Risk Premium Shocks

Informing Structural VARs with Asset Pricing Models

Gabor Pinter[*]

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Abstract

This paper integrates empirical asset pricing with structural vector autoregressions (VAR) to study the joint determinants of business cycles and risk premia. Instead of starting with a macroeconomic model and testing its asset pricing implications, I work "backwards". First I use asset prices to construct the stochastic discount factor from macroeconomic VAR innovations, and only then study its empirical relation to business cycles. This approach reveals that the two shocks that drive the level and time-variation of risk premia are mutually orthogonal, resemble conventionally-identified monetary and demand shocks and explain up to 80% of aggregate consumption fluctuations in the US.

JEL Classification*: C32, G12

1 Introduction

> *"In sum, we face two main questions. First, the equity premium question: What is there about recessions, or some other measure of economic bad times, that makes people particularly afraid that stocks will fall during those bad times—and so people require a large upfront premium to bear that risk? Second, the predictability question: What is there about recessions, or some other measure of economic bad times, that makes that premium rise—that makes people, in bad times, even more afraid of taking the same risk going forward? These are two separate questions..."*
> *(p. 947, Cochrane (2017))*

Understanding the macroeconomic forces that drive risk premia continues to be a challenge for both the macroeconomics and finance literatures. On the one hand, numerous variables have been proposed by the finance literature to explain the cross-sectional and time-series variation in expected excess asset returns. However, most of these empirical studies remain silent on how the proposed explanation of risk premia relates to the structural primitives driving business cycle fluctuations. On the other hand, the macroeconomics literature on structural vector autoregressions (VAR) has long sought to identify the drivers of business cycles. However, most structural VAR models have ignored asset price information on the cross-sectional and time-series variation in expected returns, thereby remaining silent on how the identified macroeconomic shocks relate to the determination of risk premia.

I propose a method to reduce this gap between the finance and macroeconomics literatures. Instead of starting with a macroeconomic model and testing its asset pricing implications, I avoid modelling the structure of the macroeconomy, and instead work "backwards". First I use asset prices to construct the stochastic discount factor (SDF) from macroeconomic VAR innovations, and only then study how the macroeconomic drivers of risk premia relate to business cycles. Specifically, I construct two shocks in a standard VAR that are engineered to explain the level and time-variation in risk premia, respectively, in a standard linear asset pricing model. The construction of these shock does not rely on any of the macroeconomic assumptions used by the structural VAR literature. I will show that this reverse direction should be a promising alternative to the more obvious direction – using identified structural shocks directly in asset pricing tests. The intuition is simple: identification of macroeconomic shocks may suffer from overly

restrictive identifying assumptions and from mis-measurement of macroeconomic data, which make identified shocks more likely be rejected by asset pricing models.[1]

A brief summary of the paper's economic contribution is as follows: applying the method to US data reveals that the two shocks, driving the level and time-variation of risk premia, are mutually orthogonal in the data, resemble conventionally-identified monetary and demand shocks and jointly explain up to 80% of aggregate consumption fluctuations in the US.

The paper's methodological contribution is to highlight the overlap between empirical finance models (Fama and French, 1993) and structural shocks identified by the macroeconomics literature (Sims, 1980), and to provide economists with an easily implementable VAR-based toolkit to study the link between risk premium shocks and business cycles (Christiano, Motto, and Rostagno, 2014; Tella and Hall, 2020).

Related Literature My paper is connected to the literature on finding macroeconomic factors that drive the cross-sectional and time-series variation in expected returns.[2] A subset of this literature explicitly focuses on the role of structural macroeconomic shocks such monetary policy shocks (Gurkaynak, Sack, and Swanson, 2005; Bernanke and Kuttner, 2005; Coibion, 2012; Lucca and Moench, 2015; Weber, 2015; Ozdagli and Velikov, 2016) as well as news shocks (Malkhozov and Tamoni, 2015) in affecting the macroeconomy as well as asset prices.

The proposed methodology also aims to support the growing macroeconomics and finance literature on the link between business cycles and shocks to uncertainty and risk premia (Gilchrist, Sim, and Zakrajsek, 2014; Jurado, Ludvigson, and Ng, 2015; Bloom, Floetotto, Jaimovich, Eksten, and Terry, 2018; Bekaert, Engstrom, and Xu, 2019).

The method to construct the λ-shock draws on the structural VAR literature that uses sign restrictions to identify structural shocks (Uhlig, 2005; Rubio-Ramirez, Waggoner, and Zha, 2010; Fry and Pagan, 2011; Baumeister and Hamilton, 2015). The method

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[1] A simple Monte-Carlo analysis (Appendix 4.1) will be used to highlight this point.

[2] A partial list includes Chen, Roll, and Ross (1986), Fama and French (1989), Campbell (1996), Cochrane (1996), Lettau and Ludvigson (2001a,b), Boons and Tamoni (2015), He, Kelly, and Manela (2017) and Koijen, Lustig, and Van Nieuwerburgh (2017) amongst others.

to construct the γ-shock draws on the macroeconomic literature on the identification of news shocks (Uhlig, 2004; Barsky and Sims, 2011; Kurmann and Otrok, 2013); I combine the ideas from this literature with predictive regressions of the asset pricing literature (Campbell and Shiller, 1988; Goyal and Welch, 2008; Pastor and Stambaugh, 2009).

2 A VAR Model with Asset Pricing

The method combines information on both the cross-sectional and time-series variation in expected returns with structural VAR techniques. To formalise ideas, write the dynamics of the economy as a structural VAR(1):

$$\Phi X_t = B X_{t-1} + E_t, \tag{2.1}$$

where $X_t \equiv (x_{1,t}, x_{2,t}, \ldots, x_{n,t})$ includes n vectors of macroeconomic time-series, Φ and B are matrices of parameters, and $E_t \equiv \left(\varepsilon_t^\lambda, \varepsilon_t^\gamma, \ldots, \varepsilon_{n,t} \right)$ includes n vectors of orthogonal iid disturbances that are linear combinations of the reduced-form innovations.[3] The shocks ε_t^λ and ε_t^γ are the key objects of interest that my method constructs.

It is well known that structural VARs are not identified, i.e. the elements of matrix Φ are not pinned down by the data. Since Sims (1980) had proposed a triangular structure for matrix Φ, the macroeconometrics literature has experimented with a plethora of other macroeconomic assumptions to pin down matrix Φ (or only selected columns of it) in order to construct orthogonal disturbances as candidates for causes of business cycle fluctuations.[4] The novelty of my method is to rely exclusively on asset price information (and not on macroeconomic assumptions) to construct orthogonalised disturbances from reduced-form innovations. I approach the problem from two angles, pinning down two columns of Φ that correspond to the shocks ε_t^λ and ε_t^γ.

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[3]The reduced-form VAR is written as $X_t = A X_{t-1} + U_t$ where the reduced-form innovations U_t are linked to the structural disturbances E_t by $U_t = \Phi^{-1} E_t$.

[4]See Uhlig (2005); Rubio-Ramirez, Waggoner, and Zha (2010); Stock and Watson (2012); Mertens and Ravn (2013); Baumeister and Hamilton (2015); Gertler and Karadi (2015); Ramey (2016) among others.

Explaining the Cross-Sectional Variation in Expected Returns First, I use the cross-section of asset prices, in a linear unconditional asset pricing framework, to approximate innovations in the SDF with the orthogonal shock ε_t^λ in 2.1. Specifically, I select the linear combination of reduced-form residuals (corresponding to the first column of Φ in 2.1) such that the asset pricing performance of ε_t^λ would be maximised, when using it as a "factor" in a two-pass Fama and MacBeth (1973) regression, i.e. the differential covariance of ε_t^λ with returns on, say, portfolios of small and large firms (Fama and French, 1993) best explains why these portfolios have differential mean returns.

Formally, denoting the returns on these n portfolios (in excess the risk free rate) $R_{i,t}$ ($i = 1, \ldots, n$), the first-pass involves n time-series regressions:

$$R_{i,t} = a_i + \beta_i \varepsilon_t^\lambda + \epsilon_{i,t}, \tag{2.2}$$

where a_i is a constant and $\epsilon_{i,t}$ is an error term. The second-pass involves running a cross-sectional regression with n observations:

$$\bar{R}_i = \lambda \tilde{\beta}_i + \nu_i, \tag{2.3}$$

where $\bar{R}_i$ is the mean of time-series $R_{i,t}$, $\tilde{\beta}_i$ is the OLS estimate obtained from 2.2, ν_i is an error term, and coefficient λ is the "price of risk". The method I propose is simply a maximisation problem to find a linear combination of reduced-form VAR residuals, ε_t^λ, such that the fit of regression 2.3, measured by its R^2, is maximised. This shock, which I refer to as the λ-shock, is constructed to explain the *cross-sectional variation* in expected returns.[5]

The λ-shock together with the rich machinery of the structural VAR toolbox (impulse response functions, historical decompositions etc.) can be applied to any VAR and any test portfolios, and can answer the first question of Cochrane (2017) in the opening quote.

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[5]This step connects two simple ideas: (i) β-pricing models of the cross-section of asset prices imply a linear model of the SDF (Cochrane, 2005); (ii) orthogonalised shocks in 2.1 are linear combinations of the reduced-form innovations. These two facts imply that, given the space spanned by the VAR innovations and the space spanned by the cross-section of returns, one can construct orthogonal shocks that are best linear approximations of the SDF (with other shocks in the VAR demanding zero average risk premia).

Explaining the Time-Series Variation in Expected Returns Second, I use the time-series variation in expected returns and look for an orthogonal shock in the VAR which drives the fluctuations in the macroeconomic variables that are the most relevant to predicting future excess returns. This step builds on the vast empirical evidence on the predictability of returns, by financial and macroeconomic variables, implying that expected returns vary with the business cycle (Cochrane 2011).[6]

For example, the term-spread (one of the variables in the VAR 2.1) has predictive power of future returns (Fama and French, 1989). Given that the term-spread is a reduced-form object, its predictive power could in theory be decomposed to the historical contribution of primitive economic shocks that generated fluctuations in the term-spread. I take this idea to the limit, and construct a single orthogonal shock in the VAR 2.1 which generates the part of the fluctuations in the term-spread that are the most relevant to predicting excess returns.

This method too is a maximisation problem. I select the linear combination of reduced-form residuals (corresponding to a column of Φ and associated shock series ε_t^γ in 2.1), such that the counterfactual time-series of VAR variables, denoted by $X_t^\star(\varepsilon_t^\gamma)$, induced by the shock ε_t^γ (with the historical contribution of all other orthogonalised shocks in the VAR shut down) has the following the property: it would have the best achievable predictive power (measured by the R^2 statistic) in a standard univariate return-forecasting regression:

$$r_{t+1}^H = a + \beta X_t^\star(\varepsilon_t^\gamma) + u_{t+1}, \tag{2.4}$$

where r_{t+1}^H is the cumulative log excess market return between $t+1$ and $t+H$; a is a constant, and u_{t+1} is an error term. This shock, which I refer to as the γ-shock, is constructed to explain the *time-series variation* in expected returns. To the extent that time-variation in expected returns is linked to economic booms and busts (Lettau and Ludvigson, 2010; Cochrane, 2011), the γ-shock can be thought of as the stochastic driver

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[6]This literature typically employed univariate time-series techniques to regress realised excess returns on lagged values of valuation ratios (Campbell and Shiller, 1988; Fama and French, 1988) or macroeconomic variables (Fama and French, 1989; Ferson and Harvey, 1991; Lettau and Ludvigson, 2001a), and assessed the forecasting power of the proposed predictors based on the regression R^2 statistic.

of recessions in the VAR.

The γ-shock together with the rich machinery of the structural VAR toolbox can address the second question of Cochrane (2017) in the opening quote.

Summary of Findings   As an application, I use a standard macroeconomic VAR and benchmark test assets for the US, yielding four main empirical results. First, I find that the λ-shock generates a delayed response in consumption, consistent with reduced-form models of consumption in the asset pricing literature (Bansal and Yaron, 2004; Parker and Julliard, 2005; Bryzgalova and Julliard, 2015). Extending this literature, I show that the λ-shock induces a negative comovement between the short-term interest rate and consumption, that is uncharacteristic of most recent recessions. Importantly, the λ-shock closely resembles monetary policy surprises as identified by the macroeconomics literature, often using very different methodologies (Romer and Romer 2004; Sims and Zha 2006; Gertler and Karadi 2015). This result highlights the overlap between linear pricing models of the cross-section of returns (Fama and French, 1993) and monetary shocks identified by macroeconomists (Sims, 1980).

Second, the estimated γ-shock induces a sharp response in consumption and a positive comovement between the policy rate and aggregate quantities. I find that the economic characteristics make the γ-shock resemble demand-type shocks as identified by the recent macroeconomic literature. For example, Christiano, Motto, and Rostagno (2014) used a linearised equilibrium model with financial frictions to show that exogenously fluctuating uncertainty ("risk shocks") explains 60% of US business cycle fluctuations. Though I use a VAR and rely solely on information regarding time-variation in risk premia, I find a close empirical relationship between risk shocks and the γ-shock. This result highlights the overlap between recent explanations of business cycle fluctuations, offered by macroeconomists, and the drivers of time-varying risk premia, long studied by the finance literature.

Third, even though the λ-shock and the γ-shock are not restricted to be orthogonal to each other, a key empirical finding of this paper is that they are close to being orthogonal in the data. This implies that the stochastic macroeconomic drivers of the level risk

premia are empirically orthogonal to the stochastic drivers of variation in risk premia. This is consistent with, for example, the reduced-form theoretical model of Bansal and Yaron (2004) where the level of risk premia is explained by shocks to the long-run component of consumption growth, while variation in risk premia is entirely driven by shocks to stochastic volatility.[7]

Fourth, given the orthogonality of λ-shocks to γ-shocks in the data, I compute forecast error variance (FEV) and historical decompositions to assess their contributions to business cycles. I find that γ-shocks explain most US recessions in my sample and most of the high-frequency variation in aggregate consumption. In turn, λ-shocks drive lower-frequency variation in consumption, making a large contribution only to the early 1980s recession. Importantly, while my orthogonalisation method relies exclusively on asset price information and not on macroeconomic assumptions, I find that these two shocks jointly explain up to 80% of aggregate consumption fluctuations over the 1963-2015 period.

3   Empirical Results

The VAR includes quarterly data on output, consumption, price level, the short-term interest rate, the default spread, and the term spread. As baseline test assets, I combine the standard 25 Size-B/M portfolios of Fama and French (1993) with the 30 industry portfolios (FF55) as prescribed by Lewellen, Nagel, and Shanken (2010). Returns are quarterly, in excess of the T-bill rate. Data sources are in Appendix A, and Appendix B contains further details on the construction of the λ-shock and γ-shock which may be useful for readers less familiar with VARs.

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[7]A natural interpretation of the λ-shock and the γ-shock is via the generalisation of the SDF (M_{t+1}) as in Cochrane (2017):

$$M_{t+1} = \underbrace{\beta \left(C_{t+1}/C_t\right)^{-\sigma}}_{\approx \varepsilon_t^{\lambda}} \underbrace{Y_{t+1}}_{\approx \varepsilon_t^{\gamma}}, \tag{2.5}$$

where the novel term is Y_{t+1}: the key state variable, directly related to recessions and to time-varying risk-bearing. See Appendix 4.2 for a discussion.

3.1   The λ-Shock

The Aggregate Effects of the λ-Shock   First, I present the results for the λ-shock, implied by my baseline test assets. Using the OLS estimates, I compute IRFs for the λ-shock along with the IRFs associated with interest rate innovations using Cholesky-orthogonalisation. The blue crossed lines in Figure 1a display the IRFs, normalised to induce a 100bp increase in the interest rate.[8]   The black circled lines correspond to a 100bp interest rate shock using Cholesky-orthogonalisation. This orthogonalisation method has been frequently used in the macroeconomics literature to identify monetary policy shocks (Sims 1980; Christiano, Eichenbaum, and Evans 1999). The lines in Figure 1a are difficult to tell apart, as the point estimates of the two sets of IRFs are virtually identical. The striking resemblance between the two sets of IRFs occurs in spite of the fact that constructing the λ-shock does not rely on any of the strong restrictions that Cholesky-identified monetary policy shocks rely on.[9]

The λ-shock has no effect on consumption on impact, but the effect increases substantially with the horizon, reaching a peak of -0.5% approximately 12-15 quarters after the shock. This is consistent with the consumption dynamics implied by asset pricing models that highlighted the irrelevance of short-term consumption innovations to pricing (Bansal and Yaron, 2004; Bansal, Dittmar, and Lundblad, 2005; Parker and Julliard, 2005; Bansal, Dittmar, and Kiku, 2009). The recent empirical evidence confirmed that slow moving consumption risk can explain the cross-sectional variation of average returns (Bryzgalova and Julliard, 2015; Boons and Tamoni, 2015). I add to the literature by accounting for possible general equilibrium relationships between consumption growth and other macro variables while jointly explaining the cross-section of average returns. Extending results from reduced-form consumption based asset pricing models, the λ-shock generates a *negative comovement* between the policy rate and consumption. In contrast, the recent macroeconomics literature (Smets and Wouters, 2007; Christiano, Motto, and

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[8]As shown by Figure 8 in the Appendix, the shape of these IRFs is similar when the lag length is changed or when the Great Recession is excluded from the sample.

[9]The zero-restrictions under Cholesky-identification imply that the variables ordered before the monetary policy instrument do not respond to the monetary policy shock contemporaneously. See Section 4.1 of Christiano, Eichenbaum, and Evans (1999) for a detailed discussion of this recursiveness assumption.

Figure 1: λ-Shocks and Cholesky Interest Rate Shocks

(a) Impulse Responses
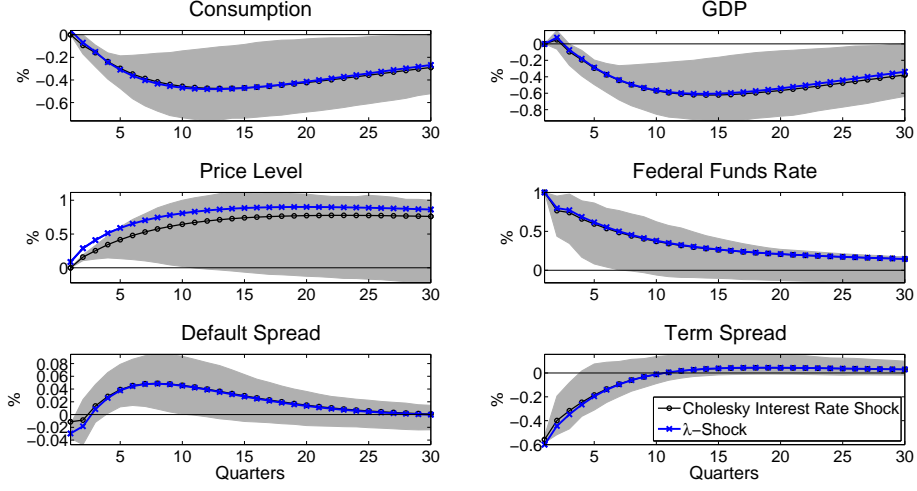
(b) Time-series of the Shocks
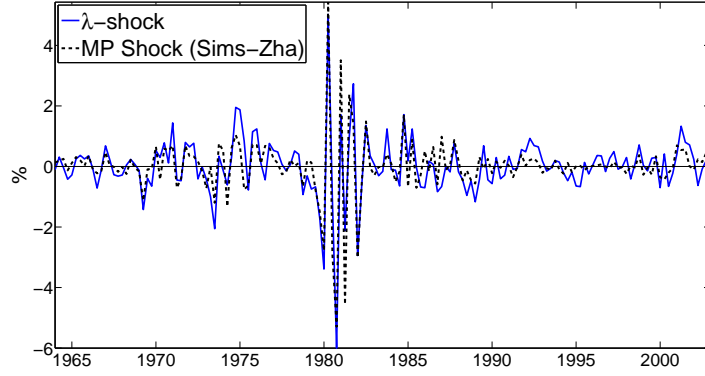
Notes: The λ-shock is from a six-variable VAR(2) which includes quarterly data on consumption, GDP, CPI, Fed Funds rate, the term spread, and the default spread. Test assets are FF55. In Panel A, the vertical axes are %-deviations from steady-state, and the horizontal axes are in quarters. The VAR is estimated on a subsample 1963Q3-2008Q3. The blue crossed lines are λ-shock, and the blacked circles lines are Cholesky-orthogonalised interest rate shock with the associated 95% confidence band (using wild-bootstrap). The IRFs are normalised to increase the interest rate by 100bp. In Panel B, The monetary policy shock series are from Sims and Zha (2006), as documented in Stock and Watson (2012), and transformed to have unit standard deviation. The correlation coefficient is 0.84.

Rostagno, 2014; Negro, Eggertsson, Ferrero, and Kiyotaki, 2017) showed that economic downturns (including the Great Recession) feature a positive comovement between the policy rate and consumption. Figure 1a is therefore important, suggesting a possible dichotomy between the macroeconomic drivers of the cross-section of average returns and the drivers of recent recessions.

Relation to Identified Monetary Policy Shocks   To highlight the relation of the λ-shock to the monetary policy literature, I compare the time-series of the λ-shock to other

benchmark estimates of monetary policy shocks. Figure 1b illustrates the correlation (0.84) against the policy shock series identified by Sims and Zha (2006). As a robustness exercise, I also check the correlation with narrative measures of monetary policy shocks: based on the overlapping estimation period 1969Q1–2007Q4, the correlation between the time-series of Romer and Romer (2004) (extended by Tenreyro and Thwaites (2016)) and the λ-shock series is 0.75.[10] Recent evidence from the empirical asset pricing literature also suggests that the short-term interest rate is relevant for explaining several dimensions of cross-sectional equity risk premia (Maio and Santa-Clara, 2017).

Despite these findings, using off-the-shelf monetary policy shock series to price the cross-section would likely lead to the rejection of the corresponding pricing model, because of restrictive identifying assumptions and mis-measurement in macroeconomic data. This is an important point which provides justification for my approach of starting with asset prices and then working "backwards". This will be further discussed in Section 4.1 below.

Relation to Other Identified Macroeconomic Shocks   The high empirical correlation between the λ-shock and conventionally-identified monetary shocks is non-trivial given that other structural shocks that the literature has identified[11] are assumed to be orthogonal to policy shocks and should also affect the SDF. Based on my investigation of these shocks as collected by Ramey (2016), the estimated λ-shock has little empirical correlation with these shocks.[12] A notable exception is news-type shocks to total factor productivity (TFP), as in Kurmann and Otrok (2013). Their Figure 4 shows IRFs for identified TFP news shocks that are similar to Figure 1. Based on the overlapping sample (1963Q4–2005Q2), the correlation between their TFP news shock series and the λ-shock series is 0.79.

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[10]As an additional check, I employ the methodology of using high frequency asset price movements around policy announcements (Gurkaynak, Sack, and Swanson, 2005) as instruments for monetary shocks in a proxy-SVAR framework (Stock and Watson, 2012; Mertens and Ravn, 2013). Using the model of Gertler and Karadi (2015) (1979m7-2012m6), the correlation between the monetary shock series corresponding to their baseline Figure 1 (p. 61 of Gertler and Karadi (2015)) and the λ-shock implied by the FF55 is 0.79.

[11]This includes technology, tax, government spending, investment shocks amongst others, as recently reviewed by Ramey (2016).

[12]For example, the delayed expansion of aggregate quantities in Figure 1 makes the shock clearly distinct from unanticipated technology shocks that would have an immediate impact on consumption and output, as studied for long by the Real Business Cycle (RBC) literature.

