Tail Risks, Investment Horizons, and Asset Prices*

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December 21, 2020

*We appreciate helpful comments from Allan Timmermann, Michal Kejak, Evžen Kočenda, Lukáš Vácha, and participants at various seminars, workshops and conferences. Support from the Czech Science Foundation under the 19-28231X (EXPRO) project is gratefully acknowledged. Note that an earlier version of this paper was circulated and cited under the title "Quantile Spectral Beta: A Tale of Tail Risks, Investment Horizons, and Asset Prices"

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Abstract

We show that the two important sources of risk – market tail risk and extreme market volatility risk – are priced in the cross-section of asset returns heterogeneously across horizons. Specifically, we find that tail risk is a short-term phenomenon whereas extreme volatility risk is priced by investors in the long-term. These risks stem from a dependence structures in the joint distribution of stochastic discount factor and asset returns at various investment horizons that are more general than usually assumed by traditional covariance-based measures. The risk premium we document suggests that investors care about the transitory as well as persistent shocks.

Keywords: Cross-sectional return variation, downside risk, tail risk, frequency, spectral risk, investment horizons

JEL: C21; C58; G12

1 Introduction

Classical result of asset pricing literature states that price of an asset should be equal to its expected discounted payoff. In the Capital Asset Pricing Model (CAPM) introduced by Sharpe (1964), Lintner (1965), Black (1972), we assume that stochastic discount factor can be approximated by return on market portfolio and thus expected excess returns can be fully described by their market betas based on covariance between asset return and market return. While early empirical evidence validated this prediction, decades of the consequent research has called the ability of traditional market beta to explain cross-sectional variation in returns to question. We aim to show that in order to understand formation of expected returns, one has to look deeper into the features of asset returns that are crucial in terms of preferences of a representative investor. We argue that the two important features are risk related to tail events, and frequency-specific (spectral) risk capturing behavior at different investment horizons. To characterize such general risks, we derive novel *quantile spectral* representation of beta that captures covariation between indicator functions capturing fluctuations of different parts of joint risky asset's and market's return distribution over various frequencies. Nesting the traditional beta, the new representation captures *tail*-specific as well as horizon-, or frequency-specific *spectral* risks.

Intuitively, covariation stemming from (extremely) negative return of the risky asset and (extremely) negative returns of market known as downside risk in the literature, should be positively compensated. While early literature (Ang et al., 2006) empirically confirms the premium for bearing downside risk, Levi and Welch (2019) concludes that estimated downside betas do not provide superior predictions compared to standard beta. More recently, Bollerslev et al. (2020) argue that we need to look at finer representation allowing combinations of positive and negative asset and market returns, and suggest how such semibetas are priced.

In this paper we argue that these attempts fail to fully account for more subtle implications arising from heterogeneity of investment horizons. An asset's drop that covary with drop of market, and at the same time is also low-frequency event with large persistence should be priced by investors differently in comparison to such extreme situation due to high frequency, transitory event. While in the first situation investors will be pricing a persistent crash resulting in the long-term fluctuations in the quantiles of the market's and risky asset's joint distribution, in the latter case the investor cares about transitory crash resulting in the short-run fluctuations. This essentially means that a covariance between the risky asset and discount factor will not only be different across all parts of the joint distribution but will be different across various investment horizons. Intuitively, these co-occurrences of tail events will have short-term or long-term effect on the marginal utility of investors. Looking on beta representation that will capture such information empirically will be also informative for the results of the rare disaster literature (Barro, 2006).

Economists have long recognized that decisions under risk are more sensitive to changes in probability of possible extreme events compared to probability of a typical event. The expected utility might not reflect this behavior since it weights probability of outcomes linearly. Consequently, CAPM beta as an aggregate measure of risk may fail to explain the cross-section of asset returns. Several alternative notions emerged in the literature. Mao (1970) presents survey evidence showing that decision makers tend to think of risk in terms of the possibility of outcomes below some target. For an expected utility maximizing investor, Bawa and Lindenberg (1977) has provided a theoretical rationale for using lower partial moment as a measure of portfolio risk. Based on the rank-dependent expected utility due to Yaari (1987), Polkovnichenko and Zhao (2013) introduce utility with probability weights and derive corresponding pricing kernel. As mentioned earlier, Ang et al. (2006); Lettau et al. (2014) argue that downside risk – risk of negative returns – is priced across asset classes and is not captured by CAPM betas. Further, Farago and Tédongap (2018) extend the results using general equilibrium model based on generalized disappointment aversion and show that downside risks in terms of market return and market volatility are priced in the cross-section of asset returns.[1]

The results described above leads us to question the role of the expected utility maximizers in asset pricing. A recent strand of literature solves the problem by considering quantile of the utility instead of its expectation. This literature complements the previously described empirical findings focusing on downside risk as it highlights the notion of economic agents particularly averse to outcomes below some threshold compared to outcomes above this threshold. The concept of a quantile maximizer and its features was pioneered by Manski (1988), and later axiomatized by Rostek (2010). Most recently, de Castro and Galvao (2019) develop a model of quantile optimizer in a dynamic setting. A different approach to emphasizing investor's aversion towards least favorable outcomes defines theory based on Choquet expectations. This approach is based on distortion function that alters probability distribution of future outcomes by accentuating probabilities associated with least desirable outcomes. This approach was utilized in finance, for example, by Bassett Jr et al. (2004).

Whereas aggregating linearly weighted outcomes may not reflect the sensitivity of

[1]In addition, it is interesting to note that equity and variance risk premium are also associated with compensation for jump tail risk (Bollerslev and Todorov, 2011). More general exploration of asymmetry of stock returns is provided by Ghysels et al. (2016), who propose a quantile-based measure of conditional asymmetry and show that stock returns from emerging markets are positively skewed. Conrad et al. (2013) use option price data and find a relation between stock returns and their skewness. Another notable approach uses high frequency data to define realized semivariance as a measure of downside risk (Barndorff-Nielsen et al., 2008). From a risk-measure standpoint, dealing with negative events, especially rare events, is highly discussed theme in both practice and academics. The most prominent example is Value-at-Risk (Adrian and Brunnermeier, 2016; Engle and Manganelli, 2004).

investors to tail risk, aggregating linearly weighted outcomes over various frequencies, or economic cycles may not reflect risk specific to different investment horizons. One can suspect that an investor cares differently about short-term and long-term risk according to their preferred investment horizon. Distinguishing between long-term and short-term dependence between economic variables was proven to be insightful since the introduction of co-integration (Engle and Granger, 1987). Frequency decomposition of risk thus uncovers another important feature of risk which cannot be captured solely by market beta which captures risk averaged over all frequencies. This recent approach to asset pricing enables to shed light on asset returns and investor's behaviour from a different point of view highlighting heterogeneous preferences. Empirical justification is brought by Boons and Tamoni (2015) and Bandi and Tamoni (2017) who show that exposure in long-term returns to innovations in macroeconomic growth and volatility of matching half-life is significantly priced in variety of asset classes. The results are based on decomposition of time series into components of different persistence proposed by Ortu et al. (2013). Piccotti (2016) further sets portfolio optimization problem into frequency domain using matching of utility frequency structure and portfolio frequency structure, and Chaudhuri and Lo (2016) present approach to constructing mean-variance-frequency optimal portfolio. This optimization yields mean-variance optimal portfolio for a given frequency band, and thus optimizes portfolio for a given investment horizon.

From a theoretical point of view, preferences derived by Epstein and Zin (1989) enables to study frequency aspects of investor's preferences, and quickly became a standard in the asset pricing literature. With the important results of Bansal and Yaron (2004), long-run risk started to enter asset pricing discussions. Dew-Becker and Giglio (2016) investigate frequency-specific prices of risk for various models and conclude that cycles longer than business cycle are significantly priced in the market. Other papers utilize frequency domain and Fourier transform to facilitate estimation procedures for parameters hard to estimate using conventional approaches. Berkowitz (2001) generalizes band spectrum regression and enables to estimate dynamic rational expectations models matching data only in particular ways, for example, forcing estimated residuals to be close to white noise. Dew-Becker (2016) proposes spectral density estimator of long-run standard deviation of consumption growth, which is a key component for determining risk premiums under Epstein-Zin preferences, and shows its superior performance compared to the previous approaches. Crouzet et al. (2017) develop model of multi-frequency trade set in frequency domain and show that restricting trading frequencies reduces price informativeness at those frequencies, reduces liquidity and increases return volatility.

The debate clearly indicates that the standard assumptions leading to classical asset pricing models do not correspond with reality. In this paper, we suggest that more general pricing models have to be defined and they should take into consideration both asymmetry of dependence structure among stock market, and relation of asymmetry to different behavior of investors at various investment horizons.

The main contribution of this paper is threefold. First, based on the frequency decomposition of covariance between indicator functions, we define the *quantile spectral beta* of an asset capturing frequency-specific tail risks and corresponding ways of measuring the beta. The newly defined notion of beta can be viewed as disaggregation of a classical beta to a frequency-, and tail- specific beta. With this notion, we describe how extreme market risks are priced in the cross-section of asset returns at various horizons. We define frequency-specific tail market risk that captures dependence between extremely low market and asset returns, as well as extreme market volatility risk that is characterized by dependence between extremely high increments of market volatility and extremely low asset returns. Second, we empirically motivate emergence of such types of risks in the cross-section of asset returns. Third, we estimate models that provide considerable improvements in explaining cross-section of asset returns. Results suggest that tail risk is priced in the cross-section of asset returns in the short-term, while extreme market volatility risk is priced mainly in the long-term. The result holds also when we control for popular factors including those moment-based that are designed to capture the asymmetric features as well as popular down side risk models (Ang et al., 2006; Lettau et al., 2014; Farago and Tédongap, 2018). We also discuss how our new beta representation relates to other risk measures. Finally, we document that the final model capturing tail specific risks across horizons significantly outperforms the other competing models that capture downside risks.

The rest of the paper has the following structure. Section 2 motivates the importance of tail risks across horizons. Section 3 introduces the concept of quantile spectral betas, Section 4 defines the empirical models used for testing significance of extreme risks, and Section 5 conducts the empirical analysis as well as robustness checks. Section 6 then concludes. In Appendix A, we detail the estimation procedure of the quantile spectral betas, and Appendix B describes specifications of the competing measures of risk, and the rest of the Appendix reports results from the robustness checks.

2 Motivation: Why Should We Care About Tail Risks across Horizons

The empirical search for explanation of why different assets earn different average returns centers around return factor models arising from the Euler equation. With the only assumption of 'no arbitrage', a stochastic discount factor m_{t+1} exists and, for the ith excess return $r_{i,t+1}$ satisfies $\mathbb{E}[m_{t+1}r_{i,t+1}] = 0$, hence

$$\mathbb{E}[r_{i,t+1}] = \frac{\mathbb{C}ov(m_{t+1}r_{i,t+1})}{\mathbb{V}ar(m_{t+1})}\left(-\frac{\mathbb{V}ar(m_{t+1})}{\mathbb{E}[m_{t+1}]}\right) = \beta_i\lambda \tag{1}$$

where loading β_i can be interpreted as exposure to systematic risk factors, and λ as the risk price associated with factors. Equation 1 assumes that risk premium of an asset or a portfolio can be explained by its covariance with some reference economic or financial variable such as consumption growth or return on market portfolio. This simple pricing relation also assumes that independent common sources of systematic risk exist in the economy, and the exposure to them can explain the cross-section of asset returns.[2] This leads to the so-called factor fishing phenomenon, which tries to identify additional risk factors beside the traditional market factor assumed by CAPM using linear combination of factors that are assumed to have non-zero covariance with an risky asset, as well as they are assumed to be independent between each other.

Covariance between the two variables of interest,

$$\gamma_{i,j}^{k} = \mathbb{C}ov\big(r_{j,t+k}, r_{i,t}\big) \equiv \mathbb{E}[(r_{j,t+k} - \bar{r}_j)(r_{i,t} - \bar{r}_i)], \tag{2}$$

that is central to the asset pricing literature, may not be sufficient in the cases in which the investor cares about different parts of the distribution of her future wealth differently, or in case an investor cares about a specific investment horizons. Empirical literature assumes silently that the risk factors aggregate information over the distribution of returns as well as investment horizons. Part of the literature tracing back to early work by Roy (1952); Markowitz (1952); Hogan and Warren (1974); Bawa and Lindenberg (1977) argue that the reason we do not empirically find the support for the above thinking is that pricing relationship is fundamentally too simplistic. If investors are averse to volatility only when it leads to losses, not gains, the total variance as a relevant measures of risk should be disaggregated.

Later work by Ang et al. (2006); Lettau et al. (2014); Farago and Tédongap (2018) show that investors require additional premium as a compensation for exposures to disappointment-related risk factors called downside risk. Recently, Lu and Murray (2019) argue that bear risk capturing the left tail outcomes is even more important, and Bollerslev et al. (2020) introduce betas based on semi-covariances. In contrast to the promising

[2]E.g., this is the cornerstone of Arbitrage pricing theory (APT) of Ross (1976).

results, Levi and Welch (2019) conclude that estimated downside betas do not provide superior predictions compared to standard aggregated beta, partially due to the difficulties of accurately determining downside betas from daily returns. With a similar argument of too simplistic pricing relation, another part of the literature looks at frequency decomposition and explores the fact that risk factors being claims on the consumption risk should be frequency dependent since consumption has strong cyclical components (Bandi et al., 2018; Dew-Becker and Giglio, 2016).

