Agency and Rising Volatility*

Mobeen Iqbal[§] **James Sefton**[†]

VERY PRELIMINARY DRAFT

PLEASE DO NOT CIRCULATE WITHOUT AUTHORS PERMISSION

*We would like to thank Marcin Kacpercyk for sharing data. The views expressed in this paper are those of the authors and do not necessarily reflect those of the authors' employers.

[§]Imperial College Business School; m.iqbal14@imperial.ac.uk.

[†]Imperial College Business School; j.sefton@imperial.ac.uk.

Agency and Rising Volatility

Abstract

We present a model of delegated money management in which benchmarked money managers, who report returns relative to a benchmark but also face a tracking error constraint, are forced to tilt their portfolios to low volatility stocks in periods of high volatility. The tilt means that low volatility, or low beta, stocks, become expensive and thereby have lower expected returns. When markets clear, this steepens the security market line (SML). We show, both through the model and empirically, that mutual funds have a tilt towards low volatility during stressed or turbulent markets, making low volatility stocks expensive and high volatility stocks cheap, and that mutual funds' betas fall during periods of high market volatility as they tilt towards low beta stocks. Therefore, low beta stocks' expected excess returns fall and high beta stocks' expected excess returns rise; equivalently, the SML steepens.

JEL Classification:

Keywords:

I. Introduction

Mutual funds typically have fixed-benchmark mandates that tend to discourage holdings in low volatility stocks. Additionally, the fact that high-risk assets tend to deliver lower expected returns on average compared to low-risk assets - or widely known as *the low volatility anomaly* that many in the finance literature have attempted to explain - goes against the very core of standard asset pricing theory: the risk-return trade-off implied by Sharpe (1964) and Lintner (1965)'s Capital Asset Pricing Model (CAPM). This can be seen by the fact that, empirically, the security market line (SML) is flatter on average compared to the average slope that the CAPM would yield, as originally documented by Black, Jensen, and Scholes (1972). However, the SML steepens during periods of high market volatility. In 'good' times, the SML is flatter than expected and can sometimes slope downwards; in 'bad' times, the SML is upward sloping and is steep (supported by evidence in several papers such as Hong and Sraer (2016) and Antoniou, Doukas, and Subrahmanyam (2016)). A large stream of academic studies, that we discuss in Section II below, has attempted to understand the drivers of the low volatility anomaly and the behaviour of the SML. This paper contributes to this literature by extending results found in Baker, Bradley, and Wurgler (2011). In their paper, the authors apply principles of behavioural finance and argue that there are two main channels through which the low volatility anomaly persists: (i) that investors are irrational and have a demand for lotteries, hence leading to higher demand for high volatility stocks; and (ii) that there exist limits to arbitrage that mean that "smart money" does not offset the price impact of irrational demand. In this paper, we present a model of delegated money management in which benchmarking acts as the main limit to arbitrage. Benchmarked money managers report returns relative to a benchmark and thus face a tracking error constraint. Baker, Bradley, and Wurgler (2011) show in a simple framework that this feature of delegated asset management means that these investors pass up the superior risk-return trade-off offered by low volatility portfolios on average. We extend the analysis by presenting a model in which higher aggregate market volatility leads to benchmarked money managers tilting towards low volatility stocks due to binding tracking error constraints. This tilt pushes the price of low volatility, or low beta, stocks up thereby depressing their expected returns. The opposite holds true for high volatility, or high beta, stocks. This causes the SML to steepen. In periods of low market volatil-

[0] The tracking error constraint can be calculated in many ways but broadly it is the standard deviation of differential returns between a portfolio and it's associated benchmark. See **?** or **?** for further details.

ity, however, these money managers aim to use up as much of their risk capacity as possible and thus tilt towards high volatility stocks. Therefore, in periods where markets are not turbulent, the low volatility anomaly persists due to benchmarked money managers' demand for high volatility stocks, encouraged by a non-binding tracking error constraint and the objective to maximise their information ratios. This paper, therefore, somewhat ties together the two channels introduced in Baker, Bradley, and Wurgler (2011) and shows how the slope of the SML fluctuates across periods of turbulent and non-turbulent financial markets.

We model two types of agents: (i) Type 1 (e.g. mutual funds) and (ii) Type 2 (e.g. hedge funds). The difference between the two types is that Type 1 investors are subject to a tracking error constraint measured against a benchmark index. The latter are not subject to such constraints and are mean-variance maximizers with fixed risk aversion. We determine equilibrium prices by imposing a market clearing condition and show that in a world without benchmarked money managers (i.e. no Type 1 investors), the slope of the SML is as implied by the CAPM; that is, high risk assets deliver a higher expected excess return. When Type 1 investors are added to the model, we find a similar expression for the SML to Baker, Bradley, and Wurgler (2011) and show that it is distorted by the amount of wealth managed by benchmarked mutual fund managers and the expected return to a minimum variance portfolio. Our model makes three main predictions: (i) as market volatility rises, the prices of volatile assets fall and thus the SML steepens more than implied by the CAPM; (ii) that as market volatility rises, beta dispersion increases; and finally, (iii) the rise in market volatility causes benchmarked money managers' tracking error constraints to bind and thus they are forced to tilt their portfolios toward low-beta assets in turn causing these assets' prices to rise and expected returns to fall. We provide empirical evidence to support our propositions. We do so in a simple way: we see that beta dispersion, as measured by the median absolute deviation of betas of the largest size quintile stocks, rises and falls with the VIX index. We also use the CRSP Mutual Fund Returns Survivor-Free database and regress fund betas on the VIX as well as other volatility indicators (such as the TED spread and the BAA-10 year corporate spread) and report regression coefficients with opposite signs for mutual funds and hedge funds. We find that the coefficient on the VIX is negative, statistically significant and economically meaningful for mutual funds, and positive, statistically significant, and roughly the same magnitude (in absolute terms) for hedge funds. This is an interesting, yet simple, empirical result and supports our model. It says that as market volatility rises, mutual funds tilt to low beta assets (thus their betas fall). But at the same time, hedge funds, who we assume have fixed risk aversion, absorb the extra risk stemming from higher aggregate market volatility. Although a simple result that future research can further explore, it shows a key difference between the two types of investors: benchmarked money managers' binding tracking error constraint forces a tilt to lower beta assets. We explore this feature further by studying mutual fund holdings to examine tilts to and away from low volatility, or low beta, stocks. We obtain holdings data from the Securities and Exchange Commission's 13F filings provided by Thomson Reuters. We construct a low volatility score for mutual funds and consider the weight of each stock in a hypothetical market portfolio. By calculating the relative difference in weights in low volatility stocks of mutual funds versus the market portfolio, we show that there is indeed a tilt to low volatility during times of high market volatility, further supporting our model's prediction and previous empirical result that higher market volatility is followed by a tilt to lower volatility assets by mutual funds. Our results shed light on a longstanding puzzle within empirical finance that has run contrary to the very core of asset pricing theory. The literature has not yet converged on a universal explanation of the low volatility anomaly and has offered many different channels of why the low volatility anomaly persists and how the slope of the SML behaves in various market episodes. We tie two promising channels that have been discussed: behavioural finance - or, the demand for high beta stocks during tranquil markets - and limits to arbitrage in a simple model and provide supporting empirical evidence.

