Estimating and Testing Long-Run Risk Models: International Evidence

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First version: February 2021; This version: June 2021

Abstract

We estimate and test long-run risk models using international macroeconomic and financial data. The benchmark model features a representative agent who has recursive preferences with a time preference shock, a persistent component in expected consumption growth, and stochastic volatility in fundamentals characterized by an autoregressive Gamma process. We construct a comprehensive dataset with quarterly frequency in the post-war period for ten developed countries and employ an efficient likelihood-based Bayesian method that exploits up-to-date sequential Monte Carlo methods to make full econometric inference. Our estimation provides international evidence in support of long-run risks, time-varying preference shocks, and countercyclicality of the stochastic discount factor.

Keywords: Consumption-based Asset Pricing, Long-Run Risks, Stochastic Discount Factor, Equity Premium Puzzle, Autoregressive Gamma Process, Projection Methods, Sequential Monte Carlo

JEL Classification: *C11, C32, C58, E44, G12*

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Really, the most natural thing to do with the consumption-based model is to estimate it and test it, as one would do for any economic model (p.267).

—— *Cochrane, John H. (2008)*

1. Introduction

The "equity premium puzzle", first documented by Mehra and Prescott (1985), states that the standard consumption-based asset pricing models with constant relative risk aversion (CRRA) would require implausibly high risk aversion to explain the historical equity premium in the US market, given low variation observed in the consumption data. Since then, a rapidly growing literature has emerged to explain the equity premium puzzle, along with other notable behaviors of asset returns such as a low and smooth risk-free rate, high equity volatility, and aggregate stock return predictability (e.g., Weil, 1989; Campbell and Cochrane, 1999; Routledge and Zin, 2010; Ju and Miao, 2012; Wachter, 2013). Among those consumption-based asset pricing models, the long-run risk model proposed by Bansal and Yaron (2004) and Bansal, Kiku, and Yaron (2012) has attracted remarkable attention and become a benchmark in the literature.

However, studies on consumption-based asset pricing models and the long-run risk models in particular so far have been confined to explaining the US market data. Analysis based on macroeconomic and financial data in other developed markets is rather limited, even though it would be interesting for reasons as follows. First, as highlighted by Campbell (2003, 2018), the equity premium puzzle is a global phenomenon that also prominently prevails in other developed countries. Second, given that the heart of the long run risk models is a slow-moving latent process driving expected consumption growth, complementing the US-based finding with evidence from other countries is one way to address questions regarding the importance of this process.

Therefore, as one contribution of our paper, we construct a comprehensive data set including quarterly macroeconomic and financial data in the post-war period for a rich set of developed countries and estimate and test long-run risk models using this data. Our sample for estimation consists of quarterly data on aggregate consumption, dividends, risk-free rates, and stock market returns for ten developed countries, including the US, the UK, Germany, France, Italy, Japan, Canada, Australia, the Netherlands, and Switzerland.

Furthermore, the vast majority of studies on consumption-based asset pricing up to now have relied on the calibration approach, i.e., choosing values of primitive parameters in a utility function and in a specification of fundamentals process to match a selected set of moments of fundamentals and asset returns. Studies on structural estimation of asset pricing models remain very limited. The main cause for the sparsity in this research is that efficient econometric estimation of consumption-based asset pricing models is challenging primarily due to that global solutions to these models are highly nonlinear functions of state variables and that data on fundamentals are often observed in very low frequencies and are hard to obtain for countries other than the US. Only a few studies have implemented econometric estimation of consumption-based models using the US data on fundamentals and asset returns; see, for example, Bansal, Gallant, and Tauchen (2007), Bansal, Kiku, and Yaron (2016), Schorfheide, Song, and Yaron (2018), Gallant, Jahan-Parvar, and Liu (2019), and Fulop et al. (2020). Most of these studies either use moment-based or indirect inference methods (e.g., Bansal, Gallant, and Tauchen, 2007; Bansal, Kiku, and Yaron, 2016; Gallant, Jahan-Parvar, and Liu, 2019), which do not fully exploit information in the likelihood function implied by the original asset pricing models, and/or crucially rely on the log-linearization method of Campbell and Shiller (1988) to solve for asset prices (e.g., Bansal, Gallant, and Tauchen, 2007; Bansal, Kiku, and Yaron, 2016; Schorfheide, Song, and Yaron, 2018). A recent paper by Pohl, Schmedders, and Wilms (2018) demonstrates that the log-linearized solutions to long-run risk models can generate significant numerical errors. They show that using projection methods to solve for global solutions to long-run risk models by accounting for higher-order effects can effectively reduce numerical errors. In this paper, we conduct full likelihood-based estimation by exploiting the global non-linear solutions and Bayesian techniques.

We consider a representative agent who has recursive preferences (Epstein and Zin, 1989; Weil, 1989) that allow for the separation between risk aversion and the elasticity of intertemporal substitution (EIS). In our model, expected consumption growth contains a slow-moving persistent component that is subject to stochastic changes, and conditional volatilities of fundamentals are stochastic, capturing time-varying economic uncertainties. Rather than using the autoregressive (AR) process to model conditional variance, as is commonly done in the long-run risk literature, we assume that conditional variances of fundamentals follow autoregressive gamma (ARG) processes to ensure positivity of conditional variances, leading to reliable solutions to the model. Furthermore, we follow Albuquerque et al. (2016) to assume that the agent's rate of time preference is subject to stochastic changes, which plays a crucial role for a consumption-based model in reconciling correlation and covariance between stock returns and fundamentals typically observed in the data. Considering the model with the time preference shock, our estimation naturally takes into account the empirical correlation between stock returns and fundamentals. As a consequence, the parameter estimates and latent states obtained in the estimation are consistent with the estimated law of motion for time preference shocks.

We rely on the collocation projection method to solve for global solutions to our models and make full econometric inference based on an efficient likelihood-based Bayesian method that exploits up-to-date sequential Monte Carlo methods. We extend the sequential Monte Carlo square (SMC^2) method used in Fulop et al. (2020) to estimate our models with more latent states. Different from moment-based methods, our SMC^2 method takes advantage of full information contained in the likelihood function, obtained from running an efficient unscented particle filter (Li, 2011), and provides us with the posterior distribution of model parameters and the smoothing distribution of latent states over time that determine fluctuations of asset prices. Different from traditional Bayesian Markov Chain Monte Carlo (MCMC) methods or particle MCMC methods (Andrieu, Doucet, and Holenstein, 2010), a tailor-made version of which is used in Schorfheide, Song, and Yaron (2018), our SMC^2 method provides us with the marginal likelihood estimates that are necessary statistics for model comparisons and can be easily parallelized, making it computationally convenient to use in estimation.

Our empirical findings can be briefly summarized as follows. First, we find that with regard to fitting asset prices, the time-varying preference shock plays much more important roles than a separate stochastic volatility process capturing time-varying idiosyncratic risks in the dividend dynamics. For almost all the countries, introducing an independent stochastic volatility process in dividend growth cannot improve the model fit on stock market returns and/or risk-free rates. Our preferred model overall is the one that features a time-varying preference shock, a persistent component in expected consumption growth, and a common stochastic volatility process that governs the dynamics of both consumption growth and dividend growth.

Second, our estimation results based on the international analysis clearly indicate values of the EIS greater than 1 (the posterior means are around 2), a presumption that has been emphasized by studies on long-run risks and more broadly, asset pricing studies based on recursive preferences; see, for example, Bansal and Yaron (2004), Bansal et al. (2014), Ai (2010), Drechsler (2013), Ju and Miao (2012), Gourio (2012), Wachter (2013), Croce (2014), and Jahan-Parvar and Liu (2014). For all the countries in our analysis, the posterior estimates of the relative risk aversion range between 5 and 10, which is reasonable and consistent with the prediction of economic theory, but are smaller than values commonly adopted in the calibration studies. We find that introducing time-varying preference shocks in the long-run risk model helps deliver economically plausible estimates of risk aversion and EIS, not only for the US but also for the other developed economies. Our estimates of the RRA and EIS over different countries provide international support to investors' preference for early resolution of uncertainty.

Third, we find that for all the countries, expected consumption growth consists of a persistent component, albeit the importance of this long-run risk component varies across countries. In the US, the long-run risk component accounts for a significant amount of time variation in consumption growth, while in other countries the long-run risk component has less importance in this aspect. Moreover, there is notable heterogeneity across countries in the level of persistence in stochastic volatility of consumption.

Fourth, for most of the countries in our sample, the stochastic discount factor under recursive utility has a countercyclical component. In addition, conditional equity premium and conditional volatility of stock returns also exhibit countercyclical variation to a certain extent. With regard to fitting time series of asset returns, for all these ten countries, our estimation generates fitted risk-free rates that closely track the historical movements of the actual risk-free rates; in contrast, fitted stock market returns remains less accurate.

Our paper is closely related to two recent papers, Schorfheide, Song, and Yaron (2018) and Fulop et al. (2020), both of which employ likelihood-based Bayesian approaches to estimate their respective versions of the long-run risk model. However, our paper differs from these two studies in important aspects. First, both studies exclusively focus on the US market, whereas ours implements empirical investigations for ten developed countries and provides international evidence in support of long-run risks. Second, Schorfheide, Song, and Yaron (2018) introduce separate volatility processes, respectively, for consumption growth, the long-run risk component, and dividend growth using log-normal processes. They rely on linearization of the log-volatility processes and the log-linearization method to find linear functions for equilibrium asset prices. Furthermore, it is worth mentioning that there exist nontrivial differences between the approximate model characterizing asset prices and the original long-run risk model. Third, Fulop et al. (2020) consider long-run risk models in which the consumption volatility process is modeled using either an ARG process or an AR process; however, the preference shock is absent in their models. Fourth, both studies have not evaluated the relative importance of different state variables that determine asset prices through model comparisons. In addition, the measurement error variance of risk-free rates is fixed in Schorfheide, Song, and Yaron (2018), and as such, the economic implications derived from their estimation results are hard to interpret.

The rest of the paper is organized as follows. Section 2 presents long-run risk models considered in the paper. Section 3 briefly describes the solution method and our econometric inference based on sequential Monte Carlo methods. Section 4 discusses the international macroeconomic and financial data used for model estimation. Section 5 presents estimation results and discusses asset pricing implications. Section 6 concludes the paper. Additional results are given in the Internet Appendix.

2. Model Framework

2.1. Preferences

We examine an endowment economy, in which a representative agent has recursive preferences as in Epstein and Zin (1989) and Weil (1989). Moreover, following Albuquerque et al. (2016) and Schorfheide, Song, and Yaron (2018), we introduce time preference shocks in the utility function. As shown in Albuquerque et al. (2016), a major role of time preference shocks is to mitigate the strong correlation between stock returns and measurable fundamentals that is typically obtained in consumption-based models without demand shocks. As a result, the agent's recursive utility function is given by

$$V_t = \left[(1-\delta)\lambda_t C_t^{\frac{1-\gamma}{\theta}} + \delta\left[\mathbb{E}_t(V_{t+1}^{1-\gamma})\right]^{\frac{1}{\theta}}\right]^{\frac{\theta}{1-\gamma}}, \tag{1}$$

where C_t is the time-t consumption, $0 < \delta < 1$ is the agent's time preference parameter, λ_t is the shock to the time rate of preference, γ is the relative risk aversion parameter, ψ is the elasticity of intertemporal substitution (EIS), $\theta = \frac{1-\gamma}{1-1/\psi}$, and $\mathbb{E}_t$ denotes conditional expectation with respect to information up to time t.

This class of preferences allows for a separation between risk aversion and the EIS. The agent prefers early (late) resolution of uncertainty when $\gamma > 1/\psi$ ($\gamma < 1/\psi$), and when $\gamma = 1/\psi$, the agent has constant relative risk aversion preferences and is neutral to the timing of resolution of uncertainty. The agent's utility maximization is subject to the following budget constraint,

$$W_{t+1} = (W_t - C_t)R_{t+1}^W, \tag{2}$$

where W_t is the agent's wealth, and R_t^W is the return on the wealth portfolio.

For any asset i with ex-dividend price $P_{i,t}$ and dividend $D_{i,t}$, the standard Euler equation holds, i.e.,

$$\mathbb{E}_t \left[M_{t+1} R_{i,t+1} \right] = 1, \tag{3}$$

where $R_{i,t+1} = (P_{i,t+1} + D_{i,t+1})/P_{i,t}$, and M_t is the stochastic discount factor (SDF). In particular, for the risk-free asset, we have $R_{f,t} = 1/\mathbb{E}_t[M_{t+1}]$. It can be shown that the stochastic discount factor for the recursive utility function defined in Equation (1) takes the form of

$$M_{t+1} = \delta \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \left(\frac{V_{t+1}}{\left[\mathbb{E}_t \left(V_{t+1}^{1-\gamma} \right) \right]^{\frac{1}{1-\gamma}}} \right)^{\frac{1}{\psi} - \gamma}, \tag{4}$$

which can be alternatively expressed as

$$M_{t+1} = \delta^{\theta} \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{\theta}{\psi}} \left(R_{t+1}^W \right)^{\theta - 1}. \tag{5}$$

Thus, the Euler equation (3) implies that for the return on the wealth portfolio, R_t^W, we have

$$\mathbb{E}_t \left[\delta^{\theta} \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{\theta}{\psi}} \left(R_{t+1}^W \right)^{\theta} \right] = 1, \tag{6}$$

and for the return on the market portfolio, $R_{m,t}$, we have

$$\mathbb{E}_t \left[\delta^{\theta} \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{\theta}{\psi}} \left(R_{t+1}^W \right)^{\theta - 1} R_{m,t+1} \right] = 1. \tag{7}$$

As in Albuquerque et al. (2016), we assume that the growth rate of preference shocks, defined as $x_{\lambda,t+1} \equiv \ln \left(\lambda_{t+1}/\lambda_t \right)$, follows an AR(1) process,

$$x_{\lambda,t+1} = \rho_{\lambda} x_{\lambda,t} + \sigma_{\lambda} \eta_{\lambda,t+1}, \tag{8}$$

where the shocks to $x_{\lambda,t}$ follow the standard normal distribution, i.e., $\eta_{\lambda,t} \sim N(0,1)$, and are independent of the shocks to consumption growth and dividend growth defined in the next subsection.

2.2. Fundamentals

We follow Bansal and Yaron (2004) and Bansal, Kiku, and Yaron (2012) and assume that the log-consumption growth, $\Delta c_{t+1} \equiv \ln(C_{t+1}/C_t)$, consists of a persistent component, x_t, and a transitory component,

$$\Delta c_{t+1} = \mu + x_t + \sigma_{c,t}\eta_{c,t+1}, \tag{9}$$

$$x_{t+1} = \rho x_t + \phi_x \sigma_{c,t}\eta_{x,t+1}, \tag{10}$$

and that dividends are imperfectly correlated with consumption and their log-growth rate, $\Delta d_{t+1} \equiv \ln\left(\frac{D_{t+1}}{D_t}\right)$, has the dynamics of

$$\Delta d_{t+1} = \mu_d + \Phi x_t + \phi_{dc}\sigma_{c,t}\eta_{c,t+1} + \phi_d \sigma_{d,t}\eta_{d,t+1}, \tag{11}$$

where $\eta_{c,t}$, $\eta_{x,t}$, and $\eta_{d,t}$ are i.i.d normal $N(0,1)$, and $\sigma_{c,t}^2$ and $\sigma_{d,t}^2$ are conditional variances of consumption growth and dividend growth, respectively.

In the standard long-run risk model (Bansal and Yaron, 2004; Bansal, Kiku, and Yaron, 2012), consumption variance is assumed to follow an AR(1) process. However, this modeling choice suggests that consumption variance can take negative values, which renders the numerical solution to the model problematic. To overcome this issue, Fulop et al. (2020) introduce an autoregressive gamma (ARG) process, proposed by Gourieroux and Jasiak (2006), to model consumption variance and show that the ARG-based long-run risk model performs better than the AR-based one in fitting US market data. To this end, we follow Fulop et al. (2020) to model the conditional variances, $\sigma_{i,t}^2$ for $i = \{c, d\}$, using ARG processes with order 1,

$$\sigma_{i,t}^2 \sim Gamma(\phi_{is} + \zeta_{i,t}, c_i), \qquad \zeta_{i,t} \sim Poisson\left(\frac{\rho_{is}\sigma_{i,t-1}^2}{c_i}\right), \tag{12}$$

where $Gamma(\cdot)$ and $Poisson(\cdot)$ denote the gamma distribution and the Poisson distribution, respectively, ρ_{is} controls the persistence of each variance process, c_i determines the scale, and to ensure positivity of conditional variances, the Feller condition, $\phi_{is} > 1$, needs to be satisfied. As shown in Gourieroux and Jasiak (2006) and Creal (2017), the transition density of $\sigma^2_{i,t}$ is a noncentral gamma distribution,[1] and its conditional mean and variance are given by $E[\sigma^2_{i,t}|\sigma^2_{i,t-1}] = \bar{\sigma}^2_i(1-\rho_{is}) + \rho_{is}\sigma^2_{i,t-1}$ and $Var[\sigma^2_{i,t}|\sigma^2_{i,t-1}] = \frac{(1-\rho_{is})\bar{\sigma}^2_i}{\phi_{is}}\left((1-\rho_{is})\bar{\sigma}^2_i + 2\rho_{is}\sigma^2_{i,t-1}\right)$, respectively. The stationary distribution of the ARG process is $Gamma(\phi_{is}, c_i/(1-\rho_{is}))$ with the long-run mean given by $\bar{\sigma}^2_i = \phi_{is}c_i/(1-\rho_{is})$. We label this long-run risk model as "LRR2SVPref".

When we assume that the same conditional variance enters into dynamics of both consumption growth and dividend growth, i.e., $\sigma_{c,t} = \sigma_{d,t} = \sigma_t$, we have a nested long-run risk model with time preference shocks, which we label as "LRR1SVPref". When we further shut down time preference shocks, we obtain the counterpart of the standard long-run risk model, which is also studied by Fulop et al. (2020), and we label this nested model as "LRR1SV".

3. Solution and Econometric Inference

3.1. Model Solution

The usually used method to solve the long-run risk models is the log-linear approximation method of Campbell and Shiller (1988); see Bansal and Yaron (2004), Bansal, Kiku, and Yaron (2012), Bansal, Kiku, and Yaron (2016), Beeler and Campbell (2012), Schorfheide, Song, and Yaron (2018), among others. In a recent paper, Pohl, Schmedders, and Wilms (2018) show that solving long-run risk models with log-linearization could yield significant numerical errors when state variables are persistent, and they advocate using projection methods that can account for higher-order effects. The higher-order effects are important for producing reliable asset pricing results. Therefore, in this paper, we solve our models

[1]The density function has the form of

$$f(\sigma^2_{i,t}|\sigma^2_{i,t-1}) = \left(\frac{\sigma^2_{i,t}}{\rho_{is}\sigma^2_{i,t-1}}\right)^{(\phi_{is}-1)/2}\frac{1}{c_i}\exp\left(-\frac{(\sigma^2_{it}+\rho_{is}\sigma^2_{i,t-1})}{c_i}\right)I_{\phi_{is}-1}\left(\frac{2\sqrt{\rho_{is}\sigma^2_{i,t-1}\sigma^2_{i,t}}}{c_i}\right),$$

where $I_\zeta(x) = (x/2)^\zeta\sum_{i=0}^\infty(x^2/4)^i/\{i!\Gamma(\zeta+i+1)\}$ denotes a modified Bessel function of the first kind.

using the collocation projection method (Judd, 1992, 1999).

To illustrate the collocation projection method, we denote the current state of the economy by z and the state of the next period by z'; for example, in the full model of LRR2SVPref, the state vector is $z = \{x_\lambda, x, \sigma_c^2, \sigma_d^2\}$. We solve the models in the following two steps.

First, we solve the Euler equation for the wealth portfolio and obtain the wealth-consumption ratio. In the projection method, the solution function to the log wealth-consumption ratio, $\varphi_w(z) \equiv \ln\left(\frac{W(z)}{C(z)}\right)$, is approximated by Chebyshev polynomials and a set of associated unknown coefficients. In particular, the approximation is given by $\hat{\varphi}_w(z) = \sum_{k=0}^{n} \alpha_{w,k}\Lambda_k(z)$, where $\Lambda_k(z)$, $k = 0, \ldots, n$, is a set of basis functions, and $\alpha_{w,k}$, $k = 0, \ldots, n$, is a set of unknown coefficients to be determined. The basis functions are constructed as products of Chebyshev polynomials for the relevant state variables. For the Euler equation, the solution function satisfies

$$\mathbb{E}\left[\exp\left(x_\lambda^{'} + \theta\left(\ln\delta + \left(1 - \frac{1}{\psi}\right)\Delta c(z'|z) + \varphi_w(z') - \ln\left(e^{\varphi_w(z)} - 1\right)\right)\right)\middle| z\right] = 1, \quad (13)$$

where the log-return on the wealth portfolio is given by

$$r_w(z'|z) \equiv \ln\left(\frac{W(z')}{W(z) - C(z)}\right) = \varphi_w(z') - \ln(e^{\varphi_w(z)} - 1) + \Delta c(z'|z). \quad (14)$$

Second, we approximate the solution function to the log price-dividend ratio by $\hat{\varphi}(z) = \sum_{k=0}^{n} \alpha_k\Lambda_k(z)$, where α_k, $k = 0, \ldots, n$, is a set of unknown coefficients to be determined. Equations (3) and (5) imply that the log price-dividend ratio, $\varphi(z) \equiv \ln\left(\frac{P(z)}{D(z)}\right)$, satisfies

$$\mathbb{E}\left[\exp\left(x_\lambda^{'} + \theta\ln\delta - \frac{\theta}{\psi}\Delta c\left(z'|z\right) + (\theta - 1)\, r_w(z'|z) + r(z'|z)\right)\middle| z\right] = 1, \quad (15)$$

where $r\left(z'|z\right)$ is the log-return on an asset with the dividend growth rate of $\Delta d(z'|z)$,

$$r(z'|z) = \ln\left(e^{\varphi(z')} + 1\right) - \varphi(z) + \Delta d(z'|z). \quad (16)$$

We apply the collocation projection method and approximate the solution functions $\varphi_w(z)$ and $\varphi(z)$ using Chebyshev polynomials. For the Gaussian innovation shocks, we can use the Gauss-Hermite quadrature to compute conditional expectations; for the ARG specification, we use the importance sampling method to compute conditional expectations. The collocation projection method leads to a square system of nonlinear equations, which can be solved using the standard nonlinear equation solvers to obtain the estimates of unknown coefficients $\alpha_{w,k}$ and α_k.[2]

3.2. Estimation

Our models can be cast into the framework of nonlinear and non-Gaussian state-space models. There are four state variables in the full model: the growth rate of preference shocks, $x_{\lambda,t}$, whose dynamics are given in Equation (8), the long-run risk component, x_t, whose dynamics are given in Equation (10), and the consumption and dividend variance processes, $\sigma^2_{i,t}$ for $i = \{c, d\}$, whose dynamics are given in Equation (12).

Moreover, there are four observables including the consumption growth rates (Δc_t), the dividend growth rates (Δd_t), the stock market returns ($r_{m,t}$), and the risk-free returns ($r_{f,t}$). The dynamics of consumption and dividend growth rates are given in Equations (9) and (11), respectively. Assuming that the stock market and risk-free returns are collected with measurement errors, their dynamics are given by

$$r_{m,t} = f(z_t, z_{t-1}, \Delta d_t, \Theta) + \sigma_m \eta_{m,t}, \tag{17}$$

$$r_{f,t} = g(\tilde{z}_t, \Theta) + \sigma_f \eta_{f,t}, \tag{18}$$

respectively, where $z_t = \{x_{\lambda,t}, x_t, \sigma^2_{c,t}, \sigma^2_{d,t}\}$, $\tilde{z}_t = \{x_{\lambda,t}, x_t, \sigma^2_{c,t}\}$, Θ denotes the set of model parameters, $r_{m,t}$ and $r_{f,t}$ are the observed market and risk-free returns, σ_m and σ_f are the standard deviations of the respective measurement errors that are assumed to follow

[2] Borovicka and Stachurski (2020) show exact necessary and sufficient conditions for existence and uniqueness of solutions to a class of models with recursive utility. In our estimation, arbitrary parameter values may be generated and do not necessarily satisfy these conditions. We impose these conditions in the estimation as additional restrictions on parameters when solving and simulating our models.

independent standard normal distributions, and $f(\cdot)$ and $g(\cdot)$ are two nonlinear functions resulted from the projection method determining the model-implied market and risk-free returns.

For T time periods, we denote all observations as $y_{1:T} = \{\Delta c_t, \Delta d_t, r_{m,t}, r_{f,t}\}_{t=1}^{T}$ and the latent states as $z_{1:T} = \{x_{\lambda,t}, x_t, \sigma^2_{c,t}, \sigma^2_{d,t}\}_{t=1}^{T}$. Our aim is to estimate the joint posterior distribution of parameters and latent states, $p(\Theta, z_{1:T}|y_{1:T})$, which can be decomposed into

$$p(\Theta, z_{1:T}|y_{1:T}) = p(z_{1:T}|\Theta, y_{1:T})p(\Theta|y_{1:T}), \tag{19}$$

where $p(z_{1:T}|\Theta, y_{1:T})$ solves state smoothing, and $p(\Theta|y_{1:T})$ addresses parameter inference.

We extend the SMC^2 method used in Fulop et al. (2020) to estimate our models. This method is based on the ideas of particle Markov chain Monte Carlo methods (PMCMC) (Andrieu, Doucet, and Holenstein, 2010) and sequential Monte Carlo samplers (Del Moral, Doucet, and Jasra, 2006). The former shows that MCMC samplers converge to the real posterior distribution of parameters even when the likelihood approximated by particle filters is used, and the latter suggests that a bridge can be built between the prior and posterior distributions of parameters by using some MCMC kernels of invariant distribution of parameters. The SMC^2 method delivers exact draws for the joint posterior distribution of parameters and latent states for any given number of the state particles.

Different from moment-based methods, our econometric method exploits full information contained in the likelihood function of the models in estimation, obtained from an efficient unscented particle filter (Li, 2011), and provides us with the posterior distribution of model parameters and the smoothing distribution of latent states over time that determine fluctuations of asset prices. Different from traditional Bayesian MCMC methods or PMCMC methods (Andrieu, Doucet, and Holenstein, 2010),[3] the SMC^2 method can directly deliver the marginal likelihood estimates that are necessary statistics for model comparisons and can be easily parallelized, making it computationally convenient to use in estimation. For more details on the SMC^2 method, we refer readers to Chopin,

[3] A tailor-made version of PMCMC is used in Schorfheide, Song, and Yaron (2018).

Jacob, and Papaspiliopoulos (2013), Fulop and Li (2013, 2019), and Fulop and Duan (2015).

4. Data

Our dataset can be viewed as an updated and extended version of the international dataset used in Campbell (1999, 2003, 2018). Specifically, we construct quarterly data on real aggregate consumption, dividends, risk-free rates, and stock market returns for each of the following ten countries: Australia (AU), Canada (CA), France (FR), Germany (DE), Italy (IT), Japan (JP), the Netherlands (NL), Switzerland (CH), the UK (UK), and USA (US). For the US and UK, the sample starts from 1947:Q2 and 1965:Q4, respectively; for all the other countries, the sample starts from 1973:Q4; and for all the countries in our analysis, our sample ends in 2019:Q3.

4.1. Macroeconomic Data

Macroeconomic data on real seasonally-adjusted aggregate consumption, population, and consumer price index (CPI) are downloaded from *Datastream*. Following Campbell (1999, 2003), for the US, we sum the seasonally adjusted real consumption (per capita) of non-durables and services obtained from the Bureau of Economic Analysis, and for the other countries, we use private final consumption expenditures to measure aggregate consumption. In specific, we take real seasonally-adjusted private final consumption expenditures from the Quarterly National Accounts of Organization for Economic Cooperation and Development (OECD) database, which are then divided by the annual population obtained from International Financial Statistics (IFS, line 99) of International Monetary Fund (IMF) to yield real seasonally-adjusted consumption per capita.[4]

The source of CPI for the US is the Treasury and Inflation database of Wharton Research Data Services (WRDS), and the source of CPI for the other countries is IFS

[4] As consumption data are time-averaged, and the level of consumption is not a point-in-time observation but a flow during a quarter, we face a timing convention problem when computing consumption growth. As such, we follow Campbell (2003) and use the 'beginning-of-quarter' timing convention to calculate the growth rate of consumption per capita.

(line 64). We construct quarterly CPI from monthly data by selecting the value of the last month in each quarter for all the countries except for Australia, as the IFS line 64 for Australia is already available at quarterly frequency. We take the first difference of log CPI to construct inflation rates.

