“Out-of-Sample Equity Premium Prediction: Combination Forecasts with Frequency-Decomposed Variables”

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Abstract

Technical trading rules are widely used by practitioners to forecast the U.S. equity premium. I decompose technical indicators into components with frequency-specific information, showing that all the predictive power comes from periodicities between 16 to 64 months, without any evidence of predictability outside of this frequency band. An investor who only forecasts with these medium-frequency components generates both statistically and economically sizable gains compared to the historical mean and the original technical indicators. The out-of-sample R^2 is significant for each of the 14 adjusted indicators in the sample. A mean-variance investor who combines individual forecasts from medium-frequency components generates a sizable utility gain of more than 350 basis points relative to the historical mean for the forecasting period from January 1966 to December 2017. This is almost twice as large as utility gains from the historical mean and more than 200 basis points larger than for combination forecasts with unadjusted technical indicators. I show that the substantial gains mainly result from an improved forecasting ability of medium-frequency components during recessions.

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1 Introduction

Predicting the equity premium has a long tradition in finance and is of interest for both academics and practitioners. Real-time forecasts are necessary to adequately allocate resources between risky assets and the risk-free rate to enhance investment performance (DeMiguel et al., 2009; Rapach and Zhou, 2013). Better estimates on both the conditional mean and conditional variance of excess stock market returns help understanding the risk-return trade-off, see, among others, Ludvigson and Ng (2007). Furthermore, the equity premium is central for the evaluation of investment performance and in the characterization of return patterns among different assets, see Ferson (2010) and Goyal (2012) for reviews. Another important point is the consistency between stock return predictability and market efficiency (Balvers et al., 1990; Cujean and Hasler, 2017).

Even though there is a voluminous literature, the overall evidence for stock return predictability is mixed. Generally, predictability is tested either over the full available sample (in-sample) or with a recursively expanding sample that mimics the real-time situation of an investor (out-of-sample).

In this paper, I contribute to the literature on out-of-sample equity premium prediction by proposing a novel forecasting method that uses frequency-decomposed predictor variables. More precisely, I apply filtering methods to split the predictor time series into a sum of frequency-specific parts. Each part depicts oscillations of different frequencies, like high, medium, and low frequencies. This allows me to analyze whether short-horizon predictability only comes from specific frequency-bands of the predictor variables. As an example, the predictability from business-cycle oscillations of a predictor can easily be buried under high-frequency noise or low-frequency movements in the trend.[2]

[2]According to Baxter and King (1999), a business-cycle has cyclical components with periodicities between 1.5 and 8 years.

I show that short-horizon predictability of technical indicators solely stems from medium-frequency components. The oscillations between 16 to 64 months have an excellent forecasting performance during recessions without much evidence of predictability in expansions. This provides further evidence that stock return predictability concentrates in bad times. In a next step, I formally test whether this state-dependent predictability is related to business-cycle expectations. The idea behind is as follows: if predictability one-month ahead depends on current recession forecasts, then an investor could form a forecasting strategy that makes use of this dependency. Due to state-dependent predictability, the sophisticated model only statistically outperforms the historical mean in recessions. Therefore, the historical mean should be used in expansions, thereby limiting forecasting errors. Indeed, I find strong evidence that predictability by medium-frequencies of technical indicators is related to recession forecasts from a simple probit model.

Building on this, I propose a nonlinear forecasting model. Whenever a recession is expected one-month ahead then the sophisticated forecasting model is applied and, otherwise, when an expansion is expected then the historical mean is used for forecasting. This automatically takes the empirical fact of state-dependent predictability into account. Obviously, recessions cannot be forecasted perfectly. Thus, it is an empirical question whether lower forecast errors in expansions offset the cost of potentially not identifying a recession in advance. My results show that the combination of frequency-decomposed predictor and nonlinear forecasting model generates sizable out-of-sample R^2 values in the range of 2%. Even further, I examine the economic significance of frequency-decomposed predictors in an asset allocation exercise. An investor would be willing to pay more than 200 basis points annually to have access to forecasts from medium-frequencies of technical indicators rather than to forecasts from the original predictors.

My work is related to several parts of the literature. Firstly, I contribute to a growing literature that applies filtering methods to cross-sectional asset pricing and forecasting exercises, see, among others, Ortu et al. (2013); Kang et al. (2017); Xyngis (2017); Faria and Verona (2018b); Bandi et al. (2019a,b). The isolated parts from the original series capture different degrees of persistence and allow a more nuanced view on dependencies between time series. As an example, Faria and Verona (2018b) highlight that the sum of frequency-decomposed parts improves on the original sum-of-the-parts forecasting method of Ferreira and Santa-Clara (2011). Secondly, my findings are related to the literature on combination forecasts (Timmermann, 2006; Rapach et al., 2010). I show that combination forecasts from medium-frequency components of both economic variables and technical indicators improve on combination forecasts from the original time series.

Thirdly, I show that medium-frequencies of technical indicators have significant forecasting power in recessions. Neely et al. (2014) find that technical indicators have a significant out-of-sample predictive power for the U.S. equity premium. I decompose these predictors into different frequencies, showing that all the predictive power comes from periodicities between 16 to 64 months. The R^2_{OS} is highly significant for each of the 14 medium-frequency components of technical indicators. Hence, the short-to-medium-end of business-cycle oscillations covers all the predictive information. Surprisingly, the results for economic variables do not reveal frequency-specific predictability. This highlights that the critique of Welch and Goyal (2008) extends to frequency-decomposed parts of individual economic variables.[3]

Fourth, I show that medium-frequency components of technical indicators are economically valuable for an investor. The gains in both certainty equivalent return (CER)

[3] I only test short-horizon predictability. So, I cannot comment on whether frequency-specific parts have forecasting power over long horizons.

and Sharpe ratio (SR) are sizable. This is an important exercise because the correlation between statistical measures of significance and economic measures of significance is only weak (Cenesizoglu and Timmermann, 2012). Combination forecasts from the 14 unadjusted technical indicators taken together increase the annualized CER by roughly 150 basis points compared to the CER for the historical mean, which is around 400 basis points. In contrast to that, combination forecasts from the respective medium-frequency parts of the technical indicators generate an increase in CER by more than 350 basis points. Hence, an investor would be willing to pay a sizable annual fee of more than 200 basis points to have access to frequency-specific combination forecasts. This is comparable in magnitude to gains from other recently proposed predictors (Rapach et al., 2016).

Fifth, I test whether one-month ahead predictability depends on business-cycle expectations (Pesaran and Timmermann, 2009). The results show that expectations in month t on the state of the economy in period $t + 1$ are closely related to out-of-sample predictability in month $t+1$. So far, most of the articles have sorted forecasting gains on ex post available NBER recession periods, finding that gains are centered in recessions. In contrast to that, I document that business-cycle expectations, which are readily available in real time, are informative for predicting stock returns. Predictability significantly depends on recession forecasts. Building on this, I propose a nonlinear forecasting model that selects between the historical mean and sophisticated forecasting models. When forecasts signal a recession then the advanced model is used, otherwise forecasts are made according to the historical mean. By including recession forecasts into a model of return forecasts, I account for state-dependent predictability (Dangl and Halling, 2012; Rapach and Zhou, 2013). Generally, there is a trade-off between lower forecast errors in expansions and potentially missed predictability in a not identified recession. My findings show that the benefits outweigh the costs, and that the increases in out-of-sample R^2 statistics are sizable.

The remainder of the paper proceeds as follows. Section 2 describes the data. The time-frequency decomposition of the original series is explained in Section 3. Section 4 outlines that frequency-decomposed predictors are statistically significant and incorporate useful forecasting information during recessions. Section 5 shows that the filtering method is economically significant. I explain in Section 6 that results are robust with respect to transaction costs, filtering methods, and parameter specifications. Section 7 concludes.

2 Data

My dataset considers the same set of 28 predictors as Neely et al. (2014). One part of the dataset comprises the 14 most commonly used economic variables like the dividend-price ratio and the term spread (Welch and Goyal, 2008; Campbell and Thompson, 2008; Rapach et al., 2010). The other part of the dataset consists of 14 technical indicators that are based on popular trend-following trading strategies. The sample covers monthly U.S. data from December 1950 to December 2017 and therefore extends the dataset of Neely et al. (2014) by six years. The variable that is to be predicted is the excess stock return as measured by continuously compounded returns on the S&P 500 index, including dividends, minus the treasury bill rate. I follow the literature and do not forecast excess returns directly but instead forecast log transformed excess returns. A description of the economic variables and technical indicators is given below.

2.1 Economic variables

Welch and Goyal (2008) provide an extensive comparison of the in-sample and out-of-sample predictability of commonly applied economic variables. Many subsequent studies have analyzed similar variables, see Rapach and Zhou (2013), Pettenuzzo et al. (2014), and Baetje and Menkhoff (2016). The set of 14 economic variables is:

1. Dividend-price ratio (DP): Difference between log of dividends and log of stock prices (S&P 500 index). Dividends are measured using a one-year moving sum.
2. Dividend yield (DY): Difference between the log of dividends and the log of lagged stock prices.
3. Earnings-price ratio (EP): Difference between the log of earnings and the log of stock prices. Earnings are measured using a one-year moving sum.
4. Dividend-payout ratio (DE): Difference between the log of dividends and the log of earnings.
5. Equity risk premium volatility (RVOL): Based on a one-year moving standard deviation estimator.[4]
6. Book-to-market ratio (BM): Ratio of book value to market value for the Dow Jones Industrial Average.
7. Net equity expansion (NTIS): One-year moving sum of net issues by NYSE listed stocks divided by the total end-of-year market capitalization of NYSE stocks.
8. Treasury bill rate (TBL): Secondary market rate on three-month treasury bill rates.
9. Long-term yield (LTY): Long-term government bond yield.
10. Long-term return (LTR): Return on long-term government bonds.
11. Term spread (TMS): Difference between the long-term yield on government bonds and the treasury bill.
12. Default yield spread (DFY): Difference between BAA- and AAA-rated corporate bond yields.

[4]RVOL is estimated as $\widehat{\mathrm{RVOL}}_t = \sqrt{\frac{\pi}{2}}\sqrt{12}\hat{\sigma}_t$, with $\hat{\sigma}_t = \frac{1}{12}\sum_{i=1}^{12}|r_{t+1-i}|$,where r_t is the excess return (no log transformation) in month t (Mele, 2007).

13. Default return spread (DFR): Difference between long-term corporate bond and long-term government bond returns.

14. Inflation (INFL): Calculated from the Consumer Price Index (All Urban Consumers).[5]

2.2 Technical indicators

The trading strategies have in common that they are all based on decision rules that either generate a buy signal ($S_{i,t} = 1$) or a sell signal ($S_{i,t} = 0$). The first trend-following strategy is a moving-average (MA) rule that compares two moving averages of different lengths. In case that the short MA is greater than or equal to the long MA then this generates a buy signal otherwise this generates a sell signal:

$$S_{i,t} = \begin{cases} 1 & \text{if } \mathrm{MA}_{s,t} \geq \mathrm{MA}_{l,t}, \\ 0 & \text{if } \mathrm{MA}_{s,t} < \mathrm{MA}_{l,t}, \end{cases} \tag{1}$$

whereby MA is defined as:

$$\mathrm{MA}_{j,t} = \frac{1}{j} \sum_{i=0}^{j-1} P_{t-i} \quad \text{for } j = s, l. \tag{2}$$

The level of the stock price index is given by P_t and s (l) refers to the short (long) MA ($s < l$). The short notation for a strategy that compares moving-averages of lengths s and l is MA(s, l). Six different MA strategies are compared with $s = 1, 2, 3$ and $l = 9, 12$.

The second strategy is based on a momentum rule that generates a buy signal if the

[5] I follow convention and lag inflation by one month to account for delayed data availability.

current stock price is higher than or equal to its m periods ago past value:

$$S_{i,t} = \begin{cases} 1 & \text{if } P_t \geq P_{t-m}, \\ 0 & \text{if } P_t < P_{t-m}. \end{cases} \tag{3}$$

I include strategies for $m = 9$ and $m = 12$, denoted as MOM(9) and MOM(12).

The third group of strategies is based on trading volume, whereby trade volume is defined as "on-balance" volume (OBV) (Granville, 1963):

$$\text{OBV}_t = \sum_{k=1}^{t} \text{VOL}_k D_k. \tag{4}$$

VOL_k is the trading volume during month k and D_k is a binary variable that equals 1 for $P_k \geq P_{k-1}$ and -1 otherwise. Therefore, trading volume has a positive effect on OBV_k if the stock price has recently increased and vice versa trading volume negatively affects the sum if the stock price has dropped. Similar to the MA strategy for price levels, one can form a MA strategy based on OBV. The intuition is straightforward, a "relatively high recent volume together with recent price increases, say, indicate a strong positive market trend and generate a buy signal" (Neely et al., 2014, p. 1775). The volume-based strategy is defined as VOL(s, l) and I analyze trading rules with $s = 1, 2, 3$ and $l = 9, 12$. Monthly volume data on the S&P 500 index are obtained from Yahoo! Finance.[6]

2.3 Descriptive statistics

Table 1 presents descriptive statistics for the monthly log excess return, the 14 economic variables, and the 14 technical indicators. The average and standard deviation of the monthly equity premium are given by 0.53% and 4.15%. The autocorrelation in excess returns is rather low with a value of 0.06. In contrast to that, the autocorrelation

[6]Further details can be found on `https://de.finance.yahoo.com/`.

for most economic variables is close to 1, with the only exceptions being LTR, DFR, and INFL. Some authors argue that the high degree of persistence results from steady state shifts in the mean, see Lettau and Van Nieuwerburgh (2008) and references therein.

The mean for all technical indicators is around 0.70, so the trend-following strategies generate buy signals for roughly 70% of the sample. Interestingly, the level of persistence is relatively high as well, with a first-order autocorrelation above 0.50 for each indicator. Baetje and Menkhoff (2016, p. 1196) interpret this as support for "the underlying assumption of the technical analysis that past price trends persist into the future".

3 Time-frequency decomposition of predictor variables

In this section I explain the wavelet multiresolution analysis (MRA) that is used to decompose a predictor into a sum of frequency-specific components. Each part captures specific frequency bands of the original series, like high, medium, and low frequencies. An advantage of this approach is that each component can be analyzed separately, thereby revealing what periodicities actually drive stock return predictability.[7]

3.1 The advantages of wavelets

A drawback of classical Fourier analysis is that a signal is assumed to be homogeneous over time (Crowley, 2007). The sine and cosine functions do not fade away and are constant over time, which is problematic when "the signal shows a different behaviour in different time periods or when the signal is localized in time as well as frequency" (Rua, 2011, p. 668). As further explained by Crowley (2007, p. 209), a refinement of standard Fourier analysis is windowed Fourier analysis that relaxes the assumption of no variation over time by transforming "short segments of the signal separately". How-

[7] It is beyond the scope of this paper to give an in-depth introduction to wavelets. For excellent introductory textbooks on wavelets I refer to Percival and Walden (2000) and Gençay et al. (2001).

ever, with this adjustment one simply splits different parts into respective sums of sine and cosine functions. So, the windowed Fourier transform "will not be able to revolve events when they happen to fall within the width of the window" (Gençay et al., 2001, p. 2). To sum this point up: standard Fourier analysis is able to detect different frequencies that are present in a time series, however, it does not provide information on when these oscillations change in the time domain, see Housworth et al. (2019) for an example.

To overcome this shortcoming, a different set of basis functions has to be applied. Rather than sine and cosine functions one needs to use wavelet functions. Wavelets are a further refinement of Fourier analysis, as they have finite energy over a compact set and provide a better resolution in the time domain. The wavelet transform gives up some frequency resolution in order to gain insights on events that are local in time (Gençay et al., 2001). Wavelets localize a signal both in frequency and time. This is especially helpful as many financial and economic series are non-stationary and exhibit structural breaks, instabilities, and volatility clusters (Faria and Verona, 2018b).

A disadvantage of discrete wavelet transform (DWT) is that it is restricted to a dyadic sample size of 2^j with j being a number from the set of positive integers. Therefore, rather than applying DWT many articles use maximum overlap discrete wavelet transform (MODWT), which is free of sample size restrictions. Recent examples of articles that apply MODWT are Kang et al. (2017), Faria and Verona (2018a), Risse (2019). I follow this literature. The MODWT MRA is explained in the next section.[8]

[8]The description mainly follows Percival and Walden (2000, Section 5).

3.2 Multiresolution analysis

A MODWT MRA applies two different types of filters, namely a MODWT wavelet filter and a MODWT scaling filter. The MODWT wavelet filter must satisfy three properties:

$$\sum_{l=0}^{L-1} \tilde{h}_l = 0, \qquad \sum_{l=0}^{L-1} \tilde{h}_l^2 = \frac{1}{2}, \qquad \text{and} \sum_{l=-\infty}^{\infty} \tilde{h}_l \tilde{h}_{l+2n} = 0. \tag{5}$$

Hence, the filter must sum to zero, have energy of $\frac{1}{2}$, and is orthogonal to even shifts. Similarly, a MODWT scaling filter must satisfy the properties:

$$\sum_{l=0}^{L-1} \tilde{g}_l = 1, \qquad \sum_{l=0}^{L-1} \tilde{g}_l^2 = \frac{1}{2}, \qquad \text{and} \sum_{l=-\infty}^{\infty} \tilde{g}_l \tilde{g}_{l+2n} = 0, \tag{6}$$

where L is the width of the filter. The last property in (5) and (6) holds for all nonzero integers n. The MODWT wavelet filter and MODWT scaling filter are defined as $\tilde{h}_l \equiv h_l/\sqrt{2}$ and $\tilde{g}_l \equiv g_l/\sqrt{2}$. Hereby, h_l and g_l are the wavelet filter and scaling filter. As an example, the Haar wavelet filter has a width of $L = 2$ and is given by $h_0 = 1/\sqrt{2}$ and $h_1 = -1/\sqrt{2}$. Additionally, the Haar scaling filter is given by $g_0 = 1/\sqrt{2}$ and $g_1 = 1/\sqrt{2}$. The relationship between MODWT wavelet and MODWT scaling filter is $\tilde{g}_l = (-1)^{l+1}\tilde{h}_{L-1-l}$.

Definitions (5) and (6) refer to scale 1, namely $\tilde{h}_{1,l} = \tilde{h}_l$ and $\tilde{g}_{1,l} = \tilde{g}_l$. The idea of MODWT is to decompose a series into components that capture different scales. Therefore, the connection between wavelet and scaling filter and MODWT wavelet and scaling filter has to be extended to different scales. Generally, for different scales j we have $\tilde{h}_{j,l} \equiv h_{j,l}/2^{j/2}$ and $\tilde{g}_{j,l} \equiv g_{j,l}/2^{j/2}$. Each of the filters has a width of $L_j \equiv (2^j - 1)(L - 1) + 1$, see Percival and Walden (2000, p. 169) for further details. As an example, the Haar wavelet filter of scale $j = 2$ is $h_{2,0} = \frac{1}{2}$, $h_{2,1} = \frac{1}{2}$, $h_{2,2} = -\frac{1}{2}$, and $h_{2,3} = -\frac{1}{2}$. The respective MODWT Haar wavelet filter is given by $\tilde{h}_{2,0} = \frac{1}{4}$, $\tilde{h}_{2,1} = \frac{1}{4}$,

$\tilde{h}_{2,2} = -\frac{1}{4}$, $\tilde{h}_{2,3} = -\frac{1}{4}$. Similarly, the (MODWT) Haar scaling filter of scale $j = 2$ is $g_{2,0} = \cdots = g_{2,3} = \frac{1}{2}$ ($\tilde{g}_{2,0} = \cdots = \tilde{g}_{2,3} = \frac{1}{4}$).[9]

Let me clarify this with an example. Suppose we want to split a series into $J = 2$ transitory components and one persistent component. Then, for a time series $\{x_t\}_{t=1}^{T}$ we can apply the scale $j = 1$ and scale $j = 2$ MODWT Haar wavelet filters to obtain:

$$t_t^{(1)} = \frac{x_t - x_{t-1}}{2}, \qquad t_t^{(2)} = \frac{x_t + x_{t-1} - x_{t-2} - x_{t-3}}{4}. \tag{7}$$

Similarly, we can apply the scale $j = 2$ MODWT Haar scale filter:

$$p_t^{(2)} = \frac{x_t + x_{t-1} + x_{t-2} + x_{t-3}}{4}. \tag{8}$$

$t_t^{(1)}$ and $t_t^{(2)}$ are transitory components, whereas $p_t^{(2)}$ is a persistent component. It is easily verifiable that x_t can be decomposed into a sum of transitory and persistent components of different scales:

$$x_t = t_t^{(1)} + t_t^{(2)} + p_t^{(2)}. \tag{9}$$

This example is similar in spirit to the one provided in Ortu et al. (2013) and Bandi et al. (2019b), and can be extended for any $J > 2$. However, a problem of this simple approach and discrete wavelet transform in general is that the sample size has to equal $T = 2^J$ observations.[10]

To make this procedure practicable for any sample size I apply MODWT. The so called

[9] The wavelet filter and scaling filter are as well known as mother wavelet and father wavelet.

[10] See footnote 6 and Appendix B in Ortu et al. (2013).

wavelet and scaling coefficients of level j are defined as:

$$W_{j,t} \equiv \sum_{l=0}^{L_j-1} \tilde{h}_{j,l} x_{t-l \bmod T}, \qquad V_{j,t} \equiv \sum_{l=0}^{L_j-1} \tilde{g}_{j,l} x_{t-l \bmod T}, \tag{10}$$

where mod is the modulo operator, see Percival and Walden (2000, p. 30). The modulo operator is needed to estimate the wavelet and scaling coefficients at the boundary of the sample. Two natural boundary conditions are to either assume that observations are "periodic" or that one should "reflect" the time series (Gençay et al., 2001, Section 4.6.3). The former takes observations from the beginning of the sample to finish computations $(x_1, x_2, \dots)$, whereas the latter takes the last observations $(\dots, x_{T-1}, x_T)$. I follow Kang et al. (2017) and Faria and Verona (2018a) and reflect data at the boundary.

In matrix notation the wavelet and scaling coefficients can be written as:

$$W_j = \tilde{W}_j x, \qquad V_j = \tilde{V}_j x, \tag{11}$$

where $\tilde{W}_j$ and $\tilde{V}_j$ contain the circularly shifted versions of periodized jth MODWT wavelet and scaling filters.[11] Finally, the level j detail and smooth are defined as:

$$D_j = \tilde{W}_j^{\intercal} W_j, \qquad S_j = \tilde{V}_j^{\intercal} V_j. \tag{12}$$

A nice property of MODWT MRA is the MODWT additive decomposition:

$$x = \sum_{j=1}^{J} D_j + S_J, \tag{13}$$

where $D_1, \dots, D_J$ are the wavelet details and S_J is the wavelet smooth.[12] So, the original series can be decomposed into parts that all represent different timescales. The sum

[11] I refer to Percival and Walden (2000) for further details on the construction of the matrices.

[12] As an example, for $j = 1$ the time series can be decomposed as $x = D_1 + S_1 = \tilde{W}_1^{\intercal} W_1 + \tilde{V}_1^{\intercal} V_1$.

of the respective components adds up to the original time series. The wavelet details and the wavelet smooth are computed with the pyramid algorithm of Mallat (1989).[13]

The different timescales can be related to certain frequency-bands (Percival and Walden, 2000, pp. 96-100). An equivalent filter for the level j detail approximates a band-pass filter with a pass-band given by $[1/2^{j+1}, 1/2^j]$. Similarly, the equivalent filter for the level J smooth approximates a low-pass filter with a pass-band given by $[0, 1/2^{J+1}]$. The inverse of the pass-band frequencies gives the approximate periodicities that are captured by the respective timescales. Hence, the level j detail reflects all movements with periodicities between 2^j and 2^{j+1}, whereas the level J smooth captures oscillations greater than 2^{J+1} periods. As an example, this means for monthly data and $J = 6$ that D_1 captures periodicities between 2 and 4 months, that D_6 represents oscillations between 64 and 128 months, and that S_6 captures periodicities greater than 128 months.

