Factor-targeted Asset Allocation: a Reverse Optimization Approach

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Abstract

When assets' expected returns follow a factor structure subject to pricing errors, we show that the mean-variance portfolio can be used to obtain a set of implied factor risk premia. We show that such implied factor risk premia result in stable factor weights. To translate these factor weights into asset portfolios, we propose an asset allocation methodology that constructs portfolios to target these factor weights while accounting for the possibility of pricing errors. The analysis shows that our "factor-targeted portfolios" exhibit higher Sharpe ratios than various allocation methodologies under a variety of scenarios in expected returns.

Keywords: Portfolio Allocation, Factors, Factor Investing, Reverse Optimization

JEL Classification: G10, G11, G12

Portfolio allocation using factors is becoming more and more popular among institutional investors. Factor investing is not only supported by studies showing that individual assets are likely driven by a common set of risk factors (Sharpe, 1964; Lintner, 1975; Ross, 1976; Berry et al., 1988; Ang, 2014; Clarke et al., 2016) but it is also considered a more stable method to design a portfolio (i.e., generating less turnover as in Bessler et. al., 2021). Indeed, the 2008 global financial crisis highlighted the limits of traditional asset allocation models, which were severely affected by the breakdown in correlations and diversification across assets. This has led academics and investment managers to seek portfolio allocation methods that are based on more stable building blocks using risk and return factors rather than assets (Fabozzi, 2020).

However, how factors can be best incorporated into the investment process is still an active research question. There are several recent studies that develop methods to translate factor weights into asset allocations (e.g., Greenberg et al., 2016; Bass et al., 2017; Bender et al., 2018; Bergeron et al., 2018; Aliaga-Diaz et al., 2020; Kolm and Ritter, 2020) but there are still questions that are largely unanswered: (1) How should investors determine the optimal exposure to the various risk factors given that asset returns are subject to idiosyncratic errors? (2) How can investors map factor weights to asset portfolios in a simple yet robust way? (3) Why would building portfolios based on factors outperform the traditional approach of building portfolios based on assets? In this study we aim to address these three questions.

First, assuming that assets' expected returns are priced by a known set of risk factors but are subject to random pricing errors, we provide a methodology that utilizes reverse optimization to calculate the implied factor expected returns given an asset portfolio.[1] Given a set of arbitrary asset portfolio weights, our reverse optimization methodology calculates the implied factor returns such that the asset portfolio would be mean-variance optimal with respect to a set of implied assets' expected returns priced by those implied factors with the least pricing errors.[2] In particular, if the asset portfolio happens to be the mean-variance tangency portfolio, we show that those implied factor returns are consistent with the risk premia estimated via a cross-sectional regression of asset expected

[1] While the "true" underlying factors driving asset returns are unobservable to the econometrician, recent studies by both investment managers and academics have agreed upon the fact that macro factors (e.g., economic growth, real rate and inflation) are important drivers of asset returns. Recent examples of articles that use a similar set of factors as the ones we employ in this article include Bass et al. (2017), Bender et al. (2018), and Gladstone et al. (2021). Such macro factors can be replicated using portfolios of traded securities such as equities, real return bonds, commodities, break-even inflation and credit.

[2] The implied assets' expected returns are reverse optimized such that they would be priced by a set of implied factor returns with the least cross-sectional square errors.

returns on factor loadings, similar to the second step of a Fama-Macbeth regression (Fama and MacBeth, 1973). [3] Using these implied factor returns, we can build a mean-variance factor portfolio. We call these portfolio weights as the portfolio's "reverse optimized" factor weights. The matrix that maps the asset weights to these reverse optimized factor weights is referred as the implied factor loading matrix.

Second, we examine the reverse optimized factor weights of a mean-variance tangency portfolio when assets' expected returns are priced by those factors but with pricing errors. Consistent with prior evidence (e.g., Michaud, 1989), we find that the mean-variance tangency portfolio weights are sensitive to asset return inputs. However, the corresponding reverse optimized factor weights are considerably more stable. The stability of factor weights stems from more stable implied factor returns and, more importantly, from less extreme values in the inverse factor covariance matrix. We demonstrate that the implied factor returns of the mean-variance tangency portfolio are equal to the true factor expected returns plus a factor return error component. The factor return errors can be expressed as estimates from the cross-sectional regression of asset pricing errors on factor loadings. [4] The sample estimates of the factor returns' errors are more stable than the assets' pricing errors themselves when the number of assets is considerably larger than the number of factors. [5] The results suggest that, while the mean-variance tangency portfolios may be prone to pricing errors, its reverse optimized factor portfolios are considerably more stable around the optimal values.

Third, using the reverse optimized factor weights, we build different asset portfolios that respect these factor weights but account for input errors of expected returns. We extend existing methodologies (Greenberg et al., 2016; Elkamhi et al., 2021) and provide a closed-form solution that translates factor weights into asset portfolios with weights that are robust and more practical for asset managers. Specifically, although Elkamhi et al. (2021) build an asset portfolio that respects the desired factor weights by targeting the portfolio factor weights, we differentiate from them by targeting the portfolio's reverse optimized factor weights instead, and by utilizing the implied factor loading matrix. [6] The factor-targeting methodology presented in this article requires both the desired

[3] In the special case where the assets' expected returns are priced by a set of factors without errors, the implied factor returns of the mean-variance tangency portfolio is equal to the true factor premia.
[4] Since the implied factor returns are consistent with estimates from a cross-sectional regression of assets' expected returns on factor loadings, so are the factor returns' errors.
[5] In our analysis using historical data, the inverse of the factor covariance matrix has much fewer extreme values than that of the asset covariance matrix. This outcome contributes to the stability of the reverse optimized factor weights.
[6] In section "Comparison between using standard and implied factor loadings", we show that factor-targeting using the reverse optimized factor weights leads to better portfolio stability and higher average Sharpe ratios.

factor weights (i.e., the reverse optimized factor weights of the mean-variance portfolio) as well as a target asset portfolio as inputs. This target asset portfolio is necessary to achieve uniqueness in the mapping between factor weights and asset weights.[7] In this article, we present our results using four sets of target asset weights calculated using traditional portfolio allocation rules: maximum diversification, minimum volatility, equally weighted (1/N) and equal risk contributions.[8] Last, we ask whether our factor-targeted portfolios improve upon the portfolios built using more traditional methods (i.e., mean-variance, etc.).

We provide an explanation for why our methodology achieves more robust portfolios that outperforms mean-variance as well as various traditional allocation rules. Previous research (e.g., Greenberg et al., 2016; Elkamhi et al., 2021) has provided methodologies for mapping factor weights to asset portfolios without considering pricing errors for expected returns.[9] We extend the existing literature by proposing a new portfolio construction methodology that accounts for pricing errors when targeting the desired factor weights. We show that failing to recognize the presence of assets' pricing errors in the portfolio construction would lead to portfolios that are inferior to those that are built using our methodology. As we translate factor weights into asset weights using our methodology with a target asset portfolio that is built using a risk-diversified approach, the resulting portfolio achieves asset risk diversification while matching the desired factor weights.

To demonstrate the usefulness of our methodology, we proceed in two steps. The first step is to demonstrate the stability of the reverse optimized factor weights of the mean-variance tangency portfolio. This result suggests that such reverse optimized factor weights can be used in practice because they are theoretically motivated (i.e., they come from mean-variance) and they are stable (i.e., more practical and less transaction costs). We conduct our analysis using a simulation approach. We simulate asset returns from a linear factor model subject to idiosyncratic errors. We evaluate the stability of the mean-variance tangency portfolio weights and the corresponding reverse optimized factor weights across different simulated scenarios.

The second step is to apply both our factor-targeting methodology as well as various portfolio allocation rules directly on assets (i.e., mean-variance, maximum diversification, equal risk

[7] It is well known that, for a given set of factor weights, there might be many portfolios that have those exact factor weights (for a discussion, see Greenberg et al., 2016).
[8] However, our methodology is flexible, and investors can use their preferred allocation rule for the target asset weights and they can also target their portfolio to different desired factor weights determined through other means.
[9] For completeness, we note there is evidence that, in the absence of pricing errors, factor-based asset allocation is not superior to asset-class based asset allocation (e.g., Idzorek and Kowara, 2013).

contributions, and 1/N) and evaluate the performance of in terms of Sharpe ratios. In our analysis, we use 17 assets covering various public asset classes. We include five macroeconomic tradable factors and build them using factor mimicking portfolios. We use historical data to estimate the asset and factor covariance matrices as well as the factor loadings needed for our analysis. We assume a set of factor risk premia consistent with industry practice and use the computed factor loadings to price the assets' expected returns. Historically, we find that the average of the pricing error standard deviations is approximately 1.5% for our set of macroeconomic factors.

The results of our simulations show that in a situation where the assets' expected returns are priced by a linear factor model but are subject to random pricing errors, the reverse optimized factor weights of the traditional mean-variance tangency portfolio are considerably more stable than those asset portfolio weights. Given a standard deviation of pricing errors of 1.5%, the average standard deviation of portfolio weights across assets using the traditional mean-variance on assets is 125%. The corresponding reverse optimized factor weights are more stable with an average standard deviation for factor weights of 29%. We also show that, when assets' expected returns are subject to pricing errors, our factor weights are stable while mean-variance portfolio weights are subject to large changes. These results suggest while mean-variance tangency weights for assets may not be reliable, their reverse optimized factor weights are more robust and closer to mean-variance optimal with respect to the true risk premia.

While the Sharpe ratios of mean-variance applied directly on assets are higher in the absence of pricing errors, they deteriorate rapidly as the pricing errors' standard deviation increases in our simulation. The factor-targeted portfolios obtained using our methodology show a much slower rate of deterioration. The results from our simulations also show that, for a pricing error standard deviation of 1.5%, our factor-targeted portfolios generate higher Sharpe ratios than that of the mean-variance portfolio with an approximate 95% probability.

Next, we compare our factor-targeted portfolios – which account for pricing errors – with the factor-targeted portfolios that do not account for pricing errors (i.e., Elkamhi et al., 2021). Our results show that the Sharpe ratios of our factor-targeted portfolios, which account for pricing errors, exhibit both higher averages and lower standard deviations across simulations.

We also compare the factor-targeted portfolios built using our methodology with the four corresponding traditional portfolios that are also used as target asset weights: maximum diversification, minimum volatility, equally weighted (1/N) and equal risk contributions. Clearly, those traditional portfolios are not subject to pricing errors in expected returns since they do not rely on them as inputs. Therefore, we proceed to analyze how all these portfolios are impacted by different risk premia settings as well as pricing errors. When there are no pricing errors in expected returns, the factor-targeted portfolios have higher Sharpe ratios than the traditional portfolios for various risk premia settings. When there are large pricing errors in assets' expected returns, factor-targeted and traditional portfolios perform similarly on average.

To summarize, we present analyses for our factor-targeted asset allocation methodology under a setting where assets' expected returns are priced by a known set of factors but are subject to pricing errors. We find that using the same assets' expected returns and covariance inputs, our factor-targeted portfolios outperform, in terms of Sharpe ratios, traditional mean-variance asset portfolios when expected returns are subject to even small pricing errors. As we evaluate portfolio performances in our analysis across a multitude of criteria, the factor-targeting methodology presented in this article creates portfolios that are stable and robust against expected return inputs and they perform well across risk premia settings.

Methodology

Reverse optimized factor weights

In this section, we (1) describe how to compute the implied factor expected returns from a given set of asset weights using reverse optimization, (2) describe how to calculate the mean-variance factor weights using the implied factor expected returns, which we labeled as "reverse optimized factor weights", and (3) show that, in the absence of pricing errors, the reverse optimized factor weights are mean-variance optimal in the factor space when the asset weights are mean-variance optimal themselves.

We start by assuming that the N asset returns follow a linear factor structure with (true) factor returns μ_f and a factor covariance matrix Σ_f for M factors. We also define B as the $N \times M$ matrix of factor loadings for the N assets with respect to the M factors such that

$$\mu_a = B\mu_f + \varepsilon \quad (1)$$

where ε is a vector of pricing errors for assets' expected excess returns with respect to the M factors.

The presence of pricing errors has been discussed extensively in the literature. For example, linear factor models are thought to be just an approximation of the true underlying relationship between factors and assets since no model is likely to be completely accurate. This is discussed, for example, in Hansen and Jagannathan (1997), Shanken (1987) and many others. Alternatively, there might be missing factors that the investors have omitted (Pastor and Stambaugh, 2000).