II. Relevant literature

As discussed, there is a vast literature on attempting to explain the persistence of the low volatility anomaly and the slope of the SML. Black (1972) attempts to reconcile a flatter SML on average than that predicted by the CAPM by relaxing one of the CAPM's core assumptions of borrowing at the risk-free rate. He shows that borrowing constraints mean that risk-tolerant investors will demand high-beta stocks as they are unable to lever up the tangency portfolio. Hong and Sraer (2016) also provide a theory for the high-risk and low-return puzzle in which investors can borrow risk-free but by relaxing other CAPM assumptions of homogeneous expectations and cost-less short-selling and are able to show that the SML may be inverted or even downward sloping, i.e. that the deviation of the SML from the CAPM is higher in times of high aggregate disagreement. The authors argue that aggregate disagreement affects high-beta assets more than low-beta assets and therefore is subject to speculative overpricing. In good times, the overpricing of high-beta assets brings down their expected returns (flatter SML). In bad times, when aggregate disagreement is high, there is less overpricing of high-beta assets and so the SML steepens. Similarly, Cohen, Polk, and Vuolteenaho (2005) argue that the SML is flatter in times of high expected inflation and Antoniou, Doukas, and Subrahmanyam (2016) relate the slope of the SML to market sentiment; they find that the SML slopes upward in pessimistic sentiment periods and downwards in optimistic periods. Frazzini and Pedersen (2014) present a model in which investors face leverage constraints and show that a tighter leverage constraint - proxied by the TED spread in their case - results in a flatter SML. Jylha (2018) provides robust empirical evidence on margin-based constraints to support the impact of tightening leverage constraints on a flat SML. Frazzini and Pedersen (2014)'s paper constructs the now well-known 'betting against beta' (BAB) factor that is long leveraged low-beta assets and short high-beta assets. This factor produces significant positive risk-adjusted returns providing support to the existence of the low volatility anomaly. Huang, Lou, and Polk (2018) study the nature of beta arbitrage and show that, using measures of speculative capital committed to betting against beta introduced in Lou and Polk (2013), arbitrage activity in exploiting the low volatility anomaly generates booms and busts in the strategy's abnormal trading profits. This is to say that institutional demand of low-beta stocks forecasts a significant portion of the time-series variation in the excess return co-movement, relative to a benchmark asset pricing model of beta-arbitrage stocks; or, somewhat equivalently, that during periods of high-beta arbitrage activity, the short-term SML slopes downwards. Studying the cross-section of expected returns, Ang, Hodrick, Xing, and Zhang (2006) examine the price of aggregate volatility risk and argue that stocks with high sensitivities to innovations in aggregate volatility have low average returns.

There is a large literature on delegated asset management or, equivalently, agency and asset pricing. Brennan (1993) is perhaps the first to explore the implications of agency within asset pricing. He considers a static mean-variance setting with constant absolute risk aversion (CARA) utility agents who are compensated based on their performance relative to a benchmark index, and shows that in equilibrium expected returns are given by a two-factor model, with the two factors being the market and the index. Baker, Bradley, and Wurgler (2011) draw upon his model to argue that benchmarking is a limit to arbitrage and can help explain the low volatility anomaly. We take inspiration from their paper and use their model as a starting point. Their paper focuses on why institutional investors such as mutual funds do not arbitrage away the low volatility anomaly. Our model, on the other hand, focuses on understanding the interaction between benchmarking and the risk-bearing capacity of mutual funds. Vayanos and Woolley (2016) offer a discussion on the curses of benchmarking and point to the interaction of benchmarking and the low volatility anomaly but do not provide a formal model. Basak and Pavlova (2013) present a model in which institutional investors care about performance relative to a certain index (their benchmark) and find that institutions tilt their portfolios towards stocks that make up the benchmark. The tilt causes upward price pressure on index-held stocks. They also show that institutional investors' demand for riskier stocks (relative to retail investors) amplifies the index stock volatilities and aggregate market volatility. This supports our model's predictions. Benchmarks as investment mandates, as mentioned, typically discourage low volatility investments. In normal times, therefore, institutional investors such as mutual funds want to use up their risk capacity and so demand riskier stocks.

There is of course an incredibly large literature on risk-based constraints being a "limit to arbitrage" in general across many asset classes. Shleifer and Vishny (1997) provide an exhaustive discussion of factors that may limit institutional investors' ability to take advantage of anomalies. Most studies, however, consider the interaction of margin- or leverage-based constraints and asset prices. Whilst those constraints may certainly be at play, we argue that benchmarking plays a significant role in understanding the cross-section of risk versus return and the puzzle that the observed SML often deviates from the CAPM.

III. Model

Consider a discrete time, one-period economy with N risky assets and a risk-free asset with exogenously given return r_f. Suppose that payoffs at the end of period $t+1$ are given by x_{t+1} with mean μ and variance $\sigma^2 V$, where σ^2 is a volatility scalar and V is the covariance matrix of payoffs. Assume that there exists a stochastic discount factor (SDF) m_{t+1}. Then, the prices of the N risky assets are given by

$$p_t = E_t(m_{t+1}x_{t+1}) \tag{1}$$

For brevity, assume a unit share of each asset. There exist two agents in the economy: Type 1 investors (mutual funds) and Type 2 investors (hedge funds). The main difference between the two is that the former maximize expected return relative to a benchmark index. Therefore, mutual funds are subject to a tracking error constraint. This means that, as we shall see, mutual funds' risk aversion must rise as market volatility increases. Hedge funds, on the other hand, have a given level of risk aversion λ and are mean-variance maximizers without any portfolio constraints (abstract from any assumptions of use of leverage and frictions restricting access to financing). Denote the shares in the risky assets held by mutual funds as n_1 and the shares held by hedge funds as n_2. Markets clear such that $n_1 + n_2 = e$, where e is a vector of ones. The value of the market, then, is $e^\top p_t$. Mutual funds manage a percentage a_1 of this wealth such that $a_1 = \frac{n_1^\top p_t}{e^\top p_t}$. The remainder, a_2, is managed by hedge funds such that $a_2 = 1 - a_1 = \frac{n_2^\top p_t}{e^\top p_t}$. Let us denote by D_{p_t} the diagonal matrix with prices p_t on the diagonal (for brevity, we also write $D_{p_t}^{-1}$ as D_{1/p_t}). Given this notation, the return vector to the investment in the risky assets is

$$r_{t+1} = D_{1/p_t} x_{t+1}. \tag{2}$$

Then, the mean and covariance of returns are given by

$$\begin{aligned} E_t(r_{t+1}) &= D_{1/p_t}\mu \qquad (3)\\ Var(r_{t+1}) &= \sigma^2 D_{1/p_t} V D_{1/p_t}. \qquad (4) \end{aligned}$$

implying that

$$\begin{aligned} w_{MKT} &= \left(\frac{D_{p_t} e}{e^\top p_t}\right)\Bigg(= \frac{1}{p_t^T e} p_t \qquad (5)\\ \sigma^2_{MKT} &= w_{MKT}^\top Var\left(r_{t+1}\right) w_{MKT} = \frac{\sigma^2 e^\top V e}{\left(p_t^\top e\right)^2} \qquad (6)\\ \beta &= \frac{1}{\sigma^2_{MKT}} Var\left(r_{t+1}\right) w_{MKT} = \frac{p_t^\top e}{e^\top V e} D_{1/p_t} V e \qquad (7)\\ \mu_{MKT} &= w_{MKT}^\top E_t\left(r_{t+1}\right) = \frac{e^\top \mu}{p_t^\top e}, \qquad (8) \end{aligned}$$

We can then define the agents' objective functions:

Definition 1 *Type 1 Investor (Mutual Fund)*
Manages a percentage a_1 of total market wealth such that $a_1 = \frac{n_1^\top p_t}{e^\top p_t}$ and has an investment 'man-

date' consistent with,

$$\max_{n_1} \left(\frac{D_{p_t} n_1}{n_1^\top p_t} - \frac{D_{p_t} e}{e^\top p_t} \right) (E_t(r_{t+1}) - r_f) \quad \textit{subject to} \tag{9}$$

$$\left(\frac{D_{p_t} n_1}{n_1^\top p_t} - \frac{D_{p_t} e}{e^\top p_t} \right)^\top Var(r_{t+1}) \left(\frac{D_{p_t} n_1}{n_1^\top p_t} - \frac{D_{p_t} e}{e^\top p_t} \right) \leq \widehat{\sigma}^2 \quad \textit{(Tracking error constraint)} \tag{10}$$

$$\left(\frac{D_{p_t} n_1}{n_1^\top p_t} - \frac{D_{p_t} e}{e^\top p_t} \right)^\top e = 0 \qquad \textit{(Fully invested constraint)} \tag{11}$$

where $\frac{D_{p_t} n_1}{n_1^\top p_t}$ *is the chosen portfolio,* $\frac{D_{p_t} e}{e^\top p_t}$ *is the market (benchmark) portfolio, and* $\widehat{\sigma}^2$ *is the tracking error upper bound.*

Definition 2 *Type 2 Investor (Hedge Fund)*
Manages a percentage $a_2 = \frac{n_2^\top p_t}{e^\top p_t}$ *and has an investment 'mandate' consistent with*

$$\max_{n_2} \left(\frac{D_{p_t} n_2}{n_2^\top p_t} \right)^\top (E_t(r_{t+1}) - r_f) - \lambda \left(\frac{D_{p_t} n_2}{n_2^\top p_t} \right)^\top Var(r_{t+1}) \left(\frac{D_{p_t} n_2}{n_2^\top p_t} \right) \tag{12}$$

where $\frac{D_{p_t} n_2}{n_2^\top p_t}$ *is the chosen portfolio and* λ *is fixed risk aversion.*

Whilst we think of Type 1 investors as mutual funds and Type 2 investors as hedge funds, in reality these can be any type of (professional) money manager that is constrained in their risk-bearing capacity relative to a benchmark (Type 1) or any unconstrained investor that is able to choose their own risk tolerance as long as it remains fixed (Type 2). So, for instance, Type 2 investors could also be sovereign wealth funds or proprietary trading desks.

