Diversifying Macroeconomic Risk Factors
— for Better or for Worse∗

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Abstract

It is widely acknowledged that asset returns are driven by common sources of risk, especially in difficult market times when the benefits from traditional portfolio diversification fail to realize. From a top down perspective, investors would be mostly concerned about shocks such as growth or inflation that ultimately govern the pricing of broad asset classes. To this extent, we propose a natural asset allocation framework to achieve a diversified exposure to orthogonal macroeconomic factors, maximizing portfolio diversification through factor risk parity strategies. We construct macro factors from several macroeconomic and financial variables and show that they significantly contribute to explain variation in asset returns. In a single-state setting, we find that a balanced allocation to macro factors reduces portfolio risk compared to an equal allocation of risk across assets while not necessarily forgoing upside potential. Furthermore, we estimate a two-state Markov Switching Regression model capturing conditional macroeconomic sensitivities. Unlike traditional risk-based strategies, factor risk parity portfolios are reactive to changes in the macro environment through state-dependent portfolio allocations. We find that they significantly outperform in the bad state and that, conditional on the inferred regime, they provide superior risk-adjusted performance.

Keywords: Macro Factors, Factor Investing, Diversification, Markov Switching Regression

JEL Classification: C34; E44; G11; G12
Introduction

Fundamental analysis recognizes asset prices to be the expected sum of future discounted cash flows, which in turn are influenced by state variables through their effect on expectations of discount rates or future cash flows. Chen, Roll and Ross (1986) posit “A rather embarrassing gap exists between the theoretically exclusive importance of systematic "state variables" and our complete ignorance of their identity. The co-movements of asset prices suggest the presence of underlying exogenous influences, but we have not yet determined which economic variables, if any, are responsible.” Indeed, macroeconomic variables, such as economic growth, inflation or interest rates, are natural candidates as state variables which define different economic regimes and can explain the time series of assets’ characteristics. For instance, Fama and French (1989) show that stocks and bonds move together and that time variation in excess returns can be predicted by common factors which are correlated to the business cycle, such as dividend yield, default and term premia, or by business cycle indicators such as output gap (Cooper and Priestley, 2009).

In this paper we study the usefulness of macro factors in portfolio allocation from a factor-based perspective by directly allocating to macro factors. Factor investing is explored as a risk management tool to achieve a diversified macroeconomic factor allocation, where the macro factors are the underlying sources of risks across assets. In a static framework, Chen et al. (1986) first adopted a multi-factor model based on macroeconomic variables to explain the cross-section of returns. Based on the Arbitrage Pricing Theory, asset returns are assumed to be driven by a common risk factor structure, where the factors represent common drivers of risk across assets. These factors can be useful in portfolio management to explain portfolio returns through a style analysis (Sharpe, 1988) or to achieve a diversified allocation to risk factors. Indeed, it is widely acknowledged that during difficult market times asset classes exhibit higher correlation, suggesting that risk and return are driven by only a few meaningful underlying forces (Ang, Goetzmann and Schaefer, 2009). Accordingly, in such environments, traditional risk-based strategies based on asset classes fail to provide effective diversification precisely when it is needed the most, and they fail to protect investors from changes in driving factors such as macroeconomic growth or risk aversion. To this extent, a factor-based portfolio approach that focuses on the primary forces of asset returns is likely to provide higher diversification benefits.

Rooted in academic theory, factor investing builds on the analysis of the assets through the lens of the underlying factors, where the latter are rewarded factors that earn a return premium over the long run, and investment decisions are directly made in terms of factors. To this date, the literature on macroeconomic factor investing is fairly narrow. As illustrated in Martellini and Milhau (2018), several drawbacks of macro
factors hamper the implementation of macro factor-based strategies. One main shortcoming of macro factors is that they have low statistical power in the regression of asset returns (Grinold and Kahn, 2000), suggesting they would have low power in explaining variation in portfolio returns. Connor (1995) compares three types of factor models of security returns and shows that fundamental and statistical factor models significantly outperform macroeconomic factor models in terms of explanatory power. The second limitation is that, unlike style factors, macro factors are not directly investable and, therefore, achieving direct exposure to macroeconomic factors would typically require constructing factor-mimicking portfolios. To this end, one needs to accurately measure the exposures of the assets to the macroeconomic factors, which, however, are subject to sampling error (Lehmann and Modest, 1988). The procedure is complicated as there is no unique and clear way to measure the macro factors, and even relying on official figures such as GDP growth suffers from delays in the release date of the figures, low frequency availability of the data (e.g. quarterly or annually) or subsequent revisions in the figures. Additionally, instability of the exposures of the assets to the macro factors might render the implementation of such factor-based strategies impractical, potentially resulting in high turnover.

In the context of factor-based investing, we study factor risk parity strategies which aim at allocating equal risk across orthogonal macro factors by imposing diversification constraints on the portfolio. These portfolios represent a natural framework to translate the diversification needs of investors into practical factor and ultimately asset allocation decisions. Our study draws from and contributes to the literature in four areas: 1) the economic relation between asset returns and macroeconomic factors, 2) the measurement of macroeconomic factors, 3) risk-based allocation and factor investing and 4) regime switching models. The main innovation is to bring together the developments and previous research in these areas in a unique, integrated study. Ultimately, we explore potential benefits from a diversified macro factor allocation compared to an equal allocation of risk across assets, whereas the traditional focus of factor investing has mostly been on allocating to investable style factors. On top of that, the analysis is extended to a conditional framework using a two-state regime switching model. The advantage of this approach is to simultaneously study the time-varying relation between asset returns and macroeconomic factors and to assess the estimated value added of macro-based factor investing in a two-regime Markov Switching model, which, to our knowledge, has not been studied before.

The advantages of relying on a factor-based view go beyond the economic rationale of focusing on the underlying sources of risk. Provided that the correlation among the risk factors is lower than among asset classes, the factor-based approach is expected to provide greater diversification benefits when forming portfolios. Additionally, the adop-
tion of a factor structure can reduce extreme allocations derived from mean-variance optimization, which suffers from instability of the covariance matrix estimator of asset returns. Indeed, the use of a risk factor model allows to shrink the covariance matrix of asset returns towards a common factor structure and to potentially reduce high dimensionality of securities and estimation error in portfolio optimization. Cocoma, Czasonis, Kritzman and Turkington (2017) question such beliefs, arguing that low correlations among factor mimicking portfolios simply arise from short positions in the underlying assets. Moreover, their findings fail to show overall superior diversification benefits of factors in terms of estimation error and noise from redundant dimensionality reduction. However, Bass, Gladstone and Ang (2017) argue that the conditions under which the asset approach is equivalent to the factor approach require that, first, the factors must be created from the asset themselves, and second, no constraints are imposed in the mapping between factors and assets; yet, they argue that these conditions are hardly observed in practice. Similarly, these conditions of equivalence do not apply in the present paper, strengthening the case for potential benefits from the factor approach: on one hand, we impose constraints on the mapping from factors to assets through an orthogonal transformation of the macro factors implicit in the factor risk parity portfolios; on the other hand, we derive the macro factors from a set of macroeconomic and financial variables. This contrasts previous literature on macro factor investing which measured macro factors directly from the investable asset returns.

The main challenge arises from appropriately measuring the macroeconomic factors to explain the cross-section of returns. In the framework of a multi-asset and global portfolio, we assume that asset returns across international markets are driven by common macroeconomic risk factors, in the spirit of international asset pricing models and, most closely, following Ferson and Harvey (1993, 1994). This relies on the assumption of international financial markets integration, from which we use a combination of global and US variables. In particular, we construct the macroeconomic factors by optimally summarizing information content from a large set of macroeconomic and financial variables. As Ludvigson and Ng (2009) point out, individual macroeconomic variables are likely to provide an inaccurate and incomplete representation of the specific macroeconomic concepts. Moreover, as argued in Huang and Shi (2011), an independent measurement of macroeconomic factors may not necessarily be the best estimate to capture the link between the macroeconomic and financial world as certain macroeconomic measures that accurately describe a macroeconomic concept may not be highly correlated with the return variables to be predicted. For this reason, the design and construction of the macro factors is here carried out by simultaneously analyzing the objective of measuring macroeconomic factors while providing a good statistical fit in explaining the time-series of returns. This follows a growing literature on dimensionality
reduction achieved through target-driven econometric methods (Huang, Jiang, Tong and Zhou, 2019) which directly address the ultimate objective of predicting asset returns. While such an approach does not address potential data-snooping biases (Lo and Craig MacKinlay, 1990), the candidate variables explored are largely supported by previous macro-finance literature.

After constructing the macro factors, we analyze the performance of corresponding factor risk parity portfolios in a single-state framework based on the contemporaneous relationship between assets and macroeconomic factors estimated via a linear factor model. We observe that asset returns are highly dependent on seven measures of macro factors: Inflation, Interest Rate, Growth, Credit, Emerging Markets, Volatility and FX, and, on average, these factors explain more than 50% of the variation in the individual asset returns. While some assets can be closely associated with individual macro factors, the majority of assets appears to be driven by similar and multiple forces, suggesting that a portfolio of such assets provides only naïve diversification compared to macro factors. With respect to portfolio performance, the factor risk parity portfolios maximize a diversification measure compared to a set of alternative risk-based portfolios and they achieve superior risk-adjusted return (with Sharpe Ratios ranging from 0.75 to 0.90), comparable to a global minimum-variance portfolio. In particular, a diversified allocation of risk across macro factors helps reducing portfolio risk vis-à-vis the traditional risk parity portfolio through lower volatility and marginally lower drawdown, while not sacrificing portfolio returns. Effectively, these portfolios represent a balanced trade-off between the global minimum-variance and Risk Parity portfolio and avoid an over-allocation to Volatility Risk which is present in the alternative risk-based strategies.

Finally, we estimate a two-state Markov-Switching regression model to capture structural breaks in asset characteristics and a time-varying relationship between assets and macroeconomic factors. Contrary to the benchmark risk-based strategies, conditional factor risk parity allocations are reactive to changes in the inferred regime and they outperform all strategies (except the GMV portfolio) in the bad state through a balanced risk allocation across macro factors. In particular, they reduce volatility risk exposure towards the Interest Rate factor in the crisis state, while tilting towards Inflation and EM-based assets in the good state. Conditional on the regime, the outperformance of the factor risk parity is slightly reduced given the prevalence of good times; however, the strategies still provide superior risk-adjusted return compared to the traditional risk-based strategies.

The paper is organized as follows. Section 1 reviews the literature on the relationship between macroeconomic variables and asset returns and on macro-based asset allocation. Sections 2 describes the data used and the construction of the macro factors. Section 3 illustrates the empirical results from the estimated factor model and Section 4 docu-
ments the implementation of macro factor risk parity portfolios. Section 5 introduces regime switching models, reviewing the literature on the corresponding asset allocation, describing the model and reporting the portfolio performance in the two-state framework. Section 6 explores alternative specifications to test robustness of the models and Section 7 concludes.

1 Literature

1.1 Factor Model and Asset Pricing Theory

Ross (1976) introduced the Arbitrage Pricing Theory (APT) positing that there are common factors which explain the main variation among asset returns and which cannot be diversified away, in the same way as the market risk is the only systematic factor in the Capital Asset Pricing Model (CAPM). Formally, he assumes that asset returns are generated by a multiple-factor model as described by:

\[ R_{it} = E[R_i] + \sum_{k=1}^{K} \beta_{ki} F_k + \varepsilon_{it}, \]

where \( R_i \) is the return on asset \( i \), \( E[R_i] \) is the unconditional, expected return on the asset, \( F_k \) is the change in the \( k^{th} \) common factor, assumed to have zero mean, and \( \beta_{ki} \) represents the sensitivity of the asset return to the specific factor. Ross assumes that the error terms have zero mean and are cross-sectionally uncorrelated, that is, a “strict factor model” as defined by Chamberlain and Rothschild (1983). In particular, he shows that, by a non-arbitrage argument, the absence of the risk-less profits implies that

\[ E[R_i] = \sum_{k=1}^{K} \beta_{ki} \Lambda_k, \]  

(1)

where \( \Lambda_k \) are the risk premia of the \( K \) common factors. Therefore, the Arbitrage Pricing Theory (APT) bridges the relationship between pricing factors, which can explain the cross of returns, and risk factors, which represent common sources of risk across assets, showing that, in equilibrium, if factors have prices, the expected return on an asset is a compensation for multiple systematic sources of risk. The implications of the APT can be further extended to a dynamic setting, supporting the empirical evidence of time-varying risk premia. This follows from Merton’s Intertemporal CAPM (1973), where the conditional expected returns on assets depend on their exposure to economic state variables which describe changes in the investment opportunity set.

Modern asset pricing theory relies on the notion of “stochastic discount factor” (SDF), which represents an index of “bad times” (Ang, 2014) and which prices all assets by discounting their expected payoffs. It can be shown that the expected risk
premium of an asset can be written as

$$E [r_i] - r_f = \frac{\text{cov} (r_i, m)}{\text{var} (m)} \left( \frac{\text{var} (m)}{E [m]} \right)$$

$$= \beta_{i,m} \times \lambda_m,$$

with $\beta_{i,m} = \frac{\text{cov} (r_i, m)}{\text{var} (m)}$ being the beta of the asset relative to the stochastic discount factor $m$, and $\lambda_m = -\frac{\text{var} (m)}{E [m]}$ being the price of risk (Cochrane, 2009). In Consumption-based asset pricing models (Lucas, 1978; Breeden, 1979), the stochastic discount factor is proportional to the representative investor’s marginal utility of consumption. Assets which have a high payoff during bad states—represented by periods of low consumption—and therefore which have high covariance $\text{cov} (r_i, m)$ with the stochastic discount factor, are more desirable and will require a lower risk premium. Conversely, assets with low payoff in bad times will sell at a ”discount” relative to the price of risk and will require a higher return. In practice, the stochastic discount factor, $m$, cannot be observed but is approximated by a combination of factors, leading to the APT multi-factor model described by Eq. 1. While continuous research in asset pricing theory investigates how to effectively measure the SDF, macroeconomic variables are intuitive candidates to measure aggregate marginal utility of consumption: they can be interpreted as state variables defining different sets of bad times, such as low growth or high inflation, and, based on the modern asset pricing theory, exposure to these factors is compensated by the assets’ risk premia. By this argument, it follows that variables which are able to describe future macroeconomic activity are important risk factors to investors.

