Frequency-domain information for active portfolio management[*]

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Abstract

We assess the benefits of using frequency-domain information for active portfolio management. To do so, we forecast the bond risk premium and equity risk premium using a methodology that isolates frequencies (of the predictors) with the highest predictive power. The resulting forecasts are more accurate than those of traditional forecasting methods for both asset classes. When used in the context of active portfolio management, the forecasts based on frequency-domain information lead to better portfolio performances than when using the original time series of the predictors. It produces higher information ratio (0.57 vs 0.45), higher CER gains (1.12% vs 0.81%), and lower maximum drawdown (19.1% vs 19.6%).

Keywords: equity risk premium, bond risk premium, predictability, multiresolution analysis, active portfolio management

JEL classification: C58, G11, G17

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1 Introduction

Active portfolio management relies on good return forecasts of asset classes under management. It is then of key interest for active asset managers to identify reliable predictors and good forecasting methods.

There is an extensive literature on the out-of-sample predictability of the equity risk premium (see the reviews of Rapach and Zhou, 2013 and Harvey, Liu, and Zhu, 2016), but the literature on the predictability of the bond risk premium is limited (notable contributions include Ludvigson and Ng, 2009; Thornton and Valente, 2012; Sarno, Schneider, and Wagner, 2016; and Gargano, Pettenuzzo, and Timmermann, 2019). The literature is dominated by time series analysis. Frequency domain techniques, like Fourier transformations, are rather new tools in finance applications (e.g. Dew-Becker and Giglio, 2016). In the context of forecasting equity returns, Faria and Verona (2018) and Bandi, Perron, Tamoni, and Tebaldi (2019) introduce models where equity returns and predictors are linear aggregates of components operating over different frequencies and predictability is frequency-specific.

The first contribution of this paper is to compare the performance of alternative predictive models of the bond risk premium (BRP) and the equity risk premium (ERP). We first use frequency-domain filtering techniques to expand an initial dataset of predictors to obtain more predictors for BRP and ERP forecasting. In particular, from each original variable we extract several time series, each corresponding to a particular frequency of the original variable and each representing a new predictor. The enlarged dataset has the same amount of information as the original dataset (we start from the same number of variables), but allows forecasting the BRP and ERP with more granular information. This allows us i) to tease out those predictor frequencies with the highest predictive power from others that bring noise to the exercise, and ii) to infer the relevance of using the frequency-domain information of the original predictors.

For both the *BRP* and the *ERP*, we find that the use of frequency-domain information significantly improves the statistical performance of forecasts over forecasts that only use original variables. While this result is not new with regards to *ERP* forecasting (see Faria and Verona 2017, 2019), it is, to the best of our knowledge, the first time frequency-domain information has been used in *BRP* forecasting. Furthermore, we find that the forecasting gains from using frequency-domain information are significantly higher when combining different frequencies from different original predictors than when combining different frequencies from the same original predictor. This finding suggests that different frequencies of different variables are useful predictors of equity and bond returns as they track different frequency components of the equity and bond risk premium. This result is in line with Fama and French (1989), who find that different financial variables track different frequency components of the equity premium.

The second contribution of this paper is an evaluation of the economic significance of frequency-domain information for active portfolio management. We adopt the perspective of a power-utility maximizing investor, whereby the *BRP* and *ERP* forecasts from the first step are treated as the investor's active views on stock and bond markets. We consider a mean-variance optimization framework and, as benchmark, a conventional allocation of 60% to stocks and 40% to bonds. We find that using frequency-domain information leads to better portfolio performances than when using the original time series of the predictors. It produces higher information ratio (0.57 vs 0.45), higher CER gains (1.12% vs 0.81%), and lower maximum drawdown (19.1% vs 19.6%). This finding is robust towards the consideration of an alternative portfolio optimization setting (Black-Litterman-type model), alternative benchmarks, and various portfolio constraint settings.

The rest of the paper is organized as follows. Section 2 sets out the data and methodology. Section 3 presents the out-of-sample results and performance of the proposed active portfolio management strategy. Section 4 documents the robustness test results. Section 5 concludes.

2 Data and methodology

Our focus is on out-of-sample (OOS) predictability of bond and equity risk premiums. The OOS exercise is relevant in evaluating effective return predictability in real time, while avoiding in-sample over-fitting, distortions from small sample size, and look-ahead bias.

Our monthly data extend from January 1973 to December 2018. *BRP* and *ERP* of month t are measured as the difference between the return on the 10-year US Treasury bond and the return on the S&P500 index in month t, respectively, and the one-month T-bill known at the beginning of month t (lagged-risk free rate). We use twelve variables taken from Goyal and Welch (2008) as the predictors: log dividend-price ratio (DP), log dividend yield (DY), log earnings-price ratio (EP), excess stock return volatility (RVOL), book-to-market ratio (BM), net equity expansion (NTIS), long-term bond yield (LTY), long-term bond return (LTR), term spread (TMS), default yield spread (DFY), default return spread (DFR), and lagged inflation rate (INFL). These predictors are briefly described in Appendix 1. Table 1 reports the summary statistics for *BRP*, *ERP* and the predictors. Figure 1 provides their time series.

The first step of our forecasting methodology is based on a wavelet multiresolution analysis as described in sub-section 2.1. The OOS procedure is explained in sub-section 2.2. The asset allocation framework in covered in sub-section 2.3.

2.1 Wavelet multiresolution analysis

Wavelet multiresolution analysis (MRA) allows decomposition of any time series into its frequency components in a way similar to bandpass filtering (e.g. Baxter and King, 1999). Given a time series y_t, its wavelet multiresolution representation can be written as

$$y_t = \sum_{j=1}^{J} y_t^{D_j} + y_t^{S_J} , \quad (1)$$

where $y_t^{D_j}$ are the J wavelet detail components and $y_t^{S_J}$ is the wavelet smooth component. Equation (1) shows that the original series y_t can be decomposed in several time series components, each capturing the fluctuation of the original time series within a specific frequency band. For small j, the j wavelet detail components represent the higher frequency components of the time series (the short-term dynamics). As j increases, the j wavelet detail components represent lower frequencies fluctuations of the series. Finally, the smooth component captures the lowest frequency component (series trend).