A. Equilibrium

We now solve for the equilibrium by taking first order conditions of both investors' maximization problems and imposing the market clearing condition that $n_1 + n_2 = e$.

Mutual fund. The mutual fund portfolio maximizes the Lagrangian,

$$
\begin{aligned}
\mathcal{L} &= \left(\frac{D_{p_t} n_1}{n_1^\top p_t} - \frac{D_{p_t} e}{e^\top p_t}\right)^\top (E_t(r_{t+1}) - r_f) \\
&+ \frac{\upsilon}{2} \quad \widehat{\sigma}^2 - \left(\frac{D_{p_t} n_1}{n_1^\top p_t} - \frac{D_{p_t} e}{e^\top p_t}\right)^\top Var(r_{t+1}) \left(\frac{D_{p_t} n_1}{n_1^\top p_t} - \frac{D_{p_t} e}{e^\top p_t}\right)\Bigg) \Bigg(\\
&- \eta \quad \left(\frac{D_{p_t} n_1}{n_1^\top p_t} - \frac{D_{p_t} e}{e^\top p_t}\right)^\top e \Bigg)
\end{aligned} \tag{13}
$$

The first order condition, then, is

$$
\left(\frac{D_{p_t} n_1}{n_1^\top p_t} - \frac{D_{p_t} e}{e^\top p_t}\right) \Bigg(= \frac{1}{\upsilon} Var^{-1}(r_{t+1})(E_t(r_{t+1}) - r_f - \eta e). \tag{14}
$$

Solving for constraints gives us the expected return to a minimum variance portfolio,

$$
\eta = \frac{(D_{p_t} e)^\top V^{-1} (\mu - r_f D_{p_t} e)}{(D_{p_t} e)^\top V^{-1} D_{p_t} e}. \tag{15}
$$

Hedge fund. Taking the first order condition for the hedge fund's maximization problem (Equation 12) gives us that

$$
\frac{D_{p_t} n_2}{n_2^\top p_t} = \frac{Var^{-1}(r_{t+1})(E_t(r_{t+1}) - r_f)}{\lambda}. \tag{16}
$$

Market clearing. All assets need to be held giving the market clearing condition $n_1 + n_2 = e$. We first re-arrange Equation (14) and Equation (16) to have expressions for n_1 and n_2:

$$
n_1 = a_1 e + \frac{a_1 e^\top p_t}{\upsilon \sigma^2} V^{-1}(\mu - (\eta + r_f) D_{p_t} e) \tag{17}
$$

$$
n_2 = \frac{(1 - a_1) e^\top p_t}{\lambda \sigma^2} V^{-1}(\mu - r_f D_{p_t} e). \tag{18}
$$

Substituting Equations (17) and (18) in the market clearing condition and noting that $D_{p_t} e = p_t$

yields

$$
\begin{aligned}
& a_1 e + \frac{n_1^\top p_t}{\upsilon\sigma^2} V^{-1}(\mu - (\eta + r_f) D_{p_t} e) + \frac{(1-a_1)e^\top p_t}{\lambda\sigma^2} V^{-1}(\mu - r_f D_{p_t} e) = e \\
\implies \quad & \frac{n_1^\top p_t}{\upsilon}(\mu - (\eta + r_f) D_{p_t} e) + \frac{(1-a_1)e^\top p_t}{\lambda}(\mu - r_f D_{p_t} e) = (1-a_1)\sigma^2 V e \\
\implies \quad & \frac{a_1 e^\top p_t}{\upsilon}(\mu - (\eta + r_f) D_{p_t} e) + \frac{(1-a_1)e^\top p_t}{\lambda}(\mu - r_f D_{p_t} e) = (1-a_1)\sigma^2 V e \\
\implies \quad & \frac{a_1}{\upsilon}(\mu - (\eta + r_f) D_{p_t} e) + \frac{(1-a_1)}{\lambda}(\mu - r_f D_{p_t} e) = \frac{(1-a_1)\sigma^2 V e}{p^\top e}
\end{aligned}
\tag{19}
$$

which gives equilibrium prices

$$
\begin{aligned}
p_t &= \left(\frac{1-a_1}{\lambda} r_f + \frac{a_1(\eta + r_f)}{\upsilon}\right)^{-1} \left(\left(\frac{a_1}{\upsilon} + \frac{1-a_1}{\lambda}\right)\mu - \frac{(1-a_1)\sigma^2 V e}{p_t^\top e}\right) \\
&= \left(\frac{r_f(1-a_1)\upsilon + a_1\lambda(\eta + r_f)}{\lambda\upsilon}\right)^{-1} \left(\left(\frac{a_1}{\upsilon} + \frac{1-a_1}{\lambda}\right)\mu - \frac{(1-a_1)\sigma^2 V e}{p_t^\top e}\right) \\
&= (r_f(1-a_1)\upsilon + (\eta + r_f)a_1\lambda)^{-1} \left(((1-a_1)\upsilon + a_1\lambda)\mu - \lambda(1-a_1)\upsilon \frac{\sigma^2 V e}{p_t^\top e}\right) \\
&= \left(r_f\left(1 + \frac{a_1\lambda}{(1-a_1)\upsilon}\right) + \eta\frac{a_1\lambda}{(1-a_1)\upsilon}\right)^{-1} \left(\mu\left(1 + \frac{a_1\lambda}{(1-a_1)\upsilon}\right) - \frac{\lambda\sigma^2 V e}{p_t^\top e}\right) \\
&= (r_f + (1-\phi)\eta)^{-1} \left(\mu - \frac{\lambda\phi\sigma^2 V e}{p_t^\top e}\right)
\end{aligned}
\tag{20}
$$

where $\phi = \frac{(1-a_1)\upsilon}{a_1\lambda + \upsilon(1-a_1)}$, $\eta = \frac{(D_{p_t} e)^\top V^{-1}(\mu - r_f D_{p_t} e)}{(D_{p_t} e)^\top V^{-1} D_{p_t} e}$ is the expected return to a minimum variance portfolio and ν is the Lagrangian multiplier on the tracking error constraint.

