A Generalized Heterogeneous Autoregressive Model using the Market Index

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This Version: December 2, 2019

Abstract

This paper shows that generalizing the heterogeneous autoregressive model (HAR) with realized (co)variances and semi-(co)variances from the index leads to more accurate volatility forecasts. To circumvent the effects of the market microstructure noise arising from using high sampling frequencies, we adopt noise-robust estimators for the realized (co)variances and develop novel noise-robust estimators for the semi-(co)variances. To explore the sampling frequency at which the forecasting gains are maximized, we adopt a mixed-sampling approach that iterates over several sampling frequencies of the stock and the index. Our analysis shows that gains are maximized at the combination of a low (high) frequency on the stock (index). We illustrate that the observed forecasting gains translates into economic gains such that a risk-averse investor is willing to pay up to 57 annual basis points by adopting a model specification that utilizes the index information.

***K*eywords:** Realized volatility; microstructure noise; pre-averaged estimators; semivariances; semicovariances; volatility forecasting; economic value; volatility-timing strategy.
***J*EL classification:** C22, C51, C53, C58, G11, G14.

◇We would like to thank Torben Andersen and Jia Li for helpful conversations on this topic. We are also grateful to participants at the 12^{th} International Conference in Computational Statistics and Financial Econometrics in PISA for helpful comments. The first author acknowledges the financial support of the Economic and Social Research Council (ESRC), Grant Number: ES/P000665/1.
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1 Introduction

The rapid growth in financial markets and the developments of new and complex financial products highlight the importance of developing tools to measure and assess the risks associated with these products. Volatility is one of the most important measures of risk, and since any evaluation procedure considers the level and riskiness of future pay-offs, it is particularly the forecasting of future volatility that is important for many investment decisions.

Over the last two decades, measures based on high-frequency data such as realized volatility (RV) were found to dominate traditional ARCH-type models, (see Bollerslev, 1986; Engle, 1982; Nelson, 1991, among others) and stochastic volatility models (see Ghysels et al., 1996; Taylor, 1986, 1994). Andersen et al. (2003) and Corsi (2009) show that RV is a more efficient estimator of return volatility, and that the long-term dependence of RV is well captured using a simple heterogeneous autoregressive (HAR) model.

In this paper, we propose a new class of realized volatility based forecasting models, by generalizing the basic HAR model with the (co)variances and semi-(co)variances from the market index (SPY-ETF) to form a more general ex-ante forecasting model. The use of the index can be motivated by a CAPM type-argument whereby the volatility of the stock is known to be driven by stock specific idiosyncratic factors as well as by systematic factors pertaining to the market portfolio.

Inspired by the success of the HAR model (Corsi, 2009) in capturing the long-term dependence feature of volatility, several studies have generalized the HAR model using endogenous and exogenous variables. For instance, Andersen et al. (2007); Corsi et al. (2010) explore the information content of jumps in volatility prediction showing decent out-of-sample improvements.[1] A bigger strand is formed by the exogenous variables, in which the literature have explored the use of implied volatility (see, Busch et al., 2011; Giot and Laurent, 2007, among others), macroeconomic variables (see, for instance Christiansen et al., 2012; Engle et al., 2007; Paye, 2012; Schwert, 1989, and the many references

[1]Other studies considering jumps as regressors are (Duong and Swanson, 2015; Hizmeri et al., 2019; Nolte and Xu, 2015; Patton and Sheppard, 2015, and the extensive references therein).

therein), and cross-sectional market volatility in Hwang and Satchell (2007), among other variables. These studies find that the use of exogenous variables provides sufficient information to improve model fit but only little or modest out-of-sample forecasting gains.

Against the unsuccessful use of external variables to aid volatility forecasting, our new HAR-X class of models allow more rapid reaction to volatility changes; i.e., HAR-X models achieve substantial forecasting gains by incorporating market index information in the form of (co)volatility.

We also extend the HAR-X class of models so that they can better capture the spontaneous changes and asymmetries in volatility by incorporating the information of negative and positive return (co)variance. We do so by estimating the so-called realized semivariances (Barndorff-Nielsen et al., 2010) and semicovariances (Bollerslev et al., 2019). Realized semivariances are decomposed using positive and negative returns, whilst the realized semicovariances are decomposed by four separate components: two based on concordant signed high-frequency returns and two based on discordant high-frequency returns. These measures allow for more refined responses to positive and negative return shocks than threshold "leverage effect" terms traditionally used in the literature. Moreover, the use of a more "flexible leverage effect" aims to provide superior asymmetric models than those found in Glosten et al. (1993) or Corsi and Renò (2012).[2]

Our empirical analysis contemplates the use of different sampling frequencies, so in an attempt to mitigate the impact of the market microstructure noise (MMSN), we use the noise-robust realized variance of Jacod et al. (2009) and Christensen et al. (2014), and the noise-robust realized covariance of Christensen et al. (2010). We also develop noise-robust measures of the realized semivariances and semicovariances by extending the work of Christensen et al. (2010, 2014); Jacod et al. (2009). This adaptation enables us to exploit the information content of the realized semi-(co)variances across sampling frequencies.[3] Previous studies (Ghysels and Sinko, 2011; Hizmeri et al., 2019) document significant

[2]Patton and Sheppard (2015) use dynamic models based on realized semivariances and shows that the information afforded by negative returns variance is superior to its positive counterpart. Bollerslev et al. (2019) shows that the elements of the semicovariances are very informative to forecasting portfolio variance.

[3]Hansen and Lunde (2006) show that standard realized measures tend to be upward biased in the presence of MMSN.

improvements in forecasting when noise-robust measures are used in the presence of MMSN relative to standard volatility measures.

Moreover, and in an effort to underscore the practical relevance of the forecasts improvements afforded by the HAR-X models, we evaluate the gains using a mean-variance portfolio allocation. The use of a mean-variance framework to evaluate the performance of forecasting models has been widely employed (see, for instance Bernales and Guidolin, 2014; Christoffersen et al., 2014; Fleming et al., 2001, 2003; Nolte and Xu, 2015) and is well-established that poor forecasting performances lead to extreme positions far away from the ex-post optimal portfolio weights.

Our main findings can be summarized as follows. First, using 20 U.S. stocks from 2000-2016 (4277 days) across various sampling frequencies, we show that the HAR-X class of models substantially outperform the basic HAR-RV model, and the HAR-Co-V models generally produce more accurate forecasts than the HAR-V models.[4] These results hold also true using Monte Carlo Simulations.

Second, models that account for volatility asymmetry render the biggest out-of-sample gains. For instance, models based on the negative semi-(co)variances dominate both the basic HAR-RV and their unsigned counterpart models. However, the best forecasting performance is achieved when the HAR-Co-V model is formed using the unsigned variances along with the positive semicovariances.

Third, the statistical improvements afforded by the HAR-X class of models also produce significant economic value. We show that the forecasts of our models react faster to changes in volatility, which results in a more accurate allocation of wealth. Thus, a risk-averse investor is willing to pay up to 57 (56) annual basis points before (after) transaction costs to switch to one of our strategies.

Finally, we show that after removing the MMSN from non-noise-robust measures, the information set across sampling frequencies is of similar magnitude, leading to more stable forecasting improvements. However, using a mixed-sampling approach we highlight the

[4]As described in Section 3, in the HAR-X class, we denote as HAR-V when the HAR-RV model is generalized by the variance of the index, and as the HAR-Co-V when the HAR-RV model is generalized by the variance and covariance of the index.

benefit of mixing the realized measures estimated at different time intervals. We find that when a higher (lower) frequency is used in the index (stock) the HAR-V model outperforms the forecasts of the benchmark HAR-RV model and the same-frequency HAR-V model. In specific, the best results are obtained using 30 second return for the index and 300 second return for the stock.

The remainder of the paper is organized as follows. Section 2 describes the theoretical background and the new noise-robust realized semivariances and semicovariances; Section 3 outlines the forecasting models and evaluation; our Monte Carlo study is outlined in Section 4; Section 5 describes our data; Section 6 presents the estimates of the HAR-X models, along with various statistical out-of-sample forecast comparisons. This section ends by illustrating the benefits from using a mixed-sampling approach in the HAR-V model; Section 7 reports the economic value of our models; conclusion is provided in Section 8.

2 Theoretical Background

A univariate log-price process evolving continuously over time is outlined as follows:

$$dp_\tau = \mu_\tau d\tau + \sigma_\tau dW_\tau, \quad 0 \leq \tau \leq T, \tag{1}$$

where μ_τ is a locally bounded and predictable drift process, σ_τ is the spot volatility which is both adapted and càdlàg, and W_τ is a standard Brownian motion.

The integrated variation (IV_t) for a subperiod (usually associated with day t) of this process is defined as:

$$IV_t = \int_{t-1}^{t} \sigma_u^2 du. \tag{2}$$

IV_t can be estimated using the so-called realized variance (RV), (see Andersen et al., 2001a,b; Barndorff-Nielsen and Shephard, 2002; Hansen and Lunde, 2006; Meddahi, 2002), defined as:[5]

$$RV_t = \sum_{j=1}^{M} r_{t,j}^2 \xrightarrow{p} \int_{t-1}^{t} \sigma_u^2 du, \tag{3}$$

where $M \equiv 1/\Delta$ refers to the total number of intra-daily return observations on day t, and the intraday returns are defined as $r_{t,j} \equiv p_{t-1+j\Delta} - p_{t-1+(j-1)\Delta}$, for $j = 1, \ldots, M$. Stock prices are influenced by market and industry factors as well as other risks, which cannot always be assumed to be contained in the individual stock price information set. This suggests that stock price fluctuations are better described using a model that incorporates both the stock and the market information as follows:

$$dp_\tau^s = \mu_\tau d\tau + \sigma_\tau^s dW_\tau^s + \sigma_\tau^m dW_\tau^m, \quad 0 \le \tau \le T \tag{4}$$

where the superscripts "s" and "m" hereafter refer to the stock and the index, respectively. μ_t is a locally bounded predictable drift process, σ_τ^s and σ_τ^m are the respective spot volatility of the stock and the index. Both spot volatilities are adapted and càdlàg. Additionally, W_τ^s and W_τ^m are the standard Brownian motion of the stock and the index, and are potentially correlated, i.e. $\langle dW_\tau^s, dW_\tau^m \rangle = \rho_\tau d\tau$.

Focusing on day t, we assume without loss of generality that the drift term in model (4) is zero, then by taking expectation to the square of the increments we obtain:

$$\begin{aligned} \mathbb{E}\left[\sum_{j=1}^{M}(p_{t+j\Delta} - p_{t+(j-1)\Delta})^2\right] &= \mathbb{E}\left[\sum_{j=1}^{M}\left(\frac{\sigma_{t,j}^s}{\sqrt{M}}\epsilon_{t,j}^s + \frac{\sigma_{t,j}^m}{\sqrt{M}}\epsilon_{t,j}^m\right)^2\right] \\ &= \sum_{j=1}^{M}\frac{(\sigma_{t,j}^s)^2}{M}\mathbb{E}[(\epsilon_{t,j}^s)^2] + \sum_{j=1}^{M}\frac{(\sigma_{t,j}^m)^2}{M}\mathbb{E}[(\epsilon_{t,j}^m)^2] + \frac{2}{M}\sum_{j=1}^{M}\sigma_{t,j}^s\sigma_{t,j}^m\mathbb{E}[\epsilon_{t,j}^s\epsilon_{t,j}^m] \\ &= \underbrace{\sum_{j=1}^{M}(\sigma_{t,j}^s)^2\Delta}_{IV_t^s} + \underbrace{\sum_{j=1}^{M}(\sigma_{t,j}^m)^2\Delta}_{IV_t^m} + 2\underbrace{\sum_{j=1}^{M}\sigma_{t,j}^s\sigma_{t,j}^m\rho_{t,j}\Delta}_{ICOV_t}. \end{aligned} \tag{5}$$

With $\epsilon_{t,j}^s\sqrt{\Delta}$ and $\epsilon_{t,j}^m\sqrt{\Delta}$ representing the Brownian increments of the stock and the index,

[5] In the presence of jumps, $RV_t \to^p \int_{t-1}^{t}\sigma_u^2 du + \sum_{0\le s\le t}(p_t - p_{t-})^2$. We ignore jumps in our DGP for ease of exposition. However, the presence of jumps does not alter the results of our models since we do not distinguish between the continuous and discontinuous part of the RV.

which are possibly correlated. The result in equation (5) suggests that the integrated variance of model (4) is best described by the sum of the integrated variance of the stock (IV_t^s), the integrated variance of the index (IV_t^m), and the integrated covariation ($ICOV_t$) between the stock and the index.

In the presence of microstructure noise, the price is observed with a measurement error, which distorts the standard volatility and covariance measures.[6] Thus, the observed price is the sum of an unobservable efficient price and a noise component due to imperfections of the trading process:

$$p_\tau^* = p_\tau + u_\tau, \tag{6}$$

where p_τ^* is the contaminated price, p_τ is the efficient price and u_τ is the observation error,[7] which is independent and identically distributed with $\mathbb{E}[u_\tau] = 0$ and $\mathbb{E}[u_\tau^2] = \omega^2$, and $p_\tau \perp u_\tau$ ($\perp$ means stochastic independence). The contaminated returns are estimated as $r_{t,j}^* \equiv p_{t-1+j\Delta}^* - p_{t-1+(j-1)\Delta}^*$. As shown by Bandi and Russell (2006); Hansen and Lunde (2006); Zhang et al. (2005), realized measures estimated from contaminated returns result in noisy measures of volatility since $\mathbb{E}[RV] = IV + 2M\omega^2$. In order to mitigate the impact of the MMSN, we consider the use of pre-averaging returns of Jacod et al. (2009). The pre-averaging returns of a day t are estimated as follows:

$$\hat{r}_{t,i} = \sum_{j=1}^{L-1} g\left(\frac{j}{L}\right) r_{t,i+j}^*, \tag{7}$$

where $g = (x \wedge 1 - x)$. Because of this pre-averaging, $\hat{r}_{t,i}$ is closer to the efficient return $r_{t,j}$.

Jacod et al. (2009) and Christensen et al. (2014) propose a noise-robust estimator for the realized variance, pre-averaging realized variance (PRV), which relies in the use of

[6]See Andersen et al. (2001a, 2003); Barndorff-Nielsen and Shephard (2002) for a more detailed exposition about estimating realized volatility in a noise-free scheme, Barndorff-Nielsen et al. (2010); Patton and Sheppard (2015) for estimating realized semivariances using standard volatility measures, and Barndorff-Nielsen and Shephard (2004); Bollerslev et al. (2019) for estimating realized covariance and semicovariances using standard realized measures.