4.2. Interest Rate Data

The short-term interest rates are downloaded from *Datastream*. Specifically, we download and construct the following nominal interest rates for each of those countries,

- Australia and Canada: 3-month or 90-day interbank rates from OECD main economic indicators;
- France: average monthly money market rates from Banque de France;
- Germany: 3-month (monthly average) Frankfurt interbank offered rates from European Banking Federation/the Financial Markets Association;
- Italy: 3-month (monthly average) interbank deposit rates from Bank of Italy;
- Japan: overnight uncollaterised call money rate (average) from Bank of Japan;
- Netherlands: average money market rates paid on bankers' call loans from IFS; missing values are replaced by the observations from call money rate from De Nederlandsche Bank (DNB);
- Switzerland: overnight Swiss franc deposit rates in international money markets from IFS; missing values are replaced by the observations from call money/interbank rate from OECD main economic indicators;
- UK: rates at which 91-day bills are allotted; weighted averages of Friday data from Bank of England;
- US: 90-day US treasury bill rates from the Treasury and Inflation database of WRDS.

To construct the real risk-free rates, we first construct the *ex post* real risk-free rates by deflating nominal interest rates using inflation rates and then regress the *ex post* real risk-free rates on one-year lagged nominal rates and one-year lagged inflation rates. The predicted values from this regression yield the *ex ante* risk-free rates, which are used in our estimation.

4.3. Stock Market Data

For the US, the stock market data are obtained from the Center for Research in Security Prices (CRSP). The market returns are the value-weighted returns on the stock portfolio of NYSE, AMEX and NASDAQ, and the dividend growth rates are constructed as the difference between the value-weighted returns including and excluding dividends. For the remaining countries, following Rangvid, Schmeling, and Schrimpf (2014), we rely on stock market data from Datastream and obtain nominal dividends by multiplying the market price index by the market dividend yield.

For all countries, as in Bansal and Yaron (2004) and Schorfheide, Song, and Yaron (2018), we smooth nominal dividends by aggregating their values of the most recent four quarters (including the current quarter). Real stock returns (real dividend growth rates) are calculated by deflating nominal stock returns (nominal dividend growth rates) using quarterly inflation rates.

4.4. Summary Statistics

Table 1 presents the summary statistics of the data used for model estimation. The annualized average real market return ranges from 3.30% (JP) to 7.32% (NL), and its annualized volatility ranges from 15.9% (CA) to 25.3% (IT). In contrast, the mean and volatility of real risk-free rates are much smaller: the annualized average rate ranges from almost 0 (CH) to 2.55% (AU) and the annualized volatility ranges from 0.90% (US and CH) to 1.44% (UK).

In general, the real dividend growth rates are larger and more volatile than the real consumption growth rates. The annualized average real consumption growth rate goes from 0.74% (CH) to 2.22% (UK) and its annualized standard deviation goes from 0.99% (US) to 2.32% (JP). The annualized average real dividend growth rate varies from 1.62% (UK) to 4.77% (CH) and its annualized standard deviation varies from 3.80% (UK) to 9.98% (IT).

The consumption growth rates across the countries have moderately positive correlations ranging from 0 (between JP and CA) to 0.48 (between AU and CH). The US consumption growth is positively correlated with consumption growth in the remaining countries, with the correlation being as high as 0.35 with the UK and France. Relative to consumption growth, the dividend growth rates across the countries show higher correlations. In particular, the dividend growth rates among the European countries have notable comovements, with correlation ranging from 0.3 to 0.5. Both the risk-free rates and stock market returns across the countries are strongly positively correlated.

5. Empirical Results and Implications

5.1. Estimation Results

We estimate three model specifications for each of the ten countries and compare performance across different models for each country. The first model (LRR1SV) considered in our estimation is the long-run risk model with one stochastic volatility process and without preference shocks. The second model (LRR1SVPref) differs from the first by taking preference shocks into account. The third model (LRR2SVPref) further allows for a separate volatility process in the dividend growth rates. Our estimation method needs to be initialized by the prior distributions of model parameters. Our choice of those prior distributions is consistent with the literature; see, e.g., Schorfheide, Song, and Yaron (2018) and Fulop et al. (2020). The exact functional forms and hyperparameters of the prior distributions are presented in the Internet Appendix.

5.1.1. Model Performance

Table 2 displays several measures that are used to assess model performance: a statistical measure, i.e., the log marginal likelihood (ML) that measure the overall goodness-of-fit of the model, and two economic measures, i.e., the standard deviations of the measurement errors in stock market returns and risk-free rates (σ_m and σ_f, respectively) that measure how far the model-implied asset returns are from the observed ones. According to the marginal likelihood estimates and the estimated standard deviations of the measurement errors, we find that the LRR1SVPref model outperforms the LRR1SV model. The estimated σ_m and σ_f in LRR1SVPref are smaller than those obtained in LRR1SV for almost all the economies under consideration. The only exceptions include the Netherlands (N-L), for which σ_f is slightly higher under LRR1SVPref, and Switzerland (CH), for which σ_m is marginally higher under LRR1SVPref. Furthermore, the marginal likelihood estimates are unanimously much higher in LRR1SVPref than those in LRR1SV for all the economies.

Turning to the comparison of performance between LRR1SVPref and LRR2SVPref, we could not find affirmative evidence of improvement of LRR2SVPref over LRR1SVPref. While the log marginal likelihood estimates improve in general when a separate dividend volatility process is introduced, there is little gain in fitting asset returns for all the economies. These results suggest that the preference shock is a very important element that leads to better performance in fitting the data, whereas allowing for independent idiosyncratic risks in dividend growth does not seem to improve the overall economic performance of the model. Thus, in what follows, we focus on estimation results and discuss asset pricing implications based on the parsimonious model of LRR1SVPref.[5]

5.1.2. Parameter Estimates

Table 3 presents posterior estimates of primitive parameters in the recursive utility function and in the dynamics of consumption and dividend growth for all the countries considered in the study resulted from the model of LRR1SVPref. The posterior mean estimates of the subjective discount factor δ are similar across most of the countries and are well above 0.99 except Germany. The standard deviation and the (5, 95)% percentiles indicate that the estimates are bounded within small intervals and consistent with low real risk-free rates observed in most of those countries.

The posterior estimates of investors' relative risk aversion (γ) for the US are largely

[5] The estimation results for the alternative models LRR1SV and LRR2SVPref, including the posterior estimates of model parameters and the smoothed latent states, are presented in the Internet Appendix.

in line with the long-run risk literature. The posterior mean of γ is around 9.8, and the (5, 95)% credible interval is (7.5, 12). These estimates are similar to those reported in Schorfheide, Song, and Yaron (2018), Gallant, Jahan-Parvar, and Liu (2019), and Fulop et al. (2020). However, for the other countries, the posterior estimates of γ are relatively small: the posterior mean ranges from 5.7 (CH) to 7.5 (IT), and their (5, 95)% percentiles are well within the interval (3,10). The values of γ lower than the upper bound of 10 are commonly considered being economically reasonable.

The long-run risk literature advocates values of the EIS parameter (ψ) greater than 1. Estimation studies such as Schorfheide, Song, and Yaron (2018), Gallant, Jahan-Parvar, and Liu (2019), and Fulop et al. (2020) find empirical support for $\psi > 1$ based on the US data. Our estimation with international data further provides support for typical values of ψ used in the calibration studies on long-run risks. Table 3 reveals that the posterior mean estimate of ψ is around 2, ranging from 1.69 (UK) to 2.32 (CH) across those ten countries in our study and slightly larger than the typical value of 1.5 used in the calibration studies. The 5% percentile estimate of ψ is consistently above 1 in all these economies. Together with estimates of relative risk aversion γ, these results suggest that investors in the developed economies have a strong preference for early resolution of uncertainty ($\psi \gg 1/\gamma$). In addition, since our estimation uses both market and consumption data jointly, the estimates of ψ are obtained to be naturally consistent with the empirical fact that the risk-free rate is not very responsive to expected consumption growth and consumption volatility.

However, when we shut down time-varying preference shocks, we obtain very different estimates of the risk-aversion and EIS parameters.[6] The posterior mean of γ varies much larger across countries, ranging from 1.3 (DE) to 8.2 (CA) and its 5% quantile is below 1 in DE, FR, and IT. Furthermore, it is also worth mentioning that the incorporation of the preference shock in the model is important for identifying the EIS parameter when long-run consumption risk is present. Absent from the preference shock, the EIS estimates

[6]For brevity, these results are not reported here. See the Internet Appendix for parameter estimates of the model of LRR1SV.

vary dramatically and become much smaller in all economies. Its posterior mean (5% quantile) is below 1 in 6 (8) out of 10 countries. These results suggest that introducing time-varying preference shocks in the long-run risk models helps deliver economically plausible estimates of risk aversion and EIS, not only for the US but also for the other developed economies.

The estimated specification of the growth rate of the preference shock exhibits high persistence for all the countries. The posterior means of the persistence parameter ρ_λ are all above 0.9 except Switzerland (0.81). Among those countries, the preference shock of the US economy has the highest level of persistence (0.99). The posterior mean, standard deviation, and (5, 95)% percentiles altogether indicate that the specification of the time preference shock is well identified from international data. These results reflect that all else being equal, investors across different economies share a common pattern in valuing future utility. The estimates of the volatility parameter ϕ_λ are small, ranging from 0.13% (US) to 0.29% (NL), and are very similar in most of those countries. The magnitude of variation in the growth rate of the preference shock implied from our estimates is in line with that obtained by Albuquerque et al. (2016).

More strikingly, our estimation based on international data provides empirical support for the presence of a persistent component in consumption growth across different countries. The posterior mean estimates of the persistence parameter (ρ_x) for the long-run risk component are above 0.9 at the quarterly frequency for the US, the UK, the Netherlands, and Switzerland. As for the US, we see that the (5, 95)% credible interval of ρ_x is (0.78, 0.97), which is very similar to that obtained by Schorfheide, Song, and Yaron (2018). The persistence of the long-run risk component is moderately lower in other countries such as Germany, France, Italy, Japan, Canada, and Australia. Nevertheless, the posterior mean of ρ_x ranges from 0.78 (IT) to 0.85 (CA) in these countries. We find that when we shut down time-varying preference shocks, the long-run consumption component becomes even more persistent in all countries (see Internet Appendix).

In addition, we find that the importance of the long-run risk component varies significantly across different countries, as is evident from the estimates of ϕ_x. The countries that feature a significant fraction of long-run risk in aggregate consumption include the US and Italy, for which the posterior mean estimates of ϕ_x are about 0.2–0.3. The consumption dynamics in other countries have moderately smaller amounts of long-run risk, with the posterior mean of ϕ_x being about 0.1–0.2. The long-run risk plays the least significant role in consumption dynamics in Japan. Together with the empirical evidence of high EIS estimates, the estimated specification implies that the long-run risk model is a convincing description of the macroeconomic and market data jointly for global developed economies.

Regarding the consumption volatility specification, the estimates of the long-run mean, $\bar{\sigma}$, are largely in line with the variation of consumption growth in different countries, respectively. However, the persistence of the stochastic volatility process varies significantly across countries as the posterior mean of ρ_s ranges from 0.55 (JP) to 0.83 (IT), which are much smaller than those values typically used in the calibration studies. Consumption volatility is more persistent in USA, the UK, France, the Netherlands, Italy, and Australia than in Germany, Switzerland, Japan, and Canada. These results therefore cast doubt on the argument usually made in the long-run risk literature (e.g., Bansal and Yaron, 2004; Bansal, Kiku, and Yaron, 2012, 2016) that a very persistent volatility process is required to explain the behavior of market returns. By fully exploiting information in the likelihood function of the asset pricing model, our study does not find evidence to support this argument.

Turning to the dividend growth process, similar to Abel (1999), our estimates of the parameter, Φ, are all well above 1 (between 3 and 12), capturing the "levered" nature of dividends, and are much higher in the other countries than in the US. This result indicates that the long-run risk component plays a more important role in depicting the dividend growth dynamics in countries excluding the US. However, we obtain very different estimates of Φ in the model absent from time-varying preference shocks: its posterior mean is around 1 in most of the countries and its 5% quantile is smaller than 1 in 7 out of 10 countries (see Internet Appendix).

In the estimation, the parameter ϕ_{dc} is primarily identified from the covariation between consumption growth and dividend growth. In the US, consumption and dividend have stronger comovement, leading to higher estimates of ϕ_{dc} than those in the other countries. The parameter ϕ_d determines the amount of variation of dividend growth due to the idiosyncratic risk. Due to the empirical result that much of the variation of dividend growth is loaded onto the long-run risk component in the countries excluding the US, the estimates of ϕ_d are moderately lower in those countries than in the US.

5.1.3. Time Series of Filtered States

Our Bayesian method can directly provide us with time series of filtered states, i.e., the growth rate of preference shocks ($x_{\lambda,t}$), the long-run consumption component (x_t), and the consumption volatility (σ_t). Those filtered time series naturally take into account both parameter and state uncertainties. Figures 1 and 2 display the posterior means of the filtered latent states for four selected countries: the US, the UK, Japan, and Australia. The plots for the posterior mean of $x_{\lambda,t}$ reflect time variation in how investors in different countries value future utility. For the US, the posterior mean of $x_{\lambda,t}$ experiences significant declines in several recession episodes such as late 1940s, early 1980s, 1990s, and the 2008 global financial crisis, albeit the average correlation with consumption growth is low.

While our model assumes that $x_{\lambda,t}$ is independent from the other latent state variables, Bayesian estimation suggests that from the perspective of posteriors the variation in $x_{\lambda,t}$ is partially associated with either the long-run risk component or stochastic volatility, or both. Interestingly, for the US, the UK and Australia, the posterior mean of $x_{\lambda,t}$ is negatively correlated with that of x_t, while for the US, the UK and Japan, the posterior mean of $x_{\lambda,t}$ is positively correlated with that of σ_t. This pattern is more noteworthy in the first half of samples of those countries discussed above. A similar pattern also holds for the other countries, whose results are reported in the Internet Appendix. In times either when expected consumption growth is low or when its conditional volatility is high, the growth rate of the time preference shock is likely to be high and as such, investors value future utility more relative to the current consumption. Because all of the three driving forces tend to induce investors to save more, asset prices therefore capture these effects altogether. As a consequence, when we use asset returns data in the estimation, our estimation strategy leads to the covariation of the posterior estimates of the latent states.

In addition, Figures 1 and 2 show that the long-run risk component plays a more significant role in driving the time variation of consumption growth for the US than for other countries. This observation echoes the parameter estimates for the long-run risk specification reported in Table 3. Not surprisingly, expected consumption growth tends to fall in recessions while rise in booms. The time series of the posterior mean of the stochastic volatility component exhibits the feature of volatility clustering for the US, the UK, Japan and Australia. In the US, the filtered volatility of consumption growth has experienced several upswings in early 1950s, mid 1970s, early 1980s and periods around the 2008 crisis, in which several episodes coincide with the NBER recessions. In the UK, significant increases in the posterior mean of σ_t occur in recession periods around 1975–1980 and 2008–2010. In Japan, the filtered volatility of consumption growth rises in late 1990s, the 1997 Asian financial crisis, and periods around 2010 and 2014, all of which have witnessed dramatic declines in consumption growth. In Australia, high consumption volatility states occur during 1980s and years around 2010, whereas consumption volatility stays at low levels recently.

5.2. Asset Pricing Implications

5.2.1. Impulse Responses

In the model, there are three shocks, the shock to the time preference, the shock to expected consumption growth, and the shock to conditional variance. Since parameter estimates differ dramatically across the countries, it is meaningful to compare impulse responses of key variables in the model, respectively, to the three shocks for different countries. For instance, does an innovation shock with a given size to expected consumption growth have larger effects on consumption growth and the SDF for the US than for the UK?

Specifically, we examine the impulse responses of consumption growth, dividend growth, the SDF, and the price-dividend ratio to shocks to x_λ, x and σ_c individually for each country. When we study the impulse response functions for one shock, we suppress the other two shocks. We assume that prior to the materialization of each shock, the economy stays at the long-run mean levels of state variables, i.e., $x_\lambda = 0$, $x = 0$, and $\sigma_c^2 = \bar{\sigma}^2$, and then each shock hits the economy in the second period. In the analysis of the impulse responses to x_λ and x shocks, we consider a negative innovation (η_λ or η_x) shock with magnitude equal to 1, while in the analysis of the impulse responses to the σ_c shock, we consider a one-standard deviation increase in σ_c. The resulting impulse response functions are plotted in Figures 3 and 4, for the four selected countries: the US, the UK, Japan, and Australia. The $y-$axis indicates the percentage change relative to the initialized state of the economy.

According to Figure 3, our structural estimation suggests that consistent with typical calibrations of the long-run risk model, the negative shock to x leads to a rise in the SDF while a reduction in P/D and hence in the realized return. Since the SDF and the realized return move in opposite directions on impact, the implied risk premium is positive. Moreover, we observe that the impulse responses of the variables differ dramatically across the countries, due to different estimates of the model parameters. As for the impact on consumption growth, the shock to x results in a significant fall in consumption growth for the US while the effect in the other countries is slightly smaller. This observation is consistent with the estimates of the parameters in the process of the persistent component x_t, reported in Table 3. In addition, as implied by the estimates of the leverage parameter Φ, dividend growth in the US is affected by the shock to the least extent, whereas dividend growth in the other countries is much more responsive to the shock. As for the impact on the SDF, the shock to x leads to more than 10% rise in the SDF for the US, but more mild effects for the other countries.

Figure 4 shows the impulse responses of the SDF and P/D to the x_λ and σ_c shocks, respectively. We observe that the impacts of the negative shock to the time preference also vary significantly across the countries. Because the estimated process of the time preference growth rate is very persistent for the US (see Table 3), the impulse responses of the SDF and P/D are the most pronounced for the US among all the countries in our sample. For instance, Panel B in Figure 4 shows that the shock to x_λ has a long-lasting effect on P/D for the US, while the effect is short-lived for Japan and Australia. Similar to the case of the shock to x, the shock to x_λ also induces movements of the SDF and P/D in opposite directions and therefore implies a positive risk premium associated with the time preference shock.

Turning to the impacts of the volatility shock, the impulse responses plots show that for the US, the UK, and Australia, the SDF and P/D move in opposite directions, whereas for Japan, the two variables move in the same direction in response to the shock to σ_c. This is primarily due to low persistence in the consumption volatility process for Japan (the posterior mean of ρ_s is about 0.55), according to our estimation. Thus, we find empirical evidence that the persistence of the volatility process can importantly alter the pricing implications of volatility risk.

5.2.2. Moment Matching and Asset Return Fitting

In the asset pricing literature, the moments matching exercise has been mostly confined to the US data so far. Few studies ever examine performance of matching moments of asset returns for other developed economies. We assess the performance of the estimated long-run risk model in matching moments (means and variances) of asset returns across the countries in our study. In particular, for a specific moment of interest we compute the model-implied analogue for a given parameter set and a latent state path under the joint posterior distribution given the data set. We then report the posterior quantiles of these model-implied moments that account for uncertainties in both the parameters and the latent states.

Table 4 presents moments of asset returns for all the countries, which are generated from the parameter and state particles in real time obtained from our SMC-based Bayesian estimation. The results reveal that the estimated model can well reconcile moments of asset returns for the developed markets in our study. First, the estimated long-run risk model can deliver mean and standard deviation of risk-free rates very close to the moments of the data across all the countries, by means of the 5%, 50% and 95% percentiles. Since the risk-free rate is the reciprocal of conditional expectation of the SDF in the model, our results imply that the behavior of the model-generated SDF is reasonable. Second, the estimated model can closely match the mean and volatility of market returns for seven out of the ten developed markets. The (5, 95)% credible intervals of $E[r_m]$ and $\sigma[r_m]$ embrace the corresponding moments of market returns in the data for the US, the UK, France, Netherlands, Japan, Canada, and Australia; for Germany, while the estimated model overstates the first moment of market returns, the (5, 95)% credible interval of $\sigma[r_m]$ well contains the true market volatility; for Switzerland, the model underestimates the equity premium but overestimates the equity volatility; and for Italy, the model overestimates both equity premium and volatility.

We also investigate how the model implied asset returns track the observed returns. Our estimation yields fitted risk-free rates that can closely track the movement of the actual risk-free rates in all the countries. The upper panels of Figures 5 and 6 display the related results for the four selected countries: the US, the UK, Japan, and Australia. In the model, either an increase in expected consumption growth or a reduction in conditional volatility leads to lower risk-free rates. As a result, our Bayesian estimation identifies the association of the variations in risk-free rates with those in x_t and σ_t for the countries in our analysis. For instance, for the UK in Figure 5, the dramatically low risk-free rates observed in 1970s are consistent with the contemporaneous high volatility of consumption growth. For Australia in Figure 6, the high risk-free rates observed in 1980s are tied to high expected consumption growth in the same period. Nevertheless, we observe that the measurement errors are significant in fitting market returns for all the countries in the analysis, a fact that can be observed from the observation error standard deviations in Table 2 as well.

5.2.3. *Cyclical Variations of SDF*

We next examine the cyclical variation of the estimated SDF. For this purpose, we decompose the SDF under the recursive utility into two components as follows,

$$M_{t+1} = \underbrace{\delta e^{x_{\lambda,t+1}} \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}}_{M_{1,t+1}} \cdot \underbrace{\delta^{\theta-1} \left(\frac{C_{t+1}}{C_t}\right)^{\gamma-\frac{\theta}{\psi}} \left(R^W_{t+1}\right)^{\theta-1}}_{M_{2,t+1}}, \tag{20}$$

where $M_{1,t+1}$ is the SDF under the CRRA utility, and $M_{2,t+1}$ arises due to the separation between risk aversion and the EIS. We compute the time series estimates of M_{t+1}, $M_{1,t+1}$ and $M_{2,t+1}$ using the posterior means of the model parameters and the filtered latent states for each country. Table 5 reports the overall correlations of the SDF and its components with consumption growth for each country. We find that the estimated SDF has a feature of countercyclicality: the correlations between the SDF and consumption growth are all negative, ranging from -0.09 (UK) to -0.41 (US). We also find that the countercyclicality of M_1 is much stronger than that of M_2 in all countries. For example, for Germany the correlation between $M_{1,t}$ ($M_{2,t}$) and consumption growth is about -0.55 (-0.13), and for Switzerland the correlation between $M_{1,t}$ ($M_{2,t}$) and consumption growth is about -0.45 (-0.27).

Figures 7 and 8 plot the time series of the estimated M_{t+1}, $M_{1,t+1}$ and $M_{2,t+1}$, along with consumption growth data, for the four selected countries: the US, the UK, Japan, and Australia. For the US, we observe that the estimated SDF has a notable countercyclical component, which rises in recessions and falls in booms. The correlation between the SDF and consumption growth is about -0.41 over the sample. A similar finding has been found for the US by Chen, Favilukis, and Ludvigson (2013). This result implies that for an asset whose payoff is procyclical, its risk premium tends to be positive in a setting where consumption growth contains a very persistent component and stochastic volatility is also persistent. Both components of the SDF account for its countercyclical variations. The SDF under the CRRA utility, $M_{1,t+1}$, is strongly countercyclical because the variation of the preference shock is low. Compared to $M_{1,t+1}$, $M_{2,t+1}$ has a dominant effect in determining the SDF, and thus, its correlation with consumption growth is also about -0.41. For the UK, the countercyclicality of the SDF is relatively weak: the correlation between the SDF and the consumption growth is only about -0.09. This is mainly driven by the low correlation of $M_{2,t+1}$ with consumption growth in the UK. Figure 8 shows that the countercyclicality of the SDF is significant in Japan while moderate in Australia, again due to respective correlations of $M_{2,t+1}$ with consumption growth in those two countries. Similar results can also be found in other countries and are reported in the Internet Appendix.

Table 5 also reports the overall correlations of consumption growth with conditional equity premium ($E_t[r_{m,t+1} - r_{f,t}]$) and conditional volatility ($\sigma_t[r_{m,t+1}]$) of equity returns implied by our long-run risk model for all the countries. The conditional equity premium and conditional volatility of equity returns are computed based on the posterior means of the model parameters and the filtered latent states. Except for France, Italy, and Germany (for equity premium), all correlations are negative: the strongest negative correlation between equity premium and consumption growth is for the US, the UK, and Switzerland, about -0.23, and the strongest negative correlation between conditional volatility and consumption growth is for the US, about -0.26. Figures 9 and 10 present conditional equity premium and conditional volatility of equity returns implied by our long-run risk model, along with the consumption growth, for the above-mentioned four countries. The plots suggest that both conditional equity premium and conditional volatility of equity returns have a countercyclical component in the US and UK. For both countries, the correlation between the conditional equity premium (conditional volatility) of equity returns and consumption growth is -0.23 (-0.26) and -0.23 (-0.23), respectively; the results for Japan and Australia are a little weak. The countercyclical variations in conditional equity premium and volatility are primarily driven by the countercyclicality of the SDF.

5.2.4. *Counterfactual Analysis*

To emphasize the importance of the preference shock and the persistent component in expected consumption growth, we perform counterfactual analyses on fitted risk-free rate and market returns generated from our estimation. In particular, we compute counterfactual risk-free rates and market returns that would be obtained either in the absence of the preference shock or the long-run risk component. For all the countries, the impacts of the preference shock and the persistent component in expected consumption growth on the risk-free rate and market returns are remarkable. Figures 11 and 12 present the corresponding results for the four selected results: the US, the UK, Japan, and Australia. Additional plots for the other countries can be found in the Internet Appendix.

It turns out that for the US the preference shock matters notably not only for the level of the risk-free rate but also for its time variation. The risk-free rate without the preference shock is too high and too smooth relative to the risk-free rate implied by the model. This finding complements the analysis of Schorfheide, Song, and Yaron (2018) who find that the preference shock mainly accounts for the time variation in the observed risk-free rate. By contrast, the impact of the preference shock on the risk-free rate is insignificant for the UK, Japan, and Australia. This is a distinct feature for most of countries other than the US, which, however, has not been documented in previous studies. The preference shock also has crucial effects on equity returns. For the US, the UK, Japan, and Australia, the equity returns implied by the model abstracted from the preference shock are too low and too smooth compared to the returns implied by the true model; such a result can also be found for the other countries. This finding is consistent with the mechanism illustrated by Albuquerque et al. (2016) that the preference shock generates additional risk premium.

Turning to the role of the persistent component in expected consumption growth, we find that the risk-free rate that would prevail without the the persistent component in expected consumption growth is too smooth compared to the fitted risk-free rate in the true model; in addition, the equity returns become lower and less volatile in the absence of the persistent component in expected consumption growth. In Figures 11 and 12, we can clearly observe such impacts for the above four selected countries.

6. Conclusions

The long-run risk model of Bansal and Yaron (2004) has attracted remarkable attention and has become a benchmark in the consumption-based asset pricing literature. Despite the success of the long-run risk models in characterizing dynamics of fundamentals and asset returns in the US market, its performance with regard to other developed countries is yet to be examined. Furthermore, the vast majority of studies on consumption-based asset pricing up to now have relied on the calibration approach, and studies on structural estimation of asset pricing models remain very limited. The main cause for the sparsity in this research is that efficient econometric estimation of consumption-based asset pricing models is challenging primarily due to that global solutions to these models are highly nonlinear functions of state variables and that data on fundamentals are often observed in very low frequencies and are hard to obtain for countries other than the US.

In this paper, we estimate and test long-run risk models by employing an efficient likelihood-based Bayesian method that exploits up-to-date sequential Monte Carlo methods for international economies. Our benchmark model features a representative agent who has recursive preferences with a time preference shock, a persistent component in expected consumption growth, and stochastic volatility in fundamentals characterized by an autoregressive Gamma process. We construct a comprehensive dataset including macroeconomic and financial data in the post-war period for ten developed countries, including the US, the UK, Germany, France, Italy, Japan, Canada, Australia, the Netherlands, and Switzerland. We use the quarterly data on consumption, dividends, and asset returns to implement estimations.

Our estimation provides international evidence in support of long-run risks in expected consumption growth and a countercyclical component in the stochastic discount factor. We find that the introduction of time-varying preference shocks in the long-run risk model helps deliver economically plausible estimates of risk aversion and EIS, not only for the US but also for the other developed economies. We also find that the importance of the long-run risk component varies significantly across the countries. In addition, our estimated stochastic volatility process, which reflects time-varying economic uncertainty, is less persistent than those postulated in the calibration studies on long-run risks. Our estimation yields model-fitted risk-free rates that closely track the historical movements of the actual risk-free rates across different countries.