So far I have only explained the Haar wavelet with a width of 2. However, there are several other wavelets like Daubechies, Coiflets, Least Asymmetric, or Fejér-Korovkin wavelets that have different widths and forms. Actually, the choice of wavelet and scaling filter is not that important for MODWT compared to DWT. Gençay et al. (2001, p. 144) write the following: "because of its added correlation between adjacent wavelet coefficients, the choice of wavelet function is not as vital when using the MODWT to decompose a given time series". I show in the Online Appendix that results are robust with respect to the choice of wavelet. In the main text I always apply the Haar wavelet and Haar scaling filter. This choice is well in line with the literature, see, Ortu et al. (2013); Xyngis (2017); Faria and Verona (2018a); Risse (2019); Bandi et al. (2019a).

Figure 1 presents a level $J = 6$ multiresolution analysis for MA(1,9). As explained

[13] I use the R package *waveslim* and the function *mra* to carry out the multiresolution analysis.

before, the sum of the seven parts $D_1, \ldots, D_6, S_6$ equals the original time series. The MODWT MRA decomposition of the technical indicator reveals dynamics that are not visible in the aggregated series. While the original series is either 0 or 1, the respective components show pronounced time-variation in the levels. As an example, D_5 approximates periodicities between 32 and 64 months, having its lowest values in 1974:2 and 2008:7 during the oil crisis and global financial crisis.

4 Empirical results: statistical significance of forecasts

The standard linear predictive regression model is:

$$r_t = \alpha_{i,t} + \beta_{i,t} x_{i,t-1} + \epsilon_t, \tag{14}$$

where r_t is the log excess return in period t, $x_{i,t-1}$ is a lagged predictor, and ϵ_t is the error term. To generate out-of-sample forecasts I split the sample into an in-sample part of M periods and an out-of-sample part of $T - M$ periods. For the one-month ahead forecast, $\hat{r}_{i,M+1}$, the investor uses data up to time M to estimate $\hat{\alpha}_{i,M}$ and $\hat{\beta}_{i,M}$. This is exactly the situation of a professional forecaster who estimates parameters based on the most recently available information. For the second forecast the investor updates information and reestimates the regression with data up to $M + 1$. In this recursively expanding manner I estimate forecasts for each of the 28 predictors and $M + 1, \ldots, T$ periods.

Following Rapach et al. (2010), I combine individual forecasts to "combination forecasts". To do so, I weight individual forecasts with $\psi_{i,t}$ to compute a weighted average:

$$\hat{r}_{c,t+1} = \sum_{i=1}^{N} \psi_{i,t} \hat{r}_{i,t+1}, \tag{15}$$

where the sum of weights equals one. In the simplest case I equally weight the N individual forecasts. Additionally, I weight predictors by the mean squared prediction error from previous forecasts. The estimated weights depend on the interval over which the past performance is evaluated. I focus on a relatively short evaluation period of 12 months to allow for sufficient time-variation in the weights.[14] The weights are:

$$\psi_{i,t} = \frac{\phi_{i,t}^{-1}}{\sum_{j=1}^{N} \phi_{j,t}^{-1}}, \qquad \phi_{j,t} = \sum_{s=0}^{h-1} \theta^s (r_{t-s} - \hat{r}_{j,t-s})^2, \tag{16}$$

and h is the holdout period. Similar to Stock and Watson (2004) I allow more recent and more distant squared prediction errors to have a different impact on the weights. For $\theta = 1$ all squared errors are weighted equally, whereas for $\theta < 1$ more distant periods are discounted. I show results for $h = 12$ and discount factors of $\theta = 1$ and $\theta = 0.9$.[15] Forecasts from the sophisticated models are compared to the historical mean. Among others, Campbell and Thompson (2008) and Welch and Goyal (2008) show that this natural benchmark cannot be consistently outperformed by economic variables. The one month ahead forecast from this model is $\bar{r}_{t+1} = \frac{1}{t} \sum_{j=1}^{t} r_j$.

4.1 Forecast evaluation: out-of-sample $\mathbf{R^2}$

Campbell and Thompson (2008) propose an out-of-sample R^2 statistic that is comparable with the in-sample R^2 statistic. The statistic is computed as

$$R^2_{\mathrm{OS}} = 1 - \frac{\sum_{t=M+1}^{T} (r_{\mathrm{t}} - \hat{r}_{\mathrm{t}})^2}{\sum_{t=M+1}^{T} (r_{\mathrm{t}} - \bar{r}_{\mathrm{t}})^2}, \tag{17}$$

where r_t is the log excess return on the S&P 500 index, $\hat{r}_t$ is the prediction from the preferred forecasting model, and $\bar{r}_t$ is the historical mean. A positive R^2_{OS} indicates that

[14] This is in contrast to Rapach et al. (2010) who consider a holdout period of 40 observations. I choose a shorter evaluation period as scale specific predictability potentially varies stronger over time.

[15] For weights with a holdout period I generate the first out-of-sample forecast for period $M + 1 - h$ rather than $M + 1$. The weights then are estimated based on the forecasting performance in periods $M + 1 - h, \ldots, M$. So, the evaluation period remains $M + 1, \ldots, T$.

the predictive regression model has lower average mean squared prediction error than the naive benchmark.

I present results relative to the historical mean, therefore I apply the Clark and West (2007) test for nested models. The null hypothesis is that the data are generated by the constant-only model. Clark and West (2007) suggest adding an adjustment term to the difference in squared prediction errors to test the null. The adjusted MSFE series is:

$$\tilde{d}_t = e_{1,t}^2 - \left[e_{2,t}^2 - (\hat{r}_t - \bar{r}_{t-1})^2\right] \quad t = M+1, \dots T, \tag{18}$$

with $e_{1,t} = r_t - \hat{r}_t$ and $e_{2,t} = r_t - \bar{r}_{t-1}$. Clark and West (2007) propose to regress $\tilde{d}_t$ for $t = M+1, \dots, T$ on a constant. The t-statistic of the constant then is the MSFE-adjusted statistic. I follow Neely et al. (2014) and use the usual least squares standard errors. The null hypothesis of the MSFE-adjusted statistic is equal forecast accuracy and the one-sided alternative is that the preferred model outperforms the historical mean.

It is important to mention that one can still reject the null in favor of the one-sided alternative even if the R_{OS}^2 value is negative. Even though this seems counter-intuitive, it is possible when comparing nested models. Let me explain the logic behind: under the null of no predictability the historical mean is expected to generate a smaller MSFE than the more sophisticated model. This simply results from the introduced noise in the larger model, thereby inflating MSFE. The more parsimonious model "gains efficiency by setting to zero parameters that are zero in population" (Clark and West, 2007, p. 292). The MSFE-adjusted statistic accounts exactly for the fact that we expect a negative difference between MSFE of the historical mean and MSFE of a larger model under the null. Therefore, one can still reject the null that the parsimonious model generates the data even if the R_{OS}^2 is negative.[16]

[16] See footnote 21 in Neely et al. (2014) for further information on this point.

Table 2 presents results of R^2_{OS} statistics for out-of-sample forecasts from 1966:1 to 2017:12. Column (1) depicts the respective predictor and column (2) shows results for the unadjusted series. In line with Welch and Goyal (2008), I find that most economic variables have a negative R^2_{OS} statistic, even though the null of no predictability is rejected for RVOL, TBL, LTY, LTR, and TMS. The last three rows in Panel A show results for combination forecasts. CF-ECON$^{\text{MEAN}}$ is the simple average of the 14 individual economic forecasts and CF-ECON$^{\text{WEIG}}_{\theta=1}$ (CF-ECON$^{\text{WEIG}}_{\theta=0.9}$) is the weighted combination with $\theta = 1$ ($\theta = 0.9$). Similar to Rapach et al. (2010), I find that combination forecasts significantly outperform the benchmark and that results are robust with respect to the choice of combination technique. An R^2_{OS} of 1.11% appears small but in light of the large unpredictable component in monthly stock returns an R^2_{OS} of 0.50% already amounts to a large increase in portfolio returns (Campbell and Thompson, 2008).

Columns (3) to (5) present results for the frequency-decomposed components of the predictors with an MODWT MRA of length $J = 6$. The number of overall components is reduced from seven to three by aggregating multiple timescales, see Ortu et al. (2013); Kang et al. (2017); Faria and Verona (2018b); Bandi et al. (2019a) for similar approaches.[17] I construct a high-frequency part as $D_H = D_1 + D_2 + D_3$, a medium-frequency component as $D_M = D_4 + D_5$, and a low-frequency series as $D_L = D_6 + S_6$. In terms of periodicities, D_H approximately captures cycles between 2 and 16 months, D_M approximates periodicities between 16 and 64 months, and D_L captures oscillations that exceed 64 months. Hence, D_M represents fluctuations at the short-to-medium end of the business-cycle.

The R^2_{OS} statistic does not reveal a general underlying pattern in the predictability

[17] This is analogous to isolating fluctuations in specific frequency bands with band-pass filters. Results for the individual timescales are shown in Table A.1 of the Online Appendix.

of economic variables that is specific to certain frequency bands. Even though there is some evidence of frequency-specific predictability, like for TBL and LTY in timescales D_H and D_M, the overall evidence is rather poor. However, the combination of frequency-decomposed forecasts reveals that the predictability only stems from D_M.

The picture is clearer for technical indicators. Interestingly, all of the predictability comes from timescales D_M without any evidence of predictability from the remaining components. This finding holds true for each of the 14 technical indicators in the sample. Even though it can be rejected that data are generated from a parsimonious model, the improvements in terms of reduced MSFE are still rather poor. The picture is the same for combined forecasts from technical indicators and from all predictors taken together. The best result is achieved for CF-ALL$^{\text{MEAN}}$, amounting to an R^2_{OS} of 1.64% for D_M, thereby almost doubling compared to forecast combinations from the unadjusted series (Basic). I conclude from these first results that most of the predictability comes from cycles with a length of 16 to 64 months.

4.2 Forecast performance and the business-cycle

The forecast performance of a model can heavily fluctuate over time and positive values for the R^2_{OS} statistic can easily be driven by a few outliers. Following Welch and Goyal (2008), I plot the differences in cumulative squared forecast errors (CDSFE) between two models to gain further insights on the potential determinants of predictability and the relative performance over time. The CDSFE in period t is estimated as:

$$\text{CDSFE}_t = \sum_{j=M+1}^{t} (r_j - \bar{r}_j)^2 - (r_j - \hat{r}_j)^2. \tag{19}$$

A consistently upward-sloping line would mean that the historical mean is outperformed in each and every period. However, this is in stark contrast to what is documented in empirical studies. Welch and Goyal (2008) show that predictability of economic variables is heavily driven by the 1973-75 oil shock. Even further, performance relative to the historical mean is extremely unstable and rather poor in the last 40 years.

Figure 2 shows CDSFE plots for selected variables. The plot for TBL nicely depicts the critique by Welch and Goyal (2008). In the 1970s TBL pronouncedly outperforms the historical mean during NBER-dated recession periods. Since then, TBL has mainly underperformed the benchmark. This is particularly visible in the last two recessions. Similarly, the solid black line for CF-ECON$^{\text{MEAN}}$ shows that combination forecasts from medium frequencies of economic variables performed extremely well in the 1970s and 1980s but since then they performed rather poor.

CDSFE plots for the technical indicators as well as for the combination forecasts CF-TECH$^{\text{MEAN}}$ and CF-ALL$^{\text{MEAN}}$ depict a different picture. Even though the basic models for MA(1,9) and VOL(2,9) perform better than their medium frequency counterparts, one can clearly see that the latter ones perform especially well during the 2008 financial crisis. Even more, most of these models have performed extremely well during the crises in the 1970s and 1980s, as well as during the dot-com crash. The only exception is the early 1990s recession, where no model was able to beat the historical mean. Hence, the figures motivate a strong relation between predictability and the state of the economy. In the next section I formally test whether return predictability depends on business-cycle expectations. If this is the case, then an investor can potentially improve forecasts by taking this dependence into account.

4.3 Testing dependence between current predictability and business-cycle expectations

Several studies document that return predictability is centered in NBER-dated recession periods, with only weak or no evidence of predictability in expansions, see Henkel et al.

(2011); Zhu and Zhu (2013); Rapach and Zhou (2013); Baetje and Menkhoff (2016). Thus, if it is possible to accurately forecast recessions then these forecasts are also good candidates for periods with predictability. Forecasting recessions on its own is a difficult task but some advances have been made in the past, see Estrella and Mishkin (1998); Kauppi and Saikkonen (2008); Liu and Moench (2016) and references therein.

I use the parametric probit model of Liu and Moench (2016) to predict recessions one-month ahead. The authors identify a model with the term spread, the term spread lagged by six months, and the one-year growth rate in the S&P 500 index to perform best in predicting recessions at the three-month horizon.[18] Let y_t be a binary variable that equals 1 for NBER-dated recession periods and 0 otherwise. The probit model is:

$$y_t = \Phi\big(\delta_0 + \delta_1 \text{TMS}_{t-1} + \delta_2 \text{TMS}_{t-7} + \delta_3 \frac{\text{SP}_{t-1} - \text{SP}_{t-13}}{\text{SP}_{t-13}}\big), \tag{20}$$

where SP is the monthly return on the S&P 500 index and TMS is the term spread. Let the one-month ahead forecast for the probability of a recession be given by $\hat{p}_t$. In order to classify $\hat{p}_t$ into a binary variable with 1 for recession and 0 for expansion one needs to specify a threshold level θ. For $\hat{p}_t \geq \theta$ a recession is predicted, $\hat{y}_t = 1$, and for $\hat{p}_t < \theta$ an expansion is predicted, $\hat{y}_t = 0$.

The threshold level θ is a critical choice as it determines the amount of predicted recessions. I make use of the Youden index to estimate the optimal cut-point in discriminating between the binary outcomes. The Youden index is a commonly applied evaluation criterion that is defined as the difference between the true positive ratio (TPR) and false

[18] Even though the forecast horizon differs, this model performs well in forecasting one month ahead.

positive ratio (FPR) (Youden, 1950). TPR and FPR are:

$$\text{TPR}(\theta) = \frac{1}{n_R}\sum_{t=1}^{T}\mathbf{I}_t^{\text{TP}}, \qquad \text{FPR}(\theta) = \frac{1}{n_E}\sum_{t=1}^{T}\mathbf{I}_t^{\text{FP}}, \tag{21}$$

where n_R is the number of recession periods ($y_t = 1$) and n_E is the number of expansionary periods ($y_t = 0$) in the sample. $\mathbf{I}_t^{\text{FP}}$ and $\mathbf{I}_t^{\text{FP}}$ are indicator functions that equal 1 for correctly and falsely predicted recessions, respectively:

$$\mathbf{I}_t^{\text{TP}} = \begin{cases} 1 & \text{if } y_t = 1 \text{ and } \hat{y}_t = 1, \\ 0 & \text{otherwise,} \end{cases} \qquad \mathbf{I}_t^{\text{FP}} = \begin{cases} 1 & \text{if } y_t = 0 \text{ and } \hat{y}_t = 1, \\ 0 & \text{otherwise.} \end{cases} \tag{22}$$

I select the optimal threshold level θ^* from an evenly spaced grid of 101 candidate values $G = \{0, 0.01, ..., 0.99, 1\}$:

$$\theta^* = \arg\max_{\theta \in G} \text{TPR}(\theta) - \text{FPR}(\theta). \tag{23}$$

For the out-of-sample exercise I estimate the optimal threshold and forecast for each period from 1966:1 to 2017:12, covering a total of 624 months..[19] The resulting one-month ahead recession forecasts are shown in Figure A.1 of the Online Appendix. During this period the NBER has classified 90 months as recessions and the probit model has forecasted 131 recession periods. From the 90 months of actual recessions the model has correctly forecasted 76 months.

Next, I formally test the hypothesis that current predictability is related to expectations on the business-cycle. Rather than using NBER-dated recession periods I utilize forecasts from the probit model. The reason for this is that recession forecasts are readily available for an investor in real-time, whereas NBER-dated recession periods are classi-

[19]NBER business-cycle data become available with an announcement delay. Thus, I only use information up to y_{t-24} to estimate the regression parameters $\hat{\delta}_0$, $\hat{\delta}_1$, $\hat{\delta}_2$, and $\hat{\delta}_3$

fied with a delay. More formally, I test whether the difference in squared forecast errors is related to expectations about the state of the economy.[20] I follow Wang et al. (2018) and apply the test of Pesaran and Timmermann (2009). While they test for "momentum of predictability", I test whether predictability depends on business-cycle expectations. Let cp_t be a binary variable that identifies periods of current predictability:

$$cp_t = \mathbf{I}[(r_t - \hat{r}_t)^2 - (r_t - \bar{r}_t)^2 < 0]. \tag{24}$$

$\mathbf{I}[.]$ as an indicator function that equals 1 if the squared forecast error of the preferred model is smaller than the squared forecast error from the historical mean, and 0 otherwise. The idea now is to test whether cp_t depends on business-cycle forecasts:

$$cp_t = c + \gamma \hat{y}_t + u_t. \tag{25}$$

The test of independence is equal to testing $\gamma = 0$. I apply the dynamically augmented reduced rank regression approach of Pesaran and Timmermann (2009) to test the null of independence. This test allows for serially correlated observations, a point that is especially present in business-cycles.

Panel A in Table 3 shows p-values for the null hypothesis that current predictability of a forecast model is independent of business-cycle expectations from the probit model. The null can be rejected for six of the 14 basic economic variables. Even further, the null is rejected at the 10% level for eight low-frequency components of economic variables. Contrarily, neither the unadjusted nor the frequency-decomposed technical indicators show any evidence of dependence between predictability and expectations.

[20]In Figure A.2 of the Online Appendix I show boxplots of $(r_t - \hat{r}_t)^2 - (r_t - \bar{r}_t)^2$ for the 14 medium-frequency components of the technical indicators (D_M). The difference in squared forecast errors is centered around zero with outliers both to the left (D_M performs better) and the right (historical mean performs better). I test whether points on the left side of the distribution depend on business-cycle expectations.

A potential explanation for the weak evidence of dependence is the definition of cp_t. The variable takes a value of 1 no matter whether the predictor marginally or substantially outperforms the benchmark. To account for this shortcoming, I redefine current predictability as:

$$cp_t^{\text{HIGH}} = \mathbf{I}[(r_t - \hat{r}_t)^2 - (r_t - \bar{r}_t)^2 < -\text{IQR} \times 1.5], \tag{26}$$

where IQR is the interquartile range. So, I assign a value of 1 only to points that are outliers. According to this new definition, I test whether periods with a pronounced forecasting improvement depend on business-cycle expectations. This rules out that the dependence is hidden under marginal forecasting improvements unrelated to the state of the economy.

Panel B in Table 3 presents results for the alternative definition. The null of independence now is rejected for 12 medium-frequencies of technical indicators; 9 of the indicators even reject the null at the 1% level. Interestingly, especially the performance of moving average strategies and volume strategies depends on expectations. Combination forecasts for the technical indicators strongly reject the null, whereas combination forecasts from the economic variables and all predictors taken together only weakly reject the null.

4.4 Nonlinear forecasting model

I have shown that forecasting gains for technical indicators are especially pronounced when a recession is expected. In a next step, I run a pseudo out-of-sample exercise that incorporates this dependence. More precisely, I propose a nonlinear model that selects between the historical mean and the sophisticated model depending on the expected state of the economy one-month ahead. The nonlinear model is:

$$\hat{r}^{\text{NL}}_{\text{t+1}} = \begin{cases} \hat{r}_{t+1} & \text{if } \hat{y}_{t+1} = 1 \\ \bar{r}_{t+1} & \text{if } \hat{y}_{t+1} = 0, \end{cases} \tag{27}$$

and nests both the linear regression model ($\hat{y}_{t+1} = 1$ for all t) and the historical mean ($\hat{y}_{t+1} = 0$ for all t). An investor that applies this model estimates in each period the probability of a recession one-month ahead and decides whether a recession is expected or not. If the investor expects a recession, then the sophisticated model is used, otherwise the historical mean is selected. This mimics the real-time situation of an investor that incorporates the empirical finding of state-dependent predictability in a forecasting model.

Table 4 shows how the R^2_{OS} statistics are affected when switching to the nonlinear forecasting model. Columns (2), (4), (6), and (8) repeat estimates from Table 2, whereas columns (3), (5), (7), and (9) state results for the nonlinear approach. As an example, the R^2_{OS} for the medium-frequency component of MA(1,9) changes from -2.16% for the linear model to 2.71% for the nonlinear model. The nonlinear models that select between medium-frequencies of technical indicators and the historical mean perform well. For each of the 14 indicators the R^2_{OS} is significant at the 5% or 1% level, with sizable gains compared to Basic and D_M. For the combined forecasts the R^2_{OS} increases from 0.45% (Basic) to 1.98% (D^{NL}_M). The improvements do not translate to economic variables (Panel A) or to combination forecasts from all predictors taken together (Panel C); this is in line with Table 3.

Figure 3 presents CDSFE plots for MA(1,9) and CF-TECH$^{\text{MEAN}}$. The patterns of the CDSFE plots for D^{NL}_M show high similarity among the different indicators. Throughout the 1970s and 1980s the plot behaves almost like a step function, with jumps in expected recessions and horizontal movement otherwise. From the mid-1980s to the end of the 1990s no gains were realized. More recently, the models outperformed the historical mean in the dot-com crisis and especially in the global financial crisis, resulting in the largest jump in the last 50 years.

5 Empirical results: economic significance of forecasts

So far I have only presented statistical measures of predictability. This does not necessarily imply that an investor would have benefited economically from a forecasting model. In this section I explain two economic measures of forecast evaluation, namely the gain in certainty equivalent representation (ΔCER) and the gain in Sharpe ratio (ΔSR). The former computes the increase in average realized utility and the latter states the ratio between reward-to-variability. Cenesizoglu and Timmermann (2012) show that R^2_{OS}, ΔCER, and ΔSR are only weakly correlated.

I follow Ferreira and Santa-Clara (2011) and Rapach et al. (2016) and perform an asset allocation exercise to estimate the economic significance of a forecasting strategy.[21] I consider a mean-variance investor that allocates wealth between the S&P 500 index and the risk-free rate based on forecasts from a predictive regression model. The maximization problem at the end of period t is:

$$\max_{\omega_t} U(r_{p,t+1}) = E_t[r_{p,t+1}] - \frac{\gamma}{2}\text{Var}_t(r_{p,t+1}) \tag{28}$$

$$\text{s.t. } r_{p,t+1} = \omega_t r_{t+1} + r^f_{t+1}, \tag{29}$$

where $U(\cdot)$ is utility and $r_{p,t+1}$ is the portfolio return. The investor decides on the share of wealth, ω_t, that is allocated to the risky asset. Solving the maximization problem,

[21] In this section I use excess returns rather than log excess returns.

the optimal weight assigned to the risky asset at the end of period t is:

$$\omega_t^* = \frac{1}{\gamma}\frac{E_t(r_{t+1})}{\text{Var}_t(r_{t+1})}, \tag{30}$$

where $E_t(r_{t+1})$ and $\text{Var}_t(r_{t+1})$ are the conditional expectation and the conditional variance of excess stock returns. γ is the coefficient of relative risk aversion and r_{t+1}^f is the risk-free rate between period t and $t+1$. The conditional expectation is estimated by the respective forecasting model, $E_t(r_{t+1}) = \hat{r}_{t+1}$. Following Campbell and Thompson (2008), I estimate the conditional variance, $\text{Var}_t(r_{t+1})$, as a rolling five-year window. Portfolio weights on stocks are constrained to lie between 0% and 150% to prevent shorting stocks and leveraging more than 50%. γ is set to 5, and the optimal portfolio return is given by $r_{p,t+1}^* = \omega_t^* r_{t+1} + r_{t+1}^f$. The average realized utility level (or certainty equivalent return (CER)) is $\hat{\upsilon} = \hat{\mu} - \frac{\gamma}{2}\hat{\sigma}^2$, where $\hat{\mu}$ and $\hat{\sigma}^2$ are the mean and variance of $r_{p,t+1}^*$. The CER can be interpreted as "the fee the investor would be willing to pay to use the information in each forecast model" (Ferreira and Santa-Clara, 2011, p. 527). It can also be interpreted as the risk-free rate that an investor is willing to accept in order not to adopt the risky portfolio (DeMiguel et al., 2009).

