Factor Models with Drifting Prices

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Abstract

Standard factor models focus on returns and leave prices undetermined. Thus, we propose a novel (co-)integrated methodology to factor modeling based on both prices and returns. Given a long-run relationship between the values of buy-and-hold portfolios and factors, we argue that a term—naturally labeled Equilibrium Correction Term (ECT)—should be included when regressing returns on factors. We also advance to validate factor models by the existence of such a term. Empirically, the ECT predicts equity portfolio returns. Furthermore, we find evidence for a common component in the asset-specific ECTs that is countercyclical and has forecasting ability for the aggregate market.

Keywords: Long-Horizon Returns, Predictability, Mispricing, Factor Models, Equilibrium Correction.
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"*We have to answer the central question, what is the source of price variation? When did our field stop being "asset pricing" and become "asset expected returning"?*"

John H. Cochrane (2011, p. 1063)

1 Introduction

It is common in the asset pricing literature to parsimoniously characterize the dynamics of returns with factors that can explain the cross-section of average returns (e.g., Fama and French, 2015a; Hou et al., 2015). However, standard factor models posit a relationship between returns on any asset and the factor returns but no relationship between the value of a buy-and-hold asset portfolio and the value of the buy-and-hold factor portfolios. This is surprising, since long-run investors (e.g., mutual funds) care whether the asset and factor portfolios share the same stochastic trend.[1] Indeed, if there is no shared trend, then deviations of asset portfolios from factor portfolios are permanent and the chosen factor model is of little use to the buy-and-hold investor. If instead the stochastic trend is common, not only the factor model provides guidance to the long-run investors, but there are also consequences for the prediction of returns and their distribution. These consequences are left unexploited when price trends and their comovements are not properly modeled. This paper proposes a new approach to factor-portfolio models based

[1]Many economic decision makers, like corporate manager, effectively face long holding periods (see discussion in Cohen et al., 2009). Also, investors with preferences for locking in long holding period returns would be consistent with the downward sloping equity term structure (see, e.g., Binsbergen et al., 2012).

on the relation between drifting prices and returns.

To provide some graphical intuition for our approach, we consider a statistical factor model composed of five principal components extracted from a large cross-section of equity portfolios, and we employ as a test asset the returns from a value strategy investing in stocks that appear to be trading for less than their book value.[2] The top panel in Figure 1 shows that the statistical factor model provides an accurate description (dashed blue line) of the returns value strategy (solid black line). However, looking at prices rather than returns paints a completely different picture. This is clearly seen in the mid left panel in Figure 1, where we overlay the cumulative returns to the value strategy and the cumulative returns implied by the factor model, and further emphasized in the bottom left panel where we plot the price difference between the (realized) buy-and-hold portfolio and the level implied by the factor model. Despite the excellent fit with respect to returns, we observe large and persistent fluctuations in prices that last for more than 40 years. These observed price deviations suggest that the buy-and-hold asset and factor portfolios follow two stochastic trends that are not related in the statistical factor model.

In this paper, we invert the standard logic of starting from returns to go to prices, and propose to start by modeling prices instead, to then obtain implica-

[2]Specifically, we use the top decile of book-to-market sorted portfolios. A detailed description of our data is provided in Section 3.1. Subsection 3.2 provides a discussion of statistical factor models estimated via principal component analysis (PCA). Here, it is just worth noticing that despite we employ standard PCA, our approach can be applied as-is to modification of principal components techniques like the risk-premium PCA of Lettau and Pelger (2020), or, in general, to any model based on tradeable factors, such as the "q-factor" model of Hou et al. (2015) and the five-factor model of Fama and French (2015a).

tions for returns. Specifically, consider constructing a buy-and-hold portfolio

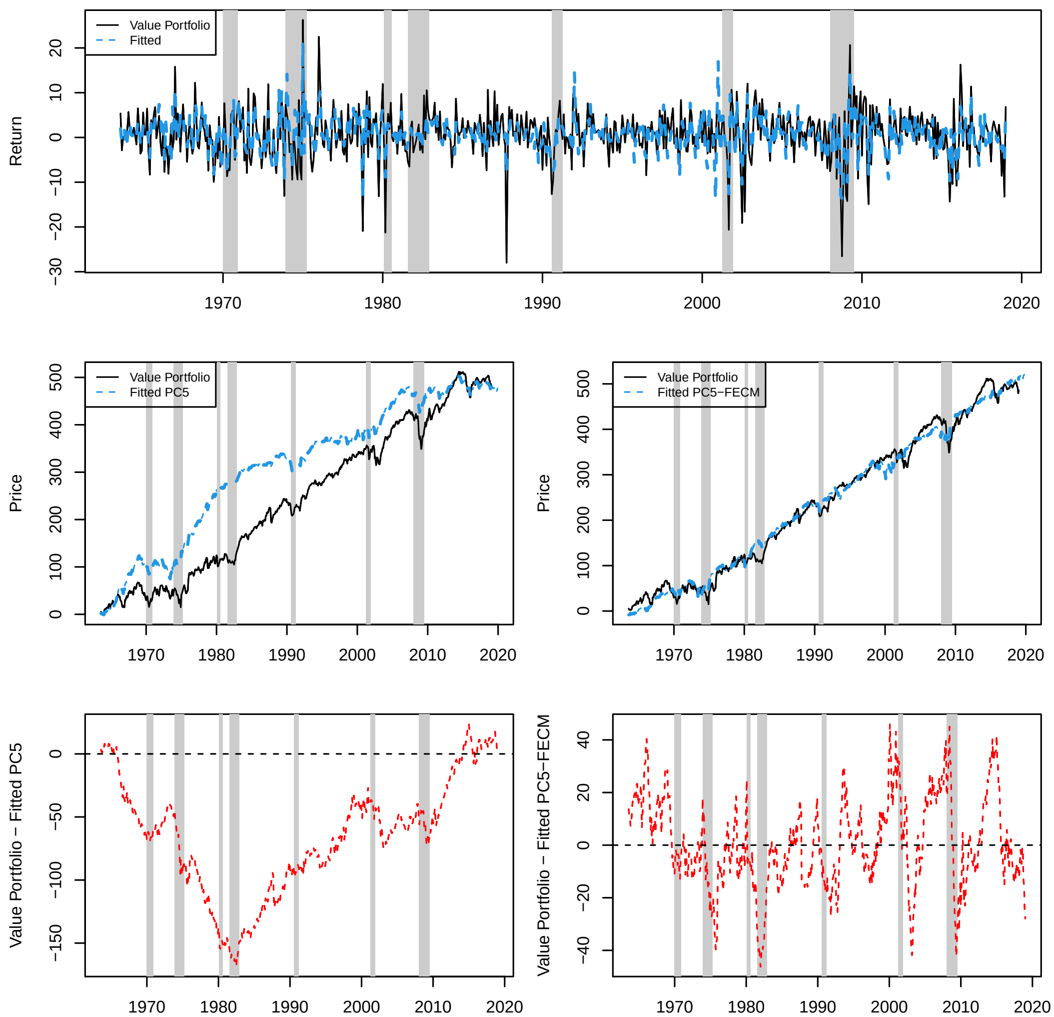


Figure 1: Price Dynamics in PC5-Factor Model and its FECM Specification. This figure shows prices for the Value Portfolio (decile 10) in the 90 anomaly portfolios constructed in Giglio et al. (2020) and Haddad et al. (2020) and reported in Appendix A.1 and fitted values for prices related to the standard factor model (1) (mid left panel) and its FECM specification (10) (mid right panel) associated to the PC5-factor model. Bottom panels show the difference between actual and fitted values for the two cases. Shaded areas are NBER recessions. The sample period is 1963 to 2019.

for the factors. There is a natural possibility for a (long-run) relationship between such a portfolio and the cumulative returns of an investment strategy (such as value, growth, etc.) over the long-run. We define price-level risk drivers as the value of buy-and-hold portfolios investing in factors (e.g., statistical factors like principal components) and prices as the value of buy-and-hold portfolios investing in any characteristic-sorted test assets.

When *asset prices* are modeled as affine in risk drivers, there are two possible scenarios. In the "bad scenario" there is no-common stochastic trend between asset prices and risk drivers so that asset prices can deviate from risk drivers for an arbitrary long time (see again Figure 1, bottom left panel).[3] On the contrary, in the "good scenario" there exists a linear combination of asset prices and risk drivers that is stationary; in this case the model captures the common long-term stochastic trend between asset prices and risk drivers. Importantly the "good scenario" has implications for the specification of the model relating *asset returns* to factors. In fact, the very presence of a common stochastic trend between risk drivers and asset prices, i.e., their cointegration, implies that a model for asset returns should include, in addition to the factors considered in the standard specification, a term that captures "disequilibria" in the long-run relationship between prices and factor-risk drivers. Hence, we naturally label this term "Equilibrium Correction Term" (henceforth, ECT), and the resultant factor model specification Factor Error Correction Model (FECM). The ECT is the mechanism through which prices converge to their equilibrium value determined by the cointegrating relation-

[3] Technically, in the bad scenario, the projection of asset prices on risk drivers delivers non-stationary residuals.

ship with risk drivers. Practically, the *ECT* constitutes a natural predictor for asset returns.

We show this exact mechanism in the mid right panel of Figure 1, where we display the resultant cumulative returns of our FECM model. It is apparent that the price level of the buy-and-hold asset successfully tracks the value of the factor portfolios; furthermore, the bottom right panel makes clear that the asset price deviations from risk drivers are temporary, since the *ECT* guarantees convergence of asset prices to the equilibrium value dictated by the risk drivers.

It is worth stressing that the misspecification in modeling price levels is not evident when the factor model is applied directly to asset returns (c.f. top panel in Figure 1). This is because the unit root in the residuals from the projection of prices on risk drivers (evident in the bottom left panel) is removed by differencing the model, i.e., by projecting returns on factors as in the traditional approach to factor asset pricing models. Thus, even a gross misspecification in modeling price levels with risk drivers goes easily undetected in a model only relating returns to factors.

