# Capital Allocation and the Market for Mutual Funds: Inspecting the Mechanism*

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**Abstract**

We analyze the effects of returns to scale on capital allocation decisions in the mutual fund market by exploiting individual heterogeneity in decreasing returns to scale across funds. We find strong evidence that steeper decreasing returns to scale attenuate flow sensitivity to performance and lead to smaller fund sizes. Our results are consistent with a rational model for active management. Using the model, we argue that a large fraction of capital allocation due to differences in decreasing returns to scale can be plausibly attributed to investors anticipating these effects of scale.

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# 1 Introduction

An important question in financial economics is whether investors efficiently allocate capital across financial assets. Under the standard neoclassical assumptions, investors compete with each other for positive present value opportunities, and by doing so, remove them in equilibrium. In the case of mutual funds, the literature has argued that decreasing returns to scale (DRS) play a key role in equilibrating the mutual fund market (Berk and Green (2004)). Because the percentage fee that mutual funds charge changes infrequently, the bulk of the equilibration process operates through the size (or Assets Under Management (AUM)) of the fund. When good news about a mutual fund arrives, rational Bayesian updating will lead investors to view the fund as a positive Net Present Value (NPV) buying opportunity at its current size. In response, flows will go to that fund. As the fund grows, the manager of the fund finds it increasingly harder to put the new inflows to good use, leading to a deterioration of the performance of the fund. The flows into the fund will stop when the fund is no longer a positive NPV investment, and the fund's abnormal return to investors has reverted back to zero.

In this paper we investigate this equilibrating mechanism more closely. In particular, if the above-mentioned equilibration process is at work, we should expect to find that the degree of decreasing returns to scale (DRS) can have implications for the flow sensitivity to performance (FSP). While there is much evidence that an active fund's ability to outperform its benchmark declines as its size increases,[1] there is surprisingly little empirical work devoted to whether investors account for the adverse effects of fund scale in making their capital allocation decisions.

We address this important question by formally deriving and empirically testing what a rational model for active management implies about the relation between returns to scale and flow sensitivity to performance. Using a theory model similar to that of Berk and Green (2004), we show that steeper decreasing returns to scale attenuates flow sensitivity to performance. In the model, investors rationally interpret high performance as evidence of the manager's superior skill, so good performance results in an inflow of funds. More importantly, the magnitude of the capital response is primarily driven by the extent of decreasing returns to scale. As a fund's returns decrease in scale more steeply, the positive net alpha is competed away with a smaller amount of capital inflows, making flows less sensitive to performance.

To test this theoretical insight, one needs a source of heterogeneity in decreasing re-

[1] See, for example, Chen et al. (2004), Yan (2008), Ferreira et al. (2013), and Zhu (2018).

turns to scale. One also needs to observe investor reactions to this heterogeneity. Indeed, we demonstrate that there is a substantial amount of heterogeneity in DRS across individual funds, with correspondingly heterogeneous flow sensitivity to performance across funds. Our approach can be interpreted as inferring how the subjective size-performance relation, perceived by investors in real time, is incorporated into the flow-performance relation going forward. We find that a steeper decreasing returns to scale parameter predicts a lower sensitivity of flows to performance, consistent with the main prediction of our model.

One of the challenges in estimating the effect of decreasing returns to scale on flow sensitivity to performance is the estimation error in fund-specific DRS. As a result, the point estimates of the DRS-FSP relation using DRS estimates from simple fund-by-fund regressions are likely to suffer from an errors-in-variables bias. We alleviate the errors-in-variables bias by relating the heterogeneity in decreasing returns to scale to a set of fund characteristics. In particular, by regressing the fund-specific DRS estimates on these characteristics, we obtain fitted values that we use as a more robust way of obtaining cross-sectional variation. We find the degree of DRS is stronger for higher-volatility funds, sole-managed funds, older funds, as well as funds that have experienced outflows in the past year. Next, we show that while the statistical significance of the DRS-FSP relation is unaffected by using characteristic-based DRS, the point estimates become substantially more negative, suggesting that the projection onto characteristics indeed has alleviated the errors-in-variables problem.

Next we turn to the economic significance of our estimates. In particular, we assess how equilibrium fund size is affected by the cross-sectional variation in decreasing returns to scale parameters. This exercise does require model assumptions. We calibrate a rational model in the spirit of Berk and Green (2004) to compute counterfactual fund sizes. We find that at least 49% of the cross-sectional variance of fund sizes can be related to cross-sectional variation in decreasing returns to scale parameters. More importantly, the uncalibrated version of our model with heterogeneous returns to scale can quantitatively reproduce capital allocation in the version calibrated to the empirical DRS-FSP relationship. We also find that the DRS-FSP relation estimates in the uncalibrated version tend to be substantially more negative than those from the data. Thus, it appears that investors in the data face a substantially harder learning problem than in our simple model, though we leave identifying additional aspects of learning to explain this gap as a question to be explored by future research.

Beyond implications for fund flows, steeper decreasing returns to scale has implications for fund size in equilibrium. In the model, fund size in equilibrium is proportional to the ratio of perceived skill over diseconomies of scale, which predicts that, all else equal, the decreasing returns to scale parameter should be lower for larger funds. This prediction is confirmed in our empirical analysis. Moreover, if investors update their beliefs about skill as in the model, their perception of optimal size ought to converge to true optimal size as funds grow older. Consistent with this argument, we find that estimates for the optimal size largely explains capital allocation across older funds in the data. We measure (log) optimal size by the average ratio of the usual net alpha that is adjusted for returns to scale over the characteristic-based DRS. We show that older fund's size continues to be significantly related to our measure of its optimal size even when we control for an alternative measure of optimal size that assumes fund scale has the same effect on performance for all funds. Again, investors seem to account not only for the presence of decreasing returns to scale, but also for the heterogeneity of decreasing returns to scale across funds.

Taken together, our results demonstrate that investors do account for the adverse effects of fund scale in making their capital allocation decisions, and that the rational expectations equilibrium does a reasonable job of approximating the observed equilibrium in the mutual fund market. In contrast, mutual fund investors were generally deemed naive return chasers because fund flows respond to past performance even though performance is not persistent.[2] Furthermore, many papers in the mutual fund literature have documented that mutual fund returns show little evidence of outperformance.[3] While these findings led many researchers to question the rationality of mutual fund investors, Berk and Green (2004) argue that they are consistent with a model of how competition between rational investors determines the net alpha in equilibrium. We contribute to this debate by presenting findings that are hard to reconcile with anything other than the existence of rational fund flows.

## 2 Definitions and hypotheses

To formally derive our hypothesis, we use the notation and setup presented in Berk and van Binsbergen (2016). Let $q_{it}$ denote assets under management (AUM) of fund $i$ at time $t$ and let $\theta_i$ denote a parameter that describes the skill of the manager of fund $i$. At time $t$, investors use the time $t$ information set $I_t$ to update their beliefs on $\theta_i$ resulting in the distribution function $g_t(\theta_i)$ implying that the expectation of $\theta_i$ at time $t$ is:

$$\overline{\theta}_{it} \equiv E[\theta_i | I_t] = \int \theta_i g_t(\theta_i)\, d\theta_i. \quad (1)$$

[2]See Chevalier and Ellison (1997) and Sirri and Tufano (1998), among others.

