s+D_{k,t})+w_t\cdot\ell_t=E_t+\sum_k\theta_{k,t+1}\cdot P_{k,t}^s.$$

The *bounded total wealth constraint* reads as

$$\sum_k\theta_{k,t+1}\cdot P_{k,t}^s\geq\underline{a}.$$

Given $\lim_{E\to0}\mathcal{D}_e v(\vec{P},E)=-\infty$, the shadow price $\nu_t^e\equiv0$. Optimal position in k-th financial security $\tilde{\theta_k}$ implies the motion equation of shadow price $\tilde{\lambda}_t$

$$\begin{aligned} & \beta^t\cdot\mu(z^t)\cdot[\lambda_t(z^t)+\eta_t(z^t)]\cdot P_{k,t}^s(z^t) \\ = & \sum_{z_{t+1}|z^t}\beta^{t+1}\cdot\mu(z^{t+1})\cdot\lambda_{t+1}(z^{t+1})\cdot[P_{k,t+1}^s(z^{t+1})+D_{k,t+1}(z^{t+1})]. \end{aligned} \tag{70}$$

Optimal consumption spending $\tilde{E}$ implies the equation for the shadow price λ_t and marginal utility of expenditure,

$$\lambda_t(z^t)=\mathcal{D}_e v(\vec{P}_t(z^t),E_t(z^t)). \tag{71}$$

Similarly, at the succeeding time-state z^{t+1}, the FOC of consumption spending also holds

$$\lambda_{t+1}(z^{t+1})=\mathcal{D}_e v(\vec{P}_{t+1}(z^{t+1}),E_{t+1}(z^{t+1})).$$

For short notation, I use $\mathcal{D}_e v_t(z^t)$ for $\mathcal{D}_e v(\vec{P}_t(z^t),E_t(z^t))$. Substituting FOCs of consumption spending into the equation of shadow price (70) yields,

$$\begin{aligned} & [\mathcal{D}_e v_t(z^t)+\eta_t(z^t)]\cdot P_{k,t}^s(z^t) \\ = & \sum_{z_{t+1}|z^t}\beta\cdot\mu(z_{t+1},z_{t+1}|z^t)\cdot\mathcal{D}_e v_{t+1}(z^{t+1})\cdot[P_{k,t+1}^s(z^{t+1})+D_{k,t+1}(z^{t+1})]. \end{aligned} \tag{72}$$

If household is unconstrained $\eta_t(z^t) = 0$, this equation is

$$1 = \beta \cdot \mathbb{E}[\frac{\mathcal{D}_e v_{t+1}(z^{t+1})}{\mathcal{D}_e v_t(z^t)} \cdot \frac{P^s_{k,t+1}(z^{t+1}) + D_{k,t+1}(z^{t+1})}{P^s_{k,t}(z^t)} | z^t]. \tag{73}$$

Denote the deflated total return for financial asset k as

$$\tilde{R}_{k,t\to t+1}(z^{t+1}) = \frac{[P^s_{k,t+1}(z^{t+1}) + D_{k,t+1}(z^{t+1})]/P_{J,t+1}(z^{t+1})}{P^s_{k,t}(z^t)/P_{J,t}(z^t)}.$$

I conclude $\mathbb{E}[\beta \cdot \frac{\tilde{M}_{t+1}}{\tilde{M}_t} \cdot \tilde{R}_{k,t\to t+1} | \mathcal{I}_t] = 1$. The similar argument can be constructed for arbitrary finite time-interval (t, t'), hence optimal financial wealth allocation implies the Euler equation for the deflated total return of financial asset k

$$\mathbb{E}[\beta^{t'-t} \cdot \frac{\tilde{M}_{t'}}{\tilde{M}_t} \cdot \tilde{R}_{k,t\to t'} | \mathcal{I}_t] = 1. \tag{74}$$

Similarly, there exists the Euler equation of financial asset f,

$$\mathbb{E}[\beta^{t'-t} \cdot \frac{\tilde{M}_{t'}}{\tilde{M}_t} \cdot \tilde{R}_{f,t\to t'} | \mathcal{I}_t] = 1. \tag{75}$$

Given the expectation operator is linear operator, the spread of deflated total return satisfies

$$\mathbb{E}[\beta^{t'-t} \cdot \frac{\tilde{M}_{t'}}{\tilde{M}_t} \cdot (\tilde{R}_{k,t\to t'} - \tilde{R}_{f,t\to t'}) | \mathcal{I}_t] = 0. \tag{76}$$

Removing the constant non-zero term $\beta^{t'-t}$ gives us the Euler equation across financial assets

$$\mathbb{E}[\frac{\tilde{M}_{t'}}{\tilde{M}_t} \cdot (\tilde{R}_{k,t\to t'} - \tilde{R}_{f,t\to t'}) | \mathcal{I}_t] = 0. \tag{77}$$

□

Corollary 1. *Given the security k and the security f, total return $R_{k,t\to t'}$ and $R_{f,t\to t'}$ satisfy*

$$\mathbb{E}[\frac{\tilde{M}_{t'}}{\tilde{M}_t} \cdot (R_{k,t\to t'} - R_{f,t\to t'}) | \mathcal{I}_t] \approx 0. \tag{9}$$

Proof. Recall the return spread across pairs of financial assets approximately equals the spread of deflated total return,

$$R_{k,t\to t'} - R_{f,t\to t'} \approx \tilde{R}_{k,t\to t'} - \tilde{R}_{f,t\to t'}, \tag{78}$$

so the equation of real current pricing kernel is written as

$$\mathbb{E}[\frac{\tilde{M}_{t'}}{\tilde{M}_t} \cdot (R_{k,t\to t'} - R_{f,t\to t'}) | \mathcal{I}_t] \approx 0. \tag{79}$$

□

E Aggregation

E.1 Proper Negishi Weight

Lemma 3. *At $(\vec{P}^*, \mathbf{E}^*)$, artificial consumer has marginal utility of expenditure equivalent with the benchmark-consumer* (1)

$$\mathcal{D}_E V(\vec{P}^*, \mathbf{E}^*; \alpha) = \mathcal{D}_E V^{(1)}(\vec{P}^*, E^{(1),*}) \tag{80}$$

Proof. By construction of $V(\vec{P}, \mathbf{E}; \alpha)$,

$$\begin{aligned} V(\vec{P}, \mathbf{E}; \alpha) = \max_{s \in \triangle^{N-1}} \quad & \frac{1}{N} \cdot \sum_{n \in \mathcal{N}} \alpha(n) \cdot V^{(n)}(\vec{P}, s(n) \cdot E) \\ s.t. \quad & \sum_{n \in \mathcal{N}} s(n) \leq 1. \end{aligned} \tag{81}$$

In addition, optimal solution satisfies $s^*(n) = \frac{E^{(n),*}}{E}$.