Let us now consider what happens in a world without Type 1 investors (i.e. a world without benchmarking and tracking error constraints). Define $\beta = Var(r_{t+1})\frac{p_t}{p_t^\top e}$ and take the limit of Equation (20) as $a_1 \to 0$ gives

$$
\begin{aligned}
p_t &= \left(\frac{r_f}{\lambda}\right)^{-1}\left(\frac{\mu}{\lambda} - \frac{\sigma^2 V e}{p_t^\top e}\right) \\
&= \frac{1}{r_f}\left(\mu - \lambda\sigma^2 V \frac{e}{p_t^\top e}\right) \\
&= \frac{1}{r_f}\left(\mu - \lambda\sigma^2 V \frac{D_{1/p_t} p_t}{p_t^\top e}\right) \\
&= \frac{1}{r_f}\left(D_{p_t} E_t(r_{t+1}) - \lambda D_{p_t} Var(r_{t+1}) D_{p_t} \frac{D_{1/p_t} p_t}{p_t^\top e}\right) \\
&= \frac{1}{r_f} D_{p_t}\left(E_t(r_{t+1}) - \lambda Var(r_{t+1}) \frac{p_t}{p_t^\top e}\right) \\
\Longrightarrow \quad p_t D_{1/p_t} &= \frac{1}{r_f}(E_t(r_{t+1}) - \lambda\beta) \qquad (21)
\end{aligned}
$$

where the last line follows as $\frac{p_t}{p_t^\top e}$ are the market weights, which can be rearranged to give the SML:

$$
E_t(r_{t+1}) = r_f + \lambda\beta. \qquad (22)
$$

Therefore, in a world without benchmarking at all, the SML as implied by the CAPM holds. Adding benchmarked investors, however, as we show below, distorts this relationship, a finding similar to that in Baker, Bradley, and Wurgler (2011). Now, going back to considering the impact of benchmarking, let us denote the market variance as

$$
\sigma^2_{MKT} = \sigma^2 \left(\frac{D_{p_t} e}{e^\top p_t}\right)^\top V \left(\frac{D_{p_t} e}{e^\top p_t}\right) \qquad (23)
$$

and thus beta can be written as

$$
\beta = \frac{\sigma^2 V}{\sigma^2_{MKT}} \left(\frac{D_{p_t} e}{e^\top p_t}\right) \qquad (24)
$$

We can then re-arrange the equation for equilibrium prices, Equation (20), to arrive at

$$
p_t = \left(\frac{1-a_1}{\lambda} r_f + \frac{a_1(\eta + r_f)}{\upsilon}\right)^{-1} \left(\left(\frac{a_1}{\upsilon} + \frac{1-a_1}{\lambda}\right) \mu - (1-a_1) D_{p_t} \sigma^2_{MKT} \beta\right) \qquad (25)
$$

Noting that $p_t = D_{p_t} e$ and that $\mu = D_{p_t} E_t(r_{t+1})$, we can express Equation (25) as

$$
E_t(r_{t+1}) = \frac{(1-a_1) r_f \upsilon + \lambda a_1(\eta + r_f)}{a_1\lambda + \upsilon(1-a_1)} + \beta \left(\frac{(1-a_1)\lambda\upsilon}{a_1\lambda + \upsilon(1-a_1)}\right) \sigma^2_{MKT}. \qquad (26)
$$

One can then express this as expected excess returns:

$$E_t(r_{t+1}) - r_f = \left(\frac{(1-a_1)\upsilon}{a_1\lambda + \upsilon(1-a_1)}\right)\Bigg(\beta\lambda\sigma^2_{MKT} + \eta\frac{\lambda a_1}{a_1\lambda + \upsilon(1-a_1)}. \tag{27}$$

Set $\phi = \frac{(1-a_1)\upsilon}{a_1\lambda+\upsilon(1-a_1)}$ and thus,

$$E_t(r_{t+1}) - r_f = \phi\beta\lambda\sigma^2_{MKT} + (1-\phi)\eta. \tag{28}$$

Note that market excess returns are given by

$$E_t(r^{MKT}_{t+1}) - r_f = \phi\lambda\sigma^2_{MKT} + (1-\phi)\eta \tag{29}$$

because, for the market, $\beta = 1$. Therefore, we can re-write Equation (28) as

$$E_t(r_{t+1}) - r_f = \beta(E_t(r^{MKT}_{t+1}) - r_f) + (1-\beta)(1-\phi)\eta, \tag{30}$$

which is the SML in a world with benchmarking. It is easy to notice that there is an additional term in Equation (30) as compared to Equation (22). This additional term effectively distorts the slope of the SML and is dependent on β, ϕ, and η. Our model makes three predictions that we summarize in Propositions 2.1 - 2.3 below.

B. Model predictions

Proposition 2.1 *[Steepening SML]* As market volatility σ^2 rises, the prices of volatile assets will fall further and so the SML steepens:

$$\frac{\partial(E_t(r^{MKT}_{t+1}) - r_f - (1-\phi)\eta)}{\partial\sigma^2} > 0. \tag{31}$$

Proof. We have that, from Equation (6)

$$\sigma^2_{MKT} = \sigma^2\left(\frac{D_{p_t}e}{e^\top p_t}\right)^\top V\left(\frac{D_{p_t}e}{e^\top p_t}\right)\Bigg(\tag{32}$$

and, from Equation (29),

$$\begin{aligned}\mathbb{E}_t(r_{t+1}^{MKT}) - r_f - (1-\phi)\eta &= \phi\lambda\sigma_{MKT}^2 \\ &= \phi\lambda\sigma^2 \left(\frac{D_{p_t}e}{e^\top p_t}\right)^\top V \left(\frac{D_{p_t}e}{e^\top p_t}\right)\end{aligned} \tag{33}$$

Then, taking the partial derivative of Equation (33) with respect to σ^2 gives

$$\frac{\partial(\mathbb{E}_t(r_{t+1}^{MKT}) - r_f - (1-\phi)\eta)}{\partial\sigma^2} = \phi\lambda \left(\frac{D_{p_t}e}{e^\top p_t}\right)^\top V \left(\frac{D_{p_t}e}{e^\top p_t}\right) > 0 \tag{34}$$

because $\phi, \lambda, Ve > 0$. ■

This finding is somewhat at odds with the suggestion in Proposition 3 of Frazzini and Pedersen (2014). In their model, the authors suggest that when leverage constraints tighten - or when markets are in distress - there is less room for arbitrage thereby causing the SML to flatten. That said, the empirical evidence presented in their paper does not align with their model's predictions giving rise to an alternative channel through which the slope of the SML is affected. Our story is less about arbitrage and more about tracking error constraints causing a tilt to low beta assets. The tilt to low volatility by benchmarked money managers means that they are taking on less risky assets.

As market volatility rises, the prices of volatile assets fall, by Proposition 2.1. Then, as the tracking error constraint on Type 1 investors binds and they tilt from high beta assets to low beta assets, the weights of high (low) beta assets in the market fall (rise). As the weights of assets become more dispersed, so do their betas.

Proposition 2.2 *[Beta dispersion]* As market volatility σ^2 rises, betas become more dispersed. That is,

$$\frac{\partial\sigma_\beta}{\partial\sigma^2} > 0, \tag{35}$$

where σ_β denotes the standard error of the betas.

Proof. We do three things in this proof. First, we show that as σ^2 rises, the value of the market $p_t e$ falls. Next, given that prices are proportional to the certainty equivalent of cash flows, and thus a function of the value of the market and market volatility, we show that the risk-adjustment to prices - that is, $\frac{\lambda\phi}{p_t^\top e}\sigma^2$ - must rise. This is because prices fall when σ^2 rises and the risk-adjustment must increase (as it is the only part of prices that is a function of market volatility). The risk-adjustment is also the only part of prices that affect relative weights, as we will show. Therefore, if the risk-adjustment increases, then the cheap (high beta) assets must get relatively cheaper and the expensive (low beta) assets must become a bigger part of the market. Therefore, beta dispersion increases.