In the reverse optimization, we look for the implied factor returns μ_f^* such that the given asset weights would be mean-variance optimal by minimizing the sum of squared pricing errors. Recalling that the asset mean-variance weights w_a are such that $w_a = (\lambda\Sigma_a)^{-1}\mu_a$, we can use Equation (1) to minimize the sum of squared errors which in matrix form can be written as

$$\underset{\mu_f}{\text{argmin}} \; (\lambda\,\Sigma_a\, w_a \; - \; B\,\mu_f)'\,(\lambda\,\Sigma_a\, w_a \; - \; B\,\mu_f\,) \tag{2}$$

where λ is the risk aversion parameter. This is equivalent to estimating the linear regression $\lambda\Sigma_a w_a = B\mu_f + \varepsilon$ which yields the following solution for μ_f^*

$$\mu_f^* = (B'B)^{-1}B'\lambda\Sigma_a\, w_a \tag{3}$$

With this implied factor return μ_f^*, we can compute the reverse mean-variance factor weights with the same risk aversion as

$$w_f^* = \left(\lambda\Sigma_f\right)^{-1}\mu_f^*$$

Substituting Equation (3) into the above expression, the mean-variance factor weights can be expressed as follows

$$w_f^* = B_i\, w_a \tag{4}$$

where

$$B_i = \Sigma_f^{-1}(B'B)^{-1}B'\Sigma_a \tag{5}$$

Note that B_i is the $M \times N$ *implied* factor loading matrix that transforms a set of given asset weights (w_a) into the reverse optimized factor weights (w_f^*). In the special case where w_a is mean-variance optimal and there are no pricing errors in expected returns (i.e., $\mu_a = B\mu_f$), w_f^* is also mean-variance optimal given μ_f. Specifically, given that $w_a \; = \; (\lambda\,\Sigma_a)^{-1}\,\mu_a$, substituting it in Equation (4) yields

$$w_f^* \; = \left(\lambda\,\Sigma_f\right)^{-1}(B'B)^{-1}B'\Sigma_a\,\Sigma_a^{-1}\,B\,\mu_f$$

$$w_f^* \; = \left(\lambda\,\Sigma_f\right)^{-1}\,\mu_f \tag{6}$$

where Equation (6) shows that w_f^* is the solution of the unconstrained mean-variance optimization using Σ_f and μ_f.

Reverse optimized factor weights for the mean-variance tangency portfolio

We present the standard unconstrained mean-variance tangency portfolio given assets' expected excess returns μ_a and covariance matrix Σ_a as

$$w_{a,tp} = \frac{\Sigma_a^{-1}\mu_a}{|\mathbf{1}_N'\Sigma_a^{-1}\mu_a|} \tag{7}$$

where μ_a is a $N \times 1$ vector of expected excess returns for N assets, Σ_a is a $N \times N$ covariance matrix and $\mathbf{1}_N$ is a $N \times 1$ vector of ones. Using Equation (7) in Equation (4), we can compute the reverse optimized factor weight for the mean-variance tangency portfolio as

$$w_{f,tp}^{*} = \frac{\Sigma_f^{-1}(B'B)^{-1}B'\mu_a}{|\mathbf{1}_N'\Sigma_a^{-1}\mu_a|} \tag{8}$$

Note that the reverse optimized factor weights do not necessarily sum up to one.

Factor-targeted asset allocation

In this section, we describe our factor-targeting methodology, which builds asset portfolios that respect a desired set of factor weights. To apply our methodology, investors need to provide two inputs: (1) the set of desired factor weights ($\overline{w}_f$) and (2) the set of "target asset weights" ($\overline{w}_a$). This latter input is required to ensure that we achieve a unique asset portfolio that is targeted to the desired factor weights. As we prove in Appendix A, for any set of desired factor weights, there might be multiple asset portfolios that respect such factor weights. Because of this non-uniqueness, in addition to targeting the portfolio's reverse optimized factor weights to the desired factor weights, we impose an extra condition to obtain a unique portfolio: we minimize the difference between the resulting portfolio weights and the target asset weights $\overline{w}_a$. For these target asset weights, investors can use any allocation rule but, in this article, we restrict our attention to traditional asset allocation rules that are risk and diversification focused (e.g., maximum diversification, minimum variance, equally weighted and equal risk contributions).[10] Formally, the optimization procedure can be written as follows

$$\underset{w}{\operatorname{argmin}}\, \gamma \underbrace{(B_i\, w - \overline{w}_f')(B_i\, w - \overline{w}_f')'}_{\substack{Deviations\ from\ Reverse \\ optimized\ factor\ weights}} + \underbrace{(1-\gamma)(w - \overline{w}_a)'\,(w - \overline{w}_a)}_{\substack{Deviations\ from \\ Target\ asset\ weights}} \tag{9}$$

where $\overline{w}_f$ is the desired target factor weights, $\overline{w}_a$ is the target asset weights and B_i is defined as per

[10] Elkamhi et al. (2021) provides a short discussion on the intuition of using traditional portfolios as the target asset weights.

Equation (5), and γ is a parameter that affects how much weight is given to deviations from the reverse optimized factor weights.

Our goal is to obtain a portfolio that respects the desired factor weights, so γ is set to an arbitrary large value in practice (i.e., 0.999) so that the squared deviations from the reverse optimized factor weights (first term in Equation (9)) are as close to zero as possible. With the implied factor weights from the chosen portfolio weights w required to be as close as possible to the reverse optimized factor weights, the optimization would then search for asset weights w such that they have the least square errors with the target asset weights $\bar{w}_a$ (second term in Equation (9)).

Solving Equation (9) yields the following solution

$$w = [\gamma\, B_i{}'B_i + (1-\gamma)I_{N\times N}]^{-1} \left(\gamma\, B_i{}'\, \bar{w}_f \;+\; (1-\gamma)\, \bar{w}_a\right) \tag{10}$$

where $I_{N \times N}$ is an $N \times N$ identity matrix. The model presented in this article follows closely the one presented in Elkamhi et al. (2021) but it extends it to the case where asset returns are generated from a factor model with pricing errors.

As for our target factor weights $\bar{w}_f$, we choose the reverse optimized factor weights of the mean-variance tangency portfolio ($w^*_{f,tp}$ in Equation (8)). We also set γ to be 0.999. We investigate four different traditional asset allocation rules (Maximum Diversification, Minimum Volatility, 1/N, and Equal Risk Contributions) to compute the target asset weights and evaluate their performances. The rules for these four traditional allocation methodologies are defined below

$$\textit{Maximum Diversification}: \quad \bar{w}_{md} = \frac{\Sigma_a^{-1} V \mathbf{1}_N}{\mathbf{1}_N' \Sigma_a^{-1} V \mathbf{1}_N} \tag{11}$$

$$\textit{Minimum Volatility}: \quad \bar{w}_{mv} = \frac{\Sigma_a^{-1} \mathbf{1}_N}{\mathbf{1}_N' \Sigma_a^{-1} \mathbf{1}_N} \tag{12}$$

$$1/N: \quad \bar{w}_{1/N} = \frac{\mathbf{1}_N}{\mathbf{1}_N{}' \mathbf{1}_N} \tag{13}$$

$$\textit{Equal Risk Contributions}: \quad \bar{w}_{ERC} = \left\{w:\ RC_i = RC_j \text{ for all assets } i,j\right\} \tag{14}$$

where V is a diagonal matrix with the asset volatilities along the diagonal. RC_i is the risk contribution for asset i, which is defined as $RC_i = w_i \cdot \frac{\partial \sigma(w)}{\partial w_i} = w_i \cdot \frac{(\Sigma\, w)_i}{\sqrt{w'\Sigma\, w}}$.[11] Finally, substituting our choices for $\bar{w}_f$ and $\bar{w}_a$ in Equation (10), the factor-targeted asset allocation rule with target asset weights w_x

[11] See Roncalli (2013) for a detailed discussion on risk contributions.

can be expressed as follows

$$w_{ft,x} = [\gamma\, B_i{}'B_i \ + \ (1-\gamma)I_{N\times N}]^{-1} \left(\gamma\, B_i{}'w_{f,tp}^* + (1-\gamma)\, \bar{w}_x\right), \ \gamma \ = \ 0.999 \tag{15}$$

where $w_{f,tp}^*$ is the vector of reverse optimized factor weights for the mean-variance tangency portfolio (see Equation (8)), and $\bar{w}_x$ is the vector of target asset weights (i.e., any of the portfolios from Equations (11) to (14)).

Analysis Setup

In this section, we describe the setup used to evaluate the performance of various allocation rules when assets follow a linear factor structure but are subject to random errors as per the following model

$$\mu_a \ = \ B\mu_f \ + \sigma_\varepsilon\, Z_N \tag{16}$$

where Z_N is a $N \times \ 1$ vector of uncorrelated random errors drawn from standard Normal distribution, σ_ε is a scalar and is the standard deviation of the errors.

We simulate 5,000 scenarios of assets' expected returns for a given true vector of risk premia μ_f using Equation (16). For each of the 5,000 scenarios, we compute the portfolio weights and Sharpe ratios using our factor-targeted methodology, the traditional mean-variance method for assets and the four asset allocation rules that we described above with the simulated assets' expected returns. We compare the average and standard deviation of the Sharpe ratios as well as the volatility of portfolio weights across the various allocation methodologies. Furthermore, we also use the distributions of Sharpe ratios from the 5,000 simulations to calculate the probabilities for our proposed methodology to outperform the mean-variance tangency portfolio. These different comparisons allow us to evaluate the robustness of those portfolio allocation rules being investigated. [12]

Values for $\boldsymbol{\Sigma_a, \Sigma_f, B_i}$

We use historical data from year 2005 to 2020 on 17 assets to compute the sample covariance matrix Σ_a for our analysis. Exhibit 1 shows the data used for the empirical analysis as well as the factor loading matrix B used for assets. Exhibit 2 shows the factor definitions as well as the assumed risk premia used in this article.

For each asset, we present the excess return calculated according to Equation (1) (the factor

[12] Analysis on the volatility of asset weights is valuable for investors because it directly affects turnover and transaction costs, which are known to be important determinants when applying an allocation rule in practice.

portfolios expected returns are provided in Exhibit 2), the factor loadings obtained by regressing the asset returns on the factor returns. We show the t-statistics of the regressions in parenthesis and report the R-squared in the furthermost right column. The factors are defined as factor mimicking portfolios.[13] All the factor returns are scaled to have 10% annualized volatility in-sample. As it is well known, uncovering the "true" underlying factors that affect asset returns is a difficult task that goes beyond the scope of this article. However, we follow the commonly used definitions of factors. For example, BlackRock and Barra developed risk models which include factors that are defined very similarly to our factor definitions.

The factor loading matrix B is obtained by regressing the assets' historical returns onto the factor mimicking portfolios returns: $r_{a,j} = r_f \, B_j' + \varepsilon$, where $r_{a,j}$ is the time series of returns for asset j, and r_f is the time series of returns for the 5 factors and B_j is the factor loading of the 5 factors for asset j (i.e., $B_j{}'$ is a row vector loadings for asset j in Exhibit 1). We use the historical sample covariance matrix for the 5 factor mimicking portfolios as Σ_f.

Values for μ_a and μ_f

Exhibit 2 shows the factor risk premia (i.e., expected returns) assumptions (i.e., μ_f). For ease of interpretability, we use factor risk premia that are consistent with those commonly assumed in practice.[14] We choose not to use historical factor mimicking portfolio returns as they are unlikely to create representative results for portfolio construction analysis in the current investing environment. Instead, we use risk premia assumptions that are in line with the most recent capital market assumptions from the industry (e.g., BlackRock and JP Morgan). The assets' expected returns are computed as $\mu_a = B \, \mu_f$. This way, the assets' expected returns (μ_a) used in our analysis would be in-line with practitioners' general expectations and the resulting portfolio weights in our analysis would be more realistic. Indeed, the assets' expected returns computed using the factor risk premia – shown in Exhibit 1 – are generally in-line with the recent capital market assumptions surveys.[15]

[Insert Exhibit 1 and Exhibit 2 here]

[13] The use of factor mimicking portfolios is common in practice (e.g., Bender et al., 2018; Greenberg et al., 2016) as they are tradable.

[14] Although our choice of factor risk premia is admittedly arbitrary, our analysis does not rely on specific estimates of risk premia. The methodology works for any set of risk premia assumptions.

[15] For example, see the capital market assumption surveys from Horizon Actuarial Services: https://www.horizonactuarial.com/blog/2020-survey-of-capital-market-assumptions

Results and Discussions

This section is organized as follows. We start by providing a discussion on why the factor-targeted portfolios discussed in this study are likely to be more robust than other factor-targeted portfolios (e.g. Elkamhi et al., 2021). After the brief discussion, we provide evidence of such robustness by demonstrating that, while mean-variance weights are highly sensitive to expected return inputs, their reverse optimized factor weights are more stable. Using these factor weights, we compare and show that our factor-targeted portfolios, which uses the implied factor loadings B_i, perform better than those that uses the standard factor loadings B.