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Table 1: **Summary Statistics**

	$E[r_m]$	$\sigma[r_m]$	$E[r_f]$	$\sigma[r_f]$	$E[\Delta c]$	$\sigma[\Delta c]$	$E[\Delta d]$	$\sigma[\Delta d]$	Sample Period
US	0.0700	0.1621	0.0058	0.0090	0.0188	0.0099	0.0254	0.0466	1947:Q2-2019:Q3
UK	0.0642	0.1873	0.0110	0.0144	0.0222	0.0219	0.0162	0.0380	1965:Q4-2019:Q3
DE	0.0575	0.2007	0.0156	0.0137	0.0153	0.0183	0.0251	0.0563	1973:Q4-2019:Q3
FR	0.0712	0.2223	0.0154	0.0139	0.0140	0.0129	0.0346	0.0499	1973:Q4-2019:Q3
NL	0.0732	0.1985	0.0128	0.0132	0.0119	0.0193	0.0229	0.0626	1973:Q4-2019:Q3
CH	0.0648	0.1888	-0.0002	0.0090	0.0074	0.0115	0.0477	0.0589	1973:Q4-2019:Q3
IT	0.0339	0.2530	0.0152	0.0169	0.0135	0.0156	0.0176	0.0998	1973:Q4-2019:Q3
JP	0.0330	0.2074	0.0083	0.0099	0.0153	0.0232	0.0211	0.0472	1973:Q4-2019:Q3
CA	0.0552	0.1587	0.0208	0.0119	0.0160	0.0166	0.0197	0.0537	1973:Q4-2019:Q3
AU	0.0666	0.1940	0.0255	0.0139	0.0172	0.0187	0.0269	0.0559	1973:Q4-2019:Q3

This table reports summary statistics of the data used for model estimation. The data are sampled at a quarterly frequency for ten developed countries, including the United States (US), the United Kingdom (UK), Germany (DE), France (FR), the Netherlands (NL), Switzerland (CH), Italy (IT), Japan (JP), Canada (CA) and Australia (AU). The sample period for each country is also shown in the table. The summary statistics consists of the mean and standard deviation of equity returns ($E[r_m]$ and $\sigma[r_m]$), the mean and standard deviation of the risk-free rate ($E[r_f]$ and $\sigma[r_f]$), the mean and standard deviation of per capita consumption growth ($E[\Delta c]$ and $\sigma[\Delta c]$), and the mean and standard deviation of dividend growth ($E[\Delta d]$ and $\sigma[\Delta d]$). All variables are in real and log terms.

Table 2: **Model Performance**

	US	UK	DE	FR	IT	JP	CA	AU	NL	CH
					Panel A. LRR1SV					
σ_m	0.0818	0.0966	0.1044	0.1076	0.1333	0.1057	0.0833	0.1017	0.1027	0.0511
	(0.0032)	(0.0047)	(0.0051)	(0.0052)	(0.0061)	(0.0050)	(0.0039)	(0.0049)	(0.0050)	(0.0062)
σ_f	1.22e-3	7.14e-4	2.13e-3	2.15e-3	1.64e-3	5.01e-4	1.34e-3	1.75e-3	1.64e-3	4.41e-3
	(1.14e-4)	(1.12e-4)	(1.19e-4)	(1.23e-4)	(1.49e-4)	(1.22e-4)	(1.54e-4)	(1.63e-4)	(1.77e-4)	(2.10e-4)
ML	3.430e3	2.364e3	1.901e3	1.965e3	1.786e3	1.989e3	1.981e3	1.915e3	1.873e3	2.010e3
					Panel B. LRR1SVPref					
σ_m	0.0759	0.0824	0.0855	0.0919	0.1129	0.0979	0.0609	0.0864	0.0807	0.0544
	(0.0042)	(0.0049)	(0.0059)	(0.0062)	(0.0078)	(0.0061)	(0.0054)	(0.0067)	(0.0085)	(0.0071)
σ_f	4.73e-4	6.21e-4	4.10e-4	6.35e-4	7.85e-4	5.12e-4	8.22e-4	1.05e-3	1.71e-3	1.37e-3
	(9.38e-5)	(1.33e-4)	(8.35e-5)	(1.44e-4)	(2.26e-4)	(1.16e-4)	(1.61e-4)	(2.18e-4)	(2.86e-4)	(2.30e-4)
ML	3.535e3	2.440e3	2.114e3	2.096e3	1.891e3	2.070e3	2.057e3	2.009e3	1.947e3	2.104e3
					Panel C. LRR2SVPref					
σ_m	0.0763	0.0841	0.0897	0.0946	0.1182	0.1014	0.0615	0.0884	0.0822	0.0705
	(0.0035)	(0.0054)	(0.0056)	(0.0058)	(0.0070)	(0.0042)	(0.0054)	(0.0058)	(0.0073)	(0.0074)
σ_f	4.84e-4	5.70e-4	3.39e-4	5.29e-4	9.60e-4	3.62e-4	6.63e-4	1.02e-3	1.61e-3	1.19e-3
	(9.31e-5)	(1.05e-4)	(6.91e-5)	(1.09e-4)	(1.89e-4)	(8.48e-5)	(1.46e-4)	(2.27e-4)	(2.54e-4)	(2.21e-4)
ML	3.544e3	2.447e3	2.141e3	2.167e3	1.916e3	2.082e3	2.133e3	2.054e3	1.968e3	2.119e3

This table shows estimation results on model performance by comparing three model specifications for the ten countries. LRR1SV refers to the long-run risk model with one stochastic volatility process but without preference shocks. LRR1SVPref refers to the model with one stochastic volatility process and preference shocks. LRR2SVPref differs from LRR1SVPref by further allowing for an independent volatility process in dividend growth. The metrics used for assessing performance of a model include the standard deviations of the measurement errors in stock returns and risk-free rates (σ_m and σ_f respectively) and the log marginal likelihood (ML).

Table 3: **Parameter Estimates**

	Mean	Std	5%	95%	Mean	Std	5%	95%
	Panel A: US				Panel B: UK			
δ	0.9983	0.0004	0.9976	0.9989	0.9915	0.0028	0.9868	0.9955
γ	9.7968	1.3593	7.5341	12.040	7.1587	1.6178	4.5554	9.8036
ψ	1.8073	0.4486	1.2165	2.5796	1.6912	0.2651	1.2625	2.1507
ρ_λ	0.9935	0.0014	0.9912	0.9956	0.9592	0.0126	0.9368	0.9770
ϕ_λ	0.0013	0.0001	0.0011	0.0014	0.0024	0.0002	0.0020	0.0028
ρ_x	0.8915	0.0614	0.7878	0.9736	0.8686	0.0277	0.8186	0.9105
ϕ_x	0.3117	0.0534	0.2173	0.3972	0.0969	0.0178	0.0716	0.1293
$\bar{\sigma}$	0.0045	0.0002	0.0043	0.0048	0.0101	0.0004	0.0095	0.0106
ρ_s	0.7343	0.0591	0.6312	0.8208	0.7009	0.0396	0.6349	0.7641
ϕ_s	1.5519	0.2304	1.1870	1.9475	1.4635	0.1993	1.1698	1.7860
Φ	3.1483	1.4505	1.5531	6.1953	7.1997	1.3257	4.9680	9.4011
ϕ_{dc}	0.5933	0.1549	0.3885	0.8920	0.1689	0.0523	0.0981	0.2617
ϕ_d	4.7024	0.4006	4.0828	5.3785	1.1288	0.0864	0.9944	1.2791
	Panel C: DE				Panel D: FR			
δ	0.9762	0.0018	0.9731	0.9789	0.9913	0.0012	0.9889	0.9930
γ	5.8102	1.2783	3.6056	7.8979	6.8733	1.4237	4.6195	9.3501
ψ	2.2741	0.2361	1.8771	2.6816	2.0050	0.3068	1.4999	2.5200
ρ_λ	0.9862	0.0026	0.9817	0.9898	0.9351	0.0111	0.9156	0.9519
ϕ_λ	0.0014	0.0001	0.0013	0.0016	0.0024	0.0002	0.0021	0.0028
ρ_x	0.8003	0.0327	0.7434	0.8494	0.8268	0.0303	0.7709	0.8723
ϕ_x	0.1310	0.0178	0.1023	0.1600	0.1738	0.0254	0.1359	0.2211
$\bar{\sigma}$	0.0080	0.0003	0.0074	0.0085	0.0059	0.0002	0.0055	0.0062
ρ_s	0.5861	0.1123	0.3914	0.7479	0.7204	0.0980	0.5378	0.8482
ϕ_s	2.3406	0.5008	1.5694	3.2263	2.4955	0.5850	1.5609	3.5179
Φ	11.760	1.4007	9.5368	14.257	10.464	1.3349	8.4773	12.754
ϕ_{dc}	0.3869	0.0957	0.2452	0.5547	0.3154	0.0822	0.2010	0.4612
ϕ_d	2.0936	0.1785	1.8259	2.3880	2.2743	0.1940	1.9695	2.6159
	Panel E: NL				Panel F: CH			
δ	0.9931	0.0020	0.9894	0.9959	0.9991	0.0004	0.9984	0.9995
γ	6.6863	1.5238	4.2142	9.3647	5.7052	1.4679	3.3705	8.3237
ψ	1.9414	0.3300	1.4101	2.4774	2.3244	0.3480	1.7603	2.9048
ρ_λ	0.9000	0.0254	0.8553	0.9382	0.8072	0.0344	0.7462	0.8628
ϕ_λ	0.0029	0.0004	0.0023	0.0036	0.0027	0.0003	0.0022	0.0034
ρ_x	0.8628	0.0365	0.7948	0.9171	0.8909	0.0231	0.8475	0.9231
ϕ_x	0.1626	0.0321	0.1099	0.2216	0.1477	0.0238	0.1136	0.1905
$\bar{\sigma}$	0.0089	0.0003	0.0083	0.0093	0.0053	0.0001	0.0050	0.0055
ρ_s	0.7235	0.0713	0.5968	0.8243	0.5532	0.1073	0.3709	0.7228
ϕ_s	1.9219	0.3733	1.3518	2.5945	1.7021	0.3033	1.2278	2.2364
Φ	6.7596	1.2632	4.9312	9.2731	10.519	1.3203	8.3768	12.689
ϕ_{dc}	0.1817	0.0730	0.0856	0.3205	0.4119	0.1253	0.2300	0.6204
ϕ_d	2.3145	0.1832	2.0409	2.6350	3.7626	0.3014	3.2595	4.2617

	Panel G: IT				Panel H: JP			
δ	0.9918	0.0016	0.9891	0.9941	0.9978	0.0009	0.9961	0.9989
γ	7.5221	1.2499	5.4424	9.4947	6.8042	1.4121	4.5437	9.2047
ψ	2.1747	0.3123	1.6527	2.6896	2.3054	0.3508	1.7030	2.8837
ρ_λ	0.9587	0.0106	0.9391	0.9742	0.9169	0.0139	0.8935	0.9391
ϕ_λ	0.0025	0.0002	0.0022	0.0030	0.0019	0.0001	0.0017	0.0022
ρ_x	0.7780	0.0352	0.7132	0.8335	0.8312	0.0276	0.7867	0.8775
ϕ_x	0.2189	0.0281	0.1775	0.2692	0.0782	0.0128	0.0574	0.0997
$\bar{\sigma}$	0.0072	0.0003	0.0066	0.0076	0.0110	0.0002	0.0105	0.0113
ρ_s	0.8358	0.0386	0.7647	0.8869	0.5488	0.0708	0.4254	0.6484
ϕ_s	1.8678	0.4083	1.2259	2.5717	1.6456	0.2327	1.2774	2.0328
Φ	12.277	1.3071	10.180	14.560	11.667	1.5718	9.3078	14.399
ϕ_{dc}	0.3099	0.1130	0.1611	0.5197	0.1953	0.0431	0.1349	0.2762
ϕ_d	3.4083	0.3744	2.8143	4.0510	1.0889	0.0811	0.9665	1.2221
	Panel I: CA				Panel J: AU			
δ	0.9933	0.0013	0.9910	0.9953	0.9946	0.0018	0.9916	0.9970
γ	6.9430	1.3927	4.5826	9.3336	6.9543	1.4374	4.6119	9.2663
ψ	1.8347	0.3134	1.3647	2.3599	2.1356	0.3174	1.5995	2.6639
ρ_λ	0.9240	0.0143	0.8983	0.9461	0.9474	0.0204	0.9097	0.9739
ϕ_λ	0.0021	0.0002	0.0018	0.0024	0.0024	0.0003	0.0020	0.0029
ρ_x	0.8519	0.0228	0.8136	0.8887	0.7903	0.0339	0.7326	0.8431
ϕ_x	0.1226	0.0204	0.0921	0.1586	0.1350	0.0212	0.1034	0.1701
$\bar{\sigma}$	0.0079	0.0003	0.0074	0.0082	0.0086	0.0003	0.0080	0.0090
ρ_s	0.6790	0.0705	0.5494	0.7817	0.7441	0.0738	0.6100	0.8489
ϕ_s	1.8653	0.3260	1.3427	2.4243	1.5239	0.3850	1.0628	2.2968
Φ	9.6001	1.4326	7.5402	12.307	9.3330	1.3703	7.2539	11.708
ϕ_{dc}	0.1832	0.0623	0.0991	0.3021	0.2282	0.0691	0.1301	0.3464
ϕ_d	2.0784	0.1475	1.8405	2.3271	1.6751	0.1662	1.4091	1.9541

This table reports posterior means, standard deviations, 5 and 95 percentiles of model parameters for the long-run risk model with one stochastic volatility process and preference shocks (the LRR1SVPref model). Parameter estimates are for preference parameters in the recursive utility function and parameters in the processes of consumption growth and dividend growth. The estimation is implemented using the Bayesian SMC^2 method, for the ten countries in our sample.

Table 4: **Asset Return Moments**

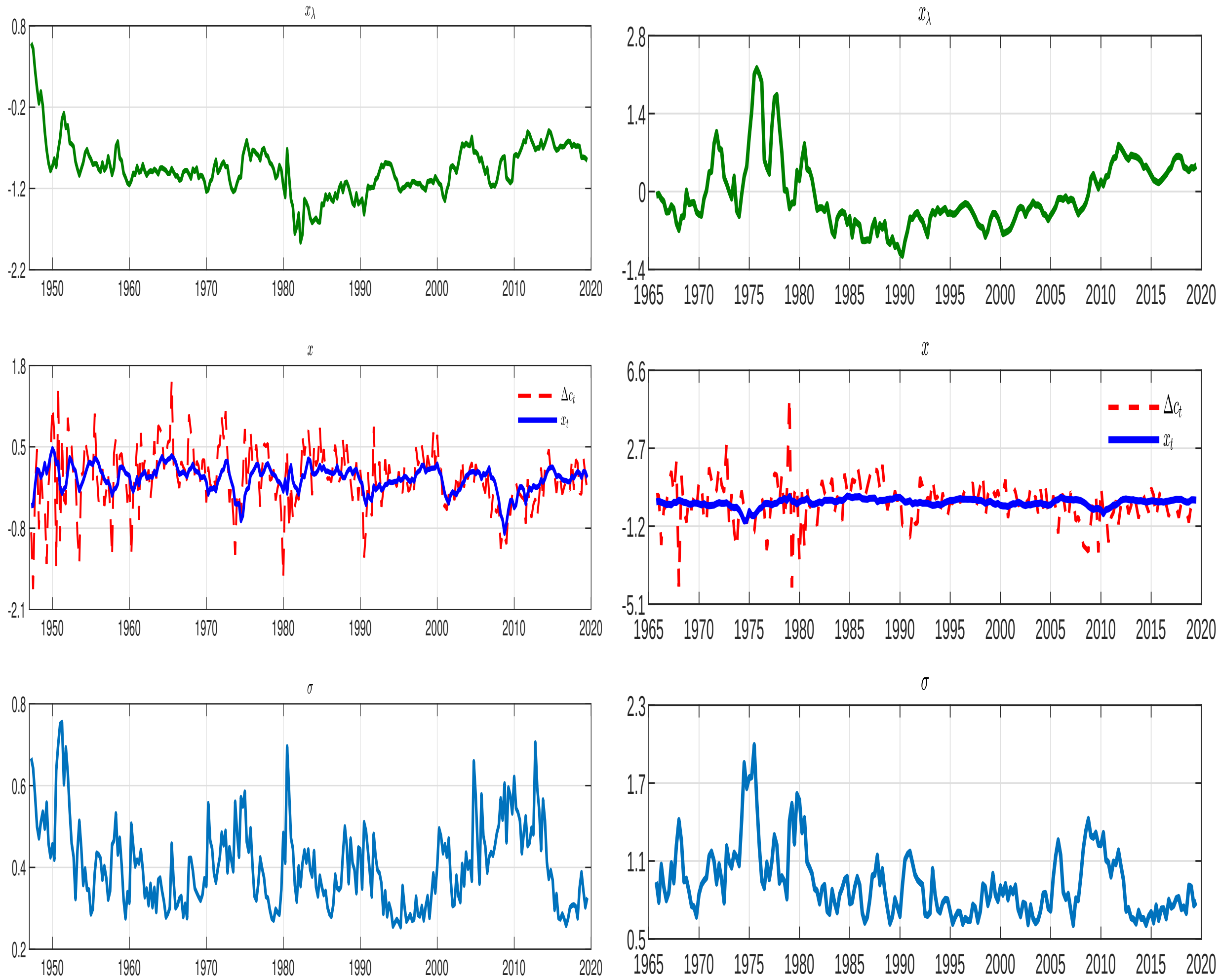
	Data	5%	50%	95%	Data	5%	50%	95%
	Panel A: US				Panel B: UK			
$E[r_m]$	7.00	4.65	10.40	11.99	6.42	7.18	9.98	12.52
$\sigma[r_m]$	16.20	13.40	18.35	25.05	18.70	15.65	19.05	23.05
$E[r_f]$	0.58	0.56	0.58	0.59	1.10	1.07	1.10	1.13
$\sigma[r_f]$	0.90	0.89	0.90	0.91	1.44	1.43	1.44	1.45
	Panel C: DE				Panel D: FR			
$E[r_m]$	5.75	10.25	11.37	12.36	7.12	5.41	6.95	8.58
$\sigma[r_m]$	20.10	18.71	21.94	25.73	22.20	18.47	22.63	27.41
$E[r_f]$	1.56	1.55	1.57	1.59	1.54	1.52	1.55	1.58
$\sigma[r_f]$	1.37	1.36	1.37	1.38	1.39	1.36	1.38	1.40
	Panel E: NL				Panel F: CH			
$E[r_m]$	7.32	4.18	6.50	9.00	6.48	4.68	4.91	5.49
$\sigma[r_m]$	19.85	19.23	24.08	29.43	18.90	21.33	24.91	28.81
$E[r_f]$	1.28	1.21	1.29	1.38	-0.02	-0.05	0.01	0.08
$\sigma[r_f]$	1.32	1.21	1.27	1.32	0.90	0.79	0.84	0.89
	Panel G: IT				Panel H: JP			
$E[r_m]$	3.39	6.88	8.66	10.29	3.30	2.15	2.72	4.31
$\sigma[r_m]$	25.30	25.55	29.70	34.53	20.70	16.47	20.09	24.18
$E[r_f]$	1.52	1.48	1.52	1.56	0.83	0.80	0.83	0.85
$\sigma[r_f]$	1.69	1.66	1.69	1.70	0.99	0.98	0.99	1.00
	Panel I: CA				Panel J: AU			
$E[r_m]$	5.52	4.70	6.30	7.90	6.66	3.90	6.48	9.65
$\sigma[r_m]$	15.90	16.87	19.91	23.14	19.40	16.24	19.61	23.54
$E[r_f]$	2.08	2.04	2.08	2.12	2.55	2.47	2.53	2.58
$\sigma[r_f]$	1.19	1.16	1.18	1.20	1.39	1.34	1.37	1.40

This table presents moments of stock returns and risk-free rates implied by the LRR1SVPref model for the ten countries. The moments of asset returns calculated from the data are also shown for each country. The moments of asset returns implied by the model are calculated from the parameter and state particles in real time obtained from the Bayesian SMC^2 method.

Table 5: **Cyclical Variations of SDF**

	US	UK	DE	FR	IT	JP	CA	AU	NL	CH
M	-0.41	-0.09	-0.15	-0.24	-0.13	-0.32	-0.34	-0.17	-0.23	-0.29
M_1	-0.68	-0.76	-0.55	-0.37	-0.48	-0.70	-0.51	-0.58	-0.56	-0.45
M_2	-0.41	-0.07	-0.13	-0.23	-0.12	-0.30	-0.33	-0.15	-0.22	-0.27
$E_t[r_{m,t+1} - r_{f,t}]$	-0.23	-0.23	0.08	0.09	0.05	-0.15	-0.15	-0.06	-0.17	-0.23
$\sigma_t[r_{m,t+1}]$	-0.26	-0.23	-0.08	0.09	0.03	-0.15	-0.15	-0.08	-0.16	-0.22

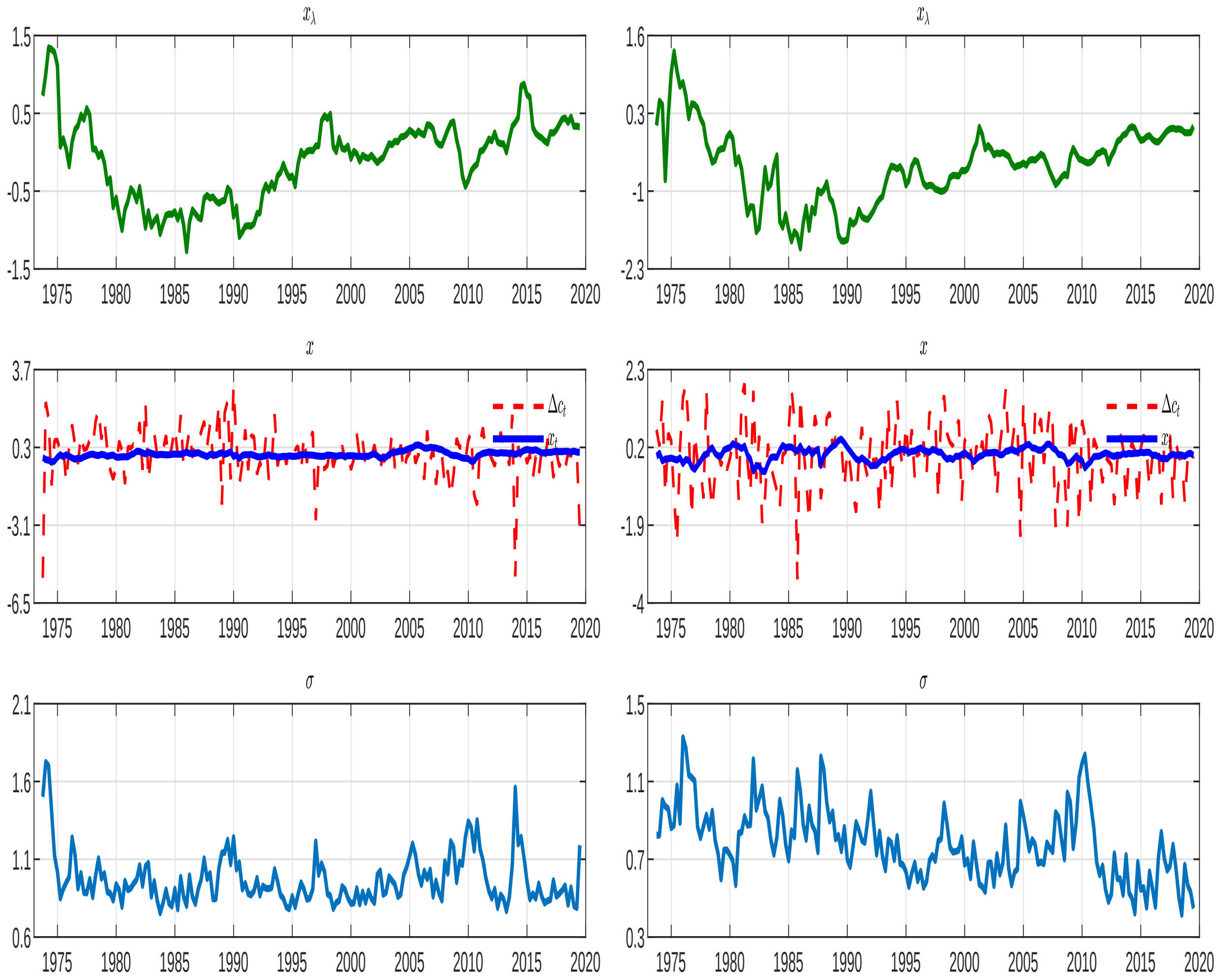
This table presents, for each of the ten countries, 1) correlations of the SDF (M_t) and its components ($M_{1,t}$ and $M_{2,t}$) with per capita consumption growth respectively, and 2) correlations of conditional equity premium ($E_t[r_{m,t+1} - r_{f,t}]$) and conditional volatility ($\sigma_t[r_{m,t+1}]$) of equity returns with per capita consumption growth respectively. The SDF and conditional moments of equity returns are computed based on the posterior means of the model parameters and the filtered latent states, both of which are estimated using the Bayesian SMC^2 method.



(a) US (b) UK

Figure 1: **Filtered Latent States: US and UK**

This figure plots the posterior means of the filtered latent states, the growth rate of preference shocks ($x_{\lambda,t}$), the long-run consumption component (x_t), and the consumption volatility state (σ_t), for the US and UK.



(a) JP (b) AU

Figure 2: **Filtered Latent States: Japan and Australia**

This figure plots the posterior means of the filtered latent states, the growth rate of preference shocks ($x_{\lambda,t}$), the long-run consumption component (x_t), and the consumption volatility state (σ_t), for Japan and Australia.

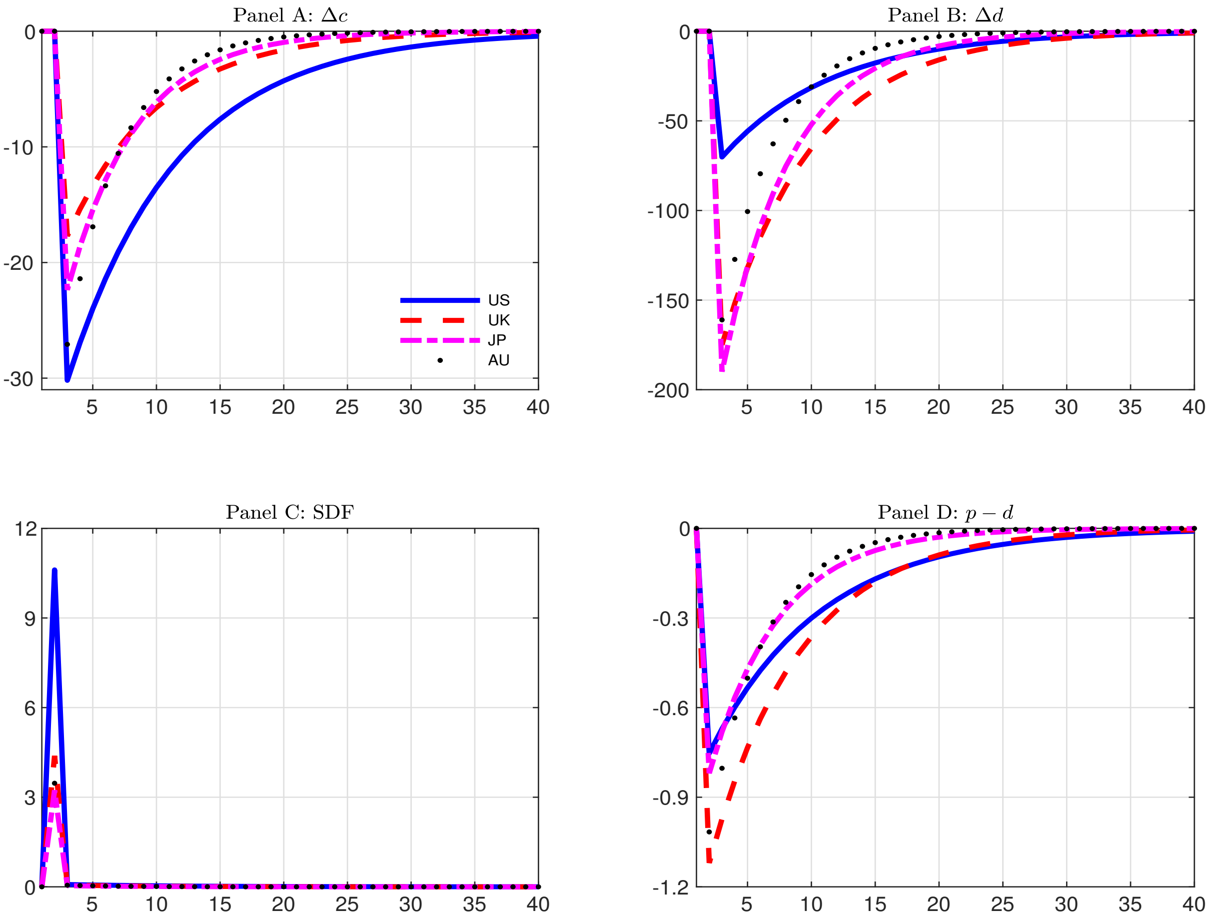


Figure 3: **Impulse responses: shock to x**

Notes: This figure plots the impulse response functions for a negative innovation shock to x with size $\eta_x = -1$. The plots include the impulse responses of consumption growth, dividend growth, the SDF and P/D.

