Macro Factor-Mimicking Portfolios [a]

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Abstract

The estimation of risk factors and their replication through mimicking portfolios are of critical importance for academics and practitioners in finance. We propose a general optimization framework to construct macro-mimicking portfolios that encompasses existing mimicking approaches, such as two-pass cross-sectional regressions (Fama and MacBeth, 1973) and maximal correlation portfolio approach (Huberman et al., 1987). We incorporate machine learning estimation improvements to mitigate the impact of estimation errors in the observed macro factors on mimicking portfolios. We provide an application to the construction of mimicking portfolios that replicate three uncorrelated global macro factors: namely growth, inflation surprises, and financial stress indicators. We show how these machine-learning mimicking portfolios can be used to improve the risk-return profile of a typical endowment asset allocation.

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1 – Introduction

Factor investing is driving a significant change in the way investors approach asset management. In a broad sense, factors are the fundamental drivers of asset returns (Ang, 2014). Investment returns are rewards that investors harvest for holding assets that expose them to systematic source of risks. The main advantage of the factor representation lies in the dimensionality reduction of the problem investors face. Providing that the estimated factor model fits reasonably well the variability (variance) of asset returns, investors can focus on a narrower set of instruments (the factors) rather than having to forecast all portfolio components individually. The factor investing approach is not new and has been present in the academic literature for a long-time, as exemplified by the CAPM or the APT models.

From theory to practice, investors need to empirically identify the underlying factors. Although of work has been devoted to statistical factors (Roll and Ross, 1980; Connor and Korajczyk, 1988) or fundamental factors (Fama and French, 1992 and 1993), macroeconomic factors have been somehow less popular. While it is clear that economic conditions are pervasive for the dynamics of financial asset returns (Chen et al., 1986; Ang and Bekaert, 2004), one major practical limitation of macro factors is that they are not tradable. A standard way to tackle this issue is to try to associate some investible asset classes to macroeconomic indicators, such as equities with growth or spread between nominal and inflation-linked bonds with inflation (Greenberg et al., 2016).

While simple and sensible, this approach remains arbitrary as the choice of asset classes aiming to replicate macroeconomic factors is not motivated by a statistical efficiency criterion. A more general solution adopted in the academic literature is to construct factor-mimicking portfolios (FMPs henceforth) which are investable portfolios replicating the underlying non-traded factors.

Although most of the existing literature has focused on the use of FMPs for pricing the cross section of equity or bond returns (Chen et al., 1986; Vassalou, 2003; and Pukthuantong et al., 2020), few studies are considering their relevance in a cross-asset allocation context. Bender et al. (2019) propose a comprehensive FMP-based multi-asset alpha framework to produce strategic and tactical asset allocations. Chousakos and Giamouridis (2020) expand the cross-asset efficient frontier with macro-mimicking portfolios (for growth, fragility, volatility). Amato and Lohre (2020) show that using a macro FMP-based multi-asset risk parity strategy leads to a better risk-adjusted performance than a traditional asset-based risk-based strategy.

This article aims to complement these recent cross-asset applications. To do so, we propose a general FMP construction framework that encompasses popular factor-mimicking portfolio approaches, such as two-pass cross-sectional regressions (Fama and MacBeth, 1973) and maximum correlation portfolio approach (Huberman et al., 1987). One of the benefits of our general FMP construction framework is that it allows investors to easily integrate practical portfolio constraints such as short-sales or liquidity restrictions.

Since the observed macro factors are measured imprecisely[1], we also develop a new machine learning estimation procedure that corrects for the errors-in-variables (EIV) bias in the OLS-FMP weight estimates. Our method can be viewed as a modified version of the estimation approach of Connor and Korajczyk (1991) or Giglio and Xiu (2020) who are using the first statistical factors of the base asset returns as instruments for the observed macro factors. The difference being that we are relying on an alternative supervised statistical extraction approach where individual asset components are selected by a LASSO regression, the target PCA (tPCA; Bai and Ng, 2008), rather

[1] Traditional macroeconomic indicators are notoriously noisy, published with some significant lag or revised ex-post. The diversity of measures or the recurrent debates between economists of how to measure GDP or define inflation illustrates how difficult it can be to measure economic variables.

than the standard PCA. This allows to take into account the predictive power of assets for each targeted macro factor.

In our empirical analysis, we first compare the ability of various FMP methodologies to replicate the characteristics of three global macro factors (growth, inflation surprises and financial stress) over nearly fifty years spanning different economic regimes - both in-sample and out-of-sample - using a set of investible assets[2] that are representative of cross-asset portfolios and can be invested through liquid and cost-efficient vehicles - such as futures/swaps derivatives or ETFs. Overall, the results prove the superiority of our machine learning (ML) macro-mimicking approach over the standard FMP approaches. We also present a case study where we illustrate how this FMP methodology can be used to hedge the macro risk exposures of an institutional investor allocation. We find that macro-hedging can improve the investor risk-return profiles significantly.

The remainder of this paper is organized as follows. In Section 2, we introduce the general FMP analytical framework. In Section 3, we discuss the estimation of the FMPs in the presence of measurement errors. In Section 4, we detail the empirical estimation of macro FMPs before illustrating their potential use for hedging a representative institutional portfolio in Section 5. We conclude in Section 6.

[2] We consider equities, nominal and inflation-linked government bonds, credit, commodities, and foreign exchange investible asset classes.

2 – Factor-Mimicking Portfolios Analytical Framework

Factor-Mimicking Portfolios have been the subject of a vast academic literature (Balduzzi and Robotti, 2008). In general terms, they consist in forming portfolios of investable assets (so-called "base assets") that replicate ("mimick") the behavior of one or several (potentially non-investable) factors. In the asset pricing literature, two main FMP approaches have been proposed: the two-pass cross-sectional regressions (CSR) and the maximum correlation portfolios (MCP).

Both approaches have in common to rely, for a set of N assets, on factor models of the type:

$$\boldsymbol{R}_t = \boldsymbol{\mu} + \boldsymbol{B}\boldsymbol{F}_{\boldsymbol{t+h}} + \boldsymbol{\varepsilon}_t \tag{1}$$

where $\boldsymbol{R}_t$ and $\boldsymbol{\mu}$ are the $(N \times 1)$ vectors of base asset returns and expected returns, respectively; $\boldsymbol{F}_{t+h}$ is the $(K \times 1)$ vector of innovations ("surprises") in the K factors that we want to replicate[3]. $\boldsymbol{B}$ is the $(N \times K)$ matrix of asset loadings to factors. $\boldsymbol{\varepsilon}_t$ is the $(N \times 1)$ vector of mean-zero disturbances.

In the two-pass CSR approach, pioneered by Fama and MacBeth (1973), the asset loadings of the factor model (1) are estimated via time-series regressions of base asset returns onto the factors. In the second step, the FMPs are obtained jointly by regressing cross-sectionally the returns of the base assets onto the estimated betas. The estimated slope coefficients are the returns of the FMPs. While in the original Fama-MacBeth approach, the second-pass cross-sectional regressions are usually performed through Ordinary Least Squares (OLS), meaning that the base asset returns are equally weighted, several authors have also recommended alternative weighting schemes to deal

[3] For macro factors, the horizon h is typically superior to 0 as financial assets tend to predict the future evolution of economic variables.

with the heteroskedasticity and cross-correlations in (1), such as Weighted Least Squares (WLS; Litzenberger and Ramaswany, 1979) or Generalized Least Squares (GLS; Lehman and Modest, 1988). In the MCP approach, pioneered by Breeden et al. (1989) and Lamont (2001), the FMPs are determined separately by regressing the base asset returns on each factor – meaning that the asset loadings of the factor model (1) are estimated for each factor through univariate regressions - or equivalently by projecting each factor on the base asset returns. The FMP weights are given by the coefficients estimated from these regressions.

Following Huberman et al. (1987), the vector of weight of the *k*-th macro factor-mimicking portfolio $\boldsymbol{w}_k$ is the solution to the variance-minimization program:

$$\text{Min}\, \frac{1}{2} \boldsymbol{w}_k^{\boldsymbol{T}} \boldsymbol{\Omega}\, \boldsymbol{w}_k \tag{2}$$

$$s.t.\, \boldsymbol{B}^{\boldsymbol{T}} \boldsymbol{w}_k = \boldsymbol{\beta}_k$$

where $\boldsymbol{\Omega}$ is the $(N \times N)$ variance-covariance matrix of the base assets and $\boldsymbol{\beta}_k$ is the $(K \times 1)$ vector of risk exposures associated with the mimicking portfolio on the *k*-th factor. By convention, we assume that each FMP has unit-beta exposure to the factor it aims to replicate and pre-specified exposures to other factors; meaning that the vector $\boldsymbol{\beta}_k$ has 1 in *k*-th entry and β_{kl} for $l \neq k$. As there are generally more investable assets than factors $(N \gg K)$, there are an infinite number of portfolios that can replicate the desired target factor exposures. Among all those portfolios, program (2) states that the FMP portfolio must be the one that maximizes the explanatory power of the factor model (1). Indeed, constraining factor exposures and minimizing variance simultaneously is equivalent to minimize the portfolio's idiosyncratic risk[4]. As a result, the *k*-th FMP is maximally correlated with the *k*-th factor. In general terms, this means that FMPs are well-

[4] Substituting the residual risk matrix $\boldsymbol{\Omega}_\varepsilon$ for the total risk matrix $\boldsymbol{\Omega}$ in the objective function (2) leads to the same FMP solution. See Melas et al. (2010) or Appendix 2.

diversified portfolios that target some specific risk exposures with respect to a set of factors of interest.