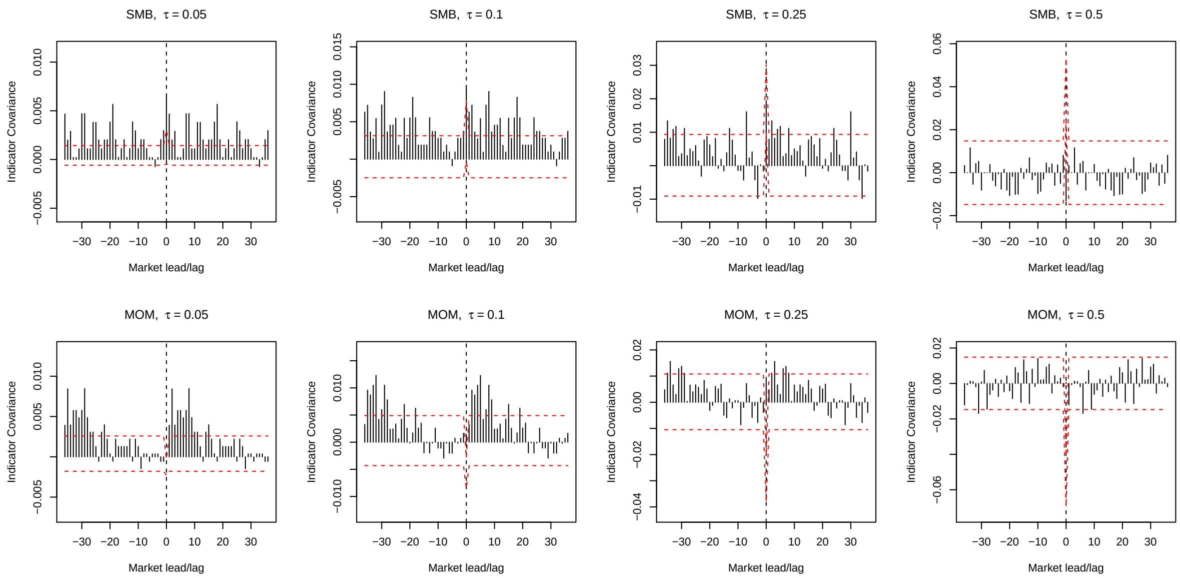
Being aware of such departures from too simplistic assumptions in the data, we need to look at more general dependence measures since a simple covariance aggregating dependence across distribution as well as investment horizons will not be a sufficient measure of (in)dependence.

To illustrate this discussion, we consider dependence between market returns and a popular small-minus-big portfolio (SMB) as well as momentum portfolio (MOM) respectively. While literature assumes these factors represent two independent sources of risk with contemporaneous correlation between them and the market being rather small, investigating the dependence in various parts of their joint distribution across different lags and leads reveals interesting relations. Instead of aggregate covariance between market return and a factor portfolio, figure 1 depicts tail, and lead/lag specific covariation for a threshold value given by τ-quantile of market return and given lead/lag k of the following form

$$\mathbb{C}ov\big(I\{r_{m,t-k} \leq q_{r_m}(\tau))\}, I\{r_{i,t} \leq q_{r_m}(\tau)\}\big), \tag{3}$$

where $r_{m,t}$ is return of market factor, $r_{i,t}$ is return of either SMB or MOM portfolio, $I\{.\}$ is an indicator function and q_{r_m} is quantile function of the market return. This simple measure captures the probability of both returns being below some threshold value in some time interval given by lead/lag k. This can be seen from the fact that

Figure 1: *Dependence Structure between Market and SMB and MOM Factor Portfolios.* Plots display covariance in tail and across horizon defined by Eq. 3 that measures general dependence between market return and SMB and MOM factor, respectively. Dashed lines represents 95% confidence intervals under the null hypothesis that the two series are jointly normally distributed correlated random variables. Data are sampled with monthly frequency.



$\mathbb{C}ov\big(I\{r_{m,t-k} \leq q_{r_m}(\tau))\}, I\{r_{i,t} \leq q_{r_m}(\tau)\}\big) = Pr\big\{r_{m,t-k} \leq q_m(\tau), r_{i,t} \leq q_m(\tau)\big\} - \tau\tau_i$. So, this dependence essentially measures additional probability over the independence copula of both variables being below some threshold value.

Looking at the median dependence of market return with SMB or MOM portfolio returns (right column of plots for $\tau = 0.5$), we observe that dependence can be fully characterized by rather weak contemporaneous covariation between market and the SMB and MOM portfolio returns, respectively since no significant relation exists at any lead or lag in the relationship.[3] Moving our attention towards the left tail of the joint distribution, more complicated dependence structures emerge. Departure from jointly Gaussian

[3]Note that the dashed lines in the figure represent confidence intervals under the null hypothesis that the two series are jointly normally distributed correlated random variables.

distribution is strongest in the left tail (left column of plots for $\tau = 0.05$). The co-occurences of large negative market returns with large negative SMB or MOM portfolio returns are significant and exist at various leads/lags.

For example, if we look at the dependence between market and SMB in the 5% tail, we observe that if market is below this threshold, there is also significant probability that the SMB portfolio will be below this threshold with some delay. Similarly, the SMB downturn precedes the market downturn with significant probability.[4] So, instead of arguing that the SMB factor proxies for an independent economic risk,[5] the results suggest that the SMB portfolio captures more complicated market tail risk at some specific horizons.

In other words, the left tail dependence shows that extreme market drop is correlated with extreme negative returns of SMB. This illustrates large negative market return being correlated with the situation when big companies largely outperform small ones in the SMB portfolio. Hence we document a joint probability of co-occurrence of the market extreme left tail event and big companies outperforming small ones situation leading to the increase of the default risk in the economy (Chan et al., 1985). An important feature of the dependence not documented by earlier studies is its persistence structure shown by autocorrelations and the same strength for leading one each other. At the same time, while momentum is negatively correlated with market, second row of the figure 1 show significant lead-lag relationship of the momentum factor and stock market pointing us to the intuition that extremely low market returns are cross-correlated with the low momentum companies outperforming high momentum.

This may lead us to the conclusion that such general dependence structures can hardly be described by the traditional contemporaneous correlation-based measures. The illustration suggests that there is no need for many factors to explain the average asset

[4]Similar lead/lag investigation regarding business cycle indicators is performed in Backus et al. (2010).

[5]E.g., Chan et al. (1985) argue that SMB proxies for default risk in the economy.

return, as a carefully measured exposure to the market risk can capture the risk the investors care about. A natural way how to summarize the dependence across these lead/lag relationships, is to employ frequency analysis and summarize precisely this joint structure for specific horizons.

From an economic perspective, it is reasonable to assume that future marginal utility is affected by realization of low quantile returns today, as this event may lead, for example, to a bankruptcy or in other way significantly shape behavior of economic agent in the future. In other words, extreme market events can have either short-run or long-run effect on the marginal utility of investors. Previous studies however fail to fully account for the horizon specific information in tails while one of the main reasons turns to be inability to measure such risks. Here we propose robust methods for measurement of such risks , and we argue that exploring the risk related to tail events as well as frequency-specific risk is crucial.

To see how tail specific risks are priced across horizons by investors, we proceed as follows. First, we define quantile risk measure based on covariance between indicator functions, which has natural economic interpretation in terms of probabilities. Second, we introduce its frequency decomposition, and combine these two frameworks into quantile spectral risk measure, which is the building block for our empirical model. This measure enables to robustly test for the presence of extreme market risks over various horizons in the asset prices. The aim is not to convince the reader that the functional form of the preferences follows precisely our model, but to show that there is a heterogeneity in the weights that investors put to the risk for different investment horizons and different parts of the distribution of their future wealth. By estimating prices of risk for short- and long-term part, we are able to identify the horizon the investor care most about. Moreover, by estimating prices of risk for various threshold values, we are able to identify the part of the joint distribution towards which is the investor the most risk averse.[6] This is done

[6]Our investigation complements work of Delikouras (2017) and Delikouras and

by controlling for CAPM beta and the influence of these new measures is measured as an incremental information over simplifying assumptions that lead to the CAPM beta asset pricing models.

3 Measuring the Tail Risks across Horizons: A Quantile Spectral Beta

As argued above, risk premium of an asset or a portfolio can be explained by its covariance with some reference economic or financial variable such as consumption growth or return on market portfolio. This measure may not be sufficient in the cases in which the investor cares about different parts of the distribution of her future wealth differently with horizon-specific preferences. Here we formalize the discussion and provide a more general measures that are able to capture the departures from normality, and will serve in defining the horizon specific tail risks.

3.1 Tail Risk

Since we are interested in pricing extreme negative events, we want to measure dependence and risk in lower quantiles of the joint distribution, and propose a quantity of the following form

$$\gamma^k_{i,m}(\tau) \equiv \mathbb{C}ov\Big(I\{m_{t+k} \leq q_m(\tau)\}, I\{r_{i,t} \leq q_m(\tau)\}\Big), \tag{4}$$

where m_t and $r_{i,t}$ are two time series of random variables, $q_m(\tau)$ is a quantile function of m_t for $\tau \in (0,1)$, and $I\{A\}$ is indicator function of event A. The measure is given by

Kostakis (2019). These studies investigate the position of the reference point of consumption growth, and show that its correct location is crucial for fit of the model based on Generalized disappointment aversion.

the covariance between two indicator functions and can fully describe joint distribution of the pair of random variables m and r_i. If distribution functions of the variables are continuous, the quantity is essentially difference between copula of pair m and r_i and independent copula, i.e. $Pr\big\{m_{t+k} \leq q_m(\tau), r_{i,t} \leq q_m(\tau)\big\} - \tau\tau_i$ where $\tau_i = F_{r_i}\{q_m(\tau)\}$ and F_{r_i} is cdf of r_i. Thus, covariance between indicators measures additional information from the copula over independent copula about how likely is that the series are jointly less or equal to a given quantile of the variable m. It enables to flexibly measure both cross-sectional structure and time-series structure of the pair of random variables.

Note that the quantity introduced in Eq. (4) can be further generalized in the way that one can replace $q_m(\tau)$ by some general threshold values derived from distribution of a reference variable. Being below the threshold value corresponds to an inconvenient event for the investor and thus this measure of dependence adequately captures the corresponding risk. In our model, we set threshold values to be equal. In case the stochastic discount factor is linear in factors and we consider the market return as a risk factor, we further look at the dependence between asset returns and market returns $r_{m,t}$, and the threshold values are based on quantiles of market returns $q_m(\tau) = q_{r_m}(\tau)$. A market beta associated with the tail risk[7] is then defined using quantity given in 4 for

[7]This is indicator beta defined in Equation 3 evaluated at lag $k = 0$. See Figure 1, which depicts the market tail risk beta for two commonly researched factors.

$k = 0$ and normalized by variance of the indicator function of the market return[8]

$$\beta_i(\tau) \equiv \frac{\mathbb{C}ov\big(I\{r_{m,t} \leq q_{r_m}(\tau))\}, I\{r_{i,t} \leq q_{r_m}(\tau)\}\big)}{\mathbb{V}ar(I\{r_{m,t} \leq q_{r_m}(\tau)\})}. \tag{5}$$

Note that $\mathbb{V}ar(I\{r_{m,t} \leq q_{r_m}(\tau)\}) = \tau(1-\tau)$.

3.2 Tail Risks across Horizons: A Quantile Spectral Beta

It is further natural to assume that economic agents care not only about different parts of the wealth distribution, but they care differently about long-, and short-term investment horizon in terms of expected returns and associated risks. Investors may be interested in long-term profitability of their portfolio and do not concern with short-term fluctuations. Frequency-dependent features of an asset return then play an important role for an investor. Bandi and Tamoni (2017) argue that covariance between two returns can be decomposed into covariance between disaggregated components evolving over different time scales, and thus the risk on these components can vary. Hence, market beta can be decomposed into linear combination of betas measuring dependence at various scales, i.e., dependence between fluctuations with various half-lives. Frequency specific risk at given time plays an important role for determination of asset prices, and the price of risk in various frequency bands may differ, this means that the expected return can be decomposed into linear combination of risks in various frequency bands.

A natural way how to decompose covariance between two assets into dependencies

[8]This risk measure is similar to the tail risk measure of Schreindorfer (2019), which is also function of τ quantile threshold of consumption growth; correlation between asset return and consumption growth is then computed conditional on realizations of consumption growth below the threshold. It is also related to the negative semibetas of Bollerslev et al. (2020), which estimates the dependence between market return and asset return conditional on the *co-occurrence* of negative events for both market and asset.

over different horizons is in the frequency domain. A frequency domain counterpart of cross-covariance is obtained as Fourier transform of the cross-covariance functions $S_{i,m}(\omega) = \frac{1}{2\pi}\sum_{k=-\infty}^{\infty}\gamma_{i,m}^{k}e^{-\mathrm{i}k\omega}$. Conversely, cross-covariance can be obtained from inverse Fourier transform of its cross-spectrum as $\gamma_{i,m}^{k} = \int_{-\pi}^{\pi} S_{i,m}(\omega)e^{\mathrm{i}k\omega}d\omega$ where $S_{i,m}(\omega)$ is cross-spectral density of random variables $r_{i,t}$ and m_t, and γ^k is cross-covariance function given by Eq. (2) and $\mathrm{i} = \sqrt{-1}$.