[7]We suppress the superscripts "s" and "m" to differentiate the price path of the stock and the market index for ease of exposition. It will be added when further clarification is required.

pre-averaging returns as follows:

$$PRV_t = \frac{M}{M-L+2}\frac{1}{L\psi_2^L}\sum_{i=0}^{M-L+1}|\hat{r}_{t,i}|^2 - \frac{\psi_1^L\hat{\omega}_t^2}{\theta^2\psi_2^L}, \tag{8}$$

where $L = \theta\sqrt{M} + o(M^{-1/4})$, $M/(M-L+2)$ is a small sample correction, while $\frac{\psi_1^L\hat{\omega}_t^2}{\theta^2\psi_2^L}$ is a bias-correction to remove a leftover effect of noise that is not eliminated by the pre-averaging estimator. ω_t^2 is estimated as in Oomen (2006): $\hat{\omega}_t^2 \equiv \hat{\omega}_{AC}^2 = -\frac{1}{M-1}\sum_{i=2}^{M} r_{t,j}^* r_{t,j-1}^*$.[8]

The constants associated with g are defined as:

$$\psi_1^L = L\sum_{j=1}^{L}\left[g\left(\frac{j}{L}\right) - g\left(\frac{j-1}{L}\right)\right]^2, \quad \psi_2^L = \frac{1}{L}\sum_{j=1}^{L-1} g^2\left(\frac{j}{L}\right). \tag{9}$$

The pre-averaged realized semi-variances are estimated as follows:

$$PRV_t^+ = \frac{M}{M-L+2}\frac{1}{L\psi_2^L}\sum_{i=0}^{M-L+1}\left(|\hat{r}_{t,i}|^2 \mathbb{1}_{\{\hat{r}_{t,i}>0\}} - \frac{1}{2}\frac{\psi_1^L\hat{\omega}_t^2}{\theta^2\psi_2^L}\right) \tag{10}$$

$$PRV_t^- = \frac{M}{M-L+2}\frac{1}{L\psi_2^L}\sum_{i=0}^{M-L+1}\left(|\hat{r}_{t,i}|^2 \mathbb{1}_{\{\hat{r}_{t,i}<0\}} - \frac{1}{2}\frac{\psi_1^L\hat{\omega}_t^2}{\theta^2\psi_2^L}\right), \tag{11}$$

where the indicator function $\mathbb{1}_{\{.\}}$ is used to obtain the required sign of the pre-averaged returns. Since the bias term on the right of the equation provides a bias-correction for all the pre-averaged returns, we scale this bias so that it affects equally the positive and negative pre-averaged returns.

We use the modulated realized covariance (MRC) of Christensen et al. (2010), which is a noise-robust estimator of the so-called realized covariance proposed by Barndorff-Nielsen and Shephard (2004), and it is estimated as follows:

$$MRC_{t,\delta} = \frac{M}{M-K_n+2}\frac{1}{\psi_2 K_n}\sum_{i=0}^{M-K_n+1}\left(\hat{r}_{t,i}^n\right)' \hat{r}_{t,i}^s. \tag{12}$$

The authors show that using $\frac{K_n}{M^{1/2+\delta}} = \theta + o(M^{-1/4+\delta/2})$ the MRC is consistent without

[8] Oomen (2006) shows that this estimator equals $(RV - RV_{AC1})/(2M)$ being very closely related to $\omega^2 = RV/(2M)$ proposed by Bandi and Russell (2006) and Zhang et al. (2005).

using a bias-correction, and recommend a $\delta = 0.1$, which results in an $M^{-1/5}$ rate of convergence. They also point out that if a bias-correction is used, the resulting estimator is not ensured to be positive semi-definite.

Following Bollerslev et al. (2019), we propose a decomposition of the MRC to enable the construction of noise-robust semicovariances:

$$MRC_{t,\delta} = MRC^{+}_{t,\delta} + MRC^{-}_{t,\delta} + MRC^{+-}_{t,\delta} + MRC^{-+}_{t,\delta}. \tag{13}$$

Each element of equation (13) is estimated as follows:

$$\begin{aligned}
MRC^{+}_{t,\delta} &= \frac{M}{M-K_n+2}\frac{1}{\psi_2 K_n}\sum_{i=0}^{M-K_n+1}\left(\hat{r}^{m}_{t,i}\mathbb{1}_{\{\hat{r}^{m}_{t,i}>0\}}\right)'\left(\hat{r}^{s}_{t,i}\mathbb{1}_{\{\hat{r}^{s}_{t,i}>0\}}\right)\\
MRC^{-}_{t,\delta} &= \frac{M}{M-K_n+2}\frac{1}{\psi_2 K_n}\sum_{i=0}^{M-K_n+1}\left(\hat{r}^{m}_{t,i}\mathbb{1}_{\{\hat{r}^{m}_{t,i}<0\}}\right)'\left(\hat{r}^{s}_{t,i}\mathbb{1}_{\{\hat{r}^{s}_{t,i}<0\}}\right)\\
MRC^{+-}_{t,\delta} &= \frac{M}{M-K_n+2}\frac{1}{\psi_2 K_n}\sum_{i=0}^{M-K_n+1}\left(\hat{r}^{m}_{t,i}\mathbb{1}_{\{\hat{r}^{m}_{t,i}>0\}}\right)'\left(\hat{r}^{s}_{t,i}\mathbb{1}_{\{\hat{r}^{s}_{t,i}<0\}}\right)\\
MRC^{-+}_{t,\delta} &= \frac{M}{M-K_n+2}\frac{1}{\psi_2 K_n}\sum_{i=0}^{M-K_n+1}\left(\hat{r}^{m}_{t,i}\mathbb{1}_{\{\hat{r}^{m}_{t,i}<0\}}\right)'\left(\hat{r}^{s}_{t,i}\mathbb{1}_{\{\hat{r}^{s}_{t,i}>0\}}\right),
\end{aligned} \tag{14}$$

where $\hat{r}^{s}_{t,i}$ and $\hat{r}^{m}_{t,i}$ represent respectively the pre-averaged returns of the stocks and the market index.

3 Forecasting Models and Evaluation

3.1 Forecasting Models

The HAR-RV model proposed by Corsi (2009) is defined as:[9]

$$RV^{s}_{t+h-1|t} = \beta_0 + \beta_d RV^{s}_{t-1} + \beta_w RV^{s}_{t-1|t-5} + \beta_m RV^{s}_{t-1|t-22} + \epsilon_t, \tag{15}$$

where $RV^{s}_{t-j|t-h} = \frac{1}{h+1-j}\sum_{i=j}^{h} RV^{s}_{t-i}$, with $j \leq h$. The popularity of the HAR-RV is explained by its easy implementation and ability to mimic long-range dynamic dependencies

[9]To simplify notation we will use $RV_t = PRV_t$ and $RC_t = MRC_{t,\delta}$ in the forecasting models.

observed in realized volatility time series.

By contrast, our new HAR-X models utilize the information from the stock and the market index, aiming to capture shocks due to new market information much quicker than previous HAR class of models.[10] The HAR-X class is defined as:

HAR-V

$$\begin{aligned} RV^s_{t+h-1|t} = \beta_0 + &\beta^s_d RV^s_{t-1} + \beta^s_w RV^s_{t-1|t-5} + \beta^s_m RV^s_{t-1|t-22} + \\ &\beta^m_d RV^m_{t-1} + \beta^m_w RV^m_{t-1|t-5} + \beta^m_m RV^m_{t-1|t-22} + \epsilon_t, \end{aligned} \tag{16}$$

HAR-Co-V

$$\begin{aligned} RV^s_{t+h-1|t} = \beta_0 + &\beta^s_d RV^s_{t-1} + \beta^s_w RV^s_{t-1|t-5} + \beta^s_m RV^s_{t-1|t-22} + \\ &\beta^m_d RV^m_{t-1} + \beta^m_w RV^m_{t-1|t-5} + \beta^m_m RV^m_{t-1|t-22} + \\ &\beta^{RC}_d RC_{t-1} + \beta^{RC}_w RC_{t-1|t-5} + \beta^{RC}_m RC_{t-1|t-22} + \epsilon_t. \end{aligned} \tag{17}$$

Both models are motivated by the non-linear dependence feature observed in asset returns. During calm periods the correlation between a stock and the index declines, often to insignificant levels, suggesting that a model ignoring the covariation such as the HAR-V model might provide better out-of-sample performance than a model incorporating the covariance information as in the HAR-Co-V model and vice versa.[11]

Previous studies have examined whether semivariances (Patton and Sheppard, 2015) and semicovariances (Bollerslev et al., 2019) provide incremental information that could aid volatility forecasting and the estimation of portfolio variance. For instance, Patton and Sheppard (2015) find that negative returns usually lead to higher level of volatility. Given the decomposition of the RV_t and RC_t previously outlined, we create extensions

[10] These models come directly from a vector HAR structure. However, here the interest is only in forecasting the stock volatility rather than the stock and the index volatility. For instance, $\boldsymbol{y_t} = \begin{pmatrix} y^s_t \\ y^m_t \end{pmatrix}$. $\boldsymbol{y_{t+h-1|t}} = \boldsymbol{\Phi_0} + \boldsymbol{\Phi_d y_{t-1}} + \boldsymbol{\Phi_w y_{t-5|t-1}} + \boldsymbol{\Phi_d y_{t-22|t-1}} + \boldsymbol{\epsilon_t} \equiv \begin{pmatrix} y^s_t \\ y^m_t \end{pmatrix} = \begin{pmatrix} \phi^s_0 \\ \phi^m_0 \end{pmatrix} + \begin{pmatrix} \phi^{s_1}_d & \phi^{m_1}_d \\ \phi^{m_2}_d & \phi^{s_2}_d \end{pmatrix} \begin{pmatrix} y^s_{t-1} \\ y^m_{t-1} \end{pmatrix} + \begin{pmatrix} \phi^{s_1}_w & \phi^{m_1}_w \\ \phi^{m_2}_w & \phi^{s_2}_w \end{pmatrix} \begin{pmatrix} y^s_{t-5|t-1} \\ y^m_{t-5|t-1} \end{pmatrix} + \begin{pmatrix} \phi^{s_1}_m & \phi^{m_1}_m \\ \phi^{m_2}_m & \phi^{s_2}_m \end{pmatrix} \begin{pmatrix} y^s_{t-22|t-1} \\ y^m_{t-22|t-1} \end{pmatrix} + \begin{pmatrix} \epsilon^s_t \\ \epsilon^m_t \end{pmatrix}$, where the first equation gives rise to the HAR-V model.

[11] We evaluate the non-linear dependence in our dataset and we find that the average correlation across all the stocks is 0.45 during the pre-crisis period, while during the crisis period the average correlation rises to 0.8. Similarly, Longin and Solnik (2001) find evidence that support an increase in correlation during bear markets, but not in bull markets.

to our previous models to account for asymmetric dependencies, or "leverage effects". Apart from the SHAR model (Patton and Sheppard, 2015) previous proposed models in the literature were extensions of the GJR (Glosten et al., 1993) threshold approach to allow the conditional variance to respond differently based on the sign of the daily returns (see, for instance Corsi and Renò, 2012). On the contrary, our models use a more flexible "continuous leverage effect" based on the semivariances for the HAR-V model and based on both semivariances and semicovariances for the HAR-Co-V model, allowing for more refine responses to positive and negative return shocks.

The asymmetric HAR-X models are outlined as follows:

Asymmetric HAR-V Models:

HAR-V$^+$

$$RV^s_{t+h-1|t} = \beta_0 + \beta_d^{s^+} RV^{s^+}_{t-1} + \beta_w^{s^+} RV^{s^+}_{t-1|t-5} + \beta_m^{s^+} RV^{s^+}_{t-1|t-22} + \beta_d^{m^+} RV^{m^+}_{t-1} + \beta_w^{m^+} RV^{m^+}_{t-1|t-5} + \beta_m^{m^+} RV^{m^+}_{t-1|t-22} + \epsilon_t.$$

HAR-V$^-$

$$RV^s_{t+h-1|t} = \beta_0 + \beta_d^{s^-} RV^{s^-}_{t-1} + \beta_w^{s^-} RV^{s^-}_{t-1|t-5} + \beta_m^{s^-} RV^{s^-}_{t-1|t-22} + \beta_d^{m^-} RV^{m^-}_{t-1} + \beta_w^{m^-} RV^{m^-}_{t-1|t-5} + \beta_m^{m^-} RV^{m^-}_{t-1|t-22} + \epsilon_t.$$

Asymmetric HAR-Co-V Models:

HAR-Co$^+$-V$^+$

$$RV^s_{t+h-1|t} = \beta_0 + \beta_d^{s^+} RV^{s^+}_{t-1} + \beta_w^{s^+} RV^{s^+}_{t-1|t-5} + \beta_m^{s^+} RV^{s^+}_{t-1|t-22} + \beta_d^{m^+} RV^{m^+}_{t-1} + \beta_w^{m^+} RV^{m^+}_{t-1|t-5} + \beta_m^{m^+} RV^{m^+}_{t-1|t-22} + \beta_d^{RC^+} RC^+_{t-1} + \beta_w^{RC^+} RC^+_{t-1|t-5} + \beta_m^{RC^+} RC^+_{t-1|t-22} + \epsilon_t.$$

HAR-Co$^-$-V$^-$

$$RV^s_{t+h-1|t} = \beta_0 + \beta_d^{s^-} RV^{s^-}_{t-1} + \beta_w^{s^-} RV^{s^-}_{t-1|t-5} + \beta_m^{s^-} RV^{s^-}_{t-1|t-22} + \beta_d^{m^-} RV^{m^-}_{t-1} + \beta_w^{m^-} RV^{m^-}_{t-1|t-5} + \beta_m^{m^-} RV^{m^-}_{t-1|t-22} + \beta_d^{RC^-} RC^-_{t-1} + \beta_w^{RC^-} RC^-_{t-1|t-5} + \beta_m^{RC^-} RC^-_{t-1|t-22} + \epsilon_t.$$

HAR-Co$^+$-V

$$RV^s_{t+h-1|t} = \beta_0 + \beta_d^{s} RV^{s}_{t-1} + \beta_w^{s} RV^{s}_{t-1|t-5} + \beta_m^{s} RV^{s}_{t-1|t-22} + \beta_d^{m} RV^{m}_{t-1} + \beta_w^{m} RV^{m}_{t-1|t-5} + \beta_m^{m} RV^{m}_{t-1|t-22} + \beta_d^{RC^+} RC^+_{t-1} + \beta_w^{RC^+} RC^+_{t-1|t-5} + \beta_m^{RC^+} RC^+_{t-1|t-22} + \epsilon_t.$$

HAR-Co$^-$-V

$$RV^s_{t+h-1|t} = \beta_0 + \beta_d^{s} RV^{s}_{t-1} + \beta_w^{s} RV^{s}_{t-1|t-5} + \beta_m^{s} RV^{s}_{t-1|t-22} + \beta_d^{m} RV^{m}_{t-1} + \beta_w^{m} RV^{m}_{t-1|t-5} + \beta_m^{m} RV^{m}_{t-1|t-22} + \beta_d^{RC^-} RC^-_{t-1} + \beta_w^{RC^-} RC^-_{t-1|t-5} + \beta_m^{RC^-} RC^-_{t-1|t-22} + \epsilon_t$$

3.2 Forecasting Evaluation

Our main interest is in the real-time out-of-sample forecasting performance of our models. We consider horizons $h = 1$, 5, and 22, which correspond to one day, one week, and one month ahead. We use an increasing window to update the coefficients, with an initial window (IW) of size 1000.