To begin, we outline a few key equations needed. From Equation (15) we have that, since $p_t = D_{p_t} e$,

$$
\begin{aligned}
\eta &= \frac{p_t^\top V^{-1}(\mu - r_f p_t)}{p_t^\top V^{-1} p_t} \\
&= \frac{p_t^\top V^{-1}\mu}{p_t^T V^{-1} p_t} - r_f
\end{aligned} \tag{36}
$$

and, from the definition of the covariance of returns in Equation (4), we have

$$
\begin{aligned}
\hat{\sigma}^2 &= \left(\frac{D_{p_t} n_1}{n_1^\top p_t} - \frac{D_{p_t} e}{e^\top p_t}\right)^\top Var(r_{t+1}) \left(\frac{D_{p_t} n_1}{n_1^\top p_t} - \frac{D_{p_t} e}{e^\top p_t}\right) \\
&= \left(\frac{1}{\upsilon} Var^{-1}(r_{t+1})(\underbrace{\mathbb{E}_t(r_{t+1})}_{=D_{1/p_t}\mu} - r_f - \eta e)\right)^\top \underbrace{Var^{-1}(r_{t+1})}_{=\frac{1}{\sigma^2} D_{p_t} V^{-1} D_{p_t}} (\underbrace{\mathbb{E}_t(r_{t+1})}_{=D_{1/p_t}\mu} - (\eta + r_f) e) \\
&= \frac{1}{\upsilon^2 \sigma^2} \left(D_{1/p_t}\mu - (\eta + r_f) e\right)^\top D_{p_t} V^{-1} D_{p_t} (D_{1/p_t}\mu - (\eta + r_f) e).
\end{aligned} \tag{37}
$$

Re-arranging for υ^2 gives

$$
\upsilon^2 = \frac{1}{\widehat{\sigma}^2 \sigma^2} \left(\mu - (\eta + r_f)\, p\right)^T V^{-1} \left(\mu - (\eta + r_f)\, p\right). \tag{38}
$$

From Equation (20), we have that

$$
p_t = \left(r_f + (1 - \phi)\,\eta\right)^{-1} \left(\mu - \frac{\lambda\phi}{p_t^T e} \sigma^2 V e\right) \tag{39}
$$

implying that

$$
\begin{aligned}
0 &= \left(r_f + (1-\phi)\,\eta\right) \left(e^T p_t\right)^2 - e^T \mu \left(p_t^T e\right) + \lambda\phi\sigma^2 e^T V e \\
\Rightarrow \left(p_t^T e\right) &= \frac{e^T \mu}{2\left(r_f + (1-\phi)\,\eta\right)} \left(1 + \left(1 - 4\frac{\left(r_f + (1-\phi)\,\eta\right)\lambda\phi\sigma^2 e^T V e}{(e^T\mu)^2}\right)^{1/2}\right)
\end{aligned} \tag{40}
$$

We want to firstly show that the value of the market, $p_t^\top e$, falls as σ^2 rises. We can do this by seeing that $\frac{\partial(p_t^\top e)}{\partial\sigma^2} < 0$. To simplify notation, let us denote the following:

$$\begin{aligned} y &= p_t^\top e && (41)\\ a &= r_f + (1-\phi)\eta && (42)\\ b &= e^\top \mu && (43)\\ c &= \lambda\phi e^\top V e. && (44) \end{aligned}$$

Then, we have

$$\begin{aligned} \frac{\partial y}{\partial \sigma^2} &= \frac{\partial}{\partial \sigma^2} \frac{b}{2a} \quad 1 + \left(1 - \frac{4ac\sigma^2}{b^2}\right)^{1/2} \Bigg)\Bigg) \\ &= \frac{-2c\sigma}{(b^2 - 4ac\sigma^2)^{1/2}} < 0. && (45) \end{aligned}$$

Now, we are interested in the change in the dispersion of beta, which can be written as,

$$\beta = \frac{1}{\sigma_{mkt}^2} Var\left(r_{t+1}\right) w_{mkt} = \frac{p_t^T e}{e^T V e} D_{1/p_t} V e. \tag{46}$$

Note that

$$\frac{1}{p_t^T e} D_{p_t}\beta = \frac{1}{e^T V e} V e \tag{47}$$

and so the weighted sum of the beta

$$\frac{e^T}{p_t^T e} D_{p_t}\beta = 1. \tag{48}$$

Hence, the variance of the weighted betas is also constant. We can write this as

$$\beta_i = \frac{1}{w_i}\frac{e_i^T V e}{e^T V e}, \tag{49}$$

where e_i is a vector with 1 in the i^{th} position and 0 elsewhere. Now we can write prices as

$$p_t = k\left(\mu - c\chi\right), \tag{50}$$

where μ and $\chi = Ve$ are constant vectors and k and c are constants. Now we know $\mu_i > 0$ for all i and $e^T\chi > 0$. Note now that $c = \phi\frac{\lambda\sigma^2}{e^\top p_t}$ and so as σ^2 increases, $\frac{\lambda\sigma^2}{e^\top p_t}$ increases because $e^\top p_t$ falls (which we have shown in Equation (45)). Thus, if c is to stay constant, then ϕ must decrease by as much as $\frac{\lambda\sigma^2}{e^\top p_t}$ increases. But as σ^2 increases, ϕ increases too. This is because ϕ increases when υ increases because υ is the Lagrange multiplier on the tracking error constraint; by the Envelope Theorem, it increases with σ^2 (which is what causes the constraint to bind). Note that a_1 doesn't move much and λ is the fixed risk aversion of Type 2 investors (hedge funds). Thus, it must be that c increases as σ^2 increases and so the relative weights of prices become more dispersed. As the relative weights become more dispersed, Equation (49) implies that betas become more dispersed too. ■

As betas become dispersed, and the fact that the prices of high (low) beta assets fall (rise), pushing up (down) their expected returns, the SML steepens. This prediction is in contrast to Proposition 4 in Frazzini and Pedersen (2014), who suggest that betas become more compressed towards one when markets are subject to funding shocks.

The beta of the investor is the average beta of the assets they hold. Thus, tilting to low beta assets during periods of rising volatility reduces (increases) the beta of mutual (hedge) funds.

Proposition 2.3 *[Fund betas]* A rise in market volatility causes the tracking error constraint of mutual funds to bind and forces them to tilt to low volatility assets pushing down their betas. Hedge funds, with fixed risk aversion, take on the riskier, high beta, assets and thus their betas rise.

Proof. Here we want to show that $\frac{\partial \beta_1}{\partial \sigma^2} < 0$, where β_1 is the beta of Type 1 investors (mutual funds). We can write,

$$\beta_1 = \frac{1}{\sigma^2_{MKT}} \sigma^2 \left(\frac{D_{p_t} n_1}{n_1^\top p_t} \right)^\top V \left(\frac{D_{p_t} e}{e^\top p_t} \right) \Bigg(\tag{51}$$

Now, by Equation (6), we have that

$$\sigma^2_{mkt} = \frac{\sigma^2 e^\top V e}{(p_t^\top e)^2}. \tag{52}$$

Substituting Equation (52) in Equation (51) yields

$$\beta_1 = \frac{(p_t^\top e)^2 \left(\frac{D_{p_t} n_1}{n_1^\top p_t} \right)^\top V \left(\frac{D_{p_t} e}{e^\top p_t} \right)}{e^\top V e}. \Bigg(\tag{53}$$

Since we have already shown in Equation (45) that

$$\frac{\partial(p_t^\top e)^2}{\partial \sigma^2} < 0, \tag{54}$$

our result follows. ■

IV. Empirical analysis

In this section, we present simple, yet compelling, empirical evidence to support our model's predictions. We namely focus on supporting Propositions 2.2 and 2.3.

A. *Beta dispersion*

Figure 1 shows that dispersion, as measured by median absolute deviation, does indeed increase when the VIX is high. Here, we select stocks in the top quintile by size (natural logarithm of market capitalization) for US common stocks (CRSP share codes 10 and 11). We only plot results for the top size quintile but results are similar for other quintiles. We don't plot quintiles together to abstract away from any effects of size and dispersion across stocks. This finding is somewhat orthogonal to that in Frazzini and Pedersen (2014) who find, in their Proposition 4, that the dispersion of betas is significantly lower when funding liquidity risk is high. Whilst the authors present compelling empirical evidence to support their result on credit constraints, we show very simply, by proxying market volatility by the VIX, that the median absolute deviation of betas for US equities increases when the VIX increases. Our result is consistent with several studies in the literature. For instance, Gomes, Kogan, and Zhang (2003) provide a theoretical model to show that beta dispersion is counter-cyclical to the business cycle and Abdymomunov and Morley (2011), amongst others, provide empirical support to show that the dispersion of betas for various factor portfolios is considerably higher in high volatility regimes. A surprising result in this plot - which is beyond the scope of the current work - is the spike in beta dispersion starting at the end of 2016 and continually rising through to the end of 2017 despite the VIX being at historically low levels. Whilst we do not aim to explain this rigorously, we discuss a few possibilities. First, from a purely econometrics point of view, betas are difficult to estimate accurately when the market is moving sideways. Since the betas we estimate are 6-month rolling betas, if markets have not moved much