From the factor model, the covariance of the asset returns can be decomposed into systematic sources of risk from the specific factor exposures and idiosyncratic risk, which—by the “law of large numbers”—can be eliminated in a sufficiently well-diversified portfolio. In particular, the covariance between asset returns is driven by the assets’ common exposure to the same systematic risk factors. We can obtain a measure of correlation between asset returns, $\rho_{i,j}$, as implied by the factor model by dividing the common factor covariance by the product of the asset returns volatilities,

$$\rho_{i,j} = \frac{\beta_i \beta_j \text{var} (F_i)}{\sqrt{\beta_i^2 \Sigma \beta_i + \sigma_{i, \varepsilon} \sqrt{\beta_j^2 \Sigma \beta_j + \sigma_{j, \varepsilon}}}}$$

(3)

Therefore, factors with larger variance contribute to higher comovement between asset returns and the sign of the correlation between assets is determined by the sign of the exposures of the individual assets to the common factors. Time-varying correlation between assets arises from either time-varying exposures to the factors or from the heteroskedastic covariance of the factors.
1.2 Common Determinants of Asset Returns

A long history of studies analyzed the determinants of stock and bond returns as well as their comovement. Campbell and Ammer (1993) attribute the variation of stocks to changes in expected real cash flows, expected real interest rates and excess stock returns and find that the latter accounts for the majority of the explained variance of unexpected stock returns. As stocks represent claims to stochastic real cash flows, dividend growth and expected output gap are commonly used as measures of the cash flow channel, whereas real interest rate is a risk factor under the discount rate channel. For instance, based on the Intertemporal Asset Pricing Theory, Merton (1973) and Cox, Ingersoll and Ross (1985) use interest rate as a state variable to describe the time-varying investment opportunity set.

Bond returns, on the other hand, provide claims to fixed nominal cash flows; therefore, they lack a cash flow risk channel and inflation is a state variable negatively influencing bond returns. Their variation is decomposed into expectations of future inflation rates, future real rates and future excess returns on long-term bonds, which can all be approximated by the nominal yield spreads. Indeed, Estrella and Hardouvelis (1991) and Stock and Watson (1989) find that yield spreads predict long-term inflation and interest rate changes as well as excess returns on long-term bonds. Campbell and Ammer (1993) find that the level of interest rates contributes to most of the variation in short-term bonds. On the other hand, variation in the long-term bonds is primarily driven by long-term interest rates, which are decomposed into real short-term interest rate, the term premium, expected inflation and an inflation risk premium.

While both stocks and bonds have negative exposure to news about real interest rates, the low volatility of real interest rates cannot account for sufficient asset comovement. On the other hand, Campbell and Ammer (1993) explain the negative stock-bond return covariance through the positive covariance between stock returns and inflation news. Indeed, while stocks are not directly exposed to inflation risks through the real cash flow channel, the Mundell-Tobin model asserts that high expected inflation increases the opportunity cost of money, driving demand for real assets up and lowering the real interest rate, which in turns suggests a positive correlation between stock returns and expected inflation. In practice, however, stocks have been found to be negatively correlated with expected inflation (Fama and Schwert, 1977).

Barsky (1989) first noted the importance of time-varying risk premia to explain common variation between stocks and bonds. Bekaert, Engstrom and Grenadier (2010) model a counter-cyclical risk aversion generated from the habit model of Campbell and Cochrane (1999). Stochastic risk aversion influences the covariance between stocks and bonds under the two opposite channels of consumption smoothing and precautionary
saving with ambiguous overall effect. Under the consumption smoothing effect, higher risk aversion requires higher short-term real interest rates. In this state, bonds and equities are risky and both require a positive risk premium, contributing to positive correlation. Conversely, under the precautionary saving effect, higher risk aversion drives up demand, and consequently prices, for bonds in a “flight to safety” as bonds are perceived as hedge assets. In this case, unlike equities, bonds have a negative risk premium, which contributes to the negative correlation.

Another stream of literature analyzes the role of economic uncertainty as risk premium channel, where uncertainty is represented by the conditional volatility of fundamentals. Bansal and Yaron (2004) introduced a time-varying volatility of consumption growth as state variable in the long-run risks model for equity returns, while Bansal and Shaliastovich (2013) rely on uncertainty on inflation and growth in a long-run risk model for bonds and currencies. Bekaert, Engstrom and Xing (2009) compare the relative importance of risk and uncertainty in an asset pricing model for equities and bonds and find that risk aversion is a main determinant of variation in the equity risk premium, while uncertainty is more important for the term structure.

Chen et al. (1986) were the first to estimate a multifactor model of US stock returns on the basis of macroeconomic factors as proxies for systematic forces of returns. They consider monthly growth rate in industrial production, changes in expected inflation, unanticipated inflation, a default premium variable defined as the difference between the returns on BAA-rated corporate bonds and long-term government bonds, and a term premium, measured as the difference between the return on long-term and short-term government bonds. Unanticipated inflation is a relevant risk factor if inflation covaries with the aggregate marginal utility, and it bears a risk premium if assets have different exposures to unanticipated changes in inflation.

Ferson and Harvey (1991) study common risk factors across bond and stock returns and consider monthly real per capital growth of personal consumption expenditures for nondurable goods, a default premium variable, the change in the slope of the yield curve, unexpected inflation and the one-month real interest rate.

Fama and French (1993) show that common variation among stock returns is driven by the market portfolio and factor-mimicking portfolios related to size and book-to-market characteristics, whereas common variation between government and corporate bonds is explained by the term and default premia. Furthermore, stocks and bonds variation is jointly driven by the two common term structure factors.

Baele, Bekaert, Inghelbrecht (2010) estimate a dynamic factor model to explain the comovement between stock and bond returns using macroeconomic variables as well as risk premium variables, including risk aversion and uncertainty about inflation and output from survey data and they find that risk and liquidity factors significantly out-
perform fundamental economic variables in fitting the time-varying stock-bond comovement.

More generally, the framework in this study is mainly consistent with previous literature on International Asset Pricing Models which investigate common sources of risks across possibly integrated international markets. Ferson and Harvey (1994) study an unconditional asset pricing model across eighteen global equity markets based on global risk factors. They include the global equity market as measure of the CAPM’s market risk, although they find that it does not significantly contribute in explaining cross-sectional differences in returns. Additionally, they use a trade-weighted US dollar index against G-10 currencies, a measure of unexpected global inflation and change in expected inflation, changes in the TED spread and global short-term real rates, the change in monthly US oil price and global industrial production growth. Finally, in the context of international markets correlations, Christoffersen, Errunza, Jacobs and Langlois (2012) show that dependence among developed and emerging markets has been increasing over time and that the average dependence with developed markets is higher than with emerging markets. However, their approach is different as they rely on a dynamic asymmetric copula model, based on evidence of time-varying, non-linear tail dependence.

1.3 Asset allocation and Macroeconomic Factors

Bass et al. (2017) analyze institutional portfolios through the lens of macroeconomic factors. They use principal component analysis on 13 global asset returns and find that around 95% of the variance can be explained by six common factors, which they identify as economic growth, real rates, inflation, credit, emerging markets and commodities. On top of that they consider a foreign exchange factor to account for the remaining risk in the multi-asset portfolios. They build factor-completion overlay portfolios which enhance the current institutional portfolios to achieve more balanced allocations in terms of the macroeconomic factors. To implement the strategy, they use a robust quadratic optimization procedure which seeks to minimize the factor exposure deviations and the tracking error between the asset allocations and a pre-determined factor allocation. The mapping procedure from factors to assets used in their study is described in more detail in Greenberg, Babu and Ang (2016). To estimate the loadings matrix they use a constrained stepwise regression of the asset returns on the macro factors, whereby each asset return is regressed on a subset of macro factors selected based on economic priors.

Blyth et al. (2016) also discuss asset allocation strategies based on macro factors and introduce an alternative asset allocation procedure to solve the indeterminateness of the mapping procedure. They also build a sparse loadings matrix based on economic priors, complemented with views from portfolio managers. Nevertheless, both Greenberg
et al. (2016) and Blyth et al. (2016) measure the macroeconomic factors in terms of asset classes, therefore failing to provide a direct mapping between assets and genuine macroeconomic variables.

2 Constructing Macro Factors

2.1 Macroeconomic Data

To measure the macroeconomic factors, we collect 26 monthly series of macroeconomic and financial variables falling into seven macro groups: Inflation, Interest Rate, Growth, Credit, Emerging Markets, Volatility and FX. All data are initially transformed to ensure stationarity. Hence, all variables are ultimately measured as changes in the original raw data to capture innovations in macroeconomic variables which are expected to explain variation in asset returns. The macroeconomic data is available for the period 1999-02-01 to 2019-06-01, on which the entire study is therefore carried out. The variables are reported at the beginning of month and, unless differently specified, they are all sourced from the FRED Macroeconomic Database (McCracken and Ng, 2016).

The Inflation variables are the monthly percent change in CPI index, the WTI oil index, and an average level of inflation expectations. The latter is obtained as the first principal component of 30 time series of Inflation Expectations over the next 1 to 30 years and collected from the Federal Reserve Bank of Cleveland. Similar to the three yield curve factors analyzed in Litterman and Scheinkman (1991), the first principal component explains 80% of the variance of the series and can be interpreted as a “level” inflation expectation, while the second principal component explains most of the remaining 20% and can be interpreted as the “slope” factor of the term structure of inflation expectations.

For the (nominal) Interest Rate variables, we use the 10-year US Treasury Bond rate, 3-month US-Treasury Bill rate and the slope of the US yield curve, measured by the difference in 10-year and 3-month rates, as commonly used in literature. We use nominal data since nonreported tests show that real interest rates only marginally contribute to risk in the asset returns, consistent with the findings of Campbell and Ammer (1993). Nevertheless, the factor risk parity strategies will ultimately allocate to orthogonalized macro factors, therefore allowing to separate the real component in the Interest Rate factor from the Inflation factor.

Growth variables include real activity variables and represent forward-looking macroeconomic measures of future economic environment. The US economic variables are collected from Bloomberg and include US PMI, Chicago Fed National Activity Index (CFNAI), Non-Manufacturing Index, Conference Board Leading Economic Index and University of Michigan Consumer Sentiment Index. OECD growth variables are col-
lected from OECD Data and are: monthly industrial production, Consumer Confidence Index (CCI), Business Confidence Index (BCI) and Composite Leading Indicator (CLI).

The variables representing the US Credit factors are the yield spread between Moody’s BAA-rated corporate bonds and the 10-year US Treasury Bond, the Option Adjusted Spread of ICE High Yield US Corporate Bonds and Bloomberg-Barclays US High Yield, and the spread of Moody’s BBB-rated corporate bonds. In practice, the use of investment grade and high yield spreads implies a combination of both credit risk and default risk.

The variables representing Emerging Market factors are the OAS on Emerging Market Corporate Bonds and the GeoPolitical Risk Index (GPR). The latter is created by Caldara and Iacoviello (2018) and it counts the occurrence of words related to geopolitical tensions in leading international newspapers. McGuire and Schrijvers (2003) show through a PCA on Emerging Market Bond spreads that the latter are highly correlated with global variables such as US interest rates, volatility and high-yield spreads, and hence can be considered as redundant factors. On the other hand, Christoffersen et al. (2012) posit that the long-term variation in emerging markets cannot be explained by macroeconomic or financial variables and further show that EMs provide additional diversification benefits when considering non-linear dependence and higher-order moments. Consequently, we include the Emerging Market factor as a separate risk factor.

The variables representing Volatility are the VIX and the MOVE Index, which measure the implied volatility in equities and bonds, respectively. We include also a variable measuring uncertainty, the Global Economic Policy Uncertainty Index, created by Baker, Bloom and Davis (2016) and available online.

Finally, the Foreign Exchange variable is collected from FRED and is the Trade-Weighted Dollar Index (a weighted average of the value of US dollar against a set of major currencies including Euro Area, Canada, Japan, UK, Switzerland, Australia and Sweden).

As concerns the dependent variables, the nine asset classes analyzed are: SP500 for US Equities, Bloomberg Barclays US Treasury Index, Bloomberg Barclays US Treasury Inflation-Linked Bond Index, Bloomberg Barclays US Corporate High Yield Total Return Index, JP Morgan Emerging Markets Bond Index, MSCI Emerging Markets, MSCI EAFE for International developed Equities excluding US, S&P GSCI for Commodities and Russell 2000 Index. The risk-free rate is the 1-month US-Treasury Bill rate. All data is sourced from Bloomberg and returns are calculated as monthly percent changes at the end of each month. Returns data is available for the period 1997-03-31 to 2019-05-31.
2.2 Macroeconomic Factors

In this thesis, the terms “macro factor” or “macro groups” refer to the seven macroeconomic factors. On the other hand, the term “macro variables” refers to the 26 macroeconomic time series, of which each is representative of one (and only one) macro factor. We group the macro variables according to the seven factors and analyze each group separately.

Initially, all macro variables are standardized in an out of sample procedure, therefore at each point in time subtracting the mean and standard deviation up to that time. The first 12 months are used for calibration and all estimates are measured in an expanding window. To construct each of the seven macro factors from the original variables, we only consider the macro variables that add significant value in explaining asset returns. To identify the optimal set of variables, we use the following model selection procedure. We regress each asset return on all possible combinations of macroeconomic variables belonging to each group, one group at a time. Therefore, if a macro group is composed of $M$ variables, we estimate $2^M$ models for each asset and for each macro group and we select the model with lowest BIC. The variables retained in each macro group are the union of the set of variables chosen from the selected models across assets. We study alternative selection criteria in the Robustness Section 6. Thus, the model selection procedure estimates for each asset return the following regression

$$R_{t,t} = \alpha + bX_t + \varepsilon_t,$$

where $X$ is the set of macroeconomic variables belonging to each macro group. We estimate the regressions for each asset out of sample using an initial window of 36 months. Therefore, the search for the optimal model and variables is made at each point in time based on the available information. After the standardization and the variable selection procedure, the out of sample period starts from February 1st 2003.

Having estimated the models, we retain only the macroeconomic variables that are picked by the model selection procedure in each month, as illustrated at each point in time in Figure 1. The procedure seems to select the same variables throughout the entire period with the following few exceptions. In the Inflation group, the level of expected inflation (ExpectedInflation Level) is selected up to the Global Financial Crisis and for a relatively short interval during the European Debt Crisis until 2013. In the Interest Rate Group, the short-term 3-month Treasury Bill rate (TB3MS) is selected in the three years prior to GFC and is almost always selected since 2011. In the Growth group, there is greater variability in the selection of variables over time. The OECD indicators, CLI (CLI.OECD) and industrial production growth (INDPROD.OECD), and the US indicators of PMI Manufacturing (PMI) and CFNAI are mainly discarded by the selection procedure, whereas the Leading Economic Index, BCI OECD, CCI OECD
and the University of Michigan Consumer Sentiment Survey (UMCSSENT\textsubscript{X}) are always selected; US growth in industrial production (INDPRO) and US PMI Non Manufacturers (ISM.Non.Manufacturers) are selected mainly after the GFC. It can be observed that popular composite indices used for nowcasting are found to have low explanatory power on returns compared to alternative measures. In the Emerging Markets group, the GeoPolitical Index is selected until the GFC, after which only the Corporate Bond OAS is chosen.