Solving program (2), the portfolio weights for the full set of the FMPs are given by the column vectors of the $(N \times K)$ weighting matrix (see Appendix 1):

$$\boldsymbol{W}_K^* = \boldsymbol{\Omega}^{-1}\boldsymbol{B}\left(\boldsymbol{B}^{\boldsymbol{T}}\boldsymbol{\Omega}^{-1}\boldsymbol{B}\right)^{-1}\boldsymbol{B}_K \tag{3}$$

where $\boldsymbol{B}_K$ is the $(K \times K)$ matrix collecting the target exposures for the various FMPs, with $\boldsymbol{B}_K = (\boldsymbol{\beta}_1 \boldsymbol{\beta}_2 \dots \boldsymbol{\beta}_K)$. The associated return of the *k*-th FMP is obtained as $FMP_{k,t} = \boldsymbol{w}_k^{*\boldsymbol{T}}\boldsymbol{R}_t$ [5].

In Table 1, we show how it is possible to recover all the traditional factor-mimicking portfolio solutions from the general formula (3). In particular, we can see that all FMP construction methods can be distinguished by three types of specifications:

(i) the covariance matrix $\boldsymbol{\Omega}$ that can be equal either to an identity matrix (OLS-CSR), a diagonal matrix (WLS-CSR), or a full matrix (GLS-CSR, Target-beta MCP and Unit-beta MCP);

(ii) the factor loading matrix $\boldsymbol{B}$ that can be obtained either by running single-factor (univariate) time-series regressions (all MCPs) or multifactor (multivariate) time-series regressions (all CSRs).

(iii) the target exposure matrix $\boldsymbol{B}_K$ that can be equal either to an identity matrix (all CSRs) or a general matrix (all MCPs).

[5] Note that while the FMP betas are by construction equal to the targeted ones, since $\boldsymbol{B}^{\boldsymbol{T}}\boldsymbol{W}_K^* = \boldsymbol{B}_K$, their covariances are generally different from the true factor covariances, since $Cov(\boldsymbol{W}_K^{*\boldsymbol{T}}\boldsymbol{R}_t) = \boldsymbol{B}_K{}^{\boldsymbol{T}}\left(\boldsymbol{B}^{\boldsymbol{T}}\boldsymbol{\Omega}^{-1}\boldsymbol{B}\right)^{-1}\boldsymbol{B}_K \neq Cov(\boldsymbol{F}_{t+h})$.

Three remarks are worth mentioning regarding the general FMP optimization program (2). First, we note that the use of a variance minimization program to obtain FMP portfolios is not new. To the best of our knowledge, Huberman et al. (1987) are the first to have advocated such approach. More recently, this has also been considered by Melas et al. (2010) and Pukthuangthong et al. (2020). However, these authors always relate it to some specific FMP cases, while we show here that we can cast all the proposed FMP methodologies into one single generic formula (3). Our minimum variance mimicking portfolio solution is also very similar to the one proposed by Roll and Srivastava (2018). The difference being that their mimicking portfolios seek to match the systematic risk exposures of a given asset or portfolio, whereas our approach is designed to reproduce the systematic exposures of the macro factors themselves. Second, in the program (2) or in the applications below, we do not impose any additional portfolio constraints other than target exposures $\boldsymbol{\beta}_k$. In particular, by default, FMPs are long-short portfolios. It is straightforward to add in the portfolio optimization program (2) various forms of constraints. For instance, the incorporation of long-only constraints, minimum or maximum per base asset, or even the addition of scores (based on liquidity, alpha etc…) are feasible as long as these constraints respect the convex nature of the optimization program. Obviously, in most of these cases, analytical solutions such as (3) are not available anymore. Third, while in the analytical expressions above or in our empirical applications, we are considering only static betas (e.g. FMP weights), our general framework can easily accommodate dynamic asset loadings through the use of lagged macroeconomic or asset specific instrumental variables as in Ferson and Harvey (1991).

3 – Machine Learning Factor-Mimicking Portfolio Estimates

In theory, expression (3) provides the Best Linear Unbiased estimator of the FMP weights. In practice, however, the individual betas are estimated imprecisely due to the presence of measurement error in the observed macroeconomic factors [6]. Since the estimated betas are used in the construction of the FMP, this introduces an error-in-variables (EIV) bias in the OLS FMP weight estimates.

To correct for this bias, we follow the instrumental variables (IV) estimation approach of Connor and Korajczyk (1991) and Giglio and Xiu (2020) that uses the first principal components (PCs) of the base asset returns as instruments for the observed macro factors to estimate the factor loadings in (1). This approach can be viewed as an application of the two-stage least squares (2SLS) regression method. In the first step, we project the observed factors ($\boldsymbol{F}_{t+h}$) against the L first PCs of the asset returns ($\boldsymbol{F}_{t+h}^{PC}$), with $L \leq N,$ to obtain the fitted values of the observed factors ($\widehat{\boldsymbol{F}}_{t+h}^{PC}$). In the second step, we regress the base asset returns against the values of the factors estimated from PCA to obtain the EIV-corrected factor loadings that are substituted in (3) to estimate the FMP weights (see Appendix 3).

A sufficient condition for this procedure to remove the measurement error in the observed factors is that the first PCs of the base asset returns span the same space as the true factors [7]. In practice,

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[6] See Ghysels et al. (2018) for an analysis of the impact of revisions on macroeconomic statistics or Rigobon and Sack (2008) for the impact of noise. More generally, following Stock and Watson (2002), a vast literature analyzes the issue of the measurement of the business cycle. The nowcasting approach presented in Beber et al. (2015) defines an alternative way to measure business cycle.

[7] Formally, consider the classical measurement error model: $\boldsymbol{F}_{t+h} = \boldsymbol{F}_{t+h}^{*} + \boldsymbol{\eta}_{t+h}$, with $\boldsymbol{F}_{t+h}^{*}$ the true factors and $\boldsymbol{\eta}_{t+h}$ the measurement errors such that $Cov\left(\boldsymbol{\eta}_{t+h,}\boldsymbol{F}_{t+h}^{*}\right) = \boldsymbol{0}$. The regression of the observed factors onto the first PCs of the asset returns yields the coefficients $\widehat{\boldsymbol{b}}_K = Cov(\boldsymbol{F}_{t+h}^{PC}, \boldsymbol{F}_{t+h})^{\boldsymbol{T}}\, Var(\boldsymbol{F}_{t+h}^{PC})^{-1}$. If the first PCs can span the same space as the true factors, this means that there exists an invertible $(L \times K)$ matrix $\boldsymbol{L}$, such that $\boldsymbol{F}_{t+h}^{PC} = \boldsymbol{L}\boldsymbol{F}_{t+h}^{*}$. We can then write $\widehat{\boldsymbol{b}}_K = \; Var(\boldsymbol{F}_{t+h}^{*})\boldsymbol{L}^{\boldsymbol{T}}[\,\boldsymbol{L}\, Var(\boldsymbol{F}_{t+h}^{*})\,\boldsymbol{L}^{\boldsymbol{T}}]^{\boldsymbol{-1}} = \boldsymbol{L}^{\boldsymbol{-1}}$. The fitted values of the observed factors are then equal to $\widehat{\boldsymbol{F}}_{t+h}^{PC} = \widehat{\boldsymbol{b}}_K\boldsymbol{F}_{t+h}^{PC} = \boldsymbol{L}^{\boldsymbol{-1}}\boldsymbol{F}_{t+h}^{PC} = \boldsymbol{L}^{\boldsymbol{-1}}\boldsymbol{L}\,\boldsymbol{F}_{t+h}^{*} = \boldsymbol{F}_{t+h}^{*}$.

however, this might not be necessarily the case if some of the latent factors are weak, i.e. affect only a small subset of base assets or affect all base assets only weakly. To mitigate this issue, we modify the IV estimation approach by replacing the traditional unsupervised PCA by an alternative supervised statistical approach, the target PCA (tPCA), that uses the predictive power of the base assets for each macro factor when extracting the statistical factors (Bair et al, 2006; Bai and Ng, 2008). Following Bai and Ng (2008), we use a "soft" thresholding LASSO penalty rule to perform for each macro factor the subset selection of the base asset returns. We then perform a traditional PCA on the selected base asset returns to extract the first L PCs onto which the macro factor of interest is projected. We finally regress the individual base asset returns against the values of the macro factors estimated from the tPCA to obtain the machine learning (ML) factor loadings estimates $\widehat{\boldsymbol{B}}^{ML}$. We refer in the rest of the article to the ML FMPs (ML-CSR) as the mimicking portfolios obtained by replacing the OLS beta estimates by the ML ones into the OLS-CSR specification [8].

[8] The ML-CSR FMP is thus given by: $\boldsymbol{W}_K^{*ML-CSR} = \widehat{\boldsymbol{B}}^{ML}\left(\widehat{\boldsymbol{B}}^{ML^{\boldsymbol{T}}}\widehat{\boldsymbol{B}}^{ML}\right)^{\boldsymbol{-1}}$, with $\widehat{\boldsymbol{B}}^{ML}$ the $(N \times K)$ matrix of the factor loadings estimated from target PCA.

4 –Empirical Analysis of Macro FMPs

As for any FMP, building macro factor-mimicking portfolios requires a set of base assets. In our case, these base assets need not only to be investable but also to be available over a long history as our empirical setting starts in the early seventies. Table 2 lists the nine selected base assets. They cover major asset classes such as equities, government bonds, credit, inflation-linked bonds[9], commodities and foreign exchange. For base assets that are not spread of indices, returns are computed as excess returns over cash rates (USD Libor 1 month). For most investors, they can be invested through liquid and cost-efficient vehicles such as derivatives (futures or swaps) or ETFs.