This ambiguity is an awkward outcome: after all, how can the λ-shock correlate so strongly with two, seemingly distinct structural disturbances? A possible explanation is that TFP news shocks and monetary policy shocks are highly correlated in the data. Appendix E.5 shows evidence for this which, to my knowledge, is not documented in the literature yet.[13]

Overall, my results so far can be interpreted as supportive of monetary policy surprises driving the cross-section of average returns (Ozdagli and Velikov, 2016). Alternatively, a negative reading is that monetary shocks are not well identified: "In the absence of an empirically useful dynamic monetary theory, at least we can require the impulse-response functions to conform to qualitative theory such as Friedman (1968). Most VARs do not conform to this standard. Prices may go down, real interest rates go up, and output may be permanently affected by an expansionary shock" (Cochrane, 1994, p. 300).

Further Discussion Two additional points are noteworthy about the analysis so far.

First, my method can separately control for the macroeconomic information set that I recover the estimated SDF innovations from, and for the aggregate risks that the given test assets proxy.[14] The λ-shock and the interest rate shock in Figure 1 are estimated on the same macroeconomic information set, allowing me to draw conclusions that Cholesky-identified interest rate shocks resemble SDF innovations. In contrast, by simply comparing SDF innovations implied by the 3-factor model of Fama and French (1993) (or any other empirical finance models of the SDF) to monetary shocks estimated by macroeconomists, it would be difficult to know the reason for any possible lack of comovement between those series. Identified monetary shocks may have little comovement with SDF innovations implied by the 3-factor model simply because this model does not span the space of the macroeconomy – needed as input for the monetary policy reaction function

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[13]My paper does not take a stand on the correct identification of monetary policy shocks or TFP news shocks, hence this issue is somewhat unrelated to my analysis. However, to help the macroeconomic interpretation of the λ-shock, Section E.5 of the Appendix comments on the possible drivers of this result, suggesting that this is a possible symptom of an identification problem in the macroeconomics literature.

[14]The separation of the space spanned by the VAR from the space of test assets allows me to control for the macroeconomic information set when changing the test assets. See Section B.1.2 of the Appendix for further discussion and Figure 7 for a pictorial illustration.

and for the identification of the non-systematic, surprise element of policy.[15]

Second, the price level response in Figure 1 is counter to how monetary policy in New Keynesian models tend to affect prices (Gali, 2008; Woodford, 2003). This is the well-known 'price puzzle' associated with Cholesky-orthogonalisation in VARs (Sims, 1992).[16] My paper does not debate the right identification of monetary policy; it only shows that monetary policy shocks, as typically identified by macroeconomists, resemble the orthogonal shock which best explains the cross-section of expected returns.[17]

3.2 The γ-Shock

I will now analyse the economic properties of the γ-shock. Recall that this is an orthogonalised shock in the VAR, engineered to explain time-variation in risk premia, i.e. the counterfactual time-series of the VAR induced by ε_t^γ (2.1) has the highest possible predictive power to forecast excess returns. Given their popularity as return predictors since at least Fama and French (1989), I will employ the last three variables in the VAR as predictors of the conditional mean of excess stock market returns: the federal funds rate (FFR), the default spread (DEF) and the term spread (TERM).

Forecasting Excess Returns Before analysing the dynamic effects of the γ-shock on the macroeconomy, I discuss the results of the predictive regression step. The results are summarised in Table 1, showing the estimation output from four sets of models:

$$r_{t+1}^H = a + \beta_1 FFR_t + \beta_2 DEF_t + \beta_3 TERM_t + \varepsilon_{t+1} \tag{3.1}$$

$$r_{t+1}^H = a + \beta_1 CAY_t + \varepsilon_{t+1} \tag{3.2}$$

$$r_{t+1}^H = a + \beta_1 \widehat{FFR}_t + \beta_2 \widehat{DEF}_t + \beta_3 \widehat{TERM}_t + \varepsilon_{t+1} \tag{3.3}$$

$$r_{t+1}^H = a + \beta_1 \widetilde{FFR}_t + \beta_2 \widetilde{DEF}_t + \beta_3 \widetilde{TERM}_t + \varepsilon_{t+1}, \tag{3.4}$$

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[15]As discussed in Appendix E.4, the pricing performance of the λ-shock implied by my baseline VAR is approximately on par with that of the 3-factor model.

[16]See Ramey (2016) for a recent discussion.

[17]Nevertheless, I further investigate the issues related to the price response in Section D.2 of the Appendix, by re-estimating the λ-shock in the context of Uhlig (2005).

where^and˜denote the counterfactual time-series of predictors that are generated by the γ-shock (ε_t^γ in 2.1) and all other shocks ($\varepsilon_t^\lambda, \varepsilon_{3,t}, \ldots, \varepsilon_{n,t}$ in 2.1), respectively. Table 1 reports the regression results for different horizons ranging from one quarter ahead up to two years ahead ($H = 1, 2, \ldots 8$). Panel A shows the results using the actual VAR variables as predictors (3.1). In Panel B, the CAY variable of Lettau and Ludvigson (2001a,b) is used as predictor (3.2). Panel C reports the results using the three counterfactual VAR variables induced by the γ-shock (3.3).[18] Panel D reports the results using the counterfactual variables induced by all other shocks that are orthogonal to the γ-shock (3.4).

Panel A and Panel B of Table 1 confirm the evidence (reviewed by Lettau and Ludvigson (2010)) on the superiority of CAY, as return predictor, over the short-term interest rate, the default spread and the term spread. For example, the last column of the table shows that CAY explains around 23% of two-year ahead excess returns. In contrast, the regression with the last three VAR variables only explains 11% of excess returns at the same horizon.

However, not all variation in the VAR variables are related to future recessions; therefore using all the variation in these reduced-form variables may not predict returns very well. Panel C of Table 1 confirms that using the counterfactual time-series, induced by the γ-shock, as predictors, substantially improves forecasting power, explaining around 40% of two-year ahead excess returns. At almost all horizons, the variation in the interest rate, default spread and the term spread, induced by the γ-shock, explains more than twice as much of future excess returns as CAY does. Panel D shows the results when all the remaining variation in the three macroeconomic variables (unexplained by the γ-shock) is used in predicting returns. As expected, this variation is not useful in predicting returns with all R^2 statistics being around zero at all forecast horizons.[19]

Appendix D.4 further illustrates the reduced-form nature of return predictors with the main point being: variations in predictors are driven by a range of macroeconomic

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[18]I construct the γ-shock by maximising the corresponding return forecasting power at 4-quarter horizon ($H = 4$).

[19]Similarly, one could analyse the macroeconomic shocks underlying time-varying bond premia (Ludvigson and Ng, 2009).

Table 1: Forecasting Excess Returns

	1Q	2Q	3Q	4Q	5Q	6Q	7Q	8Q
				Forecast Horizon H				
	Model A: Actual VAR Variables							
FFR	-0.22	-0.32	-0.27	-0.22	-0.07	0.20	0.49	0.74
	(-0.86)	(-0.76)	(-0.50)	(-0.36)	(-0.10)	(0.25)	(0.51)	(0.68)
DEF	1.53	3.46	3.92	4.15	3.80	3.44	2.75	1.83
	(0.77)	(1.18)	(1.13)	(1.10)	(0.92)	(0.72)	(0.45)	(0.25)
TERM	0.19	0.54	1.36	2.11	2.90	4.05	4.98	5.95
	(0.37)	(0.60)	(1.10)	(1.38)	(1.70)	(2.22)	(2.51)	(2.85)
	[0.01]	**[0.02]**	**[0.04]**	**[0.06]**	**[0.07]**	**[0.08]**	**[0.09]**	**[0.11]**
	Model B: CAY							
CAY	0.55	1.12	1.77	2.52	3.17	4.00	4.92	5.77
	(2.18)	(2.25)	(2.33)	(2.46)	(2.59)	(3.00)	(3.58)	(4.25)
	[0.01]	**[0.03]**	**[0.05]**	**[0.09]**	**[0.11]**	**[0.14]**	**[0.19]**	**[0.23]**
	Model C: Counterfactual VAR Variables Induced by the γ-Shock							
FFR	0.59	1.14	1.66	2.15	2.61	3.15	3.66	4.20
	(1.21)	(1.14)	(1.19)	(1.25)	(1.32)	(1.44)	(1.52)	(1.66)
DEF	-9.14	-16.87	-23.42	-29.06	-33.96	-38.78	-43.34	-48.31
	(-2.01)	(-1.73)	(-1.78)	(-1.86)	(-1.92)	(-2.04)	(-2.15)	(-2.33)
TERM	3.22	6.47	9.63	12.66	15.46	18.41	21.07	23.65
	(2.53)	(2.42)	(2.42)	(2.46)	(2.53)	(2.63)	(2.73)	(2.93)
	[0.04]	**[0.08]**	**[0.14]**	**[0.20]**	**[0.25]**	**[0.30]**	**[0.34]**	**[0.39]**
	Model D: Counterfactual VAR Variables Induced by All Other Shocks							
FFR	-0.16	-0.21	-0.13	-0.06	0.10	0.36	0.62	0.84
	(-0.64)	(-0.47)	(-0.21)	(-0.08)	(0.11)	(0.33)	(0.47)	(0.55)
DEF	1.76	3.67	3.80	3.49	2.52	1.63	0.53	-0.64
	(0.88)	(1.20)	(0.93)	(0.73)	(0.48)	(0.28)	(0.07)	(-0.07)
TERM	-0.18	-0.43	-0.27	-0.23	-0.16	0.25	0.51	0.97
	(-0.30)	(-0.39)	(-0.17)	(-0.12)	(-0.07)	(0.10)	(0.18)	(0.31)
	[-0.01]	**[-0.00]**	**[-0.00]**	**[-0.01]**	**[-0.01]**	**[-0.01]**	**[-0.01]**	**[-0.00]**

Notes: The table reports results from regressions of excess returns on lagged variables. H denotes the return horizon in quarters. The dependent variable is the sum of H log excess returns on the CRSP based S&P Composite Index. The regressors are one-period lagged values of actual time-series of the federal funds rate (FFR), the term-spread (TERM) and the default spread (DEF) in Model A, the CAY measure of Lettau and Ludvigson (2001a,b) in Model B, and the counterfactual time-series of FFR, TERM and DEF (induced by the γ-shock) from a six-variable VAR(2) estimated over 1963Q3-2015Q3. The γ-shock is constructed so that the corresponding forecast power at the four-quarter horizon is maximised. For each of the three regressions, the table reports the OLS estimates of the regressors, the t-statistics using the Hansen and Hodrick (1980) correction (as implemented in Cochrane (2011)) are in parentheses, and adjusted R^2 statistics are in the bolded square brackets. Both the CAY measure and the counterfactual predictors are treated as known variables.

forces, and not all these forces change the conditional mean of excess returns.

The Aggregate Effects of the γ-Shock and Relation to the Macroeconomic Literature   To analyse the macroeconomic properties of the γ-shock, I compute impulse responses. Recall that, to the extent that time-variation in expected returns is linked to

economic booms and busts (Lettau and Ludvigson, 2010; Cochrane, 2011), the γ-shock can be thought of as the stochastic driver of recessions in the VAR. Figure 2a shows the impulse responses for the γ-shock along with the responses for the λ-shock. Both shocks are set to be contractionary. The behaviour of the γ-shock is distinct: it causes a sharp drop in consumption and output, and generates a positive comovement among quantities and short-term interest rate. The sharp consumption response is consistent with the recent evidence from the empirical asset pricing literature on the relevance of high-frequency movements in consumption in predicting future returns (Ortu, Tamoni, and Tebaldi, 2013; Moller and Rangvid, 2015). The fall in the short-term rate is indicative of the monetary policy authority endogenously responding to the recessionary γ-shock by loosening policy (Taylor, 1993). Moreover, the default spread and the term spread immediately jump up in response to a γ-shock, whereas the λ-shock causes a delayed increase in both quantity variables and in the default spread, and generates a fall (rather than a rise) in the term spread. Overall, the dynamics generated by the γ-shock resemble features of recent US recessions.

How do these fluctuations induced by the γ-shock relate to macroeconomic explanations of the business cycle? This literature has long worked on structural models to explain the type of comovements, induced by the γ-shock (Figure 2a). The seminal paper by Smets and Wouters (2007) built a New Keynesian general equilibrium model which was among the first to explain these empirical features of the data. In this model, a large fraction of short-term fluctuations is driven by demand-type shocks including disturbances ("preference shocks") that directly distort the representative household's marginal utility (analogous to Y_{t+1} in equation 4.4).[20]

Recent macroeconomic papers such as Christiano, Motto, and Rostagno (2014) combined standard New Keynesian features with a model of financial intermediation, and used financial as well as macroeconomic data for structural estimation. While the model is linearised (thereby absent of time-varying risk premia), the driving force is exogenously

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[20]The role of demand-type preference shocks (i.e. innovations in the state variable Y_{t+1} in equation 4.4) in driving the business cycle is also important in other New Keynesian models (without a financial sector and financial shocks) such as Christiano, Eichenbaum, and Evans (2005).

Figure 2: The γ-Shock

(a) Impulse Responses
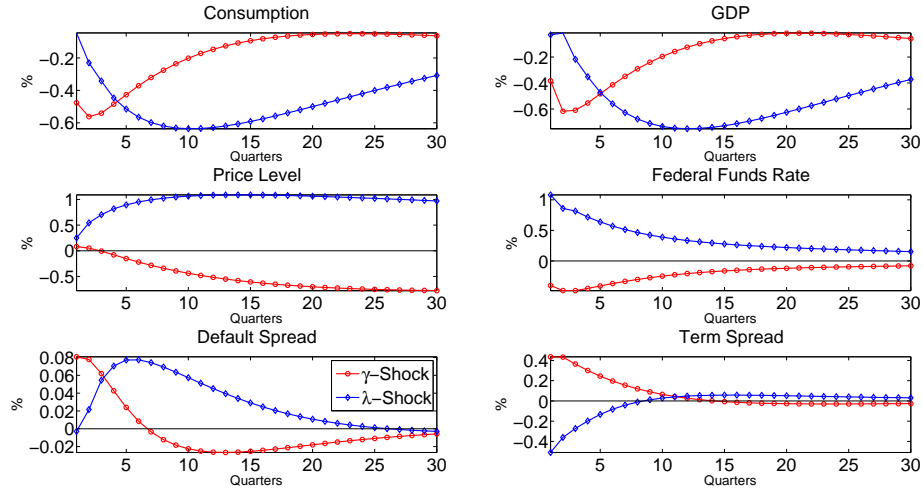
(b) γ-Shocks and Risk-shocks of Christiano, Motto, and Rostagno (2014)
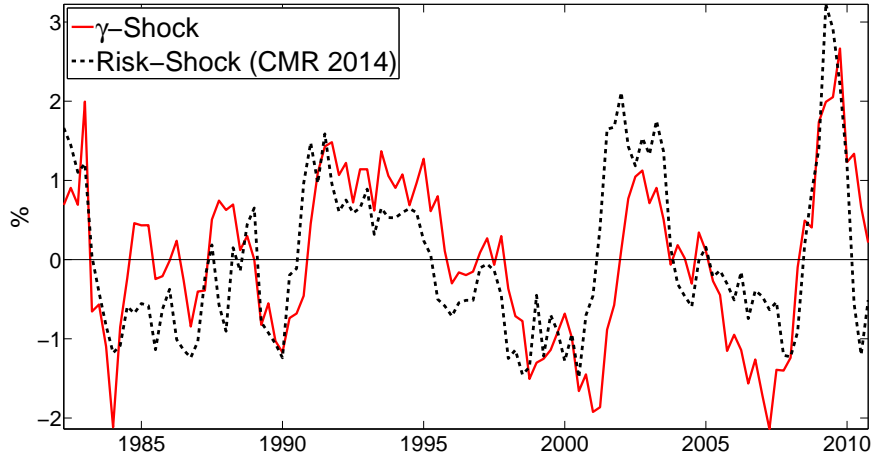
Notes: In Panel A, the vertical axes are %-deviations from steady-state, and the horizontal axes are in quarters. The VAR(2) is estimated on the sample 1963Q3-2015Q3. I use the FF55 to construct the λ-shock, and use the CRSP based aggregate SP500 stock market return (over the corresponding T-bill rate) to construct the γ-shock.
Panel B shows the four-quarter moving average of the risk shock of CMR (2014) and the (inverse of the) γ-shock, estimated from my baseline six-variable VAR over 1963Q2-2015Q3, using the CRPS based aggregate SP500 stock market return (over the corresponding T-bill rate). The correlation coefficient is 0.67.

fluctuating uncertainty related to the cross-section of idiosyncratic production risk, referred to as "risk shocks". Their results suggest that risk shocks, explaining up to 60% of US business cycle fluctuations, are the primitive macroeconomic force that is proxied by preference shocks in models without a financial sector.

Given that risk shocks explain most recent US recessions, I compare the time-series of risk shocks of Christiano, Motto, and Rostagno (2014) to the estimated time-series of

the γ-shock from my VAR model. Figure 2b shows the similarity of the two time-series, despite these two models are estimated using different information sets and very different methodologies. Both shocks made a sharp contribution to the recessions in the early 1990s, early 2000s and in the Great Recession. These results suggest that risk shocks are not only important in contributing to macroeconomic fluctuations but also drive time-variation in aggregate risk premia. More generally, these results highlight that combining predictive regressions (Campbell and Shiller 1988; Fama and French 1989; Goyal and Welch 2008; Pastor and Stambaugh 2009; Lettau and Ludvigson 2010; Cochrane 2011) with structural VARs (Sims, 1980) can be useful for business cycle analysis.

The Orthogonality of the γ-Shock to the λ-Shock So far, I have analysed the λ-shock and the γ-shock separately, without making assumptions about the covariance structure of these two shocks. The corresponding IRFs (Figure 2) suggest that they capture different macroeconomic forces. To formally check the possible orthogonality of these two shocks with respect to one another, I compare the IRFs obtained by implementing the orthogonalisation schemes separately to the IRFs obtained by implementing the orthogonalisation schemes jointly. Figure 13 in the Appendix shows that both sets of IRFs are virtually identical, confirming that the two shocks can be regarded as orthogonal to each other.

This is an important empirical result because it shows that the macroeconomic shocks that determine the *level* of risk premia are distinct from the shocks that determine *time-variation* in risk premia. This result is consistent with recent evidence from the asset pricing literature that used reduced-form empirical models to decompose consumption dynamics into high-frequency and low-frequency components and study their relations with risk premia (Ortu, Tamoni, and Tebaldi, 2013).[21]

Moreover, the orthogonality of the λ-shock and the γ-shock enables the use of structural VAR decompositions, which can help quantify the historical contribution of these shocks to business cycle fluctuations.

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[21]Ortu, Tamoni, and Tebaldi (2013) finds that the two major (high- and low-frequency) components are close to being orthogonal, similar to the orthogonality of the λ-shock and the γ-shock that I find.

3.3 Explaining the Business Cycle

To assess the contribution of the λ-shock and the γ-shock to US business cycles, I compute FEV decompositions over different forecast horizons. Table 2 shows that the λ-shock explains less than 1% of consumption fluctuations over the one-quarter horizon, but the contribution rapidly increases with the horizon and the shock explains around 60-65% of fluctuations over the longer (4-8 years) horizon. The λ-shock explains more than 70% of interest rate fluctuations over the 1-4 quarter horizon and the contribution falls only little at longer-frequency.

Table 2: The Contribution of the λ-shock and γ-shock to Business Cycles: FEV Decomposition

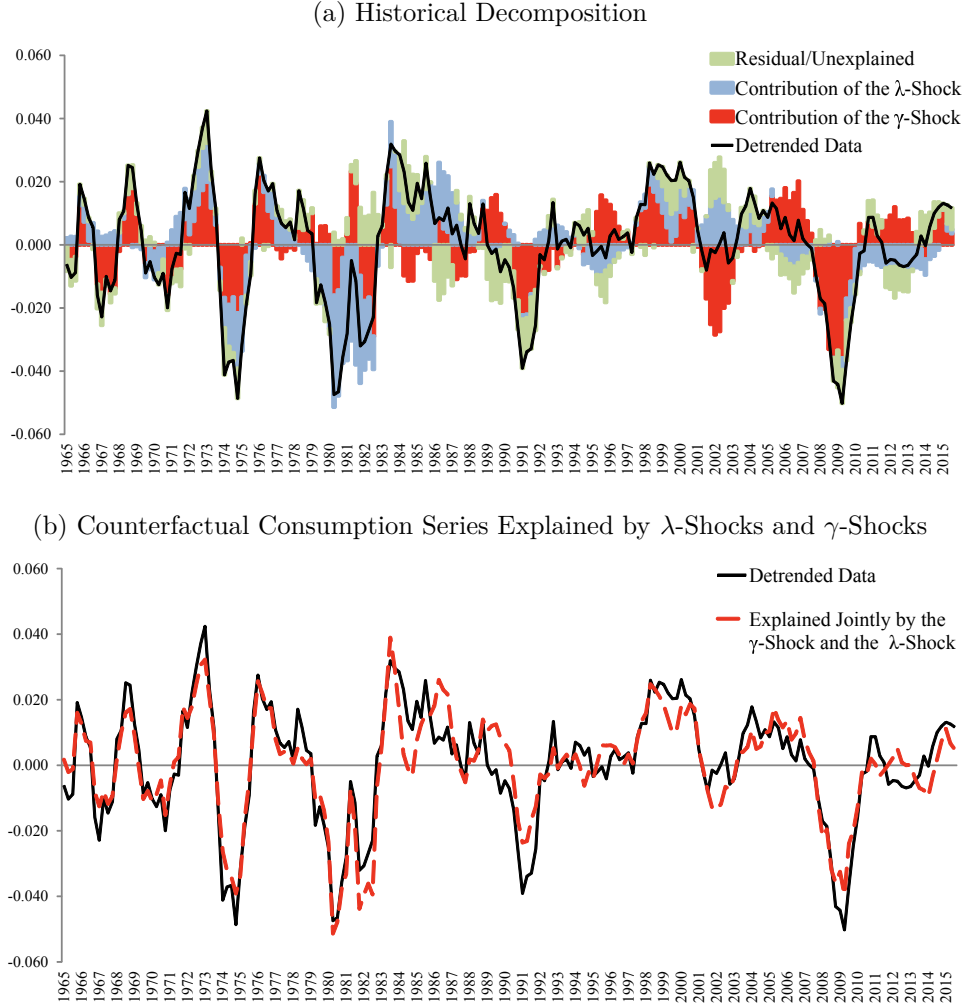|	Consumption				Federal Funds Rate			
	λ-Shock	γ-Shock	$\lambda\&\gamma$	Unexpl.	λ-Shock	γ-Shock	$\lambda\&\gamma$	Unexpl.
1Q	0.3	67.4	67.7	32.3	77.6	11.6	89.2	10.8
2Q	6.0	66.4	72.3	27.7	73.4	16.1	89.6	10.4
3Q	11.9	61.6	73.5	26.5	71.0	18.3	89.4	10.6
4Q	18.7	55.6	74.3	25.7	69.4	19.6	89.0	11.0
8Q	40.8	36.8	77.6	22.4	66.4	21.6	88.0	12.0
16Q	60.5	21.2	81.7	18.3	64.9	22.0	86.9	13.1
32Q	66.2	14.8	81.0	19.0	64.0	21.6	85.7	14.3

Notes: The table shows the % fraction of the total forecast error variance that is explained by the λ-shock and the γ-shock over different forecast horizons. The FF55 portfolios are used as test portfolios for the VAR model.