After a comparison between the two factor-targeted portfolio methodologies, we proceed to compare our methodology with traditional allocation rules. Specifically, we compare the performance of our factor-targeted portfolios with mean-variance tangency portfolios and the four other allocation rules described in Equations (11) to (14). Following that, we provide a robustness test by using different assumptions for the factor risk premia and apply them on all the portfolios describe thus far.

We also perform a robustness exercise. We compare our portfolios that target the reverse optimized factor weights of mean-variance asset portfolios against the portfolios that target mean-variance factor portfolios instead. [16] This comparison further reinforces the benefits of using reverse optimized factor weights.

Why are our factor-targeted portfolios more robust?

In this section, we provide the intuition for why the factor-targeted methodology discussed in this study is a robust method for portfolio allocation. Recall that many different asset portfolios can have the same set of optimal factor weights (Greenberg et al., 2016; Elkamhi et al., 2021). Indeed, by construction our factor-targeted portfolios have the same reverse optimized factor weights as the mean-variance portfolio. The performance difference between portfolios with the same implied factor weights lies in how well the assets are diversified in addition to the portfolio achieving the desired factor weights. There are two contributing factors to the robustness of our method: (1) our method targets the factor weights computed using the *reverse optimized* factor weights of the portfolio (i.e., $w_f^* = B_i\, w_a$), rather than using the portfolio's factor weights (i.e., $w_f^* = Bw_a$) and (2)

[16] As we describe below, "mean variance factor portfolios" are the tangency portfolios calculated using expected returns and covariances of factors. Such portfolios differ from the "reverse optimized factor weights" that are implied by mean variance asset portfolios (i.e., tangency portfolios calculated directly on assets).

we use one of the traditional allocation rules that are risk and diversification focused as our "target asset weights". This ensures that the resulting portfolio, while respecting the reverse optimized factor weights, is optimized towards one of these traditional portfolios, thus inheriting their properties..

Targeting the reverse optimized factor weights adds robustness to our portfolio when assets are generated from a known factor structure with errors. Indeed, note that in Equation (15) we use the implied factor loadings B_i rather than the standard factor loadings B, which are used in Elkamhi et al. (2021). The reason is that the standard factor loadings B are optimal only when assets' expected returns are not subject to pricing errors. In other words, using B implies the underlying factor model completely prices the assets' expected returns without the possibility of pricing errors. Thus, when the "true" model includes pricing errors, using B in a factor-targeting methodology is mis-specified leading to sub-optimal results.

Furthermore, we can decompose the solution for mean-variance portfolios into two components (sub portfolios): $w_a = (\lambda\, \Sigma_a)^{-1}\mu_a^* + (\lambda\, \Sigma_a)^{-1}\varepsilon^*$, where μ_a^* are components of assets' expected returns predicted by μ_f^* and ε^* are the corresponding residuals. In essence, the first sub portfolio – $(\lambda\, \Sigma_a)^{-1}\, \mu_a^*$ – achieves the reverse optimized factor weights of w_f^* for the mean-variance portfolio while the second sub portfolio $-$ $(\lambda\, \Sigma_a)^{-1}\, \varepsilon^*$ $-$ is a maximum Sharpe ratio portfolio using the residuals ε^*.[17] Mean-variance optimization assumes investors know the "true" assets' expected returns (i.e., there are no errors) so the second sub portfolio should not exist.

Finally, this intuition is consistent with the literature studying the trade-off between theoretical optimality and estimation error (e.g., Tu and Zhou, 2011; Kan and Zhou, 2007). On the one hand, investors can use the traditional mean-variance portfolio using assets, which is optimal in the absence of pricing errors but performs poorly in the presence of even small pricing errors. On the other hand, investors can use traditional rules which are considerably less affected by pricing errors but are sub-optimal from a theoretical perspective. Using our methodology, the decrease in theoretical portfolio optimality (i.e., deviating from asset mean-variance) is more than compensated by the reduction of the negative effects of pricing errors.

Stability comparisons between mean-variance and reverse optimized factor weights

We show in this section that the reverse optimized factor weights from a mean-variance

[17] The second sub portfolio does not have any reverse optimized factor weights because by definition $B'\varepsilon^* = 0$.

portfolio can be used as the target factor weights in Equation (9). Using the reverse optimized factor weights has two advantages: reverse optimized factor weights (1) can be calculated directly using observable asset returns, which avoids investors the difficult task to forecast or estimate factor returns, and (2) are stable even in the presence of pricing errors. The latter advantage is an unforeseen result since it is well known that that traditional asset mean-variance portfolios are sensitive to changes in expected return inputs (e.g., Michaud, 1989; Best and Grauer, 1991; Britten-Jones, 1999; DeMiguel et al., 2009). [18] Indeed, Exhibit 3 shows that when assets' expected returns follow a factor structure but are subject to pricing errors as per Equation (16), the mean-variance tangency portfolio weights, as expected, exhibit large variations.

Exhibit 3 shows that converting these asset mean-variance weights to the reverse optimized factor weights via Equation (8) yields reverse optimized factor weights that are much more stable than their assets' counterparts. For example, with a pricing error standard deviation (σ_{ε}) of 1.0%, the average of the standard deviations for tangency portfolio weights across the 5,000 simulations is 83%. Meanwhile, the average of the standard deviations for the reverse optimized factor weights is only 20%. This analysis shows that, even when assets' expected returns vary (according to Equation (16)) leading to unstable mean-variance weights, their reverse optimized factor weights exhibit less variability.

[Insert Exhibit 3 here]

There are two components that contribute to the stability of the reverse optimized factor weights for the mean-variance tangency portfolio. First, as we show below, the errors in the implied factor returns with respect to the true factor expected returns are driven by the sample covariances between the assets' pricing errors and the factor loadings. This can result in the implied factor return errors being smaller than the assets' pricing errors provided that a reasonable number of assets are involved. Second, given the smaller number of factors (i.e., 5 factors) involved than the number of assets (i.e., 17 assets), it is expected that the inverse of the factor covariance matrix has less extreme values than the inverse of the asset covariance matrix. Furthermore, absent pricing errors, these reverse optimized factor weights are mean-variance optimal. Therefore, they are not only stable but

[18] The difficulties of estimating expected returns – which ultimately affect asset allocation – are also discussed in Black (1993).

also optimal.

Using the definition of μ_a provided in Equation (1) together with the definition of μ_f^* in Equation (3), we can write the implied factor excess returns (μ_f^*) of the mean-variance tangency portfolio as

$$\mu_f^* = (B'B)^{-1} B'\mu_a$$
$$\mu_f^* = (B'B)^{-1} B'\big(B\,\mu_f + \varepsilon\big)$$
$$\mu_f^* = \mu_f + \underbrace{(B'B)^{-1}B'\varepsilon}_{\substack{Deviations\ from\ true \\ factor\ excess\ returns}} \tag{17}$$

Equation (17) shows that the implied factor excess returns are equal to the true values, μ_f, plus an error component that depends on the assets' pricing errors ε. The implied factor returns are consistent with estimates from a cross-sectional regression of asset returns on the factor loadings. Furthermore, the factor return errors ($\varepsilon_f = (B'B)^{-1}B'\varepsilon$) can also be interpreted as an estimate from the following cross-sectional regression: $\varepsilon = B\ \varepsilon_f + \epsilon$. As such, the factor returns errors ε_f only capture the covariances between the random assets' pricing errors ε and the factor loading B. Given the known factor structure (i.e., the data generating process for asset returns), these covariances have true values of zero based on the model ($\mu_a = B\mu_f + \varepsilon$). In practice, the sample estimates of the factor return errors are small when the number of assets is (considerably) larger than the number of factors. In our analysis setup, when the standard deviation of asset pricing errors (ε) is 1.00%, the average of the standard deviations for the factor returns errors' (ε_f) is 0.71%.

From Equation (17), it follows that the mean-variance factor weights are equal to

$$w_f^* = \underbrace{\left(\lambda\Sigma_f\right)^{-1}\mu_f}_{\substack{Optimal \\ Weights}} + \underbrace{\left(\lambda\Sigma_f\right)^{-1}(B'B)^{-1}B'\varepsilon}_{Error\ component} \tag{18}$$

Equation (18) shows that the mean-variance factor weights inherit the same structure as the implied factor excess returns: they are equal to the optimal weights in the absence of errors plus an error component. This error component is simply equal to the inverse of the factor covariance matrix $\left(\left(\lambda\Sigma_f\right)^{-1}\right)$ times the factor return errors $\left(\varepsilon_f = (B'B)^{-1}B'\varepsilon\right)$.

As it is well known (e.g., Fan et al., 2008), estimating high-dimensional covariance matrices is very challenging as the number of covariances to estimate grows much faster than the matrix dimension. Therefore, large covariance matrices have very large estimation errors. When the inverse of the covariance matrix is calculated, the effects of these estimation errors are exacerbated. Indeed, from linear algebra we know that the eigenvalue decomposition of the inverse covariance matrix is $\Sigma^{-1} = V\,\Lambda^{-1}\,V^{-1}$, where V is the eigenvector matrix and Λ^{-1} is a diagonal matrix with elements containing the inverse eigenvalues for each eigenvector. The inverse eigenvalues of the factor covariance matrix are less extreme than that of the asset covariance matrix, resulting in small errors in portfolio weights for the same errors in returns. For example, using the covariance matrices in our setup, the largest inverse eigenvalue of the factor covariance matrix has a value of 279, while that of the asset covariance matrix has a value of 2117, almost eight times larger.

Given the smaller values of the factor return errors (when compared with assets' pricing errors) and an inverse factor covariance matrix that has less extreme values (i.e., much less extreme inverse eigenvalues), the errors in the reverse optimized factor weights w_f^* are, as a result, more stable than the mean-variance asset weights.

Having established the benefits of using the reverse optimized factor weights in the presence of pricing errors, we proceed to compare the performances of our factor-targeted portfolios using reverse optimized factor weights against alternative methods.

Comparisons between using standard ($\boldsymbol{B}$) and implied ($\boldsymbol{B_i}$) factor loadings

The methodology presented in this study generalizes the special case provided in Elkamhi et al. (2021). While Elkamhi et al. (2021) provide a solution to build factor-targeted portfolios where assets follow a factor structure *without errors*, we generalize their approach to the more realistic case where asset returns are generated from a factor model with *pricing errors*. As discussed extensively in the literature, pricing errors arise for multiple reasons. For example, the model could be missing an important factor, or the chosen factors might only be a proxy for the "true" (and unobservable) factors underlying asset returns, etc.

In this section, we compare our factor-targeted portfolios, which uses the implied factor loadings B_i, with the factor-targeted portfolios from Elkamhi et al. (2021), which use the standard factor loadings B. Specifically, we generalize the optimization problem in Equation (9) as follows

$$\underset{w}{\operatorname{argmin}}\, \gamma \underbrace{\left(K\,w\,-\,\bar{w}_f{}'\right)\left(K\,w\,-\,\bar{w}_f{}'\right)'}_{\substack{Deviations\ from\ Reverse\\ optimized\ factor\ weights}} + (1-\gamma) \underbrace{(w\,-\,\bar{w}_a)'\,(w\,-\,\bar{w}_a)}_{\substack{Deviations\ from\\ Target\ asset\ weights}} \tag{19}$$

where $K = B$ for the standard factor loadings and $K = B_i$ for the implied factor loadings.

Empirically, B are the values provided in Exhibit 1, while the matrix B_i is computed according to Equation (5). The solution to the above problem is

$$w_{ft,x,K} = [\gamma K'K + (1-\gamma)I_{N\times N}]^{-1}\left(\gamma K' w_{f,tp}^{*} + (1-\gamma)\bar{w}_x\right), \qquad \gamma = 0.999 \tag{20}$$

When the underlying data contain pricing errors − as it is likely to be the case empirically − using $K = B_i$ results in portfolios derived from the "true" model (i.e., accounting for pricing errors). Using $K = B$ results in portfolios derived from the "wrong" model (without accounting pricing errors) and are therefore "sub-optimal". As we show below via simulations, using $K = B$ achieves weaker performances.

Exhibit 4 provides a comparison of Sharpe ratios between the factor-targeted portfolios using the standard (B) and implied factor loadings (B_i). First, Panel A of Exhibit 4 shows the average and standard deviation of the Sharpe ratios from the 5,000 simulations of asset returns. Specifically, using our 5,000 simulations of the assets expected returns as per Equation (16), we calculate the asset weights for each strategy and compute their portfolio Sharpe ratios using the "true" value of assets' expected return $\mu_a = B\mu_f$ (i.e., without pricing errors) and assets' covariance matrix Σ_a.