[3]See Jensen (1968), Malkiel (1995), Gruber (1996), Fama and French (2010), and Del Guercio and Reuter (2013), among others.

We assume throughout that $g_t(\cdot)$ is not a degenerate distribution function. Let $R_{it}^n$ denote the return in excess of the risk free rate earned by investors in fund $i$ at time $t$. This return can be split up into the excess return of the manager's *benchmark*, $R_{it}^B$, and a deviation from the benchmark $\varepsilon_{it}$:

$$R_{it}^n = R_{it}^B + \varepsilon_{it}. \tag{2}$$

Note that $q_{it}$, $R_{it}^n$ and $R_{it}^B$ are elements of $I_t$. Let $\alpha_{it}(q)$ denote investors' subjective expectation of $\varepsilon_{i,t+1}$ when investing in fund $i$ that has size $q$ between time $t$ and $t+1$, and let it be equal to:

$$\alpha_{it}(q) = \overline{\theta}_{it} - h_i(q), \tag{3}$$

where $h_i(q)$ is a strictly increasing function of $q$ that captures the decreasing returns to scale the manager faces, which can vary by fund. In equilibrium, the size of the fund $q_{it}$ adjusts to ensure that there are no positive net present value investment opportunities so $\alpha_{it}(q_{it}) = 0$ and

$$\overline{\theta}_{it} = h_i(q_{it}). \tag{4}$$

At time $t+1$, the investor observes the manager's return outperformance, $\varepsilon_{it+1}$, which is a signal that is informative about $\theta_i$. The conditional distribution function of $\varepsilon_{i,t+1}$ at time $t$, $f_t(\varepsilon_{it+1})$, satisfies the following condition in equilibrium:

$$E[\varepsilon_{it+1} | I_t] = \int \varepsilon_{it+1} f_t(\varepsilon_{it+1}) \, d\varepsilon_{it+1} = \alpha_{it}(q_{it}) = 0. \tag{5}$$

In other words, the manager's return outperformance can be expressed as follows:

$$\begin{aligned} \varepsilon_{it+1} &= \theta_i - h_i(q_{it}) + \epsilon_{it+1} \\ &= s_{it+1} - h_i(q_{it}), \end{aligned}$$

where $s_{it+1} = \theta_i + \epsilon_{it+1}$. Our hypothesis relies on the insight that good news, that is, high $s_{it}$, implies good news about $\theta_i$ and bad news, low $s_{it}$, implies bad news about $\theta_i$. The following lemma shows that this condition holds generally. That is, $\overline{\theta}_{it}$ is a strictly increasing function of $s_{it}$.

**Lemma 1** *If the likelihood ratio $f_t(s_{it+1} | \theta_i) / f_t(s_{it+1} | \theta_i^c)$ is monotone in $s_{it+1}$, increasing if $\theta_i > \theta_i^c$ and decreasing otherwise,*

$$\frac{\partial \overline{\theta}_{it+1}}{\partial s_{it+1}} > 0. \tag{6}$$

***Proof.*** *See Milgrom (1981).* ■

In addition, we assume that the costs that manager $i$ faces in expanding the fund's scale is given by:

$$h_i(q) = b_i h(q), \tag{7}$$

where $b_i > 0$ is a parameter that captures the cross sectional variation in the fund's returns to scale technology and $h(q)$ is a strictly increasing function of $q$, which essentially determines the form of decreasing returns to scale technology that is common across all funds. Using (7) to rewrite (4) now gives

$$q_{it} = h^{-1}\left(\frac{\overline{\theta}_{it}}{b_i}\right) \tag{8}$$

The following lemma shows how the size of the fund $q_{it}$ depends on the information in $s_{it}$ or the parameter $b_i$.

**Lemma 2**

$$\frac{\partial q_{it}}{\partial s_{it}} = \frac{1}{b_i h'(q_{it})} \frac{\partial \overline{\theta}_{it}}{\partial s_{it}} \tag{9}$$

*and*

$$\frac{\partial q_{it}}{\partial b_i} = -\frac{h(q_{it})}{b_i h'(q_{it})}. \tag{10}$$

***Proof.*** *First, note that $\varepsilon_{it}$ does not contain information about managerial ability that is not already contained in $s_{it}$. Because rescaling the fund's returns to scale technology (i.e., changing the parameter $b_i$) does not change the signal $s_{it}$, we can conclude that*

$$\frac{\partial \overline{\theta}_{it}}{\partial b_i} = 0. \tag{11}$$

*Now differentiating (8) with respect to $s_{it}$, and using the fact that these signals are independent of $b_i$ (i.e., $\partial b_i / \partial s_{it} = 0$), gives*

$$\frac{\partial q_{it}}{\partial s_{it}} = \frac{1}{h'\left(\overline{\theta}_{it}/b_i\right)} \frac{\partial\left(\overline{\theta}_{it}/b_i\right)}{\partial s_{it}} = \frac{1}{b_i h'\left(\overline{\theta}_{it}/b_i\right)} \frac{\partial \overline{\theta}_{it}}{\partial s_{it}} = \frac{1}{b_i h'(q_{it})} \frac{\partial \overline{\theta}_{it}}{\partial s_{it}},$$

*where the last equality follows from (8). Similarly, differentiate (8) with respect to $b_i$, and use (11) to substitute for $\partial \overline{\theta}_{it} / \partial b_i$ in this expression. Appealing, again, to (8), gives (10).* ■

Next, let the flow of capital into mutual fund $i$ at time $t$ be denoted by $F_{it}$, that is,

$$F_{it+1} \equiv \log\left(q_{it+1}/q_{it}\right).$$

Differentiating this expression with respect to $s_{it+1}$,

$$\frac{\partial F_{it+1}}{\partial s_{it+1}} = \frac{1}{q_{it+1}}\frac{dq_{it+1}}{ds_{it+1}} = \frac{1}{q_{it+1}}\frac{1}{b_i h'\left(q_{it+1}\right)}\frac{\partial \overline{\theta}_{it+1}}{\partial s_{it+1}} > 0,$$

where the second equality follows from (9) and the inequality follows from Lemma 1, so good (bad) performance results in an inflow (outflow) of funds. This result is one of the important insights from Berk and Green (2004).

Given the importance of returns to scale technology in determining the size of a fund, a natural question to ask is, what is the implication of steeper decreasing returns to scale for the flow-performance relation? We answer this question by computing the derivative of the flow-performance sensitivity with respect to $b_i$:

$$\begin{aligned}
\frac{\partial}{\partial b_i}\left(\frac{\partial F_{it+1}}{\partial s_{it+1}}\right) &= \frac{\partial}{\partial b_i}\left(\left(\frac{1}{q_{it+1}}\frac{1}{b_i h'\left(q_{it+1}\right)}\right)\left(\frac{\partial \overline{\theta}_{it+1}}{\partial s_{it+1}}\right.\right.\\
&= -\frac{q_{it+1}h'\left(q_{it+1}\right) + \frac{\partial q_{it+1}}{\partial b_i}\left(b_i h'\left(q_{it+1}\right) + q_{it+1} b_i h''\left(q_{it+1}\right)\right)}{q_{it+1}^2\left(b_i h'\left(q_{it+1}\right)\right)^2}\frac{\partial \overline{\theta}_{it+1}}{\partial s_{it+1}}\\
&= -\frac{q_{it+1}h'\left(q_{it+1}\right) - h\left(q_{it+1}\right)\left(1 + \frac{q_{it+1}h''\left(q_{it+1}\right)}{h'\left(q_{it+1}\right)}\right)}{q_{it+1}^2\left(b_i h'\left(q_{it+1}\right)\right)^2}\left(\frac{\partial \overline{\theta}_{it+1}}{\partial s_{it+1}},\right.
\end{aligned} \tag{12}$$