In contrast, the γ-shock explains a large fraction of short-term fluctuations in consumption, and only moderately contributes to the FEV in the short-term interest rate. Overall, the joint contribution of the λ-shock and the γ-shock to consumption and interest rate fluctuations amounts to 70-90% at business cycle frequency.

Another way to assess the importance of these two shocks to business cycle fluctuations is to compute historical decompositions. This is shown in Figure 3. The black solid line in Panel A shows year-on-year consumption growth after removing the deterministic trend implied by the VAR. The contribution of the λ-shocks and the γ-shocks is represented by the blue and red bars, respectively; the green bars show the contribution of the remaining residual disturbances. The results show that the λ-shock contributed largely to the recession in the early 1980s, and to a smaller extent to the recession in 1974-75. All other downturns including the Great Recession can be explained by the γ-shock.

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Moreover, consistent with the FEV results, Panel B of Figure 3 shows that most historical fluctuations in consumption growth, over the past 50 years, can be jointly explained by the λ-shock and the γ-shock.

Figure 3: Decomposing Year-on-year US Consumption Growth: the Role of λ-Shocks and γ-Shocks

(a) Historical Decomposition

(b) Counterfactual Consumption Series Explained by λ-Shocks and γ-Shocks

Notes: The figure shows the results from the historical decomposition implied by the baseline six-variable VAR(2) estimated on the sample 1963Q3-2015Q3. I use the FF55 to construct the λ-shock, and the CRSP based SP500 return (in excess of the corresponding T-bill rate) to construct the γ-shock. The contribution of the other four unrestricted orthogonalised shocks is depicted as residuals. The deterministic trend component, implied by the VAR, is removed from the time-series.

The fact that merely two orthogonal macroeconomic forces can explain the bulk of aggregate consumption fluctuations is a notable result. However, in quantitative models of the business cycle it is not atypical to have two dominant shocks explaining such a large fraction of fluctuations. This is true for more atheoretical, VAR models such as Blanchard and Quah (1989) or highly structural models such as Christiano, Motto, and Rostagno (2014). What is more important about my results is that these two dominant shocks

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are constructed based exclusively on asset price information and not on macroeconomic assumptions.

At a deeper level, these results highlight the importance of asset pricing explorations for macroeconomics (Cochrane and Hansen, 1992). Modern macroeconomic models have mainly focused on understanding the dynamics of aggregate quantities, and information on the level of and variation in expected returns has been often ignored. Since Mehra and Prescott (1985), these models have struggled to explain the level of and variation in risk premia.[22] Via its more atheoretical nature, the macroeconometric framework proposed in this paper is able to cut this Gordian knot. It continues to capture the rich dynamics of macroeconomic time-series (Sims, 1980) while connecting it with the study of risk premia.

4   Extensions and Interpretations

4.1   The Advantage of Working "Backwards": A Monte Carlo Exercise

A natural question relates to the use of established external shock series (identified by the macroeconomics literature) to price the cross-section, instead of using the cross-section of asset prices to back out the λ-shock and comparing it to external macroeconomic shocks. The reverse direction, taken in this paper, is motivated by the fact that identification of macroeconomic shocks may suffer from overly restrictive identifying assumptions and from mis-measurement of macroeconomic data. This has particularly relevant asset pricing implications, given that different ways of identifying the same macroeconomic shock can lead to different estimated time-series of the given structural shock (thereby leading to hugely different pricing performance) in spite of the fact that the given identification schemes may lead to similar impulse response functions, as discussed in the debate between Rudebusch (1998) and Sims (1998). These problems make it likely for the given macroeconomic shock to be rejected as a pricing factor, even though the shock may be truly correlated with SDF innovations.

This section presents results from a Monte-Carlo exercise to illustrate why using well-

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[22]As Cochrane (2011) explains: "The job is just hard. Macroeconomic models are technically complicated. Macroeconomic models with time-varying risk premia are even harder" (p. 1090).

known identified macroeconomic shocks to price the cross-section of returns may lead to the rejection of these shocks as valid pricing factors, even though these shocks may in fact be correlated with the true SDF innovations.

To highlight these measurement problems, I first take n test assets to construct the SDF, $x^\star$, from the corresponding payoff space (Chapter 4 of Cochrane (2005)).[23] As test assets, I use the FF55 for the sample period 1963Q3-2015Q3 as in my baseline analysis. I then define the distorted SDF, $\tilde{x}$, by introducing a noise term, ε_t:

$$\tilde{x} = x^\star + \varepsilon_t, \quad \varepsilon_t \sim N\left(0, \sigma^2\right) \tag{4.1}$$

where σ is the standard deviation of the measurement error ε_t. To assess the pricing performance of the distorted SDF, I first estimate n time series regressions, $R_{it} = \tilde{x}\beta_i + \epsilon_{it}$, $i = 1\ldots n$. Second , I estimate a cross-section regression, $\bar{R}_i = \tilde{\beta}_i \times \lambda + \alpha_i$, where $\bar{R}_i = \frac{1}{T}\sum_{t=1}^{T} R_{it}$, $\tilde{\beta}_i$ is the OLS estimate obtained in the first stage and α_i is a pricing error. The model's fit is then assessed using the following statistic (Burnside, 2011):

$$R^2 = 1 - \frac{\left(\bar{R} - \tilde{\beta}\tilde{\lambda}\right)'\left(\bar{R} - \tilde{\beta}\tilde{\lambda}\right)}{\left(\bar{R} - \widehat{R}\right)'\left(\bar{R} - \widehat{R}\right)}, \tag{4.2}$$

where $\widehat{R} = \frac{1}{n}\sum_{i=1}^{n}\bar{R}_i$ is the cross-sectional average of the mean returns in the data. Moreover, I compute the correlation between the true and distorted SDFs:

$$\rho^\star = corr\left(x^\star, \tilde{x}\right). \tag{4.3}$$

Then I explore how the cross-sectional fit 4.2 and the correlation coefficient 4.3 change as I add more noise to the SDF, i.e. I aim to estimate the derivatives $\partial R^2/\partial\sigma$ and $\partial\rho^\star/\partial\sigma$.

For the Monte-Carlo exercise, I use a grid $\sigma = [0 : 0.05 : 0.75]$ to control for the amount measurement error, and for each value of σ, I generate 5000 time-series of $\tilde{x}$, and compute the statistics using 4.2 and 4.3. Figure 4 shows the median values (solid lines)

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[23]Specifically, I follow Section 4.1. of Cochrane (2005) and construct the discount factor $x^\star$ from the payoff space using $x^\star = p'E\left(xx'\right)^{-1}x$, where x denotes the test assets with payoffs p.

Figure 4: Role of Mis-measurement in Macroeconomic Shocks: Results from a Monte-Carlo Exercise

Pricing Performance of Noisy SDFs, Implied by FF55
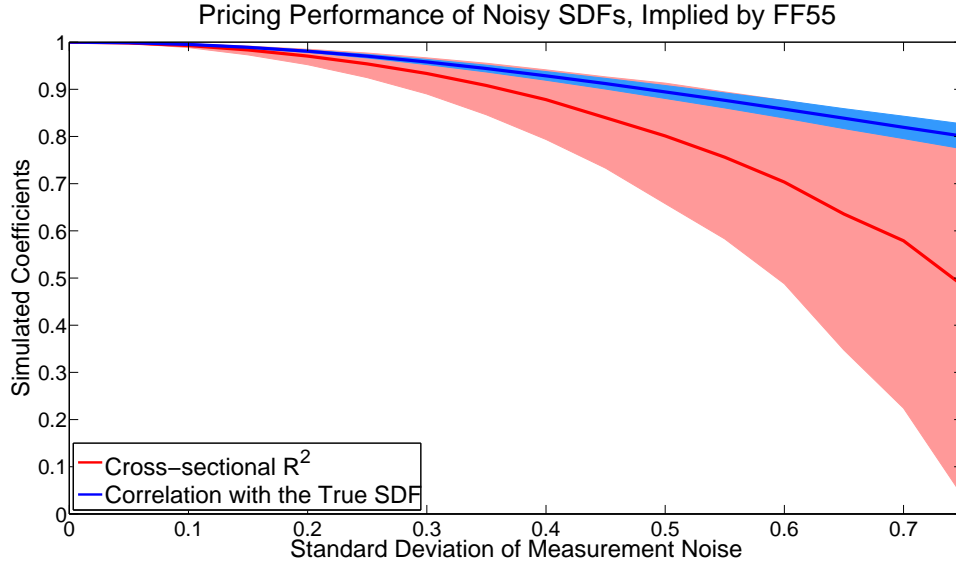
Notes: The Figure illustrates how adding noise to the SDF (constructed from the space of test assets, following 4.1. of Cochrane (2005)) changes the correlation with the true SDF (blue line) and the cross-sectional fit (red line). The cross-sectional fit is measured using the R^2 measure 4.2, and the correlation measure is based on 4.3. The shaded areas correspond to the 10-90% bands based on 5000 Monte-Carlo simulations of 4.1. The construction of the SDF is based on the FF55 portfolios, covering the sample period is 1963Q3-2015Q3.

of the statistics together with 10-90% simulation bands (shaded areas).

The results show that adding noise to the SDF deteriorates the pricing performance more quickly than it reduces the correlation between the noisy and true SDFs. Importantly, the uncertainty around the estimated R^2 increases much more rapidly than the uncertainty around the estimated $\rho^\star$. For example, for $\sigma = 0.75$, the correlation between distorted SDF and the true SDF is still above 80%, whereas the cross-sectional fit of the corresponding pricing model can result in close to zero explanatory power.

These results provide a justification for (i) why using noisy estimates of macroeconomic shocks (identified by the macroeconomics literature) directly in asset pricing tests may lead the rejection of these shocks as valid pricing factors, and (ii) why the reverse direction taken in this paper may be more successful in uncovering the empirical linkages between business cycle fluctuations and asset prices.

4.2 Macro-Finance Interpretations

One possible interpretation of the λ-shock and the γ-shock is via the generalisation of the SDF (M_{t+1}) implied by consumption-based asset pricing:

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} Y_{t+1}, \tag{4.4}$$

where C_t is consumption of the representative household, β is the subjective discount rate, σ is the risk aversion coefficient, and Y_{t+1} is a key state variable, directly related to recessions and to time-varying risk-bearing ability, as discussed in Cochrane (2017). Inspecting through the lens of this framework, the λ-shock can be thought of as innovations in the consumption growth process ($\frac{C_{t+1}}{C_t}$) which explain the level of expected returns; the γ-shock can be thought of as innovations in the recession-related state variable (Y_{t+1}) which explain time-variation in expected returns.

Numerous papers have provided theoretical explanations of the drivers of various specifications of the discount factor (4.4). Early business cycle models, aimed at explaining the level of expected returns, focused on technology shocks (Mehra and Prescott 1985; Jermann 1998) that could be thought of as reminiscent of the λ-shock. The finance literature has long studied objects that could be thought of as reminiscent of the γ-shock, including shocks to the volatility of the consumption process (Bansal and Yaron, 2004) or shocks to time-varying cross-sectional variance of individual consumption growth (Constantinides and Duffie, 1996), among other theories. In macroeconomics, γ-shocks often correspond to "preference shocks", driving short-term business cycle fluctuations in estimated New Keynesian models (Smets and Wouters, 2007).

However, the simplified framework of (4.4) is problematic for at least two reasons. First, my empirical results show that the γ-shock explains most of the short-term fluctuations in consumptions, where the λ-shock drives long-term fluctuations. A more realistic utility function is required to facilitate such horizon effects. Second, the link between the γ-shock and the SDF needs to be specified more rigorously. For example, the simplified interpretation of the γ-shock as shocks to stochastic volatility is incorrect in the light of

my finding that the γ-shock is orthogonal to the λ-shock, and in light of recent results on stochastic volatility pricing the cross-section of returns (Campbell, Giglio, Polk, and Turley 2018). An interesting extension of my paper is to formalise utility functions that can give rise to orthogonal γ- and λ-shocks.

Nevertheless, a key advantage of my empirical framework lies in its reverse engineering nature: it provides a simple and agnostic way to estimate the stochastic drivers of the discount factor, and their effects on the business cycle, without having to specify the functional form of the household's utility function (4.4) and the corresponding structural model.

4.3 Adding More Variables to the VAR

One may consider changing the state-variables in the VAR which; this can serve two purposes. First, given the proliferation of asset pricing factors in the finance literature (Harvey, Liu, and Zhu, 2016), the proposed VAR framework could model the joint dynamics of any reduced-form variables that individually have been found to price the cross-section of returns, and to link the common stochastic driver of these variables to a single or multiple orthogonal shocks. Second, one can explore how the realisation of aggregate risks proxied by the given test assets may affect different parts of the macroeconomy.

One can easily add more variables to the VAR to improve the return predicting power of the γ-shock or to improve the cross-sectional pricing performance of the λ-shock.[24] This is particularly useful, given that most pricing factors (316 of them listed in Harvey, Liu, and Zhu (2016)) are reduced-form objects and often exhibit high correlations with one another. For example, consumption innovations and output innovations extracted from individual AR(1) models have around 66% correlation, term spread innovation and federal funds rate innovations have around -82% correlation, the intermediary capital risk factor constructed by He, Kelly, and Manela (2017) and excess returns on the market

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[24]Increasing the size of the VAR introduces only computational challenges. For example, in an 8-variable (10-variable) VAR one needs to find 420 (4725) angles to span the 8-dimensional space of rotations. See section C.1 of the Appendix.

(their second pricing factor) have a 78% correlation.[25]

To illustrate these points, I will add to the VAR the aggregate capital ratio of the financial intermediary sector (constructed by He, Kelly, and Manela (2017)). I then re-estimate the VAR on a shorter sample, 1970Q1-2015Q3 (dictated by the availability of this time-series), and calculate the dynamic effects of the λ-shock and the γ-shock on the intermediary capital ratio. Note that this procedure could be applied to any other reduced-form pricing variable of interest. Figure 5 summarises the results. Panel a shows the IRFs for a one standard deviation contractionary innovation in both shocks. In response to both shocks, the intermediary capital ratio drops immediately by about 0.3% and then gradually returns to steady-state after about five years. Note that the rest of the variables in the VAR exhibit very similar dynamics to the baseline (Figure 2). While GDP is replaced with the intermediary capital ratio in the model and this VAR is estimated on a different sample, the time-series of the λ-shock and the γ-shock have a high (around 80%) correlation across the two VAR models.

Panel b of Figure 5 shows the FEV decomposition of the intermediary capital ratio along with that of aggregate consumption. The results suggest that both orthogonalised shocks are important in driving fluctuations in the capital ratio. For example, at one-year horizon, the λ-shock and the γ-shock explain about 35% and 31% of the forecast error variance in the capital ratio, respectively. The decomposition of aggregate consumption continues to be similar to my baseline VAR (Table 2). It is to note that a considerable fraction of capital ratio fluctuations is left unexplained by the λ-shock and the γ-shock. Future work could enrich this simple VAR to increase explanatory power.

This exercise highlights that unexpected changes in the balance sheet health of financial intermediaries cannot be interpreted as purely exogenous events. While the driving force in macroeconomic models with financial intermediaries (Gertler and Kiyotaki, 2010; He and Krishnamurthy, 2014; Gertler, Kiyotaki, and Prestipino, 2020) is often related to exogenous movements in the capital stock ("capital quality shock"), my results show

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[25]These number are based on estimates of individual AR(1) models on consumption, GDP, the term spread and the Federal Funds rate covering the period 1963Q3-2008Q3. The correlation between the intermediary capital risk factor and market excess returns are for 1970Q1-2012Q4 as in He, Kelly, and Manela (2017).

Figure 5: Financial Intermediary Capital Dynamics

(a) Impulse Responses to a λ-Shock and to a γ-Shock
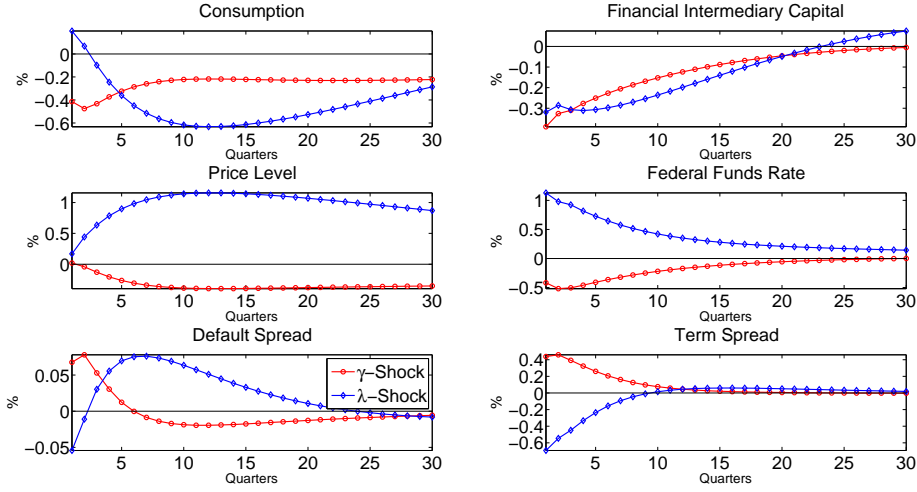
(b) The Contribution of the λ-shock and γ-shock to Capital Ratio Fluctuations

	Intermediary Capital Ratio				Consumption			
	λ-Shock	γ-Shock	λ&γ	Unexpl.	λ-Shock	γ-Shock	λ&γ	Unexpl.
1Q	31.4	34.9	66.3	33.7	4.0	55.5	59.5	40.5
2Q	33.0	34.6	67.6	32.4	2.1	59.7	61.8	38.2
3Q	33.9	33.0	66.9	33.1	5.6	58.3	63.9	36.1
4Q	34.7	31.2	65.9	34.1	12.5	53.7	66.3	33.7
8Q	35.6	26.1	61.7	38.3	41.9	34.6	76.5	23.5
16Q	34.1	21.9	56.0	44.0	61.1	21.3	82.4	17.6
32Q	32.4	20.2	52.6	47.4	60.2	19.5	79.7	20.3

Notes: In panel a, the vertical axes are %-deviations from steady-state, and the horizontal axes are in quarters. The VAR(2) is estimated on a subsample 1970Q1-2015Q3 given the availability of the intermediary capital ratio series of He, Kelly, and Manela (2017) that starts in 1970. I use the FF55 to construct the λ-shock, and use the CRSP based aggregate SP500 stock market return (over the corresponding T-bill rate) to construct the γ-shock. The table in panel b shows the % fraction of the total forecast error variance that is explained by the λ-shock and the γ-shock over different forecast horizons.

that a large fraction of the unforecastable component in the capital ratio can be explained by at least two orthogonalised macroeconomic shocks that have very different effects on business cycle fluctuations. This highlights that by purely focusing on the reduced-form unforecastable component in the capital ratio, one cannot accurately detect the nature of the macroeconomic force responsible for the observed fluctuations in intermediary balance sheets and their implications for financial markets and the wider economy.

4.4 Further Robustness Checks

I carry out a number of additional robustness checks. First, I check how the FEV decomposition changes when increasing the lag length of the VAR from two to three and four. Table 5 in the Appendix confirms robustness to these perturbations. I also recompute the historical decomposition implied by a VAR(4) model (Figure 9) and find little difference in the model's interpretation of history. Moreover, I explore the contribution of the λ-shock and the γ-shock to variation in aggregate consumption at a lower frequency. In addition to decomposing year-on-year consumption growth (Figure 3), I also compute the historical decomposition of the deviation of the level of aggregate consumption from the trend implied by my baseline VAR model. Figure 11 in Appendix D.3 shows that, consistent with the FEV results (Table 2), the λ-shock explains more of the low-frequency variation in consumption, including the persistent expansion above trend in the run-up to the Great Recession.

I present numerous additional robustness checks and extensions in Appendix E. These include (i) proposing another method to further explore the drivers of recessions and time-varying risk premia (E.2), (ii) estimating the λ-shock implied by other equity as well as government bond portfolios (E.3); (iii) documenting the asset pricing performance of my baseline model (E.4); (iv) investigating why λ-shocks resemble both monetary and TFP news shocks (E.5); (v) applying the method to UK data (E.6).

5 Conclusion

The contribution of this paper is to propose a simple empirical framework to combine structural VARs with asset price information on the cross-sectional and time-series variation in expected returns, in order to study the joint stochastic drivers of business cycle fluctuations and risk premia. The constructed VAR innovations can be thought of shocks to two key aspects of premia: one explaining the level of expected returns, the other one driving time-variation in expected returns.

Applying the method to standard macroeconomic and asset price data highlights the

overlap between empirical finance models (Fama and French, 1993) and structural shocks identified by the macroeconometrics literature (Sims, 1980). Applying the method to data from other markets and countries would provide interesting avenues for future research.

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Appendix for Online Publication

A   Data

The VAR includes quarterly data on output, consumption, price level, the short-term interest rate, the default spread, and the term spread. Consumption is total personal consumption expenditure from Greenwald, Lettau, and Ludvigson (2015). Output is seasonally adjusted real GDP (FRED code: GDPC1). Price level is the consumer price index for all urban consumers (FRED code: CPIAUCSL). Interest rate is the Federal Funds Rate (code: FEDFUNDS). Default spread is the difference between the AAA (FRED code: AAA) and BAA (FRED code: BAA) corporate bond yields. The term spread is the difference between the ten-year Treasury and T-bill rates as in Goyal and Welch (2008). The full sample period is 1963Q3-2015Q3, but I will also experiment with a shorter sample (1963Q3-2008Q3) that excludes the period of zero lower bound on the nominal interest rates. This brings my analysis closer to the information set that the monetary policy literature typically used to estimate policy shocks.

For test assets, I combine the standard 25 Size-B/M portfolios of Fama and French (1993) with the 30 industry portfolios. Returns are quarterly, in excess of the T-bill rate. Augmenting the FF25 with the 30 industry portfolios follows prescription 1 (pp. 182) of Lewellen, Nagel, and Shanken (2010), thereby relaxing the tight factor structure of Size-B/M. For robustness, I will also use the 25 portfolios double sorted on size-profitability and size-investment, as these portfolios feature prominently in recent empirical asset pricing studies (Fama and French, 2015, 2016).

As return predictors, I use the short-term interest rate (Fama and Schwert, 1977), the term spread and the default spread (Fama and French, 1989); these macroeconomic variables featured prominently in predictive regressions. To construct the γ-shock, I use CRSP based S&P 500 return over the corresponding T-bill rate (from Goyal and Welch (2008)) as the regressand in the return predictability step ins B.16. I will compare the return predictability performance (of the counterfactual time-series of the interest rate,

the term spread and the default spread, implied by the γ-shock) to the model which uses the CAY variable proposed by Lettau and Ludvigson (2001a,b).[26]

The VAR is estimated in levels as most monetary policy VARs (Sims 1980; Christiano, Eichenbaum, and Evans 1999; Uhlig 2005; Gertler and Karadi 2015), thereby I avoid making any transformation to detrend the data (Sims, Stock, and Watson, 1990). My results change very little when I include a linear trend in the VAR. The model has two lags, based on the SIC. I will also cross-check that my results are robust to changing the lag length in the VAR.