Suppose that the investor wants to compare two different forecasting models i and j with excess return forecasts $\hat{r}_{t+1}^i$ and $\hat{r}_{t+1}^j$. Then the optimal portfolio weights are $\omega^{*,i}$ and $\omega^{*,j}$ and the respective average realized utility level is $\hat{\upsilon}^i = \hat{\mu}^i - \frac{\gamma}{2}\hat{\sigma}^{2,i}$ and $\hat{\upsilon}^j = \hat{\mu}^j - \frac{\gamma}{2}\hat{\sigma}^{2,j}$. Then the difference in CER is $\Delta\text{CER} = \hat{\upsilon}^i - \hat{\upsilon}^j$, which is the fee an investor would be willing to pay to use model i rather than model j. I multiply ΔCER by 1,200 so that it can be interpreted as "the annual percentage portfolio management fee" that an investor would be willing to pay (Neely et al., 2014, p. 1788).

I as well estimate the Sharpe ratio (SR). SR is the mean portfolio return in excess of the risk-free rate divided by the standard deviation of the excess portfolio return. $SR = \frac{\tilde{\mu}}{\tilde{\sigma}}$, where $\tilde{\mu}$ and $\tilde{\sigma}$ are the mean and standard deviation of the excess portfolio return over the out-of-sample period. The annualized gain in SR of model i relative to model j then is $\Delta SR = \sqrt{12}(SR_i - SR_j)$.

To measure the statistical significance of ΔCER and ΔSR I separately estimate p-values for the null hypotheses $CER_i - CER_j \leq 0$ and $SR_i - SR_j \leq 0$. I calculate p-values according to the bootstrap approach described in DeMiguel et al. (2013). For two portfolios i and j, I obtain B=5,000 pairs of (excess) portfolio returns for CER (SR) by resampling with replacement from the observed portfolio returns. Each of the B pairs is of size $T - M$. To account for cross-correlation I resample pairs of $(r^{*,i}_{p,t+1}, r^{*,j}_{p,t+1})$. Additionally, I resample blocks of observations to control for potential autocorrelation. The length of each block has a geometric distribution with an average block length of five observations, see Politis and Romano (1994).[22]

For the $b = 1, \dots, 5,000$ bootstrap samples I estimate ΔCER_b and ΔSR_b:

$$\Delta CER_b = (\hat{\mu}^i_b - \frac{\gamma}{2}\hat{\sigma}^{2,i}_b) - (\hat{\mu}^j_b - \frac{\gamma}{2}\hat{\sigma}^{2,j}_b), \qquad \Delta SR_b = \frac{\tilde{\mu}^i_b}{\tilde{\sigma}^i_b} - \frac{\tilde{\mu}^j_b}{\tilde{\sigma}^j_b}, \tag{31}$$

whereby $\hat{\mu}^i_b$, $\hat{\mu}^j_b$, $\hat{\sigma}^i_b$, $\hat{\sigma}^j_b$ ($\tilde{\mu}^i_b$, $\tilde{\mu}^j_b$, $\tilde{\sigma}^i_b$, $\tilde{\sigma}^j_b$) are the means and volatilities of the resampled (excess) portfolio returns. Then, the respective p-value is:

$$\hat{p}_{CER} = \frac{1}{B}\sum_{b=1}^{B} \mathbf{I}[\Delta CER_b \leq 0], \qquad \hat{p}_{SR} = \frac{1}{B}\sum_{b=1}^{B} \mathbf{I}[\Delta SR_b \leq 0]. \tag{32}$$

The alternative is accepted for sufficiently small values of $\hat{p}_{CER}$ and $\hat{p}_{SR}$.

[22] For example, Goetzmann and Jorion (1993) and Maio and Santa-Clara (2012) have a fix block length of 1, whereas v. Binsbergen et al. (2012) show results for an average block length of 1,5, and 15 months.

5.1 Results for ΔCER and ΔSR

Table 5 presents the annualized difference in CER relative to the historical mean. Similar to results from the previous section the unadjusted economic variables generally perform poor. Exceptions are TBL, LTY, and TMS with utility gains of 180, 165, and 179 basis points. However, only TBL is significant at the 10% level. Combination forecasts of the unadjusted series generate significant gains, amounting to increases in the range of 170 to 190 basis points. Interestingly, each of the 14 technical indicators has a positive utility gain, with a maximum of 267 basis points for MA(2,12). An investor would be willing to pay a sizable annual fee to have access to this model. Nonetheless, the gains are well below those of the medium-frequency components.
Column (6) shows utility gains for forecasts from medium-frequency components. A comparison between columns (2) and (6) highlights that medium-frequency components generate sizable utility gains. For nine technical indicators the utility gains are larger than 300 basis points, with the largest gains amounting to 446 basis points for VOL(1,12). Contrarily, only three unadjusted indicators generate gains above 200 basis points, with none exceeding 300 basis points. Results for each of the 14 technical indicators improve.

Moreover, utility gains for combination forecasts increase when switching from baseline predictors to medium-frequencies. Panels B and C show that ΔCER more than doubles for technical indicators and for all predictors taken together. Taking the average of the 28 individual forecasts provides a gain of 431 basis points. The CER for the historical mean is 434 basis points; thus the utility gain is almost twice as large. A buy-and-hold investor that passively holds the market portfolio realizes a ΔCER of 83 basis points, well below gains from the more sophisticated models.

I have shown in Section 4 that the nonlinear models improve R^2_{OS} statistics for the medium-frequency parts of technical indicators. This is not the case for ΔCER. A comparison between columns (6) and (7) reveals that the nonlinear model does not generally boost utility gains. Hence, the additional estimation of business-cycle expectations is not necessary for improving economic measures of predictability. The only step that has to be carried out is to isolate medium-frequency parts of technical indicators; thereby capturing the short-to-medium end of business-cycle oscillations.

Table 6 reports results for the annualized gains in Sharpe ratio. The picture is qualitatively the same as for ΔCER. Results for combination forecasts generally improve when using frequency-decomposed predictors, resulting in more than twice as large Sharpe ratios for technical indicators when comparing columns (2) and (6). The SR for the historical mean is 0.28 and the ΔSR of a buy-and-hold investor is 0.12. The largest ΔSR is 0.35 for the medium-frequency component of VOL(1,12).

5.2 Portfolio weights over time

To understand what exactly drives the gains in ΔCER and ΔSR, I present the optimal shares of risky assets over time. The difference in ω_t^* among forecasting models results from varying conditional expectations of one-month ahead excess stock returns. Figure 4 shows forecasts from the historical mean (solid gray line), the combination of unadjusted technical indicators (solid black line), and the combination of medium-frequencies of technical indicators (dashed black line). While the historical mean is a slow moving object with mild time-variation, the combination forecasts show pronounced movements.

A clear pattern can be detected for D_M. A negative equity premium is predicted at the beginning of a recession and a large positive premium is predicted towards the end and directly after a recession. This finding is in line with Dangl and Halling (2012); they as well document that the predicted risk premium peaks towards the end of the recession as investors become more risk-averse. They interpret their results as follows: "Thus, we conclude that predictability reflects business cycle risk rather than market inefficiency. Therefore, it is also not surprising that predictability is not driven away over time" (Dangl and Halling, 2012, p. 169). As an example, in the beginning of the 1973-75 oil crisis and the 2008-09 global financial crisis the model predicts negative excess returns, followed by large positive excess returns directly after the recession.

Figure 5 presents the optimal weights of risky assets over time. The equity exposure of the historical mean portfolio rapidly shrinks during the crises of the 1970s and during the more recent financial crisis. However, the equity weights always remain above 40% and are rather persistent with long lasting episodes of increases and decreases. Contrarily, the weights from combination forecasts of medium-frequencies of technical indicators show substantial fluctuations. The weights often change from 0% to 150% within a few years and vice versa. This pattern is most salient around recessions. At the beginning of a recession the equity weights either run down rapidly or are already close to zero. A few months later the exposure then again is build up at the end of or shortly after a recession. The large time-variation in equity weights together with the sizable gains in ΔCER and ΔSR reveals an excellent market timing.

6 Robustness

In this section I show that the gains in CER and SR are robust with respect to transaction costs. Additionally, I outline that results remain qualitatively unchanged for several alternative specifications, like different wavelet and scaling filters.

6.1 Performance after transaction costs

So far, I have not taken transaction costs into account. Unfortunately, this holds true for many studies on the economic significance of forecasting models, see, among others

Rapach et al. (2010), Ferreira and Santa-Clara (2011), Dangl and Halling (2012), Rapach et al. (2016). This is problematic as both economic variables and technical indicators generate higher monthly turnovers (Neely et al., 2014). Hence, ignoring transaction costs results in positively biased values for ΔCER and ΔSR. Here, I focus on CF-TECH$^{\text{MEAN}}$ and only show adjusted results for this specific strategy.[23]

Suppose that an investor has an initial wealth of \$1 at the beginning of period t, then the wealth at the end of period $t+1$ is $W_{t+1} = 1 + r^f_{t+1} + \omega^*_t r_{t+1}$, and the wealth in risky assets is $W^R_{t+1} = \omega^*_t(1 + r^f_{t+1} + r_{t+1})$. At the end of period $t+1$ the investor estimates the optimal share of risky assets for the next period and the new target level of wealth; given by $W^T_{t+1} = \omega^*_{t+1} W_{t+1}$. So, the investor has to make adjustments in risky assets of $|W^T_{t+1} - W^R_{t+1}|$. The percentage of wealth traded at the end of period $t+1$ (denoted as turnover) then is:

$$\text{Turnover}_{t+1} = \frac{|W^T_{t+1} - W^R_{t+1}|}{W_{t+1}}, \tag{33}$$

whereby the numerator equals the adjustment in risky assets and the denominator equals the amount of total wealth at the end of period $t+1$. Following DeMiguel et al. (2009), I assume that the investor has to pay a proportional transaction cost of c on the turnover in each period. Therefore, the return net of transaction costs is:

$$r^{TC}_{p,t+1} = (1 + \omega^*_t r_{t+1} + r^f_{t+1})(1 - c \times \text{Turnover}_{t+1}) - 1, \tag{34}$$

in which $c \times \text{Turnover}_{t+1}$ is the transaction cost per unit of total wealth. I follow the literature and fix the parameter c to 0.005, which equals 50 basis points per transaction (Balduzzi and Lynch, 1999). The cumulative wealth after accounting for transaction costs then is $W^{TC}_{t+1} = W^{TC}_t(1 + r^{TC}_{p,t+1})$.

[23] See Section G of the Online Appendix for additional results.

Figure 6 plots the log cumulative wealth for an investor that begins with \$1 and reinvests all proceeds. Results are shown for four different forecasting models. The historical mean is depicted by the solid gray line. Clearly, this model performs especially poor during the global financial crisis in 2008-09. The other three models show the log cumulative wealth for CF-TECH$^{\text{MEAN}}$, whereby ω_t^* is estimated according to the basic linear approach (dashed gray line), the nonlinear approach with medium frequencies (dashed black line), and the linear approach with medium frequencies (solid black line), respectively. Even though all three models outperform the historical mean the frequency-decomposed forecasts perform best. D_M outperforms D_M^{NL}, implying that the combination of medium frequencies not only adds economic value during recessions but during expansions as well.

The average turnover of Basic, D_M^{NL}, and D_M relative to the average turnover of the historical mean is 3.20, 2.75, and 4.11. This is in line with Neely et al. (2014) and highlights that more sophisticated models often generate higher rates of portfolio rebalancing. The differences in annualized CER relative to the historical mean, with respective p-values (in %) in brackets, are 1.29 (14.00), 2.82 (0.88), and 3.30 (2.00). The annualized differences in SR for Basic, D_M^{NL}, and D_M are 0.08 (16.00), 0.21 (0.92), and 0.26 (1.60). Even though the average turnover for D_M is higher than for Basic, both ΔCER and ΔSR remain more than twice as large when accounting for transaction costs.

6.2 Further robustness checks

I present several additional robustness checks in the Online Appendix. Firstly, I show in Section C that the results of the asset allocation exercise are robust with respect to the choice of parameters. I repeat the analysis with the specification of Rapach et al. (2016), finding that the annualized utility gains improve even further in magnitude. Secondly, I provide in Section D separate results for the out-of-sample forecasting horizon from 1990:1 to 2017:12. Thirdly, I repeat the analysis in Section E for frequency-specific principal components. The main results translate from combination forecasts to principal component analysis. Fourth, I show in Section F that alternative wavelet and scaling filters leave results almost unchanged. Section G presents results after adjusting for transaction costs.

7 Concluding remarks

This paper examines short-horizon predictability with frequency-decomposed predictor variables. The set of predictors consists of commonly used economic variables and technical indicators (Rapach et al., 2010; Neely et al., 2014). In contrast to previous work, I do not analyze the original series but rather apply filtering methods to decompose the predictor into components with specific periodicity. This approach allows for a more nuanced view on equity premium predictability as the predictive power of some variables potentially is hidden behind high-frequency noise or low frequency trends.

I document that fluctuations at the short-to-medium end of the business-cycle incorporate the relevant information for predicting excess stock returns. The predictive power of technical indicators solely stems from periodicities of 16 to 64 months, without any evidence of predictability from other periodicities. Furthermore, I present evidence that the historical mean is mainly outperformed during recessions. This is in line with other articles and further emphasizes the role of the business-cycle (Henkel et al., 2011; Rapach and Zhou, 2013). The novel finding is that this state-dependent predictability is better captured by medium-frequency oscillations in technical indicators rather than in the original series.

The predictive power for technical indicators is both statistically and economically significant. I show that the gains in ΔCER and ΔSR more than double for combination forecasts from medium-frequencies of technical indicators compared to combination forecasts from the original series. Surprisingly, results for economic variables do not improve strongly. This extends the critique by Welch and Goyal (2008) to frequency-specific parts of economic variables. In this article I have only focused on short-horizon predictability, leaving the field of frequency-decomposed long-horizon forecasts for future research.

References

Baetje, F. and L. Menkhoff, "Equity premium prediction: Are economic and technical indicators unstable?," *International Journal of Forecasting*, 2016, *32*, 1193–1207.

Baker, S. G. and B. S. Kramer, "Peirce, Youden, and the Receiver Operating Characteristic Curves," *The American Statistician*, 2007, *61* (4), 343–346.

Balduzzi, P. and A. W. Lynch, "Transaction costs and predictability: some utility cost calculations," *Journal of Financial Economics*, 1999, *52*, 47–78.

Balvers, R. J., T. F. Cosimano, and B. McDonald, "Predicting Stock Returns in an Efficient Market," *Journal of Finance*, 1990, *45* (4), 1109–1128.

Bandi, F. M., B. Perron, A. Tamoni, and C. Tebaldi, "The scale of predictability," *Journal of Econometrics*, 2019b, *208*, 120–140.

_ , S. E. Chaudhuri, A. W. Lo, and A. Tamoni, "Spectral factor models," Johns Hopkins Carey Business School Research Paper No. 18-17. 2019a.

Baxter, M. and R. G. King, "Measuring Business Cycles: Approximate Band-Pass Filters for Economic Time Series," *Review of Economics and Statistics*, 1999, *81* (4), 575–593.

Berge, T. J. and Ò. Jordà, "Evaluating the Classification of Economic Activity into Recessions and Expansions," *American Economic Journal: Macroeconomics*, 2011, *3* (2), 246–277.

Campbell, J. Y. and S. Thompson, "Predicting the equity premium out of sample: Can anything beat the historical average?," *Review of Financial Studies*, 2008, *21*, 1509–1531.

Cenesizoglu, T. and A. Timmermann, "Do return prediction models add economic value?," *Journal of Banking & Finance*, 2012, *36*, 2974–2987.

Clark, T. E. and K. D. West, "Approximately normal tests for equal predictive accuracy in nested models," *Journal of Econometrics*, 2007, *138*, 291–311.

Crowley, P. M., "A guide to wavelets for economists," *Journal of Economic Surveys*, 2007, *21* (2), 207–267.

Cujean, J. and M. Hasler, "Why does return predictability concentrate in bad times?," *Journal of Finance*, 2017, *72* (6), 2717–2758.

Dangl, T. and M. Halling, "Predictive regressions with time-varying coefficients," *Journal of Financial Economics*, 2012, *106*, 157–181.

DeMiguel, V., L. Garlappi, and R. Uppal, "Optimal Versus Naive Diversification: How Inefficient is the $1/N$ Portfolio Strategy?," *Review of Financial Studies*, 2009, *22* (5), 1916–1953.

_ , Y. Plyakha, R. Uppal, and G. Vilkov, "Improving Portfolio Selection Using Option-Implied Volatility and Skewness," *Journal of Financial and Quantitative Analysis*, 2013, *48* (6), 1813–1845.

Estrella, A. and F. S. Mishkin, "Predicting U.S. recessions: financial variables as leading indicators," *Review of Economics and Statistics*, 1998, *80* (1), 45–61.

Faria, G. and F. Verona, "Forecasting stock market returns by summing the frequency-decomposed parts," *Journal of Empirical Finance*, 2018a, *45*, 228–242.

_ and _ , "The equity risk premium and the low frequency of the term spread," Bank of Finland Research Discussion Papers 2018b.

Ferreira, M. A. and P. Santa-Clara, "Forecasting stock market returns: The sum of the parts is more than the whole," *Journal of Financial Economics*, 2011, *100*, 514–537.

Ferson, W. E., "Investment Performance Evaluation," *Annual Review of Financial Economics*, 2010, *2*, 207–234.

Gençay, R., F. Selçuk, and B. Whitcher, *An Introduction to Wavelets and Other Filtering Methods in Finance and Economics*, Academic Press San Diego, 2001.

Goetzmann, W. N. and P. Jorion, "Testing the Predictive Power of Dividend Yields," *Journal of Finance*, 1993, *48* (2), 663–679.

Goyal, A., "Empirical cross-sectional asset pricing: a survey," *Financial Markets and Portfolio Management*, 2012, *26* (1), 3–38.

Granville, J. E., *New key to stock market profits*, New York: Prentice-Hall, 1963.

Henkel, S. J., J. S. Martin, and F. Nardari, "Time-varying short-horizon predictability," *Journal of Financial Economics*, 2011, *99*, 560–580.

Housworth, E. A., T. B. Walker, and C. Xu, "Structural asset pricing theory with wavelets," *Quantitative Finance*, 2019, *19* (10), 1659–1672.

Kang, B. U., F. In, and T. S. Kim, "Timescale betas and the cross section of equity returns: Framework, application, and implications for interpreting the Fama-French factors," *Journal of Empirical Finance*, 2017, *42*, 15–39.

Kauppi, H. and P. Saikkonen, "Predicting U.S. recessions with dynamic binary response models," *Review of Economics and Statistics*, 2008, *90* (4), 777–791.

Lettau, M. and S. Van Nieuwerburgh, "Reconciling the Return Predictability Evidence," *Review of Financial Studies*, 2008, *21* (4), 1607–1652.

Liu, W. and E. Moench, "What predicts US recessions?," *International Journal of Forecasting*, 2016, *32*, 1138–1150.

Ludvigson, S. C. and S. Ng, "The empirical risk-return relation: A factor analysis approach," *Journal of Financial Economics*, 2007, *83*, 171–222.

Maio, P. and P. Santa-Clara, "Multifactor models and their consistency with the ICAPM," *Journal of Financial Economics*, 2012, *106*, 586–613.

Mallat, S. G., "A Theory for Multiresolution Signal Decomposition: The Wavelet Representation," *IEEE Transactions on Pattern Analysis and Machine Learning*, 1989, *11* (7), 674–693.

Mele, A., "Asymmetric stock market volatility and the cyclical bevahior of expected returns," *Journal of Financial Economics*, 2007, *86*, 446–478.

Neely, C. J., D. E. Rapach, J. Tu, and G. Zhou, "Forecasting the Equity Risk Premium: The Role of Technical Indicators," *Management Science*, 2014, *60* (7), 1772–1791.

Ortu, F., A. Tamoni, and C. Tebaldi, "Long-Run Risk and the Persistence of Consumption Shocks," *Review of Financial Studies*, 2013, *26* (11), 2876–2915.

Percival, D. B. and A. T. Walden, *Wavelet Methods for Time Series Analysis*, Cambridge University Press, 2000.

Pesaran, M. H. and A. Timmermann, "Testing Dependence Among Serially Correlated Multicategory Variables," *Journal of the American Statistical Association*, 2009, *104* (485), 325–337.

Pettenuzzo, D., A. Timmermann, and R. Valkanov, "Forecasting stock returns under economic constraints," *Journal of Financial Economics*, 2014, *114*, 517–553.

Politis, D. N. and J. P. Romano, "The Stationary Bootstrap," *Journal of the American Statistical Association*, 1994, *89* (428), 1303–1313.

Rapach, D. E. and G. Zhou, "Forecasting Stock Returns," in G. Elliott and A. Timmermann, eds., *Handbook of Economic Forecasting*, Vol. 2A, Elsevier, Amsterdam, 2013, pp. 329–383.

— , **J. K. Strauss, and G. Zhou**, "Out-of-Sample Equity Premium Prediction: Combination Forecasts and Links to the Real Economy," *Review of Financial Studies*, 2010, *23* (2), 821–862.

— , **M. C. Ringgenberg, and G. Zhou**, "Short interest and aggregate stock returns," *Journal of Financial Economics*, 2016, *121*, 46–65.

Risse, M., "Combining wavelet decomposition with machine learning to forecast gold returns," *International Journal of Forecasting*, 2019, *35*, 601–615.

Rua, A., "A Wavelet Approach for Factor-Augmented Forecasting," *Journal of Forecasting*, 2011, *30*, 666–678.

Stock, J. H. and M. W. Watson, "Combination Forecasts of Output Growth in a Seven-Country Data Set," *Journal of Forecasting*, 2004, *23*, 405–430.

Timmermann, A., "Forecast Combinations," in G. Elliott, C. W.J. Granger, and A. Timmermann, eds., *Handbook of Economic Forecasting*, Vol. 1, Elsevier B.V., 2006, chapter 4.

v. Binsbergen, J., M. Brandt, and R. Koijen, "On the Timing and Pricing of Dividends," *American Economic Review*, 2012, *102* (4), 1596–1618.

Wang, Y., L. Liu, F. Ma, and X. Diao, "Momentum of return predictability," *Journal of Empirical Finance*, 2018, *45*, 141–156.

Welch, I. and A. Goyal, "A Comprehensive Look at The Empirical Performance of Equity Premium Prediction," *Review of Financial Studies*, 2008, *21*, 1455–1508.