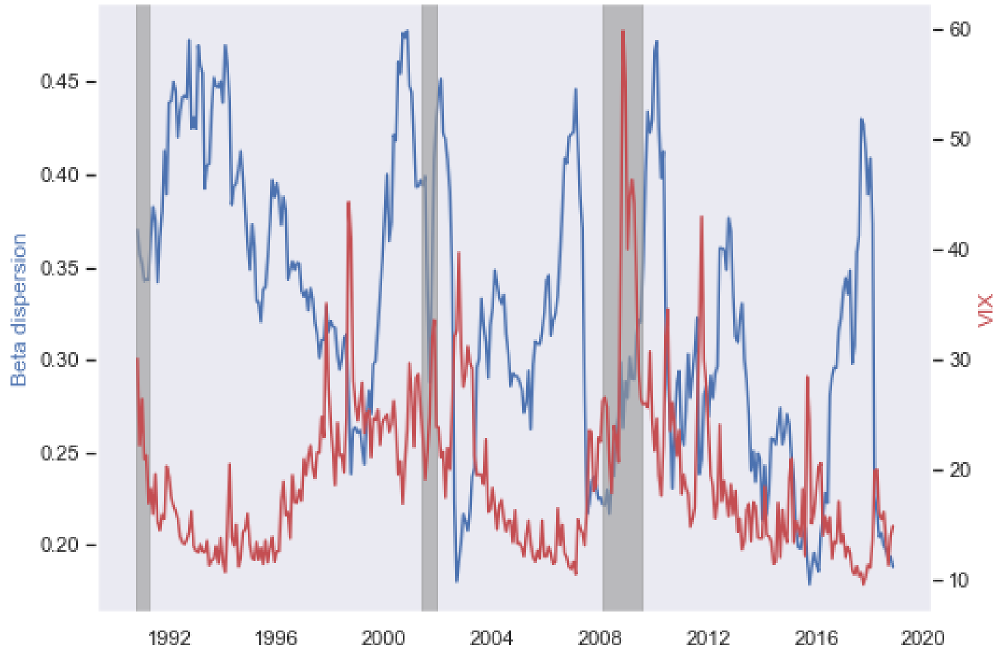

Figure 1. Beta dispersion

Notes: Dispersion of betas for top size quintile stocks in the CRSP universe measured by the median absolute deviation (blue line, LHS axis) plotted against the VIX index (red line, RHS axis) at the end of each month from 1990-2018. Grey shaded areas represent recession periods defined by the National Bureau of Economic Research. Note that there may exist a lag between the two series due to the fact that one is forward looking (VIX) and one is backward looking (beta dispersion).

in those 6-months the betas we estimate may not be accurate. Second, although the VIX was at historically low levels, geopolitical uncertainty remained. For instance, at the end of 2016, financial markets were bracing themselves for what may come of two surprise political events: the UK voting to leave the European Union ('Brexit') in June 2016 and the Presidential election of Donald Trump in November 2016. These two events are likely to impact stocks' idiosyncratic volatilities in very different ways (see Ang, Hodrick, Xing, and Zhang (2006)). Therefore, stock betas may have become dispersed simply because stocks even within the same size quintile effectively 'load' on these two surprise events in very different ways. For instance, stocks with operations in China or those with ties to trade with Mexico would be a lot more affected by Donald Trump's threats to trade deals compared to stocks that produce domestically and generate the majority of their revenue domestically. That said, this surprising result is beyond the scope of our work here and we leave it for future research.

B. Fund betas

Our model predicts that mutual funds tilt towards low volatility, or low beta, assets during stressed markets. The beta of a fund is effectively the average of the betas of the assets it holds by construction. Therefore, our model predicts that mutual fund betas fall when markets are stressed. We test whether this is the case using mutual fund returns from the CRSP Mutual Fund Returns Survivor-Bias-Free data set. As we are interested in open ended domestic (US) equity mutual funds that are benchmarked, we use the objective codes provided by CRSP. We firstly select funds that have CRSP style code 'ED' (equity - domestic). From this category, we then drop out any sector funds (i.e. funds with the CRSP style code 'EDS'). Next, we condition on Lipper objective codes: we drop funds with codes 'DSB' (dedicated short bias), 'ABR' (absolute return), and 'DL' (equity leverage). Finally, we denote funds with the CRSP style code 'EDYH' as hedge funds, and mutual funds otherwise. We ensure that no duplicate entries exist, only keep funds with at least 12 consecutive months of returns data, and winsorize returns at the 1% level. Readers familiar with literature using this database will note that our sample selection procedure is not as restrictive as prominent studies in the mutual fund literature using this data (e.g. Kacperczyk, Sialm, and Zheng (2008) or Lou (2012) and references therein). This is because most studies using this data have a focus on understanding the performance of mutual funds. Our purposes are to understand the risk-bearing capacity of mutual funds and it's impact on asset prices. Thus, restricting the sample is not necessary in this paper.

We then calculate 6-month rolling betas for each fund based on it's returns in excess of the return of the CRSP value-weighted index return. Figure 2 plots the median fund beta for mutual funds (top panel) and for hedge funds (bottom panel) against the VIX (red lines). Eye-balling these plots shows that in most instances, mutual (hedge) fund betas fall (rise) when the VIX rises. To test this result in a slightly more rigorous way, we regress the average mutual fund and hedge fund betas on proxies for market volatility. Table I shows two things. First, that the VIX seems to be the most important driver of changes in fund betas, surviving the horse-race against the TED spread, BAA-10 year corporate spread, and even realized market volatility (calculated by taking the 12-month rolling standard deviation of the S&P 500 return. This is intuitive as fund betas are related to risk. The VIX, unlike the other indicators, is a forward-looking measure of expected volatility and therefore is perhaps the best proxy for expectations of risk-bearing capacity for funds. Second, the