The out-of-sample performance is evaluated using the heteroskedastic mean square error (HMSE) and the quasi-likelihood (QLIKE) loss functions:

$$HMSE = N^{-1} \sum_{n=1}^{N} \left(1 - \frac{\widehat{RV}_n^s}{RV_n^s} \right)^2 \tag{18}$$

$$QLIKE = N^{-1} \sum_{n=1}^{N} \left(\frac{RV_n^s}{\widehat{RV}_n^s} - \log \frac{RV_n^s}{\widehat{RV}_n^s} - 1 \right) \tag{19}$$

where $\widehat{RV}_n^s$ and RV_n^s are respectively the forecasted and estimated $RV_{t+h-1|t}^s$ for the pseudo out-of-sample period, and $N = T - IW$ refers to the total number of out-of-sample observations. These loss functions are somewhat robust to outliers since the losses are evaluated as a ratio rather than a difference.

We also consider the use of the Conditional Predictive Accuracy (CPA) test, Giacomini and White (2006), to evaluate whether the propose models provide significant out-of-sample forecast improvements relative to the benchmark HAR-RV model. The CPA test is robust to nested models and its null hypothesis is of equal predictive accuracy defined as:

$$H_0 = \mathbb{E}[\Delta d_{n,i,j}] = 0, \tag{20}$$

where $\Delta d_{n,i,j} = L(\widehat{RV}_n^{s(i)}, RV_n^s) - L(\widehat{RV}_n^{s(j)}, RV_n^s)$ is the difference between two loss functions and $i \neq j$. The test statistic is then defined as:

$$\mathcal{T} = N \left(N^{-1} \sum_{n=1}^{N} \Delta d_{n,i,j} \right)' V_h^{-1} \left(N^{-1} \sum_{n=1}^{N} \Delta d_{n,i,j} \right) \sim \chi_1^2, \tag{21}$$

where V_h^{-1} is a heteroskedasticity and autocorrelation consistent (HAC) estimator of the asymptotic variance.

Finally, we evaluate whether there is a (sub)set of models that significantly outperforms the other competing models. We do this using the Model Confidence Set (MCS) of Hansen et al. (2011). We denote by $\mathcal{M}$ the set of all models (HAR-RV and HAR-X class of models). The MCS is defined as:

$$t_{i,j} = \frac{\bar{d}_{i,j}}{\sqrt{\widehat{\text{Var}}(\bar{d}_{i,j})}}, \quad \forall i, j \in \mathcal{M}, \tag{22}$$

where $\bar{d}_{i,j}$ is the average loss difference. The null of the MCS is that all the models have the same expected loss. When the null is rejected the worst performing model is eliminated, and this process is iterated until no further model can be eliminated. The surviving models are retained with a confidence level $\alpha = 0.1$.[12]

4 Monte Carlo Simulation

In this section we present a Monte Carlo study that demonstrates the expected forecasting gains from the proposed HAR-X models. We first simulate log-prices using the data generating process outlined in equation (4), and we use a stochastic correlation to allow for time-varying co-movements between the stock and the index. The stochastic volatilities and correlation are modelled using mean-reverting factors akin to Heston (1993). Equation (23) below outlines this set-up.

$$\begin{aligned}
dp_t^s &= \sqrt{\nu_t^s} dW_t^{(s)} + \sqrt{\nu_t^m} dW_t^{(m)} \\
dp_t^m &= \sqrt{\nu_t^m} dW_t^{(m)} \\
d\nu_t^s &= \kappa_s(\theta_s - \nu_t^s)dt + \sigma_s\sqrt{\nu_t^s} dZ_t^{(s)} \\
d\nu_t^m &= \kappa_m(\theta_m - \nu_t^m)dt + \sigma_m\sqrt{\nu_t^m} dZ_t^{(m)} \\
d\rho_t &= \kappa(\Theta - \rho_t)dt + \alpha\sqrt{1 - \rho_t^2} dZ_t^{(\rho)},
\end{aligned} \tag{23}$$

[12]We implement the MCS via a block bootstrap using a block length of 10 days and 5000 bootstrap replications.

where the parameters for the volatility factors are $\left\langle Z_t^{(s)}, Z_t^{(m)} \right\rangle \Big(= \rho_t$. We set $\kappa_m = 4.1741$, $\theta_m = 0.3431^2$, $\sigma_m = 0.3897$, $\kappa_s = 7.8907$, $\theta_s = 0.4985^2$, $\sigma_s = 0.4991$. The parameters of the stochastic correlation are $\kappa = 1.61$, $\Theta = 0.3505$, $\alpha = 0.608$, and $\rho_0 = \Theta$.[13] We simulate log-price increments using a sampling frequency of 300 seconds and a sample size of 1000 days. The trading hours are 6.5 hours a day, rendering 78 intraday returns. We then construct the realized measures and use an increasing window to evaluate the pseudo out-of-sample forecasts with an initial window size of 350 days. We iterate over this process 3,000 times.

Figure 1 plots the simulated 1-day ahead relative HMSE and QLIKE distribution. The relative loss is estimated as the ratio of the losses of the HAR-X models to the loss of the HAR-RV.[14] This means that values below (above) 1 indicate that our models outperform (underperform) the HAR-RV. Our aim in using a Monte Carlo simulation is to highlight the benefits of using the HAR-X class over the basic HAR-RV model. That is why we only focus on the general specifications (HAR-V and HAR-Co-V) using standard (co)volatility measures. We will then empirically evaluate the sensitivity of our models to the sampling frequency and outline the extra benefits of our model specifications when accounting for asymmetric effects.

As shown in Figure 1, the distributions of the losses are usually below 1.0 irrespective of the model in consideration. The left-tail distribution of the HAR-Co-V is generally fatter compared to the HAR-V, indicating that adding the covariance information delivers greater out-of-sample forecasting gains.

5 Data

Our sample consists of 20 individual stocks selected by trading volume over the period January 3, 2000 to December 31, 2016, a total of 4277 days, together with the S&P 500

[13]The parameters were calibrated by solving a constrained non-linear problem. The data used to calibrate the model is described in section 6. In order to ensure that the volatilities are always positive, we impose the constraint $2\kappa > \sigma^2$, and for the stochastic correlation the condition $\kappa \geq \frac{\alpha^2}{1 \pm \Theta}$ is required to remain in the set of real numbers and to ensure positive definiteness in the covariance matrix.

[14]Since we do not incorporate the effects of the MMSN here; the simulation exercise relies on standard measures to produce the realized measures and out-of-sample forecasts.

ETF (SPY) over the same time period. To gauge the sensitivity of our set-up to the sampling frequency, we consider various sampling frequencies ranging from 30 to 300 seconds.

Table 1 provides the descriptive statistics for all the stocks and the SPY. The SPY is the least volatile asset in our study, with an annualized volatility close to 15%, whereas the average level of volatility of the stocks is up to 3 times higher than the SPY volatility. Amazon displays the highest annualized return and volatility, whilst Arconic (ARNC) has the minimum annualized return and Procter & Gamble (PG) is the least volatile stock.

Table 2 reports the average correlations across the stocks for all the realized measures under analysis. Above the main diagonal the correlations are those using realized measures estimated from 30-second return. Below the main diagonal the correlations are those for 300-second return. The superscripts "s" and "m" represent the realized measures of the stock and the index, respectively. Realized measures estimated at the 30-second frequency display slightly greater relationships compared with their counterpart estimated at the 300-second frequency. Interestingly, we find that the level of correlation among the different realized measures and the stock volatility differs greatly. This finding implies that novel and important information can be utilized to further explain the behavior of stock volatility, and that using signed measures of volatility and covariances can lead to more accurate volatility forecasts.

Figure 2 depicts the average PRV and MRC with their respective components across all the stocks. The left-panel shows that negative semivariances tend to produce on average slightly greater levels of volatility than positive semivariances.[15] On the other hand, the right-panel plots the elements of the covariation; here, the $MRC^{+}+MRC^{-}$ ($MRC^{+-}+MRC^{-+}$) is positive (negative) by construction. During financial distress, the level of the positive sum (concordant elements) of the covariance elements increases more than the negative part (discordant elements) declines, confirming that during turbulent periods the correlation between stocks and indices increases. It is interesting to note that the level

[15] The importance of the negative return variance has been well documented in the literature (see, for instance Corsi and Renò, 2012; Glosten et al., 1993; Patton and Sheppard, 2015, and references therein).

of the covariation is mainly determined by the elements of the positive sum, insinuating that remaining components are of less importance and provide little information.

Figure 3 plots the autocorrelation function as the average of all the stocks for the pre-averaging realized variance, modulated covariance and their components. Compared to standard volatility measures, our results display mildly lower levels of persistence. However, this observation is expected since MMSN induces first-order autocorrelation, (Hansen and Lunde, 2006).

In line with Patton and Sheppard (2015) we find that the negative semivariance is more persistent than its positive counterpart, and that there is little difference between the unsigned and negative semivariance. On the contrary, the index is less persistent than the stocks. There is empirical evidence supporting the fact that large (finite) jumps have no persistence (see Andersen et al., 2007; Duffie et al., 2000; Duong and Swanson, 2015; Hizmeri et al., 2019, among others), and also Duong and Swanson (2015) and Hizmeri et al. (2019) find evidence that aggregation makes jumps more informative, which presumably reduces the level of persistence of the PRV^m compared to those of the stocks. The autocorrelation of the covariance and its elements show that the $MRC^{+} + MRC^{-}$ and MRC are very close and of less persistence than the $MRC^{+-} + MRC^{-+}$. As noted by Bollerslev et al. (2019), $MRC^{+-} + MRC^{-+}$ only depends on the continuous part, while $MRC^{+} + MRC^{-}$ can be formed by continuous and co-jump variation, explaining why the latter sum is less persistent.

6 Modeling and forecasting with the HAR-X Class

6.1 In-Sample Estimates

The parameter estimates obtained for each of the different models are reported in Tables 3, 4, and 5, along with the adjusted R-squares and total F-test rejections. To conserve space, we present the average of the parameter estimates and denote with *, **, and *** when the estimates are significant at the 10%, 5%, and 1% using robust standard errors.[16]

Table 3 reports the HAR-RV, HAR-V and HAR-Co-V estimates across 3 forecasting horizons, which represent one day ($h = 1$), one week ($h = 5$), and one month ($h = 22$). Similar to previous findings in the literature, the HAR-RV model estimates are usually all significant, and most of the weight is assigned to the monthly estimate, which increases with the horizon.

When the index volatility is added as regressors in the HAR-V model, we observe an improvement in the model fit relative to the benchmark model. This improvement ranges from 0.9% to 1.9% points in terms of the adjusted R-squares, and with the exception of 2 stocks at $h = 5$, the F-test rejects the null of equal fit for all the stocks across all forecasting horizons. Moreover, the HAR-Co-V model, which incorporates the variance and covariance information from the index, shows bigger model fit improvements increasing the R-squares from 1.4% to 3.4% points relative to the HAR-RV models. The F-test corroborates the increase in model fit of the HAR-Co-V model by rejecting its null of equal fit for all the stocks irrespective of the forecasting horizon.

The inclusion of the index information in the form of variance and covariance induces a decrease in the explanatory power of the stock's estimates, which is normally subsumed by daily and weekly variables of the index (co)variances. In other words, if adding the market index variables renders the stock specific volatility variables insignificant, then the stock specific volatility has little or no forecasting power in the presence of the market index.

It is noteworthy that the monthly index volatility in the HAR-V and the covariance estimates in the HAR-Co-V models are generally negative across all forecasting horizons. In the case of the HAR-V model, a negative index variance estimate reduces the weights assigned to monthly information, while increasing the weights allocated to daily and weekly information. On the contrary, negative covariance estimates have a two-fold effect in the future level of volatility. Since covariances can take either positive or negative values, a positive (negative) covariance reduces (increases) the future level of volatility.

[16]The in-sample coefficients are estimated by fitting all the models using the full sample size.

This dynamic allocation is in line with the empirical findings that negative (positive) returns induce higher (lower) levels of volatility. This phenomenon explains why the HAR-Co-V model provides a better model fit than the HAR-RV and HAR-V models.

Tables 4 and 5 report the parameter estimates along with the R-squares and F-test rejections for asymmetric HAR-V and HAR-Co-V models, respectively. Turning to the results in Table 4, we find that most of the estimates are statistically significant, and the monthly negative semivariance estimate of the index is consistently negative across all forecasting horizons. In addition to being significant, the negative estimates are of bigger magnitude than their unsigned counterparts are. This reaffirms the previous findings of Patton and Sheppard (2015) who find that negative semivariances are more important to predict future volatility. The increase in model fit is readily observed in terms of R-squares, where the HAR-V^{-} model improves between 3.4%–4.3% points relative to the HAR-RV model, and 1.5%–3.3.% points relative to the HAR-V model across all forecasting horizons. On the contrary, the positive semivariance in the HAR-V^{+} model exhibits no explanatory power resulting in the worst model fit. The F-test only rejects the null of equal fit for one stock at $h = 5$ and $h = 22$.

The asymmetric HAR-Co-V models (Table 5), as outlined in Section 3.1, are formed by 4 different models which are presented in two panels. Panel A reports the parameter estimates for the HAR-Co-V models based on unsigned volatilities and semicovariances. The main difference between these models and the unsigned HAR-Co-V model is that we have replaced the unsigned covariance by its positive (HAR-Co^{+}-V) and negative (HAR-Co^{-}-V) elements. While both models improve on the fit of the HAR-RV model across all forecasting horizons, the HAR-Co^{+}-V model on average provides a better model fit than the HAR-Co^{-}-V model. Thus, the positive semicovariance provides more explanatory power than its negative counterpart in predicting future stock volatility. In both models the estimates are generally significant, and with the exception of the HAR-Co^{-}-V model at $h = 22$, the HAR-Co^{+}-V and HAR-Co^{-}-V models improve on the fit of the HAR-Co-V model across all forecasting horizons.

Panel B reports the asymmetric HAR-Co-V models formed using the full positive

(HAR-Co$^+$-V$^+$) and negative (HAR-Co$^-$-V$^-$) structure. The HAR-Co$^-$-V$-$ model improves on the fit of the benchmark and every single model under analysis across all forecasting horizons. Notwithstanding, the fit of the HAR-V$^-$ and HAR-Co$^+$-V models are very close to the HAR-Co$^-$-V$^-$ model at medium and longer horizons. By contrast, the HAR-Co$^+$-V$^+$ model performs very poorly; however, this is expected since this model is using the positive semivariances of the stock and the index, which have no explanatory power.