B   Details on the Econometric Framework

B.1   The λ-Shock

B.1.1   The VAR Framework

Assume that the macroeconomy evolves according to a *k*-variable reduced-form VAR:

$$X_t = c + \mathbb{A}_1 X_{t-1} + \cdots + \mathbb{A}_p X_{t-p} + \eta_t, \qquad \eta_t \sim N\left(0, \Omega\right), \tag{B.1}$$

where the reduced form innovations, η_t, are related to the structural shocks e_t by an invertible matrix $\boldsymbol{B}$, $\eta_t = \boldsymbol{B} e_t$. Following Uhlig (2005), I refer to the columns of $\boldsymbol{B}$ as *impulse vectors.*

Finance papers using VARs (Campbell, 1996; Petkova, 2006; Hansen, Heaton, and Li, 2008; Boons, 2016) often used Cholesky decomposition ($\boldsymbol{B} = chol\left(\Omega\right)$) to obtain orthonormalised shocks as pricing factors as estimates of sources of aggregate risks.[27] Building on these papers, I explore the whole space of possible orthonormalisations to approximate the SDF from linear combinations of residuals η_t. Specifically, I select a

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[26]This variable measures deviations from a cointegrating relation for log consumption (C), log asset wealth (A) and log labour income (Y), and has proved to be successful in predicting excess stock market returns.

[27]Originally, Sims (1980) applied Cholesky decomposition to obtain a triangular structure in the spirit of Wold (1954). A plethora of new techniques have been proposed by the macroeconometrics literature to provide full or partial identification of $\boldsymbol{B}$, involving both point and set identification of the elements of $\boldsymbol{B}$. See Kilian and Lutkepohl (2016), Ramey (2016), and Ludvigson, Ma, and Ng (2017) for a recent review of the literature.

single or multiple impulse vectors such that the shocks associated with all other impulse vectors are *orthogonal* to the SDF. To implement the method, I first estimate the reduced-form VAR (B.1) and apply Cholesky decomposition to the estimated variance-covariance matrix $\hat{\boldsymbol{B}} = chol\left(\hat{\Omega}\right)$. One can take any orthonormal matrix Q to obtain a new structural impact matrix $\hat{\boldsymbol{B}}^{\star} = \hat{\boldsymbol{B}}Q$, thereby obtaining a new set of orthogonal shocks, which conforms to $\hat{\Omega}$, i.e. $\hat{\Omega} = \hat{\boldsymbol{B}}^{\star}\left(\hat{\boldsymbol{B}}^{\star}\right)' = \hat{\boldsymbol{B}}Q\left(\hat{\boldsymbol{B}}Q\right)' = \hat{\boldsymbol{B}}\hat{\boldsymbol{B}}'$. One could think of Q as a rotation matrix with corresponding Euler-angle(s) θ. The next proposition for the two-dimensional $\mathbb{R}^2$ case highlights how to find the rotation which generates the λ-shock.

Proposition 1 *Given the linear combination:* $m = \lambda_1 f_1 + \lambda_2 f_2$, *where* $\lambda_1, \lambda_2 \in \mathbb{R}$, $m, f_1, f_2 \in \mathbb{R}^2$, $\|f_1\| = \|f_2\| = 1$ *and* $\langle f_1, f_2 \rangle = 0$, $\exists$ *matrix* $Q = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ *such that* $m = \lambda_1^{\star} f_1^{\star} + \lambda_2^{\star} f_2^{\star}$, *where* $\lambda_1^{\star} = \|m\| \neq 0$, $\lambda_2^{\star} = 0$ *and* $f_i^{\star} = Qf_i$ *for* $i = 1, 2$.

Proof of Proposition 1. It suffices to find an angle $\theta^{\star}$ and associated rotation $r_{\theta^{\star}}$ such that m will be a scaled multiple of any one of the rotated vectors denoted by $f_1^{\star}$. If $\theta^{\star}$ exists then $\lambda_2^{\star} = 0$ because $f_1 \perp f_2$ and $r_{\theta^{\star}}$ is an orthonormal transformation. The angle $\theta^{\star} = \arctan\left(\frac{\lambda_2}{\lambda_1}\right)$ satisfies $f_1^{\star} = r_{\theta^{\star}} f_1$ so that $m = \lambda_1^{\star} f_1^{\star} + \lambda_2^{\star} f_2^{\star}$ with the associated scalars $\lambda_1^{\star} = \frac{\|m\|}{\|f_1^{\star}\|} = \|m\|$ and $\lambda_2^{\star} = 0$. $\blacksquare$

Figure 6 provides a graphical illustration via an example, whereby a linear model $m = 2f_1 + f_2$ with $f_1 \perp f_2$ is transformed to $m = \|m\| f_1^{\star} + 0 \cdot f_2^{\star}$ with $Q = \begin{bmatrix} \cos\theta^{\star} & -\sin\theta^{\star} \\ \sin\theta^{\star} & \cos\theta^{\star} \end{bmatrix}$, $\theta^{\star} = arctan\left(\frac{1}{2}\right)$, $\|f_1\| = \|f_2\| = \|f_1^{\star}\| = \|f_2^{\star}\| = 1$ and $\|m\| = \sqrt{5}$.

While the proposition is a trivial piece of linear algebra, it has important implications for using orthonormalised shocks from VARs as pricing factors in linear pricing models. Given that β-pricing models are equivalent to linear models of the SDF[28], finding the

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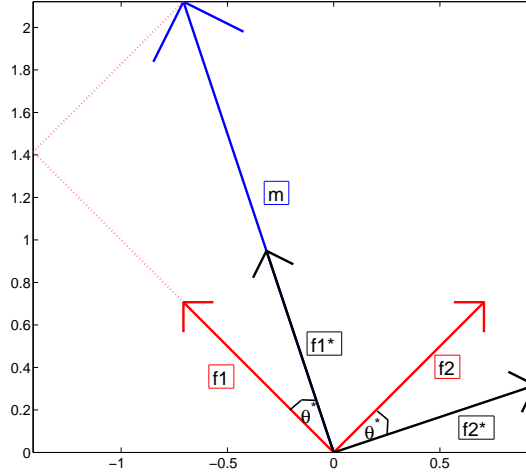[28] **Theorem 2 (Cochrane 2005)** *Denoting the SDF, the pricing factor, the excess returns and the first- and second-stage regression coefficients from a linear pricing model by* m, f, R^e, β *and* λ, *respectively, and given the model*

$$m = 1 + [f - \mathbb{E}(f)]' b$$
$$0 = \mathbb{E}(mR^e), \tag{B.2}$$

one can find λ *such that*

$$\mathbb{E}(R^e) = \beta' \lambda, \tag{B.3}$$

Figure 6: Graphical Illustration of Constructing the λ-shock

Notes: the 2-dimensional coordinate system illustrates the space spanned by reduced-form VAR innovations. The space contains vector m, which is the best linear approximation of the SDF, according to some test assets. The red perpendicular arrows ($f_1 \perp f_2$) illustrate an arbitrary orthonormalisation of the reduced-form VAR innovations, e.g. Cholesky decomposition as in Campbell (1996), Petkova (2006), Boons (2016) amongst others. Rotation and orthonormalisation do not change the spanned space, thereby leaving the information set and m unchanged. Therefore there exists $\theta^\star$ such that $m = \|m\| f_1^\star$ and $m \perp f_2^\star$.

Euler-angle θ and the associated rotation in a VAR (of *any* dimension) that delivers an orthonormalised shock with the highest price of risk ($\sqrt{5}$ and $f_1^\star$ in Figure 6) when pricing given test assets is equivalent to finding the best linear approximation of the SDF that lies in the innovation space of the VAR. By construction, all other orthonormalised shocks in the VAR ($f_2^\star$ in Figure 6) will be orthogonal the implied SDF and demand zero risk premia. Importantly, one can apply structural VAR tools to the obtained Euler-angles to study the link between the shock and macroeconomic dynamics. Given the VAR model,

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where β are the multiple regression coefficients of excess returns R^e on the factors. Conversely, given λ in B.3, we can find b such that B.2 holds.

**Proof of Theorem 2.** See p. 106-107 of Cochrane (2005). ∎

Cochrane (2005) shows that λ and b are related $\lambda = -var(f) b$. This result simplifies greatly when working with pricing factors (such as orthonormalised VAR residuals) that have zero mean and unit variance. In this case, $\lambda = -b$ and $\mathbb{E}(f) = 0$. As a result of the linearity of the pricing model and the linearity of the VAR model, finding the orthonormalised shock in a VAR of any dimension that demands the highest price of risk (λ) when pricing a given portfolio of assets is equivalent to finding a single time series that is a linear combination of the reduced form innovations of the VAR which summarises all the information relevant to pricing the given portfolio. Another way of saying this is that the cross-sectional R^2-measure associated with a pricing model that includes all the reduced-form residuals from the VAR is the same as the R^2-measure associated with the one-factor model which uses the appropriately orthonormalised shock. This will be confirmed during the empirical application of the method (Panel A and B of Tables 6–9).

the rotation Q naturally depends on the test assets that induce the SDF. The following Subsection provides further illustration and highlights the geometric nature of the idea.

B.1.2 The Geometry of the λ-shock

To highlight the geometric nature of the orthonormalisations method and the interplay between the VAR model and the test assets, I illustrate the relevant mathematical background in a three-dimensional graph (Figure 7). There is an underlying probability space, and L_2 denotes the collection of random variables with finite variances defined on that space. L_2 is a Hilbert-space with the associated norm $\|p\| = (E(p^2))^{1/2}$ for $p \in L_2$. Let P denote the space of portfolio excess returns (zero-price payoffs) that is a closed linear subspace of L^2.[29] P is represented by the red plane in Figure 7. An admissible SDF is a random variable m in L^2 such that $0 = E(mp)$ for all $p \in P$. The set of all admissible SDFs denoted by M is represented by the black line perpendicular to the red plane.[30]

Figure 7: A Simplified Geometry of Constructing the λ-shock
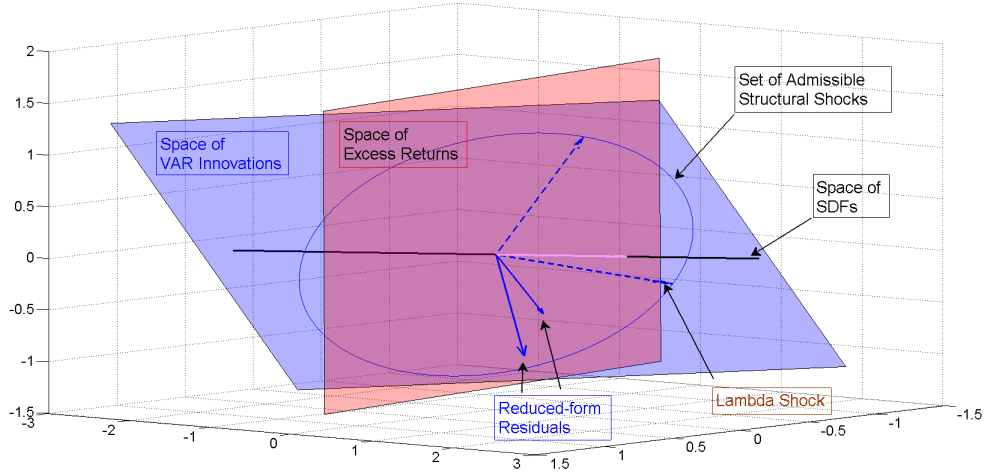
Let S denote the set of reduced-form VAR innovations (the blue solid arrows) and denote D the space spanned by these innovations. D is assumed to be a closed subspace of L_2, and it is represented by the blue plane in the Figure. The Gram-Schmidt orthogon-

---

[29]See Hansen and Jagannathan (1991, 1997) for a detailed discussion.

[30]As is well known, all SDFs can be represented as the sum of the minimum norm SDFs (the intersection of the black line and the red plane in Figure 7) and of a random variable that is orthogonal to the space P of excess returns (Hansen and Richard (1987); Cochrane (2005)).

alisation procedure allows the reduced-form innovations that span D to be transformed into a set of orthonormal vectors that also span D. The blue dashed arrows in Figure 7 represent two possible elements of the infinite sequence of orthonormalisations. The set of all admissible orthonormalisations is denoted by O and is represented by the blue circle with unit radius in the Figure.

The space of VAR innovations is unlikely to contain an SDF because of model misspecification or measurement error associated with observing SDFs (Roll, 1977). Loosely speaking, the tilted nature of the blue plane prevents all elements of O to be orthogonal to the space of excess returns, i.e. $M \cap O = \emptyset$. Yet, one can find an element in O that is closest to M in the spirit of Hansen and Jagannathan (1997) by applying the classical Projection Theorem.[31] This implies that the reduced-form VAR residuals induce *one* particular orthonormal shock, which is closer to the SDF than all the other orthonormal shocks. This is the blue arrow labelled as the λ-shock in Figure 7, whose projection onto the space of SDFs is the magenta line. This shock is the best possible approximation of the SDF: it summarises all the relevant information contained in all the reduced-form residuals of the VAR, i.e. in the blue plane.

Figure 7 highlights how the modelling of the macroeconomic dynamics (the VAR innovation space from which I recover the SDF) is somewhat disjointed from the modelling of the cross-section of asset prices (the space that induces the SDF), as mentioned in the Introduction. The link between the two spaces is the orthogonality condition $0 = E(mp)$. Changing the test portfolios can be thought of as tilting the red plane while fixing the blue plane in Figure 7. In turn, augmenting the VAR with additional state variables in order to better explain/price the given test assets can be thought of as tilting the blue plane while fixing the red plane in the Figure. For example, a VAR with good (bad) pricing performance would imply a flatter (steeper) blue plane with respect to the red plane.

---

[31]That is, assuming that O is a complete linear subspace of H, there exists a unique vector $m_0 \in O$, corresponding to any vector $x \in M$, such that $\|x - m_0\| \leq \|x - m\|$ for all $m \in O$. See pp. 50-51 of Luenberger (1969) for a classic treatment and pp. 608-609 of Hansen and Richard (1987) for a conditional version of the theorem.

B.1.3 Algebraic Illustration

To find a $k \times k$ Q matrix in the VAR model B.1, I span the space with Givens rotations to construct an orthonormal shock such that, given the test assets, the corresponding $\hat{\lambda}$ estimate in the second-pass Fama and MacBeth (1973) regression is maximised.[32] Further details about numerical implementation are found in Section C.2.1, and Example (3) connects the intuition from Figure 6 with the mechanics.

Example 3 (A Bivariate VAR Model) *Let R be a $T \times n$ matrix of excess returns of n test portfolios. Take a two-variable VAR model ($k = 2$) where the pricing factors are orthonormalised shocks given by Cholesky decomposition ($f_t = [f_{1t}|f_{2t}] = \eta_t \boldsymbol{B}^{-1} = \eta_t \left(chol\left(\Omega\right)\right)^{-1}$). The implied model of the SDF (m) is:*

$$m_t = \lambda_1 f_{1t} + \lambda_2 f_{2t}, \tag{B.4}$$

where λ_1 and λ_2 are the prices of risk associated with f_1 and f_2. Given that the factors are not persistent (Adrian, Crump, and Moench, 2015), the λs can be estimated with the two-stage procedure of Fama and MacBeth (1973).[33] Because $f_1 \perp f_2$ and $var(f_1) = var\left(f_2\right) = 1$, the variance of the SDF is simply the sum of the squared values of the estimated prices of risk associated with the two shocks:

$$var\left(\hat{m}_t\right) = \hat{\lambda}_1^2 + \hat{\lambda}_2^2. \tag{B.5}$$

Rotation does not change the information set: the volatility of the implied SDF is determined by the specification of the VAR and not by rotating the variance-covariance matrix of the residuals. The main implication of proposition 1 is that the information in the VAR residuals can be summarised by only one orthogonal shock after an appropriate rotation,

---

[32]See Section C.1 and the sign restrictions literature (Uhlig 2005; Rubio-Ramirez, Waggoner, and Zha 2010; Fry and Pagan 2011; Kilian and Murphy 2012; Baumeister and Hamilton 2015) for more information on Givens rotations and QR decompositions.

[33]First, estimate n time series regressions, $R_{it} = a_i + f_t\beta_i + \epsilon_{it}$, $i = 1 \ldots n$. Then, estimate a cross-section regression, $\bar{R}_i = \tilde{\beta}_i \times \lambda + \alpha_i$, where $\bar{R}_i = \frac{1}{T}\sum_{t=1}^{T} R_{it}$, $\tilde{\beta}_i$ is the OLS estimate obtained in the first stage and α_i is a pricing error.

i.e. there exists matrix $Q = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ *such that using* $f_i^\star = Q f_i$ *for* $i = 1, 2$ *as pricing factors, one of the estimated prices of risk is* $\lambda_1^\star = \sqrt{var\left(\hat{m}\right)}$, *as the other one is zero,* $\lambda_2^\star = 0$. *This implies that the best approximation of the SDF is found,* $f_1^\star = \hat{m}$, *and* Q *can be used to perform structural analysis in the VAR, i.e.* $\hat{\boldsymbol{B}}^\star = \hat{\boldsymbol{B}}Q$ *can be used to compute IRFs.*

It is worth noting that finding Q is not needed to find the time-series λ-shock. Applying the Fama and MacBeth (1973) procedure to any linear combinations of the VAR innovations will produce a unique time-series of the λ-shock that can be obtained as the fitted values of the second-stage regression. This is highlighted by the following lemma 4.

Lemma 4 *Suppose the SDF is modelled in an unconditional asset pricing framework as linear combinations of orthogonalised shocks from a VAR. The estimated prices of risk are dependent on identifying assumptions about* $\boldsymbol{B}^\star$, *but the estimated time-series of the SDF is independent of them.*

This statement highlights that orthogonalised shocks in a VAR are merely different linear combinations of the reduced-form residuals, thereby containing the same information set as the reduced-form innovation when pricing the cross-section of returns. In the language of empirical asset pricing: assumptions about VAR identification determine risk exposures and factor risk premia, but they do not affect the overall cross-sectional (R^2-type) fit of the transformed residuals, if all the orthogonalised shocks were to be used for pricing the cross-section of returns.

Proof of Lemma 4. The proof proceeds in three simple steps: (i) I apply arbitrary identifying assumptions to obtain a set of orthogonalised shocks (ii) I derive the estimator of the price of risk associated with the orthogonalised shocks as pricing factors (iii) and show that the implied SDF is independent of the identifying assumptions.

Let $\overline{Y}$ be an $1 \times n$ vector of average excess returns, $\tilde{Y}$ is a $T \times n$ matrix of demeaned time-series of excess returns, and η is a $T \times k$ matrix of reduced-form residuals from

a k-variable VAR of any order with variance-covariance matrix Ω. Apply Cholesky decomposition to obtain triangularised innovations as pricing factors $Z = \eta\left((\boldsymbol{B})^{-1}\right)' = \eta\left((chol\,(\Omega))^{-1}\right)'$. The estimated risk exposures are given by the first-stage βs from time-series regressions:

$$\hat{\beta} = (Z'Z)^{-1}\,Z'\tilde{Y} \quad .$$

To estimated prices of risk are obtained by the second-stage cross-sectional regression:

$$
\begin{aligned}
\hat{\lambda} &= (\beta\beta')^{-1}\,\beta\overline{Y}' \\
&= \left[(Z'Z)^{-1}\,Z'\tilde{Y}\left(\tilde{Y}'Z\right)\left((Z'Z)^{-1}\right)\right]^{-1}(Z'Z)^{-1}\,Z'\tilde{Y}\overline{Y}' \\
&= (Z'Z)\left(Z'\tilde{Y}\left(\tilde{Y}'Z\right)\right)^{-1}Z'\tilde{Y}\bar{Y}'.
\end{aligned}
\tag{B.6}
$$

Express the reduced-form innovations in terms of orthogonalised shocks, $Z = \eta\left((\boldsymbol{B})^{-1}\right)' \equiv \eta\Delta$ re-write B.6:

$$
\begin{aligned}
\hat{\lambda} &= (\Delta'\eta'\eta\Delta)\left(\Delta'\eta'\tilde{Y}\left(\tilde{Y}'\eta\Delta\right)\right)^{-1}\Delta'\eta'\tilde{Y}\overline{Y}' \\
&= \Delta'\,(\eta'\eta)\,\Delta\,(\Delta)^{-1}\left(\left(\eta'\tilde{Y}\tilde{Y}'\eta\right)\right)^{-1}(\Delta')^{-1}\,\Delta'\eta'\tilde{Y}\overline{Y}' \\
&= \Delta'\,(\eta'\eta)\left(\eta'\tilde{Y}\tilde{Y}'\eta\right)^{-1}\eta'\tilde{Y}\overline{Y}',
\end{aligned}
\tag{B.7}
$$

which proves that the estimated prices of risk depend on $\Delta \equiv \left((\boldsymbol{B})^{-1}\right)'$ which in turn depends on the identifying assumptions imposed on the structural impact matrix $\boldsymbol{B}$. The implied linear model for the SDF is written as:

$$
\begin{aligned}
m &= Z\hat{\lambda} \\
&= \eta\Delta\Delta'\,(\eta'\eta)\left(\eta'\tilde{Y}\tilde{Y}'\eta\right)^{-1}\eta'\tilde{Y}\tilde{Y}'\overline{Y}' \\
&= \eta\Omega\,(\eta'\eta)\left(\eta'\tilde{Y}\tilde{Y}'\eta\right)^{-1}\eta'\tilde{Y}\tilde{Y}'\overline{Y}',
\end{aligned}
\tag{B.8}
$$

which shows that the implied SDF depends on the *reduced-form* variance covariance matrix, Ω, and does not depend on orthogonalisation assumptions. ∎

Building on Example (3), the following proposition 5 and example 6 explain the relationship between the angle θ needed to compute the impulse vector associated with

the λ-shock.[34]

Proposition 5 *Given the VAR model in Example (3) with variance-covariance matrix*
$\Omega = \begin{bmatrix} \omega_{11} & \omega_{12} \\ \omega_{12} & \omega_{22} \end{bmatrix}$, *the column of* $\boldsymbol{B}^{\star} = \begin{bmatrix} B_{11}^{\star} & B_{12}^{\star} \\ B_{21}^{\star} & B_{22}^{\star} \end{bmatrix}$ *corresponding to the contemporaneous effect of the λ-shock is given by:* $B_{11}^{\star} = \sqrt{\omega_{11}}\cos(\theta)$ *and* $B_{21}^{\star} = \frac{\omega_{12}}{\sqrt{\omega_{11}}}\cos(\theta) + \sqrt{\omega_{22} - \omega_{12}^2/\omega_{11}}\sin(\theta)$, *with the rotation angle θ given by the prices of risks,* $\theta = \arcsin\left(\lambda_2/\sqrt{(\lambda_1^2 + \lambda_2^2)}\right)$.

Proof of Proposition 5. The proof proceeds in three simple steps: (i) I derive a general form of the structural impact matrix $\boldsymbol{B}^{\star}$ and its inverse $\boldsymbol{B}^{\star-1}$ without any reference to asset pricing; (ii) I use the linear model of the SDF (B.4) to express the elements in the row of $\boldsymbol{B}^{\star-1}$ corresponding to the λ-shock, i.e. this row determines the linear relationship between the reduced form residuals and SDF innovations; finally (iii) I match the values obtained in step (i)-(ii).