where the first equality follows from (11) because when $\overline{\theta}_{it+1}$ is solely a function of the history of realized signals and is not a function of $b_i$ then $\frac{\partial}{\partial b_i}\left(\frac{\partial \overline{\theta}_{it+1}}{\partial s_{it+1}}\right)\left(= 0\right.$ and the last equality follows from (10). What (12) combined with Lemma 1 tells us is that steeper decreasing returns to scale must lead to a smaller flow of funds response to performance if and only if

$$q_{it+1}h'\left(q_{it+1}\right) - h\left(q_{it+1}\right)\left(1 + \frac{q_{it+1}h''\left(q_{it+1}\right)}{h'\left(q_{it+1}\right)}\right)\left(> 0.\right. \tag{13}$$

Unfortunately, the left hand side of Equation 13 is not easy to sign without further assumptions. To assess whether this condition holds, we rely on the second-order approximation to the decreasing returns to scale technology:

$$h\left(q\right) \simeq h_0 + h_1 \log\left(q\right) + h_2 \log\left(q\right)^2, \tag{14}$$

where $h_i$ for $i = \{0, 1, 2\}$ are the coefficients in the second-order approximation. This approximation nests exactly specifying the technology as logarithmic, most commonly considered in empirical studies, if we set $h_1 > 0$ and $h_0 = h_2 = 0$. Going forward, we set $h_0 = 0$. This assumption is without loss of generality, because we can rewrite the skill parameter as

$\theta_i' = \theta_i - b_i h_0$, which, in turn, renders $h_0' = 0$. The following proposition shows that, under approximation (14), condition (13) holds generally. That is, steeper decreasing returns to scale leads to a weaker flow response to performance. We take this as our main hypothesis that we will take to the data.

**Proposition 3** *Under approximation (14), the derivative of the flow-performance sensitivity with respect to the decreasing returns to scale parameter is negative, that is,*

$$\frac{\partial}{\partial b_i}\left(\frac{\partial F_{it+1}}{\partial s_{it+1}}\right) < 0.$$

***Proof.*** *Under approximation (14), the left-hand side of (13) is then given by:*

$$\begin{aligned}
& h_1 + 2h_2 \log(q_{it+1}) - \left(h_1 \log(q_{it+1}) + h_2 \log(q_{it+1})^2\right)\left(1 + \frac{\frac{2h_2 - (h_1 + 2h_2 \log(q_{it+1}))}{q_{it+1}}}{\frac{h_1 + 2h_2 \log(q_{it+1})}{q_{it+1}}}\right) \\
= \; & h_1 + 2h_2 \log(q_{it+1}) - \left(h_1 \log(q_{it+1}) + h_2 \log(q_{it+1})^2\right)\frac{2h_2}{h_1 + 2h_2 \log(q_{it+1})} \\
= \; & \frac{(h_1 + 2h_2 \log(q_{it+1}))^2 - 2\left(h_1 \log(q_{it+1}) + h_2 \log(q_{it+1})^2\right)h_2}{h_1 + 2h_2 \log(q_{it+1})} \\
= \; & \frac{(h_1 + h_2 \log(q_{it+1}))^2 + h_2^2 \log(q_{it+1})^2}{h_1 + 2h_2 \log(q_{it+1})}.
\end{aligned}$$

*The numerator of this expression is the sum of two squares, so it is positive. Note that the denominator can be rewritten as the product of $q_{it+1}$ and $h'(q_{it+1})$ under the given approximation. Recall that $h(q)$ is a strictly increasing function of $q$, reflecting the fact that all mutual funds must face decreasing returns to scale in equilibrium. Requiring that, under the approximation, $h'(q_{it+1}) > 0$ is also ensured, this means that the denominator is positive as well. It then follows immediately that condition (13) holds, which completes the proof.* ■

# 3 Data

Our data come from CRSP and Morningstar. We require that funds appear in both CRSP and Morningstar, which allows us to validate data accuracy across the two databases. We merge CRSP and Morningstar based on funds' tickers, CUSIPs, and names. We then compare assets and returns across the two sources in an effort to check the accuracy of each match following Berk and van Binsbergen (2015). We refer the readers to the data appendices of that paper for the details. Our mutual fund data set contains 3,066 actively managed domestic equity-only mutual funds in the United States between 1979 and 2014.

We use Morningstar Category to categorize funds into nine groups corresponding to Morningstar's $3 \times 3$ stylebox (large value, mid-cap growth, etc.). We also use keywords in the Primary Prospectus Benchmark variable in Morningstar to exclude bond funds, international funds, target funds, real estate funds, sector funds, and other non-equity funds. We drop funds identified by CRSP or Morningstar as index funds, in addition to funds whose name contains "index." We also drop any fund observations before the fund's (inflation-adjusted) AUM reaches \$5 million.

We now define the key variables used in our empirical analysis: fund performance, fund size, and fund flows. Summary statistics are in Table 1.

## 3.1 Fund Performance

We take two approaches to measuring fund performance. First, we use the standard risk-based approach. The recent literature finds that investors use the CAPM in making their capital allocation decisions (Berk and van Binsbergen (2016)), and hence we adopt the CAPM. In this case the risk adjustment $R_{it}^{\text{CAPM}}$ is given by:

$$R_{it}^{\text{CAPM}} = \beta_{it}\text{MKT}_t,$$

where $\text{MKT}_t$ is the realized excess return on the market portfolio and $\beta_{it}$ is the market beta of fund $i$. We estimate $\beta_{it}$ by regressing the fund's excess return to investors onto the market portfolio over the sixty months prior to month $t$. Because we need historical data of sufficient length to produce reliable beta estimates, we require a fund to have at least two years of track record to estimate the fund's betas from the rolling window regressions.

Second, we follow Berk and van Binsbergen (2015) by taking the set of passively managed index funds offered by Vanguard as the alternative investment opportunity set.[4] We then define the Vanguard benchmark as the closest portfolio in that set to the mutual fund. Let $R_t^j$ denote the excess return earned by investors in the $j$'th Vanguard index fund at time $t$. Then the Vanguard benchmark return for fund $i$ is given by:

$$R_{it}^{\text{Vanguard}} = \sum_{j=1}^{n(t)} \beta_i^j R_t^j,$$

where $n(t)$ is the total number of index funds offered by Vanguard at time $t$ and $\beta_i^j$ is obtained from the appropriate linear projection of active mutual fund $i$ onto the set of Vanguard index

[4] See Table 1 of that paper for the list of Vanguard Index Funds used to calculate the Vanguard benchmark.

funds. As pointed out by Berk and van Binsbergen (2015), by using Vanguard funds as the benchmark, we ensure that this alternative investment opportunity set was marketed and tradable at the time. Again, we require a minimum of 24 months of data to estimate $\beta_i^j$'s necessary for defining the Vanguard benchmark for fund $i$.

