Return Chasing by Hedge Fund Investors

Rui Gong *

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Abstract

A stylized fact about hedge funds is that investors tend to "chase returns," meaning that they adjust their allocations substantially as a function of recent performance. I present an assessment model in which ability changes over time. In this model, chasing returns is profitable, since it moves investors' capital toward funds that happen to have ability at a particular moment in time. In addition, I present a variant of the model in which investors are subject to "representativeness bias," leading them to weigh recent returns too heavily in forming assessments of ability. On a sample of 3,697 hedge funds between 1994 and 2017, I test a number of model hypotheses. The data suggests that investors should weigh recent signals less for older funds than younger funds. In addition, persistence of managerial abilities varies substantially across types of funds. Persistence of ability is lower for "Event Driven" strategies than persistence of ability for fund managers using strategies based on skill at valuation such as "Global Macro". Thus, investors should weigh recent signals more for "Event Driven" funds and less for "Global Macro" funds. Last, investors are subject to representativeness bias in that they place more weight on recent returns than appears to be optimal. Expected returns vary considerably depending on how investors chase returns. Sorting funds by perceived ability, if investors chase returns optimally, the expected return of investing in top decile funds is 9.28% annually. The average investors with representativeness bias overreact to recent performance and only make 7.40% annually. If investors use buy and hold strategy, the expected return is 3.42% annually.

*Contact information: Rui Gong, Department of Finance, Fisher College of Business, The Ohio State University, Columbus, OH, 43210 email: gong.190@osu.edu. The author thanks Itzhak Ben-David, Justin Birru, Kewei Hou, Andrea Rossi, Nicolai Roussanov, Berk Sensoy, René Stulz, Michael Weisbach and Ingrid Werner for helpful suggestions.

1 Introduction

A stylized fact about hedge funds and other forms of delegated portfolio management is that their investors "chase" returns, meaning that inflows to the funds are strongly positively related to the funds' prior performance (see Fung et al. (2008), Agarwal et al (2004), Wang and Zheng (2008), and Baquero and Verbeek (2008)). Moreover, investors value recent performance information more than distant performance information. It can be explained both rationally and behaviorally. Rationally, abilities of hedge fund managers vary over time but display persistence. Therefore, more recent performance information is more informative of managers' abilities than distant performance, making it rational for investors to weigh recent performance more than distant returns. From a behavioral perspective, it may be that investors are instead subject to representativeness bias. In other words, investors pay too much attention to recent performance of hedge funds and ignore the prior distribution.

To disentangle these two explanation, I build a unified model of return chasing, with and without representativeness bias. Berk and Green (2004) provide a framework to study the relationship between managerial ability, fund returns and fund inflows. Ability stays constant in the Berk and Green (2004) model. However, an important aspect of hedge funds is that the ability of managers and the profitability of a strategy changes over time.

In my model, the ability to earn abnormal returns follows an AR(1) process, so that it fluctuates in every period. The variable ρ governs the persistence of a manager's ability. Unlike the constant ability model, the best estimate of a manager's ability to earn abnormal returns weighs recent returns more than distant returns. The extent to which recent returns matter more than distant returns depends on ρ. When ρ is high then ability persists, and the model approaches the standard constant-ability model in which returns are weighted equally. In contrast, when ρ is low, more recent returns are weighted more heavily. An important implication of the changing ability model is that shocks to ability make return chasing an optimal investment strategy since recent returns are informative about ability and hence future returns.

An alternative interpretation of return chasing is that investors overreact to new information and move too much of their capital in response to recent returns. This idea stems from the idea of "representativeness bias" originally proposed by Tversky and Kahneman (1972). Gennaioli and Shleifer (2010) and Bordalo et al. (2019) suggest that this bias can be thought of in terms of Bayes' Law and propose that it can be formalized by an overweighting of signals relative to priors. Since my model is based on investors forming expectations using Bayes' Law, this approach to incorporating representativeness bias is naturally applied to my model. I present such a version of the model, the key element of which is that signals are overweighted by a factor, θ. In this version of the model, overweighting of recent signals leads to larger return chasing than in the rational version since recent returns become even more important from investors' viewpoints.

Another feature of my model is that return is a function of fund size, as in Berk and Green (2004). Thus, I allow for economies of scale in hedge funds. The evidence on whether hedge funds exhibit decreasing returns to scale is mixed. Earlier works using OLS regressions find returns decrease with size. However, both size and returns are endogenous. Cao and Velthius (2017) use recursive demeaning methods and find that decreasing returns to scale do not exist at the fund level. My model allows me to quantify the scale effect in hedge funds. In addition, it is important to rule out the scale effect when studying the rationality of investors' decisions on allocating capital. This is because flows affect the size of a fund. If there exists decreasing returns to scale, then the future abnormal return of a fund with skilled managers decreases as flows come in. Without considering economies of scale, it is impossible to tell how rational investors are.

I evaluate the empirical relevance of the model in a number of ways. I first estimate the model parameters using maximum likelihood on a sample of 3,697 hedge funds between 1994 and 2017. The estimate of the key parameter ρ equals 0.9566, which suggests that ability does change in significant way over time. As a test of whether ability changes because of the temporary nature of some hedge fund strategies, I estimate ρ separately for hedge funds with different strategies. I find that "Event Driven" and "Fixed Income Arbitrage" strategies tend to have lower values of ρ relative to valuation-driven strategies such as "Global Macro." This makes sense because Event Driven strategies are based on finding idiosyncratic opportunities. The result is also consistent with the findings of Kosowski et al. (2007), who use a bootstrap method to evaluate hedge fund skills and find that skill persists for "Global Macro" funds but not so much for "Event Driven" funds.

I, next, present formal tests of the model's implications. First, the model predicts that investors should weigh recent returns less for older funds than for younger funds when forming estimates of abilities since more information is available for investors to evaluate the managerial skill of an older fund. Investors should react to recent signals of an older fund less than they should react towards a younger fund. Second, the model predicts that for funds with higher persistence of skill, ρ, it is optimal to put lower weights on recent signals since abilities do not change as much. Thus, recent signals are less informative. Third, for funds with higher values of ρ, the optimal weights on past returns are more evenly distributed. This is different from the second prediction since the weights on past performance do not sum up to 1 since there is a positive weight on the prior distribution. Finally, the model predicts that funds in different categories have different values of persistence of managerial skills, and optimal weights on signals are different across funds in different strategy categories. Each of these implications holds in the data, which suggests that the model, in which ability is being assessed by investors but changes over time, is a reasonable characterization of the hedge fund industry.

A question related to the above findings is to what extent are investors aware of the way that ability changes. Assuming capital allocation is a revealed preference of investors, flows indicate how investors actually weigh past performance of funds. I find that the weights on signals by investors do not differ across strategies. More specifically, for "Event Driven" funds and "Global Macro" funds, the weights on signals that investor put are almost identical. This result implies that investors are not aware of the difference in ability persistence across fund categories.

I present a formal test of whether investors overreact to information in the context of my model, which is determined by the parameter θ. When I estimate the model using maximum likelihood, I find a value of 8.99 for θ, with *t-stat* equal to 2.10, which is consistent with the notion that investors overreact to recent past returns.

I evaluate the return implications of my model. Expected returns vary considerably depending on how investors chase returns. The model suggests that the expected return of a fund should be positively related to the estimate of the manager's ability that comes from the model. I find that funds with a high estimated ability substantially outperform average funds. In my sample period, the average fund has an abnormal return (relative to the CAPM benchmark) of 3.42% per annum. If, instead, investors optimally adjust their measure of ability every month and take advantage of information in fund flows and prior returns to estimate ability then the annual abnormal return increases to 9.28%. If investors form estimates of ability that are subject to representativeness bias, the average top decile annual abnormal return that investors actually earn equals 7.40%, which is significantly lower than the return using optimal weights. These findings suggest that investors suffer from overreacting to signals about ability. In addition, if one uses a relatively crude measure of ability (assuming it is constant over a 2-year period) and sort expected abilities into deciles, the top decile of ability has a subsequent abnormal annual return of 9.05% in the CAPM. The difference (9.05% versus 9.28%) is not significant in CAPM. In more sophisticated models, the difference in abnormal returns is around 1%. It implies that investors should "chase returns" by putting more weights on more recent returns than more distant returns.

However, investors "chase returns" too much. They suffer from representativeness bias.

The remainder of the paper proceeds as follows. Section 2 reviews the relevant literature, including the literature on hedge fund return chasing, on "assessment based models," especially as applied to delegated portfolio management, and on representativeness bias and the approaches that have been adopted attempts to formalize it. Section 3.1 presents the rational version of the model, and 3.2 allows investors to be subject to a certain level of representativeness bias. Section 4.1 introduces the data and the procedure to clean the hedge fund data. Section 4.2 presents the estimates of risk exposure and the MLE estimates of the prior distribution. Section 4.3 presents the tests of the model's predictions about the relative importance of recent signals and how they relate to ρ. Section 4.4 tests whether investors are subject to representativeness bias, and Section 4.5 documents the way in which future returns can vary depending on the way that investors chase returns. Section 5 concludes the paper.