These findings confirm that a richer information set can be obtained after dissecting the variances and covariances by their sign. This richer information set translates in more explanatory power, resulting in a better model fit. By this far, we have shown that the HAR-X models are better specified than the HAR-RV model. However, in real time most of the attention is given to the out-of-sample forecasting test as it is more relevant for examining the genuine predictive ability. In the next two sections we comprehensively examine the predictive ability of the HAR-X models.

6.2 Out-of-Sample Forecasts

Table 6 reports the out-of-sample relative losses across forecasting horizons and sampling frequencies. The relative loss is reported as the ratio of the losses of the HAR-X models to the losses of the benchmark HAR-RV model. Thus, values below (above) 1 indicate that our proposed models outperform (underperform) the benchmark HAR-RV model. Bold numbers highlight the HAR-X models that outperform the benchmark, and the numbers in the superscript represent the total number of stocks for which the losses of the HAR-X models are significantly lower than those of the HAR-RV model. The significance of the models is evaluated using the CPA test of Giacomini and White (2006) with $\alpha = 0.05$.

Confirming the in-sample results from the previous section, we find that the HAR-X models generally outperform the forecasts of the basic HAR-RV model, and these forecasting gains are found to be significantly better than the benchmark across most of the sampling frequency and forecasting horizons. Moreover, the forecasting gains afforded by the HAR-X models tend to increase as the forecasting horizon increases, indicating that the HAR-X models better capture the volatility persistence, leading to more accurate predictions at longer horizons.

While most studies in the literature use 5-min returns in order to reduce the impact of the microstructure noise, our study relies on noise-robust measures, which enables us to explore the predictive power of our realized measures across different sampling frequencies in a less noisier environment. We find that the use of noise-robust measures yields to more stable improvements across time intervals, however, forecasts based on 300-second return provide slightly more accurate out-of-sample predictions.[17]

The asymmetric HAR-X models provide the best out-of-sample performance in our analysis. When the HAR-V models account for the so-called leverage effects using a full negative structure, we find the biggest out-of-sample improvements relative to the benchmark and the general HAR-V models. In many cases the HAR-V^- model produces forecasting gains which are significantly better than those of the HAR-RV models. For instance, using the CPA test and the QLIKE loss, we find that the HAR-V^- model significantly outperforms the benchmark in 14 stocks at $h = 1$, and 10 stocks at $h = 22$ using 300 seconds sampling frequency. By contrast, the full positive structure, HAR-V^+ model, results in the worst out-of-sample performance. This finding is not surprising as it was shown in Section 6.1 that the positive semivariances contain no predictive power.

The asymmetric HAR-Co-V models consists of four specifications: the first two are the full positive (HAR-Co^+-V^+) and negative (HAR-Co^--V^-) structures, whereas the remaining two specifications are formed using the unsigned variances plus the positive (HAR-Co^+-V) and negative (HAR-Co^--V) covariances. The full negative structure outperforms both the HAR-RV and the general HAR-Co-V models. Yet, when the negative covariance is used with the unsigned variances we still observe out-of-sample gains, however, the level of these gains are somewhat smaller compared to the full negative structure. The decrease in performance directly affects the number of significant stocks found in the HAR-Co^--V^- model; for instance, the number of significant stocks drops

[17]This implies evidence that after removing the effects of the microstructure noise the information contained in the realized (co)variances and their components is on average of similar importance.

by 4 at $h = 1$ and by 2 at $h = 5$, when the negative semivariances are replaced by the unsigned variances, i.e. from the HAR-Co^{-}-V^{-} to the HAR-Co-V^{-} model. This confirms that allowing for an asymmetric reaction results in greater out-of-sample performances, and that models based on only negative return (co)variance perform better than their counterparts.

As expected, a full positive structure in the HAR-Co-V model does not provide any out-of-sample forecasts improvements. However, the HAR-Co^{+}-V model provides one of the best out-of-sample performances across all the models, horizons and sampling frequencies. The improved forecast accuracy afforded by the HAR-Co^{+}-V model is found to be significantly different than the HAR-RV model ranging from 9 to 19 stocks using the QLIKE, and from 15 to 20 stocks based on the HMSE.

6.3 Model Selection: MCS

Hitherto we have shown that the HAR-X class of models improves on both the in- and out-of-sample performances of the HAR-RV models. This holds true across both sampling frequencies and forecasting horizons. However, all the comparisons have been made against the benchmark HAR-RV model, and we do not know whether there is a model whose losses are significantly lower relative to those of the remaining models. In order to answer this question, we use the Model Confidence Set (MCS) of Hansen et al. (2011), which evaluates the performance of all the models without targeting a benchmark. That is, it evaluates whether there is a (sub)set of models that significantly outperform all the models under analysis.

Table 7 reports MCS ranking for each individual stock and across forecasting horizons using the realized measures estimated at the 300-second sampling frequency. The numbered entries are for the retained models, while the dash-line indicates that the models have been excluded from the MCS. The MCS results are based on the QLIKE loss function.[18]

The HAR-RV along with the HAR-V^{+} and HAR-Co^{+}-V^{+} models are the most ex-

[18]Results based on the HMSE provides similar conclusions and are available upon request.

cluded models across forecasting horizons. This finding is not surprising since our previous sections show that the positive semivariance has no predictive power, and hence models containing this variable result in poor out-of-sample forecast accuracy. The HAR-RV model is only retained 3 times on average across forecasting horizons. This means that our new class of models produce significantly more accurate forecasts than the basic HAR-RV model. On the other hand, the HAR-V^-, HAR-Co^--V^-, and HAR-Co^+-V are the least excluded models and all of them are based on realized semi-(co)variances variables, which highlights the benefits of allowing for the asymmetric dependence feature that characterizes equity return data.

Table 8 reports the ranking of the MCS for $h = 1$ across sampling frequencies using the QLIKE as loss function. The results are reported in three panels, which contain results based on 30-, 60-, and 150-second return frequency. Similar to the findings in Table 7, we find that the HAR-RV model is generally excluded by the MCS irrespective of the sampling frequency, while the HAR-V^-, HAR-Co^--V^-, and HAR-Co^+-V models are the least excluded. The little variation in the ranking and performance of our models is due to the use of noise-robust realized measures, and it confirms that after accounting for microstructure noise the choice of the sampling frequency becomes insignificant, since the information set across time intervals is of similar magnitude.

6.4 Mixed Sampling Approach

While previous results indicate that our models generally outperform the HAR-RV model, their forecasting gains are of similar magnitude across sampling frequency. This finding suggests that there is no preferences nor an optimal sampling frequency when using noise-robust measures, but it is silent about whether a mixed-sampling approach can better capture the different information, if any, embedded in these sampling frequencies. Thus, motivated by this fact and by the results on Table 2 that shows slightly different levels of correlations across sampling frequencies, this section examines the impact of varying the sampling frequency on both the stock and the index on the forecasting performance of the HAR-X models.

We use the HAR-V model to construct the mixed sampling approach for two reasons. First, the HAR-V model is well-structured, thus, it facilitates the inclusion of different sampling frequencies on the variance of the stock and the variance of the index. By contrast, the HAR-Co-V model incorporates the covariation that has to be estimated at the same sampling frequency between the stock and the index. Second, the aim of this exercise is not to produce a horse-race and show which model provides the best out-of-sample performance, but rather it aims to illustrate that more accurate performances can be attained by mixing the sampling frequency when using the HAR-X class of models.

The mixed-sampling HAR-V model is then constructed by holding constant the frequency of the stock volatility while varying the frequency at which the index volatility is estimated and vice versa. In total we use 6 sampling frequencies ranging from 30- to 300-seconds. The results are then compared relative to the HAR-RV and the HAR-V models, which are based on the same sampling frequency.

Figure 4 plots the average loss ratio across all the stocks in a 3D plane. The x- and y-axis display respectively the sampling frequency of the index and the stock, while the z-axis displays the loss ratio for the QLIKE (left-panel) and HMSE (right-panel). The darker part of the figure highlights the best performance, while by contrast the lighter part indicates the worst performance.

Interesting observations can be drawn from Figure 4. First, all the sampling combinations outperform the HAR-RV model, which corroborates our previous findings that incorporating the market index information substantially improves on the forecasts of the benchmark model. Second, we observe that the surface is relatively flatter when the stock frequency is held constant.[19] However, when the index frequency is held constant, we observe significant improvements. This observation suggests that the information set of the stock varies more than the one from the index, and that the use of a mixed-sampling approach better captures these small variations in the information set of the stock and the index. Third, the mixed-sampling approach always outperforms the forecasts of the

[19] Standard measures which do not remove the variance of the MMS noise might benefit more from a mixed-sampling approach, and it would be interesting to evaluate whether the variance of the noise plays any role in a mixed-sampling approach.

same-frequency HAR-V model when the sampling frequency of the index is higher than the stock's frequency. For instance, if the stock volatility is estimated at the 300-second return, using frequencies finer than 300-seconds (i.e. 150-, ..., 30-seconds) in the index will result in out-of-sample gains relative to both the benchmark HAR-RV model and the same-frequency HAR-V model. Specifically, the best performance is achieved at a sampling frequency of 30 seconds for the stock and 300 seconds sampling frequency for the index.

7 Economic Value

In this section we conduct economic evaluations by constructing volatility timing based portfolio allocation strategies. We consider a risk-averse investor with mean-variance preferences, who allocates her wealth into one risky asset and one risk-free asset. In order to emphasize the advantages of our models, we initially focus on the daily investment horizon.[20] The underlying economic intuition for this strategy is as follows: for a certain expected return, when the volatility is high, the investor allocates more wealth into the risk-free asset. On the contrary, when the volatility level is low, the investor allocates more wealth into the risky asset.

This strategy enables us to directly evaluate whether statistical improvements in volatility forecasting can be translated into economic value. If adding the market information leads to more accurate prediction of future volatility, then we should expect the investor to improve her portfolio performance by actively rebalancing the portfolio based on the signal of the predicted volatility.

We follow Fleming et al. (2003); Marquering and Verbeek (2004) and use a mean-variance utility. Thus, the investor solves the following optimization problem:

$$\max_{w_{t+h}} U\left[E_t\left(r_{p,t+h}\right), \mathrm{Var}_t\left(r_{p,t+h}\right)\right],$$

[20]Previous studies have also considered daily re-balancing schemes, (see, for instance Fleming et al., 2001, 2003; Marquering and Verbeek, 2004, among others).

where h indicates the periods ahead, $E_t(r_{p,t+h})$ is the conditional expected portfolio return and $\text{Var}_t(r_{p,t+h}) = w_{t+h}^2 \widehat{RV}_{t+h}$ is the conditional variance of the portfolio return. The portfolio return is $E_t(r_{p,t+h}) = (1 - w_{t+h})r_{f,t+h} + w_{t+h}E_t(r_{m,t+h})$, where w_{t+h} is the portfolio weight of the risky asset, $E_t(r_{m,t+h})$ is the conditional expected return of the risky asset and $r_{f,t+h}$ is the return for the risk free asset, which we know ex-ante.[21] The mean-variance utility function is given by:

$$U\left[E_t\left(r_{p,t+h}\right), \text{Var}_t\left(r_{p,t+h}\right)\right] = E_t\left(r_{p,t+h}\right) - \frac{\gamma}{2}\text{Var}_t\left(r_{p,t+h}\right), \tag{24}$$

with γ the risk-aversion parameter. Hence, the optimal weight is given by:[22]

$$w_{t+h} = \frac{E_t\left(r_{m,t+h}\right) - r_{f,t+h}}{\gamma \text{Var}_t\left(r_{m,t+h}\right)}. \tag{25}$$

The conditional variance for the risky asset is estimated using the predicted realized variance, i.e. from the HAR-RV and the proposed HAR-X class of models. We constraint our portfolio, so short-selling and borrowing are not allowed. Then, the optimal portfolio weights become:

$$w_{t+1}^* = \begin{cases} 0 & if \quad w_{t+1} \leq 0, \\ w_{t+1} & if \quad 0 < w_{t+1} \leq 0 \\ 1 & if \quad w_{t+1} > 1. \end{cases}$$

We consider different risk aversion levels $\gamma = \{2, 6, 10\}$, and following Fleming et al. (2003); Marquering and Verbeek (2004); Nolte and Xu (2015), we estimate the sample averaged realized utility as follows:

$$\bar{U}(R_p) = \frac{1}{T}\sum_{t=0}^{T-1}\left[r_{p,t+h} - \frac{\gamma}{2}\text{Var}_t(r_{p,t+h})\right]. \tag{26}$$

[21] The risk-free rate is based on 3-month maturity Treasury Yield Curve obtained from the US-Treasury website. $E_t\left(r_{m,t+h}\right)$ is estimated using a rolling window of 1000 days.

[22] To solve for the weight function we use the FOC w.r.t. w_{t+h}. $\frac{\partial U}{\partial w_{t+h}} = 0. \leftrightarrow -r_{f,t+h} + E_t(r_{m,t+h}) - \gamma w_{t+h}\widehat{RV}_{t+h} = 0.$

The average realized utility enables us to compare the alternative investment strategies by calculating the associated average utility levels. A given utility level can be interpreted as the certain return that provides the same utility to the investor as the risky investment strategy. This way, we can determine the economic value of market timing by calculating the maximum fee, in annual basis points, that an investor is willing to pay to switch from the benchmark strategy to our strategy. This maximum fee for holding portfolio a instead of the benchmark portfolio b, Δ_γ, can be found by solving:

$$T^{-1}\sum_{t=0}^{T-1}\left[\left(r_{a,t+h}-\Delta_\gamma\right)-\frac{\gamma}{2}w^*_{a,t+h}\widehat{RV}_{a,t+h}\right]=T^{-1}\sum_{t=0}^{T-1}\left[r_{b,t+h}-\frac{\gamma}{2}w^*_{b,t+h}\widehat{RV}_{b,t+h}\right] \quad (27)$$

where a and b refer to our strategies (HAR-X class of models) and the benchmark portfolio (HAR-RV), respectively. The maximum fee, Δ_γ, can be easily estimated by taking the difference between two alternative average utilities.

Table 9 reports the economic gains of switching from the benchmark (HAR-RV) strategy to the HAR-X class of models for $h = 1$.[23] The performance fee represents the amount that an investor is willing to pay to using our new class of forecasting models. The performance is expressed in annual basis points, bold numbers highlight the models that outperform the benchmark strategy, and the starred values indicate significant gains at the 5% significance level.[24]

We show that all the strategies based on the HAR-X class of models generate positive performance fee as an average across all the stocks, but the HAR-V$^+$ and HAR-Co$^+$-V$^+$.[25] The best performance for the HAR-V and HAR-Co-V type of strategies are respectively achieved by the HAR-V$^-$, and the HAR-Co$^+$-V and HAR-Co$^-$-V$^-$ strategies. These strategies significantly outperform the benchmark strategy in at least 50% of the stocks under consideration when $\gamma = 2$. For higher levels of risk aversion ($\gamma = 6, 10$) we find that performance fee is always positive, while the number of significant stocks falls just

[23] Results for longer horizons are qualitatively similar to the $h = 1$, and are available upon request.