This representation of covariation allows us to decompose the covariance and variance into frequency components and disentangle the short-term from the long-term dependence. Then, beta for an asset i and factor m can be decomposed to a given frequency ω as

$$\beta_i \equiv \frac{\mathbb{C}ov(m_t, r_{i,t})}{\mathbb{V}ar(m_t)} = \int_{-\pi}^{\pi} w(\omega)\frac{S_{i,m}(\omega)}{S_m(\omega)}d\omega = \int_{-\pi}^{\pi} w(\omega)\beta_i(\omega)d\omega$$

where $w(\omega) = \frac{S_m(\omega)}{\int_{-\pi}^{\pi} S_m(\omega)d\omega}$ represent weights. Using similar approach, Bandi and Tamoni (2017) estimate price of risk for different investment horizons and show that investors posses heterogeneous preferences over various economic cycles instead of looking only on averaged quantities over the whole frequency spectrum.

To uncover the general dependence structures, we propose to study the Fourier transform of covariance of indicator functions $\gamma_{i,m}^{k}(\tau)$ instead. In this way, one can quantify the horizon specific risk premium across the joint distribution. To define the new beta representation that will allow us to characterize such general risks, we use the so-called quantile spectral densities introduced by Baruník and Kley (2019).

The cornerstone of the new beta representation lies in quantile cross-spectral density kernels which are defined as

$$f_{i,m}(\omega;\tau) \equiv \frac{1}{2\pi}\sum_{k=-\infty}^{\infty}\gamma_{i,m}^{k}(\tau)e^{-\mathrm{i}k\omega} = \frac{1}{2\pi}\sum_{k=-\infty}^{\infty}\mathbb{C}ov\big(I\{m_{t+k} \le q_m(\tau)\}, I\{r_{i,t} \le q_m(\tau)\}\big)e^{-\mathrm{i}k\omega} \tag{6}$$

with $= \tau \in (0,1)$ and . A quantile cross-spectral density kernel is obtained as a Fourier transform of covariances of indicator functions defined in Equation 4, and will allow us to define beta that will capture the tail risks as well as spectral risks.

A *quantile spectral* (QS) betas for a given τ quantile of market returns are defined as

$$\beta_i(\omega;\tau) \equiv \frac{f_{i,m}(\omega;\tau)}{f_m(\omega;\tau)} \equiv \frac{\sum_{k=-\infty}^{\infty} \gamma_{i,m}^k(\tau) e^{-\mathrm{i}k\omega}}{\sum_{k=-\infty}^{\infty} \gamma_m^k(\tau) e^{-\mathrm{i}k\omega}}. \tag{7}$$

QS betas for given asset quantify the dependence between ith asset and market factor m for a given frequency ω at chosen quantiles $\tau \in (0,1)$ of the joint distribution.

For better interpretability, we construct quantile spectral beta for a given frequency band corresponding to reasonable economic cycles as

$$\beta_i(\Omega;\tau) \equiv \int_\Omega \frac{f_{i,m}(\omega;\tau)}{f_m(\omega;\tau)} d\omega \tag{8}$$

where $\Omega \equiv [\omega_1, \omega_2)$, $\omega_1, \omega_2 \in [-\pi, \pi]$, $\omega_1 < \omega_2$ is a frequency band corresponding to an investment horizon. This definition is important since it allows to define short-run, or long-run bands covering corresponding frequencies, and hence disaggregate beta based on the specific demands of a researcher.

Finally, we note that for serially uncorrelated variables (no matter of their joint or marginal distributions), the Frećhet/Hoeffding bounds gives the limits that QS beta can attain $\frac{\max\{\tau+\tau_i-1,0\}-\tau\tau_i}{\tau(1-\tau)} \leq \beta_i(\omega;\tau) \leq \frac{\min\{\tau,\tau_i\}-\tau\tau_i}{\tau(1-\tau)}$ where τ_i is derived as described above.

4 Pricing Model for Extreme Risks across Frequency Domain

Quantile spectral betas defined in the previous sections will be the cornerstone of our empirical models. We assume that QS betas for low threshold values will be significant determinants of risk priced heterogeneously across investment horizons. We will employ

QS betas to study two kinds of risk related to the market return. First, we will investigate *tail market risk* (TR), a risk representing dependence between extreme negative events of both market as well as asset return at a given horizon. Our notion of tail risk relates to the downside risk of Ang et al. (2006); Lettau et al. (2014). While downside risk stems from covariation of asset returns and market return under some threshold, our notion stems from joint probability of the co-occurrence of extreme negative returns in both asset's as well as market's returns. This is more in line with approach of semibetas (Bollerslev et al., 2020), but with important feature of persistence structure of such risks across investment horizons. Second, we will examine *extreme market volatility risk* (EVR), a risk capturing unpleasant situations in which extremely high increments of market volatility are linked to the extremely low asset asset returns, again with respect to the investment horizon. We argue that both these concepts capture important features of risk of an asset faced by the investor, and thus should be priced in cross-section of asset returns.

In each of the models defined in the paper, we control for CAPM beta as a baseline measure of risk. This ensures that if the QS betas are proven to be significant determinants of risk premium, they do not simply duplicate the information contained in the CAPM beta. Moreover, in case of tail market risk, we define relative betas that explicitly capture the additional information over CAPM beta, only.

4.1 Tail Market Risk

We expect the dependence between market return and asset return during extreme negative joint events will be priced across assets positively. The stronger the relationship, the higher the risk premium required by investors. In addition, we expect this risk to be priced heterogeneously across different investment horizons.

To capture the tail market risk measuring the probability of co-occurrence between (extreme) negative events of both market as well as asset return at a given horizon, we define

$$\beta_i^{\mathrm{TR}}(\Omega;\tau) \equiv \sum_{\Omega\equiv[\omega_1,\omega_2)} \left(\frac{\sum_{k=-\infty}^{\infty} \mathbb{C}ov\Big(I\{r_{m,t+k} \leq q_{r_m}(\tau)\}, I\{r_{i,t} \leq q_{r_m}(\tau)\}\Big) e^{-\mathrm{i}k\omega}}{\sum_{k=-\infty}^{\infty} \mathbb{C}ov\Big(I\{r_{m,t+k} \leq q_{r_m}(\tau), I\{r_{m,t} \leq q_{r_m}(\tau)\}\Big) e^{-\mathrm{i}k\omega}} \right). \quad (9)$$

The numerator of Eq. (9) captures probability of co-occurence of the negative events at a given horizon, and denominator captures information related to probability of the market tail events at a given horizon which is related to variation of market returns.

Similarly to Ang et al. (2006) and Lettau et al. (2014), we define relative betas which capture additional information not contained in the classical CAPM beta. This way we can test the significance of tail market risk decomposed into the long- and short-term components in order to obtain their prices of risk separately. Because we want to quantify risk which is not captured by CAPM beta, we propose to test significance of tail market risk via differences of the QS beta and QS beta implied by the Gaussian white noise assumption. We call it *relative* QS beta and compute it for a given frequency band Ω_j and given market τ-quantile level as

$$\beta_i^{\mathrm{rel}}(\Omega_j;\tau) \equiv \beta_i^{\mathrm{TR}}(\Omega_j;\tau) - \beta_i^{\mathrm{Gauss}}(\Omega_j,\tau), \quad (10)$$

where $\beta_i^{\mathrm{Gauss}}(\omega;\tau) = \frac{C^{\mathrm{Gauss}}(\tau,\tau_i;\rho)-\tau\tau_i}{\tau(1-\tau)}$ with C^{Gauss} being Gaussian copula with correlation ρ between market return and an asset's return.[9]

Assuming that all the relevant pricing information is contained in the CAPM beta, contemporaneous covariance between two time series should capture all the priced information. Moreover, if the series are jointly normally distributed and independent through time, CAPM beta contains all the available information regarding the dependence. Hence

[9]This stems from the fact that quantile cross-spectral density corresponds to a difference of probabilities $Pr\big\{r_{i,t} \leq q_{r_m}(\tau), r_{m,t} \leq q_{r_m}(\tau)\big\} - \tau\tau_i$, where τ and τ_i are probability levels under Gaussian distribution, and τ_i is obtained as $\tau_i = F_{r_i}\{q_m(\tau)\}$.

under the hypothesis that market and asset returns are correlated Gaussian noises, the $\beta_i^{\mathrm{rel}}(\Omega_j;\tau)$ will not carry any additional information, and CAPM characterizes the risks well. Note that $\beta_i^{\mathrm{Gauss}}(\Omega_j,\tau)$ is constant across frequencies and depends only on chosen quantile and correlation coefficient. On the other hand, if the investors price an information not captured by the CAPM beta, QS beta estimated without any restriction may identify additional dimension of risk not contained in the CAPM beta. More specifically, we can identify whether dependence in specific part of the joint distribution and/or over specific horizon is significantly priced.

In case that CAPM beta captures all the information regarding risk priced in the cross-section, risk premium corresponding to relative QS beta will be insignificant. Moreover, if the returns are Gaussian, the relative QS beta will be zero at all frequencies and quantiles.[10]

Our first model is hence a tail market risk (TR) model which is defined as

$$\mathbb{E}[r_{i,t+1}^{e}] = \sum_{j=1}^{2} \beta_i^{\mathrm{rel}}(\Omega_j;\tau)\lambda^{\mathrm{TR}}(\Omega_j;\tau) + \beta_i^{\mathrm{CAPM}}\lambda^{\mathrm{CAPM}}, \tag{11}$$

[10]Here, we briefly note that we set the threshold values in the covariance between indicators measure of dependence as a τ quantile of market return. In case of TR betas, the threshold for market and asset return are the same and is given by τ quantile of market return. In case of EVR betas, threshold for increments of market volatility is given by τ quantile of the series of increments of market volatility, and threshold for asset return is given by τ quantile of market return. Note that one could flexibly choose the thresholds based on the best model fit specific to our datasets. For example, we may choose the threshold value to be asset specific by corresponding to the τ quantile of the asset return. We do not follow this approach because we do not explicitly care about dependence between quantiles in the cross-section. We rather care about dependence in extreme market situations.

where $r^e_{i,t+1}$ is excess return of asset i,[11] β_i^{CAPM} is an aggregate CAPM beta, λ^{CAPM} is price of aggregate risk of market captured by the classical beta, and $\lambda^{\text{TR}}(\Omega_j, \tau)$ is price of tail risk (TR) for given quantile and horizon (frequency band). We specify our models to include disaggregation of risk into two horizons – long and short. Long horizon is defined by corresponding frequencies of cycles of 3 years and longer, and short horizon by frequencies of cycles shorter than 3 years.[12] Procedure how to obtain these betas is explained below.

4.2 Extreme Volatility Risk

Assets with high sensitivities to innovations in aggregate volatility have low average returns (Ang et al., 2006). We further focus on extreme events in volatility and investigate whether dependence between extreme market volatility and tail events of assets is priced across assets. Because of the fact that time of high volatility within the economy is perceived as a time with high uncertainty, investors are willing to pay more for the assets that yield high returns during these turmoils and thus positively covary with innovations in market volatility. This drives the prices of these assets up and decreases expected returns. This notion is formally anchored in the intertemporal pricing model, such as intertemporal CAPM model of Merton (1973) or Campbell (1993). According to these models, market volatility is stochastic and causes changes in the investment opportunity set by changing the expected market returns, or by changing the risk-return trade-off. Market volatility thus determines the systematic risk and should determine expected returns of individual assets or portfolios. Moreover, we assume that extreme events in

[11]Note that all the risk measures (in line with the literature) present in the paper are calculated using excess returns.

[12]In Appendix C we perform robustness check by defining the horizons using 1.5 year as a threshold and the results do not qualitatively differ. Different specifications are available upon request.

the market volatility play significant role in the perception of systematic risk, and that the exposure to them affects the risk premium of the assets.

In addition, decomposition of volatility into short-run and long-run when determining asset premium was proven to be useful as well (Adrian and Rosenberg, 2008). Moreover, Bollerslev et al. (2020) incorporated notion of downside risk into concept of volatility risk and showed that stocks with high negative realized semivariance yield higher returns. Farago and Tédongap (2018) examine downside volatility risk in their five-factor model and obtain model with negative prices of risk of volatility downside factor yielding low returns for assets that positively covary with innovations of market volatility during disappointing events. Thus, we want to investigate which horizon and part of the joint distribution of market volatility and asset returns generate these findings.

We assume that assets that yield highly negative returns during times of large innovations of volatility are less desirable for investors and thus holding these assets should be rewarded by higher risk premium. In addition, we assume such risk will be horizon specific. To measure the extreme volatility risk, we define the beta that will capture the joint probability of co-occurrences of negative asset returns and extreme increment of market volatility across horizons. Because of the nature of covariance between indicator functions, we work with negative market volatility innovations $-\Delta\sigma_t^2 = -(\sigma_t^2 - \sigma_{t-1}^2)$, where we estimate σ_t with a popular GARCH(1,1). Then the high volatility increments correspond to low quantiles of distribution of the negative differences. If an asset positively covary with increments of market volatility, the extreme volatility risk beta will be small, and vice versa. This is in contrast to most of the measures employed in the similar analyses. We define the beta that captures extreme volatility risk across horizons as

$$\beta_i^{\Delta\sigma^2}(\Omega_j;\tau) \equiv \sum_{\Omega\equiv[\omega_1,\omega_2)} \left(\frac{\sum_{k=-\infty}^{\infty} \mathbb{C}ov\Big(I\{-\Delta\sigma_{t+k}^2 \leq q_{-\Delta\sigma_t^2}(\tau)\}, I\{r_{i,t} \leq q_{r_m}(\tau)\}\Big)e^{-ik\omega}}{\sum_{k=-\infty}^{\infty} \mathbb{C}ov\Big(I\{-\Delta\sigma_{t+k}^2 \leq q_{-\Delta\sigma_t^2}(\tau), I\{-\Delta\sigma_t^2 \leq q_{-\Delta\sigma_t^2}(\tau)\}\Big)e^{-ik\omega}} \right) \tag{12}$$

Threshold values for asset returns are obtained in the same manner as for tail market risk and are derived from the distribution of the market returns, which means that $q_{r_m}(\tau)$ is used as an asset threshold value. For example, for model with $\tau = 0.05$, when computing extreme market volatility beta, as a threshold for negative innovations of market squared volatility, we use 5% quantile of its distribution (corresponding to 95% quantile of the original distribution), and threshold for asset return is set to 5% quantile of distribution of market returns.