2 Literature Review

2.1 Return Chasing Literature

Return chasing behavioral has been studied both theoretically and empirically. Theory works on return chasing focus on explaining the behavior in the stock market. DeLong et al. (1990) builds a model to explain that return chasing activities causes destabilization of the market with the existence of rational speculators. Rational speculators anticipate stock movements. They buy future winners which pushes prices away from fundamentals. Barberis and Shleifer (2003) documents style investing. The authors claim that many investors allocate funds based on relative past performance, moving into styles that have performed well in the past. Hong and Stein (1999) introduces a unified theory of under-reaction, momentum trading, and over-reaction in asset market. Under the assumption that information diffuses gradually, momentum traders profit from trend-chasing. Rabin (2002) models investors who believe in law of small numbers.

Some empirical works document the behavior of return chasing in mutual funds and hedge funds. Baquero and Verbeek (2015) finds flows react to performance streaks. The authors also find that by chasing performance streaks, investors earn lower return than by using rational models using historical performance. Fung et al. (2008) finds evidence of flow being positively correlated with performance ranks of hedge funds. My work differs from theirs by studying how should investors weigh recent performance and distant performance differently. In my paper, the past performance is not treated as one single variable. Instead, past performances are treated heterogeneously based on how recent they are. In my model, investors should put more weight on recent performance since managerial skills change over time.

In addition, Wang and Zheng (2008) finds a significant and positive relation between aggregate flows and past aggregate hedge fund performance. Agarwal et al. (2004) finds flows chase recent good performance. The authors also find that the performance-flow relation is convex. Moreover, they find that money-flows are significantly higher (lower) for funds that are persistent winners (losers). Last, they observe a positive relation between flows and delta after controlling for recent performance. The larger is the value of delta, the greater are the managerial incentives to deliver superior performance. Ding et al. (2009) finds hedge funds exhibit a convex flow-performance relation in the absence of share restrictions (similar to mutual funds), but exhibit a concave relation in the presence of restrictions. The authors also find that money is "smart," that is, fundpredict future hedge fund performance. Getmansky et al. (2019) further studies the relation between share restriction on flow and claim that investors rationally endogenizing the expected future binding restrictions when investing their money. Baquero and Verbeek (2009) explores the flow-performance relationship by separating inflows and outflows using a regime-switching model. The authors find a weak positive response of fund inflows to past performance at quarterly horizons but a very pronounced positive response of fund outflows to past performance. However, this pattern is reversed at an annual horizon. They do not find evidence that hedge fund flows can predict future returns, i.e., the "smart money" effect.

2.2 Performance Evaluation Literature

My work relates to the literature on evaluating managerial skills and fund performance. My model essentially extends Berk and Green (2004) to a model which allows managerial skill change over time. When managers' abilities vary over time, the perceived ability is modeled in Holmström (1999). The model has been applied in research on corporate governance, for example, Hermalin and Weisbach (2017). Investors update their belief on managerial ability following the Bayes' law. The expected ability is a weighted average of the prior distribution and past performance information. The persistence of managerial ability governs how should investors weigh past performance. The model of Berk and Green (2004) has been widely applied in the study of assessing managerial skills in portfolio management industry. Stambaugh (2019) introduces a general equilibrium model on managerial skill and fee revenue. Roussanov et al. (2019a) applies the model on mutual funds and study how does the level of advertisement is endogenously determined. Cavagnaro et al. (2019) applies MCMC Bayesian learning model of skills into private equity industry. Korteweg and Sorensen (2017) uses Bayesian method to quantify the effect of skill on LP returns. Baks et al. (2001) examines track records of active mutual funds and find that extremely skeptical prior beliefs about skill would be required to produce zero investment in all funds. The authors solve the Bayesian portfolio problem fund by fund.

There are also a number of empirical works to study managerial skills in hedge funds. Fung and Hsieh (2001) provides 7 factors to evaluate performance. Agarwal and Naik (2004) finds option prices predict hedge fund returns. Giglio et al. (2018) adds latent factors into traditional factor models (CAPM, Fama and French three-factor model, Agarwal and Naik (2004) option-based factor model) to evaluate managers' abilities. Avramov et al. (2011) examines whether are hedge fund managers able to deliver alpha in the context of macroeconomic conditions. The authors begin from the observation that some strategies, such as global macro, perform better in times of crisis than others, such as equity long/short. They apply the Bayesian framework of Avramov and Wermers (2006) to compare groups of hypothetical hedge fund investors with varying beliefs about the predictability of managerial skill. Cao, Farnsworth, and Zhang (2014) uses differences in the environments in which new hedge funds are launched to identify managerial skill. The authors find the skill-driven inceptions outperform the demand-driven inceptions by approximately 4 - 5% per year.

Researchers also study hedge fund performance using different statistical approaches. Kosowski et al. (2007) uses a bootstrap method to examine hedge fund performance. The objective is to test whether hedge fund alpha can be explained by luck alone. The results indicate that the performance of the top hedge funds (ranked by the t-statistic of the alpha) cannot be attributed to chance alone. Chen et al. (2012) uses the Expectation-Maximization algorithm to infer managerial skill. The authors assume managers fall into a discrete number of skill categories. The findings suggest that approximately 50% of hedge fund managers possess skill. Jagannathan, Malakhov, and Novikov (2010) also evaluates whether "hot hands" exist among hedge fund managers. The authors use relative fund performance to predict future relative performance. They model individual fund performance as a function of overall equity market and fund style index performance using both generalized method of moments (GMM) and weighted least squares (WLS) techniques to minimize measurement error. Their results suggest that hedge fund performance is persistent at a 3-year horizon and that this persistence is largely explained by persistence in top performers.

My study is also related to the growing body of literature that has investigated how investors evaluate fund managers' abilities. Berk and van Binsbergen (2016) shows that the Capital Asset Pricing Model (CAPM) is the closest model to the model that investors use to make their capital allocation decision in the mutual fund industry. Barber et al. (2016) shows mutual fund investors react most to market risk (beta) when evaluating funds and treat returns attributable to size, value, momentum, and industry factors as alpha. As for hedge funds, the question has been studied by Agarwal et al (2018), which shows that CAPM alpha explains hedge fundflows better than alphas from more sophisticated factor models. Ben-David et al. (2018) shows Morning Star ratings predict in flows better than CAPM alpha in the mutual fund industry. Baquero and Verbeek (2015) explores the role of information when investors allocate capital among hedge funds. The authors find that investors react to performance streaks. They show that investors invest in funds when they outperform some benchmark in consecutive months.

2.3 Behavioral Literature

My results relate closely to the findings in the literature from psychology and economics that document the behavioral biases of investors. It can be traced back to Tversky and Kahneman (1971; 1972), which introduces representatives bias. Representativeness bias is also referred as "law of small number." Tversky and Kahneman defined representativeness as "the degree to which [an event] (i) is similar in essential characteristics to its parent population, and (ii) reflects the salient features of the process by which it is generated." It leads to two well-known biases in pattern recognition: the "gambler's fallacy" and "hot-hand fallacy." The belief that, if something happens more frequently than normal during a given period, it will happen less frequently in the future (or vice versa) is termed the "gambler's fallacy." Those, on the other hand, who attribute causal significance to a series of signals inferred (mistakenly) to be too long to be random (i.e., the coin is not fair, the manager is talented, the player has a hot hand), expect continuation. This is the rationale for the so-called "hot-hand fallacy." In the context of hedge funds, investors who chase recent performance signals and ignore both the prior skill distribution and the distant performances of a fund suffer from the "hot-hand fallacy."

Gennaioli and Shleifer (2010) and Bordalo et al. (2019) provide a model of investors picking stocks when they are subject to representativeness bias. My work builds upon their model and apply their model in hedge funds. My result suggests that hedge fund investors have "representativeness bias." They react too strongly to recent performance signals.

3 A Bayesian Learning Model of Ability

3.1 Rational Model

Let return be a linear function of ability ($\alpha_{i,t}$), risk exposure ($\sum_{k=1}^{K}\beta_{i,k,t}F_{k,t}$) and luck ($\epsilon_{i,t}$). I.e.,

$$R_{i,t} - R_{rf,t} = \alpha_{i,t} + \sum_{k=1}^{K}\beta_{i,t}F_{k,t} - D(A_{i,t}) + \epsilon_{i,t}, \tag{1}$$

where $\{F_{k,t}\}_{k=1}^{K}$ are risk factors. I use 4 different risk models, including the CAPM, the Carhart (1997) four-factor model (Carhart4), the Agarwal and Naik (2004) option-factor model (AN), and the Fung and Hsieh (2001; 2004) seven-factor model (FH7). For example, for the Carhart four-factor model (Carhart4) model, the risk factors $\{F_{k,t}\}_{k=1}^{K}$ include market, size, book-to-market, and momentum. $D(A_{i,t})$ captures the effect of scale. In Berk and Green (2004), return decreases when the assets under management ($A_{i,t}$) is large. Following Berk and Green (2004), I allow return change with the amount of assets under management.