Motivated by this evidence, this paper shows that the *ECT* and our FECM approach have profound implications for time-series predictability of characteristic-sorted portfolios, and validation of (multi-)factor models.

We start by showing that the cointegration analysis between risk drivers and asset prices is useful to assess the validity of factor models to describe long- and short-run dynamics of assets. Risk drivers should explain the long-run performance of any given portfolio. As risk drivers and prices are non-stationary variables, the validity of a given set of risk drivers to explain portfolio prices is naturally investigated by assessing if there exists a stationary linear combination of them (i.e., if they are cointegrated). Importantly, the presence of a stationary linear combination of prices and risk drivers rules out the possibility of an omitted risk driver, because the omission of a relevant one would prevent cointegration. Thus, our framework linking asset prices to risk drivers can be employed to select the relevant (number of) factors in a model since a non-stationary deviation of prices from risk drivers is an argument for long-run misspecification of the factor model. Applying our FECM framework, we show that five principal components are needed to deliver uniform evidence of cointegration between prices and the equilibrium value implied by risk drivers. Since the existence of cointegration discards the possibility of an omitted risk driver, we conclude that it is unlikely that more than five factors are needed to get (portfolio) prices right. In this respect, our FECM approach is informative about the dimensionality of the (zoo of) factors' space (Cochrane, 2011).

Having determined the right number of factors, our analysis proceed by documenting the pervasiveness of the ECT for equity portfolios. To this end, we consider a large cross-section of anomaly portfolios used in several recent studies (Giglio et al., 2020; Haddad et al., 2020; Kozak et al., 2020). Our analysis shows that the sign of the portfolio loading on the ECT is negative. This is consistent with our economic intuition of the ECT mechanism: when asset prices are higher (lower) than the long-run equilibrium implied by their relationship with risk drivers, we expect lower (higher) returns in the next period so that the "disequilibrium" is corrected. In our sample, a positive 1% log price deviation is associated with a log return over the next year which is lower by 45 basis points across value-sorted portfolios.

Importantly, the ECT is not subsumed by the book-to-market of the test assets, suggesting that the ECT captures an additional, new dimension of return predictability. Interestingly, we also find that the comovement in Equilibrium Correction Terms (proxied by the first principal component extracted from the cross-section of ECTs) predicts the market. The ECT contains asset-specific information that aggregates to generate market-return predictability. A battery of out-of-sample tests confirm the robustness of the ability of the common component in the ECTs to predict aggregate stock market returns.

The rest of the paper is organized as follows. We discuss the related literature next. Section 2 lays out the joint model for prices, returns, factors, and risk drivers, and provides some simulation-based evidence on the relevance of including price dynamics in factor-portfolio models. Section 3.1 describes our data. We discuss the usage of our framework to detect (long-run) misspecification in factor models in Section 3.2. Section 3.3 documents the pervasiveness of the existence of the Equilibrium Correction Term in the equity space, and illustrates the ability of our FECM model to forecast portfolio returns. Section 3.4 benchmarks the predictive ability of the ECT to that of other portfolio-specific and aggregate predictors. Section 4 concludes.

Related Literature. Our paper speaks to the vast literature on factor models. Factor models are widely used in asset pricing (see Ang, 2014 for an overview). Despite the great popularity of these models, the literature on the relationship between the choice of factors and the investment horizon has been much less developed. Specifically, the factor-based approach to portfolio allocation and risk management has concentrated almost exclusively on modeling one-period returns while not devoting enough attention to the relationship between the long-run performance of assets and factors. A notable exception is Hansen, Heaton and Li (2008). These authors are among the first to provide evidence on the importance of understanding long-run dynamics for equity returns and assets valuation. Moreover, the factor models literature has traditionally focused on factor-representation of stationary variables and only recently the factor framework has been extended to non-stationary cointegrated factors (e.g., Barigozzi et al., 2020; Banerjee et al., 2017). Interestingly, the dynamic dividend growth model (Campbell and Shiller, 1988) is built under the null that (log) prices are cointegrated with (log) dividends. As a consequence the model-consistent relationship between returns and dividend growth contains an ECT term that guarantees that (log) prices and (log) dividends share the same stochastic trend. It is somewhat curious that this feature of the dynamic dividend growth model is not shared by any of the factor models for returns available in the literature.

In a seminal contribution, Bauer and Rudebusch (2020) propose a dynamic term structure model with an embedded stochastic interest rate trend. Our cointegrating framework for equity factors shares several features with their approach. Furthermore, our analysis of the time-series implications of mispricing in the cross-section of equity complements their quantification of the importance of time-varying macroeconomic trends for interest rates. Overall, Bauer and Rudebusch (2020) and our work attest the need for (empirical and theoretical) models featuring long-run trends that are shared across prices.

Our paper fits into the literature on the relationship between cointegrated variables and error correction models (see, for example, Hendry, 1986; Engle and Yoo, 1987; Johansen, 1995; Pesaran and Shin, 1998; Liu and Timmermann, 2013). Lettau and Ludvigson (2001) are a first notable example of the use of cointegration analysis in macro-finance. They show that aggregate consumption, asset holdings, and labor income share a common long-term trend and temporary trend-deviations successfully predict short- and medium-term expected stock returns. In a series of papers, Bansal et al. (2007) and Bansal and Kiku (2011) show that the cointegrating relationship between dividends and consumption, a measure of long-run consumption risks, is a key determinant of expected equity excess returns, particularly at long investment horizons. While these works focus on economic variables, we are interested in financial markets' dynamics and the relation between asset values and risks. Overall, our work provides new supportive evidence that cointegration analysis can play a relevant role for financial markets and long-term investors.

Our paper relates to the large empirical literature that studies temporary deviations of asset values from fundamentals. In an early contribution, Poterba and Summers (1988) find positive autocorrelation in returns over short horizons and negative autocorrelation over longer horizons which can be explained by persistent, but transitory, divergences between prices and fundamental values. Concurrently, Fama and French (1988) argue that the U-shaped pattern in first-order autocorrelation observed in U.S. stock returns is consistent with the view that prices have a slowly decaying stationary component. Cohen et al. (2009) show that cash-flow or long-horizon CAPM betas perform satisfactorily in explaining cross-sectional variation in the level of asset prices. They conclude that, while the CAPM may fail to explain short-term expected returns on dynamic trading strategies, it is effective in "getting" stock prices and expected long-term returns approximately "right." On the theoretical side, Bossaerts and Green (1989) derive a dynamic pricing model with time-varying risk premia in which the risk of individual securities and equilibrium risk premia change predictably, featuring the inverse relationship between prices and returns. Brennan and Wang (2010) show that a premium in average returns is created as a result of Jensen's inequality when stock prices diverge from fundamental values because of stochastic pricing errors, even when the mispricing has an average of zero. Van Binsbergen and Opp (2019) investigate the implications of financial market mispricing for the real economy and find that pricing errors can cause allocative distortions. Although the literature on mispricing (Chernov et al., 2018; Cho and Polk, 2020) is rapidly growing, our cointegrating methodology and its usage to detect transitory asset-specific price deviations is novel to the literature. Furthermore, our approach implies a new definition of (long-term) misspecification of a factor model based on the magnitude of cointegration in prices, or absence thereof.

2 From Factors for Asset Returns to Risk Drivers for Asset Prices

2.1 Traditional Factor Models for Returns

Factor models are commonly used to characterize parsimoniously the predictive distribution of asset returns. Specifically, multi-factor models in which k factors characterize in a lower parametric dimension the distribution of n asset returns, have the following general form:

$$r_{i,t+1} = \alpha_i + \beta_i' \mathbf{f}_{t+1} + v_{i,t+1}, \tag{1}$$

$$\mathbf{f}_{t+1} = E\left(\mathbf{f}_{t+1} \mid I_t\right) + \epsilon_{t+1} \qquad \text{with } \epsilon_{t+1} \sim \mathcal{D}\left(\mathbf{0}, \boldsymbol{\Sigma}\right) \tag{2}$$

where $Cov\left(v_{i,t+1}, v_{j,t+1}\right) = 0$ for $i \neq j$, $\mathbf{f}_{t+1}$ is a k-dimensional vector of factors at time $t+1$, $r_{i,t+1}$ is the return on the i-th of the n assets at time $t+1$, and the vector β_i' contains the loadings for asset i on the k factors. Equation (1) specifies the conditional distribution of returns on factors, while equation (2) specifies the predictive distribution for factors at time $t+1$ conditioning on information available at time t. A baseline specification for this system assumes away factors predictability thus implying that conditional expectations of factors have no variance (i.e., $E\left(\mathbf{f}_{t+1} \mid I_t\right) = \mu$).

In equation (1) it is often assumed without further qualification that returns and factors are stationary variables.[4] Our paper is a reappraisal of this seem-

[4] The unconditional moments of returns are constant, although the conditional ones might be time varying.

ingly harmless hypothesis. In fact, next we show that the standard framework relating asset returns to stationary factors leaves asset prices undetermined.

Consider a test asset i and denote its log one-period return by $r_{i,t}$. We define the log price of this asset as:

$$\ln P_{i,t} = \ln P_{i,t-1} + \mathbf{r}_{i,t} \ , \tag{3}$$

i.e., prices of any test asset are cumulative returns. The analogous of the (log) price for an asset can be constructed for any given factor. We define as price-level risk driver the cumulative returns of a portfolio investing in standard factors (e.g., the aggregate market return).[5] The generic risk driver associated to a factor with a log period return of $\mathbf{f}_t$ evolves according to the following process:

$$\ln \mathbf{F}_t = \ln \mathbf{F}_{t-1} + \mathbf{f}_t \ . \tag{4}$$

If test assets returns and factors are stationary, then prices and risk drivers are non-stationary. In fact, imagine now to simulate data using the model given by equations (1), (2), (3), (4). The simulated data will deliver a linear relationship between returns and factors but no relationship between prices and risk-drivers. Prices and risk-drivers will follow two *unrelated* stochastic trends. In technical jargon, the model given by (1)–(4) rules out the hypothesis of cointegration between prices and risk drivers by assumption. The presence of co-integration which is borne out by the data (c.f. Section 3) therefore

[5] Meucci (2011) introduces the concept of risk drivers of any given security as a set of random variables that completely specifies the security price and that follow a stochastic process homogeneous across time. We adopt the terminology here with some adaptation.

prevents it from being a valid representation.

