Diversifying Macroeconomic Factors —for Better or for Worse[*]

LIVIA AMATO[†]
HEC Paris

HARALD LOHRE[‡]
Invesco
CEAM, Cambridge Judge Business School
EMP, Lancaster University Management School

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[†]HEC Paris, 1 rue de la Libération, 78350 Jouy-en-Josas, France; `livia.amato@hec.edu`; Phone: +39 3495944872

[‡]Harald Lohre is Director of Research at Invesco Quantitative Strategies, An der Welle 5, 60322 Frankfurt/Main, Germany; a Fellow of the Centre for Endowment Asset Management (CEAM), Cambridge Judge Business School, University of Cambridge, UK; and a Visiting Research Fellow at the Centre for Financial Econometrics, Asset Markets and Macroeconomic Policy (EMP), Lancaster University Management School, Bailrigg, Lancaster LA1 4YX, United Kingdom; `harald.lohre@invesco.com`

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Abstract

It is widely acknowledged that asset returns are driven by common sources of risk, especially in challenging times when the benefits from traditional portfolio diversification fail to realize. From a top-down perspective, investors are mostly concerned about shocks in growth or inflation that ultimately govern the pricing of broad asset classes. To this extent, we propose a natural asset allocation framework to achieve a diversified exposure to orthogonal macro risk factors and to harvest the associated long-term premia. We examine the role and usefulness of different types of macroeconomic variables, as systematic sources of risk or state variables that drive time variation in the asset returns, and compare their diversification potential across different states of the world.

Keywords: Macro Factors, Factor Investing, Diversification, Markov Switching
JEL Classification: C34; E44; G11; G12

It is generally accepted that asset classes exhibit higher correlation during difficult markets, suggesting that risk and return are driven by only a few meaningful underlying forces (Longin and Solnik, 2002; Ang and Bekaert, 2002; Ang and Chen, 2001, Ang, Goetzmann and Schaefer, 2009). Accordingly, traditional risk-based strategies based on asset classes fail to provide effective diversification precisely when it is needed most, and they fail to protect investors from changes in key driving factors such as economic growth or risk aversion. Thus, a factor-based portfolio that focuses on the primary drivers of asset returns is likely to provide higher diversification benefits for portfolio performance.

In this paper we study the relevance of macroeconomic factors in multi-asset allocation and present an allocation framework to achieve a diversified exposure to orthogonal macroeconomic factors based on factor risk parity portfolios. We examine the usefulness of macro factors as common drivers of assets from both a risk perspective to manage portfolio risk exposures and from a return perspective to harvest rewarded premia. Indeed, based on modern asset pricing theory, macroeconomic factors represent state variables describing different sets of bad times and exposure to these factors is then compensated by the assets' risk premia earned during the good states. First, we study which economic factors are useful in explaining variation in asset returns, building on the large literature of multi-factor models based on macroeconomic variables and initiated with the Arbitrage Pricing Theory of Ross (1976). As Chen, Roll and Ross (1986) note, *"A rather embarrassing gap exists between the theoretically exclusive importance of systematic "state variables" and our complete ignorance of their identity. The co-movement of asset prices suggests the presence of underlying exogenous influences, but we have not yet determined which economic variables, if any, are responsible"*. In particular, we separately analyse the different role of macroeconomic risk factors and state variables in asset allocation and explore potential benefits from a diversified macro factor allocation versus an asset class approach. Additionally, we study a novel implementation of factor risk parity strategies where we derive orthogonal macro risk factors using a recursive identification scheme following the extensive macroeconomic literature on VAR, allowing us to give a structural interpretation to the relevant macroeconomic shocks. Ultimately, we extend the analysis to a conditional framework to assess the diversification properties of macroeconomic factors across states and we use a Markov Switching VAR model to estimate macroeconomic risks and identify economic regimes.

The literature on factor investing has generally focused on style factors to explain cross-sectional variation in securities, based on fundamental characteristics or behavioral biases observed in financial markets. In contrast, an earlier literature relied on factor analysis applied to the covariance matrix of asset returns to identify the implicit factors that explain cross-sectional variation (Roll and Ross, 1980; Chen, 1983; Connor and

Korajzyck, 1988, and Lehman and Modest, 1988). Nevertheless, as Fama (1991, p.1594) states, such factor models *"leave one hungry for economic insights about how the factors relate to uncertainties about consumption and portfolio opportunities that are of concern to investors, that is, the hedging arguments for multifactor models of Fama (1970) and Merton (1973)."* In this vein, economic variables represent shocks to future expected cash flows or discount rates and are natural candidates to explain the time series and cross-sectional variation in asset returns and to bridge the gap between asset prices and the real economy.

Despite the economic relevance of macroeconomic factors in asset returns, the literature on macro factor investing is relatively narrow. As illustrated in Martellini and Milhau (2018), several drawbacks of macro factors hamper the implementation of macro factor-based strategies. For example, macro factors tend to have low statistical power in explaining variation in asset returns (Grinold and Kahn, 2000). Connor (1995) compares three types of factor models of security returns and shows that fundamental and statistical factor models significantly outperform macroeconomic factor models in terms of explanatory power. Another limitation is that –unlike style factors– macro factors are not directly investable and, therefore, achieving direct exposure to macroeconomic factors would typically require constructing factor-mimicking portfolios. However, one would need to accurately measure the exposures of the assets to the macroeconomic factors, which is subject to sampling error (Lehmann and Modest, 1988). The procedure is complicated as there is no unique and clear way to measure the macro factors, and even relying on official figures such as GDP growth suffers from delays in their release date, low frequency data availability (e.g. quarterly or annually) or subsequent revisions. Additionally, instability of the exposures of the assets to the macro factors may render the implementation of such factor-based strategies impractical, potentially resulting in high turnover.

This study draws from and contributes to the literature in three areas centered on the economic relationship between asset returns and the macroeconomy as well as on asset allocation: 1) risk-based portfolio allocation, 2) macro factor-mimicking portfolios, and 3) the econometric literature on VAR and Markov-Switching models. Our main innovation is to bring together the recent developments and previous research in these areas into a unique, integrated study by examining the implications of macroeconomic factors for portfolio diversification.

Our results confirm the joint importance of financial and macroeconomic risk variables in a multi-factor model of asset returns. A balanced exposure to five risk factors, deemed as the primary sources of risk across assets, enhances the risk-return profile by explicitly harvesting multiple sources of expected returns. Nevertheless, such portfolios provide only partial diversification benefits due to common variation in the risk factors at times when diversification is needed the most. On the contrary, when relying on macroeconomic state variables which help explaining time-variation in asset returns and represent the ultimate long-term economic drivers, factor risk parity strategies offer effective diversification, especially during states of high economic uncertainty, while not forgoing higher risk-adjusted returns. On the one hand, they achieve higher upside potential by explicitly harvesting long-term macro premia; on the other hand, they reduce portfolio drawdowns through a diversified exposure to macro factors representing different states of the world, resulting in a strategy that is robust across different economic cycles.

This paper is organized as follows. Section 1 presents the theory behind multi-factor models and asset pricing and reviews the literature on macro-based asset allocation. Section 2 introduces factor risk parity and details the methodology used for the empirical analysis and portfolio implementation. Section 3 describes the data, while Section 4 presents the results from the empirical analysis and the performance of the strategies. Section 5 extends the analysis of the strategies under an MS-VAR model. Section 6 presents robustness tests and Section 7 concludes the paper.

1 Literature

1.1 Factor Models and Asset Pricing Theory

Identifying the relevant factors that explain risk and return variation in asset returns is an open question in empirical finance. In practice, the notion of factors refers to multiple interconnected concepts, as noted in Martellini and Milhau (2018). *Risk factors* represent common underlying sources of risk across assets. The covariance of the asset returns can be decomposed into systematic sources of risk from the specific factor exposures and idiosyncratic risk, which—by the "law of large numbers"—can be eliminated in a sufficiently well-diversified portfolio. These factors may not carry a premium but are useful in portfolio management to explain portfolio returns through a style analysis (Sharpe, 1988) or for risk budgeting in the portfolio construction.

A related concept is that of *pricing factors*, which are systematic sources of risk that can explain differences in the cross-section of returns. Ross (1976) introduced the Arbitrage Pricing Theory (APT), which posits that there are common factors that explain the main variation across asset returns and that cannot be diversified away, in the same way as the market risk is the only systematic factor in the Capital Asset Pricing Model (CAPM). Formally, Ross (1976) assumes that asset returns are generated by a multiple-factor model described by:

$$R_{it} = E\left[R_i\right] + \sum_{k=1}^{K} \beta_{ik} F_{kt} + \varepsilon_{it}, \tag{1}$$

where R_i is the return on asset i, $E[R_i]$ is the unconditional, expected return on the asset, F_k is the change in the k^{th} common factor, assumed to have zero mean, and β_{ki} represents the sensitivity of the asset return to the specific factor. Ross (1976) assumes that the error terms have zero mean and are cross-sectionally uncorrelated, that is, a "strict factor model" as defined by Chamberlain and Rothschild (1983). In particular, he shows, by a no-arbitrage argument, that the absence of risk-less profits implies that

$$E[R_i] = \sum_{k=1}^{K} \beta_{ki}\Lambda_k, \tag{2}$$

where Λ_k are the risk premia of the K common factors. Therefore, the Arbitrage Pricing Theory (APT) bridges the relationship between pricing factors, which can explain the cross-section of returns, and risk factors, which represent common sources of risk across assets. This shows that, in equilibrium, if factors have prices, the expected return on an asset is a compensation for multiple systematic sources of risk.

Modern asset pricing theory relies on the notion of a "stochastic discount factor" (SDF), which represents an index of "bad times" (Ang, 2014) and which prices all assets by discounting their expected payoffs. It can be shown that the expected risk premium of an asset can be written as

$$\begin{aligned} E[r_i] - r_f &= \frac{cov(r_i, m)}{var(m)}\left(-\frac{var(m)}{E[m]}\right) \\ &= \beta_{i,m} \times \lambda_m, \end{aligned} \tag{3}$$

with $\beta_{i,m} = \frac{cov(r_i,m)}{var(m)}$ being the beta of the asset relative to the stochastic discount factor m, and $\lambda_m = -\frac{var(m)}{E[m]}$ being the price of risk (Cochrane, 2009). In consumption-based asset pricing models (Lucas, 1978; Breeden, 1979), the stochastic discount factor is proportional to the representative investor's marginal utility of consumption. Assets with a high payoff during bad states (e.g. periods of low consumption and high marginal utility) have a high covariance $cov(r_i, m)$ with the stochastic discount factor; they are more desirable and therefore will require a lower risk premium. Conversely, assets with a low payoff during bad states will sell at a "discount" relative to the price of risk and will require a higher return. In practice, the stochastic discount factor, m, cannot be observed but is approximated by a combination of factors, leading to the APT multi-factor model described by Eq. 2.

Therefore, the APT starts with a statistical decomposition of the covariance matrix of the asset returns and derives an equilibrium relationship between risk and expected returns (or equivalently an expected return-beta relationship). However, as Cochrane (2009, p.171) points out, *"A factor structure in the covariance of returns or high R^2 in regressions of returns on factors can imply factor pricing but factors can price returns* *without describing their covariance matrix."*

Indeed, a separate concept is that of *state variables*, which are factors describing shifts in the investment opportunity set and which can explain the *time series* of expected risk and returns on assets, supporting the empirical evidence of time-varying risk premia. For example, Fama and French (1989) show that stocks and bonds move together and that time variation in excess returns can be predicted by common factors that are correlated with the business cycle, such as dividend yield, default and term premia, or by business cycle variables such as output gap (Cooper and Priestley, 2009). The Intertemporal CAPM by Merton (1973) bridges the relationship between the time-series and cross-section of expected returns arguing that state variables which are of hedging concerns to investors and help predict the stochastic discount factor can be pricing risk factors in the cross-section of expected returns.

While continuous research in asset pricing theory investigates how to effectively measure the SDF, macroeconomic variables are intuitive candidates to measure aggregate marginal utility. They can be interpreted as state variables defining different sets of bad times, such as low growth or high inflation, against which investors want to be hedged. Thus, based on the modern asset pricing theory, exposure to these factors is compensated by the assets' risk premia. By this argument, it follows that macroeconomic variables which describe the conditional distribution of future income and asset returns are important risk factors to investors.

1.2 Macro Factor Allocation

Bass, Gladstone and Ang (2017) analyze institutional portfolios through the lens of macroeconomic factors. They use principal component analysis on thirtheen global asset class returns and find that approximately 95% of the variance can be explained by six common factors, which they identify as economic growth, real rates, inflation, credit, emerging markets and commodities. Furthermore, to account for the remaining risk in the multi-asset portfolios, Bass et al. (2017) consider a foreign exchange factor. They investigate factor completion overlay portfolios to achieve more balanced allocations in terms of the macroeconomic factors. In particualr, they use a robust quadratic optimization procedure to minimize the factor exposure deviations and the tracking error between the asset allocations and a pre-determined factor allocation. To derive a mapping from factors to assets, Greenberg, Babu and Ang (2016) describe a sparse mapping procedure whereby each asset return is regressed on a subset of macro factors selected on the basis of economic priors and constrained stepwise regressions.

Blyth, Szigety and Xia (2016) study asset allocation strategies based on macro factors and introduce an alternative asset allocation procedure to solve the indeterminateness of the mapping procedure relying on a sparse loadings matrix based on economic priors similarly to Greenberg et al. (2016). Nevertheless, both studies ultimately measure the macroeconomic factors in terms of asset returns or financial variables. For example, economic growth is generally associated with a broad equity market index, or the inflation factor with a long-short position in nominal and real government bonds. While such an approach can successfully explain short-term variation in asset returns with real-time measures, it fails to provide a direct mapping between assets and genuine macroeconomic variables.

Conversely, Brooks (2017) explicitly considers a global macro strategy based on four macroeconomic fundamentals: business cycle, international trade, monetary policy and risk sentiment. In particular, Brooks (2017) constructs macro momentum portfolios based on improving macroeconomic trends across asset classes. He finds that a combination of the four macro momentum strategies is weakly correlated with traditional asset classes and alternative risk premia and provides effective diversification across various economic regimes.

A different strain of literature focuses on building investable portfolios that replicate macroeconomic factors, although the topic was studied extensively in early research on asset pricing in the cross-section of expected returns. Jurczenko and Teiletche (2019), for example, compare several methodologies to build factor-mimicking portfolios that replicate three macroeconomic variables: economic growth, inflation surprises and financial stress. They show that these portfolios can help hedge macroeconomic risks in a reference endowment portfolio. Chousakos and Giamouridis (2020) build macro-mimicking portfolios that track three economic variables: macro-growth (using a wide range of economic activity indicators), fragility (as a measure of firm financial health), and volatility (based on the second moment of a large cross-section of stock returns). Similarly, Ang (2014) argues that the three most relevant macro factors for asset allocation are growth, inflation and volatility, and that unexpected changes in these variables are of greater concern to investors than their level.