Step 1 Apply the Cholesky algorithm to the reduced form variance covariance matrix
$\Omega = \begin{bmatrix} \omega_{11} & \omega_{12} \\ \omega_{12} & \omega_{22} \end{bmatrix}$ to obtain a candidate for $\boldsymbol{B}$. Because Ω is positive definite, $\boldsymbol{B}$ exists
and can be written as:

$$\boldsymbol{B} = chol\left(\Sigma\right) = \begin{bmatrix} \sqrt{\omega_{11}} & 0 \\ \frac{\omega_{12}}{\sqrt{\omega_{11}}} & \sqrt{\omega_{22} - \left(\frac{\omega_{12}}{\sqrt{\omega_{11}}}\right)^2} \end{bmatrix}. \tag{B.9}$$

It is known (Fry and Pagan, 2011) that one can take any orthonormal matrix Q to obtain a new structural impact matrix $\boldsymbol{B}^{\star} = \boldsymbol{B}Q$, with the associated set of orthogonalised shocks $e_t^{\star} = \eta_t\left(\boldsymbol{B}^{\star-1}\right)'$, which conforms to the reduced-form variance covariance matrix, i.e. $\Omega = \boldsymbol{B}^{\star}\left(\boldsymbol{B}^{\star}\right)' = \boldsymbol{B}Q\left(\boldsymbol{B}Q\right)' = \boldsymbol{B}Q\left(Q'\boldsymbol{B}'\right) = \boldsymbol{B}\boldsymbol{B}'$. Let Q be a rotation $r_{\theta} =$

---

[34]A linear model of the SDF, that uses arbitrarily orthonormalised VAR residuals, uniquely pins down one of the rows of the matrix $\left(\hat{\boldsymbol{B}}^{\star}\right)^{-1}$. However, this is not sufficient to carry out structural VAR analysis, because to do so one needs to know the column in the structural impact matrix $\hat{\boldsymbol{B}}^{\star}$.

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}, \text{ so } \boldsymbol{B}^{\star} = \boldsymbol{B}Q = \boldsymbol{B}r_{\theta} \text{ implies:}$$

$$\boldsymbol{B}^{\star} = \begin{bmatrix} \sqrt{\omega_{11}}\cos(\theta) & -\sqrt{\omega_{11}}\sin(\theta) \\ \frac{\omega_{12}}{\sqrt{\omega_{11}}}\cos(\theta) + \xi\sin(\theta) & -\frac{c}{\sqrt{\omega_{11}}}\sin(\theta) + \cos(\theta)\xi \end{bmatrix}, \tag{B.10}$$

where $\xi = \sqrt{\omega_{22} - \omega_{12}^2/\omega_{11}}$. Matrix inversion yields:

$$\boldsymbol{B}^{\star-1} = \frac{1}{\omega_{11}\psi} \begin{pmatrix} -\omega_{12}\sin(\theta) + \sqrt{\omega_{11}}\cos(\theta)\,\psi & \omega_{11}\sin(\theta) \\ -\omega_{12}\cos(\theta) - \sqrt{\omega_{11}}\sin(\theta)\,\psi & \omega_{11}\cos(\theta) \end{pmatrix}, \tag{B.11}$$

where $\psi \equiv \omega_{11}\sqrt{\frac{\omega_{11}\omega_{22}-\omega_{12}^2}{\omega_{11}}}$.

Step 2 The linear model of the SDF (B.4) can be re-written in terms of the reduced form residuals, $\eta_t = [\eta_{1t}|\eta_{2t}]$, by using the identity $f_t = [f_{1t}|f_{2t}] = \eta_t \boldsymbol{B}^{-1} = \eta_t \left(chol\left(\Omega\right)\right)^{-1}$ and the definition B.9:

$$\begin{aligned} m_t &= \lambda_1 f_{1t} + \lambda_2 f_{2t} \\ &= \left(\lambda_1 \frac{1}{\sqrt{\omega_{11}}} - \lambda_2 \frac{\omega_{12}}{\psi}\right)\eta_{1t} + \left(\lambda_2 \frac{1}{\psi}\right)\eta_{2t}. \end{aligned} \tag{B.12}$$

Applying proposition 1 implies that the SDF can be expressed by a single orthogonalised shock, $e_{1t}^{\star}$, where $e_t^{\star} = [e_{1t}^{\star}|e_{2t}^{\star}] = \eta_t \boldsymbol{B}^{\star-1}$:

$$\begin{aligned} m_t &= \lambda_1 f_{1t} + \lambda_2 f_{2t} \\ &= \left(\sqrt{\lambda_1^2 + \lambda_2^2}\right)e_{1t}^{\star} + 0 \cdot e_{2t}^{\star}. \end{aligned} \tag{B.13}$$

[Note that designating the λ-shock to be the first column of $e_t^{\star}$ is arbitrary, but this does not play a role given the orthogonality of the columns of $e_t^{\star}$.] Hence B.12 together with B.13 determines the first row of $\boldsymbol{B}^{\star-1}$ written as:

$$\boldsymbol{B}_{1,1:1,2}^{\star-1} = \begin{bmatrix} \frac{\lambda_1 \frac{1}{\sqrt{\omega_{11}}} - \lambda_2 \frac{\omega_{12}}{\psi}}{\sqrt{\lambda_1^2 + \lambda_2^2}} & \frac{\lambda_2 \frac{1}{\psi}}{\sqrt{\lambda_1^2 + \lambda_2^2}} \end{bmatrix}. \tag{B.14}$$

Step 3 Matching values of the top right elements of B.11 and B.14 yields:

$$\theta = \arcsin\left(\frac{\lambda_2}{\sqrt{(\lambda_1^2 + \lambda_2^2)}}\right).$$

∎

As an empirical illustration of Proposition 5, consider the following example.

Example 6 (A Bivariate VAR and the Consumption-CAPM) *Let the two vari-ables in a quarterly VAR(2) be the log of consumption and the term spread, and let the test assets be the 25 Fama-French (FF25) portfolios. An OLS regression using data from 1970Q1 to 2012Q2 yields an estimated variance-covariance matrix* $\hat{\Omega} = \begin{bmatrix} 0.38 & -0.07 \\ -0.07 & 0.78 \end{bmatrix}$.
Given the reduced-form residuals, u_t^C and u_t^{Term}, the implied linear model of the SDF is $\hat{m}_t = 0.21 u_t^C + 1.14 u_t^{Term}$, implying that term-spread innovations load more on the SDF than consumption innovations. Consider the contemporaneous impact of the shock on the variables. Using the appropriate angle θ, the elements of the $\hat{B}^\star$ are:

$$
\begin{aligned}
\hat{B}_{11}^\star &= 0.006 & \hat{B}_{12}^\star &= 0.61 \\
\hat{B}_{21}^\star &= 0.88 & \hat{B}_{22}^\star &= -0.12.
\end{aligned}
\tag{B.15}
$$

The first column shows that a s.d. λ-shock induces a large (0.88pp) jump in the term spread, but has virtually no contemporaneous effect (<0.01%) on consumption. The second column shows the contemporaneous effect of the shock that is by construction orthogonal to the implied SDF, thus demanding zero risk premia. This shock has a large (0.61%) contemporaneous effect on consumption which implies that virtually all of the one period ahead FEV in consumption is explained by a shock, exposure to which demands zero risk compensation according to the FF25.

This simple example highlights the empirical relevance of the Consumption-CAPM literature which emphasises that news about current consumption growth are irrelevant to determining the level of risk premia.[35]

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[35]For recent contributions, see Bryzgalova and Julliard (2015) and Boons and Tamoni (2015) amongst others.

To estimate the effect of the λ-shock at longer horizons, one needs to compute impulse response functions, defined as follows.

Definition 7 (Impulse Response Functions) *Consider a VAR(2) model $X_t = c + \mathbb{A}_1 X_{t-1} + \mathbb{A}_2 X_{t-2} + \eta_t$ with response matrices $\Phi_0 = I$, $\Phi_1 = \Phi_0 \mathbb{A}_1$, $\Phi_2 = \Phi_1 \mathbb{A}_1 + \Phi_0 \mathbb{A}_2$, ..., $\Phi_h = \Phi_{h-1}\mathbb{A}_1 + \Phi_{h-2}\mathbb{A}_2$ for h-period ahead. Given the impact matrix $\boldsymbol{B}$, the associated structural IRFs at horizon h are given by $\Gamma_h = \Phi_h \boldsymbol{B}$.*

To compute IRFs for the λ-shock, define a $k \times 1$ vector $\boldsymbol{e}_\lambda$, all of whose elements are zero except for a unit corresponding to the λ-shock. Given the impact matrix $\boldsymbol{B}^\star$ corresponding to the λ-shock, the impulse vector is $a_\lambda = \boldsymbol{B}^\star \boldsymbol{e}_\lambda$ and the associated IRFs at horizon h are given by $\Gamma_{\lambda,h} = \Phi_h \boldsymbol{a}_\lambda$.

B.2 The γ-Shock

To construct the γ-shock, I integrate the return-forecasting framework of the empirical finance literature (see Lettau and Ludvigson (2010); Cochrane (2011) for a review) with the VAR model B.1. To estimate variation in the conditional mean of excess returns, the finance literature typically estimated the following predictive regression model:

$$r_{t+1}^H = a + \beta X_t^\star + \varepsilon_{t+1}, \tag{B.16}$$

where r_{t+1}^H is the cumulative log excess market return between $t+1$ and $t+H$; $X_t^\star$ is a vector of variables at the end of t used to predict the excess returns, and ε_{t+1} is a zero-mean disturbance term. Horse race among predictors is typically assessed using the R^2-statistic of the estimated regression B.16. In my empirical model, $X_t^\star$ will be a subset of the state vector, X_t in B.1. I partition the state vector as $X_t = \begin{bmatrix} \bar{X}_t; & X_t^\star \end{bmatrix}$ (in the spirit of Adrian, Crump, and Moench (2015)), where $\bar{X}_t$ are the remaining variables in the VAR that are not used in the predictive regression B.16. Constructing the γ-shock is then based on the historical decomposition of $X_t^\star$ in the VAR.

Definition 8 (Historical Decomposition) *Consider a covariance stationary VAR of the form B.1. Given a structural impact matrix $\boldsymbol{B}$ and corresponding orthogonal shocks*

$f_t = \eta_t \boldsymbol{B}^{-1}$, *the historical decomposition of* X_t *can be computed as follows:*

$$X_t = \sum_{s=0}^{t-1} \Gamma_s f_{t-s} + \sum_{s=t}^{\infty} \Gamma_s f_{t-s}, \tag{B.17}$$

where Γ *is a* $k \times k$ *matrix of IRFs as in definition 7.*

The γ-shock is constructed to be an orthogonalised shock which generates counterfactual time-series $\hat{X}_t^\star$ in the predictor variables $X_t^\star$ with the following property: when $\hat{X}_t^\star$ is used in the predictive regression B.16, the associated R^2-statistic is maximised. Moreover, $\tilde{X}_t^\star$ will denote the counterfactual time-series in $X_t^\star$ that are generated by all remaining shocks, but the γ-shock, in the VAR.[36] In essence, my method finds the γ-shock by using the return-predictability step to restrict the historical decomposition of $X_t^\star$. Further details about numerical implementation are found in Appendix C.2.2. I now briefly discuss how the method of constructing the γ-shock can be linked to the macroeconomic and finance literatures.

In terms of the link to the macroeconometrics literature, the method is similar to the identification of news shocks (Uhlig 2004; Barsky and Sims 2011; Kurmann and Otrok 2013). These papers identify news shocks about future economic fundamentals, based on maximising the contribution of these shocks to the forecast error variance of a selected variable in the VAR over a pre-specified future horizon. My orthogonlisation scheme is based on finding counterfactual variation in selected variables in the VAR that have maximal forecasting power as return predictors. In essence, the return-predictability step serves as an external reference point which restricts the entire sequence of historical decompositions.

In terms of the link to the finance literature, the method aims to uncover the stochastic macroeconomic drivers of time-variation in risk premia. Since Fama and French (1989) and Ferson and Harvey (1991), growing empirical evidence points to the countercyclical nature of expected excess returns, implying that risk premia are high in recessions and

---

[36]I include the deterministic/trend component ($T_t^\star$), implied by the VAR, when constructing both sets of counterfactual time-series of the predictors ($\hat{X}_t^\star$ and $\tilde{X}_t^\star$). This means that the decomposition of the time-series of the predictors can be written as: $X_t^\star = \hat{X}_t^\star + \tilde{X}_t^\star - T_t^\star$.

low in expansions. While reduced-form macroeconomic variables such as the term spread (Fama and French, 1989) or the short-term interest rate (Fama and Schwert, 1977) have been found to forecast excess returns, it is clear that not all time-series variation in the term spread or the interest rate is driven by unexpected macroeconomic shocks that would ultimately lead to recessions and to spikes in risk premia. By constructing the γ-shock, the aim is to use a non-restrictive way to find the portion of variation in predictor variables that can be directly attributed to macroeconomic disturbances which cause recessions.

C   Numerical Implementation

C.1   Rotation Matrices

To select matrix Q in a n-variable VAR model (Section B), one needs to span the n-dimensional space of rotations. See Golub and Loan (1996) for a textbook treatment and Zhelezov (2017) for a recent algorithm to generate n-dimensional rotation matrices. As an example, consider the case of a four-variable VAR model, I write Q as the product of three auxiliary Givens matrices:

$$Q = Q_1 \times Q_2 \times Q_3, \tag{C.1}$$

where:

$$Q_1 = \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 & 0 \\ 0 & 0 & \cos(\theta_2) & -\sin(\theta_2) \\ 0 & 0 & \sin(\theta_2) & \cos(\theta_2) \end{bmatrix}$$

$$Q_2 = \begin{bmatrix} \cos(\theta_3) & & -\sin(\theta_3) & 0 \\ 0 & \cos(\theta_4) & 0 & -\sin(\theta_4) \\ \sin(\theta_3) & 0 & \cos(\theta_2) & 0 \\ 0 & \sin(\theta_4) & 0 & \cos(\theta_4) \end{bmatrix}$$

$$Q_3 = \begin{bmatrix} \cos(\theta_5) & & & -\sin(\theta_5) \\ 0 & \cos(\theta_6) & -\sin(\theta_6) & 0 \\ & \sin(\theta_6) & \cos(\theta_6) & 0 \\ \sin(\theta_5) & 0 & 0 & \cos(\theta_5) \end{bmatrix}.$$

The six Euler-angles θ_1, θ_2, θ_3, θ_4, θ_5, θ_6 are then chosen appropriately so that the objective function (described in Section C.2) is satisfied. A similar construction can be used for higher dimensions. However, the number of angles needed to span the space rapidly increases as we add more variables to the VAR.[37]

C.2 Numerical Algorithm to Find the λ-shock and the γ-shock

To estimate the λ-shock and the γ-shock, I start with a reduced-form k-variable VAR B.1, written as:

$$X_t = c + \mathbb{A}_1 X_{t-1} + \cdots + \mathbb{A}_p X_{t-p} + \eta_t, \qquad \eta_t \sim N\left(0, \Omega\right),$$

where η the reduced form innovations, and Ω is the reduced-form covariance matrix.

C.2.1 Constructing the λ-shock

The algorithm, that uses a random search approach to construct the λ-shock is as follows:

- Step 1: Draw a $k \times k$ random matrix L from the multivariate standard normal distribution.

- Step 2: Compute the orthonormal matrix $Q^\star$ from the QR decomposition of L. (Orthonormality of Q means that $Q'Q = I$).

- Step 3: Compute the Cholesky decomposition of the reduced-form covariance matrix, $\boldsymbol{B} = chol\left(\Omega\right)$, to obtain a structural impact matrix $\boldsymbol{B}$.

---

[37]For example, while a 4-variable VAR requires merely six angles (C.1), an 8-variable VAR requires 420 angles.

- Step 4: Combine results from Step 2 and Step 3 to obtain a new structural impact matrix $\boldsymbol{B}^\star = \boldsymbol{B}Q$.

- Step 5: Construct the time-series of the orthogonalised shocks corresponding to $\boldsymbol{B}^\star$ by computing $f^\star = \eta \left(\boldsymbol{B}^\star \right)^{-1}$.

- Step 6: Use each orthogonalised shock $f_j^\star$ $(j = 1, \dots k)$ as a factor, and estimate the first stage of the Fama and MacBeth (1973) regression, $R_{i,t} = a_{i,j} + f_j^\star \beta_{i,j} + \epsilon_{i,j,t}$, $i = 1 \dots n$, given n time-series of test portfolios.

- Step 7: Estimate a cross-section regression, $\bar{R}_i = \tilde{\beta}_{i,j} \times \lambda_j + \alpha_{i,j}$, where $\bar{R}_i = \frac{1}{T} \sum_{t=1}^{T} R_{it}$, $\tilde{\beta}_{i,j}$ is the OLS estimate obtained in the first stage (Step 6) and $\alpha_{i,j}$ is a pricing error.

- Step 8: For each of the k one-factor models (Steps 6-7), compute the statistic $R_j^2 = 1 - \frac{\left(\bar{R} - \tilde{\beta}_{i,j} \lambda_j \right)' \left(\bar{R} - \tilde{\beta}_{i,j} \lambda_j \right)}{\left(\bar{R} - \widehat{R} \right)' \left(\bar{R} - \widehat{R} \right)}$, where $\widehat{R} = \frac{1}{n} \sum_{i=1}^{n} \bar{R}_i$ is the cross-sectional average of the mean returns in the data. Select the one-factor model with the best fit, $\dot{R}^2 = \max \left(R_1^2, R_2^2, \dots, R_k^2 \right)$, and save the corresponding impulse vector $\dot{a}_\lambda$.

- Step 9: Re-run steps 1-8 N times.

- Step 10: From the set of N impulse vectors, chose the one which corresponds to the largest $\dot{R}^2$ values, and compute impulse responses.

While the random search approach outlined above makes the description of the method transparent, in practice, it can be transformed into a more efficient numerical optimisation problem: instead of using the QR decomposition of random matrices (Step 2), I use Givens rotations (Section C.1) and chose the corresponding Euler-angles directly to maximise the R^2 statistic (Step 8).

C.2.2 Constructing the γ-shock

The algorithm, that uses a random search approach to construct the γ-shock is as follows (steps 1-5 are the same as in Section C.2.1):

- Step 1: Draw a $k \times k$ random matrix L from the multivariate standard normal distribution.

- Step 2: Compute the orthonormal matrix $Q^\star$ from the QR decomposition of L. (Orthonormality of Q means that $Q'Q = I$).

- Step 3: Compute the Cholesky decomposition of the reduced-form covariance matrix, $\boldsymbol{B} = chol(\Omega)$, to obtain a structural impact matrix $\boldsymbol{B}$.

- Step 4: Combine results from Step 2 and Step 3 to obtain a new structural impact matrix $\boldsymbol{B}^\star = \boldsymbol{B}Q$.

- Step 5: Construct the time-series of the orthogonalised shocks corresponding to $\boldsymbol{B}^\star$ by computing $f^\star = \eta \left(\boldsymbol{B}^\star \right)^{-1}$.

- Step 6: Use each orthogonalised shock $f_j^\star$ $(j = 1, \ldots k)$ to produce k sets of historical decompositions of the data matrix X_t (using B.17) and k sets of counterfactual time-series $\hat{X}_{j,t}$.

- Step 7: Partition the counterfactual time-series of the state vector as $\hat{X}_{j,t} = \left[\hat{\hat{X}}_{j,t}; \ \hat{X}_{j,t}^\star \right]$. In my application, $\hat{X}_{j,t}^\star$ will include the counterfactual time-series of the federal funds rate (FFR), the default spread (DEF) and the term spread (TERM).

- Step 8: Estimate the predictive regression $r_{t+1}^H = a_j + \beta_j \hat{X}_{j,t}^\star + \varepsilon_{j,t+1}$ for each $j = 1, \ldots k$ and save the regression R_j^2. Select the orthogonalised shock $f_j^\star$ with the best fit, $\dot{R}^2 = \max \left(R_1^2, R_2^2, \ldots, R_k^2 \right)$, and save the corresponding impulse vector $\dot{a}_\lambda$.

- Step 9: Re-run steps 1-8 N times.

- Step 10: From the set of N impulse vectors, chose the one which corresponds to the largest $\dot{R}^2$ values, and compute impulse responses.

In practice, the random search approach outlined above can be transformed into a numerical optimisation problem: instead of using the QR decomposition of random matrices
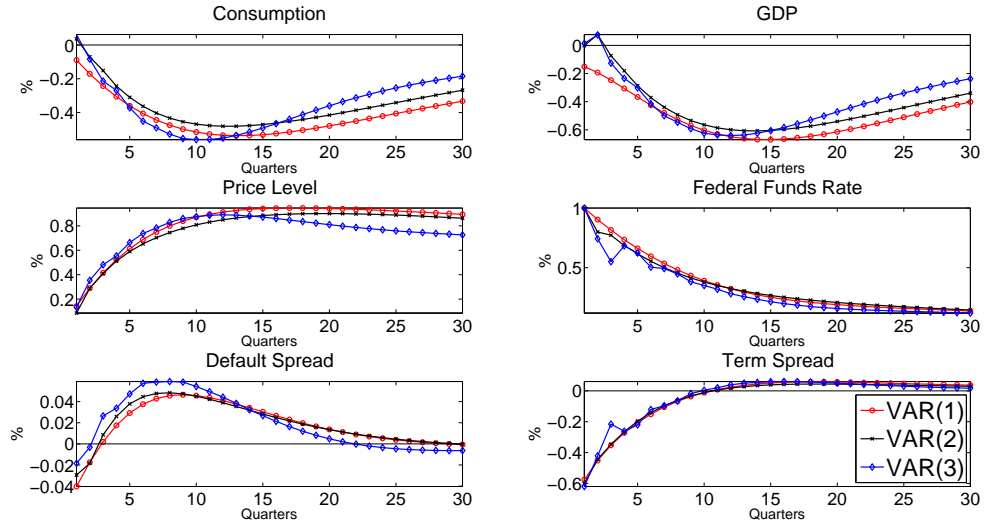
(Step 2), I use Givens rotations (Section C.1) and chose the corresponding Euler-angles directly to maximise the R^2 statistic (Step 8).
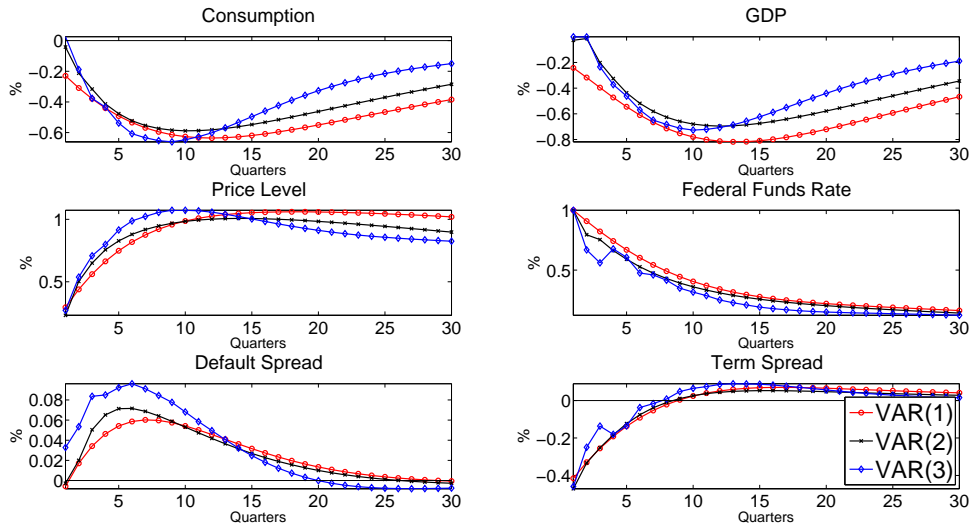
D   Additional Empirical Results

D.1   *Robustness to Alternative Lag Structure of the VAR*

Figure 8: Impulse Responses to a λ-shock: Robustness to Lag Structure

(a) Sample: 1963Q3-2008Q3

(b) Sample: 1963Q3-2015Q3

Notes: The vertical axes are %-deviations from steady-state, and the horizontal axes are in quarters. The IRFs are computed from VAR(1), VAR(2) and VAR(3) models. In all cases, the FF55 portfolios were used as test assets, and the IRFs are normalised to increase the federal funds rate by 100bp.