Our second model, Extreme volatility risk (EVR) model will test the significance of EVR betas and is defined as

$$\mathbb{E}[r^e_{i,t+1}] = \sum_{j=1}^{2} \beta_i^{\Delta\sigma^2}(\Omega_j;\tau)\lambda^{\mathrm{EV}}(\Omega_j;\tau) + \beta_i^{\mathrm{CAPM}}\lambda^{\mathrm{CAPM}}, \tag{13}$$

where, as in the case of TR model, we include CAPM beta to control for the corresponding risk premium. In line with results of current literature (e.g, Boons and Tamoni (2015), Boguth and Kuehn (2013), or Adrian and Rosenberg (2008)), we expect positive prices of risk corresponding to EVR betas. This is because EVR betas measures dependence between extremely high increments of market volatility (i.e., low values of negative innovations of market volatility) and low values of asset returns. So, if an asset yields low returns in times of high market volatility, investor requires high premium in order to hold it.

4.3 Full model

Finally, to show the independence of the two horizon specific tail market risks, we also combine them into the third model that includes both tail market risk and extreme market volatility risk for both short- and long-run horizons, again controlling for a traditional

CAPM beta. Model possesses the following form

$$\mathbb{E}[r_{i,t+1}^e] = \sum_{j=1}^{2} \beta_i^{rel}(\Omega_j;\tau)\lambda^{\mathrm{TR}}(\Omega_j;\tau) + \sum_{j=1}^{2} \beta_i^{\Delta\sigma^2}(\Omega_j;\tau)\lambda^{\mathrm{EV}}(\Omega_j;\tau) + \beta_i^{\mathrm{CAPM}}\lambda^{\mathrm{CAPM}}. \quad (14)$$

We denote this model as *Full model*. Assuming that TR and EVR are priced, using this model, we will investigate whether these risks are subsumed by each other or whether they describe independent dimensions of priced risk.

Throughout the paper, we focus on results for τ equal to 1%, 5%, 10%, 15%, 20%, and 25%. The choice of 1%, 5% and 10% quantiles is natural and arises in many economic and finance applications. Probably the most prominent example is Value-at-Risk, which is a benchmark measure of risk widely used in practice and studied among academics. Remaining values of τ, i.e. 15%, 20%, and 25%, capture general downside risk and thus more probable negative joint events.

4.4 Estimation

Estimation of QS betas (for both TR and EVR) relies on proper estimation of quantile cross-spectral densities using rank-based copula cross-periodograms, which are then smoothed in order to obtain consistency of the estimator. For this, we extend the results of Baruník and Kley (2019). Technical details are provided in the Appendix A.

To test our models, we use the standard Fama and MacBeth (1973) cross-sectional regressions. In the first stage, we estimate all required QS betas, relative QS betas, and CAPM betas for all assets. We define two non-overlapping horizons: short and long. Horizon is specified by the corresponding frequency band. We specify long horizon by frequencies with corresponding cycles 3 years and longer, and short horizon by frequencies with corresponding cycles below 3 years.[13] QS betas for these horizons are obtained by averaging QS betas over corresponding frequency bands.

[13]For robustness check of using 1.5 year as a threshold value, see the Appendix C

In the second stage, we use these betas as explanatory variables and regress average asset returns on them and obtain the model fit. We assess significance of a given risk by significance of its corresponding estimated price [14]. In case of the full model, we obtain the statistical inference on the estimated prices of risk by repeating cross-sectional regression in every time point, i.e., in every month $t = 1, \ldots, T$, we estimate model of the following form

$$r^e_{i,t} = \sum_{j=1}^{2} \widehat{\beta}_i^{\text{rel}}(\Omega_j;\tau)\lambda_t^{\text{TR}}(\Omega_j;\tau) + \widehat{\beta}_i^{\text{CAPM}}\lambda_t^{\text{CAPM}} + \sum_{j=1}^{2} \widehat{\beta}_i^{\Delta\sigma^2}(\Omega_j;\tau)\lambda_t^{\text{EV}}(\Omega_j;\tau) + e_{t,i}. \quad (15)$$

We obtain T cross-sectional estimates of lambdas for each of the corresponding beta. Then, we estimate the prices of risk by time series averages of the lambdas over the whole period

$$\widehat{\lambda}^k(\Omega_j;\tau) = \frac{1}{T}\sum_{t=1}^{T} \widehat{\lambda}_t^k(\Omega_j;\tau), \quad j = 1, 2, \quad k = \text{TR}, \text{EVR}. \quad (16)$$

Standard errors and corresponding t-statistics are computed from $\sigma^2\Big(\widehat{\lambda}^k(\Omega_j;\tau)\Big) = \frac{1}{T^2}\sum_{t=1}^{T}\Big(\hat{\lambda}_t^k\Omega_j;\tau) - \widehat{\lambda}^k(\Omega_j;\tau)\Big)^2$ for both horizons $j = \{1, 2\}$ and risks $k = \{\text{TR}, \text{EVR}\}$.

The same estimation logic applies to other studied models. To take into account multiple hypothesis testing, we follow Harvey et al. (2016) and report t-statistics of estimated parameters (below the actual estimates). Overall fit of the model is measured from the OLS regression of average returns of the assets on their betas. Throughout the paper, we use the root mean squared error (RMSPE) metric, which is widely used metric for assessing model fit in asset pricing literature, to asses the overall model performance.

As mentioned earlier, we estimate our models for various values of threshold given by τ quantile of market return. Further, we compare our newly proposed measures with

[14]As shown in Shanken (1992), if the betas are estimated over the whole period, the second stage regression is T-consistent.

i) classical CAPM ii) downside risk model of Ang et al. (2006) (DR1) iii) downside risk model of Lettau et al. (2014) (DR2) iv) 3-factor model of Fama and French (1993) v) GDA3 and GDA5 models of Farago and Tédongap (2018) vi) and coskewness and cokurtosis measures. Details regarding the estimation of the risk measures of the competing models is summarized in Appendix B. All the models are estimated for comparison purposes without any restrictions in two stages similarly as in the case of our three- and five-factor models. Thus, GDA3 and GDA5 are, despite their theoretical background, estimated without setting any restriction to their coefficients and are also estimated in two stages.

5 Quantile Spectral Risk and the Cross-Section of Expected Returns

Here we discuss how extreme risks are priced in the cross-section of asset returns across horizons. We begin our empirical investigation by briefly discussing the data we use in our investigation and then presenting the results from standard Fama and MacBeth (1973) cross-sectional predictive regressions. We focus on the results from the three maim models, look at a summary statistics of the distributional features of the resulting beta estimates and finally investigate how the new features of priced risk relate to other competing risk measures.

5.1 Data

We collect our data from the Center for Research in Securities Prices (CRSP) database on monthly basis. The sample begins in July 1926 and ends in December 2015. We select stocks with long enough history in order to obtain precise estimates of our measures of risk. While the main results are presented with a sample of stocks with available history of 60 years, to study the robustness of our results on larger cross-section of data, we report also results based on the stocks with shorter history of 50 years. On the other hand, one can argue that a precision of the estimated measures of risk relies on number of observations available in the tail, hence we also report results based on stocks with 70 years of available history as well. Market return is computed using value-weight average return on all CRSP stocks. As a risk-free rate we use Treasury bill rate from Ibbotson Associates.[15]

5.2 TR Model

We report estimation results of the TR model in the left panel of Table 1. We present results for three different samples based on different number of minimum years of history an asset has to posses in order to be included in a sample. Models are estimated for different values of the threshold value given by the τ market quantile to capture the different probability of event co-occurrences.

Results show that relative TR beta for short-horizon is more significant for low values of τ corresponding 0.01, 0.05 and 0.10, while for $\tau \geq 0.15$, relative TR beta becomes significant for long horizon. This pattern is observed across all three samples, but it is weaker among stocks with history of 50 years, especially regarding the prices of risk corresponding to the long relative TR betas. This result may be caused by the fact that long relative TR betas require longer history of data to obtain precise estimates in comparison to the short TR betas.

Signs of the estimated prices of risk are intuitive. More extreme dependence between market and asset return in both horizons leads to the higher risk premium as we may expect. If an asset is likely to deliver poor performance when the market is in downturn, this asset is not desirable from the point of view of an investor, and in order to hold such asset, she requires significant risk premium. From the magnitude of the coefficients we infer that investors price the tail risk in short-term more than in long-term. Moreover,

[15]All the data were obtained from Kenneth French's online data library.

Table 1: *Estimated Coefficients of the TR, EVR and Full Models.* Prices of risk estimated on monthly stock data from CRSP database sampled between July 1926 and December 2015. Models are estimated for various values of thresholds given by τ. We employ three samples with varying number of minimum years. Long horizon is given by frequencies corresponding to 3-year cycle and longer. Below the coefficients, we include Fama-MacBeth t-statistics.

		Tail market risk				Extreme volatility risk				Full model					
	τ	λ^{TR}_{long}	λ^{TR}_{short}	λ^{CAPM}	RMSPE	λ^{EV}_{long}	λ^{EV}_{short}	λ^{CAPM}	RMSPE	λ^{TR}_{long}	λ^{TR}_{short}	λ^{EV}_{long}	λ^{EV}_{short}	λ^{CAPM}	RMSPE
	0.01	-0.059	0.657	0.754	26.683	-0.086	-0.275	0.960	28.542	0.125	0.620	-0.210	-0.168	0.834	26.387
		-0.660	3.309	3.911		-0.972	-0.887	5.001		0.901	3.087	-1.485	-0.539	4.196	
	0.05	0.102	1.319	0.717	26.807	0.232	0.380	0.682	28.192	0.080	1.295	0.009	0.303	0.721	26.757
		0.500	3.520	3.871		1.394	0.753	3.028		0.221	3.534	0.030	0.595	2.978	
	0.1	0.368	1.203	0.739	27.121	0.474	0.472	0.558	27.212	-0.064	1.037	0.422	0.311	0.555	26.555
70 years		1.483	2.304	4.144		2.532	0.677	2.533		-0.158	1.973	1.384	0.448	2.312	
(142 assets)	0.15	0.544	0.895	0.733	26.672	0.509	0.552	0.602	27.016	0.242	0.882	0.326	0.538	0.618	26.289
		2.327	1.511	4.075		2.783	0.778	2.903		0.730	1.487	1.223	0.752	2.875	
	0.2	0.665	0.279	0.784	26.995	0.702	-0.272	0.605	25.796	-0.041	0.848	0.693	-0.094	0.586	25.431
		2.566	0.400	4.454		3.665	-0.348	3.070		-0.116	1.250	2.601	-0.120	2.868	
	0.25	0.746	-0.132	0.812	27.244	0.823	-0.543	0.648	25.768	0.009	0.444	0.805	-0.397	0.643	25.676
		2.805	-0.181	4.662		3.816	-0.678	3.366		0.026	0.616	2.810	-0.498	3.284	
	0.01	-0.044	0.439	0.759	29.725	-0.090	0.271	0.939	30.494	0.170	0.387	-0.241	0.255	0.865	29.442
		-0.633	2.693	4.126		-1.293	1.007	5.144		1.431	2.391	-2.010	0.939	4.659	
	0.05	0.189	1.219	0.660	28.674	0.243	0.614	0.645	29.945	0.115	1.271	0.027	0.653	0.661	28.600
		1.068	3.653	3.573		1.471	1.505	2.850		0.379	3.903	0.103	1.594	2.759	
	0.1	0.315	1.000	0.718	29.243	0.503	0.420	0.511	29.201	-0.161	0.939	0.509	0.443	0.493	28.676
60 years		1.388	2.450	4.022		2.471	0.830	2.281		-0.489	2.282	1.737	0.875	2.058	
(267 assets)	0.15	0.500	0.779	0.709	29.257	0.441	0.550	0.603	29.608	0.297	0.819	0.233	0.443	0.626	29.099
		2.188	1.546	3.961		2.248	1.004	2.889		1.042	1.630	0.958	0.805	2.972	
	0.2	0.484	0.287	0.765	29.764	0.630	-0.198	0.595	28.769	-0.234	0.735	0.742	-0.253	0.561	28.620
		2.002	0.537	4.359		3.153	-0.338	3.024		-0.824	1.438	3.007	-0.430	2.808	
	0.25	0.487	0.242	0.785	30.045	0.619	-0.463	0.661	29.389	-0.053	0.574	0.629	-0.443	0.657	29.341
		2.017	0.427	4.513		3.084	-0.764	3.505		-0.203	1.037	2.768	-0.737	3.481	
	0.01	-0.089	0.441	0.823	29.727	-0.099	0.478	0.970	30.281	0.001	0.410	-0.096	0.439	0.873	29.683
		-1.655	2.953	4.631		-1.783	2.508	5.420		0.009	2.787	-1.077	2.337	4.816	
	0.05	-0.022	1.185	0.760	29.233	0.061	0.649	0.800	30.059	0.019	1.268	-0.059	0.780	0.781	29.039
		-0.152	3.949	4.233		0.484	1.971	3.776		0.067	4.400	-0.258	2.421	3.439	
	0.1	0.153	0.820	0.786	29.762	0.289	0.093	0.680	29.700	-0.104	0.801	0.288	0.182	0.665	29.417
50 years		0.794	2.450	4.436		1.785	0.223	3.224		-0.333	2.450	1.167	0.436	2.971	
(528 assets)	0.15	0.348	0.862	0.767	29.600	0.148	0.023	0.783	30.112	0.476	0.830	-0.149	0.101	0.813	29.570
		1.832	2.034	4.307		0.921	0.051	3.891		1.733	1.984	-0.670	0.222	3.945	
	0.2	0.272	0.683	0.810	29.973	0.346	-0.267	0.735	29.791	-0.002	0.743	0.314	-0.298	0.735	29.704
		1.410	1.437	4.605		2.011	-0.566	3.789		-0.009	1.605	1.374	-0.622	3.695	
	0.25	0.257	0.822	0.821	30.035	0.322	-0.051	0.765	30.004	0.034	0.946	0.281	-0.081	0.764	29.885
		1.316	1.594	4.704		1.885	-0.106	4.082		0.152	1.848	1.388	-0.167	4.070	

it is important to note that these features are not subsumed by the CAPM beta as we explicitly control for it in the model, and also report TR betas relative to the CAPM beta as discussed above.