[24] To evaluate the performance of our strategies we create a null hypothesis that examines whether the performance fee is equal to zero. In other words, $H_0 : \Delta_\gamma = 0$ and $H_1 : \Delta_\gamma > 0$. We follow Bandi et al. (2008); Engle and Colacito (2006); Nolte and Xu (2015), among others, and apply a one-sided t-test with a robust variance-covariance estimator.

[25] The negative performance fee from these two models is expected as the positive semivariance is found to be uninformative producing very weak out-of-sample performances.

below 50%.

In terms of performance fee, we find that an investor is willing to pay ranging from 57 ($\gamma = 2$) to 11 ($\gamma = 10$) basis points to switch from the benchmark strategy to the HAR-Co$^+$-V strategy. Similarly and without considering the performance of the HAR-V$^+$ nor the HAR-Co$^+$-V$^+$, we find that at $\gamma = 2$ the HAR-V type of strategies produce an average performance fee across all the stocks of 5.627 basis points, while the HAR-Co-V type of strategies achieve an average performance fee of 9.357 basis points. These results highlight the economic benefits of incorporating the covariance information, and that strategies accounting for the asymmetric dependencies of the data far exceed the performance of the general and benchmark strategies.

Figure 6 plots the weights of 4 different models across time using Procter & Gamble (PG) results.[26] We use the benchmark strategy, the HAR-Co$^+$-V and HAR-V$^-$, which are the best performing strategies from the HAR-V and HAR-Co-V family, and the HAR-V$^+$ strategy. They grey lines in the background of each subplot represent the weights of all remaining models. It is noteworthy that during more volatile periods all models allocate similar weights to the risky asset. However, when the level of volatility is low, we find that the HAR-Co$^+$-V and the HAR-V$^-$ models usually allocate more weight to the risky asset, while the HAR-RV and HAR-V$^+$ strategies fail to do so and allocate weights that are of similar magnitude during turmoil periods. Thus, our more responsive forecasts are able to estimate the weights more dynamically and adjust faster to the different economic conditions, implying that their weights are closer to the fundamental weights.

Figure 5 shows the behavior of the HAR-Co$^+$-V strategy across the sampling frequency and forecasting horizon as an average of all the stocks. The 300-second frequency produce the biggest average realized utility irrespective of the forecasting horizon. This result is in line with our previous findings, which indicate that the 300-second frequency produces slightly better forecasting performance. However, as the risk-aversion level increases the difference in performance fee shrinks drastically, indicating that the choice of the sampling frequency is only relevant when the level of risk-aversion is low.

[26]We use PG as its expected return using the rolling scheme is always positive and greater than the risk-free rate. This ensures a nice dynamics of the weight distribution.

Finally, we evaluate the performance of our volatility-timing portfolio strategies in the presence of transaction costs. Following standard arguments (see, DeMiguel et al., 2014) we define the transaction cost adjusted portfolio return as:

$$\bar{r}_{p,t+1} = r_{p,t+1} - \pi \underbrace{\left(w_{t+1} - w_t \frac{1 + r_{p,t+1}}{1 + w_t r_{p,t+1}} \right)}_{\text{Turnover}}, \tag{28}$$

where $\bar{r}_{p,t+1}$ is the transaction cost adjusted portfolio return, and π is the transaction cost parameter. We follow Nolte and Xu (2015) and set π to 0.0025, corresponding to a 2.5 cent half spread on a 10 dollar stock. Results reported in Table 10 correspond to the average across all the stocks for 1-day ahead forecasts. The average performance fees of our strategies are slightly smaller compared to the results in Table 9. This indicates that transaction costs only have a marginal effect on our strategies, which is expected since our comparison is based on two dynamic strategies both using high-frequency information. We find that a higher performance fee is directly associated with a bigger turnover. The higher turnover observed in our strategies comes from the fact they react faster to new information, while the HAR-RV strategy is smoother. This adjustment to new information implies more dynamic changes in weights, producing a significantly bigger performance fee at the minimal cost of a slightly bigger trading turnover.

8 Conclusions

This paper extends the popular structure of the heterogeneous autoregressive (HAR) model of Corsi (2009) introducing the HAR-X class of models, which incorporates the information from the market index in the form of (co)variances. We show that whereas the HAR-RV model assigns more weight to monthly volatility, the HAR-X models rely on more recent information to predict future volatility. This new distribution of information renders more responsive forecasts improving significantly the forecast accuracy compared to the benchmark model. These forecasting gains hold true both in simulation and in- and out-of-sample comparisons on the volatility of 20 individual stocks.

The HAR-X class of models also account for the observed asymmetry in volatility utilizing the novel noise-robust semi-(co)variances extensions that were developed in this paper following the work of Christensen et al. (2010, 2014); Jacod et al. (2009). We show that the use of negative semivariances in the HAR-V model far exceeds the performance of its unsigned counterpart and the benchmark model. Moreover, a full negative structure in the HAR-Co-V (HAR-Co^{-}-V^{-}) also outperforms the benchmark model and its unsigned counterpart, however, the HAR-Co^{+}-V model achieves the best performance across all the HAR-X model classifications.

The previous statistical improvements afforded by the HAR-X class of models directly translates in significant economic gains. We find by using a volatility timing strategy that a risk-averse investor can substantially increase her portfolio performance by using the information from the market index, and that she is willing to pay up to 57 (56) annual basis points prior (after) transaction costs for switching to our HAR-Co^{+}-V model strategy.

Finally, while forecasting gains are found to be more stable across sampling frequencies after accounting for the presence of microstructure noise, a mixed sampling frequencies approach indicates that bigger out-of-sample improvements are attained using a low (high) frequency for the stock (index). This finding suggests that mixing the sampling frequencies better captures the index information signal.

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A Tables and Figures

Table 1: Summary statistics

Stock/Index	Ticker	T	Annualized Returns (%)	Annualized Volatility (%)	Min PRV	Mean PRV	Median PRV	Max PRV
Amazon.com, Inc.	AMZN	4277	21.708	**41.356**	0.073	6.787	2.465	268.493
Arconic Inc.	ARNC	4277	-35.512	32.069	0.117	4.081	2.220	133.436
Boeing Co.	BA	4277	6.458	23.621	0.031	2.214	1.147	61.611
Bank of America Corporation	BAC	4277	-22.745	32.518	0.011	4.196	1.255	442.475
Caterpillar Inc	CAT	4277	-0.022	26.106	0.046	2.704	1.466	77.442
China Mobile Ltd.	CHL	4277	-0.485	19.626	0.034	1.529	0.664	69.936
Costco Wholesale Corporation	COST	4277	16.141	24.721	0.040	2.425	1.008	206.367
Cisco Systems, Inc.	CSCO	4277	-5.333	29.355	0.038	3.420	1.386	210.661
The Walt Disney Company	DIS	4277	13.642	23.794	0.048	2.247	0.996	198.043
DowDuPont Inc.	DOW	4277	-2.152	28.021	0.038	3.116	1.533	179.970
Exelon Corporation	EXC	4277	2.232	22.909	0.049	2.083	1.017	194.428
Freeport-McMoRan Inc.	FCX	4277	-24.247	39.345	0.137	6.143	3.313	181.682
Halliburton Company	HAL	4277	-16.572	35.905	0.142	5.116	2.667	612.552
Honeywell International Inc.	HON	4277	-4.752	25.453	0.017	2.571	1.202	130.178
International Business Machines Corporation	IBM	4277	15.874	20.014	0.032	1.590	0.711	56.868
The Coca-Cola Co.	KO	4277	12.537	17.227	0.015	1.178	0.565	52.832
The Procter & Gamble Company	PG	4277	14.640	**16.486**	0.019	1.078	0.514	94.868
Southern Co.	SO	4277	4.054	17.326	0.034	1.191	0.587	67.818
Wells Fargo & Company	WFC	4277	-0.646	28.594	0.025	3.244	0.937	254.379
Xerox Corporation	XRX	4277	7.218	33.763	0.076	4.524	1.903	439.853
SPDR S&P 500 ETF	SPY	4277	-0.056	14.863	0.010	0.877	0.410	61.442

Note: The table reports the descriptive statistics for all the stocks and the SPY. The realized measures presented are estimated at the 300 second frequency. The annualized volatility is estimated as $\sigma_T\sqrt{252}$, where σ_T is the average daily pre-averaged realized volatility, and the annualized return is $\mu \times 252$, where μ is the average daily return. PRV is the pre-averaged realized volatility defined as in equation (8). The bold numbers represent the highest and lower annualized volatility, while the blue and red font highlight the highest and lowest annualized return.

Table 2: Average correlations across sampling frequency and realized measures

	PRV^s	PRV^m	MRC	MRC^+	MRC^-	PRV^{s^+}	PRV^{s^-}	PRV^{m^+}	PRV^{m^-}
PRV^s	–	0.688	0.769	0.730	0.736	0.931	0.919	0.647	0.650
PRV^m	0.611	–	0.924	0.878	0.822	0.689	0.586	0.959	0.918
MRC	0.721	0.897	–	0.933	0.874	0.766	0.659	0.886	0.848
MRC^+	0.665	0.820	0.897	–	0.659	0.805	0.539	0.924	0.691
MRC^-	0.632	0.733	0.789	0.456	–	0.620	0.755	0.674	0.914
PRV^{s^+}	0.854	0.586	0.702	0.788	0.404	–	0.714	0.694	0.586
PRV^{s^-}	0.832	0.460	0.530	0.333	0.701	0.439	–	0.500	0.625
PRV^{m^+}	0.553	0.926	0.836	0.897	0.492	0.615	0.322	–	0.767
PRV^{m^-}	0.528	0.833	0.741	0.481	0.887	0.383	0.537	0.563	–

Note: The table reports the correlation among all the realized measures under analysis. The entries report the average correlation for the 20 stocks. Entries above the main diagonal are estimated using 30-second returns, while below the main diagonal the entries are estimated using 300-second returns.

Table 3: HAR-X prediction regression results

	HAR-RV	HAR-V	HAR-Co-V	HAR-RV	HAR-V	HAR-Co-V	HAR-RV	HAR-V	HAR-Co-V
		$h=1$			$h=5$			$h=22$	
β_0	0.368***	0.355***	0.286***	0.502***	0.488***	0.412***	0.785***	0.752***	0.674***
β_d^s	0.191***	0.158***	0.162***	0.117***	0.085**	0.103	0.065	0.046	0.057
β_w^s	0.335***	0.237*	0.259	0.291***	0.212***	0.196	0.200**	0.137	0.199
β_m^s	0.349***	0.406***	0.439***	0.421***	0.461***	0.524***	0.464***	0.501***	0.490***
β_d^m		0.139	0.151		0.146*	0.242*		0.090	0.145
β_w^m		0.406**	0.395*		0.342*	0.045		0.180*	0.404
β_m^m		−0.311	0.396		−0.257	0.773*		−0.081	0.648*
β_d^{RC}			−0.030			−0.128*			−0.077
β_w^{RC}			−0.067*			0.259			−0.353
β_m^{RC}			−0.729			−1.105			−0.718*
R^2_{adj}	0.389	**0.398**	**0.403**	0.551	**0.566**	**0.575**	0.564	**0.583**	**0.598**
F-test	–	20	20	–	18	20	–	20	20

Note: The table reports the coefficients for the average across all the stocks at the 300 seconds. *, **, and *** represent the significant of the coefficients at the 10%, 5%, and 1% level using the Newey-West HAC correction allowing for serial correlation up to order 5 ($h = 1$), 10 ($h = 5$), and 44 ($h = 22$). Bold numbers highlight the HAR-X models that outperform the benchmark HAR-RV. The bottom panel reports average values across all the stocks for the R^2_{adj}, and the number of rejections of the F-test. The F-test has a null hypothesis of equal fit, and hence its rejection indicates that HAR-X models are a better model fit than HAR-RV models.

Table 4: Asymmetric HAR-V prediction regression results

	HAR-V$^+$	HAR-V$^-$	HAR-V$^+$	HAR-V$^-$	HAR-V$^+$	HAR-V$^-$
	$h=1$		$h=5$		$h=22$	
β_0	0.551***	0.326***	0.650***	0.480***	0.886***	0.757***
β_d^{s+}	0.149		0.083		0.041	
β_w^{s+}	0.408		0.328		0.200	
β_m^{s+}	0.888***		0.963***		1.004***	
β_d^{m+}	−0.035		0.052		0.041	
β_w^{m+}	0.780		0.663		0.414	
β_m^{m+}	−0.276		−0.232		−0.026	
β_d^{s-}		0.209***		0.112*		0.061
β_w^{s-}		0.429*		0.371*		0.263*
β_m^{s-}		0.884***		0.963***		0.980***
β_d^{m-}		0.995**		0.627**		0.357*
β_w^{m-}		0.857*		0.887*		0.407
β_m^{m-}		−0.897*		−0.652		−0.072
R^2_{adj}	0.335	**0.431**	0.497	**0.594**	0.526	**0.598**
F-test	0	19	1	20	1	20
HAR-RV						
R^2_{adj}	0.389		0.551		0.564	

Note: See notes to Table 3. The bottom panel of the Table reports the 1-day ($h=1$), 5-day ($h=5$), and 22-day ($h=22$) ahead HAR-RV's adjusted R-squares for the average across all the stocks.

Table 5: Asymmetric HAR-Co-V prediction regression results

Panel A: The table reports the prediction regression results for HAR-Co-V based on unsigned volatilities and semicovariances.

	HAR-Co$^+$-V	HAR-Co$^-$-V	HAR-Co$^+$-V	HAR-Co$^-$-V	HAR-Co$^+$-V	HAR-Co$^-$-V
	$h=1$		$h=5$		$h=22$	
β_0	0.160	0.311***	0.332**	0.445***	0.614***	0.745***
β_d^s	0.216**	0.102	0.120***	0.051	0.063***	0.027
β_w^s	0.326**	0.121	0.281**	0.114	0.194***	0.109
β_m^s	0.417***	0.481***	0.489***	0.540***	0.551***	0.502***
β_d^m	0.537**	−0.149	0.377**	−0.041	0.197*	−0.007
β_w^m	0.859*	−0.278	0.692	−0.267	0.472	−0.059
β_m^m	0.149	−0.081	0.295	0.145	0.643	−0.005
β_d^{RC+}	−0.727**		−0.447**		−0.232*	
β_w^{RC+}	−1.159*		−0.882*		−0.751	
β_m^{RC+}	−0.908		−1.103		−1.436*	
β_d^{RC-}		0.896**		0.532**		0.309*
β_w^{RC-}		1.789*		1.617*		0.585
β_m^{RC-}		−0.719		−1.099		−0.237
R^2_{adj}	**0.426**	**0.427**	**0.593**	**0.591**	**0.607**	**0.594**
F-test	20	20	20	20	20	20

Panel B: The table reports the prediction regression results for HAR-Co-V using semi-(co)variances.