Although each asset has its own exposure to different macro variables, the objective is to derive single macro factors that explain the cross section of asset returns. As such, the macro factors are created as a weighted average of the selected macro variables. While a more straightforward approach could be to use a simple average, this does not take into consideration the relative importance of each variable in explaining asset returns, which would result in underweighting variables that are found to significantly contribute to the variation in one or more asset returns and vice versa. As such, the cross-sectional approach requires an appropriate weighting of the macro variables which is conditioned on the asset returns. The weights in the averaging procedure are measured as follows. For each macro group we calculate the contributions to risk of the macro variables in each month and the final macro factors are obtained by weighting each macro variable in proportion to its total percent risk contribution across assets, where risk is measured by the convex measure of standard deviation. By Euler’s Homogenous Function Theorem, the contributions to the risk of asset \(i\), denoted \(\mathcal{R}C_i\), can easily be expressed as the sum of the contributions to risks of each macro variable \(m\), that is

\[
\mathcal{R}C_i = \sum_{m=1}^{M} \left( b_m \frac{\partial \mathcal{R}C_i}{\partial b_m} + \epsilon_m \frac{\partial \mathcal{R}C_i}{\partial \epsilon_m} \right),
\]

where \(\frac{\partial \mathcal{R}C_i}{\partial b_m} = \frac{\partial \sigma_m}{\partial b_m} = \frac{(\Sigma \times b)_m}{\sigma_m} \) and \(\Sigma_X\) is the covariance matrix of the macro variables.

The coefficients \(b\) are the ones derived from the OLS regressions estimated through the out of sample model selection procedure described before. The absolute value of the percent risk contributions is taken to account for instances of negative contributions to risk. Moreover, to avoid including variables which only marginally contribute to total risk, which would only contribute to larger volatility in the combined macro factor, we discard all variables with contribution to risk below 5% and rescale the weights. Figure 2 reports the weights of the individual macroeconomic variables in each macro factor over time.

In practice, implementing the model selection procedure in each month is costly in terms of implementation time in the optimal search and because an initial window of 36 observations would be consumed to estimate the models. As such, a trade-off is made between, on one hand, a pure out-of-sample selection, and, on the other hand,
larger sample length. Consequently, we select the variables which are retained from the in-sample model selection procedure and use the final estimated weights to average the macro variables throughout the entire sample.

The Inflation factor is largely composed of the oil price variable, whereas the Credit factor is composed of the ICE US High-Yield Option Adjusted Spread (63%) and Moody’s US BBB-rated corporate bond spreads (37%). The Emerging Market and FX factors include one single variable, respectively, the OAS on EM Corporate Bonds and the trade-weighted US dollar index, while the Volatility factor is composed by the VIX (93%) and Global Economic Policy Uncertainty (7%). The Interest Rate factor is made up by the 10-year US Treasury bond yield and the slope of the yield curve, measured by the difference between the 10-year and 3-month rates, with relative weights of 7% and 13%. Interestingly, the weights match the proportion of variance explained by the first two principal components derived from a PCA of the covariance matrix of changes in key Treasury rates (2,5,7,10,20 year). This is consistent with the findings of Litterman and Scheinkman (1991), who showed that the first two principal components can be interpreted as the level and slope of the yield curve, respectively. Finally, the Growth factor is largely composed by BCI OECD (83.3%), and a combination of CCI OECD and US PMI Non Manufacturers. As expected, growth is mostly measured by international economic indicators, as they have a larger risk contribution across the set of international assets.

The final macro factors, which are a combination of standardized variables, are then winsorized to mitigate the effect of spurious outliers in the estimation of the factor model. We apply a threshold of 5 in absolute value to all factors, which we found to be binding for the Credit ad Emerging Market factors. The time-series of the constructed macro factors is illustrated in Figure 3.

3 Mapping Asset Returns on Macroeconomic Factors

Once constructed the macro factors, we estimate a factor model using OLS by regressing each return on the seven macro factors. We estimate the contemporaneous regressions out of sample, therefore in each month $t$ using only information available up to $t - 1$ and using an expanding window with initial window of 36 months. The model is described by

$$R_{i,t} = a_i + \sum_{k=1}^{7} \beta_{ik} F_{k,t} + \varepsilon_i$$

\(^1\)Sample length is extremely important for the estimation of the regime switching model since the calibration period would start from 2006 using the purely out of sample procedure and the reduced sample length would make it difficult to accurately estimate the model given the large number of parameters to estimate. Moreover, the performance of the strategies would be biased by the bullish market performance experienced for most of the reduced out-of-sample period.
where $F_k$ are the seven combined macro factors and $R_{t,i}$ are the excess returns on each of the nine assets.

The estimated parameters and results using the entire sample period are displayed in Table 1. Asset returns are negatively related to the Volatility factor and the relationship is statistically significant at 1% level for all assets except for US_Treasury and US_TIPS, which supports the attractiveness of US government bonds as safe haven in scenarios with high volatility. These variables are also positively exposed to the Credit factor and the relationship is statistically significant at 1% level.

$S&P500$, US_HighYield, MSCI_EM, MSCI_EAFE and RUSSELL2000 have positive and statistically significant exposure to the Growth factor, which confirms the positive relationship of Equities and Corporate bonds with economic growth. On the other hand, the relationship with the Credit factor is negative and statistically significant.

The Inflation factor was constructed with dominant weight given by the oil price variable. Accordingly, the OLS regression shows a positive, statistically and economically significant relationship between GSCI and the Inflation factor. A positive and significant relationship holds also for US_TIPS and EMBI, although the level of significance is lower in terms of economic magnitude and statistical significance.

The international return indices, MSCI_EM and MSCI_EAFE are negatively related to the FX factor, which measures the US dollar against a basket of currencies, and the relationship is statistically significant. As the index returns are measured in dollar terms, the result supports the presence of indirect negative effects of the dollar strength on international non-US markets. Finally, US_HighYield, EMBI, and US_TIPS are negatively related to the Emerging Market factor, while an increase in Emerging Markets spreads leads to higher returns on the S&P500 and RUSSELL2000. Similarly, this can be attributed to substitution effects across international equity markets.

Figure 4 shows the risk contributions of the macro factors over time. While the same factors seem to contribute to risk over time, the magnitude of the contributions by Volatility, Credit and Emerging Markets factors shifted upward during the GFC and remained at these higher levels thereafter. For S&P500, MSCI_EM, MSCI_EAFE and RUSSELL2000, the largest contributions to risk are driven by the Volatility Factor, followed by Credit and Growth, whereas for US_HighYield the Credit factor accounts alone for the more than 70% of risk. For GSCI, EMBI and US_Treasury, risk contribution is mostly driven, respectively, by the Inflation, Emerging Markets and Interest Rate factors.
4 Macro Factor Allocation Strategies

4.1 Implementation

Once the macroeconomic sensitivities are estimated, the next aim is to achieve a balanced and well-diversified allocation to macro factors in a multi-asset portfolio. Several risk budgeting techniques have been developed to achieve a target or balanced allocation to risk in the total portfolio. Maillard, Roncalli and Teiletche (2010) analyze equally-weighted risk contribution portfolios which equalize the contribution to risk of each component in the portfolio where the individual assets are the building blocks and based on the Risk-Parity approach coined by Qian (2005). They find that the portfolio is equivalent to a minimum-variance portfolio subject to a norm constraint on the weights and that the total portfolio volatility lies between that of a minimum-variance and an equally-weighted portfolio.

Choueifaty and Coignard (2008) introduce the Most Diversified Portfolio (MDP) which maximizes a measure of portfolio diversification, represented by the ratio of the weighted average volatility of the portfolio to the overall volatility.

A more recent risk budgeting technique, here denoted Factor Risk Parity, calls for allocating equal amount of risk to uncorrelated factors rather than to assets. Factor Risk Parity was introduced by Meucci (2009) and aims at maximizing diversification across uncorrelated risk factors. The uncorrelated sources of risks are generated using a principal component analysis of the covariance matrix of the portfolio asset returns. The diversification distribution is represented by the contributions of the principal portfolios, normalized by total portfolio variance, and expressed by $p_k = \frac{\tilde{w}_k^2 \lambda_k}{\text{var}(R_w)}$ where $\lambda_k$ are the variances of the uncorrelated principal components and $\tilde{w}_k$ are the loadings weights derived from the principal component analysis. Portfolio Diversification is then measured as the exponential of the entropy of the principal portfolios’ diversification distribution and is denoted as the Effective Number of Bets,

$$ENB = \exp \left( - \sum_{k=1}^{K} p_k \log p_k \right)$$

This measure indeed represents the actual number of uncorrelated risk exposures in the portfolio, such that, for example, for $p_k = 1$ and $p_j = 0$ with $k \neq j$, the portfolio has maximum concentration (lowest degree of diversification) and $ENB = 1$. On the other hand, for $p_k = p_j = 1/K$ for all $k, j$, all risk contributions are equal and the effective number of bets is maximized at $ENB = K$.

The optimization problem involves equalizing the relative contributions of each factor.
$p_k$ to the total portfolio variance:

$$p_k = \frac{1}{K} \leftrightarrow \hat{w}_k^2 \lambda_k = \frac{1}{K} w' \Sigma w \text{ for } k = 1, \ldots, K$$

As explained in Deguest, Martellini and Meucci (2013) the optimal factor weights can be written as

$$\hat{w}_k = \pm \frac{\gamma}{\sigma_k} = \gamma \Sigma_k^{-1/2} \begin{pmatrix} \pm 1 \\ \vdots \\ \pm 1 \end{pmatrix}$$

It can be observed, therefore, that all Factor Risk Parity allocations are inversely proportional to the factor variances, with $\Sigma_k$ being the diagonal covariance matrix of the principal components. However, the ENB optimization problem is not uniquely defined but has $2^K$ different solutions, depending on the signs of the uncorrelated risk factors. After imposing full investment on the assets, there are $2^K - 1$ solutions left, which all satisfy:

$$w_{FRP} = B^{-1} \Sigma_k^{-1/2} \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}$$

It shall be pointed out that the signs do not represent the directional exposures of the factors as the factor risk parity takes into consideration only the contributions to risk of the factors, which are given by the squared factor exposures.

Lohre, Opfer and Ország (2014) adopt this strategy to develop a Diversified Risk Parity (DRP) portfolio in their study of multi-asset allocation and compare their strategy with traditional risk-based strategies. They find superior risk-adjusted performance and a well-diversified allocation to the underlying principal portfolios contrary to alternative strategies such as equal weight and minimum-variance which result being heavily concentrated in few principal portfolios.

The use of PCA, however, suffers from several shortcomings which make the diversified risk parity strategy difficult to implement in practice. As pointed out in Meucci, Santangelo and Deguest (2015), principal components are likely to be statistically unstable and difficult to interpret from an economic perspective. Moreover, they are very sensitive to scalar multiplications, which implies that any simple scalar transformation will lead to extreme changes in the principal component bets. Finally, the principal components bets are not unique as there are $2^K$ possible combinations arising by changing the sign of any of the eigenvectors.
Alternatives to the PCA have therefore been explored to achieve factor risk parity through statistical techniques to extract factors. Lassance, DeMiguel and Vrins (2019) use Independent Component Analysis (ICA) to extract principal components which are maximally independent. They point out that principal components, while uncorrelated by construction, are not independent when returns do not follow a Gaussian distribution. Conversely, independence of the ICs allows to account also for higher-order moments and to reduce the portfolio’s kurtosis. Therefore, they create factor risk parity strategies based on ICs where risk is measured by modified VaR and show out-of-sample out-performance compared to a FRP portfolio based on PCs in terms of Sharpe Ratio, tail risk and turnover.

To overcome the issues from the PCA method, we implement the alternative FRP strategy which was proposed by Meucci et al. (2015) and relying on a Minimum-Torsion transformation, $t^2$. The minimum torsion bets, $F_{MT}$, are found as the orthogonal factors that minimize the tracking error relative to the original factors, $F_k$, therefore benefiting from their uncorrelated nature and a clearer economic interpretation.

$$t = \arg \min_{\text{corr}(tF)=I_{K} \times K} \left\{ \frac{1}{K} \sum_{k=1}^{K} \text{Var} \left( \frac{t^t F_k - F_k}{\sigma_{F_k}} \right) \right\}$$

Bernardi, Leippold and Lohre (2018) analyze diversified risk parity strategies using a set of commodity assets, where the uncorrelated risks are measured using both the principal components from PCA and the minimum torsion factors, allowing for a direct comparison of the two procedures. They find that the minimum torsion factors are relatively stable over time and can be clearly identified as specific commodity factors, while the principal portfolios are much more unstable resulting in excessive turnover in the diversified risk parity portfolio based on the statistical factors.

In this study, the underlying constituents of the torsion transformation are the correlated macro factors that feature in the factor model of returns estimated before, which differs from existing applications of the PCA or MLT method directly to the asset returns. The asset returns are represented by $R_t = B^t F_t + \varepsilon_t$, where $F$ are the seven macro factors, $\varepsilon$ is the residual term, and $B$ is the 7x9 loadings matrix. Using the minimum torsion transformation, the uncorrelated minimum torsion bets are obtained as $F_{MT} = t F$ and the diagonal covariance matrix of the torsion bets is $\Sigma_{MT} = t \Sigma_F t^t$. Then, the factor model of returns is represented as

$$R = B^t F + \varepsilon = B^t_{MT} F_{MT} + \varepsilon \text{ and } B_{MT} = t^{-1} B$$

\footnote{We thank Attilio Meucci for publicly sharing the code for the torsion transformation, https://it.mathworks.com/matlabcentral/fileexchange/43245-portfolio-diversification-based-on-optimized-uncorrelated-factors?s_tid=prof_contribblnk. For more technical details on the transformation, the reader is referred to the original paper by Meucci et al. (2015)}
The portfolio returns with asset weights $w$ can then be expressed in terms of the uncorrelated torsion bets as $R_w = w' R = w'_{MT} F_{MT}$ and the weights of the torsion bets are

$$w_{MT} = t^{-1} B w.$$  \hfill (5)

The covariance matrix of portfolio returns is decomposed as $\Sigma = B' \Sigma_F B + \Sigma_\varepsilon$, where $\Sigma_\varepsilon$ is the diagonal matrix containing the variance of the error terms, and the factors and error terms are uncorrelated by construction.

Using the minimum torsion transformation, the portfolio variance in terms of torsion bets is

$$\Sigma = B't^{-1} \Sigma_{MT} t'^{-1} B + \Sigma_\varepsilon$$

Deguest et al. (2015) then implement factor risk parity along minimum torsion bets by maximizing the effective number of bets, defined as

$$ENB = \exp \left( \sum_{k=1}^{K} \log \frac{p_{MT,k} \sigma_{MT,k}^2}{\sigma_{MT}^2} \right)$$  \hfill (6)

where $p_{MT,k}$ represents the diversification distribution, for which $p_{MT,k}$ is always positive and the sum across the uncorrelated bets is always equal to 1, and $\sigma_{MT,k}^2$ refers to the variance of the minimum torsion bets. The asset weights are then backed out by inverting the torsion transformation and the coefficient matrix estimated from the factor model, such that $w = B^{-1}_{MT} w_{MT} = B^{-1} t' w_{MT}$. With 9 assets and 7 factors, the inverse of the loadings matrix $B$ is estimated using the Moore-Penrose inverse throughout this study.