2 Factor Risk Parity

2.1 Introduction

Our primary objective is to derive portfolio strategies with a well-diversified exposure to macroeconomic factors. In the portfolio risk budgeting literature several techniques have been developed to achieve a target or balanced allocation to risk in the total portfolio. Risk Parity portfolios, as first denoted by Qian (2005), allocate risk equally across the asset classes or securities that compose the portfolio. Maillard, Roncalli and Teïletche (2010) thus analyze equally-weighted risk contribution portfolios and find that these portfolios are equivalent to a minimum-variance portfolio subject to a norm constraint on the weights and that the total portfolio volatility lies between that of a minimum-variance and an equally-weighted portfolio. Bridgewater Associates (Prince, 2011) shows that balancing risk exposures from different asset classes provides effective diversification across different economic environments, based on the assumption that different asset classes perform well at different times. Consequently, they refer to the risk parity portfolio as an “All-Weather” approach which is robust to shifts in economic growth or inflation regimes known to determine cross-sectional differences in asset returns. In a related vein, Choueifaty and Coignard (2008) introduce the “Most Diversified Portfolio” which maximizes portfolio diversification as represented by the ratio of the weighted average volatility of assets to the overall portfolio volatility.

In this study we refer to a more recent risk budgeting technique, here denoted Factor Risk Parity, which calls for allocating equal risk to uncorrelated factors rather than to the assets that ultimately compose the portfolio. Factor Risk Parity was introduced by Meucci (2009) and is obtained by maximizing a measure of diversification across uncorrelated risk factors. In Meucci (2009)'s original work, the latter are generated through a principal component analysis of the covariance matrix of asset returns and represent the main building blocks, or principal portfolios, explaining cross-sectional variation in a portfolio of assets. The diversification distribution of the portfolio is represented by the contributions of the principal portfolios, normalized by total portfolio variance, and is expressed by

$$p_k = \frac{\widetilde{w}_k^2 \lambda_k}{var\left(R_w\right)} \text{for } k = 1, \ldots, K \tag{4}$$

where λ_k are the variances of the uncorrelated principal components, $\widetilde{w}_k$ are the weights in the principal portfolios and K is the total number of principal portfolios. Therefore, a well-diversified portfolio implies that the principal portfolios' contributions p_k are “*approximately equal and the diversification distribution is close to uniform*” (Meucci, 2009), equivalently to the concept of risk parity.

Portfolio diversification is then measured as the exponential of the entropy of the diversification distribution and is denoted as the Effective Number of Bets,

$$ENB = \exp\left(-\sum_{k=1}^{K} p_k \log p_k\right) \tag{5}$$

This measure represents the actual number of uncorrelated risk exposures in the portfolio, such that, for example, for $p_k = 1$ and $p_j = 0$ with $k \neq j$, the portfolio has maximum concentration (lowest degree of diversification) and $ENB = 1$. Vice versa, for $p_k = p_j = 1/K$ for all k, j, all risk contributions are equal and the effective number of bets is maximized at $ENB = K$.

Therefore, maximizing the effective number of bets is equivalent to equalizing the relative contributions of each factor p_k to the total portfolio variance:

$$p_k = \frac{1}{K} \Leftrightarrow \widetilde{w}_k^2 \lambda_k = \frac{1}{K} w' \Sigma w \text{ for } k = 1, \ldots, K$$

As explained in Deguest, Martellini and Meucci (2013) the solution to the maximization problem yields the optimal factor risk parity weights (in terms of factors)

$$\widetilde{w} = \Sigma_k^{-1/2} \begin{pmatrix} \pm 1 \\ \vdots \\ \pm 1 \end{pmatrix} \tag{6}$$

Note that all Factor Risk Parity allocations are inversely proportional to the factor variances, with Σ_k being the diagonal covariance matrix of the principal components. In practice, the ENB optimization problem is not uniquely defined but has 2^K different solutions, depending on the signs attributed to the uncorrelated risk factors in Equation 6 and additional constraints must be imposed to derive a unique portfolio solution associated with factor risk parity.

Lohre, Opfer and Ország (2014) adopt this strategy to develop a Diversified Risk Parity (DRP) portfolio in a multi-asset allocation study and find superior risk-adjusted performance and a well-diversified allocation to the underlying principal portfolios contrary to alternative risk-based strategies.

However, the use of principal component analysis to derive uncorrelated risk factors suffers from several shortcomings which make the diversified risk parity strategy difficult to implement in practice. As pointed out in Meucci, Santangelo and Deguest (2015), principal components are likely to be statistically unstable and difficult to interpret from an economic perspective. Moreover, they are very sensitive to scalar multiplications, which lead to extreme changes in the principal component bets. Finally, the principal component bets are not unique as there are 2^K possible combinations depending on the sign of the underlying eigenvectors.

Several researchers have therefore explored alternative statistical techniques to extract uncorrelated factors and achieve factor risk parity. For example, Lassance, DeMiguel and Vrins (2019) implement an Independent Component Analysis (ICA) to extract principal components which are independent. They point out that principal components that are uncorrelated by construction are not independent when returns do not follow a Gaussian distribution. Conversely, they note that the independence of the ICs allows to account for higher-order moments and to reduce the portfolio's kurtosis. Therefore, they create factor risk parity strategies based on independent components where risk is measured by modified VaR and they show out-of-sample outperformance in terms of

Sharpe Ratio, tail risk and turnover, as compared to a factor risk parity portfolio based on principal components.

To overcome the issues from the PCA method, Meucci et al. (2015) implement factor risk parity through an alternative orthogonalization approach which relies on a *Minimum-Torsion transformation*, t.[1] The minimum torsion bets, F_{MT}, are found as the orthogonal factors that minimize the tracking error relative to the original factors, F_k, therefore benefiting from their uncorrelated nature and a clearer economic interpretation.

$$t = \arg \min_{corr(tF)=I_{KxK}} \sqrt{\frac{1}{K}\sum_{k=1}^{K} Var\left(\frac{t'F_k - F_k}{\sigma_{F_k}}\right)}$$

Bernardi, Leippold and Lohre (2018) thus analyze diversified risk parity strategies using a set of commodity assets, where the uncorrelated risks are measured using either the principal components from PCA or the minimum torsion factors, allowing for a direct comparison of the two procedures. They find that, unlike the principal portfolios, the minimum torsion factors are relatively stable over time and can be clearly identified as specific commodity factors.

2.2 Methodology

This study is based on a linear multi-factor model describing the contemporaneous relationship between asset returns and several economic factors. Hence, the model is represented by $R_t = B'F_t + \varepsilon_t$, where R are the excess returns on the asset classes, F are the economic factors, and B is the loadings matrix describing the exposures of the assets to the macro factors. Because we determine the portfolio allocation from the allocation to uncorrelated risk factors, the first part of our analysis derives the latter by applying the minimum linear torsion transformation, t, to the original macroeconomic factors. This differs from existing factor risk parity approaches in which the PCA or MLT was directly applied to the asset returns that composed the effective investment universe of the portfolio. Therefore, the uncorrelated minimum torsion bets are defined as $F_{MT} = tF$ and the diagonal covariance matrix of the torsion bets is $\Sigma_{MT} = t\Sigma_F t'$. Consequently, the factor model can be expressed in terms of the uncorrelated risk factors as

$$R = B'F + \varepsilon = B'_{MT}F_{MT} + \varepsilon \text{ where } B_{MT} = t'^{-1}B \tag{7}$$

and the portfolio return with asset weights w can then be expressed in terms of the uncorrelated torsion bets as $R_w = w'R = w'_{MT}F_{MT}$.

[1]The code for the torsion transformation is available at: https://it.mathworks.com/matlabcentral/fileexchange/43245-portfolio-diversification-based-on-optimized-uncorrelated-factors?s_tid=prof_contriblnk For more technical details, readers are referred to the original paper by Meucci et al. (2015).

The covariance matrix of the asset returns is decomposed as $\Sigma = B'\Sigma_F B + \Sigma_\varepsilon$, where Σ_ε is the diagonal matrix containing the variance of the error terms, and the factors and error terms are, by construction, uncorrelated. Using the minimum torsion transformation, the variance of the asset returns in terms of torsion bets is as follows:

$$\Sigma = B' t^{-1} \Sigma_{MT} t^{'-1} B + \Sigma_\varepsilon$$

We propose two alternative approaches to achieve factor risk parity in terms of uncorrelated macro factors, differing in the orthogonalization procedure as well as in the choice of macro factors.

2.2.1 Macro Risk Factors and Minimum Linear Torsion

The first approach includes macroeconomic variables as risk factors which are useful in explaining variation within and across assets. By achieving a diversified allocation to these risk factors, we aim at directly managing risk exposures to the factors that most likely explain total portfolio volatility. Equalizing the contributions to risk of the selected factors yields the optimal factor risk parity weights, w^{MT}, as inverse volatility allocations in the uncorrelated factors:

$$w^{MT} = \Sigma_{MT}^{-1/2} \begin{pmatrix} \pm 1 \\ \vdots \\ \pm 1 \end{pmatrix}$$

where Σ_{MT} denotes the covariance matrix of the uncorrelated risk factors estimated using the minimum linear torsion (MT).

For a unique factor risk parity solution, we set the signs in the vector of 1s consistent with a long position in the premium generated by each factor. Beceause these factors are not directly investable, however, we first need to derive the implied factor-mimicking portfolios to determine the respective macro factor premia. We follow Deguest et al. (2013) and estimate the macro factor-mimicking portfolios by inverting the relationship described from the factor model in Eq. 7. Therefore, we obtain the implied mimicking portfolio returns as:

$$R_{MT} = t B^{'-1} R, \tag{8}$$

where R are the excess returns on the test assets, that is, the investment universe, and B^{-1} is the inverse of the OLS coefficient matrix and is estimated using the Moore-Penrose inverse given that the number of assets exceeds the number of macro factors. We follow the asset pricing theory to ensure we have a positive directional exposure to the rewarded macro-mimicking portfolios. Precisely, we use the underlying macro factors as observable proxies for the stochastic discount factor, and we impose that the covariance between the return on the mimicking portfolio and the factor defining bad states of the world be negative. In this way, in equilibrium, the mimicking portfolio will earn a positive premium as a compensation for underperforming during times of high marginal utility.

Having identified the directional exposures in the factor risk parity weights, we then back out the final asset weights by inverting the minimum torsion and the coefficient matrix estimated from the factor model, so that $w = B_{MT}^{-1} w^{MT} = B^{-1} t' w^{MT}$, and we normalize the final weights as:

$$w^{FRP} = \frac{B^{-1} t' \Sigma_{MT}^{-\frac{1}{2}} 1_K^{RP}}{1_K' B^{-1} t' \Sigma_{MT}^{-\frac{1}{2}} 1_K^{RP}} \tag{9}$$

where 1_K^{RP} contains the signs that guarantee positive macro factor exposure.

In addition, because the resulting portfolio allocations generally require short positions and leverage, we investigate constrained factor risk parity allocations as well. We follow Dichtl, Drobetz, Lohre and Rother (2020) and solve a traditional mean-variance optimization using implied views from the optimal factor risk parity allocations w^{FRP}. The views are calculated as $v = \gamma \Sigma_N w^{FRP}$, where Σ_N is the covariance matrix of the asset returns and γ is the risk aversion coefficient. Our empirical analysis imposes full investment constraints and a 40% upper boundary allocation in each asset. Hence, we will refer to the unconstrained factor risk parity portfolio (obtained with Equation 9) as the *FRP Opt* portfolio, whereas we denote the long-only mean-variance factor risk parity portfolio as *FRP LO*.

2.2.2 Macro State Variables and Choleski Decomposition

Our second approach considers an alternative set of macroeconomic state variables that are priced sources of risk and therefore earn a positive premium over the long run. Unlike the risk factors used in the previous section, we focus on a narrower set of pure macroeconomic variables as the ultimate drivers of time-series comovement in asset returns.

Since investors are ultimately concerned only about unexpected shocks, we measure the macroeconomic factors as the innovations from a VAR model governed by the relevant state variables. Rather than using the minimum linear torsion to orthogonalize the factors, we rely on the macroeconomic literature that identifies orthogonal macro shocks using a recursive scheme based on a Choleski decomposition (Christiano, Eichenbaum and Evans, 1999; Sims, 1980), which requires specifying the "causal" ordering of the variables in the system. While a complete structural identification is beyond the scope of our study, a recursive identification fits our objective of deriving orthogonal macro factors with a clear economic interpretation. Although the method relies on an arbitrary choice of recursive order, we are able to obtain well-defined uncorrelated factors given that our chosen state variables are well-known macroeconomic variables for which the recursive identification scheme has been widely adopted in the macroeconomic literature and is justified by economic theory.

Given K time series state variables $y_1, \ldots, y_T$ with $y_t = (y_{1t}, \ldots, y_{Kt})'$, we estimate the VAR(p) process[2]

$$y_t = A_1 y_{t-1} + \ldots + A_p y_{t-p} + u_t, \tag{10}$$

where u_t are the white-noise reduced-form macroeconomic shocks, so that $u_t \sim (0, \Sigma_u)$. These residuals do not reveal the underlying relationship between the macro state variables because they are likely instantaneously correlated, or in other words, Σ_u is not a diagonal matrix of uncorrelated shocks. Therefore, we use a Choleski decomposition of the covariance matrix $\Sigma_u = PP'$, where P is a lower-triangular matrix with positive elements on the main diagonal. We then define $A := P^{-1}$ and from the reduced-form VAR in (10), obtaining the structural form VAR process as follows:

$$Ay_t = A_1^* y_{t-1} + \ldots + A_p^* y_{t-p} + \epsilon_t, \tag{11}$$

where $A_i^* := AA_i$, for $i = 1, \ldots, p$, and $\epsilon_t = (\epsilon_{1t}, \ldots, \epsilon_{Kt})' := Au_t$

The final orthogonal shocks of interest to us are therefore ϵ_t with unit variance and with diagonal covariance matrix defined by

$$\Sigma_\epsilon = E[\epsilon_t \epsilon_t'] = AE[u_t u_t']A' = A\Sigma_u A' \tag{12}$$

Hence, we estimate the linear factor model using the correlated innovations u_t as regressors (similarly to the correlated macro risk factors F_t in the previous analysis) and we replace the minimum linear torsion t with the matrix A (therefore replacing the minimum torsion bets $F_{MT,t}$ with the orthogonal macro shocks ϵ_t), leaving the remaining steps unchanged.

Given this set of state variables and the associated uncorrelated factors, we derive the optimal and constrained Factor Risk Parity portfolios following the procedure described previously in 2.2.1.