Our measures of fund performance are then $\widehat{\alpha}_{it}^{\text{CAPM}}$ and $\widehat{\alpha}_{it}^{\text{Vanguard}}$, the realized return for the fund in month $t$ less $R_{it}^{\text{CAPM}}$ and $R_{it}^{\text{Vanguard}}$. The average of $\widehat{\alpha}_{it}^{\text{CAPM}}$ is +1.5 bp per month, whereas the average $\widehat{\alpha}_{it}^{\text{Vanguard}}$ is −1.4 bp per month.

## 3.2 Fund Size and Flows

We adjust all AUM numbers by inflation by expressing all numbers in January 1, 2000 dollars. Adjusting AUM by inflation reflects the notion that the fund's real (rather than nominal) size is relevant for capturing decreasing returns to scale in active management. That is, lagged real AUM corresponds to $q_{it-1}$ in the previous section. There is considerable dispersion in real AUM: the inner-quartile range is from \$45 million to \$618 million.

We calculate flows for fund $i$ in month $t$ as:

$$F_{it} = \frac{AUM_{it} - AUM_{it-1}\left(1 + R_{it}\right)}{AUM_{it-1}\left(1 + R_{it}\right)},$$

where $AUM_{it}$ is the (nominal) AUM of fund $i$ at the end of month $t$, and $R_{it}$ is the total return of fund $i$ in month $t$.[5] So flows represent the change in the fund's net assets not attributable to its return gains or losses. The flow of fund data contains some implausible outliers, so we winsorize flows at its 1st and 99th percentiles. Median $F_{it}$ is −0.2% per month.

# 4 Methodology

Our analysis relies on a theoretical link between decreasing returns to scale and flow sensitivity to returns. We discuss how we estimate each part in the following sections.

[5] Note that we use $AUM_{it-1}\left(1 + R_{it}\right)$ in the denominator rather than $AUM_{it-1}$, which is typically used in much of the existing literature on fund flows. Unfortunately, this definition distorts the flow for very large negative returns. For example, liquidition of a fund, i.e., $AUM_{it} = 0$, implies a flow of $-\left(1 + R_{it}\right)$. Our measure of the flow of funds is equal to, and correctly so, −1 in this case. Regardless, our findings are unaffected by using the more common definition of the flow.

## 4.1 Fund-Specific Decreasing Returns to Scale (DRS)

Empirically, we assume that the net alpha that manager $i$ generates by actively managing money is given by:

$$\alpha_{it} = a_i - b_i \log(q_{it-1}) + \epsilon_{it}, \tag{15}$$

where $a_i$ is the fund fixed effect, $b_i$ captures the size effect, which can vary by fund, and $q_{it-1}$ is the dollar size of the fund.

The simple regression model in equation (15) corresponds to the model in Section 2. This model further assumes the form of the fund's decreasing returns to scale technology is logarithmic, which is often used to empirically analyze the nature of returns to scale due to severe skewness in dollar fund size.

We depart from much of the literature describing the size-performance relation by taking the size-performance relation to vary across funds. Indeed, the effect of scale on a fund's performance is unlikely to be constant across funds. For example, a fund's returns should be decreasing in scale more steeply for those that have to invest in small and illiquid stocks, which are likely to face lower liquidity.

Given that it is not clear a priori why and how the size-performance relation depends on which fund characteristics, we prefer to estimate fund-specific $a$ and $b$ parameters in our main analysis. For each fund $i$ at time $t$, we run the time-series regression of $\alpha_{i\tau}$ on $\log(q_{i\tau-1})$ using sixty months of its data before time $t$. Estimating $b$ fund by fund leads to imprecise estimates especially for funds with short track records, so we require at least three years of data to estimate fund-specific returns to scale of a mutual fund.

The estimate of $b_i$, $\widehat{DRS}_{it}^{m}$, is obtained from (15) using sixty months of the data for fund $i$ prior to time $t$, where the alpha can be estimated under model $m \in \{\text{CAPM}, \text{Vanguard}\}$. Intuitively, these estimates represent, for investors who use model $m$ in making capital allocation decisions, their perception of the effect of size on performance for fund $i$ at time $t$ based on information prior to time $t$.

Panel A of Figure 1 shows how the cross-sectional distribution of $\widehat{DRS}_{it}$ using the CAPM alpha varies over time. For each month in 1991 through 2014, the figure plots the average as well as the percentiles of the estimated fund-specific $b$ parameters across all funds operating in that month. The plot shows considerable heterogeneity in decreasing returns to scale across funds. For example, the interquartile range is more than 3 times larger than the estimates' cross-sectional median in a typical month; in fact, this ratio can be almost as large as 22 in some months. We find that, for the average fund, one percent increase in fund size is typically associated with a sizeable decrease in fund performance of about 0.9 basis points (bp) per month. This evidence suggests that the subjective size-performance relation, perceived by investors in real time, provides an ideal identifying variation in the extent of decreasing returns to scale.

Panel B of Figure 1 shows the time evolution of $\widehat{DRS}_{it}$ when we take Vanguard index funds as the alternative investment opportunity set. Similar to our main measure in Panel A, the alternative measure exhibits a clear heterogeneity in diseconomies of scale across funds, though these estimates typically indicate milder decreasing returns to scale.

## 4.2 Fund-Specific Flow Sensitivity to Performance (FSP)

We estimate the fund-specific flow sensitivities to past performance by estimating the following regression fund by fund:

$$F_{it} = c_i + \gamma_i P_{it-1} + \upsilon_{it}, \tag{16}$$

where $P_{it-1}$ is annual alpha for the year leading to month $t-1$, computed by compounding the monthly alphas as follows:

$$P_{it-1} = \prod_{s=t-12}^{t-1} \left(1 + R_{is}^{n} - R_{is}^{B}\right) - 1.$$

This regression is consistent with empirical evidence that investors do not respond immediately. For example, Berk and van Binsbergen (2016) and Barber, Huang, and Odean (2016) show that flows respond to recent returns, as well as distant returns. Parameter $\gamma_i > 0$ captures the positive time-series relation between performance and fund flows, which can vary by fund. Again, this is where we depart from much of the literature describing the flow-performance relation.

At time $t$, we calculate the fund's flow sensitivity to performance by estimating (16) using its data over the subsequent 5 years. For fund $i$, let $\left(\widehat{FSP}_{it}^{m}\right)$ be the estimated flow-performance regression coefficient of that model, where the performance can be estimated under model $m \in \{\text{CAPM}, \text{Vanguard}\}$. To avoid using imprecise estimates, we require these coefficient estimates to be obtained from at least three years of data. For the average fund, we observe that an increase of 1% in the monthly CAPM alpha is associated with an increase of 1.3% in monthly flows next month.

Figure 2 displays the evolution of the distribution of $\widehat{FSP}_{it}$ by plotting the average as well as the percentiles of the estimated flow sensitivities to performance at each point of time. Panel A shows the results using the CAPM alpha, and Panel B shows the results when net alpha is computed using Vanguard index funds as benchmark portfolios. Note that the results are very similar across the two panels, manifesting considerable heterogeneity in the flow-performance relation across funds. More importantly, Figure 2 shows that while both the mean and median $\widehat{FSP}_{it}$ do not exhibit any obvious trend, these are certainly time varying. As the red dashed lines in the figure make clear, the distribution has remained roughly the same over our sample period, conditional on the median.