The sample period spans from January 1974 to March 2020 (555 monthly observations). These near-50-years data window covers different business cycles with various macroeconomic events such as the oil shock of the 70's or several economic recessions and well-known episodes of market crashes.

Regarding macro factors, we consider as in Ang (2014) three global macroeconomic variables: growth, inflation and financial stress[10]. Following the academic literature, we rely on macroeconomic innovations rather than levels. Growth is measured through the OECD Composite Leading Indicator (CLI), which is a popular predictive measure for global economic activity, as it focuses on indicators able to capture turning points in the global business cycle. Furthermore, the indicator removes the effect of long-term trends, providing a measure of innovations in future

[9] U.S. Treasury inflation-linked bonds (TIPS) have been launched only in 1997. To replicate the return of TIPS before that date, we follow the methodology developed by Swinkels (2018) based on real rates. Consistently with the author, we obtain a correlation of 0.6 between the simulated returns and observed returns over the common sample post-1997.

growth for a large part of the world economy. Inflation is proxied by the six-month ahead difference between the realized and lagged past year OECD year-on-year inflation rates, hence assuming sticky expectations. Finally, for financial stress, we use an equal-weight combination of two popular indicators, the Chicago Fed's National Financial Conditions Index (NFCI) and the turbulence index developed by Chow et al. (1999)[11].

To facilitate their interpretation, the global growth, inflation and financial stress factors are first normalized to z-scores by subtracting their full-sample average from each observation and dividing by their full-sample standard deviations. To further improve their interpretability and their statistical properties, the three macro factors are orthogonalized following the approached described in Klein and Chow (2013). This "egalitarian" approach leads to orthogonal factors without being dependent on a specific order of variables (see Appendix 4 for more details). In Table 3, we report various correlation metrics involving original macro factors and orthogonalized ones. On the upper side of the matrix, we report correlation figures among original macro factors. Consistently with expectations, growth and inflation surprises are positively correlated, while growth and financial stress factors are negatively correlated. The correlation between inflation surprises and financial stress is close to zero. On the lower part of the matrix, we report the correlation among the orthogonalized macro factors which are zero by construction. On the diagonal, we report the correlation between the original and orthogonalized factors: the high correlation numbers (all above 0.96) shows that the orthogonalization structure does not change the economic nature of the factors, just renders them more statistically easy to handle. As a last

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[11] The NFCI index is a weighted average of a large spectrum of individual measures of financial conditions, mixing measures of liquidity and volatility in financial markets (such as VIX, swaptions or TED spread) with commercial banking conditions (such as FRB Senior Loan Officer Survey or Consumer Credit Outstanding) or health of financial institutions. The turbulence indicator is computed as the Mahalanobis distance metric applied to the monthly returns of the base asset listed in Table 1, and smoothed using a six-months rolling windows.

step, the resulting individual orthogonalized macro factors are finally scaled to a 1% monthly volatility.

To construct macro FMPs, we first need to estimate a multifactor model linking base asset returns to the macro factors we want to mimick. For this, we use returns over 12-month rolling samples in order to account for the fact that financial assets can react to economic events ahead of their advent. In total, we regress the base assets excess returns from *t*-12 to *t* on the macro variables observed in month *t*.

Table 4 reports the results of the OLS estimates of these multivariate regressions over the full sample (555 observations). T-statistics are corrected for the effect of overlapping due to the rolling windows through Newey-West estimates with a lag of twelve. All regressions are statistically significant, with R^2 ranging from 6% to 32%. The set of base assets are reacting very differently to the various orthogonalized macro factors. As emphasized by Lehman and Modest (1988), it is critical that the base assets used to form the macro factor-mimicking portfolios display sufficient dispersion in their macro betas to replicate them efficiently. Higher growth benefits to equities, credit, and industrial metals while it impacts negatively nominal bonds performance. Inflation surprise is beneficial to inflation-linked bonds versus nominal bonds, gold and energy and negatively impacts the U.S. dollar. Financial stress negatively impacts growth assets (equities, credit) and benefits to gold, energy and U.S. dollar.

As explained in Section 3, poorly estimated factor models can undermine the quality of the FMPs. This is particularly relevant for macro factor models since they are inherently noisy. To correct for the classic errors-in-variables problem, we use the machine learning regression estimation framework described in the previous section. For each orthogonalized observed macro factor, we first extract the relevant statistical factors by performing a target PCA, where individual base asset return components are selected by a LASSO regression[12], and estimate the fitted value of the macro factor accordingly (see Section 3). We then regress the individual asset excess returns against the orthogonal macro factors estimated by the target PCA. The regression results of the machine learning factor model are displayed in Table 5. We observe that the explanatory power of the machine learning factor model is much larger than the equivalent OLS model in Table 5, with R^2 doubled or more for each base asset. This confirms the importance of the considered macro factors for the base asset returns.

Equipped with factor loading estimates, one can then build macro FMPs based on the various methodologies presented above[13]. We consider four specific unit-beta FMP candidates: (i) **OLS**: the OLS-CSR Fama-McBeth model; (ii) **WLS**: FMP with WLS correction to estimate the variance-covariance matrix of factor model residuals; (iii) **MCP**: unit-beta MCP; (iv) **ML**: the ML-CSR where we use machine learning beta estimates in the Fama-McBeth model.

In Figure 1, we present the composition of these factor-mimicking portfolios for the three global macro factors. Some base assets display consistent exposures across the different FMP approaches. Growth FMPs are long equities, industrial metals, FX-carry strategies and U.S. dollar and are short nominal treasuries and gold. Inflation FMPs are short U.S. dollar and industrial metals, and long inflation-linked bonds, credit, energy and gold. Financial stress FMPs are long U.S. dollar, industrial metals and energy, and short equities, credit and inflation-linked bonds. Other capital exposures vary according to the FMP approach considered.

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[12] The LASSO shrinkage parameter is automatically selected according to the Bayesian Information Criterion (BIC).

[13] To build these macro FMPs, we use monthly excess returns of base assets. These returns are consistent with returns an investor would get in unfunded instruments (e.g. futures, swaps,…) replicating associated indices.

In Table 6, we report different characteristics of these macro FMPs. Through leverage (measured as sum of absolute positions), we observe that ML FMPs are much less extreme for all macro factors. As leverage is often associated to risk or limited for some investors, this is a key advantage. At the opposite, WLS FMP method, that corrects for the variance-covariance matrix of assets, lead to highly leveraged portfolios. The (full sample) volatility of the mimicking portfolios varies significantly across specifications. Here again, the ML approach leads to more reasonable numbers but also closer to the original volatility of the macro factors (set at 1% per annum by construction). We then display a set of statistics aiming at evaluating the goodness of fit of the FMPs relatively to macro factors over the full sample, and more precisely the correlation of macro FMPs with their underlying orthogonal macro factors, RMSE (Root Mean Square Errors) and MAE (Mean Absolute Error). While correlation metrics do not lead to a clear hierarchy, measures of average errors consistently point to the superiority of the ML approach, followed by MCP, while OLS (Fama and McBeth) is frequently offering the worst fit.

In Figure 2, we represent the time-series evolution of the realized returns of the growth, inflation surprises and financial stress ML FMPs, jointly with the respective underlying orthogonalized observed macro factors. We use one-year moving average of realized ML FMPs to reduce the inherent noise in monthly returns. At such horizon, ML FMPs seem to track well the evolution of the orthogonalized observed macro factors while short-term (monthly) macro factors will be noisier.

So far, the results are based on estimation over the full sample. To gauge how the different mimicking methodologies fare on an out-of-sample basis, we run two different types of tests. In the first out-of-sample test, we estimate the macro FMPs over the first half of the sample (from January 1974 to March 1996) and then analyze the goodness-of-fit metrics over the second half of the sample. In the second test, starting from April 1996, we re-estimate the model every month by expanding the sample and apply the new estimated macro FMPs to the next month. The results for these two out-of-sample tests are summarized in Table 7. For almost all methodologies and all macro factors, we observe as expected a deterioration out-of-sample in all the goodness-of-fit metrics. Indeed, some correlations even turn negative, while RMSEs and MAEs are larger. Looking at the different methodologies, the ML FMP approach seems to be the most robust across the different out-of-sample tests, as volatility, RMSE or MAE remain frequently the lowest. This confirms the dominance of our methodology already observed over the full sample.

5 – Practical use: Hedging macro risks for an endowment portfolio

One of the main potential advantage of investable macro factor-mimicking portfolios is to allow investors to hedge macro risks they represent, i.e. recession, inflation surprise and financial stress in our case. We provide such an illustration with a representative endowment portfolio, with the following portfolio allocation [14]: 35% Global Equities (MSCI World), 10% Global Treasuries (Bloomberg Barclays Global Treasuries), 5% US High Yield (Bloomberg Barclays US High Yield), 5% Commodities (Bloomberg Commodities Total Return), 20% Hedge Funds (Hedge Fund Research Fund of Funds), 10% Real Estate (NCREIF Property index) and 15% Private Equity (Cambridge Associates Private Equity index).

To analyze this portfolio through a macro angle, we report in Table 8 regressions of the endowment portfolio excess returns on the three macro ML FMPs. The macro model explains more than 60%

[14] This allocation is inspired by the 2018 version of Nacubo-TIAA study of endowments.

of the total variance of the endowment portfolio returns. We also see that this portfolio has an alpha equivalent to close to 4% per year. In Figure 3, we represent the breakdown of the portfolio in terms of risk contributions, risk being defined in terms of volatility. The portfolio is dominated by growth factor then inflation surprises and financial stress, while the idiosyncratic risk is near 40% of the total portfolio risk.