We study several specifications to implement factor risk parity. The following three strategies represent three risk parity portfolios with precise economic interpretation using the analytical solution from Deguest et al. (2013) reported above in 4.

- The **Factor Risk Parity Minimum-Variance** is derived using the closed-end formula proved in Deguest et al. (2013). In particular, the FRP-MV portfolio is the portfolio that is solution to max $ENB$ and with the lowest variance among all the FRP solutions and the weights are:

$$w^{FRP-MV} = \frac{B^{-1} t' \Sigma_{MT}^{-\frac{1}{2}} 1_M \Sigma_{MT}^{-\frac{1}{2}}}{1_K' B^{-1} t' \Sigma_{MT}^{-\frac{1}{2}} 1_M}$$

where $1_M$ contains the signs of the corresponding entries in $B_{MT}^{-1} = tB'^{-1}$ and represents the signs of the total exposures of each factor to the assets.

- The **Factor Risk Parity Inverse-Volatility** represents the portfolio solution to
max ENB and in which the torsion weights follow an inverse volatility strategy. The weights are found as in Dichtl et al. (2019).

\[ w_{MT,k}^{\text{MT}} = \frac{\sigma_{MT,k}^{-1}}{\sum_k \sigma_{MT,k}^{-1}} \]

\[ w_{\text{FRP-IV}}^{\text{FRP}} = \frac{B^{-1}t'w_{MT,k}}{1_K'B^{-1}t'w_{MT,k}1_K'} \]

- The Factor Risk Parity Maximum Sharpe Ratio is derived using the closed-end formula proved in Deguest et al. (2013) with weights given by:

\[ w_{\text{FRP-MSR}}^{\text{FRP}} = \frac{B^{-1}t'\sum_{MT,k}^{-1}1_k^{MSR}}{1_K'B^{-1}t'\sum_{MT,k}^{-1}1_k^{MSR}} \]

where \( 1_k^{MSR} = \text{sign}(\lambda_{MT}) \) contains the signs of the implied Sharpe Ratios of the minimum torsion bets. Since the volatilities of the torsion bets are positive, the sign of the Sharpe Ratios simply corresponds to the signs of the expected returns on the bets. Deguest et al. (2013) propose retrieving the signs from the excess returns of the assets as implied from the estimated coefficients of the factor model and after applying the torsion rotation. Accordingly, we define \( 1_k^{MSR} = \text{sign}(\mu_{MT}) \), where \( \mu_{MT} \) are the expected excess returns of the factors estimated using an expanding window. We present a summary of the derived implied returns on the macro factors in Appendix. However, Deguest et al. (2013) specify that the Factor Risk Parity Portfolio coincides with the maximum Sharpe Ratio portfolio only if the factors' Sharpe Ratios are equal in absolute value.

Additionally, we study three strategies which have been proposed in Deguest et al. (2013) and incorporate ENB constraints in the optimization problem in terms of the assets, in a similar manner as the norm constraints imposed in DeMiguel, Garlappi, Nogales and Uppal (2009).

- The constrained MV-FRP portfolio involves minimizing the total portfolio variance under long-only constraints, in the same way as the Global Minimum Variance portfolio. An additional constraint is imposed, requiring that the ENB be equal to the maximum ENB \((ENB_{\text{max}})\) that can be achieved under the specified investment constraints. To derive \( ENB_{\text{max}} \), therefore, we first solve the optimization problem of maximizing the ENB under full investment and shortsale constraints and obtain the resulting \( ENB_{\text{max}} \) that can be achieved at each point in time.

\[ w = \arg\min_w w'\Sigma w \]
The constrained MV-ENB involves minimizing the total portfolio variance under long-only constraints, in the same way as the Global Minimum Variance portfolio. In addition, a lower boundary constraint of 6.5 Effective Number of Bets is imposed to achieve sufficient diversification in terms of torsion bets. We arbitrarily choose the target ENB of 6.5 instead of the maximum ENB to balance factor diversification and a more flexible allocation.

\[
w = \arg\min_w w'\Sigma w
\]

\[
\begin{align*}
\text{s.t.} & \quad w'1 = 1 \\
& \quad w \geq 0 \\
& \quad \text{ENB}(w) = \text{ENB}_{\text{max}}
\end{align*}
\]

The constrained MSR-ENB involves maximizing the Sharpe Ratio the total portfolio variance under long-only constraints and with a lower boundary constraint of 6.5 Effective Number of Bets. The difference with respect to the previous MV-ENB constrained portfolio lies in the optimization function.

\[
w = \arg\max_w \frac{w'\mu}{\sqrt{w'\Sigma w}}
\]

\[
\begin{align*}
\text{s.t.} & \quad w'1 = 1 \\
& \quad w \geq 0 \\
& \quad \text{ENB}(w) \geq 6.5
\end{align*}
\]

The ENB-constrained strategies are compared with the alternative Minimum Variance, Equal Risk Contribution, Inverse Volatility and Equal Weight strategies. The portfolio allocations are found using the formulas as described in Appendix.

### 4.2 Asset Allocation performance

To implement the factor risk parity strategies, we estimate the minimum torsion bets \(^3\) using the transformation as described before and find that they have a clear economic interpretation, each corresponding precisely to one of the seven macro factors. The strategies presented are analyzed out of sample using expanding window (with initial

---

\(^3\)In unreported results we tested the use of PCA on the covariance matrix of the correlated macro factors instead of using the torsion transformation but the principal components failed to have an intuitive economic interpretation and the loadings were more volatile over time.
window length of 36 monthly observations). As such, the out of sample period consists in February 1st 2003 to June 1st 2019.

The results are reported in Table 2 and reveal large differences in the degree of diversification across the strategies. All Figures (5, 6 and 7) are reported in Appendix. The Minimum Variance Portfolio is the most concentrated with approximately 2.5 ENB throughout the sample period. In terms of risk contributions, it is clear that the total portfolio variance is dominated by the Interest Rate Factor, which is expected as it features the lowest volatility. In turn, this leads to an overallocation to the US Treasury Bond Index, which has the largest exposure to the Interest Rate Factor.

The Inverse Volatility and ERC portfolios are fairly similar in terms of diversification and asset allocation. The strategies appear overall to be sufficiently diversified starting from 4 and 5 ENB, respectively, and reaching 5.5 and 5.8 ENB by the end of the sample period. Consistently, the Risk Contributions are relatively balanced among most factors, although the Volatility factor prevails over the others.

The Equal Weight portfolio starts with less than 3 ENB and maintains approximately 5 ENB since the Global Financial Crisis. Therefore, despite equal weighting of the assets, the strategy has lower degree of diversification compared to the Inverse Volatility and ERC portfolios. Indeed, the equal weights assigned to the riskier asset classes result in large Risk Contribution by the Volatility bet and zero contribution from the Interest Rate factor since the GFC, and overall provide only naive diversification in nominal terms.

Finally, the three ENB-constrained strategies have the largest ENB by construction. In practice, the MV-FRP strategy which was constrained to achieve the maximum ENB at each point in time, appears to be quite unstable. The ENB ranges throughout the period between 5.5 and 7 ENB (with two exceptions in 2005 and in the GFC, where the ENB fell to 4 and 3, respectively) and the risk contributions are well balanced across torsion bets but all measures are very volatile. The portfolio has large allocations to the US Treasury Bond Index and US High Yield Index, but the instability in the asset weights renders the strategy difficult to implement in practice. The MV-ENB and MSR-ENB constrained portfolios, which involve a lower boundary constraint of 6.5 ENB, feature the same balanced characteristics in terms of risk contributions and asset allocation, although the MSR-ENB portfolio has an overallocation to US_TIPS before the financial crisis. However, they avoid any of the instability in the parameters from the optimization procedure, appearing therefore to be an effective tool to achieve close to factor risk parity. The ENB is constant at 6.5 throughout the entire period.

In terms of portfolio performance, the three factor-based portfolios are the best performing in terms of Sharpe Ratio (by order, equal to 0.89, 0.87 and 0.75). However, as pointed out in Lee (2011), caution should be taken in analyzing performance based
on Sharpe Ratios as returns are not taken into account in the factor risk parity portfolio construction. The MV-FRP and MV-ENB constrained portfolios have portfolio volatility (4.86% and 4.54%, respectively) and maximum drawdown in between that of the Minimum Variance Portfolio (3.65%) and the Risk Parity portfolio (5.51%), and the return on the MV-ENB constrained portfolio is only marginally below that of the Risk Parity portfolio (3.93% against 3.99%); on the other hand, the MSR-ENB constrained portfolio has both higher volatility and portfolio return (5.82% and 4.32%, respectively) compared to the Risk Parity portfolio.

Overall, the MV-ENB-constrained strategy appears to provide an attractive trade-off between risk and return and to be a valuable alternative to Equal Weight, Inverse Volatility and ERC portfolios. Achieving a balanced allocation across macro factors results in lower portfolio risk, as measured by standard deviation, maximum drawdown and skewness, while delivering higher average return than a GMV portfolio. In particular, compared to the ERC strategy, equal allocation of risk across macro factors, instead of assets, has lower risk and relatively low sacrifice in average annualized return (MV-ENB portfolio). Compared to the other two factor-based portfolios, the MV-ENB constrained is more stable in the allocations over time, not being subject to turnover (MV-FRP portfolio) nor in large failure to meet the full investment constraint (MSR-ENB portfolio).

Finally, we examine the performance of the three factor-risk parity strategies introduced in the previous section obtained from the closed-end formulas. While this approach achieves perfect factor risk parity, the main limitation is the lack of flexibility due to the closed formula solution. This results in large, long and short allocations as well as larger volatility and drawdown, which make the strategies not implementable for constrained investors. The three FRP portfolios identified as the FRP-MV, FRP-IV and FRP-MSR strongly underperform both the ENB-constrained portfolios and the traditional risk-based portfolios under all metrics.

5 Asset Returns and Macro Factors in a Regime-Switching Model

5.1 Literature

In the present section we extend the analysis to a two-state multivariate Markov Switching model which allows to capture the presence of structural breaks in time series assuming non-linearity in the underlying dynamics which cannot be represented by a single-state process. Markov Switching models build on the pioneering work of Hamilton (1990), who uses a two-state hidden Markov Model (HMM) to predict US business cycles based on US Gross National Product. In the context of asset returns, Markov
Switching models can incorporate differences in mean and covariance of asset returns (as well as non-normality in higher-order moments), which are driven by latent, or un-observable, states and where the states follow a Markov chain switching process.

There is large empirical evidence on the presence of structural breaks in financial time series and that modelling non-linearity and departures from normality in the assets’ characteristics adds value to asset allocation performance. Schwert (1989) and Hamilton and Lin (1996) first showed that stock returns volatility depends on the underlying state of the economy and is higher during economic recessions. Pástor and Stambaugh (2001) find evidence of structural breaks in the US equity premium, and Turner, Startz and Nelson (1989) show evidence of time-varying predictability in stock returns and variances conditional on accurately learning the state. Ang and Chen (2001) illustrate that downside correlations in US equity markets are significantly larger than upside correlations and that neglecting the presence of asymmetric correlations will overestimate the benefits from portfolio diversification in down markets and will results in over allocations to riskier stocks.

Ang and Bekaert (2004) model time-varying correlations between international stocks through a two-state Markov switching model characterized by a normal and persistent regime and a bear regime featuring lower mean returns, high volatility and higher correlations across assets. They further analyze mean-variance asset allocation developing a market-timing model with conditional estimates from the Regime Switching model and show out of sample outperformance of the latter vis-à-vis an unconditional asset allocation.

Ammann and Verhofen (2006) study dynamic allocation of style risk premia based on the Carhart four-factor model (Carhart, 1997) and estimate a two-state multivariate regime-switching model. They find evidence of two regimes, a High-Variance Regime where value stocks outperform, and a Low Variance Regime in which the market portfolio and momentum stocks outperform. Consequently, they test a tactical asset allocation strategy that switches between value stocks in the bear regime and momentum stocks during the bull regime and find that the dynamic allocation outperforms a simple buy and hold strategy out of sample. They use Markov Chain Monte Carlo (MCMC) to estimate the parameters of the regime switching model and use Gaussian quadrature to obtain the optimal portfolio weights. The prevailing regime is identified as in Ang and Bekaert (2004), as the one with highest probability of occurrence.

Guidolin and Timmermann (2006) estimate a Markov Switching model to model the joint distribution of US stock and bond returns and find that, individually, stocks and bonds are best represented by a two-state and three-state model, respectively. However, they show that the correlation between the smoothed state probabilities across stocks and bonds is negative or close to zero, suggesting that the assets are driven by very
different processes. Consequently, they find that a four-state model is needed to capture the joint dynamics of stocks and bonds and interpret the four states as crash, slow growth, bull and recovery regimes. Consistent with these findings, Guidolin and Timmerman (2007) analyze asset allocation under a four-state Markov Switching model of equities and bonds and quantitatively measure the economic costs of neglecting the presence of regimes on portfolio performance, where the model is estimated using Markov Chain Monte Carlo. They assume that the state is unobservable and use Monte Carlo simulations for integral approximation to compute the mean variance portfolio weights, extending the analysis to study learning effects about the state.

Guidolin and Ono (2006) study the dynamics of the relationship between asset returns and macroeconomic variables in a regime switching framework comparing different specifications of heteroskedastic Markov Switching VAR models. They jointly model eight variables represented by excess stocks and bond returns, the real Treasury Bill yield, the default spread and dividend yield, and pure macroeconomic variables being inflation, industrial production growth and real money growth. They find that the best model is a four-state Markov model where the VAR loadings matrix is time invariant (MSIH(4,0)-VAR(1)). Accordingly, they reject the hypothesis of a time-varying relationship between asset returns and macroeconomic variables and show that their model provides superior out-of-sample forecasting performance.

5.2 Model Specification

The estimation of the parameters for the Regime Switching model is performed through maximum likelihood based on the Expectation Maximization algorithm. We report a brief technical explanation of the Markov Switching procedure in the Appendix. The inputs in the model are the time series of the nine asset classes’ excess returns and the seven macro factors as constructed in Section 2. The data generating process for each asset excess return is represented by

\[ R_t = \alpha_S + \beta^S F_t + \Sigma^{1/2} \varepsilon_t \] with \( \varepsilon_t \sim IID(0, I_N) \)

The model is a MSIH(2) featuring a homogenous Markov chain, therefore with constant transition state probabilities, while all regression parameters are regime-switching, including the intercepts (I), slopes and heteroskedastic covariance matrix (H). The transition probability from state \( i \) to state \( j \) is indicated by \( p_{ij} = Pr[S_t = i|S_{t-1} = j] \) and all transition probabilities are collected in the transition matrix

\[
P = \begin{pmatrix}
p_{11} & p_{12} \\
p_{21} & p_{22}
\end{pmatrix} = \begin{pmatrix}
p_{11} & 1 - p_{11} \\
1 - p_{22} & p_{22}
\end{pmatrix}
\]
The total number of parameters to be estimated in the model is

\[ K \left[ N + \frac{N(N-1)}{2} + (N \times M) + (K - 1) \right] \]

with \( K = 2 \) being the number of regimes, \( N = 9 \) is the number of assets, \( M = 7 \) is the number of macro factors. This amounts to 218 parameters to be estimated against \( T = 232 \) monthly observations, and a saturation ratio of 9.6, where Guidolin and Pedio (2018) define the saturation ratio as the ratio of the total number of available observations (NT) to the total number of parameters to estimate. They recommend as a rule-of-thumb to have a saturation ratio greater than 20, implying that at least 20 observations per parameter are needed to perform any type of econometric analysis. In this case, the imposition of the factor structure largely increases the curse of dimensionality due to the additional \( (N \times M) \) regression slopes to be estimated. Consequently, the resulting saturation ratio of 9.6 is remarkably low and the model is very prone to estimation error. For this reason, we restrict the analysis to a two-state Regime Switching model, although a deeper study on the relationship between asset returns and macroeconomic factors may require considering additional states as well as modelling the regime-switching dynamics of the macro factors.

