Collective Learning about Systematic Risk

Yongjin Kim and Kai Li [1]

Abstract

We present an investment-based asset pricing model in which firms' exposure to systematic risk is uncertain. The parameter is updated from collective observations of firms' peers, causing an endogenous fluctuation in the discount rate that enters firms' real decisions and the market valuation. We empirically show that the revision of the risk exposure through this collective learning negatively predicts investment-capital ratio and market-to-book ratio and positively predicts the implied cost of capital. Data also support the model predictions that the precision of the parameter beliefs lowers the cost of capital, in turn raising the capital investment. In contrast, an alternative risk-estimate from firms' individual history cannot explain the observables, revealing the collective nature of the learning.

JEL Codes: E2, E3, G12

Keywords: Parameter uncertainty, Collective learning, Systematic risk, Implied cost of capital

[1]Yongjin Kim (yongjkim@cityu.edu.hk) is an assistant professor of finance at City University of Hong Kong, and Kai Li (kaili@ust.hk) is an assistant professor of finance at Hong Kong University of Science and Technology. We thank George Pennacchi and Hai Lin, as well as participants at the City University of Hong Kong Finance Conference and the Wellington Finance Summit for their helpful comments. Yongjin Kim gratefully acknowledges the financial support from the Research Grants Council of the Hong Kong Special Administrative Region, China (Project No. 9048132).

1. Introduction

Systematic risk is arguably one of the most fundamental quantities in finance. Various decisions ranging from portfolio choice to corporate investment require the expected return or the discount rate as an essential input, which is often accompanied by the measurement of the risk exposure. This study examines the importance of accounting for parameter uncertainty regarding firms' systematic risk. We propose that this parameter is continuously updated from collective observations of firms' peers, causing an endogenous fluctuation in the firms' discount rate. We provide supportive empirical evidence that the learning-induced shift in discount rate has a widespread impact on firms' real decisions and market valuations.

Firms' typical characteristics make a quick identification of the systematic-risk exposure elusive. The relevant observations that are informative about the parameter tend to contain substantial noises. As a rough estimate, a simple regression finds that only 6.8% of the variance of firms' productivity growth is explained by macroeconomic shocks that constitute the systematic risk. Given the dominant noise that hampers identifying the risk exposure, collective learning is particularly conducive. Benefiting from a larger dataset from industry peers, which likely have similar exposure to the systematic risk as Fama and French (1997) posit, decision-makers can update the parameter more efficiently than if they relied solely on firms' individual observations. We document that it is this particular estimate of risk exposure through collective learning that empirically drives both firms' real decisions and market valuations.

To elucidate the connection between the parameter learning and the firm observables, we build a collective-learning framework in the context of the investment-based asset pricing model. The new feature of the model is that firms' exposure to systematic risk – covariance between firm-level productivity and the macroeconomic shocks that affect the stochastic discount factor – is unknown at the beginning. Instead, decision-makers gradually update their beliefs about this parameter as they observe newly realized productivity and macroeconomic shocks. Interestingly, this learning process causes the systematic-risk estimate to fluctuate, although the true parameter is constant. The model, in turn, provides a theoretical link from the evolution of the parameter beliefs to the discount rate, capital investment, and market valuation. As the posterior estimate of systematic risk (i.e., the mean of parameter beliefs) is revised upward (downward), both the investment-capital ratio and the market-to-book ratio should decrease (increase). Simultaneously, the cost of capital should increase (decrease). Intuitively, perceiving a greater exposure to the systematic risk, investors require a higher return on firms' assets and thus evaluate lower the present value of future cash flows from new and existing capital.

Beyond the posterior estimate of systematic risk, the precision of parameter beliefs should affect investment and valuation. The model predicts that the uncertainty about the systematic-risk parameter creates the additional risk arising from the correlation between macroeconomic shocks and the revision of the risk exposure. Improvement in the precision reduces this risk, thereby decreasing the cost of capital. Likewise, we expect both investment-capital ratio and market-to-book ratio to respond positively to the precision.

The key empirical finding of this study is that these learning-related variables have a concerted impact on firms' real decisions and market valuation, as the theory predicts. The capital investment negatively reacts to the posterior estimate of the systematic risk with a t-statistic of -5.20. This association is also economically significant. A rise in the systematic-risk estimate by one standard deviation decreases the investment by 9.5% (e.g., the annual investment-capital ratio changes from 0.217 to 0.196). Simultaneously, the precision of parameter beliefs positively predicts the investment with a t-statistic of 5.13, and a one-standard-deviation increase in the precision raises the investment by 7.2%. We observe these strong associations after we control for other determinants of capital investment noted in the literature, including firm's size, age, Tobin's q, leverage ratio, cash flow, an indicator of financial constraints, and the amount of competition within an industry. Furthermore, these findings are robust to different ways of classifying industries. Irrespective of whether we define industry peers based on SIC, four-digit NAICS, or Hoberg and Phillips (2016)'s text-based industry classification, most of the associations continue to be significant at the 1% level.

This parameter learning also influences the market valuation in practice. The empirical data on the market-to-book ratio exhibit the predicted regularities caused by the learning. Specifically, the market evaluates firms lower when a new signal indicates that the systematic risk is higher than previously thought. Also, the market value increases as the parameter beliefs become more precise. Most of these connections are significant at the 5% level in the regressions, which are designed to reveal within-firm changes through time. These time-series patterns in investment and valuation suggest that decision-makers, in practice, constantly update beliefs about systematic risk.

Furthermore, this learning about systematic risk creates a cross-sectional dispersion in the cost of capital. Using the implied measure of the cost of capital from accounting information, as suggested by Hou et al. (2012), we find that the implied cost is positively related to the posterior estimate of risk exposure; an increase in the risk estimate by one standard deviation raises the annualized cost of capital by 0.9%. This evidence corroborates the previous findings of Bansal et al. (2005) and Da (2009) that the consumption beta is a determinant of a cross-section of expected return. Moreover, we find that the precision of the parameter belief is also priced in the cross section. The market requires a lower return on assets for which systematic risk becomes less ambiguous. This finding supports the model prediction that the learning alleviates the risk associated with parameter uncertainty.

We try further to define the extent to which the exact form of learning is collective. One may consider an alternative form: individual learning in which each firm's systematic risk is identified from its history only without reference to peers' observations. This idea is worth considering because even peer firms in the same industry might have different business profiles and therefore have different risk exposures. In such a case, focusing on each firm's own history would lead to a better estimate of the risk. In turn, we conduct an alternative measurement of the risk exposure based solely on individual history.

The empirical analysis, however, shows that this alternative risk estimate from individual learning is only insignificant for predicting the investment, the market valuation, and the cost of capital. We conjecture that this insignificance is due to the scarcity of datasets that inherently follows the individual learning, as compared to the collective learning that uses rich industry-wide observations. When updating systematic risks, acquiring a large dataset is particularly crucial because the sources for learning have a remarkably low signal-to-noise ratio. Specifically, the volatility of the idiosyncratic shock (noise) to productivity is approximately 30 times as large as the volatility of the systematic component (signal) in the calibrated model. This substantial noise impedes the updating of the parameter, especially when relying on a few observations that the individual learning offers. As a result, the risk estimate from the individual history is incapable of explaining corporate decisions and valuations.

In a robustness test, we explore the possibility that the true exposure to systematic risk is dynamic, contrary to our model assumption. In the model, despite the true exposure being constant, the risk estimate fluctuates in the course of leaning from growing observations. However, one might cast doubt on this assumption and argue that the learning-based risk estimate may misleadingly capture dynamics of the true risk characteristics. To address these concerns, we employ rolling-window-based estimations, an alternative approach to measuring risk exposure, in contrast to expanding-window-based estimations that parameter learning inspires. From the design, the rolling-window approach is able to better detect time variations with respect to true risk, if any exists. However, we find that for various choices of the estimation window, the risk estimate derived from the rolling-window approach has a weaker, often insignificant association with corporate decisions and valuations than does our baseline estimate. This finding lets us conclude that critical dimensions of firm decisions and valuations are empirically more responsive to learning-induced changes in the risk estimate than fluctuations in the true risk profile.

Literature Review This study builds on a growing body of literature that studies the implications of parameter learning with respect to market valuations and corporate decisions. Pastor and Veronesi (2009) provides a comprehensive review of these learning models. In the context of a corporate setting, prior studies have considered parameter uncertainty regarding a long-term mean of productivity (Pastor and Veronesi (2003) and Alti (2003)) or return-to-scale parameters (Johnson (2007)). Distinctively, our paper studies the implications of uncertainty with respect to systematic-risk exposure, which is an equally fundamental parameter. In this regard, our focus is similar to Ai et al. (2018), who consider ambiguity regarding risk exposure and document that this learning can rationalize seemingly puzzling facts in the term structure of equity returns. We complement this study by elaborating upon the learning mechanism; we let decision-makers learn from the history of realized output instead of noisy independent signals in the prior study. This setting generates a path dependence of firms' decisions and market valuations, which we use to test the empirical presence of the learning. Moreover, Ai et al. (2018) reflects the learning of firm-specific exposure, while our paper is specifically about learning about a common exposure to systematic risk, which suggests collective learning. This idea of learning from peers is similar to Foucault and Fresard (2014),as they document that a firm's investment responds to its peers' Tobin's q due to its informativeness about future cash flows. In comparison, we consider learning about systematic risk and show that collective learning influences much broader dimensions of firm observables, including investment.

This study is also related to the literature that examines the role of consumption risk in financial markets, including Lettau and Ludvigson (2001), Bansal et al. (2005), Da (2009), and Boguth and Kuehn (2013). These studies successfully relate a cross-sectional dispersion in expected stock returns to the covariance between firms' cash flows or returns with the macroeconomic shocks affecting consumption. We expand the scope of this analysis and document that capital investments and valuation ratios also respond to the consumption beta. Furthermore, we reveal that the precision of the risk-exposure estimate is priced in the cross-section of returns, highlighting that decision-makers engage in learning about the parameter.

More broadly, our work is also related to dynamic investment models that investigate the implications of firms' optimal decisions on asset returns. Prior studies, including Berk et al. (1999), Gomes et al. (2003), Carlson et al. (2004), Zhang (2005), and Kuehn and Schmid (2014), show that observed regularities in stock and bond returns can arise as a result of corporate investment policy. We complement the literature by documenting new regularities in both the investment and return for which the parameter learning accounts.

The remainder of this paper is organized as follows. In section 2, we describe the theoretical model. In section 3, we explain the calibration and put forward testable predictions from the model. In section 4, we provide empirical tests and discusses the main findings. We conclude this paper with section 5.

2. Model

We consider peer firms that belong to one industry $\mathbb{I}$. These firms are heterogeneous ex-post but have a common characteristic: identical exposure to systematic risk. To define systematic risk in a tractable way, we specify a consumption-based stochastic discount factor. The representative agent has recursive preferences over exogenous consumption. Given the stochastic discount factor, each firm makes optimal investment decisions with reference to all up-to-date information. Our model only offers partial equilibrium since we do not connect the sum of production outputs in the economy back to aggregate consumption.

2.1. Stochastic Discount Factor

Preferences of the representative agent are recursive as in Epstein and Zin (1989). The preferences are characterized by the standard parameters, including the rate of time preference β, the elasticity of intertemporal substitution ψ, and the coefficient of relative risk aversion γ.

We assume that consumption growth conditional on time t is normally distributed as in Kuehn and Schmid (2014):

$$\ln\left(\frac{C_{t+1}}{C_t}\right) = g + \mu_c(\omega_t) + \sigma_c(\omega_t)\eta_{t+1} \tag{1}$$

where η_{t+1} is standard normal innovation, and the mean $\mu_c(\omega_t)$ and the volatility $\sigma_c(\omega_t)$ of the growth depend on the state of the economy ω_t. The economic state shifts over time following a Markov chain with transition matrix P.

The stochastic discount factor is

$$M_{t,t+1} = \beta^{\theta}\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left(\frac{S_{t+1}+1}{S_t}\right)^{-(1-\theta)} \tag{2}$$

where S_t denotes the wealth-consumption ratio and $\theta = \frac{1-\gamma}{1-1/\psi}$. The wealth-consumption ratio is determined solely by the state of the economy, so $S_t = S(\omega_t)$. This ratio can be solved through the Euler equation described in Appendix A.

2.2. Firm's Production and Investment

Consider firm i that employs a production technology with decreasing return-to-scale. Its output in time t is

$$A_{i,t}K_{i,t}^{\alpha} \tag{3}$$

for which $A_{i,t}$ is the productivity shock, $K_{i,t}$ is the capital stock, and $0 < \alpha < 1$ is the capital share of the production. The capital stock accumulates according to

$$K_{i,t+1} = I_{i,t} + (1-\delta)K_{i,t} \tag{4}$$

where $I_{i,t}$ is investment and δ is the constant depreciation rate. We assume that capital installments are not frictionless as in the literature, so the firm incurs convex adjustment costs in addition to purchasing costs. The adjustment costs are $\phi\left(I_{i,t}/K_{i,t}\right)^2 K_{i,t}$ where ϕ is a positive constant.

The firm's productivity growth is stochastic and correlated with consumption growth. Due to the tight connection between the consumption shock and the stochastic discount factor, this correlation engenders the firm's exposure to the systematic risk. Specifically, the productivity growth is

$$\ln\left(\frac{A_{i,t+1}}{A_{i,t}}\right)\Bigg(= g+\mu_c(\omega_t)-\frac{(b^{\mathbb{I}})^2\sigma_c(\omega_t)^2}{2}+b^{\mathbb{I}}\sigma_c(\omega_t)\eta_{t+1}+\nu\epsilon_{i,t+1} \tag{5}$$

where $\epsilon_{i,t+1}$ is a firm-specific standard normal innovation, and ν controls the magnitude of this idiosyncratic shock.