By the envelope theorem,

$$\begin{aligned} \mathcal{D}_E V(\vec{P}^*, \mathbf{E}^*; \alpha) =& \mathcal{D}_E \left[\frac{1}{N} \cdot \sum_{n \in \mathcal{N}} \alpha(n) \cdot V^{(n)}(\vec{P}^*, s^*(n) \cdot E) \right] \\ =& \frac{1}{N} \cdot \sum_{n \in \mathcal{N}} \alpha(n) \cdot \mathcal{D}_E V^{(n)}(\vec{P}^*, s^*(n) \cdot E) \\ =& \frac{1}{N} \cdot \sum_{n \in \mathcal{N}} \alpha(n) \cdot \mathcal{D}_E V^{(n)}(\vec{P}^*, E^{(n),*}) \end{aligned} \tag{82}$$

By construction of Negishi-weight α,

$$\frac{1}{N} \cdot \sum_{n \in \mathcal{N}} \alpha(n) \cdot \mathcal{D}_E V^{(n)}(\vec{P}^*, E^{(n),*}) = \mathcal{D}_E V^{(1)}(\vec{P}^*, E^{(1),*}). \tag{83}$$

Therefore, I conclude

$$\mathcal{D}_E V(\vec{P}^*, \mathbf{E}^*; \alpha) = \mathcal{D}_E V^{(1)}(\vec{P}^*, E^{(1),*}).$$

□

E.2 Absolute Share

Lemma 4. *At $(\vec{P}^*, \mathbf{E}^*)$, absolute expenditure share of artificial consumer $V(\vec{P}^*, \mathbf{E}^*; \alpha)$ is identical with aggregate expenditure share on the equilibrium path,*

$$\vec{\omega}(\vec{P}^*, \mathbf{E}^*; \alpha) = \sum_{n \in \mathcal{N}} \frac{E^{(n),*}}{\sum_{m \in \mathcal{N}} E^{(m),*}} \cdot \vec{\omega}^{(n)}(\vec{P}^*, E^{(n),*}) \tag{84}$$

Proof. Recall the Roy identity,

$$\omega_j^{(n)}(\vec{P}, E^{(n)}) = -\frac{\mathcal{D}_j V^{(n)}(\vec{P}, E^{(n)})}{\mathcal{D}_E V^{(n)}(\vec{P}, E^{(n)})} \cdot \frac{P_j}{E^{(n)}}. \tag{85}$$

Hence, I can simplify the formula as below

$$\begin{aligned}
&\sum_{n\in\mathcal{N}} \frac{E^{(n),*}}{\sum_{m\in\mathcal{N}} E^{(m),*}} \cdot \omega_j^{(n)}(\vec{P}^*, E^{(n),*}) \\
=& -\sum_{n\in\mathcal{N}} \frac{E^{(n),*}}{\sum_{m\in\mathcal{N}} E^{(m),*}} \cdot \frac{\mathcal{D}_j V^{(n)}(\vec{P}^*, E^{(n),*})}{\mathcal{D}_E V^{(n)}(\vec{P}^*, E^{(n),*})} \cdot \frac{P_j^*}{E^{(n),*}} \\
=& -\sum_{n\in\mathcal{N}} \frac{P_j^*}{\sum_{m\in\mathcal{N}} E^{(m),*}} \cdot \frac{\mathcal{D}_j V^{(n)}(\vec{P}^*, E^{(n),*})}{\mathcal{D}_E V^{(n)}(\vec{P}^*, E^{(n),*})}
\end{aligned} \tag{86}$$

By the Roy Identity, absolute expenditure share of artificial consumer $V(\vec{P}^*, \mathbf{E}^*; \alpha)$ is

$$\omega_j(\vec{P}^*, \mathbf{E}^*; \alpha) = -\frac{\mathcal{D}_j V(\vec{P}^*, \mathbf{E}^*; \alpha)}{\mathcal{D}_E V(\vec{P}^*, \mathbf{E}^*; \alpha)} \cdot \frac{P_j}{E^*}. \tag{87}$$

By the envelope theorem,

$$\mathcal{D}_j V(\vec{P}^*, \mathbf{E}^*; \alpha) = \frac{1}{N} \cdot \sum_{n\in\mathcal{N}} \alpha(n) \cdot \mathcal{D}_j V^{(n)}(\vec{P}^*, E^{(n),*}). \tag{88}$$

I derive the formula $\frac{\alpha(n)}{\mathcal{D}_E V(\vec{P}^*, \mathbf{E}^*; \alpha)}$ as

$$\begin{aligned}
\frac{\alpha(n)}{\mathcal{D}_E V(\vec{P}^*, \mathbf{E}^*; \alpha)} =& \frac{\mathcal{D}_E V^{(1)}(\vec{P}^*, E^{(1),*})}{\mathcal{D}_E V^{(n)}(\vec{P}^*, E^{(n),*})} \cdot \frac{1}{\mathcal{D}_E V(\vec{P}^*, \mathbf{E}^*; \alpha)} \\
=& \frac{\mathcal{D}_E V^{(1)}(\vec{P}^*, E^{(1),*})}{\mathcal{D}_E V^{(n)}(\vec{P}^*, E^{(n),*})} \cdot \frac{1}{\mathcal{D}_E V^{(1)}(\vec{P}^*, E^{(1),*})} \\
=& \frac{1}{\mathcal{D}_E V^{(n)}(\vec{P}^*, E^{(n),*})}.
\end{aligned} \tag{89}$$

The first equality comes from the definition of Proper Negeishi weight $\alpha^*(n) = \frac{\mathcal{D}_E V^{(1)}(\vec{P}^*, E^{(1),*})}{\mathcal{D}_E V^{(n)}(\vec{P}^*, E^{(n),*})}$. The second equality comes from Lemma 3, $\mathcal{D}_E V(\vec{P}^*, \mathbf{E}^*; \alpha) = \mathcal{D}_E V^{(1)}(\vec{P}^*, E^{(1),*})$.

I simplify the formula below

$$\begin{aligned}
\frac{\mathcal{D}_j V(\vec{P}^*, \mathbf{E}^*; \alpha)}{\mathcal{D}_E V(\vec{P}^*, \mathbf{E}^*; \alpha)} =& \frac{\frac{1}{N} \cdot \sum_{n\in\mathcal{N}} \alpha(n) \cdot \mathcal{D}_j V^{(n)}(\vec{P}^*, E^{(n),*})}{\mathcal{D}_E V^{(1)}(\vec{P}^*, E^{(1),*})} \\
=& \frac{1}{N} \cdot \sum_{n\in\mathcal{N}} \frac{1}{\mathcal{D}_E V^{(n)}(\vec{P}^*, E^{(n),*})} \cdot \mathcal{D}_j V^{(n)}(\vec{P}^*, E^{(n),*}).
\end{aligned} \tag{90}$$

The first equality substitutes the formula $\mathcal{D}_j V(\vec{P}^*, \mathbf{E}^*; \alpha)$ with equation (88). The second equality substitutes the formula $\frac{\alpha(n)}{\mathcal{D}_E V(\vec{P}^*, \mathbf{E}^*; \alpha)}$ with equation (89).