5.3 EVR Model

Estimation results for the EVR model are captured in the middle panel of Table 1. In this case, parameters are not significant for low values of τ, but starting with $\tau \geq 0.1$, long EVR becomes significantly priced in the cross-section. On the other hand, short-horizon EVR risk is not significantly priced for any values of τ.

Significant prices of risk corresponding to long-horizon EVR betas for $\tau \geq 0.10$ posses intuitive positive signs as we expected. The EVR betas capture dependence between extremely high increments of market volatility[16] and extremely low asset returns, and the results are consistent with current literature (Boons and Tamoni, 2015; Boguth and Kuehn, 2013; Adrian and Rosenberg, 2008). Moreover, results are in line with conclusions of long-run risk models, as well. We observe few instances of unintuitive negative signs of prices of risk, but these coefficients are insignificant and observed mostly for low values of τ, which may be caused by the measurement error for the corresponding betas. We may conclude that EVR betas, especially their long-term component, provide priced information regarding risk, which is moreover orthogonal to the information featured in the CAPM beta.

In terms of the RMSPE, the TR model delivers better results as the EVR model for low values of τ, as short TR betas are significantly priced for these values of τ. On the other hand, for higher values of τ, EVR model delivers improved values of RMSPE, as the long EVR betas for these τ values deliver a significant dimension of risk priced in the cross-section and TR betas posses higher explanatory power for lower values of τ.

[16]Note that we work with negative of increments of market volatility when we estimate the QS betas.

Moreover, we identify the fact that there is a complex interplay between the horizons and parts of the joint distribution priced in the cross-section. Extreme TR is mostly short-run phenomenon, and TR associated with more probable joint events (higher values of τ) is priced with respect to long-term dependence between market and assets. On the other hand, EVR is not significantly priced in cases of extreme joint events, but as the unpleasant events become more probable, the joint dependence between increments of market volatility and asset return in the long-run become significant determinant of risk premium. In Table 7 in Appendix C.1, we present the results for 1.5-year being the threshold in the definition of the long horizon. Results are qualitatively very similar and all the findings from the 3-year horizon hold for this case, as well.

5.4 Full Model

From the results above, we can conclude that the tail market and extreme market volatility risks are priced in the cross-section of stock returns across different horizons. Natural question arises whether these risks capture different information or one measure can subsume the other. For this purpose, we test the Full model, which contains both risks for given τ level at the same time.

Estimated parameters of the full model can be found in the right panel of Table 1. We observe results mostly consistent with the outcomes of the separate TR and EVR models. Significantly priced determinants of the risk are short-term tail risk for low values of τ, and long-term extreme volatility risk for the higher values of τ, both priced across assets with expected positive signs. Tail risk is more significant for lower values of τ meaning that dependence between market return and asset return during extremely negative events is a significant determinant of the risk premium. On the other hand, long-term extreme volatility risk is significant for higher values of τ - around 0.2. This finding suggests that investors price downside dependence between asset returns and market volatility, but focus on more probable market situations. We can deduce that price of long-run risk of Bansal and Yaron (2004) is hidden in this coefficient.

The main deviation of the Full model from the results of the separate TR and EV models is that the long TR betas for higher values of τ become insignificant, in contrast with the conclusions from the TR model. One potential explanation for this result is that only a small fraction of the market return fluctuations is due to its long-term component in comparison to the short-term one, and thus the risk premium for this risk is only small. Other explanation is that the long-term aspect of the market tail risk may by fully captured by the extreme volatility risk, namely the long TR betas are subsumed by the long EVR betas. This make sense since variance is much more persistent than the market return (high portion of variance due to the long-term component) and thus the investors fear the fluctuation in long-term variance much more than the variance in the short term.

5.5 Summary Statistics about Quantile Spectral Betas

Table 2: *Descriptive Statistics.* Table summarizes basic descriptive statistics and correlation structure for all betas from our full model for the two choices of the quantile levels. Betas are computed using CRSP database sampled between July 1926 and December 2015. Presented results are computed on our largest sample, i.e., using stocks with at least 50 years of history. Long horizon is given by frequencies corresponding to 3-year cycle and longer.

	$\tau = 0.05$					$\tau = 0.10$				
	β^{CAPM}	β^{rel}_{long}	β^{rel}_{short}	β^{EV}_{long}	β^{EV}_{short}	β^{CAPM}	β^{rel}_{long}	β^{rel}_{short}	β^{EV}_{long}	β^{EV}_{short}
Mean	1.068	0.310	0.098	0.726	0.016	1.068	0.197	0.051	0.632	0.015
Median	1.084	0.324	0.096	0.715	0.016	1.084	0.191	0.048	0.634	0.016
St. Dev.	0.372	0.208	0.083	0.296	0.065	0.372	0.164	0.064	0.212	0.051
β^{CAPM}	1.000	0.234	-0.188	0.595	0.041	1.000	-0.040	-0.100	0.435	0.066
β^{rel}_{long}	0.234	1.000	0.147	0.688	0.032	-0.040	1.000	0.275	0.595	0.055
β^{rel}_{short}	-0.188	0.147	1.000	-0.062	-0.053	-0.100	0.275	1.000	0.104	-0.073
β^{EV}_{long}	0.595	0.688	-0.062	1.000	0.025	0.435	0.595	0.104	1.000	0.112
β^{EV}_{short}	0.041	0.032	-0.053	0.025	1.000	0.066	0.055	-0.073	0.112	1.000

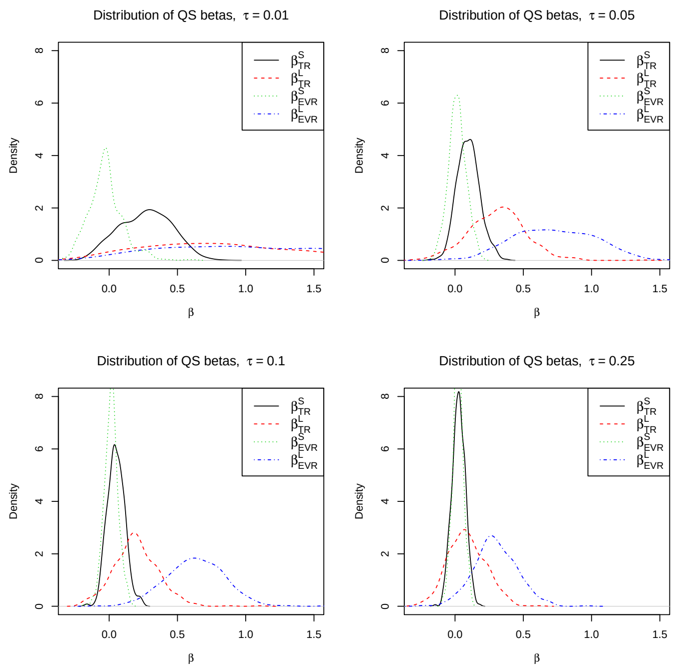
Further we are interested to see what distributions of estimated quantile spectral betas reveal, and so we display the unconditional distribution of the estimated betas used in the TR, EVR and Full models. Table 2 summarizes descriptive statistics for all estimated betas. We focus on two values of τ - 0.05, and 0.10, and present cross-sectional means, medians and standard deviations of the estimated parameters in the top panel. We observe that all the betas are on average positive. This is particularly interesting for relative TR betas, which means that, roughly speaking, average stock posses higher tail dependence with market than suggested by the simple covariance based measures. Bottom panel of Table 2 presents correlation structure of TR, EV and CAPM betas. We observe higher values of correlation between long-term betas, and also between long-term EV and CAPM betas. Nevertheless, all these correlation are far below 1, which suggests that all the variables may posses different and potentially important information regarding the risk associated with the assets. Another interesting observations is that the relative TR betas, both long- and short-term, are almost uncorrelated with the CAPM betas, which is exactly what we want to see given their definition.

To further visualize the distributional features, Figure 2 presents unconditional distributions of the betas for four different threshold value for quantile levels. We observe the highest dispersion of betas for the lowest values of τ corresponding the the most extreme case. As we move to higher values of τ, the distributions exhibit less and less variance. Moreover, the distribution of long-term betas is wider than the distribution of the short-term betas for the respective risks.

5.6 Robustness Checks: Tail Risk across Horizons and other risk factors

Large number of other risk factors and firm characteristics have been documented by the literature as significant drivers of the cross-sectional variation in equity returns (Harvey et al., 2016). While we do not attempt to include the whole exhaustive set of all controls, we would like to see if our newly defined risk factors are not subsumed by a subset of prominent variables, as well as variables related to the tails and moments of the return distribution. Hence we naturally focus on the downside measures and we use downside risks proposed by Ang et al. (2006), downside risk beta specification of Lettau

Figure 2: *Distribution of TR and EVT betas at different tails.* Plots displays kernel density estimates of the unconditional distribution of the short-term and long-term TR and EVT betas. Presented results are computed on our largest cross-section, i.e., using stocks with at least 50 years of history. Long horizon is given by frequencies corresponding to 3-year cycle and longer.



et al. (2014) as well as recently proposed five factor generalized disappointment aversion (GDA5) model by Farago and Tédongap (2018). Further we use coskewness and cokurtotis measures, as well as size, book-to-market and momentum factors used by Fama and French (1993). Moreover, we present all these results with long horizon being 1.5 years in Appendix C.

To investigate whether our newly proposed measures of risk can be driven out by other determinants of risk proposed earlier in the literature, we include these risks as control variables in the previous regressions. First, we focus on the GDA5 model proposed by Farago and Tédongap (2018) as these are the risks most closely related to ours. It contains two measures of tail market risk as well as two measures of extreme volatility risk, but focuses on various specifications of downside dependence and does not take into consideration frequency aspect of the risks. Based on these competing measures, we compare risk measures associated with market return, and market volatility increments separately. The aim of this analysis is to decide which measures of risk better capture the notion of extreme risks associated with risk premium. The detailed specification of the corresponding betas can be found in Appendix B.

Table 3 reports the risk premium of our quantile spectral risks controlled for the GDA5 risks. In case of tail market risk presented in left panel, we see that GDA5 measures of risk (λ_D and λ_{WD}) do not drive out our measures for any value of τ and remain insignificant when we include our TR measures. Moreover, the pattern of prices of risk corresponding to TR betas remain the same as in the TR and Full model specifications. This clearly suggests that our measures captures the asymmetric features of risk priced in the cross-section of assets.

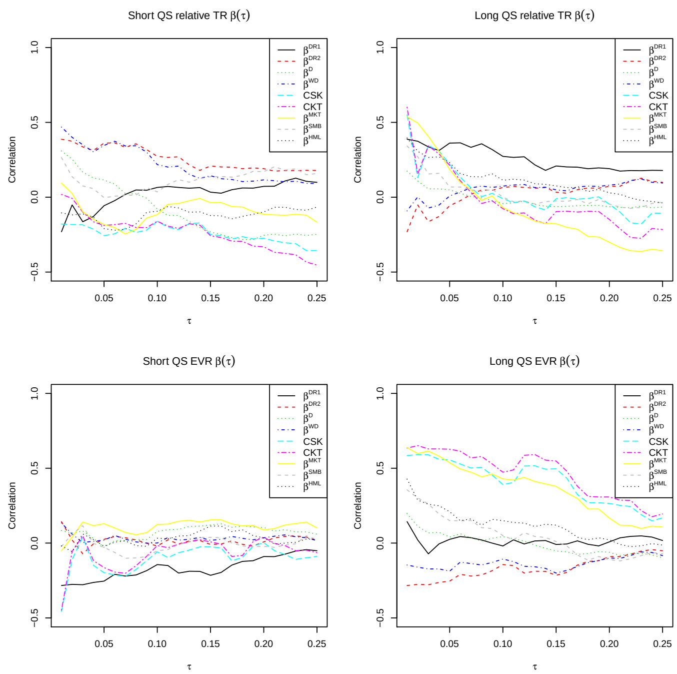
In case of extreme volatility risk, we see from the right panel of Table 3 that the situation is similar. Especially, the price of risk for long-term EVR betas stays significantly strong for higher values of quantile. In addition, short-term EVR betas emerge as a significant predictors for the lower values of τ. On the other hand, GDA5 measures of volatility risk remain insignificant in all of the cases. All the results suggest that our model brings an improvement in terms of identifying form of asymmetric risk which is priced in the cross-section of asset returns.