# 5 Results

## 5.1 DRS and Flow Sensitivity to Performance

To investigate whether investors pay attention to the fund's decreasing returns to scale technology in making their capital allocation decisions, we run panel regressions of fund $i$'s flow sensitivity to performance going forward in month $t$, $\widehat{FSP}_{it}$, on the fund's returns to scale estimated as of the previous month-end, $\widehat{DRS}_{it}$. We test the null hypothesis that the slope on $\widehat{DRS}_{it}$ is zero.[6] We consider two approaches: plain OLS and OLS with fixed effects (OLS FE), as detailed below. We report the results in Table 2.[7] In Panel A, we report the results using the CAPM as the benchmark; in Panel B, we use Vanguard index funds as the benchmark.

We show results based on raw estimates in the first three columns. Across these three columns, we gradually saturate with month and fund fixed effects to focus on variation coming from the market equilibrating mechanism beyond differences in sensitivity across funds and over time. The fund fixed effects absorb the cross-sectional variation in flow/performance sensitivity that is due to differences in investor clientele across funds, while the time fixed effects soak up any variation in flow/performance sensitivity due to investor attention allocation over time. Indeed, there is evidence of clientele differences because some investors tend to update faster than others,[8] and Figure 2 shows how the average as well as the median of flow-performance dynamics vary considerably over time.

In the first column, we include no fixed effects to include all variation in flow sensitivities. Consistent with the main prediction of our model, the estimated coefficient on $\widehat{DRS}_{it}$ is

[6] Surely, not only the independent variable, but the dependent variable are measured imprecisely. The measurement error in $DRS_{it}$ will bias the OLS estimator toward zero. While the measurement error in $FSP_{it}$ will not induce bias in the OLS coefficients, it will render their variance larger. For now, we do not worry, as the errors-in-variables problem will work against us from finding a statistically significant relation that the model predicts.

[7] Table 2 reports the double clustered (by fund and time) $t$-statistics.

[8] See Berk and Tonks (2007).

significantly negative using the CAPM benchmark. This finding is unaffected by controlling for month and/or fund fixed effects. In the second column, we include month fixed effects. The third column further adds fixed effects for funds. The negative coefficients on $\widehat{DRS}_{it}$ in the CAPM-adjusted result are highly statistically significant, with $t$-statistic that are smaller than $-3$. While the estimated coefficient on $\widehat{DRS}_{it}$ using the Vanguard benchmark in column 1 is marginally insignificant (with $t$-statistic of $-1.6$), including month and/or fund fixed effects in this case causes the $t$-statistics to grow substantially in magnitude. Thus, the estimates in the next two columns of Panel B are significantly negative at the 10% and 5% confidence levels, respectively.

The last two columns in Table 2 repeat this exercise with percentile ranks in each month based on $\widehat{DRS}_{it}$ and $\widehat{FSP}_{it}$. In this case, we do not use month fixed effects, as percentile ranks already soak up any time variation in the flow-performance relation. In column 4 of each panel, the estimated plain OLS coefficient on $\widehat{DRS}_{it}$ is significantly negative at the 1% confidence level. We then allow for differences in clientele across funds by adding fund fixed effects (see column 5 of Table 2). Again, the evidence for our main prediction becomes only stronger: the estimated coefficients on $\widehat{DRS}_{it}$ roughly double, while the $t$-statistic more than double to $-7.8$ in Panel A and to $-6.9$ in Panel B.

To summarize, we find a strong negative relation between decreasing returns to scale and flow sensitivity to performance. This relation, which is statistically significant, is consistent with the presence of investors rationally accounting for the adverse effects of fund scale in making their capital allocation decisions. Unfortunately, these coefficient values are not easily interpretable in economic terms, as they represent the effect of one regression coefficient on another regression coefficient. In Section 5.4.1, we propose a way of assessing the economic magnitude of such relation by computing counterfactual fund sizes.

## 5.2 DRS and Fund Size in Equilibrium

While the main implication of our model is that steeper decreasing returns to scale attenuate flow sensitivity to performance, another immediate implication is that steeper decreasing returns to scale shrink fund size. Recall that fund size in equilibrium is proportional to the ratio of perceived skill over diseconomies of scale (see equation (8)). Are large funds characterized by relatively flat decreasing returns to scale technology? To address this question, we run panel regressions of fund $i$'s log real AUM in month $t$ on the fund's returns to scale estimated as of the previous month-end, $\widehat{DRS}_{it}$. We test the null hypothesis that the slope on $\widehat{DRS}_{it}$ is zero.[9] We consider two approaches: plain OLS and OLS with fixed effects (OLS FE), as detailed below. We report the results in Table 4.[10] In Panel A, we report the results using the CAPM as the benchmark; in Panel B, we use Vanguard index funds as the benchmark.

Across the first three columns, we gradually saturate with month and fund fixed effects to focus on variation coming from the market equilibrating mechanism beyond differences in size across funds and over time. The fund fixed effects absorb the cross-sectional variation in fund size due to differences in investors' perception of skill across funds, while the time fixed effects soak up any variation in fund size due to the arrival of news that commonly affect fund performance.

In the first column, we include no fixed effects to include all variation in fund sizes. Consistent with the above prediction of our model, the estimated coefficients on $\widehat{DRS}_{it}$ are significantly negative. This finding is unaffected by using the CAPM or the Vanguard benchmark, as well as controlling for month and/or fund fixed effects. In the second column, we include month fixed effects. The third column further adds fixed effects for funds. The negative coefficients on $\widehat{DRS}_{it}$ in the CAPM-adjusted result are highly statistically significant, with $t$-statistic that are smaller than $-2.33$. The estimated coefficient on $\widehat{DRS}_{it}$ using the Vanguard benchmark in column 1 is marginally significant, with $t$-statistic of $-1.8$. However, including month and/or fund fixed effects in this case cause the $t$-statistics to grow substantially in magnitude, so the estimate in column 2 (3) of Panel B are significantly negative at the 5% (1%) confidence level. Finally, the last column in Table 4 shows that these findings are unaffected by further including controls that are plausibly correlated with the fund size: family size, fund age, and turnover.

## 5.3 Determinants of Returns to Scale

In this subsection, we investigate what drives heterogeneity in returns to scale by analyzing how it depends on fund characteristics. We explore a number of characteristics that seem relevant a priori for heterogeneity in returns to scale: volatility, a multi-manager indicator, a redemption indicator, fund age, and risk exposures.[11]

[9] Again, the independent variable is measured imprecisely. The measurement error in $DRS_{it}$ will bias the OLS estimator toward zero. We will address the estimation error in scale effects in Section 5.4.

[10] Table 4 reports the double clustered (by fund and time) $t$-statistics.