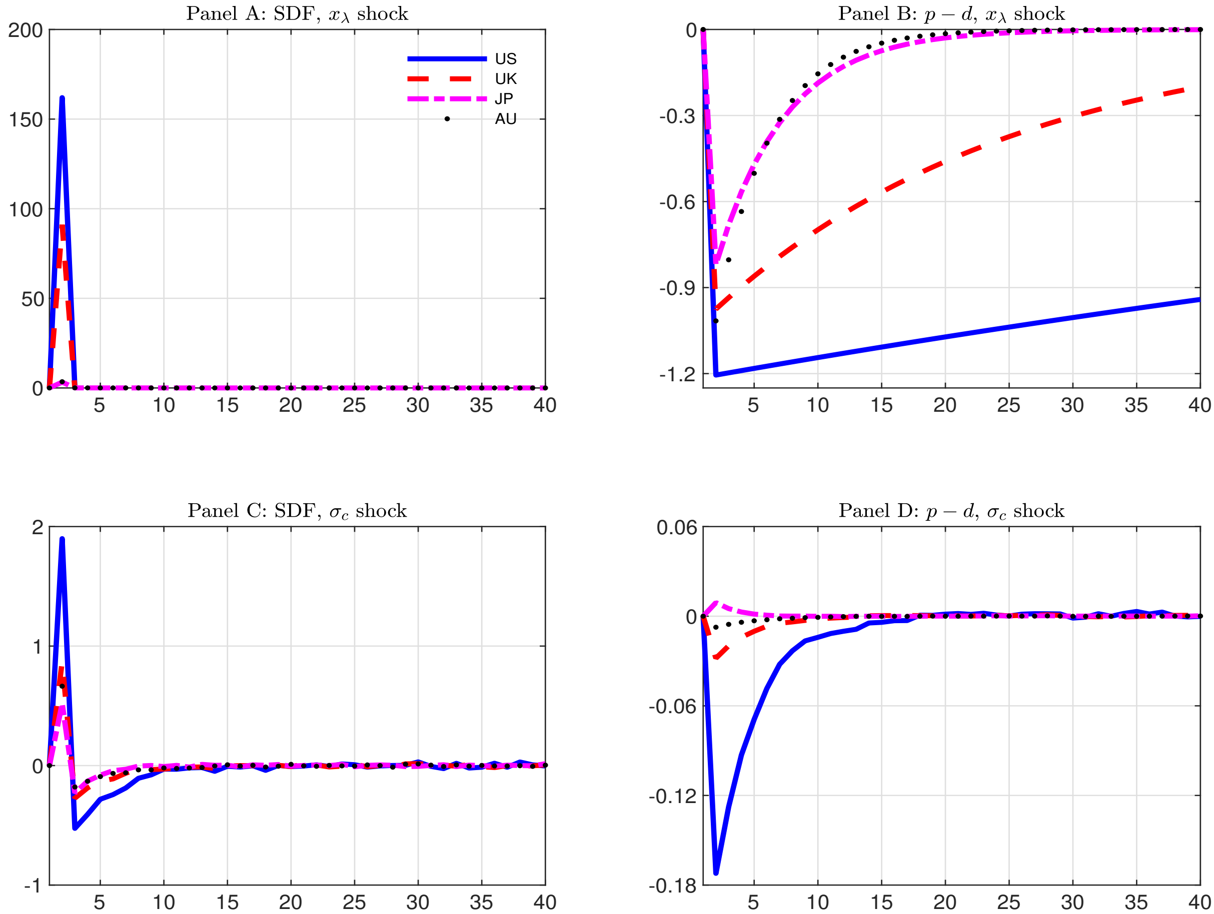
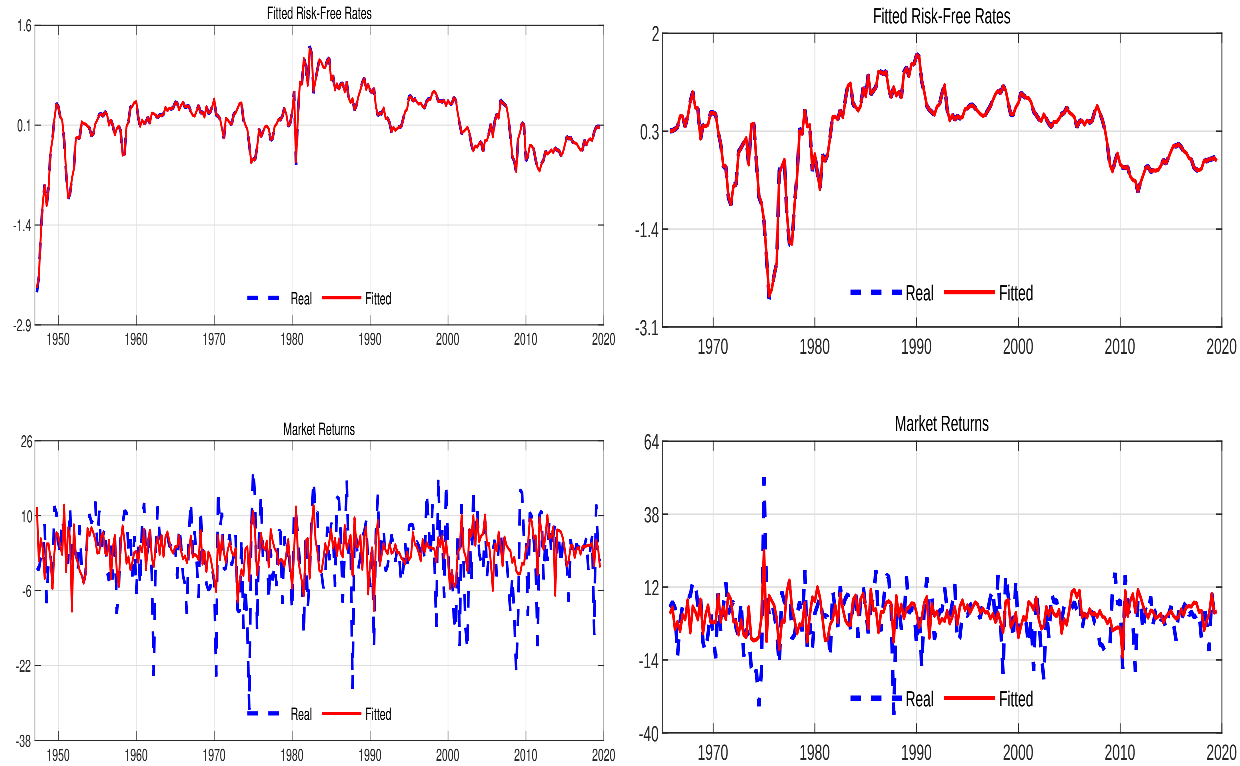


Figure 4: **Impulse responses: shocks to x_λ and σ_c**

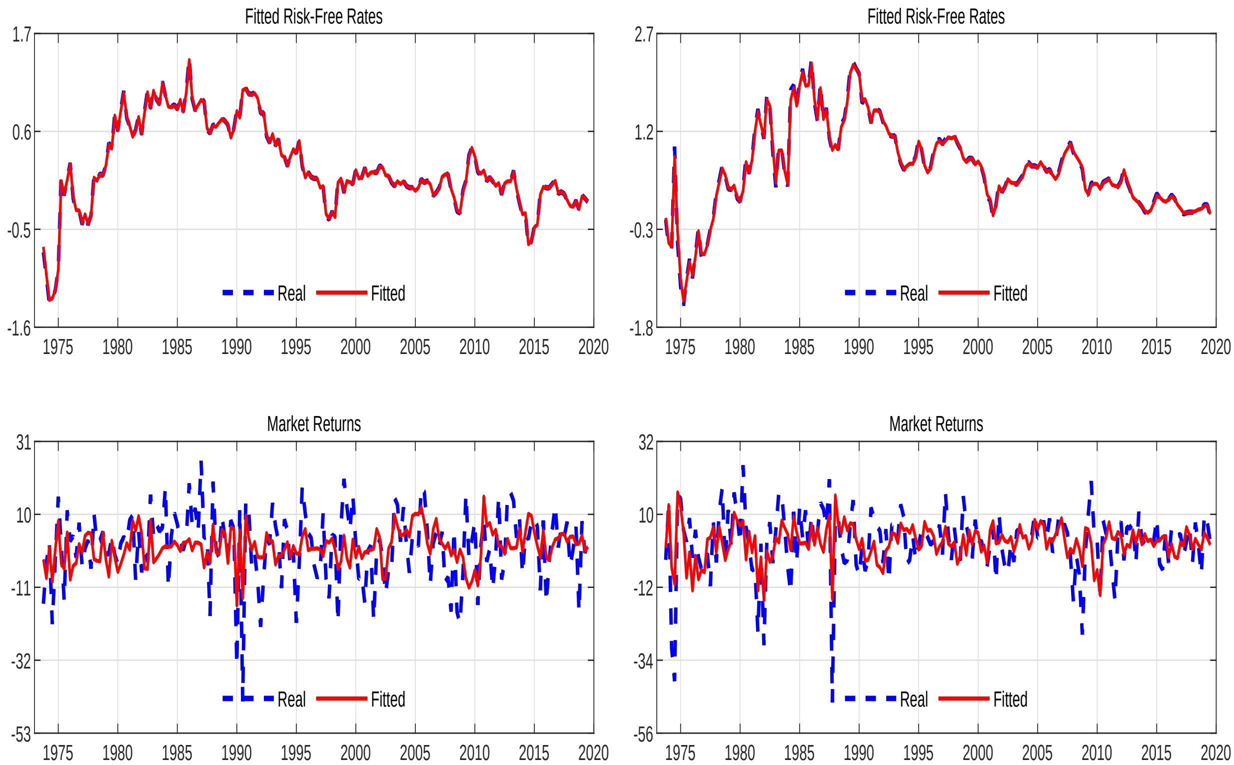
Notes: Panel A and Panel B plot the impulse response functions for a negative innovation shock to x_λ with size $\eta_\lambda = -1$. Panel C and Panel D plot the impulse response functions for a one-standard deviation increase in σ_c. The plots include the impulse responses of the SDF and P/D.



(a) US (b) UK

Figure 5: **Fitted Risk-Free Rates and Market Returns: US and UK**

This figure plots the model-implied risk-free rates and market returns together with the actual returns in the data for the US and UK respectively. For each country, the model-implied risk-free rates and market returns are computed from the posterior means of the model parameters and the posterior means of the filtered latent states.



(a) JP (b) AU

Figure 6: **Fitted Risk-Free Rates and Market Returns: Japan and Australia**

This figure plots the model-implied risk-free rates and market returns together with the actual returns in the data for Japan and Australia respectively. For each country, the model-implied risk-free rates and market returns are computed from the posterior means of the model parameters and the posterior means of the filtered latent states.

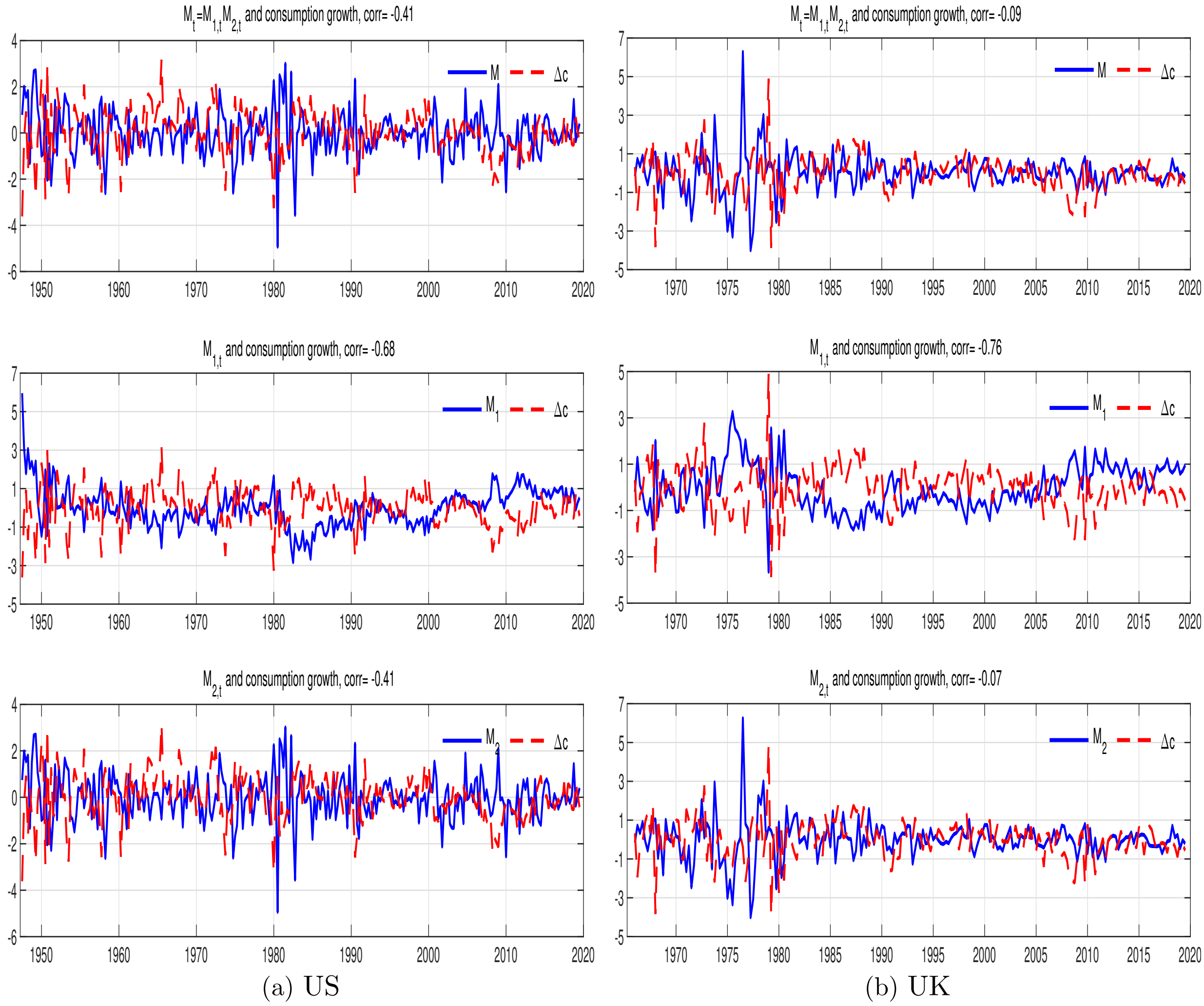
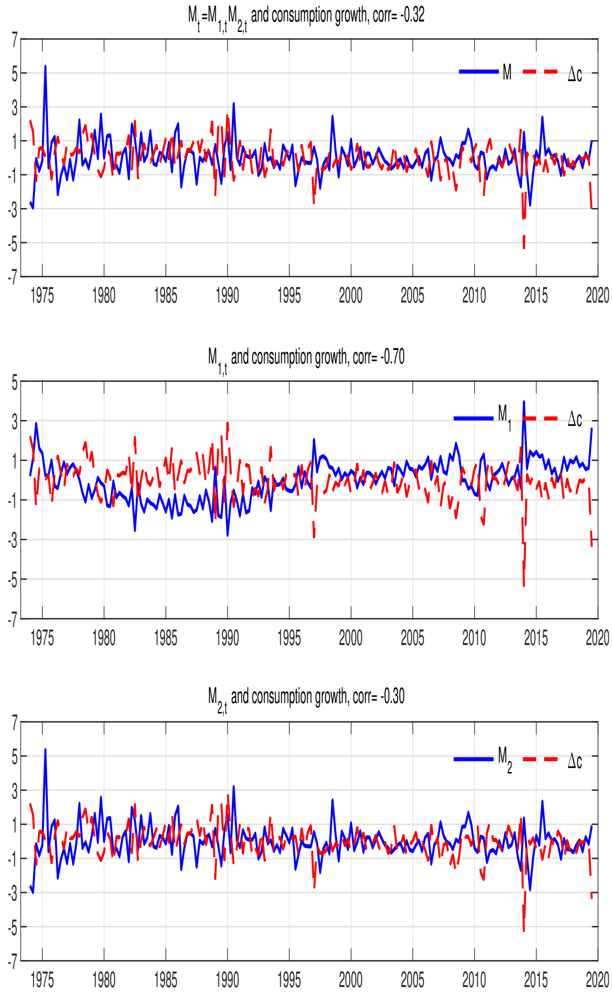
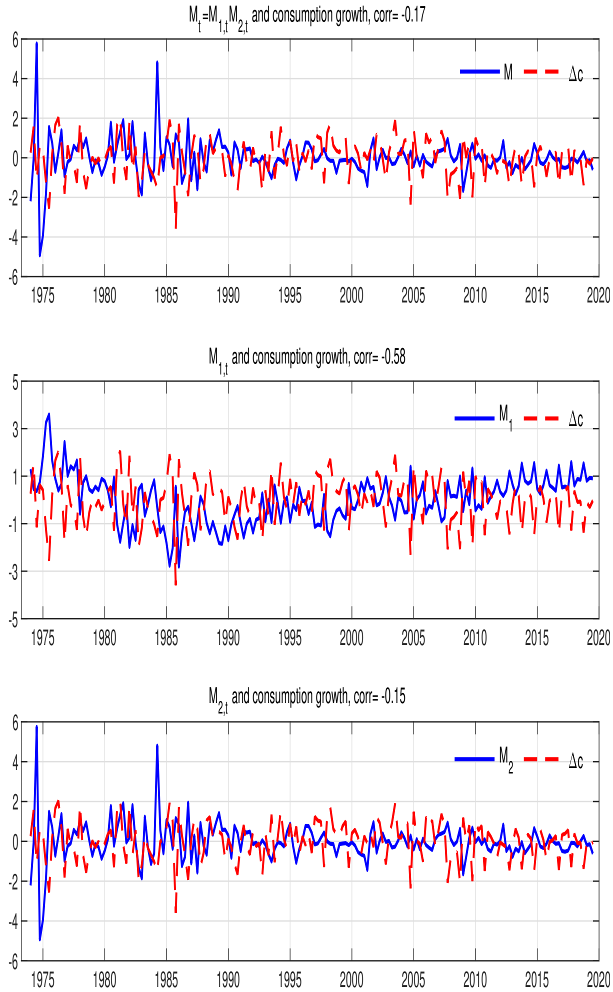


Figure 7: **Stochastic discount factor: US and UK**

This figure plots the model-implied SDF and its components, together with per capita consumption growth for the US and UK respectively. For each country, the model-implied SDF is computed from the posterior means of the model parameters and the posterior means of the filtered latent states.



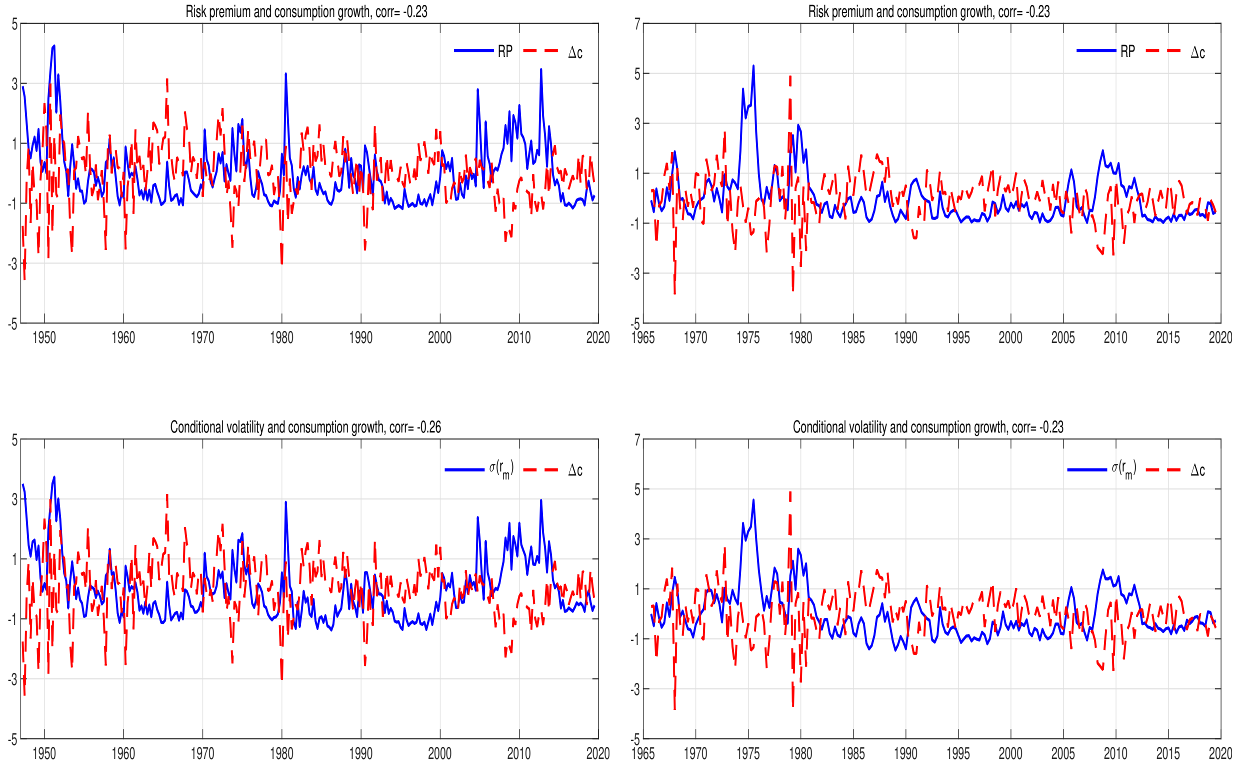
(a) Japan (JP)



(b) Australia (AU)

Figure 8: **Stochastic discount factor: Japan and Australia**

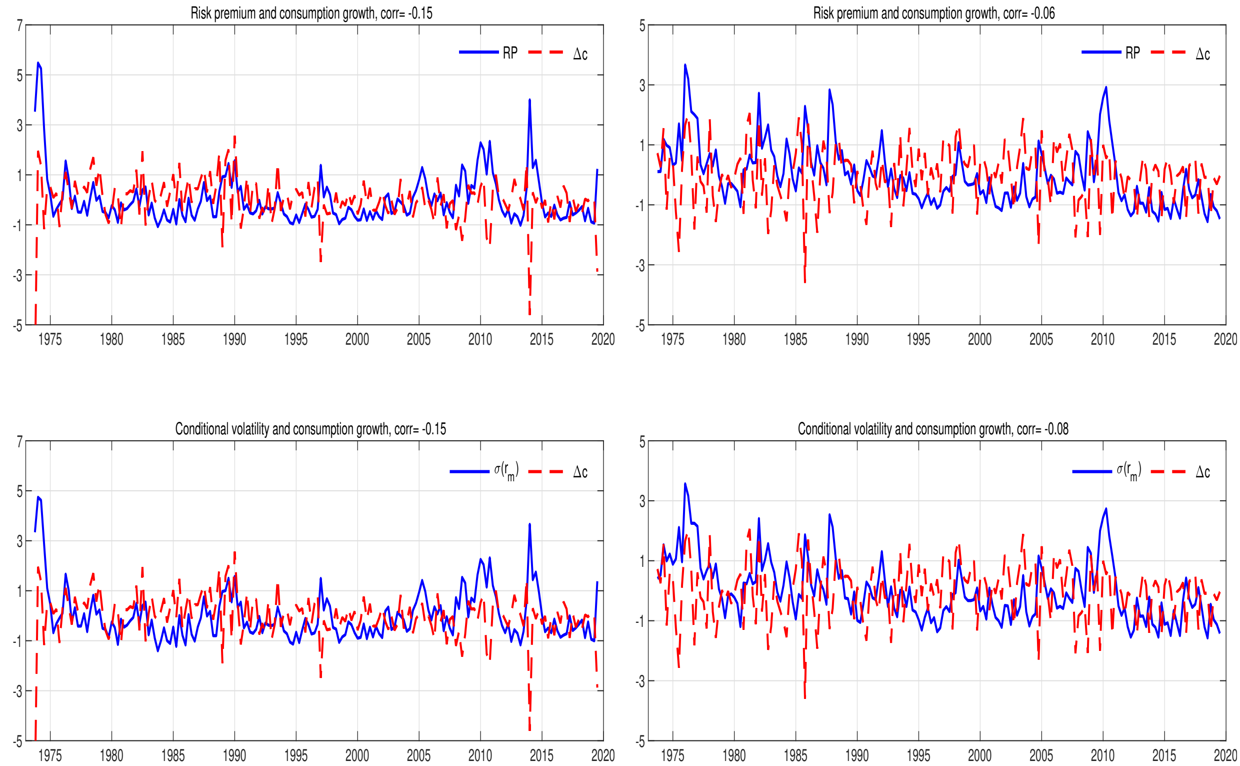
This figure plots the model-implied SDF and its components, together with per capita consumption growth for Japan and Australia respectively. For each country, the model-implied SDF is computed from the posterior means of the model parameters and the posterior means of the filtered latent states.



(a) US (b) UK

Figure 9: **Conditional risk premium and volatility: US and UK**

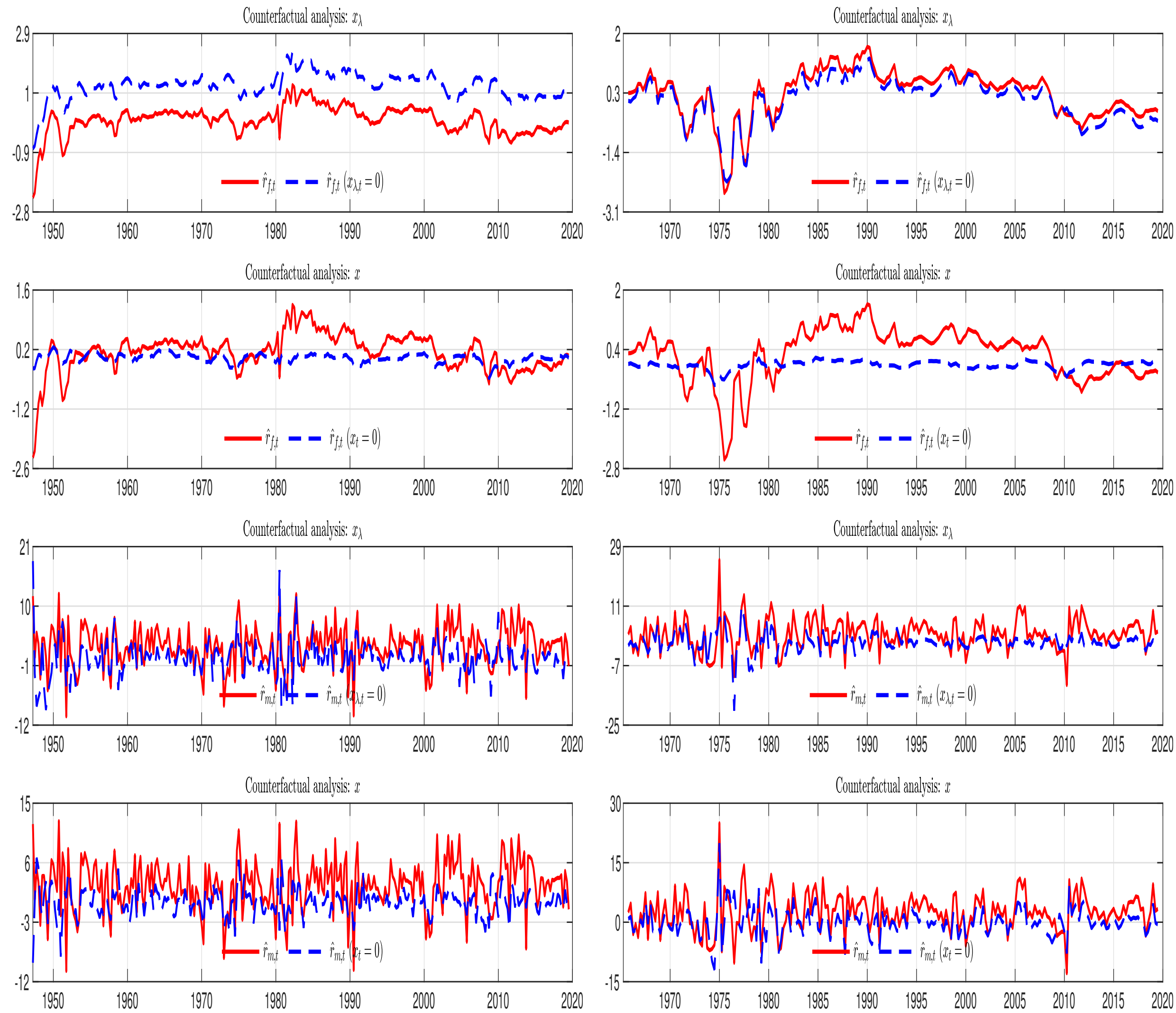
This figure plots the model-implied conditional equity premium and conditional volatility of equity returns, together with per capita consumption growth for the US and UK respectively. For each country, the conditional equity premium and conditional volatility of equity returns are computed from the posterior means of the model parameters and the posterior means of the filtered latent states.