Xyngis, G., "Business-cycle variation in macroeconomic uncertainty and the cross-section of expected returns: Evidence for scale-dependent risks," *Journal of Empirical Finance*, 2017, *44*, 43–65.

Youden, W. J., "Index for Rating Diagnostic Tests," *Cancer*, 1950, *3*, 32–35.

Zhu, X. and J. Zhu, "Predicting stock returns: A regime-switching combination approach and economic links," *Journal of Banking & Finance*, 2013, *37*, 4120–4133.

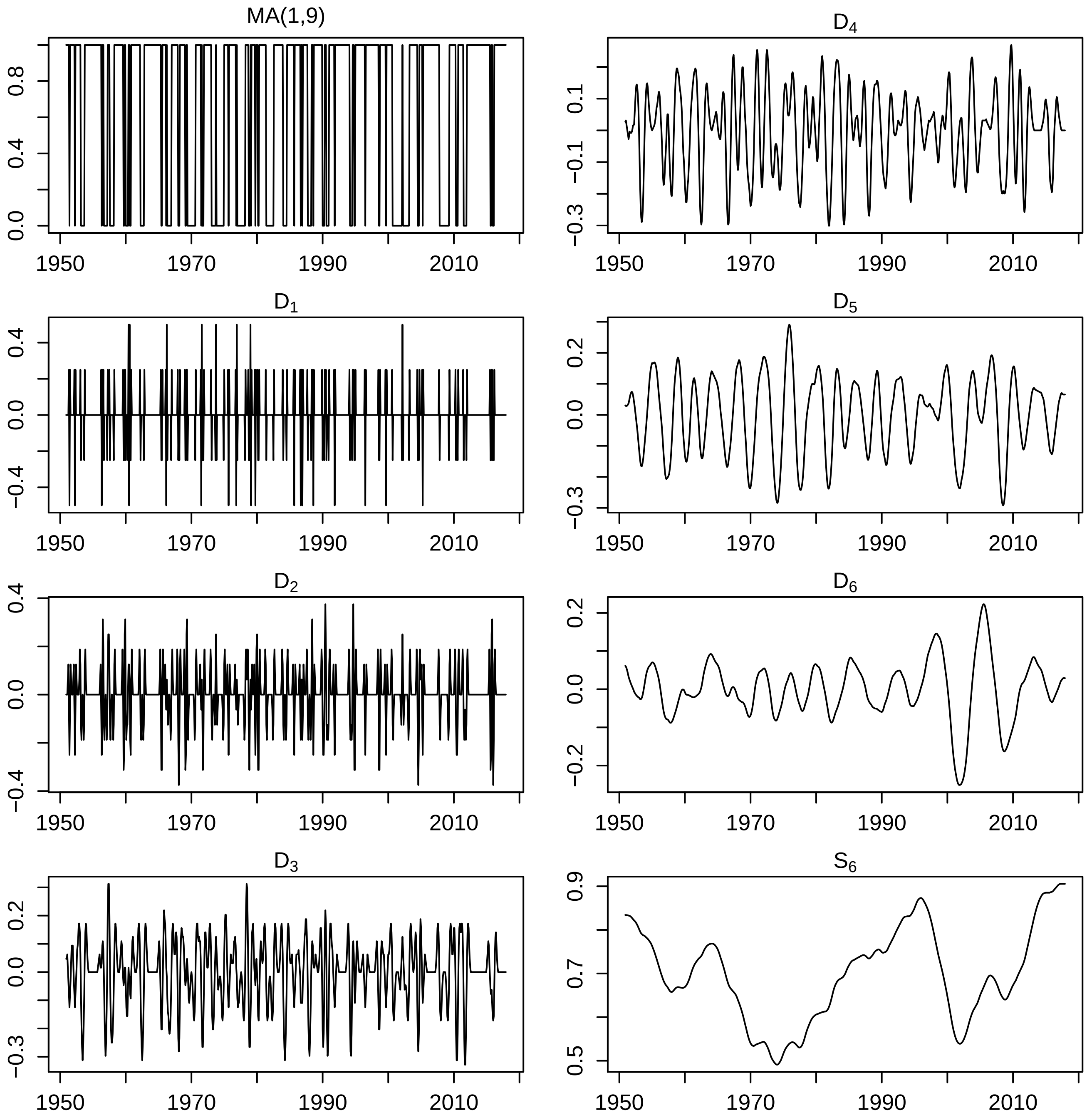

Figure 1

Multiresolution analysis of MA(1,9)

This figure presents the different timescale components for MA(1,9). The time series is decomposed with a MODWT MRA of level $J = 6$ using the Haar filter and data at the boundary are reflected. D_j refers to the level j wavelet detail and S_6 is the wavelet smooth. The sample is 1950:12 to 2017:12.

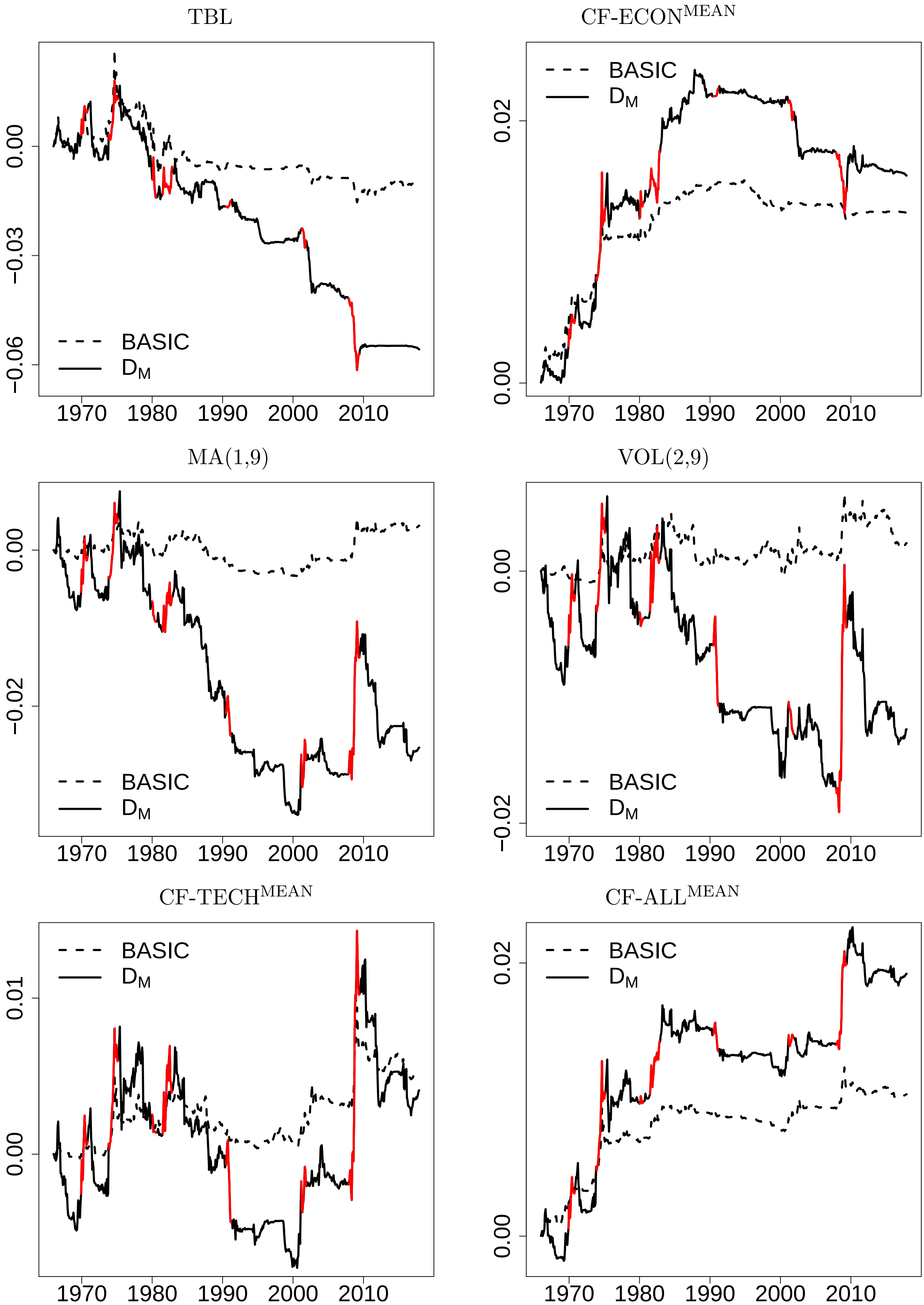

Figure 2

Out-of-sample performance for selected predictors

This figure plots the out-of-sample performance of different forecasting models relative to the historical mean. The dashed black line shows performance of the model with unadjusted predictors, whereas the solid black line presents results for the medium-frequency components. The out-of-sample period runs from 1966:1 to 2017:12 and NBER recession periods are colorized in red for the medium-frequency models.

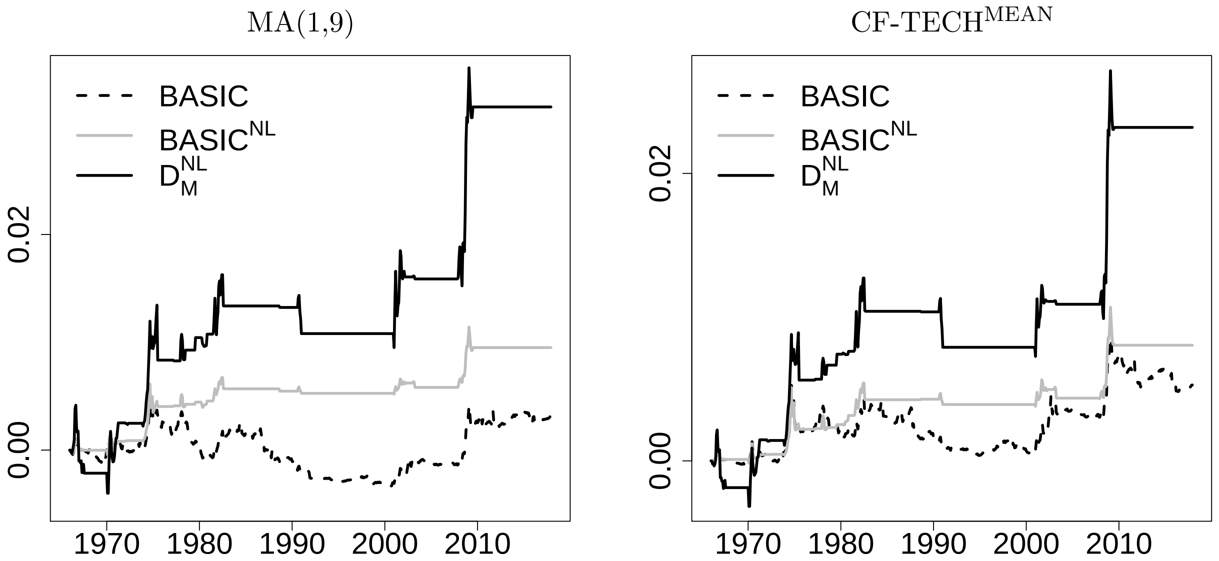

Figure 3
Out-of-sample performance of the nonlinear model
This figure plots the out-of-sample performance of different forecasting models relative to the historical mean. The solid black line shows performance of the nonlinear forecasting model with medium-frequency components, the solid gray line shows performance of the nonlinear forecasting model with unadjusted series, and the dashed black line shows performance of the linear forecasting model with unadjusted series. The relative performance is shown for MA(1,9) and CF-TECH^{MEAN}. The out-of-sample period runs from 1966:1 to 2017:12.

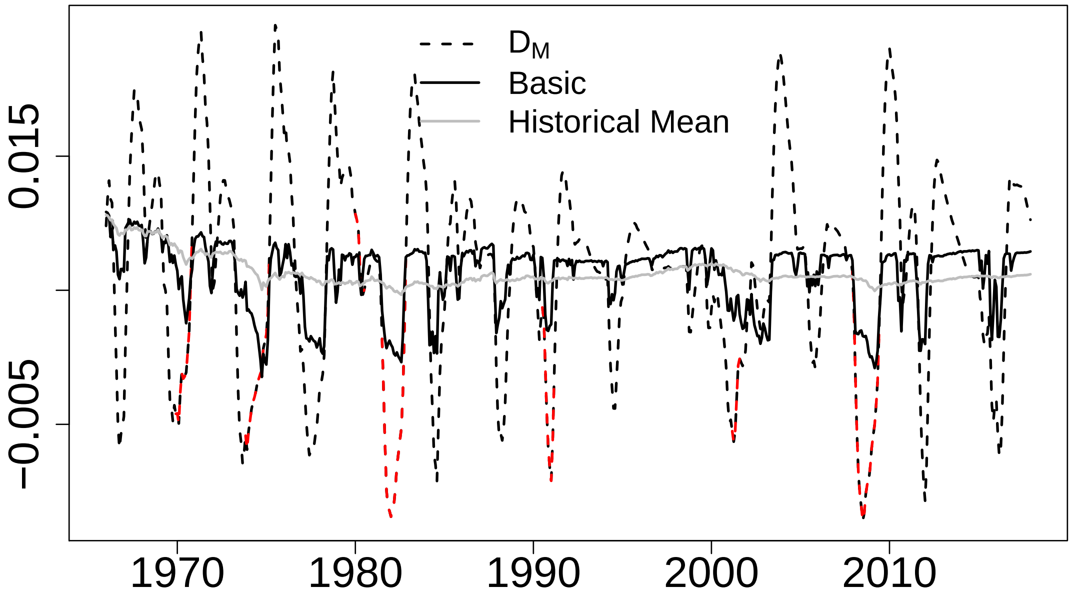

Figure 4

Excess return forecasts from CF-TECH$^{\text{MEAN}}$ and from the historical mean

This figure presents out-of-sample forecasts of excess stock returns from the historical mean (solid gray line), from the combination of forecasts from unadjusted technical indicators (solid black line), and from the combination of forecasts from medium-frequency components of technical indicators (dashed back line). The out-of-sample period runs from 1966:1 to 2017:12.

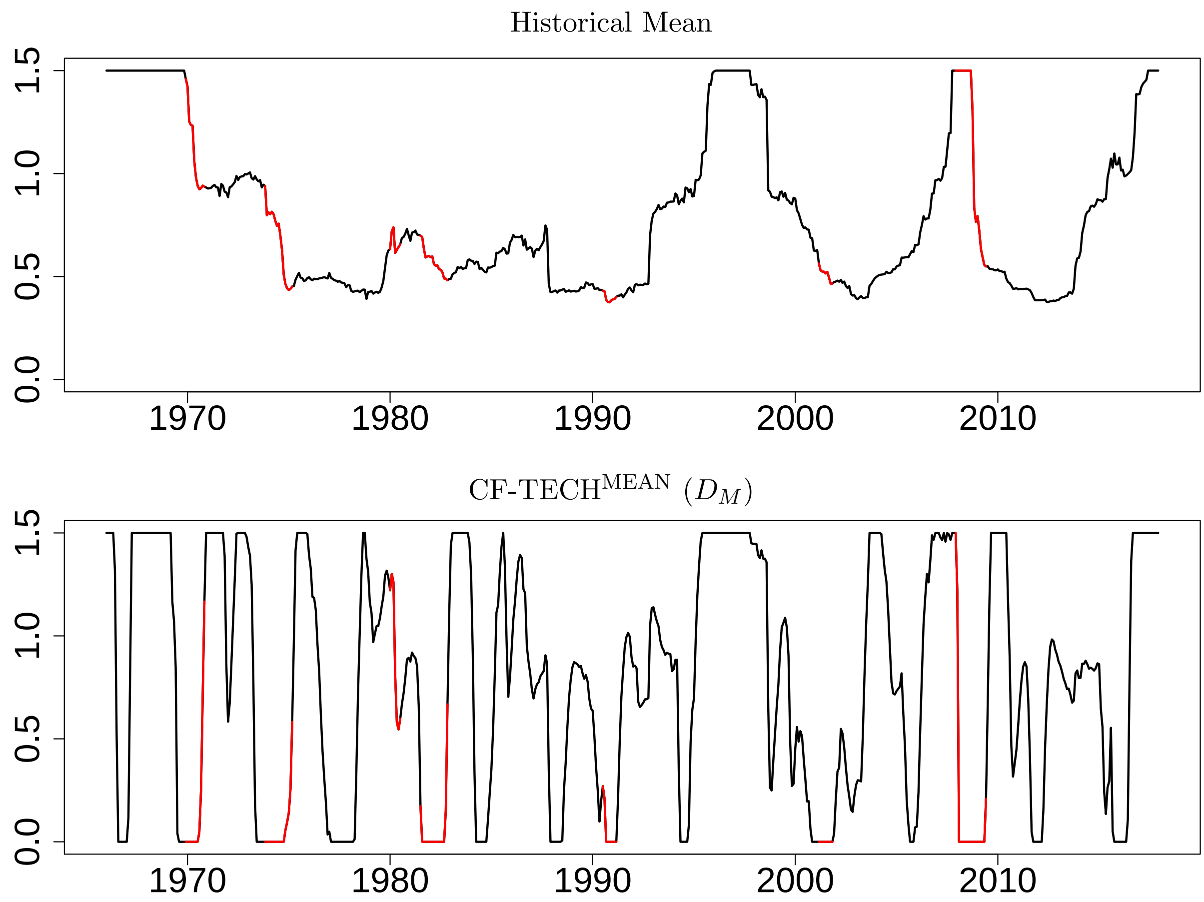

Figure 5
Optimal share in risky assets over time
This figure presents the optimal portfolio weights in risky assets (ω_t^*) for different forecasting models over time. Results are shown for the historical mean (top graph), as well as for the combination of medium-frequency components from technical indicators (bottom graph). The optimal weight of risky assets is restricted to lie between 0 and 1.5. The out-of-sample periods runs from 1966:1 to 2017:12. NBER recession periods are colorized in red.

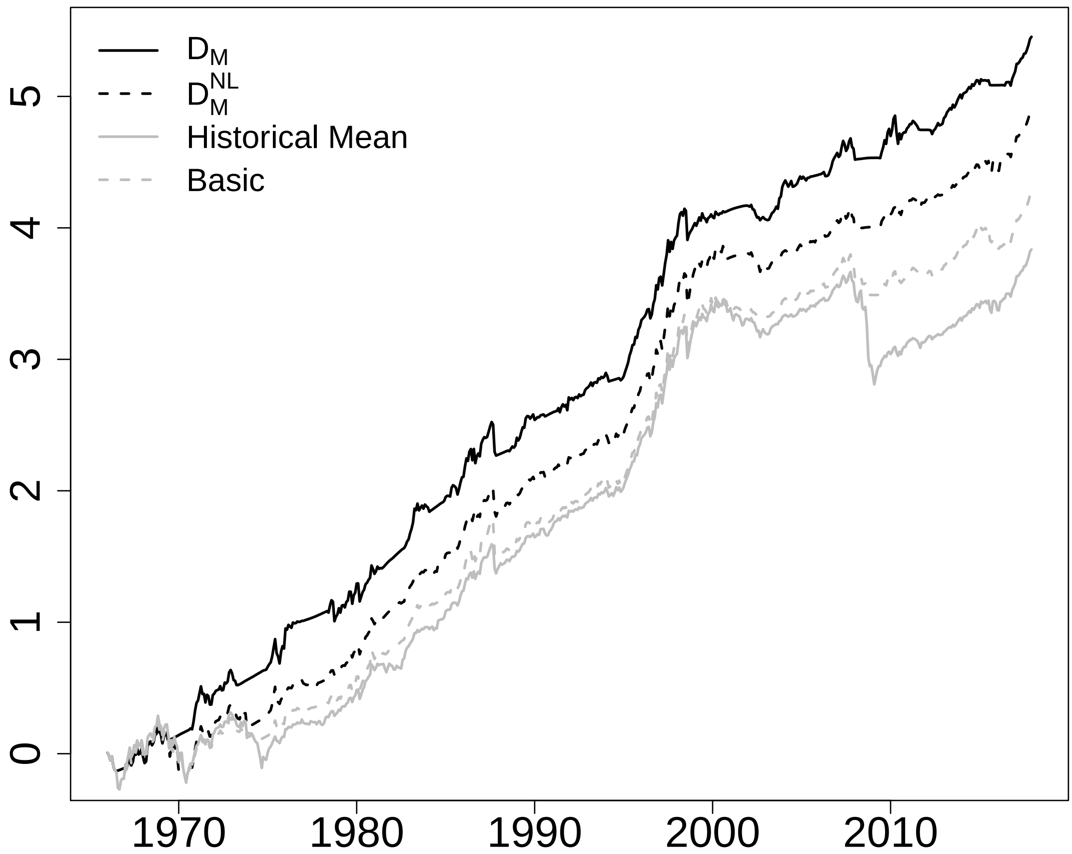

Figure 6

Log cumulative wealth for CF-TECH$^{\mathbf{MEAN}}$ after taking transaction costs into account

This figure presents the log cumulative wealth of four different forecasting models after accounting for transaction costs of 50 basis points per transaction. The sample covers the period from 1966:1 to 2017:12. The solid gray line shows results for the historical mean forecasting model. The dashed gray line (Basic) depicts the log wealth development for CF-TECH$^{\text{MEAN}}$ with a simple average of forecasts from 14 technical indicators. The dashed black line (D_M^{NL}) shows results for the nonlinear forecasting model that combines both combination forecasts from medium range frequencies and the historical mean. D_M depicts combination forecasts from the medium range frequencies of the 14 technical indicators.

Table 1

Summary statistics

This table reports summary statistics of the monthly log equity premium, the 14 economic variables, and the 14 technical indicators. The statistics include the mean (Mean), standard deviation (Std. dev.), skewness (Skew.), kurtosis (Kurt.), minimum (Min.), maximum (Max.), and the first-order autocorrelation (ρ_1). The sample period is 1950:12 to 2017:12.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
			1950:12 to 2017:12				
Variable	Mean	Std. dev.	Skew.	Kurt.	Min.	Max.	ρ_1
Log excess return							
r_t	0.53	4.15	-0.67	5.47	-24.84	14.87	0.06
Economic variables							
DP	-3.53	0.41	-0.21	2.37	-4.52	-2.60	0.99
DY	-3.52	0.42	-0.21	2.40	-4.53	-2.59	0.99
EP	-2.80	0.42	-0.74	5.88	-4.84	-1.90	0.99
DE	-0.73	0.29	2.60	18.78	-1.24	1.38	0.99
RVOL	0.14	0.05	0.83	3.89	0.05	0.32	0.96
BM	0.52	0.25	0.59	2.64	0.12	1.21	0.99
NTIS	0.01	0.02	-0.92	3.72	-0.06	0.05	0.98
TBL	4.28	3.09	0.88	4.08	0.01	16.30	0.99
LTY	5.99	2.76	0.82	3.19	1.75	14.82	0.99
LTR	0.53	2.75	0.51	6.26	-11.24	15.23	0.05
TMS	1.71	1.39	-0.14	2.89	-3.65	4.55	0.96
DFY	0.96	0.44	1.82	7.65	0.32	3.38	0.97
DFR	0.03	1.39	-0.39	9.91	-9.75	7.37	-0.08
INFL	0.29	0.36	0.14	5.71	-1.92	1.81	0.55
Technical indicators							
MA(1,9)	0.70	0.46	-0.86	1.74	0.00	1.00	0.69
MA(1,12)	0.72	0.45	-0.99	1.99	0.00	1.00	0.78
MA(2,9)	0.70	0.46	-0.88	1.77	0.00	1.00	0.76
MA(2,12)	0.72	0.45	-0.98	1.96	0.00	1.00	0.82
MA(3,9)	0.71	0.46	-0.90	1.81	0.00	1.00	0.79
MA(3,12)	0.72	0.45	-0.99	1.97	0.00	1.00	0.82
MOM(9)	0.72	0.45	-0.96	1.92	0.00	1.00	0.76
MOM(12)	0.73	0.44	-1.06	2.12	0.00	1.00	0.80
VOL(1,9)	0.69	0.46	-0.80	1.64	0.00	1.00	0.56
VOL(1,12)	0.71	0.45	-0.94	1.88	0.00	1.00	0.66
VOL(2,9)	0.68	0.47	-0.77	1.59	0.00	1.00	0.73
VOL(2,12)	0.70	0.46	-0.89	1.80	0.00	1.00	0.79
VOL(3,9)	0.69	0.46	-0.84	1.71	0.00	1.00	0.75
VOL(3,12)	0.70	0.46	-0.89	1.80	0.00	1.00	0.82

Table 2

Out-of-sample R^2 statistics (in %) for aggregated timescales

This table presents statistics on the out-of-sample predictability of one month ahead log excess returns on the S&P 500 index. Panel A (Panel B) shows results for economic variables (technical indicators). In addition to the individual forecasts, I display results for three different combination forecasting methods. Panel C shows results when combining both sets of predictors. For each model the out-of-sample R^2 (in %) is displayed (Campbell and Thompson, 2008). *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively according to the Clark and West (2007) MSFE-adjusted statistic. The null hypothesis is equal MSFE and the alternative is that the more sophisticated model has smaller MSFE than the historical mean benchmark. Column (1) shows the respective predictor and column (2) shows results for the unadjusted predictors. Columns (3) to (5) present results for the frequency-decomposed predictors. D_H refers to components with periodicities between 2 to 16 months, D_M refers to components with periodicities between 16 to 64 months, and D_L captures oscillations above 64 months.