Next, we propose a novel testable framework that, starting from the relationship between asset prices and risk drivers, derives the mapping of asset returns into factors.

2.2 A (Co-)Integrated Approach to Factor Modeling

If factor-risk drivers are the non-stationary variables that drive the non-stationary dynamics of prices, then a linear combination of prices and risk drivers should be stationary; i.e., prices and risk drivers should be cointegrated.

Consider the following model describing the exposure of a given portfolio $P_{i,t}$ to risk drivers:

$$\ln P_{i,t} = \alpha_{0,i} + \alpha_{1,i} t + \beta_i' \ln \mathbf{F}_t + u_{i,t} \ .$$

The estimation of such regression delivers stationary residuals $u_{i,t}$ anytime the chosen set of risk drivers captures the stochastic trend that determines the long-run dynamics of prices. In this case, the linear combination of the left-hand side variables of the equation defines the long-run cointegrating relationship and $u_{i,t}$ captures temporary deviations of prices from the long-run equilibrium value determined by risk drivers. Thus, it is natural to refer to the residuals $u_{i,t}$ as the "Equilibrium Correction Term" (henceforth, ECT) associated with asset i at time t. Formally, we define the residual from the long-run cointegrating relationship:

$$ECT_{i,t} \equiv \ln P_{i,t} - \hat{\alpha}_{0,i} - \hat{\alpha}_{1,i} t - \hat{\beta}_i' \ln \mathbf{F}_t \ .$$

For expository purposes, it is useful to specify the error term $u_{i,t}$ as an AR(1) process. In sum, we model the joint distribution of portfolio prices, factors, and risk drivers as follows:

$$\begin{aligned} \ln P_{i,t+1} &= \alpha_{0,i} + \alpha_{1,i} t + \beta_i' \ln \mathbf{F}_{t+1} + u_{i,t+1} \\ u_{i,t+1} &= \rho_i u_{i,t} + v_{i,t+1} \\ \mathbf{f}_{t+1} &= E\left(\mathbf{f}_{t+1} \mid I_t\right) + \epsilon_{t+1} \\ \ln P_{i,t} &= \ln P_{i,t-1} + r_{i,t} \\ \ln \mathbf{F}_t &= \ln \mathbf{F}_{t-1} + \mathbf{f}_t \end{aligned} \tag{5}$$

where $\epsilon_{t+1} \sim \mathcal{D}\left(\mathbf{0}, \mathbf{\Sigma}\right)$, $u_{i,t+1}$ and $v_{i,t+1}$ have zero mean and variance $\sigma^2_{u,i}$ and $\sigma^2_{v,i}$, respectively, and $Cov\left(v_{i,t+1}, v_{j,t+1}\right) = 0$ for $i \neq j$.

By taking first differences of our model in (5) we obtain a novel relationship between returns and factors:

$$r_{i,t+1} = \alpha_{1,i} + \beta_i' \mathbf{f}_{t+1} + \underbrace{(\rho_i - 1)}_{\delta_i} \underbrace{u_{i,t}}_{\equiv ECT_{i,t}} + v_{i,t+1}. \tag{6}$$

Eq. (6) represents the Factor Error Correction Model.[6]

[6] The equilibrium correction representation (6) of cointegrated time-series (see the system of equations in (5)) is warranted by the Engle and Granger (1987) representation theorem.

Two comments are in order. First, we include a linear trend in Eq. (5) since it allows to recover the standard short-run specification—returns are regressed on factors plus a constant—when taking first-differences. In other words, a positive α_1 in the long-run relation (5) generates "alpha" in returns.[7]

Second, when $ECT_{i,t}$ is stationary, then prices and risk drivers are cointegrated. The stationarity of $ECT_{i,t}$ implies that, in the relation (6) linking returns to factors, this term appears with a coefficient δ_i capturing the speed with which the system eliminates disequilibria with respect to the long-run relationship. Indeed, δ_i is related to the persistence ρ_i of $ECT_{i,t}$, see Eq. (6).

When risk drivers explain the buy-and-hold value of a portfolio, cointegration implies that portfolio returns respond to the Equilibrium Correction Term so far omitted in the empirical asset pricing literature. The inclusion of the ECT ensures that the specification for returns is consistent with the long-run relationship between risk drivers and portfolio prices. The omission of the ECT leads to a misspecification of the factor model, in the sense that the factor model leaves price dynamics undetermined.

Interestingly, a traditional factor model would not be affected by omitting the disequilibrium term only if risk drivers and prices are not cointegrated (i.e., when $\mid \rho_i \mid = 1$). However, this case also implies that a given factor model is unable to price the buy-and-hold portfolios since prices do not track risk drivers in the long-run. In Section 3 we provide strong evidence in support of the disequilibrium correction mechanism that operates through the ECT and,

[7]Moreover, as discussed by Engle and Yoo (1987) and MacKinnon (2010), the inclusion of a trend is a simple way to avoid the dependence of the distribution of test statistics for residuals on α_1.

thus, we reject the extreme case of $\mid \rho_i \mid = 1$.

2.3 Evidence From Monte-Carlo Simulation of a Simple DGP

We run a simple, but informative, simulation exercise to gauge the importance of modeling price dynamics in factor models. Let's assume the following Data Generating Process (DGP):

$$
\begin{aligned}
\ln P_{t+1} &= \alpha_0 + \alpha_1 t + \beta \ln F_{t+1} + u_{t+1} \qquad (7) \\
u_{t+1} &= \rho u_t + \sigma_1 \sqrt{1-\rho^2} v_{t+1} \\
f_{t+1} &= \mu + \sigma_2 \epsilon_{t+1} \\
\ln P_t &= \ln P_{t-1} + r_t \\
\ln F_t &= \ln F_{t-1} + f_t \\
\begin{pmatrix} u_t \\ \epsilon_t \end{pmatrix} &\sim N.I.D. \left[\mathbf{0}, \mathbf{I} \right]
\end{aligned}
$$

In this simplified DGP, we consider only one test asset and one factor. The cointegrating relationship between the price and the risk driver associated respectively to the return and the factor is controlled by the parameter ρ. We assume no predictability for the factor. We calibrate μ and σ_2 to 5.33 and 15.37 respectively such that f_t features the annualized mean and the standard deviation of U.S. stock market excess returns (in percentage) over the risk-free rate in the period 1963–2019. Similarly, we calibrate parameters in the long-run regression (7) by considering the values delivered by projecting (log) cumulative returns for the value portfolio (decile 10 of portfolios formed on Book-to-Market) on the risk driver associated with our single-factor, the market. Thus, α_0 is 12.08, α_1 is 5.61, and β is 0.64. Finally, we set $\sigma_1 = 26.53$, that equals the standard deviation of equation (6) for the value portfolio.

We study the effect of estimating a traditional factor model when the DGP is given by the FECM in (7). The traditional factor model is specified as follows:

$$\begin{aligned} r_{t+1} &= \alpha + \beta f_{t+1} + v_{t+1} \\ f_{t+1} &= \gamma + \epsilon_{t+1}. \end{aligned} \tag{8}$$

First, consider the case in which cointegration holds, i.e., we simulate the DGP by calibrating $\rho = 0.5$. The top panel in Figure 2a reports—for one run of the Monte-Carlo simulation—the simulated returns along with the returns predicted respectively by the traditional (i.e., without ECT) factor model and by the factor model that includes the ECT (i.e., the FECM). The figure illustrates that when the (misspecified) traditional model (8) is used to predict future returns, the predicted value (red solid line) has zero variance despite the predictability implied by the DGP. This predictability, which can be relevant for risk measurement, risk management, and asset allocation, is instead exploited by the inclusion of the Equilibrium Correction Term in the FECM specification (blue dotted line).[8] As shown in the bottom panel of Figure 2a,

[8]Interestingly, our approach delivers predictability through the ECT also when a con-

price deviations from the long-run equilibrium implied by the risk driver are mean-reverting towards zero.

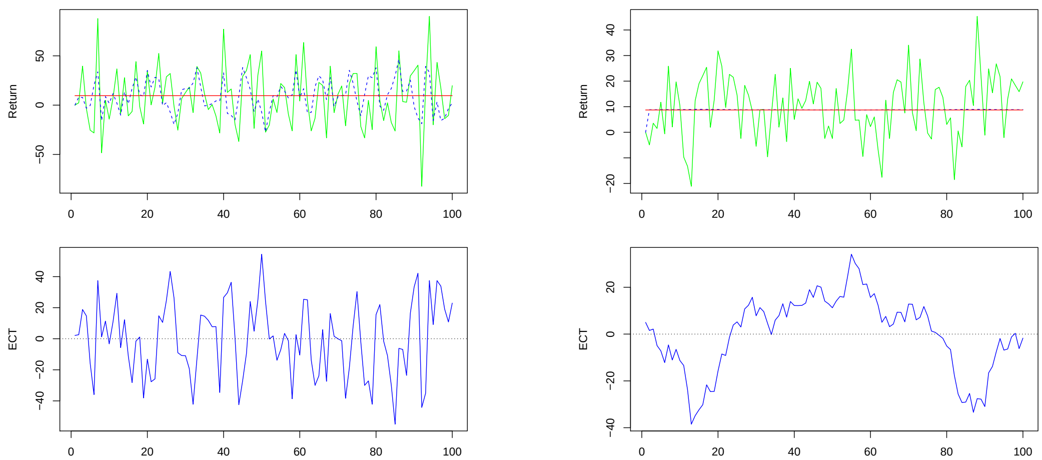
Next, we consider the case of no-cointegration by simulating the DGP under $\rho = 1$. In this case, the FECM specification collapses to the traditional factor model (8), and the ECT does not have power in forecasting returns. Figure 2b illustrates simulated returns, and predicted returns by the standard factor model. The figure makes clear that the model fits well the relation between returns and factors (i.e., the solid red and dashed blue lines overlap, see top panel), but price deviations from long-run equilibrium implied by the risk driver are non-stationary (see bottom panel). Indeed, the single-factor model is not able to track the value of the buy-and-hold portfolio.