The previous results suggest that an investor might be seeking to hedge the macro risks to improve her risk-return profile. A practical way to do such optimal hedging is to determine a combination of the endowment portfolio with macro FMPs that minimizes the variance of the combined portfolio. More precisely, we assume that the investor determines the weights $\boldsymbol{\omega}$ so that:

$$\min_{\boldsymbol{\omega}} T^{-1} \sum_{t=1}^{T} H_t^2, \quad (5)$$

with:

$$H_t \equiv ENDO_t - \boldsymbol{\omega^T FMP}_t,$$

where $ENDO_t$ is the endowment portfolio return and $\boldsymbol{FMP}_t$ are the macro ML FMPs returns[15]. To make the exercise more practically orientated, we run it as an out-of-sample exercise as follows. For each quarter, the optimization problem (5) is solved over the previous 10 years (40 quarterly observations) for FMP estimated on an expanding basis up (but excluding) the current quarter. The first out-of-sample hedged portfolio H_1 is obtained in 2000Q1 (with data from 1990Q1 to 1999Q4) and the last one in 2020Q1, leading to 81 observations.

In Figure 4, we jointly represent the quarterly returns of the endowment portfolio and its (minimum variance) macro-hedged counterpart on the top of the chart, while we represent their respective maximum drawdowns on the bottom chart. Table 9 displays descriptive statistics on both

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[15] Note that the macro FMPs are self-financed as the base assets are adjusted for cash returns when they are not defined as return spreads.

portfolios. While the average returns are slightly reduced, the macro-hedging allows the investor to achieve more consistent returns over time, with a much lower volatility or lower maximum drawdown. Overall, the risk-adjusted metrics (Sharpe and Calmar ratios) are significantly improved thanks to macro hedging. This corroborates the findings of Herskovic et al. (2020) in a different set-up.

6 – Conclusion

Investors frequently form views on macro factors. Still, acting on the basis of these views remains a challenge as macro factors are not directly investable. In the asset pricing literature, the issue of non-tradability of economic factors has been handled through a variety of different methodologies grouped under the common label of "factor-mimicking portfolios" (FMPs). In this article, we introduce a general FMP framework that encompasses existing factor-mimicking approaches, such as the two-pass cross-sectional regressions (CSR) and maximum correlation portfolio (MCP) approach.

We also show how investors can improve the estimation of macro FMPs by combining machine learning methods such as supervised principal component analysis and LASSO regressions. We apply our macro factor-mimicking framework to three orthogonal global macro factors (growth, inflation surprises and financial stress) over the period 1974-2018, and show how the estimated machine learning macro FMPs can be used to improve a typical endowment portfolio risk-return profile through macro-hedging.

Empirical applications of our methodological framework can naturally be extended to deal with macro factor correlation dynamics or practically relevant portfolio constraints such as transaction costs, liquidity or regulatory guidelines. We leave these extensions to future research.

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Appendix 1. General FMP Portfolio Optimization Program

For each macro factor k, the factor-mimicking portfolio (FMP) can be obtained as the solution of the following variance minimization problem:

$$\min_{\boldsymbol{w}_k} \frac{1}{2}\ \boldsymbol{w}_k^{\boldsymbol{T}} \Omega\, \boldsymbol{w}_k \tag{A.1.1}$$

$$s.t. \boldsymbol{B}^{\boldsymbol{T}} \boldsymbol{w}_k = \boldsymbol{\beta}_k$$

where $\boldsymbol{\Omega}$ is the positive definite $(N \times N)$ covariance matrix, $\boldsymbol{B}$ is the $(N \times K)$ factor loading matrix (univariate or multivariate) and $\boldsymbol{\beta}_k$ is an $(K \times 1)$ vector of portfolio factor exposures, with the k-th entry equal to 1 and all others equal to β_{kl}.

The Lagrangian of the system (A.1.1) is given by

$$L(\boldsymbol{w}_k; \lambda_k) = \frac{1}{2} \boldsymbol{w}_k^{\boldsymbol{T}} \Omega\, \boldsymbol{w}_k - \lambda_k^{\boldsymbol{T}} (\boldsymbol{B}^{\boldsymbol{T}} \boldsymbol{w}_k - \boldsymbol{\beta}_k) \tag{A.1.2}$$

where $\boldsymbol{\lambda}_k$ is the $(K \times 1)$ vector of the Lagrangian multipliers. The first-order condition of (A.1.2) is given by:

$$\frac{\partial L(\boldsymbol{w}_k; \lambda_k)}{\partial \boldsymbol{w}_k} = \Omega\, \boldsymbol{w}_k - \boldsymbol{B} \lambda_k = 0 \tag{A.1.3}$$

Pre-multiplying by the inverse matrix $\boldsymbol{\Omega}^{-1}$ and solving for the optimal weights, we have:

$$\boldsymbol{w}_k = \Omega^{-1} \boldsymbol{B} \lambda_k \tag{A.1.4}$$

Premultiplying (A.1.4) by $\boldsymbol{B}^{\boldsymbol{T}}$ leads to:

$$\boldsymbol{B}^{\boldsymbol{T}} \boldsymbol{w}_k = \boldsymbol{B}^{\boldsymbol{T}} \Omega^{-1} \boldsymbol{B} \lambda_k = \boldsymbol{\beta}_k \tag{A.1.5}$$

We infer:

$$\lambda_k = \left(\boldsymbol{B}^{\boldsymbol{T}} \Omega^{-1} \boldsymbol{B}\right)^{-1} \boldsymbol{\beta}_k \tag{A.1.6}$$

Equation (A.1.4) can then be rewritten as:

$$\boldsymbol{w}_k^* = \Omega^{-1} \boldsymbol{B} \left(\boldsymbol{B}^{\boldsymbol{T}} \Omega^{-1} \boldsymbol{B}\right)^{-1} \boldsymbol{\beta}_k \tag{A.1.7}$$

Taken together, the factor-mimicking portfolio weight matrix is:

$$\boldsymbol{W}_K^* = \boldsymbol{\Omega}^{-1}\boldsymbol{B}\left(\boldsymbol{B}^T\boldsymbol{\Omega}^{-1}\boldsymbol{B}\right)^{-1}\boldsymbol{B}_K \qquad \text{(A.1.8)}$$

where $\boldsymbol{B}_K$ is the $(K \times K)$ target factor beta matrix, with $\boldsymbol{B}_K = (\boldsymbol{\beta}_1\, \boldsymbol{\beta}_2 \dots \boldsymbol{\beta}_K)$.

Appendix 2. Equivalence between Specific Risk and Total Risk FMP Solutions

Following the approach of Grinblatt and Titman (1987), define:

$$\boldsymbol{A} = \boldsymbol{\Omega}^{-1}\boldsymbol{B}\left(\boldsymbol{B}^T\boldsymbol{\Omega}^{-1}\boldsymbol{B}\right)^{-1} \qquad \text{(A.2.1)}$$

which implies:

$$\begin{cases} \boldsymbol{B}^T\boldsymbol{A} = \boldsymbol{I}_N \\ \Omega\, \boldsymbol{A}\left(\boldsymbol{B}^T\boldsymbol{\Omega}^{-1}\boldsymbol{B}\right) = \boldsymbol{B} \end{cases} \qquad \text{(A.2.2)}$$

Substituting $\boldsymbol{\Omega} = \boldsymbol{B}\,\boldsymbol{\Omega}_K\,\boldsymbol{B}^T + \boldsymbol{\Omega}_{\boldsymbol{\varepsilon}}$, where $\boldsymbol{\Omega}_K$ is the covariance matrix of factors and $\boldsymbol{\Omega}_\varepsilon$ is the covariance matrix of idiosyncrasic risks, into the latter equation of (A.2.2) and using (A.2.1) yields:

$$\boldsymbol{B}\,\boldsymbol{\Omega}_K\left(\boldsymbol{B}^T\boldsymbol{\Omega}^{-1}\boldsymbol{B}\right) + \boldsymbol{\Omega}_{\boldsymbol{\varepsilon}}\boldsymbol{A}\left(\boldsymbol{B}^T\boldsymbol{\Omega}^{-1}\boldsymbol{B}\right) = \boldsymbol{B} \qquad \text{(A.2.3)}$$

That is:

$$\boldsymbol{A} = \boldsymbol{\Omega}_{\boldsymbol{\varepsilon}}^{-1}\boldsymbol{B}\left[\left(\boldsymbol{B}^T\boldsymbol{\Omega}^{-1}\boldsymbol{B}\right)^{-1} - \boldsymbol{\Omega}_K\right] \qquad \text{(A.2.4)}$$

Premultiplying by $\boldsymbol{B}^T$ and using (A.2.1) yields:

$$\left(\boldsymbol{B}^T\boldsymbol{\Omega}_{\boldsymbol{\varepsilon}}^{-1}\boldsymbol{B}\right)^{-1} = \left[\left(\boldsymbol{B}^T\boldsymbol{\Omega}^{-1}\boldsymbol{B}\right)^{-1} - \boldsymbol{\Omega}_K\right] \qquad \text{(A.2.5)}$$

Substituting this into (A.2.4) leads to the desired result, that is:

$$\boldsymbol{\Omega}^{-1}\left(\boldsymbol{B}^T\boldsymbol{\Omega}^{-1}\boldsymbol{B}\right)^{-1} = \boldsymbol{\Omega}_{\boldsymbol{\varepsilon}}^{-1}\boldsymbol{B}\left(\boldsymbol{B}^T\boldsymbol{\Omega}_{\boldsymbol{\varepsilon}}^{-1}\boldsymbol{B}\right)^{-1} \qquad \text{(A.2.6)}$$

So that:

$$\boldsymbol{W}_K^* = \boldsymbol{\Omega}^{-1}\boldsymbol{B}\left(\boldsymbol{B}^T\boldsymbol{\Omega}^{-1}\boldsymbol{B}\right)^{-1}\boldsymbol{B}_K = \boldsymbol{\Omega}_{\boldsymbol{\varepsilon}}^{-1}\boldsymbol{B}\left(\boldsymbol{B}^T\boldsymbol{\Omega}_{\boldsymbol{\varepsilon}}^{-1}\boldsymbol{B}\right)^{-1}\boldsymbol{B}_K \qquad \text{(A.2.7)}$$

Appendix 3. Principal Components Instrumental Variables FMP Estimator

We assume that the individual asset returns have a macro factor structure that can be represented as:

$$\boldsymbol{R}_t = \boldsymbol{\mu} + \boldsymbol{B}\boldsymbol{F}_{t+h} + \boldsymbol{\varepsilon}_t \tag{A.3.1}$$

where $\boldsymbol{R}_t$ and $\boldsymbol{\mu}$ are the $(N \times 1)$ vector of the base asset returns and associated expected returns, respectively. $\boldsymbol{B}$ is the $(N \times K)$ matrix of asset loadings to factors; $\boldsymbol{F}_{t+h}$ is the $(K \times 1)$ vector of zero-mean observable factors; and $\boldsymbol{\varepsilon}_t$ is the $(N \times 1)$ vector of zero-mean disturbances.