(1)	(2)	(3)	(4)	(5)
		1966:1 to 2017:12		
Predictor	Basic	D_H	D_M	D_L
		Panel A: Economic variables		
DP	-0.28	-40.91	-0.24	0.32*
DY	-0.24	-25.39	-0.83	0.32*
EP	-0.60	-42.64	-0.69	-0.30
DE	-0.86	-10.49	-2.13	-0.86
RVOL	-0.07*	-2.89	-1.76	-0.47
BM	-1.25	-27.11	-0.10	-0.71
NTIS	-0.88	0.28*	-3.09	-0.72
TBL	-0.81**	0.63**	-4.75**	-0.64
LTY	-0.71**	1.50***	-2.16***	-0.69
LTR	0.32**	-0.41	-0.77**	0.09
TMS	-0.86**	-0.82	-6.64	-0.54
DFY	-0.63	-8.72	-3.71*	-1.33
DFR	-0.48	-0.68	-3.83**	-2.04
INFL	-0.36	0.28	-0.70*	-0.22
$\text{CF-ECON}^{\text{MEAN}}$	1.11***	-2.06	1.35***	0.11
$\text{CF-ECON}^{\text{WEIG}}_{\theta=1}$	1.11***	-1.00	1.31***	0.12
$\text{CF-ECON}^{\text{WEIG}}_{\theta=0.9}$	1.13***	-0.92	1.25***	0.12
		Panel B: Technical indicators		
MA(1,9)	0.27	-3.47	-2.16**	-7.41
MA(1,12)	0.63*	-2.86	-0.06**	-8.07
MA(2,9)	0.29	-3.35	-0.97**	-6.46
MA(2,12)	0.69**	-2.27	0.45***	-7.44
MA(3,9)	0.39*	-1.78	-0.57**	-5.92
MA(3,12)	0.02	-2.07	0.49**	-7.17
MOM(9)	0.10	-2.71	0.66**	-6.67
MOM(12)	0.12	-2.11	0.82**	-6.16
VOL(1,9)	0.15	-1.53	-2.46**	-2.82
VOL(1,12)	0.46*	-2.10	-0.20***	-3.19
VOL(2,9)	0.19	-2.31	-1.07**	-4.18
VOL(2,12)	0.24	-3.63	-0.04***	-2.52
VOL(3,9)	0.00	-2.24	-0.99**	-2.71
VOL(3,12)	0.64**	-1.98	0.26**	-3.30
$\text{CF-TECH}^{\text{MEAN}}$	0.45*	-1.65	0.35***	-4.71
$\text{CF-TECH}^{\text{WEIG}}_{\theta=1}$	0.45*	-1.64	0.33**	-4.66
$\text{CF-TECH}^{\text{WEIG}}_{\theta=0.9}$	0.46*	-1.65	0.31**	-4.66
		Panel C: All predictors taken together		
$\text{CF-ALL}^{\text{MEAN}}$	0.89***	-1.53	1.64***	-1.37
$\text{CF-ALL}^{\text{WEIG}}_{\theta=1}$	0.89**	-1.09	1.59***	-1.38
$\text{CF-ALL}^{\text{WEIG}}_{\theta=0.9}$	0.90**	-1.05	1.55***	-1.32

Table 3

Testing dependence between current predictability and business-cycle expectations

This table reports p-values for the null hypothesis that current predictability is independent of business-cycle expectations. Current predictability in Panel A is defined as $cp_t = \mathbf{I}[(r_t - \hat{r}_t)^2 - (r_t - \bar{r}_t)^2 < 0]$, whereas current predictability in Panel B is defined as $cp_t^{\mathrm{HIGH}} = \mathbf{I}[(r_t - \hat{r}_t)^2 - (r_t - \bar{r}_t)^2 < -\mathrm{IQR} \times 1.5]$. IQR is the interquartile range. Business-cycle expectations are estimated from the Liu and Moench (2016) probit model with an optimal threshold according to the maximum Youden index. The p-values are estimated with the dynamically augmented reduced rank regression approach of Pesaran and Timmermann (2009). The out-of-sample period is 1966:1 to 2017:12.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Predictor	Basic	D_H	D_M	D_L	Predictor	Basic	D_H	D_M	D_L
Panel A: $cp_t = \mathbf{I}[(r_t - \hat{r}_t)^2 - (r_t - \bar{r}_t)^2 < 0]$									
DP	6.53*	1.85**	96.71	9.09*	MA(1,9)	33.97	30.75	82.38	26.39
DY	4.95**	40.10	53.12	8.73*	MA(1,12)	84.36	77.49	38.78	41.38
EP	6.51*	2.87**	84.75	41.17	MA(2,9)	33.84	33.29	17.40	19.31
DE	57.72	79.79	93.20	14.94	MA(2,12)	41.59	42.87	35.40	16.02
RVOL	47.68	61.64	74.08	72.72	MA(3,9)	28.62	41.57	21.83	19.29
BM	0.22***	21.72	26.28	9.78*	MA(3,12)	52.02	44.16	16.94	46.43
NTIS	35.01	25.54	62.29	63.71	MOM(9)	72.66	97.59	77.17	8.24*
TBL	10.66	78.63	95.18	5.15*	MOM(12)	33.43	87.19	4.95**	28.51
LTY	4.18**	50.07	9.22*	5.27*	VOL(1,9)	73.40	36.37	32.04	92.54
LTR	33.99	61.88	44.40	8.93*	VOL(1,12)	14.16	21.25	18.95	89.39
TMS	49.55	39.47	95.84	81.42	VOL(2,9)	52.38	25.19	22.43	97.22
DFY	12.71	79.98	45.75	9.22*	VOL(2,12)	33.48	28.69	64.24	97.17
DFR	72.86	98.22	36.88	82.45	VOL(3,9)	12.51	16.70	26.81	92.53
INFL	0.86***	8.94*	83.84	6.04*	VOL(3,12)	7.93*	67.74	23.20	94.06
CF-ECON$^{\mathrm{MEAN}}$	22.83	31.52	50.93	19.63	CF-TECH$^{\mathrm{MEAN}}$	13.93	10.05	31.36	75.01
CF-ECON$^{\mathrm{WEIG}}_{\theta=1}$	48.25	27.45	55.21	20.95	CF-TECH$^{\mathrm{WEIG}}_{\theta=1}$	14.04	10.05	31.74	74.85
CF-ECON$^{\mathrm{WEIG}}_{\theta=0.9}$	71.29	26.11	62.71	21.13	CF-TECH$^{\mathrm{WEIG}}_{\theta=0.9}$	13.81	9.79*	50.36	74.85
CF-ALL$^{\mathrm{MEAN}}$	53.58	12.39	28.03	32.09					
CF-ALL$^{\mathrm{WEIG}}_{\theta=1}$	39.16	23.36	14.60	69.76					
CF-ALL$^{\mathrm{WEIG}}_{\theta=0.9}$	39.86	27.29	15.34	74.10					
Panel B: $cp_t^{\mathrm{HIGH}} = \mathbf{I}[(r_t - \hat{r}_t)^2 - (r_t - \bar{r}_t)^2 < -\mathrm{IQR} \times 1.5]$									
DP	40.31	97.53	58.01	24.87	MA(1,9)	4.34**	61.50	0.00***	33.78
DY	91.84	40.01	80.97	20.72	MA(1,12)	20.00	26.82	1.31**	35.06
EP	19.28	85.52	18.01	37.29	MA(2,9)	5.53*	12.97	0.02***	32.64
DE	82.65	65.81	82.87	42.69	MA(2,12)	30.40	3.40**	7.08*	50.90
RVOL	2.09**	10.86	18.58	55.48	MA(3,9)	9.06*	20.61	0.25***	45.92
BM	0.11***	83.68	39.90	32.39	MA(3,12)	98.50	42.02	4.55**	52.12
NTIS	20.54	52.18	97.22	59.02	MOM(9)	5.38*	68.23	37.13	69.60
TBL	16.98	14.03	2.20**	93.56	MOM(12)	63.64	87.97	33.80	23.07
LTY	12.21	7.52*	28.53	12.09	VOL(1,9)	67.52	83.95	0.03***	73.28
LTR	15.22	6.52*	73.41	10.87	VOL(1,12)	11.16	10.42	0.00***	66.30
TMS	11.08	54.58	1.38**	1.79**	VOL(2,9)	90.78	31.93	0.19***	48.63
DFY	1.87**	62.70	0.50***	10.42	VOL(2,12)	53.13	86.92	0.02***	66.18
DFR	7.56*	2.07**	21.51	85.55	VOL(3,9)	0.05***	14.46	0.01***	66.50
INFL	1.25**	69.95	83.07	65.84	VOL(3,12)	9.15*	42.46	0.05***	32.16
CF-ECON$^{\mathrm{MEAN}}$	31.09	38.11	6.95*	98.08	CF-TECH$^{\mathrm{MEAN}}$	13.25	74.78	0.01***	86.80
CF-ECON$^{\mathrm{WEIG}}_{\theta=1}$	28.61	54.40	5.46*	88.07	CF-TECH$^{\mathrm{WEIG}}_{\theta=1}$	13.15	72.26	0.00***	89.74
CF-ECON$^{\mathrm{WEIG}}_{\theta=0.9}$	47.24	51.38	4.53**	62.66	CF-TECH$^{\mathrm{WEIG}}_{\theta=0.9}$	13.15	72.26	0.00***	89.74
CF-ALL$^{\mathrm{MEAN}}$	22.04	61.19	6.46*	88.34					
CF-ALL$^{\mathrm{WEIG}}_{\theta=1}$	31.65	51.16	10.01	72.55					
CF-ALL$^{\mathrm{WEIG}}_{\theta=0.9}$	24.34	51.16	9.96*	73.95					

Table 4

Out-of-sample R^2 statistics (in %) for the nonlinear model

This table presents statistics on the out-of-sample predictability of one month ahead log excess returns on the S&P 500 index. Panel A (Panel B) shows results for economic variables (technical indicators). In addition to the individual forecasts, I display results for three different combination forecasting methods. Panel C shows results when combining both sets of predictors. For each model the out-of-sample R^2 (in %) is displayed (Campbell and Thompson, 2008). *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively according to the Clark and West (2007) MSFE-adjusted statistic. The null hypothesis is equal MSFE and the alternative is that the more sophisticated model has smaller MSFE than the historical mean benchmark. Column (1) shows the respective predictor and column (2) shows results for the unadjusted series. Columns (3) to (9) present results for the frequency-decomposed predictors. D_H refers to components with periodicities between 2 to 16 months, D_M refers to components with periodicities between 16 to 64 months, and D_L captures oscillations above 64 months. The superscript NL indicates that the nonlinear forecasting model is applied in columns (3), (5), (7), and (9).

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
				1966:1 to 2017:12				
Predictor	Basic	Basic^{NL}	D_H	D_H^{NL}	D_M	D_M^{NL}	D_L	D_L^{NL}
				Panel A: Economic variables				
DP	-0.28	0.56*	-40.91	-24.64	-0.24	-0.17	0.32*	0.49***
DY	-0.24	0.80**	-25.39	-13.39	-0.83	-0.30	0.32*	0.53***
EP	-0.60	-0.41	-42.64	-22.67	-0.69	-0.06	-0.30	-0.07
DE	-0.86	-0.42	-10.49	-6.05	-2.13	-0.08	-0.86	-0.42
RVOL	-0.07*	0.40*	-2.89	-1.38	-1.76	-0.43	-0.47	-0.19
BM	-1.25	-0.46	-27.11	-15.53	-0.10	-0.03	-0.71	-0.54
NTIS	-0.88	-1.27	0.28*	0.04	-3.09	-2.76	-0.72	-0.12
TBL	-0.81**	-1.62	0.63**	-0.61	-4.75**	-1.57	-0.64	-1.27
LTY	-0.71**	-1.56	1.50	0.29*	-2.16***	-1.91	-0.69	-1.10
LTR	0.32**	1.00**	-0.41	0.75**	-0.77**	-0.38	0.09	0.22
TMS	-0.86**	0.19**	-0.82	-0.76	-6.64	-1.66	-0.54	-0.09
DFY	-0.63	0.07	-8.72	-6.53	-3.71*	-0.74*	-1.33	-0.95
DFR	-0.48	-0.14	-0.68	-0.16	-3.83**	0.72*	-2.04	-1.16
INFL	-0.36	-1.11	0.28	0.65**	-0.70*	-0.87	-0.22	-1.22
CF-ECON^{MEAN}	1.11***	0.37*	-2.06	-2.19	1.35***	0.46	0.11	-0.13
CF-ECON$^{WEIG}_{\theta=1}$	1.11***	0.43*	-1.00	-1.35	1.31***	0.60*	0.12	-0.13
CF-ECON$^{WEIG}_{\theta=0.9}$	1.13***	0.48*	-0.92	-1.33	1.25***	0.62*	0.12	-0.11
				Panel B: Technical indicators				
MA(1,9)	0.27	0.81**	-3.47	-1.89	-2.16**	2.71***	-7.41	-3.23
MA(1,12)	0.63*	0.86**	-2.86	-1.50	-0.06**	1.93***	-8.07	-3.46
MA(2,9)	0.29	0.95**	-3.35	-2.05	-0.97**	2.45***	-6.46	-2.68
MA(2,12)	0.69**	0.97**	-2.27	-1.31	0.45***	1.84***	-7.44	-3.08
MA(3,9)	0.39*	1.03**	-1.78	-1.21	-0.57**	2.17***	-5.92	-2.85
MA(3,12)	0.02	0.44*	-2.07	-1.71	0.49**	1.47***	-7.17	-3.20
MOM(9)	0.10	0.30	-2.71	-1.65	0.66**	1.44**	-6.67	-3.02
MOM(12)	0.12	0.20	-2.11	-0.85	0.82**	1.01**	-6.16	-2.59
VOL(1,9)	0.15	0.63**	-1.53	-0.96	-2.46**	1.70***	-2.82	-1.12
VOL(1,12)	0.46*	0.78**	-2.10	-1.35	-0.20***	1.79***	-3.19	-1.54
VOL(2,9)	0.19	0.46*	-2.31	-1.33	-1.07**	1.75***	-4.18	-2.12
VOL(2,12)	0.24	0.28	-3.63	-2.15	-0.04***	1.56***	-2.52	-1.21
VOL(3,9)	0.00	0.48*	-2.24	-1.53	-0.99**	1.75***	-2.71	-1.20
VOL(3,12)	0.64**	0.85**	-1.98	-1.39	0.26**	1.44***	-3.30	-1.62
CF-TECH^{MEAN}	0.45*	0.69**	-1.65	-1.32	0.35***	1.98***	-4.71	-2.24
CF-TECH$^{WEIG}_{\theta=1}$	0.45*	0.69**	-1.64	-1.32	0.33**	1.97***	-4.66	-2.23
CF-TECH$^{WEIG}_{\theta=0.9}$	0.46*	0.69**	-1.65	-1.32	0.31**	1.97***	-4.66	-2.22
				Panel C: All predictors taken together				
CF-ALL^{MEAN}	0.89***	0.56**	-1.53	-1.61	1.64***	1.52***	-1.37	-0.82
CF-ALL$^{WEIG}_{\theta=1}$	0.89**	0.60**	-1.09	-1.24	1.59***	1.59***	-1.38	-0.84
CF-ALL$^{WEIG}_{\theta=0.9}$	0.90**	0.63**	-1.05	-1.23	1.55***	1.61***	-1.32	-0.77

Table 5

Annualized gains in CER

This table reports the annualized gain in certainty equity return (CER) relative to the CER from the historical mean (in percent). ΔCER is estimated for a mean-variance investor with a relative risk aversion of five who allocates each month between the S&P 500 index and the risk-free rate. The optimal weight is estimated according to forecasts of one-month ahead excess returns from predictive regression models. The optimal weight in risky assets is constrained to lie between 0 and 1.5 to prevent shorting stocks and leveraging more than 50%. The out-of-sample period runs from 1966:1 to 2017:12.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
			1966:1 to 2017:12					
Predictor	Basic	Basic^{NL}	D_H	D_H^{NL}	D_M	D_M^{NL}	D_L	D_L^{NL}
			Panel A: Economic variables					
DP	-0.67	0.53	-3.05	-3.18	0.19	0.13	0.65	0.94**
DY	-0.24	1.08	-3.93	-2.50	-1.34	-0.56	0.70	1.06**
EP	0.25	0.44	-0.83	-0.49	0.28	0.83*	-0.24	0.08
DE	-0.32	0.18	-0.12	-0.52	-1.01	0.18	0.25	0.56
RVOL	-1.04	0.00	-0.45	-0.64	-1.45	-0.58	-0.24	0.02
BM	-1.25	-0.13	-3.46	-3.06	0.15	0.15	-0.60	-0.26
NTIS	0.14	-0.89	0.76	0.12	0.00	-1.55	-0.51	0.58
TBL	1.80*	-0.01	1.16*	-0.33	1.11	0.70	1.52	-0.00
LTY	1.65	-0.01	3.13***	1.05*	2.39*	0.04	1.15	-0.11
LTR	0.87	0.83*	0.20	0.63	1.48	0.80	0.52	0.37
TMS	1.79	0.93	-1.24	-0.92	-0.61	0.66	1.10	0.58
DFY	-0.78	-0.21	-0.31	-0.44	-1.03	0.35	0.18	0.01
DFR	0.16	0.17	-0.65	-0.38	1.03	1.23	0.81	1.21
INFL	0.27	-0.53	0.74	1.19	1.79	0.90	1.67	0.08
CF-ECON^{MEAN}	1.72***	0.67*	-1.47	-2.01	2.32**	0.69	0.99	0.59
CF-ECON$^{WEIG}_{\theta=1}$	1.82***	0.83*	-0.88	-1.46	2.39**	0.90*	1.03	0.63
CF-ECON$^{WEIG}_{\theta=0.9}$	1.83***	0.89*	-0.86	-1.48	2.39**	0.94*	1.05	0.66
			Panel B: Technical indicators					
MA(1,9)	1.55*	1.96***	-1.92	-1.00	2.92*	3.53***	-0.05	1.17
MA(1,12)	2.62**	2.57***	-1.64	-0.89	3.56**	2.97***	-0.18	1.11
MA(2,9)	1.75*	2.41***	-1.76	-1.13	2.99**	3.36***	0.36	1.28
MA(2,12)	2.67**	2.71***	-1.45	-0.85	3.58***	2.89***	-0.07	1.11
MA(3,9)	2.18*	2.58***	-1.45	-0.88	2.77**	2.94***	0.05	0.95
MA(3,12)	1.15	1.50**	-0.55	-0.78	2.85**	2.31**	-0.25	0.98
MOM(9)	1.23	1.31*	-0.86	-0.80	3.14**	2.32**	-0.32	0.96
MOM(12)	1.14	1.22*	-0.79	-0.46	2.00*	2.03**	-0.25	1.20
VOL(1,9)	1.18	1.65**	-0.62	-0.60	3.32**	3.15***	0.96	1.68*
VOL(1,12)	1.79*	2.27***	-0.98	-0.70	4.46***	3.16***	0.81	1.65*
VOL(2,9)	0.88	1.44**	-1.21	-0.77	3.66**	2.86***	0.31	1.34
VOL(2,12)	0.89	1.17*	-1.39	-0.92	3.74***	2.67**	0.74	1.57*
VOL(3,9)	0.61	1.27**	-1.07	-0.73	3.26**	2.87***	0.61	1.46
VOL(3,12)	1.88*	2.27***	-0.94	-0.77	3.34**	2.49**	0.51	1.36
CF-TECH^{MEAN}	1.59*	1.85**	-1.02	-0.89	3.71***	3.05***	0.39	1.21
CF-TECH$^{WEIG}_{\theta=1}$	1.60*	1.86**	-1.00	-0.89	3.71***	3.06***	0.41	1.23
CF-TECH$^{WEIG}_{\theta=0.9}$	1.60*	1.86**	-1.01	-0.89	3.71***	3.06***	0.41	1.23
			Panel C: All predictors taken together					
CF-ALL^{MEAN}	1.71***	1.24**	-1.90	-2.12	4.31***	2.92***	0.71	1.19
CF-ALL$^{WEIG}_{\theta=1}$	1.80***	1.35**	-1.39	-1.60	4.17***	2.95***	0.74	1.23
CF-ALL$^{WEIG}_{\theta=0.9}$	1.82***	1.40**	-1.37	-1.57	4.15***	2.96***	0.76	1.27