Assume $\epsilon_{i,t}$ are independent and normally distributed. I.e., let

$$\epsilon_{i,t} \sim \mathbb{N}(0, 1/\omega).$$

Let $\alpha_{i,t}$ evolves based on the following.

$$\alpha_{i,t+1} = (1-\rho)\mu + \rho\alpha_{i,t} + \sqrt{1-\rho^2}\nu_{i,t+1},$$

where $\nu_{i,t+1}$ are independent and normally distributed with mean zero and precision η. The initial ability levels are drawn from a normal distribution with mean μ and precision η. I.e., for any fund i, at any time t,

$$\alpha_{i,1} \sim \mathbb{N}(\mu, 1/\eta),$$
$$\nu_{i,t+1} \sim \mathbb{N}(0, 1/\eta).$$

Define the expected ability $\phi_{i,t}$, and the variance of ability σ_t^2 to be the following.

$$\phi_{i,t} := \mathbb{E}[\alpha_{i,t}|R_{i,t-1}, \cdots, R_{i,1}].$$

The expected ability is a function of the prior distribution and past returns. It can be written as the following recursive form. $\forall t \geq 2$,

$$\begin{aligned}
\phi_{i,t} &= \rho[\phi_{i,t-1} + \frac{\sigma_{t-1}^2}{\sigma_{t-1}^2 + 1/\omega}(R_{i,t-1} - R_{rf,t-1} - \sum_{k=1}^{K} \beta_{i,k,t-1}F_{k,t-1} + D(A_{i,t-1}) - \phi_{i,t-1})] + (1-\rho)\mu, \\
\sigma_t^2 &= \rho^2 \frac{1}{\omega + 1/\sigma_{t-1}^2} + (1-\rho^2)\frac{1}{\eta}.
\end{aligned}$$

When $t = 1$, $\phi_{i,1} = \mu$ and $\sigma_1^2 = 1/\eta$.

Assume

$$D(A_{i,t}) = \xi \frac{A_{i,t}^{\gamma} - 1}{\gamma}, \gamma \in [0, 1].$$

When $\gamma = 1$, the return decreases linearly with the amount of asset under management of a fund i. And when $\gamma = 0$, then the return decreases with the log of $A_{i,t}$. When $\xi > 0$, then there is a decreasing return to scale.

Implications from the model:

1. The weight on recent signal is lower for an older fund than that for a younger fund.

Proof:

$$\frac{\sigma_{T-1}^2}{\sigma_{T-1}^2 + 1/\omega} \text{ decreases in } T.$$

2. For a fund with higher ρ, the weight on recent signal is lower.

Funds with higher ρ have more persist managerial skills. They tend to be funds with lower managerial turnover rate. Investors should weigh performance signals less than the funds with lower ρ. In addition, the property implies that the weights on signal are different across funds adopting different strategies because funds using different strategies have different ρ parameters. In section 4, I show that managerial skills in "Event Driven" funds vary more than the managerial skills in "Global Macro" funds. And the weight on signal for "Event Driven" funds is higher than that for "Global Macro" funds. It is consistent with the prediction above.

3. For a fund with higher ρ, the curvature of weights is lower. In other words, weights are more evenly distributed. The curvature is defined as follows.

Curvature = weight on last s^{th} month information/weight on the last $(s+1)^{\text{th}}$ month information.

$$curvature = 1 + \frac{\sigma_{t-s}^2}{\rho\sigma_{t-s-1}^2/(1/\omega + \sigma_{t-s-1}^2)}$$

Notice that this implication is not equivalent to implication 2 because the weights on past performance information do not add up to 1. There is positive weight on the prior belief from the Bayesian model. Implication 2 is about the absolute weight on the performance signal, while implication 3 is about the relative weight on a recent signal to the fund's distant performance information.

3.2 A Model of Representativeness Bias

As in Gennaioli and Sheifer (2010), consider a decision maker assessing the distribution $h(T = \tau|G)$ of a variable T in a group G. Gennaioli and Sheifer (2010) define the representativeness of the specific type τ for G as:

$$R(\tau, G) \equiv \frac{h(T = \tau|G)}{h(T = \tau| - G)}.$$

A type is more representative if it is relatively more frequent in G than in the comparison group $-G$. To capture overestimation of representative types, Bordalo et al. (2018) assume that probability judgements are formed using the representativeness-distorted density:

$$h^\theta(T = \tau|G) = h(T = \tau|G)\left[\frac{h(T = \tau|G)}{h(T = \tau| - G)}\right]^\theta Z,$$

where $\theta \geq 0$ and Z is a constant ensuring that the distorted density $h^\theta(T = \tau|G)$ integrates to 1. The extent of probability distortions increases in θ. $\theta = 0$ capturing the rational benchmark.

Here, $h(\alpha_{i,t}|R_{i,t})$ is the distribution of $\alpha_{i,t}$ after observing $R_{i,t}$, and $h(\alpha_{i,t}|-R_{i,t})$ is the distribution of $\alpha_{i,t}$ before observing $R_{i,t}$. We have

$$h(\alpha_{i,t}|R_{i,t}) = \frac{1}{\sqrt{2\pi/(\omega+1/\sigma_t^2)}} \exp\left(-\frac{\left(\alpha_{i,t}-\phi_{i,t}-\frac{\sigma_t^2}{\sigma_t^2+1/\omega}(\tilde{R}_{it}-\phi_{i,t})\right)^2}{2/(\omega+1/\sigma_t^2)}\right),$$

where $\tilde{R}_{i,t} = R_{it} - R_{rf,t} - \sum_{k=1}^{K}\beta_{i,k,t}F_{k,t} + D(A_{i,t})$. And

$$h(\alpha_{i,t}|-R_{i,t}) = \frac{1}{\sqrt{2\pi\sigma_t^2}}\exp(-\frac{(\alpha_{i,t}-\phi_{i,t})^2}{2\sigma_t^2}).$$

Thus, the distorted belief of distribution of $\alpha_{i,t}$ is the following.

$$h^\theta(\alpha_{i,t}|R_{i,t}) = constant \cdot \exp\left(-(1+\theta)\frac{\left(\alpha_{i,t}-\phi_{i,t}-\frac{\sigma_t^2}{\sigma_t^2+1/\omega}(\tilde{R}_{it}-\phi_{i,t})\right)^2}{2/(\omega+1/\sigma_t^2)} + \theta\frac{(\alpha_{i,t}-\phi_{i,t})^2}{2\sigma_t^2}\right).$$

$$h^\theta(\alpha_{i,t}|R_{i,t}) = C\cdot\exp\left(-(\frac{1+\theta}{2/(\omega+1/\sigma_t^2)} - \frac{\theta}{2\sigma_t^2})\alpha_{i,t}^2 + \left(\frac{(1+\theta)(\phi_{i,t}+\frac{\sigma_t^2}{\sigma_t^2+1/\omega}(\tilde{R}_{it}-\phi_{i,t}))}{1/(\omega+1/\sigma_t^2)} - \frac{\theta\phi_{i,t}}{\sigma_t^2}\right)\alpha_{i,t}\right),$$

where C is some constant.

Therefore, with representativeness bias, the distorted belief on $\alpha_{i,t}$ follows a normal distribution. Let $\phi^{\theta}_{i,t+1}$ denote the expectation of $\alpha_{i,t}$ with representativeness bias. We have

$$\begin{aligned}
\phi^{\theta}_{i,t} &= \frac{\frac{(1+\theta)(\phi_{i,t}+\frac{\sigma_t^2}{\sigma_t^2+1/\omega}(\tilde{R}_{it}-\phi_{i,t}))}{1/(\omega+1/\sigma_t^2)} - \frac{\theta\phi_{i,t}}{\sigma_t^2}}{\frac{1+\theta}{2/(\omega+1/\sigma_t^2)} - \frac{\theta}{2\sigma_t^2}}, \\
&= \begin{cases} \phi_{i,t} + \frac{\rho\frac{\theta}{\omega+1/\sigma_t^2}\frac{\sigma_t^2}{\sigma_t^2+1/\omega}(\tilde{R}_{i,t}-\phi_{i,t})}{(1+\theta)\sigma_t^2-\frac{\theta}{\omega+1/\sigma_t^2}} & \text{if } \theta > 0, \\ \phi_{i,t} & \text{if } \theta = 0. \end{cases}
\end{aligned}$$

3.3 Estimation Procedure

I provide an outline for the estimation below. The more detailed description of the estimation process in presented in Section 4.