Therefore, absolute expenditure share of artificial consumer $V(\vec{P}^*, \mathbf{E}^*; \alpha)$ is simplified as

$$\omega_j(\vec{P}^*, \mathbf{E}^*; \alpha) = -\frac{1}{N} \cdot \sum_{n \in \mathcal{N}} \frac{\mathcal{D}_j V^{(n)}(\vec{P}^*, E^{(n),*})}{\mathcal{D}_E V^{(n)}(\vec{P}^*, E^{(n),*})} \cdot \frac{P_j}{E^*}. \tag{91}$$

By construction of E^*,

$$\sum_{m \in \mathcal{N}} E^{(m),*} = N \cdot E^*,$$

Replacing $\omega_j^{(n)}(\vec{P}, E^{(n)}) = -\frac{\mathcal{D}_j V^{(n)}(\vec{P}, E^{(n)})}{\mathcal{D}_E V^{(n)}(\vec{P}, E^{(n)})} \cdot \frac{P_j}{E^{(n)}}$, I close the proof with

$$\begin{aligned}
\omega_j(\vec{P}^*, \mathbf{E}^*; \alpha) &= -\sum_{n \in \mathcal{N}} \frac{P_j^*}{\sum_{m \in \mathcal{N}} E^{(m),*}} \cdot \frac{\mathcal{D}_j V^{(n)}(\vec{P}^*, E^{(n),*})}{\mathcal{D}_E V^{(n)}(\vec{P}^*, E^{(n),*})} \\
&= \sum_{n \in \mathcal{N}} \frac{P_j^*}{\sum_{m \in \mathcal{N}} E^{(m),*}} \cdot \omega_j^{(n)}(\vec{P}^*, E^{(n),*}) \cdot \frac{E^{(n),*}}{P_j^*} \\
&= \sum_{n \in \mathcal{N}} \frac{E^{(n),*}}{\sum_{m \in \mathcal{N}} E^{(m),*}} \cdot \omega_j^{(n)}(\vec{P}^*, E^{(n),*}).
\end{aligned}$$

□

E.3 Effective Representative Consumer

Theorem 2. *In the economy where price system (P, M) and quantity system $(\{\tilde{c}^{(n)}\}_{n \in \mathcal{N}}, \{\tilde{\ell}^{(n)}\}_{n \in \mathcal{N}})$ constitute a Competitive Equilibrium for N heterogeneous consumers with preference $\{\succeq^{(n)}\}$, there exists a* ***Representative Consumer*** *with preference $\succeq^{\mathcal{N}}$ such that*

- *price system (P, M) and quantity system $(\sum_{n \in \mathcal{N}} \tilde{c}^{(n)}, \sum_{n \in \mathcal{N}} \tilde{\ell}^{(n)})$ constitute a Competitive Equilibrium for N homogeneous consumers with preference $\succeq^{\mathcal{N}}$.*

The indirect utility function of the Representative Consumer is $V(\vec{P}, \mathbf{E}; \alpha)$ with the Negishi weight constructed in equation (39). Along the equilibrium path, the Representative Consumer has identical marginal utility of expenditure with the Benchmark Consumer

$$\mathcal{D}_E V(\vec{P}^*, \mathbf{E}^*; \alpha) = \mathcal{D}_E V^{(1)}(\vec{P}^*, E^{(1),*}), \tag{40}$$

and the absolute expenditure share of artificial consumer $V(\vec{P}^*, \mathbf{E}^*; \alpha)$ is identical with observed aggregate expenditure share,

$$\vec{\omega}(\vec{P}^*, \mathbf{E}^*; \alpha) = \sum_{n \in \mathcal{N}} \frac{E^{(n),*}}{\sum_{m \in \mathcal{N}} E^{(m),*}} \cdot \vec{\omega}^{(n)}(\vec{P}^*, E^{(n),*}) \tag{41}$$

Proof. The construction of representative consumer is completed after I verify the Lemma 3 and the Lemma 4. □

Corollary 2. *Given invariant distribution of Negishi-weight $\{\alpha^*(n)\}_n$ along the equilibrium path, the log-change in real marginal utility of expenditure for the representative consumer approximately equals*

$$\mathrm{d}\tilde{m} = -\sum_{j=1}^{J} b_j(\alpha)\cdot\omega_j\cdot(\mathrm{d}p_j - \mathrm{d}p_J) - b_e(\alpha)\cdot(\mathrm{d}\boldsymbol{e} - \mathrm{d}p_J) + o(h). \tag{42}$$

where α is the artificial Negishi-weight, $\vec{\omega}$ is the aggregate expenditure share, $\boldsymbol{e}$ is the (log) aggregate total consumption expenditure, and the vector $b(\alpha)$ is written with aggregate expenditure share $\vec{\omega}$ and representative consumer's price-elasticity η

$$\begin{aligned} b_j(\alpha) &= -\,[\frac{\mathcal{D}_E V(\alpha)\cdot E}{V(\alpha)} + 1] + \sum_{i=1}^{J}\eta_{j,i}(\alpha),\\ b_e(\alpha) &= [\frac{\mathcal{D}_E V(\alpha)\cdot E}{V(\alpha)} + 1] + 1 - \sum_{j=1}^{J}\omega_j\cdot\sum_{i=1}^{J}\eta_{j,i}(\alpha). \end{aligned} \tag{43}$$

Proof. Recall Lemma 3,

$$\mathcal{D}_E V(\vec{P}^*, \mathbf{E}^*; \alpha) = \mathcal{D}_E V^{(1)}(\vec{P}^*, E^{(1),*}).$$

Recall the benchmark consumer's interior expenditure decision implies,

$$\mathcal{D}_E V^{(1)}(\vec{P}^*, E^{(1),*}) = M.$$

Therefore, the financial market SDF $\{M_t(z^t)\}$ can be measured by the marginal utility of constructed aggregate consumer,

$$M_t(z^t) = \mathcal{D}_E V(\vec{P}_t^*(z^t, \{z_{(n)}^t\}), \mathbf{E}_t^*(z^t, \{z_{(n)}^t\}); \alpha)$$

Recall the definition of real SDF,

$$\mathrm{d}\log(\tilde{M}) = \mathrm{d}\log[\mathcal{D}_E V(\alpha)\cdot P_J]. \tag{92}$$