3 Empirical Results

3.1 Data

We focus on U.S. data–NYSE, AMEX, and Nasdaq stocks from the Center for Research in Security Prices (CRSP) and Compustat data required for sorting – for the sample 1963–2019. Throughout we use monthly observations but we focus on 1-year holding-period excess return. Indeed, our co-integrated approach to modeling asset prices and returns is designed for low-frequency fluctuations in returns. Specifically, the horizon over which asset returns are computed must be sufficiently long to allow for a reaction of returns at time

stant expected return specification for factors is assumed as in the DGP described by (7).



(a) DGP with cointegration. **(b)** DGP without cointegration.

Figure 2: Monte-Carlo Simulation. All simulations are based on the DGP in the system of equation (7). The green line is simulated returns, the red solid line is the predicted returns by the single-factor model (i.e., CAPM), the blue dashed line is the predicted returns by FECM, and the solid blue line is the ECT. Panel (a) shows simulated data when $\rho = 0.5$, i.e., with cointegration between prices and risk drivers. Panel (b) shows simulated data when $\rho = 1$, i.e., without cointegration between prices and risk drivers.

$t + 1$ to disequilibrium in the relationship between risk drivers and prices at time t. Without further qualification, then, r_{t+1} will always denote the one-year-ahead excess returns.[9]

Our empirical setting with monthly observations of annual returns requires extra care when computing standard errors. To this end, we follow Ang and Bekaert (2007) and rely on conservative standard errors from reverse regressions proposed by Hodrick (1992). To remove the overlap in the error term, this approach exploits the covariance of one-period returns with an h-period

[9] In our sample, there is evidence that returns react to disequilibrium within a quarter, i.e., $\hat{u}_{i,t}$ manifests forecasting ability for $r_{i,t+3/12}$. We leave open the question of the economic determinants of the timing of reaction to disequilibrium, and decide to focus on 1-year holding-period returns in line with recent empirical studies on time-variation in anomaly returns (Haddad et al., 2020; Lochstoer and Tetlock, 2020).

sum of the predictor ($h = 12$ for monthly data and annual returns).[10,11]

To investigate the validity and performance of our factor model augmented with the ECT, dubbed FECM, we consider as test assets a large cross-section of anomaly portfolios based on single-sorts of 45 different characteristics. These test assets, or a subset of it, have been used by Kozak et al. (2020), Giglio et al. (2020), Haddad et al. (2020), and Lettau and Pelger (2020), among others.[12] Focusing on such a large cross-section allows us to confirm the ubiquitous presence of disequilibria in the long-run relationship between portfolio prices and price-level risk drivers.

With regard to the empirical model for return, we employ principal components analysis (PCA) to extract factors from the 45 long-short portfolios. Despite our approach being applicable to classical factor models like the one proposed by Fama and French (1993), we prefer to use PCA for three main reasons. First, PCA uses a purely statistical criterion to derive factors, and has the advantage of requiring no ex-ante knowledge of the structure of average returns. Second, PCA is grounded in Ross (1976) seminal Arbitrage

[10] Ang and Bekaert (2007) show reverse regressions standard errors to have superior small-sample properties compared with the commonly used Hansen-Hodrick or Newey-West errors, both of which overreject the null hypothesis of no predictability in small sample. Similarly, in our setting, we find Hodrick (1992) standard error to be more conservative than Newey-West or GMM ones.

[11] Yearly frequencies remove issues associated with overlapping observations; moreover the exploitable predictability in the mean is usually paired with a constant volatility specification. Importantly, our conclusions are unchanged when we employ only returns at the end of December (i.e., 57 annual observations).

[12] We kindly thank Serhiy Kozak for making his data available at https://sites.google.com/site/serhiykozak/data?authuser=0. Appendix Table A.1 lists the categories and the portfolios included in each category. Note that we follow the convention of labelling decile 10 the one with higher (than decile one) average return. E.g., decile 10 for size corresponds to small stocks, whereas decile one to large stocks. In other words, characteristic are signed so that they predicts returns with a positive sign.

Pricing Theory (APT) and it is by far the most popular technique in finance to analyze latent factor models for returns with key empirical contributions dating back to Chamberlain and Rothschild (1983) and Connor and Korajczyk (1986, 1988), and, more recently, Kozak et al. (2020).[13] Third, at the managed-portfolio level, the fit of PCA in terms of both cross-sectional and time-series R^2 are generally excellent and superior to, e.g., the Fama and French (1993) three-factor model and the Fama and French (2015a) five-factor model (c.f., Table 2 in Kelly et al., 2019).

We proceed as follows. We first use our co-integrated approach for asset prices to select the number of factors. Given the number of factors, we then show the pervasiveness of a "disequilibrium" long-run relationship between prices and factor-risk drivers in our cross-section of anomaly portfolios.

3.2 ECT and the Detection of (Buy–and–Hold) Misspecification in Factor Models

Every factor structure that generates cointegration between risk drivers and prices rules out permanent deviations of portfolio prices from their projection on risk drivers, and it naturally leads to an Equilibrium Correction Term. The ECT would not be significant to explain returns only in absence of cointegration between risk drivers and portfolio prices. The existence of cointegration

[13]Recently, Kelly et al. (2019) and Lettau and Pelger (2020) have extended classical PCA along important dimensions. Kelly et al. (2019) propose instrumental principal component analysis (IPCA) where the factor loadings are dynamic and can be instrumented with observable portfolio characteristics. Lettau and Pelger (2020) propose risk-premium PCA to extract factors that fits not only the time-series but also the cross-sectional variation in asset returns. For simplicity, we stick to the most classical PCA technique.

discards the possibility of an omitted risk driver. Equivalently, the omission of a factor whose associated risk driver is relevant to determine the price dynamics of a given portfolio would prevent cointegration between portfolio prices and any set of risk drivers that omits the relevant one. Non-stationarity of the residual in Eq. (5) could in fact be taken as evidence of the factor model to be misspecified. Simply put, the presence of cointegration, or lack thereof, is revealing about the number of relevant (price-level) risk drivers and, hence, factors for returns.

In this section, we exploit the presence of mis-specification (or absence thereof) about long-run dynamics in factor models to select the relevant number of factors to be used.[14] Specifically, we estimate by ordinary least squares the following regression:[15]

$$\ln P_{i,t} \quad = \quad \alpha_{0,i} + \alpha_{1,i} t + \beta_i' \ln \mathbf{F}_t + u_{i,t} \qquad i = 1, ..., 90 \tag{9}$$

where $\ln P_{i,t}$ is the (log) price for the characteristic-sorted portfolio i at time t, $\ln \mathbf{F}_t$ are the risk drivers, and t is a time trend. The risk drivers consist of principal components that are added sequentially until we find evidence for portfolio prices reverting to their equilibrium levels.

Table 1 reports summary statistics about the speed of mean reversion of

[14]Our long-run type of misspecification for *prices* is different from and complementary to the large literature on misspecification of factor models for *returns*, see, e.g., Hansen and Jagannathan (1997); Kan and Zhang (1999); Lewellen et al. (2010); Kan et al. (2013); Gospodinov et al. (2014, 2017).

[15]Using the dynamic ordinary least squares (DOLS) technique (see Stock and Watson, 1993) for the long-run specification, as in e.g., Lettau and Ludvigson (2001), leaves the evidence of cointegration and the estimates for the cointegrating relationship unaltered.

the ECTs for the large cross-section of portfolios. Each row refers to a specific factor model. The models are ranked by increasing complexity as proxied by the number of principal components employed. It is immediately apparent that the ECTs implied by a model using only the first principal component fails to deliver cointegration with one-fourth of the portfolios requiring more than five years for the disequilibrium to disappear, and one portfolio almost never reverting back to the equilibrium dictated by the sole risk driver (the half-life of 173 months corresponds essentially to prices permanently drifting away from their equilibrium level). Adding principal components improves the evidence for cointegration by reducing the half-life. However, even with three principal components we continue to observe quite persistent deviations with half of the equilibrium error being absorbed only after two and a half years for 25% of the portfolios. Selecting five principal components seems necessary in order to have portfolio prices that revert quickly toward equilibrium, with an half life that is around one year for 75% of the portfolios, and never greater than two years.[16,17] Since the existence of cointegration discards the possibility of an omitted risk driver, we conclude that it is unlikely that more than five factors are needed. Hence, we set

$$\mathbf{F}_t' = \begin{bmatrix} PC1_t & PC2_t & PC3_t & PC4_t & PC5_t \end{bmatrix} .$$

[16]The cumulative proportion of variance explained by PC1, PC2, PC3, PC4, and PC5 is respectively 25.2%, 45.3%, 58.7%, 62.8%, and 66.6%.

[17]More formally, one can test for cointegration following the methodology proposed by Engle and Granger (1987). In that case, using quarterly observations and simulated critical values proposed by MacKinnon (2010) we reject the null of no-cointegration for PC1, PC2, PC3, PC4, and PC5 respectively for the 76.7%, 93.3%, 92.2%, 98.9%, and 100% of portfolios.