	HAR-Co$^+$-V$^+$				HAR-Co$^-$-V$^-$		
	$h=1$	$h=5$	$h=22$		$h=1$	$h=5$	$h=22$
β_0	0.472***	0.583***	0.832***	β_0	0.265**	0.400***	0.698***
β_d^{s+}	0.238	0.155	0.085	β_d^{s-}	0.192***	0.123***	0.071
β_w^{s+}	0.570	0.435	0.316	β_w^{s-}	0.393***	0.293	0.394
β_m^{s+}	0.874**	0.995**	1.027**	β_m^{s-}	1.067***	1.237***	1.056***
β_d^{m+}	0.273	0.337	0.202	β_d^{m-}	0.816	0.587*	0.367*
β_w^{m+}	1.478	1.007	0.831	β_w^{m-}	−0.005	−0.238	0.827
β_m^{m+}	0.684	0.938	1.085	β_m^{m-}	0.749*	1.755*	1.200
β_d^{RC+}	−0.424	−0.377	−0.219	β_d^{RC-}	0.122	0.004	−0.023
β_w^{RC+}	−0.968	−0.541	−0.618	β_w^{RC-}	0.772	1.026	−0.645
β_m^{RC+}	−0.920	−1.200	−1.144	β_m^{RC-}	−1.711*	−2.616*	−1.412*
R^2_{adj}	0.346	0.513	0.544	$\overline{R}^2_{adj}$	**0.436**	**0.603**	**0.609**
F-test	0	3	4	F-test	19	20	20
HAR-RV							
R^2_{adj}	0.389	0.551	0.564				

Note: See notes to Table 3 for details. The bottom panel of the Table reports the 1-day ($h=1$), 5-day ($h=5$), and 22-day ($h=22$) ahead HAR-RV's adjusted R-squares for the average across all the stocks.

Table 6: Out-of-sample ranking performance

	QLIKE $h=1$	Rank	QLIKE $h=5$	Rank	QLIKE $h=22$	Rank	Avg.	HMSE $h=1$	Rank	HMSE $h=5$	Rank	HMSE $h=22$	Rank	Avg.
	30-second return							30-second return						
HAR-RV	1.000	6	1.000	7	1.000	7	1.000	1.000	7	1.000	7	1.000	7	1.000
HAR-V	**0.980**^{11}	4	**0.963**^{9}	6	**0.976**^{3}	6	**0.973**	**0.958**^{13}	6	**0.955**^{9}	6	**0.985**^{6}	6	**0.966**
HAR-V$^+$	1.170^{0}	9	1.154^{0}	9	1.086^{0}	9	1.137	1.363^{0}	9	1.281^{0}	9	1.160^{1}	9	1.268
HAR-V$^-$	**0.939**^{14}	1	**0.895**^{17}	1	**0.937**^{6}	3	**0.924**	**0.841**^{19}	3	**0.868**^{17}	3	**0.940**^{9}	5	**0.883**
HAR-Co-V	**0.982**^{11}	5	**0.958**^{9}	5	**0.956**^{8}	5	**0.965**	**0.922**^{15}	5	**0.902**^{13}	5	**0.912**^{12}	3	**0.912**
HAR-Co$^+$-V$^+$	1.149^{1}	8	1.132^{2}	8	1.059^{4}	8	1.113	1.279^{3}	8	1.206^{5}	8	1.096^{6}	8	1.194
HAR-Co$^-$-V$^-$	**0.970**^{10}	3	**0.900**^{13}	2	**0.923**^{9}	2	**0.931**	**0.835**^{19}	2	**0.831**^{18}	2	**0.885**^{12}	2	**0.850**
HAR-Co$^+$-V	**0.965**^{10}	2	**0.905**^{17}	3	**0.909**^{11}	1	**0.926**	**0.791**^{19}	1	**0.802**^{19}	1	**0.848**^{15}	1	**0.813**
HAR-Co$^-$-V	1.001^{8}	7	**0.928**^{9}	4	**0.951**^{6}	4	**0.960**	**0.887**^{15}	4	**0.896**^{11}	4	**0.936**^{9}	4	**0.906**
	60-second return							60-second return						
HAR-RV	1.000	6	1.000	7	1.000	7	1.000	1.000	7	1.000	7	1.000	7	1.000
HAR-V	**0.976**^{13}	4	**0.957**^{10}	6	**0.972**^{6}	6	**0.968**	**0.942**^{15}	6	**0.939**^{13}	6	**0.977**^{8}	6	**0.953**
HAR-V$^+$	1.176^{0}	9	1.168^{0}	9	1.098^{0}	9	1.147	1.377^{0}	9	1.302^{1}	9	1.179^{0}	9	1.286
HAR-V$^-$	**0.945**^{14}	1	**0.895**^{16}	1	**0.935**^{6}	3	0.925	**0.840**^{17}	3	**0.866**^{17}	3	**0.942**^{9}	5	**0.883**
HAR-Co-V	**0.973**^{15}	3	**0.949**^{10}	5	**0.952**^{9}	5	**0.958**	**0.899**^{16}	5	**0.881**^{15}	5	**0.902**^{12}	3	**0.894**
HAR-Co$^+$-V$^+$	1.147^{1}	8	1.133^{4}	8	1.061^{3}	8	1.114	1.263^{3}	8	1.201^{4}	8	1.095^{3}	8	1.186
HAR-Co$^-$-V$^-$	**0.964**^{13}	2	**0.901**^{13}	3	**0.921**^{7}	2	**0.929**	**0.820**^{19}	2	**0.822**^{19}	2	**0.881**^{12}	2	**0.841**
HAR-Co$^+$-V	**0.984**^{9}	5	**0.897**^{17}	2	**0.902**^{12}	1	**0.927**	**0.763**^{20}	1	**0.779**^{20}	1	**0.833**^{15}	1	**0.792**
HAR-Co$^-$-V	1.024^{7}	7	**0.924**^{10}	4	**0.944**^{8}	4	**0.964**	**0.869**^{15}	4	**0.876**^{13}	4	**0.921**^{10}	4	**0.889**
	300-second return							300-second return						
HAR-RV	1.000	7	1.000	7	1.000	7	1.000	1.000	7	1.000	7	1.000	7	1.000
HAR-V	**0.966**^{16}	3	**0.943**^{16}	6	**0.963**^{8}	6	**0.957**	**0.902**^{15}	6	**0.909**^{14}	6	**0.961**^{10}	6	**0.924**
HAR-V$^+$	1.174^{0}	9	1.198^{2}	9	1.138^{1}	9	1.170	1.408^{0}	9	1.384^{2}	9	1.260^{1}	9	1.350
HAR-V$^-$	**0.963**^{14}	2	**0.901**^{15}	2	**0.930**^{10}	3	**0.931**	**0.818**^{17}	3	**0.855**^{15}	4	**0.939**^{12}	5	**0.871**
HAR-Co-V	**0.959**^{16}	1	**0.923**^{16}	4	**0.936**^{8}	4	**0.940**	**0.823**^{15}	4	**0.817**^{17}	3	**0.857**^{14}	3	0.833
HAR-Co$^+$-V$^+$	1.135^{2}	8	1.142^{4}	8	1.085^{4}	8	1.121	1.249^{4}	8	1.220^{4}	8	1.138^{3}	8	1.202
HAR-Co$^-$-V$^-$	**0.983**^{12}	4	**0.911**^{14}	3	**0.909**^{12}	2	**0.934**	**0.765**^{17}	2	**0.781**^{18}	2	**0.842**^{15}	2	**0.796**
HAR-Co$^+$-V	**0.985**^{12}	5	**0.881**^{19}	1	**0.885**^{14}	1	**0.917**	**0.693**^{19}	1	**0.729**^{20}	1	**0.806**^{16}	1	**0.743**
HAR-Co$^-$-V	**0.995**^{8}	6	**0.931**^{12}	5	**0.944**^{8}	5	**0.957**	**0.868**^{15}	5	**0.880**^{1}4	5	**0.931**^{12}	4	**0.893**

Note: The table reports the average relative loss for QLIKE and HMSE across all the stocks. The relative losses are estimated as the ratio of the losses of the HAR-X models to the losses of the benchmark HAR-RV. Entries in bold indicate that our proposed models outperform the HAR-RV. Avg. column reports for each model the average relative loss across all forecasting horizons. The number in the superscript represents the number of stocks for which the losses of the HAR-X models are significantly lower than the losses of the benchmark model. We use the CPA test of Giacomini and White (2006) at the 5% significance level.

Table 7: Model confidence set ranking – QLIKE

	AMZN	ARCN	BA	BAC	CAT	CHL	COST	CSCO	DIS	DOW	EXC	FCX	HAL	HON	IBM	KO	PG	SO	WFC	XRX
										$h = 1$										
HAR-RV	–	4	–	–	7	–	–	–	–	–	2	–	–	–	–	–	–	–	–	–
HAR-V	–	2	–	–	6	–	–	–	–	–	1	–	4	–	–	2	4	1	5	–
HAR-V$^+$	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
HAR-V$^-$	1	6	–	–	4	3	1	2	–	2	–	–	3	3	3	5	3	3	1	–
HAR-Co-V	3	3	2	3	3	4	–	–	2	–	–	1	2	–	–	1	1	2	–	2
HAR-Co$^+$-V$^+$	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
HAR-Co$^-$-V$^-$	–	–	–	4	2	1	–	1	4	3	–	–	5	4	1	3	2	–	2	–
HAR-Co$^+$-V	2	5	1	1	1	2	–	–	1	1	–	–	1	1	2	–	–	4	3	1
HAR-Co$^-$-V	–	1	3	2	5	–	–	–	3	–	–	–	–	2	–	4	–	–	4	–
										$h = 5$										
HAR-RV	–	–	–	–	–	–	–	–	–	–	3	–	–	–	–	–	–	–	–	–
HAR-V	–	–	–	–	–	–	–	–	–	–	1	4	–	–	–	6	–	–	–	–
HAR-V$^+$	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
HAR-V$^-$	–	3	–	2	3	–	1	1	–	3	2	6	–	3	3	3	4	3	2	–
HAR-Co-V	–	4	2	–	–	–	–	–	2	–	4	1	2	–	–	2	2	2	4	2
HAR-Co$^+$-V$^+$	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
HAR-Co$^-$-V$^-$	–	–	4	3	1	2	2	2	–	1	–	5	–	4	1	1	1	–	3	–
HAR-Co$^+$-V	1	1	1	1	2	1	–	–	1	2	–	3	1	1	2	4	3	1	1	1
HAR-Co$^-$-V	–	2	3	4	–	–	–	–	–	–	–	2	–	2	–	5	–	–	–	3
										$h = 22$										
HAR-RV	2	3	–	6	4	–	–	–	–	–	–	7	–	–	2	–	–	–	–	–
HAR-V	–	4	–	3	–	–	–	–	–	–	–	5	–	–	–	–	–	–	–	–
HAR-V$^+$	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
HAR-V$^-$	–	2	–	4	3	2	1	1	–	3	2	6	–	2	–	4	–	–	2	5
HAR-Co-V	3	6	–	5	5	–	–	–	2	4	–	4	1	–	–	5	2	–	–	4
HAR-Co$^+$-V$^+$	–	–	–	–	–	–	–	–	4	–	–	–	–	–	–	–	–	–	–	–
HAR-Co$^-$-V$^-$	–	5	–	2	1	3	2	–	3	1	3	3	–	–	1	1	1	–	–	3
HAR-Co$^+$-V	1	1	1	1	2	1	–	–	1	2	1	1	2	1	3	2	3	1	1	1
HAR-Co$^-$-V	–	7	–	7	6	–	3	–	–	–	–	2	3	–	4	3	–	–	–	2

Note: The table reports the ranking based on the Model Confidence Set (MCS) proposed by Hansen et al. (2011). The entries are the ranking of the models across each stock and forecasting horizon based on noise-robust measures estimated using 300-second return frequency. We use the QLIKE as loss function, and the MCS is estimated using a block bootstrap with a window equal to 10 days, 5,000 replications, and a significance level of 10%. The dash-line indicates that the model has been excluded from the MCS.

Table 8: Model confidence set across sampling frequencies – QLIKE

	AMZN	ARCN	BA	BAC	CAT	CHL	COST	CSCO	DIS	DOW	EXC	FCX	HAL	HON	IBM	KO	PG	SO	WFC	XRX
									30-second return											
HAR-RV	–	1	–	–	4	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
HAR-V	–	2	–	–	5	–	–	–	–	–	–	2	–	–	–	2	–	–	–	3
HAR-V^+	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
HAR-V^-	1	3	–	1	1	2	1	1	3	1	–	1	5	1	2	1	2	2	1	2
HAR-Co-V	2	4	3	–	3	–	–	–	–	–	1	3	3	–	–	3	4	–	–	6
HAR-Co^+-V^+	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
HAR-Co^--V^-	–	7	–	4	–	3	2	2	2	4	–	4	4	2	–	–	3	–	–	4
HAR-Co^+-V	3	6	1	3	2	1	3	–	1	2	–	–	1	–	1	4	1	1	–	5
HAR-Co^--V	–	5	2	2	–	–	–	–	–	3	–	–	2	3	–	–	–	–	–	1
									60-second return											
HAR-RV	–	2	4	–	5	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
HAR-V	–	1	–	–	–	5	–	–	–	–	–	3	–	–	–	3	–	4	–	3
HAR-V^+	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
HAR-V^-	1	5	–	3	2	1	1	1	–	1	–	2	5	3	2	1	1	3	1	–
HAR-Co-V	3	3	2	–	3	4	–	–	–	–	1	1	3	–	–	2	4	2	–	2
HAR-Co^+-V^+	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
HAR-Co^--V^-	–	–	–	4	4	3	2	2	–	4	–	5	4	2	3	4	2	–	–	–
HAR-Co^+-V	2	–	1	1	1	2	3	–	1	2	–	4	1	4	1	–	3	1	–	–
HAR-Co^--V	–	4	3	2	–	–	–	–	–	3	–	–	2	1	–	–	–	–	–	1
									150-second return											
HAR-RV	–	4	4	–	–	–	–	–	–	–	3	–	–	–	–	–	–	–	–	–
HAR-V	–	2	–	–	–	–	–	–	–	–	1	–	–	–	–	2	4	2	–	–
HAR-V^+	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
HAR-V^-	1	6	–	–	4	2	1	1	–	2	–	–	–	–	–	4	3	4	1	–
HAR-Co-V	2	5	2	–	3	3	–	–	–	–	2	1	2	–	–	1	2	3	–	2
HAR-Co^+-V^+	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
HAR-Co^--V^-	–	–	–	3	2	1	–	2	–	3	–	–	4	3	1	3	1	–	3	–
HAR-Co^+-V	–	3	1	1	1	4	2	–	1	1	–	–	1	2	2	–	–	1	2	1
HAR-Co^+-V	–	1	3	2	–	–	–	–	–	4	–	–	3	1	–	5	5	–	–	–

Note: The table reports the ranking based on the Model Confidence Set (MCS) proposed by Hansen et al. (2011). The entries are the ranking of the models across sampling frequencies based on 1-day ahead pseudo out-of-sample forecasts. We use the QLIKE as loss function, and the MCS is estimated using a block bootstrap with a window equal to 10 days, 5000 replications, and a significance level of 10%. The dash-line indicates that the model has been excluded from the MCS.