Table 3: Forecasting Excess Returns: Results from a <u>VAR(3)</u> Model

	1Q	2Q	3Q	4Q	5Q	6Q	7Q	8Q
				Forecast Horizon H				
	Model A: CAY							
CAY	0.55	1.12	1.77	2.52	3.17	4.00	4.92	5.77
	(2.18)	(2.25)	(2.33)	(2.46)	(2.59)	(3.00)	(3.58)	(4.25)
	[0.01]	**[0.03]**	**[0.05]**	**[0.09]**	**[0.11]**	**[0.14]**	**[0.19]**	**[0.23]**
	Model B: Counterfactual VAR Variables Induced by the γ-Shock							
FFR	0.47	1.00	1.50	1.87	2.24	2.75	3.31	3.80
	(1.14)	(1.19)	(1.37)	(1.45)	(1.56)	(1.77)	(2.03)	(2.29)
DEF	-7.76	-14.53	-20.40	-24.42	-28.36	-32.55	-37.35	-41.47
	(-1.84)	(-1.68)	(-1.91)	(-2.04)	(-2.19)	(-2.42)	(-2.70)	(-2.94)
TERM	3.01	6.41	9.72	12.40	14.99	17.87	20.81	23.16
	(2.71)	(2.72)	(2.90)	(2.96)	(3.09)	(3.27)	(3.53)	(3.87)
	[0.04]	**[0.10]**	**[0.17]**	**[0.23]**	**[0.28]**	**[0.33]**	**[0.38]**	**[0.42]**
	Model C: Counterfactual VAR Variables Induced by All Other Shocks							
FFR	-0.18	-0.28	-0.25	-0.21	-0.09	0.11	0.32	0.50
	(-0.70)	(-0.64)	(-0.43)	(-0.30)	(-0.11)	(0.11)	(0.25)	(0.34)
DEF	1.82	3.71	3.91	3.71	2.94	2.27	1.32	0.27
	(0.90)	(1.23)	(0.97)	(0.79)	(0.57)	(0.39)	(0.18)	(0.03)
TERM	-0.22	-0.64	-0.58	-0.45	-0.28	0.17	0.38	0.96
	(-0.37)	(-0.58)	(-0.36)	(-0.23)	(-0.12)	(0.07)	(0.14)	(0.32)
	[-0.01]	**[-0.00]**	**[-0.00]**	**[-0.01]**	**[-0.01]**	**[-0.01]**	**[-0.01]**	**[-0.01]**

Notes: The table reports results from regressions of excess returns on lagged variables. H denotes the return horizon in quarters. The dependent variable is the sum of H log excess returns on the CRSP based S&P Composite Index. The regressors are one-period lagged values of actual time-series of the federal funds rate (FFR), the term-spread (TERM) and the default spread (DEF) in Model A, the CAY measure of Lettau and Ludvigson (2001a,b) in Model B, and the counterfactual time-series of FFR, TERM and DEF (induced by the γ-shock) from six-variable VAR(3) and VAR(4) models, estimated over 1963Q3-2015Q3. The γ-shock is constructed so that the corresponding forecast power at the four-quarter horizon is maximised. For each of the three regressions, the table reports the OLS estimates of the regressors, the t-statistics using the Hansen and Hodrick (1980) correction (as implemented in Cochrane (2011)) are in parentheses, and adjusted R^2 statistics are in the bolded square brackets. Both the CAY measure and the counterfactual predictors are treated as known variables.

Table 4: Forecasting Excess Returns: Results from a <u>VAR(4)</u> Model

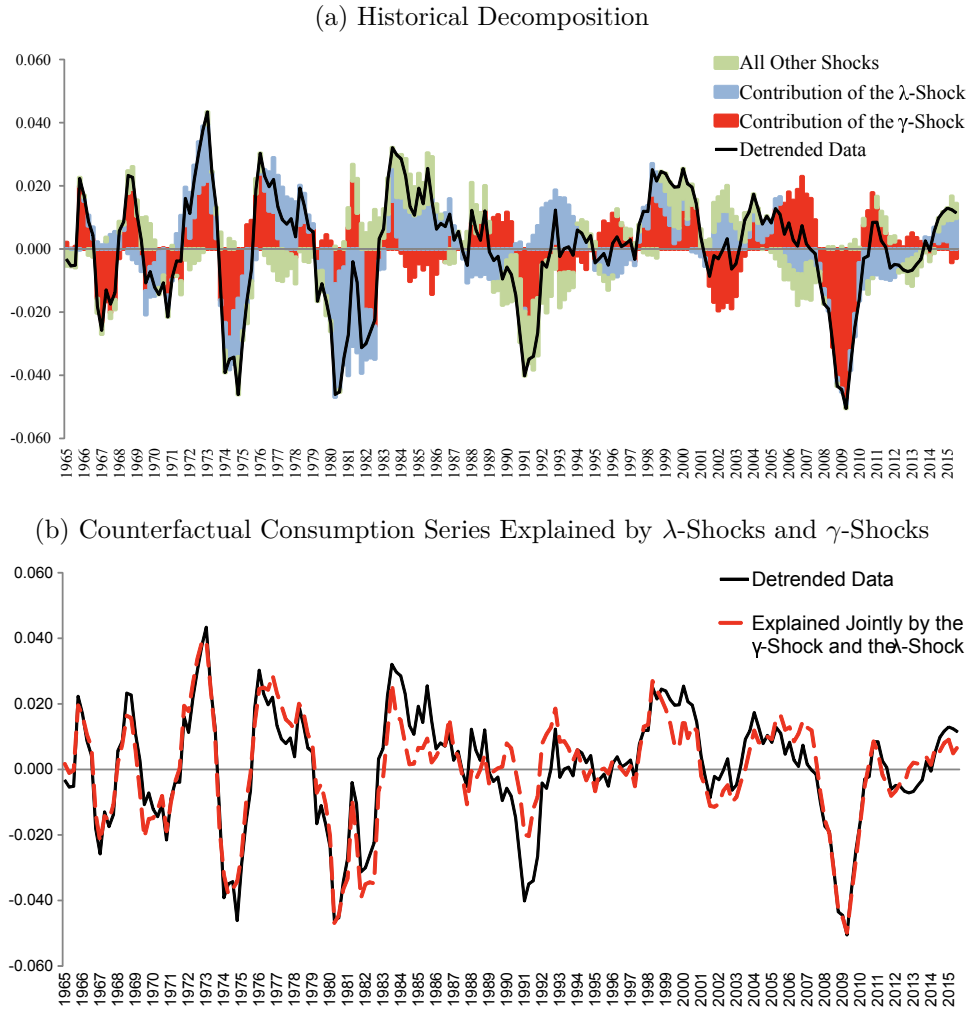|				Forecast Horizon H				
	1Q	2Q	3Q	4Q	5Q	6Q	7Q	8Q
				Model A: CAY				
CAY	0.55	1.12	1.77	2.52	3.17	4.00	4.92	5.77
	(2.18)	(2.25)	(2.33)	(2.46)	(2.59)	(3.00)	(3.58)	(4.25)
	[0.01]	**[0.03]**	**[0.05]**	**[0.09]**	**[0.11]**	**[0.14]**	**[0.19]**	**[0.23]**
			Model B: Counterfactual VAR Variables Induced by the γ-Shock					
FFR	0.40	0.74	1.07	1.35	1.64	2.02	2.47	2.98
	(1.06)	(0.94)	(1.02)	(1.09)	(1.20)	(1.39)	(1.62)	(1.92)
DEF	-7.01	-12.02	-16.29	-19.46	-22.33	-24.99	-28.18	-32.25
	(-1.88)	(-1.52)	(-1.65)	(-1.77)	(-1.94)	(-2.15)	(-2.38)	(-2.68)
TERM	2.80	5.66	8.55	11.08	13.47	16.03	18.63	21.04
	(2.71)	(2.54)	(2.68)	(2.75)	(2.90)	(3.10)	(3.38)	(3.77)
	[0.04]	**[0.09]**	**[0.15]**	**[0.20]**	**[0.25]**	**[0.30]**	**[0.34]**	**[0.39]**
			Model C: Counterfactual VAR Variables Induced by All Other Shocks					
FFR	-0.20	-0.29	-0.23	-0.18	-0.04	0.20	0.44	0.63
	(-0.78)	(-0.67)	(-0.40)	(-0.25)	(-0.04)	(0.19)	(0.34)	(0.41)
DEF	2.38	4.59	4.86	4.69	3.86	2.98	1.84	0.84
	(1.15)	(1.55)	(1.25)	(1.01)	(0.73)	(0.50)	(0.23)	(0.09)
TERM	-0.18	-0.45	-0.26	-0.05	0.20	0.79	1.13	1.77
	(-0.31)	(-0.42)	(-0.16)	(-0.03)	(0.10)	(0.37)	(0.46)	(0.65)
	[-0.00]	**[0.00]**	**[0.00]**	**[-0.00]**	**[-0.01]**	**[-0.01]**	**[-0.01]**	**[-0.00]**

Notes: The table reports results from regressions of excess returns on lagged variables. H denotes the return horizon in quarters. The dependent variable is the sum of H log excess returns on the CRSP based S&P Composite Index. The regressors are one-period lagged values of actual time-series of the federal funds rate (FFR), the term-spread (TERM) and the default spread (DEF) in Model A, the CAY measure of Lettau and Ludvigson (2001a,b) in Model B, and the counterfactual time-series of FFR, TERM and DEF (induced by the γ-shock) from six-variable VAR(3) and VAR(4) models, estimated over 1963Q3-2015Q3. The γ-shock is constructed so that the corresponding forecast power at the four-quarter horizon is maximised. For each of the three regressions, the table reports the OLS estimates of the regressors, the t-statistics using the Hansen and Hodrick (1980) correction (as implemented in Cochrane (2011)) are in parentheses, and adjusted R^2 statistics are in the bolded square brackets. Both the CAY measure and the counterfactual predictors are treated as known variables.

Table 5: The Contribution of the λ-shock and γ-shock to Business Cycles: FEV Decomposition from VAR(3) and VAR(4) Models

	VAR(3) Model							
	Consumption				Federal Funds Rate			
	λ-Shock	γ-Shock	$\lambda\&\gamma$	Unexpl.	λ-Shock	γ-Shock	$\lambda\&\gamma$	Unexpl.
1Q	0.5	72.3	72.8	27.2	64.1	3.2	67.3	32.7
2Q	3.9	72.1	76.0	24.0	54.1	9.0	63.1	36.9
3Q	10.7	70.4	81.1	18.9	51.4	11.0	62.4	37.6
4Q	15.1	66.5	81.6	18.4	50.7	14.1	64.8	35.2
8Q	32.4	44.5	76.9	23.1	47.1	20.8	67.9	32.1
16Q	43.0	25.9	68.9	31.1	43.9	24.1	68.0	32.0
32Q	42.2	20.7	62.8	37.2	42.8	24.5	67.3	32.7
	VAR(4) Model							
	Consumption				Federal Funds Rate			
	λ-Shock	γ-Shock	$\lambda\&\gamma$	Unexpl.	λ-Shock	γ-Shock	$\lambda\&\gamma$	Unexpl.
1Q	0.2	75.8	76.0	24.0	73.3	2.4	75.7	24.3
2Q	4.0	75.8	79.8	20.2	62.3	8.2	70.5	29.5
3Q	10.9	74.0	84.8	15.2	60.0	10.1	70.1	29.9
4Q	13.7	70.3	84.0	16.0	60.7	12.5	73.3	26.7
8Q	33.5	46.3	79.8	20.2	58.3	18.1	76.4	23.6
16Q	48.7	27.2	75.9	24.1	56.8	18.7	75.5	24.5
32Q	47.6	24.0	71.7	28.3	56.3	18.3	74.6	25.4

Notes: The table shows the % fraction of the total forecast error variance that is explained by the λ-shock and the γ-shock over different forecast horizons. The FF55 portfolios are used as test portfolios for the VAR models. The estimation period is 1963Q3-2015Q3.

Figure 9: Results from a VAR(4) – Decomposing Annual US Consumption Growth: the Role of λ-Shocks and γ-Shocks

(a) Historical Decomposition

(b) Counterfactual Consumption Series Explained by λ-Shocks and γ-Shocks

Notes: The figure shows the results implied by the historical decomposition from a VAR(4) estimated on the sample 1963Q3-2015Q3. I use the FF55 to construct the λ-shock, and the CRSP based SP500 return (in excess of the corresponding T-bill rate) to construct the γ-shock. The deterministic trend component, implied by the VAR, is removed from the time-series.

D.2 The λ-shock in the Monthly VAR of Uhlig (2005)

As robustness check, I investigate the price response of the λ-shock further, and use the monthly VAR of Uhlig (2005). Following his paper, I impose sign restrictions on the impulse responses of prices, nonborrowed reserves and the Federal Funds Rate in response to a monetary policy shock, thereby fixing the price puzzle anomaly while remaining agnostic about the effect of monetary policy shocks on other macrovariables of interest. The black lines and the associated error bands in Figure 10 replicate Figure 6 of Uhlig (2005), using his dataset. The blue line shows the response to a Cholesky-orthogonalised innovation in the federal funds rate that is ordered before nonborrowed and total reserves (Figure 5 of Uhlig (2005)). The red and purple lines show the responses to a λ-shock that is constructed using the FF55 and FF25 portfolios, respectively.

Figure 10: The VAR Model of Uhlig (2005): Monetary Policy Shocks and λ-shocks
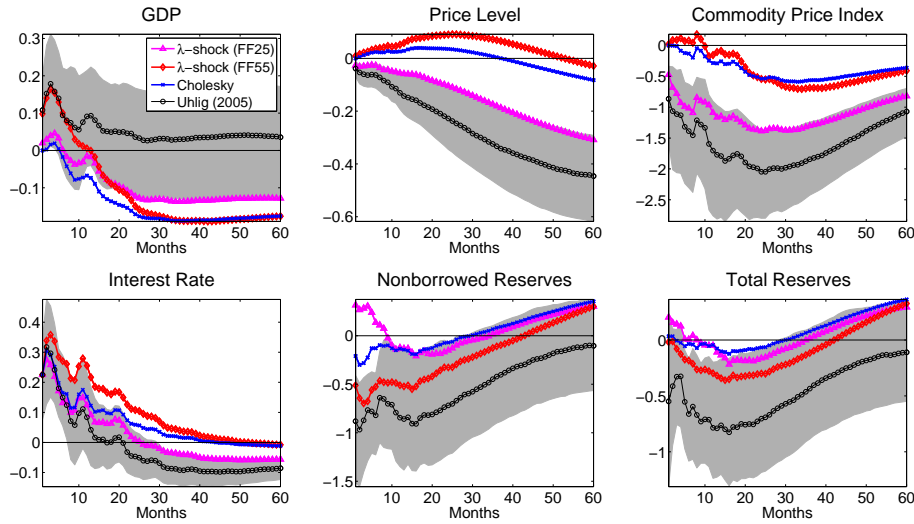
Notes: The vertical axes are %-deviations from steady-state, and the horizontal axes are in months. The monthly VAR(12) is estimated over 1965m1-2003m12, using the dataset of Uhlig (2005). The black lines and associated 16-84 bands are identical to Figure 6 of Uhlig (2005) that shows the responses to a monetary policy shock identified by the 'pure sign restriction approach'. To construct the λ-shock, I use the FF25 (purple lines) and the FF55 (red lines) portfolios. The blue lines are the are Cholesky-orthogonalised (Sims, 1980) interest rate shock. The IRFs are normalised to have the same contemporaneous effect on the interest rate.

While using a different dataset at monthly (instead of quarterly) frequency, the estimated λ-shock continues to induce business cycles similar to that caused by monetary policy shocks. Note, however that the λ-shocks generate a more contractionary GDP response than the monetary policy shock identified by Uhlig (2005). However, recent papers (Antolin-Diaz and Rubio-Ramirez, 2018; Arias, Rubio-Ramirez, and Waggoner,

2018) have argued that the original identification of Uhlig (2005) retains many structural parameters with improbable implications for the systematic response of monetary policy to output, and the updated empirical evidence delivers more contractionary impulse responses of output. Overall, Figure 10 suggests that the dynamics generated by the λ-shock are in-between those generated by sign restrictions and Cholesky-orthogonalisation. The 'price puzzle' is present in the case of the Cholesky-shock and λ-shock induced by the FF55 portfolios, but it is absent in the case of sign restrictions (by construction) and the λ-shock induced by the FF25 portfolios.

D.3   Decomposing Lower Frequency Variation in Consumption

This subsection explores the contribution of the λ-shock and the γ-shock to variation in aggregate consumption at a lower frequency. Instead of decomposing annual consumption growth (as in Figure 3 of the main text), I now decompose the deviation of the level of aggregate consumption from the trend implied by the baseline six-variable VAR model. The results are shown in Figure 11. Consistent with FEV decomposition, the λ-shock contributes to the lower frequency dynamics much more than to the higher frequency dynamics proxied by annual consumption growth.

Specifically, in addition to largely affecting the early 1980s consumption decline, the λ-shock made a sizeable contribution to the persistent expansion of consumption in the 2000s above the long-run trend.

D.4   The γ-Shock and Time-varying Risk Premia

Figure 12 plots the time-series of the term spread along with the counterfactual time-series of the term spread induced by the γ-shock – the variation most relevant to predicting excess returns. While the two series are correlated (0.75), the correlation clearly breaks down in certain periods such as the 1980s. In fact, the historical decomposition will show that aggregate fluctuations during this period were mainly driven by the other orthogonal force in the model, the λ-shock. Overall, Figure 12 illustrates that, while the term spread "tends to be low near business-cycle peaks and high near troughs" (Fama and French,

Figure 11: Decomposing Level Deviations of US Consumption: the Role of λ-Shocks and γ-Shocks
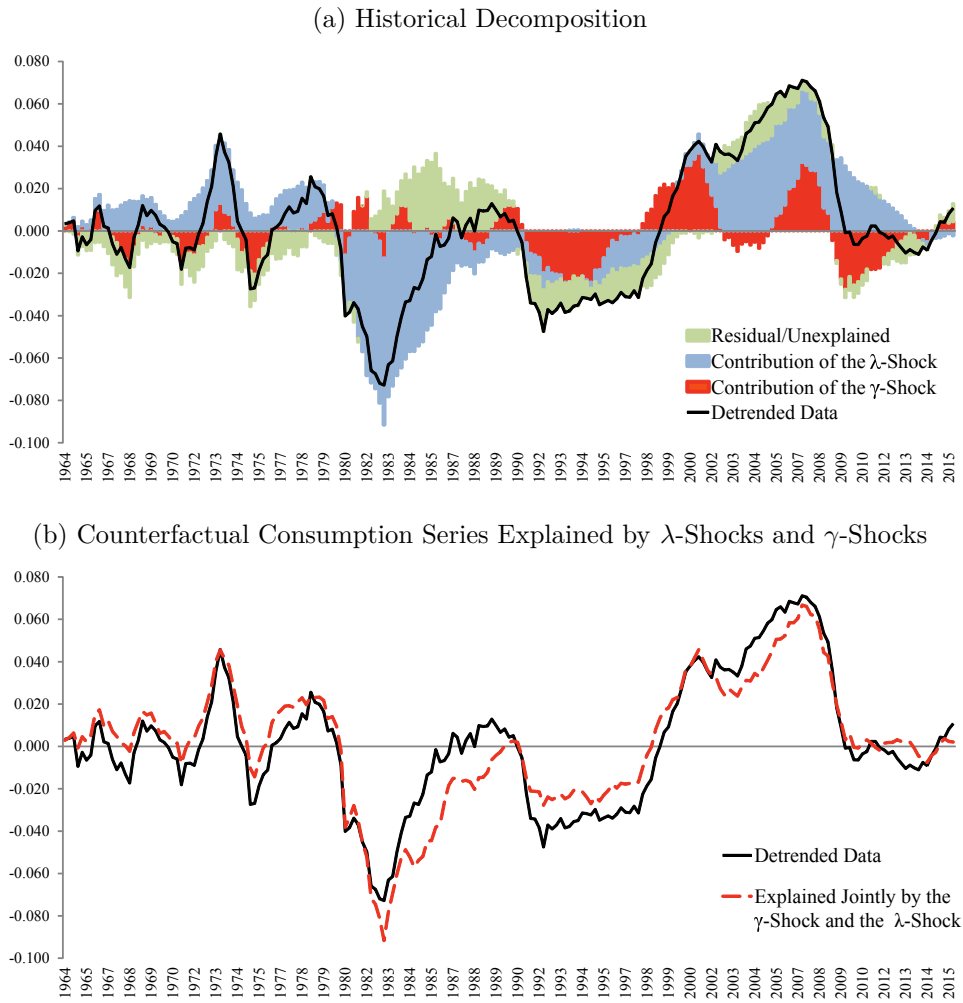
(a) Historical Decomposition

(b) Counterfactual Consumption Series Explained by λ-Shocks and γ-Shocks
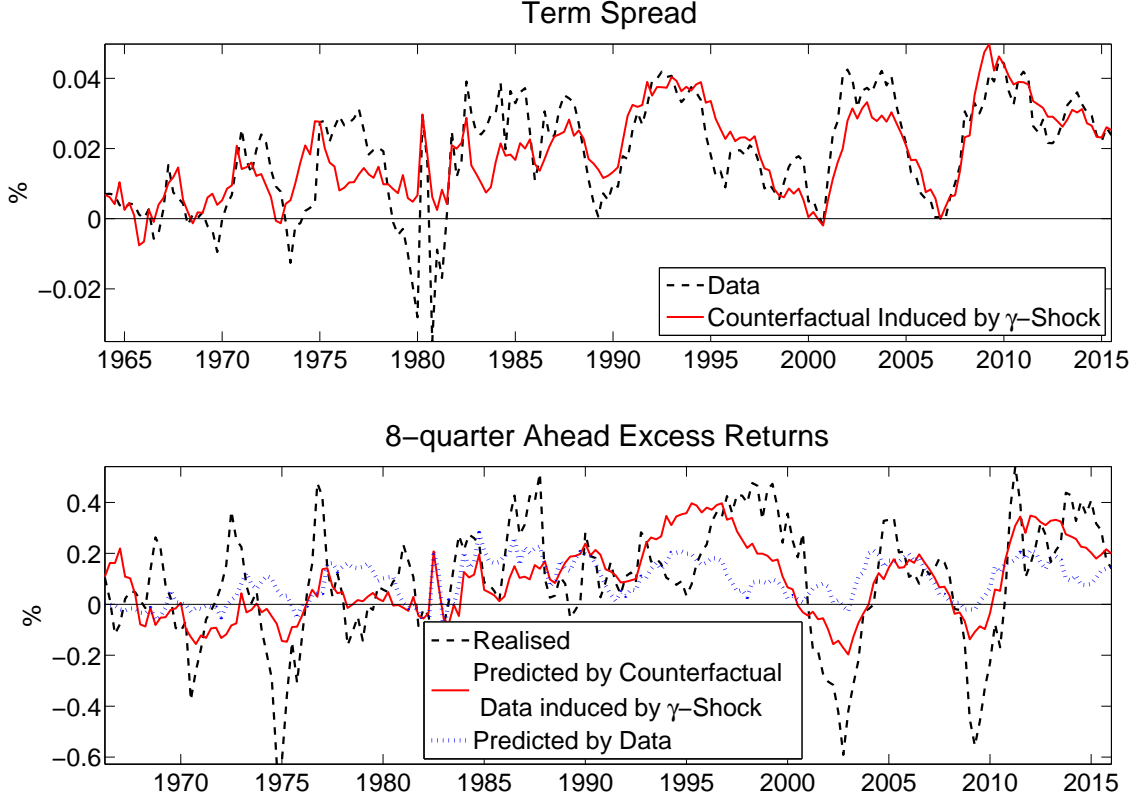

Notes: The figure shows the results implied by the historical decomposition from a VAR(2) estimated on the sample 1963Q3-2015Q3. I use the FF55 to construct the λ-shock, and the CRSP based SP500 return (in excess of the corresponding T-bill rate) to construct the γ-shock. The deterministic trend component, implied by the VAR, is removed from the time-series.