Here, we perform our wavelet decomposition analysis using the Haar wavelet filter[1] and the maximal overlap discrete wavelet transform (MODWT) MRA. This methodology is not restricted to a particular sample size and is not sensitive to the choice of starting point for the examined time series. Moreover, it does not introduce phase shifts in the wavelet coefficients, i.e. peaks and troughs of the original time series are perfectly aligned with similar events in the MODWT MRA.[2]

Given the length of the data series under analysis, we consider a J=6 level MRA for each of the original predictors, so that the decomposition delivers seven time-frequency series: six wavelet detail components ($y_t^{D_1}$ to $y_t^{D_6}$) and a wavelet smooth component ($y_t^{S_6}$).[3] As we use monthly data, the first detail component $y_t^{D_1}$ captures oscillations between 2 and 4 months, while detail components $y_t^{D_2}$, $y_t^{D_3}$, $y_t^{D_4}$, $y_t^{D_5}$ and $y_t^{D_6}$ capture oscillations with a period of 4-8, 8-16, 16-32, 32-64 and 64-128 months, respectively. Finally, the smooth component $y_t^{S_6}$

[1] Besides its simplicity and wide use, the Haar filter makes a neat connection to temporal aggregation as the wavelet coefficients are simply differences of moving averages (see Bandi, Perron, Tamoni, and Tebaldi, 2019 and Lubik, Matthes, and Verona, 2019).

[2] This section provides a brief description of the theory directly relevant to our empirical analysis. A more detailed analysis of wavelet methods is provided in Appendix 2 and in Percival and Walden (2000). Recent papers using the MODWT MRA decomposition are Bekiros and Marcellino (2013), Gallegati and Ramsey (2013), Barunik and Vacha (2015), Crowley and Hughes Hallett (2015), Berger (2016), and Faria and Verona (2018), among others. See Verona (2019) for a description of the advantages of wavelet filters over other band-pass filtering techniques.

[3] As regards the choice of J, the number of observations dictates the maximum number of frequency bands that can be used. In particular, if t_0 is the number of observations in the in-sample period, then J has to satisfy the constraint $J \leq \log_2 t_0$.

(re-denoted as $y_t^{D_7}$ in our later discussion) captures oscillations with a period longer than 128 months (10.6 years).[4]

To illustrate the rich set of dynamics aggregated (and therefore hidden) in the original time series, Figure 2 plots the time series of one of the predictors used (term spread) and its seven time-frequency series components. As expected, the lower the frequency, the smoother the resulting filtered time series. As can be seen, the time-frequency series components exhibit different time series properties and dynamics, so one can expect that only some are good *ERP* and *BRP* predictors. As Faria and Verona (2019) show, the lowest frequency component of the term spread (TMS^{D_7}) is a strong OOS predictor of the *ERP*, whereas the other frequency components of the term spread have much worse forecasting performances.

2.2 Out-of-sample forecasts

The OOS forecasts of the *BRP* and *ERP* are generated using a sequence of expanding windows. We use an initial sample (1973:01 to 1989:12) to make our first one-step-ahead OOS forecast. The sample is then increased by one observation and a new one-step-ahead OOS forecast is produced. We proceed this way until the end of the sample, ultimately obtaining a sequence of 348 one-step-ahead OOS forecasts. The full OOS period spans the period from 1990:01 to 2018:12.

As the MODWT MRA is a two-sided filter, we recompute the frequency components of the original predictors recursively at each iteration of the OOS forecasting process using data from the start of the sample through the month of forecast formation. This important step ensures that our method does not have a "look-ahead" bias, as the forecasts are made with current and past information only. The literature suggests several types of boundary

[4] In the MODWT, each wavelet filter at frequency j approximates an ideal high-pass filter with passband $f \in \left[1/2^{j+1}\,,\,1/2^{j}\right]$, while the smooth component is associated with frequencies $f \in \left[0\,,\,1/2^{j+1}\right]$. The level j wavelet components are therefore associated to fluctuations with periodicity $\left[2^{j}\,,\,2^{j+1}\right]$ (months, in our case).

3.2 Active portfolio management performance

We use the *BRP* and *ERP* forecasts from the respective *multi_ wav* model to feed the vector of active views $\hat{\boldsymbol{R}} = \left(\hat{R}_{b,t+1},\ \hat{R}_{e,t+1}\right)$ driving the *APM* strategy. This is denoted as *APM_ WAV*. As mentioned, the benchmark is the conventional allocation of 60% to stocks and 40% to bonds (denoted $Benchmark_{60-40}$). For comparison purposes, we also report the performance of an *APM* strategy based on the *BRP* and *ERP* forecasts obtained with the original time series of the predictors (*multi_ ts*). We denote this as *APM_ TS*.

Figure 3 presents the *APM_ WAV, APM_ TS*, and $Benchmark_{60-40}$ portfolio weights (solid, dashed, and dotted lines, respectively). Both active strategies (*APM_ WAV* and *APM_ TS*) strongly deviate from the 60-40 benchmark throughout the entire sample period. Moreover, the *APM_ WAV* weights seem to oscillate around the trends defined by the *APM_ TS* weights. Interestingly, with the exception of the mid-nineties period, the differences between *APM_ WAV* and *APM_ TS* weights are most evident around and during recessions. In particular, the *APM_ WAV* has relatively lower exposure to equity immediately before and during recessions. This suggests an improved equity market timing of the *APM_ WAV* strategy compared to that of the *APM_ TS* strategy.