Table 1: Half-Life

This table reports the half-life (in months) for different specifications of the long-run regression (9). For 1-PC, we employ only the first principal component extracted from the 45 long-short anomaly portfolio returns constructed in Giglio et al. (2020) and Haddad et al. (2020) and reported in Appendix A.1; for 2-PC, we employ the first two principal components; similarly for 3-PC, 4-PC, and 5-PC (we employ the first five principal components). The half-life is calculated as $\log(0.5)/\log(|\ \rho\ |)$, where ρ is the estimated first-order autoregressive parameter for the ECT related to the different specifications. We compute ECT as in equation (10). Test assets are 90 decile 1 and decile 10 anomaly portfolios. Monthly observations. The sample period is August 1963 to December 2019.

Half-Life	Mean	Min	Pctl(25)	Median	Pctl(75)	Max
1-PC	52.2	22.0	36.1	49.2	62.7	172.9
2-PC	32.8	16.6	25.6	30.5	35.7	76.7
3-PC	28.0	12.7	19.6	26.3	31.2	86.3
4-PC	19.3	9.1	13.8	17.9	22.6	57.4
5-PC	13.8	9.0	12.0	13.1	14.7	25.3

In this respect, our FECM approach is informative about the dimensionality of the (zoo of) factors' space (Cochrane, 2011).

With regard to the last point, it is interestingly to observe that the state-of-the-art q-factor model of Hou et al. (2015) and the Fama and French (2015a) model employ four and five factors, respectively. These models have become the benchmarks in empirical finance since they have been shown to perform well in the space of portfolio *returns*, e.g. Fama and French (2015b). We reach a similar conclusion about the number of factors using a very difference criteria which is based on a metric based on *prices*, not returns.

3.3 The Statistical Evidence on the Equilibrium Correction Term

Given the evidence of cointegration when five risk drivers are used, in the rest of the paper we employ a factor model with five principal components (PCs).

Specifically, for each portfolio returns, we specify a system of equations that includes— in addition to the five PCs —the Equilibrium Correction Term derived from the estimation of the long-run cointegrating relationship:

$$\begin{aligned} r_{i,t+1} &= \alpha_{1,i} + \beta_i' \mathbf{f}_{t+1} + \delta_i ECT_{i,t} + v_{i,t+1} \\ ECT_{i,t} &= \ln P_{i,t} - \hat{\alpha}_{0,i} - \hat{\alpha}_{1,i} t - \hat{\beta}_i' \ln \mathbf{F}_t \\ \ln P_{i,t} &= \ln P_{i,t-1} + r_{i,t} \\ \ln \mathbf{F}_t &= \ln \mathbf{F}_{t-1} + \mathbf{f}_t \\ \mathbf{F}_t' &= \begin{bmatrix} PC1_t & PC2_t & PC3_t & PC4_t & PC5_t \end{bmatrix} \end{aligned} \tag{10}$$

where $r_{i,t+1}$ is the one-year ahead excess return of test asset i and $ECT_{i,t}$ is the Equilibrium Correction Term for test asset i observed at month t, estimated as the residual from equation (9).[18]

To show the pervasiveness of a "disequilibrium" long-run relationship between prices and factor-risk drivers, we employ a cross-section of anomaly portfolios that is based on single-sorts of 45 different characteristics. For ease of exposition, we group anomalies within categories (Table A.1 lists the port-

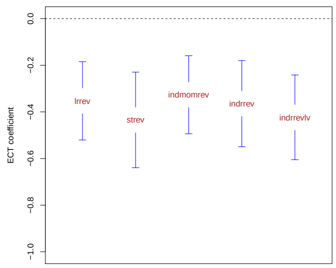
[18] Practically, we cumulate (log) returns on asset i in excess of the risk-free rate to remove inflation-related trends in asset prices.

folios included in each category).

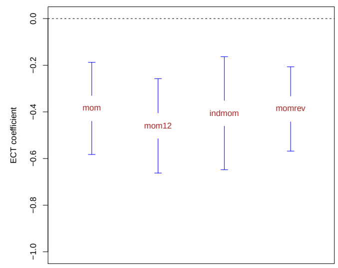
Figures 3 and 4 display the estimated loading $\widehat{\delta}_i$ of portfolio i on the ECT_i, see equation (10), with their associated 95% confidence intervals. Figure 3 refers to decile one, whereas Figure 4 to decile ten. Two comments are in order. First, for each category, the estimates of $\hat{\delta}_i$ are negative and statistically significant. This is consistent with our economic intuition of the ECT mechanism: When asset prices are higher (lower) than their long-run equilibrium value implied by risk drivers, we expect lower (higher) returns in the next period so that the "disequilibrium" disappears. Second, recall that the magnitude of the $\hat{\delta}_i$ is inversely related to the speed of adjustment of the correction term (c.f. Eq. (6)). In this respect, Figures 3 displays value anomalies at one extreme (high delta, and fast mean reversion to equilibrium) and momentum and trading frictions at the other of the spectrum (low delta, and low mean reversion to equilibrium).[19] This evidence is consistent with the idea that frictions make it difficult for feedback traders to close the disequilibrium, whereas arbitrage activity is generally successful at stabilizing anchored strategies, like value (see Stein, 2009; Lou and Polk, 2013).

Table 2 zooms in onto the value anomaly (decile ten of book-to-market sorted portfolios). In the first column, we collect the estimates of the long-run cointegrating relation (c.f., equation (9)), and in the second column we report the estimates for the short-run factor error correction model (c.f., equation (10)). Value returns are negatively related to PC1, and positively related to

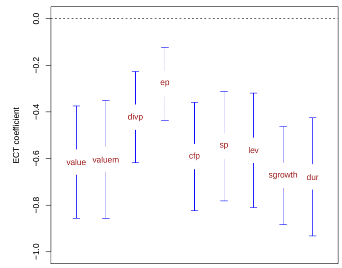
[19] Specifically, for decile one, the median half life of strategies in the value category is about 10 months, whereas that of portfolios in the momentum and trading frictions categories is about 16 months.



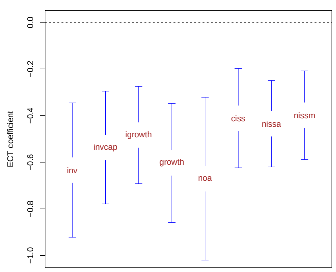
(a) reversal



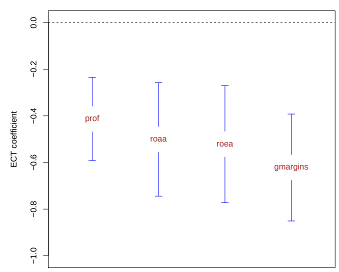
(b) momentum



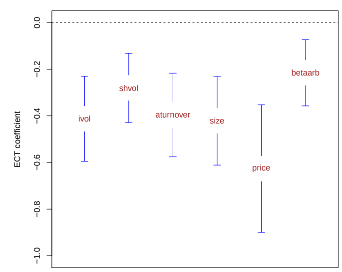
(c) value



(d) investment

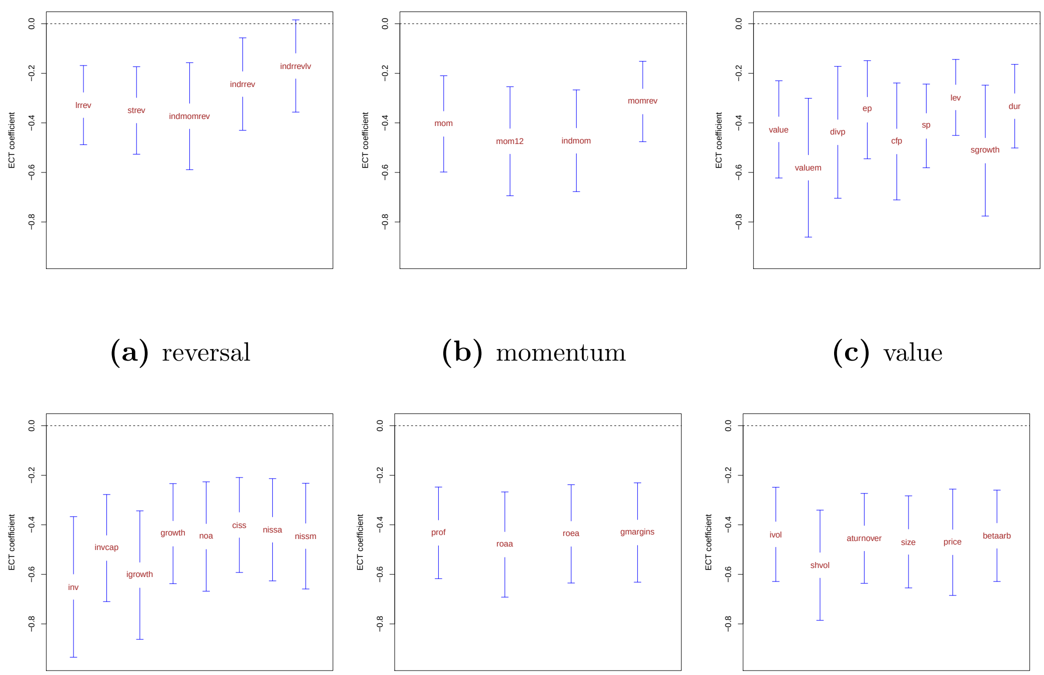


(e) profitability



(f) trading frictions

Figure 3: Anomaly Portfolios (Decile 1) and ECT. This figure shows the estimated *ECT* coefficients for the FECM specification (10) for the decile 1 anomaly portfolios constructed in Giglio et al. (2020) and Haddad et al. (2020) in the Categories reversal, momentum, value, investment, profitability, and trading frictions with respective confidence intervals at 5% level of significance. Categories are reported in Appendix A.1. We employ the first five principal components extracted from the 45 long-short anomaly portfolio returns as factors. Standard errors are computed as in Hodrick (1992). Monthly observations of annual returns. The sample period is August 1963 to December 2019.