1. Estimate $\beta_{i,k,t}$ using rolling window regression.

2. Estimate skill distribution parameters.

   Let $\Gamma := (\mu, \omega, \eta, \rho, \xi, \gamma)$. Estimate Γ using MLE. More formally, Γ maximizes the likelihood of future risk-adjusted returns equals to expected abilities, which are the summations of belief in ability and the future risk exposure. Let

$$R_{i,t} - R_{rf,t} = \phi_{i,t}(\Gamma) + \sum_{k=1}^{K} \beta_{i,k,t}F_{k,t} - D(A_{i,t}) + u_{i,t},$$

where $u \sim \mathbb{N}(0, 1/\omega)$. Then $R_{i,t} - R_{rf,t} \sim \mathbb{N}(\phi_{i,t}(\Gamma) + \sum_{k=1}^{K} \beta_{i,k,t}F_{k,t} - D(A_{i,t}), 1/\omega)$. Define $\hat{\Gamma}$ as follows.

$$\begin{aligned}
\hat{\Gamma} &:= argmax_{\Gamma} log(L(\Gamma; R)) \\
&= argmax_{\Gamma} \sum_{i,t} log(\omega) - \left(R_{i,t} - R_{rf,t} - (\phi_{i,t}(\Gamma) + \sum_{k=1}^{K} \beta_{i,k,t}F_{k,t} - D(A_{i,t})) \right)^2.
\end{aligned}$$

3. Estimate θ using MLE.

Let θ be the parameter value that investors use to allocate their capital. Notice that flows and returns are not in the same scale, we cannot equate flows to the beliefs on managers' ability. In Berk and Green (2004), flow in equilibrium is linear in belief. I follow the convention. Let

$$flow_{i,t} = \kappa\phi_{i,t} + v_{i,t},$$

where κ is a constant and $v \sim \mathbb{N}(0, \sigma_v^2)$. Then $flow_{i,t} \sim \mathbb{N}(\kappa\phi_{i,t}(\theta), \sigma_v^2)$. $(\hat{\kappa}, \hat{\eta})$ is defined to be the pair of parameters that maximizes the likelihood of flows equal to expected ability times a constant. I.e.,

$$\begin{aligned}(\hat{\kappa}, \hat{\theta}) &:= argmax_{\kappa,\theta} log(L(\kappa, \theta; flow)) \\ &= argmin_{\kappa,\theta} \sum_{i,t} \left(flow_{i,t} - \kappa\phi_{i,t}(\theta)\right)^2.\end{aligned}$$

4 Empirical Analysis

My empirical analysis uses the Lipper Trading Advisor Selection System (TASS) database. These data include monthly information on performance and assets under management (AUM) for hedge funds. Because the Lipper TASS dataset is subject to several potential biases, I follow closely the bias correction procedures of Sinclair (2018)[1] and Getmansky et al. (2015).

TASS database is subject to "backfill bias." It exists because funds have an incentive to start to report to TASS after a period of outperformance. More precisely, hedge funds choose when to start reporting to TASS. They have the option of providing backfilled data, which is data prior to the date they joined the database. Fung and Hsieh (2000) estimate that backfill

[1]Many thanks to Andrew Sinclair for sharing his code with me.

bias of 1.4% per year in TASS using a sample from 1994 to 1998. As in Sinclair (2018), I use performance data after a fund joins TASS. Most hedge funds that joined TASS before March 2011 report their "DateAddedtoTASS" information. However, TASS discontinued this field after March 2011. For funds added after that date, the field is blank. For funds joined TASS after 2011, I estimate the date that a fund joined TASS using the methodology proposed by Jorion and Schwarz (2019). I delete fund-month observations that are prior to estimated dates of the funds.

I further clean the data as recommended by Getmansky et al. (2015) and Sinclair (2018). Whenever possible, I compute the returns using the changes in NAV; when NAV is not available, I use reported returns. I remove funds that do not report monthly, and funds that do not report net-of-fee returns. I also remove funds with suspicious returns: monthly returns above 200% or below -100%, stale returns (equal to the past two monthly returns), cases where AUM is reported as zero, and cases where within 6 months, funds display a 10,000% increase immediately reversed, because these are likely to be data entry errors. If a fund stops reporting return or AUM information and then resumes at a future date, I use only the first sequence of uninterrupted data.

I require a history of 24 months of abnormal returns to calculate expected ability. For calculating expected ability, I need 23-month return history for estimating risk exposure and the monthly abnormal return. Moreover, I need 12-month window to estimate future returns and flows. I exclude funds with less than 59 months of monthly returns data. This process leads to a sample of 3,697 funds from 1994 to 2017. It is smaller than the sample size of 8,456 funds in Sinclair (2018) because I exclude funds with less than 59 months of return data.

I define *Fund Flow* as:

$$flow_{i,t} = \frac{AUM_{i,t}}{AUM_{i,t-1}} - (1 + R_{i,t}).$$

Table 1 reports the summary statistics. The summary statistics of hedge fund returns, flows, assets under management, and characteristics of funds are similar to the ones provided in the existing literature (e.g., Agarwal et al. (2018)).

[Table 1 here]

A number of risk factors have been shown by prior work to predict hedge fund returns. However, there is no consensus on which factor model should be used to explain hedge fund returns. For this reason, I use four different models, including the CAPM, the Carhart (1997) four-factor model (Carhart4), the Agarwal and Naik (2004) option-factor model (AN), and the Fung and Hsieh (2001; 2004) seven-factor model (FH7). The risk factors in the CAPM and Carhart4 models include market (MKTRF) and the factors - size (SMB), value (HML), and momentum (UMD) from Fama and French (1993). The AN model includes these four factors plus an out-of-the-money call option factor (OTM CALL) and an out-of-the-money put option factor (OTM PUT). Size (SMB), value (HML), and momentum (UMD) factors are pulled from Kenneth French's data library. The Option factors are constructed by using European call and put option prices from Option-Metric dataset. The FH7 model includes three trend-following risk factors constructed from portfolios of look-back straddle options on currencies (PTFSFX), commodities (PTFSCOM), and bonds (PTFSBD); two equity-oriented risk factors capturing excess market returns (SNPMRF) and the size premium (SCMLC); and two bond-oriented risk factors constructed using 10-year Treasury constant maturity bond yields (BD10RET) and the difference in yields of Moody's BAA bonds and 10-year Treasury constant maturity bonds (BAAMTSY).

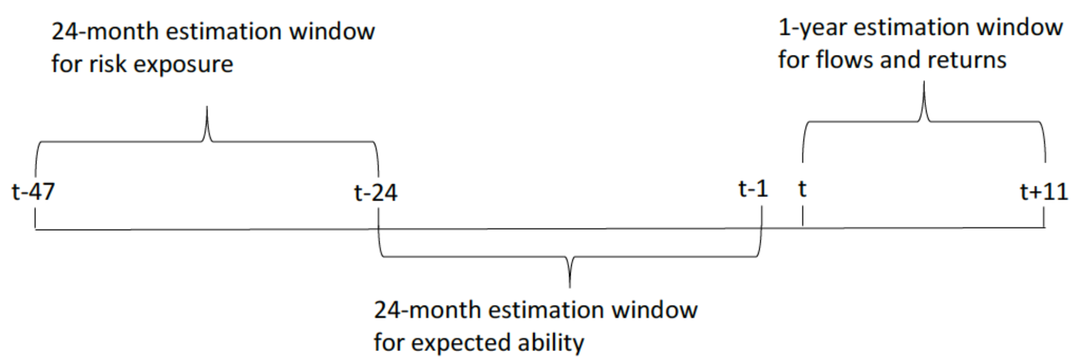
4.1 Estimates of Risk Exposure and the Prior Distribution

In each risk model, I estimate risk exposure by running rolling window regressions using 24 months of return data for each fund. For example, in the CAPM model, where the only risk factor is excess market return, I obtain the factor loading on market by estimating the following model for fund i from months $t-23$ to t:

$$R_{i,s} - R_{rf,s} = \alpha_{i,t} + \beta_{i,t}(R_{MKT,s} - R_{rf,s}) + \epsilon_{i,s}, s = t-23, \cdots, t.$$

The timeline of the estimation process is shown in Figure 1. The same as Agarwal et al. (2018), a 24-month window is required for estimating risk exposure. I multiply the betas with factor returns to get the benchmark returns for each fund at each point in time. We subtract the benchmark return from the net return to get the monthly realized alpha (true alpha plus idiosyncratic shock minus size effect). The above procedure to is the same as Roussanov et al. (2019b). The authors then aggregate the monthly realized alpha to annual level. I simply use the monthly realized alphas due to the fact the hedge funds have shorter lives than mutual funds. The same as Agarwal et al. (2018) and Roussanov et al. (2019b), I use 1-year window for future flows and future returns.

Figure 1: Timeline of the estimation of performance and flow for the flow-performance relation.



I estimate the distribution parameters $\Gamma := (\mu, \omega, \eta, \rho, \xi, \gamma)$ using maximum likelihood. More precisely, Γ maximizes the likelihood that future risk-adjusted returns equals to expected abilities, which are the summations of belief in ability and the future risk exposure.