Notice that the industry-specific parameter $b^{\mathbb{I}}$ controls the productivity's exposure to the consumption shock η_{t+1}. An increase in $b^{\mathbb{I}}$ would amplify the covariance between productivity and consumption, thereby increasing the systematic risk. Meanwhile, the change in $b^{\mathbb{I}}$ would leave the the mean of future productivity unchanged. This is because the third term in the right hand side, $(b^{\mathbb{I}})^2\sigma_c(\omega_t)^2/2$, adjusts for the Jensen's inequality effect; otherwise, without this adjustment, a rise in b would result in a higher future productivity, on average. In sum, the specification of equation (5) means that a rise in b leads to the mean-preserving spread of future productivity that the agent penalizes.

Importantly, we assume that the parameter $b^{\mathbb{I}}$ is unobservable for the agent and must be estimated from realized productivity. Learning about $b^{\mathbb{I}}$ is non-trivial because the productivity is also subject to the unobservable idiosyncratic shock, which the agent cannot distinguish from the systematic component. Moreover, because of identical risk exposures among all firms in the same industry, the realized productivity of its industry peers is as informative of the parameter as firm i's own productivity. In the next section, we describe in more detail the learning about $b^{\mathbb{I}}$.

2.3. Collective Learning About Systematic Risk

The industry $\mathbb{I}$'s risk exposure $b^{\mathbb{I}}$ is unknown at the beginning. The agent starts with prior beliefs about $b^{\mathbb{I}}$ that are normally distributed with mean $m^{\mathbb{I}}_{b,0}$ and standard deviation $\sigma^{\mathbb{I}}_{b,0}$. Since then, the agent updates the beliefs after receiving new information, in particular, the realized productivity of every industry constituent and consumption growth.

To formulate the learning process, we let Y_t denote the $(1 \times n)$ vector of the time-t productivity growth for n constituents. In addition, X_t denotes the $(1 \times n)$ vector with all elements equal to the time-t consumption growth. Specifically,

$$
\begin{aligned}
Y_t &= \left[\ln\left(\frac{A_{1,t}}{A_{1,t-1}}\right) - g - \mu_c(\omega_{t-1}) \quad \cdots \quad \ln\left(\frac{A_{i,t}}{A_{i,t-1}}\right) - g - \mu_c(\omega_{t-1}) \quad \cdots \quad \ln\left(\frac{A_{n,t}}{A_{n,t-1}}\right) - g - \mu_c(\omega_{t-1}) \right] \\
X_t &= \left[\ln\left(\frac{C_t}{C_{t-1}}\right) - g - \mu_c(\omega_{t-1}) \quad \cdots \quad \ln\left(\frac{C_t}{C_{t-1}}\right) - g - \mu_c(\omega_{t-1}) \quad \cdots \quad \ln\left(\frac{C_t}{C_{t-1}}\right) - g - \mu_c(\omega_{t-1}) \right]
\end{aligned} \tag{6}
$$

where we subtract the conditional mean from productivity and consumption to simplify the following analysis. Using all of the information obtained so far, including both the new information in time t and the old information from the past, the agent updates the beliefs by performing a least-square estimation. In other words, the agent, who is aware of the structure given by equation (5), searches for the new estimate $m^{\mathbb{I}}_{b,t}$ that minimizes the sum of squared errors as follows:

$$
m^{\mathbb{I}}_{b,t} = \underset{\lambda}{\operatorname{argmin}} \sum_{s=1}^{t} (Y_s - \lambda X_s)(Y_s - \lambda X_s)^T. \tag{7}
$$

The resulting $m^{\mathbb{I}}_{b,t}$ is a consistent estimator of b. See Appendix B for the proof.

As new realizations of productivity arrive over time, the agent continuously updates the estimation. This recursive estimation implies the following update of the parameter beliefs:

$$
\begin{aligned}
m^{\mathbb{I}}_{b,t} &= m^{\mathbb{I}}_{b,t-1} + \frac{\left(\sigma^{\mathbb{I}}_{b,t}\right)^2}{\nu^2} X_t \left(Y_t^T - m^{\mathbb{I}}_{b,t-1} X_t^T\right) \\
\frac{1}{\left(\sigma^{\mathbb{I}}_{b,t}\right)^2} &= \frac{1}{\left(\sigma^{\mathbb{I}}_{b,t-1}\right)^2} + \frac{1}{\nu^2} X_t X_t^T,
\end{aligned}
\tag{8}
$$

where $\sigma^{\mathbb{I}}_{b,t}$ denotes the conditional standard error of the estimator. The derivation is provided in the Appendix Appendix B. Intuitively, having more observations over time improves the precision of beliefs, as measured by $1/\sigma^{\mathbb{I}}_{b,t}$. In addition, observing a higher-than-expected covariance between productivity and consumption, $X_t \left(Y_t^T - m_{b,t-1} X_t^T\right) > 0$, leads the agent to revise the estimate upward. This revision is more sensitive to new observations when the agent is more uncertain about the parameter, high $\sigma^{\mathbb{I}}_{b,t}$.

We choose to model the parameter learning through this least-square estimation rather than the Bayesian update that is often employed in the literature. In our setting, the posterior distribution of $b^{\mathbb{I}}$ is difficult to obtain in a closed form,[2] which is required in the Bayesian update. Despite the distinction in formulation, this least-square approach updates the parameter in a nearly equivalent way to the Bayesian approach under a certain condition: if the standard deviation ν of the idiosyncratic shock to productivity is significantly larger than the standard deviation $\sigma_c(\omega_t)$ of consumption shock. In this case, the posterior distribution of $b^{\mathbb{I}}$ is close to being normally distributed. Our calibrated model satisfies this parameter condition, so we approximate the posterior beliefs as normally distributed with mean $m^{\mathbb{I}}_{b,t}$ and standard deviation $\sigma^{\mathbb{I}}_{b,t}$.

2.4. Valuation

Suppose that the financial markets are frictionless. Firms' financing choices are then irrelevant to firm value, so we assume for simplicity's sake that firms are entirely financed

[2] Although the prior distribution of b is assumed to be normal, the posterior distribution is not normal. This is because the productivity growth contains the non-linear term of $b^{\mathbb{I}}$ that adjusts for the Jensen's effect.

with equity. In this setting, firm i chooses investment to maximize its market value:

$$V_{i,t} = \max_{I_{i,t}} \left\{ A_{i,t} K_{i,t}^{\alpha} - I_{i,t} - \phi \left(\frac{I_{i,t}}{K_{i,t}} \right)^2 K_{i,t} + \mathbb{E}_t \left[M_{t,t+1} V_{i,t+1} \right] \right\}. \tag{9}$$

The state variables in this firm's problem are the aggregate state of the economy ω_t, the productivity shock $A_{i,t}$, capital stock $K_{i,t}$ and the distribution of posterior beliefs about the systematic risk: the mean $m_{b,t}^{\mathbb{I}}$ and the standard error $\sigma_{b,t}^{\mathbb{I}}$. These state variables evolve according to the law of motion described by equations (4), (5) and (8).

A distinctive feature of this problem is that firm i's investment and valuation are influenced by the history of the realized productivity of peer firms that shape the beliefs about the systematic risk. Instead of keeping track of the entire history, however, it is sufficient to see the industry-wide statistics, $m_{b,t}^{\mathbb{I}}$ and $\sigma_{b,t}^{\mathbb{I}}$, for firm i to make the optimal decisions.

As in the q-theory literature, one of the major determinants of the investment policy is the marginal value of capital. To analyze the capital value, let us define the "ex-dividend" value of unit capital as $P_{i,t} \equiv E_t \left[M_{t,t+1} \frac{\partial V_{i,t+1}}{\partial K_{i,t+1}} \right]$. The following proposition characterizes the capital value.

Proposition 1: *The ex-dividend value of firm i's unit capital satisfies the recursive equation*

$$P_{i,t} = \mathbb{E}_t \left[M_{t,t+1} \left(\alpha A_{i,t+1} K_{i,t+1}^{\alpha-1} + \phi \left(\frac{I_{i,t+1}}{K_{i,t+1}} \right)^2 + (1-\delta) P_{i,t+1} \right) \right]. \tag{10}$$

The proof is provided in Appendix C. Intuitively, the marginal value of capital consists of the present value of production output in the next period and the continuation value of capital after the production. Using this recursive structure, we numerically solve the capital value.

Once the capital value is obtained, firm i's realized gross return from time t to $t+1$ is

$$R_{i,t+1} = \frac{\alpha A_{i,t+1} K_{i,t+1}^{\alpha-1} + \phi \left(\frac{I_{i,t+1}}{K_{i,t+1}} \right)^2 + (1-\delta) P_{i,t+1}}{P_{i,t}}, \tag{11}$$

because stock return on an all-equity-financed firm is identical to return on capital investment (Cochrane (1991) and Liu et al. (2009)).

3. Asset Pricing Implications

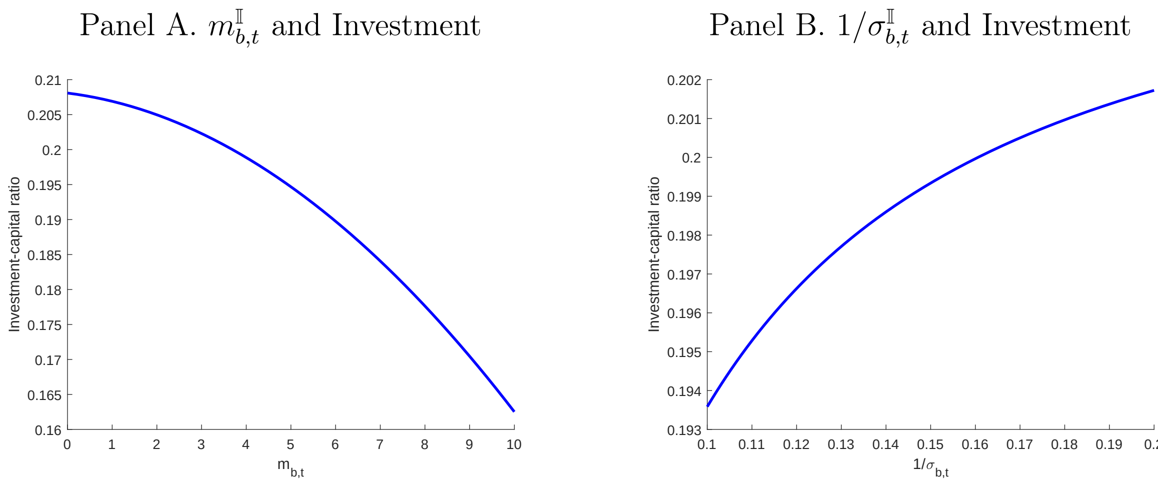
This section presents our model predictions that parameter uncertainty and subsequent learning about systematic risk cause unique patterns in firms' investment, valuation, and cost of capital. Considering that the predictions may depend on model parameters, we first calibrate the theoretical model so that it matches relevant moments of the empirical data. After putting forward these model-implied predictions, we test them empirically in the following section.

3.1. Calibration

Table 1 presents the calibration results. We first calibrate parameters characterizing the stochastic discount factor, following the procedure described by Kuehn and Schmid (2014). The preference parameters are set within the range of common values used in the literature; the rate of time preference is 0.996, the elasticity of intertemporal substitution is 2, and the coefficient of relative risk aversion is 10. Next, in the specification of the consumption process, we assume that the state space of the Markov chain consists of five different states of the economy. These five states have different conditional means and volatilities of consumption growth. We calibrate these consumption parameters and the transition matrix such that the Markov chain approximates the continuous-state consumption process of Bansal and Yaron (2004). The resulting states are named arbitrarily: state 1 denotes the economic state with the lowest mean and the highest volatility, while state 5 denotes the state with the highest mean and the lowest volatility. Under these parameter choices, the simulated moments for consumption dynamics and risk-free returns align with the empirical counterparts.

Regarding firm-level parameters, we set the capital share of production to be 0.65, based on evidence by Cooper and Ejarque (2003). Also, the capital depreciation rate is 3% per

Figure 1: The Learning-Related Variables and Firms' Investment



This figure presents the investment-capital ratio for different values of the learning-related variables: the posterior estimate $m^{\mathbb{I}}_{b,t}$ of the systematic risk and the precision $1/\sigma^{\mathbb{I}}_{b,t}$ of the estimate.

quarter as in Cooley and Prescott (1995). We set the productivity's exposure to the systematic risk to be 1.98. This number is the average of the risk exposure across industries from our estimation.[3] Note that this true value of the exposure is not observable by firms in our model, but crucial for generating the data that form the parameter learning. Finally, we choose the volatility of idiosyncratic shock to productivity to match the standard deviation of the investment-capital ratio.

3.2. Testable Model Predictions

Having calibrated the model, we now put forward testable predictions. The first two corollaries describe the regularities in firms' investment that the learning causes.

Corollary 1: *A firm's investment-capital ratio decreases with $m^{\mathbb{I}}_{b,t}$, the posterior estimate of the systematic risk for the industry to which the firm belongs.*

[3] We approximate the most up-to-date estimate as the true risk exposure for each industry. This is based on the theoretical property that the posterior estimate converges to the true parameter as the number of observations becomes infinitely large.