1989), not all business cycles (and variation in return predictors) have been caused by the macroeconomic force which drives time-variation in expected returns.

Naturally, the same logic applies to the estimated time-series of expected excess returns implied by the given predictors. The lower panel of Figure 12 shows the time-series of realised cumulative 8-quarter excess returns (dashed line) against the return forecast implied by the actual time-series of the predictors (dotted line) and the forecast implied by the counterfactual time-series of the predictors induced by the γ-shock (solid line). The coefficients of correlation between realised returns and the data-based forecast and the counterfactual-based forecast are 0.36 and 0.63, respectively. This is another way of

conveying the message summarised in Table 1: variations in return predictors are driven by a range of macroeconomic forces, and not all these forces change the conditional mean of excess returns.

Figure 12: The γ-Shock Induced Counterfactual Term Spread and Forecasting Excess Returns

Notes: The upper panel of the figure shows the term spread (measured as the difference between the 10-year yield and the short-term Treasury bill rate, taken from Goyal and Welch (2008)) along with the counterfactual term spread implied by the γ-shock from a VAR(2) estimated on the sample 1963Q3-2015Q3. The lower panel shows realised 8-quarter cumulative excess returns along with the fitted values from the regression (based on B.16) $r_{t+1}^H = a + \beta_1 FFR_t + \beta_2 DEF_t + \beta_3 TERM_t + \varepsilon_{t+1}$ with $H = 8$ (blue dotted line), and also the fitted values from the regression $r_{t+1}^H = a + \beta_1 \widehat{FFR}_t + \beta_2 \widehat{DEF}_t + \beta_3 \widehat{TERM}_t + \varepsilon_{t+1}$ with $H = 8$ (red solid line), where $\widehat{}$ denotes the counterfactual time-series implied by the γ-shock. The correlation between realised returns and the data-based predicted series and the γ-shock-based predicted series are 0.36 and 0.63, respectively.
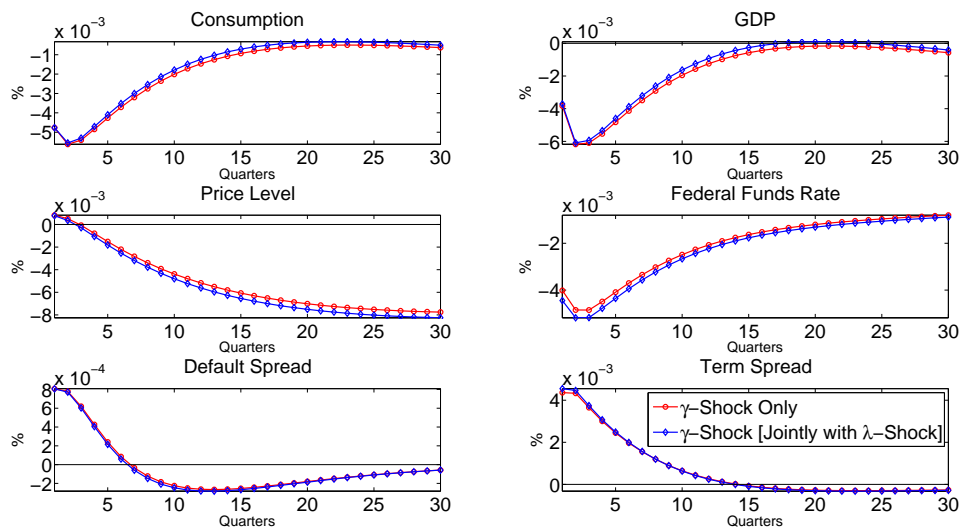
65

E   Robustness and Extensions

E.1   *The Orthogonality of the λ-Shock to the γ-Shock*

Figure 13: Illustrating the Orthogonality of the λ-Shock with respect to the γ-Shock

(a) The λ-Shock
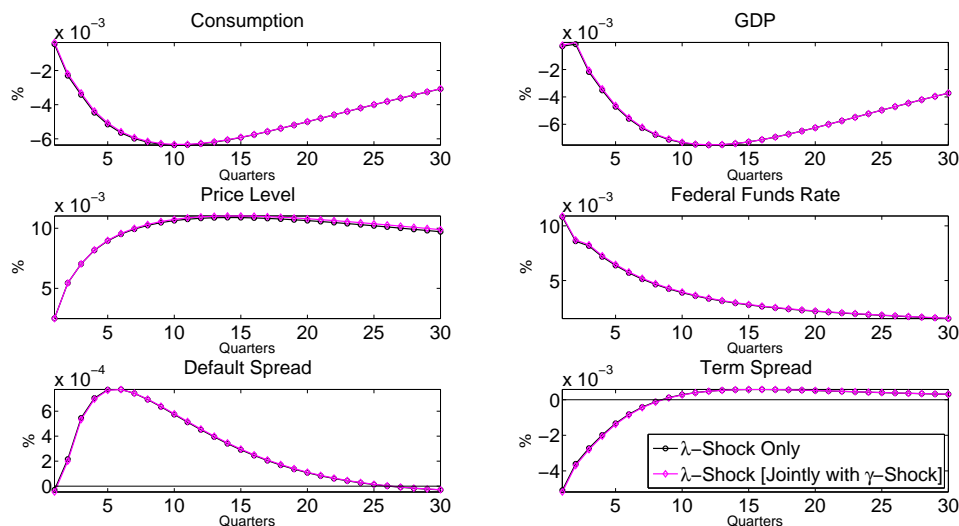
(b) The γ-Shock

Notes: The Figure illustrates the orthogonality of the λ-Shock with respect to the γ-Shock. In the upper panel, the λ-Shock is constructed with and without simultaneously constructing the γ-Shock. In the lower panel, the γ-Shock is constructed with and without simultaneously constructing the λ-Shock. The vertical axes are %-deviations from steady-state, and the horizontal axes are in quarters. The FF55 portfolios were used as test assets, and the sample period is 1963Q3-2015Q3.

E.2 The Recession-Shock and the γ-Shock

As mentioned in the main text, finding the γ-shock which drives time-variation in expected returns may uncover the macroeconomic drivers of recessions. As a robustness exercise, I propose a third orthogonalisation scheme which directly looks for the drivers of recessions without using information on the time-variation in risk premia. The obtained Recession-shock is assumed to be the sole orthogonal macroeconomic contributor to a given historical event, e.g. to the Great Recession. One can then check how this macroeconomic force compares to the γ-shock, i.e. the driver of time-varying risk premia.

Methodologically, the identification of the Recession-shock is based on finding a rotation matrix Q such that one of the impulse vectors associated with the structural impact matrix $\boldsymbol{B}$ will deliver an orthogonal shock with the following property: the historical contribution of this shock to a variable of interest in the VAR is as close as possible to the realised path of this variable over a given horizon. More formally, let Θ denote the set of possible rotations, Y_j is one of the variables in the VAR whose realised path the identified shock is to explain over a pre-specified period, with t_1 and t_2 denoting the start and end of the period. Let $\hat{Y}_{j,t_1:t_2}^{Recession-Shock}$ denote the counterfactual path of variable Y_j which would have realised between time t_1 and t_2, if the only source of business cycle fluctuations had been the Recession-shock. The algorithm is then written as:

$$Q_{opt} = \underset{Q \in \Theta}{\arg\min} \left| Y_{j,t_1:t_2} - \hat{Y}_{j,t_1:t_2}^{Recession-Shock}(Q) \right|. \tag{E.1}$$

In my application, Y_j will be aggregate consumption growth and the period of interest will be t_1=2008Q1, t_2=2009Q4. The problem E.1 can be solved using numerical optimisation techniques. The method draws on the technique of matching impulse response functions (Christiano, Eichenbaum, and Evans, 2005), as well as on recent orthogonalisation schemes that use event-related restrictions (Ludvigson, Ma, and Ng, 2015, 2017; Antolin-Diaz and Rubio-Ramirez, 2018; Ben Zeev, 2018). The proposed orthogonalisation scheme is general, and could be used to explore whether the driving force of one particular historical event can account for the causes of other, seemingly similar, historical events

as well.

This identification theme provides an agnostic way (i) to explore the dynamic effects of the shock that triggered the Great Recession without directly restricting the impact of the shock on macroeconomic variables, and (ii) to check whether previous recessions had been caused by the same macroeconomic force that triggered the Great Recession. It remains an empirical question whether the Recession-shock behaves similarly to the γ-shock. Both the IRF analysis (Figure 14) and the historical decomposition (Figure 15) confirm that the two shocks proxy the same macroeconomic force.

Figure 14: Impulse Responses to a Recession-Shock, γ-Shock and to a λ-Shock
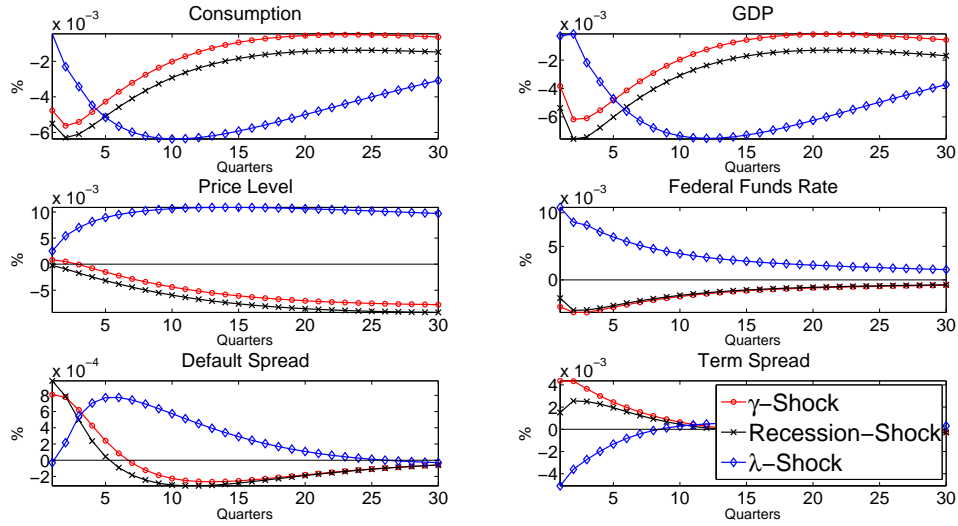
Notes: The vertical axes are %-deviations from steady-state, and the horizontal axes are in quarters. The VAR(2) is estimated over the 1963Q3-2015Q3 period. The shocks are 1 sd.The FF55 portfolios were used as test assets.

Figure 15: Decomposing Level Deviations of US Consumption: the Role of λ-Shocks and Recession-Shocks
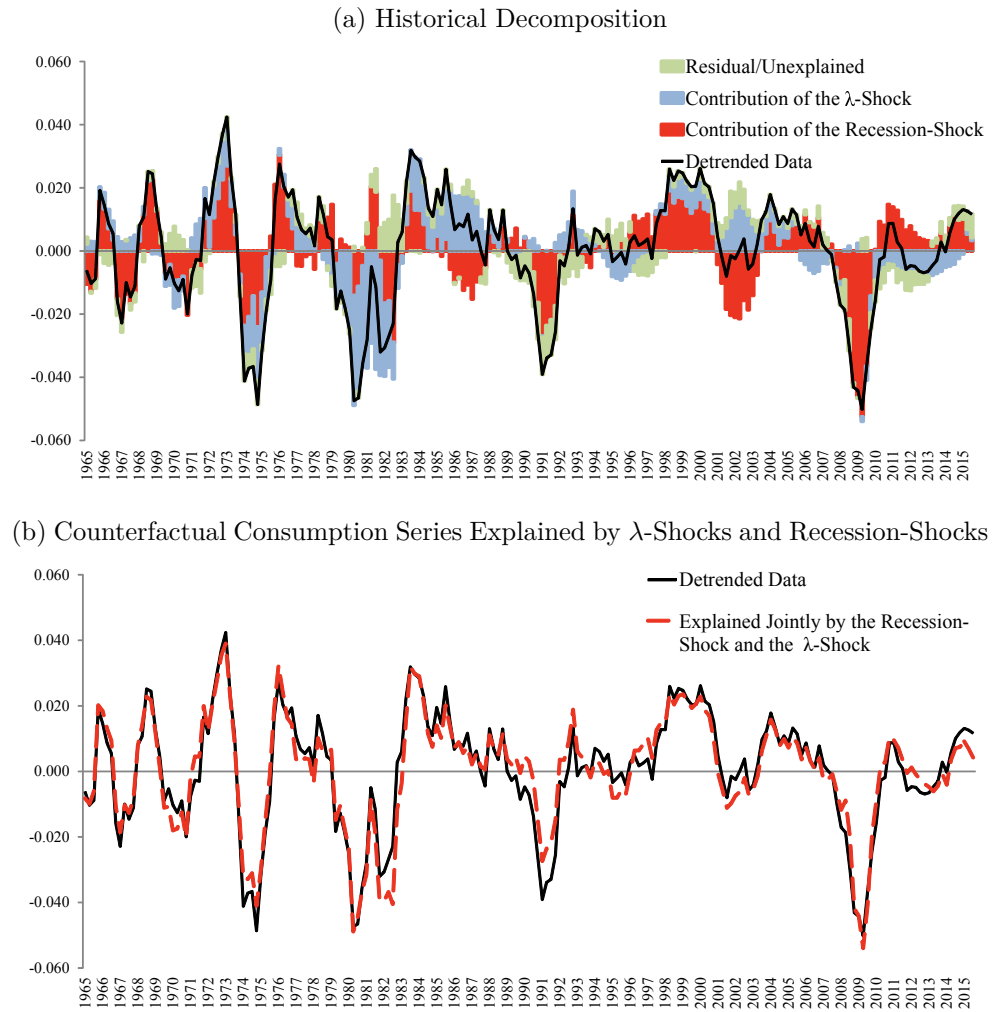
(a) Historical Decomposition

(b) Counterfactual Consumption Series Explained by λ-Shocks and Recession-Shocks

Notes: The figure shows the results implied by the historical decomposition from a VAR(2) estimated on the sample 1963Q3-2015Q3. I use the FF55 to construct the λ-shock. The maximisation of the price of risk and the minimisation of E.1 are done jointly. The blue (red) line is the contribution of the λ-shock (Recession-shock) to the data. The purple line in the bottom panel is the sum of the contributions of the λ-shock and the Recession-shock to the data. The deterministic trend component, implied by the VAR, is removed from the time-series.

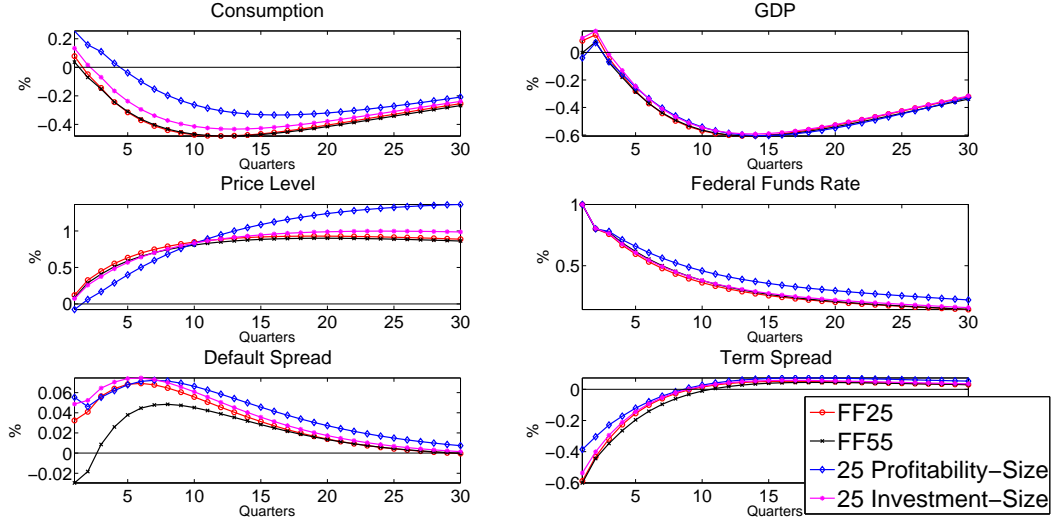## E.3   Other Equity Portfolios and Government Bond Returns

To check the robustness of the baseline results I explore how the behaviour of the λ-shock changes when the same VAR model and the orthogonalisation method are applied to other test assets. A natural choice is the 25 portfolios double sorted on size-profitability and size-investment. These portfolios feature prominently in the most recent empirical asset pricing studies (Fama and French, 2015, 2016). In addition, I also compute the IRFs for the λ-shock implied by the benchmark FF25 portfolios, sorted on size-B/M, that have been the most studied test assets to date.

The upper panel of Figure 16 shows the IRFs for these three sets of equity portfolios along the benchmark FF55 used in the main text. The results suggest that the economic behaviour of the λ-shock implied by these portfolios is very similar to each other. The only quantitative difference is that the baseline results imply a larger peak effect on consumption and a more delayed effect on the default spread compared to Figure 16.

Moreover, I also use government bond returns that are calculated using the zero coupon yield data constructed by Gurkaynak, Sack, and Wright (2007) that fit Nelson-Siegel-Svensson curves on daily data. The parameters for backing out the cross-section of yields are published on their website. The sample period is 1975Q2-2008Q3 so that I have sufficiently large cross-section of yields. I use maturities for $n = 18, 24, \ldots, 120$ months and compute one-month holding period excess returns which I then transform into quarterly series. The resulting 18 bond portfolios are used to construct the λ-shock. The lower panel of Figure 16 shows the results, confirming that the shock responsible for pricing equities is virtually identical to the shock that prices government bonds. This is consistent with the relatively small but growing literature on the joint pricing of stocks and bonds (Lettau and Wachter 2011; Bryzgalova and Julliard 2015; Koijen, Lustig, and Van Nieuwerburgh 2017).

Figure 16: Impulse Responses to a λ-shock, Implied by other Equities vs Bonds

(a) *FF25, FF55, 25 Profitability-Size and 25 Investment-Size* Portfolios

(b) *FF25* and *US Government Bond Returns*
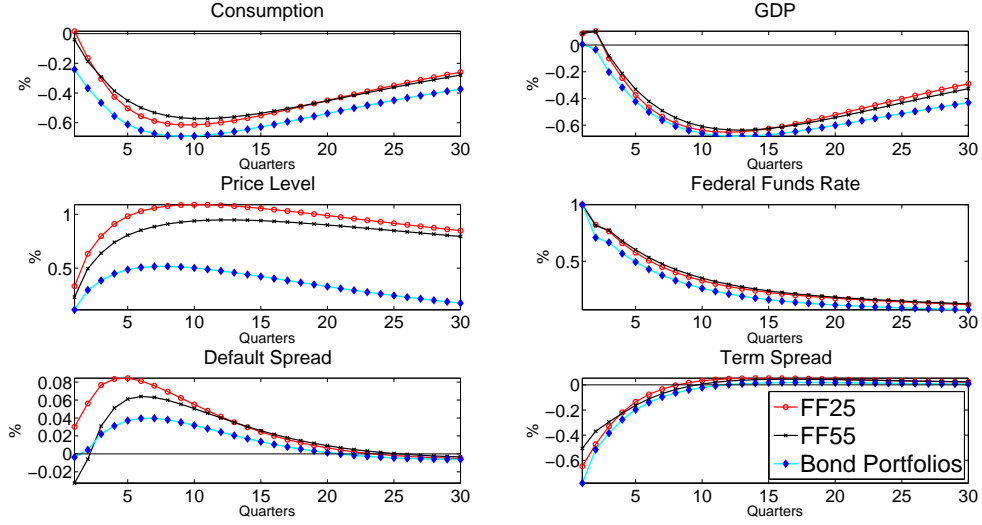
Notes: The vertical axes are %-deviations from steady-state, and the horizontal axes are in quarters. The upper panel uses the VAR(2), as estimated in Subsection 3, and employs alternative equity portfolios to the construction of the λ-shock. The lower panel estimates the same VAR(2) on a subsample 1975Q2-2008Q3, and constructs the λ-shock implied by the FF25 and FF25 portfolios as well as by quarterly holding period excess returns on 18 US treasury bonds with maturities $n = 18, 24, \ldots, 120$ months. I all IRFs are normalised to increase the federal funds rate by 100bp.

E.4   Pricing the Cross-section of Stock Returns

It is worth noting that the focus of this paper is not the asset pricing performance of the λ-shock. The pricing performance of the given λ-shock can easily be improved by changing the specification of the VAR (e.g. including additional variables such as the

excess return on the market[38]). Checking the asset-pricing performance of the λ-shock is therefore only a test as to whether the variables included in the VAR contain information relevant to pricing the given portfolios. Tables 6–9 present the results from the two-pass regression technique of Fama and MacBeth (1973). During this exercise, I treat the uncovered λ-shock as a known factor when estimating the two-pass regression model. To estimate the risk premium associated with the λ-shock, I apply the GMM procedure described in Cochrane (2005) and implemented by Burnside (2011).

Overall, the pricing performance of the VAR (or equivalently, the λ-shock) is comparable with the 3-factor model of Fama and French (1993).[39] Moreover, as explained in the main text (Section B.1), finding the λ-shock implies that the other four orthogonalised shocks have zero covariance with the implied SDF, and therefore the associated estimated prices of risk are numerically zero, as shown in panel B of Tables 6–9. Relatedly, the R^2 statistic (computed based on 4.2) associated with the one-factor model using the λ-shock is identical to the R^2 for the model using any set of five orthogonalised shocks or in fact the model which uses the five reduced-form VAR residuals.

Moreover, the results are also consistent with Lewellen, Nagel, and Shanken (2010) who pointed out the strong factor structure of the FF25 portfolios which makes it relatively easy to find factors that generate high cross-sectional R^2s. Hence, they prescribed to augment the FF25 with the 30 industry portfolios of Fama-French to relax the tight factor structure of the FF25. Indeed, the cross-sectional R^2 drops drastically from 0.82 to 0.19 for the 1-factor model without a common constant, and it drops from 0.65 to 0.09 for the 3-factor model of Fama-French without a common constant. This can be interpreted as the relevant information content of the VAR being much smaller for pricing the FF55 portfolios than for pricing the FF25 portfolios. Nevertheless, augmenting the VAR to improve pricing performance is unnecessary: the macroeconomic shock that captures all relevant information for pricing the cross section (irrespective of whether the information content is relatively small or large) bears virtually the same economic characteristics as

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[38]These results are available upon request.

[39]Applying the 3-factor model to the FF25 portfolios (Table 7) yields similar results to those obtained in the literature (e.g. Petkova (2006)).

the λ-shock using the FF25 portfolios. The IRFs are similar for the λ-shock using the FF25 and the FF55 (Figures 1 and 16), and the time-series of the shocks implied by the two portfolios have a 0.89 correlation coefficient on the 1964-2015Q3 sample.