In Panel A of Table 3, we report the performance measurements of the strategies. Both *APM* strategies outperform the $Benchmark_{60-40}$, with the *APM_ WAV* strategy outperforming the *APM_ TS*. Compared with the $Benchmark_{60-40}$ performance, both *APM* strategies improve the average annual return while decreasing the maximum drawdown. This translates to higher annualized Sharpe ratios. The fact that the active deviations from the 60-40 benchmark (as illustrated in Figure 3) add value to the active investor is reflected in the annualized information ratios of 0.57 (*APM_ WAV*) and 0.45 (*APM_ TS*). From an utility perspective, this also translates to annualized CER gains of 1.12% (*APM_ WAV*) and 0.81% (*APM_ TS*).

The fact that the *APM_ WAV* strategy outperforms the *APM_ TS* strategy implies that there are economic gains from using frequency-domain information in active portfolio management. To disentangle the distribution of those gains across the asset classes traded (bonds and equity), we report the performance metrics for two additional active portfolio management strategies in Panel B of Table 3. *APM_Equity_WAV* is based on the forecast of the *multi_wav* (*multi_ts*) model for equity (bond) return. *APM_Bond_WAV* is based on the forecast of the *multi_wav* (*multi_ts*) model for bond (equity) return. By comparing *APM_Equity_WAV* (*APM_Bond_WAV*) with *APM_TS*, we can assess the gains from using frequency-domain information in the forecast of equity (bond) return.

We highlight two main results. First, there are gains for both asset classes when using frequency-domain information in the forecast of their returns. The gains are quite similar in magnitude. Second, the gains are more expressive when using frequency-domain information to forecast both the return of bonds and equities.

Figure 4 shows the cumulative wealth of an investor who invests 1$ in January 1990 and reinvests all proceeds adopting the *APM_WAV* strategy (solid line), the *APM_TS* strategy (dashed line), and the $Benchmark_{60-40}$ strategy (dotted line). From a cumulative return perspective, the active strategy *APM_WAV* clearly outperforms the others. By December 2018, the investor has obtained $38.6 with the *APM_WAV* strategy, instead of $28.7 with the *APM_TS* strategy, or $12.4 with the $Benchmark_{60-40}$.

The strong performance of the *APM_WAV* strategy is not without its caveats. In the upper panel of Figure 5, we report the dynamics of the 3-year moving average information ratio of the *APM_WAV* strategy (solid line). The 3-year moving average information ratio is positive for most of the sample period, but there are periods when it is negative (i.e. generating utility losses). However, the figure also shows that the *APM_WAV* strategy dominates the *APM_TS* strategy (dotted line), as its 3-year moving average information ratio is either higher (for most of the sample) or similar. From the utility perspective, similar conclusions can be drawn by looking at the dynamics of the 3-year moving average CER gains of the

APM_ WAV and *APM_ TS* strategies (reported in the lower panel of Figure 5).

Overall, these results demonstrate the usefulness of frequency-domain information for active portfolio management. In the next section, we test the robustness of our findings by considering an alternative portfolio-optimization framework and other changes in the settings used so far.

4 Robustness

4.1 Alternative portfolio optimization framework

We test the robustness of the results reported so far by using the Black-Litterman model (*BLM*), which is a framework often considered in the context of *APM*. The objective of the *BLM* is to outperform the benchmark portfolio within a certain tracking error.

We use the same *BRP* and *ERP* forecasts from previous sections as the active views on stock and bond markets, treat them as inputs in a version of the *BLM* (as proposed by Da Silva, Lee, and Pornrojnangkool, 2009 and Almadi, Rapach, and Suri, 2014 and described in Appendix 4) to obtain optimal weights across assets.

We consider a power-utility maximizing investor with $\gamma = 2$ and $Benchmark_{60-40}$ as the benchmark strategy. For simplicity, we assume the investor will neither leverage nor short-sell available assets ($h = 1$ and $l = 0$). The target level of the annualized tracking error of the investor is assumed to be 5.80%, i.e. the same tracking error of the *APM_ WAV* strategy for an investor with $\gamma = 2$, $h = 1$ and $l = 0$. *APM_ BLMWAV* and *APM_ BLMTS* denote the active portfolio management strategies based on asset return forecasts from *multi_ wav* and *multi_ ts* methodologies used in the context of a Black-Litterman optimization framework.

The results, which are reported in Panel C of Table 3, are qualitatively similar to those in the mean-variance setting. Both *APM* strategies based on the BLM outperform the

$Benchmark_{60-40}$, achieving positive information ratios and CER gains. Similarly, using frequency-domain information in the context of the BLM still improves the performance of the strategy over the scenario where only the original time series of the predictors are used (*APM_ BLMWAV* versus *APM_ BLMTS*). Both the annualized information ratio and the annualized CER gain are higher (0.44 versus 0.11 and 0.68% versus 0.12%, respectively). Finally, the portfolio weights (reported in Figure 6) of the strategy using frequency-domain information are much more stable than those of the strategy using time-series information only.

4.2 Other robustness tests

In this sub-section, we briefly comment on additional robustness tests that were implemented.[6]

4.2.1 Alternative benchmarks

Instead of $Benchmark_{60-40}$, we consider two alternative benchmarks: a naive diversification rule 1/N (50% equity and 50% bonds) and an allocation of 40% equity and 60% bonds. In both cases, the information ratios and CER gains of the *APM_ WAV* and *APM_ TS* strategies are still positive, and the information ratios and CER gains of the *APM_ WAV* strategy are higher than those of *APM_ TS* strategy. Qualitatively, these results confirm that our findings are robust towards alternative benchmarks.

4.2.2 Alternative set of portfolio constraints and investor risk aversion

For a given level of risk aversion of the representative investor, the *APM_ WAV* strategy outperforms the *APM_ TS* strategy (and the $Benchmark_{60-40}$) in alternative scenarios with

[6] The results are not reported here, but available upon request from the authors.