[11] We also explore whether high-turnover funds exhibit steeper decreasing returns to scale and whether there is a weaker negative size-performance relation for funds with a significant degree of international exposure in unreported results. We find a negative relation between returns to scale and international exposure, although the relation is mostly statistically insignificant. The relation between returns to scale and turnover is usually insignificant and flips to negative when we add other fund characteristics. More importantly, our results in Tables 5 and 6 are unaffected by including these characteristics as controls. All of these results are available

The first characteristic, $Std\,(Alpha)$, is the standard deviation of a fund's alphas, which we calculate over the prior 1 year. The second characteristic, $1\,(MultiMgr)$, is a dummy variable that is equal to one if the fund is managed by many managers. About 56% of our funds are multi-manager funds. The third characteristic, $1\,(Outflows)$, is an indicator for whether the fund experienced outflows over the prior 1 year. The fourth characteristic we examine is fund age, measured by the natural logarithm of years since the fund's first offer date (from CRSP) or, if missing, since the fund's inception date (from Morningstar). As is common in the mutual fund literature, we measure the riskiness of the mutual fund using its risk exposures to the factors identified by Fama and French (1995) and Carhart (1997).[12]

Why do we expect these characteristics to affect how scale impacts performance? Theoretically, a fund's portfolio can be interpreted as a combination of investing in the passive benchmark and investing in the actively managed portfolio that is independent of the benchmark returns. Since the cost of managing benchmark exposure is relatively small, the costs of operating the fund are primarily determined by the amount of funds under active management. A reasonable hypothesis is funds that manage a greater proportion of their assets actively are likely to face larger trading costs and, thus, steeper decreasing returns to scale. These behaviors manifest themselves as higher volatility of benchmark-adjusted returns.

Funds experiencing investor outflows might also exhibit steeper decreasing returns to scale. The reason is that funds experiencing redemptions are forced to decrease existing positions, which creates price pressure against these mutual funds.[13] On the other hand, younger funds might exhibit milder decreasing returns to scale. This hypothesis is motivated by Chevalier and Ellison (1999), who find that younger managers hold less risky and more conventional portfolios because they are more likely to be fired for bad performance. In turn, it suggests that younger funds tend to be less aggressive in their trading, perhaps due to fund managers' career concerns. Such incentives, if present, would mitigate the performance erosion associated with fund size. In addition, the division of labor within a fund might alleviate the negative impact of size on performance, so it is the fund's assets under management on a per-manager basis that matters for capturing decreasing returns to scale. If so, a multi-manager fund would be able to deploy capital more easily and, consequently, exhibit milder decreasing returns to scale.

Surely, the extent of decreasing returns to scale is likely to be affected by the stock characteristics chosen by the funds. For example, Carhart (1997) finds that funds with high

upon request.

[12]We estimate these risk exposures by regressing the fund's return on the factors over the prior sixty months.

[13]See Coval and Stafford (2007).

past performance repeat their abnormal performance not because fund managers successfully follow momentum strategies, but probably because some mutual funds accidentally end up holding last year's winners. In turn, these funds capture short-term momentum effect in stock returns virtually without transaction costs. This logic suggests that momentum funds are likely to exhibit steeper decreasing returns to scale. In analyzing the dependence of returns to scale on fund characteristics, we thus control for the contribution of fund style and risk using the loadings on the four Fama-French-Carhart factors.

We examine these hypotheses by running panel regressions of the scale effect computed using only fund $i$'s observations prior to month $t-1$, $\widehat{DRS}_{it}$, on the fund's characteristics at the end of the previous month. Table 5 shows the estimation results.[14] Panel A reports the results using the CAPM as the benchmark; Panel B uses Vanguard index funds as the benchmark. In both panels, we find significant relations between $\widehat{DRS}$ and three characteristics: volatility (column 1), the multi-manager indicator (column 2), and the redemption indicator (column 3). We also find that the slope on fund age is positive (column 4). This result is marginally significant for the CAPM (with $t$-statistic of 1.63 in Panel A of Table 5), but it is statistically significant using the Vanguard benchmark. These results lend strong support to the narrative from the previous paragraphs.

When all four fund characteristics are added at the same time (column 5), the estimated slopes on volatility, multi-manager indicator, and redemption indicator are robust, indicating steeper decreasing returns to scale for higher-volatility funds, sole-manager funds, and funds experiencing outflows. Finally, fund age continues to enter with a positive slope, as in column 4, and it now does so significantly regardless of how one defines the benchmark, indicating that decreasing returns to scale are more pronounced for old funds. To summarize, the same conclusions continue to hold when we jointly assess the dependence of returns to scale on fund characteristics.

## 5.4 Characteristic-Based DRS

We have estimated fund-specific $b$ parameter based on a rolling estimation window. As noted earlier, estimating $b$ fund by fund leads to imprecise estimates especially for funds with short track records. Instead of using the coefficient estimates $\widehat{DRS}$ as before, we use the estimates from column 5 of Table 6 to obtain an economically interpretable component of $\widehat{DRS}$ based on fund characteristics. This implementation choice assumes that all the funds with the same fund characteristics share the same $b$ value. While ignoring variation might potentially lead to inaccuracy in quantifying fund-specific $b$, this method actually seems to increase the

[14] Standard errors of these regressions are two-way clustered by fund and time.

accuracy of the $b$ estimate by dramatically reducing estimation errors. While 27% (30%) of the funds in our sample end up with negative $\widehat{DRS}$ using the CAPM (Vanguard benchmark), less than 1% and 2% of their predicted values based on fund characteristics, denoted by $\widetilde{DRS}$, are negative using the CAPM and Vanguard benchmark, respectively. These results seem sensible since, theoretically, all mutual funds must face decreasing returns to scale in equilibrium.

Figure 3 shows how the cross-sectional distributions of $\widetilde{DRS}_{it}$ varies over time. Panel A shows the results using the CAPM alpha, and Panel B shows the results when net alpha is computed using the Vanguard benchmark. While these distributions are naturally tighter than those of $\widehat{DRS}_{it}$, they remain quite disperse, confirming the presence of considerable heterogeneity in DRS. Interestingly, the cross-sectional distributions over our sample period are stable, net of time-series variation in median $\widehat{DRS}$, which themselves are similar to those in Figure 1.

To assess the robustness of our results regarding the effect of returns to scale on capital flows and sizes, we replace $\widehat{DRS}$ by $\widetilde{DRS}$ and rerun the regressions in Tables 2 and 3, whose results are tabulated in Tables 5 and 6, respectively. When we rerun our analysis in Table 2 with characteristic-based DRS, we obtain similar and even stronger results indicating that steeper decreasing returns to scale attenuates flow sensitivity. Table 5 shows that $\widetilde{DRS}$ has significantly negative slopes throughout, but the coefficients' estimated values become substantially more negative than in Table 2: the estimated coefficients based on raw estimates are more than 7 times larger (compare the first three columns of Tables 2 and 5).

The results from Table 3 are also very similar when capital flows are replaced with log real size: the slopes on $\widetilde{DRS}$ are significantly negative in Table 6, except for the first two columns in Panel B before controlling for fund fixed effects. These estimates of the size-DRS relation are likely to suffer from an omitted-variable bias; in equilibrium, the size of a fund is driven not only by its decreasing returns to scale technology, but also by its raw skill. Consistent with this argument, we find that the slopes on $\widetilde{DRS}$ turn significant (in the last two columns in Panel B) after controlling for fund fixed effects. Again, the coefficients' estimates values become substantially more negative than in Table 3: the estimated coefficients in regressions with fund fixed effects are typically more than 30 times larger.

To summarize, when we conduct the analysis using cleaner measures of decreasing returns to scale, our conclusions on the effects of decreasing returns to scale on capital allocation only become stronger. These results suggest that the attenuation bias due to using $\widehat{DRS}$ to conduct the analysis is quite severe, so we assess the economic magnitude of the DRS-FSP relation estimated using $\widetilde{DRS}$ in the following subsection.