Let

$$R_{i,t} - R_{rf,t} = \phi_{i,t}(\Gamma) + \sum_{k=1}^{K} \beta_{i,k,t} F_{k,t} - D(A_{i,t}) + u_{i,t},$$

where $u_{i,t} \sim \mathbb{N}(0, 1/\omega)$. Then $R_{i,t} - R_{rf,t} \sim \mathbb{N}(\phi_{i,t}(\Gamma) + \sum_{k=1}^{K} \beta_{i,k,t} F_{k,t} - D(A_{i,t}), 1/\omega)$. Define $\hat{\Gamma}$ as follows.

$$\begin{aligned} \hat{\Gamma} &:= argmax_{\Gamma} log(L(\Gamma, R)) \\ &:= argmax_{\Gamma} \sum_{i,t} log(\omega) - (R_{i,t} - R_{rf,t} - (\phi_{i,t}(\Gamma) + \sum_{k=1}^{K} \beta_{i,k,t} F_{k,t} - D(A_{i,t})))^2 \end{aligned}$$

I present the estimation results in Table 2. The table shows the estimates of the distribution parameters. Across models, the distribution parameters are similar. In addition, it appears that the precision of the ability distribution - η is significantly larger than the precision of idiosyncratic shocks to returns - ω. Moreover, the persistence of the ability is 0.9566 monthly, in the CAPM. It is smaller than the persistence of abilities of mutual fund managers. Roussanov et al. (2019a, 2019b) estimate the persistence of mutual fund managers' skill and find it is about 0.95 annually. Last, the estimate implies that there is no evidence for decreasing returns to scale in hedge funds. This is another difference between hedge funds and mutual funds. In mutual funds, there is evidence for decreasing returns to scale.

[Table 2 here]

4.2 Tests for the Implications from the Rational Model

There are a number of implications of the rational model I presented in 3.1. In particular, the model implies that: 1) the optimal weight an investor should place on a performance signal should vary inversely with the fund's age; 2) the optimal weight on a performance signal should vary inversely with the persistence of managerial ability of a fund; 3) weights should be distributed more evenly when the persistence of managerial ability of a fund is low. Notice that prediction 3 is not implied by prediction 2 since the weights on past returns do not sum to one. There is a positive weight on the prior distribution implied by the Bayes' law.

In this section, I test each of these predictions.

4.2.1 How does the optimal weight on signal change with the age of a fund?

For each fund-month observation, I define *Age* as the age (in months) of the fund up to that point in time. Observations are grouped into portfolios based on the Age variable. For each portfolio of funds, I estimate and optimal weights on each of the past 24 months using an autoregressive model. More precisely, for each portfolio, I regress future abnormal returns on abnormal returns of the past 24 months. Take the CAPM as an example. The abnormal return for fund i at month t is $R_{i,t} - R_{rf.t} - \beta_{i,t} R_{MKT,t}$, where the risk exposures are defined in section 4.2. The abnormal returns for the portfolio are calculated by weighting each fund equally. Let $\tilde{R}_{p,t}$ denote the abnormal return for the portfolio p at month t. The weights $\{w_{p,1}, \cdots, w_{p,24}\}$ are estimated as follows.

$$\tilde{R}_{p,t} = \sum_{s=1}^{24} w_{p,s} \tilde{R}_{p,t-s} + \epsilon_{p,t}.$$

For portfolio p, define the weight on signal as $w_{p,1}$. I then estimate the way these weights are affected by *Age* of the portfolios. In particular, I estimate:

$$w_{p,t} = \beta^{age} Age_p + \epsilon_p^{age}.$$

[Table 3 here]

I present estimates of this equation in Table 3. Consistent with the model's prediction, the estimate of the coefficient β^{age} is significantly negative in all models. For example, in the CAPM model, the coefficient is -0.0043. It implies that for a fund (Fund A) that is 3 years older than another fund (Fund B), the optimal weight on abnormal return of the past quarter for Fund A should be 1.2 percentage point lower than that for Fund B.

4.2.2 How does the weight on a signal change with the persistence of managerial ability for a fund?

In this subsection, I test the hypothesis that the optimal weight on signal should be negatively correlated with the persistence of managerial ability ρ.

First, I construct portfolios based on fund strategies and measure the correlation between optimal weights and the persistence of managerial ability ρ in each strategy category.

More precisely, I estimate ρ for funds in each strategy by MLE. The MLE estimation is the same as described in Table 2, using subsamples consisting of funds employing a particular strategy instead of the full sample. I rank the categories by ρ from low to high. The estimates of ρ are shown in column 2 of Table 4. In addition, for each fund category, I estimate the optimal weights on each of the past 24 months using a autoregressive model. More precisely, for each portfolio, I regress future abnormal returns on abnormal returns of past 24 months. The optimal weight on signal is defined to be the summation of coefficients on the abnormal returns of the previous 3 months. The analysis is based on CAPM model. I present these estimates in Table 4, which shows the persistence of managerial ability ρ across strategies, as well as estimates of the relation between the weight on the signal and the persistence - ρ for each strategy.

[Table 4 here]

The estimates of ρ provide a rough way of gauging the usefulness of the changing ability model. An important feature of the hedge fund industry is that funds are constantly developing new trading strategies to earn abnormal returns. However, these strategies often are profitable for some period of time and less profitable during other period of times. For "Event Driven" funds or funds that exploit particular arbitrage opportunities, it is likely that opportunities will be relatively short-lived. Therefore, persistence of managerial skill appear to be lower since the profitability of strategies is likely to vary substantially over time. In contrast, for funds that take advantage of skills that are likely to be long-lived, such as an ability to value companies well or an understanding of the macroeconomy, profit opportunities are likely to last longer. For these strategies, the persistence of managerial ability -ρ is likely to be high.

The estimates of ρ presented in Table 4 are consistent with this idea. The smallest estimates are for "Fixed Income Arbitrage" and "Event Driven", for which profits are likely to be relatively short-lived. In contrast, "Global Macro Equity" and "Multi-Strategy", for which profitability depends on valuation skills of hedge fund managers that do not vary that much over time, have relatively large estimates of ρ.

In addition, when skills are relatively short-lived, more recent performance should be more informative about skill and future profitability. Therefore, we expect that when ρ is relatively low, investors should place higher weights on more recent performance when evaluating managers' skills. Moreover, when ρ is relatively high, investors should weigh recent signals less.

Empirically, the weights on signals presented in Table 4 decrease almost monotonically as ρ increases. Take "Event Driven" versus "Global Macro" strategy as an example. Compared to "Global Macro" funds, a lower estimate of ρ for "Event Driven" strategy implies a lower persistence of ability for fund managers using "Event Driven" strategy. It implies that the performance history for "Event Driven" funds is less informative. Thus, the weight on signal should be higher for "Event Driven" funds than for "Global Macro" funds. The autoregressive estimates confirm this hypothesis. The weight on recent performance in the autoregressive model for "Event Driven" funds is 13.10%, which is significantly higher than that of the "Global Macro" funds. p -value of the weights being the same equals 0.

Interesting, the weights on signals by investors do not differ across strategies. More precisely, for each strategy category, I regress future flows of funds on the past 24 month abnormal returns. The weights on recent signal for "Event Driven" funds and "Global Macro" funds are similar. It implies that investors are unaware of the difference in ability persistence across strategy categories.

Next, I formally test the hypothesis that the optimal weight on recent signal should be negatively correlated with the persistence of managerial ability ρ. For each fund, I estimate the optimal weights on the past 24 months abnormal returns using an autoregressive model. More precisely, for each fund, I regress future abnormal returns on abnormal returns of the fund of the past 24 months. The optimal weight on signal is defined in two ways. It is defined either to be the summation of coefficients on the last 3 months abnormal returns or to be the coefficient on the last month abnormal return. In addition, for each fund, I estimate ρ using MLE. The MLE estimation is the same as described in Table 2, but using samples in each fund instead of using the full sample. I regress the optimal weights on signal on ρ across funds. I.e.,

$$w_{i,1} = \beta^{\rho} \rho_i + \epsilon_i^{\rho}.$$

The coefficient β^{ρ} is expected to be negative.

[Table 5 here]

I present estimates of this equation in Table 5. Consistent with the predictions from the model, the coefficients on β^{ρ} are negative and statistically significantly different from zero. For example, using the CAPM as the benchmark risk model, consider a fund (Fund A) with $\rho = 0.6649$, the same as a "Fixed Income Arbitrage" fund, and a fund (Fund B) with $\rho = 1$, the same as a "Global Macro" fund. Then the optimal weight on performance in last quarter for Fund A is 8.38 percentage points higher than that of Fund B. In other words, funds with higher values of ρ have lower weights on recent return signals. It is consistent with the notion that performance farther back in time is more informative about future ability for these types of funds.

4.2.3 How does the curvature of weights change with the persistence of managerial ability for a fund?

In this subsection, I test the hypothesis that the curvature of weights decreases with the persistence of managerial ability of a fund. The curvature captures the relative importance of the recent performance information to the distant performance information. When curvature is higher, more weight should be placed on the recent performance signal relative to distance performance. Therefore, for funds with more persistent managerial skills, the weights on past performance information should be more evenly distributed. The procedure to test the hypothesis is as follows.

Similar to prior tests, For each fund, I estimate and optimal weights on the past 24 month abnormal returns using an autoregressive model. More precisely, for each fund, I regress future abnormal returns on abnormal returns of the past 24 months. The optimal weights are defined to be the coefficients on the abnormal returns of the past 24 months. Curvature is defined to be the weight on the previous month abnormal return divided by the weight on the abnormal return of the second previous month. Formally, for fund i, the curvature is defined by:

$$curvature_i = \frac{w_{i,1}}{w_{i,2}},$$

where $w_{i,1}$, $w_{i,2}$ are weights from the autoregressive model estimation. In addition, for each fund, I estimate ρ using MLE. The MLE estimation is the same as described in Table 2, but using samples in each fund instead of using the full sample. I regress the curvature on ρ across funds. I.e.,

$$curvature_i = \beta^{cur} \rho_i + \epsilon_i^{cur}.$$

I present the results in Table 6.