[Insert Exhibit 4 here]

The mean-variance tangency portfolio Sharpe ratio statistics are provided as a reference in Panel A since we know that in the absence of pricing errors, they are the best performance that an investor could achieve. However, we also note that the performance of the mean-variance tangency portfolio sharply deteriorates in the presence of pricing errors (e.g., Michaud, 1989).

Before discussing the results in the presence of pricing errors, we examine the standard deviation for expected returns' pricing errors that we can expect empirically. Using the historical data and the factor loading presented in Exhibit 1, we compute the historical weekly returns for the factor mimicking portfolios. Together with the factor loadings B, we calculate (or price) the asset returns based on the realized factor returns and Equation (1). We then compute the empirically observed pricing errors as the difference between the realized asset returns and the asset returns priced by the realized factor returns. We compute the standard deviation for these pricing errors for each asset and apply the square root of time rule to compute the standard deviation for the average annualized pricing errors for a 30-year time period. This time window represents a typical time period for long-term expected return forecasts. Averaging those standard deviations across assets yield a value of approximately 1.5%. Therefore, our analysis shows that a value of 1.5% is a representative of the standard deviation of pricing errors of assets' expected return for a 30-year horizon under our factor model.

Panel B and Panel C of Exhibit 4 show the statistics for the Sharpe ratio of the factor-targeted portfolios using standard (B) and implied (B_i) factor loadings, respectively. To ensure the robustness of our results, we employ four-different target asset weights: Maximum Diversification, Minimum Volatility, 1/N and Equal Risk Contributions, which are described in Equations (11) to (14). Consistent with the intuition that we provided above, the average Sharpe ratios are lower when using the standard factor loadings (B) as opposed to the implied factor loadings (B_i) for any level of pricing errors σ_ε and for any choice of target asset weights. This confirms that not only B is sub-optimal theoretically but it also performs worse than B_i.

[Insert Exhibit 5 here]

To gain further insights into the performance of our factor-targeted portfolios, Exhibit 5 plots the distribution of Sharpe ratios from the 5,000 simulations using a value of $\sigma_\varepsilon = 1.25\%$. Panel A of Exhibit 5 plots the distribution of Sharpe ratios for the mean-variance tangency portfolio (red bars), the factor-targeted portfolios using standard factor loadings B (yellow bars) and the factor-targeted portfolios using implied factor loadings B_i (green bars). For the factor-targeted portfolios, we use the Maximum Diversification portfolio as target asset weights. The figure clearly shows that the performance of the factor-targeted portfolios using implied factor loadings B_i vastly supersedes the other two methodologies: the distribution is shifted to the right (i.e., it has a higher average) and it is also much more "narrow" around its mean (i.e., it has less variability). Thus, this provides evidence that our factor-targeted portfolios, which use the implied factor loadings, are less affected by pricing errors. Panel B, C and D of Exhibit 5 repeat the same analysis provided in Panel A but use different target asset weights. Specifically, Panel B uses the Minimum Volatility Portfolio as target asset weights, while Panel C and D use the 1/N and the Equal Risk Contribution portfolios, respectively.

[Insert Exhibit 6 here]

In Panel A of Exhibit 6, we report the probabilities that the factor-targeted portfolios outperform the mean-variance tangency portfolio in terms of Sharpe ratios. We run our analysis for various levels of σ_ε. First, as expected, in the absence of estimation error, the mean-variance tangency portfolio is the optimal portfolio with certainty. Interestingly, as soon as there are small pricing errors ($\sigma_\varepsilon \geq 0.5\%$), the probabilities of outperforming mean-variance for the four factor-targeted portfolios quickly increase above 50% and reach approximately 90% ($\sigma_\varepsilon = 1.0\%$), thus showing a strong outperformance with respect to the tangency portfolio.

This result provides further evidence that our factor-targeted portfolios perform better than the tangency portfolio in the presence of even small pricing errors. Panel B provides the same analysis for the factor-targeted portfolios using standard factor loadings. There is one main takeaway from Panel B of Exhibit 6: the probability of outperforming the mean-variance tangency portfolio is much lower when factor-targeted portfolios are built using the standard factor loadings B rather than built using the implied factor loadings B_i. For example, using the Maximum Diversification portfolio as target asset weights and σ_ε = 0.5%, the probability of outperforming the mean-variance tangency portfolio is 87% using implied factor loadings B_i, while the probability is only 41.7% when using standard factor loadings.

Last, we would like to comment on the effect that the target asset weights have on the factor-targeted portfolios. Our methodology is similar in spirit to the shrinkage estimators pioneered by Stein (1956) and to the methodology of Elkamhi et al. (2021). Shrinkage estimators generate an estimate of a model's coefficient by shrinking the original raw estimate toward a common value. Similarly, our methodology finds the factor-targeted portfolios by "shrinking" the portfolio towards the target asset weights. In other words, our methodology chooses the factor-targeted portfolios such that (1) they respect the desired factor weights and (2) they are as close as possible to the chosen target asset weights. Throughout this article, we present four allocation rules (Maximum Diversification, Minimum Volatility, 1/N and Equal Risk Contributions) but these are just for illustrative purposes. Our proposed methodology – which ultimately is summarized in Equation (9) – can be applied to any allocation rule. [19]

[19] In untabulated results, we find that using inverse volatility and inverse variance allocation rules yields qualitatively similar results to those discussed for the 4 allocation rules presented in this article. That is, using implied factor loadings leads to factor-targeted portfolios that outperform portfolios built using standard factor loadings.

Comparisons between factor-targeted portfolios and traditional portfolios

Exhibit 7 shows the weights for the target asset portfolios $\overline{w}_x$, their corresponding factor-targeted portfolios $w_{ft,x}$ and their reverse optimized factor weights for this analysis.[20] For each of the 5,000 simulation, we first calculate the reverse optimized factor weights $w^*_{f,tp}$ based on the mean-variance tangency portfolio. Next, we use these reverse optimized factor weights $w^*_{f,tp}$ and apply them in Equation (15) for the four target asset portfolios as per Equations (11) to (14) to calculate the factor-targeted portfolio weights $w^*_{ft,x}$.

[Insert Exhibit 7 and Exhibit 8 here]

Exhibit 7 shows that the reverse optimized weights for the four factor-targeted portfolios are the same when $\mu_a = B\ \mu_f$ (with no errors) and they are equal to the reverse optimized factor weights from the mean-variance tangency portfolio shown in Panel B of Exhibit 3.[21] In other words, these factor-targeted portfolios imply the same factor risk premia as the mean-variance portfolio would for the same assets' expected return inputs.

Using our simulated data, we calculate the standard deviations of these factor-targeted asset weights across the simulation and report them in Exhibit 8. There are two main results from Exhibit 8. First, we note that the asset weight standard deviations are the same for each asset across all four factor-targeted asset portfolios. As shown in Equation (15), only $w^*_{f,tp}$ is affected by pricing errors to μ_a while $\overline{w}_x$ is not. This is because the four traditional allocation rules used in Equation (15) do not depend on μ_a and therefore their weights $\overline{w}_x$ are not affected by the pricing errors in μ_a.

The covariance matrix for the factor-targeted portfolio weights can be written as

$$\Sigma_{w_{ft,x}} = P\ \Sigma_f\ P' \tag{21}$$

where $P = [\gamma\ B_i{}'B_i + (1-\gamma)I_{N\times N}]^{-1}\ \gamma B_i{}'$. The asset weight standard deviations shown in Exhibit 8 across the 5,000 simulations are estimates of the square root of the diagonal elements of $\Sigma_{w_{ft,x}}$. Equation (21) is applicable for all four target asset portfolios (Equations (11) to (14)). Therefore, their factor-targeted asset weights standard deviations are only affected by the variation

[20] It is worth noting that in Exhibit 4, the reverse optimized factor weights for the traditional portfolios can be quite arbitrary as they are affected only by Σ_a and what assets are included in the optimization (we explore this point later in the results and discussions section).

[21] This result is by design as it can be derived from Equation (15).

in the reverse optimized factor weights, which is the same across all four target portfolios (e.g., Maximum Diversification, etc.).

Second, the weight standard deviations from Exhibit 8 are considerably more stable than those for the mean-variance tangency portfolios shown in Exhibit 3. For example, with $\sigma_\varepsilon = 1\%$, the average of the weight standard deviations across assets for the mean-variance tangency portfolio is 83% (see Exhibit 3). For the factor-targeted portfolios, the average of the weight standard deviations across assets is only 2.8%. Another way to analyze the stability of the factor-targeted portfolios is to evaluate their risk exposures. In the interest of brevity, we report such results in Appendix B where we show that, indeed, the factor-targeted portfolios are stable in terms of volatility contributions as well as standalone volatility risk.

Comparisons under different risk premia assumptions

How would different risk premia assumptions affect the factor-targeted and traditional portfolios? We address the question in this section by considering four alternative risk premia assumptions, which are shown in Exhibit 9. Set-1 is the same set of assumptions as those provided in Exhibit 2. Set-2 to Set-4 represent scenarios where risk premiums are concentrated on selected factors: Set-2 is a scenario whereby only the equity and credit factors have risk premia; Set-3 represents a scenario whereby only bonds have risk premia; Set-4 is a scenario whereby only commodities have risk premia.

Exhibit 10 shows the averages and standard deviations of portfolio Sharpe ratios across the 5,000 simulations for the four sets of risk premia settings and for $\sigma_\varepsilon = 0\%$ (no errors), 1% and 2%.[22] Specifically, Panel A contains the results for the Mean-variance portfolio and Panel B contains the results for the four traditional portfolios that we also used as target asset weights. Panel C and D contain the results for the factor target portfolios using implied (B_i) and standard (B) factor loadings, respectively.

[Insert Exhibit 9 and Exhibit 10 here]

There are several observations on the results shown in Exhibit 10. First, for each set of risk premia, the Sharpe ratio of the tangency portfolios (Panel A) when $\sigma_\varepsilon = 0\%$ is the highest when

[22] We do not present the Sharpe ratio standard deviations for the traditional portfolios (Panel B) since they are not affected by pricing errors (σ_ε).

compared with all other asset allocation rules (Panels B, C and D). This is intuitive since, in the absence of pricing errors, the tangency portfolio is, by definition, the portfolio with the highest Sharpe ratio.

Second, we compare the mean-variance tangency and the factor-targeted portfolios, both using implied (B_i) and standard (B) factor loadings. Panel C shows that the performance of the factor-targeted portfolios becomes superior to that of the tangency portfolios when assets' expected returns are subject to errors. When $\sigma_\varepsilon = 1\%$ or 2%, the factor-targeted portfolio Sharpe ratios using implied (B_i) factor loadings become higher than those of the tangency portfolios. This is because, as the error standard deviation increases, the factor-targeted Sharpe ratios deteriorate at a much slower pace than those of the tangency portfolios (Panel A). Panel D shows that the factor-targeted portfolio Sharpe ratios using standard (B) factor loadings are also higher than those of the tangency portfolios in the presence of pricing errors; thus, showing that the use of factor-targeted portfolios outperforms the use of mean-variance asset tangency portfolios. However, consistent with our discussion above, the Sharpe ratios shown in Panel C, which use implied factor loadings, are higher than those shown in Panel D, which use standard factor loadings.

Third, we compare the factor-targeted and other traditional portfolios (Maximum Diversification, Minimum Volatility, 1/N and Equal Risk Contributions). Although traditional portfolios are not affected by pricing errors in the expected returns as they utilize only the covariance matrix (or they are naïve such as 1/N and use no information), their performances vary noticeably across different risk premia scenarios. When $\sigma_\varepsilon = 0\%$, the factor-targeted portfolios have higher Sharpe ratios than those of the traditional portfolios across the four different sets of risk premia assumptions. When $\sigma_\varepsilon = 2\%$, most of the factor-targeted portfolios have higher Sharpe ratios across the risk premia assumptions labelled Set-1, Set-2 and Set-4. For Set-3 (where only bonds have risk premia), the traditional portfolios have higher Sharpe ratios. The reason why traditional portfolios outperform for Set-3 is due to their high real return bonds factor weights in our examples, as shown in Panel B from Exhibit 7. These high real return bonds factor weights depend on the choice of the asset universe used in the portfolio construction (i.e., the 17 assets we used in this article).

Using weights from mean-variance on factors as target factor weights

The results presented thus far use the reverse optimized factor weights for tangency portfolios described in Equation (8) as the target factor weights to build factor-targeted portfolios. These desired factor weights are derived by taking into account that there are pricing errors they are equivalent to the mean-variance optimal factor weights in the absence of pricing errors (see Equation (6)).