3 Data Description

3.1 Asset Returns

For asset allocation we consider twelve assets that are associated with four broad asset classes: equities, government bonds, credit and commodities. The equity markets are the

[2]This description of the VAR model follows Lüktepohl (2005, p.58)

S&P500, MSCI EAFE (for international developed markets excluding the U.S.), MSCI Emerging Markets and RUSSELL2000 (for small-cap U.S. stocks). The government bond returns are the ten-year government bond indices for the U.S., Germany, the U.K. and Japan. The credit assets are the Bloomberg Barclays US High Yield Index and Bloomberg Barclays US Corporate Bond (Investment Grade). The commodity asset classes are the S&P Goldman Sachs Commodity Index and the S&P GSCI Gold Index. We use the former because its composition is tilted in favor of energy-related commodities, which allows us to separate its role from a pure gold index. The series are measured as total return indices and are available for the 1989-01-31 to 2020-01-31 period. Excess returns are calculated as logarithmic returns in excess of the one-month US Treasury bill rate.

3.2 Macroeconomic Risk Factors

Regarding macroeconomic factors, we consider innovations to macroeconomic variables that are likely to explain variation in the asset returns through their effect on cash flows or discount rates, where innovations in the variables are measured using log returns or first differences. The economic risk factors used in the empirical analysis are common choices in the literature and are described as follows.

MARKET is the return on the world market portfolio in excess of the one-month US Treasury bill rate, where the market portfolio is measured by the log return on the MSCI World Index in local currency. Following the asset pricing literature, the market portfolio represents a proxy for the aggregate wealth of investors and is a priced source of risk in the CAPM. Although empirical studies on asset allocation conveniently associate the market portfolio with economic growth, we interpret the market index strictly as a measure of equity risk and we consider the role of pure economic variables after controlling for the market portfolio.

TERM is the first difference in the term spread, defined as the difference in yields between a global measure of ten-year government bonds and three-month government instruments. The long-term and short-term global yields are a weighted average of the respective government yields across the G7 countries, where the G7 weights are constructed using the relative shares of GDP from the OECD outlook data. For countries with shorter history of the three-month government yield, we measure the initial series using the local interbank rate. As in Ferson and Harvey (1991), this variable is included to measure the risk from unexpected changes in the slope of the yield curve, and ultimately in changes in expectations of future interest rates. To the extent that monetary policy decisions on the short end of the yield curve influence the term structure of interest rates by a no-arbitrage argument, changes in the slope capture unexpected changes in the monetary policy stance and can be a priced source of risk. While the macroeconomic literature identifies monetary policy shocks by changes in the short-term interest rate, the low volatility of this variable makes the short-term instrument ineffective at capturing monthly variation in the asset returns. Indeed, Campbell and Ammer (1993) find that the level of interest rates contributes to most of the variation in short-term bonds, while the variation in the long-term bonds is primarily driven by long-term interest rates, which are decomposed into the real short-term interest rate, the term premium, expected inflation and an inflation risk premium.

USD is the log return on the US trade-weighted dollar index and is sourced from FRED. The asset returns are measured in local currency, so the USD variable represents a measure of exchange risk following the international CAPM literature (see Adler and Dumas, 1983; Solnik,1983), and Ferson and Harvey (1993), where the trade-weighted dollar index is a systematic risk factor in a global multi-factor model across national equity markets.

OIL is the log return of the WTI oil benchmark index. Chen et al. (1986) consider this variable as a source of economic risk in the cross-section of U.S. stock returns, whereas Ferson and Harvey (1993) study whether the oil risk factor is a priced source of risk across national equity markets. The traditional macroeconomic literature considers oil price shocks as a risk variable that influences macroeconomic outcomes, although the impact has been decreasing over the past two decades (Blanchard and Gali, 2009). Hamilton (2009) illustrates that high oil prices lead to lower purchasing power, a fall in aggregate spending and consumer sentiment and have been a significant contributor to all major U.S. recessions. Huang and Miao (2016) point out that oil prices can be perceived as proxies for the price of inputs or raw material. Hence, high oil prices translate into higher costs of production for firms and for oil-importing countries, including the world's largest economies, which induces lower investments and spending. As such, high oil prices are traditionally viewed as signifying a bad state of the world, and assets with high exposure to the oil factor should earn lower returns in equilibrium as they hedge against the common risk factor. The last decade provided apparently contradictory evidence to this theory due to the observed positive correlation between the stock market and oil prices. In practice, Kilian (2009) shows that there are three primary determinants of oil price shocks: oil supply shocks, shocks related to global demand for commodities, and oil-specific precautionary shocks. To this extent, the puzzling, spurious positive correlation can be attributed to the role of oil prices as a proxy for global economic demand (Bernanke, 2016; Hamilton, 2014) as well as to stock market herding due to higher uncertainty or risk aversion (Blanchard, 2016), while it does not weaken the conventional negative relationship between *exogenous* oil price shocks and the economy or future stock returns.

DEF is the change in the difference between Moody's Baa and AAA- rated corporate bond yields. The variable is widely used as a measure of default risk (Ferson and Harvey, 1991), while other studies include the difference in yields between low-rated corporate bonds and the long-term government bond (Chen et al., 1989). Fama and French (1989, 1993) find that the default spread explains common variation among stocks, government and corporate bonds and is correlated with the business cycle. In particular, the variable captures the part of expected returns that is high during recessions and low during favorable business environments.

3.3 Macroeconomic State Variables

Separately, we also consider macroeconomic state variables that are of hedging concerns for investors, and consequently are likely to carry risk premia that describe the expected returns on all assets. In the choice of our state variables we follow Baele, Bekaert and Inghelbrecht (2010), who estimate a dynamic factor model with macroeconomic state variables and risk premium variables to model the time-varying comovement between stocks and bonds. We consider three macro state variables which are standard in a small-scale New-Keynesian model and which influence stocks and bonds through either cash flow or discount rate effects: a measure of output gap, inflation and the short-term interest rate. Furthermore, we include a volatility measure to capture common risk aversion or an uncertainty shock. Throughout this study we assume that these variables describe time-series comovement in the asset returns as suggested in previous work, and we leave a formal, econometric proof of the hypothesis to others.

Output. As stocks represent claims to stochastic real cash flows, one commonly uses dividend growth and expected output gap as measures of the cash flow channel. While all standard macroeconomic studies are generally carried out at a quarterly frequency (at which output or consumption variables are available), our study requires appropriate proxies for the cash flow channel on a monthly frequency. We therefore use the log difference in the composite leading indicator (CLI) for G7 countries issued by the OECD. This variable is constructed by aggregating several economic variables to measure economic fluctuations around the long-term output level and is useful at detecting turning points in the business cycle.

Inflation is measured by a weighted average of the inflation rates for the G7 countries, where inflation is measured as the log difference in the respective monthly CPI measures. As bond returns provide claims to fixed nominal cash flows, inflation is a state variable that negatively affects bond returns. On the other hand, the Fisher hypothesis predicts that stock returns are not directly exposed to inflation risks through the real cash flow channel. However, the Mundell-Tobin model asserts that high expected inflation increases the opportunity cost of money, thus inducing a portfolio rebalancing from real money to financial assets. This leads to a lower real interest rate, and in turn to a negative correlation between real stock returns and expected inflation. Conversely, Fama (1981) points to an alternative “proxy effect” which arises from the combination of a negative relation between inflation and real economic activity and a positive relation between real stock returns and real economic activity. In practice, however, stocks have been found to be negatively correlated with expected and unexpected inflation (Fama and Schwert, 1977).

Interest Rate risk is a risk factor under the discount rate channel. For example, based on Intertemporal Asset Pricing Theory, Merton (1973) and Cox, Ingersoll and Ross (1985) use the interest rate as a state variable to describe the time-varying investment opportunity set. We use the change in a global measure of the three-month government bond yield across the G7 countries as a proxy for the monetary policy rate set by central banks. In standard VAR models, the policy rate is usually measured in levels, whereas we use the first difference to make the variable stationary and to allow the factor model to capture sufficient variability in the interest rate (especially given the stability of policy rates following the global financial crisis).[3]

Risk Aversion/Volatility. Baele, Bekaert, Inghelbrecht (2010) include several risk premium variables, including risk aversion and uncertainty about inflation and output from survey data and they find that risk and liquidity factors significantly outperform fundamental macroeconomic variables in fitting the time-varying stock-bond comovement. Consequently, we include the VIX as a measure of Risk Aversion and volatility in financial markets. However, the variable is an imperfect proxy as it is a measure of implied volatility and includes both a component of conditional, expected volatility on the stock market as well as a variance risk premium component. Barsky (1989) first noted the importance of time-varying risk premia to explain common variation between stocks and bonds. Stochastic risk aversion influences the covariance between stocks and bonds under the two opposite channels of consumption smoothing and precautionary saving with an overall ambiguous effect. Under the consumption smoothing effect, higher risk aversion requires higher short-term real interest rates. In this state, bonds and equities are risky and both require a positive risk premium, which contributes to positive correlation. Conversely, under the precautionary saving effect, higher risk aversion drives up demand, and thus prices, for bonds. This results in a “flight to safety” as bonds are perceived as hedge assets. In this case, unlike equities, bonds have a negative risk premium, which contributes to the negative correlation.

Given our four state variables, the recursive ordering for the Choleski decomposition is output, inflation, interest rate and volatility.[4] This assumes that each endogenous

[3]Alternatively, we could include all variables in levels and increase the number of lags in the VAR model. In unreported results, we found that the performance of the factor risk parity strategies is robust to the specification of the short-term rate in level.

[4]Note that the results are robust to different ordering of the variables.

variable in the VAR system has contemporaneous effects only on itself and the variables below it in the order. Following the terminology of Bernanke, Boivin and Eliasz (2004), we can therefore consider output and inflation as "slow-moving" variables which do not react contemporaneously to monetary policy shocks, whereas we can consider volatility as a "fast-moving" variable which reacts contemporaneously to the interest rate shocks.

4 Diversified Macro Factor Investing

4.1 Macro Risk Factor Model

In this section we study the contemporaneous relationship between the excess returns on each asset and the macroeconomic risk factors by estimating the following multi-factor model via OLS

$$R_{i,t} = a_i + \beta_{i1}TERM_t + \beta_{i2}MARKET_t + \beta_{i3}USD_t + \beta_{i4}OIL_t + \beta_{i5}DEF_t + \varepsilon_{it}$$

where $R_{i,t}$ are the excess returns on each of the twelve assets. The macroeconomic variables have been previously normalized, by subtracting the trailing twelve-month mean and dividing by the trailing twelve-month standard deviation.[5]

For each month t, the regressions are estimated out-of-sample in an expanding window, using information up to time *t-1* and with an initial calibration period of sixty months. Table 1 reports the results from the full-sample regressions. As expected, the coefficient of TERM for all bond indices is negative and statistically significant, indicating an inverse relationship between interest rates and bond prices. We also find a negative relationship between the gold index and TERM, which is significant at the 5% level. Since TERM is constructed as the difference in nominal yields, it is measured in real terms. It can thus capture the negative relationship between gold prices and real interest rates, whereby a higher real yield increases the opportunity cost of real money and requires a lower gold price to balance demand and supply. MARKET has a positive and statistically significant effect at the 1% level on the four equity markets and on the two credit indices. The coefficient of the USD index is negative and statistically significant at the 1% level for US.IG and gold and at the 5% level for US.Bond. OIL has a positive and statistically significant impact on the two commodity indices, with a larger economic magnitude for the S&P GSCI Commodity Index. Additionally, the coefficient is negative and significant at the 5% level for DE.Bond and UK.Bond, which is attributable to the role of the oil prices as an inflation proxy. Finally, the default risk variable (DEF) has a negative and statistically significant effect on the two credit

[5] This procedure is required in order to allow the minimum linear torsion to accurately find orthogonal macro factors which can be associated with specific macro variables, given the different scales of the original variables.

indices and the gold index, with larger magnitude for US.HY. On average, the model explains more than 50% of the variation in the asset returns, as measured by the adjusted R^2, with the MARKET and TERM factors contributing to explain most of the total variation in asset returns.

Given the set of risk factors, we derive uncorrelated torsion factors using the minimum linear torsion. Figure 1 shows the loadings of each torsion factor on the five risk factors using the full-sample (although in implementing the factor risk parity strategies all estimates are obtained out-of-sample and using an expanding window).[6] Clearly, each minimum torsion factor can be closely identified as one and only one risk factor, highlighting the advantage of the Minimum Linear Torsion transformation over PCA in achieving uncorrelated factors with clear economic interpretation. Figures 2 and 3 illustrate the exposures of the implied macro factor-mimicking portfolios to the asset returns as well as their cumulative performance. The exposures are measured using the Pearson correlation coefficients. As expected, the portfolio mimicking the TERM factor has large and positive loadings on all government bonds and credit indices. The portfolio replicating the MSCI World Index (MARKET) has large positive exposures to all equity indices, followed by the two credit indices, whereas the USD-mimicking portfolio is almost exclusively associated with the gold Index. The OIL-mimicking portfolio has a large negative exposure to the S&P GSCI Commodity Index, and a positive exposure to the MSCI EM Equity Index. The latter can be explained by observing that the largest constituents of the emerging markets index (China, South Korea, Taiwan and India) are commodity-importing countries. Thus, overall, the value of the EM Equity Index moves inversely with upward oil price shocks. Finally, the DEF-mimicking portfolio has the largest positive exposure to the credit indices and the MSCI EM Equity Index, reflecting their higher credit risk.

4.2 Macro State Variables Factor Model

We next repeat the analysis by replacing the five risk factors with the four previously defined macro state variables. In particular, we estimate the linear multi-factor model using innovations in the macroeconomic state variables from the reduced-form VAR model and obtain the associated uncorrelated factors using a Choleski decomposition (see Figure 1 for the respective loadings). We estimate the VAR model by including a constant, and the number of lags is chosen to be one according to the Schwartz information criterion. Table 2 reports the estimates from the OLS factor model. Notably, the goodness of fit of the model is considerably lower than that of the factor model based on the previous five risk factors. The average adjusted R^2 is 15% across the twelve assets,

[6] In unreported results we analyze the loadings estimated using an expanding and rolling window and find that the stability and interpretation of the torsion factors remain unchanged.

despite strong evidence of statistical significance in the estimated coefficients. Overall, the result confirms the empirical evidence that macroeconomic variables fail to explain significant variation in asset returns, unlike market-based risk factors. Additionally, this allows us to contrast the significant yet different role played by the previous five economic risk factors and by the pure macroeconomic state variables. The former reflect the primary sources of risk that explain variation in the asset returns; the latter represent the ultimate long-term economic fundamentals that the academic literature suggests are the primary drivers of asset return comovement and long-term premia.