Corollary 2: *A firm's investment-capital ratio increases with $1/\sigma^{\mathbb{I}}_{b,t}$, the precision of beliefs about the systematic risk for the industry to which the firm belongs.*

These model predictions are depicted in Figure 1. The intuition behind our results is as follows. The estimate $m^{\mathbb{I}}_{b,t}$ captures the average of posterior beliefs about the systematic risk. Thus, a higher value of $m^{\mathbb{I}}_{b,t}$ means that future production outputs from new capital have, on average, a larger exposure to systematic risk, such that the marginal value of capital falls. Firms, therefore, invest less in equilibrium, as shown in Corollary 1 and Panel A of the figure.

Corollary 2 describes how the precision of the beliefs influences the investment. As illustrated in Panel B of the figure, a greater uncertainty $\sigma^{\mathbb{I}}_{b,t}$, or lower precision of the parameter beliefs, discourages the investment. Interestingly, this negative association is the product of the updating mechanism. Equation (8) shows that in cases of greater uncertainty about the systematic risk (high $\sigma^{\mathbb{I}}_{b,t}$), the new estimate is more sensitive to new observations of shocks to productivity and consumption. Accordingly, the risk estimate in the next period, $m^{\mathbb{I}}_{b,t+1}$, is more dispersed ex ante. This greater dispersion then lowers the average value of capital, due to the concave dependence of the capital value on the risk estimate as shown in Panel A.[4]

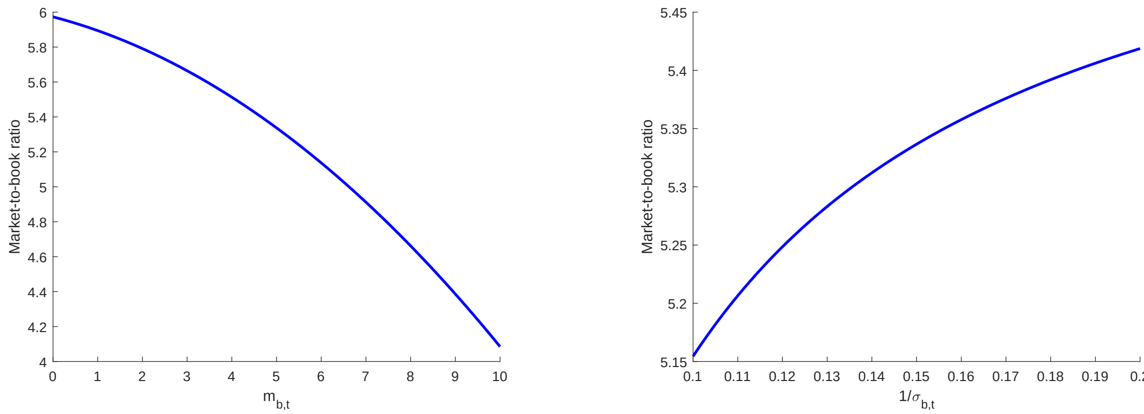
This learning mechanism also generates a unique pattern in investment-cash flow association. Conventionally, cash flows have been expected to influence the investment because a lack of cash flows tends to constrain firms financially (Fazzari et al. (1988) Hoshi et al. (1991)). On the contrary, in our framework, even unconstrained firms should respond to cash flows, or productivity in our setting, because they are informative about firms' systematic risk. In particular, our model predicts that the investment should respond negatively to a part of cash flow growth for which the systematic component accounts.

To elaborate upon this prediction, we revisit equation (8), which formulates the updating of the systematic risk. Recall that Y_t is growth in productivity, which is equivalent to cash

[4] In the model, the investment-capital ratio is a linear function of the ex-dividend price of capital. Thus, we can readily infer the relationship between the capital value and $m^{\mathbb{I}}_{b,t}$ from Panel A of Figure 1.

Figure 2: The Learning-Related Variables and Firms' Valuation

Panel A. $m^{\mathbb{I}}_{b,t}$ and Market-to-Book Ratio   Panel B. $\sigma^{\mathbb{I}}_{b,t}$ and Market-to-Book Ratio



This figure presents the market-to-book ratio for different values of the learning-related variables: the posterior estimate $m^{\mathbb{I}}_{b,t}$ of the systematic risk and the precision $1/\sigma^{\mathbb{I}}_{b,t}$ of the estimate.

flows scaled by capital stock, and X_t is the systematic component in this growth. This equation can be rewritten as:

$$m^{\mathbb{I}}_{b,t} - m^{\mathbb{I}}_{b,t-1} = \frac{\left(\sigma^{\mathbb{I}}_{b,t}\right)^2}{\nu^2} \sum_{i \in \mathbb{I}} Y^2_{t,i} \left(f^s_{i,t} - m^{\mathbb{I}}_{b,t-1} (f^s_{i,t})^2 \right) \approx \frac{\left(\sigma^{\mathbb{I}}_{b,t}\right)^2}{\nu^2} \sum_{i \in \mathbb{I}} Y^2_{t,i} \cdot f^s_{i,t} \tag{12}$$

where $Y_{t,i}$ is time-t growth in firm i's cash flow, and $f^s_{i,t}$ is the ratio of the systematic component to $Y_{t,i}$. This last approximation is based on the fact that usual values of $f^s_{i,t}$ are much smaller than one because the idiosyncratic component in cash-flow growth far outweighs the systematic component in the absolute size.[5] According to equation (12), a larger fraction of systematic component in the latest growth in cash flows leads firms to revise their respective risk estimates upward. This will, in turn, cause firms to reduce investment, resulting in the following corollary.

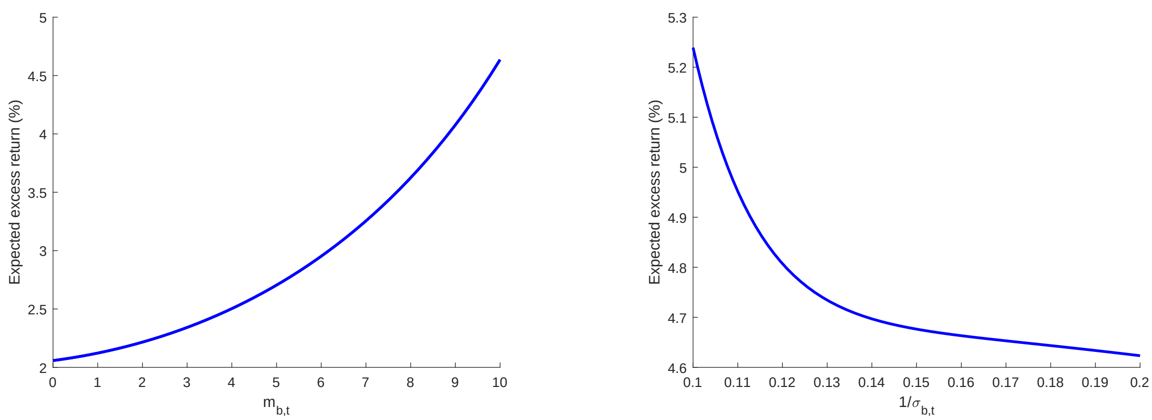
Corollary 3: *A firm's investment-capital ratio decreases with the fraction of cash-flow growth for which the systematic component accounts.*

[5]In the data, 10th percentile of $f^s_{i,t}$ is -0.133 and 90th percentile is 0.130.

Figure 3: The Learning-Related Variables and Stock Return

Panel A. $m^{\mathbb{I}}_{b,t}$ and Expected Return

Panel B. $1/\sigma^{\mathbb{I}}_{b,t}$ and Expected Return



This figure presents the expected return on stock in excess of the risk-free rate for different values of the learning-related variables: the posterior estimate $m^{\mathbb{I}}_{b,t}$ of the systematic risk and the precision $1/\sigma^{\mathbb{I}}_{b,t}$ of the estimate.

Notice that this prediction is particularly distinct from the conventional understanding that cash flows help investment due to alleviating financial constraints. In this conventional logic, cash-flow growth should be positively linked to the investment, whether the growth is systematic or idiosyncratic.

Next, we turn to the implications for valuation. The rationale we employ in Corollaries 1 and 2 can also be applied to the existing capital stock in the firm beyond the new capital. This leads to the following two hypotheses regarding the market-to-book ratio.

Corollary 4: *A firm's market-to-book ratio decreases with* $m^{\mathbb{I}}_{b,t}$.

Corollary 5: *A firm's market-to-book ratio increases with* $1/\sigma^{\mathbb{I}}_{b,t}$.

Figure 2 illustrates these predictions. Notably, the positive relationship in Corollary 5 between the market-to-book ratio and the precision of the beliefs is distinct from Pastor and Veronesi (2003)'s prediction that the valuation ratio increases with uncertainty with respect to a model parameter: mean profitability. This difference arises because the impact of parameter uncertainty differs depending on the form of the association between the model parameter and the firm value (i.e., whether the association is convex or concave). In the prior study, the firm value is a convex function of the mean profitability, so a rise in the uncertainty increases the average firm value due to Jensen's inequality effect. As to the parameter of systematic risk, however, our calibrated model predicts a concave dependence of the firm value on the parameter as shown in Panel A of figure 2. Thus, a mean-preserving spread of parameter beliefs lowers the average value.

The last dimension that we consider as evidence of the learning impact is the expected return, or cost of capital. Because the risk-exposure estimate is updated through the realized productivity that is unique for each industry, we conjecture that the risk estimate varies substantially across industries; we later confirm this observation empirically in Figure 5. The heterogeneity will naturally lead to a cross-sectional difference in the expected returns.

Corollary 6: *The expected return on stock in excess of the risk-free rate increases with* $m^{\mathbb{I}}_{b,t}$.

Corollary 7: *The expected return on stock in excess of the risk-free rate decreases with* $1/\sigma^{\mathbb{I}}_{b,t}$.

In Figure 3, we describe these model predictions. In Corollary 6, the return dependence on $m^{\mathbb{I}}_{b,t}$ is intuitive. When perceiving a greater average of systematic-risk exposure, market participants require a higher return on firms' assets.

With respect to Corollary 7, why is the expected return positively associated with uncertainty about the systematic risk? We find that this parameter uncertainty creates the additional risks that the market penalizes. Specifically, the updating mechanism in equation (8) uncovers that revisions in the risk estimate correlate to consumption shock as described in the following lemma.

Lemma 1: *The time-t covariance between the change in the risk estimate,* $m^{\mathbb{I}}_{b,t+1} - m^{\mathbb{I}}_{b,t}$*, and consumption shock,* $\sigma_c(\omega_t)\eta_{t+1}$*, is* $-\frac{n(\sigma^{\mathbb{I}}_{b,t})^2\sigma_c(\omega_t)^4}{2\nu^2}E_t\left[(b^{\mathbb{I}})^2\right]$.

The proof is provided in Appendix D. This negative covariance between an increase in the risk estimate and consumption shock eventually enlarges the systematic risk of the capital return. Recall that the capital price decreases with the risk estimate. Hence, a decrease in the risk estimate, $m^{\mathbb{I}}_{b,t+1} - m^{\mathbb{I}}_{b,t} < 0$, which is likely to happen when $\sigma_c(\omega_t)\eta_{t+1} > 0$, results in a positive return on capital, all else being equal. According to Lemma 1, this positive covariance between the return and the consumption is amplified by the standard error of the systematic risk $\sigma^{\mathbb{I}}_{b,t}$. Therefore, a larger uncertainty about the parameter causes market participants to require a higher return.

4. Empirical Analysis

4.1. Data

Our data consist of annual observations for non-financial and non-utilities firms on Compustat and CRSP for years 1952-2017. We choose the annual frequency to minimize a seasonality impact on corporate investment in our analysis. Among our firm-year observations, we exclude data points with negative or missing sales revenue or total assets lower than \$10 millions, which results in a total of 121,279 observations.

Firm-level variables are measured in standard ways in the literature. Firm size is defined as the natural log of total assets (AT). The investment-capital ratio is capital expenditures (CAPX) divided by the beginning-of-period capital stock (PPENT). We measure productivity as the calibrated model implies; the productivity is operating profits (OIBDP) divided by the capital stock raised to the power of 0.65. The book leverage ratio is the sum of debt in current liabilities (DLC) and long-term debt (DLTT) divided by AT. Cash flow is operating profits (OIBDP) divided by the beginning-of-period AT. For Tobin's q and the market-to-book ratio, we follow the measurement of Erickson et al. (2014). The numerator of Tobin's q is DLTT plus DLC plus market equity minus current assets (ACT), in which the market equity is the product of common shares outstanding (CSHO) and stock price (PRCC). The denominator of Tobin's q is gross capital stock (PPEGT). The numerator of the market-to-book ratio is AT plus the market equity minus book common equity (CEQ) minus deferred taxes (TXCB). The denominator of the market-to-book ratio is AT. For return on equity, we follow the measurement of Pastor and Veronesi (2003). Earnings are income before extraordinary items available to shareholders (IBCOM), plus deferred taxes from income statements (TXDI), plus investment tax credits (ITCI). Book equity value is stockholders' equity (SEQ), plus deferred taxes and investment tax credit from balance sheets (TXDITC), minus the book value of the preferred stock (PSTKRV). A firm's age is measured by the log of the number of years since the firm's stock price first appeared on CRSP.

We measure firms' financial constraints as in Whited and Wu (2006). Each firm's cash flows, dividend, leverage ratio, total assets, and sales growth are aggregated to generate the composite index (hereafter, referred to as the WW-index).