Table 6

Annualized gains in SR

This table reports the annualized gain in the Sharpe ratio (SR) relative to the SR from the historical mean. ΔSR is estimated for a mean-variance investor with a relative risk aversion of five who allocates each month between the S&P 500 index and the risk-free rate. The optimal weight is estimated according to forecasts of one-month ahead excess returns from predictive regression models. The optimal weight in risky assets is constrained to lie between 0 and 1.5 to prevent shorting stocks and leveraging more than 50%. The out-of-sample period runs from 1966:1 to 2017:12.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
				1966:1 to 2017:12				
Predictor	Basic	Basic^{NL}	D_H	D_H^{NL}	D_M	D_M^{NL}	D_L	D_L^{NL}
				Panel A: Economic variables				
DP	-0.11	0.06	-0.08	-0.13	-0.01	0.01	0.03	0.06**
DY	-0.08	0.09*	-0.17	-0.09	-0.10	0.00	0.03	0.07**
EP	-0.01	0.03	0.03	-0.01	0.03	0.05	-0.03	-0.00
DE	-0.06	0.01	0.04	0.01	0.00	0.04	-0.02	0.02
RVOL	0.01	0.05	0.02	0.01	-0.01	0.02	-0.03	-0.01
BM	-0.08	0.00	-0.14	-0.15	0.01	0.02	-0.02	-0.03
NTIS	0.06	-0.06	0.06	0.01	0.05	-0.07	-0.04	0.02
TBL	0.12	-0.02	0.10**	-0.00	0.14	0.08	0.09	-0.03
LTY	0.10	-0.03	0.25***	0.09*	0.19**	-0.01	0.04	-0.04
LTR	0.11*	0.08**	0.04	0.06*	0.15**	0.08*	0.05	0.02
TMS	0.19**	0.08*	-0.09	-0.06	0.02	0.08	0.10	0.02
DFY	-0.02	0.01	0.04	0.01	0.01	0.08*	-0.03	-0.03
DFR	0.01	0.01	-0.04	-0.02	0.11	0.10	0.05	0.07
INFL	0.01	-0.05	0.05	0.09**	0.14*	0.07	0.13*	-0.02
CF-ECON^{MEAN}	0.11**	0.04	-0.05	-0.08	0.19***	0.07*	0.05	0.01
$\text{CF-ECON}^{WEIG}_{\theta=1}$	0.12**	0.05*	-0.03	-0.06	0.19***	0.08*	0.05	0.02
$\text{CF-ECON}^{WEIG}_{\theta=0.9}$	0.12**	0.06*	-0.03	-0.06	0.19***	0.08**	0.05	0.02
				Panel B: Technical indicators				
MA(1,9)	0.10*	0.13***	-0.10	-0.08	0.24**	0.27***	0.03	0.06
MA(1,12)	0.19**	0.18**	-0.09	-0.07	0.28**	0.22***	0.04	0.06
MA(2,9)	0.12*	0.17***	-0.10	-0.09	0.24**	0.26***	0.07	0.07
MA(2,12)	0.20**	0.20***	-0.09	-0.06	0.28**	0.21***	0.04	0.06
MA(3,9)	0.16*	0.19***	-0.09	-0.07	0.22**	0.22***	0.05	0.05
MA(3,12)	0.07	0.09*	-0.02	-0.06	0.22**	0.16**	0.03	0.05
MOM(9)	0.07	0.08	-0.04	-0.06	0.24**	0.16**	0.03	0.05
MOM(12)	0.07	0.07	-0.04	-0.03	0.14*	0.14**	0.04	0.07
VOL(1,9)	0.07	0.11**	-0.01	-0.05	0.26**	0.24***	0.09	0.11
VOL(1,12)	0.13*	0.16***	-0.05	-0.06	0.35***	0.24***	0.08	0.11
VOL(2,9)	0.06	0.09**	-0.06	-0.06	0.29***	0.21***	0.05	0.08
VOL(2,12)	0.06	0.07	-0.08	-0.07	0.30***	0.20**	0.08	0.10
VOL(3,9)	0.04	0.08**	-0.05	-0.05	0.26**	0.21***	0.07	0.09
VOL(3,12)	0.14*	0.16***	-0.06	-0.06	0.26***	0.18**	0.06	0.08
CF-TECH^{MEAN}	0.11*	0.12**	-0.05	-0.07	0.29***	0.23***	0.07	0.07
$\text{CF-TECH}^{WEIG}_{\theta=1}$	0.11*	0.12**	-0.05	-0.07	0.29***	0.23***	0.07	0.07
$\text{CF-TECH}^{WEIG}_{\theta=0.9}$	0.11*	0.13**	-0.05	-0.07	0.29***	0.23***	0.07	0.07
				Panel C: All predictors taken together				
CF-ALL^{MEAN}	0.11***	0.08**	-0.09	-0.11	0.34***	0.22***	0.06	0.06
$\text{CF-ALL}^{WEIG}_{\theta=1}$	0.12***	0.08**	-0.07	-0.09	0.33***	0.22***	0.07	0.07
$\text{CF-ALL}^{WEIG}_{\theta=0.9}$	0.12***	0.09**	-0.07	-0.09	0.33***	0.22***	0.07	0.07

Online Appendix for
"Out-of-Sample Equity Premium Prediction: Combination Forecasts with Frequency-Decomposed Variables"

Not for Publication

A $\mathbf{R^2_{OS}}$ for individual timescales

In the main article I have only provided results for the aggregated timescales D_H, D_M, and D_L. Table A.1 shows the R^2_{OS} statistics for the individual timescales D_1 to D_6, and S_6. Panel B highlights that predictability by technical indicators solely stems from D_4 and D_5, which approximates periodicities between 16 to 32 months and 32 to 64 months.

B Recession forecasts and differences in squared forecasting errors

The so called receiver operating characteristic (ROC) curve plots the entire set of possible combinations of TPR(θ) and FPR(θ) (Berge and Jordà, 2011). Figure A.1 presents the ROC curve for the grid of candidate values for the full sample from 1950:12 to 2017:12. Firstly, I generate in-sample predictions for the probability of a recession one-month ahead. Secondly, I estimate the respective values of TPR(θ) and FPR(θ) for the 101 candidate values. Then, the value is selected that maximizes the difference between both ratios. Graphically, the optimal point is the point with the largest distance to the diagonal line (Baker and Kramer, 2007). The dashed diagonal line is the equivalent of a random guess and a model with optimal accuracy "would have a ROC curve that hugged the top left corner" (Liu and Moench, 2016, p. 1141). The ROC curve shows that the probit model has an excellent classification ability for the full sample. For the in-sample exercise the optimal threshold level equals $\theta^* = 0.19$. The one-month ahead recession forecasts from the probit model are shown in the right panel of Figure A.1.

Figure A.2 shows boxplots of $(r_t - \hat{r}_t)^2 - (r_t - \bar{r}_t)^2$ for the 14 medium-frequency components of the technical indicators (D_M). The difference in squared forecast errors is centered around zero with outliers both to the left (D_M performs better) and the right (historical mean performs better). A predictor that consistently outperforms the naive benchmark would only have observations to the left of zero. I show in the main article that points on the left side of the distribution depend on business-cycle expectations.

C Asset allocation exercise with alternative choice of parameters

In this section I set the parameters of the asset allocation exercise identical to Rapach et al. (2016). I restrict the share of risky assets to lie between −0.5 and 1.5, allowing for a short position in risky assets of 50%. The coefficient of relative risk aversion is set to three and the volatility forecast is estimated according to a ten-year moving window of past excess returns. The results for ΔCER and ΔSR under this specification are shown in Table A.2 and Table A.3.

D Subsample analysis: 1990:1 to 2017:12

Tables A.4 to A.6 present results for the out-of-sample period from 1990:1 to 2017:12. Results remain qualitatively the same. For the more recent sample the combination forecasts of economic variables perform rather poor. Contrarily, combination forecasts of medium frequencies from technical indicators provide a sizable utility gain of 357 basis points relative to the historical mean, and a gain of 143 basis points relative to combination forecasts from the unadjusted indicators. The R^2_{OS} for the nonlinear forecasting model with combination forecasts from medium frequencies of technical indicators is 2.25%. This is more than three times larger than for the simple combination forecasts.

E Principal component analysis

Similar to Neely et al. (2014), I analyze the forecasting performance of principal components. Firstly, I split each predictor into frequency-specific components and then group the respective components for all predictors. Thus, the frequency-specific information of all predictors is saved in respective matrices. Secondly, I normalize the predictors to have a mean of zero and a standard deviation of one. Thirdly, I estimate the first, second, and third principal components of the frequency-specific matrices. Table A.7 shows the R^2_{OS} values, whereas Table A.8 and Table A.9 present results for ΔCER and ΔSR.

F Alternative choice of wavelet filter

In the main text I have only applied the Haar wavelet filter to decompose time series. The choice of the Haar filter is often justified by the fact that "the wavelet coefficients are simply differences of moving averages" (Faria and Verona, 2018a). Ortu et al. (2013) show that the Haar filter is a simple method to decompose time series along the persistence dimension. Bandi et al. (2019a, p. 17) write that "alternative nonparametric filters, like the Daubechies filter, could have been used instead without affecting the empirical results". Likewise, Kang et al. (2017, p. 24) use the "least asymmetric" wavelet filter, writing that their findings "are not specific to the particular wavelet filter used". In line with this, Rua (2011, p. 671) states that results do not change much when using Daubechies and Coiflets rather than a symmlet 4 wavelet. Similarly, Risse (2019) explains that Daubechies wavelets do not lead to superior performance compared to the Haar wavelet. So, the choice of wavelet seems to be a "technical note" rather than a crucial choice (Kang et al., 2017, p. 24). Percival and Walden (2000, p. 197) write the following:

> "To summarize, as compared to DWT-based MRAs, a MODWT-based MRA is less dependent upon our choice of wavelet filter, but not so much so that we can recommend always using a particular filter. A careful study of the differences between MRAs based on different wavelet filters is still needed to see which filter is best matched to a particular application."

Hence, there is no clear guidance on the choice of wavelet. Therefore, I simply repeat the analysis from the previous sections with four different wavelets. Table A.10 presents results for R^2_{OS}, ΔCER, and ΔSR, when using combination forecasts and alternative wavelets to isolate the medium-frequency components of predictors. It can be seen that results are qualitatively the same, with different wavelets only having a minor effect. The wavelets with a lower width (D(4) and FK(4)) seem to perform slightly better in terms of R^2_{OS} compared to wavelets with a width of 8 or 16 observations.[1] However, the overall role of wavelet choice is subordinate. Thus, my findings are in line with other articles documenting that the choice of filter is of minor importance.

G Performance after transaction costs

Table A.11 presents the average turnover of portfolios resulting from advanced forecasting models relative to the average turnover of the portfolio based on forecasts from the historical mean. The average turnover of a portfolio based on the historical mean forecast is 2.13%. None of the portfolios from more advanced forecasting models has a relative average turnover below one. Each of these portfolios has higher transaction costs, therefore it is important to analyze whether the gains in ΔCER and ΔSR remain significant after accounting for these costs. Table A.13 and Table A.14 show results after accounting for a proportional transaction cost of 50 basis points per transaction. As an example, the combination forecasts from medium frequencies of all predictors generate a utility gain of 396 basis points relative to the historical mean, and a utility gain of 250 basis points relative to the unadjusted predictors. The results remain sizable after accounting for transaction costs.

[1] The Haar wavelet has a width of 2 and as well belongs to discrete Daubechies wavelets. Therefore, the Haar wavelet is also called D(2) wavelet.

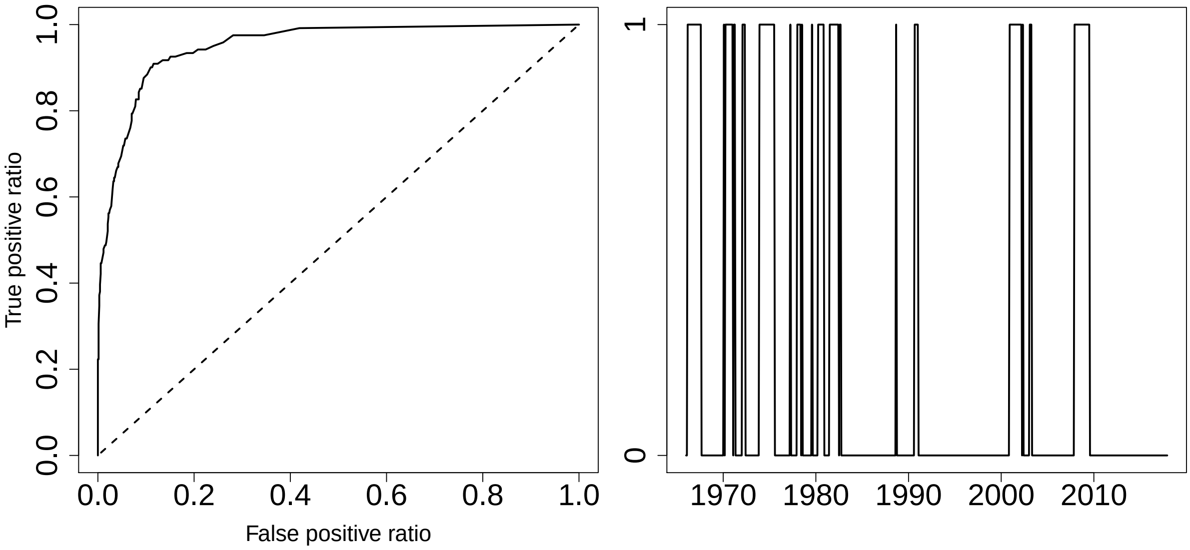

Figure A.1

ROC curve and business-cycle forecasts

This figure plots the ROC curve for the in-sample period from 1950:12 to 2017:12 (left panel) and the out-of-sample recession forecasts for the period from 1966:1 to 2017:12 (right panel). The recession periods are classified according to the maximum Youden index and the probability forecasts are based on the probit model.

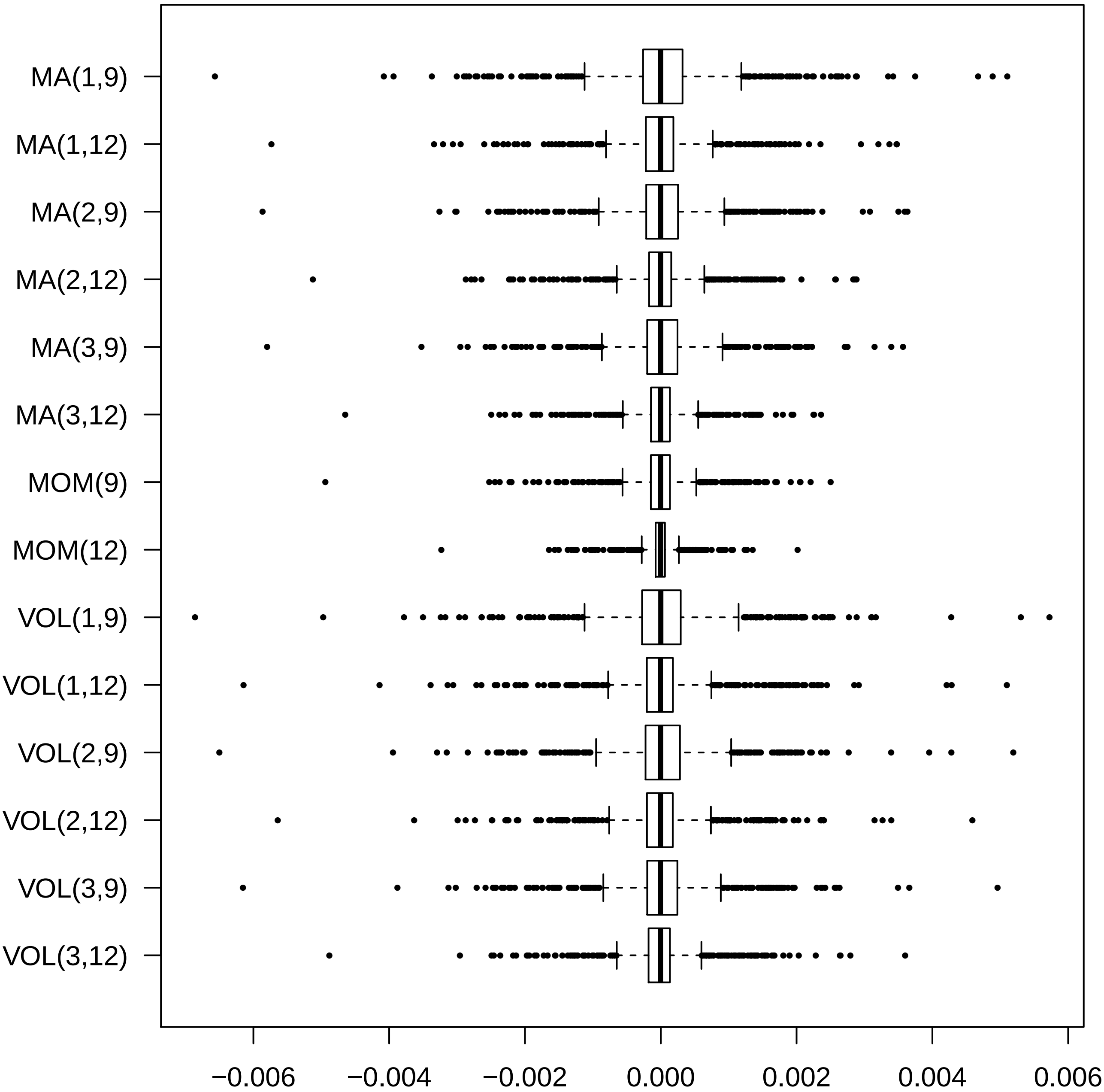

Figure A.2

Boxplots for differences in squared forecasting errors

This figure presents boxplots for differences in squared forecasting errors. The squared errors from forecasts with medium-frequency components of technical indicators are subtracted from squared forecasting errors of the historical mean, $(r_t - \hat{r}_t)^2 - (r_t - \bar{r}_t)^2$. The out-of-sample forecasting period is 1966:1 to 2017:12. The ends of the box are the upper and lower quartiles, so the box spans the interquartile range (IQR). The whiskers are the dashed lines that extend from both sides of the box up to the highest or lowest value within $\pm 1.5 \times$ IQR. The dots represent points that are outside of this range.

Table A.1

Out-of-sample R^2 statistics (in %)

This table presents statistics on the out-of-sample predictability of one month ahead log excess returns on the S&P 500 index. Panel A (Panel B) shows results for economic variables (technical indicators). In addition to the individual forecasts, I display results for three different combination forecasting methods. Panel C shows results when combining both sets of predictors. For each model the out-of-sample R^2 (in %) is displayed (Campbell and Thompson, 2008). *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively according to the Clark and West (2007) MSFE-adjusted statistic. The null hypothesis is equal MSFE and the alternative is that the more sophisticated model has smaller MSFE than the historical mean benchmark. Column (1) shows the respective predictor and column (2) shows results for the unadjusted series. Columns (3) to (9) present results for the frequency-decomposed predictors. D_1 refers to the component with the highest frequency and S_6 refers to the component with the lowest frequency.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
				1966:1 to 2017:12				
Predictor	Basic	D_1	D_2	D_3	D_4	D_5	D_6	S_6
				Panel A: Economic variables				
DP	-0.28	-29.86	-26.21	-4.30	-0.49	-0.20	-0.80	0.18
DY	-0.24	-0.13	-53.15	-20.07	-2.62	-0.19	-0.56	0.16
EP	-0.60	-38.56	-28.80	-0.71	-0.30	-1.09	-1.63	-0.11
DE	-0.86	-6.23	-18.18	-5.27	-2.34	-2.19	-1.24	-0.17
RVOL	-0.07*	-0.47	-4.34	-7.66	-5.23	-0.65	-0.49	-0.31
BM	-1.25	-17.58	-17.07	-3.05	-0.71	0.05	-1.11	-0.55
NTIS	-0.88	-0.44	0.25	-1.66	-3.50	-2.20	-0.62	-0.51
TBL	-0.81**	-1.96	-0.27	-0.94**	-7.82**	-2.77**	-2.10	-0.20
LTY	-0.71**	-0.47	0.76**	-2.23***	-2.37***	-1.56*	-1.09	-0.41
LTR	0.32**	0.38*	-2.22*	-3.45	-1.40**	0.02**	0.04	-1.31
TMS	-0.86**	-1.39	-1.18	-1.34	-8.75	-3.92*	-1.83	-0.18
DFY	-0.63	-1.20	-5.02	-17.45	-8.42*	-1.55	-0.93	-1.13
DFR	-0.48	-0.49	-0.71	-3.54	-3.79*	-2.55	-1.35	-1.41
INFL	-0.36	0.16	0.32*	-1.06	-2.26	0.51**	-0.10	-0.09
CF-ECON^{MEAN}	1.11***	-1.82	-1.91	-0.59	1.05***	0.95**	-0.38	0.08
CF-ECON$^{WEIG}_{\theta=1}$	1.11***	-1.70	-1.19	-0.49	0.91**	0.99**	-0.35	0.09
CF-ECON$^{WEIG}_{\theta=0.9}$	1.13***	-1.70	-1.11	-0.47	0.85**	0.98**	-0.33	0.09
				Panel B: Technical indicators				
MA(1,9)	0.27	-2.88	-2.55	-0.15	-2.42*	-1.87**	-9.30	-3.66
MA(1,12)	0.63*	-2.82	-2.64	0.09	-0.15**	-0.54**	-8.39	-4.06
MA(2,9)	0.29	-1.85	-6.89	0.08	-1.10*	-1.08**	-7.80	-3.16
MA(2,12)	0.69**	-0.90	-5.41	-0.25	0.32**	-0.17**	-6.96	-4.34
MA(3,9)	0.39*	-0.43	-4.80	-0.18	-0.60*	-0.89**	-6.57	-2.91
MA(3,12)	0.02	-0.26	-4.61	-1.14	0.42**	-0.30**	-6.77	-3.89
MOM(9)	0.10	-1.21	-1.58	-0.75	0.58**	-0.27**	-6.57	-3.65
MOM(12)	0.12	-0.78	-1.89	-1.43	0.31*	0.36**	-5.05	-3.74
VOL(1,9)	0.15	-1.15	-1.26	-0.56	-2.81*	-1.53**	-3.40	-1.20
VOL(1,12)	0.46*	-0.65	-2.44	0.10	-0.39**	-0.32***	-3.51	-1.36
VOL(2,9)	0.19	-0.52	-5.02	-0.09	-0.92*	-1.16***	-4.82	-1.51
VOL(2,12)	0.24	-1.06	-6.23	-0.60	0.20**	-0.87**	-4.10	-0.93
VOL(3,9)	0.00	-0.60	-3.71	-0.39	-0.74*	-1.12**	-3.51	-1.10
VOL(3,12)	0.64**	-0.20	-4.05	-1.05	0.35**	-0.61**	-3.35	-1.41
CF-TECH^{MEAN}	0.45*	-0.52	-2.24	-0.10	0.13**	-0.07**	-4.97	-2.38
CF-TECH$^{WEIG}_{\theta=1}$	0.45*	-0.53	-2.28	-0.12	0.12**	-0.10**	-4.83	-2.35
CF-TECH$^{WEIG}_{\theta=0.9}$	0.46*	-0.53	-2.29	-0.11	0.12**	-0.11**	-4.82	-2.34
				Panel C: All predictors taken together				
CF-ALL^{MEAN}	0.89***	-0.99	-1.72	-0.10	1.17***	1.33***	-1.59	-0.76
CF-ALL$^{WEIG}_{\theta=1}$	0.89**	-0.97	-1.45	-0.09	1.07***	1.36***	-1.49	-0.73
CF-ALL$^{WEIG}_{\theta=0.9}$	0.90**	-0.97	-1.41	-0.08	1.03***	1.34***	-1.44	-0.73