First-order approximation of $\log[\mathcal{D}_E V(\alpha)\cdot P_J]$ is similar with the analysis of representative consumer under the special case $\gamma = 1$. From Lemma 4, it is legitimate to replace the expenditure share implied by the artificial-consumer with

$$\vec{\omega}(\vec{P}^*, \mathbf{E}^*; \alpha) = \sum_{n\in\mathcal{N}} \frac{E^{(n),*}}{\sum_{m\in\mathcal{N}} E^{(m),*}}\cdot\vec{\omega}^{(n)}(\vec{P}^*, E^{(n),*}),$$

at each time-state node along the equilibrium path. So I close the proof. □

F Online Figure Appendix

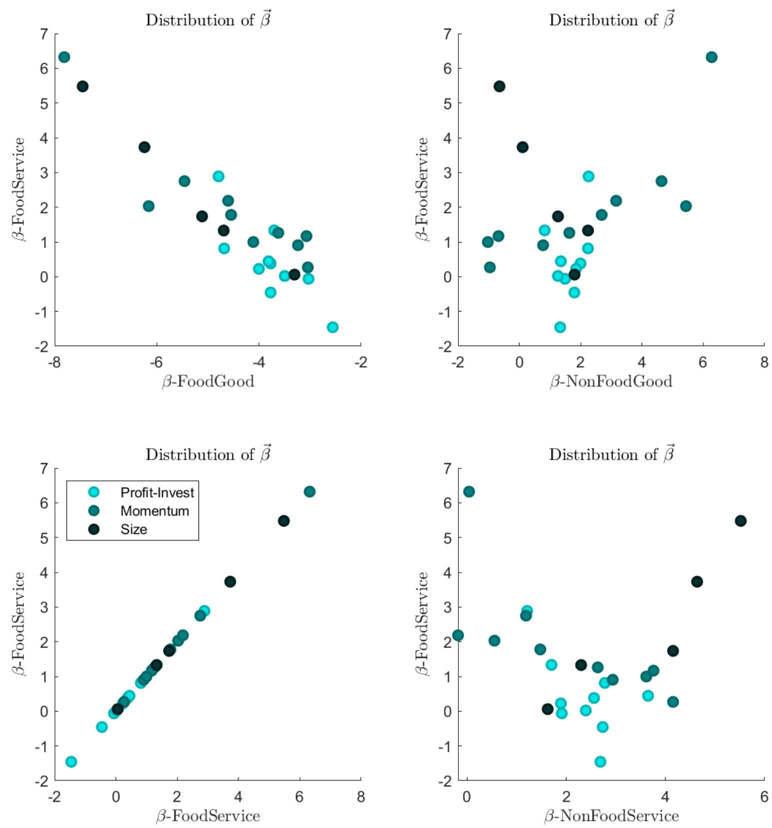

Figure A.1: Factor-Loading of Detailed Consumption Sectors

Description: The X-axis is risk-exposure from the Fama-Macbeth 2-step regression $\vec{\beta}$, for estimation "All" with deflator $\mathrm{d}e$ in Table (8). The Y-axis is the risk-exposure toward the Food-Service sector $\mathrm{d}p_{s,f} - \mathrm{d}e$. The dark-green dots are Size-BM portfolios. The green dots are Profitability-Investment portfolios. The light-green dots are Momentum portfolios.

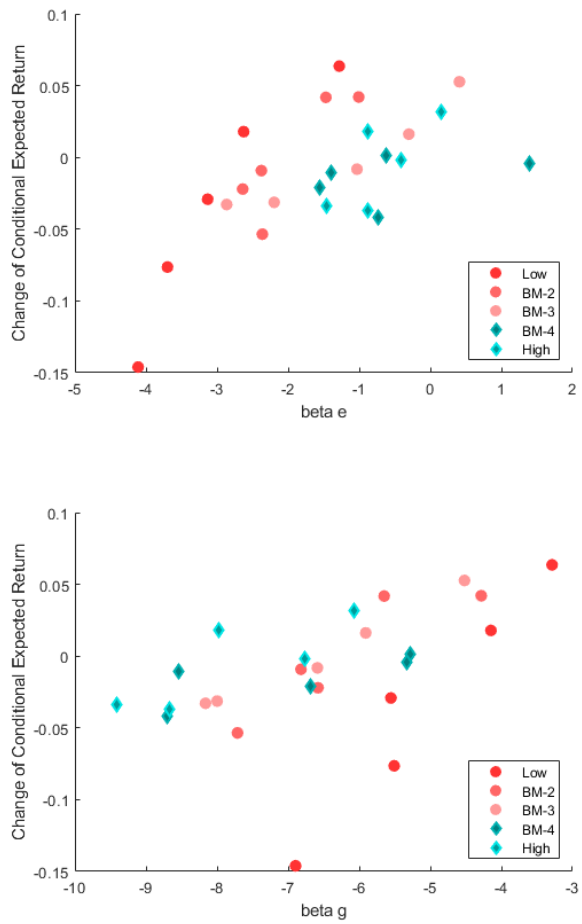

Figure A.2: Conditional Expected Return: Size-BM 25 Portfolios

Description: The X-axis is risk-exposure from the Fama-Macbeth 2-step regression, $\vec{\beta}$. The Y-axis is the change of 15-year average excess return between the sample start and the sample end, $\mathbb{E}_T[R^e_{k,t+1}] - \mathbb{E}_0[R^e_{k,t+1}]$. Portfolios are grouped into 5 groups, based on the Book-to-Market ratio. The red dots are Growth portfolios. The cyan dots are Value portfolios. Other colors are explained in the legend box.

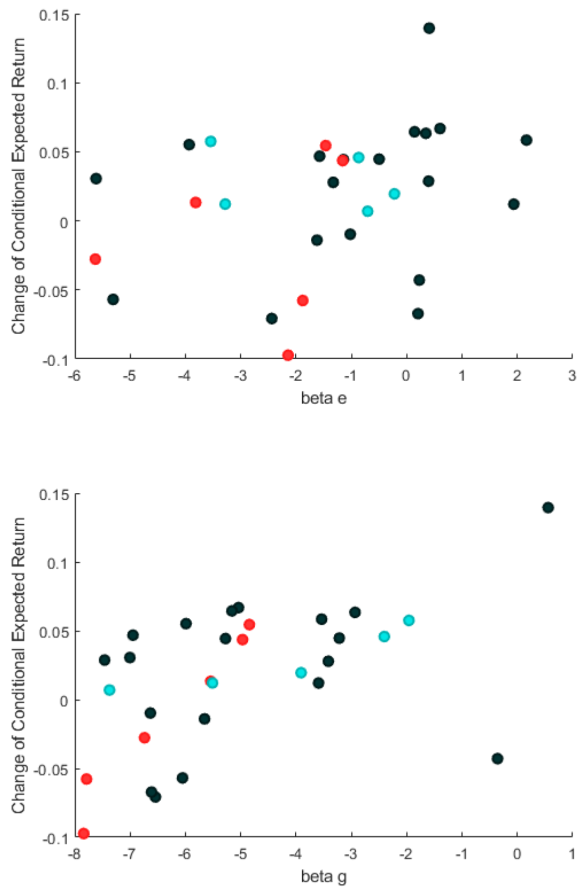

Figure A.3: Conditional Expected Return: Fama-Frech 30 Industry Portfolios

Description: The X-axis is risk-exposure from the Fama-Macbeth 2-step regression, $\vec{\beta}$. The Y-axis is the change of 15-year average excess return between the sample start and the sample end, $\mathbb{E}_T[R^e_{k,t+1}] - \mathbb{E}_0[R^e_{k,t+1}]$. The red dots are Final Consumption sectors: "Food","Beer","Games","Clths","Hlth","Servs","Meals". The light-green dots are Intermediate sectors: "Chems","Txtls","FabPr","Carry","Trans". Other industry sectors are dark-green dots.