Our analysis requires an industry classification. Following the standard in the literature, we identify industries using either four-digit SIC or four-digit NAICS codes. In addition, we employ the text-based classification recently developed by Hoberg and Phillips (2010). This classification is based on product similarity among firms that is measured through a text-based analysis of 10-K filings. This text-based network industry classification system (hereafter, referred to as TNIC) is obtained from the Hoberg-Phillips Data Library.

Once industries are defined, we calculate the Herfindahl index to measure the level of competition among industry constituents. To obtain the index, we first calculate the market share of each constituent using sales revenue (SALE) and sum the squared shares across the constituents. The final index is the reciprocal of the sum, so a higher value indicates a higher level of competition.

We estimate the time-t belief about industry $\mathbb{I}$'s systematic risk through the regression analysis, as formulated in equation (7). The dependent variable of the regression is the collection of the up-to-date realized productivity for every industry constituent. Regressing the dependent variable onto the corresponding consumption shocks, we obtain the regression coefficient $m^{\mathbb{I}}_{b,t}$, which is approximately equal to the posterior mean of beliefs about the systematic risk. The precision of the parameter beliefs $1/\sigma^{\mathbb{I}}_{b,t}$ is computed recursively as the equation suggests. The parameter uncertainty before the first observation, $\sigma^{\mathbb{I}}_{b,0}$, is arbitrarily set at 20.

We also consider an alternative estimation of firms' systematic risk that uses each firm's own history only for reasons we discuss in section 4.3.4. To determine the risk estimate $m^i_{b,t}$ and its precision $1/\sigma^i_{b,t}$ implied by this individual learning, we regress a firm's up-to-date productivity to consumption shock, similarly to the collective learning. The only difference from the collective learning is that the individual learning does not use peers' observation to update the parameter.

These regressions require the time-series of the consumption shock. As the aggregate state of the economy shifts according to the Markov chain, we first need to identify the economic state for each time. For the identification, we use the quarterly time-series of consumption on non-durables and services from the Federal Research Economic Data. We then apply the Bayesian filtering technique as in Cappe et al. (2011) to these observations, so we may obtain the probability distribution of consumption shock for each quarter. By aggregating into annual frequency and computing the average, we are finally able to estimate the annual time series of consumption shock.

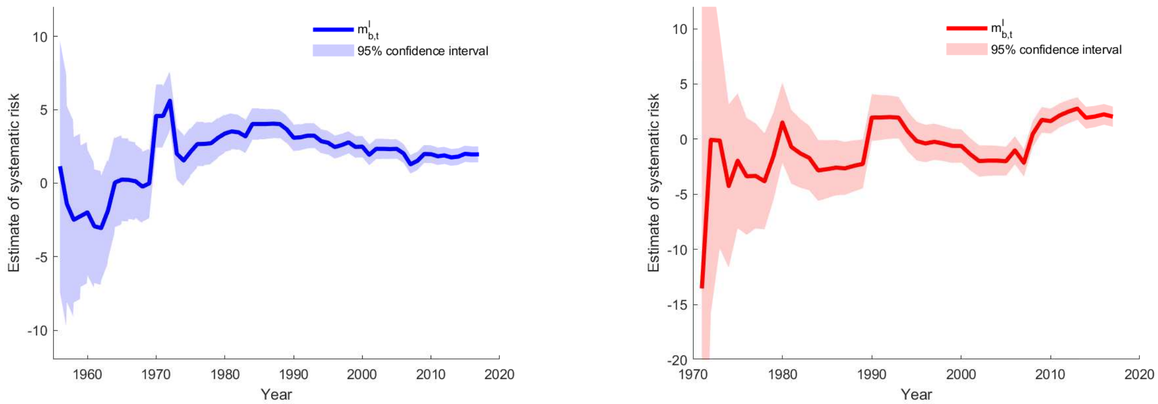
In addition, we measure firms' exposures to risk factors identified by Fama and French (2015). To determine each firm's time-t risk exposures, we regress all of a firm's monthly returns up to time t onto the time-series of the risk factors from Kenneth French's website. Using an expanding-window estimation allows us to make the measurement compatible to that of $m^{\mathbb{I}}_{b,t}$.

4.2. A Look at Posterior Estimates of Systematic Risk

Prior to the empirical tests, we first present the properties of the systematic-risk estimates. In Figure 4, we plot the time-series of the posterior estimate of the risk for these selected industries: Aircraft Engines and Engine Parts (SIC 3724) and Air Transportation (SIC 4522). We choose these two industries as examples because their risk exposures are almost the same, according to the most recent estimate in 2017: 1.96 for Aircraft Engines and 2.02 for Air Transportation. Despite the similarity in these recent values, their historical paths of the

Figure 4: Examples of the Estimated Systematic Risk

Panel A. Aircraft Engines (SIC 3724) Panel B. Air Transportation (SIC 4522)



This figure presents the time-series of the estimated systematic risk for selected industries (in this case, Aircraft Engines and Engine Parts (SIC 3724) and Air Transportation (SIC 4522)). The shaded areas indicate the 95% confidence interval of the estimates from the standard error of the posterior estimates as described in equation (8).

parameter updates are strikingly different, especially in the early years of the industries. For example, the first estimate of risk exposure is 0.05 for Aircraft Engines, whereas it is -13.5 for Air Transportation. This distinction, however, is not surprising because the parameter is updated through the realized productivity that is unique to each industry. As a result, the learning process is remarkably idiosyncratic, although the two industries may have similar true exposures as the recent estimates suggest.

Focusing on each industry, we find that the estimates display substantial time-variations. The systematic-risk estimate for Aircraft Engines changes from -3.05 to 5.60. Quantitatively, the time series have a standard deviation of 1.90, which is similar to its mean of 1.91. In the entire sample, each industry has, on average, a standard deviation of 4.33, confirming variations in the risk estimate through time. Furthermore, as the confidence interval in the plot indicates, the precision of the estimate improves gradually due to the growing dataset that serves as reference for the learning.

Moreover, we find that the risk estimates vary significantly in a cross-section. Figure 5 shows the distribution of the estimates across industries. For the purpose of highlighting the learning impact on the cross-sectional dispersion, we plot two histograms: One shows the distribution of the risk estimates from five-years worth of data for each industry, and the other shows the distribution of the estimates from fifty-years' worth of data.

The figure reveals that the early estimate with respect to the five-years' worth of data varies significantly across industries. In this case, the parameter learning is based on just a few observations, so the resulting estimator has a large standard error; we therefore observe rather extreme values such as 40 or -40. On the other hand, once a large number of observations are obtained, the parameter estimator becomes more precise. This makes the distribution of the risk estimate more concentrated, as documented by the histogram of the estimates from the fifty-years' worth of data. Nonetheless, the risk exposure from the long data still has non-trivial dispersion. In the cross-section, the standard deviation is 3.86, comparable to the mean of 2.35. This finding underscores the necessity of learning about the parameter at the industry level.

In summary, we find that each industry has uniquely updated its respective risk exposure. Considering the uniqueness of the learning path, a finding of firm observables responding to this industry-specific history offers strong evidence that the learning takes place in practice. We empirically test our model predictions in the following section.

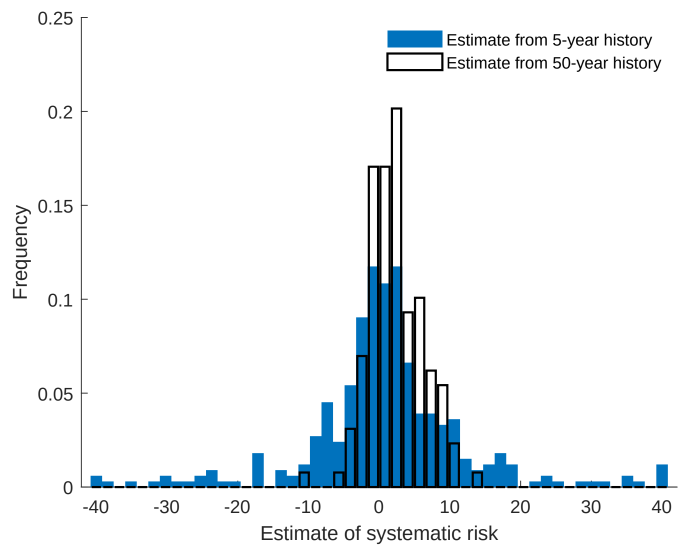
4.3. Empirical Tests of Model Implications

4.3.1. Implications for Investment

We now test whether capital investment exhibits the regularities caused by learning. If firms, in practice, learn about their exposure to systematic risk, we expect their respective investments to respond to the learning-related variables as in Corollaries 1 and 2. To test this hypothesis, we conduct the following predictive regression:

$$\text{INVEST}_{i,t} = \alpha_i + \beta_1 \times m^{\mathbb{I}}_{b,t-1} + \beta_2 \times \left(1/\sigma^{\mathbb{I}}_{b,t-1}\right) + \gamma \times \text{Controls} + \epsilon_{i,t} \tag{13}$$

Figure 5: Industry Age and the Distribution of Estimated Systematic Risk



This figure presents the cross-sectional distribution of the systematic-risk estimates across industries. Plotted are two histograms: one shows the distribution of the estimates from five-years' worth of data since the inception of each industry, and the other shows the distribution of the estimates from fifty-years' worth of data.

for which $\text{INVEST}_{i,t}$ denotes the firm i's investment-capital ratio. Other controls include variables that have been found in the literature to affect the investment: namely, firms' size, age, Tobin's q, cash flow, leverage ratio, and the indicator of financial constraints and industries' Herfindahl index.

In Table 3, we report our regression results. In specification (1), we use four-digit SIC codes to identify industries and calculate the learning-related variables accordingly. Our main finding is that capital investment responds negatively to shifts in $m_{b,t}^{\mathbb{I}}$, with a strong significance documented by the t-statistic of -5.20.

Furthermore, this association is economically significant, as the coefficient estimate suggests that a rise in the systematic-risk estimate by one standard deviation decreases investment by 9.5% (i.e., the annual investment-capital ratio changes from 0.217 to 0.196). Considering that the firm fixed effect is taken into account in this regression, this negative coefficient reveals the time-series response of firms to changes in $m_{b,t}^{\mathbb{I}}$. We therefore interpret this as evidence that firms reduce (raise) capital investment when they update their beliefs about the systematic risk upward (downward).

Moreover, this negative association persists under alternative industry classifications. In specifications (2) and (3), we refer to NAICS or TNIC, instead of SIC, to identify industry peers and estimate industries' risk exposure accordingly. When we do so, we find that $m^{\mathbb{I}}_{b,t}$ continues to be a negative predictor of the investment at a 1% level of significance; the t-statistic of NAICS (TNIC) measure is -4.63 (-2.91). All of these findings strongly support Corollary 1.

Beyond the point estimate, the precision of the parameter beliefs also predicts the investment, confirming Corollary 2. In all of the specification (1) through (3), we find that the coefficient on $1/\sigma^{\mathbb{I}}_{b,t}$ is positive and statistically significant with t-statistics that range from 1.69 to 5.13. In the economic magnitude, an improvement in precision of the beliefs by one standard deviation raises investment by 7.2% (e.g., the annual investment-capital ratio increases from 0.217 to 0.233). Consistent with the model prediction, firms indeed invest more in practice as their beliefs about the risk exposure become more precise.

One might suspect that the response to the parameter precision is due to effects distinct from the learning. According to equation (8), the precision increases monotonically with the number of constituent firms in an industry. This number of firms may then reflect the amount of competition within the industry, which potentially influences capital investment as found by Ghosal and Loungani (1996). To address this alternative explanation, we include the Herfindahl index in the regression. The coefficient on the Herfindahl index is found positive, suggesting that firms increase investment when facing heightened competition. More importantly, after controlling for this competition mechanism, the precision is significant for predicting investment. This finding corroborates the model prediction, apart from the force of competition, that firms factor in the parameter precision in making investment decisions.

Next, we examine another regularity that is dictated by the mechanism of updating systematic risk. As described in Corollary 3, this learning mechanism causes a peculiar pattern in investment-cash flow association; firms reduce investment when the systematic component accounts for a larger part in cash-flow growth. Interestingly, this prediction counters the conventional understanding that cash flow-growth, whether it is systematic or idiosyncratic, alleviates financial constraints so thus helps investment.

To test this model prediction, we slightly modify the regression equation (13). We include both the two-year lagged estimate of cash flow and its growth rate from $t-2$ to $t-1$. This setting is equivalent to controlling for the one-year lagged estimate of cash flow.

Table 4 reports the regression results. First, specification (1) confirms that the investment is positively associated with the overall growth in cash flow, consistent with the literature. Next, specifications (2) through (4) uncover the distinctive impact of the systematic component in the cash-flow growth. The fraction of systematic component negatively predicts the investment with 1% level of significance for all of the specifications. Notice that this empirical pattern is uniquely predicted by the learning mechanism and cannot be explained by the other cash-flow effects noted in the literature. This novel regularity further strengthens our proposition that decision-makers engage in learning about the systematic risk.

4.3.2. Implications for Valuation

We now shift our focus to firms' valuation as another dimension that is likely to manifest the learning process. Specifically, we test Corollaries 4 and 5 using the regression specification,

$$\mathrm{MB}_{i,t} = \alpha_i + \beta_1 \times m^{\mathbb{I}}_{b,t-1} + \beta_2 \times \left(1/\sigma^{\mathbb{I}}_{b,t-1}\right) + \gamma \times \text{Controls} + \epsilon_{i,t} \tag{14}$$

where $\mathrm{MB}_{i,t}$ is the firm i's market-to-book ratio. Controls variables are firms' size, age, return on equity, and leverage ratio and industries' Herfindahl index, which the literatures has noted to influence the valuation ratio.