(a) reversal **(b)** momentum **(c)** value

(d) investment **(e)** profitability **(f)** trading frictions

Figure 4: Anomaly Portfolios (Decile 10) and ECT. This figure shows the estimated *ECT* coefficients for the FECM specification (10) for the decile 10 anomaly portfolios constructed in Giglio et al. (2020) and Haddad et al. (2020) in the Categories reversal, momentum, value, investment, profitability, and trading frictions with respective confidence intervals at 5% level of significance. Categories are reported in Appendix A.1. We employ the first five principal components extracted from the 45 long-short anomaly portfolio returns as factors. Standard errors are computed as in Hodrick (1992). Monthly observations of annual returns. The sample period is August 1963 to December 2019.

Table 2: FECM for the Value Portfolio

This table reports the estimated coefficients for the specifications in equations (9) and (10) where P and r are respectively log prices and returns for the Value Portfolio (decile 10) in the 90 anomaly portfolios constructed in Giglio et al. (2020) and Haddad et al. (2020) and reported in Appendix A.1. We employ the first five principal components extracted from the 45 long-short anomaly portfolio returns as factors. Column (1) reports results for the long-run specification (log prices on log risk drivers). Column (2) reports least squares estimates for the short-run specification (log returns on log factors plus the ECT). Column (3) reports the semi-partial R^2 for each regressor in Column (2). Constant estimates are not tabulated. Values in parenthesis are heteroskedasticity and autocorrelation consistent (HAC) standard errors using Newey and West (1987) with automatic bandwidth selection procedure as described in Newey and West (1994). The standard error for ECT is computed as in Hodrick (1992). Monthly observations of annual returns. The sample period is August 1963 to December 2019.

	Long-Run	FECM	$Sp\mathrm{R}^2$
	(1)	(2)	(3)
Trend	11.060		
	(6.796)		
PC1	−0.182	−0.182	0.26
	(0.075)	(0.029)	
PC2	0.027	0.047	0.02
	(0.157)	(0.036)	
PC3	−0.239	0.050	0.01
	(0.068)	(0.049)	
PC4	0.245	0.113	0.02
	(0.121)	(0.069)	
PC5	0.111	−0.032	0
	(0.078)	(0.114)	
ECT(-1)		−0.426	0.12
		(0.100)	
Observations	677	665	
Adjusted R^2	0.987	0.616	

PC4 factors. The ECT coefficient is economically and statistically significant, and negative: a positive deviation of (log) prices for the value portfolio from their long-term relation with the risk drivers in this period implies a lower expected return for the next period, with an order of magnitude of 43 basis points per unit of deviation.

Further supporting evidence for the importance of the ECT for understanding the time-series dynamics of returns is provided by the analysis of the semi-partial R^2. The semi-partial R^2 is defined as the difference between the overall regression R^2 and the R^2 of the regression that includes all regressors except the i^{th}-regressor for which the semi-partial R^2 is computed. We show the results for the semi-partial R^2 associated to each factor in the last column of Table 2. The ECT is the second most relevant factor after the PC1, explaining by itself about 10% of the total variance of returns. We find similar evidence for the other portfolios with the partial R^2 associated to the ECT ranging from 8% to 16%.

We conclude that not only the cointegrating relationship between risk drivers and asset prices is present in a large cross-section of anomaly portfolios, but also the inclusion of the ECT improves the description of returns dynamics.

3.4 The Information in ECT

3.4.1 The ECT and Other Return Predictors

How is the ECT related to other return predictors? To address this question, we compare the predictive power of ECT to that of valuation ratios. Valuation ratios like the dividend-price ratio, are often used in forecasting return regressions (e.g., Cochrane, 2005; Campbell, 2017) since they represent a natural predictor: The Campbell-Shiller log-linear present value model implies that the dividend-price ratio is a good proxy for expected returns, if dividend growth is unpredictable.

Table 3 reports the results from the regression:

$$r_{i,t+1} = \alpha_i + \beta_i \mathbf{f}_{t+1} + \delta_i ECT_{i,t} + \gamma_i BM_{i,t} + \epsilon_{i,t},$$

where $r_{i,t+1}$ denotes the log excess return of portfolio i, $BM_{i,t}$ is the log book-to-market ratio for portfolio i at time t, and $ECT_{i,t}$ is the ECT for portfolio i at time t.

We consider the top and the bottom decile portfolios for five well known anomalies, each one representative of a different category (c.f., Table A.1): value (Rosenberg et al., 1984), leverage (Bhandari, 1988), investment (Chen et al., 2011), gross profitability (Novy-Marx, 2013), and size (Fama and French, 1993). The first and third rows report the estimates for δ_i and γ_i, whereas the second and last row displays the partial R^2 associated with the ECT.

We start commenting on Book-to-Market sorted portfolios. These assets constitute a natural playing field since value strategies are predictable by (the spread in) valuation multiples (Asness et al., 2000; Cohen et al., 2003; Baba-Yara et al., 2020). We see that the statistical significance of the ECT continue to be strong even after controlling for the book-to-market ratio.[20] Turning to other anomalies, we continue to find a statistically significant and negative loading on the ECT across portfolios. Moreover, the ECT captures a substantial amount of variability in future portfolio returns, with (partial) R^2 ranging from 10% to 18%.

This evidence suggests that the ECT conveys information about the time-series dynamics of returns for a wide range of portfolios. Importantly the informational content of the ECT is not subsumed by the book-to-market ratio of the portfolios.

3.4.2 Common vs Portfolio Specific Components of ECT

Is the predictive ability of the ECT portfolio specific? Or does the ECT predict common co-movement across portfolios? The answer to this question is not trivial. It may well be the case that the prices of each portfolio deviate from the risk drivers in a synchronous way. Indeed, we find that the first principal component (PC) extracted from the $ECT_{i,t}$ explains about 73.4% of the overall ECT variability.[21]

[20] In a simple regression, the sign of book-to-market is positive as implied by the classical Campbell-Shiller decomposition. For example, value returns load on B/M with a coefficient of about 0.19.

[21] The first three PCs explain 86.7% of the ECTs associated with the cross-section of anomaly portfolios. More importantly, the percentage of variance explained by the PCs is stable across time; e.g., we find that the first PC (the first three PCs) extracted from the

Table 3: ECT and the Book-to-Market Ratio

This table reports the ordinary least squares estimate for δ_i, γ_i, and the semi-partial R^2 from the regression: $r_{i,t+1} = \alpha_i + \beta_i \mathbf{f}_{t+1} + \delta_i ECT_{i,t} + \gamma_i BM_{i,t} + \epsilon_{i,t}$, where $r_{i,t+1}$ is the test asset i log excess return at time $t+1$, $\mathbf{f}$ is a vector containing the first five principal components extracted from the 45 long-short anomaly portfolio returns constructed in Giglio et al. (2020) and Haddad et al. (2020) and reported in Appendix A.1, and $ECT_{i,t}$ and $BM_{i,t}$ are respectively the ECT and the book-to-market ratio for the test asset i at time t. Test assets are a selection from the 90 decile 1 and decile 10 anomaly portfolios. Values in parenthesis are standard errors computed as in Hodrick (1992). Monthly observations of annual returns. The sample period is August 1963 to December 2019.

	Value		Leverage		Investment		Profitability		Size	
	(dec 1)	(dec 10)	(dec 1)	(dec 10)	(dec 1)	(dec 10)	(dec 1)	(dec 10)	(dec 1)	(dec 10)
ECT(-1)	−0.646	−0.634	−0.605	−0.396	−0.623	−0.725	−0.562	−0.482	−0.467	−0.621
	(0.123)	(0.117)	(0.131)	(0.089)	(0.167)	(0.157)	(0.106)	(0.102)	(0.107)	(0.112)
$Sp\mathrm{R}^2_{ECT}$	0.150	0.181	0.127	0.139	0.125	0.175	0.166	0.138	0.133	0.104
BM(-1)	−0.366	−0.132	−0.244	−0.164	0.027	−0.100	−0.136	−0.212	−0.086	−0.186
	(0.221)	(0.041)	(0.233)	(0.078)	(0.143)	(0.076)	(0.061)	(0.203)	(0.099)	(0.060)
$Sp\mathrm{R}^2_{ECT}$	0.012	0.065	0.005	0.035	0.000	0.013	0.038	0.007	0.006	0.024

To answer this question we decompose the ECT of portfolio i in a common component and a portfolio-specific component. Given our set of portfolios, we extract the first PC from the time series of the $ECT_{i,t}$ and denote it with $PC1\_ECT$. We then run the following regression:

$$ECT_{i,t} = \beta_i PC1\_ECT_t + e_{i,t}.$$

We call $\beta_i PC1\_ECT_t$ the common ECT component, and $e_{i,t}$ the portfolio specific component. We denote these two components by ECT_t^{COM} and $ECT_{i,t}^{SPEC}$.

Table 4 reports the results from the regression:

$$r_{i,t+1} = \alpha_i + \beta_i \mathbf{f}_{t+1} + \delta_i ECT_t^{COM} + \epsilon_{i,t}, \tag{11}$$

in Panel A, and the results from

$$r_{i,t+1} = \alpha_i + \beta_i \mathbf{f}_{t+1} + \delta_i ECT_{i,t}^{SPEC} + \epsilon_{i,t}, \tag{12}$$

in Panel B. We let $r_{i,t+1}$ denote the log excess return of portfolio i. Since $ECT_{i,t}^{SPEC}$ is by construction orthogonal to ECT_t^{COM}, the sum of R^2 from regressions (11) and (12) is equal to the R^2 obtained from a multiple regression of $r_{i,t+1}$ on both $ECT_{i,t}^{SPEC}$ and ECT_t^{COM}. For easy of exposition, we focus on the same anomalies investigated in Table 3; however, our results extend to the larger cross-section of anomalies.[22] Table 4 shows that the statistical evidence of the common ECT component is strong across anomalies and deciles. On the

ninety ECTs explain 84% (90%) of the variance in the sample before January 2000, and 89% (94%) afterwards. Similarly, when we exclude the financial crisis and stop our sample

Table 4: ECT: Common and Idiosyncratic Component?