(i) no leverage or short-selling possibilities ($h = 1$ and $l = 0$), (ii) no leverage possibilities, but short-selling allowed ($h = 1$ and $l = -0.5$) and (iii) both leverage and short-selling possibilities ($h = 1.5$ and $l = -0.5$). The higher is the level of leverage and short-selling allowed, the higher is the outperformance of the APM_WAV strategy versus the APM_TS strategy. Finally, the lower is the level of risk aversion of the representative investor, the higher is the outperformance of the APM_WAV strategy versus the APM_TS strategy (everything else constant).

5 Concluding remarks

Fama and French (1989) find that different financial variables can be useful in predicting equity returns as they track different frequency components of the equity premium. In this paper, we show that using information from different frequencies of different predictors helps improve forecasts of bond and equity returns. When used in the context of active portfolio management, these forecasts lead to superior portfolio performances.

We envision several interesting research avenues related with the use of frequency-domain information for active portfolio management. Here, we only used twelve variables as possible predictors of bond and equity returns, but the same methodology can be readily applied to larger datasets, and even combined with large dimensional statistical models. It could also be worthwhile to explore the statistical and economic gains from the use of frequency-domain information in the context of forecasting models with time-varying parameters and stochastic volatility.

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	Average return	CAGR	Sharpe ratio	Maximum drawdown	Tracking error	Information ratio	CER gain
Panel A: baseline							
APM_ WAV	14.2%	13.4%	1.28	19.1%	7.4%	0.57	1.12%
APM_ TS	13.0%	12.3%	1.18	19.6%	7.2%	0.45	0.81%
$Benchmark_{60-40}$	9.5%	9.1%	1.13	29.1%	-	-	-
Panel B: different forecasting inputs							
APM_ Equity_ WAV	13.7%	13.0%	1.24	19.3%	7.3%	0.52	0.99%
APM_ Bond_ WAV	13.5%	12.7%	1.22	19.4%	7.3%	0.49	0.94%
Panel C: different portfolio optimization framework							
APM_ BLMWAV	12.4%	11.7%	1.23	24.2%	5.8%	0.44	0.68%
APM_ BLMTS	10.2%	9.6%	1.05	26.1%	5.8%	0.11	0.12%

Table 3: Portfolio performance statistics

This table reports the performance statistics of different portfolio strategies. The performance statistics are: average return, which is the annualized first moment of returns time series; CAGR, which is the composite annual growth rate of returns time series; Sharpe ratio, measured as the 1-year moving average of portfolio's annualized Sharpe ratio; maximum drawdown, measured as the maximum percentage reduction in the portfolio's cumulative return; tracking error, measured as the annualized standard deviation of the *APM* monthly excess return (towards the benchmark); the information ratio, measured as the annualized average *APM* monthly excess return (towards the benchmark) divided by the tracking error; CER gain, measured as the annualized increase in certainty equivalent return that a power-utility maximizing investor with relative risk aversion $\gamma = 2$ would have by having access to the *APM* portfolio instead of the benchmark portfolio. The benchmark portfolio is 60% allocation to stocks and 40% to bonds. In Panel A are presented the performance statistics for the strategies *APM_ WAV* and *APM_ TS*, which are the active portfolio management strategy based on asset return forecasts from *multi_ wav* and *multi_ ts* methodologies, respectively. In Panel B are presented the performance statistics for the strategy *APM_ Equity_ WAV*, which is an active portfolio management strategy based on equity (bond) return forecasts from *multi_ wav* (*multi_ ts*) methodology, and for the strategy *APM_ Bond_ WAV*, which is an active portfolio management strategy based on bond (equity) return forecasts from *multi_ wav* (*multi_ ts*) methodology. In Panel C are presented the performance statistics for the strategies *APM_ BLMWAV* and *APM_ BLMTS*, which are the active portfolio management strategy based on asset return forecasts from *multi_ wav* and *multi_ ts* methodologies used in the context of a Black-Litterman portfolio-optimization framework, respectively. The sample period is from 1973:01 to 2018:12. The out-of-sample forecasting period is from 1990:01 to 2018:12, monthly frequency.