Table A.2

Annualized gains in CER

This table reports the annualized gain in certainty equity return (CER) relative to the CER from the historical mean (in percent). ΔCER is estimated for a mean-variance investor with a relative risk aversion of three who allocates each month between the S&P 500 index and the risk-free rate. The optimal weight is estimated according to forecasts of one-month ahead excess returns from predictive regression models. The optimal weight in risky assets is constrained to lie between -0.5 and 1.5. The out-of-sample period runs from 1966:1 to 2017:12.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
				1966:1 to 2017:12				
Predictor	Basic	Basic^{NL}	D_H	D_H^{NL}	D_M	D_M^{NL}	D_L	D_L^{NL}
				Panel A: Economic variables				
DP	-1.17	0.95	-3.58	-3.06	-0.13	0.05	0.63	1.12**
DY	-1.10	1.17*	-5.19	-2.56	-1.20	-0.31	0.73	1.25**
EP	0.26	0.72	-0.46	-0.52	0.38	1.01	-0.80	-0.07
DE	-0.75	0.08	0.77	0.30	-0.78	0.59	-0.01	0.74
RVOL	-0.55	0.59*	0.23	0.01	-1.28	0.07	-0.39	-0.13
BM	-0.66	0.35	-3.55	-3.66	0.06	0.28	-0.99	-0.83
NTIS	0.51	-0.54	1.51*	0.80**	0.93	-0.81	-1.12	-0.02
TBL	1.99	-0.51	1.06	-0.68	2.65	1.39	0.81	-0.78
LTY	1.21	-0.84	4.09***	1.58	2.43	-0.47	0.40	-0.89
LTR	2.06*	1.47*	0.26	0.75	1.82	1.36	0.58	0.51
TMS	3.66**	1.28	-1.55	-0.89	0.54	1.15	1.87	0.60
DFY	-1.03	-0.51	-0.98	-0.63	0.11	0.82	-1.75	-1.06
DFR	0.54	0.04	-0.44	-0.40	0.19	1.06	1.11	0.91
INFL	0.49	-0.59	0.51	1.19***	2.60*	0.70	2.09	-0.28
$\text{CF-ECON}^{\text{MEAN}}$	1.73**	0.86	-1.23	-1.66	2.62**	0.98	0.64	0.47
$\text{CF-ECON}^{\text{WEIG}}_{\theta=1}$	1.93**	1.11*	-0.54	-1.03	2.62**	1.21	0.72	0.51
$\text{CF-ECON}^{\text{WEIG}}_{\theta=0.9}$	1.98**	1.22*	-0.46	-1.00	2.51**	1.21	0.74	0.55
				Panel B: Technical indicators				
MA(1,9)	1.55	1.92**	-3.26	-1.65	3.24	4.26**	-0.28	0.39
MA(1,12)	2.78*	2.29*	-2.69	-1.30	4.51**	3.64**	0.24	0.43
MA(2,9)	1.74	2.17**	-2.62	-1.64	3.32	4.09**	0.15	0.46
MA(2,12)	2.91*	2.45*	-2.11	-1.21	4.55**	3.57**	-0.04	0.54
MA(3,9)	2.29	2.44**	-2.10	-1.17	3.22*	3.62**	0.15	0.32
MA(3,12)	0.96	1.28*	-1.97	-1.25	3.44*	2.94**	-0.04	0.43
MOM(9)	1.16	1.06	-1.66	-1.04	4.12**	2.93**	-0.08	0.45
MOM(12)	1.09	0.93	-1.54	-0.27	2.80**	2.55**	-0.09	0.81
VOL(1,9)	1.21	1.77**	-1.01	-0.44	3.59*	3.41**	1.71	1.57
VOL(1,12)	1.89	2.06**	-1.27	-0.87	5.21**	3.60**	1.47	1.45
VOL(2,9)	1.32	1.41**	-1.90	-1.01	4.42**	3.49**	0.71	0.67
VOL(2,12)	1.52*	1.13*	-2.15	-1.23	4.72**	3.19**	1.47	1.48
VOL(3,9)	0.72	1.33**	-1.84	-0.99	4.28**	3.58**	1.39	1.39
VOL(3,12)	2.22*	2.18**	-1.81	-1.30	4.39**	3.04**	0.99	1.18
$\text{CF-TECH}^{\text{MEAN}}$	1.77*	1.78**	-2.22	-1.40	4.94**	4.00**	0.76	0.83
$\text{CF-TECH}^{\text{WEIG}}_{\theta=1}$	1.78*	1.79**	-2.19	-1.39	4.94**	3.99**	0.79	0.84
$\text{CF-TECH}^{\text{WEIG}}_{\theta=0.9}$	1.78*	1.79**	-2.19	-1.40	4.96**	4.01**	0.78	0.85
				Panel C: All predictors taken together				
$\text{CF-ALL}^{\text{MEAN}}$	1.72**	1.40**	-2.07	-1.76	5.00***	3.48***	0.75	0.58
$\text{CF-ALL}^{\text{WEIG}}_{\theta=1}$	1.80**	1.50**	-1.74	-1.48	5.09***	3.65***	0.85	0.62
$\text{CF-ALL}^{\text{WEIG}}_{\theta=0.9}$	1.83**	1.56**	-1.74	-1.47	5.07***	3.65***	0.88	0.67

Table A.3

Annualized gains in SR

This table reports the annualized gain in the Sharpe ratio (SR) relative to the SR from the historical mean. ΔSR is estimated for a mean-variance investor with a relative risk aversion of three who allocates each month between the S&P 500 index and the risk-free rate. The optimal weight is estimated according to forecasts of one-month ahead excess returns from predictive regression models. The optimal weight in risky assets is constrained to lie between -0.5 and 1.5. The out-of-sample period runs from 1966:1 to 2017:12.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
				1966:1 to 2017:12				
Predictor	Basic	Basic$^{\text{NL}}$	D_H	D_H^{NL}	D_M	D_M^{NL}	D_L	D_L^{NL}
				Panel A: Economic variables				
DP	-0.11	0.06*	-0.19	-0.15	-0.02	0.00	0.03	0.06**
DY	-0.11	0.07*	-0.29	-0.12	-0.08	-0.00	0.03	0.07**
EP	0.00	0.04	-0.03	-0.03	0.02	0.05	-0.05	-0.00
DE	-0.07	-0.00	0.05	0.03	-0.04	0.04	-0.02	0.03
RVOL	-0.01	0.05**	0.02	0.01	-0.05	0.02	-0.03	-0.01
BM	-0.05	0.02	-0.21	-0.19	-0.00	0.02	-0.05	-0.06
NTIS	0.05	-0.02	0.08	0.05*	0.05	-0.04	-0.06	-0.01
TBL	0.11	-0.04	0.06	-0.03	0.16	0.08	0.03	-0.06
LTY	0.05	-0.07	0.24***	0.09	0.14	-0.04	-0.01	-0.07
LTR	0.12*	0.09*	0.02	0.04	0.11	0.08	0.03	0.02
TMS	0.22**	0.08	-0.09	-0.05	0.04	0.07	0.11	0.02
DFY	-0.03	-0.02	-0.05	-0.02	0.01	0.06	-0.16	-0.08
DFR	0.03	0.00	-0.03	-0.03	0.01	0.06	0.05	0.04
INFL	0.02	-0.05	0.03	0.07***	0.15*	0.04	0.12	-0.03
CF-ECON$^{\text{MEAN}}$	0.09**	0.04	-0.07	-0.08	0.15**	0.06	0.02	0.01
CF-ECON$^{\text{WEIG}}_{\theta=1}$	0.11**	0.06	-0.03	-0.05	0.15**	0.07	0.02	0.01
CF-ECON$^{\text{WEIG}}_{\theta=0.9}$	0.11**	0.07*	-0.03	-0.05	0.15**	0.07	0.03	0.02
				Panel B: Technical indicators				
MA(1,9)	0.08	0.11**	-0.19	-0.10	0.19	0.26**	-0.03	0.00
MA(1,12)	0.16*	0.13*	-0.15	-0.08	0.27**	0.22**	0.01	0.01
MA(2,9)	0.10	0.12**	-0.16	-0.10	0.20	0.25**	0.00	0.01
MA(2,12)	0.17*	0.14*	-0.12	-0.07	0.28**	0.22**	-0.01	0.02
MA(3,9)	0.13	0.14**	-0.12	-0.07	0.19*	0.22**	0.00	0.00
MA(3,12)	0.05	0.06	-0.12	-0.07	0.20*	0.17**	-0.01	0.01
MOM(9)	0.06	0.05	-0.09	-0.06	0.25**	0.17**	-0.01	0.01
MOM(12)	0.06	0.04	-0.09	-0.01	0.16**	0.15**	-0.00	0.04
VOL(1,9)	0.06	0.10**	-0.06	-0.03	0.21*	0.20**	0.10	0.08
VOL(1,12)	0.11	0.12**	-0.07	-0.05	0.32***	0.22**	0.08	0.07
VOL(2,9)	0.07	0.07*	-0.11	-0.06	0.27**	0.21**	0.03	0.02
VOL(2,12)	0.08*	0.06*	-0.12	-0.07	0.29**	0.19**	0.08	0.08
VOL(3,9)	0.04	0.07**	-0.11	-0.06	0.26**	0.22**	0.08	0.07
VOL(3,12)	0.13*	0.12**	-0.11	-0.08	0.26**	0.18**	0.05	0.06
CF-TECH$^{\text{MEAN}}$	0.10*	0.10**	-0.12	-0.08	0.30**	0.24**	0.04	0.03
CF-TECH$^{\text{WEIG}}_{\theta=1}$	0.10*	0.10**	-0.12	-0.08	0.30**	0.24**	0.04	0.03
CF-TECH$^{\text{WEIG}}_{\theta=0.9}$	0.10*	0.10**	-0.12	-0.08	0.30**	0.25**	0.04	0.04
				Panel C: All predictors taken together				
CF-ALL$^{\text{MEAN}}$	0.09**	0.07**	-0.11	-0.09	0.31***	0.21***	0.03	0.02
CF-ALL$^{\text{WEIG}}_{\theta=1}$	0.10**	0.08**	-0.09	-0.07	0.32***	0.22***	0.04	0.02
CF-ALL$^{\text{WEIG}}_{\theta=0.9}$	0.10**	0.08**	-0.09	-0.07	0.31***	0.22***	0.04	0.02

Table A.4

Out-of-sample R^2 statistics (in %) - 1990:1 to 2017:12

This table presents statistics on the out-of-sample predictability of one month ahead log excess returns on the S&P 500 index. Panel A (Panel B) shows results for economic variables (technical indicators). In addition to the individual forecasts, I display results for three different combination forecasting methods. Panel C shows results when combining both sets of predictors. For each model the out-of-sample R^2 (in %) is displayed (Campbell and Thompson, 2008). *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively according to the Clark and West (2007) MSFE-adjusted statistic. The null hypothesis is equal MSFE and the alternative is that the more sophisticated model has smaller MSFE than the historical mean benchmark. Column (1) shows the respective predictor and column (2) shows results for the unadjusted series. Columns (3) to (9) present results for the frequency-decomposed predictors. D_H refers to components with periodicities between 2 to 16 months, D_M refers to components with periodicities between 16 to 64 months, and D_L captures oscillations above 64 months. The superscript NL indicates that the nonlinear forecasting model is applied in columns (3), (5), (7), and (9).

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
				1990:1 to 2017:12				
Predictor	Basic	Basic^{NL}	D_H	D_H^{NL}	D_M	D_M^{NL}	D_L	D_L^{NL}
				Panel A: Economic variables				
DP	-1.71	0.79**	-44.16	-25.71	-0.49	-0.18	-0.48	0.24
DY	-1.87	1.04**	-27.84	-10.93	-2.01	-0.53	-0.49	0.29
EP	-0.58	-0.27	-39.36*	-21.91	-0.66	0.15	-0.85	-0.29
DE	-2.04	-0.25	-24.28	-13.21	-4.03	-0.63	-0.57	0.25
RVOL	-0.52	0.08	-3.27	-1.76	-1.26	-0.58	-0.92	-0.55
BM	-0.41	0.17	-16.75	-8.07	0.01	0.16*	-0.59	-0.44
NTIS	-1.86	-1.36	0.10	-0.06	-4.89	-3.59	-0.81	0.05
TBL	-0.57	-0.86	-1.14	-1.16	-6.83	-3.17	0.21	-0.16
LTY	0.11	-0.34	-0.37	-0.58	-0.71	-0.47	0.06	-0.18
LTR	-0.52	0.21	-0.32	0.36	-2.14	-0.63	0.15	0.01
TMS	-1.46	-0.51	-1.03	-0.67	-7.04	-2.62	0.26*	0.21**
DFY	-0.85	-0.52	-13.21	-11.02	-6.05	-2.72	-1.04	-0.47
DFR	-0.49	-0.53	-0.83	-0.81	-7.84	0.69	-0.10	-0.01
INFL	-1.17	-1.40	0.85**	1.30***	-1.03	-0.60	0.55*	-0.02
CF-ECON^{MEAN}	-0.31	-0.13	-2.19	-2.02	-1.07	-0.50	-0.06	0.01
CF-ECON$^{WEIG}_{\theta=1}$	-0.27	-0.11	-1.32	-1.18	-0.95	-0.36	-0.06	0.00
CF-ECON$^{WEIG}_{\theta=0.9}$	-0.28	-0.11	-1.28	-1.23	-0.99	-0.36	-0.06	0.01
				Panel B: Technical indicators				
MA(1,9)	0.69*	0.71*	-4.66	-1.56	-1.36*	3.27**	-6.71	-1.87
MA(1,12)	0.77	0.96*	-2.75	-1.15	0.49*	2.57**	-7.22	-2.27
MA(2,9)	0.24	0.87*	-3.31	-1.62	-1.29	2.65**	-5.73	-1.76
MA(2,12)	0.81	1.04*	-2.09	-0.89	0.91**	2.48**	-6.61	-2.10
MA(3,9)	-0.16	0.62	-1.45	-0.86	-0.39	2.47**	-5.25	-1.97
MA(3,12)	0.01	0.38	-1.40	-0.79	0.90*	2.09**	-6.22	-2.32
MOM(9)	0.37	0.53	-2.10	-1.14	1.46**	2.15**	-5.69	-2.07
MOM(12)	0.43	0.48	-2.25	-1.12	1.12*	1.50**	-5.24	-1.87
VOL(1,9)	0.29	0.54	-1.30	-0.76	-2.38*	1.71*	-0.72	-0.17
VOL(1,12)	0.59	0.67	-1.23	-0.77	0.15**	1.90*	-1.11	-0.52
VOL(2,9)	0.25	0.37	-1.01	-0.35	-1.08*	1.56*	-1.04	-0.63
VOL(2,12)	0.90*	0.54	-2.62	-0.93	0.63**	1.59*	-0.44	-0.52
VOL(3,9)	0.14	0.34	-2.09	-0.77	-0.96	1.77*	-0.93	-0.41
VOL(3,12)	1.11*	0.86*	-1.71	-1.05	0.71**	1.61*	-0.74	-0.56
CF-TECH^{MEAN}	0.59	0.66	-1.15	-0.83	0.69*	2.25**	-3.20	-1.25
CF-TECH$^{WEIG}_{\theta=1}$	0.60	0.66	-1.12	-0.82	0.65*	2.25**	-3.12	-1.25
CF-TECH$^{WEIG}_{\theta=0.9}$	0.60	0.66	-1.12	-0.83	0.65*	2.25**	-3.10	-1.23
				Panel C: All predictors taken together				
CF-ALL^{MEAN}	0.24	0.30	-1.41	-1.35	0.70*	1.24**	-0.56	-0.34
CF-ALL$^{WEIG}_{\theta=1}$	0.26	0.31	-1.02	-0.95	0.78*	1.37**	-0.52	-0.39
CF-ALL$^{WEIG}_{\theta=0.9}$	0.26	0.32	-0.99	-0.97	0.75*	1.38**	-0.49	-0.35

Table A.5

Annualized gains in CER - 1990:1 to 2017:12

This table reports the annualized gain in certainty equity return (CER) relative to the CER from the historical mean (in percent). ΔCER is estimated for a mean-variance investor with a relative risk aversion of five who allocates each month between the S&P 500 index and the risk-free rate. The optimal weight is estimated according to forecasts of one-month ahead excess returns from predictive regression models. The optimal weight in risky assets is constrained to lie between 0 and 1.5. The out-of-sample period runs from 1990:1 to 2017:12.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
				1990:1 to 2017:12				
Predictor	Basic	Basic$^{\text{NL}}$	D_H	D_H^{NL}	D_M	D_M^{NL}	D_L	D_L^{NL}
				Panel A: Economic variables				
DP	-1.86	1.73**	-3.46	-1.48	-0.32	-0.15	-0.40	0.84
DY	-1.58	2.18**	-3.85	-0.52	-2.31	-0.45	-0.38	1.00
EP	1.77	2.25*	3.15	2.56*	0.53	0.97*	-0.72	0.12
DE	-2.05	-0.19	-3.32	-1.68	-2.07	0.38	-0.19	0.58
RVOL	-2.42	-0.60	-0.86	-0.69	-1.38	-0.44	-0.66	-0.46
BM	-0.53	0.70	-4.39	-1.51	-0.24	-0.04	-0.47	-0.37
NTIS	-0.62	-0.59	0.55	0.06	-3.02	-2.01	0.47	1.33
TBL	-0.00	-0.84	-1.39	-1.28	-3.18	-0.88	0.27	-0.34
LTY	-0.04	-0.47	-0.33	-0.34	-0.84	-0.84	-0.12	-0.29
LTR	-0.70	0.28	-0.47	0.21	0.02	0.34	0.15	-0.02
TMS	-0.44	-0.53	-1.26	-0.70	-3.81	-0.77	0.79**	0.13
DFY	-1.16	-0.72	-1.59	-1.13	-3.69	-0.54	0.02	0.57
DFR	0.42	0.47	-1.12	-1.05	-0.76	1.28	1.83	1.90
INFL	-1.07	-1.13	1.83	2.21***	1.30	1.54	0.80	0.16
CF-ECON$^{\text{MEAN}}$	-0.59	-0.29	-1.17	-0.81	-1.17	-0.71	0.47	0.55
CF-ECON$_{\theta=1}^{\text{WEIG}}$	-0.51	-0.24	-0.34	-0.21	-0.92	-0.49	0.46	0.55
CF-ECON$_{\theta=0.9}^{\text{WEIG}}$	-0.53	-0.24	-0.33	-0.26	-0.89	-0.47	0.47	0.56
				Panel B: Technical indicators				
MA(1,9)	2.06*	2.04*	-2.47	-0.37	3.11	3.77**	1.97	2.47
MA(1,12)	3.20*	3.19*	-1.14	-0.35	3.34*	3.44*	1.75	2.19
MA(2,9)	1.94	2.67*	-1.75	-0.46	1.97	3.46**	1.97	2.21
MA(2,12)	3.20*	3.28*	-0.89	-0.24	3.41*	3.39**	1.83	2.17
MA(3,9)	1.69	2.42*	-0.94	-0.29	2.12	2.98*	1.56	1.94
MA(3,12)	1.52	1.82	-0.12	-0.20	2.72	2.91*	1.54	1.97
MOM(9)	2.07	2.08	-0.33	-0.47	3.34*	2.97*	1.34	2.05
MOM(12)	1.93	1.96	-0.95	-0.59	2.41	2.91*	1.76	2.02
VOL(1,9)	1.55	1.94*	-0.45	-0.21	3.38	3.25*	2.89*	2.51
VOL(1,12)	2.39	2.63*	-0.61	-0.13	4.76***	3.40**	2.52	2.44
VOL(2,9)	1.39	1.83	-0.64	-0.11	3.21	2.66	2.23	2.20
VOL(2,12)	2.11	2.06*	-1.09	-0.43	4.08**	2.75*	2.76*	2.18
VOL(3,9)	1.02	1.34	-0.94	-0.13	2.84	2.86	2.22*	2.25
VOL(3,12)	2.80*	2.75*	-0.70	-0.42	3.90**	2.83*	2.71*	2.24
CF-TECH$^{\text{MEAN}}$	2.14	2.25*	-0.37	-0.21	3.57*	3.26**	2.23	2.21
CF-TECH$_{\theta=1}^{\text{WEIG}}$	2.15	2.26*	-0.33	-0.20	3.55*	3.27**	2.25	2.23
CF-TECH$_{\theta=0.9}^{\text{WEIG}}$	2.15	2.26*	-0.33	-0.20	3.56*	3.27**	2.25	2.23
				Panel C: All predictors taken together				
CF-ALL$^{\text{MEAN}}$	0.96	0.99	-1.02	-1.01	2.98*	2.96**	2.18	1.89
CF-ALL$_{\theta=1}^{\text{WEIG}}$	1.04	1.05	-0.79	-0.75	2.95*	3.01**	2.17	1.87
CF-ALL$_{\theta=0.9}^{\text{WEIG}}$	1.04	1.06	-0.83	-0.80	2.95*	3.01**	2.19	1.89

Table A.6

Annualized gains in SR - 1990:1 to 2017:12

This table reports the annualized gain in the Sharpe ratio (SR) relative to the SR from the historical mean. ΔSR is estimated for a mean-variance investor with a relative risk aversion of five who allocates each month between the S&P 500 index and the risk-free rate. The optimal weight is estimated according to forecasts of one-month ahead excess returns from predictive regression models. The optimal weight in risky assets is constrained to lie between 0 and 1.5. The out-of-sample period runs from 1990:1 to 2017:12.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
				1990:1 to 2017:12				
Predictor	Basic	Basic$^{\text{NL}}$	D_H	D_H^{NL}	D_M	D_M^{NL}	D_L	D_L^{NL}
				Panel A: Economic variables				
DP	-0.32	0.14**	-0.17	-0.07	-0.04	-0.01	-0.05	0.06
DY	-0.28	0.18**	-0.22	-0.01	-0.20	-0.01	-0.05	0.07
EP	0.19*	0.19*	0.24	0.21*	0.04	0.07*	-0.06	0.00
DE	-0.20	-0.01	-0.19	-0.09	-0.13	0.03	-0.04	0.04
RVOL	-0.15	-0.02	-0.05	-0.03	-0.08	-0.01	-0.05	-0.03
BM	-0.07	0.05	-0.26	-0.10	-0.02	0.00	-0.00	-0.03
NTIS	-0.04	-0.04	0.04	0.01	-0.22	-0.13	0.03	0.10
TBL	0.04	-0.05	-0.09	-0.08	-0.15	-0.03	0.04*	-0.02
LTY	0.02	-0.03	-0.01	-0.02	-0.02	-0.04	-0.00	-0.02
LTR	-0.04	0.03	-0.03	0.02	0.02	0.03	0.02	0.00
TMS	0.00	-0.03	-0.10	-0.05	-0.23	-0.03	0.07**	0.01
DFY	-0.10	-0.05	-0.09	-0.07	-0.22	-0.02	-0.00	0.04
DFR	0.03	0.03	-0.08	-0.07	-0.07	0.10	0.14	0.15
INFL	-0.06	-0.08	0.14	0.17***	0.10	0.12	0.08*	0.01
CF-ECON$^{\text{MEAN}}$	-0.06	-0.02	-0.07	-0.04	-0.07	-0.03	0.04	0.04
CF-ECON$_{\theta=1}^{\text{WEIG}}$	-0.05	-0.02	-0.03	-0.01	-0.06	-0.02	0.03	0.04
CF-ECON$_{\theta=0.9}^{\text{WEIG}}$	-0.05	-0.02	-0.03	-0.01	-0.06	-0.02	0.04	0.04
				Panel B: Technical indicators				
MA(1,9)	0.17*	0.17**	-0.15	-0.03	0.25	0.33**	0.16	0.21
MA(1,12)	0.27*	0.28**	-0.06	-0.02	0.27	0.31**	0.14	0.18
MA(2,9)	0.16	0.23**	-0.11	-0.03	0.16	0.31	0.16**	0.18
MA(2,12)	0.27*	0.29**	-0.05	-0.02	0.29*	0.30**	0.15	0.18
MA(3,9)	0.13	0.20*	-0.05	-0.02	0.17	0.26*	0.13	0.15
MA(3,12)	0.12	0.15	0.01	-0.01	0.22	0.25*	0.13	0.16
MOM(9)	0.17	0.17*	-0.01	-0.03	0.28*	0.26*	0.12	0.17
MOM(12)	0.16	0.16*	-0.06	-0.04	0.20	0.25*	0.15	0.16
VOL(1,9)	0.12	0.16*	-0.01	-0.01	0.27	0.28*	0.23	0.21
VOL(1,12)	0.20	0.22*	-0.03	-0.01	0.40**	0.30**	0.20	0.20
VOL(2,9)	0.11	0.15	-0.03	-0.00	0.26	0.22	0.18	0.18
VOL(2,12)	0.17	0.17*	-0.06	-0.03	0.33**	0.23*	0.22	0.18
VOL(3,9)	0.08	0.10*	-0.05	-0.01	0.23	0.24*	0.18	0.18
VOL(3,12)	0.23*	0.23**	-0.05	-0.03	0.32**	0.24*	0.21	0.18
CF-TECH$^{\text{MEAN}}$	0.17	0.19*	-0.01	-0.01	0.30*	0.28**	0.18	0.18
CF-TECH$_{\theta=1}^{\text{WEIG}}$	0.18	0.19*	-0.01	-0.01	0.30*	0.29**	0.18	0.18
CF-TECH$_{\theta=0.9}^{\text{WEIG}}$	0.18	0.19*	-0.01	-0.01	0.30*	0.29**	0.18	0.18
				Panel C: All predictors taken together				
CF-ALL$^{\text{MEAN}}$	0.07	0.07*	-0.06	-0.06	0.25*	0.25**	0.17	0.15
CF-ALL$_{\theta=1}^{\text{WEIG}}$	0.08	0.08*	-0.05	-0.05	0.25*	0.26**	0.17	0.15
CF-ALL$_{\theta=0.9}^{\text{WEIG}}$	0.08	0.08*	-0.05	-0.05	0.25*	0.26**	0.17	0.15

Table A.7

Out-of-sample R^2 statistics (in %) - principal component analysis

This table presents statistics on the out-of-sample predictability of one month ahead log excess returns on the S&P 500 index. Panel A (Panel B) shows results for economic variables (technical indicators). Panel C shows results for all predictors taken together. I display results for the first, second, and third principal component. For each model the out-of-sample R^2 (in %) is displayed (Campbell and Thompson, 2008). *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively according to the Clark and West (2007) MSFE-adjusted statistic. The null hypothesis is equal MSFE and the alternative is that the more sophisticated model has smaller MSFE than the historical mean benchmark. Column (1) shows the respective predictor and column (2) shows results for the unadjusted series. Columns (3) to (9) present results for the frequency-decomposed predictors. D_H refers to components with periodicities between 2 to 16 months, D_M refers to components with periodicities between 16 to 64 months, and D_L captures oscillations above 64 months. The superscript NL indicates that the nonlinear forecasting model is applied in columns (3), (5), (7), and (9).