This table reports the ordinary least squares estimate for γ_i and the semi-partial R^2 from the regression: $r_{i,t+1} = \alpha_i + \beta_i \mathbf{f}_{t+1} + \gamma_i ECT\_Comp_{i,t} + \epsilon_{i,t}$, where $r_{i,t+1}$ is the test asset i log excess return at time $t+1$ and $\mathbf{f}$ is a vector containing the first five principal components extracted from the 45 long-short anomaly portfolio returns constructed in Giglio et al. (2020) and Haddad et al. (2020) and reported in Appendix A.1. Test assets are a selection from the 90 decile 1 and decile 10 anomaly portfolios. In Panel A, ECT_Comp is the fitted value from the regression $ECT_{i,t} = \beta_i PC\_ECT_t + e_{i,t}$, where PC_ECT_t is the first principal component extracted from the 90 decile 1 and decile 10 anomaly portfolio returns' ECTs. In Panel B, ECT_Comp is the residual from regression $ECT_{i,t} = \beta_i PC\_ECT_t + e_{i,t}$, i.e., the orthogonal component to PC_ECT. Values in parenthesis are standard errors computed as in Hodrick (1992). Monthly observations of annual returns. The sample period is August 1963 to December 2019.

Panel A: The ECT common component

	Value		Leverage		Investment		Profitability		Size	
	(dec 1)	(dec 10)	(dec 1)	(dec 10)	(dec 1)	(dec 10)	(dec 1)	(dec 10)	(dec 1)	(dec 10)
ECT^{COM}(-1)	−0.635	−0.466	−0.619	−0.368	−0.548	−0.656	−0.474	−0.509	−0.448	−0.476
	(0.143)	(0.122)	(0.137)	(0.101)	(0.158)	(0.162)	(0.096)	(0.107)	(0.103)	(0.119)
$Sp\text{R}^2$	0.108	0.112	0.121	0.102	0.074	0.112	0.156	0.173	0.151	0.048

Panel B: The ECT portfolio-specific component

	Value		Leverage		Investment		Profitability		Size	
	(dec 1)	(dec 10)	(dec 1)	(dec 10)	(dec 1)	(dec 10)	(dec 1)	(dec 10)	(dec 1)	(dec 10)
ECT^{SPEC}(-1)	−0.917	−0.230	−0.615	−0.188	−0.975	−0.711	0.155	0.203	0.330	−0.633
	(0.277)	(0.185)	(0.351)	(0.133)	(0.271)	(0.244)	(0.222)	(0.282)	(0.402)	(0.184)
$Sp\text{R}^2$	0.053	0.008	0.015	0.014	0.103	0.058	0.003	0.004	0.004	0.044

other hand, the variation in expected returns captured by the portfolio-specific component is generally weak. Only for firms that are small (size portfolios, decile 10), that have high growth (book-to-market, decile 1) and high investment (investment, decile 1) we do find that the asset-specific component of the ECT is statistically strong. However, even in these cases, the common component yields R^2s that are on par or stronger than those implied by the asset specific component.

The evidence in Panel A of Table 4 suggests that the common variation in ECT may not be entirely diversified away, and thus it may help predicting the aggregate market. To address this question, we run the following regression:

$$MKT_{t+1} = \gamma_0 + \gamma_1 \mathbf{PC\_ECT}_t + \varepsilon_{t+1}, \tag{13}$$

where MKT_{t+1} is the log excess return on the market and $\mathbf{PC\_ECT}$ is a vector that contains the first three principal components extracted from the ECTs. Inspired by the work of Engleberg et al. (2019), we claim that $\mathbf{PC\_ECT}$ form a systematic return predictor if γ_1 is statistically different from zero.

Table 5 reports the estimate of $\gamma_{1,i}$ and the R^2 for regression (13). Overall, we find strong evidence for (the systematic component of) the ECTs to forecast the market. This result contrasts with the analysis of Engleberg et al. (2019) who find that cross-sectional predictors are not good time-series predictors. Comparing column (1) to (2), we see that all the information content

in 2007 the first PC still captures 75% of the ECTs variability.

[22] Specifically, the semi-partial R^2 for the common component range from 1.7% to 21.6% with an average of 9.9% across portfolios, whereas the semi-partial R^2 for the asset-specific component range from 0% to 14.1% with an average of 2.7%.

Table 5: The Aggregate Market and Portfolio ECTs

This table reports the ordinary least squares estimate for γ_1 and the adjusted R^2 from the regression: $MKT_{t+1} = \gamma_0 + \gamma_1 \mathbf{x}_t + \epsilon_{t+1}$, where MKT_{t+1} is the log market return in excess of the risk-free rate at time $t+1$ and $\mathbf{x}$ is a vector containing potential market predictors at time t. In Column (1), we employ as market predictor the first principal component extracted from the 90 decile 1 and decile 10 anomaly portfolio returns' ECTs constructed in Giglio et al. (2020) and Haddad et al. (2020) and reported in Appendix A.1. In Column (2), we employ as market predictors the first three principal components extracted from the 90 decile 1 and decile 10 anomaly portfolio returns' ECTs. In Column (3), we employ as market predictor the variable cyclical comsumption (cc) constructed in Atanasov et al. (2019). In Column (4), we employ as market predictor the first principal component extracted from the 90 decile 1 and decile 10 anomaly portfolio returns' ECTs plus cc. In Column (5), we employ as market predictor the first three principal components extracted from the 90 decile 1 and decile 10 anomaly portfolio returns' ECTs plus cc. Values in parenthesis are standard errors computed as in Hodrick (1992). Monthly observations of annual returns. The sample period is August 1963 to December 2019.

	MKT				
	(1)	(2)	(3)	(4)	(5)
PC1_ECT(-1)	−0.457	−0.451		−0.373	−0.365
	(0.135)	(0.135)		(0.133)	(0.134)
PC2_ECT(-1)		0.100			−0.332
		(0.489)			(0.530)
PC3_ECT(-1)		−0.657			−0.450
		(0.558)			(0.575)
cc(-1)			−1.736	−1.313	−1.429
			(0.548)	(0.537)	(0.594)
Adjusted R^2	0.165	0.187	0.135	0.236	0.255

is conveyed by the first principal component.

In our market return predictive regressions, the sign of the loadings on the common ECT component continues to be negative. This implies that when portfolios prices are simultaneously below their long-run value determined by the risk drivers, i.e., when the common ECT component is negative, we expected higher aggregate market returns going forward. Interestingly, the common component of the ECT is positively correlated at 26.4% with industrial production growth; the countercyclical behavior is also apparent in Figure 5 with the common component decreasing sharply in correspondence of NBER recessions.

Given this evidence, it is then natural to control for fluctuations in aggregate consumption that, in agreement with economic theory, are well known predictors of aggregate stock market (see Lettau and Ludvigson, 2001; Atanasov et al., 2019). Indeed, if investors in the economy exhibit external habit formation as in Campbell and Cochrane (1999) then, in bad times (consumption below its trend and high marginal utility), expected returns need to be high to induce investors to postpone the valuable present consumption. Furthermore, Atanasov et al. (2019) show that cyclical (i.e., business cycle) fluctuations in consumption, referred to as cc_t, has information content above and beyond that of many alternative economic variables that are popular in the literature, such as the consumption–wealth ratio of Lettau and Ludvigson (2001), and the labor-income-to-consumption ratio of Santos and Veronesi (2006). Thus, next we run multiple regressions controlling for cc_t.[23]

[23]Furthermore, at our annual horizon, few predictors are known to perform well. Indeed,

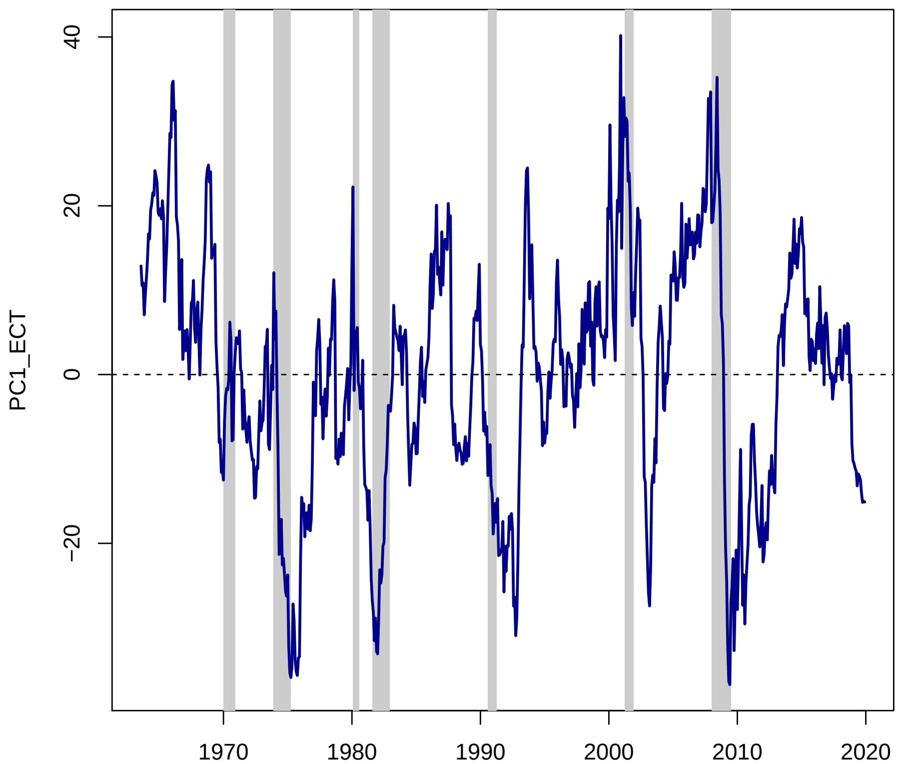


Figure 5: Countercyclicality of PC1_ECT. This figure shows the dynamics of the first principal component extracted from the 90 decile 1 and decile 10 anomaly portfolio returns' *ECT*s constructed in Giglio et al. (2020) and Haddad et al. (2020) and reported in Appendix A.1. Shaded areas are NBER recessions. The sample period is August 1963 to December 2019.