Table 5 reports the regression results. We find that the market-to-book ratio also responds to the learning-related variables as hypothesized. The posterior estimate $m^{\mathbb{I}}_{b,t-1}$ of systematic risk negatively predicts $\mathrm{MB}_{i,t}$ at 1% level for all of the industry classifications. At the same time, the precision of the parameter beliefs, $1/\sigma^{\mathbb{I}}_{b,t}$, is a positive predictor of the valuation ratio at the 5% significance level in most specifications except for specification (1). These findings indicate that an upward or downward revision of the systematic-risk estimate through the learning leads to the opposite fluctuation in the firm value. In addition, the market evaluates firms higher as the learning alleviates the uncertainty about the systematic risk. This evidence corroborates Corollaries 4 and 5.

In summary, we establish that empirical data for both investment and valuation display the time-series regularities caused by the collective learning about the systematic risk. Our evidence strongly suggests that decision-makers, both inside and outside firms, constantly update their beliefs about the firms' risk exposure.

4.3.3. Implications for Implied Cost of Capital

The beliefs about systematic risk influence investment and valuation in a concerted way due to its impact on the discount rate. To see this connection more directly, we here examine the empirical link from the parameter beliefs to the cost of capital. In the spirit of the empirical asset pricing literature, we focus on whether the learning-related variables can explain the cross-section of the expected returns, testing Corollaries 6 and 7.

In this test, we measure the cost of capital utilizing accounting data as suggested by Hou et al. (2012). The rationale for choosing the implied cost of capital over realized returns is two fold. First, realized returns are a noisy proxy for the discount rate, as pointed out by Blume and Friend (1973) and Elton (1999). Second, it is more sensible to link the time-t parameter beliefs to the implied cost of capital that we can also measure in a snapshot at time t. On the contrary, the realized returns from time t to $t+1$ are from the valuation of firms at two different points in time, which reflect different beliefs. This fact complicates the empirical test.

Let $\text{ICC}_{i,t}$ denote the annualized implied cost of capital for firm i. We cross-sectionally regress the time-t estimate of the implied cost of capital onto the learning-related variables:

$$\mathrm{ICC}_{i,t} = \lambda_{0,t} + \lambda_{1,t} \times m^{\mathbb{I}}_{b,t} + \lambda_{2,t} \times \left(1/\sigma^{\mathbb{I}}_{b,t}\right) + \epsilon_{i,t}. \tag{15}$$

This cross-sectional regression is repeated every year and we calculate time-series average of coefficient estimates as in Fama and MacBeth (1973).

Table 6 presents the regression results. In specifications (1) through (3), the cost of capital is regressed only on the posterior estimate of risk exposure and its precision from the collective learning. We first find that the parameter precision $1/\sigma^{\mathbb{I}}_{b,t}$ is strongly negatively linked to the cost of capital at 1% or 5% significance level. Consistent with Corollary 7, market participants require a lower return on a firm's assets, when they are more certain about the firm's systematic risk. Economically, an increase in the parameter precision by one standard deviation reduces the annualized cost of capital by 0.4%.

Furthermore, we find that the systematic-risk estimate is positively associated with the cost of capital. For all of the industry classifications, this positive connection is statistically significant at 1% or 5% level, lending strong support for Corollary 6. This association is also economically significant; an increase in the risk estimate by one standard deviation raises the annualized cost of capital by 0.9%. This finding reveals that market participants respond to a cross-sectional variation in the systematic-risk estimate and they indeed require a higher return on firms with a larger risk estimate.

In specifications (5) through (7), we additionally control for other risk factors noted by Fama and French (2015). It appears that the predictive power of the learning-related variables is robust to the inclusion of additional factors in most specifications except for the precision impact in specification (5). This robustness implies that the learning variables convey return-relevant information that are not captured by the risk factors that the literature has noted so far.

Although the above result may seem obvious, prior studies have reported mixed empirical results on whether the consumption beta can explain cross-sectional differences in stock returns. The association is found positive in Bansal et al. (2005) and Da (2009), whereas it is insignificant in Lettau and Ludvigson (2001) and Boguth and Kuehn (2013). We conjecture that this inconsistency arises due to difficulty in the reliable identification of the consumption beta. Firms' cash flows, or productivity in our setting, are exposed to idiosyncratic shock that tends to outweigh the systematic component in magnitude. Considering this substantial noise that hampers the parameter identification, we can reliably estimate the risk exposure only when we use a sufficient number of observations, such as analyzing at portfolio level (Bansal et al. (2005)) or using a very long firm-level data including future earnings (Da (2009)).

Our finding supports this interpretation. Note that the risk estimates, which predict returns in specifications (1) through (3) and (5) through (7), are obtained from the collective learning where firms make use of their peers' observations so thus learn from larger dataset than otherwise. To contrast, we let firms learn from their own history only and test whether this alternative risk-estimate from the individual learning can explain the cost of capital. Specifications (4) and (8) report that the risk estimate through individual learning is incapable of predicting the cost of capital in the cross section. The discrepancy in the predictability highlights that utilizing collective observations is critical to determination of the systematic risk. Further comparison between collective and individual learning is provided in the following section.

4.3.4. Is the Learning Collective or Individual?

We have documented, theoretically and empirically, that the parameter beliefs about firms' systematic risk have a widespread impact on firms' decisions and valuations. In obtaining these findings, we make an assumption as to which data is relevant for the learning about a firm's risk exposure. Specifically, we assume that a target firm's peer observations are also informative about its risk profile, as constituents in one industry tend to have similar, if not identical, exposure to the systematic risk. Accordingly, this learning takes place at the collective level.

Meanwhile, one may conjecture another form of learning: that of individual learning, in which each firm's own history only is used without reference to peers. This alternative form is worth considering because the employed systems of industry classifications might be only loosely defined. In other words, even firms in the same industry might have differential business profiles upon a closer look, so peers' observations might not accurately reflect each other's systematic risk. If this is indeed the case, focusing instead on individual history would result in a more precise estimate. Considering this possibility, we test whether the parameter beliefs $m^i_{b,t}$ and $1/\sigma^i_{b,t}$ from individual learning predict the investment and the valuation.

In Tables 3 and 5, we use specification (4) to report our regression results. Surprisingly, it turns out that $m^i_{b,t}$ is insignificant for predicting both the investment-capital ratio and the market-to-book ratio with t-statistics of 0.69 and -0.31, respectively. This is in stark contrast to the 1% level of significance of $m^{\mathbb{I}}_{b,t}$ that is associated with collective learning.

Why do firms not respond to the risk exposure that is estimated from their own history? Certainly, the insignificance is not because of the theoretical design of the estimator; both individual and collective learning update the parameter through the least-squared estimation. Instead, the main difference between the two forms is the number of observations that enter the learning process. When we include peers' observations as the information source, this collective learning offers decision-makers much richer data from which to learn than does the individual learning. This seemingly mechanical difference plays a crucial role in this context of identifying the systematic risk, as the primary source of information, the realized growth in productivity, contains a substantial noise; in the calibrated model, the volatility of idiosyncratic shock (noise) to productivity is approximately 30 times as large as the volatility of the systematic component (signal). Due to the remarkably low signal-to-noise ratio, the reliable identification of the parameter requires a fairly large number of observations. Failing to do so, individual learning leads to an inaccurate risk-estimate that is incapable of explaining firms' decisions.

In summary, we confirm that updating beliefs about systematic risk is a collective process.

It is the particular estimate of risk exposure through collective learning, rather than the estimate that would otherwise emerge from individual history, that drives various empirical dimensions with respect to firm investment and market valuation.

5. Robustness Tests

5.1. Time-Variation in the True Value of Systematic Risk

One of the stylized assumptions in our model is that each industry's true exposure to the systematic risk is constant. What causes the risk estimate to fluctuate is not changes in the true risk profile but the parameter learning from growing observations. We conjecture that, if the true risk exposure itself fluctuates contrary to our assumption, then it is possible that the learning-based risk estimate may misleadingly capture a variation in the true exposure. To address this concern, we conduct alternative estimations that are designed to better detect dynamics, if they exist, in the true exposure. Provided this true risk exposure is time-varying, this new estimate based on dynamic risk would outperform our baseline estimate in predicting investment and valuation.

Recall that the baseline estimation of the systematic risk refers to the entire history of productivity since industry inception (expanding window); if true risk exposure is constant, all observations are equally informative. In contrast, if the true risk exposure varies, then recent observations would be more informative about the current level of risk than would old observations. As an approach focusing on this recent information, a rolling-window estimation would measure the risk more effectively. Accordingly, we try the rolling-window approach with alternative choices of the estimation window: three, five, and seven years. $m_{b,t}^{\text{3-yr rolling}}$, $m_{b,t}^{\text{5-yr rolling}}$, and $m_{b,t}^{\text{7-yr rolling}}$ are the resulting estimates of the systematic risk.

In addition, we also consider a rather extreme form of rolling-window estimation, an approach that uses the latest observations only; for example, this estimation only uses year-t productivity to measure the systematic risk in year t. This measurement is possible because each industry usually has a cross-section of realized productivity from multiple constituents.

Specifically, if there are sufficiently many constituents in an industry, taking a cross-sectional average of firm-level productivities will diversify away idiosyncratic components. What then remains in the cross-sectional average is the systematic component multiplied by the risk exposure. Dividing the average by the consumption shock, we obtain another risk exposure $m_{b,t}^{\text{cross-section in year } t}$.

In Table 7, we report our regression results. In the panel regressions reported in specifications (1) through (10), we find that the rolling-window-based risk estimates perform conspicuously poorly compared to the expanding-window-based risk estimates. The associations between investment and the rolling-window-based estimates are statistically weaker – t-statistics range from -1.97 to 0.78 – than the expanding-window-based estimate that is significant with the t-statistic of -4.24. We find a similar underperformance when we use the rolling-window-based estimates to explain the market-to-book ratio. Consistently, the alternative risk estimate from the cross-section, $m_{b,t}^{\text{cross-section in year } t}$, is only a weak predictor for investment and the market-to-book ratio, with t-statistics of -0.34 and -0.71, respectively.

Specifications (11) through (15) report the Fama-MacBeth regressions of the implied cost of capital on the alternative risk measures. We find that the cost of capital is only insignificantly connected to these measures that are designed to capture dynamic risk exposures; t-statistics range from 0.12 to 1.05. This finding further substantiates the outperformance of the expanding-window-based estimate.

Nevertheless, we do not interpret this evidence as a reason to rule out the possibility that the true risk exposure is dynamic. Specifically, we do not formally formulate any dynamics of the true risk exposure and indirectly infer about the risk profile from observed moments. Rather, we argue that this finding highlights that critical dimensions of firm observables, including capital investment, market-to-book ratio, and the cost of capital, are empirically more responsive to learning-induced changes in the risk estimate than changes in the true risk profile.

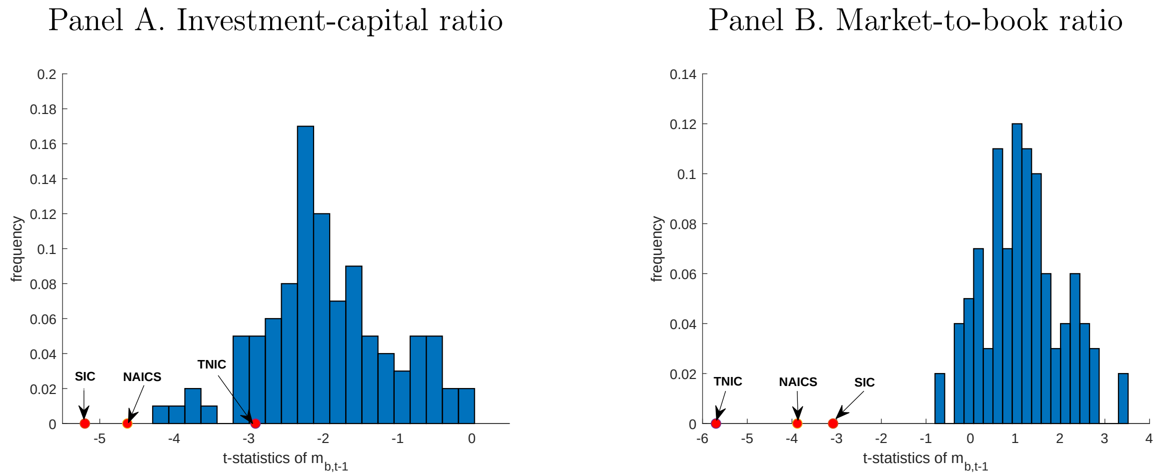
5.2. Does Industry Classification Matter?

In our main analysis, the identification of each firm's peers depends on the particular systems of industry classifications. In turn, we question the extent to which these classification systems help identify the best sources of information with respect to systematic risk. To be critical on the role of these industry classifications is reasonable for a couple of reasons. First, the industry definition derived from either the SIC or NAICS system might not be as robust as possible because their classification criteria are chosen in a rather arbitrary way [see Bhojraj et al. (2003) and Weiner (2005)]. Second, if the risk exposure is similar for all firms in the economy irrespective of industry,[6] any group of firms would be informative about the parameter. If this is indeed the case, the industry codes would contribute nothing in identifying the best information source.