Figure 2 illustrates the exposures of the estimated macro-mimicking portfolios on each asset from the full sample. The first mimicking portfolio tracks the growth variable and has the highest loading on the MSCI EM Equity Index. The second mimicking portfolio tracks the inflation state variable. It has the highest and most negative loading on the S&P GSCI Commodity Index, followed by the Gold Index. This is expected because the commodity indices tend to be good hedges against high inflation and therefore should earn a negative premium if inflation is priced in the cross-section. The negative exposure to the MSCI EM Equity Index suggests an inflation-hedging property which can be attributed to low correlation between the G7 countries' inflation rates (which comprise the inflation state variable) and emerging markets' inflation. The third mimicking portfolio tracks the short-term interest rate. As expected, it has the highest loadings on all the bond indices as these assets are the most sensitive to unexpected changes in the monetary policy rate. We also find a positive loading of the interest rate factor on gold, consistent with the economic explanation illustrated in the previous section. Finally, the last mimicking portfolio tracks the VIX and has the highest positive loadings in, by order, the S&P500, RUSSELL2000, MSCI EAFE, US.HY and the S&P GSCI Commodity Index. This is consistent with the interpretation of the VIX as a measure of risk aversion as the academic literature finds that equity returns are primarily driven by risk premium (instead of the cash flow channel), and that commodities also underperform during times of high volatility. The negative exposure to the gold and bond indices reflects the role of the latter as safe havens during times of high risk aversion or uncertainty.

Figure 3 shows the cumulative performance of the mimicking portfolios using the full-sample estimates while Figure 4 shows the correlations among the four mimicking portfolios and the innovations in the macro state variables. The sign of the correlations with the original state variables is consistent with a positive exposure to the respective good states and therefore to the risk premia, whereas the average pairwise correlation between the macro-mimicking portfolios is -0.18. The orthogonalization procedure (either Choleski or minimum linear torsion) implies that the estimated mimicking portfolios track the underlying uncorrelated factors. This is in constrast to the traditional construction of factor mimicking portfolios, which influences the resulting loadings on the test assets. For example, we expect the mimicking portfolios that track economic growth and volatility to have positive exposures to the equity indices. However, after purging the VIX from the CLI variable and enforcing orthogonality, the growth-mimicking portfolios only loads positively on the MSCI EM (the most growth-sensitive asset), while the VIX mimicking portfolio loads positively on the remaining equity indices which have higher relative exposure to the volatility factor.

4.3 Portfolio Performance

Table 3 and Figure 5 report the performance of the Factor Risk Parity strategies. The first two strategies (*FRP 5RF*) are based on the five risk factors and they achieve the highest average annualized portfolio returns (5.1% and 3.6%, respectively for the optimal and constrained long-only version) by explicitly harvesting the five different economic risk premia. However, this is accompanied by higher portfolio volatility, resulting in Sharpe Ratios of 0.77 and 0.61, respectively. Remarkably, the strategies exhibit severe maximum drawdowns of 17.5% and 18.43%, which are second only to that of an equal-weighted portfolio (26.46%). Figure 6 illustrates the asset allocations and shows that the strategies are overall well balanced across the four asset classes (credit, government bonds, equities and commodities), with relatively larger weights on US.Bond, S&P GSCI Gold Index, US.HY and MSCI.EM. In the unconstrained optimal version, the portfolio initially has negative weights in RUSSELL2000 and UK.Bond, followed by short positions since early 2000 in the S&P GSCI Commodity Index. In terms of risk contributions (column 2 and 3 of Figure 6), the strategies are balanced across the five risk factors. By construction, the unconstrained version achieves perfect factor risk parity and the maximum ENB of 5, whereas imposing short-constraints results in a relatively higher risk allocation to the MARKET factor. Interestingly, when examining the risk contributions in terms of the four macro state variables, Figure 6 (column 3) shows that most of total portfolio volatility is attributed to exposure to the growth (CLI) and risk aversion (VIX) factors.

Next, we examine the factor risk parity strategies based on the four macro state variables (*FRP 4SV*). The portfolios have average annualized returns of 3.43% and 3.20% and Sharpe Ratios of 0.67 and 0.73 for the optimal and long-only versions, respectively. Additionally, they exhibit the lowest maximum drawdowns among all the alternative strategies (13.62% and 10.67%, respectively). In terms of asset allocations, the portfolios seem to overallocate to the government bond asset class, which is relatively evenly split across the four countries, followed, in order, by an allocation to the US.HY and an initially higher allocation to the MSCI.EAFE and S&P GSCI Gold Indices. The increasing allocation to the government bonds is likely attributable to the lower volatility of the G7.Short.Rate factor in the later part of the sample, recalling that the factor risk parity solutions are inverse volatility allocations in the underlying macro factors. As expected, the risk contributions are balanced across the four macro state variables. They are exactly equal in the optimal unconstrained version, with the maximum ENB of 4, as the short position in the S&P GSCI Commodity Index allows us to fill the risk allocation to the G7.INFLATION factor, although the results are broadly similar to those from the long-only version. Conversely, when examining the risk contributions in terms of the five risk factors, Figure 6 suggests that the long-only portfolio (*FRP 4SV LO*) is composed exclusively of exposures to the TERM and MARKET factors, while the unconstrained (*FRP 4SV Opt*) factor is primarily exposed to the TERM, MARKET and OIL factors. This results in a lower Effective Number of Bets of approximately 3 out of 5.

Thus, our analysis of both *FRP 5RF* and *FRP 4SV* sheds light on the non-negligible portfolio consequences of allocating in terms of risk factors versus macro state variables. In particular, investors who are primarily concerned with managing different risk exposures are likely to care more about the five macroeconomic risk factors as building blocks that explain most of the monthly portfolio volatility. Conversely, they would strongly dislike the *FRP 4SV* based on the lower Effective Number of Bets (given overall similar risk-adjusted performance). In practice, this could lead to misleading conclusions about the effective degree of diversification if one were to assume that the true drivers of asset returns are the four macro state variables. Given the large maximum drawdowns experienced in the *FRP 5RF*, compared to the lowest drawdowns in the *FRP 4SV*, the result seems to lean towards the latter assumption on the nature of the true drivers. This would be especially true if some of the economic risk factors tend to move together and be driven by further underlying forces, which we study in the next section.

Finally, we compare the results with those from the performance of benchmark risk-based strategies following an asset-class approach. The Equal Risk Contribution *ERC* – which involves risk parity along the asset classes– features allocations which are balanced across the four broad asset classes, whereas the Minimum-Variance portfolio exhibits an overallocation to JP.Bond, DE.Bond and US.IG as they have the lowest return volatility. In terms of portfolio performance, the two portfolios achieve lower average portfolio returns than the Factor Risk Parity portfolios (2.90% and 2.34%, respectively) and overall similar portfolio volatility (4.49% and 3.69%, respectively for the ERC and MV portfolios) vis-à-vis *FRP 4SV LO* (4.37). This results in overall lower risk-adjusted returns of 0.65, and 0.63 and higher maximum drawdowns of 14.52 and 10.76%. Hence, the higher allocation across the government bonds in *FRP 4SV LO* contributes to a lower maximum drawdown and higher returns that more than compensates for the relatively higher allocations to equities and commodities in the *ERC* portfolio. In turn, the latter results in risk contributions that are largely dominated by the VIX factor, resulting in a lower Effective Number of Bets of approximately 2 (Figure 7).

5 Dynamic Macro Factor Allocation

5.1 Regime-Switching Introduction and Model Specification

Based on the four macro state variables defined in the previous section, the relationship between asset returns and macro factors is described by the factor model

$$R_t = E[R_t] + \beta' F_t + \varepsilon_t, \tag{13}$$

where $R_t = (r_1, \dots, r_N)'$ contains the excess returns on the assets, $\beta = (\beta_1, \dots, \beta_N)$ is a $K \times N$ matrix of the loadings of the assets returns on the K factors, and $\varepsilon_t = (\varepsilon_{1,t}, \dots, \varepsilon_{N,t})$ represents the return innovations not explained by the factors. F_t is the $K \times 1$ vector of macroeconomic factors, obtained as innovations from a VAR process with the state variables, that is,

$$F_t = X_t - E[X_t],$$

where $F_t \sim (0, \Sigma_t)$. From the factor model, the systematic comovement between any two assets is described by

$$cov(r_{it}, r_{jt}) = \beta_i \Sigma_t \beta_j',$$

for $i \neq j$. It follows that time-variation in the comovement between asset returns is driven by either time variation in the sensitivities of the assets to the common macroeconomic shocks, that is the betas are not constant, or by heteroskedasticity in the macroeconomic factors.

In this study, we restrict the betas to be constant for the sake of simplicity and to avoid the curse of dimensionality in the estimation of the parameters. Instead, we examine the role of heteroskedasticity in the macro factors. This is supported by empirical evidence of time-varying macroeconomic volatilities, such as the observed downward shift in output and inflation volatilities characterizing the Great Moderation. To that extent we use a regime-switching model to estimate the covariance matrix of the macro shocks conditional on the states.

Markov-Switching models build on the pioneering work of Hamilton (1989), who uses a two-state hidden Markov model to study U.S. business cycles based on Gross National Product. Given macroeconomic shocks based on a VAR process, we estimate a Markov-Switching Vector Autoregression to model non-linearities or structural breaks in the dynamics of the macro factors. MS-VAR models are examined at length in Krolzig

(1997), as well as in Hamilton (2008) and Guidolin and Pedio (2018) and have been widely adopted in the macroeconomic literature to study structural breaks in monetary or fiscal regimes. For example, Sims and Zha (2006) use a Bayesian MS-VAR to detect shifts in US monetary policy regimes, while Ehrmann, Ellison and Valla (2003) estimated regime-dependent impulse response functions in an MS-VAR model to study the different effects of oil price shocks on the economy. Lanne and Lüktepohl (2008) and Lanne, Lüktepohl and Maciejowska (2010) estimate Structural Vector Autoregressions with Markov Switching based on time-varying shock volatilities and provide the conditions for identification of the shocks and useful macroeconomic applications.

We report technical details about MS-VAR models and their implementation in the Appendix (8.2). In our MS-VAR specification, the parameters of the VAR process are constant. We only allow the intercepts and the covariance matrix of the shocks to vary across states, since ultimately we are only interested in the heteroskedasticity of the macro factors. Additionally, we assume the existence of two regimes defined by a single Markov process. Hence, we follow the conventional terminology and define such model as MSIH(2,0)-VAR(1). In practice, we could consider each of the four macro variables as independent shocks and estimate four different Markov chains (one for each macro variable) which can take on two values, but that would result in a total of sixteen different regimes. In a similar vein, Baele et al. (2010) and Bikbov and Chernov (2008) consider three separate regimes: 1) a common regime variable shifting the volatility of both exogenous output and inflation, 2) a process shifting the systematic monetary policy rule, and 3) a process shifting the volatility of the risk aversion shock or of the monetary policy shock. Conversely, we consider a single Markov process which drives common variation across all the macro variables in order to retain model simplicity and avoid parameter proliferation. Ultimately, the total number of parameters to be estimated in the model is

$$M\left[K + pK^2 + \frac{K\,(K+1)}{2} + (M-1)\right],$$

where $M = 2$ is the number of regimes, $K = 4$ is the number of macro state variables and $p = 1$ is the number of lags in the VAR process. As a result, we estimate sixty-two parameters against $T = 364$ full-sample monthly observations, resulting in a saturation ratio of 24. The latter is defined as the ratio of the total number of available observations (KT) to the total number of parameters to be estimated. In particular, Guidolin and Pedio (2018) recommend as a rule-of-thumb to have a saturation ratio greater than 20, implying that at least 20 observations per parameter are needed to conduct any type of econometric analysis.

5.2 Model Estimation

We estimate the model by maximum likelihood based on an Expectation Maximization algorithm.[7] For our implementation of the strategies, we replace the static covariance matrix of the macro shocks with the conditional expected covariance matrix estimated using the MS-VAR model, and based on the one-step-ahead predicted states. In the case of the first moment, the one-step-ahead predicted value of state variables is :

$$E\left[y_{t+1}|\mathscr{F}_t\right] = X_{t+1}\hat{A}(\hat{\xi}_{t+1|t} \otimes \iota_K),$$

where $X_{t+1} = [1y'_t \dots y'_{t-p+1}] \otimes \iota_K$, $\hat{A}$ contains the estimated conditional parameters of the VAR system, and $\hat{\xi}_{t+1|t}$ are the one-step-ahead forecasted state probabilities which are filtered from the currently available information set $\mathscr{F}_t$. In particular, the predicted probability vector is obtained from the transition equation:

$$E_t[\xi_{t+1}|\mathscr{F}_t] \equiv E_t[\xi_{t+1}] = \hat{\xi}_{t+1|t} = P'\hat{\xi}_{t|t},$$

where P′ is the transpose of the estimated transition probability matrix. Therefore, we obtain predicted values for the variables for each state and then estimate the conditional value as a weighted average of the state-dependent moments using the predicted probabilities as weights. In a similar manner and related to the scope of our research, we estimate the conditional covariance matrix of the macro factor innovations:

$$E\left[\Sigma_{t+1}|\mathscr{F}_t\right] = \Sigma_{MS}(\hat{\xi}_{t+1|t} \otimes \iota_K), \tag{14}$$

where Σ_{MS} includes the estimated covariance matrix of the innovations for each state.[8] Hence, we derive the orthogonal macro shocks by applying a Choleski decomposition to the estimated conditional covariance matrix and we obtain the factor risk parity allocations using the procedure described in the previous section.[9]

For robustness, because the former MS-VAR model is estimated in-sample, we compare the results from an alternative MSIAH(2,0)-VAR(1) specification estimated out-of-sample. In this model, we allow all parameters (the intercept, autoregressive parameters

[7]To estimate this model we use the MSVARlib package from Benoit Bellone. Due to high costs in terms of computing time, we estimate the model in-sample.

[8]If we were interested in the conditional second moment of the endogenous state variables, then we would also have to include the covariance component from the VAR process and an additional term for the presence of switches in the first moment; however, as we are only interested in the innovations we rely on Equation 14 for the conditional covariance matrix.

[9]In unreported results we also test an alternative procedure that is similar to that in Oliveira and Pereira (2018): we derive the factor risk parity weights for each state independently, based on the regime-dependent covariance matrix of the macro innovations and Choleski decompositions. Then, we obtain the final factor risk parity weights as a weighted average of the regime-dependent allocations, weighted by the predicted state probabilities. We find that the results from the two approaches are effectively equivalent.

and the variance and covariances) to switch between states.