G Online Table Appendix

Table A.1: **Variable Definition**

Variable Name	Definition	Time-Span	Data Source
		NIPA	
Non-Durable Expenditure	Current-Price Amount	1965-2019, Annual	Table 2.3.4
Good Price, Service Price	Implied sector-level deflator	same as above	Table 2.3.4, 2.3.5
Food Prices	Line 27-29, Line 83	same as above	Table 2.4.4, 2.4.5
Wage, Labor Hour, Output	Private industries	same as above	Table 6.6, 6.9, 6.1
		Financial Data	
Equity Return	-	1965-2019, Annual, Quarterly	Kenneth French's website
VIX	-	1991-2019, Annual	Compustat CBOE
EP-10Y	1/PE - 10 Year Yield	1965-2019, Annual	Campbell Shiller's website
CaY	-	1965-2017, Annual, Quarterly	Martin Lettau's website
		Synthetic Households	
Expenditure	item "PQ"+"CQ"	1997-2019, Quarterly	CEX FIMD
Consumption Price	Implied sector-level deflator	1997-2019, Quarterly	CEX FIMD, BLS's CPI dataset
		Instrument for Consumption Price	
Innovation of Expectation	4-M & 1-M Forward	1997-2019, Quarterly	Datastream - Future

Table A.2: **Pairwise Correlation**

Description: Time span of sample is during 1965-2019. Correlation coefficient are computed for the relative price x and the relative good price, with respect to the service price. Standard error is in parenthesis.

	Business Cycle - Correlation		
	PCE	Good	Dur Good
Corr($\mathrm{d}p_{x/s}, \mathrm{d}p_{g/s}$)	0.58	0.55	0.14
($s.e$)	(0.10)	(0.11)	(0.08)
	Vehicle	Furniture	Rec Vehicle
Corr($\mathrm{d}p_{x/s}, \mathrm{d}p_{g/s}$)	0.06	0.27	0.12
($s.e$)	(0.18)	(0.08)	(0.12)
	Other DurGood	(NIPA) NDur Good	Food
Corr($\mathrm{d}p_{x/s}, \mathrm{d}p_{g/s}$)	0.30	0.61	0.92
($s.e$)	(0.12)	(0.12)	(0.05)
	Clothes	Gasoline	Other NDurGood
Corr($\mathrm{d}p_{x/s}, \mathrm{d}p_{g/s}$)	0.32	0.15	0.36
($s.e$)	(0.09)	(0.15)	(0.13)
	(NIPA) Serv	House	Util
Corr($\mathrm{d}p_{x/s}, \mathrm{d}p_{g/s}$)	0.10	0.05	-0.04
($s.e$)	(0.06)	(0.07)	(0.11)
	Health	Transport	Recreation
Corr($\mathrm{d}p_{x/s}, \mathrm{d}p_{g/s}$)	-0.13	0.27	0.23
($s.e$)	(0.08)	(0.12)	(0.17)
	FoodAway	Finance	Other Service
Corr($\mathrm{d}p_{x/s}, \mathrm{d}p_{g/s}$)	0.45	-0.23	0.28
($s.e$)	(0.09)	(0.12)	(0.12)

Table A.3: **Dicker-Fuller Test**

Description: Time span of sample is during 1965-2019. Dicker-Fuller test is implemented for the relative price with respect to the service sector price. P-value is reported in brackets.

	Dicker-Fuller Test		
	NDur Good	Wage	Dur Good
$p_{x/s}$	1.00	1.00	0.00
$[p]$	[0.00]	[0.01]	[0.17]

Table A.4: Parameters, 1935-2019

Description: This table reports the estimation during the time-interval 1935-2019. In the 2nd column and the 3rd column, teststing assets **Mix 20** uses the size, BM ratio, and momentum portfoliors. Panel (A-C) reports the estimate of GMM estimation. Other description of statistics in Table (2) and Table 3 applies.

	Specification of Testing Assets					
	Mix 20		Size-BM 25		Industry 30	
	Panel (A): Risk Price					
	1st-Stage	2nd-Stage	1st-Stage	2nd-Stage	1st-Stage	2nd-Stage
b_e	22.67	22.76	31.40	37.30	35.72	32.54
$[t]$	[1.54]	[6.72]	[3.17]	[10.06]	[3.71]	[12.06]
b_g	-89.34	-87.75	-68.76	-69.77	-69.03	-66.67
$[t]$	[-1.96]	[-9.75]	[-3.14]	[-12.89]	[-4.48]	[-19.39]
	Panel (B): Stats of GMM					
MAPE	0.33		0.58		0.92	
RMSE	0.45		0.75		1.25	
CV-R^2	0.98		0.92		0.47	
J-pval		66.95		78.78		84.30
	Panel (C): Test Statistic					
Test-t		[-5.94]		[-5.10]		[-9.76]

Table A.4: Parameters, 1935-2019

Description: This table reports the estimation during the time-interval 1935-2019. Panel (D-E) reports the estimate of Fama-Macbeth two-step regression. In the 2nd column and the 3rd column, teststing assets **Mix 20** uses the size, BM ratio, and momentum portfoliors. Other description of statistics in Table (2) and Table (3) applies.

	Specification of Testing Assets					
	Mix 20		Size-BM 25		Industry 30	
	Panel (D): Risk Premium					
	without	with	without	with	without	with
λ_e	-0.43	0.66	0.63	1.68	-1.13	-0.09
$[t]$	[-0.52]	[1.09]	[1.02]	[2.84]	[-2.57]	[-0.21]
λ_g	-5.33	-3.37	-4.21	-0.71	-4.38	-0.49
$[t]$	[-4.12]	[-2.93]	[-4.04]	[-0.96]	[-4.70]	[-0.83]
α	-	3.31	-	8.36	-	9.37
$[t]$	-	[1.20]	-	[3.91]	-	[4.48]
	Panel (E): Stats of Two-step Regression					
OLS-R^2	0.76		-0.01		-7.61	
GLS-R^2	-0.06		-0.32		-0.48	
COLS-R^2		0.79		0.68		0.09
CGLS-R^2		0.11		0.04		0.07

Table A.5: Parameters, Detrended Share

Description: This table reports the estimation using the detrended expenditure share $\tilde{\omega}_g$. The log expenditure share of good sector, $\log(\omega_g)$, is detrended in annual frequency during the sample period 1965-2019. Cyclical term is recovered as with exponential calculation. Detrended expenditure share $\tilde{\omega}_g$ use the cyclical term multiplied by the in-sample average of expenditure share. Panel (A-C) reports the estimate of GMM estimation. Other description of statistics in Table (2) and Table (3) applies.

	Specification of Testing Assets					
	Mix 30		Size-BM 25		Industry 30	
	Panel (A): Risk Price					
	1st-Stage	2nd-Stage	1st-Stage	2nd-Stage	1st-Stage	2nd-Stage
b_e	27.15	26.96	24.11	25.02	30.05	29.81
$[t]$	[1.51]	[25.18]	[1.72]	[12.50]	[4.14]	[24.53]
b_g	-77.02	-76.45	-77.65	-73.80	-76.74	-74.06
$[t]$	[-2.37]	[-27.79]	[-3.01]	[-18.10]	[-3.36]	[-24.13]
	Panel (B): Stats of GMM					
MAPE	0.37		0.35		0.81	
RMSE	0.45		0.46		1.00	
CV-R^2	0.97		0.96		0.78	
J-pval		92.08		79.50		94.56
	Panel (C): Test Statistic					
Test-t		[-15.64]		[-9.39]		[-11.63]

Table A.6: Parameters, Constant Share

Description: This table reports the estimation using the in-sample average of expenditure share ω_g, during the sample period 1965-2019. Panel (A-C) reports the estimate of GMM estimation. Other description of statistics in Table (2) and Table (3) applies.