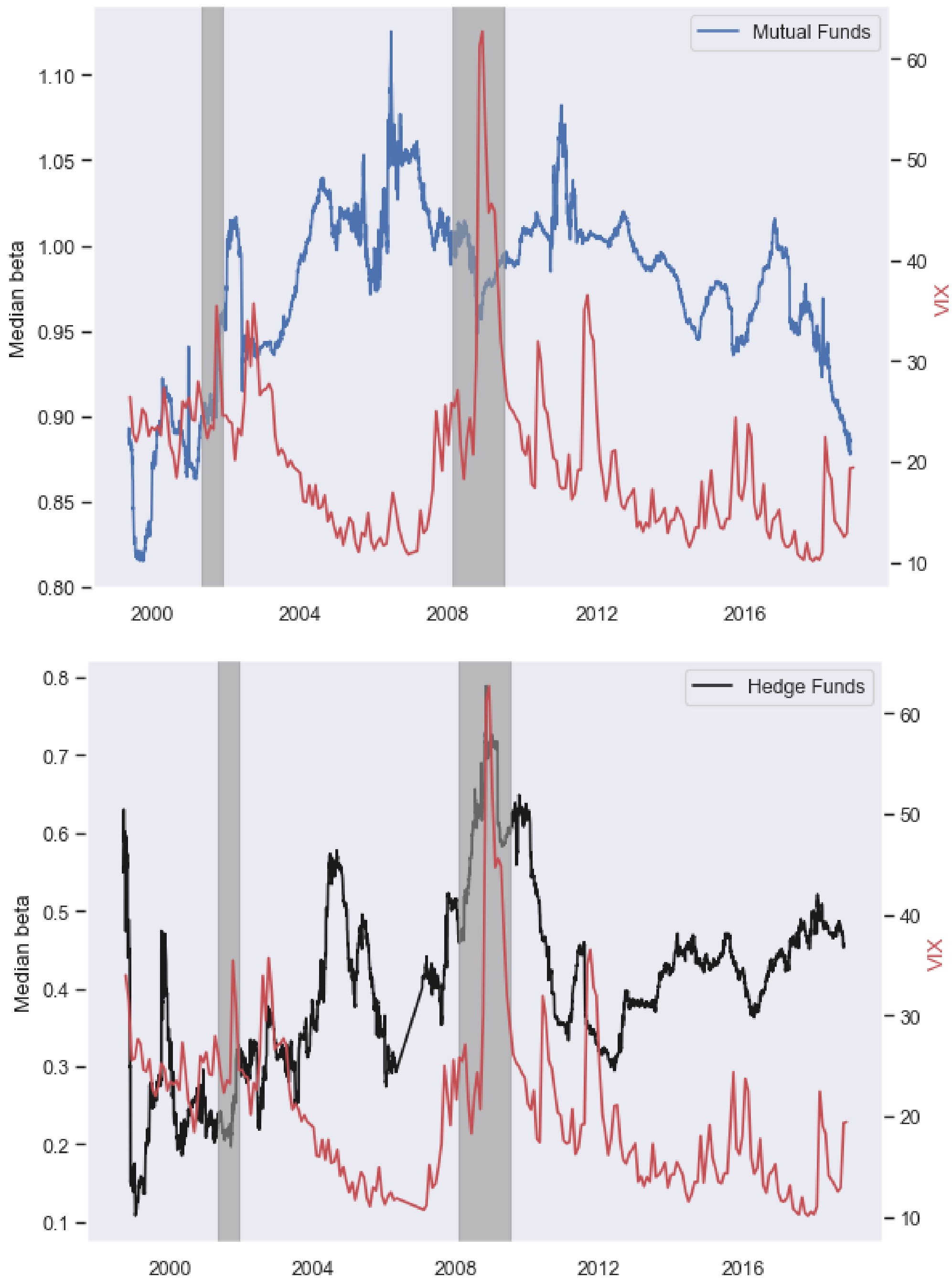

Figure 2. Fund betas

Notes: 6-month rolling fund betas for domestic equity funds by fund type: non-hedged funds (top panel) and hedged funds (bottom panel). Sample is from 1998-2018. Grey shaded areas represent recession periods defined by the National Bureau of Economic Research.

coefficient on the VIX is approximately the same magnitude and *opposite* in sign for mutual fund betas versus hedge fund betas. This is precisely what our model predicts, at least qualitatively:

that mutual fund betas fall and hedge fund betas rise as market volatility rises. This is true in both the univariate and multivariate regressions (models (1) and (5) in both cases). This result is simple yet intuitive and supports our model. It highlights a key difference between benchmarked and non-benchmarked money managers. The former face a binding tracking error constraint when aggregate market volatility is high. This induces a tilt away from high volatility assets and a tilt to lower volatility, or lower beta, assets by such managers. As such, the beta of the returns, which is approximately equivalent to the beta of the assets the manager holds, falls as well. On the other hand, the latter investor - that is, hedge funds or non-benchmarked money managers - who does not face a binding tracking error constraint and instead has fixed risk aversion, according to our model, absorbs this excess risk that stems from rising market volatility. The tilt to higher beta assets by these investors, similarly, results in a higher beta of their returns as shown in Figure 2.

Table I. Fund betas and stressed markets

The dependent variable in the regressions estimated in this table is the first difference of average fund beta between months $t-1$ and t. The independent variables are the one-month lagged values (at $t-1$) of the VIX, the TED spread, the Moody's BAA corporate spread (BAA - 10 year US government yield), and realized market volatility (estimated by the 12-month rolling standard deviation of the return on the S&P500 composite index).

Panel A: Mutual funds	(1)	(2)	(3)	(4)	(5)
VIX	-0.00334***				-0.00534**
	(0.0009)				(0.00198)
TED Spread		-0.0111			0.00976
		(0.0127)			(0.0172)
BAA 10yr			-0.0130		0.0291*
			(0.00944)		(0.0136)
12-month S&P500 Std.				-0.00285**	-0.00661
				(0.00996)	(0.0229)
R-squared	0.124	0.008	0.019	0.079	0.167

Panel B: Hedge funds	(1)	(2)	(3)	(4)	(5)
VIX	0.00343***				0.00414**
	(0.000619)				(0.00123)
TED Spread		0.0232			0.0104
		(0.0120)			(0.0129)
BAA 10yr			0.0272***		-0.000535
			(0.00774)		(0.0104)
12-month S&P500 Std.				0.003169***	-0.001324
				(0.815)	(1.499)
R-squared	0.243	0.038	0.114	0.136	0.251

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

C. *Tilt to low volatility*

We showed through fund returns that mutual fund betas fall during periods of high volatility, i.e. that there is a tilt to low volatility. The argument there was that mutual funds' betas fall because they tilt to lower volatility, or lower beta, stocks during periods of high market volatility. To study whether this tilt indeed takes place, we next study holdings data. All institutions, including mutual funds, are required to report their holdings greater than $100 million in US-listed stocks on a quarterly-basis no later than 45 days after the quarter-end under a form known as 13F.[1] We use the holdings data to measure mutual funds' tilt to low volatility versus the market portfolio as shown in Figure 3.

To construct this tilt, we use mutual funds holding data from the SEC 13F filings provided by the CDA/Spectrum data base via Thomson Reuters. In particular, we use the 'S12' file which contains the filings under 13F of mutual funds.[2] This data covers almost all historic domestic mutual funds and roughly 3,000 global funds that hold a fraction of assets in stocks traded in US exchanges. It is free of survivor-bias, that tends to be a challenge in mutual fund research, as it keeps nearly all US-based mutual funds since 1980 (see **?** for more on survivor-bias in mutual fund research). To construct a basket of low volatility stocks, we first calculate the standard deviation of each stock in our sample over the preceding 6 months using daily data. We multiply each stock's standard deviation by -1 - so that a high value indicates low volatility - and then assign a percentile score to each stock according to the resulting value at each date. This is then the stock's low volatility score. Next, we calculate the total weight of each stock in the aggregate mutual fund portfolio. That is, we calculate first the total dollar amount held in the stock (as per the quarter-end filing) and divide it by the total value of all filings at each quarter-end. We then multiply the stock's aggregate mutual fund portfolio weight by it's low volatility score. We do the same for the market portfolio. In this case, we compute the market portfolio by taking the largest 500 stocks at each quarter-end as per their market capitalizations at that date. We do not use the S&P 500 index so as to not drop stocks that are not contained in the index; however, in unreported results, we find that the results are not materially affected. We then multiply each stock's weight in the market portfolio with it's low volatility score. To compute the tilt to low volatility, we calculate

[1] The nature of SEC 13F filings is discussed in more detail in paper 3, in which we study the uses of publically available institutional holdings data more closely.

[2] Note that the 'S34' file has known errors which we allude to more in paper 3.

Figure 3. Tilt to low volatility

the difference between the resulting value of the stock weight multiplied by the low volatility score for the aggregate mutual fund portfolio and the associated value for the hypothetical value market portfolio. A positive (negative) difference implies that the mutual fund portfolio was overweight (underweight) low volatility - or had a strong tilt to (away from) low volatility - compared to the market portfolio.

From Figure 3, it is evident that during periods of high volatility - for instance, during the grey shaded areas that represent NBER recessions - the tilt to low volatility rises for the aggregate mutual fund portfolio. During the 2008/09 financial crisis, this tilt is largest, unsurprisingly, at 6 percentiles higher than the market portfolio. This supports our model's predictions of large tilts to low volatility. In particular, with the aggregate mutual fund portfolio tilting - meaning a very large segment of the market - the low beta stocks' expected returns get pushed down as they become more expensive. This, in turn, causes the SML to flatten, as we have argued.