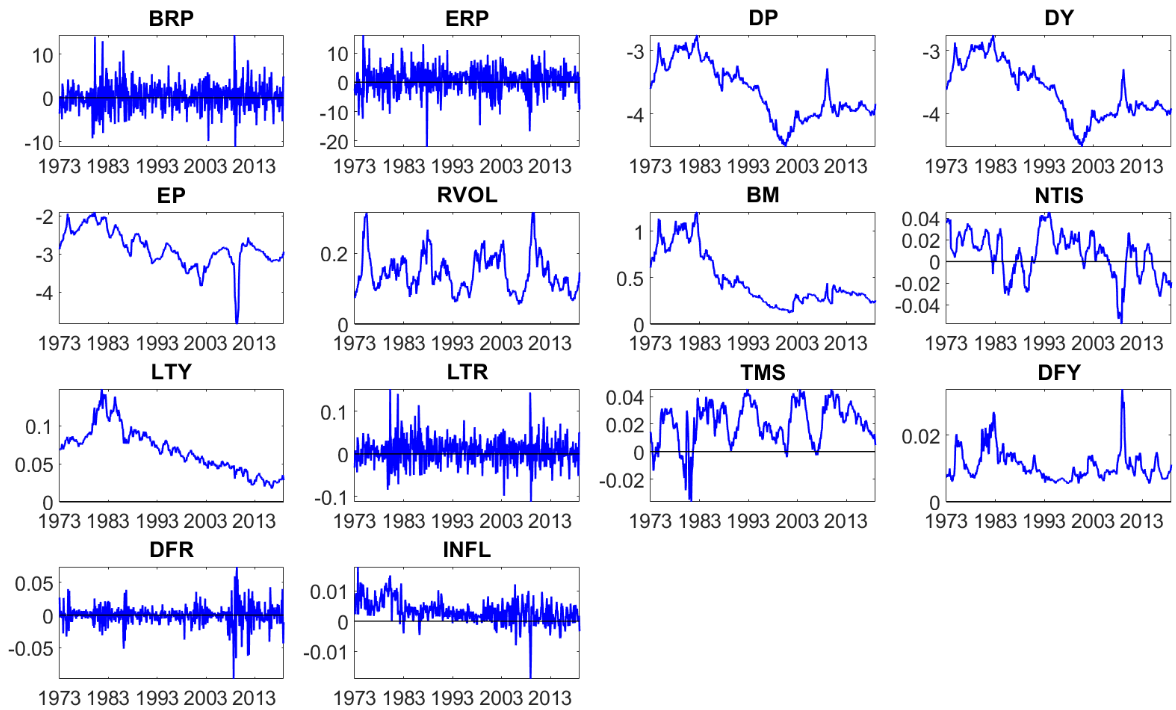

Figure 1: Monthly time series of the *BRP*, the *ERP*, and their predictors

This figure plots the time series of the bond risk premium (*BRP*), equity risk premium (*ERP*), and of each of the predictors. The *BRP* and *ERP* are measured as the difference between the return on the 10-year US Treasury bond and the return on the S&P500 index, respectively, and the return on a one-month T-bill. The set of predictors is described in Appendix 1. The sample period extends from 1973:01 to 2018:12.

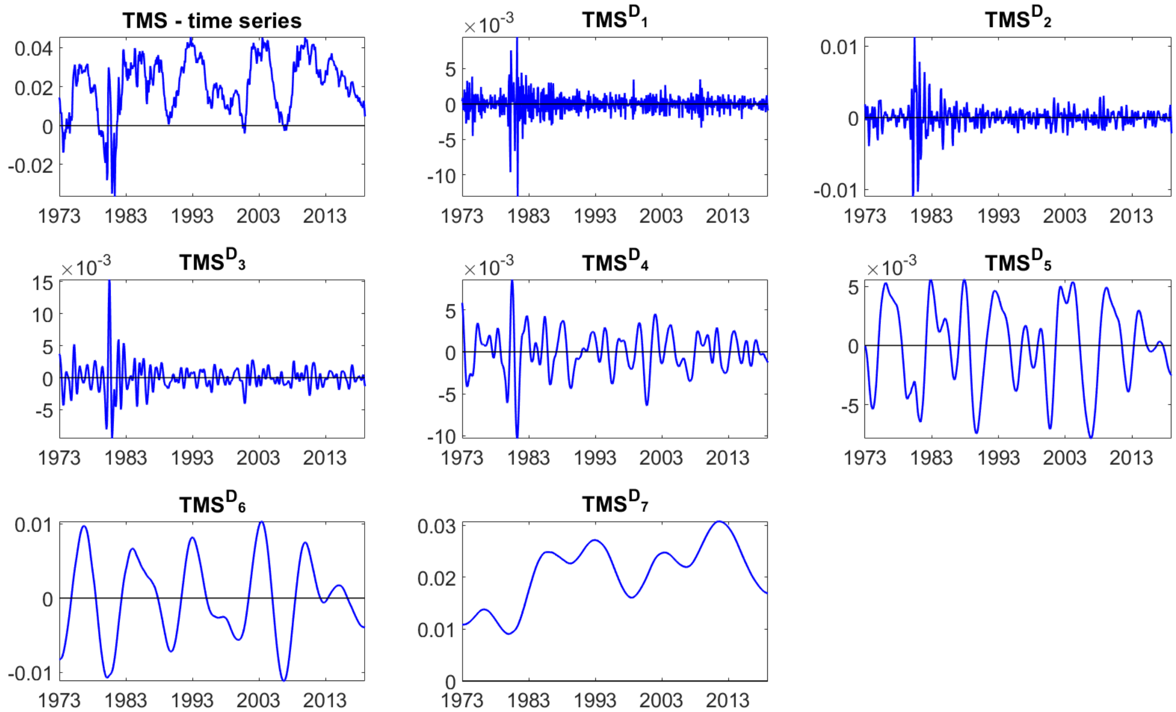

Figure 2: Term spread time series and wavelet decomposition

This figure plots the time series of the term spread (TMS) and the seven frequency components into which the time series is decomposed. It is applied a $J = 6$ level wavelet decomposition, which produces six wavelet details (D_1, $D_2, \ldots,$ D_6), each representing higher-frequency characteristics of the series, as well as a wavelet smooth (D_7), which captures the low-frequency dynamics of the series. The sample period runs from 1973:01 to 2018:12 (monthly frequency).

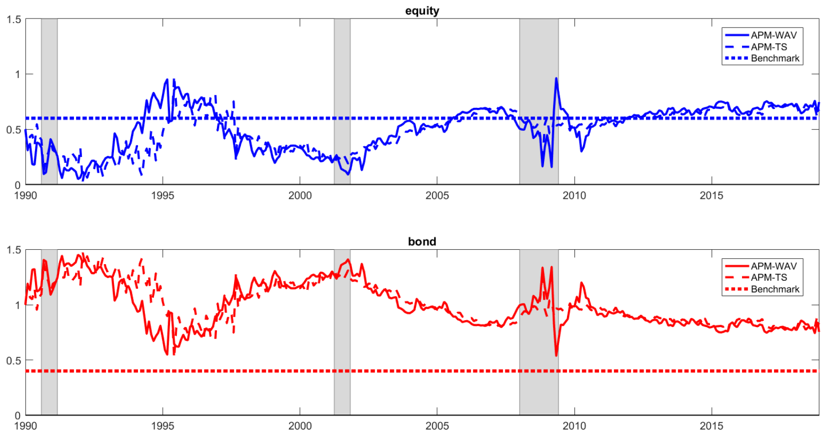

Figure 3: APM_WAV, APM_TS, and balanced $Benchmark_{60-40}$ portfolio weights

This figure plots the APM_WAV, APM_TS, and balanced $Benchmark_{60-40}$ portfolio weights (solid, dashed and dotted lines, respectively), rebalanced on a monthly basis. APM_WAV and APM_TS stand for the active portfolio management strategy based on asset return forecasts from $multi\_wav$ and $multi\_ts$ methodologies, respectively. The sample period is from 1973:01 to 2018:12. The out-of-sample forecasting period runs from 1990:01 to 2018:12 (monthly frequency). Gray bars denote NBER-dated recessions.