It is natural to ask: how would the results change if one uses the mean-variance optimal factor weights, rather than using the reverse optimized factor weights for a tangency portfolio, as the target factor weights for our methodology? We know that in the absence of pricing errors, using the mean-variance factor weights is equivalent to using the reverse optimized factor weights of a tangency portfolio but it is relevant to investigate how pricing errors would affect the results.[23]

In Exhibit 11, we provide a comparison between factor-targeted portfolios built using reverse optimized factor weights of a tangency portfolio (i.e., Equation (8)) and factor-targeted portfolios built using mean-variance factor weights. Similar to our analysis above, we simulate asset returns according to Equation (1). For each of the 5000 simulations, we calculate the expected returns for each of the 5 macroeconomic factors using the simulated assets' expected returns and the definition of macroeconomic tradeable factors provided in Exhibit 2. We then use mean-variance optimization (i.e., Equation (6)) to obtain the "mean-variance optimal" factor weights for each simulation. We use these mean-variance optimal factor weights as target factor weights to build a version of factor-targeted portfolios that we compared against those that uses the reverse optimized factor weights of a tangency portfolio.

[Insert Exhibit 11 and Exhibit 12]

Panel A provides the average and standard deviation of Sharpe ratios using mean-variance directly on assets, which provides a benchmark. Panel B and Panel C show the average and standard deviation of Sharpe ratios for the factor-targeted portfolio built using reverse optimized factor weights of tangency portfolios and using mean-variance optimal factor portfolios (i.e., portfolios built using the mean-variance optimal factor weights), respectively. When there are no pricing errors, Panels B and C generate the same results as expected.

However, for pricing errors σ_ε that are greater than zero, Panels B and C exhibit differences. Specifically, consistent with the fact that reverse optimized factor weights account for pricing errors while mean-variance factor weights do not, the former methodology outperforms the latter more and more as pricing errors become larger. Consider the row with target asset weights equal to the maximum diversification portfolio (w_A: $Max.Div$); for $\sigma_\varepsilon = 2\%$, the average Sharpe ratio ($\mu(SR)$)

[23] We thank the anonymous referee for suggesting us to conduct this analysis using mean-variance factor weights.

is equal to 28.1% using reverse optimized factor weights of tangency portfolios while it is lower at 22.6% using mean-variance factor weights. Exhibit 11 shows that, as pricing errors become larger, the standard deviation of the Sharpe ratios increases more for portfolios built using mean-variance factor weights than those built using reverse optimized factor weights. For $\sigma_\varepsilon = 2\%$, the standard deviation of Sharpe ratios ($\sigma(SR)$) is equal to 5.1% using reverse optimized factor weights of tangency portfolio while it is much higher at 12.8% using mean-variance factor weights.

Exhibit 12 shows the probabilities of outperforming mean-variance for factor-targeted portfolios built using the two sets of target factor weights. As we did in Exhibit 6, we measure such probability as the percentage of cases when a factor-targeted portfolio has higher Sharpe ratio than the mean-variance tangency portfolio. The blue solid line shows the probability when we use the reverse optimized factor weights while the dashed orange line shows the probability when we use mean-variance factor weights. Each of the four panels in Exhibit A contains the result for one of the target asset weights that we used throughout this article: Maximum Diversification in Panel A, Minimum Volatility in Panel B, 1/N in Panel C, Equal Risk Contributions in Panel D. Consistent with Exhibit 11, Exhibit 12 shows that as pricing errors increase, the portfolios built using reverse optimized factor weights outperform those built using mean-variance factor weights. This result is robust across all 4 target-asset weights.

CONCLUSIONS

In this article, we show that when assets' expected returns follow a factor structure subject to pricing errors, the traditional asset mean-variance portfolio can be reverse optimized to obtain implied factor premia and associated mean-variance factor portfolio weights that are considerably less volatile than the asset weights. This is due to (1) the implied factor expected returns likely having less errors than the assets' expected returns and (2) the inverse factor covariance matrix having fewer extreme values than the inverse asset covariance matrix. This means that the reverse optimized factor weights from the mean-variance tangency portfolio can be used as the desired factor weights to build a portfolio.

We exploit this result by developing a methodology to build what we call "factor-targeted" portfolios that target the factor weights implied by the traditional mean-variance tangency portfolio. Our methodology achieves a unique mapping between factor weights and asset weights by using a set of target asset weights as an additional portfolio optimization objective. In our analysis, we use four traditional, risk and diversification focused portfolio allocation rules to compute target asset weights and show that they all yield similar results in terms of Sharpe ratios. Our factor-targeted portfolios not only have higher average Sharpe ratios than those of the mean-variance tangency portfolios when assets' expected returns are subject to errors, but they also generate such higher Sharpe ratios with a high probability even when pricing errors are small. Furthermore, the factor-targeted portfolio weights are more robust than the mean-variance portfolio weights when asset expected return inputs vary, which is an important consideration for investors in practice. We provide robustness tests to confirm the validity of our results by using (1) alternative factor risk premia assumptions, and (2) mean-variance factor weights as our target factor weights.

We provide an explanation for why our methodology achieves robust results when applied. The intuition is as follows. Other methodologies, such that those from Greenberg et al. (2016) and Elkamhi et al. (2021), build portfolios that target a desired set of factor weights under an implicit modelling assumption that assets' expected returns have no pricing errors. We show in this article that, in the presence of pricing errors, the optimal factor weights differ from those that one would obtain by assuming no pricing errors. Therefore, when there are pricing errors but investors do not account for them, they would build sub-optimal portfolios. Also, our methodology guarantees that the portfolio would achieve a high level of asset risk diversification in addition to matching the desired factor weights by targeting the portfolio to traditional allocation rules that are risk and diversification focused.

In conclusion, our factor-targeted portfolios combine useful elements from both mean-variance and traditional methodologies. By targeting the portfolios' reverse optimized factor weights to those implied by the mean-variance tangency portfolio, the resulting portfolio has the same factor optimality as the mean-variance portfolio but is much more robust against errors in assets' expected returns.

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DISCLAIMER

This article is for informational purposes only and should not be construed as legal, tax, investment, financial, or other advice. The views and opinions expressed here are those of the authors alone and do not necessary reflect the views of their employers and their affiliates.

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Exhibit 1: Historical data and factor loadings

Assets	Bloomberg ticker	Expected Excess returns	Factor loading matrix B (t-stats in parenthesis)					R-squared
			Equities	Real Return Bonds	Commodities	Break-even Inflation	Credit	
Equity - U.S.	ES1 Index	5.0%	1.60	0.00	-0.01	-0.12	0.20	84.59%
			(46.49)	(0.14)	(-0.43)	(-3.64)	(6.15)	
Equity - Europe	VG1 Index	5.7%	2.02	-0.08	-0.15	0.23	-0.06	86.13%
			(50.80)	(-2.46)	(-4.64)	(5.83)	(-1.54)	
Equity - U.K.	Z 1 Index	5.1%	1.67	0.10	0.02	-0.06	0.04	85.97%
			(51.00)	(3.81)	(0.64)	(-1.94)	(1.27)	
Equity - Japan	TP1 Index	4.9%	1.78	-0.05	-0.27	0.15	-0.13	61.51%
			(27.61)	(-1.05)	(-5.32)	(2.36)	(-2.05)	
Equity - E.M.	EEM US Equity*	6.8%	2.13	0.01	0.48	-0.17	-0.12	84.59%
			(37.56)	(0.19)	(10.63)	(-3.16)	(-2.24)	
Fixed Income - U.S.	TY1 Comdty	0.4%	-0.01	0.48	0.03	-0.34	-0.04	81.46%
			(-1.16)	(53.67)	(3.33)	(-30.99)	(-3.87)	
Fixed Income - Europe	RX1 Comdty	0.5%	0.00	0.50	-0.03	-0.39	0.03	86.95%
			(0.47)	(66.50)	(-3.69)	(-42.97)	(3.36)	
Fixed Income - U.K.	G 1 Comdty	0.6%	0.01	0.58	0.00	-0.43	0.01	86.60%
			(0.90)	(66.98)	(-0.28)	(-40.40)	(1.24)	
Credit - I.G.	IBOXIG Index*	0.9%	0.04	0.57	-0.01	-0.30	0.34	84.59%
			(1.71)	(32.44)	(-0.45)	(-14.17)	(16.22)	
Credit - H.Y.	IBOXHY Index*	1.1%	-0.01	0.19	0.01	-0.14	0.91	98.86%
			(-1.16)	(53.67)	(3.33)	(-30.99)	(208.98)	
Inflation Linked - U.S.	LBUTTRUU Index*	0.5%	0.01	0.51	0.07	-0.07	0.00	68.93%
			(0.31)	(39.99)	(5.16)	(-4.48)	(0.08)	
Inflation Linked - Europe	BEIG1T Index*	0.3%	0.05	0.38	-0.06	0.09	-0.01	61.39%
			(3.06)	(29.32)	(-4.28)	(5.79)	(-0.53)	
Inflation Linked - U.K.	FTRFILA Index*	0.3%	-0.10	0.84	-0.01	-0.05	0.01	84.59%
			(-3.51)	(35.71)	(-0.37)	(-1.87)	(0.48)	
Commodity - Oil	CL1 Comdty	2.2%	0.04	-0.30	2.39	0.57	0.07	47.49%
			(0.29)	(-2.81)	(22.04)	(4.35)	(0.56)	
Commodity - Gold	GC1 Comdty	0.7%	-0.31	0.40	1.36	-0.15	-0.10	50.16%
			(-4.99)	(8.15)	(27.26)	(-2.51)	(-1.75)	
Commodity - Copper	HG1 Comdty	3.4%	0.63	-0.22	1.70	0.05	-0.06	56.28%
			(7.18)	(-3.17)	(24.25)	(0.65)	(-0.66)	
Commodity - Corn	C 1 Comdty	1.4%	-0.20	-0.18	1.92	-0.25	0.17	84.59%
			(-1.86)	(-2.11)	(22.21)	(-2.35)	(1.65)	

* series converted to excess returns using the local 3-month risk free rate

This table shows the 17 assets used to compute the assets' covariance matrix Σ_a. Expected excess returns are computed using the formula $\mu_a = B\mu_f$. B is the 17 × 5 factor loadings' matrix (t-stats in parenthesis). The factor definitions and their assumed risk premia μ_f are provided in Exhibit 2. Data are obtained from Bloomberg for the period between January 1, 2005 and December 31, 2019. Non-overlapping weekly excess returns are used. We regress asset returns on the 5 factors defined in Exhibit 2 to calculate the factor loadings B shown above.

Exhibit 2: Factor definitions and risk premium assumptions

Factors	Risk Premiums μ_f	Definitions
Equities	3.00%	Inverse volatility weighted equity basket (U.S., Europe, U.K., Japan, Emerging Markets), scaled to 10% annualized volatility.
Real Return Bonds	0.75%	Inverse volatility weighted real return bond basket (U.S., Europe, U.K.), scaled to 10% annualized volatility.
Commodities	1.00%	Inverse volatility weighted commodity basket (Oil, Gold, Copper, Corn), scaled to 10% annualized volatility.
Break-even Inflation	-0.25%	Long 1 unit on the Real Return Bonds factor, and short 1 unit on the inverse volatility weighted fixed income basket (U.S., Europe, U.K.) scaled to 10% annualized volatility. The break-even inflation factor is then scaled to 10% annualized volatility.
Credit	1.00%	Long 1 unit on the U.S. High Yield Credit in excess returns and short 0.4 units on the U.S. 10-year Treasury futures for a neutral interest rate duration credit factor portfolio.

This table contains the definitions for the 5 factors used in the analysis and their assumed risk premia. Factors are defined by factor mimicking portfolios. The weekly return series for assets listed in Exhibit 1 are weighted according to the definitions above to create the factor return series. The historical factor return series are used to compute the factor covariance matrix Σ_f.