Let $\overline{\boldsymbol{R}}$ correspond to the $(N \times T)$ matrix of asset centered returns, $\boldsymbol{F}$ denote the $(K \times T)$ matrix of observed factors and $\boldsymbol{\varepsilon}$ be the $(N \times T)$ matrix of disturbances. The factor model can then be represented in matrix form as:

$$\overline{\boldsymbol{R}} = \boldsymbol{B}\boldsymbol{F} + \boldsymbol{\varepsilon} \tag{A.3.2}$$

Given the base asset returns and the factors of interest, the principal components Instrumental Variables (IV) FMP weight estimator proceeds as follows:

1. **PC extraction step:** Extract the first L principal components of base asset returns by conducting the PCA of the covariance matrix $\widehat{\boldsymbol{\Omega}} = (T)^{-1}\overline{\boldsymbol{R}}\overline{\boldsymbol{R}}^{\boldsymbol{T}}$, so that:

$$\boldsymbol{F}^{PC} = \boldsymbol{E}_L^{\boldsymbol{T}}\,\overline{\boldsymbol{R}} \tag{A.3.3}$$

where $\boldsymbol{F}^{PC}$ is the $(L \times T)$ matrix of PCs and $\boldsymbol{E}_L$ corresponds to the $(N \times L)$ eigenvector matrix associated with the largest L eigenvalues of $\widehat{\boldsymbol{\Omega}}$, with $L \leq N$.

2. **Fitted factor estimation step:** Regress the K observed economic factors $\boldsymbol{F}_{t+h}$ onto the L statistical factors extracted from the PCA to obtain the fitted values of the observed factors and their multivariate factor exposures, that is

$$\widehat{\boldsymbol{F}}^{PC} = \widehat{\boldsymbol{b}}_K\boldsymbol{F}^{PC} \text{ and } \widehat{\boldsymbol{b}}_K = \boldsymbol{F}\boldsymbol{F}^{PC^{\boldsymbol{T}}}\left(\boldsymbol{F}^{PC}\boldsymbol{F}^{PC^{\boldsymbol{T}}}\right)^{-1} \tag{A.3.4}$$

The observable fitted factors and their exposures can be written equivalently more compactly as:

$$\widehat{\boldsymbol{F}}^{PC} = \boldsymbol{F}\boldsymbol{P} \tag{A.3.5}$$

where $\boldsymbol{P}$ is a $(T \times T)$ projection matrix, with $\boldsymbol{P} = \boldsymbol{F}^{PC^{\boldsymbol{T}}} \left(\boldsymbol{F}^{PC}\boldsymbol{F}^{PC^{\boldsymbol{T}}}\right)^{-\boldsymbol{1}} \boldsymbol{F}^{PC}$.

3. **Times-series regression step:** Regress the excess asset returns $\overline{\boldsymbol{R}}$ onto the K fitted values of the observable factors $\widehat{\boldsymbol{F}}^{PC}$ to obtain the Instrumental Variables (IV) macro beta estimates, that is:

$$\widehat{\boldsymbol{B}}^{PCIV} = \overline{\boldsymbol{R}}\, \widehat{\boldsymbol{F}}^{PC^{\boldsymbol{T}}} \left(\widehat{\boldsymbol{F}}^{PC}\widehat{\boldsymbol{F}}^{PC^{\boldsymbol{T}}}\right)^{-\boldsymbol{1}} \tag{A.3.6}$$

Substituting in (A.3.6) $\widehat{\boldsymbol{F}}^{PC}$ by its expression (A.3.5) and rearranging leads to:

$$\widehat{\boldsymbol{B}}^{PCIV} = \overline{\boldsymbol{R}}\, \boldsymbol{P}^{\boldsymbol{T}}\boldsymbol{F}^{\boldsymbol{T}}(\boldsymbol{F}\boldsymbol{P}\boldsymbol{P}^{\boldsymbol{T}}\boldsymbol{F}^{\boldsymbol{T}})^{-\boldsymbol{1}} \tag{A.3.7}$$

where $\boldsymbol{P}$ is the $(T \times T)$ projection matrix defined in step 2.

The instrumental variables estimator of the FMP weights is then obtained by substituting in (A.1.8) $\widehat{\boldsymbol{B}}$ by $\widehat{\boldsymbol{B}}^{PCIV}$. That is:

$$\boldsymbol{W}_K^{*PCIV} = \widehat{\boldsymbol{\Omega}}^{-\boldsymbol{1}}\widehat{\boldsymbol{B}}^{PCIV}\left(\widehat{\boldsymbol{B}}^{PCIV^T}\widehat{\boldsymbol{\Omega}}^{-\boldsymbol{1}}\widehat{\boldsymbol{B}}^{PCIV}\right)^{-\boldsymbol{1}} \boldsymbol{B}_K \tag{A.3.8}$$

where $\boldsymbol{B}_K$ is the $(K \times K)$ target beta matrix. One can remark that the IV estimator (A.3.8) provides an unbiased estimator of the FMP weights when the first PCs of the base asset returns can recover the same space as the true latent macro factors (Bai and Ng, 2002).

Appendix 4. Egalitarian Orthogonalization Procedure

To orthogonalize the original macro factors, we follow the "Egalitarian" procedure described in Klein and Chow (2013). The Egalitarian orthogonalization method presents a variety of benefits relative to other orthogonalization approaches such as Principal Component Analysis (PCA), Gram-Schmidt (GS) process or Minimum Torsion (MT) transformation. First, it produces orthogonalized factors that are the closest in the least squares sense to the original factors, while PCA is struggling to maintain a meaningful one-to-one correspondence from the original factors to the orthogonalized ones. Second, it treats all factors equally, while the GS process is sequential and therefore dependent of the ordering of the factors. Finally, the Egalitarian orthogonalization approach leads to a simple analytical closed-form solution, while the MT transformation solution can only be obtained numerically.

Formally, let $\boldsymbol{F}$ corresponds to the $(K \times T)$ matrix of observed original factors and $\widehat{\boldsymbol{\Omega}}_K = (T-1)^{-1}\boldsymbol{F}\boldsymbol{F}^{\boldsymbol{T}}$ be the covariance matrix of the factors' returns. To derive the set of uncorrelated and variance-preserving macro factors, denoted by $\boldsymbol{F}^{\perp}$, we define the following linear transformation:

$$\boldsymbol{F}^{\perp} = \boldsymbol{L}\boldsymbol{F} \tag{A.4.1}$$

where $\boldsymbol{L}$ is a $(K \times K)$ invertible matrix. The new basis $\boldsymbol{F}^{\perp}$ will be orthonormal if:

$$\boldsymbol{F}^{\perp}\boldsymbol{F}^{\perp\boldsymbol{T}} = (T-1)\,\boldsymbol{L}\,\widehat{\boldsymbol{\Omega}}_K\boldsymbol{L}^{\boldsymbol{T}} = \boldsymbol{I} \tag{A.4.2}$$

or equivalently:

$$\boldsymbol{L}^{\boldsymbol{T}}\boldsymbol{L} = \frac{1}{(T-1)}\widehat{\boldsymbol{\Omega}}_K^{-\boldsymbol{1}} \tag{A.4.3}$$

The general solution of (A.4.3) is:

$$\boldsymbol{L} = \frac{1}{\sqrt{T-1}}\widehat{\boldsymbol{\Omega}}_K^{-\frac{1}{2}} \tag{A.4.4}$$

From the spectral decomposition of the inverse square root of the matrix $(T-1)\,\widehat{\boldsymbol{\Omega}}_K$, we obtain:

$$\boldsymbol{L} = \boldsymbol{E}_K\,\boldsymbol{\Lambda}_K^{-\frac{1}{2}}\,\boldsymbol{E}_K^{\boldsymbol{T}} \tag{A.4.5}$$

where $\boldsymbol{E}_K$ and $\boldsymbol{\Lambda}_K$ are the $(K \times K)$ matrix of the eigenvectors and the $(K \times K)$ diagonal matrix of the eigenvalues of $(T-1)\widehat{\boldsymbol{\Omega}}_K$, respectively.