### 5.4.1 Simulation Exercise

In this section, we use our model to ask how much capital is allocated the way it is because of these differences in decreasing returns to scale. Specifically, we compute counterfactual fund sizes by assuming the investors believe a priori that returns are decreasing in scale at the same (average) rate for all funds.

Two factors fully determine the magnitude of capital response to performance in a rational model — the degree of decreasing returns to scale, and the prior and posterior beliefs about managerial skill. This means that, for a given value of $b$ in equation (15), the prior uncertainty about $a$, $\sigma_0$, can be inferred from the flow-performance relation, as long as investors update their posteriors with the history of returns as Bayesians.

We simulate benchmark-adjusted fund returns from equation (15). It is straightforward to show that the mean of investors' posteriors will satisfy the following recursion:

$$\theta_{it} = \theta_{it-1} + \frac{\sigma_{i0}^2}{\sigma^2 + t\sigma_{i0}^2} r_{it},$$

where $\theta_{i0}$ is the mean of the initial prior. Using (8), we compute fund size as follows:

$$q_{it} = \exp\left(\frac{\theta_{it}}{b_i}\right)\Bigg($$

We begin by tying down the model parameters that can be set directly. Following Berk and Green (2004), we set $\mathrm{Std}(\varepsilon) = 20\%$ per year, or 5.77% per month. Investors' prior on a fund's ability is that $\theta_i$ is normally distributed with mean $\theta_0$ and standard deviation $\sigma_0^2$. Since investors are assumed to have rational expectations, this is also the distribution from which we draw each fund's skill. We shall also assume that funds shut down the first time $\theta_{it} < \overline{\theta}$, where we set $\overline{\theta} = 0$.[15] These parameter values are summarized in the top panel of Table 3. It is straightforward to see that the only remaining parameters that we need to set for simulating data are $b$, $\theta_0$ and $\sigma_0$.

The empirical distribution of $b$ is generally well approximated by a geometric distribution, from which we draw $b$ randomly. In that case, assuming that $\theta_0$ is independent of $b$ gives rise to distributions of fund size considerably more disperse than in our actual sample. Specifically, the simulated fund sizes tend to be too big for funds whose returns decrease in

[15] Intuitively, managers incur fixed costs of operation each period. These costs can be, for example, overhead, back-office expenses, and the opportunity cost of the manager's time. Managers will optimally choose to exit when they cannot cover their fixed costs.

scale more gradually, while the simulated fund sizes tend to be too small for those that exhibit steeper decreasing returns to scale. In turn, we model prior mean as a quadratic function of $b$. Our approach is to fit the parameters governing this function such that the simulated mean and standard deviation of log fund size essentially match the empirical benchmark values of 5.12 and 1.89, respectively.[16] The prior mean as a function of $b$ that we use in our simulation analysis is plotted in Panel A of Figure 4.

Recall from Table 5 that steeper decreasing returns to scale imply less flow sensitivity to performance. For example, as shown in column 3 of Panel A,

$$\widehat{FSP} = 0.117 - 2.40 \times DRS. \tag{17}$$

We consider five plausible values of $b$: 0.00357, 0.00531, 0.00770, 0.0106, and 0.0140. These values correspond to the 10th, 25th, 50th, 75th, and 90th percentiles of fund-specific $b$ estimates, respectively. For each value of $b$, we construct 2,500 samples of simulated panel data for 100 funds over 100 months. In each sample, we estimate the flow-performance sensitivity by running the following regression:

$$\log\left(q_{it}/q_{it-1}\right) = c + \gamma r_{it} + \upsilon_{it}.$$

Given $b$, we set $\sigma_0$ so that the median of the $\gamma$ estimates across simulated samples matches the flow-performance relation implied by (17). Panel B of Table 7 contains the values of $\sigma_0$ for all five values of $b$ that resulted from this process. Panel B of Figure 4 also plots the prior uncertainty as a function of $b$ that we use in our simulation analysis. Column 3 shows the target flow-performance sensitivities computed using (17), while the resulting median of the $\gamma$ estimates across simulated samples are reported in the last column of Panel B. Note that the relation between flow and performance in the model is a close match to the target relation.

Matching the flow-performance sensitivities for funds at different levels of decreasing returns to scale requires the distribution of skills across these funds to be quite different than the skill distribution for funds whose returns decrease with fund size at a median rate. Panel C of Table 7 shows the median of the $\gamma$ estimates across simulated samples for each $b$, but using the counterfactual value of $\sigma_0 = 0.162\%$ per month instead of its calibrated value.

[16] Note that there generally exist multiple ways prior mean as a function of $b$ for which the simulated mean and standard deviation of log fund size can match the empirical benchmark values. To pick a single function, we impose the additional constraint that the simulated mean of log fund size is decreasing in $b$. This constraint is motivated by empirical evidence presented earlier in Section 5.2: steeper decreasing returns to scale shrink fund size.

If we assume that prior uncertainty is constant across different levels of decreasing returns to scale, the model produces much smaller (larger) flow sensitivities to performance for funds that exhibit relatively steeper (flatter) decreasing returns to scale than those implied by (17).

To quantitatively assess the role of heterogeneity in returns to scale in capital allocation, and to assess the economic magnitude of equation (17), we must construct a counterfactual. We construct two counterfactuals. We construct the first counterfactual by assuming investors who learn about skill based on distorted beliefs that the fund exhibits median decreasing returns to scale and its skill is drawn from a normal distribution with the correspondingly calibrated standard deviation. Specifically, the first set of counterfactual investors assume that $b = 0.00770$ and $\sigma_0 = 0.162\%$ per month. We then construct the other counterfactual by assuming investors who learn about skill based on distorted beliefs that the fund's skill is drawn from a distribution corresponding to those facing median decreasing returns to scale, i.e., they assume that $\sigma_0 = 0.162\%$ per month. The second set of counterfactual investors differ from the first set in that they know the true $b$. Then, updating investors' beliefs with the history of its returns under the counterfactual assumptions, we compute what the size of the fund would have been.

We construct 2,500 samples of simulated panel data for $10,000$ funds over 100 months. To simulate a given sample, we first draw each fund's DRS $b_i$ randomly from a geometric distribution consistent with the distribution of fund-specific $b$ estimates, while we draw the fund's skill $\theta_i$ from a normal distribution with mean $\theta_0(b_i)$ and standard deviation $\sigma_0(b_i)$. Next, we draw the random values of $\varepsilon_{it}$, building up the panel data of $r_{it}$ and $q_{it}$. For every $i$ and $t$, we compute the fund's size under the counterfactual, $q_{it}^C$, as detailed above. Finally, for each sample, we calculate the R-squared from a regression of $\log(q_{it})$ on $\log(q_{it}^C)$ to check the goodness of fit by counterfactuals. Under the first counterfactual, 1 minus the R-squared can be interpreted as the fraction of capital allocation explained by individual heterogeneity in decreasing returns to scale, coupled with difference between the empirical and model DRS-FSP relations. On the other hand, 1 minus the R-squared under the second counterfactual can be interpreted as the fraction of capital allocation explained solely by this difference between the empirical and model DRS-FSP relations.