Exhibit 3: Stability of mean-variance and reverse optimized factor weights

Assets / Factors	Portfolio weights	Std. dev. of portfolio weights for different σ_ε			
		0.5%	1.0%	1.5%	2.0%
Panel A: Mean-variance tangency portfolio					
Equity - U.S.	10.0%	15%	29%	44%	58%
Equity - Europe	8.2%	17%	34%	51%	68%
Equity - U.K.	10.1%	18%	37%	55%	73%
Equity - Japan	8.5%	5%	9%	14%	18%
Equity - E.M.	7.3%	6%	13%	19%	25%
Fixed Income - U.S.	50.4%	135%	271%	406%	542%
Fixed Income - Europe	47.3%	134%	268%	402%	536%
Fixed Income - U.K.	41.1%	116%	231%	347%	463%
Credit - I.G.	0.0%	50%	101%	151%	202%
Credit - H.Y.	-8.9%	32%	63%	95%	126%
Inflation Linked - U.S.	-27.6%	84%	168%	252%	336%
Inflation Linked - Europe	-30.2%	57%	114%	171%	228%
Inflation Linked - U.K.	-15.8%	27%	54%	80%	107%
Commodity - Oil	-0.1%	2%	3%	5%	7%
Commodity - Gold	-0.2%	5%	9%	14%	18%
Commodity - Copper	-0.1%	3%	5%	8%	11%
Commodity - Corn	-0.1%	2%	4%	6%	8%
Average of Std. dev. across assets		42%	83%	125%	166%
Panel B: Reverse optimized factor weights from the mean-variance tangency portfolio					
Factor - Equities	80.1%	9%	18%	28%	37%
Factor - Real Return Bonds	31.6%	6%	13%	19%	25%
Factor - Commodities	-0.7%	3%	7%	10%	14%
Factor - Break-even Inflation	-52.3%	16%	32%	48%	64%
Factor - Credit	-8.2%	14%	28%	43%	57%
Average of Std. dev. across factors		10%	20%	29%	39%

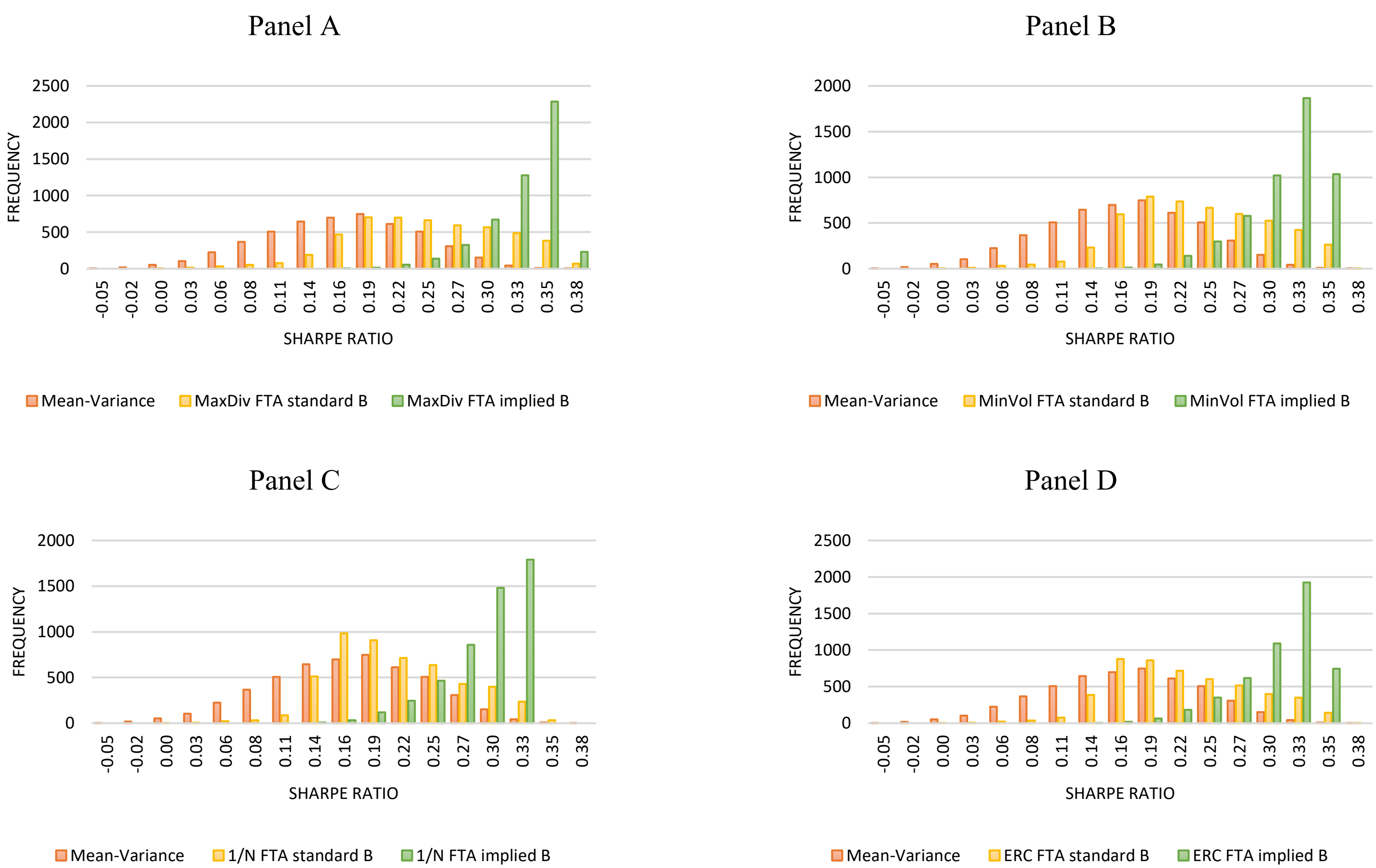
Panel A shows the portfolio weights ($w_{a,tp}$) for the mean-variance tangency portfolio in column "Portfolio Weights". The four rightmost columns show the standard deviations of $w_{a,tp}$ across 5,000 simulations for different values of σ_ε according to Equation (16). The averages of the asset weight standard deviations are provided in the last row. Panel B provides the same type of information for the reverse optimized factor weights ($w^*_{f,tp}$) obtained from the asset weights $w_{a,tp}$ in Panel A. For each simulation, the mean-variance tangency weights are converted to the reverse optimized weights using Equation (8) and the standard deviations of those factor weights across simulations are calculated for different σ_ε.

Exhibit 4: Sharpe ratio comparisons for different factor-targeted portfolios

Portfolio allocation methods	Sharpe ratio statistics	Std. dev. of portfolio weights for different σ_ε					
		0.0%	0.5%	1.0%	1.25%	1.5%	2.0%
		Panel A: Mean-variance portfolio					
Tangency	$\mu(SR)$	40.8%	29.1%	18.8%	15.8%	13.5%	10.4%
	$\sigma(SR)$		4.9%	6.6%	6.9%	7.0%	7.1%
		Panel B: Factor-Targeted portfolios using the standard factor loadings B					
w_A: Max. Div.	$\mu(SR)$	28.3%	27.9%	24.9%	23.0%	21.2%	18.1%
	$\sigma(SR)$		4.6%	6.2%	6.8%	7.3%	7.9%
w_A: Min. Vol.	$\mu(SR)$	25.9%	25.9%	23.7%	22.1%	20.5%	17.6%
	$\sigma(SR)$		4.4%	5.9%	6.4%	6.9%	7.5%
w_A: 1/N	$\mu(SR)$	20.8%	21.3%	20.7%	19.8%	18.7%	16.4%
	$\sigma(SR)$		3.7%	5.3%	5.7%	6.1%	6.7%
w_A: ERC	$\mu(SR)$	22.3%	22.8%	21.9%	20.8%	19.5%	16.9%
	$\sigma(SR)$		4.1%	5.7%	6.2%	6.5%	7.1%
		Panel C: Factor-Targeted portfolios using the implied factor loadings B_i					
w_A: Max. Div.	$\mu(SR)$	35.4%	34.6%	32.8%	31.7%	30.5%	28.1%
	$\sigma(SR)$		1.1%	2.6%	3.3%	3.9%	5.1%
w_A: Min. Vol.	$\mu(SR)$	32.8%	32.2%	30.7%	29.7%	28.6%	26.4%
	$\sigma(SR)$		1.5%	2.9%	3.5%	4.1%	5.1%
w_A: 1/N	$\mu(SR)$	31.3%	30.7%	28.9%	27.8%	26.7%	24.4%
	$\sigma(SR)$		1.6%	3.0%	3.6%	4.2%	5.2%
w_A: ERC	$\mu(SR)$	32.8%	32.1%	30.3%	29.2%	28.0%	25.7%
	$\sigma(SR)$		1.5%	3.0%	3.7%	4.3%	5.3%

This table shows the averages ($\mu(SR)$) and the standard deviations ($\sigma(SR)$) of Sharpe ratio across 5,000 simulations for different values of σ_ε. The analysis involves the mean-variance tangency portfolio and the two sets of factor-targeted portfolios using different factor loadings.

Exhibit 5: Distribution of Sharpe ratios



These figures show the distribution of Sharpe ratios across 5,000 simulations for $\sigma_\varepsilon = 1.25\%$. Panel A to D involve the use of the four traditional allocation rules: Maximum Diversification (MaxDiv), Minimum Volatility (MinVol), 1/N and Equal Risk Contributions (ERC), respectively, for the factor-targeted portfolios. For each panel, the chart shows the distribution of Sharpe ratios for the mean-variance tangency portfolio (Mean-Variance) and the two factor-targeted asset (FTA) portfolios using standard factor loadings (standard B) and implied factor loadings (implied B).

Exhibit 6: Probability of outperforming mean-variance for various σ_ε

Panel A: Probabilities using implied factor loadings B_i for different σ_ε

	σ_ε								
	0.00%	0.25%	0.50%	0.75%	1.00%	1.25%	1.50%	1.75%	2.00%
MaxDiv	0.0%	26.1%	87.0%	96.6%	98.2%	98.5%	98.4%	98.1%	97.8%
MinVol	0.0%	8.3%	70.3%	90.1%	94.7%	96.2%	96.4%	96.4%	96.2%
1/N	0.0%	3.7%	58.0%	84.0%	91.2%	93.2%	94.0%	94.0%	93.7%
ERC	0.0%	8.1%	69.1%	89.1%	93.7%	95.4%	95.7%	95.6%	95.2%

Panel B: Probabilities using standard factor loadings B for different σ_ε

	σ_ε								
	0.00%	0.25%	0.50%	0.75%	1.00%	1.25%	1.50%	1.75%	2.00%
MaxDiv	0.0%	3.2%	41.7%	63.4%	71.6%	75.2%	76.9%	77.4%	77.5%
MinVol	0.0%	1.1%	31.8%	56.3%	67.5%	72.4%	74.9%	75.6%	76.4%
1/N	0.0%	0.0%	13.5%	38.1%	54.6%	63.3%	68.3%	71.3%	72.6%
ERC	0.0%	0.1%	18.9%	44.4%	59.0%	67.0%	70.7%	73.1%	73.9%

Panel A shows the probability that the factor-targeted portfolios using implied factor loadings generate Sharpe ratios higher than those of the mean-variance tangency portfolio. We measure such probability as the percentage of cases where the factor-targeted portfolios exhibit Sharpe ratios than those of the tangency portfolio in the 5,000 simulations. We provide the results for different levels of pricing error standard deviation (σ_ε). Panel B reports results from the same analysis using standard factor loadings for the factor-targeted portfolios.

Exhibit 7: Factor-targeted portfolio weights for various target asset portfolios

	Target asset weights for different portfolios				Factor-Targeted portfolio weights for different target asset weights using B_i			
	Max. Div.	Min. Vol	1/N	ERC	Max. Div.	Min. Vol	1/N	ERC
Panel A: Asset weights								
Equity - U.S.	4.7%	4.8%	5.9%	2.9%	7.4%	7.1%	3.9%	3.5%
Equity - Europe	10.9%	1.2%	5.9%	2.6%	15.3%	5.5%	5.2%	5.5%
Equity - U.K.	-8.9%	-0.6%	5.9%	2.7%	-6.1%	2.4%	4.4%	3.6%
Equity - Japan	7.2%	2.7%	5.9%	3.2%	12.1%	7.9%	7.3%	7.8%
Equity - E.M.	-1.5%	-2.0%	5.9%	2.0%	3.1%	4.8%	5.6%	6.1%
Fixed Income - U.S.	50.7%	47.0%	5.9%	14.3%	49.7%	46.2%	5.3%	12.0%
Fixed Income - Europe	34.7%	27.7%	5.9%	14.8%	33.8%	26.9%	5.3%	12.5%
Fixed Income - U.K.	12.7%	10.2%	5.9%	11.6%	11.6%	9.4%	5.2%	8.9%
Credit - I.G.	-17.0%	-22.4%	5.9%	5.9%	-18.5%	-24.2%	3.5%	1.5%
Credit - H.Y.	11.1%	19.3%	5.9%	5.5%	10.0%	17.0%	2.4%	1.6%
Inflation Linked - U.S.	-23.4%	-3.4%	5.9%	7.8%	-24.5%	-4.2%	4.6%	5.0%
Inflation Linked - Europe	-2.8%	20.0%	5.9%	10.0%	-3.2%	19.6%	5.1%	8.2%
Inflation Linked - U.K.	5.8%	-8.6%	5.9%	5.7%	3.9%	-10.1%	4.0%	0.6%
Commodity - Oil	3.5%	0.5%	5.9%	1.9%	-3.0%	-4.1%	-4.7%	-4.2%
Commodity - Gold	5.1%	0.7%	5.9%	3.7%	3.0%	2.0%	4.2%	0.5%
Commodity - Copper	3.2%	1.8%	5.9%	2.4%	5.3%	7.8%	5.3%	6.9%
Commodity - Corn	4.0%	1.1%	5.9%	3.0%	-0.4%	-2.6%	-2.0%	-1.6%
Panel B: Reverse optimized factor weights								
Factor - Equities	15.1%	8.3%	46.5%	24.1%	80.1%	80.1%	80.1%	80.1%
Factor - Real Return Bonds	26.5%	26.3%	18.0%	37.8%	31.7%	31.7%	31.7%	31.7%
Factor - Commodities	22.3%	5.0%	43.9%	19.3%	-0.7%	-0.7%	-0.7%	-0.7%
Factor - Break-even Inflation	-12.2%	-19.7%	6.4%	-20.4%	-52.3%	-52.3%	-52.3%	-52.3%
Factor - Credit	2.1%	10.2%	9.0%	9.2%	-8.2%	-8.2%	-8.2%	-8.2%

Max. Div.: Maximum diversification portfolio; Min. Vol.: Minimum volatility portfolio; 1/N: Equally weighted portfolio; ERC: Equal Risk Contributions Portfolio

In Panel A, the first four columns report the target asset weights for the four traditional portfolios defined in Equations (11) to (14). The rightmost four columns show the four factor-targeted portfolios using implied factor loadings (Equation (15)) and the target asset weights reported in the first four columns. The table uses $\gamma = 0.999$. Panel B shows the reverse optimized factor weights for the portfolios in Panel A using Equation (4).