Motivated by these possibilities, we examine whether the reference to industry classification is critical for the learning. Essentially, we compare the actual systems of industry classification (i.e., SIC, NAICS, TNIC) to random grouping of firms. We design the test as follows. First, we create 392 hypothetical industries, so we may match the total number of industries according to the four-digit SIC code. Second, each firm in Compustat is randomly classified into an industry, and each of these counter-factual industries has 38 constituents (i.e., the average number of firms for SIC industries). Once assigned, the industry code is fixed for each firm across time. Next, we let firms learn from past observations of their counter-factual peers and update the systematic risk accordingly. Importantly, firms here use the actual productivity data from Compustat, and we only simulate the grouping of firms is simulated. Lastly, we conduct regressions as in equations (13) and (14) to see whether the investment and valuation respond to this risk estimate from counter-factual peers. These steps constitute one simulated case, and we simulate 100 cases to obtain 100 regression coefficients for both investment and valuation. Our conjecture is that the risk estimate from the

[6] It may seem wild guess. However, we cannot rule this out because we do not observe the true values of systematic risk.

Figure 6: The Risk Estimates from Counter-Factual Peers and Firms' Investment and Valuation



This figure shows histograms of 100 estimates of the t-statistics of alternative measures of the systematic risk in investment and valuation regressions. The systematic risk is estimated from the observations of the counter-factual peers that are randomly grouped. The regression specifications are equations (13) and (14). The dots on the x-axis indicate t-statistics found in the regressions when we use actual industry classifications (i.e., SIC, NAICS, TNIC).

simulated pairs would also predict the firm observables with a similar significance our the baseline estimates, if the actual systems of industry classifications are of no use.

In Figure 5.2, we present the histograms of these 100 estimates of t-statistics for the risk estimate in our regressions. We find that the predictive power of the risk estimate from the counter-factual peers is noticeably lower than the baseline estimates. In the investment regression, in which the risk exposure is supposed to predict negatively, the t-statistics for the SIC-based and NAICS-based estimates are lower than all of the counter-factual estimates (which are larger in the absolute magnitude). Similarly, the t-statistic for the TNIC-based estimates are lower than 89 out of the 100 counter-factual estimates.

The significance of the actual industry classifications is even more pronounced in predicting the market-to-book ratio. The estimate from every actual classification outperforms all counter-factual estimates. In contrast, if firms learn from hypothetical industries, the resulting estimates only lead to an insignificant relation between the risk exposure and the market value; the median of the t-statistics of the counter-factual estimates is 1.11, which is inconsistent with the model prediction. These comparisons confirm that the actual systems of industry classification help determine which observations are informative about the systematic risk. As documented in Figure 5, industries are indeed heterogeneous with respect to the risk exposure. Therefore, without reference to the classification, firms are destined to use observations that are actually irrelevant to the parameter of their interest, thus rendering them incapable of updating the parameter properly.

These results emphasize that our main findings are coincidental. Despite some drawbacks that industry classification codes might have as noted in the literature, the classification helps firms identify their peers with similar risk exposure. With guidance, decision-makers can learn about firms' exposure to systematic risk much more efficiently than they could otherwise.

6. Conclusion

Parameter uncertainty is present for virtually any decision in financial markets. Among many parameters, firms' exposure to systematic risk is particularly hard to identify quickly. The realized productivity, which is informative of the parameter in the production economy, is primarily driven by idiosyncratic innovation that acts as noise hampering the parameter identification. Accordingly, a precise understanding of risk exposure requires a substantial number of observations. In this context, we propose a collective-learning framework in which decision-makers learn about a target firm's risk from its peers.

We present time-series and cross-sectional regularities caused by this learning. The empirical data strongly supports our model predictions. First, investment-capital ratio and the market-to-book ratio respond negatively to upward or downward revision in the systematic-risk estimate. Simultaneously, these two ratios increase with the precision of parameter beliefs. Second, the learning mechanism induces capital investment to respond negatively to a part of cash-flow growth for which the systematic component accounts. Lastly, the learning mechanism creates a cross-sectional dispersion in the cost of capital; the market commands a higher return on firms with a larger estimate of risk exposure or a lower precision of parameter beliefs. We further show that that it is this particular risk-estimate derived from the collective learning, rather than individual learning, that predicts the crucial dimensions of firm observables. Furthermore, we find that firms' real decisions and market valuations appear to be more responsive to learning-induced changes in the risk estimate than fluctuations in the true risk profile.

Nevertheless, our study does not provide a complete picture of how learning interacts with firms' decisions. A possible extension of our study is to incorporate firms' endogenous entry or exit. The decision on entry or exit, which will change the dataset for the learning, is likely to correlate with the aggregate state of the economy. This correlation, in turn, could amplify or mitigate the risk associated with the parameter uncertainty. We leave the exploration of this question to a future study.

Table 1: Calibration Results

This table presents the calibrated model parameters and the resulting moments on the simulated firm panel. The parameters in Panel A are used to simulate the quarterly time-series of the aggregate state of the economy and the firm-level variables. All moments in panel B are annualized.

Panel A. Parameters

Description	Parameter	Value
Rate of time preference	β	0.996
Relative risk aversion	γ	10
Elasticity of intertemporal substitution	ψ	2
Unconditional mean of consumption growth	g	0.005
Capital share of production	α	0.65
Depreciation rate	δ	0.03
Exposure to systematic risk	b	1.98
Volatility of idiosyncratic shock to productivity	ν	0.285
Coefficient for investment-adjustment costs	ϕ	6.5

Panel B. Moments

Moment	Data	Model
Average consumption growth	0.019	0.020
Volatility of consumption growth	0.022	0.025
Average risk-free rate	0.009	0.019
Volatility of risk-free rate	0.010	0.003
Average investment-capital ratio	0.211	0.204
Volatility of investment-capital ratio	0.261	0.354
Average stock return	0.072	0.054
Volatility of stock return	0.429	0.360

Table 2: Summary Statistics

This table presents the descriptive statistics of the annualized variables. $m^{\mathbb{I}}_{b,t}$ is the time-t estimate of industry $\mathbb{I}$'s systematic risk, and $1/\sigma^{\mathbb{I}}_{b,t}$ is the precision of the risk estimate. $m^{i}_{b,t}$ and $1/\sigma^{i}_{b,t}$ are the learning-related variables from individual learning. $\text{ICC}_{i,t}$ is firm i's implied cost of capital at time t. $\text{HHI}_{\mathbb{I},t}$ is industry $\mathbb{I}$'s Herfindahl index. $\beta^i_{\text{SMB},t}$, $\beta^i_{\text{HML},t}$, $\beta^i_{\text{RMW},t}$ and $\beta^i_{\text{CMA},t}$ are firm i's exposures to the Fama and French (2015)'s risk factors.

Variable	Mean	Std.Dev.	25%	50%	75%
$m^{\mathbb{I}}_{b,t}$ (SIC)	1.694	5.958	-0.911	1.551	4.355
$m^{\mathbb{I}}_{b,t}$ (NAICS)	1.773	4.987	-0.355	1.571	3.979
$m^{\mathbb{I}}_{b,t}$ (TNIC)	2.307	5.968	-0.915	1.807	5.266
$m^{i}_{b,t}$	2.186	16.45	-4.829	0.851	8.027
$1/\sigma^{\mathbb{I}}_{b,t}$ (SIC)	1.305	0.805	0.743	1.146	1.641
$1/\sigma^{\mathbb{I}}_{b,t}$ (NAICS)	1.702	0.960	0.953	1.611	2.290
$1/\sigma^{\mathbb{I}}_{b,t}$ (TNIC)	0.676	0.557	0.265	0.517	0.921
$1/\sigma^{i}_{b,t}$	0.235	0.132	0.116	0.220	0.328
Investment-capital $\text{ratio}_{i,t}$	0.217	0.242	0.130	0.211	0.368
Market-to-book $\text{ratio}_{i,t}$	1.569	1.375	0.951	1.207	1.736
$\text{ICC}_{i,t}$	0.121	0.385	0.033	0.065	0.126
$\text{Size}_{i,t}$	5.599	1.192	4.202	5.405	6.852
$\text{Cashflow}_{i,t}$	0.116	0.065	0.069	0.105	0.152
$\text{Leverage}_{i,t}$	0.247	0.198	0.091	0.231	0.364
$\text{Q}_{i,t}$	2.618	4.619	0.341	0.867	2.457
$\text{Age}_{i,t}$	2.404	1.042	1.792	2.485	3.178
$\text{ROE}_{i,t}$	0.033	0.860	-0.021	0.100	0.188
WW $\text{index}_{i,t}$	-0.084	1.464	-0.226	-0.142	-0.046
$\text{HHI}_{\mathbb{I},t}$ (SIC)	6.985	7.254	2.948	4.806	8.400
$\text{HHI}_{\mathbb{I},t}$ (NAICS)	17.387	33.079	3.971	7.071	12.494
$\beta^i_{\text{SMB},t}$	0.733	0.894	0.157	0.641	1.200
$\beta^i_{\text{HML},t}$	0.103	1.152	-0.388	0.153	0.651
$\beta^i_{\text{RMW},t}$	-0.144	1.416	-0.666	$-9.49e^{-3}$	0.503
$\beta^i_{\text{CMA},t}$	-0.031	0.625	0.557	0.914	1.224

Table 3: Capital Investment and Learning-Related Variables

This table presents panel regressions of capital investment on its determinant. The regression specification is:

$$\text{INVEST}_{i,t} = \alpha_i + \beta_1 \times m^{\mathbb{I}}_{b,t-1} + \beta_2 \times \left(1/\sigma^{\mathbb{I}}_{b,t-1}\right) + \gamma \times \text{Controls} + \epsilon_{i,t}$$

for which $\text{INVEST}_{i,t}$ is firm i's investment-capital ratio. Industry $\mathbb{I}$'s systematic risk $m^{\mathbb{I}}_{b,t}$ and the precision of the parameter beliefs $1/\sigma^{\mathbb{I}}_{b,t}$ in specifications (1) through (3) are calculated based on four-digit SIC codes, four-digit NAICS codes, or the text-based classification system (TNIC). In specification (4), the systematic risk $m^{i}_{b,t}$ and its precision $1/\sigma^{i}_{b,t}$ are alternatively estimated through individual learning. The additional controls include firms' age, Tobin's q, size, leverage, cash flow, and the WW-index and industries' Herfindahl index. The standard errors are clustered by firms. The t-statistics are presented in parentheses below the parameter estimates. *, **, *** denote significance at the 10%, 5%, and 1% levels, respectively.

	(1)	(2)	(3)	(4)
$m^{\mathbb{I}}_{b,t-1}$(SIC)	-0.00345***			
	(-5.20)			
$1/\sigma^{\mathbb{I}}_{b,t-1}$(SIC)	0.0195***			
	(5.13)			
$m^{\mathbb{I}}_{b,t-1}$(NAICS)		-0.00351***		
		(-4.63)		
$1/\sigma^{\mathbb{I}}_{b,t-1}$(NAICS)		0.00538*		
		(1.69)		
$m^{\mathbb{I}}_{b,t-1}$(TNIC)			-0.000801***	
			(-2.91)	
$1/\sigma^{\mathbb{I}}_{b,t-1}$(TNIC)			0.00878**	
			(2.37)	
$m^{i}_{b,t-1}$				0.0000253
				(0.62)
$1/\sigma^{i}_{b,t-1}$				0.0740***
				(3.94)
$\text{Age}_{i,t-1}$	-0.0156***	-0.00715***	-0.0114***	-0.0104***
	(-9.38)	(-3.63)	(-4.58)	(-4.58)
$\text{Q}_{i,t-1}$	0.0157***	0.0149***	0.0167***	0.0149***
	(38.08)	(31.04)	(33.56)	(31.03)
$\text{Size}_{i,t-1}$	-0.0474***	-0.0433***	-0.0469***	-0.0443***
	(-27.17)	(-22.73)	(-20.84)	(-23.31)
$\text{Leverage}_{i,t-1}$	-0.143***	-0.116***	-0.121***	-0.115***
	(-17.07)	(-12.98)	(-11.33)	(-12.89)
$\text{Cashflow}_{i,t-1}$	0.647***	0.629***	0.543***	0.636***
	(37.47)	(32.84)	(24.79)	(33.22)
$\text{WW index}_{i,t-1}$	-0.000940*	-0.000100	-0.000217	-0.0000509
	(-1.71)	(-0.11)	(-0.22)	(-0.05)
$\text{HHI}_{\mathbb{I},t-1}$	0.00158***	0.00177***	0.00163**	0.00183***
	(3.64)	(3.87)	(2.95)	(4.02)
N	121,055	96,995	66,787	96,995
adj. R^2	0.172	0.142	0.169	0.142

Table 4: Capital Investment and Systematic Components in Cash-Flow Growth

This table presents panel regressions of firms' investment. The regression specification is:

$$\text{INVEST}_{i,t} = \alpha_i + \beta_1 \times g_{i,t-1}^{\text{cash flow}} + \beta_2 \times \left(\text{Systematic component in } g_{i,t-1}^{\text{cash flow}}\right) + \gamma \times \text{Controls} + \epsilon_{i,t}$$

for which $\text{INVEST}_{i,t}$ is firm i's investment-capital ratio. $g_{i,t-1}^{\text{cash flow}}$ is the growth rate from $t-1$ to t of firm i's cash flow. Systematic component in $g_{i,t-1}^{\text{cash flow}}$ is the fraction of the systematic component in the growth. The controls include firms' age, Tobin's q, size, leverage, cash flow, and the WW-index and industries' systematic risk $m_{b,t}^{\mathbb{I}}$, the precision of the parameter beliefs $1/\sigma_{b,t}^{\mathbb{I}}$, and the Herfindahl index. The standard errors are clustered by firms. The t-statistics are presented in parentheses below the parameter estimates. *, **, *** denote significance at the 10%, 5%, and 1% levels, respectively.