We derive the expected mean and covariance matrix of the assets in each state based on the regression parameters estimated from the Regime Switching model. The expected return in state \( s_i \) is calculated as:

\[ \hat{E}[r]_{s_i,t} = E \left[ \alpha_{s_i} + \beta_{s_i}' F_t | s_t = s_i \right]. \]  

(8)

In particular, to measure the expected return in state \( s_i \), we calculate the fitted values from the MS regression for that state and average across the values realized in state \( s_i \), which we infer as the state with largest filtered probability at each point in time. Therefore, we take the expectation considering only the values attributed to that state since, provided that the regime is correctly inferred, the parameters of the MS regression for state \( s_i \) apply only to the values of the macro factors in the LHS realized in the same state. The binary approach used at this stage to infer the regime (with 50% probability as switching threshold) is not necessarily restrictive since there are only few instances where the difference in the filtered state probabilities between states is negligible and therefore where there is high uncertainty on the prevailing regime (Figure 8). Similarly, the covariance matrix of the returns in each state is calculated as:

\[ \hat{E}[\Sigma]_{s_i,t} = B'_{s_i} \Sigma_{F_t|s_t=s_i} B_{s_i} + \Sigma_{e,s_i}, \]  

(9)

where \( B_{s_i} \) and \( \Sigma_{e,s_i} \) are the regime-switching slopes and covariance matrix of the error
term estimated from the MS Regression model, respectively. \( \Sigma_{F|s_t=s_i} \) represents the covariance matrix of the macro factors in state \( s_i \) and we calculate it by considering only the values of the macro factors in state \( s_i \), following the reasoning above. This approach allows to implicitly derive in-sample regime-dependent estimates for the macro factors which are needed to implement the factor risk parity portfolios. In particular, we use this method to overcome the inability to explicitly—and more rigorously—jointly model the regime-switching dynamics of the macro factors and the assets due to the extremely large number of parameters that would be required to estimate.

5.3 Model Estimates

The outputs of the MS Regression model estimated are presented as follows. Figure 8 plots the smoothed and filtered state probabilities, whereas Table 3 illustrates the expected performance of the asset classes in the two states. The smoothed probabilities represent the full-time inference of the state probabilities, conditional on information based on the entire sample period, whereas the filtered probabilities are the real-time inference of the state probabilities conditional to information up to each point in time (Guidolin and Pedio, 2018). The transition probability matrix suggests that both states are highly persistent: the probability of remaining in the same State in the next period is 90\% and 96\% for State 1 and State 2, respectively. The expected average duration of each state is calculated as \( \frac{1}{1-p_{ii}} \) and is equal to 10 months for State 1 and 22 months for State 2.

We classify State 1 as the crisis state, which is inferred at the beginning of the sample until mid 2004 and during the Global Financial Crisis. The model successfully foresees the beginning of the GFC as recognized by NBER dating, while it fails to recognize the successive OECD crisis, which may also hint at higher potential for fitting a United-States only economy than a global economy. In this regime, all equities and the Commodity index have negative returns and all assets have higher volatility compared to State 2. Additionally, \textit{US.Treasury} and \textit{US.TIPS} exhibit positive returns and have twice as higher Sharpe Ratio in the bad state than in the normal state. The first finding suggests that these assets are valuable diversifiers in the bad state, where all other equities generate negative returns. In addition, the second finding reveals that the assets’ outperformance in the bad state holds not only in the cross-section, when compared to the other assets, but also in absolute terms, compared to their respective performance in the good state, which suggests that the added value of these assets lies precisely in their role as safe-havens during crisis.

On the other hand, we classify State 2 as the normal regime, as it has higher average expected duration and it matches the bullish market and high-growth environment experienced in the after GFC. Moreover, all assets exhibit positive average returns and
lower volatility compared to State 1. The interpretation of the states confirms the majority of studies on two-state Regime Switching models applied to financial markets in which the two states are identified as, respectively, the crisis state characterized by high volatility and a bearish market environment, and the normal state featuring a bullish market outlook.

The estimation results indicate overall higher correlations in the bad state among equities and in particular for the Emerging market assets. This is consistent with empirical evidence that correlations among riskier asset classes are significantly higher in the crisis state (Longin and Solnik, 2002; Ang and Bekaert, 2002; Ang and Chen, 2001, Ang et al., 2009), although the difference in correlation coefficients herein observed is not extremely large and the correlation of the S&P P500 with US_HighYield and RUSSEL2000 is higher in the normal state. Finally, US_Treasury is negatively correlated with US_HighYield and almost uncorrelated with GSCI in the bad state, whereas the relations are inverted in the normal state. Furthermore, we find the negative correlation of US_Treasury with S&P P500, MSCI_EAFE and RUSSELL2000 to be more pronounced in the good state.

5.4 Regime Switching Portfolio Allocation

In this section, we study the performance of the asset allocation strategies based on the in-sample Regime Switching estimates, where the portfolios analyzed are the seven strategies, including the three ENB-constrained factor risk parity strategies illustrated in the previous section. The implementation of the strategies requires three steps, following a method we define as the “weights-mixture approach”. First, for each strategy considered, we calculate the optimal weights in each state. We refer to these weights as the “absolute” weights, since the portfolios are created using as inputs the expected means and covariance observed in each state separately. Secondly, we estimate the portfolio allocations conditional on the state as a weighted average of the absolute allocations in the two states expected in the next period (hence the “weights-mixture” approach), where the weights are given by the probability of transitioning to each state. Lastly, following from the second step, we find the final weights by conditioning on the state that is inferred at each point in time based on the Markov Switching filtered probabilities.

For the first step, we find the “absolute weights” by taking into consideration the behavior of the assets in each state independently. In practice, we need only the estimates for the regime-dependent covariance matrix for the risk-based portfolios (Eq. 9), whereas we use the estimates for the regime-dependent means simply to measure the expected performance of the portfolios in each state. For the factor risk parity portfolios, we assume that the ENB constraints are dependent on the state through the regime-switching slopes from the MS Regression and the regime-dependent covariance
matrix of the macro factors. Therefore, the state-dependent ENB is defined by:

\[
ENB_{s_i} = \exp \left( -\sum_{k=1}^{K} \left( b_i^{s_i} \log p_{MT,k}^{s_i} \right) \right)
\]

where

\[
p_{MT,k}^{s_i} = \frac{w_{MT,k,s_i}^2 \sigma_{MT,k,s_i}^2}{\sum_{k=1}^{K} w_{MT,k,s_i}^2 \sigma_{MT,k,s_i}^2},
\]

- \(\sigma_{MT,k,s_i}^2\) are the diagonal entries from the regime-dependent covariance matrix of the minimum torsion factors calculated as \(\Sigma_{MT} = t \Sigma_{F^{i|s_i=t'}} \) and \(\Sigma_{F^{i|s_i=t'}}\) is estimated as discussed in Section 5.2;
- \(w_{MT,k,s_i}^2\) derive from the minimum torsion weights which are backed out from the asset weights, \(w\), that are being optimized. They are calculated as \(w_{MT,S_i} = t'^{-1} B_{s_i} w\).

The performance results are summarized as follows and are illustrated in Table 4. All the benchmark risk-based strategies achieve similar performance across states and indeed the weights of these portfolios are almost unchanged compared to the unconditional allocation. Conversely, the performance and weights of the factor risk parity portfolios vary across state and the factor-based portfolios outperform in the bad state. The result suggests that neither the regime-switching slopes nor the covariance matrix of the assets leads to significantly different performance across states, as observed in the benchmark strategies. On the other hand, the use of the regime-dependent covariance matrix of the macro factors in the factor-risk parity portfolios generates different results across states. The hypothesis is confirmed by here unreported results that control for a joint hypothesis critique.

In terms of relative performance across strategies, the Minimum Variance Portfolio remains the best performing strategy in terms of Sharpe Ratio and lower volatility. In the bad state, the three factor risk parity portfolios outperform the Inverse Volatility, Equal Weight and Risk Parity portfolios in terms of Sharpe Ratio (0.58, 0.62 and 0.89 for the MV-FRP, MV-ENB and MSR-ENB constrained portfolios, respectively), followed by the Risk Parity (Sharpe Ratio of 0.41). The MV FRP and MV-ENB constrained portfolios have the lowest volatility (5.71% and 5.54%, respectively) compared to the other three risk-based strategies, whereas the MSR-ENB constrained portfolio achieves a volatility of 7.73%, higher than for the Risk Parity portfolio (6.89%). In terms of returns, the MV-FRP and MV-ENB constrained portfolios are in between the Minimum Variance and Risk Parity portfolio, whereas the MSR-ENB constrained portfolio achieves the largest annualized average return of 6.87%. The results can be explained by observing in Fig. 10 that the weights of the MSR-ENB constrained portfolio change significantly from the
unconditional allocation by tilting away from the overallocation to US.Treasury towards US.TIPS, which features both higher risk and reward in the bad state as analyzed in Table 3.

In the good state, the MV-ENB and MSR-ENB constrained portfolios underperform the Risk Parity (ERC) portfolio in terms of Sharpe Ratio (0.78 and 0.84, respectively, against ERC with 1.02) due to higher volatility (5.35% and 6.08% for the ENB-constrained portfolios, respectively, against 4.06% for the Risk Parity portfolio). The MV-FRP constrained portfolio has similar Sharpe Ratio (1.03), as the lower portfolio return is compensated by lower risk (3.46%). In terms of weights, the allocations in the MV-ENB and MSR-ENB constrained portfolios change significantly compared to both the unconditional and the bad state allocations, favoring US.TIPS at the expense of the overallocation to US.Treasury and increasing the allocation towards EMBI and MSCI.EM.

Finally, Figure 12 illustrates the macro factor risk contributions for the seven strategies in each state. The results confirm the findings observed in the static analysis. The Minimum Variance Portfolio has the lowest ENB below 3 as it is driven almost exclusively by the Interest Rate factor in both states. The Inverse Volatility, Equal Weight and Risk Parity portfolios are over concentrated in the Volatility factor during the bad state (with 40% risk contribution); in the good state, they maintain a relatively high risk allocation to the Volatility factor, whereas they tilt the risk allocation to the Growth Factor from the bad state towards either the Credit or EM factor. As observed in the previous section, the ERC portfolio has higher ENB compared to the alternative benchmark strategies (5.85 and 5.50 in State 1 and State 2, respectively). Conversely, the three factor-based portfolios have the most balanced allocation to the seven macro factors in both states and the highest ENB of bets (of minimum 6.5 in the bad state and below 6 in the good state). Most importantly, during the bad state they reduce the risk exposure to the Volatility factor in favor of the less volatile Interest Rate factor (and partly to EM factor in the MSR ENB-constrained portfolio); in the good state, the exposure to the Volatility factor does not increase, whereas the risk allocations to the Interest Rate and Growth factors are reduced to benefit the Inflation and Credit factors.

In the second step, we estimate the portfolio allocations conditioning on each state based on the weights-mixture approach. In particular, for any strategy, conditional on state \( i \), the allocations are calculated as

\[
\tilde{w}_t = (\tilde{w}_{t+1}|s_t = i) = \sum_{j=1}^{2} p_{i,j} \pi_j, \tag{10}
\]

where \( p_{i,j} \) represents the probability of transitioning from state \( i \) to state \( j \), and \( \pi_j \)
represents the “absolute” optimal weights in state $j$ (illustrated in 10). Finally, we find the ultimate portfolio weights by conditioning on the state inferred in the previous month at each point in time. Since the actual state cannot be observed in practice, we derive the final allocation as the weighted average of the conditional portfolio weights $\hat{w}_t$ from the previous step, using as weights the filtered state probabilities.

The final performance of the seven strategies is reported in Table 5 and is consistent with the results from the state-dependent analysis. The best performing strategies, in terms of Sharpe Ratio and lowest risk, are the Minimum Variance and MV-FRP portfolios. These are followed by the MV-ENB and MSR-ENB constrained portfolios, which, despite the underperformance in the persistent good state, achieve an overall risk-reward ratio at least as high as that of the Risk Parity portfolio.

6 Robustness

We consider in this section alternative specifications in the creation of the macro factors and report the associated performance of the factor risk parity strategies. First, following Greenberg et al. (2016), we estimate a simplified factor model by imposing constraints on the mapping from assets to macro factors based on economic priors. In particular, for each asset we select a subset of macro factors to which the assets have exposure while shrinking the coefficients of the remaining macro factors to zero. For instance, we assume that US Treasury has only exposure to the Interest Rate and Inflation Factors, while S&P500 has exposure to the Growth and Volatility factors. The groups included a priori are consistent with the macro factors which were found to be statistically significant in the Factor Model estimated.

Secondly, we consider two alternative models where the macroeconomic variables within each group are chosen following the model selection procedure based on the AIC criterion or the highest adjusted R squared, contrary to the BIC-based model selection adopted in the core section.

Fourth, a useful alternative to combine the macroeconomic variables is to extract common information among the set of macro variables. Consequently, we perform a Principal Component Analysis on the covariance matrix of the standardized macro variables within each group and we retain the first principal component. The factor model is then estimated as before by regressing each return on all the macro factors, here measured by the first principal components.

Finally, we also estimate a six-factor macro model. As shown in Figure 13, the high correlation between the Credit and EM factor (0.83) calls for considering a single source of risk common to both factors. We therefore combine the two factors into a single Credit factor, since they are both created with the use of Corporate bond spreads. The aggregation is made using a simple average.
Table 6 below reports the results of the ENB-constrained and FRP portfolios under the alternative specifications where the macro variables are selected from the full-sample model. The original specification, with model selection based on BIC, achieves the best performance in terms of Sharpe Ratios and standard deviation, whereas the model with R squared selection and with principal components result in the highest standard deviation. However, all the specifications are fairly similar in terms of investment performance, which strongly supports the robustness of the model and of the factor strategies. As concerns the pure factor risk parity portfolios, the alternative specifications confirm their underperformance due to larger standard deviations and large long-short positions. Using a six-factor macro model, the factor risk parity strategies achieve marginally lower standard deviations and maximum drawdown compared to the seven factor model, suggesting potential benefits from additional shrinkage towards fewer factors. We report in Table 7 the out of sample performance derived from the models where the variables are selected at each point in time in an expanding window. The shorter out-of-sample sample period supports the results obtained from the in-sample selection in all specifications.