From these results, we can infer that our QS measures may potentially provide an additional information not captured by other risk measures. To further investigate this hypothesis, we present correlation structure of our QS measures with all other highly discussed asset pricing risk measures in Figure 3. Details regarding their specifications are contained in the Applendix B. We plot dependence between them and the QS measures with respect to the value of quantile of the threshold value. Generally, our measures posses the highest correlation with coskewness and cokurtosis and market beta (computed using FF3 specification) in the extreme left tail and long horizon, while they show high correlation with downside risk measures in extreme left tail at short horizon. This suggests that downside risk measures capture short-term risk while moment-based risk

Table 3: *Estimated Coefficients of the TR, EVR and Full Models controlled for GDA5 Measures.* Table reports coefficients and their t-statistics from the horse race estimations. Displayed are prices of risk of three-factor models also including the GDA5 measures for corresponding risks. We use CRSP database between July 1926 and December 2015. Models are estimated for various values of thresholds given by τ. We employ 3 samples with varying number of minimum years. Long horizon is given by frequencies corresponding to 3-year cycle and longer. Below the coefficients, we include Fama-MacBeth t-statistics.

		Tail market risk						Extreme volatility risk					
	τ	λ_D	λ_{WD}	λ^{TR}_{long}	λ^{TR}_{short}	λ^{CAPM}	RMSPE	λ_X	λ_{XD}	λ^{EV}_{long}	λ^{EV}_{short}	λ^{CAPM}	RMSPE
70 years (142 assets)	0.01	-0.027	0.118	-0.034	0.628	0.760	26.377	0.848	0.775	-0.096	0.773	0.735	28.313
		-0.987	0.597	-0.364	3.231	3.879		1.326	1.450	-1.155	3.892	3.781	
	0.05	-0.024	0.207	0.050	1.149	0.779	26.434	0.108	0.285	0.122	1.294	0.704	27.698
		-0.908	1.037	0.234	3.066	4.153		0.169	0.526	0.612	3.431	3.805	
	0.1	-0.017	0.253	0.305	0.795	0.799	26.688	0.157	0.343	0.388	1.220	0.724	26.560
		-0.622	1.279	1.243	1.488	4.436		0.246	0.633	1.580	2.344	4.066	
	0.15	-0.010	0.214	0.449	0.746	0.779	26.326	0.219	0.367	0.572	0.786	0.720	26.627
		-0.357	1.090	1.992	1.210	4.297		0.344	0.678	2.422	1.325	4.018	
	0.2	-0.010	0.244	0.526	0.301	0.824	26.532	0.302	0.492	0.723	0.176	0.766	25.345
		-0.324	1.258	2.115	0.401	4.630		0.473	0.900	2.732	0.257	4.374	
	0.25	-0.018	0.268	0.587	-0.064	0.856	26.693	0.377	0.554	0.850	-0.332	0.793	25.364
		-0.587	1.423	2.300	-0.086	4.878		0.587	1.007	3.089	-0.468	4.576	
60 years (267 assets)	0.01	-0.014	0.141	-0.011	0.338	0.765	29.578	0.367	0.032	-0.060	0.495	0.753	30.288
		-0.584	0.809	-0.151	2.269	4.168		0.631	0.068	-0.950	3.104	4.069	
	0.05	-0.009	0.110	0.187	1.094	0.681	28.606	-0.050	-0.139	0.166	1.218	0.663	29.764
		-0.364	0.627	1.030	3.437	3.719		-0.086	-0.297	0.949	3.679	3.608	
	0.1	0.000	0.164	0.283	0.850	0.740	29.013	-0.044	-0.192	0.284	1.004	0.721	28.928
		0.002	0.946	1.277	2.166	4.166		-0.076	-0.408	1.253	2.453	4.071	
	0.15	0.001	0.182	0.464	0.627	0.733	28.962	-0.076	-0.234	0.479	0.774	0.713	29.459
		0.022	1.067	2.097	1.279	4.108		-0.131	-0.500	2.104	1.570	4.020	
	0.2	-0.007	0.244	0.449	0.092	0.796	29.317	-0.032	-0.147	0.473	0.240	0.765	28.616
		-0.259	1.414	1.906	0.173	4.541		-0.056	-0.314	1.966	0.458	4.401	
	0.25	-0.005	0.237	0.425	0.199	0.813	29.566	-0.028	-0.143	0.477	0.182	0.784	29.268
		-0.205	1.399	1.814	0.367	4.691		-0.049	-0.305	1.995	0.337	4.557	
50 years (528 assets)	0.01	-0.028	0.114	-0.054	0.396	0.823	29.452	0.254	-0.016	-0.108	0.489	0.822	29.913
		-1.399	0.799	-0.968	3.126	4.643		0.465	-0.037	-2.154	3.272	4.611	
	0.05	-0.028	0.118	0.009	1.097	0.778	29.023	0.016	-0.098	-0.049	1.191	0.762	29.791
		-1.389	0.781	0.061	4.134	4.373		0.030	-0.229	-0.359	4.120	4.279	
	0.1	-0.026	0.204	0.161	0.503	0.826	29.567	-0.056	-0.165	0.120	0.843	0.788	29.443
		-1.261	1.338	0.821	1.614	4.726		-0.104	-0.390	0.652	2.559	4.499	
	0.15	-0.025	0.224	0.347	0.452	0.809	29.393	-0.122	-0.219	0.331	0.868	0.769	29.895
		-1.172	1.490	1.827	1.077	4.610		-0.227	-0.518	1.772	2.131	4.378	
	0.2	-0.031	0.268	0.279	0.153	0.860	29.664	-0.107	-0.203	0.260	0.679	0.811	29.577
		-1.357	1.786	1.450	0.307	4.945		-0.200	-0.476	1.350	1.516	4.685	
	0.25	-0.024	0.246	0.250	0.478	0.863	29.723	-0.110	-0.203	0.251	0.804	0.822	29.759
		-1.125	1.673	1.287	0.922	5.015		-0.204	-0.472	1.284	1.672	4.795	

Figure 3: *Correlations with Other Risk Measures.* Plots display correlations between the QS betas and various other risk measures widely used in the asset pricing literature using CRSP database between July 1926 and December 2015. Presented results are computed on our largest sample, i.e., using stocks with at least 50 years of history. Long horizon is given by frequencies corresponding to 3-year cycle and longer.



measures are more related to the extreme volatility in the long-term. Although the correlations in few cases exceed 0.5 in absolute value, all the values are well below 1 suggesting potentially important additional information regarding the risk.

Next, we check whether these measures can drive out our QS measures in the cross-sectional estimation. Table 4 reports the results of risk prices controlled for coskewness and cokurtosis risks. We first include coskewness into our Full model and check whether it can drive out our risk measures. We can see that although the coskewness is significant, it does not drive out our QS measures, which follow the same pattern as in the case of previous specifications of the models. Table 4 also reports in the right panel horse race regression including cokurtosis. We observe that cokurtosis does not bring any new explanatory information when included in our full model, as the corresponding estimated coefficients for cokurtosis are insignificant for all specifications.

Table 4: *Estimated Coefficients of the TR, EVR and Full Models controlled for Coskewness and Cokurtosis.* Displayed are prices of risk of full models also including either coskewness or cokurtosis. We use CRSP database between July 1926 and December 2015. Models are estimated for various values of thresholds given by τ. We employ 3 samples with varying number of minimum years. Long horizon is given by frequencies corresponding to 3-year cycle and longer. Below the coefficients, we include Fama-MacBeth t-statistics.

		Coskewness							Cokurtosis						
	τ	λ^{CSK}	$\lambda^{\text{TR}}_{\text{long}}$	$\lambda^{\text{TR}}_{\text{short}}$	$\lambda^{\text{EV}}_{\text{long}}$	$\lambda^{\text{EV}}_{\text{short}}$	λ^{CAPM}	RMSPE	λ^{CKT}	$\lambda^{\text{TR}}_{\text{long}}$	$\lambda^{\text{TR}}_{\text{short}}$	$\lambda^{\text{EV}}_{\text{long}}$	$\lambda^{\text{EV}}_{\text{short}}$	λ^{CAPM}	RMSPE
70 years (142 assets)	0.01	-0.255	0.122	0.493	-0.122	-0.320	0.790	25.630	-0.020	0.113	0.676	-0.197	-0.241	0.928	26.332
		-1.321	0.879	2.420	-0.854	-1.021	4.152		-0.942	0.820	3.374	-1.390	-0.777	4.217	
	0.05	-0.316	0.016	0.983	0.093	-0.179	0.747	25.911	-0.009	0.017	1.288	0.094	0.236	0.725	26.756
		-1.701	0.044	2.674	0.327	-0.343	3.279		-0.452	0.049	3.492	0.330	0.467	2.874	
	0.1	-0.345	-0.147	0.633	0.470	-0.453	0.610	25.532	-0.022	-0.294	0.952	0.642	0.165	0.575	26.478
		-1.833	-0.368	1.183	1.582	-0.642	2.684		-0.988	-0.709	1.817	1.921	0.242	2.310	
	0.15	-0.344	-0.028	0.301	0.584	-0.205	0.616	25.214	-0.021	0.106	0.647	0.541	0.425	0.663	26.262
		-1.644	-0.084	0.475	2.165	-0.280	3.040		-0.903	0.319	1.062	1.926	0.595	2.721	
	0.2	-0.281	-0.152	0.327	0.743	-0.395	0.630	24.616	-0.023	-0.135	0.509	0.845	-0.052	0.668	25.353
		-1.429	-0.443	0.466	2.925	-0.510	3.234		-0.993	-0.389	0.720	3.111	-0.067	2.735	
	0.25	-0.279	0.001	-0.310	0.754	-0.761	0.709	24.895	-0.017	-0.005	0.021	0.896	-0.477	0.721	25.653
		-1.417	0.004	-0.402	2.835	-0.960	3.752		-0.738	-0.015	0.027	3.089	-0.607	2.921	
60 years (267 assets)	0.01	-0.342	0.159	0.218	-0.091	0.021	0.776	28.728	-0.011	0.169	0.409	-0.229	0.222	0.908	29.442
		-1.988	1.343	1.308	-0.751	0.074	4.427		-0.531	1.424	2.462	-1.857	0.798	4.486	
	0.05	-0.340	-0.071	0.767	0.252	0.144	0.636	28.063	-0.014	0.026	1.225	0.152	0.519	0.672	28.600
		-2.095	-0.233	2.466	0.950	0.338	2.772		-0.749	0.091	3.818	0.629	1.236	2.697	
	0.1	-0.377	-0.385	0.559	0.597	-0.272	0.545	27.857	-0.034	-0.499	0.805	0.812	0.163	0.544	28.572
		-2.406	-1.164	1.441	2.017	-0.522	2.338		-1.632	-1.518	2.029	2.625	0.325	2.188	
	0.15	-0.368	-0.026	0.067	0.505	-0.079	0.624	28.124	-0.018	0.176	0.618	0.397	0.281	0.668	29.093
		-2.246	-0.092	0.139	1.963	-0.139	3.121		-0.860	0.645	1.259	1.707	0.514	2.848	
	0.2	-0.350	-0.266	-0.058	0.715	-0.562	0.639	27.744	-0.025	-0.321	0.360	0.876	-0.254	0.661	28.553
		-2.185	-0.941	-0.114	2.973	-0.954	3.313		-1.211	-1.161	0.701	3.568	-0.433	2.815	
	0.25	-0.361	-0.034	-0.471	0.553	-1.001	0.747	28.311	-0.014	-0.065	0.252	0.681	-0.524	0.725	29.336
		-2.241	-0.132	-0.845	2.564	-1.665	4.073		-0.691	-0.252	0.448	2.990	-0.889	3.127	
50 years (528 assets)	0.01	-0.406	0.046	0.184	-0.014	0.116	0.820	29.347	-0.020	-0.003	0.438	-0.070	0.365	0.940	29.666
		-2.735	0.522	1.259	-0.156	0.599	4.720		-0.973	-0.035	2.858	-0.757	1.906	4.605	
	0.05	-0.398	-0.112	0.754	0.159	0.273	0.746	28.705	-0.026	-0.100	1.215	0.123	0.569	0.815	29.028
		-2.693	-0.403	2.862	0.686	0.837	3.383		-1.404	-0.396	4.236	0.602	1.721	3.413	
	0.1	-0.447	-0.292	0.320	0.417	-0.181	0.682	28.936	-0.039	-0.412	0.656	0.603	0.009	0.716	29.371
		-3.130	-0.933	1.042	1.660	-0.433	3.115		-1.955	-1.420	2.093	2.378	0.022	3.058	
	0.15	-0.419	0.169	0.139	0.143	-0.323	0.793	29.185	-0.018	0.362	0.660	0.008	-0.067	0.852	29.531
		-2.872	0.619	0.363	0.620	-0.697	4.066		-0.920	1.416	1.691	0.036	-0.148	3.734	
	0.2	-0.412	-0.077	-0.056	0.365	-0.522	0.776	29.196	-0.030	-0.120	0.332	0.498	-0.382	0.828	29.700
		-2.871	-0.309	-0.129	1.636	-1.091	4.030		-1.516	-0.509	0.772	2.256	-0.798	3.596	
	0.25	-0.414	0.043	-0.123	0.263	-0.607	0.828	29.387	-0.023	0.020	0.475	0.367	-0.263	0.854	29.880
		-2.905	0.191	-0.259	1.361	-1.262	4.514		-1.142	0.089	1.016	1.872	-0.547	3.746	

In addition, Table 5 reports the results controlled for the two specification of relative downside betas. In the left panel, we report results with downside risk specification of Ang et al. (2006). We observe that the downside risk beta does not capture any additional important dimension of risk when included in our full model specification. The same is true for the downside risk model of Lettau et al. (2014), which is captured in the right panel.