Using the full sample estimate of the main MSIH(2,0)-VAR(1) model, the transition probability matrix indicates that both regimes are highly persistent: the probabilities of remaining in the same state in the next period are 93% and 84% for State 1 and State 2, respectively. The expected average duration of each state is calculated as $\frac{1}{1-p_{ii}}$ and is equal to approximately 15 months for State 1 and 6 months for State 2. To asses the quality of our model we report the Regime Classification Measure (RCM) of Hamilton (1988), calculated as:

$$RCM = 100\frac{M^2}{T}\sum_{t=1}^{T}\prod_{m=1}^{M}\hat{\xi}_{t|t}^{m}$$

The intuition is that, if the model is able to clearly identify a given regime m at each point in time, then the filtered state probability for state m $\hat{\xi}_{t|t}^{m}$ will be close to 1, while the probabilities associated with the remaining states will be lower and $\prod_{m=1}^{M}\hat{\xi}_{t|t}^{m} \approx 0$. Therefore, with RCM $\in [0, 100]$, lower values will suggest better regime identification. We find an RCM value of 14.5, which suggests a relatively clear classification of the states over time. Table 4 reports the regime-specific estimates of the macro variables and reveals that all volatilities of the macro factors in State 2 are higher than those in State 1. We interpret this to mean that the second state, which occurs less frequently, is a bad state of the world which captures higher economic uncertainty as reflected by the higher variance of all macro shocks. In Figure 8 we illustrate the smoothed state probabilities (that is, estimated using information from the full-sample) against the underlying time series of the macro state variables. The bad state appears to broadly match the recession periods as identified by the US NBER recession indicator, including the entire GFC period. Nevertheless, unlike the NBER recession dates, the state probabilities from the MS-VAR model identify more occurrences of the bad state. This reflects periods of larger realized shocks common to all macro factors, including the late 1990s and the early 2000s, the VIX shocks of 2011 and 2016.

5.3 Performance of Dynamic Macro Factor Allocation Strategies

Table 5 compares the performance of the FRP portfolios based on the two MS-VAR specifications and the static FRP portfolio analyzed in the previous section. The results indicate that the overall performance is largely unchanged compared to the original model and the asset allocations (unreported) remain effectively unaltered. The result therefore seems to suggest that accounting for heteroskedasticity of the state variables, in the form of regime-switching variation, does not significantly contribute to higher (or lower) portfolio performance. One explanation for this is that in the second state the volatilities of the macro shocks are higher in absolute terms, but the relative importance of the four variables is unaffected. This does not in turn affect the relative allocations to the macro factors. Naturally, the results rely on the accuracy of the selected Markov Switching model as more elaborate models could account for separate Markov-switching processes for each state variable and could distinguish among the relative variance of the macro factors across different states. An alternative hypothesis is that the time-varying or state-dependent exposures (betas) of the assets to the macro factors play a more active role in influencing the factor risk parity allocations than macroeconomic heteroskedasticity.

Next, in Figure 10, we extend the conditional analysis by studying the correlations within asset classes, macro risk factors and macro state variables across the two MS-VAR regimes. Given the full-sample smoothed probabilities from the MSIH(2,0)-VAR(1) model, we classify the regimes using a 50% probability threshold, which is supported by the clear identification of the states as suggested by the RCM measure. In both states, the correlations between macro risk factors or state variables are lower than those between asset classes. During the high economic uncertainty state, the correlation between the OIL and DEF factors increases in absolute value from -0.06 to -0.33; similarly, the correlation between USD and OIL doubles from -0.22 to -0.4, while that between USD and DEF increases from 0.13 to 0.21. This suggests larger comovement among high credit spreads, low oil prices and a higher U.S. dollar during times of higher uncertainty. The correlations between the macro-mimicking portfolios tracking the risk factors or state variables are of great interest as they ultimately guide the factor risk parity allocations. During normal times, the MARKET-mimicking portfolio is negatively correlated with the OIL- and DEF-mimicking portfolios (with correlations of -0.32 and -0.54, respectively). This highlights substantial diversification benefits from harvesting different risk premia, while the negative correlation completely disappears in the bad state.[10] In a similar vein, the positive correlation between the USD- and OIL-mimicking factors in the good state (0.31) vanishes in the high uncertainty state. The results suggest that energy commodities provide only limited diversification benefits as, during times of high uncertainty, oil prices fall together with credit and equity indices, and are offset only by higher dollar and gold. In particular, the usefulness of oil as a portfolio diversifier is sensitive to the specific determinants of oil price shocks at a given point in time, to the extent that global demand and precautionary shocks are related to the macroeconomic environment and to risk preferences that drive other asset returns as well. In turn, the correlations between the macro risk factors help reconcile the change in correlation patterns between asset classes across states. In the first state, credit and equities have an average correlation of 0.33, while the correlation between credit and government bonds is 0.56, reflecting the common exposure to interest rate risk during normal times. Con-

[10] In unreported results we repeated the correlation analysis using US business cycle regimes as identified by the NBER recession reference dates. Not only does the negative correlation between the mimicking portfolios disappears, but it also breaks into positive territory.

versely, in the high uncertainty state, the correlation between equities and credit spikes upward to 0.70, while that between credit and government bonds is effectively zero, as in this state the risk premium channel prevails over the normal interest rate channel, driving opposite comovement in the bond groups. On the contrary, the correlations between the mimicking portfolios tracking the four macro state variables are effectively unchanged across the two states and broadly offset each other. This points to the diversification benefits and robustness of the *FRP 4SV* portfolios across times of both high and low economic uncertainty.

Finally, we compare the performance of the investment strategies analyzed in the previous section across the two regimes as identified by the MS-VAR model. Hence, we group the realized portfolio returns for each strategy by the prevailing regime at each point in time. Table 6 reports the regime-dependent performance and Figure 9 illustrates the regime-dependent mean-variance portfolio combinations. In the good state, the factor risk parity portfolios achieve lower risk-adjusted returns as well as marginally higher drawdowns compared to the benchmark ERC and equal-weighted portfolios. Following DeMiguel, Garlappi and Uppal (2009) we also report the Certainty Equivalent (CEQ) returns for the strategies within each regime. These represent the risk-free rate a mean-variance investor would require in order to forego the investment in the given strategy. The CEQ is calculated as

$$r_{CE} = E[r_p] - \frac{\gamma}{2} Var[r_p],$$

where r_p is the annualized excess portfolio return and γ is the risk-aversion parameter (here set to an average value of 5). For example, comparing the risk parity portfolio along the asset classes (*ERC*) with the long-only factor risk parity along the four state variables (*FRP 4SV LO*), the differential CEQ suggests that the mean-variance investor would be willing to pay an extra annualized 0.5% to switch from *FRP 4SV LO* to *ERC* in the good state.

In the bad state, the entire portfolio frontier shifts rightward as all strategies have higher portfolio volatility. However, the same asset-based strategies (ERC and EW) become the worst-performing, whereas the factor risk parity portfolios, together with the Minimum Variance Portfolio, exhibit higher average portfolio returns in the bad state –lying in the upper part of the portfolio frontier–, with the unconstrained factor risk parity portfolios having the highest risk-adjusted performance. Nevertheless, as observed in the previous section, it is in this state that the *FRP 5RF* portfolios suffer the worst drawdown. Based on the findings in the previous paragraph, we attribute this to the increase in correlations between the macro factor-mimicking portfolios in the bad state, illustrating the fallacy of effective diversification based on the five risk factors as they underperform at the same time. Conversely, the *FRP 4SV* portfolios have the lowest drawdowns (7.74% and 9.57%, respectively for the optimal unconstrained and long-only) and also the lowest tail risk, as measured by higher skewness, thus offering greater protection during the bad state through a balanced allocation across separate return-generating factors. Following the previous CEQ gain example, the same mean-variance investor would now be willing to pay an annualized 2.5% fee to switch back from *ERC* to *FRP 4SV LO*. Naturally, the extra value of switching to the factor risk parity portfolio must be weighted by the frequency of the respective regimes since a higher return earned during the bad state may not compensate for giving up higher returns from the asset-class approach during the good state if the bad state occurs only rarely. Hence, we weigh the state-dependent CER gains by the estimated ergodic probabilities of 70% and 30%, respectively for the normal and high uncertainty state. The resulting 0.5% CER gain would suggest that the relative loss from the asset-based approach in the bad state is sufficiently large that, on average, the investor would be willing to pay a (marginal) positive amount to switch to *FRP 4SV LO*. While the gain is likely not statistically significant and is specific to the given scenario, investors featuring higher risk aversion or incorporating higher order moments in their utility function are likely to gain larger benefits from the factor risk parity approach (notwithstanding the rarity of the bad state).

6 Robustness

6.1 Choleski Decomposition and Minimum Linear Torsion

In this section, we repeat the analysis of the state variables' factor risk parity using the Minimum Linear Torsion instead of the Choleski decomposition to orthogonalize the macro factors. Panel A of Table 7 reports the results when the torsion is applied directly to the original macro factors (MLT) as well as to the standardized macro factors (MLTnorm), which is achieved by subtracting the twelve-month rolling window mean and dividing by the trailing twelve-month standard deviation. The performance of the MLT portfolios is consistent with the results from the Choleski procedure, confirming the robustness of the latter methodology, and the asset allocations and risk contributions are effectively unchanged (hence we do not report them). When we standardize the macro factors before estimating the VAR model, the resulting minimum linear torsions can be easily attributed to each state variable. However, the resulting performance (MLTnorm) slightly deteriorates compared to the alternative specifications, although the maximum drawdown is still below that of the alternative asset-based strategies observed in the main section.

6.2 Real Time and Alternative Output Measures

The CLI state variable used as a measure of output and issued by the OECD tends to undergo significant adjustments as an aggregation of several economic variables, and the values reported in the official statistics are usually subsequently revised and not real-time measures. Indeed, the first in-sample bias arises because the CLI data for the G7 countries is available only two months after the reference period. Secondly, at each revision date, the entire CLI series is adjusted, due, for example, to changes in pre-selection (i.e. the reference/component series included in the aggregate indicator) and filtering (periodicity, seasonal adjustments, outlier detection, cycle identification and normalization) (OECD, 2012). In this section we study the robustness of the FRP 4SV performance against the two in-sample biases mentioned above. First, to control for reference-publication lag bias, we lag the reported (revised) CLI values by two months and use the adjusted measure in the portfolio implementation (*FRP lag*). Secondly, to control for subsequent revisions, we use the initial vintages of the CLI variable as originally published by the OECD Main Economic Indicators Original Release Data and Revisions Database and we match the reported values with the publication date (instead of the reference date) to ensure real-time measurement. Since the first vintage series dates back to February 1999, effective real-time analysis is only possible thereafter. Nevertheless, the vintage dataset suffers from several shortcomings which make an accurate, real-time measurement challenging. For example, on some dates the revised series did not report any values for the prior period. In those cases, we carry forward the values reported from the previous period. Additionally, the OECD skips the publication of the CLI in cases of "significant unforeseen or unexpected events, for example natural disasters" (OECD, 2016). With reference to our CLI series, the OECD suspended the release of the indicator from July to September 2016, following the U.K. referendum, as the event was "affecting the underlying expectation and outturn indicators used to construct the CLIs". Similarly, for those months, we simply carried forward the last reported observations from the most recently available vintage series. Hence, the real-time series is used to construct the *FRP vintage* portfolio.

Panel B of Table 7 reports the performance results of the different specifications. The performance of *FRP lag* does not change significantly compared to the revised CLI-based FRP (original) although it exhibits marginally higher maximum drawdown. When we replace the CLI series with the vintage data in *FRP vintage*, we observe that both volatility and maximum drawdown increase, while the risk-adjusted performance and allocations are only marginally affected (with higher –unreported– allocations to gold and US.HY). The higher portfolio volatility can be partly attributed to the continuous revisions and gaps in the underlying CLI series. The overall results therefore suggest that while the reporting lag bias plays a minor role in the portfolio performance, the series revisions account for most of the real-time/in-sample performance differential. We relied on the revised series for our main analysis as our primary objective was to present applications of factor risk parity based on macroeconomic state variables, whereas we leave an analysis or creation of the best, real-time output measures to other studies.

Finally, we compare the results based on an alternative measure of output, that is the IHS Markit US Manufacturing PMI, using a log difference transformation. The advantage of the PMI is that it provides real-time and monthly measures of economic activity and closely tracks the changes in the industrial production series. Under this specification, the performance decreases to a greater extent, resulting in higher portfolio volatility and maximum drawdown. The allocations change primarily because of higher allocations to the gold index. However, a comparison of the CLI and PMI reveals that the two series are weakly correlated (0.34) as they appear to track different aspects of the economic cycle: while the PMI tracks the volatile, monthly measures of economic activity, the CLI, by construction, tracks the long-term cyclical output fluctuations and turning points. Therefore, beyond the in-sample considerations, the underperformance of the PMI-based portfolio may also be due to the fact that asset returns are driven mainly by the long-run, persistent output fluctuations (Bansal and Yaron, 2004) rather than by short-term economic activity, as captured by the PMI indicator.

7 Conclusions

The relationship between asset returns and the macroeconomy has been a subject of long-standing interest in empirical and academic studies and macroeconomic variables play a vital role as pricing factors driving common variation in the cross-section and time-series of asset returns. The investment management world acknowledges that allocating to factors which are the underlying sources of risk and return across assets can provide superior risk-adjusted performance and diversification benefits compared to an asset class approach. Yet, few studies to date have examined the role of macroeconomic factors as useful diversifiers in asset allocation or which types of macro factors are most relevant for asset allocation. We presented an asset allocation framework to achieve a diversified exposure to macro factors. In particular we distinguish between two sets of macro variables: we consider five macroeconomic risk factors (TERM, MARKET, USD, OIL and DEF) as the primary sources of risk, explaining more than 50% of monthly variation, while we define four macroeconomic variables (CLI, G7.INFLATION, G7.Short.Rate and VIX) as the ultimate long-term sources of expected returns and as state variables driving time-series comovement in the asset returns. Generally, we find that the correlation between macroeconomic variables is lower than that between asset returns, suggesting that a balanced allocation to macroeconomic risks has diversification potential. We also find that a factor risk parity strategy that aims at explicitly harvesting macroeconomic premia enhances the risk-adjusted return compared to alternative risk-based strategies. However, a conditional regime-dependent analysis reveals that the five macroeconomic risk factors provide only limited diversification benefits as these factors all underperform and exhibit larger comovement during bad states of the world –which is then reflected in the higher correlations between assets during crisis. Moreover, an explicit exposure to the associated macro premia during times of high economic uncertainty results in large drawdowns. In constrast, the four macroeconomic state variables represent, unconditionally, separate sources of expected returns. Consequently, a diversified allocation across these four macro premia truly enhances the risk-return profile through greater upside potential and effective diversification across different economic regimes and –most importantly– during states of high economic uncertainty, when diversification is needed the most.

The results of this paper provide useful considerations in favor of adopting a macro factor-based approach in portfolio management to achieve a diversified exposure to the true sources of risk and return and to protect investors from difficult market and economic times. Using a simple two-state Markov-Switching model, we assume the existence of a normal state and a high economic uncertainty state. In practice, however, investors are likely to be concerned with multiple states of the world. For example, our analysis begins with the early 1990s and is necessarily conditioned on an overall low-inflation and declining interest rate regime. Hence, a useful extension of this paper would be to examine the diversifying role of macroeconomic factors over a longer history to account for different macroeconomic regimes and structural changes in both macroeconomic and asset returns as documented in the literature. Moreover, the slow-dynamics of the macroeconomic variables call for examining the relationship between asset returns and macro factors at a lower frequency. Finally, an additional extension would be to examine regime-based asset allocation by timing the macroeconomic regimes and altering the allocations to the individual macro risk premia during the realization of the bad states of the world, following the hedging motives of investors.