7 Conclusion

We designed seven measures of macro factors through a model selection approach to summarize information from a wide range of macroeconomic and financial variables. A single-state linear factor model reveals that assets returns are significantly exposed to the seven combined measures of macro factors: Inflation, Interest Rate, Growth, Credit, Emerging Markets, Volatility and FX. We propose the use of factor risk parity strategies to achieve a balanced exposure to orthogonal macro risk factors by imposing diversification constraints on traditional asset allocation strategies. The results suggest that diversification in terms of macro risk factors reduces portfolio risk, as measured by portfolio volatility and maximum drawdown, compared to alternative risk-based asset allocation strategies, where the latter are found to be over exposed to the Volatility factor which drives portfolio performance. In particular, we find that factor risk parity portfolios provide a good trade-off between risk and reward in between the Global Minimum Variance and Equal Risk Contribution portfolios. The strategies are robust to alternative methods for constructing the macro factors as well as to the use of a six-factor model. Finally, we study the performance of the factor risk parity portfolios in a two-state setting by estimating a Regime Switching Regression model to capture structural breaks in asset returns and conditional macroeconomic sensitivities. In this framework, we find that, unlike the traditional risk-based portfolios, the factor risk parity strategies are reactive to changes in the economic environment by varying their allocations conditional on the state. Most importantly, we show that these portfolios
significantly outperform the alternative strategies in the bad state by maintaining a diversified allocation across the macro factors. When conditioning on the inferred state, the outperformance of the factor risk parity portfolios is reduced, although the strategies continue to provide superior risk-adjusted return and lower drawdown when compared to ERC portfolios.

The results of this paper provide useful considerations in favor of adopting a macro factor-based approach in portfolio management to achieve a diversified exposure to true sources of risk and to protect investors from difficult market and economic times. Several recommendations are illustrated for possible future studies as follows. First, an improvement on this work is to more thoroughly select macro factors as building blocks in the portfolio risk allocation. In practice, we recommend a more parsimonious as well as economically-motivated model by taking into consideration the presence of rewarded macro factor premia as well as fewer macro factors. Secondly, a natural extension is to explore optimal timing of macro factors conditional on the state to capture potential time-variation in macro risk premia instead of achieving a balanced allocation at all times. Moreover, one may argue that the slow-dynamics of macroeconomic variables call for examining a macro factor allocation under lower rebalancing frequencies, consistent with the longer investment horizons of many institutional investors and sovereign wealth funds who are concerned with macroeconomic sensitivities. Finally, it would be useful to study the out-of-sample performance of the factor risk parity strategies in the Regime Switching model. In particular, interesting extensions could be to compare different Regime Switching models by considering more states and jointly modelling the time-variation in asset returns and macroeconomic factors.
References


### Table 1: Factor Model Results

The table below reports the estimated coefficients and R squared from the OLS regressions of each return on the seven macro factors. For illustrative purposes only, the results here refer to the regression estimated at the end of the sample period, using all observations.

<table>
<thead>
<tr>
<th></th>
<th>Intercept</th>
<th>Inflation</th>
<th>InterestRate</th>
<th>Growth</th>
<th>Credit</th>
<th>EM</th>
<th>Volatility</th>
<th>FX</th>
<th>Adj. R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>SP500</td>
<td>0.0032</td>
<td>0.0006</td>
<td>0.0021</td>
<td>0.0095</td>
<td>-0.0085</td>
<td>0.0097</td>
<td>-0.0271</td>
<td>-0.0024</td>
<td>0.63</td>
</tr>
<tr>
<td>US_HighYield</td>
<td>0.0058</td>
<td>0.0015</td>
<td>-0.0074</td>
<td>0.0026</td>
<td>-0.0180</td>
<td>-0.0052</td>
<td>-0.0021</td>
<td>-0.0015</td>
<td>0.81</td>
</tr>
<tr>
<td>GSCI</td>
<td>0.0047</td>
<td>0.0421</td>
<td>-0.0001</td>
<td>-0.0022</td>
<td>-0.0099</td>
<td>0.0056</td>
<td>-0.0089</td>
<td>-0.0049</td>
<td>0.49</td>
</tr>
<tr>
<td>EMBI</td>
<td>0.0081</td>
<td>0.0024</td>
<td>-0.0129</td>
<td>-0.0013</td>
<td>0.0001</td>
<td>-0.0185</td>
<td>-0.0062</td>
<td>-0.0010</td>
<td>0.67</td>
</tr>
<tr>
<td>US_Treasury</td>
<td>0.0022</td>
<td>0.0003</td>
<td>-0.0088</td>
<td>0.0005</td>
<td>0.0051</td>
<td>-0.0017</td>
<td>0.0005</td>
<td>-0.0005</td>
<td>0.53</td>
</tr>
<tr>
<td>MSCI_EM</td>
<td>0.0073</td>
<td>0.0031</td>
<td>-0.0036</td>
<td>0.0106</td>
<td>-0.0118</td>
<td>-0.0093</td>
<td>-0.0240</td>
<td>-0.0104</td>
<td>0.61</td>
</tr>
<tr>
<td>MSCI_EAFE</td>
<td>0.0028</td>
<td>-0.0004</td>
<td>0.0020</td>
<td>0.0083</td>
<td>-0.0042</td>
<td>-0.0049</td>
<td>-0.0234</td>
<td>-0.0100</td>
<td>0.63</td>
</tr>
<tr>
<td>RUSSEL2000</td>
<td>0.0048</td>
<td>-0.0001</td>
<td>0.0002</td>
<td>0.0121</td>
<td>-0.0200</td>
<td>0.0169</td>
<td>-0.0312</td>
<td>-0.0006</td>
<td>0.57</td>
</tr>
<tr>
<td>US_TIPS</td>
<td>0.0043</td>
<td>0.0039</td>
<td>-0.0107</td>
<td>-0.0007</td>
<td>0.0036</td>
<td>-0.0063</td>
<td>-0.0005</td>
<td>-0.0016</td>
<td>0.36</td>
</tr>
</tbody>
</table>

***: p-value ≤ 0.001; **: p-value ≤ 0.01; *: p-value ≤ 0.05; .: p-value ≤ 0.10
Table 2: Strategies Performance Summary. This table reports the average return and standard deviation of the ten strategies, including Sharpe Ratio, Maximum Drawdown and skewness. All figures are annualized. The strategies are studied using expanding window. MV-FRP, MV-ENB and MSR-ENB refer to the factor risk parity strategies that impose ENB constraints in the optimization, whereas the last three FRP-MV, FRP-IV, FRP-MSR are the factor risk parity portfolios derived using the closed-end formulas.

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Average Return (%)</th>
<th>Standard Deviation (%)</th>
<th>Sharpe Ratio</th>
<th>Maximum Drawdown (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum Variance</td>
<td>3.15</td>
<td>0.86</td>
<td>0.08</td>
<td>-0.8</td>
</tr>
<tr>
<td>Inverse Volatility</td>
<td>4.64</td>
<td>0.69</td>
<td>0.25</td>
<td>-1.57</td>
</tr>
<tr>
<td>Equal Weight</td>
<td>5.19</td>
<td>0.52</td>
<td>0.39</td>
<td>-1.27</td>
</tr>
<tr>
<td>Risk Parity (ERC)</td>
<td>3.99</td>
<td>0.72</td>
<td>0.20</td>
<td>-1.68</td>
</tr>
<tr>
<td>MV-FRP (constr)</td>
<td>4.32</td>
<td>0.89</td>
<td>0.18</td>
<td>-1.69</td>
</tr>
<tr>
<td>MV-ENB (constr)</td>
<td>3.93</td>
<td>0.87</td>
<td>0.17</td>
<td>-1.17</td>
</tr>
<tr>
<td>MSR-ENB (constr)</td>
<td>4.38</td>
<td>0.75</td>
<td>0.22</td>
<td>-2.03</td>
</tr>
<tr>
<td>FRP-MV</td>
<td>2.06</td>
<td>0.16</td>
<td>0.46</td>
<td>0.06</td>
</tr>
<tr>
<td>FRP-IV</td>
<td>3.29</td>
<td>0.19</td>
<td>0.43</td>
<td>-0.23</td>
</tr>
<tr>
<td>FRP-MSR</td>
<td>8.48</td>
<td>0.24</td>
<td>0.77</td>
<td>0.40</td>
</tr>
</tbody>
</table>

Table 3: Descriptive Summary-Regime Switching Model. The figures below are all annualized and represent the average excess return, standard deviation of excess returns and Sharpe Ratio for the nine asset classes as estimated from the Regime Switching Model. The left-hand part refers to the estimated coefficients in the first state, while the right-hand part in the second state. The mean returns are calculated using the fitted values (intercept and beta coefficients) from the MS factor model.

<table>
<thead>
<tr>
<th>Regime 1</th>
<th>Regime 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Average Return (%)</td>
</tr>
<tr>
<td>SP500</td>
<td>-11.70</td>
</tr>
<tr>
<td>US_HighYield</td>
<td>3.29</td>
</tr>
<tr>
<td>GSCI</td>
<td>-4.31</td>
</tr>
<tr>
<td>EMBI</td>
<td>8.86</td>
</tr>
<tr>
<td>US_Treasury</td>
<td>5.86</td>
</tr>
<tr>
<td>MSCI_LEM</td>
<td>-8.72</td>
</tr>
<tr>
<td>MSCI_EAFE</td>
<td>-15.46</td>
</tr>
<tr>
<td>RUSSELL2000</td>
<td>-3.63</td>
</tr>
<tr>
<td>US_TIPS</td>
<td>8.34</td>
</tr>
</tbody>
</table>
Table 4: Portfolio Performance in each State, Weights Mixture Approach.
The figures below are all annualized and represent the average return, standard deviation, Sharpe Ratio and ENB for the seven strategies in each state. The conditional weights procedure is described in 5.4 and the inputs are the conditional estimates for the mean and covariance matrix of the assets in each State, as derived from the MS Regression model. All strategies are implemented in-sample. Panel A reports the performance in Regime 1, while Panel B in Regime 2.

<table>
<thead>
<tr>
<th>Panel A: Regime 1</th>
<th>Average Return (%)</th>
<th>Standard Deviation (%)</th>
<th>Sharpe Ratio</th>
<th>ENB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum Variance</td>
<td>4.42</td>
<td>4.63</td>
<td>0.95</td>
<td>2.57</td>
</tr>
<tr>
<td>Inverse Volatility</td>
<td>1.69</td>
<td>9.21</td>
<td>0.18</td>
<td>5.55</td>
</tr>
<tr>
<td>Equal Weight</td>
<td>-1.94</td>
<td>13.77</td>
<td>-0.14</td>
<td>4.84</td>
</tr>
<tr>
<td>Risk Parity (ERC)</td>
<td>2.83</td>
<td>6.89</td>
<td>0.41</td>
<td>5.85</td>
</tr>
<tr>
<td>MV-FRP (constr)</td>
<td>3.31</td>
<td>5.71</td>
<td>0.58</td>
<td>6.67</td>
</tr>
<tr>
<td>MV-ENB (constr)</td>
<td>3.43</td>
<td>5.54</td>
<td>0.62</td>
<td>6.50</td>
</tr>
<tr>
<td>MSR-ENB (constr)</td>
<td>6.87</td>
<td>7.73</td>
<td>0.89</td>
<td>6.50</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Regime 2</th>
<th>Average Return (%)</th>
<th>Standard Deviation (%)</th>
<th>Sharpe Ratio</th>
<th>Maximum Drawdown (%)</th>
<th>Skewness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum Variance</td>
<td>3.02</td>
<td>2.79</td>
<td>1.08</td>
<td>2.01</td>
<td></td>
</tr>
<tr>
<td>Inverse Volatility</td>
<td>5.02</td>
<td>5.08</td>
<td>0.99</td>
<td>5.14</td>
<td></td>
</tr>
<tr>
<td>Equal Weight</td>
<td>6.25</td>
<td>8.03</td>
<td>0.78</td>
<td>4.46</td>
<td></td>
</tr>
<tr>
<td>Risk Parity (ERC)</td>
<td>4.14</td>
<td>4.06</td>
<td>1.02</td>
<td>5.50</td>
<td></td>
</tr>
<tr>
<td>MV-FRP (constr)</td>
<td>3.56</td>
<td>3.46</td>
<td>1.03</td>
<td>5.83</td>
<td></td>
</tr>
<tr>
<td>MV-ENB (constr)</td>
<td>4.18</td>
<td>5.35</td>
<td>0.78</td>
<td>5.93</td>
<td></td>
</tr>
<tr>
<td>MSR-ENB (constr)</td>
<td>5.09</td>
<td>6.08</td>
<td>0.84</td>
<td>5.96</td>
<td></td>
</tr>
</tbody>
</table>

Table 5: Regime Switching Portfolio Performance - Weights Mixture Approach. This table reports the average return, standard deviation, Sharpe Ratio, Maximum Drawdown and skewness of the seven strategies. The weights-mixture approach estimates the conditional weights by combining the absolute weights found in each state separately and using as weights the probability of transitioning to the two states in the next period. The final weights are derived by conditioning on the inferred state, by weighting the conditional weights by the filtered state probabilities in the previous month.

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Average Return (%)</th>
<th>Standard Deviation (%)</th>
<th>Sharpe Ratio</th>
<th>Maximum Drawdown (%)</th>
<th>Skewness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum Variance</td>
<td>3.35</td>
<td>3.46</td>
<td>0.97</td>
<td>-0.38</td>
<td>5.96</td>
</tr>
<tr>
<td>Inverse Volatility</td>
<td>3.88</td>
<td>6.61</td>
<td>0.59</td>
<td>-1.36</td>
<td>24.09</td>
</tr>
<tr>
<td>Equal Weight</td>
<td>3.55</td>
<td>10.13</td>
<td>0.35</td>
<td>-1.11</td>
<td>38.70</td>
</tr>
<tr>
<td>Risk Parity (ERC)</td>
<td>3.62</td>
<td>5.09</td>
<td>0.71</td>
<td>-1.19</td>
<td>16.77</td>
</tr>
<tr>
<td>MV-FRP (constr)</td>
<td>3.40</td>
<td>4.27</td>
<td>0.80</td>
<td>-1.12</td>
<td>13.24</td>
</tr>
<tr>
<td>MV-ENB (constr)</td>
<td>3.91</td>
<td>5.42</td>
<td>0.72</td>
<td>-0.77</td>
<td>13.97</td>
</tr>
<tr>
<td>MSR-ENB (constr)</td>
<td>5.10</td>
<td>6.62</td>
<td>0.77</td>
<td>-1.08</td>
<td>19.04</td>
</tr>
</tbody>
</table>
Table 6: ENB-constrained strategies under Alternative Specifications (In Sample Selection). This table reports the summary performance of the ENB-constrained and FRP strategies. The strategies are implemented using expanding window. The first column refers to the alternative specification used to either select or combine the macro variables to create the macro factors. Only the selection/combination is made in sample, using the optimal specification found at the end of the sample period. The last row refers to the specification analyzed in the core section. The out of sample period for evaluation of the strategies is February 1st 2003 to June 1st 2019.