Finally, Table 6 reports regressions including additional betas from the three-factor model of Fama and French (1993).[17] This model is not explicitly related to the asymmetric features of market or volatility risk, but as we show in the Section 3, these factors may be just capturing market risk in different horizons in specific parts of the joint distribution of market and asset returns, so we should check whether they are not superior in describing these kind of risks. As in the case of other horse race regressions, the additional risk factors do not drive out the QS measures, which repeat the same pattern as in the cases without the additional variables.

Although the QS measures are correlated with some of the other variables discussed previously in the literature, they do not drive out the QS measures of risk. Moreover, these variables are, in most of the cases, subsumed by the variables from the full model. Our results are in agreement with recent results of Bollerslev et al. (2020), which show that the dependence characterized by the *co-occurrence* of negative asset and negative market returns posses the highest explanatory power on formation of asset returns among all specifications of disaggregated conventional beta. Importantly, we explicitly show that premium for this risk is generated by the dependence in the extreme left tail and by its short-term component. In addition, we extend the analysis to extreme volatility risk and show that the investors focus on more probable joint negative outcomes which unfold over the long horizon.

[17]We have to include only 2 additional betas as the market beta is already included in our full model.

Table 5: *Estimated Coefficients of the TR, EVR and Full Models controlled for Downside Risk Betas.* Displayed are prices of risk of full models also including either downside risk beta of Ang et al. (2006) or downside risk beta specification of Lettau et al. (2014). We use CRSP database between July 1926 and December 2015. Models are estimated for various values of thresholds given by τ. We employ 3 samples with varying number of minimum years. Long horizon is given by frequencies corresponding to 3-year cycle and longer. Below the coefficients, we include Fama-MacBeth t-statistics.

		DR beta of Ang et al. (2006)							DR of Lettau et al. (2014)						
	τ	λ^{DR1}	$\lambda^{\text{TR}}_{\text{long}}$	$\lambda^{\text{TR}}_{\text{short}}$	$\lambda^{\text{EV}}_{\text{long}}$	$\lambda^{\text{EV}}_{\text{short}}$	λ^{CAPM}	RMSPE	λ^{DR2}	$\lambda^{\text{TR}}_{\text{long}}$	$\lambda^{\text{TR}}_{\text{short}}$	$\lambda^{\text{EV}}_{\text{long}}$	$\lambda^{\text{EV}}_{\text{short}}$	λ^{CAPM}	RMSPE
	0.01	-0.017	0.129	0.629	-0.210	-0.194	0.826	26.387	-0.017	0.129	0.629	-0.210	-0.194	0.826	26.387
		-0.044	0.922	3.240	-1.484	-0.631	4.104		-0.044	0.922	3.240	-1.484	-0.631	4.104	
	0.05	0.149	0.042	1.250	-0.005	0.285	0.754	26.736	0.149	0.042	1.250	-0.005	0.285	0.754	26.736
		0.410	0.119	3.448	-0.017	0.559	3.216		0.410	0.119	3.448	-0.017	0.559	3.216	
	0.1	0.082	-0.086	0.997	0.412	0.212	0.573	26.548	0.082	-0.086	0.997	0.412	0.212	0.573	26.548
70 years		0.235	-0.216	1.895	1.414	0.306	2.451		0.235	-0.216	1.895	1.414	0.306	2.451	
(142 assets)	0.15	0.128	0.211	0.864	0.325	0.367	0.636	26.259	0.128	0.211	0.864	0.325	0.367	0.636	26.259
		0.363	0.638	1.433	1.272	0.512	3.036		0.363	0.638	1.433	1.272	0.512	3.036	
	0.2	0.121	-0.098	0.840	0.717	-0.207	0.594	25.397	0.121	-0.098	0.840	0.717	-0.207	0.594	25.397
		0.352	-0.283	1.242	2.780	-0.269	2.962		0.352	-0.283	1.242	2.780	-0.269	2.962	
	0.25	0.135	-0.030	0.454	0.802	-0.475	0.656	25.627	0.135	-0.030	0.454	0.802	-0.475	0.656	25.627
		0.391	-0.090	0.631	2.895	-0.598	3.399		0.391	-0.090	0.631	2.895	-0.598	3.399	
	0.01	0.107	0.161	0.345	-0.211	0.244	0.852	29.418	0.107	0.161	0.345	-0.211	0.244	0.852	29.418
		0.368	1.350	2.368	-1.760	0.912	4.611		0.368	1.350	2.368	-1.760	0.912	4.611	
	0.05	0.135	0.093	1.171	0.041	0.612	0.671	28.588	0.135	0.093	1.171	0.041	0.612	0.671	28.588
		0.486	0.308	3.716	0.167	1.493	2.934		0.486	0.308	3.716	0.167	1.493	2.934	
	0.1	0.192	-0.205	0.839	0.516	0.345	0.509	28.620	0.192	-0.205	0.839	0.516	0.345	0.509	28.620
60 years		0.711	-0.637	2.096	1.836	0.674	2.183		0.711	-0.637	2.096	1.836	0.674	2.183	
(267 assets)	0.15	0.240	0.252	0.657	0.257	0.362	0.638	28.943	0.240	0.252	0.657	0.257	0.362	0.638	28.943
		0.867	0.888	1.313	1.101	0.656	3.119		0.867	0.888	1.313	1.101	0.656	3.119	
	0.2	0.296	-0.236	0.536	0.721	-0.344	0.588	28.443	0.296	-0.236	0.536	0.721	-0.344	0.588	28.443
		1.055	-0.827	1.053	3.062	-0.589	3.021		1.055	-0.827	1.053	3.062	-0.589	3.021	
	0.25	0.313	-0.060	0.404	0.598	-0.605	0.686	29.091	0.313	-0.060	0.404	0.598	-0.605	0.686	29.091
		1.105	-0.233	0.733	2.763	-1.035	3.705		1.105	-0.233	0.733	2.763	-1.035	3.705	
	0.01	0.110	0.003	0.367	-0.086	0.418	0.873	29.683	0.110	0.003	0.367	-0.086	0.418	0.873	29.683
		0.471	0.038	2.938	-0.951	2.285	4.883		0.471	0.038	2.938	-0.951	2.285	4.883	
	0.05	0.142	0.005	1.167	-0.045	0.753	0.790	29.039	0.142	0.005	1.167	-0.045	0.753	0.790	29.039
		0.610	0.018	4.423	-0.206	2.372	3.612		0.610	0.018	4.423	-0.206	2.372	3.612	
	0.1	0.275	-0.120	0.628	0.276	0.164	0.695	29.396	0.275	-0.120	0.628	0.276	0.164	0.695	29.396
50 years		1.192	-0.385	1.978	1.168	0.397	3.189		1.192	-0.385	1.978	1.168	0.397	3.189	
(528 assets)	0.15	0.284	0.446	0.642	-0.134	0.034	0.831	29.516	0.284	0.446	0.642	-0.134	0.034	0.831	29.516
		1.228	1.631	1.568	-0.643	0.076	4.176		1.228	1.631	1.568	-0.643	0.076	4.176	
	0.2	0.304	-0.002	0.546	0.296	-0.332	0.760	29.626	0.304	-0.002	0.546	0.296	-0.332	0.760	29.626
		1.305	-0.007	1.206	1.381	-0.695	3.923		1.305	-0.007	1.206	1.381	-0.695	3.923	
	0.25	0.299	0.033	0.781	0.255	-0.133	0.789	29.797	0.299	0.033	0.781	0.255	-0.133	0.789	29.797
		1.271	0.147	1.549	1.356	-0.276	4.293		1.271	0.147	1.549	1.356	-0.276	4.293	

Table 6: *Estimated Coefficients of the TR, EVR and Full Models controlled for Fama and MacBeth (1973) factors.* Displayed are prices of risk of full models also including either HML and SMB betas of Fama and French (1993). We use CRSP database between July 1926 and December 2015. Models are estimated for various values of thresholds given by τ. We employ 3 samples with varying number of minimum years. Long horizon is given by frequencies corresponding to 3-year cycle and longer. Below the coefficients, we include Fama-MacBeth t-statistics.

	τ	λ^{SMB}	λ^{HML}	$\lambda^{\text{TR}}_{\text{long}}$	$\lambda^{\text{TR}}_{\text{short}}$	$\lambda^{\text{EV}}_{\text{long}}$	$\lambda^{\text{EV}}_{\text{short}}$	λ^{CAPM}	RMSPE
70 years (142 assets)	0.01	0.035	-0.050	0.133	0.636	-0.217	-0.174	0.838	26.281
		0.266	-0.280	0.960	3.276	-1.558	-0.569	4.206	
	0.05	-0.073	-0.181	0.401	1.108	-0.238	0.093	0.893	26.349
		-0.550	-1.034	1.238	3.072	-0.989	0.186	4.007	
	0.1	-0.009	-0.200	0.187	0.888	0.276	0.076	0.674	26.119
		-0.066	-1.188	0.562	1.777	1.101	0.112	3.093	
	0.15	0.001	-0.165	0.376	0.612	0.231	0.289	0.702	25.969
		0.010	-0.949	1.232	1.068	1.044	0.416	3.504	
	0.2	0.090	-0.150	0.016	0.535	0.712	-0.259	0.609	25.052
		0.685	-0.869	0.050	0.827	2.965	-0.341	3.107	
	0.25	0.067	-0.150	0.118	0.171	0.753	-0.731	0.691	25.349
		0.517	-0.873	0.359	0.251	2.759	-0.962	3.604	
60 years (267 assets)	0.01	-0.149	0.040	0.198	0.389	-0.233	0.281	0.851	29.218
		-1.184	0.252	1.657	2.622	-1.979	1.053	4.481	
	0.05	-0.175	-0.016	0.291	1.216	-0.204	0.552	0.827	28.377
		-1.434	-0.101	1.035	3.859	-1.018	1.410	3.858	
	0.1	-0.121	-0.051	0.063	0.996	0.255	0.341	0.650	28.578
		-0.978	-0.333	0.216	2.589	1.152	0.676	3.045	
	0.15	-0.143	-0.052	0.414	0.837	0.025	0.442	0.751	28.833
		-1.182	-0.340	1.561	1.831	0.137	0.821	3.926	
	0.2	-0.060	-0.054	-0.151	0.709	0.640	-0.261	0.618	28.464
		-0.492	-0.359	-0.559	1.550	3.120	-0.448	3.258	
	0.25	-0.102	-0.059	0.014	0.623	0.505	-0.534	0.726	29.051
		-0.850	-0.388	0.055	1.243	2.506	-0.940	4.000	
50 years (528 assets)	0.01	-0.087	0.006	-0.005	0.457	-0.081	0.493	0.874	29.354
		-0.761	0.041	-0.063	3.577	-0.921	2.582	4.805	
	0.05	-0.088	-0.051	0.135	1.172	-0.213	0.708	0.905	28.909
		-0.806	-0.361	0.604	4.502	-1.349	2.251	4.504	
	0.1	-0.044	-0.096	0.022	0.791	0.137	0.120	0.774	29.182
		-0.394	-0.697	0.092	2.507	0.781	0.289	3.917	
	0.15	-0.095	-0.126	0.612	0.730	-0.347	0.153	0.950	29.086
		-0.854	-0.919	2.645	1.926	-2.262	0.344	5.144	
	0.2	-0.042	-0.084	0.091	0.682	0.201	-0.306	0.804	29.402
		-0.376	-0.613	0.422	1.710	1.202	-0.652	4.429	
	0.25	-0.055	-0.090	0.101	0.914	0.163	-0.166	0.835	29.519
		-0.499	-0.662	0.476	2.004	1.033	-0.354	4.793	

6 Conclusion

We introduce a novel approach how to isolate effects of various risk dimensions on formation of expected returns. Until this point, studies focused either on exploring downside features of risk, or on investigating its horizon-specific properties. We define novel measures that estimates risk in specific part of the joint distribution over specific horizon, and we show that extreme risks are priced in cross-section of asset returns heterogeneously across horizons. Further, we argue that it is important to distinguish between tail market risk and extreme volatility risk. Tail market risk is characterized by the dependence between highly negative market and asset events. Extreme volatility risk is defined as co-occurrence of extremely high increases of market volatility and highly negative asset returns. Negative events are derived from distribution of market returns and its respective quantile is used for determining threshold values for computing quantile spectral betas.

Based on these results, we show that the price of risk varies over horizons and different parts of the joint distribution of asset and market return. More specifically, our TR model identifies that premium for tail market risk is mostly featured in its short-term component in the extreme left tail of the joint distribution. On the other hand, we discover that the premium for extreme volatility risk is mostly associated with its long-term component and higher values of threshold.

In order to consistently estimate the models, data with long enough history has to be employed. But if the data are available, our measures of risk are able to outperform competing measures and their performance is best for low threshold values suggesting that investors require risk premium for holding assets susceptible to extreme risks. Moreover, we show that the state-of-the-art downside risk measures does not capture the information contained in our newly proposed ones. Our results have important implications for asset pricing models. We show that taking into account only contemporaneous dependence averaged over the whole distribution when measuring risk exposure leads to omitting important information regarding the risk.