	(1)	(2)	(3)	(4)
$g_{i,t-1}^{\text{cash flow}}$	0.0634***	0.0634***	0.0621***	0.0551***
	(39.43)	(39.37)	(36.98)	(27.74)
Systematic component in $g_{i,t-1}^{\text{cash flow}}$		-0.0000285***	-0.0000310***	-0.0000290***
		(-3.36)	(-3.06)	(-3.72)
$\text{Cashflow}_{i,t-2}$	0.511***	0.511***	0.500***	0.452***
	(26.90)	(26.76)	(25.45)	(19.09)
$m_{b,t-2}^{\mathbb{I}}$ (SIC)		-0.000879 ***		
		(-3.00)		
$1/\sigma_{b,t-2}^{\mathbb{I}}$ (SIC)		0.0155***		
		(3.98)		
$m_{b,t-2}^{\mathbb{I}}$ (NAICS)			-0.00117***	
			(-3.84)	
$1/\sigma_{b,t-2}^{\mathbb{I}}$ (NAICS)			0.0129***	
			(3.97)	
$m_{b,t-2}^{\mathbb{I}}$ (TNIC)				-0.000702**
				(-2.50)
$1/\sigma_{b,t-2}^{\mathbb{I}}$ (TNIC)				0.00328
				(0.85)
$\text{Age}_{i,t-1}$	-0.00750***	-0.0113***	-0.0123***	-0.00811***
	(-4.83)	(-6.25)	(-6.31)	(-2.91)
$\text{Q}_{i,t-1}$	0.0158***	0.0158***	0.0159***	0.0165***
	(35.23)	(35.15)	(35.19)	(31.10)
$\text{Size}_{i,t-1}$	-0.0466***	-0.0483***	-0.0474***	-0.0456***
	(-25.86)	(-26.20)	(-25.00)	(-19.35)
$\text{Leverage}_{i,t-1}$	-0.143***	-0.144***	-0.134***	-0.121***
	(-17.88)	(-18.00)	(-16.36)	(-10.76)
$\text{WW index}_{i,t-1}$	-0.000830	-0.000819	-0.000811	-0.000144
	(-0.86)	(-0.84)	(-0.84)	(-0.15)
$\text{Herfindahnl}_{i,t-1}$	0.00232***	0.00192***	0.00198***	0.00190***
	(5.20)	(4.28)	(4.27)	(3.37)
N	108,383	108,163	98,114	61,710
adj. R^2	0.154	0.154	0.158	0.155

Table 5: Market-to-Book Ratio and Learning-Related Variables

This table presents panel regressions of the market-to-book ratio on its determinant. The regression specification is:

$$\text{MB}_{i,t} = \alpha_i + \beta_1 \times m^{\mathbb{I}}_{b,t-1} + \beta_2 \times \left(1/\sigma^{\mathbb{I}}_{b,t-1}\right) + \gamma \times \text{Controls} + \epsilon_{i,t}$$

for which $\text{MB}_{i,t}$ is firm i's market-to-book ratio. Industry $\mathbb{I}$'s systematic risk $m^{\mathbb{I}}_{b,t}$ and the precision of the parameter beliefs $1/\sigma^{\mathbb{I}}_{b,t}$ in specifications (1) through (3) are calculated based on four-digit SIC codes, four-digit NAICS codes, or the text-based classification system (TNIC). In specification (4), the systematic risk $m^{i}_{b,t}$ and its precision $1/\sigma^{i}_{b,t}$ are alternatively estimated through the individual learning. The additional controls include firms' age, size, leverage ratio, and return on equity and industries' Herfindahl index. The standard errors are clustered by firms. The t-statistics are presented in parentheses below the parameter estimates. *, **, *** denote significance at the 10%, 5%, and 1% levels, respectively.

	(1)	(2)	(3)	(4)
$m^{\mathbb{I}}_{b,t-1}$(SIC)	-0.0137***			
	(-3.08)			
$1/\sigma^{\mathbb{I}}_{b,t-1}$(SIC)	0.0399			
	(1.44)			
$m^{\mathbb{I}}_{b,t-1}$(NAICS)		-0.00688***		
		(-3.88)		
$1/\sigma^{\mathbb{I}}_{b,t-1}$(NAICS)		0.0571**		
		(2.35)		
$m^{\mathbb{I}}_{b,t-1}$(TNIC)			-0.00950***	
			(-5.70)	
$1/\sigma^{\mathbb{I}}_{b,t-1}$(TNIC)			0.0586**	
			(2.63)	
$m^{i}_{b,t-1}$				-0.0000614
				(-0.31)
$1/\sigma^{i}_{b,t-1}$				0.177
				(1.29)
$\text{Size}_{i,t-1}$	-0.190***	-0.172***	-0.203***	-0.169***
	(-16.38)	(-13.93)	(-14.18)	(-13.85)
$\text{Age}_{i,t-1}$	-0.0145	0.0372***	0.0142	0.0387***
	(-1.49)	(2.96)	(0.89)	(2.69)
$\text{ROE}_{i,t-1}$	0.105***	0.125***	0.0962***	0.126***
	(12.26)	(11.34)	(8.82)	(11.44)
$\text{Leverage}_{i,t-1}$	-0.353***	-0.377***	-0.358***	-0.378***
	(-6.73)	(-6.22)	(-5.05)	(-6.25)
$\text{HHI}_{\mathbb{I},t-1}$	-0.00387	-0.00445	0.00161	-0.00408
	(-1.25)	(-1.40)	(0.44)	(-1.30)
N	118458	94643	65770	95339
adj. R^2	0.040	0.030	0.037	0.029

Table 6: Cross-Section of the Implied Cost of Capital and Learning-Related Variables

This table presents the Fama-MacBeth regressions of firms' implied cost of capital. In the cross-section, we regress annualized cost of capital in year t ($\mathrm{ICC}_{t,i}$) on the estimate of systematic risk $m^{\mathbb{I}}_{b,t}$ and the precision of parameter beliefs $1/\sigma^{\mathbb{I}}_{b,t}$. Industry $\mathbb{I}$'s systematic risk $m^{\mathbb{I}}_{b,t}$ and the precision of the parameter beliefs $1/\sigma^{\mathbb{I}}_{b,t}$ are calculated based on four-digit SIC codes, four-digit NAICS codes, or the text-based classification system (TNIC). In specifications (5) through (8), we additionally control for the exposures to the risk factors identified by Fama and French (2015). They are the exposure to the size factor $<\beta^{\mathbb{I}}_{\mathrm{SMB},t}>$, the exposure to the value factor $<\beta_{\mathrm{HML},t}>$, the exposure to the profitability factor $<\beta_{\mathrm{RMW},t}>$ and the exposure to the investment factor $<\beta_{\mathrm{CMA},t}>$. *, **, *** denote significance at the 10%, 5% and 1% levels, respectively.

	(1) $\mathrm{ICC}_{i,t}$	(2) $\mathrm{ICC}_{i,t}$	(3) $\mathrm{ICC}_{i,t}$	(4) $\mathrm{ICC}_{i,t}$	(5) $\mathrm{ICC}_{i,t}$	(6) $\mathrm{ICC}_{i,t}$	(7) $\mathrm{ICC}_{i,t}$	(8) $\mathrm{ICC}_{i,t}$
$m^{\mathbb{I}}_{b,t-1}$ (SIC)	0.0015***				0.0010**			
	(2.95)				(2.22)			
$1/\sigma^{\mathbb{I}}_{t-1}$ (SIC)	-0.0044***				-0.0015			
	(-2.67)				(-1.37)			
$m^{\mathbb{I}}_{b,t-1}$ (NAICS)		0.0014***				0.0008**		
		(3.15)				(2.12)		
$1/\sigma^{\mathbb{I}}_{t-1}$ (NAICS)		-0.0071***				-0.0057***		
		(-5.22)				(-5.42)		
$m^{\mathbb{I}}_{b,t-1}$ (TNIC)			0.0010**				0.0008**	
			(2.55)				(2.58)	
$1/\sigma^{\mathbb{I}}_{t-1}$ (TNIC)			-0.0033**				-0.0038***	
			(-2.04)				(-2.85)	
$m^{i}_{b,t-1}$				0.0004				0.0003
				(1.55)				(1.14)
$1/\sigma^{i}_{t-1}$				-0.0528**				-0.0316**
				(-2.39)				(-2.46)
$\beta_{\mathrm{SMB},t-1}$					0.0071***	0.0074***	0.0059**	0.0085***
					(2.85)	(2.85)	(2.35)	(3.50)
$\beta_{\mathrm{HML},t-1}$					0.0169***	0.0183***	0.0193***	0.0250***
					(9.81)	(11.20)	(9.26)	(11.51)
$\beta_{\mathrm{RMW},t-1}$					-0.0051***	-0.0063***	-0.0073***	-0.0071***
					(-3.97)	(-4.95)	(-4.45)	(-4.93)
$\beta_{\mathrm{CMA},t-1}$					0.0084***	0.0096***	0.0097***	0.0107***
					(6.73)	(8.32)	(7.59)	(8.18)
N	71,544	77,481	55,664	67,396	71,544	77,481	55,664	67,396
adj. R^2	0.004	0.006	-0.002	-0.021	0.023	0.023	0.015	0.013

Table 7: Comparison of Expanding-Window with Rolling-Window Estimations

In the table, specifications (1) through (10) present panel regressions of firms' investment and valuation. The regression specification is:

$$\text{Dependent Variable}_{i,t} = \alpha_i + \beta \times m_{b,t-1} + \gamma \times \text{Controls} + \epsilon_{i,t}$$

for which the dependent variable is either firm i's investment-capital ratio or market-to-book ratio. The posterior estimate of systematic risk $m_{b,t}$ is measured by different approaches. $m_{b,t}^{\mathbb{I},\text{expanding}}$ is obtained from the expanding-window estimation, which uses the entire history since the industry inception. $m_{b,t}^{\mathbb{I},\text{3-yr rolling}}$ is obtained from the rolling-window estimation, which uses the industry history from year $t-2$ to t. Similarly, $m_{b,t}^{\mathbb{I},\text{5-yr rolling}}$ and $m_{b,t}^{\mathbb{I},\text{7-yr rolling}}$ are obtained from the history over the last five and seven years, respectively. $m_{b,t}^{\mathbb{I},\text{cross-section in year } t}$ is the cross-sectional mean of industry constituents' productivity in year t divided by the consumption shock. For each of investment and valuation regression, controls in equation (13) and (14) are included; they are firms' age, size, Tobin's q, leverage ratio, cash flow, the WW-index, and return on equity, and industries' Herfindahl index. The standard errors are clustered by firms. Specifications (11) through (15) present Fama-MacBeth regression of firms' implied cost of capital. For each year t, a cross-sectional regressions is conducted with the specification:

$$\text{ICC}_{i,t} = \lambda_{0,t} + \lambda_{1,t} \times m_{b,t}^{\mathbb{I}} + \gamma \times \text{Controls} + \epsilon_{i,t}.$$

We then report the time-series average of these coefficients. The t-statistics are presented in parentheses below the parameter estimates. *, **, *** denote significance at the 10%, 5%, and 1% levels, respectively.

	(1) $\text{INVEST}_{i,t}$	(2) $\text{INVEST}_{i,t}$	(3) $\text{INVEST}_{i,t}$	(4) $\text{INVEST}_{i,t}$	(5) $\text{INVEST}_{i,t}$	(6) $\text{MB}_{i,t}$	(7) $\text{MB}_{i,t}$	(8) $\text{MB}_{i,t}$	(9) $\text{MB}_{i,t}$	(10) $\text{MB}_{i,t}$	(11) $\text{ICC}_{i,t-1}$	(12) $\text{ICC}_{i,t-1}$	(13) $\text{ICC}_{i,t-1}$	(14) $\text{ICC}_{i,t-1}$	(15) $\text{ICC}_{i,t-1}$
$m_{b,t-1}^{\text{expanding}}$	-0.00206***					-0.00725**					0.0010***				
	(-4.24)					(-2.47)					(2.50)				
$m_{b,t-1}^{\text{3-yr rolling}}$		-0.000197**					-0.0000710					0.0001			
		(-1.97)					(-0.15)					(0.57)			
$m_{b,t-1}^{\text{5-yr rolling}}$			0.0000777					-0.000917*					$0.204e^{-4}$		
			(0.62)					(-1.69)					(0.12)		
$m_{b,t-1}^{\text{7-yr rolling}}$				0.000155					0.00182					0.0001	
				(0.78)					(1.59)					(0.44)	
$m_{b,t-1}^{\text{cross-section in year } t-1}$					-0.00000315					-0.0000309					0.0002
					(-0.34)					(-0.71)					(1.05)
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
N	63,238	63,238	63,238	63,238	63,238	63,102	63,102	63,102	63,102	63,102	54,180	54,180	54,180	54,180	54,180
adj. R^2	0.183	0.182	0.182	0.182	0.182	0.035	0.035	0.035	0.035	0.035	0.037	0.036	0.036	0.036	0.037

Appendix A. Wealth-Consumption Ratio

Let W_t denote the time-t wealth of the representative agent. The wealth satisfies the Euler equation,

$$W_t = \mathbb{E}_t \left[\beta^\theta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \left(\frac{S_{t+1}+1}{S_t} \right)^{-(1-\theta)} (C_{t+1} + W_{t+1}) \right]. \tag{A.1}$$

Dividing equation A.2 by time-t consumption and letting S_t denote W_t/C_t, we obtain the equation for the wealth-consumption ratio,

$$S_t^\theta = \mathbb{E}_t \left[\beta^\theta \left(\frac{C_{t+1}}{C_t} \right)^{1-\gamma} (S_{t+1}+1)^\theta \right]. \tag{A.2}$$

Appendix B. Least Square Estimator of b

The solution in the least-square estimation, equation (7), is

$$m_{b,t} = \left[\sum_{s=1}^{t} X_s X_s^T \right]^{-1} \left[\sum_{s=1}^{t} X_s Y_s^T \right] \tag{B.1}$$

As the number of observations becomes infinitely large, the estimates converges in probability to b as follows:

$$\begin{aligned}
m_{b,t} &\xrightarrow{p} \left[\mathbb{E}\left[X_t X_t^T \right] \right]^{-1} \mathbb{E}\left[X_t Y_t^T \right] && \text{(B.2)} \\
&\xrightarrow{p} \left[\mathbb{E}\left[X_t X_t^T \right] \right]^{-1} \mathbb{E}\left[b^{\mathbb{I}} X_t X_t^T + \sigma_c(\omega_{s-1}) \eta_s \quad \sum_{i=1}^{n} \left(\epsilon_{i,s} - \frac{(b^{\mathbb{I}})^2 \sigma_c(\omega_{s-1})^2}{2} \right) \right] && \text{(B.3)} \\
&\xrightarrow{p} b^{\mathbb{I}} && \text{(B.4)}
\end{aligned}$$

where equations (1), (5) and (6) are used to expand X_t and Y_t. Hence $m_{b,t}$ is a consistent estimator of $b^{\mathbb{I}}$.