<table>
<thead>
<tr>
<th>Specification</th>
<th>Strategy</th>
<th>Average Return (%)</th>
<th>Standard Deviation (%)</th>
<th>Sharpe Ratio</th>
<th>Maximum Drawdown (%)</th>
<th>Skewness</th>
</tr>
</thead>
<tbody>
<tr>
<td>AIC</td>
<td>MV-FRP</td>
<td>4.00</td>
<td>4.97</td>
<td>0.80</td>
<td>0.18</td>
<td>-1.63</td>
</tr>
<tr>
<td></td>
<td>MV-ENB</td>
<td>4.18</td>
<td>4.74</td>
<td>0.88</td>
<td>0.17</td>
<td>-1.07</td>
</tr>
<tr>
<td></td>
<td>MSR-ENB</td>
<td>4.34</td>
<td>5.86</td>
<td>0.74</td>
<td>0.22</td>
<td>-2.04</td>
</tr>
<tr>
<td></td>
<td>FRP-MV</td>
<td>1.72</td>
<td>11.99</td>
<td>0.14</td>
<td>0.47</td>
<td>-0.07</td>
</tr>
<tr>
<td></td>
<td>FRP-IV</td>
<td>2.95</td>
<td>16.66</td>
<td>0.18</td>
<td>0.49</td>
<td>-0.17</td>
</tr>
<tr>
<td>R²</td>
<td>MV-FRP</td>
<td>3.99</td>
<td>5.19</td>
<td>0.77</td>
<td>0.20</td>
<td>-2.05</td>
</tr>
<tr>
<td></td>
<td>MV-ENB</td>
<td>4.14</td>
<td>4.72</td>
<td>0.88</td>
<td>0.17</td>
<td>-1.11</td>
</tr>
<tr>
<td></td>
<td>MSR-ENB</td>
<td>4.18</td>
<td>6.01</td>
<td>0.70</td>
<td>0.23</td>
<td>-2.08</td>
</tr>
<tr>
<td></td>
<td>FRP-MV</td>
<td>1.98</td>
<td>12.21</td>
<td>0.16</td>
<td>0.48</td>
<td>-0.06</td>
</tr>
<tr>
<td></td>
<td>FRP-IV</td>
<td>3.27</td>
<td>17.64</td>
<td>0.19</td>
<td>0.49</td>
<td>-0.19</td>
</tr>
<tr>
<td>Principal Components</td>
<td>MV-FRP</td>
<td>4.40</td>
<td>8.14</td>
<td>0.54</td>
<td>0.20</td>
<td>-0.73</td>
</tr>
<tr>
<td></td>
<td>MV-ENB</td>
<td>4.05</td>
<td>8.31</td>
<td>0.49</td>
<td>0.21</td>
<td>-0.68</td>
</tr>
<tr>
<td></td>
<td>MSR-ENB</td>
<td>4.91</td>
<td>7.38</td>
<td>0.67</td>
<td>0.24</td>
<td>0.28</td>
</tr>
<tr>
<td>BIC with economic priors</td>
<td>MV-FRP</td>
<td>3.84</td>
<td>4.81</td>
<td>0.80</td>
<td>0.15</td>
<td>-0.98</td>
</tr>
<tr>
<td></td>
<td>MV-ENB</td>
<td>4.01</td>
<td>5.14</td>
<td>0.78</td>
<td>0.19</td>
<td>-1.13</td>
</tr>
<tr>
<td></td>
<td>MSR-ENB</td>
<td>4.41</td>
<td>5.82</td>
<td>0.78</td>
<td>0.21</td>
<td>-1.30</td>
</tr>
<tr>
<td></td>
<td>FRP-MV</td>
<td>6.91</td>
<td>10.08</td>
<td>0.69</td>
<td>0.19</td>
<td>-0.53</td>
</tr>
<tr>
<td>Original Model (BIC)</td>
<td>MV-FRP</td>
<td>4.32</td>
<td>4.86</td>
<td>0.89</td>
<td>0.18</td>
<td>-1.69</td>
</tr>
<tr>
<td></td>
<td>MV-ENB</td>
<td>3.93</td>
<td>4.54</td>
<td>0.87</td>
<td>0.17</td>
<td>-1.17</td>
</tr>
<tr>
<td></td>
<td>MSR-ENB</td>
<td>4.38</td>
<td>5.82</td>
<td>0.75</td>
<td>0.22</td>
<td>-2.03</td>
</tr>
<tr>
<td></td>
<td>FRP-MV</td>
<td>2.06</td>
<td>12.62</td>
<td>0.16</td>
<td>0.46</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>FRP-IV</td>
<td>3.29</td>
<td>17.07</td>
<td>0.19</td>
<td>0.43</td>
<td>-0.23</td>
</tr>
<tr>
<td>Six Factor Model</td>
<td>MV-FRP</td>
<td>3.63</td>
<td>4.24</td>
<td>0.86</td>
<td>0.15</td>
<td>-0.83</td>
</tr>
<tr>
<td></td>
<td>MV-ENB</td>
<td>3.80</td>
<td>4.45</td>
<td>0.85</td>
<td>0.16</td>
<td>-1.04</td>
</tr>
<tr>
<td></td>
<td>MSR-ENB</td>
<td>4.16</td>
<td>6.02</td>
<td>0.69</td>
<td>0.22</td>
<td>-2.03</td>
</tr>
</tbody>
</table>

42
Table 7: ENB-constrained strategies under Alternative Specifications (Out of Sample). This table reports the average return and standard deviation of the ENB-constrained and FRP strategies, including Sharpe Ratio, Maximum Drawdown and skewness. All figures are annualized. The strategies are implemented using expanding window and the out of sample period is February 1st 2006 to June 1st 2019. The first column refers to the alternative specification used to either select or combine the macro variables to create the macro factors. The selection/combination is made out of sample, therefore iteratively estimating the optimal model in each month. The fifth specification refers to the model analysed in the core section, while the last case reports the performance of the benchmark risk-based strategies out of sample.

<table>
<thead>
<tr>
<th>Specification</th>
<th>Strategy</th>
<th>Average Return (%)</th>
<th>Standard Deviation (%)</th>
<th>Sharpe Ratio</th>
<th>Maximum Drawdown (%)</th>
<th>Skewness</th>
</tr>
</thead>
<tbody>
<tr>
<td>AIC</td>
<td>MV-FRP</td>
<td>2.90</td>
<td>4.93</td>
<td>0.59</td>
<td>0.18</td>
<td>-1.65</td>
</tr>
<tr>
<td></td>
<td>MV ENB</td>
<td>3.09</td>
<td>4.55</td>
<td>0.68</td>
<td>0.17</td>
<td>-1.58</td>
</tr>
<tr>
<td></td>
<td>MSR-ENB</td>
<td>3.17</td>
<td>6.82</td>
<td>0.46</td>
<td>0.28</td>
<td>-3.48</td>
</tr>
<tr>
<td></td>
<td>FRP-MV</td>
<td>2.55</td>
<td>9.73</td>
<td>0.26</td>
<td>0.34</td>
<td>-0.73</td>
</tr>
<tr>
<td>R2</td>
<td>MV-FRP</td>
<td>3.10</td>
<td>5.08</td>
<td>0.61</td>
<td>0.19</td>
<td>-1.48</td>
</tr>
<tr>
<td></td>
<td>MV ENB</td>
<td>2.99</td>
<td>4.50</td>
<td>0.66</td>
<td>0.17</td>
<td>-1.57</td>
</tr>
<tr>
<td></td>
<td>MSR-ENB</td>
<td>3.10</td>
<td>6.85</td>
<td>0.45</td>
<td>0.28</td>
<td>-3.59</td>
</tr>
<tr>
<td></td>
<td>FRP-MV</td>
<td>1.71</td>
<td>8.67</td>
<td>0.20</td>
<td>0.32</td>
<td>-0.40</td>
</tr>
<tr>
<td>Principal Components</td>
<td>MV-FRP</td>
<td>2.25</td>
<td>6.61</td>
<td>0.34</td>
<td>0.17</td>
<td>-1.24</td>
</tr>
<tr>
<td></td>
<td>MV ENB</td>
<td>2.20</td>
<td>7.18</td>
<td>0.31</td>
<td>0.18</td>
<td>-1.10</td>
</tr>
<tr>
<td></td>
<td>MSR-ENB</td>
<td>3.66</td>
<td>6.41</td>
<td>0.57</td>
<td>0.23</td>
<td>-1.23</td>
</tr>
<tr>
<td></td>
<td>FRP-MV</td>
<td>2.36</td>
<td>8.21</td>
<td>0.29</td>
<td>0.36</td>
<td>-0.36</td>
</tr>
<tr>
<td>BIC with economic priors</td>
<td>MV-FRP</td>
<td>2.70</td>
<td>6.60</td>
<td>0.41</td>
<td>0.25</td>
<td>-1.84</td>
</tr>
<tr>
<td></td>
<td>MV ENB</td>
<td>2.73</td>
<td>5.13</td>
<td>0.53</td>
<td>0.21</td>
<td>-1.61</td>
</tr>
<tr>
<td></td>
<td>MSR-ENB</td>
<td>2.66</td>
<td>7.18</td>
<td>0.37</td>
<td>0.30</td>
<td>-3.04</td>
</tr>
<tr>
<td></td>
<td>FRP-MV</td>
<td>3.63</td>
<td>11.36</td>
<td>0.32</td>
<td>0.53</td>
<td>-1.27</td>
</tr>
<tr>
<td>Principal Components</td>
<td>MV-FRP</td>
<td>3.13</td>
<td>4.78</td>
<td>0.65</td>
<td>0.17</td>
<td>-1.58</td>
</tr>
<tr>
<td></td>
<td>MV ENB</td>
<td>3.09</td>
<td>4.48</td>
<td>0.69</td>
<td>0.17</td>
<td>-1.51</td>
</tr>
<tr>
<td>Benchmark Portfolios</td>
<td>Global MV</td>
<td>2.72</td>
<td>3.65</td>
<td>0.75</td>
<td>0.10</td>
<td>-1.13</td>
</tr>
<tr>
<td></td>
<td>Inverse Volatility</td>
<td>3.41</td>
<td>7.25</td>
<td>0.47</td>
<td>0.27</td>
<td>-1.62</td>
</tr>
<tr>
<td></td>
<td>Equal Weight</td>
<td>3.18</td>
<td>10.59</td>
<td>0.30</td>
<td>0.39</td>
<td>-1.22</td>
</tr>
<tr>
<td></td>
<td>Risk Parity (ERC)</td>
<td>2.92</td>
<td>5.94</td>
<td>0.49</td>
<td>0.22</td>
<td>-2.00</td>
</tr>
</tbody>
</table>
Figures

Figure 1: Macro Variables retained by the model selection procedure (Out of Sample). Each figure, associated with one macro group, reports on the vertical axis the original variables belonging to each group. The figures illustrate which variables are picked by the model selection procedure in each month, where the model with lowest BIC is selected. Areas in green indicate that the variable has been selected, whereas areas in white indicate that opposite. Only six of the seven factors are shown since the seventh factor (FX) is composed by only one variable.
Figure 2: Macro Factor Weightings by underlying Macro Variables. The figure shows the weights of each macro variable in the combined macro factors. The weights are determined from the percent risk contribution of each macro variable across assets, where the risk contributions are derived using the coefficients from the selected OLS model. Variables with percent contribution to risk below 5% were neglected. The selected models, and in turn the weights, are estimated out of sample using expanding window with a window of 36 months.
**Figure 3: Time Series of Macro Factors.** The final macro factors are obtained as a weighted average of the variables picked by the model selection procedure documented in this study, with weights proportional to the percent contributions to risk of each macro variable summed across assets. The variables are expressed in difference, following the transformations applied as specified in Appendix A.2. The factors are a combination of standardized variables and are winsorized to avoid the spurious effect of outliers.
Figure 4: Risk Contributions from Factor Model (Out of Sample). The figure illustrates the time series of the percentage risk contributions of the macro factors for each asset class. The risk contributions shown are measured out of sample by estimating the regression in each month based on expanding window. Risk contributions are calculated as the product of the marginal contribution to risk and the loading in the factor model, \( RC_{i,k} = b_i \left( \frac{\sum_j b_j}{\sigma(R_i)} \right) \), and the Percentage Contribution to Risk illustrated is equal to \( b_i \left( \frac{\sum_j b_j}{\sigma(R_i)} \right) \).
Figure 5: Risk Contributions of Asset Allocation Strategies. The plot shows the percent risk contributions of the out of sample strategies. The last three rows refer to the factor risk parity strategies constructed imposing ENB constraints on a Minimum Variance and Maximum Sharpe Ratio Portfolio, respectively, with max ENB and with ENB larger than 6.5. Risk contributions are expressed in terms of the orthogonal torsion bets.
Figure 6: Asset Weights of Asset Allocation Strategies. The plots show the asset weights for each of the seven strategies. The last three rows refer to the factor risk parity strategies constructed imposing ENB constraints on a Minimum Variance and Maximum Sharpe Ratio portfolio.
Figure 7: Effective Number of Bets. The plots show the asset weights for each of the seven strategies. The last three rows refer to the factor risk parity strategies constructed imposing ENB constraints on a Minimum Variance Portfolio and Maximum Sharpe Ratio portfolio.
**Figure 8: Regime Switching State Probabilities.** The plot illustrates the smoothed (solid line) and filtered (dotted line) probabilities estimated from a two-state multivariate regression MS model. The smoothed probabilities represent the full-time inference of the state probabilities, conditional on information based on the entire sample period, whereas the filtered probabilities are the real-time inference of the state probabilities conditional to information up to each point in time. The grey areas represent the OECD based Recession periods for OECD and Non-member Economies.

**Figure 9: Correlation Matrix in each Regime.** The figure on the left refers to the correlation matrix in State 1 (crisis state), while that on the right in State 2 (normal state).
Figure 10: “Absolute” Portfolio Weights in each State, Weights Mixture Approach. The weights of the portfolios are obtained using the formulas in Section 4; however, the inputs are now the conditional estimates for the mean and covariance matrix of the assets in each State, as derived from the MS Regression model.
Figure 11: Factor Risk Contributions in each State, Weights Mixture approach. The figure shows the risk contributions for the seven strategies in terms of the macro torsion factors. The asset weights of the portfolios are obtained using the formulas in Section 4.1; however, the inputs are now the conditional estimates for the mean and covariance matrix of the assets in each State, as derived from the MS Regression model.
### A Appendix

#### A.1 Asset Returns Descriptive Analysis

**Table 8: Descriptive Statistics Asset Returns.** This table shows performance statistics of the nine asset classes. All figures are annualized, returns are multiplied by 12 and standard deviation is multiplied by $\sqrt{12}$. Returns are calculated using arithmetic monthly returns and refer to the excess returns, net of the 1-month US Treasury Bill rate. The time period is March 31st 1997 to May 31st 2019.