Table 9: Volatility-timing portfolio performance fee

	AMZN	ARNC	BA	BAC	CAT	CHL	COST	CSCO	DIS	DOW	EXC	FCX	HAL	HON	IBM	KO	PG	SO	WFC	XRX	$\bar{\Delta}_\gamma$
											$\gamma = 2$										
HAR-V	**2.428***	–	−0.190	**3.822***	−0.123	**0.053**	**4.152***	**0.192**	−1.909	**1.069**	**1.530**	**0.138**	**0.246**	−0.360	**1.366**	**0.025**	**4.878***	**0.200**	**0.045**	**45.464***	**3.151**
HAR-V$^+$	−22.394	–	−1.356	−4.885	−1.226	−1.211	−10.435	−2.541	−5.006	−3.453	**0.703**	**0.051**	−0.028	−2.990	−37.601	−5.667	−1.377	−0.569	−1.018	**8.553***	−4.622
HAR-V$^-$	**23.481***	–	−0.116	**12.288***	**0.330**	**1.358**	**20.431***	**5.117***	−2.118	**3.329***	**3.418***	−0.020	**0.413**	**0.688**	**29.539***	**4.330***	**7.985***	**1.017**	**0.893**	**49.685***	**8.102**
HAR-Co-V	**10.060***	–	**0.132**	**10.004***	**0.112**	**0.333**	**4.343***	**1.413**	**6.891***	**3.034***	**2.565***	**0.234**	**0.386**	−0.477	**14.087***	**2.384***	**19.123***	**1.213**	**0.648**	**50.339***	**6.341**
HAR-Co$^+$-V$^+$	−8.855	–	−0.666	−1.433	−1.024	−0.973	−11.203	−2.404	**4.982***	−2.110	**2.044***	**0.128**	**0.203**	−1.716	−28.601	−3.457	**8.035***	**0.718**	−0.761	**13.699***	−1.670
HAR-Co$^-$-V$^-$	**27.332***	–	**0.044**	**16.833***	**1.099**	**1.750**	**30.348***	**5.467***	**3.548***	**6.911***	**3.482***	**0.010**	**0.299**	**0.704**	**40.720***	**10.123***	**16.826***	**1.292**	**0.904**	**49.144***	**10.842**
HAR-Co$^+$-V	**46.439***	–	**0.846**	**12.778***	**0.412**	**3.157***	**8.474***	**9.682***	**12.136***	**4.258***	**4.125***	**0.221**	**0.622**	**2.614***	**31.659***	**7.670***	**30.873***	**2.092***	**3.546***	**57.437***	**11.952**
HAR-Co$^-$-V	**46.039***	–	**0.320**	**10.732***	−0.001	−0.369	**35.017***	**3.799***	**0.525**	1.527	1.886	**0.196**	**0.242**	**2.277***	**14.745***	**0.794**	**2.783***	**0.444**	**0.192**	**44.705***	**8.293**
											$\gamma = 6$										
HAR-V	**0.809**	–	−0.063	**1.274**	−0.041	**0.018**	**1.384**	**0.064**	−0.636	**0.356**	**0.510**	**0.046**	**0.082**	−0.120	**0.455**	**0.008**	**1.626**	**0.067**	**0.015**	**15.155***	**1.050**
HAR-V$^+$	−7.465	–	−0.452	−1.628	−0.409	−0.404	−3.478	−0.847	−1.669	−1.151	**0.234**	**0.017**	−0.009	−0.997	−12.534	−1.889	−0.459	−0.190	−0.339	**2.851***	−1.541
HAR-V$^-$	**7.827***	–	−0.039	**4.096***	**0.110**	**0.453**	**6.810***	**1.706**	−0.706	**1.110**	**1.139**	−0.007	**0.138**	**0.229**	**9.846***	**1.443**	**2.662***	**0.339**	**0.298**	**16.562***	**2.701**
HAR-Co-V	**3.353***	–	**0.044**	**3.335***	**0.037**	**0.111**	**1.448**	**0.471**	**2.297***	**1.011**	**0.855**	**0.078**	**0.129**	−0.159	**4.696***	**0.795**	**6.374***	**0.404**	**0.216**	**16.780***	**2.114**
HAR-Co$^+$-V$^+$	−2.952	–	−0.222	−0.478	−0.341	−0.324	−3.734	−0.801	**1.661**	−0.703	**0.681**	**0.043**	**0.068**	−0.572	−9.534	−1.152	**2.678***	**0.239**	−0.254	**4.566***	−0.557
HAR-Co$^-$-V$^-$	**9.111***	–	**0.015**	**5.611***	**0.366**	**0.583**	**10.234***	**1.822**	**1.183**	**2.304***	**1.161**	**0.003**	**0.100**	**0.235**	**13.573***	**3.374***	**5.609***	**0.431**	**0.301**	**16.381***	**3.620**
HAR-Co$^+$-V	**15.587***	–	**0.282**	**4.259***	**0.137**	**1.052**	**2.825***	**3.233***	**4.045***	**1.419**	**1.375**	**0.074**	**0.207**	**0.871**	**10.553***	**2.557***	**10.515***	**0.697**	**1.185**	**19.146***	**4.001**
HAR-Co$^-$-V	**15.914***	–	**0.107**	**3.577***	−0.000	−0.123	**11.672***	**1.266**	**0.175**	**0.509**	**0.629**	**0.065**	**0.081**	**0.759**	**4.915***	**0.265**	**0.928**	**0.148**	**0.064**	**14.902***	**2.793**
											$\gamma = 10$										
HAR-V	**0.486**	–	−0.038	**0.764**	−0.025	**0.011**	**0.830**	**0.038**	−0.382	**0.214**	**0.306**	**0.028**	**0.049**	−0.072	**0.273**	**0.005**	**0.976**	**0.040**	**0.009**	**9.093***	**0.630**
HAR-V$^+$	−4.479	–	−0.271	−0.977	−0.245	−0.242	−2.087	−0.508	−1.001	−0.691	**0.141**	**0.010**	−0.006	−0.598	−7.520	−1.133	−0.275	−0.114	−0.204	**1.711**	−0.924
HAR-V$^-$	**4.696***	–	−0.023	**2.458***	**0.066**	**0.272**	**4.086***	**1.023**	−0.424	**0.666**	**0.684**	−0.004	**0.083**	**0.138**	**5.908***	**0.866**	**1.597**	**0.203**	**0.179**	**9.937***	**1.620**
HAR-Co-V	**2.012***	–	**0.026**	**2.001**	**0.022**	**0.067**	**0.869**	**0.283**	**1.378**	**0.607**	**0.513**	**0.047**	**0.077**	−0.095	**2.817***	**0.477**	**3.825***	**0.243**	**0.130**	**10.068***	**1.268**
HAR-Co$^+$-V$^+$	−1.771	–	−0.133	−0.287	−0.205	−0.195	−2.241	−0.481	**0.996**	−0.422	**0.409**	**0.026**	**0.041**	−0.343	−5.720	−0.691	**1.607**	**0.144**	−0.152	**2.740***	−0.334
HAR-Co$^-$-V$^-$	**5.466***	–	**0.009**	**3.367***	**0.220**	**0.350**	**6.141***	**1.093**	**0.710**	**1.382**	**0.696**	**0.002**	**0.060**	**0.141**	**8.144***	**2.025***	**3.365***	**0.258**	**0.181**	**9.829***	**2.172**
HAR-Co$^+$-V	**9.352***	–	**0.169**	**2.556***	**0.082**	**0.631**	**1.695**	**1.940***	**2.427***	**0.852**	**0.825**	**0.044**	**0.124**	**0.523**	**6.332***	**1.534**	**6.342***	**0.418**	**0.711**	**11.487***	**2.402**
HAR-Co$^-$-V	**9.549***	–	**0.064**	**2.146***	−0.000	−0.074	**7.003***	**0.760**	**0.105**	**0.305**	**0.377**	0.039	**0.048**	**0.455**	**2.949***	**0.159**	**0.557**	**0.089**	**0.038**	**8.941***	**1.676**

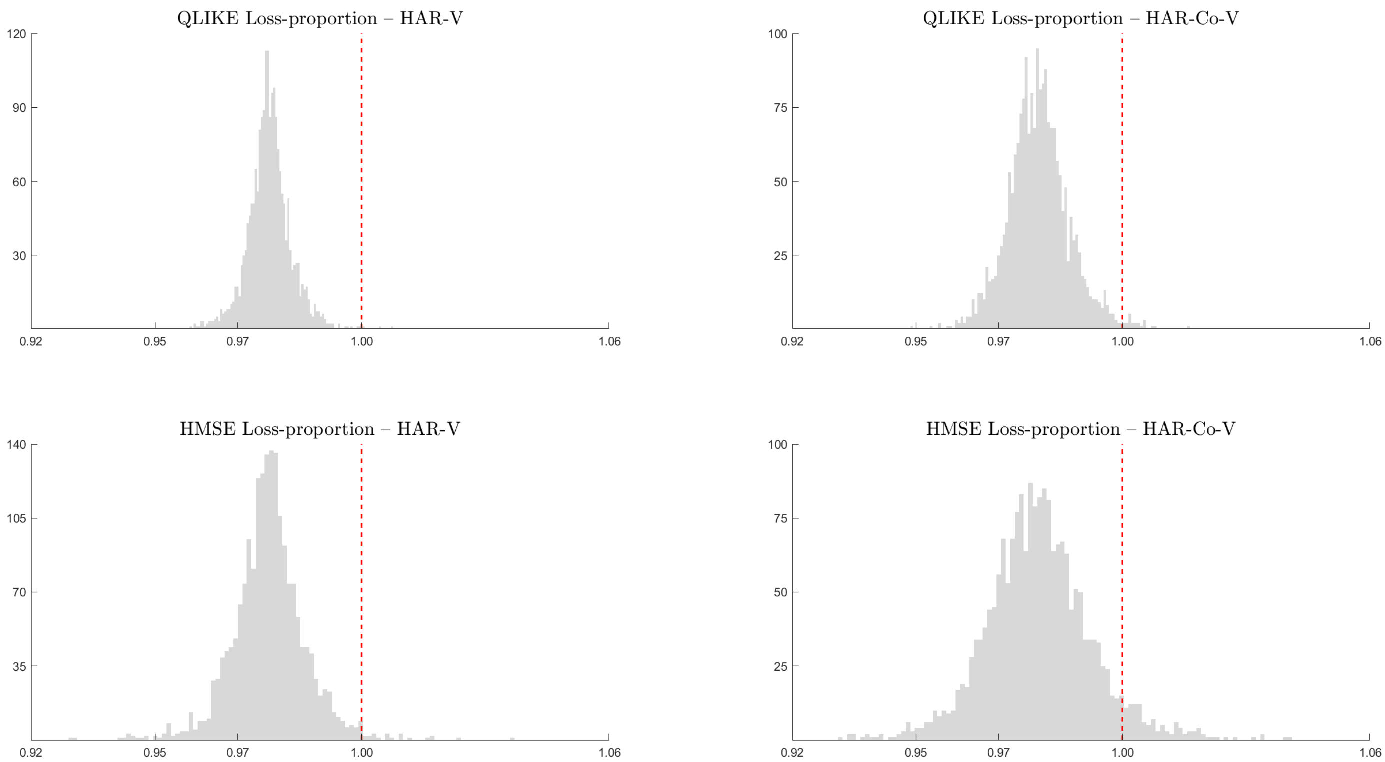
Note: The table shows results for the volatility-timing portfolio strategy using 1-day ahead forecasts. We report the economic gains of switching from the HAR-RV model to the HAR-X models in annual basis points Δ_{ab}. The last column, $\bar{\Delta}_\gamma$, reports the average performance fee for each model across all the stocks under analysis. Bold numbers highlight the HAR-X strategies that outperform the benchmark HAR-RV strategy, and the starred values indicate that our strategy is significantly better than the benchmark in terms of performance fee evaluated at the 5% significance level. The empty columns for ARNC are because its expected return is always negative. Since short selling is not allowed and negative returns produce negative weights, an investor in this scenario can only select the risk-free rate.

Table 10: Volatility-timing portfolio performance fee with transaction costs

	$\bar{\Delta}_2$	TO	$\bar{\Delta}_6$	TO	$\bar{\Delta}_{10}$	TO
HAR-RV		0.0041		0.0014		0.0008
HAR-V	**3.135**	**0.0047**	**1.045**	**0.0016**	**0.627**	**0.0009**
HAR-V^+	-4.595	0.0031	-1.532	0.0010	-0.919	0.0006
HAR-V^-	**7.985**	**0.0083**	**2.663**	**0.0028**	**1.598**	**0.0017**
HAR-Co-V	**6.300**	**0.0056**	**2.101**	**0.0019**	**1.260**	**0.0011**
HAR-Co^+-V^+	-1.661	0.0038	-0.554	0.0013	-0.332	0.0008
HAR-Co^--V^-	**10.691**	**0.0095**	**3.571**	**0.0032**	**2.143**	**0.0019**
HAR-Co^+-V	**11.805**	**0.0095**	**3.953**	**0.0032**	**2.373**	**0.0019**
HAR-Co^--V	**8.178**	**0.0083**	**2.755**	**0.0028**	**1.653**	**0.0017**

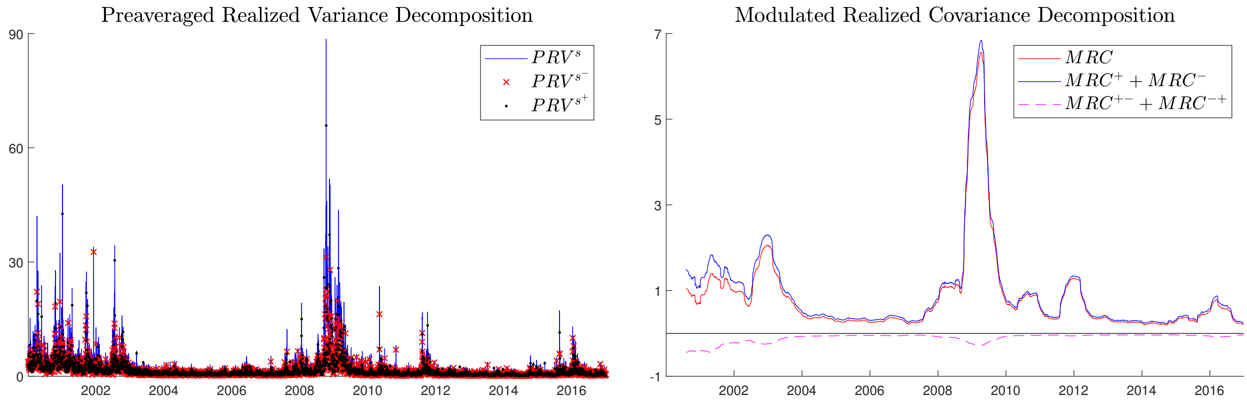
Note: The table reports average performance fee across all the stocks using 1-day ahead forecasts based on 300-second frequency. $\bar{\Delta}_\gamma$ represents the average performance, while TO is the turnover estimated as in equation (10). Bold face numbers indicate that the average performance fee is positive and that the turnover of the strategy is greater than the benchmark's turnover.