(a) Japan (JP) (b) Australia (AU)

Figure 10: **Conditional risk premium and volatility: Japan and Australia**

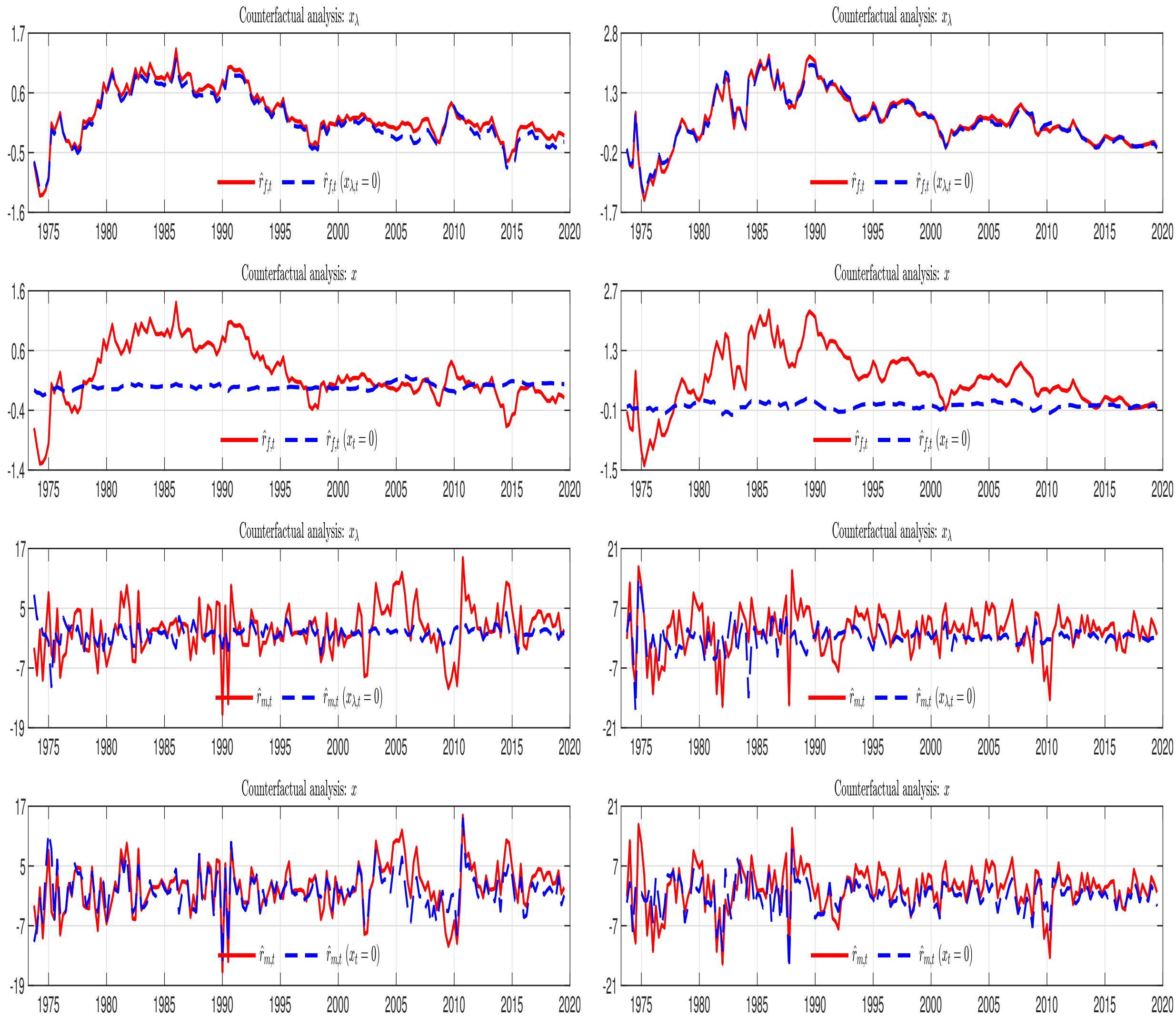
This figure plots the model-implied conditional equity premium and conditional volatility of equity returns, together with per capita consumption growth for Japan and Australia respectively. For each country, the conditional equity premium and conditional volatility of equity returns are computed from the posterior means of the model parameters and the posterior means of the filtered latent states.



(a) US (b) UK

Figure 11: **Counterfactual analysis: US and UK**

This figure plots counterfactual risk-free rates and market returns for the US and UK that would be obtained either in the absence of the preference shock ($x_{\lambda,t}$) or the long-run risk component (x_t). For each country, the results are computed from the posterior means of the model parameters and the posterior means of the filtered latent states.



(a) Japan (JP) (b) Australia (AU)

Figure 12: **Counterfactual analysis: Japan and Australia**

This figure plots counterfactual risk-free rates and market returns for Japan and Australia that would be obtained either in the absence of the preference shock ($x_{\lambda,t}$) or the long-run risk component (x_t). For each country, the results are computed from the posterior means of the model parameters and the posterior means of the filtered latent states.

Internet Appendix to
"Risks for the Long Run: International Evidence"

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First version: February 2021; This version: March 2021

Summary of Contents

- Section 1 describes the log-linear solutions to the long-run risk models with preference shocks and the ARG volatility process.
- Section 2 presents the prior distributions of key parameters in the model.
- Section 3 presents parameter estimates for the alternative models LRR1SV and LRR2SVPref.
- Section 4 contains additional plots for international economies under the benchmark model LRR1SVPref.

1. Log-linear Solutions to the Long-run Risk Models with Preference Shocks and the ARG Volatility Process

In general, log-linear solutions to long-run risk models rely on the approximation to the log-return on the wealth portfolio, $r_t^W = \ln\left(R_t^W\right)$,

$$r_{t+1}^W = wc_{t+1} + \Delta c_{t+1} - \kappa_0 - \kappa_1 wc_t,$$

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where

$$wc_t = \ln\left(\frac{W_t}{C_t}\right),\ \kappa_1 = \frac{e^{\overline{wc}}}{e^{\overline{wc}}-1},\ \kappa_0 = \ln\left(e^{\overline{wc}}-1\right) - \kappa_1\overline{wc},$$

$\overline{wc}$ is the long-run mean of the log wealth-consumption ratio, and the wealth portfolio return is defined as

$$R^W_{t+1} = \frac{W_{t+1}/C_{t+1}}{W_t/C_t - 1}\frac{C_{t+1}}{C_t}.$$

The stochastic discount factor in the model is

$$M_{t+1} = \delta^\theta e^{\theta x_{\lambda,t+1}}\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\theta}{\psi}}\left(R^W_{t+1}\right)^{\theta-1}.$$

The Euler equation for the wealth portfolio is

$$\mathbb{E}_t\left[\exp\left(\theta\ln\delta + \theta x_{\lambda,t+1} - \frac{\theta}{\psi}\Delta c_{t+1} + \theta r^W_{t+1}\right)\right] = 1.$$

The dynamics of the preference shock, consumption growth, dividend growth and the two volatility processes is given by

$$\begin{aligned}
\Delta c_{t+1} &= \mu + x_t + \sigma_t\eta_{c,t+1}, \\
x_{t+1} &= \rho x_t + \phi_x\sigma_t\eta_{x,t+1}, \\
x_{\lambda,t+1} &= \rho_\lambda x_{\lambda,t} + \sigma_\lambda\eta_{\lambda,t+1}, \\
\Delta d_{t+1} &= \mu_d + \Phi x_t + \phi_{dc}\sigma_t\eta_{c,t+1} + \phi_d\sigma_t\eta_{d,t+1},
\end{aligned}$$

where the conditional variance σ_t^2 follows an ARG process with the scale parameter c, persistence parameter ν and degree of freedom ϕ_s. As usual, we first solve for the wealth-consumption ratio and determine the stochastic discount factor. We then solve for the price-dividend ratio.

We conjecture that the solution to the log wealth-consumption ratio is $wc_t = A_0 + A_1 x_t + A_{1\lambda}x_{\lambda,t} + A_2\sigma_t^2$, where A_0, A_1, $A_{1\lambda}$ and A_2 are coefficients to be determined. We substitute the conjectured solution into the Euler equation and obtain

$$
\begin{aligned}
1 &= \mathbb{E}_t\left[\exp\left\{\theta\ln\delta+\theta x_{\lambda,t+1}+\theta\left(1-1/\psi\right)\left(\mu+x_t+\sigma_t\eta_{c,t+1}\right)\right.\right.\\
&\quad -\theta\kappa_0+\theta\left(A_0+A_1x_{t+1}+A_{1\lambda}x_{\lambda,t+1}+A_2\sigma_{t+1}^2\right)\\
&\quad \left.\left.-\theta\kappa_1\left(A_0+A_1x_t+A_{1\lambda}x_{\lambda,t}+A_2\sigma_t^2\right)\right\}\right]\\
&= \mathbb{E}_t\left[\exp\left\{\theta\ln\delta+\theta\left(1+A_{1\lambda}\right)\left(\rho_\lambda x_{\lambda,t}+\sigma_\lambda\eta_{\lambda,t+1}\right)-\theta\kappa_1A_{1\lambda}x_{\lambda,t}\right.\right.\\
&\quad \theta\left(1-1/\psi\right)\left(\mu+x_t+\sigma_t\eta_{c,t+1}\right)-\theta\kappa_0+\theta A_0\\
&\quad \left.\left.+\theta A_1\left(\rho x_t+\phi_x\sigma_t\eta_{x,t+1}\right)-\theta\kappa_1\left(A_0+A_1x_t+A_2\sigma_t^2\right)\right\}\right]\\
&\quad \mathbb{E}_t\left[\exp\left\{\theta A_2\sigma_{t+1}^2\right\}\right],
\end{aligned}
$$

where the second equality follows from the condition independence of σ_{t+1}^2 with σ_t^2 and other innovation shocks. According to Lemma 1 in the Appendix of Gourieroux and Jasiak (2006), the last conditional expectation term is given by

$$
\mathbb{E}_t\left[\exp\left\{\theta A_2\sigma_{t+1}^2\right\}\right]=\left(1-c\theta A_2\right)^{-\phi_s}\exp\left(\frac{\nu\theta A_2}{1-c\theta A_2}\sigma_t^2\right),
$$

where $c=\bar{\sigma}^2\left(1-\nu\right)/\phi_s$; $\bar{\sigma}^2$ is the long-run mean of σ_t^2.

Collecting and matching coefficients yields the following equations

$$
\begin{aligned}
A_1 &= \frac{1-1/\psi}{\kappa_1-\rho},\\
A_{1\lambda} &= \frac{\rho_\lambda}{\kappa_1-\rho_\lambda},
\end{aligned}
$$

$$
\begin{aligned}
\theta\left(1-\kappa_1\right)A_0 &= \phi_s\ln\left(1-c\theta A_2\right)+\theta\kappa_0-\theta\ln\delta\\
&\quad -\theta\left(1-1/\psi\right)\mu-\frac{1}{2}\theta^2\sigma_\lambda^2\left(1+A_{1\lambda}\right)^2,
\end{aligned}
$$

$$
\begin{aligned}
\kappa_1c\theta A_2^2+&\left[-\frac{1}{2}c\theta^2\left(1-1/\psi\right)^2-\frac{1}{2}c\left(\theta A_1\phi_x\right)^2-\kappa_1+\nu\right]A_2\\
&+\frac{1}{2}\theta\left(1-1/\psi\right)^2+\frac{1}{2}\theta\left(A_1\phi_x\right)^2=0.
\end{aligned}
$$

The equation for A_2 is quadratic and has two real roots if its discriminant $Disc=\left(B_\sigma^2-4A_\sigma C_\sigma\right)>$

0, where

$$
\begin{aligned}
A_\sigma &= \kappa_1 c\theta, \\
B_\sigma &= -\frac{1}{2}c\theta^2\left(1-1/\psi\right)^2-\frac{1}{2}c\left(\theta A_1\phi_x\right)^2-\kappa_1+\nu, \\
C_\sigma &= \frac{1}{2}\theta\left(1-1/\psi\right)^2+\frac{1}{2}\theta\left(A_1\phi_x\right)^2.
\end{aligned}
$$

We choose the root that satisfies the requirement of stochastic stability (Hansen, 2012)

$$
A_2=\frac{-B_\sigma+\operatorname{sign}\left(B_\sigma\right)\sqrt{B_\sigma^2-4A_\sigma C_\sigma}}{2A_\sigma}.
$$

The log risk-free rate is given by

$$
r_{f,t}=-\ln\left(\mathbb{E}_t\left[M_{t+1}\right]\right)
$$

and

$$
\mathbb{E}_t\left[M_{t+1}\right]=\mathbb{E}_t\left[\exp\left(\theta\ln\delta-\frac{\theta}{\psi}\Delta c_{t+1}+\left(\theta-1\right)r_{t+1}^W\right)\right].
$$

We can substitute the log-linear approximation to r_{t+1}^W into the Euler equation and obtain the solution to $r_{f,t}$. The solution to $r_{f,t}$ is

$$
r_{f,t}=B_{f0}+B_{f1\lambda}x_{\lambda,t}+B_{f1}x_t+B_2\sigma_t^2
$$

where

$$
B_{f0}=-A_{f0},\ B_{f1\lambda}=-A_{f1\lambda},\ B_{f1}=-A_{f1},\ B_{f2}=-A_{f2},
$$

and

$$
\begin{aligned}
A_{f0} &= \theta \ln(\delta) + \frac{1}{2}\left((\theta-1) A_{1\lambda} + \theta\right)^2 \sigma_\lambda^2 + \left(\theta - 1 - \frac{\theta}{\psi}\right)\mu \\
&\quad + (\theta-1)(A_0 - \kappa_0 - \kappa_1 A_0) - \phi_s \ln\left(1 - c(\theta-1) A_2\right) \\
A_{f1\lambda} &= (\theta-1) A_{1\lambda}(\rho_\lambda - \kappa_1) + \theta\rho_\lambda \\
A_{f1} &= \left(\theta - 1 - \frac{\theta}{\psi}\right) + (\theta-1) A_1 (\rho - \kappa_1) \\
A_{f2} &= \frac{1}{2}\left(\theta - 1 - \frac{\theta}{\psi}\right)^2 + \frac{1}{2}\left((\theta-1) A_1 \phi_x\right)^2 \\
&\quad - (\theta-1) A_2 \kappa_1 + \frac{\nu(\theta-1) A_2}{1 - c(\theta-1) A_2}.
\end{aligned}
$$

The log return on the market portfolio, $r_{m,t} = \ln(R_{m,t})$, is

$$
r_{m,t+1} = \kappa_{m0} + \kappa_{m1} pd_{t+1} - pd_t + \Delta d_{t+1},
$$

where

$$
pd_t = \ln\left(\frac{P_t}{D_t}\right), \ \kappa_{m1} = \frac{e^{\overline{pd}}}{e^{\overline{pd}} + 1}, \ \kappa_{m0} = \ln\left(e^{\overline{pd}} + 1\right) - \kappa_{m1}\overline{pd},
$$

$\overline{pd}$ is the long-run mean of the log price-dividend ratio, and the market portfolio return is defined as

$$
R_{m,t+1} = \frac{P_{t+1}/D_{t+1} + 1}{P_t/D_t} \frac{D_{t+1}}{D_t}.
$$

The Euler equation for the market portfolio is

$$
\mathbb{E}_t\left[\exp\left(\theta \ln\delta + \theta x_{\lambda,t+1} - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta-1) r_{t+1}^W + r_{m,t+1}\right)\right] = 1.
$$

We conjecture that the solution to the log price-dividend ratio is $pd_t = A_{m0} + A_{m1}x_t + A_{m1\lambda}x_{\lambda,t} + A_{m2}\sigma_t^2$. By substituting the conjectured solution into the Euler equation, collecting and matching coefficients, we obtain the following system of equations

$$
\begin{aligned}
A_{m1} &= \frac{\Phi - 1/\psi}{1 - \kappa_{m1}\rho}, \\
A_{m1\lambda} &= \frac{\rho_\lambda}{1 - \kappa_{m1}\rho_\lambda},
\end{aligned}
$$

$$\begin{aligned}(1-\kappa_{m1})\,A_{m0} &= \theta\ln\delta+(\theta-1-\theta/\psi)\,\mu+(\theta-1)\,(A_0-\kappa_1A_0-\kappa_0)\\ &\quad+\kappa_{m0}+\mu_d+\frac{1}{2}\left(\theta+(\theta-1)\,A_{1\lambda}+\kappa_{m1}A_{m1\lambda}\right)^2\sigma_\lambda^2\\ &\quad-\phi_s\ln\left(1-c\left((\theta-1)\,A_2+\kappa_{m1}A_{m2}\right)\right)\end{aligned}$$

$$\begin{aligned}0 &= \frac{1}{2}\left(\theta-1-\theta/\psi+\phi_{dc}\right)^2+\frac{1}{2}\left((\theta-1)\,A_1+\kappa_{m1}A_{m1}\right)^2\phi_x^2\\ &\quad+\frac{1}{2}\phi_d^2-(\theta-1)\,\kappa_1A_2-A_{m2}+\frac{\nu\left[(\theta-1)\,A_2+\kappa_{m1}A_{m2}\right]}{1-c\left[(\theta-1)\,A_2+\kappa_{m1}A_{m2}\right]}.\end{aligned}$$

The equation for A_{m2} is quadratic and can be written as

$$A_\sigma^m A_{m2}^2+B_\sigma^m A_{m2}+C_\sigma^m=0.$$

If its discriminant $Disc=(B_\sigma^m)^2-4A_\sigma^m C_\sigma^m>0$, we choose the root that maintains stochastic stability:

$$A_{m2}=\frac{-B_\sigma^m+\operatorname{sign}(B_\sigma^m)\sqrt{(B_\sigma^m)^2-4A_\sigma^m C_\sigma^m}}{2A_\sigma^m}.$$

It can be shown that conditional risk premium is given by

$$\begin{aligned}\mathbb{E}_t\left[r_{m,t+1}\right]-r_{f,t} &= \kappa_{m0}+\kappa_{m1}A_{m0}-A_{m0}+\mu_d-B_{f0}+\kappa_{m1}A_{m2}\bar{\sigma}^2(1-\nu)\\ &\quad+\left(\kappa_{m1}A_{m1}\rho-A_{m1}+\Phi-B_{f1}\right)x_t\\ &\quad+\left(\kappa_{m1}A_{m1\lambda}\rho_\lambda-A_{m1\lambda}-B_{f1\lambda}\right)x_{\lambda,t}\\ &\quad+\left(\kappa_{m1}A_{m2}\nu-A_{m2}-B_{f2}\right)\sigma_t^2\end{aligned}$$

The expression of conditional volatility of market returns is too cumbersome and thus omitted.

An extended model We also allow for a separate ARG volatility process in the dividend growth process

$$\Delta d_{t+1}=\mu_d+\Phi x_t+\phi_{dc}\sigma_t\eta_{c,t+1}+\phi_d\sigma_{d,t}\eta_{d,t+1},$$

in which the conditional variance $\sigma_{d,t}^2$ follows an ARG process with the scale parameter c_d, persistence parameter ν_d and degree of freedom $\phi_{d,s}$. The process $\sigma_{d,t}^2$ is independent of σ_t^2. In such a case, we conjecture that the solution to the log price-dividend ratio is $pd_t=A_{m0}+A_{m1}x_t+A_{m1\lambda}x_{\lambda,t}+A_{m2}\sigma_t^2+A_{m2d}\sigma_{d,t}^2$. By substituting the conjectured solution into the Euler equation, collecting and matching coefficients, we obtain the following system of equations

$$
\begin{aligned}
A_{m1} &= \frac{\Phi - 1/\psi}{1 - \kappa_{m1}\rho}, \\
A_{m1\lambda} &= \frac{\rho_\lambda}{1 - \kappa_{m1}\rho_\lambda},
\end{aligned}
$$

$$
\begin{aligned}
0 &= \theta \ln \delta + (\theta - 1 - \theta/\psi)\,\mu + (\theta - 1)\,(A_0 - \kappa_1 A_0 - \kappa_0) \\
&\quad + \kappa_{m0} + \kappa_{m1} A_{m0} - A_{m0} + \mu_d \\
&\quad + \frac{1}{2}\left(\theta + (\theta - 1)\,A_{1\lambda} + \kappa_{m1} A_{m1\lambda}\right)^2 \sigma_\lambda^2 \\
&\quad - \phi_s \ln\left(1 - c\left[(\theta - 1)\,A_2 + \kappa_{m1} A_{m2}\right]\right) \\
&\quad - \phi_{d,s} \ln\left(1 - c_d \kappa_{m1} A_{m2d}\right)
\end{aligned}
$$

$$
\begin{aligned}
0 &= \frac{1}{2}\left(\theta - 1 - \theta/\psi + \phi_{dc}\right)^2 + \frac{1}{2}\left((\theta - 1)\,A_1 + \kappa_{m1} A_{m1}\right)^2 \phi_x^2 \\
&\quad - (\theta - 1)\,\kappa_1 A_2 - A_{m2} + \frac{\nu\left[(\theta - 1)\,A_2 + \kappa_{m1} A_{m2}\right]}{1 - c\left[(\theta - 1)\,A_2 + \kappa_{m1} A_{m2}\right]}.
\end{aligned}
$$

$$
\frac{1}{2}\phi_d^2 - A_{m2d} + \frac{\nu_d \kappa_{m1} A_{m2d}}{1 - c_d \kappa_{m1} A_{m2d}} = 0
$$

Similarly, the equation for A_{m2d} is quadratic, and we choose the root that maintains stochastic stability.

2. The Prior Distributions

The table below presents the exact distributional form, the support, and the hyper-parameters of the prior distribution for each parameter. We assume normal distributions as priors; however, if a parameter under consideration has a bounded support, we choose a truncated normal distribution or a uniform distribution as its prior. In the full model with two stochastic volatility processes, the parameter of ϕ_d is normalized to 1.

Table 1: **Prior Distributions**

Θ	D. Form	Support	Hyper	Θ	D. Form	Support	Hyper
δ	Uniform	(0, 1)	(0.80, 1.00)	ϕ_s	Tr. Normal	(1, ∞)	(2.00, 4.00)
γ	Tr. Normal	(0, ∞)	(6.00, 2.00)	μ_d	Normal	(-∞, ∞)	($\bar{\mu}_d$, 1e-8)
ψ	Tr. Normal	(0, ∞)	(2.00, 0.50)	Φ	Tr. Normal	(0, ∞)	(3.00, 6.00)
ρ_λ	Uniform	(-1, 1)	(-1.00, 1.00)	ϕ_{dc}	Tr. Normal	(0, ∞)	(3.00, 6.00)
σ_λ	Tr. Normal	(0, ∞)	(0.002, 0.01)	ϕ_d	Tr. Normal	(0, ∞)	(5.00, 6.00)
μ	Normal	(-∞, ∞)	($\bar{\mu}$, 1e-8)	$\bar{\sigma}_{sd}$	Tr. Normal	(0, ∞)	(0.006, 0.005)
ρ	Uniform	(-1, 1)	(-1.00, 1.00)	ρ_{sd}	Uniform	(-1, 1)	(-1.00, 1.00)
ϕ_x	Uniform	(-1, 1)	(-1.00, 1.00)	σ_m	Tr. Normal	(0, ∞)	(0.03, 0.10)
$\bar{\sigma}$	Tr. Normal	(0, ∞)	(0.004, 0.005)	σ_f	Tr. Normal	(0, ∞)	(0.003, 0.01)
ρ_s	Uniform	(-1, 1)	(-1.00, 1.00)				

3. Parameter Estimates for Alternative Models

Table 2 presents the posterior estimates of parameters in the model abstracted from the preference shock for all the ten developed countries. This model is analyzed by Fulop et al. (2020), who perform the SMC-Bayesian estimation on the US market. The present analysis extends theirs to other developed markets. A comparison of the results to related results based on estimation of the model with the preference shock suggests that the preference shock is important mainly for identifying the risk aversion and EIS parameters. Without the preference shock, the posterior estimates of γ become lower and have higher standard deviations than those for the benchmark estimation. In particular, for Germany, France and Australia, the 5% percentile of the γ estimates is around 1. In addition, the EIS estimates for developed markets other than the US, UK and Switzerland are significantly below 1. These findings are inconsistent with values of ψ that are typically assumed in the calibration studies on long-run risks (e.g., Bansal and Yaron (2004) and Bansal et al. (2012)). For the dynamics of consumption and dividend growth rates in developed countries, the estimation is able to identify a persistent component in expected consumption and a less persistent stochastic volatility process.

Table 3 presents the posterior parameter estimates for the LRR2SVPref model, which incorporates the preference shock and an independent stochastic volatility process in the dividend growth process. We find that the persistence of conditional volatility in the consumption growth dynamics decreases when a separate stochastic volatility process is included in the dividend growth dynamics. For most of developed countries such as Germany, France, Japan and Australia, the idiosyncratic shock to dividend growth does not show persistent variation in conditional volatility.