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A Estimation of quantile spectral betas

Estimation of QS betas defined in our paper is based on the smoothed quantile cross-periodograms studied in Baruník and Kley (2019). For a strictly stationary time series $X_{0,j}, \ldots, X_{n-1,j}$, we define $I\{\hat{F}_{n,j}(X_{t,j}) \leq \tau\} = I\{R_{n;t,j} \leq n\tau\}$ where $\hat{F}_{n,j}(x) \equiv n^{-1} \sum_{t=0}^{n-1} I\{X_{t,j} \leq x\}$ is the empirical distribution function of $X_{t,j}$ and $R_{n;t,j}$ denotes the rank of $X_{t,j}$ among $X_{0,j}, \ldots, X_{n-1,j}$. We have seen that the cornerstone of quantile spectral beta is quantile cross-spectral density defined in Equation 6. Its population counterpart is called rank-based copula cross-periodogram, CCR-periodogram, and is defined as

$$I_{n,R}^{j_1,j_2}(\omega; \tau_1, \tau_2) \equiv \frac{1}{2\pi n} d_{n,R}^{j_1}(\omega; \tau_1) d_{n,R}^{j_2}(-\omega; \tau_2) \tag{17}$$

where

$$d_{n,R}^{j}(\omega; \tau) \equiv \sum_{t=0}^{n-1} I\{\hat{F}_{n,j}(X_{t,j}) \leq \tau\} e^{-i\omega t} = \sum_{t=0}^{n-1} I\{R_{n;t,j} \leq n\tau\} e^{-i\omega t}, \quad \tau \in [0, 1]. \tag{18}$$

As discussed in Baruník and Kley (2019), CCR-periodogram is not a consistent estimator of quantile cross-spectral density. Consistency can be achieved by smoothing CCR-periodogram across frequencies. Following Baruník and Kley (2019), we employ the following

$$\hat{G}_{n,R}^{j_1,j_1}(\omega; \tau_1, \tau_2) \equiv \frac{2\pi}{n} \sum_{s=0}^{n-1} W_n(\omega - 2\pi s/n) I_{n,R}^{j_1,j_2}(2\pi s/n, \tau_1, \tau_2) \tag{19}$$

where W_n is defined in Section 3 of Baruník and Kley (2019). Estimator of quantile spectral beta is defined as

$$\hat{\beta}_{n,R}^{j_1,j_2}(\omega; \tau_1, \tau_2) \equiv \frac{\hat{G}_{n,R}^{j_1,j_2}(\omega; \tau_1, \tau_2)}{\hat{G}_{n,R}^{j_2}(\omega; \tau_2)}. \tag{20}$$

Consistency of the estimator can be proven using exactly same logic as in Theorem 3.4 in Baruník and Kley (2019) by replacing quantile coherency with quantile spectral beta.

B Specification of the Competing Models

In this section, we briefly describe the specification of the models we use in the main part of the paper. We denote market excess return as r_m and its mean and variance as μ_m and σ_m^2, respectively. Excess return of an asset is denoted as r_i with mean μ_i and variance σ_i^2.

We present how we estimate betas in the first-stage regression. The second-stage regression is the same for all the models and is performed via OLS by regressing the average asset returns on their betas. This then leads to the estimated values of RMSPE.

B.1 Downside Risk Models

We follow two specifications of the downside risk models. First, we use specification of Ang et al. (2006) and estimate their relative downside risk betas as

$$\beta_i^{DR1} \equiv \beta_{i,\mu_m}^{-} - \beta_i = \frac{\mathbb{C}ov(r_i, r_m | r_m < \mu_m)}{\mathbb{V}ar(r_m | r_m < \mu_m)} - \frac{\mathbb{C}ov(r_i, r_m)}{\mathbb{V}ar(r_m)}. \tag{21}$$

Downside risk beta specification of Lettau et al. (2014) is then obtained as

$$\beta_i^{DR2} \equiv \beta_{i,\delta}^{-} - \beta_i = \frac{\mathbb{C}ov(r_i, r_m | r_m < \delta)}{\mathbb{V}ar(r_m | r_m < \delta)} - \frac{\mathbb{C}ov(r_i, r_m)}{\mathbb{V}ar(r_m)} \tag{22}$$

where we define the threshold value as $\delta \equiv \mu_m - \sigma_m$.

B.2 Generalized Disappointment Aversion Models

We employ specification of Generalized Disappointment Aversion (GDA) models of Farago and Tédongap (2018) and estimate two main versions of their cross-sectional models. Their models are based on disappointment events $\mathcal{D}_t$.

B.2.1 GDA3

First model is their three-factor model, which does not contain volatility-related factors. The betas posses the following form

$$\beta_{i,m} \equiv \frac{\mathbb{C}ov(r_i, r_m)}{\mathbb{V}ar(r_m)} \tag{23}$$

$$\beta_{i,\mathcal{D}} \equiv \frac{\mathbb{C}ov(r_i, I(\mathcal{D}))}{\mathbb{V}ar(I(\mathcal{D}))} \tag{24}$$

$$\beta_{i,m\mathcal{D}} \equiv \frac{\mathbb{C}ov(r_i, r_m I(\mathcal{D}))}{\mathbb{V}ar(r_m I(\mathcal{D}))} \tag{25}$$

where we follow the specification and set $\mathcal{D}_t = \{r_{m,t} < b\}$ where $b = -0.03$ and I is an indicator function.

B.2.2 GDA5

Five-factor specification of the GDA model contains, in addition to the betas from the three-factor model, the following betas

$$\beta_{i,X} \equiv \frac{\mathbb{C}ov(r_i, \Delta\sigma_m^2)}{\mathbb{V}ar(\Delta\sigma_m^2)} \tag{26}$$

$$\beta_{i,X\mathcal{D}} \equiv \frac{\mathbb{C}ov(r_i, \Delta\sigma_m^2 I(\mathcal{D}))}{\mathbb{V}ar(\Delta\sigma_m^2 I(\mathcal{D}))} \tag{27}$$

where the disappointment events are given by $\mathcal{D}_t = \left\{ r_{m,t} - a\frac{\sigma_m}{\sigma_X}\Delta\sigma_{m,t}^2 < b \right\}$ where $\Delta\sigma_{m,t}^2$ are increments of market volatility, $\sigma_X^2 = \mathbb{V}ar(\Delta\sigma_m^2)$, $a = 0.5$ and $b = -0.03$.

B.3 Coskewness and Cokurtosis

Following work of Kraus and Litzenberger (1976); Harvey and Siddique (2000); Dittmar (2002); Ang et al. (2006), we estimate the coskewness and cokurtosis as

$$CSK_i \equiv \frac{\mathbb{E}[(r_i - \mu_i)(r_m - \mu_m)^2]}{\sqrt{\mathbb{E}[(r_i - \mu_i)^2]}\mathbb{E}[(r_m - \mu_m)^2]}, \tag{28}$$

$$CKT_i \equiv \frac{\mathbb{E}[(r_i - \mu_i)(r_m - \mu_m)^3]}{\sqrt{\mathbb{E}[(r_i - \mu_i)^2]}\mathbb{E}[(r_m - \mu_m)^{3/2}]}. \tag{29}$$

B.4 Fama-French Three-Factor Model

Betas of the three-factor model of Fama and French (1993) are estimated via time-series regression of excess asset return on three factors: SMB (obtained by sorting stocks based on their size), HML (obtained by sorting stocks based on their book-to-market vale) and MKT (market factor)

$$r_{i,t} = \alpha_i + \beta_i^{SMB} SMB_t + \beta_i^{HML} HML_t + \beta_i^{MKT} MKT_t + e_{i,t}. \tag{30}$$

Factor data were obtained from Kenneth French's online data library.

C Different Definition of Long horizon - 1.5 years

C.1 TR and EVR Models

Table 7: *Estimated Coefficients of the TR, EVR and Full Models.* Prices of risk estimated on monthly stock data from CRSP database sampled between July 1926 and December 2015. Models are estimated for various values of thresholds given by τ. We employ three samples with varying number of minimum years. Long horizon is given by frequencies corresponding to 1.5-year cycle and longer. Below the coefficients, we include Fama-MacBeth t-statistics.

		Tail market risk				Extreme volatility risk				Full model					
	τ	λ^{TR}_{long}	λ^{TR}_{short}	λ^{CAPM}	RMSPE	λ^{EV}_{long}	λ^{EV}_{short}	λ^{CAPM}	RMSPE	λ^{TR}_{long}	λ^{TR}_{short}	λ^{EV}_{long}	λ^{EV}_{short}	λ^{CAPM}	RMSPE
70 years (142 assets)	0.01	-0.045	0.644	0.751	26.672	-0.084	-0.258	0.946	28.576	0.135	0.610	-0.213	-0.150	0.832	26.436
		-0.476	3.303	3.910		-0.883	-0.853	4.885		0.896	3.085	-1.366	-0.495	4.151	
	0.05	0.149	1.272	0.716	26.801	0.242	0.378	0.689	28.212	0.160	1.258	-0.024	0.297	0.738	26.755
		0.699	3.485	3.861		1.413	0.774	3.047		0.431	3.519	-0.081	0.602	3.046	
	0.1	0.432	1.153	0.735	27.109	0.519	0.441	0.549	27.226	-0.005	1.004	0.442	0.287	0.553	26.567
		1.673	2.265	4.119		2.588	0.649	2.461		-0.011	1.964	1.357	0.425	2.269	
	0.15	0.611	0.842	0.730	26.651	0.557	0.522	0.592	27.025	0.312	0.832	0.344	0.501	0.615	26.280
		2.493	1.453	4.050		2.836	0.756	2.817		0.898	1.436	1.210	0.719	2.816	
	0.2	0.732	0.224	0.780	26.944	0.750	-0.313	0.590	25.765	0.050	0.753	0.709	-0.139	0.580	25.417
		2.726	0.326	4.432		3.604	-0.411	2.950		0.137	1.127	2.463	-0.183	2.784	
	0.25	0.786	-0.165	0.808	27.241	0.869	-0.563	0.631	25.703	0.045	0.418	0.844	-0.420	0.628	25.608
		2.898	-0.230	4.631		3.695	-0.727	3.238		0.130	0.589	2.751	-0.544	3.165	
60 years (267 assets)	0.01	-0.036	0.430	0.759	29.723	-0.083	0.263	0.934	30.514	0.168	0.380	-0.227	0.250	0.865	29.496
		-0.491	2.685	4.130		-1.105	1.002	5.093		1.342	2.388	-1.784	0.948	4.630	
	0.05	0.225	1.187	0.661	28.681	0.256	0.611	0.655	29.996	0.175	1.241	0.016	0.646	0.675	28.617
		1.211	3.659	3.574		1.493	1.545	2.878		0.562	3.921	0.060	1.626	2.819	
	0.1	0.368	0.958	0.715	29.239	0.550	0.371	0.501	29.193	-0.109	0.906	0.538	0.402	0.490	28.678
		1.549	2.407	3.999		2.516	0.753	2.200		-0.318	2.265	1.730	0.815	2.015	
	0.15	0.559	0.729	0.706	29.244	0.488	0.495	0.593	29.614	0.358	0.770	0.250	0.398	0.622	29.089
		2.326	1.482	3.940		2.349	0.923	2.805		1.191	1.573	0.968	0.739	2.918	
	0.2	0.538	0.236	0.762	29.736	0.677	-0.251	0.581	28.726	-0.166	0.657	0.757	-0.296	0.555	28.607
		2.139	0.450	4.343		3.146	-0.440	2.913		-0.560	1.309	2.870	-0.517	2.739	
	0.25	0.523	0.208	0.782	30.035	0.669	-0.526	0.645	29.286	-0.041	0.539	0.680	-0.506	0.641	29.244
		2.114	0.375	4.491		3.065	-0.895	3.382		-0.154	0.996	2.755	-0.870	3.356	
50 years (528 assets)	0.01	-0.087	0.436	0.825	29.726	-0.088	0.463	0.969	30.286	-0.007	0.407	-0.073	0.422	0.871	29.689
		-1.535	2.981	4.634		-1.507	2.500	5.396		-0.082	2.829	-0.798	2.305	4.787	
	0.05	0.001	1.162	0.762	29.233	0.071	0.636	0.807	30.075	0.060	1.242	-0.055	0.765	0.790	29.054
		0.004	3.980	4.238		0.542	1.991	3.795		0.209	4.422	-0.236	2.446	3.474	
	0.1	0.195	0.783	0.784	29.756	0.316	0.069	0.672	29.669	-0.060	0.770	0.302	0.155	0.662	29.400
		0.972	2.401	4.418		1.836	0.168	3.152		-0.187	2.416	1.165	0.381	2.932	
	0.15	0.404	0.811	0.765	29.583	0.168	-0.003	0.776	30.089	0.532	0.782	-0.150	0.084	0.810	29.552
		2.010	1.968	4.288		0.991	-0.006	3.817		1.840	1.923	-0.637	0.187	3.893	
	0.2	0.324	0.631	0.808	29.957	0.377	-0.316	0.725	29.730	0.038	0.692	0.332	-0.342	0.727	29.650
		1.599	1.363	4.594		2.045	-0.687	3.694		0.144	1.535	1.359	-0.731	3.611	
	0.25	0.303	0.775	0.819	30.024	0.373	-0.116	0.752	29.926	0.046	0.912	0.335	-0.136	0.750	29.817
		1.509	1.546	4.690		2.019	-0.247	3.980		0.199	1.833	1.536	-0.287	3.958	