Substituting in (A.4.1) $\boldsymbol{L}$ by its expression (A.4.5) and rescaling the orthogonal factors to the original factor variances, leads to:

$$\boldsymbol{F}^{\perp} = \sqrt{T-1}\, Diag(\widehat{\boldsymbol{\Omega}}_K)^{\frac{1}{2}} \left(\boldsymbol{E}_K\, \boldsymbol{\Lambda}_K^{-\frac{1}{2}} \boldsymbol{E}_K^{\boldsymbol{T}}\right) \boldsymbol{F} \tag{A.4.6}$$

One can verify that the orthogonalized factors are uncorrelated and retains the same variances as the original factors since:

$$\begin{aligned} \widehat{\boldsymbol{\Omega}} &= Diag(\widehat{\boldsymbol{\Omega}}_K)^{\frac{1}{2}} \left(\boldsymbol{E}_K\, \boldsymbol{\Lambda}_K^{-\frac{1}{2}} \boldsymbol{E}_K^{\boldsymbol{T}}\right) (T-1)\, \widehat{\boldsymbol{\Omega}}_K \left(\boldsymbol{E}_K\, \boldsymbol{\Lambda}_K^{-\frac{1}{2}} \boldsymbol{E}_K^{\boldsymbol{T}}\right) Diag(\widehat{\boldsymbol{\Omega}}_K)^{\frac{1}{2}} \\ &= Diag(\widehat{\boldsymbol{\Omega}}_K)^{\frac{1}{2}} \left(\boldsymbol{E}_K\, \boldsymbol{\Lambda}_K^{-\frac{1}{2}} \boldsymbol{E}_K^{\boldsymbol{T}}\right) \boldsymbol{E}_K \boldsymbol{\Lambda} \boldsymbol{E}_K^{\boldsymbol{T}} \left(\boldsymbol{E}_K\, \boldsymbol{\Lambda}_{\boldsymbol{K}}^{-\frac{1}{2}} \boldsymbol{E}_K^{\boldsymbol{T}}\right) Diag(\widehat{\boldsymbol{\Omega}}_K)^{\frac{1}{2}} \\ &= Diag(\widehat{\boldsymbol{\Omega}}_K) \end{aligned} \tag{A.4.7}$$

where we are using in (A.4.7) the spectral decomposition of $(T-1)\,\widehat{\boldsymbol{\Omega}}_K$ and the orthonorm²al property of the eigenvectors, e.g $(T-1)\,\widehat{\boldsymbol{\Omega}}_K = \boldsymbol{E}_K \boldsymbol{\Lambda}_K \boldsymbol{E}_K^{\boldsymbol{T}}$ and $\boldsymbol{E}_K^{\boldsymbol{T}} \boldsymbol{E}_K = \boldsymbol{I}$.

Table 1. Factor-Mimicking Portfolio Construction Methodologies

Methodology	Properties	Specifications	Portfolio Weight Matrix W_K
Two-Pass Cross-Sectional Regression Approach (CSR)			
OLS-CSR (Fama and MacBeth, 1973)	Minimum variance portfolio with unit beta to the k-th factor and zero betas with respect to all other factors	$\boldsymbol{\Omega} = \sigma^2 \boldsymbol{I}_N$: Uncorrelated assets with constant variance $\boldsymbol{B}$: Multivariate $\boldsymbol{B}_K = \boldsymbol{I}_K$	$\boldsymbol{B}(\boldsymbol{B}^{\boldsymbol{T}}\boldsymbol{B})^{-\mathbf{1}}$
WLS-CSR (Litzenberger and Ramaswany, 1979)	Same	$\boldsymbol{\Omega} = Diag(\boldsymbol{\sigma}^2)$: Uncorrelated assets $\boldsymbol{B}$: Multivariate $\boldsymbol{B}_K = \boldsymbol{I}_K$	$Diag(\boldsymbol{\sigma}^{-2})\boldsymbol{B}(\boldsymbol{B}^{\boldsymbol{T}} Diag(\boldsymbol{\sigma}^{-2})\boldsymbol{B})^{-\mathbf{1}}$
GLS-CSR (Lehman and Modest, 1988)	Same	$\boldsymbol{\Omega}$: Unconstrained $\boldsymbol{B}$: Multivariate $\boldsymbol{B}_K = \boldsymbol{I}_K$	$\boldsymbol{\Omega}^{-\mathbf{1}}\boldsymbol{B}(\boldsymbol{B}^{\boldsymbol{T}}\boldsymbol{\Omega}^{-\mathbf{1}}\boldsymbol{B})^{-\mathbf{1}}$
Maximum Correlation Portfolio Approach (MCP)			
Unit-beta MCP (Grinold and Kahn, 2000)	Minimum variance portfolio with unit beta to the k-th factor and non-null associated betas with respect to all other factors	$\boldsymbol{\Omega}$: Unconstrained $\boldsymbol{B}$: Univariate $\boldsymbol{B}_K = (\boldsymbol{B}^{\boldsymbol{T}}\boldsymbol{\Omega}^{-\mathbf{1}}\boldsymbol{B})Diag(\boldsymbol{B}^{\boldsymbol{T}}\boldsymbol{\Omega}^{-\mathbf{1}}\boldsymbol{B})^{-1}$	$\boldsymbol{\Omega}^{-\mathbf{1}}\boldsymbol{B}\ Diag(\boldsymbol{B}^{\boldsymbol{T}}\boldsymbol{\Omega}^{-\mathbf{1}}\boldsymbol{B})^{-1}$
Target-beta MCP (Huberman et al., 1987, Breeden et al., 1989)	Minimum variance portfolio with pre-specified beta to the k-th factor and non-null associated betas with respect to all other factors	$\boldsymbol{\Omega}$: Unconstrained $\boldsymbol{B}$: Univariate $\boldsymbol{B}_K = \boldsymbol{B}^{\boldsymbol{T}}\boldsymbol{\Omega}^{-\mathbf{1}}\mathbf{B}$	$\boldsymbol{\Omega}^{-\mathbf{1}}\boldsymbol{B}$
MCP (Lamont, 2001)	Minimum variance portfolio with beta to the k-th factor equals to the R^2 in the regression of the k-th factor on the base asset returns and non-null associated betas with respect to all other factors	$\boldsymbol{\Omega}$: Unconstrained $\boldsymbol{B}$: Univariate $\boldsymbol{B}_K = (\boldsymbol{B}^{\boldsymbol{T}}\boldsymbol{\Omega}^{-\mathbf{1}}\boldsymbol{B})Diag(\boldsymbol{\Omega}_K)$	$\boldsymbol{\Omega}^{-\mathbf{1}}\boldsymbol{B}\ Diag(\boldsymbol{\Omega}_K)$

Notes. The table summarizes the various specifications of the FMPs. $\boldsymbol{\Omega}$ is the $(N \times N)$ covariance matrix of the base asset returns, $\boldsymbol{\Omega}_K$ is the $(K \times K)$ covariance matrix of the factors, $\boldsymbol{B}$ is the $(N \times K)$ factor loading matrix, $\boldsymbol{B}_K$ is the $(K \times K)$ matrix of target risk exposures, $\boldsymbol{I}_K$ is the $(K \times K)$ identity matrix.

Table 2 – Definition of base assets

Base asset	Acronym	Indices used
Equities	WEQ	MSCI World in USD
Treasuries (nominal)	GLT	Bloomberg Barclays U.S. Treasury
Credit	CRE	Bloomberg Barclays U.S. Credit Baa index vs Aaa index
Inflation-Linked Bonds	ILB	From April 1997 onwards, Bloomberg Barclays U.S. TIPS vs US Treasury All maturities. Before that date, spread return based on estimated real yields changes
Gold	GOLD	Gold
Industrial Metals	INM	From February 1977 onwards, S&P GSCI Industrial Metals. Before that date, equally-weighted basket made of Aluminum, Copper, Lead, and Zinc
Energy commodity	ENG	From February 1983 onwards, S&P GSCI Energy. Before that date, equally-weighted basket made of Crude Oil and Natural Gas
U.S. Dollar	DXY	US dollar trade-weighted index
Commodity vs safe heaven currencies	FXCS	Spread of returns (against USD) between commodity currencies (equally-weighted basket made of Canadian dollar, Norwegian krona, Australian dollar) vs safe-heaven currencies (equally-weighted basket made of Japanese yen, Swiss franc)

Notes. Sources for data are Bloomberg and World Bank.

Table 3 – Original and orthogonalized macro factors: Correlation structure

	Growth	Inflation Surprises	Financial Stress
Growth	0.96	0.38	-0.39
Inflation Surprises	0.00	0.98	0.01
Financial Stress	0.00	0.00	0.98

Notes. The table displays correlation metrics in different dimensions: correlation among original macro factors above the diagonal; correlation among orthogonalized macro factors below the diagonal; correlation between original and orthogonalized factors on the diagonal. Sample period: January 1974-March 2020.