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Tables

Table 1: Macroeconomic Risk Factor Model This table reports the estimates from an OLS regression of the excess asset returns on the five macro risk factors. Significance levels are adjusted for heteroskedasticity and autocorrelation using the Newey-West method and are reported below the coefficient estimates. The sample period is 1993-12-31 to 2019-12-31.

	Intercept	TERM	MARKET	USD	OIL	DEF	adj. R2 (%)
US.HY	0.0056 ***	-0.0010	0.0096 ***	-0.0009	0.0013	-0.0051 ***	37.42
US.IG	0.0037 ***	-0.0076 ***	0.0033 ***	-0.0019 ***	0.0004	-0.0009 *	46.71
US.Bond	0.0027 **	-0.0121 ***	-0.0007	-0.0017 *	-0.0009	0.0006	48.92
DE.Bond	0.0028 ***	-0.0077 ***	0.0000	0.0003	-0.0015 *	0.0003	34.27
UK.Bond	0.0039 ***	-0.0091 ***	0.0017 *	0.0000	-0.0016 *	0.0015 .	32.92
JP.Bond	0.0009	-0.0047 ***	0.0004	-0.0012 .	-0.0010	0.0001	17.19
SP500	0.0081 ***	0.0002	0.0268 ***	-0.0022 *	0.0000	-0.0012	65.20
MSCI.EAFE	0.0040 *	0.0008	0.0285 ***	0.0017	0.0003	-0.0013	68.34
MSCI.EM	0.0166 ***	0.0064 .	0.0175 ***	-0.0024	0.0026	-0.0059 .	15.53
RUSSELL2000	0.0067 ***	0.0036 .	0.0298 ***	0.0006	0.0030	-0.0032	50.20
S&P.GSCI.Commodity	0.0012	-0.0008	0.0027 .	0.0005	0.0384 ***	-0.0045 *	61.32
GSCI.Gold	0.0013	-0.0041 *	-0.0014	-0.0075 ***	0.0043 *	-0.0012	6.59

p-value legend: ***: <0.001, **:<0.001, *:<0.05, .: <0.10

Table 2: Macroeconomic State Variables Factor Model This table reports the estimates from OLS regression of the excess asset returns on the four macro state variables. The independent variables are measured as innovations from a VAR(1) process with intercept. Significance levels are adjusted for heteroskedasticity and autocorrelation using the Newey-West method and are reported below the coefficient estimates. The sample period is 1993-12-31 to 2019-12-31.

	Intercept	CLI	INFLATION	Short.Rate	VIX	adj. R2 (%)
US.HY	0.0041	0.1591	0.0045	-0.0066	-0.2822	28.51
.	**	**			***	
US.IG	0.0032	0.0249	-0.0070	-0.0168	-0.0924	8.17
	***		*	*	*	
US.Bond	0.0026	-0.0531	-0.0171	-0.0336	0.0236	7.02
	**	*	**	***		
DE.Bond	0.0027	-0.0152	-0.0096	-0.0315	0.0477	8.29
	**		*	***		
UK.Bond	0.0037	-0.0073	-0.0141	-0.0306	0.0116	4.80
	***		**	**		
JP.Bond	0.0008	-0.0147	-0.0046	-0.0213	-0.0151	2.81
				***		
SP500	0.0059	0.1092	-0.0057	-0.0112	-0.7153	46.91
	***	.			***	
MSCI.EAFE	0.0016	0.1629	-0.0044	0.0036	-0.6306	37.09
		***			***	
MSCI.EM	0.0146	0.3530	0.0312	0.0500	-0.4638	14.75
	***	***	.		***	
RUSSELL2000	0.0041	0.1520	-0.0008	-0.0113	-0.8731	40.98
		*			***	
S&P.GSCI.Commodity	-0.0006	-0.0468	0.1076	0.0183	-0.2637	15.44
			***		**	
GSCI.Gold	0.0012	-0.0053	0.0180	-0.0062	-0.0069	-0.36

p-value legend: ***: <0.001, **:<0.001, *:<0.05, .: <0.10

Table 3: Portfolio Performance This table reports the main portfolio results from the empirical analysis. The portfolios in Panel A are Factor Risk Parity strategies along the five macro risk factors (5RF) or four macro state variables (4SV). The FRP LO are long-only factor risk parity portfolios obtained using views from macro factor-mimicking portfolios in a mean-variance optimization setting. The last two columns refer to the average Effective Number of Bets in terms of the five risk factors and four state variables, respectively. Panel B refers to the benchmark strategies based on an asset-class approach. The out-of-sample period is 1994-01-31 through 2020-01-31.

	AverageRet	StDev	SharpeRatio	MDD	Skewness	ENB_{5RF}	ENB_{4SV}
Panel A: Macro strategies							
FRP 5RF Opt	5.10	6.62	0.77	17.5	-0.16	5.00	2.56
FRP 5RF LO	3.60	5.92	0.61	18.43	-0.73	3.85	2.67
FRP 4SV Opt	3.43	5.13	0.67	13.62	0.15	2.81	4.00
FRP 4SV LO	3.20	4.37	0.73	10.67	-0.47	2.14	3.61
Panel B: Benchmark strategies							
MinimumVariance	2.34	3.69	0.63	10.76	-0.48	2.57	2.44
ERC	2.90	4.49	0.65	14.52	-1.27	2.52	2.38
EqualWeight	3.40	6.48	0.52	26.47	-1.58	1.82	2.24

Table 4: MS-VAR Summary Statistics This table reports the regime-specific moments of the macro factors estimated from the MSIH(2,0)-VAR(1) model. The covariance matrix is multiplied by 100 for ease of illustration. The autoregressive parameters are not reported here as they are not regime-switching in the model.

	CLI	INFLATION	Short.Rate	VIX
Intercepts:				
State 1	-0.007	0.153	0.006	0.041
State 2	-0.007	0.191	-0.030	0.067
Covariance Matrix:				
State 1				
CLI	0.047			
G7.INFLATION	-0.060	3.266		
G7.Short.Rate	-0.043	-0.049	0.577	
VIX	-0.002	-0.009	0.005	0.071
State 2				
CLI	0.432			
G7.INFLATION	0.160	5.769		
G7.Short.Rate	0.010	0.052	3.217	
VIX	-0.069	-0.097	-0.180	0.309

Table 5: MS Portfolio Performance This table reports the portfolio performance based on the Markov-Switching model. The covariance matrix of the macro factors used in the implementation of the FRP portfolios is the conditional Markov-Switching estimate. $MSVAR_1$ refers to the MSIH(2,0)-VAR(1) model estimated in-sample, whereas $MSVAR_2$ refers to the MSIAH(2,0)-VAR(1) model estimated out-of-sample. FRP (static) refers to the original portfolio based a single-state linear model. The out-of-sample period is 1994-01-31 through 2020-01-31.

	AverageRet	StDev	SharpeRatio	MDD	Skewness
FRP Opt (static)	3.43	5.13	0.67	13.62	0.15
FRP LO (static)	3.20	4.37	0.73	10.67	-0.47
FRP Opt (MSVAR_1)	3.42	5.03	0.68	15.22	0.33
FRP LO (MSVAR_1)	3.11	4.30	0.72	10.31	-0.36
FRP Opt (MSVAR_2)	3.55	5.23	0.68	13.31	0.38
FRP LO (MSVAR_2)	3.43	4.44	0.77	10.76	-0.43

Table 6: Regime Dependent Performance This table illustrates the performance of the portfolio strategies in each regime, which is identified based on the smoothed state probabilities estimated from the MSIH(2,0)-VAR(1) model and using a 50% probability threshold. The values reported are monthly estimates, except for the CEQ which is an annualized measure.

	Mean	StDev.	SharpeRatio	MDD	Skewness	CEQ
Panel A: Regime 1						
FRP 5RF Opt	0.37	1.72	0.22	9.84	-0.13	3.60
FRP 5RF LO	0.28	1.49	0.19	8.49	-0.19	2.66
FRP 4SV Opt	0.21	1.36	0.15	13.68	-0.43	1.95
FRP 4SV LO	0.25	1.13	0.22	8.51	-0.38	2.65
MV	0.17	0.93	0.18	6.68	-0.15	1.76
ERC	0.28	1.06	0.26	6.30	-0.28	3.02
EW	0.43	1.40	0.31	7.43	-0.27	4.63
Panel B: Regime 2						
FRP 5RF Opt	0.58	2.42	0.24	17.50	-0.29	5.25
FRP 5RF LO	0.37	2.29	0.16	18.43	-1.18	2.90
FRP 4SV Opt	0.53	1.79	0.30	7.74	0.79	5.38
FRP 4SV LO	0.31	1.61	0.19	9.57	-0.56	2.93
MV	0.28	1.41	0.20	10.76	-0.84	2.79
ERC	0.12	1.86	0.07	14.52	-1.51	0.43
EW	-0.19	2.86	-0.07	33.11	-1.38	-4.78

Table 7: Robustness Performance This table reports the performance of the FRP 4SV strategies under alternative specifications. Panel A explores different orthogonalization procedures: MLT refers to the Minimum Linear Torsion applied to the macro factor innovations while in MLTnorm the Minimum Linear Torsion is applied to the previously demeaned and standardized macro factor innovations. The original specification is based on a Choleski decomposition. Panel B compares different specifications of the output state variable. The first (original) is based on the revised, in-sample measure of the CLI series. The second (lag) uses the revised CLI series lagged by two months to account for the publication-reference period lag. The third (vintage) uses the lagged, vintage series of the CLI. The last (PMI) replaces the CLI measure with the US PMI index, measured in log difference.

	AverageRet	StDev	SharpeRatio	MDD	Skewness
Panel A: Choleski vs Minimum Linear Torsion					
FRP LO (Choleski)	3.20	4.37	0.73	10.67	-0.47
FRP Opt (Choleski)	3.43	5.13	0.67	13.62	0.15
FRP LO (MLT)	3.18	4.43	0.72	11.06	-0.57
FRP Opt (MLT)	3.56	5.37	0.66	14.64	0.31
FRP LO (MLTnorm)	2.98	4.72	0.63	12.11	-0.16
FRP Opt (MLTnorm)	3.48	6.86	0.51	33.38	0.16
Panel B: Alternative Output Measures					
FRP LO (original)	3.20	4.37	0.73	10.67	-0.47
FRP Opt (original)	3.43	5.13	0.67	13.62	0.15
FRP LO (lag)	3.66	4.96	0.74	15.16	-0.82
FRP Opt (lag)	4.19	5.67	0.74	15.52	0.34
FRP LO (vintage)	4.26	6.03	0.71	22.35	-0.82
FRP Opt (vintage)	4.16	7.29	0.57	22.75	-1
FRP LO (PMI)	3.66	6.78	0.54	23.13	-0.79
FRP Opt (PMI)	3.87	8.03	0.48	31.31	-0.13

Figures

Figure 1: Uncorrelated Macro Factor Loadings This figure illustrates the full-sample mappings of the uncorrelated macro factors to the original macro variables. The first row refers to the minimum linear torsion loadings for the five macro risk factors; the second row refers to the loadings of the four macro state variables based on the Choleski decomposition. The sample period is 1993-12-31 to 2019-12-31.

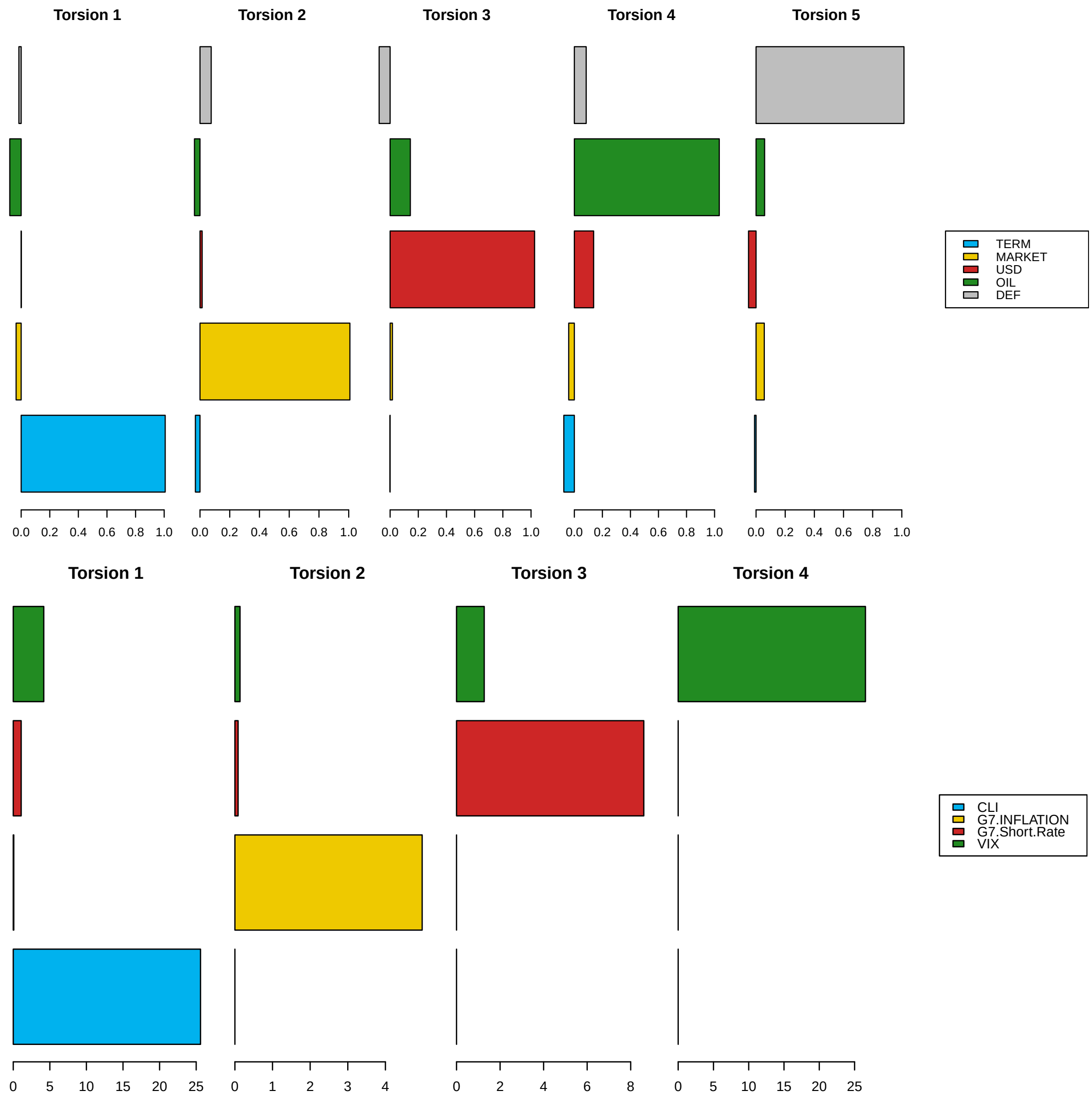


Figure 2: Macro Factor-Mimicking Portfolio Loadings This figure illustrates the exposures of the mimicking portfolios that track the five macro risk factors (first row) and four macro state variables (second row) to the test assets. The loadings are measured as the correlation coefficients between the macro factor-mimicking portfolios and the asset returns over the full-sample period.