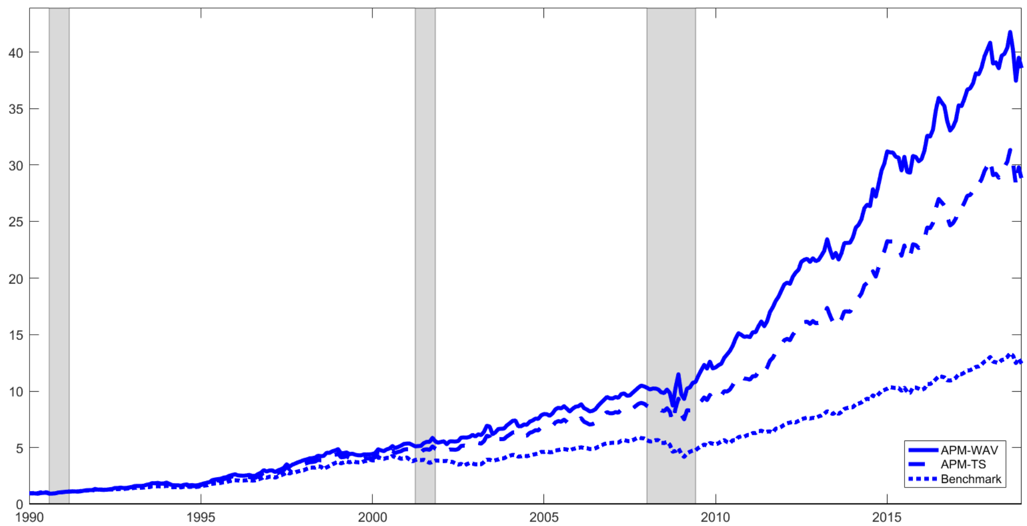

Figure 4: Cumulative wealth for APM_WAV, APM_TS, and $Benchmark_{60-40}$ investors

This figure represents the cumulative wealth of an investor who begins with $1 and reinvests all proceeds on a monthly basis, adopting an APM_WAV, APM_TS, and $Benchmark_{60-40}$ strategy (solid, dashed, and dotted lines, respectively). The APM_WAV and APM_TS active portfolio management strategies are based on asset return forecasts from *multi_wav* and *multi_ts* methodologies, respectively. The sample period extends from 1973:01 to 2018:12. The out-of-sample forecasting period runs from 1990:01 to 2018:12 (monthly frequency). Gray bars denote NBER-dated recessions.

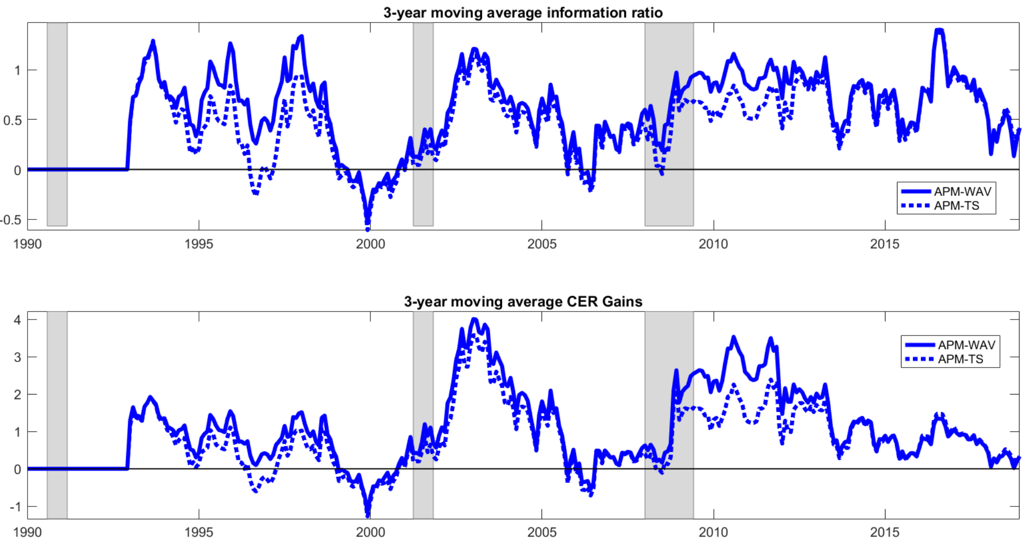

Figure 5: 3-year moving average information ratios and CER gains

The upper figure plots the 3-year moving average information ratio for the APM_WAV and APM_TS strategies relative to the $Benchmark_{60-40}$. The lower figure plots the 3-year moving average annualized CER gain for the APM_WAV and the APM_TS strategies. The sample period is from 1973:01 to 2018:12. The out-of-sample forecasting period runs from 1990:01 to 2018:12 (monthly frequency). Gray bars denote NBER-dated recessions.