Table 6: Results from the Two-pass Regressions, FF55 Portfolios

Factor Prices							R^2
Panel A: 1-factor Model with the λ-Shock							
Constant	λ-Shock						
0.78	0.54						0.28
(0.56) [0.64]	(0.23) [0.26]						
	0.84						0.19
	(0.27) [0.35]						
Panel B: 5-factor Model with the λ- and Other VAR Shocks							
	λ-Shock	Shock2	Shock3	Shock4	Shock5	Shock6	
	0.84	0.00	0.00	0.00	0.00	0.00	0.19
	(0.20) [0.25]						
Panel C: The Fama-French 3-factor Model							
Constant	MKT	HML	SMB				
3.13	-1.44	0.73	0.66				0.40
(0.73) [0.76]	(0.92) [0.95]	(0.42) [0.43]	(0.40) [0.40]				
	1.66	0.75	0.50				0.09
	(0.60) [0.60]	(0.42) [0.43]	(0.40) [0.40]				

Notes: This table reports the cross-sectional regressions using the excess returns on the FF55 portfolios. The coefficients are expressed as percentage per quarter. Panel A presents results for the 1-factor model where the identified λ-shock is used as the sole pricing factor. Panel B presents the results for five-factor model using all orthogonalised shocks from the VAR(2). Panel C presents results for the Fama-French 3-factor model. MKT is the market factor, HML is the book-to-market factor and SMB is the size factor. OLS standard errors are in parentheses, whereas standard errors using the Shanken (1992) procedure are in brackets. The R^2 statistic is computed based on 4.2. The sample period is 1964Q1-2015Q3.

Table 7: Results from the Two-pass Regressions, FF25 Portfolios

Factor Prices							R^2
Panel A: 1-factor Model with the λ-Shock							
Constant	λ-Shock						
0.22	1.10						0.83
(0.71) [1.07]	(0.25) [0.38]						
	1.21						0.82
	(0.35) [0.56]						
Panel B: 5-factor Model with the λ- and Other VAR Shocks							
	λ-Shock	Shock2	Shock3	Shock4	Shock5	Shock6	
	1.21	0.00	0.00	0.00	0.00	0.00	0.82
	(0.22) [0.34]						
Panel C: The Fama-French 3-factor Model							
Constant	MKT	HML	SMB				
3.26	-1.62	1.14	0.65				0.73
(0.95) [1.00]	(1.12) [1.16]	(0.41) [0.41]	(0.39) [0.40]				
	1.55	1.21	0.69				0.65
	(0.60) [0.60]	(0.41) [0.42]	(0.39) [0.40]				

Notes: See notes under Table 6.

Table 8: Results from the Two-pass Regressions, 25 Profitability-Size Portfolios

Factor Prices							R^2
Panel A: 1-factor Model with the λ-Shock							
Constant	λ-Shock						
0.03	1.46						0.61
(0.65) [1.17]	(0.49) [0.88]						
	1.48						0.61
	(0.47) [0.86]						
Panel B: 5-factor Model with the λ- and Other VAR Shocks							
	λ-Shock	Shock2	Shock3	Shock4	Shock5	Shock6	
	1.48	0.00	0.00	0.00	0.00	0.00	0.61
	(0.44) [0.79]						
Panel C: The Fama-French 3-factor Model							
	MKT	HML	SMB				
2.78	-1.09	1.03	0.65				0.65
(1.00) [1.04]	(1.17) [1.20]	(0.64) [0.65]	(0.40) [0.40]				
	1.50	1.97	0.63				0.56
	(0.60) [0.60]	(0.73) [0.78]	(0.40) [0.41]				

Notes: See notes under Table 6.

Table 9: Results from the Two-pass Regressions, 25 Investment-Size Portfolios

Factor Prices							R^2
Panel A: 1-factor Model with the λ-Shock							
Constant	λ-Shock						
0.41	1.01						0.59
(0.61) [0.89]	(0.31) [0.45]						
	1.21						0.56
	(0.37) [0.58]						
Panel B: 5-factor Model with the λ- and Other VAR Shocks							
	λ-Shock	Shock2	Shock3	Shock4	Shock5	Shock6	
	1.21	0.00	0.00	0.00	0.00	0.00	0.56
	(0.20) [0.31]						
Panel C: The Fama-French 3-factor Model							
	MKT	HML	SMB				
1.68	0.06	1.77	0.46				0.76
(1.05) [1.11]	(1.19) [1.24]	(0.52) [0.53]	(0.40) [0.40]				
	1.68	2.10	0.43				0.74
	(0.60) [0.60]	(0.49) [0.51]	(0.40) [0.40]				

Notes: See notes under Table 6.

E.5   The λ-shock, Monetary Policy Shocks and TFP News Shocks

An application of the proposed orthogonalisation strategy to the standard equity portfolios lead to the result that the estimated λ-shock bears a close empirical relationship both with TFP news shocks and with monetary policy shocks. As briefly discussed in the main text, a simple explanation for such an ambiguity is that TFP news shocks and monetary policy shocks are highly correlated in the data.

To provide evidence for this, I use the VAR model of Kurmann and Otrok (2013) to identify a monetary policy shock using Cholesky orthogonalisation as done by Sims (1980), Christiano, Eichenbaum, and Evans (1999) and many others in the monetary policy literature. In this case, I deliberately use exactly the same VAR specification as used by Kurmann and Otrok (2013) when they identified a TFP news shock so that I can learn about differences and similarities across the two identification themes without changing the information set. The upper panel of Figure 17 plots the estimated time-series of the TFP news shocks (black dashed line) against the monetary policy shock series identified with Cholesky orthogonalisation (red solid line). The correlation between the two series is strikingly high (0.96), raising serious questions about the orthogonality of these shocks with respect to one another.

Of course, the identification of monetary policy shocks with Cholesky orthogonalisation is only one of the many possible identification strategies. Therefore, I provide additional evidence from the structural model of Smets and Wouters (2007) which is a dynamic stochastic general equilibrium (DSGE) model estimated with Bayesian methods. Monetary policy shocks in this framework are the estimated innovations in a Taylor-type monetary policy rule. The estimated time-series of these structural innovations from the DSGE model are plotted in the lower panel of Figure 17 (blue solid line) against the TFP news shocks (black dashed line) of Kurmann and Otrok (2013). The correlation between these two series is still remarkably high (0.81).

I interpret these findings that the ambiguous characterisation of the estimated λ-shock does not reflect the weakness of my orthogonalisation theme, but is a result of the high empirical correlation between the two, well-known structural disturbances that the

Figure 17: Comparing TFP News Shocks against Monetary Policy Shocks: Results from Kurmann and Otrok (2013)'s VAR and from Smets and Wouters (2007)'s DSGE Model.
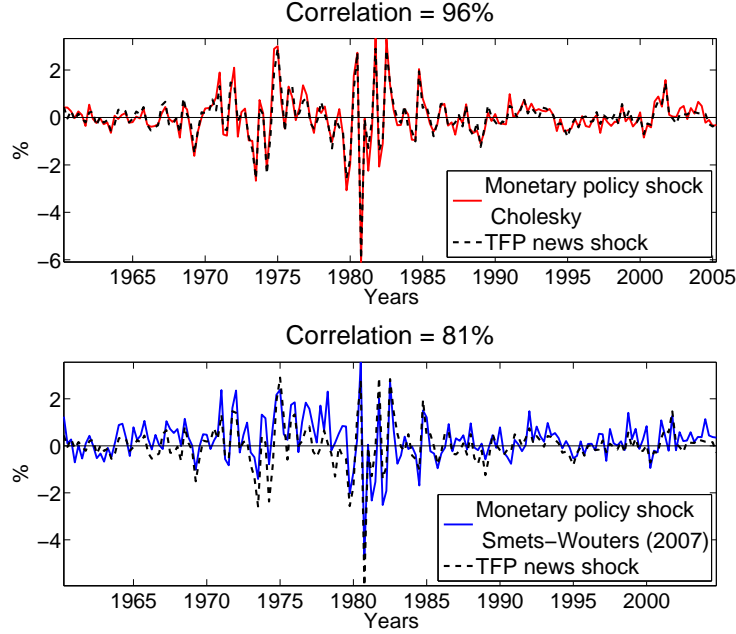
Notes: The TFP news shock series (black dashed line) are the ones plotted in Figure 5 on pp. 2625 of Kurmann and Otrok (2013) who apply the method of Uhlig (2004) to identify a TFP news shock over the period 1959Q2-2005Q2. The monetary policy shock series in the upper panel (red solid line) are identified with Cholesky identification as in Christiano, Eichenbaum, and Evans (1999), using the same variables and lag length as Kurmann and Otrok (2013). The monetary policy shock series in the lower panel (blue solid line) are the estimated time-series of innovations in the Taylor-rule in the DSGE model of Smets and Wouters (2007).

λ-shock resembles. To the best of my knowledge, this empirical regularity has not been documented in the literature yet, and it could be subject to further research. For example, the high empirical correlation may be because of the true correlatedness of these structural disturbances (Curdia and Reis, 2010). An alternative, negative reading of this finding is that it is an identification problem in the literature. To provide some suggestive evidence for this, it is instructive to first review the main assumption of Kurmann and Otrok (2013)'s identification, which builds on the premise that observed technology follows the exogenous process:

$$\log TFP_t = v(L)\,\varepsilon_t^{current} + d(L)\,\varepsilon_t^{news}, \qquad (E.2)$$

which assumes that technology is driven by two uncorrelated innovations: one related to current innovations affecting TFP in t ($\varepsilon_t^{current}$), and the other one (ε_t^{news}) which affects TFP only in $t+1$ onwards. The exogeneity assumption E.2 is used together

with zero restrictions on contemporaneous movement on observed TFP. They implement the identification theme following Barsky and Sims (2011) which in turn builds on Uhlig (2004). This entails searching for a structural shock in the VAR which (i) does not move TFP on impact, and (ii) explains the maximal amount of the forecast error variance in TFP over some forecast horizon (40 quarters).

The question then is whether a small amount of violation of assumption E.2 could deliver "TFP news shocks" that can act as monetary policy shocks. I find that assumption E.2 does seem to be violated empirically. For example, the observed utilisation-adjusted TFP measure of Fernald (2012) that Kurmann and Otrok (2013) uses is considerably cyclical in the data. They use vintages of TFP growth that can have about 0.4–0.5 correlations with output growth, compared to most recent vintages that have a lower contemporaneous correlation (Sims, 2016). Of course, correlation coefficients are only crude measures of cyclicality, and it is more instructive to analyse the conditional dynamic relationship in a VAR.

Figure 18: Assessing the Conditional Cyclicality of the Observed TFP measure: The Effect of a Monetary Policy Shock Using the Romer and Romer (2004) Narrative Measure
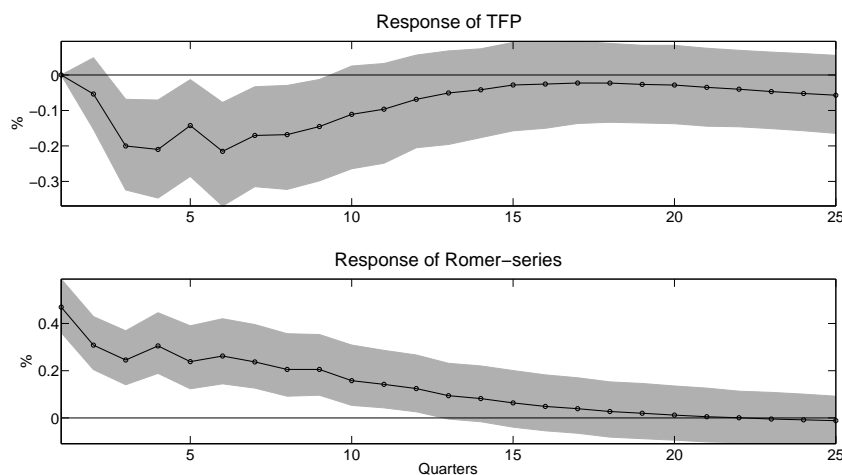
Notes: The Figure shows the IRFs for TFP and the cumulative sum of the Romer and Romer (2004) series (updated by Tenreyro and Thwaites (2016)) from a VAR(4) model. The sample period is 1969Q1-2005Q2. The model includes a constant, but results with a constant and a linear/quadratic trend are very similar. The shaded areas show 95% wild-bootstrapped confidence bands.

Therefore I re-estimate the five-variable VAR model of Kurmann and Otrok (2013) after replacing the short-term interest rate with the cumulative sum of the monetary policy innovations of Romer and Romer (2004) and apply Cholesky orthogonalisation in

order to measure the cyclicality of TFP conditional on exogenous monetary policy shocks. Thereby I follow the most recent practice of estimating monetary policy effects in VAR models using narrative measures (Cloyne and Hurtgen, 2016). To focus the attention to the response of TFP, Figure 18 shows only two of the five sets of the IRFs in response to a one standard deviation contractionary shock to monetary policy. Just like in the case of TFP news shock, the monetary policy shock induces a delayed response in TFP. Moreover, the peak response (based on the point estimate) is around 0.2% in absolute value, which is also very similar to the peak effect of a TFP news shock on TFP. The endogenous reaction of TFP to monetary policy shocks displayed by Figure 18 (i) can make it difficult to apply assumption E.2 to identifying a TFP news shock, and as a result (ii) it may be that the 'identified' TFP news shock (ε_t^{news}) is actually picking up some of these monetary policy effects. This could be one of the explanations behind the large empirical correlations displayed by Figure 17.

E.6   Results from the UK

To check whether the results are similar when looking at countries other than the US, I apply the proposed VAR methodology to UK data, covering the period 1970Q1-2012Q4. One advantage of using data for the UK is related to the availability of both comparable monetary policy shock series and comparable test assets across the two countries in question. To estimate the λ-shock, I use the cross-section of 16 equity portfolios (FF16UK), constructed by Dimson, Nagel, and Quigley (2003). Their portfolio formation closely follows Fama and French (1993), by creating portfolios sorted on size-B/M, whereby breakpoints were applied to the 40th, 60th and 80th percentiles of market capitalisation and to the 25th, 50th and 75th percentiles of book-to-market. To estimate the γ-shock, I use excess returns on the FTSE All-Share index from Chin and Polk (2015), and I also use their series of the Price-Earnings (PE) ratio, as an alternative predictor (given the lack of available CAY measure for the UK).

To keep the empirical model close to the US counterpart presented above, I estimate a VAR(2) model with five macroeconomic variables: log of consumption, log of GDP, log

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of CPI, the Bank of England policy rate, and the term spread defined as the difference between the ten-year and one-year constant maturity Gilt rates. Given the open-economy nature of the UK and also the lack of available time-series for the default spread, I use the dollar-sterling exchange rate as the sixth variable in the VAR.

Forecasting Excess Returns in the UK   As in my baseline model for the US (Table 1), I construct the γ-shock for the UK by maximising the corresponding return forecasting power at 4-quarter horizon, and using the same γ-shock, I compute the results for different horizons ranging from one quarter ahead up to two years ahead. Panel A reports the results using the actual VAR variables as predictors; Panel B shows the results using the Price-Earnings (PE) ratio used by Chin and Polk (2015); Panel C reports the results using the three counterfactual VAR variables induced by the γ-shock; Panel D reports the results using the counterfactual variables induced by all other shocks that are orthogonal to the γ-shock.

Panel A and Panel B of Table 10 are consistent with my baseline results for the US (Table 1) and also corroborate previous evidence for the US on the relevance of valuation ratios to predicting excess results. For example, the last column of the table shows that the PE variable explains around 26% of two-year ahead excess stock market returns; whereas the regression that includes the last three variables of my baseline VAR only explains 16% of excess returns at the same horizon. In contrast, variation in the same VAR variables that is induced by the γ-shock explains about 29% of excess returns at the two-year horizon.

Impulse Response for the UK   The upper panel of Figure 19 shows the IRFs for the λ-shock and for Cholesky-orthogonalised interest rate innovations, implied by the UK data. The results are quantitatively very similar to my baseline Figure 19, implied by the US data, with the IRFs of λ-shock being virtually identical to Cholesky interest rate innovations. The lower panel of Figure 19 shows the results for the γ-shock along with the λ-shock. The dynamics are qualitatively very similar to those found the US. An additional finding is that a contractionary λ-shock causes an appreciation of the nominal

Table 10: Forecasting Excess Returns in the UK

				Forecast Horizon H				
	1Q	2Q	3Q	4Q	5Q	6Q	7Q	8Q
				Model A: Actual VAR Variables				
BoE	0.20	0.44	0.70	0.93	1.19	1.50	1.78	2.11
	(0.84)	(1.03)	(1.26)	(1.51)	(1.73)	(1.84)	(2.03)	(2.30)
TERM	1.07	2.29	3.39	4.23	4.73	5.26	6.03	6.75
	(1.32)	(1.66)	(1.94)	(2.16)	(2.21)	(2.27)	(2.46)	(2.59)
EXCH	-0.06	-0.14	-0.21	-0.27	-0.32	-0.38	-0.45	-0.51
	(-1.41)	(-1.32)	(-1.25)	(-1.15)	(-1.10)	(-1.22)	(-1.43)	(-1.70)
	[0.01]	**[0.04]**	**[0.07]**	**[0.08]**	**[0.09]**	**[0.11]**	**[0.14]**	**[0.16]**
				Model B: PE				
PE	-0.37	-0.79	-1.23	-1.68	-2.10	-2.50	-2.95	-3.30
	(-1.33)	(-1.58)	(-2.13)	(-2.99)	(-3.80)	(-3.80)	(-3.69)	(-3.77)
	[0.02]	**[0.04]**	**[0.07]**	**[0.11]**	**[0.14]**	**[0.18]**	**[0.22]**	**[0.26]**
			Model C: Counterfactual VAR Variables Induced by the γ-Shock					
BoE	0.18	0.31	0.47	0.61	0.75	0.94	1.14	1.35
	(0.76)	(0.69)	(0.75)	(0.79)	(0.84)	(0.95)	(1.10)	(1.30)
TERM	1.28	2.75	4.20	5.59	6.70	7.83	8.97	10.02
	(1.13)	(1.43)	(1.83)	(2.27)	(2.52)	(2.70)	(2.98)	(3.22)
EXCH	-0.12	-0.24	-0.35	-0.46	-0.56	-0.67	-0.78	-0.88
	(-2.00)	(-1.53)	(-1.32)	(-1.23)	(-1.23)	(-1.36)	(-1.58)	(-2.03)
	[0.02]	**[0.06]**	**[0.09]**	**[0.13]**	**[0.16]**	**[0.20]**	**[0.24]**	**[0.29]**
			Model D: Counterfactual VAR Variables Induced by All Other Shocks					
BoE	0.11	0.24	0.39	0.49	0.63	0.86	1.08	1.39
	(0.50)	(0.58)	(0.69)	(0.72)	(0.81)	(0.93)	(1.05)	(1.23)
TERM	0.23	0.39	0.47	0.00	-0.74	-1.14	-0.93	-0.64
	(0.31)	(0.29)	(0.27)	(0.00)	(-0.26)	(-0.31)	(-0.24)	(-0.16)
EXCH	-0.02	-0.04	-0.07	-0.07	-0.06	-0.09	-0.13	-0.17
	(-0.29)	(-0.34)	(-0.38)	(-0.29)	(-0.21)	(-0.29)	(-0.41)	(-0.52)
	[-0.02]	**[-0.01]**	**[-0.01]**	**[-0.01]**	**[-0.00]**	**[0.01]**	**[0.02]**	**[0.03]**

Notes: The table reports results from regressions of excess returns on lagged variables. H denotes the return horizon in quarters. The dependent variable is the sum of H log excess returns on the CRSP based S&P Composite Index. The regressors are one-period lagged values of actual time-series of the Bank of England base rate (BoE), the term-spread (TERM) and the dollar-sterling nominal exchange rate (EXCH) in Model A, the Price-Earnings (PE) ratio of Chin and Polk (2015) in Model B, and the counterfactual time-series of BoE, TERM and EXCH (induced by the γ-shock) from a six-variable VAR(2) estimated over 1970Q1-2012Q4. The γ-shock is constructed so that the corresponding forecast power at the four-quarter horizon is maximised. For each of the three regressions, the table reports the OLS estimates of the regressors, the t-statistics using the Hansen and Hodrick (1980) correction (as implemented in Cochrane (2011)) are in parentheses, and adjusted R^2 statistics are in the bolded square brackets. Both the PE measure and the counterfactual predictors are treated as known variables.

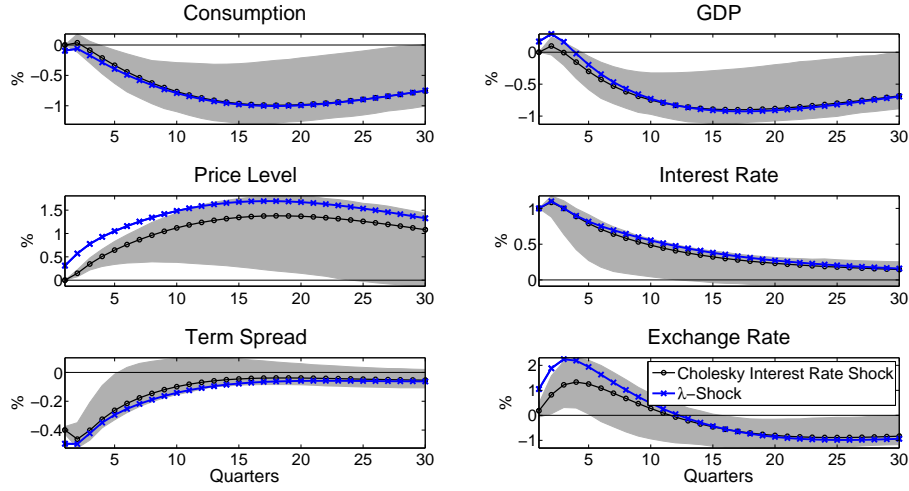exchange rate, whereas a negative γ-shock causes a depreciation.

Moreover, similar to the US case, the estimated time-series of the λ-shock is empirically related to monetary policy shocks. The monetary policy shock series of Cloyne and Hurtgen (2016) and the estimated λ-shock series have around 60% correlation on

the overlapping sample (1975Q1-2007Q4).[40] Overall, the results obtained for the UK are similar to those obtained for the US.
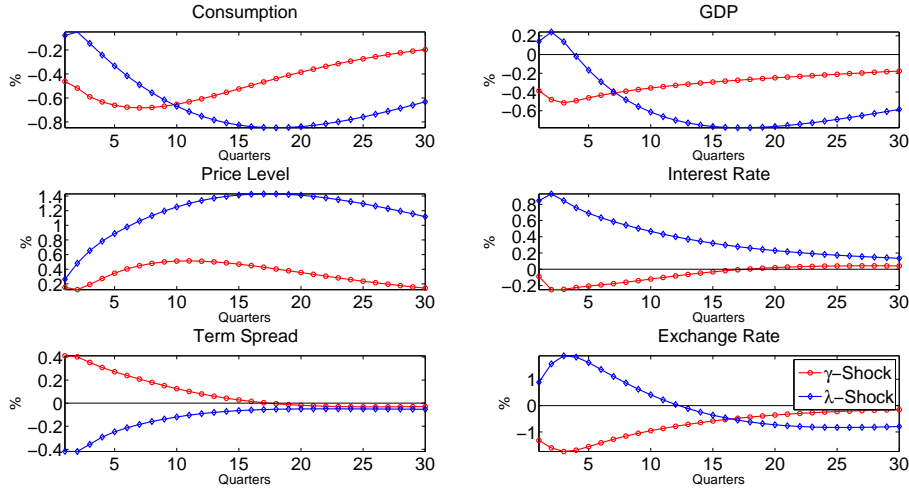
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[40]The methodology of Cloyne and Hurtgen (2016) follows that of Romer and Romer (2004) by trying to eliminate much of the endogenous movement between the interest rate and other macroeconomic variables as well as to control for the effects related to current expectations of future economic conditions.

Figure 19: Results from the UK

(a) Impulse Responses to a λ-shock and to an Interest Rate Shock

(b) Impulse Responses to a γ-Shock and to a λ-Shock