Exhibit 8: Stability of the factor-targeted portfolios

Assets	Std. dev. of Factor-Targeted portfolio weights using implied factor loadings B_i for different σ_ε			
	0.5%	1.0%	1.5%	2.0%
Equity - U.S.	1.3%	2.7%	4.0%	5.3%
Equity - Europe	0.8%	1.5%	2.3%	3.0%
Equity - U.K.	0.8%	1.7%	2.5%	3.4%
Equity - Japan	0.6%	1.2%	1.7%	2.3%
Equity - E.M.	1.2%	2.4%	3.6%	4.8%
Fixed Income - U.S.	0.7%	1.4%	2.1%	2.8%
Fixed Income - Europe	0.7%	1.4%	2.1%	2.8%
Fixed Income - U.K.	0.9%	1.7%	2.6%	3.4%
Credit - I.G.	1.7%	3.4%	5.2%	6.9%
Credit - H.Y.	2.4%	4.7%	7.1%	9.4%
Inflation Linked - U.S.	0.9%	1.8%	2.7%	3.6%
Inflation Linked - Europe	0.6%	1.3%	1.9%	2.6%
Inflation Linked - U.K.	1.6%	3.3%	4.9%	6.6%
Commodity - Oil	1.1%	2.3%	3.4%	4.5%
Commodity - Gold	2.6%	5.1%	7.7%	10.2%
Commodity - Copper	3.0%	6.0%	9.1%	12.1%
Commodity - Corn	2.5%	5.1%	7.6%	10.1%
Average of Std. dev. across assets	1.4%	2.8%	4.1%	5.5%

This table shows the standard deviations of the factor-targeted portfolio weights when using any of the four target asset portfolios shown in Exhibit 4. The standard deviations of the factor-targeted portfolio weights are computed using 5,000 simulations for different values of σ_ε according to Equation (16).

Exhibit 9: Alternative risk premia assumptions

Factors	Set-1	Set-2	Set-3	Set-4
Equities	3.00%	3.00%	0.00%	0.00%
Real Return Bonds	0.75%	0.00%	3.00%	0.00%
Commodities	1.00%	0.00%	0.00%	3.00%
Break-even Inflation	-0.25%	0.00%	-1.00%	0.00%
Credit	1.00%	1.00%	0.00%	0.00%

This table shows the four sets of risk premia assumptions used in the Sharpe ratio analysis. Set-1 uses the same assumptions as shown in Exhibit 3 and Exhibit 5. Set-2 represents a scenario where only the equity and credit factors have risk premiums. Set-3 represents a scenario where only bonds have risk premiums. Set-4 represents a scenario where only commodities have risk premiums.

Exhibit 10: Sharpe ratio analysis for different risk premia assumptions

Portfolios	Sharpe ratio statistics	σ_ε: 0%				σ_ε: 1%				σ_ε: 2%			
		Factor premia assumption				Factor premia assumption				Factor premia assumption			
		Set-1	Set-2	Set-3	Set-4	Set-1	Set-2	Set-3	Set-4	Set-1	Set-2	Set-3	Set-4
Panel A: Mean-variance portfolio													
Tangency	$\mu(SR)$	0.41	0.38	0.41	0.33	0.19	0.24	0.10	0.07	0.10	0.15	0.05	0.04
	$\sigma(SR)$	*0.00*	*0.00*	*0.00*	*0.00*	*0.07*	*0.05*	*0.07*	*0.03*	*0.07*	*0.06*	*0.07*	*0.02*
Panel B: Target Asset portfolios													
Max. Div.	$\mu(SR)$	0.27	0.16	0.27	0.16	0.27	0.16	0.27	0.16	0.27	0.16	0.27	0.16
Min. Vol.	$\mu(SR)$	0.22	0.12	0.34	0.05	0.22	0.12	0.34	0.05	0.22	0.12	0.34	0.05
1/N	$\mu(SR)$	0.27	0.20	0.08	0.15	0.27	0.20	0.08	0.15	0.27	0.20	0.08	0.15
ERC	$\mu(SR)$	0.28	0.17	0.27	0.12	0.28	0.17	0.27	0.12	0.28	0.17	0.27	0.12
Panel C: Factor-Targeted portfolios using implied factor loadings B_i for different σ_ε													
w_A: Max. Div.	$\mu(SR)$	0.35	0.34	0.36	0.33	0.33	0.33	0.30	0.25	0.28	0.30	0.22	0.17
	$\sigma(SR)$	*0.00*	*0.00*	*0.00*	*0.00*	*0.03*	*0.02*	*0.05*	*0.05*	*0.05*	*0.04*	*0.10*	*0.08*
w_A: Min. Vol.	$\mu(SR)$	0.33	0.32	0.37	0.33	0.31	0.31	0.30	0.25	0.26	0.29	0.22	0.17
	$\sigma(SR)$	*0.00*	*0.00*	*0.00*	*0.00*	*0.03*	*0.02*	*0.06*	*0.05*	*0.05*	*0.04*	*0.10*	*0.08*
w_A: 1/N	$\mu(SR)$	0.31	0.32	0.34	0.33	0.29	0.31	0.26	0.25	0.24	0.28	0.18	0.17
	$\sigma(SR)$	*0.00*	*0.00*	*0.00*	*0.00*	*0.03*	*0.02*	*0.06*	*0.05*	*0.05*	*0.04*	*0.11*	*0.08*
w_A: ERC	$\mu(SR)$	0.33	0.32	0.37	0.33	0.30	0.31	0.29	0.25	0.26	0.29	0.20	0.17
	$\sigma(SR)$	*0.00*	*0.00*	*0.00*	*0.00*	*0.03*	*0.02*	*0.06*	*0.05*	*0.05*	*0.04*	*0.11*	*0.08*
Panel D: Factor-Targeted portfolios using standard factor loadings B for different σ_ε													
w_A: Max. Div.	$\mu(SR)$	0.28	0.27	0.36	0.33	0.25	0.26	0.19	0.14	0.18	0.22	0.11	0.08
	$\sigma(SR)$	*0.00*	*0.00*	*0.00*	*0.00*	*0.06*	*0.05*	*0.09*	*0.07*	*0.08*	*0.07*	*0.10*	*0.06*
w_A: Min. Vol.	$\mu(SR)$	0.26	0.26	0.33	0.33	0.24	0.25	0.19	0.14	0.18	0.21	0.11	0.08
	$\sigma(SR)$	*0.00*	*0.00*	*0.00*	*0.00*	*0.06*	*0.05*	*0.09*	*0.07*	*0.07*	*0.06*	*0.09*	*0.06*
w_A: 1/N	$\mu(SR)$	0.21	0.22	0.23	0.32	0.21	0.23	0.17	0.14	0.16	0.20	0.10	0.08
	$\sigma(SR)$	*0.00*	*0.00*	*0.00*	*0.00*	*0.05*	*0.05*	*0.08*	*0.07*	*0.07*	*0.06*	*0.09*	*0.06*
w_A: ERC	$\mu(SR)$	0.22	0.23	0.27	0.32	0.22	0.23	0.18	0.14	0.17	0.20	0.11	0.08
	$\sigma(SR)$	*0.00*	*0.00*	*0.00*	*0.00*	*0.06*	*0.05*	*0.08*	*0.07*	*0.07*	*0.06*	*0.09*	*0.06*

Panel A, B, C and D show the Sharpe ratio statistics for the mean-variance tangency portfolio, the traditional portfolios and the factor-targeted portfolios using implied (B_i) and standard (B) factor loadings, respectively. For each panel, both the averages ($\mu(SR)$) and the standard deviations ($\sigma(SR)$) of Sharpe ratio across 5,000 simulations are reported. For each level of σ_ε as per Equation (16), we report the results for four different sets of factor premia assumptions (Set-1 to Set-4). The details of the four sets of risk premia assumptions are provided in Exhibit 9.

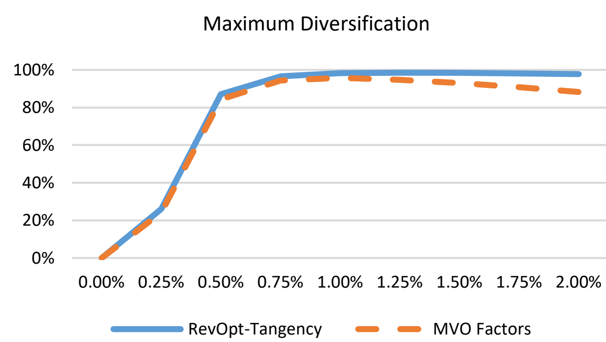
Exhibit 11: Sharpe ratio analysis using the mean-variance factor portfolio as target factor weights

Portfolio allocation methods	Sharpe ratio statistics	Std. dev. of portfolio weights for different σ_ε					
		0.0%	0.5%	1.0%	1.25%	1.5%	2.0%
Panel A: Mean-variance portfolio							
Tangency	$\mu(SR)$	40.8%	29.1%	18.8%	15.8%	13.5%	10.4%
	$\sigma(SR)$		4.9%	6.6%	6.9%	7.0%	7.1%
Panel B: Factor-Targeted portfolios using reverse optimized factor weights of the tangency portfolio as target factor weights							
w_A: Max. Div.	$\mu(SR)$	35.4%	34.6%	32.8%	31.7%	30.5%	28.1%
	$\sigma(SR)$		1.1%	2.6%	3.3%	3.9%	5.1%
w_A: Min. Vol.	$\mu(SR)$	32.8%	32.2%	30.7%	29.7%	28.6%	26.4%
	$\sigma(SR)$		1.5%	2.9%	3.5%	4.1%	5.1%
w_A: 1/N	$\mu(SR)$	31.3%	30.7%	28.9%	27.8%	26.7%	24.4%
	$\sigma(SR)$		1.6%	3.0%	3.6%	4.2%	5.2%
w_A: ERC	$\mu(SR)$	32.8%	32.1%	30.3%	29.2%	28.0%	25.7%
	$\sigma(SR)$		1.5%	3.0%	3.7%	4.3%	5.3%
Panel C: Factor-Targeted portfolios using the mean-variance factor weights as target factor weights							
w_A: Max. Div.	$\mu(SR)$	35.4%	34.2%	31.3%	29.2%	27.0%	22.6%
	$\sigma(SR)$		1.5%	4.3%	7.1%	9.5%	12.8%
w_A: Min. Vol.	$\mu(SR)$	32.8%	31.8%	29.3%	27.4%	25.4%	21.1%
	$\sigma(SR)$		1.7%	4.2%	6.9%	9.2%	12.5%
w_A: 1/N	$\mu(SR)$	31.3%	30.3%	27.8%	25.9%	23.8%	19.5%
	$\sigma(SR)$		1.8%	4.2%	6.8%	9.0%	12.3%
w_A: ERC	$\mu(SR)$	32.8%	31.7%	29.0%	27.1%	25.0%	20.6%
	$\sigma(SR)$		1.9%	4.3%	7.0%	9.3%	12.6%

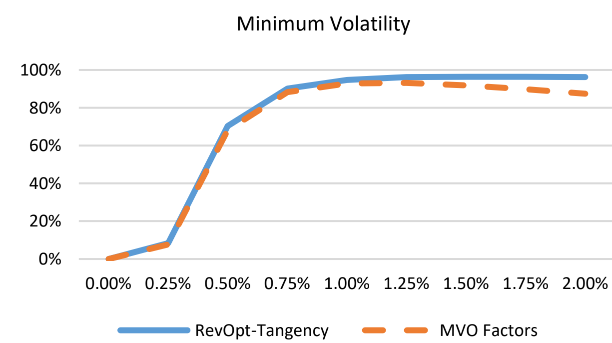
This table shows the averages ($\mu(SR)$) and the standard deviations ($\sigma(SR)$) of Sharpe ratio across 5,000 simulations for different values of σ_ε. The analysis involves the mean-variance tangency portfolio (Panel A), the factor-targeted portfolios using reverse optimized factor weights of the tangency portfolio (Panel B) and the factor-targeted portfolios using the mean-variance factor weights. The implied factor loadings (B_i) are used for the factor-targeted portfolios.