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
				1966:1 to 2017:12				
Predictor	Basic	Basic^{NL}	D_H	D_H^{NL}	D_M	D_M^{NL}	D_L	D_L^{NL}
				Panel A: Economic variables				
PC-ECON$_1$	1.24***	0.75**	-29.85	-15.51	0.16**	0.62*	-0.48	-0.81
PC-ECON$_2$	-6.80	-1.98	-33.10	-12.46	-12.49	-4.12	-1.59	-0.83
PC-ECON$_3$	-15.37	-5.25	-9.75	-3.58	-3.69	-1.92	-3.32	0.35
				Panel B: Technical indicators				
PC-TECH$_1$	0.52*	0.88**	-5.56	-3.41	-0.04***	2.04***	-6.86	-3.34
PC-TECH$_2$	0.24	0.39	-0.17	0.65***	-2.96**	1.41***	-0.00*	-0.95
PC-TECH$_3$	-5.64	-3.56	-0.45	-0.21	-188.19	-74.03	-64.72	-28.94
				Panel C: All predictors taken together				
PC-ALL$_1$	0.35	0.52	-11.77	-6.28	-0.22**	1.57**	-5.76	-3.06
PC-ALL$_2$	-3.16	-1.25	-3.13	-0.70	-18.43	-8.67	-3.01	-0.96
PC-ALL$_3$	-2.75	-1.43	-10.20	-1.62*	-3.27	-0.37	-1.63	-0.48

Table A.8

Annualized gains in CER - principal component analysis

This table reports the annualized gain in certainty equity return (CER) relative to the CER from the historical mean (in percent). ΔCER is estimated for a mean-variance investor with a relative risk aversion of five who allocates each month between the S&P 500 index and the risk-free rate. The optimal weight is estimated according to forecasts of one-month ahead excess returns from predictive regression models. The optimal weight in risky assets is constrained to lie between 0 and 1.5 to prevent shorting stocks and leveraging more than 50%. The out-of-sample period runs from 1966:1 to 2017:12.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
				1966:1 to 2017:12				
Predictor	Basic	Basic$^{\text{NL}}$	D_H	D_H^{NL}	D_M	D_M^{NL}	D_L	D_L^{NL}
				Panel A: Economic variables				
PC-ECON$_1$	2.09**	1.23**	-2.24	-0.75	1.04	0.64	1.18	0.11
PC-ECON$_2$	-0.65	0.40	-2.34	-0.60	-0.52	-0.22	0.54	-0.12
PC-ECON$_3$	-2.53	-1.23	-1.00	-0.22	-0.48	0.37	-0.22	2.27**
				Panel B: Technical indicators				
PC-TECH$_1$	2.07*	2.35***	-1.49	-1.03	3.75***	3.00***	0.33	1.22
PC-TECH$_2$	1.57*	1.32**	0.40	0.76***	3.56**	2.99***	1.43	0.31
PC-TECH$_3$	-2.37	-2.28	-0.26	-0.02	-3.09	-1.13	1.97	1.72
				Panel C: All predictors taken together				
PC-ALL$_1$	1.94*	2.03**	-2.35	-2.02	3.26**	2.68**	0.36	1.10
PC-ALL$_2$	0.00	1.15	-0.74	-0.06	-1.00	-1.24	-1.94	0.15
PC-ALL$_3$	1.36	1.15*	-0.36	0.47	-0.58	-0.06	-0.03	-0.28

Table A.9

Annualized gains in SR - principal component analysis

This table reports the annualized gain in the Sharpe ratio (SR) relative to the SR from the historical mean. ΔSR is estimated for a mean-variance investor with a relative risk aversion of five who allocates each month between the S&P 500 index and the risk-free rate. The optimal weight is estimated according to forecasts of one-month ahead excess returns from predictive regression models. The optimal weight in risky assets is constrained to lie between 0 and 1.5 to prevent shorting stocks and leveraging more than 50%. The out-of-sample period runs from 1966:1 to 2017:12.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
				1966:1 to 2017:12				
Predictor	Basic	Basic$^{\text{NL}}$	D_H	D_H^{NL}	D_M	D_M^{NL}	D_L	D_L^{NL}
				Panel A: Economic variables				
PC-ECON$_1$	0.14**	0.09*	-0.05	-0.02	0.11*	0.07*	0.06	-0.02
PC-ECON$_2$	-0.00	0.04	-0.07	-0.02	-0.02	-0.00	0.04	-0.01
PC-ECON$_3$	-0.14	-0.06	-0.01	0.02	-0.04	0.03	-0.09	0.17**
				Panel B: Technical indicators				
PC-TECH$_1$	0.15*	0.17**	-0.06	-0.08	0.29***	0.23***	0.07	0.07
PC-TECH$_2$	0.10	0.08*	0.02	0.06***	0.28***	0.23***	0.11	0.01
PC-TECH$_3$	-0.15	-0.10	-0.01	0.00	-0.10	-0.02	0.16	0.13
				Panel C: All predictors taken together				
PC-ALL$_1$	0.14*	0.14**	-0.09	-0.11	0.26**	0.20**	0.06	0.06
PC-ALL$_2$	-0.03	0.07	-0.02	-0.00	-0.04	-0.07	-0.13	0.01
PC-ALL$_3$	0.12	0.09*	0.02	0.05	-0.00	0.01	0.07	0.02

Table A.10

Results for different wavelet filters from combined forecasts of medium-frequency components

This table presents results on the out-of-sample performance of combination forecasts from economic variables and technical indicators for different wavelets. Panel A presents results for R^2_{OS}, whereas Panel B (Panel C) shows the annualized ΔCER (monthly ΔSR). The respective wavelets are the Daubechies wavelet with width 4 and 8 (D(4) and D(8)), the Fejér-Korovkin wavelet with width 4 (FK(4)), and the least asymmetric wavelet with width 16 (LA(16)).The out-of-sample period runs from 1966:1 to 2017:12.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
				Panel A: R^2_{OS}				
	D_M				D_M^{NL}			
Predictor	D(4)	D(8)	FK(4)	LA(16)	D(4)	D(8)	FK(4)	LA(16)
CF-ECON$^{\text{MEAN}}$	1.13***	1.02***	1.26***	0.95***	0.39	0.34	0.43	0.32
CF-TECH$^{\text{MEAN}}$	0.29***	0.19***	0.33***	0.07***	1.93***	1.85***	1.97***	1.76***
CF-ALL$^{\text{MEAN}}$	1.53***	1.43***	1.60***	1.32***	1.46***	1.38***	1.49***	1.31***
				Panel B: ΔCER				
	D_M							
Predictor	D(4)	D(8)	FK(4)	LA(16)				
CF-ECON$^{\text{MEAN}}$	2.07**	1.93**	2.23**	1.91**				
CF-TECH$^{\text{MEAN}}$	3.93***	3.84***	3.83***	3.70***				
CF-ALL$^{\text{MEAN}}$	4.31***	4.07***	4.34***	3.85***				
				Panel C: ΔSR				
	D_M							
Predictor	D(4)	D(8)	FK(4)	LA(16)				
CF-ECON$^{\text{MEAN}}$	0.05**	0.05**	0.05***	0.05**				
CF-TECH$^{\text{MEAN}}$	0.09***	0.09***	0.09***	0.08***				
CF-ALL$^{\text{MEAN}}$	0.10***	0.09***	0.10***	0.09***				

Table A.11

Relative average turnover

This table reports the average turnover of a portfolio based on a sophisticated forecasting model relative to the average turnover of a portfolio based on the historical mean forecast. Turnover is defined as the percentage of wealth traded at the end of each period. The out-of-sample period runs from 1966:1 to 2017:12.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
				1966:1 to 2017:12				
Predictor	Basic	Basic^{NL}	D_H	D_H^{NL}	D_M	D_M^{NL}	D_L	D_L^{NL}
				Panel A: Economic variables				
DP	2.03	2.45	16.87	4.54	1.77	1.62	1.56	1.77
DY	2.79	2.90	16.92	4.39	2.66	2.26	1.62	1.83
EP	1.63	2.08	13.21	4.55	2.44	2.01	1.46	1.62
DE	2.02	2.21	3.86	2.34	2.93	3.13	1.65	1.87
RVOL	4.07	2.44	11.44	3.51	3.08	2.06	1.87	1.57
BM	2.32	2.40	15.37	4.61	1.87	1.77	1.68	1.82
NTIS	3.09	2.22	6.58	1.94	3.30	3.03	2.16	1.59
TBL	1.44	2.52	6.16	3.37	2.93	3.51	1.13	2.46
LTY	1.03	2.53	10.46	4.21	3.14	2.75	1.10	2.41
LTR	22.89	6.12	20.18	5.92	10.83	4.20	3.34	2.14
TMS	4.15	3.79	5.90	2.44	4.03	3.45	2.21	2.45
DFY	2.52	2.13	11.84	4.34	3.22	3.01	1.33	2.12
DFR	10.04	4.30	8.25	4.12	12.45	5.08	5.67	2.76
INFL	7.59	3.52	5.39	1.74	8.01	3.40	1.59	2.41
CF-ECON^{MEAN}	3.94	2.47	9.05	3.51	3.56	2.96	1.94	1.99
CF-ECON$^{WEIG}_{\theta=1}$	4.21	2.59	8.54	3.41	3.95	3.03	2.04	2.02
CF-ECON$^{WEIG}_{\theta=0.9}$	4.32	2.63	8.42	3.35	4.21	3.08	2.11	2.03
				Panel B: Technical indicators				
MA(1,9)	4.30	2.34	8.44	2.82	5.75	3.17	3.51	2.92
MA(1,12)	4.00	2.54	5.95	2.60	4.12	2.97	3.24	2.97
MA(2,9)	4.39	2.30	7.37	2.65	5.01	2.98	3.06	2.91
MA(2,12)	3.74	2.47	4.94	2.31	3.83	2.74	3.03	2.97
MA(3,9)	4.54	2.62	5.98	2.49	4.91	2.99	3.14	3.01
MA(3,12)	2.73	2.10	5.56	2.61	3.68	2.62	3.11	2.96
MOM(9)	2.62	1.82	6.53	2.76	3.48	2.50	3.09	2.97
MOM(12)	2.38	1.72	5.56	2.20	2.78	1.99	2.72	2.75
VOL(1,9)	5.67	2.33	11.36	3.25	5.71	3.65	3.05	2.76
VOL(1,12)	5.37	2.23	8.57	2.47	4.02	3.01	2.81	2.78
VOL(2,9)	3.70	1.94	7.67	2.66	4.93	3.22	3.21	2.88
VOL(2,12)	2.74	1.76	5.83	2.52	4.04	2.97	2.60	2.60
VOL(3,9)	2.91	1.83	6.91	2.76	4.60	3.25	2.88	2.73
VOL(3,12)	3.24	2.14	4.94	2.34	3.75	2.74	2.77	2.79
CF-TECH^{MEAN}	3.20	2.01	6.36	2.60	4.11	2.75	2.70	2.67
CF-TECH$^{WEIG}_{\theta=1}$	3.21	2.02	6.45	2.61	4.09	2.74	2.73	2.69
CF-TECH$^{WEIG}_{\theta=0.9}$	3.22	2.02	6.47	2.61	4.11	2.74	2.75	2.69
				Panel C: All predictors taken together				
CF-ALL^{MEAN}	2.95	2.03	7.61	3.07	3.63	2.79	2.40	2.46
CF-ALL$^{WEIG}_{\theta=1}$	3.14	2.13	7.13	2.89	3.79	2.82	2.51	2.47
CF-ALL$^{WEIG}_{\theta=0.9}$	3.24	2.17	7.09	2.88	3.94	2.87	2.57	2.46

Table A.12

Annualized gains in CER after transaction costs

This table reports the annualized gain in certainty equity return (CER) relative to the CER from the historical mean (in percent). Results are net of a proportional transaction cost of 50 basis points per transaction. ΔCER is estimated for a mean-variance investor with a relative risk aversion of five who allocates each month between the S&P 500 index and the risk-free rate. The optimal weight is estimated according to forecasts of one-month ahead excess returns from predictive regression models. The optimal weight in risky assets is constrained to lie between 0 and 1.5 to prevent shorting stocks and leveraging more than 50%. The out-of-sample period runs from 1966:1 to 2017:12.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
				1966:1 to 2017:12				
Predictor	Basic	Basic^{NL}	D_H	D_H^{NL}	D_M	D_M^{NL}	D_L	D_L^{NL}
				Panel A: Economic variables				
DP	-0.78	0.35	-4.85	-3.58	0.09	0.05	0.57	0.84**
DY	-0.46	0.83	-5.97	-2.93	-1.54	-0.71	0.62	0.95**
EP	0.17	0.31	-2.22	-0.91	0.09	0.70	-0.31	-0.00
DE	-0.46	0.02	-0.47	-0.68	-1.26	-0.10	0.17	0.44
RVOL	-1.42	-0.17	-1.77	-0.96	-1.71	-0.70	-0.36	-0.06
BM	-1.41	-0.32	-5.10	-3.47	0.04	0.05	-0.69	-0.37
NTIS	-0.13	-1.06	0.05	0.00	-0.29	-1.81	-0.67	0.50
TBL	1.74	-0.21	0.49	-0.64	0.87	0.37	1.51	-0.20
LTY	1.65	-0.22	1.90**	0.63	2.12*	-0.20	1.14	-0.30
LTR	-1.98	0.16	-2.29	-0.02	0.21	0.39	0.22	0.21
TMS	1.39	0.56	-1.87	-1.11	-0.99	0.35	0.94	0.38
DFY	-0.97	-0.35	-1.70	-0.86	-1.30	0.09	0.13	-0.13
DFR	-1.02	-0.25	-1.58	-0.78	-0.47	0.69	0.20	0.97
INFL	-0.58	-0.86	0.17	1.09**	0.89	0.59	1.60*	-0.11
CF-ECON^{MEAN}	1.34**	0.47	-2.44	-2.30	1.99**	0.44	0.87	0.45
CF-ECON$^{WEIG}_{\theta=1}$	1.41**	0.62	-1.82	-1.76	2.01**	0.64	0.89	0.49
CF-ECON$^{WEIG}_{\theta=0.9}$	1.40**	0.67	-1.77	-1.77	1.97**	0.67	0.90	0.51
				Panel B: Technical indicators				
MA(1,9)	1.09	1.79**	-2.78	-1.23	2.27	3.24***	-0.38	0.92
MA(1,12)	2.19*	2.36**	-2.21	-1.09	3.13**	2.71**	-0.47	0.84
MA(2,9)	1.29	2.24***	-2.53	-1.34	2.47*	3.10***	0.09	1.02
MA(2,12)	2.29*	2.51**	-1.94	-1.02	3.22**	2.66**	-0.32	0.85
MA(3,9)	1.70	2.37***	-2.07	-1.07	2.26	2.67**	-0.22	0.68
MA(3,12)	0.92	1.36*	-1.12	-0.99	2.50*	2.10**	-0.52	0.72
MOM(9)	1.01	1.20	-1.52	-1.03	2.81**	2.12**	-0.58	0.70
MOM(12)	0.95	1.12	-1.35	-0.62	1.76*	1.90**	-0.47	0.97
VOL(1,9)	0.54	1.48**	-1.88	-0.89	2.67*	2.81**	0.69	1.44
VOL(1,12)	1.19	2.11**	-1.90	-0.89	4.06***	2.90***	0.57	1.41
VOL(2,9)	0.52	1.32**	-2.05	-0.99	3.14**	2.57**	0.03	1.09
VOL(2,12)	0.66	1.07	-2.00	-1.12	3.34**	2.41**	0.54	1.35
VOL(3,9)	0.36	1.16**	-1.81	-0.95	2.80*	2.57**	0.37	1.23
VOL(3,12)	1.57	2.12**	-1.44	-0.94	2.98**	2.26**	0.28	1.13
CF-TECH^{MEAN}	1.29	1.72**	-1.68	-1.09	3.30**	2.82***	0.17	0.98
CF-TECH$^{WEIG}_{\theta=1}$	1.30	1.73**	-1.68	-1.09	3.30**	2.83***	0.19	1.00
CF-TECH$^{WEIG}_{\theta=0.9}$	1.30	1.73**	-1.69	-1.09	3.30**	2.83***	0.18	1.00
				Panel C: All predictors taken together				
CF-ALL^{MEAN}	1.46***	1.10**	-2.71	-2.37	3.96***	2.69***	0.53	1.00
CF-ALL$^{WEIG}_{\theta=1}$	1.52**	1.20**	-2.15	-1.83	3.81***	2.71***	0.54	1.03
CF-ALL$^{WEIG}_{\theta=0.9}$	1.53**	1.24**	-2.13	-1.81	3.76***	2.71***	0.56	1.07

Table A.13

Annualized gains in SR after transaction costs

This table reports the annualized gain in the Sharpe ratio (SR) relative to the SR from the historical mean. Results are net of a proportional transaction cost of 50 basis points per transaction. ΔSR is estimated for a mean-variance investor with a relative risk aversion of five who allocates each month between the S&P 500 index and the risk-free rate. The optimal weight is estimated according to forecasts of one-month ahead excess returns from predictive regression models. The optimal weight in risky assets is constrained to lie between 0 and 1.5 to prevent shorting stocks and leveraging more than 50%. The out-of-sample period runs from 1966:1 to 2017:12.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
				1966:1 to 2017:12				
Predictor	Basic	Basic^{NL}	D_H	D_H^{NL}	D_M	D_M^{NL}	D_L	D_L^{NL}
				Panel A: Economic variables				
DP	-0.12	0.05	-0.21	-0.16	-0.02	0.01	0.02	0.05*
DY	-0.11	0.08	-0.29	-0.11	-0.12	-0.01	0.02	0.06*
EP	-0.02	0.02	-0.07	-0.04	0.01	0.04	-0.03	-0.01
DE	-0.07	-0.00	0.02	-0.00	-0.01	0.02	-0.03	0.01
RVOL	-0.01	0.04	-0.07	-0.01	-0.02	0.01	-0.04	-0.01
BM	-0.09	-0.01	-0.26	-0.18	-0.00	0.01	-0.03	-0.04
NTIS	0.04	-0.07	-0.00	0.00	0.03	-0.08	-0.06	0.01
TBL	0.12	-0.03	0.05	-0.02	0.13	0.05	0.08	-0.04
LTY	0.09	-0.04	0.16**	0.05	0.17**	-0.03	0.04	-0.05
LTR	-0.08	0.03	-0.14	0.01	0.06	0.05	0.02	0.01
TMS	0.17**	0.05	-0.14	-0.07	0.00	0.06	0.09	0.00
DFY	-0.03	-0.00	-0.05	-0.01	-0.01	0.06	-0.04	-0.04
DFR	-0.09	-0.02	-0.11	-0.05	0.00	0.06	-0.01	0.04
INFL	-0.05	-0.08	0.01	0.08**	0.07	0.05	0.12*	-0.03
CF-ECON^{MEAN}	0.07*	0.02	-0.12	-0.10	0.16**	0.05	0.03	0.00
CF-ECON$^{WEIG}_{\theta=1}$	0.08*	0.03	-0.10	-0.08	0.16**	0.06	0.04	0.00
CF-ECON$^{WEIG}_{\theta=0.9}$	0.08*	0.04	-0.10	-0.08	0.16**	0.06	0.04	0.01
				Panel B: Technical indicators				
MA(1,9)	0.06	0.12**	-0.17	-0.09	0.19*	0.25***	0.01	0.04
MA(1,12)	0.16*	0.16**	-0.14	-0.08	0.24**	0.20**	0.02	0.03
MA(2,9)	0.08	0.15**	-0.16	-0.10	0.19*	0.23***	0.05	0.05
MA(2,12)	0.17*	0.18**	-0.12	-0.08	0.25**	0.19**	0.03	0.04
MA(3,9)	0.12	0.17 **	-0.14	-0.08	0.18*	0.19**	0.03	0.02
MA(3,12)	0.05	0.08*	-0.06	-0.07	0.19**	0.14*	0.02	0.03
MOM(9)	0.06	0.07	-0.09	-0.08	0.21**	0.14*	0.01	0.03
MOM(12)	0.05	0.06	-0.09	-0.04	0.12	0.12*	0.03	0.05
VOL(1,9)	0.02	0.09*	-0.10	-0.07	0.22*	0.21**	0.07	0.09
VOL(1,12)	0.08	0.15**	-0.12	-0.07	0.32***	0.21***	0.07	0.09
VOL(2,9)	0.03	0.08*	-0.13	-0.08	0.25**	0.19**	0.03	0.06
VOL(2,12)	0.04	0.06	-0.12	-0.09	0.26***	0.17**	0.07	0.08
VOL(3,9)	0.02	0.07**	-0.10	-0.07	0.22**	0.19**	0.06	0.07
VOL(3,12)	0.11	0.15**	-0.10	-0.07	0.24**	0.16**	0.05	0.06
CF-TECH^{MEAN}	0.08	0.11**	-0.10	-0.09	0.26**	0.21***	0.05	0.05
CF-TECH$^{WEIG}_{\theta=1}$	0.08	0.11**	-0.10	-0.09	0.26**	0.21***	0.06	0.05
CF-TECH$^{WEIG}_{\theta=0.9}$	0.08	0.11**	-0.10	-0.09	0.26**	0.21***	0.06	0.05
				Panel C: All predictors taken together				
CF-ALL^{MEAN}	0.09**	0.06**	-0.14	-0.12	0.31***	0.20***	0.05	0.04
CF-ALL$^{WEIG}_{\theta=1}$	0.09**	0.07**	-0.12	-0.10	0.30***	0.20***	0.05	0.05
CF-ALL$^{WEIG}_{\theta=0.9}$	0.10**	0.07**	-0.12	-0.10	0.29***	0.20***	0.05	0.05