Table 2: **Parameter Estimates: LRR1SV**

	Mean	Std	5%	95%	Mean	Std	5%	95%
	US				UK			
δ	0.9980	0.0003	0.9974	0.9985	0.9976	0.0004	0.9970	0.9982
γ	7.6463	0.9752	6.0667	9.3653	7.3833	1.2468	5.4272	9.4749
ψ	1.2599	0.0538	1.1760	1.3618	1.1452	0.1026	0.9757	1.3114
ρ_x	0.9865	0.0026	0.9819	0.9903	0.9082	0.0203	0.8739	0.9408
ϕ_x	0.2480	0.0268	0.2044	0.2905	0.1916	0.0199	0.1605	0.2231
$\bar{\sigma}$	0.0048	0.0001	0.0046	0.0049	0.0106	0.0002	0.0102	0.0109
ρ_s	0.7347	0.0365	0.6737	0.7909	0.6855	0.0388	0.6149	0.7449
ϕ_s	2.0541	0.2821	1.6095	2.5523	1.6543	0.2323	1.2704	2.0613
Φ	0.9188	0.0681	0.8140	1.0417	1.0210	0.1238	0.8308	1.2332
ϕ_{dc}	0.8286	0.2074	0.5143	1.1942	0.5807	0.0965	0.4274	0.7462
ϕ_d	4.6438	0.2458	4.2374	5.0646	1.7533	0.1202	1.5625	1.9600
	DE				FR			
δ	0.9971	0.0005	0.9963	0.9978	0.9988	0.0004	0.9981	0.9993
γ	1.3155	0.6662	0.3724	2.5567	2.6441	1.6320	0.5009	5.7076
ψ	0.4066	0.0553	0.3221	0.5074	0.4284	0.0447	0.3576	0.5050
ρ_x	0.9730	0.0058	0.9626	0.9809	0.9627	0.0145	0.9352	0.9805
ϕ_x	0.0788	0.0130	0.0611	0.1039	0.1319	0.0177	0.1061	0.1625
$\bar{\sigma}$	0.0086	0.0002	0.0083	0.0089	0.0059	0.0002	0.0054	0.0062
ρ_s	0.2539	0.1158	0.1031	0.4623	0.6286	0.1141	0.4160	0.7897
ϕ_s	2.7211	0.3662	2.1402	3.3472	2.6117	0.5249	1.7633	3.5443
Φ	0.8816	0.2921	0.4773	1.3818	1.3773	0.3408	0.8791	1.9757
ϕ_{dc}	1.9088	0.2307	1.5400	2.2949	0.8916	0.2110	0.5606	1.2509
ϕ_d	3.8611	0.2623	3.4734	4.3254	4.1831	0.2849	3.7166	4.6635
	NL				CH			
δ	0.9986	0.0003	0.9981	0.9990	0.9990	0.0001	0.9988	0.9992
γ	5.6440	1.7308	3.1758	8.6140	7.1346	1.3203	4.9078	9.2125
ψ	0.5135	0.0588	0.4238	0.6124	2.0838	0.2510	1.6715	2.4806
ρ_x	0.9173	0.0223	0.8792	0.9497	0.8918	0.0201	0.8553	0.9214
ϕ_x	0.1537	0.0232	0.1203	0.1957	0.1731	0.0295	0.1277	0.2226
$\bar{\sigma}$	0.0085	0.0003	0.0080	0.0089	0.0053	0.0002	0.0050	0.0056
ρ_s	0.6362	0.0798	0.4920	0.7549	0.5489	0.1271	0.3331	0.7606
ϕ_s	2.1903	0.4422	1.4885	2.9360	1.7563	0.3520	1.2488	2.3959
Φ	1.9021	0.3438	1.4294	2.5362	9.2183	1.5216	6.8527	11.8925
ϕ_{dc}	0.8651	0.1946	0.5708	1.2177	0.5072	0.1578	0.2949	0.8067
ϕ_d	3.5780	0.2943	3.1492	4.0967	3.8270	0.2813	3.3576	4.3249

	IT				JP			
δ	0.9985	0.0004	0.9978	0.9992	0.9985	0.0003	0.9980	0.9988
γ	3.3115	1.4161	0.9397	5.6125	6.7542	1.2677	4.7539	8.8883
ψ	0.5120	0.0529	0.4242	0.6004	1.1837	0.1147	0.9899	1.3627
ρ_x	0.9349	0.0149	0.9073	0.9563	0.9169	0.0173	0.8845	0.9428
ϕ_x	0.1756	0.0224	0.1411	0.2152	0.1776	0.0187	0.1483	0.2083
$\bar{\sigma}$	0.0070	0.0003	0.0065	0.0074	0.0112	0.0002	0.0108	0.0114
ρ_s	0.6898	0.0707	0.5640	0.7967	0.5884	0.0539	0.4990	0.6671
ϕ_s	2.3072	0.5835	1.4028	3.2675	1.6282	0.2211	1.2786	2.0057
Φ	1.9817	0.4524	1.3407	2.7670	0.6740	0.1624	0.4328	0.9467
ϕ_{dc}	1.3130	0.3284	0.8462	1.9205	0.5149	0.1061	0.3550	0.7061
ϕ_d	6.2528	0.4534	5.5431	7.0474	2.2594	0.1647	2.0073	2.5457
	CA				AU			
δ	0.9979	0.0002	0.9975	0.9982	0.9986	0.0004	0.9979	0.9992
γ	8.2064	1.3342	5.9940	10.4040	4.0802	1.8065	1.2320	7.2552
ψ	0.6783	0.0672	0.5687	0.7915	0.6491	0.0771	0.5263	0.7788
ρ_x	0.9098	0.0152	0.8847	0.9340	0.9393	0.0152	0.9121	0.9620
ϕ_x	0.1788	0.0193	0.1505	0.2108	0.1620	0.0218	0.1310	0.2021
$\bar{\sigma}$	0.0072	0.0002	0.0067	0.0076	0.0083	0.0002	0.0079	0.0087
ρ_s	0.5829	0.0940	0.4092	0.7272	0.7188	0.0642	0.6057	0.8161
ϕ_s	2.0539	0.2886	1.6018	2.5259	2.7810	0.6892	1.6475	3.9095
Φ	1.0550	0.2029	0.7557	1.4049	1.0366	0.2263	0.6831	1.4284
ϕ_{dc}	0.7960	0.1104	0.6225	0.9805	0.8450	0.1186	0.6512	1.0377
ϕ_d	3.7989	0.2313	3.4403	4.2079	2.8244	0.1849	2.5295	3.1422

Note:

Table 3: **Parameter Estimates: LRR2SVPref**

	Mean	Std	5%	95%	Mean	Std	5%	95%
	US				UK			
δ	0.9989	0.0003	0.9983	0.9994	0.9916	0.0021	0.9880	0.9946
γ	8.7056	1.1621	6.8000	10.7410	8.4978	1.8846	5.4266	11.4865
ψ	1.4363	0.1763	1.2159	1.7486	2.0736	0.3746	1.4692	2.7051
ρ_λ	0.9949	0.0013	0.9925	0.9967	0.9502	0.0131	0.9264	0.9689
ϕ_λ	0.0012	0.0001	0.0011	0.0014	0.0020	0.0003	0.0016	0.0024
ρ_x	0.8352	0.0415	0.7635	0.8990	0.9003	0.0236	0.8619	0.9356
ϕ_x	0.3239	0.0506	0.2442	0.4122	0.1712	0.0323	0.1193	0.2206
$\bar{\sigma}$	0.0045	0.0001	0.0043	0.0047	0.0102	0.0003	0.0097	0.0106
ρ_s	0.7555	0.0517	0.6594	0.8262	0.7146	0.0672	0.5920	0.8052
ϕ_s	1.6278	0.3224	1.1478	2.1767	1.0385	0.0216	1.0076	1.0765
Φ	3.6711	0.8821	2.4351	5.2824	2.6162	0.8965	1.6488	4.4492
ϕ_{dc}	0.6424	0.1030	0.4864	0.8151	0.1669	0.0463	0.1025	0.2507
$\bar{\sigma}_d$	0.0216	0.0008	0.0201	0.0226	0.0157	0.0011	0.0137	0.0173
ρ_{ds}	0.7196	0.0381	0.6504	0.7758	0.4938	0.1205	0.2919	0.6820
ϕ_{ds}	1.0956	0.0448	1.0255	1.1734	2.9758	0.7519	1.7705	4.2189
	DE				FR			
δ	0.9778	0.0015	0.9755	0.9803	0.9921	0.0014	0.9898	0.9941
γ	4.1250	1.0082	2.3869	5.7332	7.5785	1.2159	5.3230	9.5004
ψ	2.2398	0.2055	1.9120	2.5824	1.9733	0.2438	1.5902	2.3348
ρ_λ	0.9870	0.0023	0.9827	0.9904	0.9363	0.0125	0.9153	0.9556
ϕ_λ	0.0014	0.0001	0.0012	0.0015	0.0022	0.0002	0.0019	0.0025
ρ_x	0.8284	0.0275	0.7810	0.8732	0.8387	0.0275	0.7937	0.8821
ϕ_x	0.1120	0.0142	0.0897	0.1373	0.1772	0.0170	0.1536	0.2050
$\bar{\sigma}$	0.0081	0.0002	0.0077	0.0085	0.0060	0.0002	0.0057	0.0062
ρ_s	0.3517	0.0957	0.1991	0.5193	0.5715	0.1502	0.2693	0.7810
ϕ_s	1.0751	0.0400	1.0158	1.1457	4.0402	0.8134	2.6086	5.2349
Φ	10.8839	1.4056	8.8415	13.4907	9.4696	0.9357	7.9291	10.9558
ϕ_{dc}	0.3226	0.0776	0.2034	0.4706	0.3488	0.0805	0.2334	0.5006
$\bar{\sigma}_d$	0.0200	0.0018	0.0169	0.0230	0.0191	0.0015	0.0162	0.0212
ρ_{ds}	0.6412	0.1385	0.3643	0.8354	0.3205	0.1000	0.1713	0.4983
ϕ_{ds}	3.0189	0.8029	1.7592	4.5020	1.0221	0.0128	1.0035	1.0451
	NL				CH			
δ	0.9934	0.0018	0.9902	0.9960	0.9985	0.0006	0.9973	0.9993
γ	7.3222	1.3084	5.1554	9.3284	6.1003	1.3874	3.7463	8.4475
ψ	1.8558	0.3183	1.2440	2.3674	2.0128	0.3933	1.3198	2.7097
ρ_λ	0.8893	0.0281	0.8382	0.9319	0.8118	0.0320	0.7583	0.8604
ϕ_λ	0.0030	0.0004	0.0025	0.0037	0.0030	0.0003	0.0025	0.0035
ρ_x	0.8960	0.0254	0.8537	0.9333	0.9082	0.0216	0.8679	0.9376
ϕ_x	0.1559	0.0272	0.1198	0.2079	0.1391	0.0241	0.1011	0.1795
$\bar{\sigma}$	0.0088	0.0003	0.0083	0.0092	0.0054	0.0001	0.0051	0.0056
ρ_s	0.6847	0.0874	0.5331	0.8106	0.4657	0.1097	0.2896	0.6515
ϕ_s	2.4009	0.5461	1.5656	3.3217	2.7533	0.6831	1.6641	3.9883
Φ	5.4118	0.9176	4.0843	6.8676	9.2821	1.1893	7.5629	11.2998
ϕ_{dc}	0.2433	0.0692	0.1526	0.3721	0.3686	0.1036	0.2249	0.5506
$\bar{\sigma}_d$	0.0248	0.0016	0.0220	0.0271	0.0220	0.0014	0.0195	0.0242
ρ_{ds}	0.5734	0.0864	0.4184	0.7040	0.6352	0.1024	0.4534	0.7904
ϕ_{ds}	1.0430	0.0229	1.0073	1.0821	1.0571	0.0305	1.0114	1.1116

	IT				JP			
δ	0.9928	0.0014	0.9904	0.9949	0.9957	0.0006	0.9947	0.9966
γ	5.6355	1.0796	3.9269	7.5098	8.2829	0.9962	6.6580	9.9429
ψ	1.9906	0.3085	1.4418	2.4958	3.0091	0.3031	2.4948	3.4771
ρ_λ	0.9536	0.0101	0.9358	0.9685	0.9731	0.0056	0.9639	0.9813
ϕ_λ	0.0026	0.0002	0.0022	0.0030	0.0015	0.0001	0.0013	0.0017
ρ_x	0.8327	0.0357	0.7729	0.8877	0.8790	0.0125	0.8574	0.9000
ϕ_x	0.2428	0.0316	0.1935	0.2968	0.1021	0.0083	0.0893	0.1164
$\bar{\sigma}$	0.0072	0.0002	0.0068	0.0075	0.0110	0.0002	0.0106	0.0113
ρ_s	0.3344	0.0844	0.1935	0.4722	0.1976	0.0627	0.1101	0.3097
ϕ_s	1.0317	0.0179	1.0065	1.0626	3.2011	0.4110	2.5182	3.8986
Φ	7.2422	1.4924	5.0678	9.8900	7.8430	0.6614	6.8183	9.0248
ϕ_{dc}	0.7080	0.1788	0.4582	1.0350	0.2181	0.0365	0.1668	0.2888
$\bar{\sigma}_d$	0.0403	0.0025	0.0359	0.0441	0.0132	0.0009	0.0117	0.0146
ρ_{ds}	0.6540	0.0934	0.4750	0.7786	0.4868	0.0992	0.3189	0.6449
ϕ_{ds}	1.9554	0.4978	1.1817	2.8379	1.0503	0.0283	1.0093	1.1010
	CA				AU			
δ	0.9930	0.0011	0.9911	0.9946	0.9947	0.0014	0.9920	0.9966
γ	6.8916	1.0649	5.1643	8.5118	6.6876	1.1798	4.6250	8.5975
ψ	2.0127	0.2978	1.4975	2.4958	2.0672	0.2523	1.6594	2.5051
ρ_λ	0.9138	0.0139	0.8890	0.9354	0.9392	0.0181	0.9079	0.9631
ϕ_λ	0.0023	0.0002	0.0021	0.0026	0.0025	0.0003	0.0021	0.0030
ρ_x	0.8570	0.0190	0.8259	0.8857	0.7918	0.0320	0.7390	0.8418
ϕ_x	0.1290	0.0165	0.1041	0.1583	0.1313	0.0190	0.1025	0.1640
$\bar{\sigma}$	0.0077	0.0002	0.0073	0.0080	0.0087	0.0002	0.0084	0.0090
ρ_s	0.4825	0.1316	0.2592	0.6837	0.5645	0.1305	0.3324	0.7527
ϕ_s	3.4762	0.6117	2.5385	4.4915	3.6629	0.7081	2.5589	4.8724
Φ	8.5161	0.9456	7.1073	10.0714	8.3752	1.1984	6.7460	10.4095
ϕ_{dc}	0.1552	0.0455	0.0971	0.2360	0.2686	0.0614	0.1741	0.3733
$\bar{\sigma}_d$	0.0183	0.0013	0.0162	0.0203	0.0204	0.0014	0.0181	0.0226
ρ_{ds}	0.6348	0.0761	0.5072	0.7625	0.3696	0.0784	0.2509	0.4990
ϕ_{ds}	1.0478	0.0272	1.0068	1.0953	1.0263	0.0143	1.0051	1.0512

Note:

4. Additional Plots for International Economies

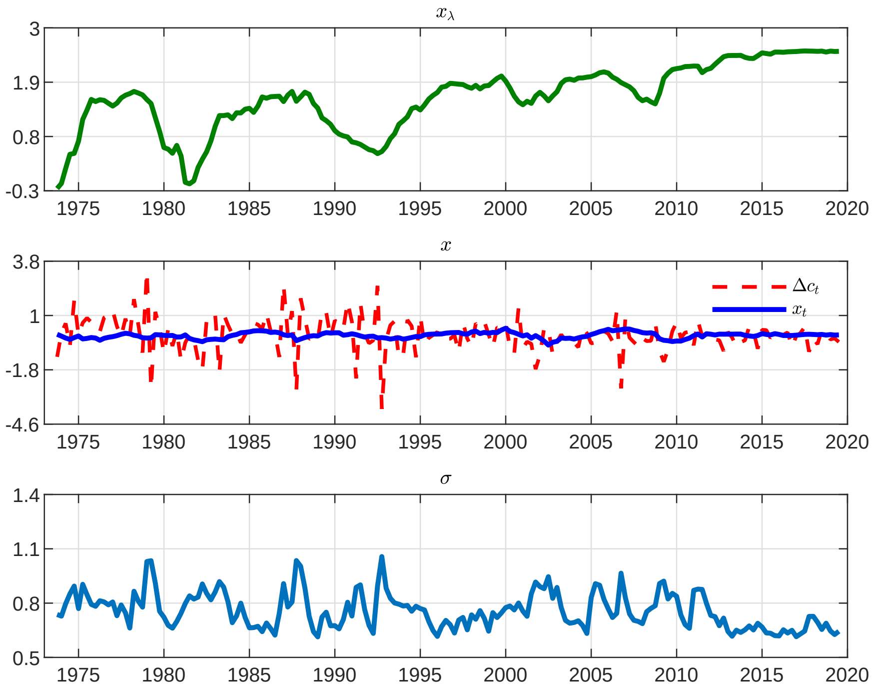
Figures 1–3 display the posterior means of the filtered latent states, including the growth rate of preference shocks ($x_{\lambda,t}$), the long-run consumption component (x_t), and the consumption volatility (σ_t), for Germany, France, Netherlands, Switzerland, Italy and Canada. The posterior mean of $x_{\lambda,t}$ falls in early to mid-1980s, early 1990s and the 2008 global financial crisis. Nevertheless, comovement in the preference shock across different countries is not strong on average but becomes more significant in the recent two decades. In addition, the preference shock has more variation in the first half of the sample than in the second. For France, Italy, Netherlands and Switzerland, the posterior mean of $x_{\lambda,t}$ is negatively correlated with that of x_t.

Figures 1–3 also show that a persistent component in expected growth exists in all countries' consumption dynamics. The filtered expected consumption growth rises in good times while decreases in bad times, though the extent of its correlation with the actual consumption growth data differs across countries significantly. The correlation between the filtered x_t and the actual consumption growth data is about 0.3–0.35 for Italy and Netherlands, 0.2–0.25 for France, Canada and Switzerland, and 0.07 for Germany. The plots of the filtered stochastic volatility component σ_t indicate that European countries including Germany, France, Italy, Netherlands and Switzerland have similar time variation of σ_t, where notable rises in σ_t occurred in early and late 1980s, periods around the formation of EU, and periods around the 2008 global financial crisis as well as the European sovereign debt crisis.

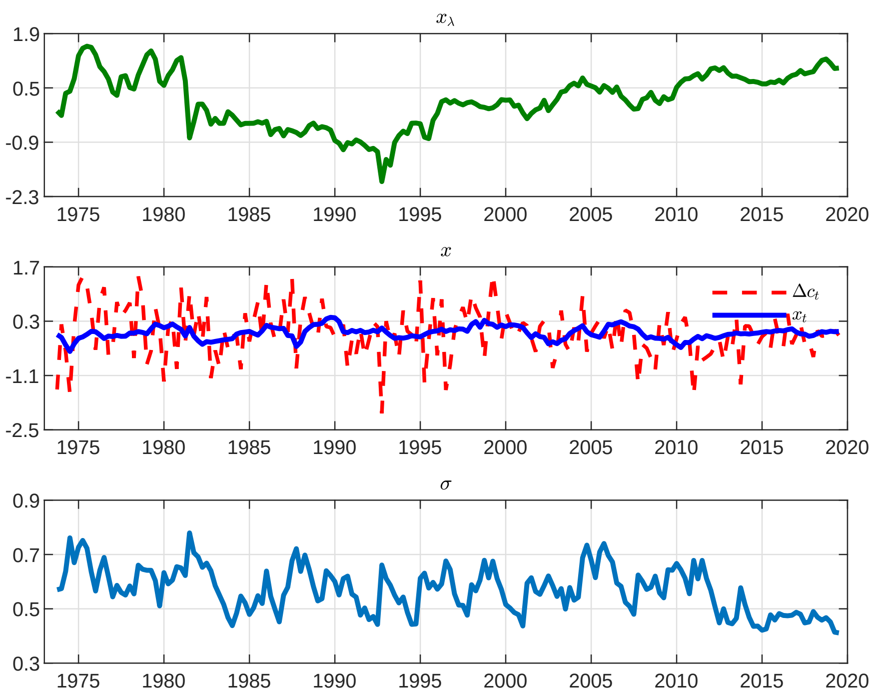
For those countries in the above-mentioned analysis, our estimation produces the time series of the filtered risk-free rates that fits the data considerably well. Because in the model the risk-free rate is the reciprocal of conditional expectation of the stochastic discount factor (SDF), the result implies that our estimation delivers reasonable dynamics of the SDF. Moreover, our estimation generates fitted risk-free rates and market returns that well capture strong comovements in the actual risk-free rates and market returns across different countries.

For the six countries discussed in the Appendix, we observe in Figures 7–9 that for each country the estimated SDF has a countercyclical component that covaries negatively with the actual consumption growth. Among them, the correlation of the model-implied SDF with the actual consumption growth is between -0.3 and -0.2 for France, Netherlands, Switzerland and Canada and is very modest for Germany and Italy. On the other hand, results are mixed across countries with reference to the cyclicality of conditional equity premium and conditional volatility of equity returns, both of which are implied by the model's parameter estimates and the filtered latent states. The conditional equity premium and conditional volatility of equity returns of Netherlands, Switzerland and Canada have clear countercyclical components, whereas the cyclicality is very weak for the remaining countries.

Figures 13–18 display counterfactual risk-free rate and market returns that would be obtained either in the absence of the preference shock or the long-run risk component for the six above-mentioned countries. We observe that except for Switzerland and Italy, the preference shock exerts significant impacts on the risk-free rate in all countries. The risk-free rate without the preference shock becomes lower than the model-fitted risk-free rate. Furthermore, the impact of the preference shock on market returns is noteworthy for all countries. This suggest that the valuation risk explains a significant proportion of market risks and leads to non-negligible risk premium. Similar to the results in the main paper, the highly persistent component in expected consumption growth also matters substantially for explaining the behavior of the risk-free rate and market returns in each country.

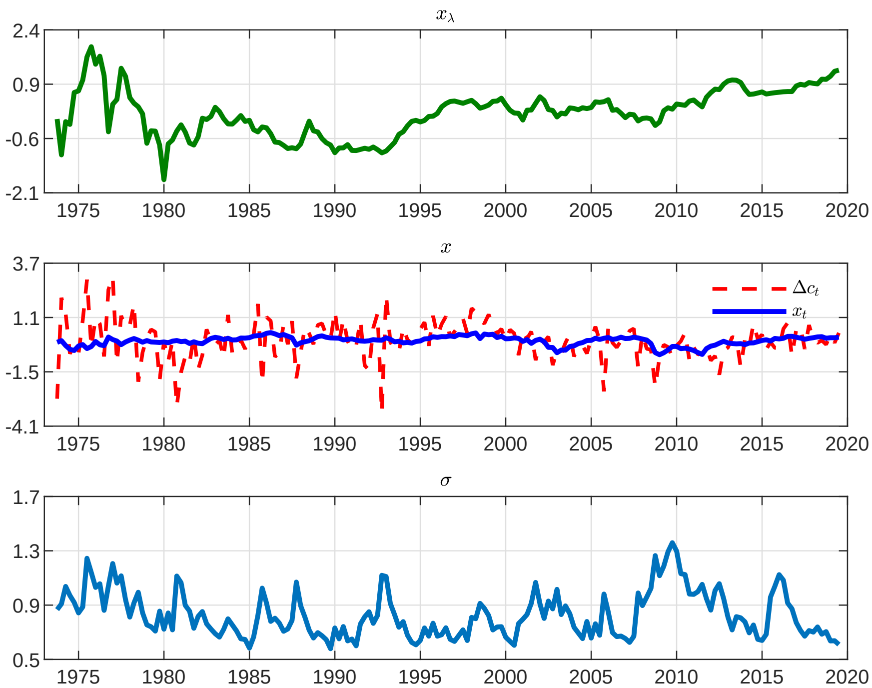


(a) Germany (DE)

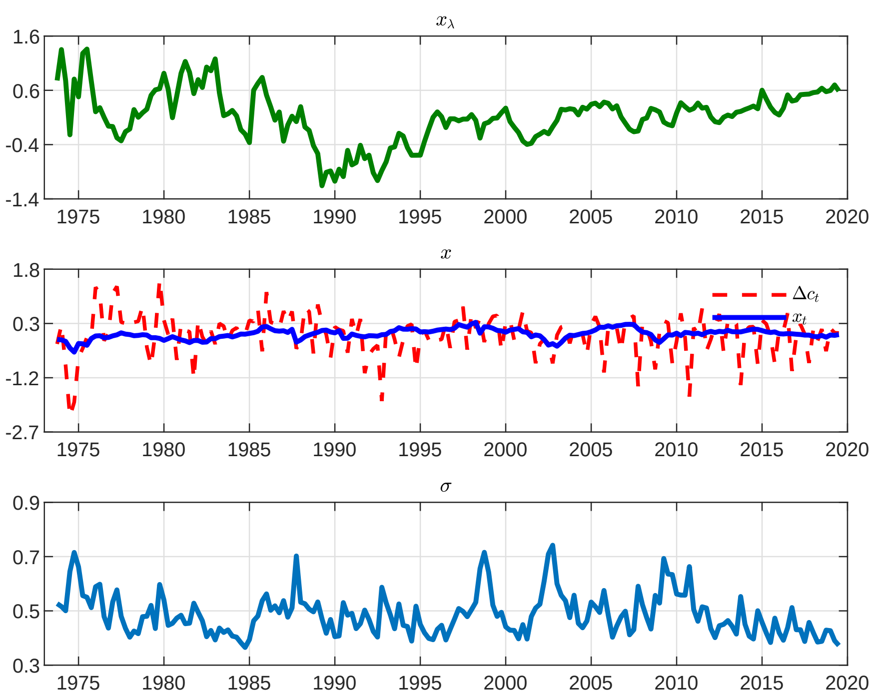


(b) France (FR)

Figure 1: **Filtered states: Germany and France**

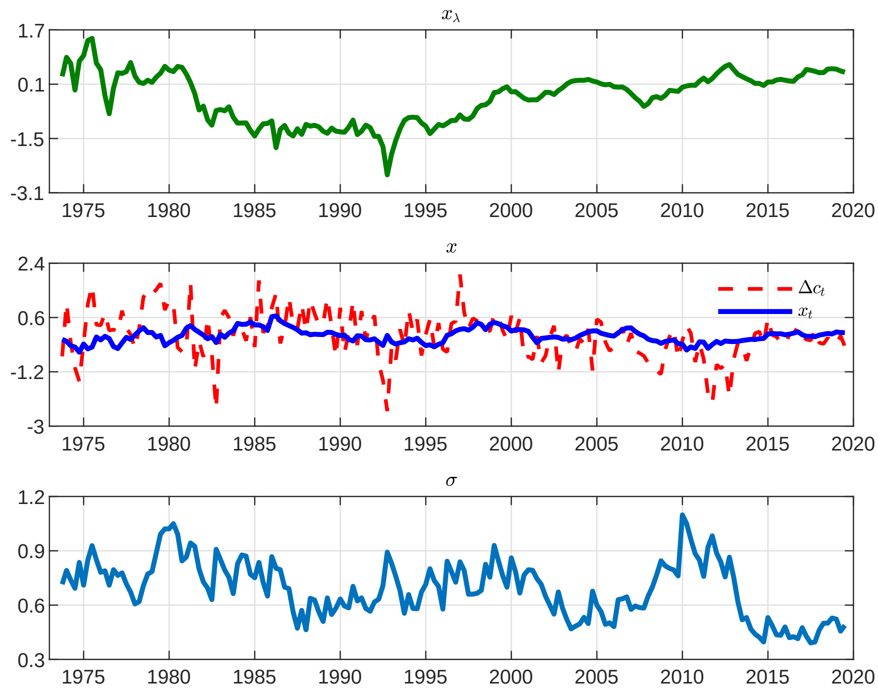


(a) Netherlands (NL)

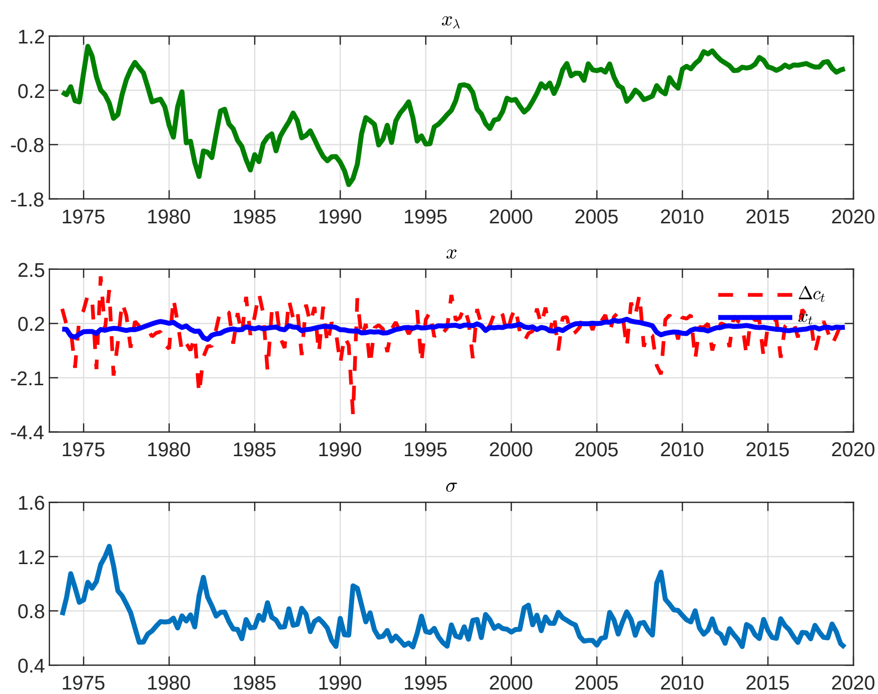


(b) Switzerland (CH)

Figure 2: **Filtered states: Netherlands and Switzerland**

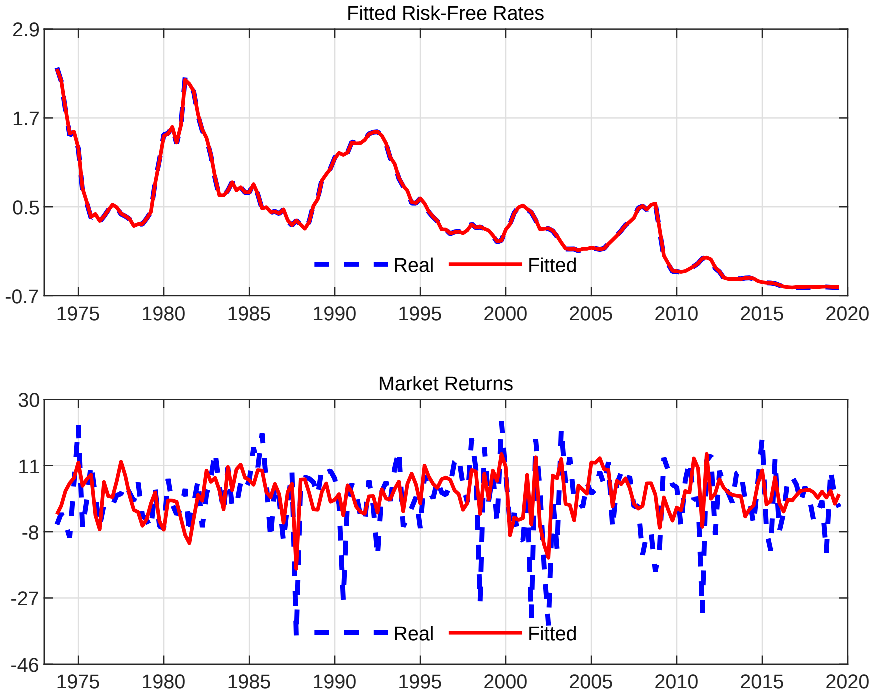


(a) Italy (IT)