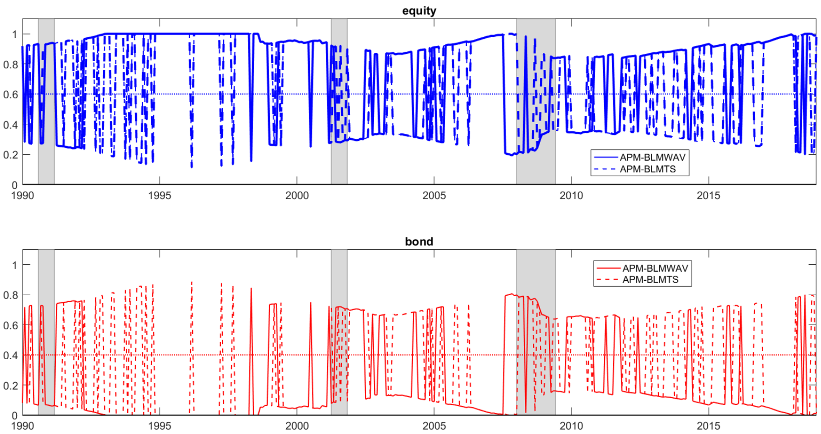

Figure 6: APM_BLMWAV, APM_BLMTS, and balanced $Benchmark_{60-40}$ portfolio weights

This figure plots the APM_BLMWAV, APM_BLMTS, and balanced $Benchmark_{60-40}$ portfolio weights (solid, dashed, and dotted lines, respectively), rebalanced on a monthly basis. The APM_BLMWAV and APM_BLMTS active portfolio management strategies are based on asset return forecasts from $multi\_wav$ and $multi\_ts$ methodologies used in the context of a Black-Litterman portfolio-optimization framework. The sample period extends from 1973:01 to 2018:12. The out-of-sample forecasting period runs from 1990:01 to 2018:12 (monthly frequency). Gray bars denote NBER-dated recessions.

Appendix 1. Predictors of equity and bond risk premiums

- Log dividend-price ratio (DP): difference between the log of dividends (12-month moving sums of dividends paid on S&P 500) and the log of prices (S&P 500 index).
- Log dividend yield (DY): difference between the log of dividends (12-month moving sums of dividends paid on S&P 500) and the log of lagged prices (S&P 500 index).
- Log earnings-price ratio (EP): difference between the log of earnings (12-month moving sums of earnings on S&P 500) and the log of prices (S&P 500 index price).
- Excess stock return volatility (RVOL): calculated using a 12-month moving standard deviation estimator.
- Book-to-market ratio (BM): ratio of book value to market value for the DJIA.
- Net equity expansion (NTIS): ratio of 12-month moving sums of net equity issues by NYSE-listed stocks to the total end-of-year NYSE market capitalization.
- Long-term yield (LTY): long-term government bond yield.
- Long-term return (LTR): long-term government bond return.
- Term spread (TMS): difference between the long-term government bond yield and the T-bill.
- Default yield spread (DFY): difference between Moody's BAA- and AAA-rated corporate bond yields.
- Default return spread (DFR): difference between long-term corporate bond and long-term government bond returns.
- Inflation rate (INFL): calculated from the Consumer Price Index (CPI) for all urban consumers.

Appendix 2. Maximal overlap discrete wavelet transform

Discrete wavelet transform (DWT) multiresolution analysis (MRA) allows the decomposition of a time series into its constituent multiresolution (frequency) components. There are two types of wavelets: father wavelets (ϕ), which capture the smooth and low frequency part of the series, and mother wavelets (ψ), which capture the high frequency components of the series, where $\int \phi(t)\,dt = 1$ and $\int \psi(t)\,dt = 0$.

Given a time series y_t with a certain number of observations N, its wavelet multiresolution representation is given by

$$y_t = \sum_k s_{J,k}\phi_{J,k}(t) + \sum_k d_{J,k}\psi_{J,k}(t) + \sum_k d_{J-1,k}\psi_{J-1,k}(t) + \cdots + \sum_k d_{1,k}\psi_{1,k}(t) \quad , \qquad (5)$$

where J represents the number of multiresolution levels (or frequencies), k defines the length of the filter, $\phi_{J,k}(t)$ and $\psi_{j,k}(t)$ are the wavelet functions, and $s_{J,k}$, $d_{J,k}$, $d_{J-1,k}, \ldots, d_{1,k}$ are the wavelet coefficients.

The wavelet functions are generated from the father and mother wavelets through scaling and translation as follows

$$\begin{aligned} \phi_{J,k}(t) &= 2^{-J/2}\phi\left(2^{-J}t - k\right) \\ \psi_{j,k}(t) &= 2^{-j/2}\psi\left(2^{-j}t - k\right) \quad , \end{aligned}$$

while the wavelet coefficients are given by

$$\begin{aligned} s_{J,k} &= \int y_t \phi_{J,k}(t)\,dt \\ d_{j,k} &= \int y_t \psi_{j,k}(t)\,dt \quad , \end{aligned}$$

where $j = 1, 2, ..., J$.

Due to the practical limitations of DWT in empirical applications, we perform wavelet decomposition analysis here by applying the maximal overlap discrete wavelet transform (MODWT).

Appendix 3. Out-of-sample R-squares for all predictors

Predictor	*ERP*			*BRP*		
	single_ts	*single_wav*	Frequency	*single_ts*	*single_wav*	Frequency
DP	-1.87	-0.59	D_6	-0.75	2.89**	D_1
DY	-1.96	-0.24	D_1	-0.42	-0.01	D_1
EP	-1.05	0.77**	D_3	-0.60	-0.50	D_7
RVOL	-0.73	-0.30	D_1	-0.26	-0.09	D_4
BM	-0.53	0.50**	D_5, D_6	-0.15	5.45***	D_1, D_2, D_5, D_7
NTIS	-3.05	0.15	D_1	-3.19	0.53	D_1
LTY	-0.32	0.08	D_6	-1.81	0.76**	D_4
LTR	-0.29	0.85**	D_7	-0.36	-4.16	D_7
TMS	-0.76	1.70***	D_7	1.70**	1.35**	D_5
DFY	-2.82	-0.86	D_6	-1.14	-0.25	D_7
DFR	-1.84	0.07	D_1	-1.13	-0.82	D_6
INFL	-0.61	1.77**	D_2, D_5	-0.77	-0.44	D_1