Next, we derive the recursive updating equation (8). The time-t estimate can be expanded as

$$m_{b,t} = \left[\sum_{s=1}^{t} X_s X_s^T \right]^{-1} \left[b^{\mathbb{I}} \sum_{s=1}^{t} X_s X_s^T + \sum_{s=1}^{t} \sigma_c(\omega_{s-1}) \eta_s \quad \sum_{i=1}^{n} \left(\epsilon_{i,s} - \frac{(b^{\mathbb{I}})^2 \sigma_c(\omega_{s-1})^2}{2} \right) \right] \tag{B.5}$$

The expectation of the estimate conditional on time t is

$$E_t\left[m_{b,t}\right] = b^{\mathbb{I}} + \left[\sum_{s=1}^{t} X_s X_s^T\right]^{-1} \sum_{s=1}^{t} \left(\sigma_c(\omega_{s-1})\eta_s \quad \sum_{i=1}^{n} \left(-\frac{(b^{\mathbb{I}})^2 \sigma_c(\omega_{s-1})^2}{2} \right) \right) \tag{B.6}$$

where we use the fact the past consumption innovations η_s are observable, whereas the idiosyncratic shocks to productivity $\epsilon_{i,s}$ are not. The conditional variance is

$$\begin{aligned} \sigma_{b,t}^2 &= E_t\left[\left(m_{b,t} - E_t[m_{b,t}]\right)^2\right] \\ &= \left[\sum_{s=1}^{t} X_s X_s^T\right]^{-2} E_t\left[\left(\sum_{s=1}^{t} \sigma_c(\omega_{s-1})\eta_s \quad \sum_{i=1}^{n} \left(\nu\epsilon_{i,s} \right) \right)^2 \right] \\ &= \left[\sum_{s=1}^{t} X_s X_s^T\right]^{-2} \left[\sum_{s=1}^{t} X_s X_s^T\right] \nu^2 \\ &= \left[\sum_{s=1}^{t} X_s X_s^T\right]^{-1} \nu^2. \end{aligned} \tag{B.7}$$

The variance can be expressed recursively

$$\frac{1}{\sigma_{b,t}^2} = \frac{1}{\sigma_{b,t-1}^2} + \frac{X_t X_t^T}{\nu^2}. \tag{B.8}$$

To derive the updating equation for $m_{b,t}$, let Φ_t denote $\sum_{s=1}^{t} X_t X_t^T$ and Ψ_t denote $\sum_{s=1}^{t} X_t Y_t^T$. It then follows that $\Phi_t = \Phi_{t-1} + X_t X_t^T$ and that $\Psi_t = \Psi_{t-1} + X_t Y_t^T$. In addition, denoting the inverse of Φ_t by P_t, we can rewrite time-t estimate as

$$
\begin{aligned}
m_{b,t} &= P_t \Psi_t \\
&= P_t \left[\Psi_{t-1} + X_t Y_t^T\right] \\
&= P_t \left[m_{b,t-1}\Phi_{t-1} + X_t Y_t^T\right] \\
&= P_t \left[m_{b,t-1}(\Phi_t - X_t X_t^T) + X_t Y_t^T\right] \\
&= m_{b,t-1} - P_t X_t X_t^T m_{b,t-1} + P_t X_t Y_t^T \\
&= m_{b,t-1} + P_t X_t \left(Y_t^T - m_{b,t-1} X_t^T\right) \\
&= m_{b,t-1} + \frac{\sigma_{b,t}^2}{\nu^2} X_t \left(Y_t^T - m_{b,t-1} X_t^T\right)
\end{aligned} \tag{B.9}
$$

where we use the fact that $\sigma_{b,t}^2 = \left[\Phi_t\right]^{-1} \nu^2$.

Appendix C. The Ex-Dividend Price of Unit Capital

The first-order condition of the firm's problem given by equation (9) is

$$
-1 - 2\phi\left(\frac{I_t}{K_t}\right) + \underbrace{\mathbb{E}_t\left[M_{t,t+1}\frac{\partial V_{t+1}}{\partial K_{t+1}}\right]}_{\equiv P_{i,t}} = 0 \tag{C.1}
$$

In addition, from the Envelope theorem,

$$
\frac{\partial V_t}{\partial K_t} = \alpha A_t K_t^{\alpha-1} + \phi\left(\frac{I_t}{K_t}\right)^2 + (1-\delta)\underbrace{\mathbb{E}_t\left[M_{t,t+1}\frac{\partial V_{t+1}}{\partial K_{t+1}}\right]}_{\equiv P_{i,t}} \tag{C.2}
$$

Using equation (C.2), we can obtain the ex-dividend value of unit capital

$$
\begin{aligned}
P_{i,t} &= \mathbb{E}_t\left[M_{t,t+1}\frac{\partial V_{t+1}}{\partial K_{t+1}}\right] \\
&= \mathbb{E}_t\left[M_{t,t+1}\left(\alpha A_{t+1} K_{t+1}^{\alpha-1} + \phi\left(\frac{I_{t+1}}{K_{t+1}}\right)^2 + (1-\delta)P_{i,t+1}\right)\right] \\
&= \mathbb{E}_t\left[M_{t,t+1}\left(\alpha A_{t+1} K_{t+1}^{\alpha-1} + \frac{P_{i,t+1}^2}{4\phi} + \left(1-\delta-\frac{1}{2\phi}\right)P_{i,t} + \frac{1}{4\phi}\right)\right]
\end{aligned} \tag{C.3}
$$

where the last equality is obtained from substituting the investment-capital ratio from equation (C.1) for the marginal value of capital.

Appendix D. Covariance Between a Revision in the Risk Estimate and Consumption Shock

Rewriting the equation (8) to update time-$t+1$ estimate leads to

$$m^{\mathbb{I}}_{b,t+1} = m^{\mathbb{I}}_{b,t} + \frac{\left(\sigma^{\mathbb{I}}_{b,t+1}\right)^2}{\nu^2} X_{t+1}\left(Y^T_{t+1} - m^{\mathbb{I}}_{b,t} X^T_{t+1}\right).$$

To see how the updating equation relates to consumption shock, let's write more explicitly as follows:

$$\begin{aligned}
m^{\mathbb{I}}_{b,t+1} - m^{\mathbb{I}}_{b,t} &= \frac{\left(\sigma^{\mathbb{I}}_{b,t+1}\right)^2}{\nu^2}(\sigma_c\eta_{t+1})\sum_{i=1}^{n}\left(b^{\mathbb{I}}\sigma_c\eta_{t+1} - \frac{(b^{\mathbb{I}})^2\sigma_c^2}{2} + \nu\epsilon_{t+1} - m^{\mathbb{I}}_{b,t}\sigma_c\eta_{t+1}\right)\\
&= \frac{\left(\sigma^{\mathbb{I}}_{b,t+1}\right)^2}{\nu^2}(\sigma_c\eta_{t+1})\sum_{i=1}^{n}\left((b^{\mathbb{I}} - m^{\mathbb{I}}_{b,t})\sigma_c\eta_{t+1} - \frac{(b^{\mathbb{I}})^2\sigma_c^2}{2} + \nu\epsilon_{i,t+1}\right)\\
&= \frac{\left(\sigma^{\mathbb{I}}_{b,t+1}\right)^2}{\nu^2}\left(n(b^{\mathbb{I}} - m^{\mathbb{I}}_{b,t})(\sigma_c\eta_{t+1})^2 - n\frac{(b^{\mathbb{I}})^2\sigma_c^3\eta_{t+1}}{2} + (\sigma_c\eta_{t+1})\nu\sum_{i=1}^{n}\epsilon_{i,t+1}\right).
\end{aligned}$$

Next, let's calculate the covariance between $m^{\mathbb{I}}_{b,t+1} - m^{\mathbb{I}}_{b,t}$ and consumption shock $\sigma_c\eta_{t+1}$.

$$\begin{aligned}
cov_t\left(m^{\mathbb{I}}_{b,t+1} - m^{\mathbb{I}}_{b,t}, \sigma_c\eta_{t+1}\right) &= E_t\left[(m^{\mathbb{I}}_{b,t+1} - m^{\mathbb{I}}_{b,t})\sigma_c\eta_{t+1}\right] - E_t\left[m^{\mathbb{I}}_{b,t+1} - m^{\mathbb{I}}_{b,t}\right]\underbrace{E_t\left[\sigma_c\eta_{t+1}\right]}_{=0}\\
&= E_t\left[\frac{\left(\sigma^{\mathbb{I}}_{b,t+1}\right)^2}{\nu^2}\left(n(b^{\mathbb{I}} - m^{\mathbb{I}}_{b,t})(\sigma_c\eta_{t+1})^3 - n\frac{(b^{\mathbb{I}})^2\sigma_c^4\eta_{t+1}^2}{2} + (\sigma_c\eta_{t+1})^2\nu\sum_{i=1}^{n}\epsilon_{i,t+1}\right)\right]\\
&= E_t\left[\frac{\left(\sigma^{\mathbb{I}}_{b,t+1}\right)^2}{\nu^2}\left(n(b^{\mathbb{I}} - m^{\mathbb{I}}_{b,t})(\sigma_c\eta_{t+1})^3 - n\frac{(b^{\mathbb{I}})^2\sigma_c^4\eta_{t+1}^2}{2}\right)\right]\\
&\quad + \underbrace{E_t\left[\frac{\left(\sigma^{\mathbb{I}}_{b,t+1}\right)^2}{\nu^2}(\sigma_c\eta_{t+1})^2\nu\sum_{i=1}^{n}\epsilon_{i,t+1}\right]}_{=0}
\end{aligned}$$

where we utilize the fact $\epsilon_{i,t+1}$ is independent of η_{t+1} and $E_t\left[\sum_{i=1}^{n}\epsilon_{i,t+1}\right]=0$ in the last line. From the equation of updating precision,

$$\sigma_{b,t+1}^2=\frac{\sigma_{b,t}^2\nu^2}{\nu^2+n\sigma_{b,t}^2\sigma_c^2\eta_{t+1}^2}$$

Plugging this into the above covariance,

$$\begin{aligned}
cov_t\left(m_{b,t+1}^{\mathbb{I}}-m_{b,t}^{\mathbb{I}},\sigma_c\eta_{t+1}\right) &= E_t\left[\frac{\sigma_{b,t}^2}{\nu^2+n\sigma_{b,t}^2\sigma_c^2\eta_{t+1}^2}\left(n(b^{\mathbb{I}}-m_{b,t}^{\mathbb{I}})(\sigma_c\eta_{t+1})^3-n\frac{b^2\sigma_c^4\eta_{t+1}^2}{2}\right)\right]\\
&= E_t\left[n(b^{\mathbb{I}}-m_{b,t}^{\mathbb{I}})\frac{\sigma_{b,t}^2(\sigma_c\eta_{t+1})^3}{\nu^2+n\sigma_{b,t}^2\sigma_c^2\eta_{t+1}^2}\right]-E_t\left[\frac{n}{2}\frac{(b^{\mathbb{I}})^2\sigma_{b,t}^2\sigma_c^4\eta_{t+1}^2}{\nu^2+n\sigma_{b,t}^2\sigma_c^2\eta_{t+1}^2}\right]\\
&\approx \underbrace{E_t\left[n(b^{\mathbb{I}}-m_{b,t}^{\mathbb{I}})\frac{\sigma_{b,t}^2(\sigma_c\eta_{t+1})^3}{\nu^2}\right]}_{=0}-E_t\left[\frac{n}{2}\frac{(b^{\mathbb{I}})^2\sigma_{b,t}^2\sigma_c^4\eta_{t+1}^2}{\nu^2}\right]\\
&= -\frac{n\sigma_{b,t}^2\sigma_c^4}{2\nu^2}E_t\left[(b^{\mathbb{I}})^2\right]
\end{aligned}$$

where the approximation in the second last line is based on the fact that the size of idiosyncratic shock ν far outweighs the size of systematic shock σ_c.

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