[Table 6 here]

Table 6 shows that the coefficient is significantly negative for all the four models. For example, in the CAPM, β^{cur} being negative implies that the higher the persistence of managerial ability, the more evenly weights should be distributed across past performance.

4.3 Estimate of the degree of representativeness bias – θ

The main difference between the rational and behavioral models presented above is that in the behavioral model, agents overreact to information. The model follows the formalization suggested by Bordalo et al. (2018) and characterizes this behavior through a parameter θ. In this subsection, I estimate θ using MLE.

I follow Berk and Green (2004) and assume that flow in equilibrium is linear in belief, so:

$$flow_{i,t} = \kappa \phi_{i,t} + \nu_{i,t},$$

where κ is a constant and $\nu \sim \mathbb{N}(0, \sigma_\nu^2)$. Then $flow_{i,t} \sim \mathbb{N}(\kappa \phi_{i,t}, \sigma_\nu^2)$. $(\hat{\kappa}, \hat{\theta})$ is defined to be the pair of parameters that maximizes the likelihood of flows equal to expected ability times a constant. I.e.,

$$
\begin{aligned}
(\hat{\kappa}, \hat{\theta}) &:= argmax_{\kappa,\theta} log(L(\kappa, \theta; flow)) \\
&= argmin_{\kappa,\theta} \sum_{i,t} (flow_{i,t} - \kappa \phi_{i,t}(\theta))^2,
\end{aligned}
$$

The estimates for θ are shown in Table 7.

[Table 7 here]

The estimates in Table 7 indicate that θ is positive and statistically significantly different from zero. The degree of representativeness bias - θ being significantly positive implies that there is positive degree of representativeness bias among hedge fund investors. The investors react too strongly to recent return signals and place less weight than they should on the prior distribution and distant performance information.

Notice that θ is estimated separately from the distribution parameters $\Gamma = (\mu, \omega, \eta, \rho, \xi, \gamma)$. Another way is to estimate θ together with the distribution parameters Γ. The estimates are almost identical in both ways.

I also estimate distribution parameters $\Gamma = (\mu, \omega, \eta, \rho, \xi, \gamma)$ and the level of representativeness bias θ using Bootstrapping, as a way to control and check the stability of my results in Table 2 and Table 7. More specifically, I estimate the 95% confidence intervals of the above parameters using Bootstrap. DiCiccio and Efron (1996) has shown that bootstrap is asymptotically more accurate than the standard intervals obtained using sample variance and assumptions of normality. The estimation procedures and results are shown in the Appendix. The estimates from Bootstrapping are close to the estimates from the standard MLE method.

4.4 Return Implications

In the model presented in Section 3, investors chase past returns because past abnormal returns are indicative for the managers' abilities, which in turn is likely to affect future abnormal returns. If, as suggested by the model, investors infer the ability of fund managers to earn future returns from their past returns, then it should be the case that funds whose expected ability is high should outperform other funds. An empirical prediction of my model is that investors who chase past returns should outperform investors who buy and hold funds long-term.

In addition, the model provides explicit formulas that investors should use for weighting past returns to obtain the best estimate of managers' abilities. Therefore, a prediction of the model is that returns that come from a measure of ability derived from the optimal weights should be higher than returns from a simpler strategy of return chasing.

Finally, the estimates of θ are positive. It implies that investors overweight more recent returns when estimating managers' abilities relative to the optimal weighting scheme. I quantify the cost of the representativeness bias to investors. More precisely, to evaluate how this behavior by investors affects the returns they receive, I compare the future returns one would get under an optimal weighting scheme to the one investors receive if they form expectations given they are subject to representativeness bias.

I evaluate these implications in this subsection. Expected abilities can be estimated using different weighting methods. When investors are rational, they use the weights derived from optimal return chasing scheme. To show the returns from optimal return chasing, in each month, I sort funds into 10 portfolios based on their expected ability. The future abnormal returns from investing in the 10 portfolios are reported in A of Table 8.

[Table 8 here]

This table reports the future risk-adjusted returns of funds sorted by their expected abilities. The future risk-adjusted returns are reported in percent per year. The future risk-adjusted returns are annualized from future monthly returns. The future monthly risk-adjusted returns are the time-series averages of future monthly returns adjusted for risk. The t-statistics are the average future returns divided by the time-series standard errors.

Table 8 shows the return implications in all 4 models. I compare the future returns one would get under an optimal weighting scheme to the return investors would receive if they form expectations given representativeness bias. Take the CAPM model as an example, from investing in the top decile funds when investors over react to signals, the annual risk-adjusted return is 7.40%. Comparing the monthly time-series returns investors would receive from optimal return chasing to the returns with representativeness bias, investors lose 1.88% return per annum due to the behavioral bias. Investors should make 7.40% per annum if they chase returns optimally.

Next, I compare the returns from optimal return chasing to the returns from naïve chasing strategy. By putting optimal weights on a fund's past performance information, investors earn an annual risk-adjusted return of 9.28%, based on the CAPM model. This return is significantly higher than the return from using the naïve return chasing strategy, which is putting equal weights on past performance information. The return from investing into the top funds using the naïve return chasing strategy is only 9.05%. The differences in returns between optimal return chasing and naïve return chasing strategy are bigger in more sophisticated models. It implies that ability is evaluated more accurately by putting more weights on recent performance information and less weights on distant return information. The results are shown in Table 9.

[Table 9 here]

In conclusion, expected returns vary considerably depending on how investors chase returns. The model suggests that funds returns should be positively related to the estimates of ability that come from the model. I find that funds with a high estimated ability substantially outperform average funds. In my sample period, the average fund has an abnormal return (relative to the CAPM benchmark) of 3.42% per annum. In contrast, if we use a relatively crude measure of ability (assuming it is constant over a 2-year period), the top decile of ability has a subsequent abnormal annual return of 9.05%. If, instead, investors optimally adjust their measure of ability every month and take advantage of information in fund flows and prior returns to estimate investors' perceptions of ability, then the annual abnormal return increases to 9.28%. However, investors actually earn only 7.40% by investing in funds that they believe are in the top decile. The flow data implies that investors are subject to representativeness bias. The bias cost investors 1.88% loss in returns.

5 Conclusion

The paper answers the question of how rational it is to chase hedge fund returns. It is a stylized fact that investors react to recent performance of hedge funds. I study how rational it is for investors to do that. More precisely, I quantify the optimal level of return chasing. The persistence of managerial skill is significantly less than 1. It implies that it is rational to "chase" returns. Or more formally, the optimal pattern for assessing the managerial ability based on past return information is to weigh recent return information more than distant return information. However, investors appear to chase returns too much. The weighting scheme that investors use is irrational and it cost them significant return lose.

In the paper, I present a model of accessing managerial skills and test its implications. The model is an extension of the Berk and Green (2004) model, in which investors are continually assessing the "ability" of fund managers to earn abnormal returns. To capture the dynamic of strategies being more or less profitable over time, I depart from the Berk and Green (2004) setup and allow ability to change over time, with the rate of change a function of the type of strategy the fund employs. I also present a version of the model in which investors exhibit representativeness bias and overreact to information.

I estimate this model on a large sample of hedge funds. A novel feature of this estimation is that estimates of ρ, the persistence of managerial ability, can be interpreted as the extent to which the profitability of funds' strategies persists over time. The variable, as well as the other characteristics of funds, affects the pattern of return chasing in the manner suggested by the model. For example, the estimates indicate that funds using strategies of Event driven or Fixed income arbitrage have faster changing skills/profit than funds that using Global macro strategy. Interestingly, the estimates suggest that investors have not realize the fact. Investors weigh performance signals similarly for Event Driven funds and Global macro funds. In addition, for younger funds, the weight on the performance signals appears to be higher than that for an older fund, again, consistent with the model's implications. Finally, for funds with more persistent managerial skill, the weights investors place on past performance information appear to be more evenly distributed.

In addition, I find that investors are subject to a certain degree of representativeness bias when making investment decisions. Maximum likelihood estimates indicate that investors respond to recent performance by even more than the model suggests is optimal. Take the CAPM model as an example, investor lose 1.88% because of the representativeness bias. Investors should earn 9.28% annually if they chase returns in the optimal way. However, investors earn only 7.40% because of the representativeness bias. Consider a strategy that long the top funds and short the bottom funds. Investors can earn 11.73% abnormal return if they use optimal weights. However, they earn only 8.96% because they are subject to representativeness bias.

Unlike other asset classes such as mutual funds, return chasing in hedge funds increases investor performance. While a buy and hold strategy in average hedge funds earns 3.42% more than investing in public equity markets in the CAPM model, even a "naïve" return chasing strategy can do better: investing in the top decile of funds based on past performance outperforms a buy and hold strategy by 5.63%. However, investors can do still better. I find investors should value past month performance 3.04 times more than the past 24th month performance when evaluating fund managers' abilities. By investing in the top decile funds chosen by optimal return chasing outperform naïve return chasing by 0.23% per annum.