	Specification of Testing Assets					
	Mix 30		Size-BM 25		Industry 30	
	Panel (A): Risk Price					
	1st-Stage	2nd-Stage	1st-Stage	2nd-Stage	1st-Stage	2nd-Stage
b_e	27.16	26.97	24.10	25.00	29.98	29.78
$[t]$	[1.51]	[25.09]	[1.71]	[12.49]	[4.11]	[24.30]
b_g	-77.10	-76.59	-77.79	-73.95	-76.91	-74.20
$[t]$	[-2.37]	[-27.34]	[-3.00]	[-17.69]	[-3.35]	[-23.38]
	Panel (B): Stats of GMM					
MAPE	0.37		0.35		0.82	
RMSE	0.45		0.46		1.00	
CV-R^2	0.97		0.96		0.78	
J-pval		92.07		79.55		94.52
	Panel (C): Test Statistic					
Test-t		[-15.50]		[-9.23]		[-11.33]

Table A.7: Parameters, Fama-French 5-factor Model

Description: This table reports the estimation using Fama-French 5-Factor Model "FF-5" during the time-interval 1965-2019. Panel (A-C) reports the estimate of GMM estimation. Other description of statistics in Table (2) and Table (3) applies.

	Specification of Testing Assets					
	Mix 30		Size-BM 25		Industry 30	
	Panel (A): Risk Price					
	1st-Stage	2nd-Stage	1st-Stage	2nd-Stage	1st-Stage	2nd-Stage
b_{MKT}	2.54	2.65	2.51	2.65	2.64	2.78
[t]	[3.53]	[7.00]	[4.41]	[10.07]	[4.02]	[7.94]
b_{Size}	0.41	0.58	1.28	1.21	0.88	0.68
[t]	[0.29]	[0.67]	[1.32]	[2.95]	[0.69]	[1.45]
b_{BM}	-2.13	-2.34	-2.33	-1.87	-5.86	-4.88
[t]	[-0.94]	[-3.14]	[-1.11]	[-3.03]	[-2.13]	[-6.31]
b_{Profit}	0.22	0.65	5.80	6.29	5.18	5.30
[t]	[0.09]	[0.88]	[2.40]	[9.29]	[2.96]	[10.63]
b_{Invest}	7.49	7.61	7.13	7.47	9.36	8.21
[t]	[2.78]	[9.42]	[3.21]	[10.86]	[2.05]	[6.91]
	Panel (B): Stats of GMM					
MAPE	1.68		0.65		1.09	
RMSE	2.68		0.81		1.37	
CV-R^2	-0.19		0.89		0.59	
J-pval		81.78		60.15		84.45

Table A.7: Parameters, Fama-French 5-factor Model

Description: This table reports the estimation using Fama-French 5-Factor Model "FF-5" during the time-interval 1965-2019. Panel (D-E) reports the estimate of Fama-Macbeth two-step regression. Other description of statistics in Table (2) and Table (3) applies.

	Specification of Testing Assets					
	Mix 30		Size-BM 25		Industry 30	
	Panel (D): Risk Premium					
	without	with	without	with	without	with
λ_{MKT}	7.58	-18.13	7.05	-4.52	8.33	1.09
[t]	[2.97]	[-3.18]	[2.73]	[-1.01]	[3.19]	[0.20]
λ_{Size}	1.25	3.31	3.22	2.91	0.31	1.19
[t]	[0.65]	[1.75]	[1.68]	[1.52]	[0.13]	[0.45]
λ_{BM}	1.83	1.25	4.03	4.15	-3.69	-3.42
[t]	[0.90]	[0.61]	[2.06]	[2.12]	[-1.52]	[-1.39]
λ_{Profit}	1.53	0.31	2.34	-0.30	2.06	2.14
[t]	[1.15]	[0.23]	[1.28]	[-0.15]	[0.92]	[0.96]
λ_{Invest}	-4.50	-0.26	3.15	3.75	-0.42	-0.68
[t]	[-2.64]	[-0.18]	[1.93]	[2.28]	[-0.17]	[-0.28]
α	-	24.39	-	11.26	-	6.75
[t]	-	[4.98]	-	[3.30]	-	[1.58]
	Panel (E): Stats of Two-step Regression					
OLS-R^2	-0.38		0.70		0.12	
GLS-R^2	0.01		-0.18		0.09	
COLS-R^2		0.66		0.74		0.43
CGLS-R^2		0.03		0.10		0.13

Table A.8: Subgroup of Testing Assets, Fama-French 5-factor Model

Description: This table reports the point estimate for the vector $\vec{b}$ using different setups of testing assets. "Size-BM" uses the 5 size, 5 BM ratio portfoliors. "Profit-IK" uses the 5 profitability, and 5 investment portfoliors. "Momentum 10" uses the 10 momentum portfoliors. All testing assets are from the Data Library of Kenneth French. Other description in Table (3) applies.

	Specification of Testing Assets					
	Size-BM		Profit-IK		Momentum	
	Panel (A): Risk Price					
	1st-Stage	2nd-Stage	1st-Stage	2nd-Stage	1st-Stage	2nd-Stage
b_{MKT}	2.27	2.40	2.71	2.55	7.79	2.48
$[t]$	[2.76]	[4.48]	[3.24]	[4.44]	[1.26]	[3.00]
b_{Size}	0.24	0.91	1.28	1.16	-7.88	-0.83
$[t]$	[0.19]	[1.29]	[0.58]	[0.74]	[-0.39]	[-0.38]
b_{BM}	2.89	0.64	-7.79	-7.44	-42.66	-7.29
$[t]$	[1.01]	[0.40]	[-1.58]	[-2.07]	[-2.82]	[-2.38]
b_{Profit}	1.71	3.79	5.94	5.59	-0.60	5.17
$[t]$	[0.54]	[1.47]	[4.24]	[4.85]	[-0.02]	[2.25]
b_{Invest}	-2.14	2.64	12.55	11.88	68.70	14.64
$[t]$	[-0.44]	[1.19]	[2.20]	[2.94]	[2.42]	[3.31]
	Panel (B): Stats of GMM					
MAPE	0.35		0.11		2.36	
RMSE	0.40		0.14		2.89	
CV-R^2	0.93		0.99		0.36	
J-pval		14.51		91.86		1.90

Table A.8: Subgroup of Testing Assets, Fama-French 5-factor Model

Description: This table reports the point estimate for the vector $\vec{b}$ using different setups of testing assets. "Size-BM" uses the 5 size, 5 BM ratio portfoliors. "Profit-IK" uses the 5 profitability, and 5 investment portfoliors. "Momentum 10" uses the 10 momentum portfoliors. All testing assets are from the Data Library of Kenneth French. Other description in Table (3) applies.