Table 4: Out-of-sample R-squares (R^2_{OS})

This table reports the out-of-sample R-squares as percentages for bond risk premium (*BRP*) and equity risk premium (*ERP*) forecasts at monthly frequencies of regressions using one original predictor at a time (*single_ts*) and regressions using the frequencies of one original predictor at a time (*single_wav*). The list of predictors is described in Appendix 1. The out-of-sample R-squares $\left(R^2_{OS}\right)$ measures the proportional reduction in the mean squared forecast error for the predictive model relative to the forecast based on the historical mean (HM). The one-month-ahead out-of-sample forecast of the *BRP* and the *ERP* is generated using a sequence of expanding windows. The sample period is from 1973:01 to 2018:12. The out-of-sample forecasting period is from 1990:01 to 2018:12, monthly frequency. Asterisks denote the significance of the out-of-sample *MSFE*-adjusted statistic of Clark and West (2007). *** and ** denote significance at the 1% and 5% levels, respectively.

Appendix 4. Implemented version of the Black-Litterman model

There are N assets and K active investment views ($N = K = 2$: bonds and stocks). $\boldsymbol{\mu}$ is an $N\times$ vector of expected excess returns: BRP and ERP forecasts for bonds and stocks, respectively. τ is a scaling parameter (which we set to unity as in Almadi, Rapach, and Suri, 2014), $\boldsymbol{\Sigma}$ is an $N \times N$ covariance matrix, P is a $K \times N$ matrix whose elements in each row represent the weight of each asset in each of the $K-$view portfolios, Ω is a matrix representing the confidence in each view, Q is a $K \times 1$ vector of expected excess returns of the $K-$view portfolios, and Π is a $N\times$ vector of the equilibrium excess returns of the assets.

The original Black-Litterman model (BLM) of expected excess returns in Black and Litterman (1992) is given by:

$$\mu = \left[(\tau\Sigma)^{-1} + P'\Omega^{-1}P\right]^{-1}\left[(\tau\Sigma)^{-1}\Pi + P'\Omega^{-1}Q\right] \quad ,$$

which by applying the Matrix Inversion Lemma can be rewritten as follows (Da Silva, Lee, and Pornrojnangkool, 2009):

$$\mu = \Pi + \Sigma P'\left[\frac{\Omega}{\tau} + P\Sigma P'\right]^{-1}(Q - P\Pi) = \Pi + G \quad , \tag{6}$$

where G is the term that captures the deviations of expected excess returns from the equilibrium due to active investment views. Equation (6) summarizes the key idea behind the BLM model: the expected excess return will be different from the equilibrium excess return if and only if investor views differ from equilibrium views.

The construction of the actively managed portfolios consists in two steps. First, we compute the posterior expected excess return vector, μ_{t+1}, and posterior return covariance matrix, Σ_{t+1}. We start from the selected vector of excess return forecasts ($\widehat{BRP}_{t+1}$ and $\widehat{ERP}_{t+1}$) obtained from predictive regression models explained in section 2.2. We generate an exponentially weighted moving average estimate of the monthly return covariance matrix $V = \hat{\Sigma}_{t+1}$. We set the decay parameter to 0.97, which is frequently used for monthly series.

To set matrix Ω, we follow the suggestion of Da Silva, Lee, and Pornrojnangkool (2009) and use:

$$\frac{\Omega}{\tau} = diag(diag(PVP')) \ .$$

Adopting the approach of Idzorek (2004), the posterior return covariance matrix is given by:

$$\Sigma_{t+1} = \left[(\tau V)^{-1} + \left(P'\Omega^{-1}P\right)\right]^{-1} \ .$$

From expression (6) for the expected returns, by setting (i) the vector of the equilibrium excess returns of assets as $\Pi = 0$, as in Da Silva, Lee, and Pornrojnangkool (2009), (ii) using the vector of BRP and ERP forecasts as matrix Q and (iii) using the posterior return covariance matrix Σ_{t+1}, it is obtained the posterior expected excess return vector, μ_{t+1}.

The second step for the construction of the portfolio consists in using μ_{t+1} and Σ_{t+1} to obtain the portfolio weights. Recall that the objective function of an active asset manager is to maximize the return of the portfolio with a penalty on the square of tracking error towards the relevant benchmark:

$$\begin{aligned} & max \ \ (\boldsymbol{\varpi_A} + \boldsymbol{\varpi_B})' \boldsymbol{\mu} - \lambda \boldsymbol{\varpi'_A} \boldsymbol{\Sigma} \boldsymbol{\varpi_A} \\ & s.t. \ \ \boldsymbol{\varpi'_A} 1 = 0 \end{aligned} \tag{7}$$

where $\boldsymbol{\varpi_A}$ and $\boldsymbol{\varpi_B}$ are the vectors of active positions and benchmark portfolio weights, respectively. The parameter λ is given by $\lambda = \frac{1}{2TE}\sqrt{\Theta'\boldsymbol{\Sigma}\Theta}$, with TE representing the tracking error (set to a constant annualized value of 5.80% as explained in section 4.1) and matrix Θ

is:

$$\Theta = \mathbf{\Sigma}^{-1}\left(I - 1\frac{1'\mathbf{\Sigma}^{-1}}{1'\mathbf{\Sigma}^{-1}1}\right)\boldsymbol{\mu}\ .$$

The active weights $\boldsymbol{\varpi_A}$ are given by $\boldsymbol{\varpi}_A = \frac{\Theta}{2\lambda}$. Thus, total weights are $\boldsymbol{\varpi} = \boldsymbol{\varpi_A} + \boldsymbol{\varpi_B}$. We assume the investor will neither leverage nor short-sell available assets (following the notation in the paper, $h = 1$ and $l = 0$). We further assume that the investor rebalances the portfolio at the same monthly frequency as the forecast horizon.