In conclusion, the paper provides a framework to understand return chasing behavior of hedge fund investors. On one hand, performance information indicates managerial skills. And managerial skills change over time. More recent performance is more indicative of managerial skills. Thus, it is rational to weigh recent performance information more than distant performance information. On the other hand, investors sub-optimally chase recent returns more than they should because of the representativeness bias they are subject to. My model quantifies the weights that investors should put on past performance information to best evaluate skills and predict future returns.

6 Reference

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Table 1. Summary Statistics.

This table summarizes the statistics for the sample of 3,697 hedge funds and funds of hedge funds from 1994 to 2017. Panel A reports the statistics for the panel data on annual flows, annual performance and annual assets under management (AUM) using fund-year observations. All variable are winsorized at the 1% and 99% levels. Panel B reports the statistics for the panel data on monthly flows, monthly performance and monthly AUM using fund-month observations. All variable are winsorized at the 1% and 99% levels. Panel C presents the statistics for funds' contractual characteristics using one observation for each fund. Reported statistics include the numver of ovseravtions (N), average (Mean), 25th Percentile (25th Pctl), median (Median), 75th Percentile (75th Pctl), and standard deviation (SD).

Variables	N	Mean	25^{th} Pctl.	Median	75^{th} Pctl.	SD
Panel A: Annual flows, performance, age, and assets under management						
Annual flow	19,221	0.0406	-0.2604	-0.0285	0.2152	0.5805
Annual return	26,035	0.0841	0.0028	0.0726	0.1451	0.1912
AUM ($M)	22,918	165.03	15.75	50.99	158.00	307.20
Panel B: Monthly flows, performance, age, and assets under management						
Monthly flow	310,418	0.0002	-0.0163	0.0000	0.0143	0.0902
Monthly returns	310,418	0.0055	-0.0069	0.0060	0.0180	0.0357
AUM ($M)	310,418	158.38	13.96	47.00	149.00	302.54
Panel C: Characteristics of funds						
Management fee (%)	3,650	1.39	1.00	1.50	1.85	0.61
Incentive fee (%)	3,450	15.09	10.00	20.00	20.00	7.77
High water mark	3,694	0.6381	0	1	1	0.4859
Lockup (days)	3,697	98.61	0	0	0	210.23
Restriction (days)	3,252	125.58	60	90	180	96.26
Leverage	3,697	0.52	0	1	1	0.50

Table 2. Estimate of distribution parameters

For each risk model, I estimate the distribution parameters using MLE. More precisely, the expected ability $\phi_{i,t}$ of fund i and time t is a function of distribution parameters (μ, ω, η, ρ, ξ, γ) and the past 24 month abnormal returns. The distribution parameters maximize the likelihood of observing future abnormal returns. I.e, the distribution parameters minimize the error term: $(R_{i,t} - \phi_{i,t}(\mu, \omega, \eta, \rho, \xi, \gamma))^2$. The sample period is from January 1994 to December 2017.

	CAPM	Car4	AN	FH7
μ(%)	-0.2343	-0.2355	-0.2882	-0.2387
s.e.	(0.4998)	(0.4351)	(0.4265)	(0.4795)
ω(%)	0.0251	0.0443	0.0708	0.0777
s.e.	(0.0196)	(0.0306)	(0.0416)	(0.0377)
η(%)	0.8548	1.1155	1.0820	1.1311
s.e.	(0.1802)	(0.2044)	(0.1804)	(0.2010)
ρ	0.9566	0.9281	0.8915	0.9241
s.e.	(0.0623)	(0.0744)	(0.0814)	(0.0510)
ξ(%)	-0.0720	-0.0698	-0.0808	-0.0796
s.e.	(0.1816)	(0.1581)	(0.1491)	(0.1535)
γ	1.2240E-07	1.1663E-07	1.5062E-08	1.0228E-06
s.e.	(1.0095)	(0.9001)	(0.7706)	(0.8698)

Table 3. Relation between the optimal weight on signal and fund age.

Using fund-month observations, I define Age variable as the age (in months) of the fund up to that month. Fund-month observations are grouped into portfolios based on the Age variable. For each portfolio of funds, I estimate and optimal weights on each of the past 24 months using an autoregressive model. More precisely, for each portfolio, I regress future abnormal returns on abnormal returns of the past 24 months. The weights on the past abnormal returns are the coefficients on abnormal returns of the past 24 months. Signal is defined to be the last quarter abnormal return or the last month return based on 4 risk models. The sample period is from January 1994 to December 2017.

	weight on last quarter (CAPM)	weight on last quarter (Carhart 4-factor)	weight on last quarter (AN option-factor)	weight on last quarter (FH 7-factor)
age (months)	-0.0043	-0.0040	-0.0065	-0.0028
	(t=-6.78)	(t=-5.33)	(t=-3.88)	(t=-2.70)
N	158	160	134	149

	weight on last month (CAPM)	weight on last month (Carhart 4-factor)	weight on last month (AN option-factor)	weight on last month (FH 7-factor)
age (months)	-0.0020	-0.0027	-0.0036	-0.0021
	(t=-3.67)	(t=-5.34)	(t=-7.10)	(t=-3.71)
N	158	160	134	149

Table 4. Relation between the weight on signal and the persistence of ability - ρ across strategies.

I estimate ρ for funds in each strategy category by MLE. The MLE estimation is the same as described in Table 2, but using samples in each portfolio instead of using the full sample. I rank the categories by ρ from low to high. For each category of funds, I estimate the optimal weights on each of the past 24 months using an autoregressive model. More precisely, for each portfolio, I regress abnormal returns on abnormal returns of past 24 months. The optimal weights are defined as the coefficients on the abnormal returns of the past 24 months. The weight on signal is defined as the sum of the coefficients on the last quarter. The analysis is based on CAPM model.

ρ rankings	ρ	weight on signal	strategy
1 (low)	0.6649	15.04%	Fixed Income Arbitrage
2	0.8087	13.10%	Event Driven
3	0.8621	9.03%	Equity Market Neutral
4	0.8952	5.47%	Long/Short Equity Hedge
5	0.9008	5.62%	Convertible Arbitrage
6	0.9698	8.19%	Emerging Markets
7	0.9708	7.92%	Funds of Funds
8	0.9905	5.83%	Multi-Strategy
9 (high)	1.0000	1.73%	Global Macro

Table 5. Relation between optimal weight on signal and the persistence of ability - ρ across funds

For each fund, I estimate the optimal weights on each of the past 24 months using an autoregressive model. More precisely, for each fund, I regress abnormal returns on abnormal returns of past 24 months. The optimal weights are defined as the coefficients on the abnormal returns of the past 24 months. Signal is defined to be either the last month abnormal return or the last quarter abnormal return. In addition, for each fund, I estimate ρ using MLE. The MLE estimation is the same as described in Table 2. The sample period is from January 1994 to December 2017.

	weight on last quarter (CAPM)	weight on last quarter (Carhart 4-factor)	weight on last quarter (AN option-factor)	weight on last quarter (FH 7-factor)
ρ	-3.0034	-1.5408	-1.9620	-1.6483
	(t=-4.59)	(t=-2.97)	(t=-4.05)	(t=-3.05)
N	3,697	3,697	3,697	3,697

	weight on last month (CAPM)	weight on last month (Carhart 4-factor)	weight on last month (AN option-factor)	weight on last month (FH 7-factor)
ρ	-1.2184	-0.7418	-1.0490	-0.9390
	(t=-4.82)	(t=-3.27)	(t=-5.17)	(t=-3.80)
N	3,697	3,697	3,697	3,697

Table 6. Relation between curvature of weights and the persistence of ability ρ across funds

For each fund, I estimate the optimal weights on each of the past 24 months using an autoregressive model. More precisely, for each fund, I regress abnormal returns on abnormal returns of past 24 months. The optimal weights are defined as the coefficients on the abnormal returns of the past 24 months. Curvature is defined to be the weight on the last 1 month performance divided by the weight on the last 2 month performance. In addition, for each fund, I estimate ρ using MLE. The MLE estimation is the same as described in Table 2, but using samples in each fund instead of using the full sample. The sample period is from January 1994 to December 2017.

	curvature (CAPM)	curvature (Car4)	curvature (AN)	curvature (FH7)
ρ	-1.4087	-1.5080	-1.0337	-1.6483
	(t=-2.10)	(t=-2.97)	(t=-2.65)	(t=-3.05)
N	3,697	3,697	3,697	3,697

Table 7. Estimate of θ - the degree of the representativeness bias.

For each risk model, I estimate the θ parameter using MLE. More precisely, with investors having representativeness bias, the perceived ability $\phi^{\theta}_{i,t}$ of i and time t is a function of θ - the degree of the representativeness bias. θ maximizes the likelihood of observing future inflows. I.e, θ minimizes the error term: $(flow_{i,t} - \phi^{\theta}_{i,t})^2$. Notice that the estimates are the same when I estimate θ jointly with parameters - (μ, ω, η, ρ, ξ, γ). The sample period is from January 1994 to December 2017.