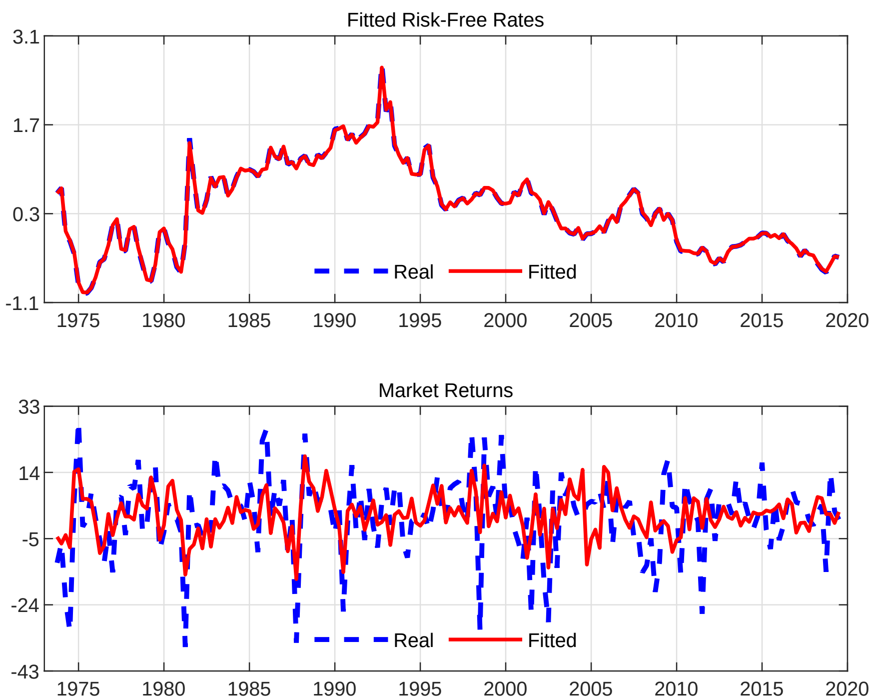


(b) Canada (CA)

Figure 3: **Filtered states: Italy and Canada**

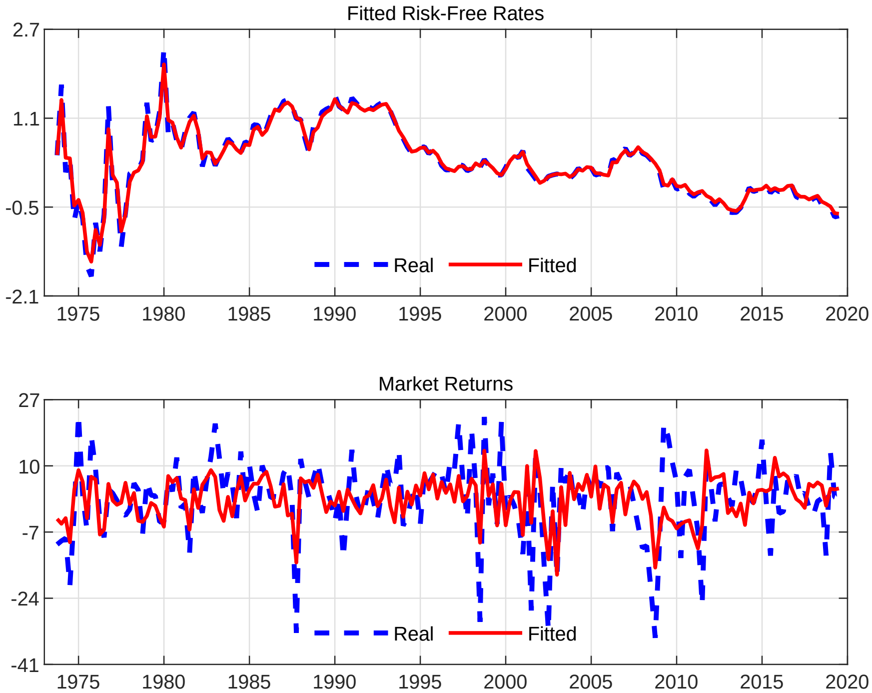


(a) Germany (DE)

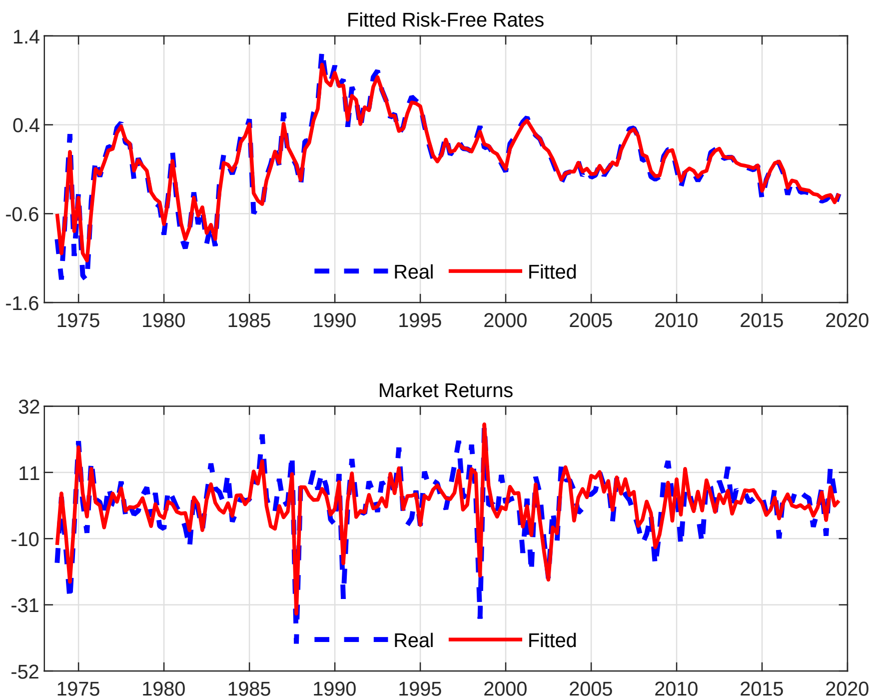


(b) France (FR)

Figure 4: **Fitted risk-free rates and market returns: Germany and France**

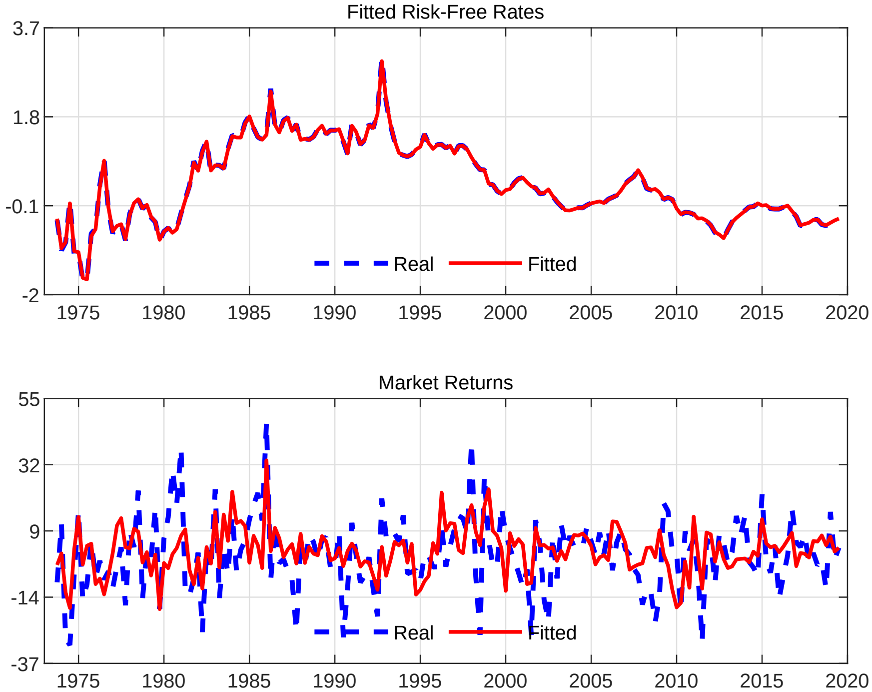


(a) Netherlands (NL)

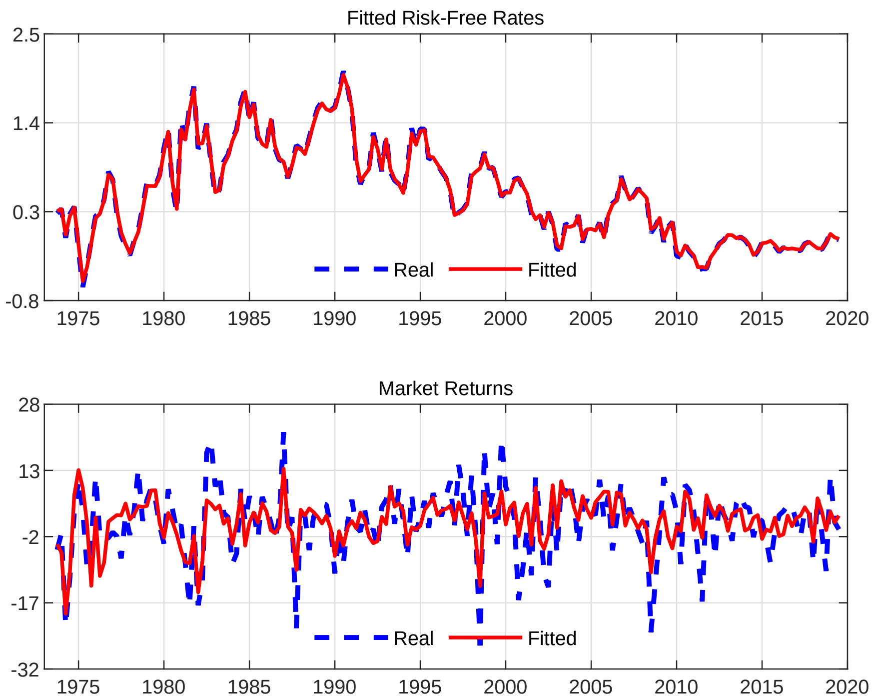


(b) Switzerland (CH)

Figure 5: **Fitted risk-free rates and market returns: Netherlands and Switzerland**

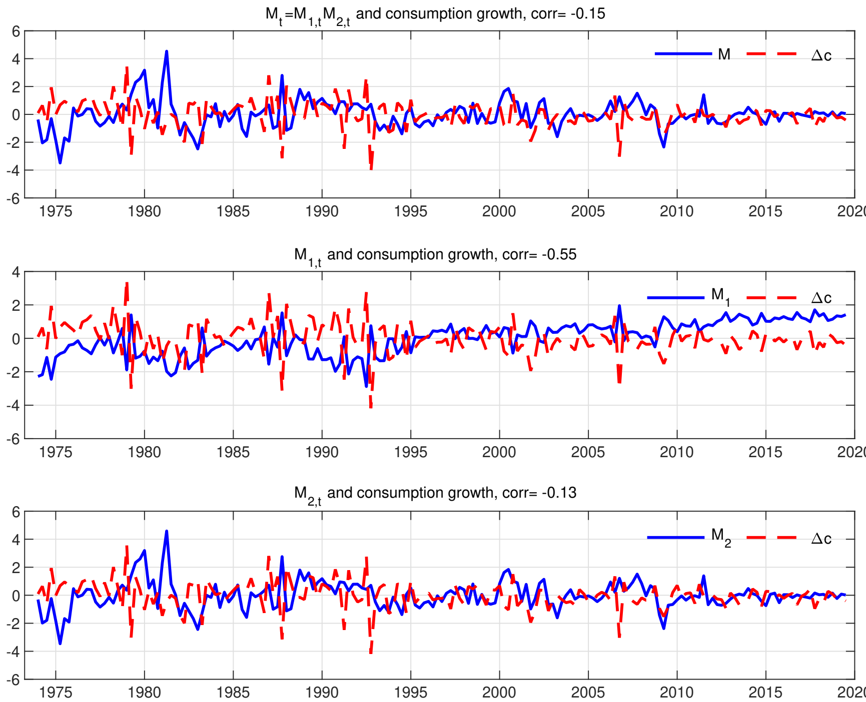


(a) Italy (IT)

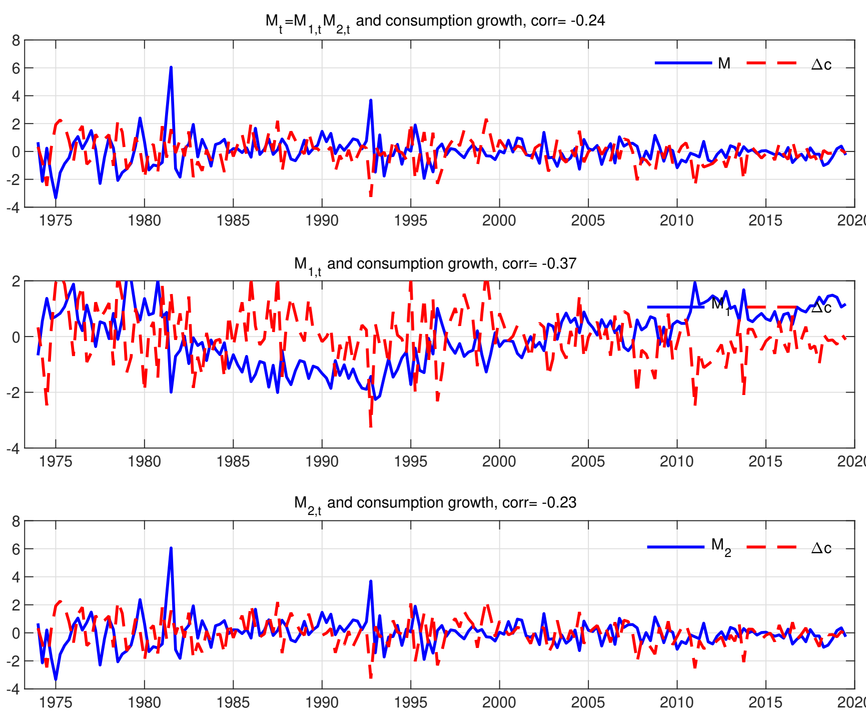


(b) Canada (CA)

Figure 6: **Fitted risk-free rates and market returns: Italy and Canada**

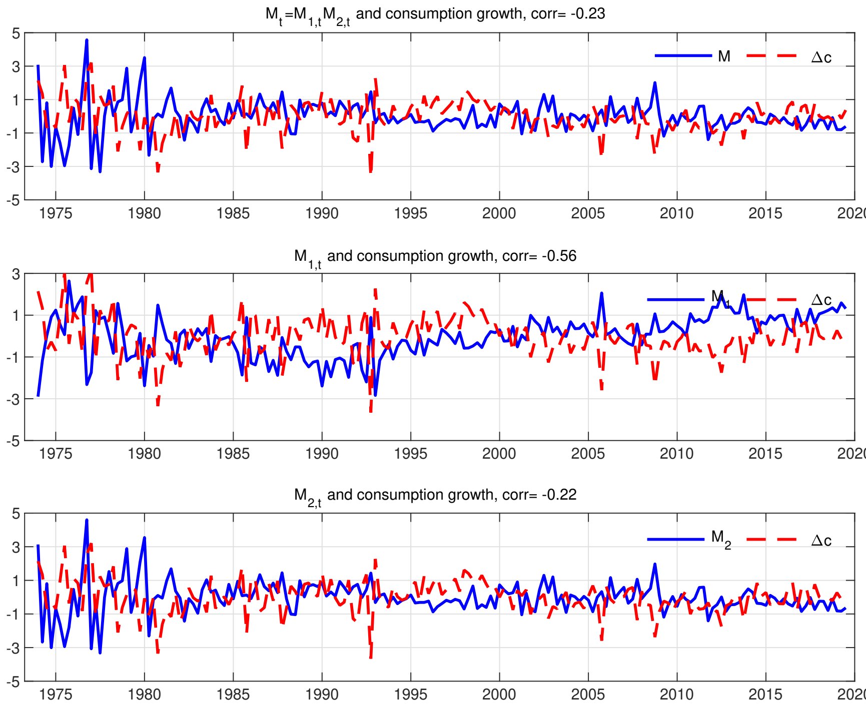


(a) Germany (DE)

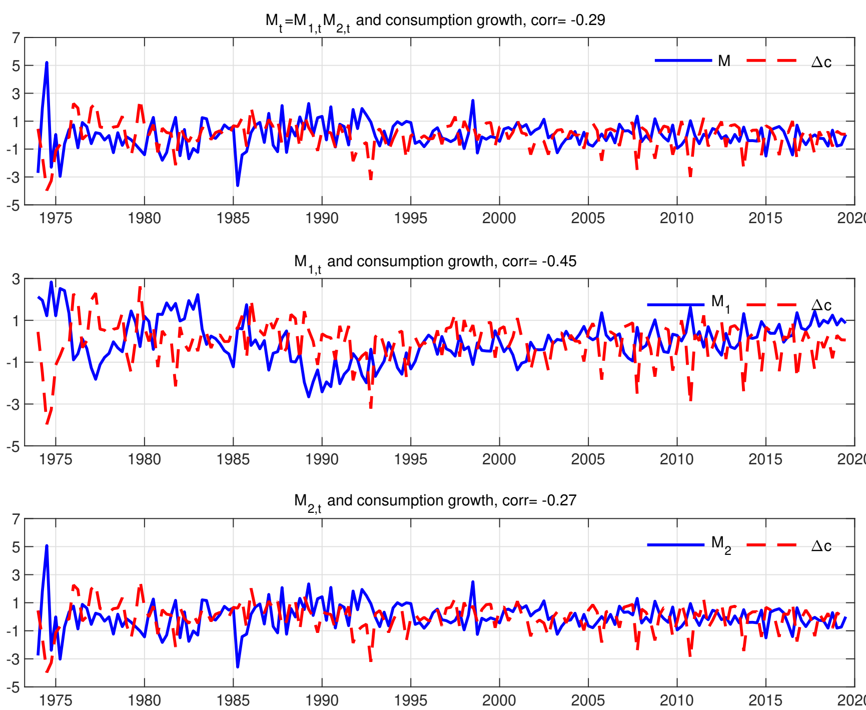


(b) France (FR)

Figure 7: **Stochastic discount factor: Germany and France**

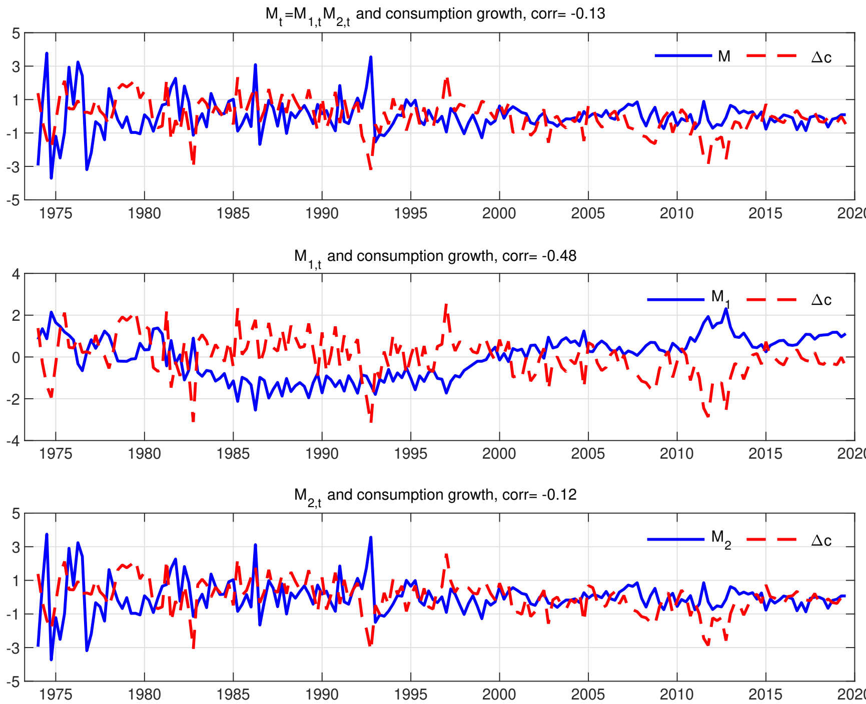


(a) Netherlands (NL)

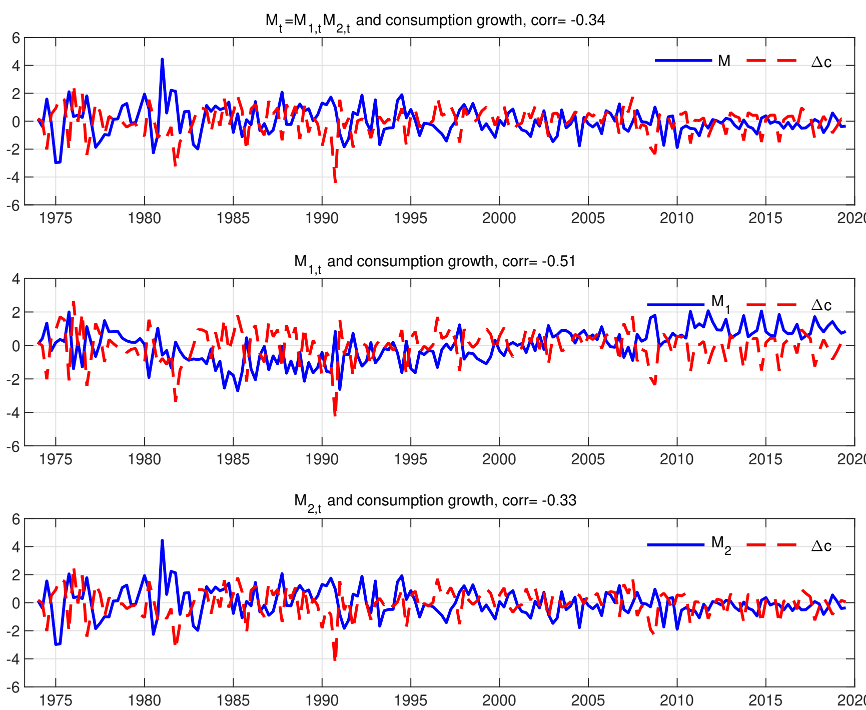


(b) Switzerland (CH)

Figure 8: **Stochastic discount factor: Netherlands and Switzerland**

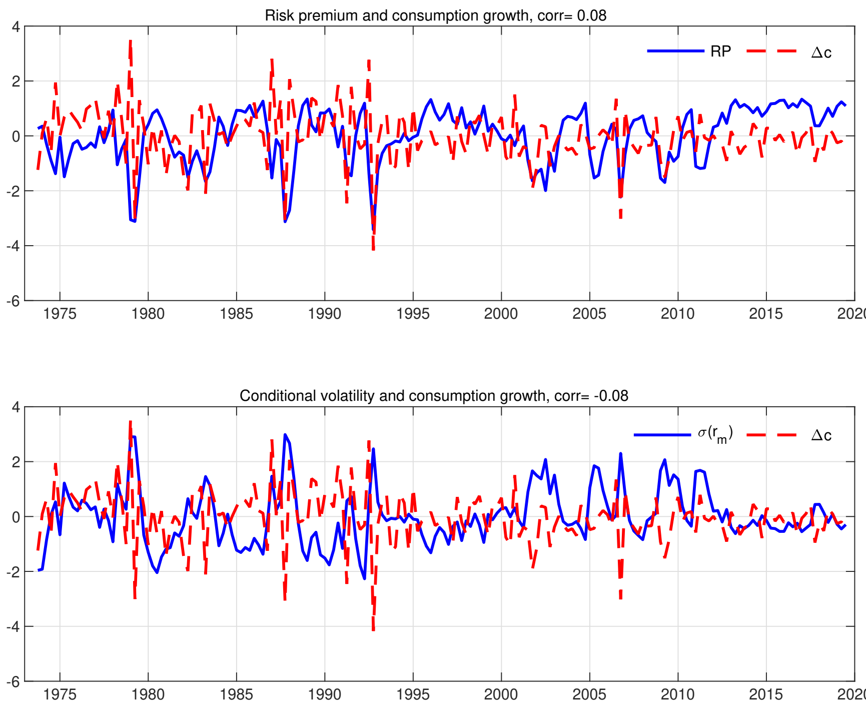


(a) Italy (IT)

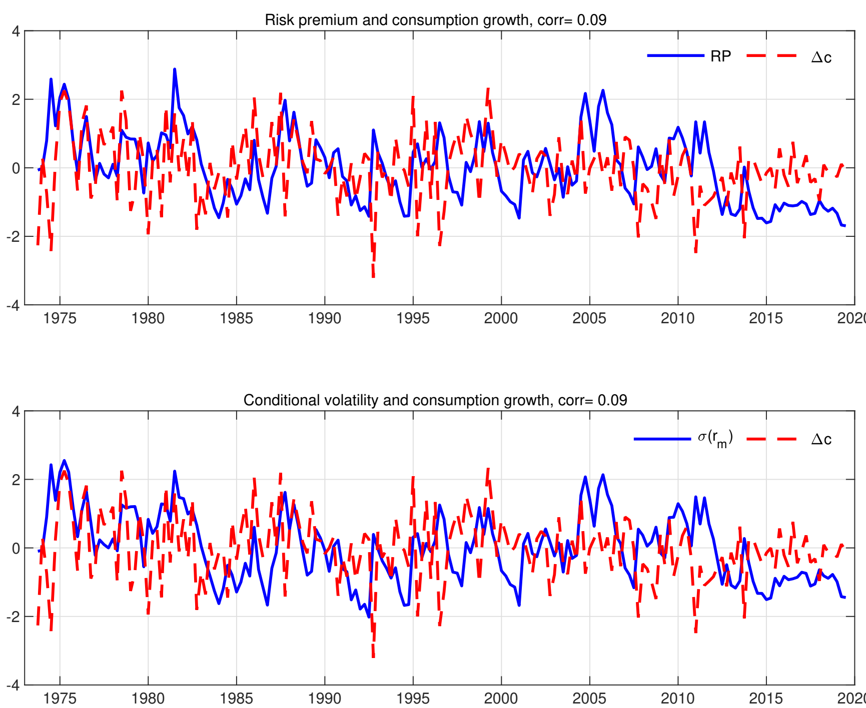


(b) Canada (CA)

Figure 9: **Stochastic discount factor: Italy and Canada**

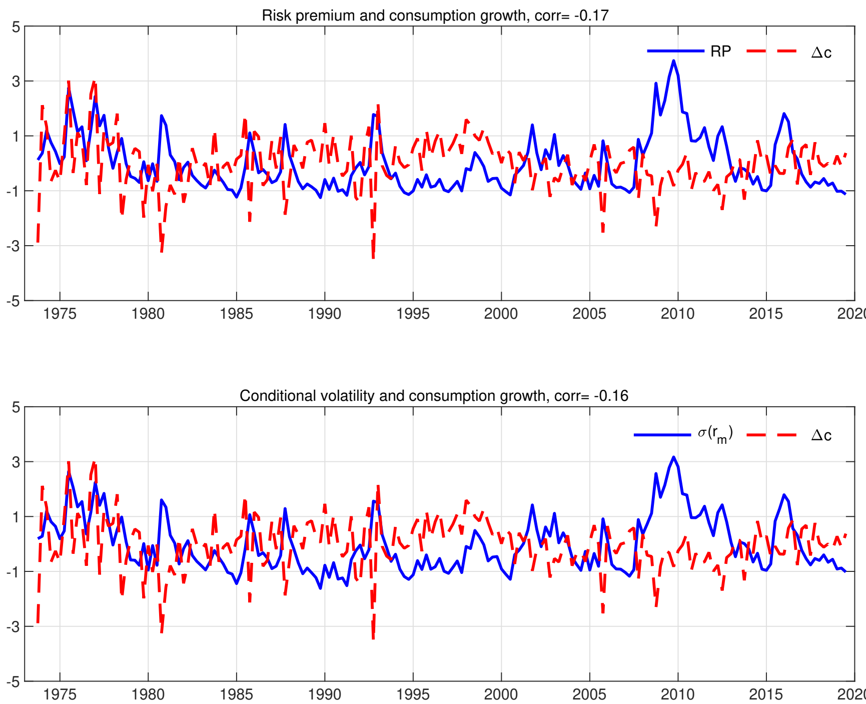


(a) Germany (DE)