First, in column (3), we confirm the finding of Atanasov et al. (2019) in our extended sample: the estimated coefficient on cc_t is negative and strongly statistically significant, and the associated R^2 is 14%, just below the R^2 in column (1) when we employ our common ECT component. Column (4) shows the results from a multiple regression where we control for cc_t. Importantly, we observe that our common ECT component continues to have a statistically and economically sizable predictive effect on future excess stock market returns. Together, ECT^{COM} and cc explain about 20% of the overall market variability.

Taking stock of the evidence in Tables 4 and 5, we conclude that the predictability of portfolio returns induced by the ECT is driven by the common ECT component and, to a lesser extent by the idiosyncratic component. Moreover, the common variation in ECTs aggregates to generate market return predictability, suggesting that there is portfolio-specific information in the ECT that cannot be diversified away.

3.4.3 Out-of-Sample Analysis

The main objective of this subsection is to examine the validity and stability of the in-sample evidence that the ECT predicts future returns. In the interest of space, we focus on the predictability for the aggregate equity market.

the variance risk premium and its variants (see Bollerslev et al., 2009; Martin, 2017) are know to predict the market at monthly, hence shorter, horizon. On the other hand, the dividend-price ratio captures longer term market fluctuations, often in the range of 3 to 7 years ahead. In line with this argument, Table X in Atanasov et al. (2019) register that at the yearly horizon 17 out of 19 traditional predictive variables have negative out-of-sample R^2, and only the investment-to-capital ratio (i/k) proposed by Cochrane (1991) is statistically significant. Table B.1 reports the results for the predictive regression (13) controlling for i/k. Given that i/k is subsumed by PC1_ECT at the one-year horizon, in the remaining analysis we control only for cc.

We proceed as follows. We follow Lettau and Ludvigson (2001) and consider a scenario in which the parameters in the ECT (i.e., the long-run cointegrating relationship) are fixed at their values estimated over the full sample. This technique might be advantageous because it does not induce sampling error in the estimation of parameters in ECT. Next, we employ the ECT values in recursive predictive regressions for stock returns to form out-of-sample forecasts. We use an expanding estimation window where the coefficients in the return forecasting regression are estimated recursively using only the information available through time t in forecasting over the next $h = 12$ months. To ensure that our results are not sensitive to the choice of evaluation period, we perform out-of-sample tests for three different out-of-sample forecasting periods: 1980:1 to 2019:12, 1990:1 to 2019:12, and 2000:1 to 2019:12.

We compare the forecasting error from a series of out-of-sample return forecasts obtained from a prediction equation that includes a constant and PC1 from the cross-section of the ECTs (the unrestricted model) to that from a prediction equation that includes a constant as the sole forecasting variable (the restricted model).

Table 6 displays our results. The out-of-sample R^2 statistics are all positive and significant. Importantly, the statistically significant out-of-sample predictive power for aggregate stock market returns holds regardless of whether the out-of-sample forecasting starts in 1980, 1990, or 2000. Furthermore, the MSE-F test of McCracken (2007) rejects the null hypothesis that the mean squared errors from the unrestricted model are greater than or equal to those from the historical average return. These results stand out since Goyal and Welch

(2007) conclude instead that a long list of popular business cycle predictor variables has been unsuccessful out-of-sample over the last few decades.

Table 6: Out-of-Sample Tests

This table reports results of out-of-sample forecasts of 1-year-ahead log market excess returns. A time-varying excpected returns model with PC1_ECT as predictor is compared against a constant expected returns model. The parameters used to calculate PC1_ECT are estimated over the full sample. R^2_{OOS} is the out-of-sample R^2 in percent. MSE-F is the F-statistic of McCracken (2007). DM p-values are the one-sided p-values for testing the null hypothesis of equal forecast accuracy against the alternative that the time-varying excpected returns model is more accurate, using the method of Diebold and Mariano (2002). p-values for R^2_{OOS} are computed as in Clark and West (2007). The first observation in the out-of-sample period is January 1980 in Column (1), January 1990 in Column (2), and January 2000 in Column (3), and the predictive model is estimated recursively until December 2019. Monthly observations of annual returns. The sample period is August 1963 to December 2019.

	From 1980 (1)	From 1990 (2)	From 2000 (3)
R^2_{OOS}	20.95	26.59	31.54
p-value	0.01	0.01	0.01
MSE-F	130.12	134.36	115.64
DM p-value	0.02	0.02	0.03

4 Conclusions

This paper has proposed a novel co-integrated approach to model factors and asset returns *and* their prices. We find that focusing on both prices and returns, rather than just returns, naturally leads to an "Equilibrium Correction Term" that conveys new relevant information about the time-series dynamics of assets. Using a large cross-section of anomalies, we find that the ECT significantly forecasts portfolio returns. We also document the presence across portfolios of a common ECT component. This component turns out to be the main driver of the predictability at the portfolio level. Furthermore, the common component is countercyclical, and it displays predictive information content about future expected market returns above and beyond state-of-the-art economic predictive variables. Overall, accounting for dis-equilibria between portfolio prices and (price-level) risk drivers has important consequences for our understanding of the predictive distribution of returns.

We also argue that the existence of a long-run relationship between prices and associated drivers of risk can be used to validate the ability of any factor models to explain short- and long-run asset performances. Furthermore, our model is flexible about the choice of factors and test assets, and it would be natural to apply our framework to other asset classes like bonds and foreign currencies.

Finally, in this paper we have taken a local perspective by focusing on U.S. risk drivers. Future research could explore models based on the simultaneous utilization of local and global factors to model asset returns (e.g., Griffin, 2002). Cointegration among local and global risk drivers has an obvious potential for explaining the dynamics of local factors as determined by the response to an Equilibrium Correction Term in which global risk drivers determine the local ones.

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Online Appendix

A Anomaly Portfolios

Table A.1: Categories

We follow Freyberger et al. (2020) and Lettau and Pelger (2020) to group anomaly portfolios constructed in Giglio et al. (2020) and Haddad et al. (2020). This table lists the categories and the portfolios that we include in each category. In total, we consider 8 categories and 45 anomaly portfolios. Anomalies are defined in Kozak and Santosh (2019), Giglio et al. (2020), Haddad et al. (2020), and Kozak et al. (2020).

Category	Anomaly Portfolios
reversal	indmomrev, indrrev, indrrevlv, lrrev, strev
value interaction	valmom, valmomprof, valprof
momentum	indmom, mom, mom12, momrev
value	cfp, divp, dur, ep, lev, sgrowth, sp, value, valuem
investment	ciss, inv, invcap, igrowth, growth, nissa, nissm, noa
profitability	gmargins, prof, roaa, roea
trading frictions	aturnover, betaarb, ivol, price, shvol, size
others	accruals, age, divg, exchsw, fscore, season

Notes: lrrev is long-term reversal. strev is short-term reversal. indmomrev is industry momentum-reversal. indrrev is industry relative reversal. indrrevlv is industry relative reversal (low volatility). valmom is value-momentum. valmomprof is value-momentum-profitability. valprof is value-profitability. mom is 6-months momentum. mom12 is 12-months momentum. indmom is long-term reversal. momrev is momentum-reversal. value is annual value. valuem is monthly value. divp is dividend yield. ep is earnings/price. cfp is cash-flow/market value of equity. sp is sales-to-price. lev is leverage. sgrowth is sales growth. dur is cash-flow duration. inv is investment. invcap is investment-to-capital. igrowth is investment growth. growth is asset growth. noa is net operating asset. ciss is composite issuance. nissa is annual share issuance. nissm is monthly share issuance. prof is gross profitability. roaa is annual return on assets. roea is annual return on equity. gmargins is gross margins. ivol is idiosyncratic volatility. shvol is share volume. aturnover is asset turnover. size is size. price is size. betarb is size. accruals is accruals. age is firm age. divg is dividend growth. fscore is Piotroski's F-score. season is seasonality.

B Alternative Market Predictors

Table B.1: The Aggregate Market and Portfolio ECTs

This table reports the ordinary least squares estimate for γ_1 and the adjusted R^2 from the regression: $MKT_{t+1} = \gamma_0 + \gamma_1 \mathbf{x}_t + \epsilon_{t+1}$, where MKT_{t+1} is the log market return in excess of the risk-free rate at time $t+1$ and $\mathbf{x}$ is a vector containing potential market predictors at time t. In Column (1), we employ as market predictor investment-to-capital ratio (i/k) constructed in Campbell (1991). In Column (2), we employ as market predictor the first principal component extracted from the 90 decile 1 and decile 10 anomaly portfolio returns' ECTs plus i/k. In Column (3), we employ as market predictor the first three principal components extracted from the 90 decile 1 and decile 10 anomaly portfolio returns' ECTs plus i/k. Values in parenthesis are standard errors computed as in Hodrick (1992). Monthly observations of annual returns. The sample period is August 1963 to December 2019.

		MKT	
	(1)	(2)	(3)
PC1_ECT(-1)		−0.396	−0.367
		(0.157)	(0.153)
PC2_ECT(-1)			0.151
			(0.492)
PC3_ECT(-1)			−0.771
			(0.545)
i/k(-1)	−14.306	−6.219	−8.306
	(6.716)	(7.756)	(7.636)
Adjusted R^2	0.078	0.175	0.206