Table 4– OLS regressions of base asset returns on macro factors

	Growth (t-stat) (% variance)	Inflation Surprises (t-stat) (% variance)	Financial Stress (t-stat) (% variance)	Explained variance
WEQ	0.67	0.11	-0.46	32.4%
	(6.34)	(0.66)	(-2.96)	
	(21.8%)	(0.5%)	(10.1%)	
GLT	-0.17	-0.07	0.03	12.8%
	(-3.28)	(-1.88)	(-0.23)	
	(10.8%)	(1.6%)	(0.4%)	
CRE	0.10	0.00	-0.14	17.5%
	(2.84)	(0.03)	(-2.62)	
	(5.6%)	(0.0%)	(11.9%)	
ILB	0.21	0.27	-0.05	17.9%
	(1.70)	(2.94)	(-0.53)	
	(6.4%)	(11.1%)	(0.4%)	
GOLD	0.03	0.94	0.67	27.9%
	(-0.01)	(4.96)	(1.72)	
	(0.0%)	(18.5%)	(9.3%)	
INM	1.36	0.57	0.23	27.9%
	(3.99)	(1.30)	(0.65)	
	(23.2%)	(4.0%)	(0.7%)	
ENG	0.17	1.51	1.57	21.3%
	(0.24)	(2.18)	(2.08)	
	(0.1%)	(10.2%)	(11.0%)	
DXY	-0.07	-0.25	0.10	12.4%
	(-0.76)	(-4.25)	(2.14)	
	(0.8%)	(10.1%)	(1.5%)	
FXCS	0.19	0.04	0.00	5.9%
	(1.79)	(0.42)	(0.72)	
	(5.7%)	(0.2%)	(0.0%)	

Notes. The table represents individual monthly multivariate OLS regressions of base assets past yearly returns over realized macro factors. For each base asset, large-font numbers represent beta estimates and adjusted R^2, while small-font numbers represent t-stat. T-stats are based on heteroscedasticity and autocorrelation consistent Newey-West standard errors with twelve lags. Sample period: January 1974-March 2020.

Table 5 – Machine learning macro factor models

	Growth (t-stat) (% variance)	Inflation Surprises (t-stat) (% variance)	Financial Stress (t-stat) (% variance)	Explained variance
WEQ	1.31	0.22	-1.03	57.2%
	(8.22)	(1.00)	(-4.89)	
	(38.0%)	(0.6%)	(18.6%)	
GLT	-0.36	-0.14	0.03	24.7%
	(-4.98)	(-1.54)	(0.31)	
	(22.7%)	(1.8%)	(0.1%)	
CRE	0.21	-0.03	-0.36	38.5%
	(4.27)	(-0.35)	(-3.98)	
	(11.8%)	(0.1%)	(26.5%)	
ILB	0.35	1.14	-0.24	59.3%
	(3.56)	(7.64)	(-2.43)	
	(8.4%)	(47.7%)	(3.2%)	
GOLD	0.08	3.44	1.36	77.5%
	(0.54)	(9.38)	(3.63)	
	(0.1%)	(63.0%)	(14.4%)	
INM	3.09	1.74	1.10	69.8%
	(7.73)	(5.22)	(3.66)	
	(54.7%)	(9.5%)	(5.6%)	
ENG	0.90	4.98	3.87	54.0%
	(1.73)	(4.20)	(3.12)	
	(1.7%)	(27.9%)	(24.5%)	
DXY	-0.01	-1.13	0.45	65.4%
	(-0.14)	(-10.66)	(4.16)	
	(0.0%)	(53.4%)	(12.0%)	
FXCS	0.47	-0.01	0.11	16.2%
	(3.94)	(-0.08)	(0.79)	
	(15.6%)	(0.0%)	(0.6%)	

Notes: The table represents individual multivariate OLS regressions of base assets over target PCA-fitted macro factors. For each base asset, large-font numbers represent beta estimates and adjusted R^2, while small-font numbers represent t-statistics. T-stats are based on heteroscedasticity and autocorrelation consistent Newey-West standard errors with twelve lags. Sample period: January 1974-March 2020.

Table 6 – In-sample characteristics of FMPs

	Growth				Inflation Surprises				Financial Stress			
Method:	**OLS**	**WLS**	**MCP**	**ML**	**OLS**	**WLS**	**MCP**	**ML**	**OLS**	**WLS**	**MCP**	**ML**
Leverage	184%	418%	317%	66%	406%	493%	298%	77%	427%	607%	449%	116%
Volatility	4.8%	5.3%	4.0%	1.9%	6.9%	6.2%	4.9%	1.5%	6.6%	5.4%	4.1%	2.1%
Correlation with original factor	0.16	0.12	0.11	0.13	0.21	0.21	0.26	0.25	0.10	0.17	0.27	0.10
RMSE	4.78%	5.31%	3.98%	2.06%	6.74%	6.11%	4.75%	1.55%	6.60%	5.34%	3.98%	2.22%
MAE	3.59%	4.11%	2.99%	1.55%	5.08%	4.75%	3.53%	1.17%	5.10%	4.07%	2.89%	1.70%

Notes. The table represents the characteristics of the macro FMPs estimated through four methodologies: OLS, WLS, MCP, and ML. Leverage is estimated as the sum of absolute weights. Volatility is the full sample (monthly) volatility. Correlation is the correlation between each FMP and the underlying orthogonal observed macro factor. RMSE and MAE are the Root Mean Squared Error and Mean Absolute Error statistics. In both cases, errors are defined as the difference between each macro FMP and underlying observed orthogonal macro factor. Sample period: January 1974 to March 2020.

Table 7 – Out-of-sample characteristics of FMPs

	Growth				Inflation Surprises				Financial Stress			
Method:	**OLS**	**WLS**	**MCP**	**ML**	**OLS**	**WLS**	**MCP**	**ML**	**OLS**	**WLS**	**MCP**	**ML**
OOS #1 - Two subperiods												
Correlation	0.02	-0.02	-0.09	-0.03	0.08	0.14	0.18	0.12	-0.22	-0.18	0.11	-0.20
Volatility	3.7%	5.2%	5.6%	2.4%	5.3%	6.0%	4.7%	1.4%	5.0%	5.0%	4.3%	1.6%
RMSE	0.05%	0.04%	0.23%	0.11%	0.45%	0.13%	0.10%	0.07%	0.05%	0.02%	0.04%	0.18%
MAE	2.90%	3.72%	3.55%	1.93%	4.13%	4.81%	3.61%	1.11%	3.90%	3.90%	3.00%	1.34%
OOS #2 – Expanding sample												
Correlation	-0.01	-0.04	-0.07	-0.02	0.23	0.18	0.18	0.23	-0.21	0.09	0.15	-0.20
Volatility	4.3%	5.3%	4.3%	2.4%	4.5%	3.9%	4.8%	1.3%	4.1%	3.7%	4.6%	1.4%
RMSE	0.12%	0.02%	0.06%	0.07%	0.02%	0.06%	0.22%	0.02%	0.22%	0.01%	0.07%	0.26%
MAE	3.35%	3.69%	3.13%	1.94%	3.36%	2.96%	3.61%	0.99%	3.27%	2.68%	3.14%	1.33%
Turnover	9.13%	19.98%	43.31%	6.82%	2.89%	20.85%	52.21%	1.24%	4.90%	32.14%	51.83%	2.19%

Notes. The table displays the out-of-sample metrics associated to the macro FMPs estimated through four methodologies: OLS, WLS, MCP, and ML. We run two types of out-of-sample tests. In the first test (OOS #1), the macro FMP models are estimated over the first half of the sample and maintained over the second subperiod where we estimate the metrics. In the second test (OOS #2), we re-estimate the model every month over a rolling window of half the sample (278 months) and applying the estimated weights to the next month. Correlation is the correlation between each macro FMP and the underlying orthogonal observed macro factor. Volatility is the volatility of FMP. RMSE is the Root Mean Squared Errors statistics where errors are defined as the difference between each macro FMP and underlying observed macro factor. MAE is the Mean Absolute Error statistics where errors are defined as the difference between each macro FMP and underlying orthogonal observed macro factor. Sample period: January 1974 to March 2020.

Table 8 – Exposure of the endowment portfolio to ML macro FMPs

	Coefficients	t-stat
Intercept	1.05%	4.27
Growth	0.89	11.95
Inflation	0.83	6.69
Stress	-0.17	-2.74
Adjusted R^2 0.63		F-stat 66.36 (p-val: 0.00)

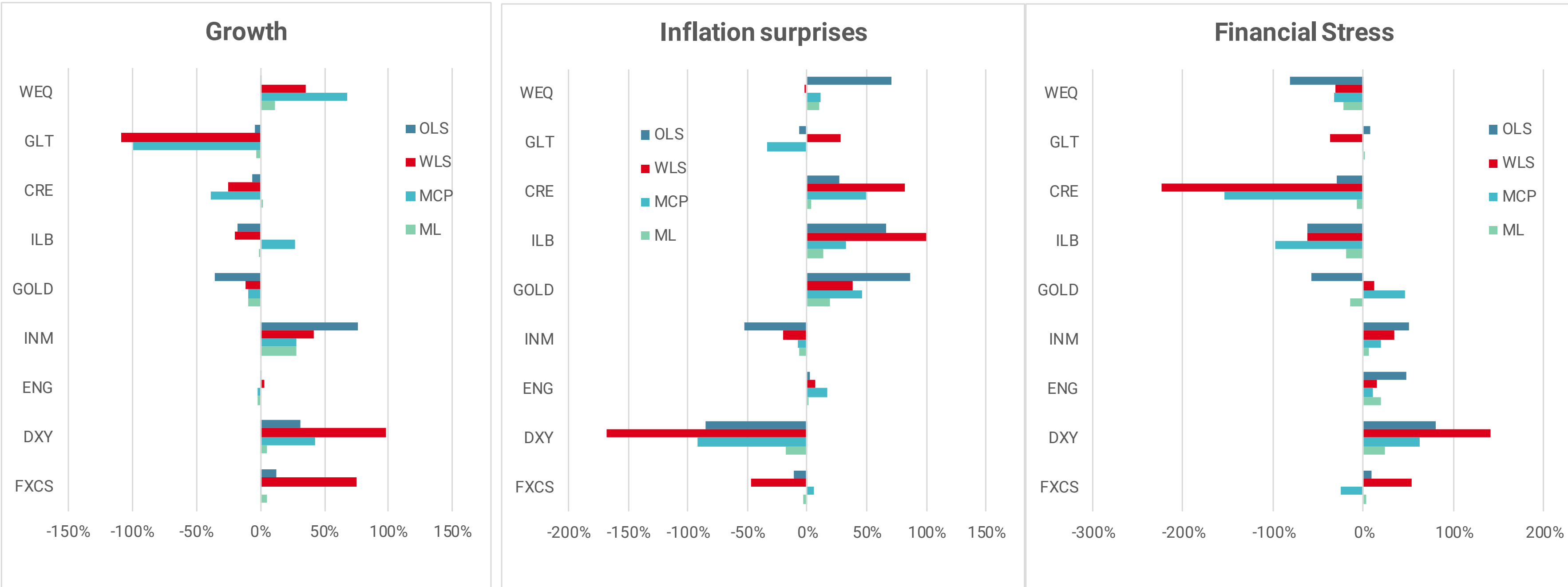
Notes. The table summarizes the regression results of the returns of the endowment portfolio on the returns of the ML macro FMPs. Sample period is 2000Q1 –2020Q1.