	Specification of Testing Assets					
	Size-BM		Profit-IK		Momentum	
	Panel (D): Risk Premium					
	without	with	without	with	without	with
λ_{MKT}	7.58	-18.13	7.05	-4.52	8.33	1.09
[t]	[2.97]	[-3.18]	[2.73]	[-1.01]	[3.19]	[0.20]
λ_{Size}	1.25	3.31	3.22	2.91	0.31	1.19
[t]	[0.65]	[1.75]	[1.68]	[1.52]	[0.13]	[0.45]
λ_{BM}	1.83	1.25	4.03	4.15	-3.69	-3.42
[t]	[0.90]	[0.61]	[2.06]	[2.12]	[-1.52]	[-1.39]
λ_{Profit}	1.53	0.31	2.34	-0.30	2.06	2.14
[t]	[1.15]	[0.23]	[1.28]	[-0.15]	[0.92]	[0.96]
λ_{Invest}	-4.50	-0.26	3.15	3.75	-0.42	-0.68
[t]	[-2.64]	[-0.18]	[1.93]	[2.28]	[-0.17]	[-0.28]
α	-	24.39	-	11.26	-	6.75
[t]	-	[4.98]	-	[3.30]	-	[1.58]
	Panel (E): Stats of Two-step Regression					
OLS-R^2	0.91		0.96		0.48	
GLS-R^2	0.47		0.92		0.05	
COLS-R^2		0.91		0.98		0.81
CGLS-R^2		0.51		0.96		0.18

Table A.9: Detailed Consumption Sector: “Industry 30” portfolios

Description: This table reports the point estimate for the vector $\vec{b}$ using multiple consumption sectors . “Good” considers the Food-good category and non-Food good category. “Service” considers the Food-service category and non-Food service category. “All” considers the four categories: Food-good, non-Food good, Food-service and the non-Food service. All estimation use the “Industry 30” portfolios as the testing assets. Other description in Table (8) applies.

	Risk Price							
	Good			Service			All	
$b_{g,f}$	-79.68	-80.43	b_g	-103.82	-103.72	$b_{g,f}$	-119.13	-125.14
$[t]$	[-2.63]	[-2.64]	$[t]$	[-3.70]	[-3.70]	$[t]$	[-2.77]	[-3.13]
$b_{g,n}$	-90.33	-88.11				$b_{g,n}$	-83.63	-65.14
$[t]$	[-1.42]	[-1.36]				$[t]$	[-1.74]	[-1.38]
b_s	14.09		$b_{s,f}$	204.61	204.25	$b_{s,f}$	198.67	188.83
$[t]$	[0.34]		$[t]$	[1.28]	[1.28]	$[t]$	[1.34]	[1.32]
			$b_{s,n}$	-10.72		$b_{s,n}$	-12.47	
			$[t]$	[-0.52]		$[t]$	[-0.34]	
b_e		25.76	b_e		21.31	b_e		23.59
$[t]$		[1.78]	$[t]$		[2.93]	$[t]$		[3.35]
	Deflator							
	de	dp_s		de	$dp_{s,n}$		de	$dp_{s,n}$
	GMM statistic							
MAE	0.82	0.82		0.63	0.64		0.69	0.69
RMSE	1.01	1.02		0.77	0.78		0.83	0.84
Cohen-R^2	0.77	0.77		0.87	0.87		0.85	0.85
J-pval	90.38	90.42		91.87	91.86		88.52	87.65

Table A.10: Detailed Consumption Sector: "MIX 25" portfolios

Description: This table reports the point estimate for the vector $\vec{b}$ using multiple consumption sectors. "Good" considers the Food-good category and non-Food good category. "Service" considers the Food-service category and non-Food service category. "All" considers the four categories: Food-good, non-Food good, Food-service and the non-Food service. Estimation Estimation use the 5-size, 5-profitability, 5-investment and 10-momentum portfoliors. The expenditure in each sector is not detrended. Other description in Table (8) applies.

	Risk Price							
	Good			Service			All	
$b_{g,f}$	-84.13	-74.16	b_g	-74.08	-84.57	$b_{g,f}$	-76.20	-100.17
$[t]$	[-1.56]	[-1.52]	$[t]$	[-3.21]	[-3.40]	$[t]$	[-1.07]	[-2.29]
$b_{g,n}$	-36.13	-79.03				$b_{g,n}$	-27.72	-54.46
$[t]$	[-0.47]	[-1.47]				$[t]$	[-0.33]	[-0.81]
b_s	-22.36		$b_{s,f}$	15.95	183.49	$b_{s,f}$	-84.23	204.28
$[t]$	[-0.56]		$[t]$	[0.07]	[1.25]	$[t]$	[-0.42]	[1.55]
			$b_{s,n}$	-7.27		$b_{s,n}$	-13.97	
			$[t]$	[-0.11]		$[t]$	[-0.23]	
b_e		26.10	b_e		30.23	b_e		33.64
$[t]$		[2.56]	$[t]$		[4.10]	$[t]$		[2.81]
	Deflator							
	de	dp_s		de	$dp_{s,n}$		de	$dp_{s,n}$
	GMM statistic							
MAPE	0.41	0.35		0.42	0.28		0.40	0.29
RMSE	0.51	0.44		0.52	0.38		0.50	0.36
Cohen-R^2	0.96	0.97		0.96	0.98		0.96	0.98
J-pval	71.30	74.68		76.37	73.07		67.67	70.57

Table A.11: Detailed Consumption Sector: "Size-BM 25" portfolios

Description: This table reports the point estimate for the vector $\vec{b}$ using multiple consumption sectors. "Good" considers the Food-good category and non-Food good category. "Service" considers the Food-service category and non-Food service category. "All" considers the four categories: Food-good, non-Food good, Food-service and the non-Food service. All estimation use the "Size-BM 25" portfolios as the testing assets. Other description in Table (8) applies.

	Risk Price							
	Good			Service			All	
$b_{g,f}$	-97.85	-97.74	b_g	-105.11	-104.85	$b_{g,f}$	9.99	-1.99
$[t]$	[-1.54]	[-1.54]	$[t]$	[-4.18]	[-4.17]	$[t]$	[0.07]	[-0.01]
$b_{g,n}$	-39.07	-39.44				$b_{g,n}$	-121.33	-116.76
$[t]$	[-0.40]	[-0.41]				$[t]$	[-0.73]	[-0.76]
b_s	-3.85		$b_{s,f}$	285.61	283.18	$b_{s,f}$	-416.25	-354.44
$[t]$	[-0.08]		$[t]$	[2.01]	[1.96]	$[t]$	[-0.97]	[-0.91]
			$b_{s,n}$	-42.14		$b_{s,n}$	122.69	
			$[t]$	[-1.83]		$[t]$	[1.08]	
b_e		28.52	b_e		27.09	b_e		17.10
$[t]$		[1.55]	$[t]$		[4.34]	$[t]$		[0.98]
	Deflator							
	de	dp_s		de	$dp_{s,n}$		de	$dp_{s,n}$
	GMM statistic							
MAE	0.36	0.36		0.32	0.32		0.65	0.61
RMSE	0.48	0.48		0.41	0.41		0.85	0.79
Cohen-R^2	0.96	0.96		0.97	0.97		0.88	0.90
J-pval	76.15	76.28		79.53	79.51		73.36	74.78

Table A.11: Detailed Consumption Sector: "Size-BM 25" portfolios

Description: This table reports the estimate of risk premium (without intercept term) using multiple consumption sectors. "Good" considers the Food-good category and non-Food good category. "Service" considers the Food-service category and non-Food service category. "All" considers the four categories: Food-good, non-Food good, Food-service and the non-Food service. All estimation use the "Size-BM 25" portfolios as the testing assets. Other description in Table (8) applies.