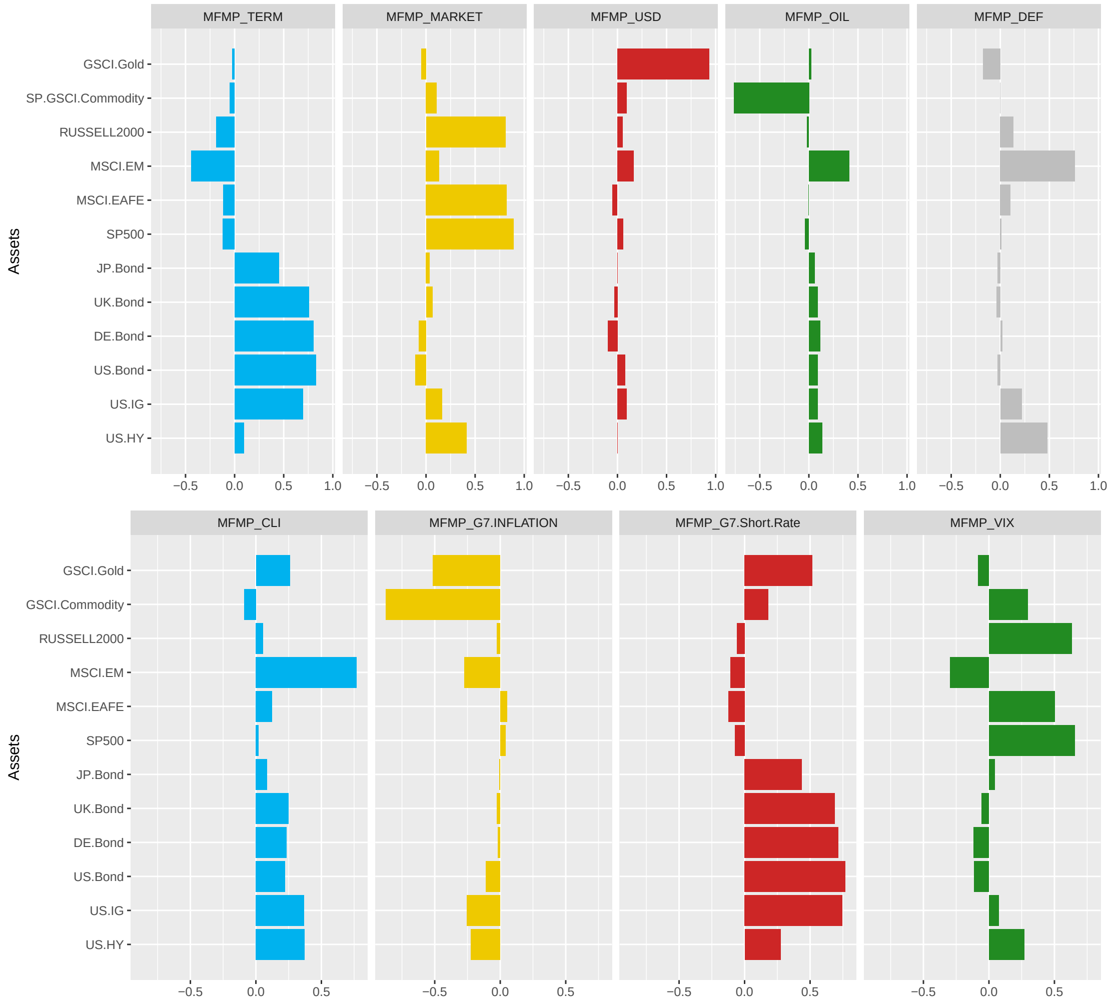


Figure 3: Performance of Macro Factor-Mimicking Portfolios This figure illustrates the full-sample cumulative returns on the estimated macro factor-mimicking portfolios tracking the five uncorrelated risk factors (first row) and four state variables (second row).

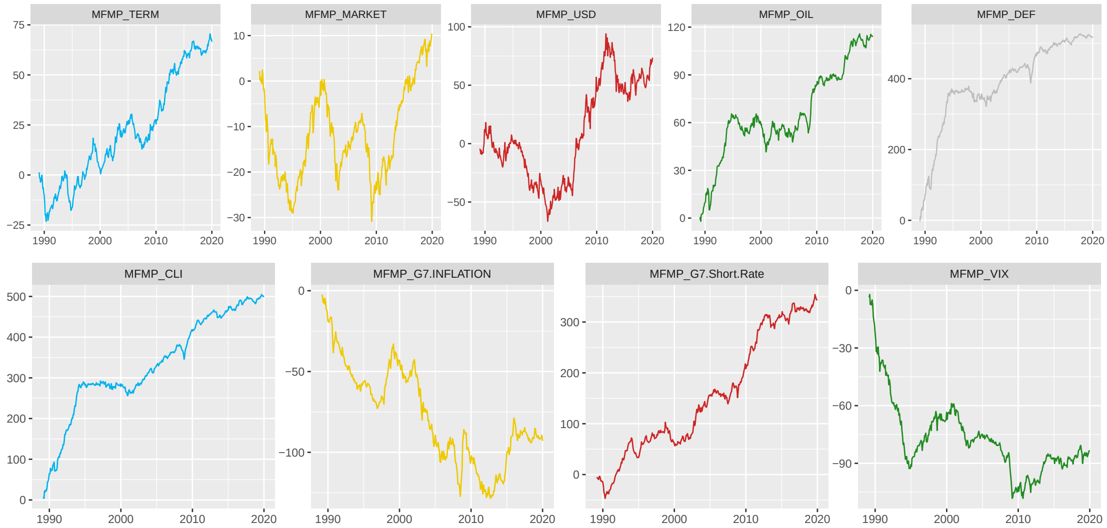


Figure 4: Macro Factor-Mimicking Portfolios Correlations This figures illustrates the correlation coefficients between the macro factor-mimicking portfolios (top left quadrant) and the correlations between the macro factor-mimicking portfolios and the (correlated) macro factor innovations (bottom left quadrant)

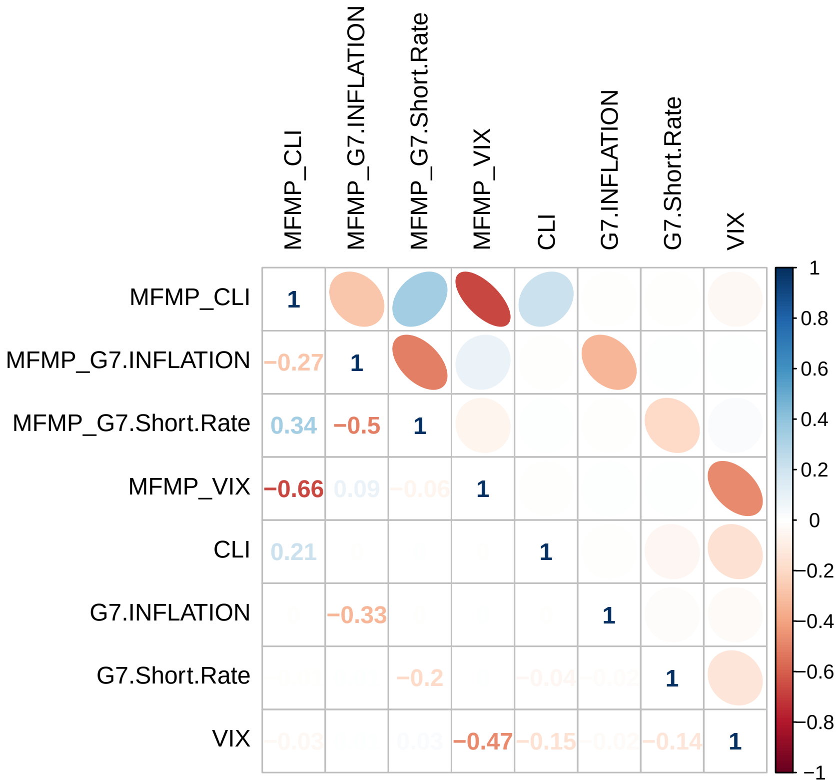


Figure 5: Cumulative Strategy Performance This figure illustrates the cumulative returns for the out-of-sample period 1994-01-31 to 2020-01-31.

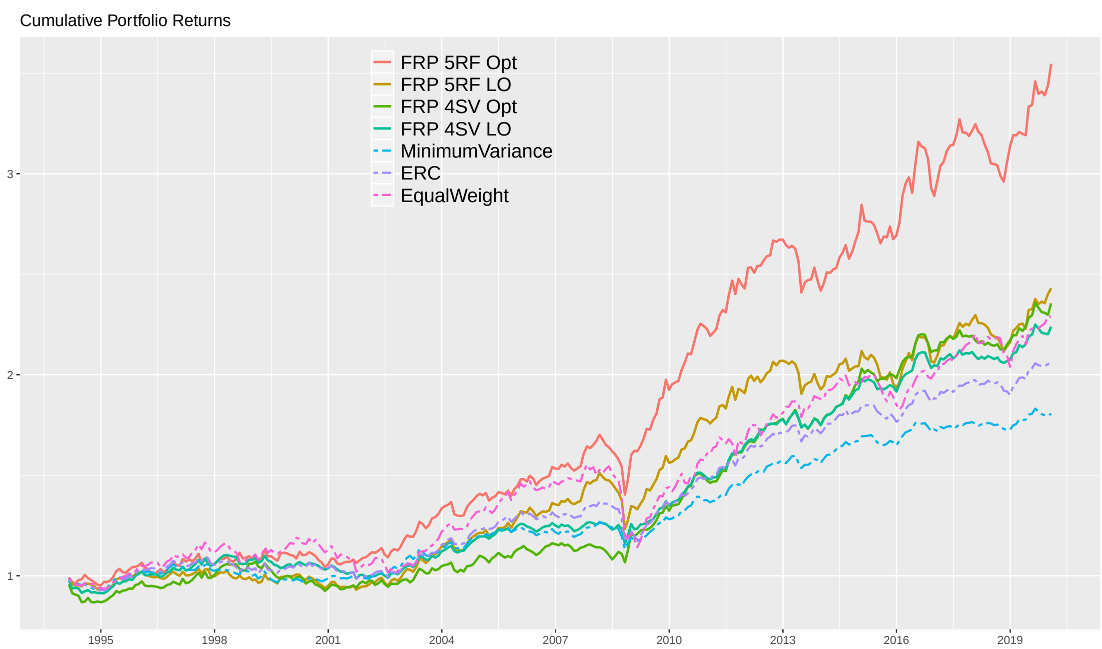


Figure 6: Asset Allocations This figure illustrates the asset weights (first column) and risk contributions (columns (2) and (3)) for the portfolio strategies. The risk contributions are expressed in terms of the five uncorrelated macro risk factors (second column) and four macro state variables (third column). The factors have been orthogonalized using a Minimum Linear Torsion and a Choleski decomposition, respectively.

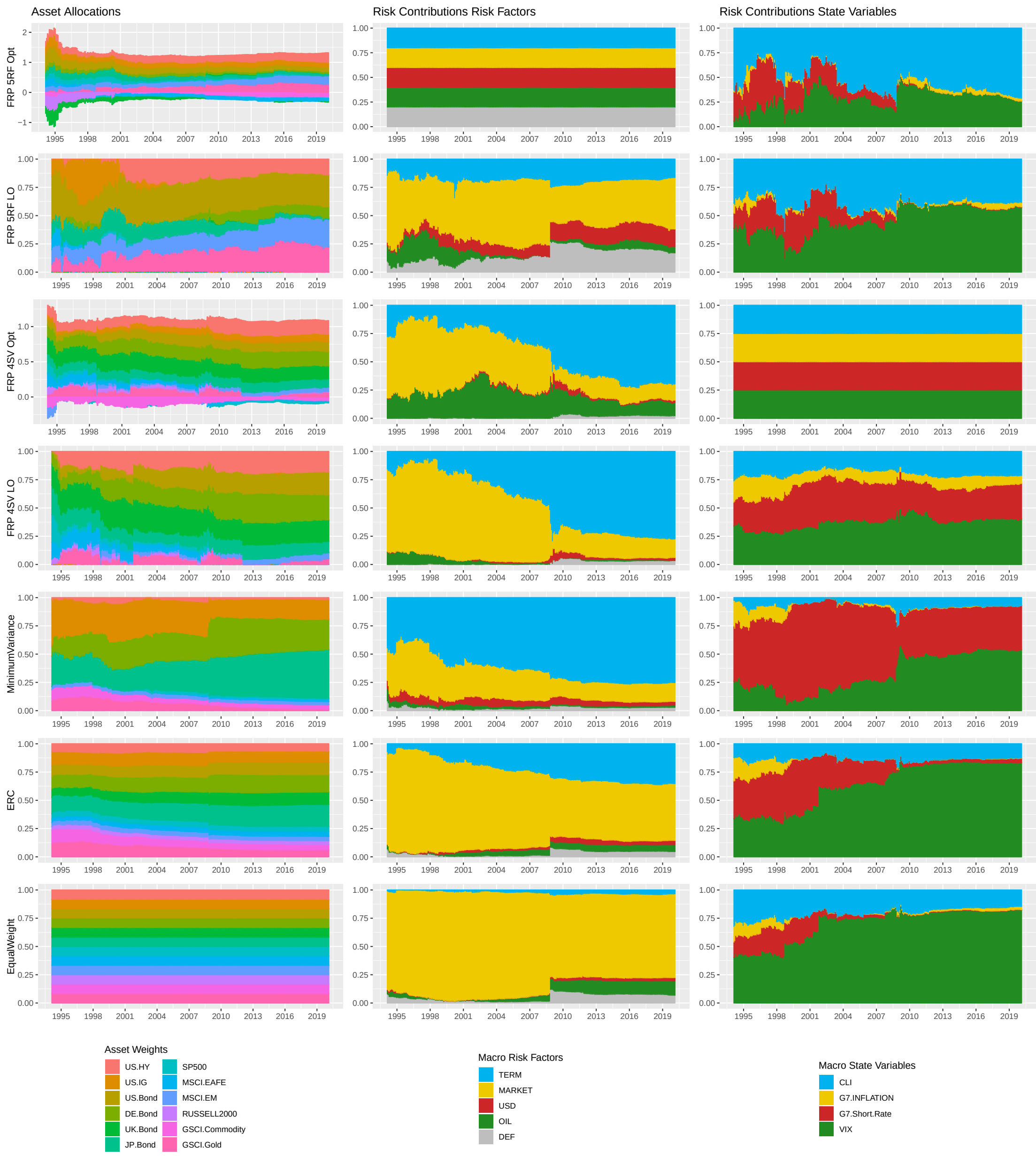


Figure 7: Effective Number of Bets This figure illustrates the Effective Number of Bets for the portfolio strategies in terms of the five macroeconomic risk factors (first row) and four macro state variables (second row).