Exhibit 12 Comparison of probabilities: using reverse optimized factor weights of tangency portfolios versus using mean-variance factor weights

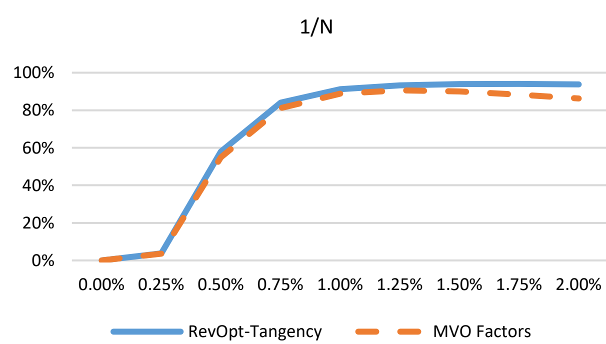
Panel A



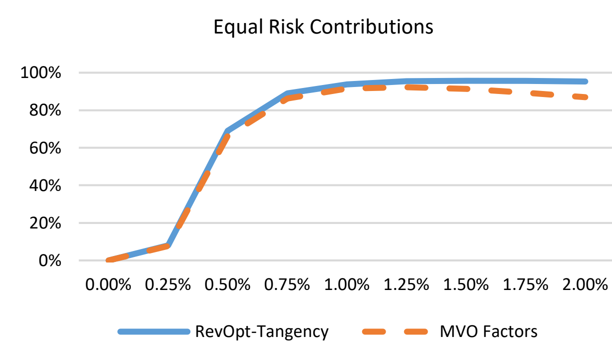
Panel B



Panel C



Panel D



For each Panel, the line shows the probability that a factor-targeted portfolio shows a higher Sharpe ratio than the mean-variance tangency portfolio. The blue solid line shows the factor-targeted portfolios built using the reverse optimized factor weights of tangency portfolios ("RevOpt-Tangency" in legend) as the desired target factor weights. The orange dashed line shows the factor-targeted portfolios built using the mean-variance factor weights ("MVO Factors" in legend) as the desired target factor weights. As in Exhibit 6, we measure such probability as the percentage of cases when the factor-targeted portfolios exhibit a higher Sharpe ratio than the tangency portfolio in the 5,000 simulations. Panel A shows the results for the Maximum Diversification portfolio, Panel B for the Minimum Volatility Portfolio, Panels C and D plot the results for the 1/N and Equal Risk Contributions portfolios. We provide the results for different levels of pricing errors' standard deviation (σ_ε), which vary along the x-axis.

Appendix A

In this appendix, we prove that there exist multiple asset portfolios that respect a desired set of factor weights; therefore, additional constraints or objectives are required to obtain a unique mapping as discussed in this article as well as in Greenberg et al. (2016) and Elkamhi et al. (2021). To achieve a unique mapping between factor weights and asset portfolios, investors are required to provide at least one additional constraint or objective. This is discussed in Greenberg et al. (2016) and Elkamhi et al. (2021) and we provide a formal derivation in this appendix to prove that there does not exist a unique mapping between factor weights and asset portfolios in the absence of such additions.

Formally, investors seeking a desired target factor weight – without additional constraints or objectives – build an asset portfolio with the following optimization procedure

$$\underset{w}{\operatorname{argmin}} \underbrace{(B_i\, w \;-\; \overline{w}_f')(B_i\, w \;-\; \overline{w}_f')'}_{\substack{Deviations\ from\ Reverse \\ optimized\ factor\ weights}} \tag{A.1}$$

where $\overline{w}_f$ is the desired target factor weights, and B_i is the $M \times N$ *implied* factor loading matrix defined in Equation (5). Note that Equation A.1 is a special case of Equation (9), where the parameter γ is set to 1. N is the number of assets and M is the number of factors.

The solution to the problem in Equation (A.1) would be

$$w = [B_i{}'B_i\,]^{-1}\, B_i{}'\, \overline{w}_f \tag{A.2}$$

and it is unique if and only if $B_i{}'B_i$ – which is a $N \times N$ matrix – is an invertible matrix. However, $B_i{}'B_i$ is not invertible since its rank is equal to M (the number of factors), which is less than N (the number of assets). In other words, as shown in Equation (1), the N assets are spanned by the M factors and it follows that only M columns are linearly independent. Therefore, the matrix $B_i{}'B_i$ cannot be inverted since it does not have full rank. This implies that there does not exist a unique solution to Equation (A.2).

For investors, this means that, in the absence of additional constraints or objectives, there does not exist a unique portfolio of assets that achieves the desired factor weights, rather there are potentially many of them. In Equation (9), we add a second objective to achieve a unique mapping between factor weights and asset portfolios by using the target asset weights.

Appendix B

In this Appendix, we address the question: do factor-targeted portfolios provide stability in terms of risk exposures? We answer this question by analyzing the standard deviations of volatility risk contributions and standalone volatility risks across the 5,000 simulations. First, for each simulation we calculate the risk contribution of each asset to total volatility of the portfolio as well as the assets' standalone risks. Given a vector of asset weights w and a variance covariance matrix Σ, it is well known that the volatility of the portfolio can be written as $\sigma(w) = \sqrt{w' \Sigma w}$. The risk contribution for asset i (RC_i) is defined as

$$RC_i = w_i \cdot \frac{\partial \sigma(w)}{\partial w_i} = w_i \cdot \frac{(\Sigma w)_i}{\sqrt{w' \Sigma w}} \quad \text{(B.1)}$$

where w_i is the weight of asset i, and $(\Sigma w)_i$ is the i-th element of the vector Σw.[24] The standalone risk of asset i (SA_i) is defined as the asset weight multiplied by its volatility

$$SA_i = w_i \times \sigma_i \quad \text{(B.2)}$$

where σ_i is the volatility of asset i.

Exhibit B.1 shows the standard deviation of risk contributions across 5,000 simulations for a standard deviation of pricing errors of 1% ($\sigma_\varepsilon = 1\%$). We present our results for the mean-variance factor portfolio as well as for the four factor-targeted portfolios defined in Equations (11) to (14). The standard deviations of risk contributions for the mean-variance tangency portfolio are considerably larger than those for the four factor-targeted portfolios. Indeed, the average of the standard deviations of risk contribution for the tangency portfolio is 1.5% while it is less than 0.3% for those of the factor-targeted portfolios. This confirms that our factor-targeted portfolios provide investors with more stable risk diversification compared to the tangency portfolio. Furthermore, the results also show that the standard deviations of risk contributions between the four factor-targeted portfolios are all very similar. This result shows that our choice of traditional rules used to build a factor-targeted portfolio with our methodology (Equation (15)) does not affect the stability of the risk contributions.

[Insert Exhibit B.1 and Exhibit B.2 here]

[24] For a textbook treatment of risk contributions, we refer the interested reader to Roncalli (2013).

Exhibit B.2 presents the analysis on standalone risks. The table shows the standard deviation of standalone risk across 5,000 simulations for a standard deviation of pricing errors of 1% ($\sigma_\varepsilon = 1\%$). Similarly, to the results on risk contributions, we present our findings for the mean-variance factor portfolio as well as for the four factor-targeted portfolios defined in Equations (11) to (14). The table shows that the standard deviations of standalone risks for the four factor-targeted portfolios are lower than those of the mean-variance tangency portfolio. This finding provides further evidence that our factor-targeted portfolios exhibit stability across a multitude of characteristics: asset weights as shown in Exhibit 8, risk contributions as shown in Exhibit B.1 and standalone risks as shown in Exhibit B.2.

Exhibit B.1: Stability of Risk Contributions

		Factor-targeted Portfolios			
	MVO	MaxDiv	MinVol	1/N	ERC
Equity - U.S.	1.8%	0.4%	0.4%	0.4%	0.3%
Equity - Europe	2.2%	0.4%	0.3%	0.3%	0.3%
Equity - U.K.	2.3%	0.2%	0.2%	0.3%	0.2%
Equity - Japan	0.6%	0.3%	0.2%	0.2%	0.2%
Equity - E.M.	1.1%	0.4%	0.4%	0.4%	0.4%
Fixed Income - U.S.	3.2%	0.3%	0.3%	0.0%	0.1%
Fixed Income - Europe	3.3%	0.2%	0.1%	0.0%	0.1%
Fixed Income - U.K.	2.9%	0.1%	0.1%	0.0%	0.1%
Credit - I.G.	1.7%	0.1%	0.2%	0.1%	0.1%
Credit - H.Y.	1.0%	0.3%	0.4%	0.2%	0.2%
Inflation Linked - U.S.	2.1%	0.1%	0.0%	0.1%	0.1%
Inflation Linked - Europe	1.5%	0.0%	0.1%	0.0%	0.1%
Inflation Linked - U.K.	0.7%	0.1%	0.1%	0.1%	0.1%
Commodity - Oil	0.1%	0.1%	0.1%	0.2%	0.1%
Commodity - Gold	0.1%	0.3%	0.2%	0.3%	0.2%
Commodity - Copper	0.2%	0.9%	1.1%	1.0%	1.1%
Commodity - Corn	0.1%	0.4%	0.3%	0.3%	0.3%
Average	1.5%	0.3%	0.3%	0.2%	0.2%

This table shows the standard deviations of risk contributions across 5,000 simulations for $\sigma_\varepsilon = 1\%$. The definition of risk contributions is provided in Equation (20). Column MVO shows the standard deviations of risk contributions for the mean-variance tangency portfolio. The remaining columns contain the standard deviation of risk contributions for the factor-targeted portfolios: MaxDiv is the factor-targeted portfolio with asset target weights equal to the Maximum Diversification portfolio, MinVol uses the Minimum Volatility portfolio as target asset weights, 1/N uses the equally weighted portfolio and ERC uses the equal risk contribution portfolio as target asset weights. Equation (15) provides the solution for the factor-targeted portfolios.

Exhibit B.2: Stability of Standalone Risk

		Factor-targeted Portfolios			
	MVO	MaxDiv	MinVol	1/N	ERC
Equity - U.S.	5.3%	0.5%	0.5%	0.5%	0.5%
Equity - Europe	7.5%	0.3%	0.3%	0.3%	0.3%
Equity - U.K.	6.6%	0.3%	0.3%	0.3%	0.3%
Equity - Japan	2.0%	0.2%	0.2%	0.2%	0.2%
Equity - E.M.	3.2%	0.6%	0.6%	0.6%	0.6%
Fixed Income - U.S.	14.7%	0.1%	0.1%	0.1%	0.1%
Fixed Income - Europe	14.5%	0.1%	0.1%	0.1%	0.1%
Fixed Income - U.K.	14.3%	0.1%	0.1%	0.1%	0.1%
Credit - I.G.	7.4%	0.3%	0.3%	0.3%	0.3%
Credit - H.Y.	5.5%	0.4%	0.4%	0.4%	0.4%
Inflation Linked - U.S.	9.9%	0.1%	0.1%	0.1%	0.1%
Inflation Linked - Europe	6.1%	0.1%	0.1%	0.1%	0.1%
Inflation Linked - U.K.	5.5%	0.3%	0.3%	0.3%	0.3%
Commodity - Oil	1.3%	0.9%	0.9%	0.9%	0.9%
Commodity - Gold	1.6%	0.9%	0.9%	0.9%	0.9%
Commodity - Copper	1.5%	1.7%	1.7%	1.7%	1.7%
Commodity - Corn	1.1%	1.5%	1.5%	1.5%	1.5%
Average	6.4%	0.5%	0.5%	0.5%	0.5%

This table shows the standard deviations of standalone risks across 5,000 simulations for $\sigma_\varepsilon = 1\%$. The definition of standalone risks is provided in Equation (21). Column MVO shows the standard deviations of standalone risks for the mean-variance tangency portfolio. The remaining columns contain the standard deviation of standalone risks for the factor-targeted portfolios: MaxDiv is the factor-targeted portfolio with asset target weights equal to the Maximum Diversification portfolio, MinVol uses the Minimum Volatility portfolio as target asset weights, 1/N uses the equally weighted portfolio and ERC uses the equal risk contribution portfolio as target asset weights. Equation (15) provides the solution for the factor-targeted portfolios.