Variable	CAPM	Car4	AN	FH7
θ	8.9867	10.5488	10.4914	2.7725
(s.e.)	(4.28)	(5.19)	(5.77)	(1.55)

Table 8. Return Implications - the comparison between returns from optimal return chasing and returns with the representativeness bias.

This table reports the risk-adjusted returns of funds sorted by their expected abilities. The risk-adjusted returns are reported in percent per year. The risk-adjusted returns are annualized from monthly returns. The monthly risk-adjusted returns are the time-series averages of monthly returns adjusted for risk. The t-statistics are the average returns divided by the time-series standard errors. In A, funds are sorted into 10 deciles based on their expected ability with the optimal level of return chasing. In B, funds are sorted into 10 deciles based on their expected abilities with representativeness bias. In C, the difference between returns from optimal return chasing and returns with the representativeness bias are shown. The risk-adjusted returns of funds in all 10 deciles are reported. The sample period is from January 1994 to December 2017.

	A: Returns with optimal weights				B: Returns with representativeness bias				C: Difference			
	CAPM	Car4	AN	FH7	CAPM	Car4	AN	FH7	CAPM	Car4	AN	FH7
1	-2.45	-3.78	-5.13	-5.39	-1.56	-2.14	-2.85	-4.36	-0.88	-1.64	-2.28	-1.03
									(t=-2.65)	(t=-6.43)	(t=-8.33)	(t=-3.69)
2	1.30	-0.49	-0.87	-0.51	0.53	0.14	-0.05	-0.42				
3	1.33	0.21	0.16	0.07	1.75	0.95	0.03	0.23				
4	1.82	0.65	0.56	0.87	1.97	1.16	0.85	1.08				
5	2.12	0.99	1.12	1.59	2.21	1.42	1.10	1.62				
6	2.39	2.08	1.85	2.54	2.63	1.78	1.85	2.13				
7	2.69	2.49	2.60	3.01	2.93	2.20	2.55	2.76				
8	3.31	3.58	3.58	3.41	4.21	3.05	3.09	4.20				
9	5.06	4.26	4.68	5.49	4.68	3.67	4.17	4.75				
10	9.28	8.23	9.74	10.57	7.40	6.15	7.62	9.56	1.88	2.08	2.12	1.01
									(t=4.51)	(t=7.25)	(t=8.22)	(t=4.97)
10-1	11.73	12.01	14.87	15.96	8.96	8.29	10.47	13.92	2.67	3.72	4.40	1.85
									(t=4.57)	(t=8.97)	(t=9.70)	(t=5.80)

Table 9. Return Implications - the comparison between returns from optimal return chasing and returns assuming abilities stay constant.

This table reports the risk-adjusted returns of funds sorted by their expected abilities. The risk-adjusted returns are reported in percent per year. The risk-adjusted returns are annualized from monthly returns. The monthly risk-adjusted returns are the time-series averages of monthly returns adjusted for risk. The t-statistics are the average returns divided by the time-series standard errors. In A, funds are sorted into 10 deciles based on their expected ability with the optimal level of return chasing. In B, funds are sorted into 10 deciles based on their expected abilities assuming ability do not change over time. In other words, the weights on historical return information are the same. C shows the difference between returns from optimal return chasing and returns when putting equal weights on past returns. The risk-adjusted returns of funds in all 10 deciles are reported. The sample period is from January 1994 to December 2017.

	A: Returns with optimal weights				B: Returns with equal weights				C: Difference			
	CAPM	Car4	AN	FH7	CAPM	Car4	AN	FH7	CAPM	Car4	AN	FH7
1	-2.45	-3.78	-5.13	-5.39	-2.01	-3.38	-4.01	-4.83	-0.44	-0.40	-1.12	-0.56
									(t=-3.04)	(t=-2.08)	(t=-5.84)	(t=-2.99)
2	1.30	-0.49	-0.87	-0.51	0.92	-0.05	-0.27	0.07				
3	1.33	0.21	0.16	0.07	1.38	0.40	0.39	0.52				
4	1.82	0.65	0.56	0.87	1.51	0.63	0.84	1.17				
5	2.12	0.99	1.12	1.59	2.13	0.98	1.43	1.41				
6	2.39	2.08	1.85	2.54	2.59	2.03	1.81	1.97				
7	2.69	2.49	2.60	3.01	3.05	2.61	2.51	2.73				
8	3.31	3.58	3.58	3.41	3.64	3.35	3.15	3.88				
9	5.06	4.26	4.68	5.49	4.46	4.18	4.32	5.24				
10	9.28	8.23	9.74	10.57	9.05	7.47	8.10	9.31	0.23	0.76	1.64	1.26
									(t=0.95)	(t=3.36)	(t=7.31)	(t=5.42)
10-1	11.73	12.01	14.87	15.96	11.06	10.85	12.11	14.14	0.67	1.16	2.76	1.82
									(t=2.33)	(t=4.19)	(t=7.89)	(5.50)

Appendix

Bootstrap Confidence Intervals

In the Appendix, I estimate the mean and 95% confidence intervals for the distribution parameters $\Gamma = (\mu, \omega, \eta, \rho, \xi, \gamma)$ and the level of representativeness bias θ using Bootstrapping. Returns across time for each fund are obviously dependent. Assume that returns across funds are independent. I treat time-series returns for one fund as one observation from the sample. Denote R^i as the time-series returns for fund i. Let N be the number of funds. The bootstrap procedure is described as follows.

I sample N series $\{R^{1*}, \cdots, R^{N*}\}$ with replacement from $\{R^1, \cdots, R^N\}$. I estimate the distribution of parameters $\Gamma = (\mu, \omega, \eta, \rho, \xi, \gamma)$ and the level of representativeness bias θ by MLE using the resampled date. I resample 1,000 times. Thus, I have 1,000 estimates for Γ and θ. The means and confidence intervals are obtained from the sample of 1,000 estimates. More precisely, the mean of μ is the average of the 1,000 estimates of μ. The 95% confidence interval of μ is the range between the 2.5 percentile and the 97.5 percentile of the 1,000 estimates of μ.

Table A1. Estimate of distribution parameters from Bootstrapping

I resample with replacement 1,000 times. For each resampling and for each model, I estimate the distribution parameters using MLE. More precisely, the expected ability $\phi_{i,t}$ of fund i and time t is a function of distribution parameters (μ, ω, η, ρ, ξ, γ) and the past 24 month abnormal returns. The distribution parameters maximize the likelihood of observing future abnormal returns. I.e, the distribution parameters minimize the error term: $(R_{i,t} - \phi_{i,t}(\mu, \omega, \eta, \rho, \xi, \gamma))^2$. The means and 95% confidence intervals are obtained from the sample of 1,000 estimates. The sample period is from January 1994 to December 2017.

	CAPM	Car4	AN	FH7
$\mu(\%)$	-0.2353	-0.2359	-0.2883	-0.2368
95% C.I.	[-0.3106, -0.1687]	[-0.2967, -0.1706]	[-0.3488, -0.2237]	[-0.3065, -0.1692]
$\omega(\%)$	0.0251	0.0444	0.0709	0.0775
95% C.I.	[0.0191, 0.0308]	[0.0366, 0.0529]	[0.0603, 0.8199]	[0.0665, 0.0892]
$\eta(\%)$	0.8579	1.1141	1.0819	1.1321
95% C.I.	[0.7772, 0.9400]	[1.0177, 1.2157]	[0.9972, 1.1747]	[1.0310, 1.2455]
ρ	0.9564	0.9279	0.8912	0.9241
95% C.I.	[0.9363, 0.9746]	[0.9068, 0.9474]	[0.8651, 0.9159]	[0.9079, 0.9393]
$\xi(\%)$	-0.0722	-0.0697	-0.0808	-0.0783
95% C.I.	[-0.0879, -0.0572]	[-0.0827, -0.0568]	[-0.0931, -0.0676]	[-0.0942, -0.0604]
γ	0.0004	0.0003	0.0002	0.0050
95% C.I.	[2.78E-08, 1.78E-05]	[1.10E-08, 3.33E-05]	[4.13E-10, 1.49E-06]	[5.33E-08, 0.0526]

Table A2. Estimate of θ - the degree of the representativeness bias, from Bootstrapping.

I resample with replacement 1,000 times. For each resampling and for each model, I estimate the θ parameter using MLE. More precisely, with investors having representativeness bias, the perceived ability $\phi^{\theta}_{i,t}$ of i and time t is a function of θ - the degree of the representativeness bias. θ maximizes the likelihood of observing future inflows. I.e, θ minimizes the error term: $(flow_{i,t} - \phi^{\theta}_{i,t})^2$. Notice that the estimates are the same when I estimate θ jointly with parameters - (μ, ω, η, ρ, ξ, γ). The means and 95% confidence intervals are obtained from the sample of 1,000 estimates. The sample period is from January 1994 to December 2017.

Variable	CAPM	Car4	AN	FH7
θ	10.4508	11.9453	11.8725	3.5572
95% C.I.	[1.3532, 50.8857]	[1.3839, 63.4305]	[1.2083, 59.8521]	[0.5303, 14.0571]