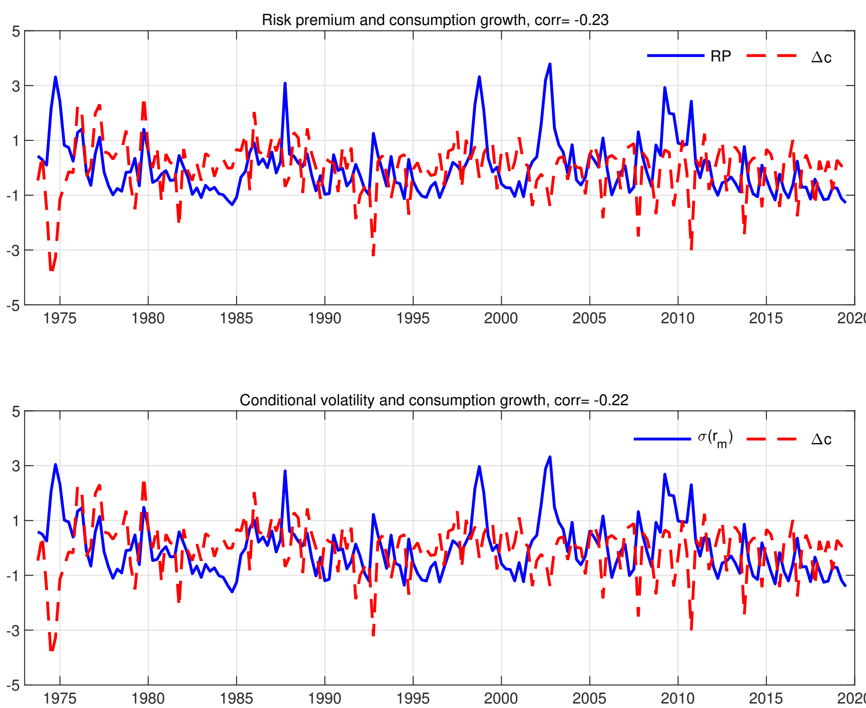


(b) France (FR)

Figure 10: **Conditional risk premium and volatility: Germany and France**

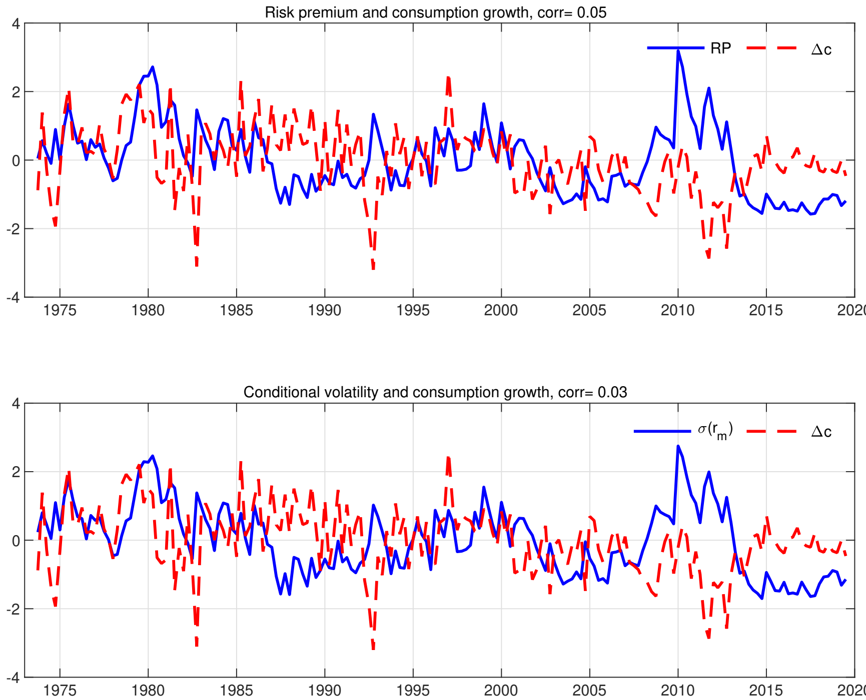


(a) Netherlands (NL)

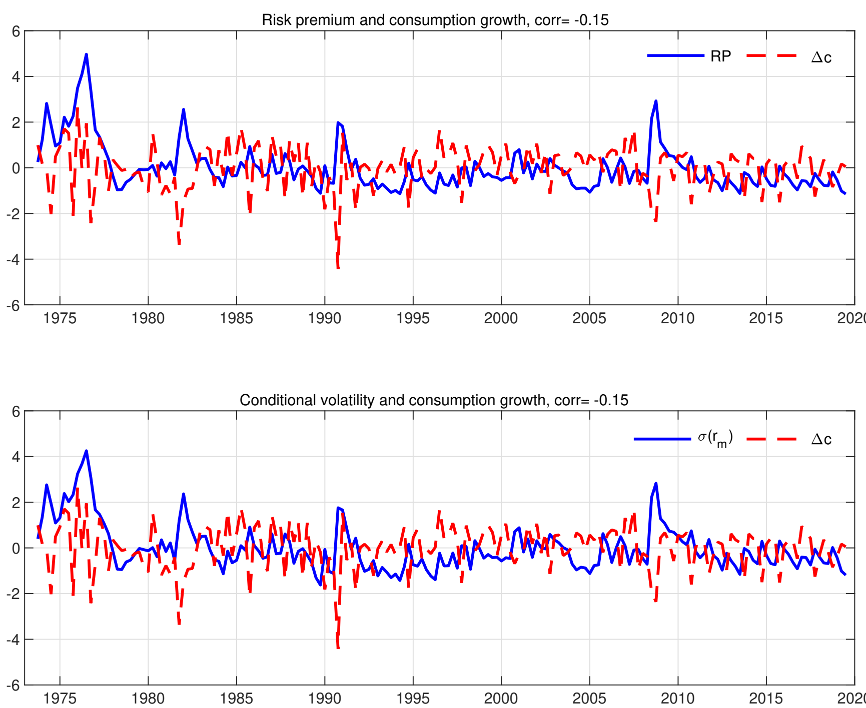


(b) Switzerland (CH)

Figure 11: **Conditional risk premium and volatility: Netherlands and Switzerland**



(a) Italy (IT)



(b) Canada (CA)

Figure 12: **Conditional risk premium and volatility: Italy and Canada**

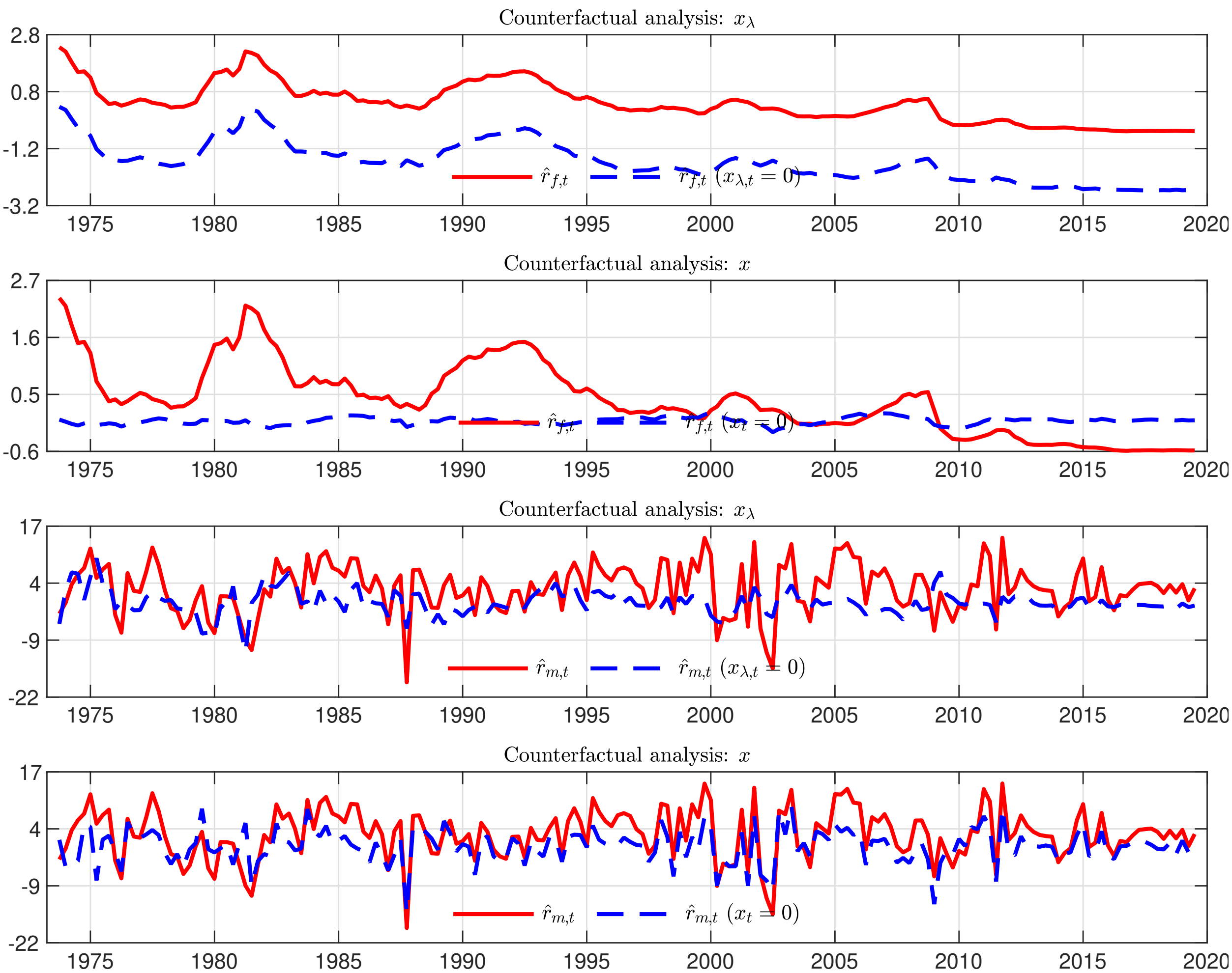


Figure 13: **Counterfactual analysis: Germany**

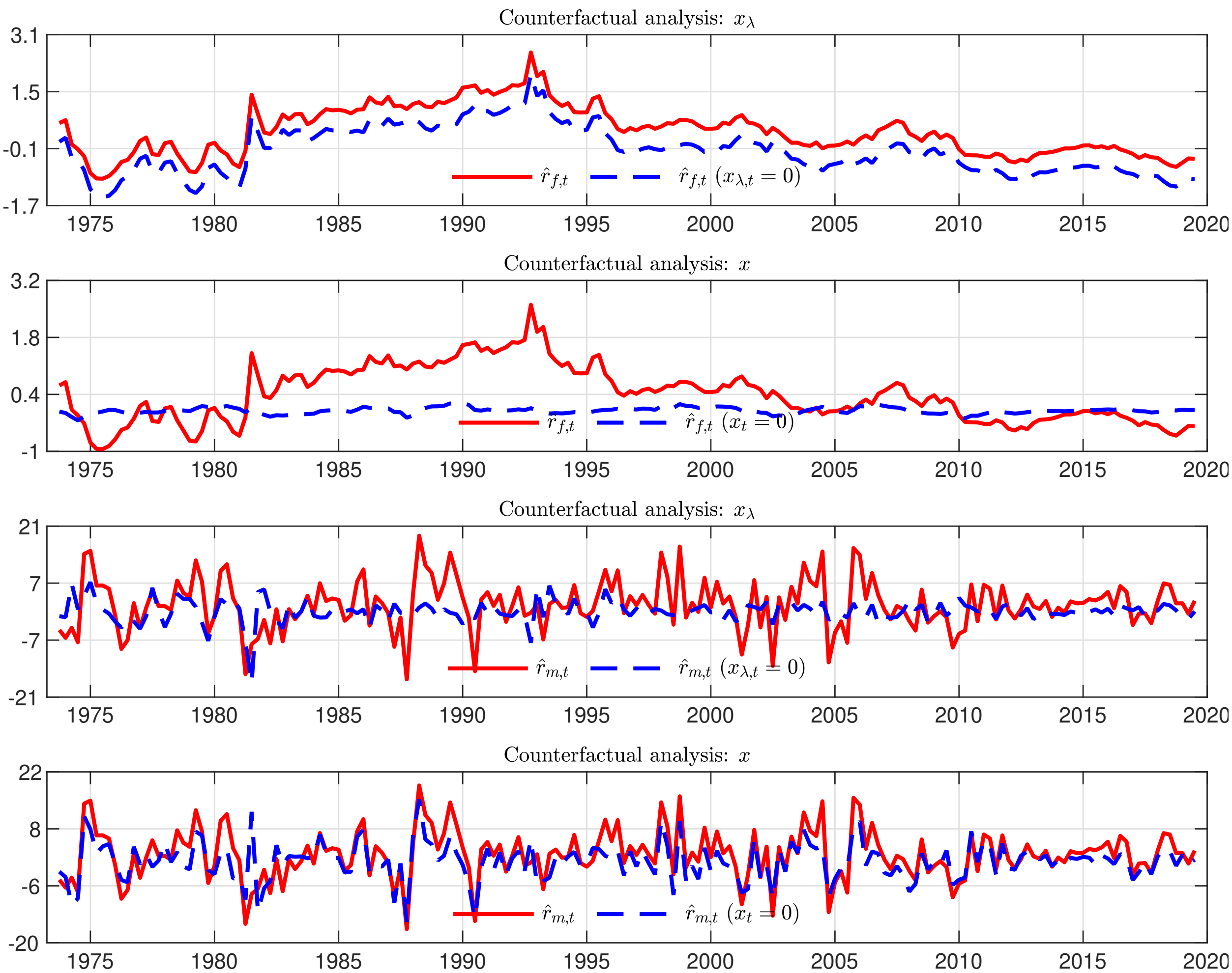


Figure 14: **Counterfactual analysis: France**

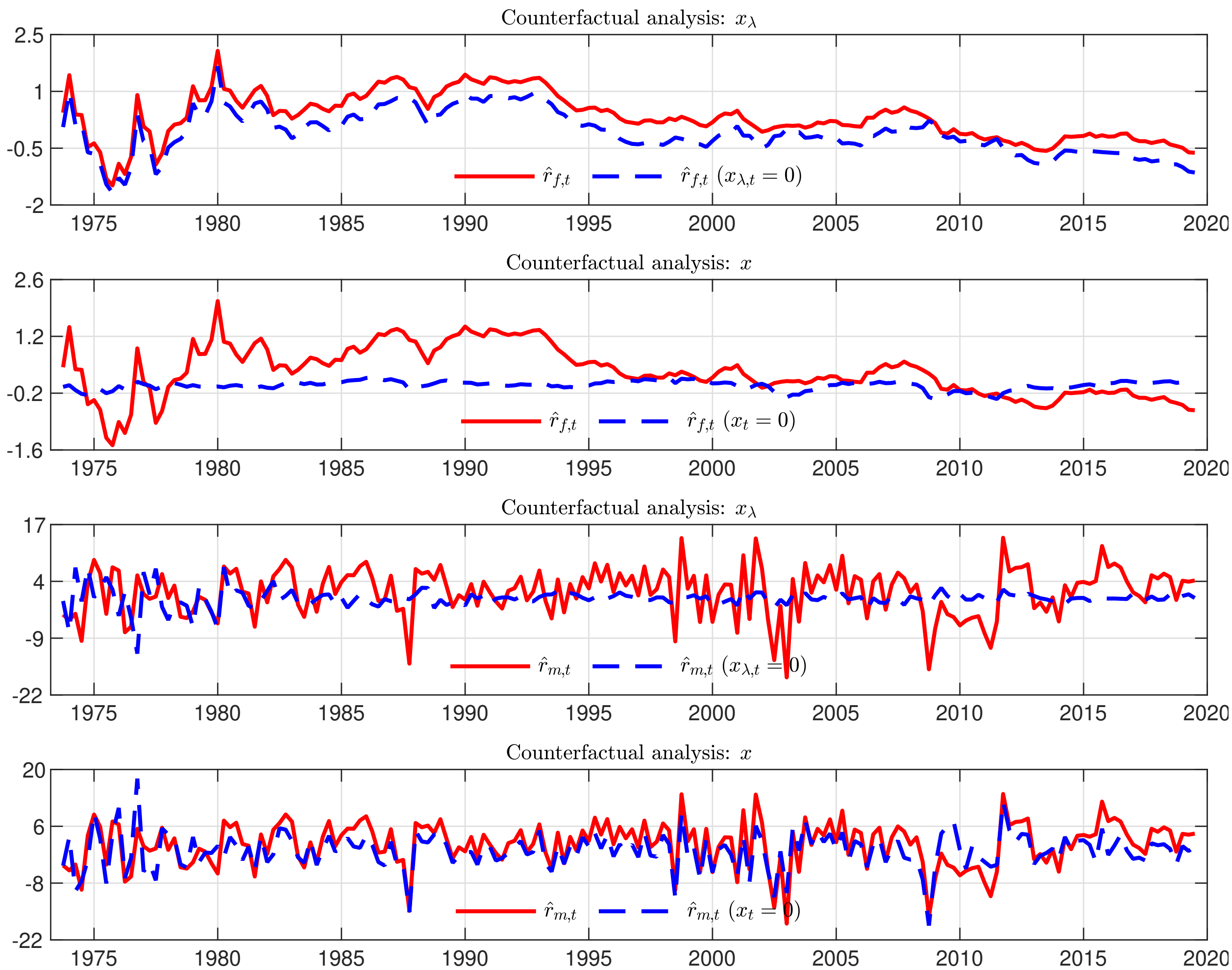


Figure 15: **Counterfactual analysis: Netherlands**

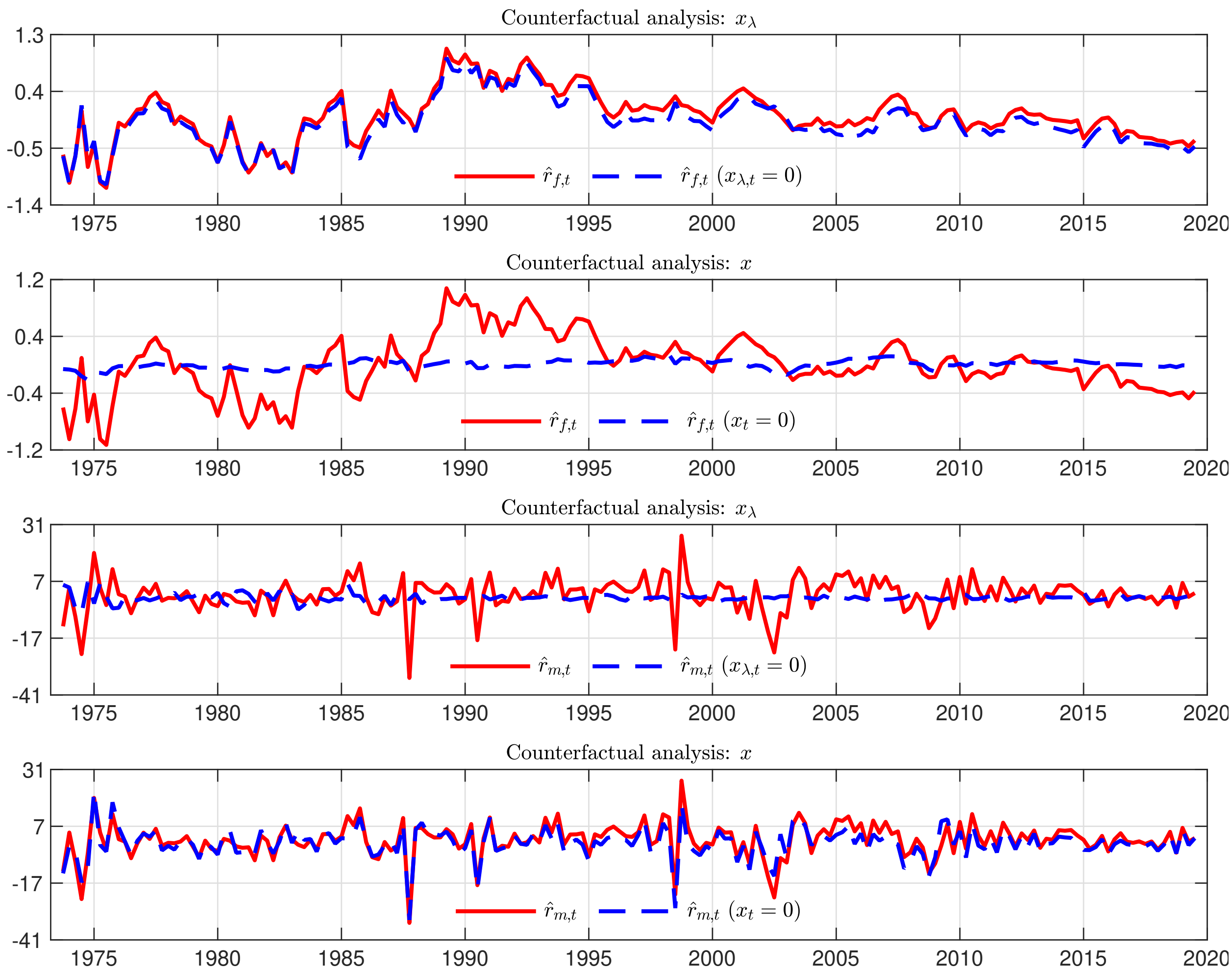


Figure 16: **Counterfactual analysis: Switzerland**

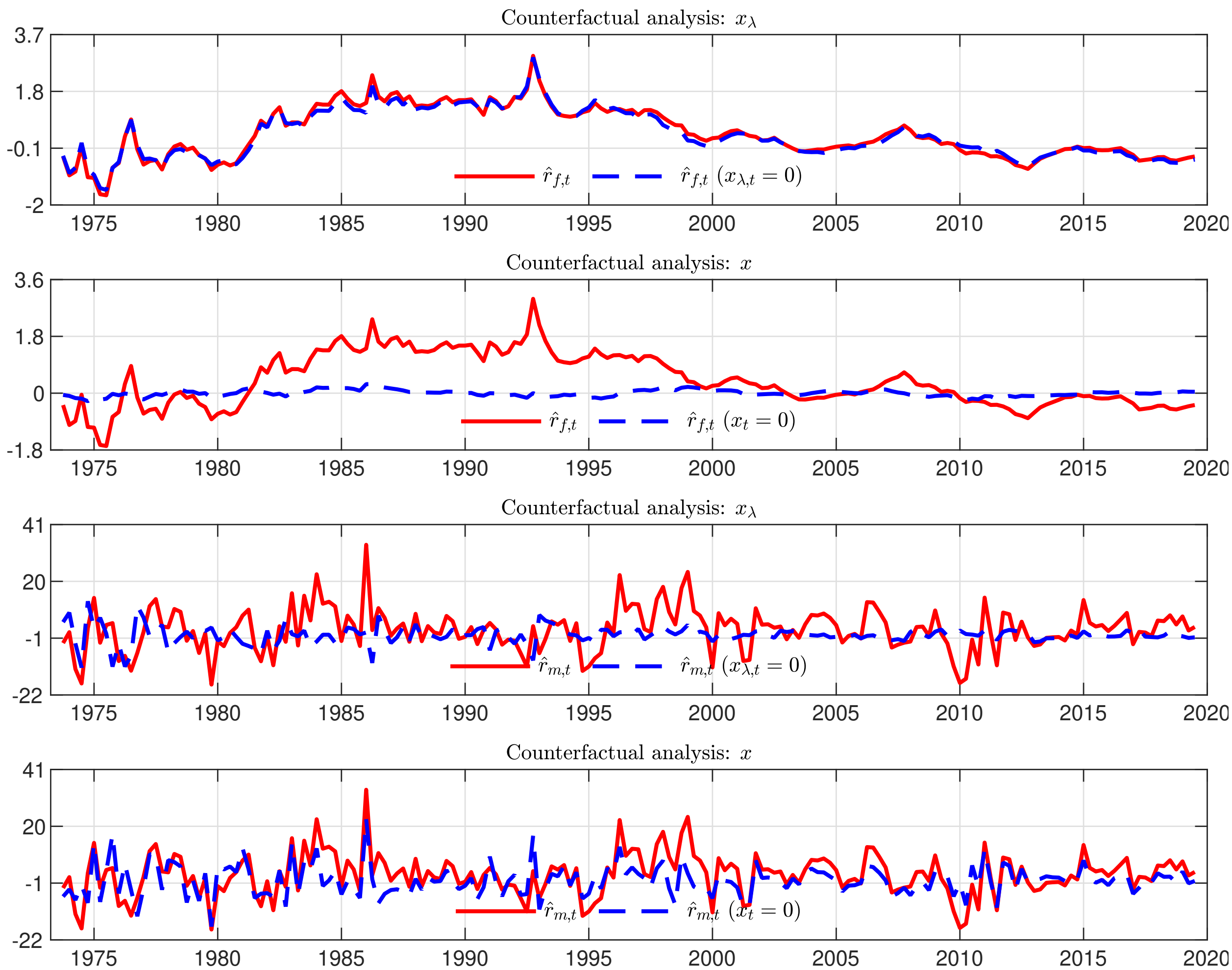


Figure 17: **Counterfactual analysis: Italy**

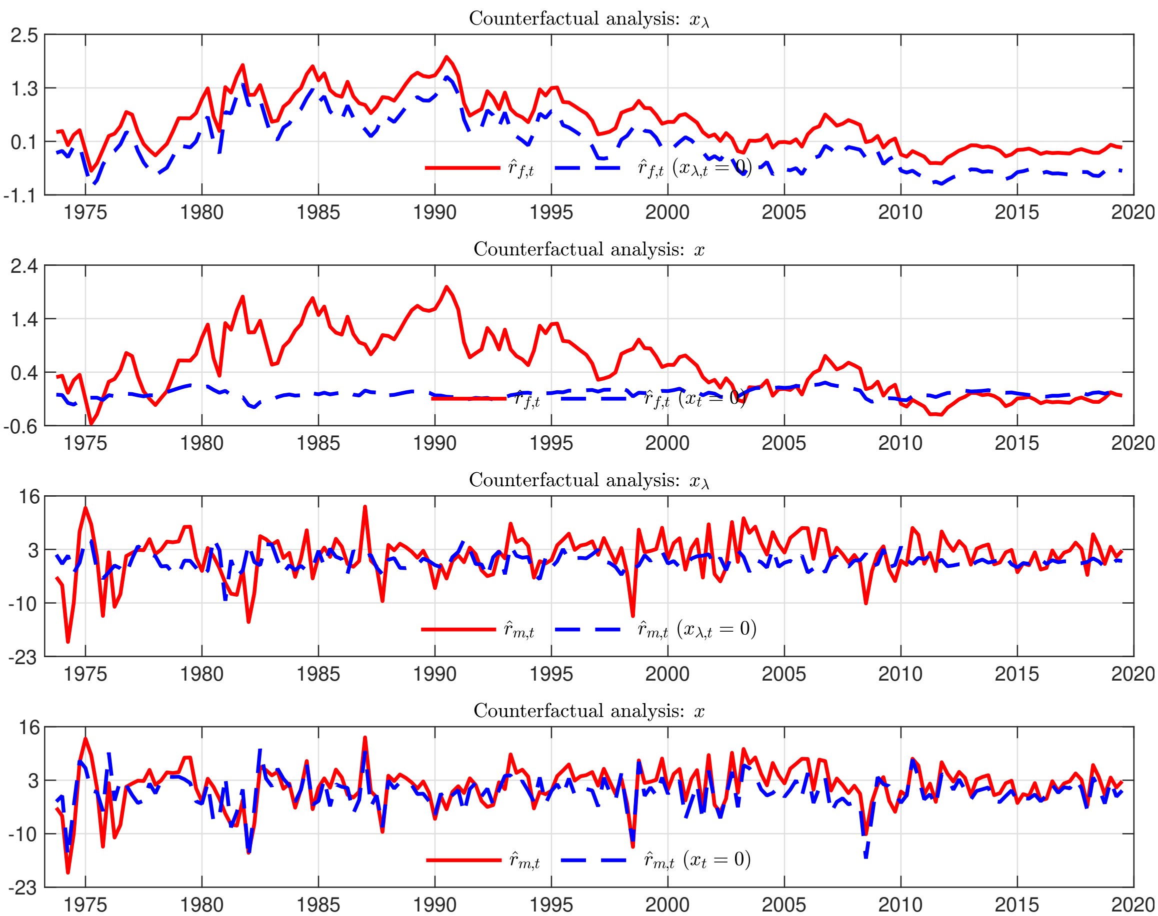


Figure 18: **Counterfactual analysis: Canada**

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