				Risk Premium				
	Good			Service			All	
$\lambda_{g,f}$	-3.03	-2.20	λ_g	-1.21	-1.76	$\lambda_{g,f}$	-2.26	-2.57
$[t]$	[-3.36]	[-4.16]	$[t]$	[-1.74]	[-4.06]	$[t]$	[-2.63]	[-4.46]
$\lambda_{g,n}$	-1.11	-0.29				$\lambda_{g,n}$	-0.39	-0.70
$[t]$	[-1.46]	[-0.68]				$[t]$	[-0.63]	[-1.34]
λ_s	-0.82		$\lambda_{s,f}$	-1.47	-2.02	$\lambda_{s,f}$	-1.54	-1.85
$[t]$	[-1.16]		$[t]$	[-2.30]	[-4.83]	$[t]$	[-2.45]	[-4.73]
			$\lambda_{s,n}$	0.55		$\lambda_{s,n}$	0.31	
			$[t]$	[0.91]		$[t]$	[0.58]	
λ_e		0.82	λ_e		-0.55	λ_e		-0.31
$[t]$		[1.16]	$[t]$		[-0.91]	$[t]$		[-0.58]
				Deflator				
	de	dp_s		de	$dp_{s,n}$		de	$dp_{s,n}$
				Fama-Macbeth Two-step Statistic				
OLS-R^2	0.68	0.68		0.78	0.78		0.79	0.79
GLS-R^2	-0.22	-0.22		-0.28	-0.28		-0.17	-0.17

Table A.11: Detailed Consumption Sector: “Size-BM 25” portfolios

Description: This table reports the estimate of risk premium (with intercept term) using multiple consumption sectors. “Good” considers the Food-good category and non-Food good category. “Service” considers the Food-service category and non-Food service category. “All” considers the four categories: Food-good, non-Food good, Food-service and the non-Food service. All estimation use the “Size-BM 25” portfolios as the testing assets. Other description in Table (8) applies.

	Risk Premium							
	Good			Service			All	
$\lambda_{g,f}$	-2.83	-2.08	λ_g	-1.17	-1.66	$\lambda_{g,f}$	-2.29	-2.59
$[t]$	[-2.65]	[-2.58]	$[t]$	[-1.51]	[-2.81]	$[t]$	[-2.20]	[-3.07]
$\lambda_{g,n}$	-1.09	-0.33				$\lambda_{g,n}$	-0.39	-0.69
$[t]$	[-1.40]	[-1.11]				$[t]$	[-0.63]	[-1.75]
λ_s	-0.75		$\lambda_{s,f}$	-1.43	-1.91	$\lambda_{s,f}$	-1.56	-1.86
$[t]$	[-1.16]		$[t]$	[-1.96]	[-4.01]	$[t]$	[-2.16]	[-0.58]
			$\lambda_{s,n}$	0.49		$\lambda_{s,n}$	0.30	
			$[t]$	[1.01]		$[t]$	[0.59]	
λ_e		0.75	λ_e		-0.49	λ_e		-0.30
$[t]$		[1.16]	$[t]$		[-1.01]	$[t]$		[-0.59]
α	0.67	0.67	α	0.60	0.60	α	-0.13	-0.13
$[t]$	[0.21]	[0.21]	$[t]$	[0.19]	[0.19]	$[t]$	[-0.04]	[-0.04]
	Deflator							
	de	dp_s		de	$dp_{s,n}$		de	$dp_{s,n}$
	Fama-Macbeth Two-step Statistic							
COLS-R^2	0.68	0.68		0.79	0.79		0.79	0.79
CGLS-R^2	0.07	0.07		0.05	0.05		0.13	0.13

Table A.12: Long-run Shift of Risk Premium: with Detailed Service Sector

Description: This table reports the estimate for the risk premium in different time blocks, for three categories: good, Food-service and the non-Food service. All estimation use the "Momentum-10" and "Industry-30" portfolios as the testing assets. Expenditure share is detrended.

	Risk Premium					
	1950-2019		1950-2005		1965-2019	
	Panel (A): Parameters					
	without	with	without	with	without	with
λ_e	-1.05	0.17	-0.72	-0.09	0.05	0.18
[t]	[-2.14]	[0.42]	[-1.38]	[-0.18]	[0.15]	[0.55]
λ_g	-3.45	-1.24	-3.06	-0.42	-1.88	-0.77
[t]	[-5.30]	[-2.66]	[-3.87]	[-0.79]	[-3.80]	[-1.93]
λ_{fserv}	-1.93	-0.49	-1.54	0.16	-0.91	-0.42
[t]	[-4.61]	[-1.25]	[-3.03]	[0.44]	[-2.88]	[-1.23]
α	-	7.70	-	8.52	-	5.25
[t]	-	[3.85]	-	[4.14]	-	[2.46]
	Panel (C): Stats of Two-step Regression					
OLS-R^2	-1.21		-1.15		-0.28	
GLS-R^2	-0.14		0.07		-0.10	
COLS-R^2		0.35		0.13		0.18
CGLS-R^2		0.11		0.13		0.08

Table A.12: Long-run Shift of Risk Premium: with Detailed Service Sector

Description: This table reports the point estimate for the vector $\vec{b}$ in different time blocks, for three categories: good, Food-service and the non-Food service. All estimation use the "Momentum-10" and "Industry-30" as the testing assets. Expenditure share is detrended.

	Risk Price					
	1950-2019		1950-2005		1965-2019	
	Panel (A): Parameters					
	1st-Stage	2nd-Stage	1st-Stage	2nd-Stage	1st-Stage	2nd-Stage
b_e	19.57	19.87	23.77	23.91	18.33	18.29
$[t]$	[3.15]	[35.96]	[6.93]	[64.45]	[2.68]	[45.73]
b_g	-91.97	-88.80	-64.60	-64.00	-91.69	-89.29
$[t]$	[-2.93]	[-25.45]	[-5.43]	[-29.44]	[-2.54]	[-38.46]
b_{fserv}	204.89	181.78	115.05	108.14	174.12	166.36
$[t]$	[1.23]	[8.29]	[1.24]	[7.06]	[0.98]	[16.74]
	Panel (B): Stats of GMM					
MAPE	0.43		0.36		0.48	
RMSE	0.52		0.45		0.61	
CV-R^2	0.95		0.98		0.95	
J-pval		96.43		99.50		99.45