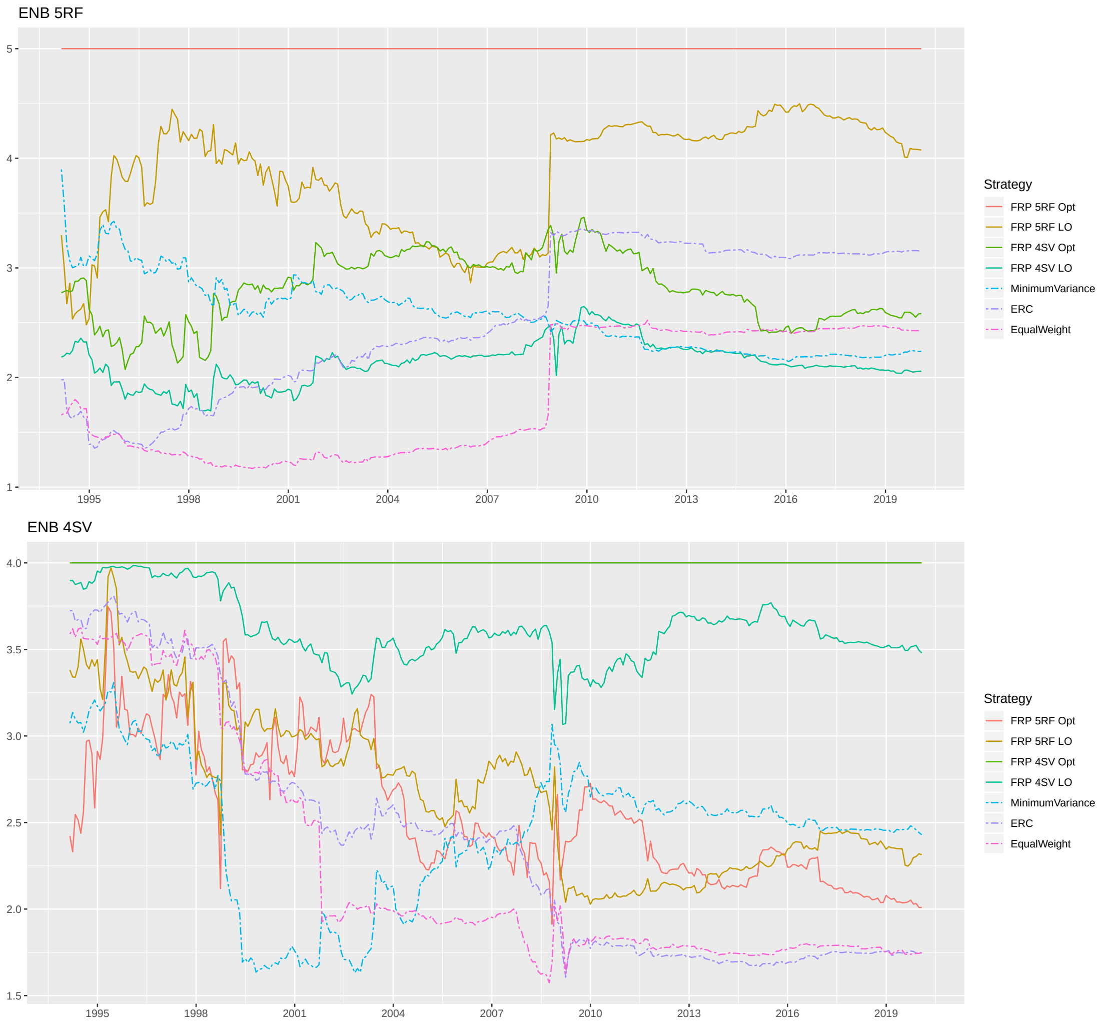


Figure 8: Regime-Switching State Probabilities This figure illustrates smoothed state probabilities estimated in-sample from the MSIH(2,0)-VAR(1) model and the time series of the four macro state variables. The blue line indicates the US recession periods as identified by the NBER U.S. recession reference dates.

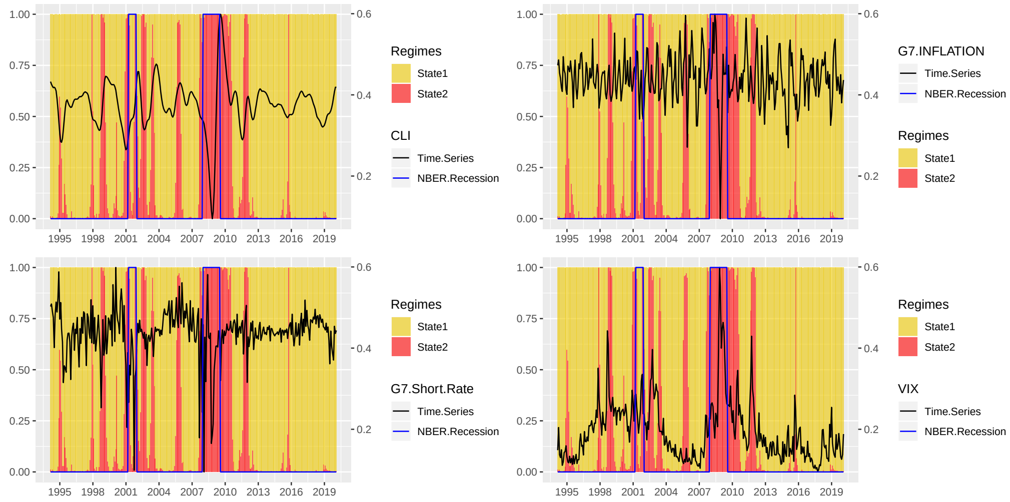


Figure 9: State-Dependent Portfolio Frontier This figure illustrates the mean-variance combinations for the original portfolio strategies for each state. The state-dependent performance is derived by subsetting the realized performance based on the states as identified from the in-sample MSIH(2,0)-VAR(1). The portfolios labelled in bold are the factor risk parity strategies.

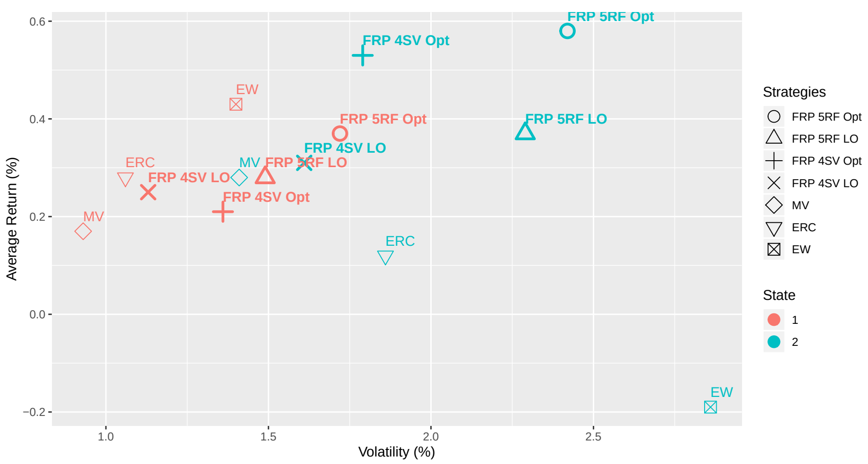
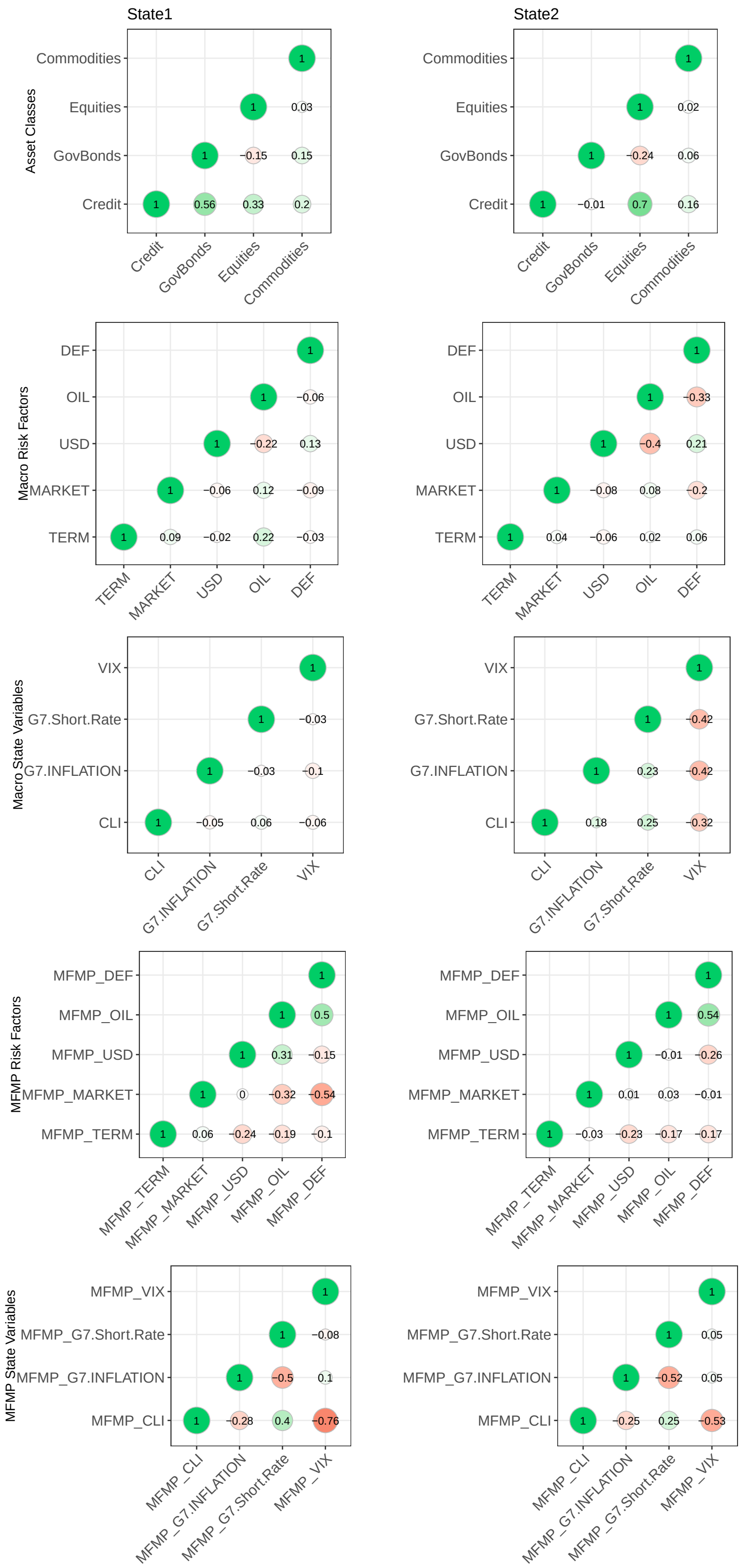


Figure 10: State-Dependent Asset and Macro Factor Correlations This figure illustrates the correlations within asset classes, macro risk factors and state variables, including the associated factor-mimicking portfolios, during normal times (first column) and during regimes of high economic uncertainty (second column). The states are identified based on the in-sample MSIH(2,0)-VAR(1) model.



8 Appendix

8.1 Benchmark Allocation Strategies

- The **Equal Weight** strategy assigns equal weights to the assets at each monthly rebalancing date and provides naïve diversification in nominal terms.

$$w_i = \frac{1}{N} \text{ for } i = 1, \ldots, 12.$$

- The **Minimum Variance** strategy minimizes the total variance of the portfolio subject to full investment and positivity constraints

$$w = \underset{w}{\operatorname{argmin}}\, w' \Sigma w$$

$$s.t. \; w' 1 = 1 \text{ and } w \geq 0$$

- The **Equal Risk Contribution** of Maillard et al. (2010) is obtained numerically as the solution of the following optimization problem, which minimizes the difference in contribution to risks of the assets

$$w = \underset{w}{\operatorname{argmin}} \sum_{i=1}^{N} \sum_{j=1}^{N} (\mathcal{R}C_i - \mathcal{R}C_j)^2$$

$$s.t. \; w' 1 = 1 \text{ and } w \geq 0$$

8.2 Regime Switching Model

Given a VAR(p) model for the macro state variables $y_t = (y_{1t}, \ldots, y_{Kt})'$, the parameters of the process depend on an unobserved regime variable $s_t \in (1, \ldots, M)$, which represents the probability of being in a given state. In particular, the data generating process is described by

$$y_t = v(s_t) + A_1(s_t)y_{t-1} + \ldots + A_p(s_t)y_{t-p} + u_t, u_t|s_t \sim NID(0, \Sigma(s_t)) \tag{15}$$

The regime variable is governed by an M-state stochastic Markov process, defined by the transition probabilities

$$p_{ij} = Pr(s_{t+1} = j|s_t = i), \sum_{j=1}^{M} p_{ij} = 1 \; \forall i, j \in 1, \ldots, M$$

and collected in the transition matrix

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} = \begin{bmatrix} p_{11} & 1 - p_{11} \\ 1 - p_{22} & p_{22} \end{bmatrix}$$

The latent states, s_t, follow a discrete, irreducible, ergodic first-order Markov Chain process. The first-order chain indicates that the probability of occurrence of state i at time t depends only on the state at time $t - 1$. The ergodic property means that there is a stationary vector of probabilities ξ such that $\xi = P'\xi$, while irreducibility indicates that all latent states can occur, that is, $\xi > 0$.

The MSIAH(2,0)-VAR(1) model analyzed for robustness is estimated out-of-sample using an expanding window and an initial calibration period of five years. The model is estimated by maximum likelihood and using the Hamilton filter to derive the regimes.[11] In particular, the MLE is based on the blockwise algorithm of Sims, Waggoner and Zha (2008): the parameters to be estimated are grouped into four separate blocks: the transition matrix, the intercepts, the autoregressive parameters and the error covariances. The blockwise algorithm then conducts four separate optimizations for each block and over several iterations. In practice, there are concerns with the blockwise optimization that, from an economic perspective, the classification of the regimes may be less intuitive and the labelling of the regimes may differ in each period in the out-of-sample analysis. Hence, we rely on the in-sample MSIH(2,0)-VAR(1) model introduced in the main section to study, ex-post, the conditional performance of the strategies.

[11] We are grateful to Patrick Brandt and W. Ryan Davis for providing the R package MSBVAR to estimate the model (Brandt and Davis, 2016). Unlike the main Bayesian scope of the package, we used the msvar function for inference.