<table>
<thead>
<tr>
<th>Asset</th>
<th>Name</th>
<th>Average Return (%)</th>
<th>Standard Deviation (%)</th>
<th>Sharpe Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>SP500</td>
<td>SP500</td>
<td>4.74</td>
<td>15.03</td>
<td>0.32</td>
</tr>
<tr>
<td>Bloomberg Barclays US High Yield</td>
<td>US_HighYield</td>
<td>4.93</td>
<td>8.89</td>
<td>0.55</td>
</tr>
<tr>
<td>S&amp;P GSCI</td>
<td>GSCI</td>
<td>-0.90</td>
<td>22.21</td>
<td>-0.04</td>
</tr>
<tr>
<td>J.P. Morgan Emerging Markets Bond Index</td>
<td>EMBI</td>
<td>6.46</td>
<td>11.12</td>
<td>0.58</td>
</tr>
<tr>
<td>Bloomberg Barclays US Treasury Index</td>
<td>US_Treasury</td>
<td>2.76</td>
<td>4.26</td>
<td>0.65</td>
</tr>
<tr>
<td>MSCI Emerging Markets</td>
<td>MSCI_EM</td>
<td>3.57</td>
<td>23.04</td>
<td>0.15</td>
</tr>
<tr>
<td>MSCI EAFE</td>
<td>MSCI_EAFE</td>
<td>1.38</td>
<td>16.43</td>
<td>0.08</td>
</tr>
<tr>
<td>Russell 2000</td>
<td>RUSSELL2000</td>
<td>6.29</td>
<td>19.78</td>
<td>0.32</td>
</tr>
<tr>
<td>Bloomberg Barclays US Treasury</td>
<td>Bloomberg</td>
<td>6.29</td>
<td>19.78</td>
<td>0.32</td>
</tr>
<tr>
<td>Inflation-Linked Bond Index</td>
<td>US_TIPS</td>
<td>3.26</td>
<td>5.50</td>
<td>0.59</td>
</tr>
</tbody>
</table>
A.2 Macroeconomic data transformation and descriptive analysis

Table 9: Stationarity Transformations of macro variables. Transformation Codes: 1) No change, 2) Difference, 3) Difference of logarithm. All variables are transformed according to the Transformation codes reported in the fourth column. The transformation is necessary to ensure stationarity of the variables. Unit root tests are performed on the variables before and after transformation, comparing results from the Augmented Dickey Fuller test (ADF), Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test and Phillips-Perron (PP) tests. After the transformation, the variables are found to be all stationary.

<table>
<thead>
<tr>
<th>Name</th>
<th>Macro Group</th>
<th>Transformation</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPI.Growth</td>
<td>Inflation</td>
<td>3</td>
</tr>
<tr>
<td>ExpectedInflation_Level</td>
<td>Inflation</td>
<td>1</td>
</tr>
<tr>
<td>OILPRICEx</td>
<td>Inflation</td>
<td>3</td>
</tr>
<tr>
<td>T10Y3M</td>
<td>Interest Rate</td>
<td>1</td>
</tr>
<tr>
<td>GS10</td>
<td>Interest Rate</td>
<td>2</td>
</tr>
<tr>
<td>CFNAI</td>
<td>Growth</td>
<td>1</td>
</tr>
<tr>
<td>INDPRO</td>
<td>Growth</td>
<td>3</td>
</tr>
<tr>
<td>PMI Manufacturing</td>
<td>Growth</td>
<td>2</td>
</tr>
<tr>
<td>ISM.NonManufacturers</td>
<td>Growth</td>
<td>2</td>
</tr>
<tr>
<td>Leading.Economic.Index</td>
<td>Growth</td>
<td>2</td>
</tr>
<tr>
<td>BCI</td>
<td>Growth</td>
<td>2</td>
</tr>
<tr>
<td>UMCSENTx</td>
<td>Growth</td>
<td>3</td>
</tr>
<tr>
<td>INDPROD.OECD</td>
<td>Growth</td>
<td>2</td>
</tr>
<tr>
<td>CCI.OECD</td>
<td>Growth</td>
<td>2</td>
</tr>
<tr>
<td>BCI.OECD</td>
<td>Growth</td>
<td>2</td>
</tr>
<tr>
<td>CLI.OECD</td>
<td>Growth</td>
<td>2</td>
</tr>
<tr>
<td>BAAT10Y.Spread</td>
<td>Credit</td>
<td>2</td>
</tr>
<tr>
<td>ICE.Corporate.Spread</td>
<td>Credit</td>
<td>2</td>
</tr>
<tr>
<td>BBB.Spread</td>
<td>Credit</td>
<td>2</td>
</tr>
<tr>
<td>EMBI.Spread</td>
<td>Emerging Markets</td>
<td>2</td>
</tr>
<tr>
<td>EM.Corporate.OAS</td>
<td>Emerging Markets</td>
<td>2</td>
</tr>
<tr>
<td>GRP</td>
<td>Emerging Markets</td>
<td>2</td>
</tr>
<tr>
<td>VIX</td>
<td>Volatility</td>
<td>2</td>
</tr>
<tr>
<td>GlobalEPU</td>
<td>Volatility</td>
<td>2</td>
</tr>
<tr>
<td>MOVE.Index</td>
<td>Volatility</td>
<td>2</td>
</tr>
<tr>
<td>TWEXMMTH</td>
<td>FX</td>
<td>3</td>
</tr>
</tbody>
</table>
Figure 12: Macro Variables Correlation Matrix. The figure reports the correlation matrix of all 26 macro variables estimated in sample using all observations.

Figure 13: Correlation Matrix of Macro Factors. This figure shows the Pearson correlation coefficients between Macro factors at the end of the sample period, using all observations.
A.3 Macro Factor Mimicking Portfolios

This Section derives factor mimicking portfolios for the uncorrelated torsion bets. While the factor-based strategies are implemented using the macro factors, it is useful to study the implied factor mimicking portfolios in order to test the presence of risk premia among the uncorrelated macro factors. Failure to find a positive premium does not negate the usefulness of the factors as they can be relevant as risk factors in the factor model. In particular, the sign of the implied factor mimicking portfolios will be used in the MSR-FRP portfolio, for which the sign of the Sharpe Ratios of the uncorrelated factors is needed.

We derive the implied returns of the uncorrelated macro factors from the expected returns of the asset classes at each point in time, which are estimated as the expanding window average returns. Then, we back out the uncorrelated factor returns by inverting the coefficient matrix and applying the minimum torsion transformation, resulting in $B_{MT}^{-1}E[R]$, where $B_{MT}^{-1} = tB^{-1}$, and the out-of-sample coefficients and torsion matrix are used for estimation. The expected returns of the orthogonal macro bets are illustrated in Figure 14.

Figure 14: Implied Average Returns on Uncorrelated Macro Factors. The figure shows the expected returns on the torsion bets at each point in time, which are backed out from the expected returns on the assets by inverting the factor model coefficients and applying the minimum torsion transformation. The expected asset returns are measured at each point in time as the average return up to the previous month using expanding window and initial calibration period of 36 months.
Alternatively, we estimate the implied returns using the two-step Fama-MacBeth regression, although the derived signs of the factor returns fail to add value to the MSR-FRP portfolio. The details of the procedure are described as follows. In the first step of the Fama-MacBeth regression, we regress the asset returns on all the seven macro factors and the time series of the factor coefficients, $\hat{\beta}s$, are determined. These correspond precisely to the OLS coefficients from the factor model estimated. In the second step, we regress the cross section of asset returns on the macro factor coefficients at each point in time and we obtain the time series of the macro risk premia for each macro factor. This corresponds to estimating $T$ cross-sectional regressions of the nine returns on the seven estimates of the $\beta$s:

$$R_{i,1} = \gamma_{1,0} + \gamma_{1,1}\hat{\beta}_{i,F_1} + \gamma_{1,2}\hat{\beta}_{i,F_2} + \cdots + \gamma_{1,K}\hat{\beta}_{i,F_K} + \varepsilon_{i,1}$$

$$\ldots$$

$$R_{i,T} = \gamma_{T,0} + \gamma_{T,1}\hat{\beta}_{i,F_1} + \gamma_{T,2}\hat{\beta}_{i,F_2} + \cdots + \gamma_{T,K}\hat{\beta}_{i,F_K} + \varepsilon_{i,T}$$

where $R$ are the excess returns on the nine asset classes, $\gamma$ are the regression coefficients estimated with the cross-sectional regression, $\hat{\beta}_{i,F_1}, \ldots, \hat{\beta}_{i,F_K}$ are the coefficients estimated from the time-series regressions of each asset $i$ on the $K$ factors. The time series of the macro risk premia is given by the estimated $\gamma_k$, for $k$ that goes from 1 to 7 and is illustrated in Figure 15.

Figure 15: Time Series of Macro Mimicking Portfolio Returns (Fama-MacBeth). The figure illustrates the returns of the macro mimicking portfolios which are constructed using the two-step Fama-MacBeth regression procedure.

While it is well acknowledged that the OLS CSR procedure suffers from errors-in-variables (EIV) problem, for purposes of simplicity we here use the OLS procedure instead of valid alternatives such as GLS or WLS estimators. Finally, the risk premia
of the uncorrelated macro factors are derived by rotating the macro factor premia using the minimum torsion loadings. With $F_{MT} = tF$, the implied torsion bets premia are $t\gamma_k$.

**Table 10: Average Returns of the Factor Mimicking Portfolios.** The table reports the monthly average return of the factor mimicking portfolios. The first row is related to the implied torsion factors calculated as documented at the beginning of the section from the expected assets returns in an expanding window and inverting the mapping. The second row refers to the macro factor mimicking portfolios derived from the two-pass Fama-MacBeth procedure, whereas the third row refers to the torsion factor mimicking portfolios obtained by applying the minimum torsion transformation to the macro mimicking portfolios in the second row.

<table>
<thead>
<tr>
<th>Implied Torsion Factors</th>
<th>Inflation</th>
<th>Interest Rate</th>
<th>Growth</th>
<th>Credit</th>
<th>EM</th>
<th>Volatility</th>
<th>FX</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fama-MacBeth Macro Factors</td>
<td>-0.13</td>
<td>-0.35*</td>
<td>-0.06</td>
<td>-0.17</td>
<td>-0.03</td>
<td>-0.18</td>
<td>0.07</td>
</tr>
<tr>
<td>Fama-MacBeth Torsion Factors</td>
<td>-0.04</td>
<td>-0.39*</td>
<td>0.00</td>
<td>-0.33</td>
<td>0.10</td>
<td>-0.13</td>
<td>0.12</td>
</tr>
</tbody>
</table>

***: p – value ≤ 0.001; **: p – value ≤ 0.01; *: p – value ≤ 0.05; .: p – value ≤ 0.10
A.4 Benchmark Allocation Strategies

- The **Equal Weight** strategy assigns equal weights to the assets at each monthly rebalancing date and provides naive diversification in nominal terms.

\[
    w_i = \frac{1}{N} \quad \text{for } i = 1, \ldots, 9.
\]

- The **Minimum Variance** strategy minimizes the total variance of the portfolio subject to full investment and positivity constraints

\[
    w = \arg\min_w w' \Sigma w \\
    s.t. \quad w' 1 = 1 \quad \text{and} \quad w \geq 0
\]

- The **Equal Risk Contribution** of Maillard et al. (2010) is found numerically as the solution of the following optimization problem that minimizes the difference in contribution to risks of the assets

\[
    w = \arg\min_w \sum_{i=1}^{N} \sum_{j=1}^{N} (RC_i - RC_j)^2 \\
    s.t. \quad w' 1 = 1 \quad \text{and} \quad w \geq 0
\]

- The **Inverse Volatility** strategy is a type of risk parity whose weights are inversely proportional to the volatility of the assets

\[
    w_i = \frac{1/\sigma_i}{\sum_{i=1}^{N} 1/\sigma_i} \quad \text{for } i = 1, \ldots, 9
\]
A.5 Markov Switching model estimation procedure

The latent states, $S_{t,i}$, in the Regime Switching model follow a discrete, irreducible, ergodic first-order Markov Chain process. The first-order chain indicates that the probability of occurrence of state $i$ at time $t$ depends only on the state at time $t - 1$.

The ergodic property means that there is a stationary vector of probabilities $\xi$ such that $\xi = P^t \xi$, while irreducibility indicates that all latent states can occur, that is, $\xi > 0$.

Markov Switching models can generally be estimated using two procedures, a Bayesian approach based on Markov Chain Monte Carlo (MCMC) or through Maximum Likelihood Estimation, here analyzed.

The MLE of the regime switching model is performed using the Expectation-Maximization algorithm, introduced by Dempster, Laird and Rubin (1977) and adapted by Hamilton (1990), which allows to derive the latent states from the observable information of the model. The EM algorithm consists in two steps.

The first step is the Expectation step which takes as given the parameters of the model $\theta$ and estimates the filtered probabilities $\{\hat{\xi}_{i,t}\}_{t=1}^T$ based on a prior distribution for the probabilities $\xi$. The filtered probabilities $\{\hat{\xi}_{i,t}\}_{t=1}^T$ represent the real-time inferences of the state probabilities conditional on information of the time series $\{y_t\}_{t=1}^T$ up to time $t$, or $I_t$, that is, $\hat{\xi}_{i,t|I_t} \equiv \Pr(\xi_{i,t}|I_t)$, (Guidolin and Pedio, 2018).

This step is performed by applying Bayes’ rule as

$$\Pr (\xi_t | I_{t-1}, y_t) = \frac{\Pr (y_t | \xi_t, I_{t-1}) \Pr (\xi_t | I_{t-1})}{\Pr (y_t | I_{t-1})}$$

Then, the one-step ahead predicted state probabilities are derived recursively as $\hat{\xi}_{t+1|t} \equiv \Pr (\xi_{t+1}|I_t)$ based on the current information. Once the entire time series of filtered probabilities is estimated, the vector of smoothed probabilities $\{\hat{\xi}_{i,T}\}_{t=1}^T$ is obtained by iterating the algorithm forward. The smoothed probabilities $\{\hat{\xi}_{i,T}\}_{t=1}^T$ represent the full-time inference of the state probabilities conditional on the information on the entire time series $\{y_t\}_{t=1}^T$, up to time $T$, or $I_T$, that is, $\hat{\xi}_{i|T} \equiv \Pr (\xi_{i}|I_T)$.

The second step is the Maximization step which takes as given the smoothed probabilities estimated in the previous step and maximizes the log-likelihood function of the model $L (\{y_t\}_{t=1}^T \{\hat{\xi}_{i,T}\}_{t=1}^T, \theta, \rho)$, where $\rho$ is the vector of transition probabilities. This gives an estimate of the parameters $\theta, \rho$.

The two steps are then combined in the EM algorithm, where the values of $\xi, \theta, \rho$ are updated iteratively based on the estimates from the previous step until convergence.