Figure 1: Distribution of the standardized out-of-sample losses



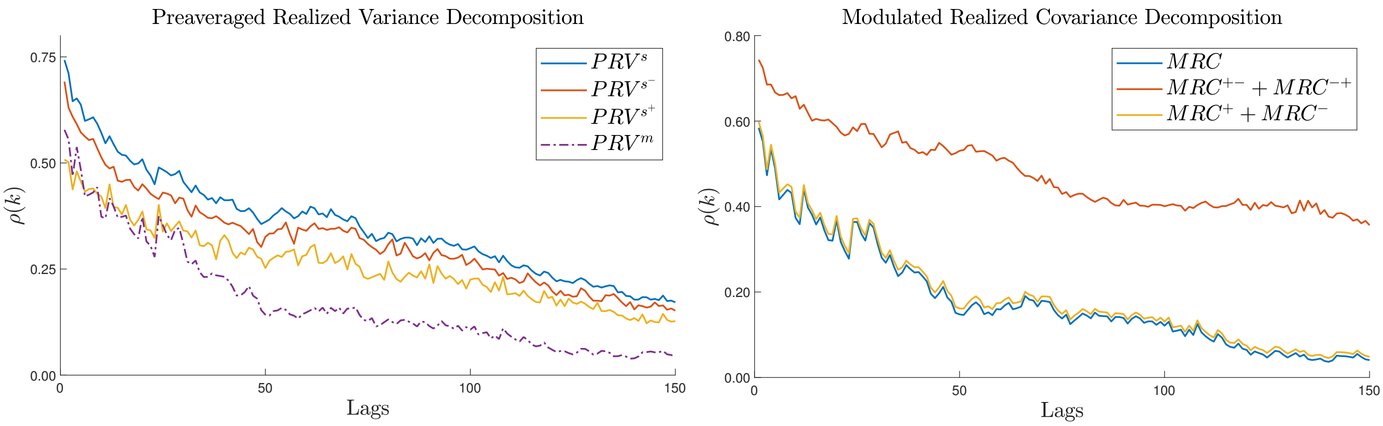
Note: The plot depicts the distribution of the 1-day ahead standardized losses. We forecast 1,000 simulated days, and repeat this process 3,000 times. The losses are standardized by the HAR-RV loss function. The top panel plots the distribution of the QLIKE for the HAR-V (top-left) and HAR-Co-V (top-right), whilst the bottom panel shows the HMSE distribution for the HAR-V (bottom-left) and HAR-Co-V (bottom-right).

Figure 2: Realized variance/covariance and their elements



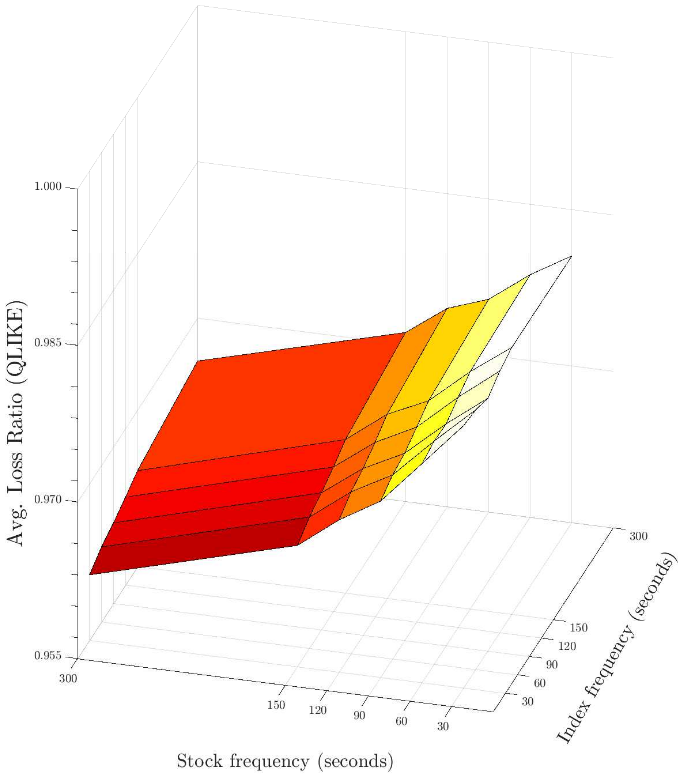
Note: The graph plots the variance and covariance decomposition based on the average of the 20 stocks. The realized measures are estimated at the 300 seconds.

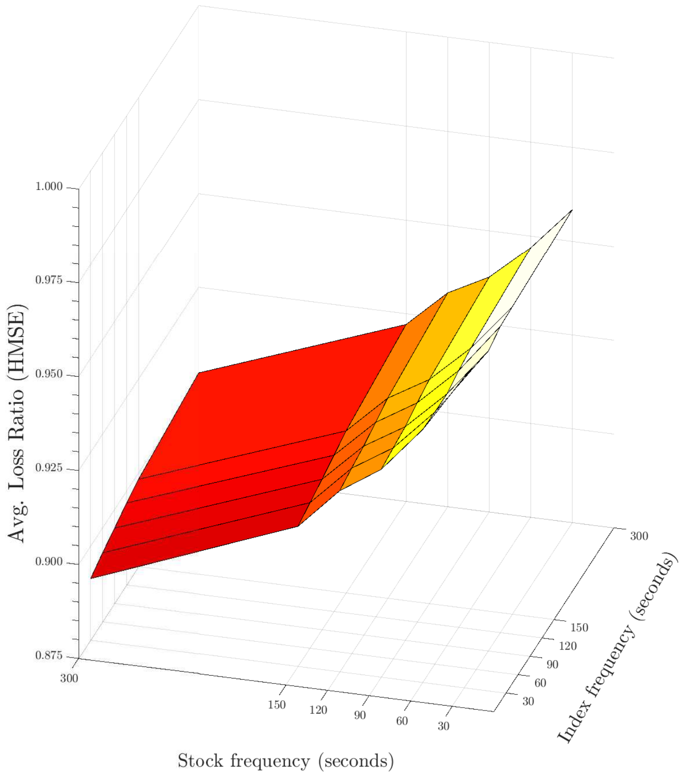
Figure 3: Autocorrelation function



Note: The figure graphs the autocorrelation function for the different realized variances and covariances elements. The results presented are for the average across the stocks.

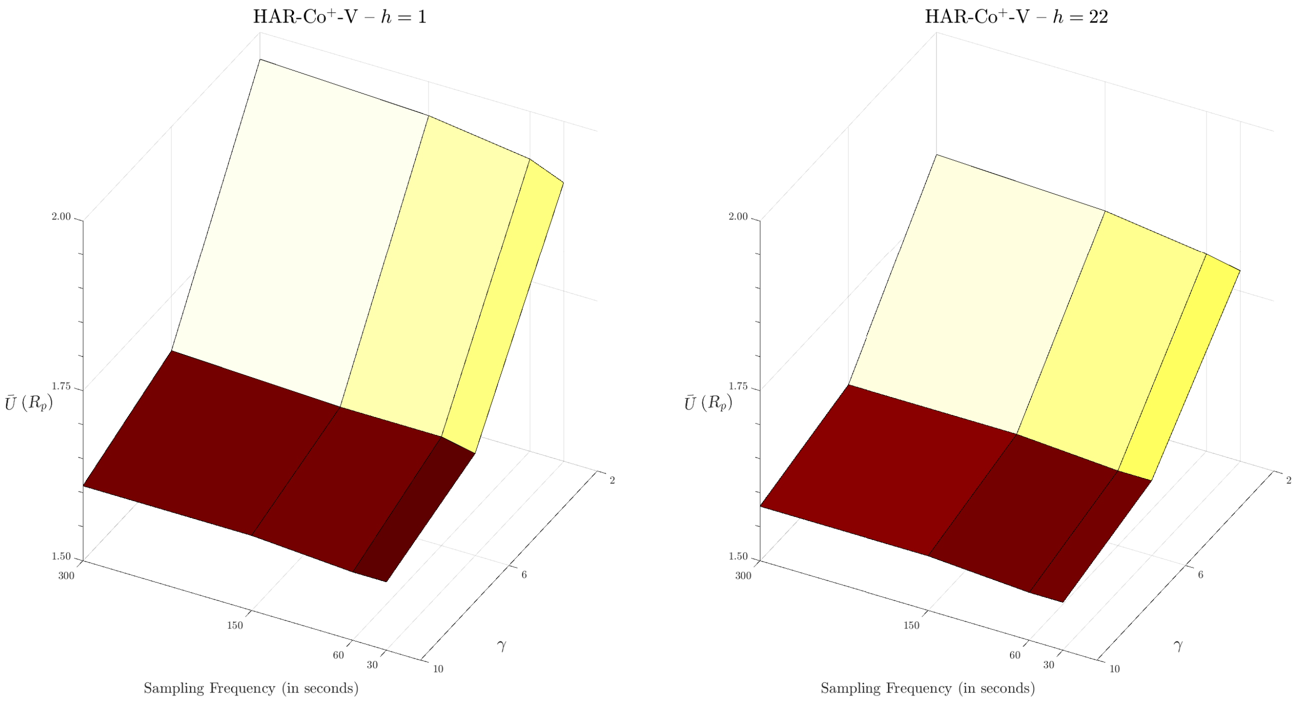
Figure 4: Mixed sampling HAR-V model





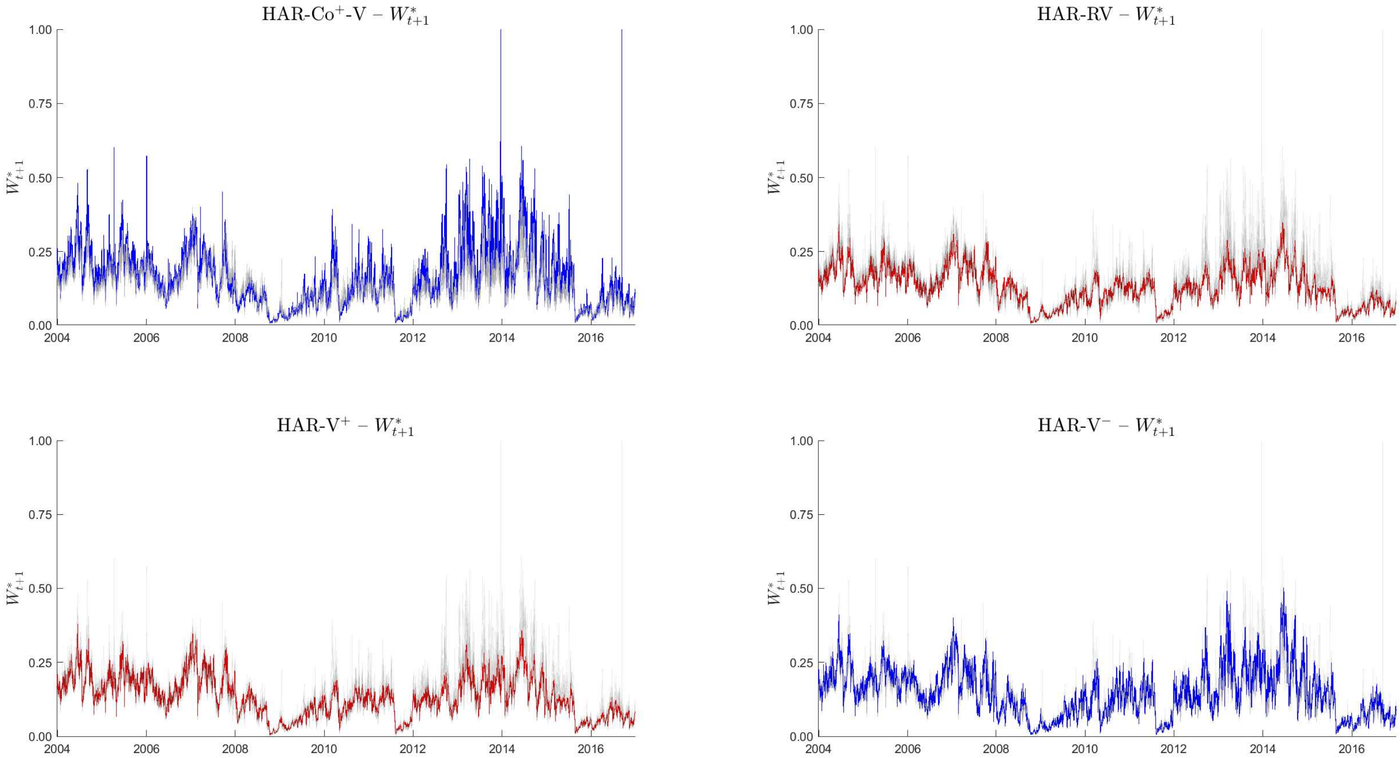
Note: The figure depicts the out-of-sample average relative loss for a mixed sampling HAR-V model. The model is estimated by varying the sampling frequency used to estimate the stock and index volatility. The left-panel plots the QLIKE loss ratio surface, and the right-panel plots the HMSE loss ratio surface.

Figure 5: Average realized utility across sampling frequency



Note: The figure plots the average realized utility function across the sampling frequency and level of risk aversion for all the stocks under analysis using the HAR-Co^{+}-V at $h = 1$ (left panel) and $h = 22$ (right panel).

Figure 6: Weight time series for different models



Note: The figure illustrates the weights behavior of Procter & Gamble (PG) for 4 different models across the out-of-sample period under analysis. The grey lines are for all the remaining models, while the blue/red lines are for the models specified in the title of each subplot. The risk aversion parameter is set to $\gamma = 2$.