Table 9 – Endowment portfolio and its macro-hedged version: Portfolio performance metrics

	Original	**Macro-Hedged**
Annualized return	5.3%	4.9%
Volatility	9.2%	4.6%
Maximum Drawdown	29.5%	9.4%
Sharpe Ratio	0.32	0.57
Calmar ratio	0.07	0.18

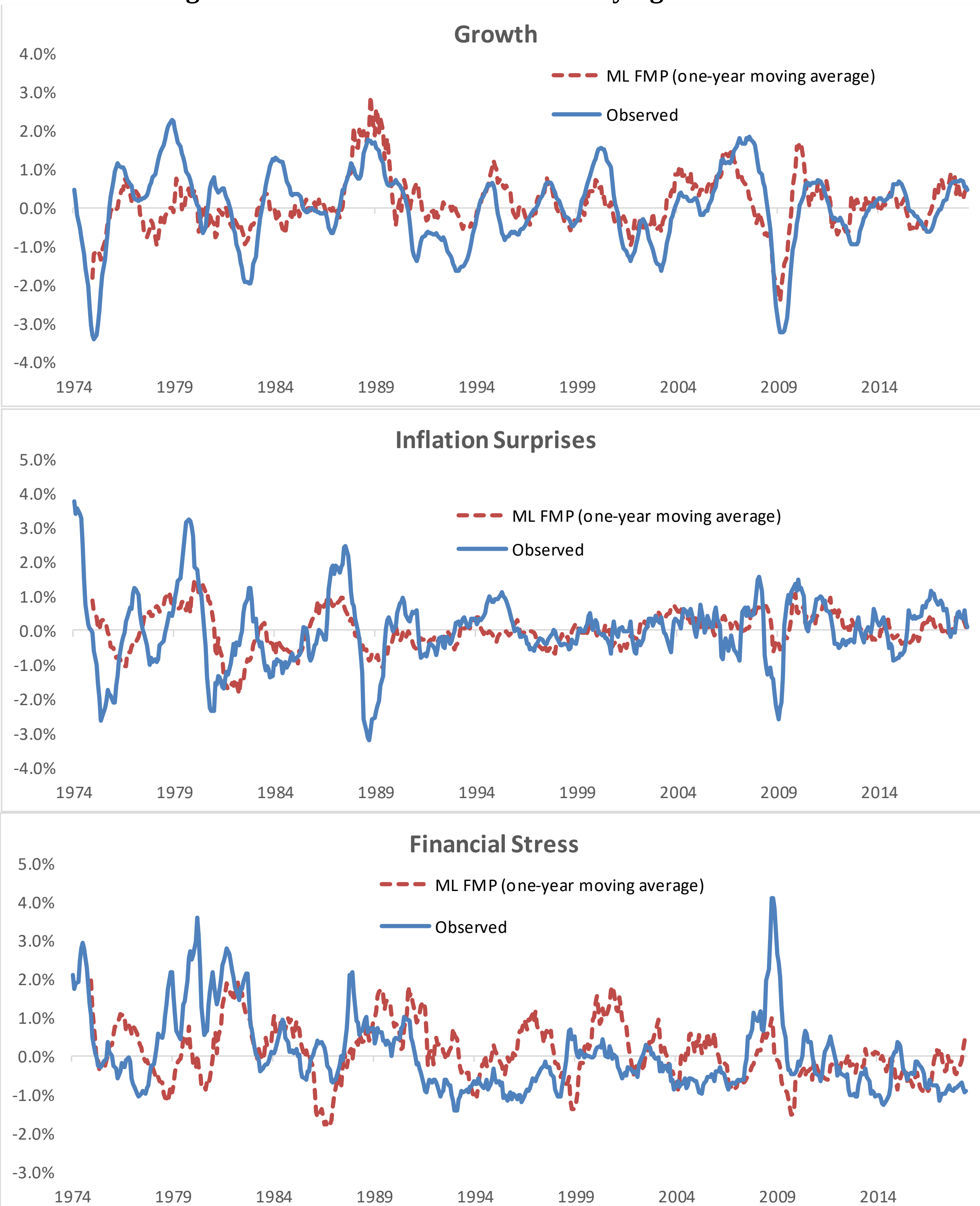
Notes. The table represents the performance metrics of the endowment portfolio and of its macro-hedged version. Sample period is 2000Q1 –2020Q1.

Figure 1 – Macro FMP composition as estimated by different methods



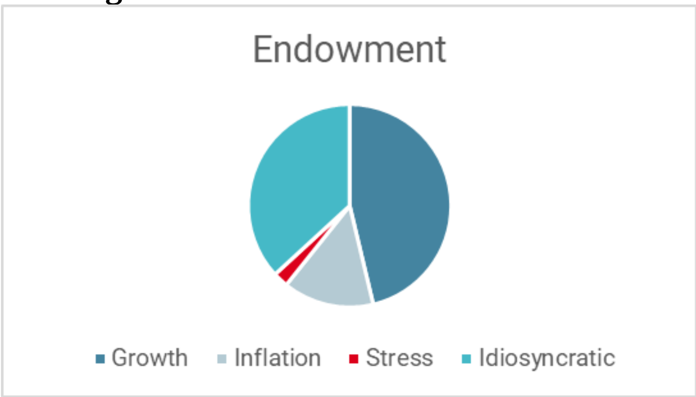
Notes: The figure represents the weights of the macro FMPs estimated through four methodologies: OLS, GLS, MCP, and ML. Sample period: January 1974 to March 2020.

Figure 2 – ML macro FMPs vs underlying macro factors



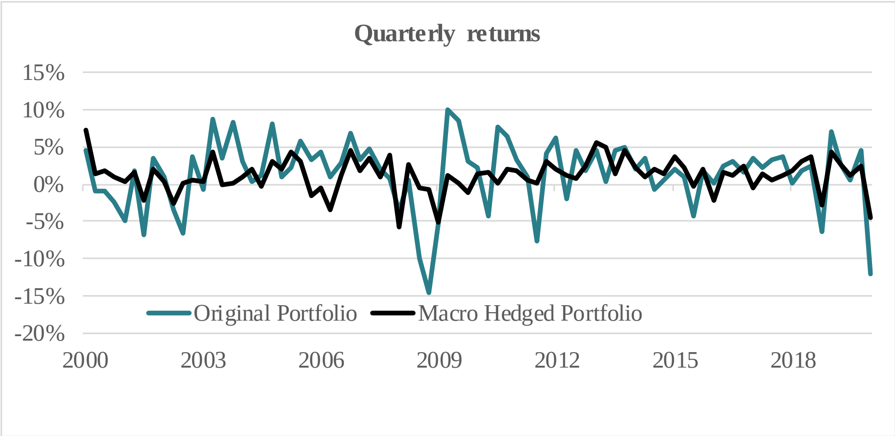
Source: OECD, Chicago Fed, Bloomberg. ML-CSR are machine learning macro factor-mimicking portfolios where macro factor models are estimated by the machine learning factor model presented in Section 3. Macro factors are orthogonalized by using an Egalitarian procedure (see Appendix 4) and studentized to a 1% (monthly) volatility.

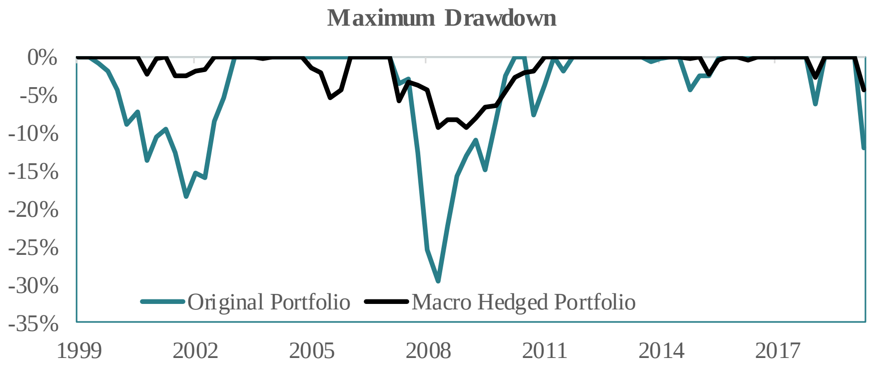
Figure 3 – Macro Risk Contributions



Notes. The figure displays the contributions to the volatility of the endowment portfolio, as obtained through the regression of the endowment returns on ML macro FMPs. Sample period is 2000Q1 – 2020Q1.

Figure 4 – Endowment portfolio and its macro-hedged version: Quarterly returns and Maximum Drawdowns





Notes. The figure jointly represents the endowment portfolio returns and its (minimum variance) macro hedged version. Sample period is 2000Q1 –2020Q1.