D. Low volatility rotation

Our model has argued that the prices of low (high) beta assets rise (fall) in periods of high market volatility due to the tilt by benchmarked money managers to low volatility assets. Low volatility

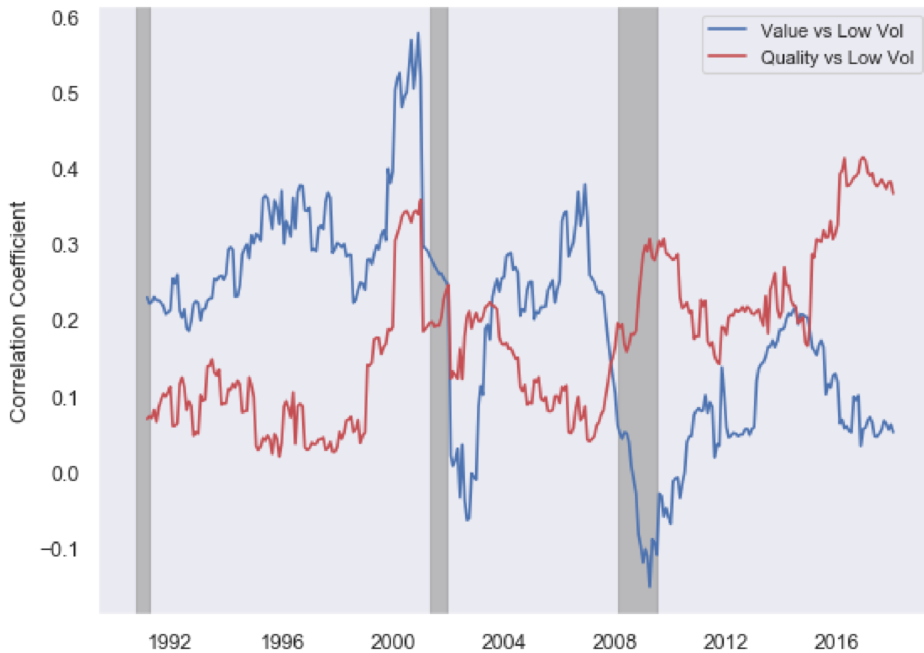

Figure 4. Correlation between Value and Low Volatility (blue line) versus Quality and Low Volatility (red line)

assets then become expensive and vice versa for high volatility assets. One way to look at this - although not explicitly shown in our model - is to look at the low volatility, value (cheap), and quality (expensive) scores of stocks in the cross-section. Figure 4 plots the cross-sectional correlation of value and quality against low volatility. We see that the correlation of low volatility against value is inverse to the correlation of low volatility against quality. This is indicative that low volatility looks a lot like value during good times and like quality in bad times. A proof or rigorous explanation of this effect is beyond the scope of this paper; however, this simple empirical result sheds light on the phenomenon that the SML steepens during bad times: high beta stocks look cheap (like value) and low beta stocks look expensive (like quality).

V. Conclusion

This paper contributes to the large and still growing literature on understanding the cross-section of stock returns as well as the low volatility anomaly that goes against the CAPM that high risk assets should deliver higher returns relative to low risk assets on average. We do this by presenting a model of delegated asset management in which benchmarked money managers face a constraint on their risk-taking ability known as a tracking error constraint. That is, they must keep volatility under a threshold relative to their benchmark. Our model consists of two types of agents: Type 1 (e.g. mutual funds) that are benchmarked and Type 2 (e.g. hedge funds) that are not benchmarked, are mean-variance maximizers, and have a given level of risk aversion. The model shows several things and makes three predictions. Firstly, we show that the SML is as implied by the CAPM in a world without benchmarking. That is, without mutual funds (Type 1 investors), or without any tracking error constraints at play, the SML would hold perfectly. Adding Type 1 investors, however, distorts the SML by factors that are dependent on the amount of the market managed by Type 1 investors - a_1 in the model - and the Lagrangian multipliers from solving Type 1 investors' optimization problem (η and υ). Next, we show that as market volatility σ^2 rises, the prices of volatile assets fall thereby causing the SML to steepen. This is a result of Type 1 investors tilting to low volatility assets. If the prices of high beta assets fall relative to low beta assets, the high beta assets' relative weights in the market fall. This means that beta dispersion increases. We prove this by showing that the risk-adjustment to prices must rise. Our model argues that Type 1 investors tilt to low beta assets as market volatility rises. That is that the beta of Type 1 investors' portfolios falls as market volatility rises. We next present simple empirical evidence that support our model's predictions. We find that beta dispersion, measured by the median absolute deviation of betas in the cross-section, rises with the VIX. This is contrary to the prediction and evidence shown in Frazzini and Pedersen (2014) but supported by our model as well as other studies in the literature mentioned. Next, by looking at mutual fund returns and proxying for hedge funds by looking at those mutual funds that behave like 'market-neutral' funds, we show that the betas of mutual (hedge) funds fall (rise) during stressed markets. By horse-racing several variables indicative of stressed markets, we find that the VIX wins the horse race with a negative (positive) and statistically significant coefficient for mutual (hedge) funds. This fits in with our argument: it is risk capacity that drives the changes in the betas. Our entire model is centred around the view that Type 1 investors tilt to low volatility assets during turbulent markets. By constructing a measure of their tilt to low volatility using holdings data from the SEC's 13F filings and comparing their holdings in low beta assets to the market portfolio, we show that mutual funds are overweight low volatility assets during periods of crisis. Our results highlight the importance of delegated money management in understanding asset pricing anomalies as well as addressing still outstanding questions on the low volatility anomaly. We contribute another channel through which the SML's slope is not in line with what the CAPM predicts.

References

Abdymomunov, A., and J. Morley (2011): "Time variation of CAPM betas across market volatility regimes," *Applied Financial Economics*, 21, 1463–1478.

Ang, A., R. J. Hodrick, Y. Xing, and X. Zhang (2006): "The Cross-Section of Volatility and Expected Returns," *Journal of Finance*, 61(1), 259–299.

Antoniou, C., J. A. Doukas, and A. Subrahmanyam (2016): "Investor Sentiment, Beta, and the Cost of Equity Capital," *Management Science*, 62(2).

Baker, M., B. Bradley, and J. Wurgler (2011): "Benchmarks as Limits to Arbitrage: Understanding the Low-Volatility Anomaly," *Financial Analysts Journal*, 67(1), 1–15.

Basak, S., and A. Pavlova (2013): "Asset Prices and Institutional Investors," *American Economic Review*, 103(5).

Black, F. (1972): "Capital market equilibrium with restricted borrowing," *Journal of Business*, 4, 444–455.

Black, F., M. C. Jensen, and M. Scholes (1972): *The Capital Asset Pricing Model: Some Empirical Tests*, vol. 81.

Brennan, M. (1993): "Agency and asset pricing," Working paper, University of California, Los Angeles.

Cohen, R. B., C. Polk, and T. Vuolteenaho (2005): "Money Illusion in the Stock Market: The Modigliani-Cohn Hypothesis," *The Quarterly Journal of Economics*, 70, 639–668.

Frazzini, A., and L. H. Pedersen (2014): "Betting against beta," *Journal of Financial Economics*, 111, 1–25.

Gomes, J., L. Kogan, and L. Zhang (2003): "Equilibrium Cross Section of Returns," *Journal of Political Economy*, 111(4).

HONG, H., AND D. A. SRAER (2016): "Speculative Betas," *The Journal of Finance*, 71(5), 2095–2144.

HUANG, S., D. LOU, AND C. POLK (2018): "The Booms and Busts of Beta Arbitrage," Working paper.

JYLHA, P. (2018): "Margin Requirements and the Security Market Line," *The Journal of Finance*, 73(3), 1281–1321.

KACPERCZYK, M., C. SIALM, AND L. ZHENG (2008): "Unobserved Actions of Mutual Funds," *Review of Financial Studies*, 21(6), 2379–2416.

LINTNER, J. (1965): "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets," *The Review of Economics and Statistics*, 47(1), 13–37.

LOU, D. (2012): "A Flow-Based Explanation for Return Predictability," *Review of Financial Studies*, 25(12), 3457–3489.

LOU, D., AND C. POLK (2013): "Comomentum: Inferring Arbitrage Activity from Return Correlations," Working paper.

SHARPE, W. F. (1964): "Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk," *Journal of Finance*, 19(3), 425–442.

SHLEIFER, A., AND R. VISHNY (1997): "The Limits of Arbitrage," *Journal of Finance*, 52(1), 35–55.

VAYANOS, D., AND P. WOOLLEY (2016): "Curse of the Benchmarks," Financial Markets Group Discussion Paper No 747 https://bit.ly/2GEwwnY, London School of Economics.