We report the results in Table 8. Panel A reports the results from our first counterfactual; Panel B focuses on the second counterfactual. The first two rows in each panel show summary statistics of the coefficient estimates from the regression of $\log(q_{it})$ on $\log\left(q_{it}^C\right)$ across simulated samples; the last row shows summary statistics of the R-squared from this regression across simulated samples.

Even under the first counterfactual where investors believe that all funds are subject to the same decreasing returns to scale technology and their skills are drawn from the same distribution, the counterfactually computed fund sizes explain about 51% of the variation of simulated fund sizes. Perhaps surprisingly, counterfactual sizes are negatively related to actual sizes. This is because a fund whose returns decrease in scale more steeply (gradually) is typically small (big) in equilibrium, but its counterfactual size tend to be bigger (smaller), as investors underestimate (overestimate) the effects of scale on performance under the first counterfactual. Thus, the counterfactuals ignoring heterogeneity in DRS are very different than the actual size. In this sense, we can interpret 1 minus the R-squared as a lower bound on the role of heterogeneity in returns to scale on capital allocation: at least 49% of the cross-sectional variance of fund sizes can be related to cross-sectional variation in decreasing returns to scale parameters, which is economically significant.

Under the second counterfactual, investors account for heterogeneity in returns to scale. These counterfactual investors differ from the actual investors in that they ignore how the distribution of skill changes across different levels of $b$. Not only do the counterfactually computed fund sizes explain almost completely the variation in simulated fund sizes, they are quantitatively very similar to the actual sizes, i.e., $\widehat{\log(q_{it})} = 0.0006 + 0.9999 \log\left(q_{it}^C\right) \approx \log\left(q_{it}^C\right)$. Thus, the uncalibrated version of our model with heterogeneous returns to scale does a great job at explaining capital allocation in the fully calibrated version. On the other hand, recall that conditional on higher (lower) $b$, the uncalibrated model produces much smaller (larger) flow sensitivities to performance (see Panel C of Table 7), indicating a much stronger DRS-FSP relation than in the data. In this sense, the DRS-FSP relation estimated in the uncalibrated model puts a theoretical upper bound on its magnitude to be found in the data, which allows us assess the economic magnitude of equation (17). Panel C of Table 8 shows summary statistics of the DRS-FSP relations in both the fully calibrated model and the uncalibrated model.

As expected, the DRS-FSP relation estimates in the fully calibrated model tend to be closely related to those in the data: $-2.6$, compared to $-2.4$ in the data (column 3 of Panel A of Table 5). On the other hand, the DRS-FSP relation estimates in the uncalibrated version tend to be substantially more negative, about $-11.2$. Thus, it appears that the magnitude of the DRS-FSP relation estimates from the data is much smaller than what the model predicts. While this confirms how severe the errors-in-variables problem, it might also suggest that our simple model does not fully capture how investors react to the effects of scale. For example, investors in our simple model know precisely the fund-specific effects of scale, but investors in the data might be learning about returns to scale, which would help explain the small magnitude of empirical DRS-FSP relation estimates.[17] We leave bridging this gap to future research.

To summarize, Table 8 shows that a significant fraction of how capital is allocated in equilibrium is explained because of investor response to differences in decreasing returns to scale. While fund sizes in the data are quantitatively consistent with what our simple model predicts they should be, the magnitude of empirical DRS-FSP relation estimates are much smaller than were our simple model to hold perfectly in the data.

## 5.5 DRS and Optimal Fund Size

Thus far, we have used heterogeneity in decreasing returns to scale across funds and over time to test whether investors respond to the adverse effects of fund scale in making their capital allocation decisions. If investors update their beliefs about skill as in the model, their perception of optimal size ought to converge to true optimal size over a fund's lifetime. This idea predicts that the sizes of older funds should be more closely related to their optimal sizes based on the model than those of younger funds are. In this section, we test this prediction and find empirical support for it.

We estimate fund-specific $a$ and $b$ parameters to compute the optimal fund size $q_i^*$. Using the $b$ estimates based on fund characteristics, the parameter $a$ for fund $i$ can be estimated as:

$$\widehat{a}_i = \frac{1}{T_i}\sum_{t=1}^{T_i}\left(\widehat{\alpha}_{it} + \widehat{b}_{it}\log\left(q_{it-1}\right)\right)$$

where $\widehat{\alpha}_{it}$ is the risk-adjusted net return, and $T_i$ is the number of observations for fund $i$. We exponentiate the average value of the ratios $\widehat{a}_i/\widehat{b}_{it}$ over a fund's lifetime to get an estimate for $q_i^*$, $\widehat{q}_i^*$. Of course, if investors ignore heterogeneity in decreasing returns to scale, our measure of optimal fund size might be irrelevant. To allow for investor learning about optimal fund size based on a simple return model, we construct an alternative measure of optimal log fund size, assuming that fund size has the same effect on performance for all funds. Using a recursive demeaning procedure of Zhu (2018), we estimate the average fund-level decreasing returns to scale parameter in our sample, denoted by $\widehat{b}_{RD2}$.[18] Measuring performance using the CAPM, the estimated coefficient is statistically significant, indicating

[17] For formal models that relate capital allocation to learning about returns to scale, see Pastor and Stambaugh (2012) and Kim (2017).

[18] Pástor, Stambaugh, and Taylor (2015) analyze the nature of returns to scale by developing a recursive demeaning procedure. They find coefficients indicative of decreasing returns to scale both at the fund level and at the industry level, though only the latter is statistically significant. Zhu (2018) improves upon the empirical strategy in PST (by using more recent fund sizes as the instrument) and establishes strong evidence of fund-level diseconomies of scale.

that an 1% increase in fund size is associated with a decrease in the fund's CAPM alpha of 0.0042% per month, or 5.1 bp per year.[19] We can then estimate the parameter $a$ for fund $i$ as:

$$\widehat{a}_{iRD2} = \frac{1}{T_i} \sum_{t=1}^{T_i} \left( \widehat{\alpha}_{it} + \widehat{b}_{RD2} \log\left(q_{it-1}\right) \right)$$

The alternative measure of optimal fund size $\widehat{q}^*_{iRD2}$ is calculated as $\exp\left(\widehat{a}_{iRD2}/\widehat{b}_{RD2}\right)$.[20]

To test the above prediction, we examine how the relation between log real AUM and our measures of optimal fund size depends on fund age. Specifically, we assign funds to one of three samples based on fund age: $[0, 5]$, $(5, 10]$, and $> 10$ years. In each age-sorted sample, we run panel regressions of fund $i$'s log real AUM in month $t$ on the fund's log optimal fund size estimate, $\log\left(\widehat{q}^*_i\right)$. We report the results in the first three columns of Table 9.[21] In Panel A, we report the results using the CAPM as the benchmark; in Panel B, we use Vanguard index funds as the benchmark.

Across all age-sorted samples, the estimated coefficients on $\log\left(\widehat{q}^*_i\right)$ are positive, with $t$-statistics of more than 13 using the CAPM as the benchmark and $t$-statistics around 8 using the Vanguard benchmark. More importantly, the coefficient values increases over a typical fund's lifetime, indicating that this positive relation between the fund's size and its optimal size is stronger for older funds. As the fund ages, investors learn about its optimal size, so a fund's optimal size has a larger effect that that fund's equilibrium size if it is older. In addition, the $R^2$ of the regressions are consistent. The $R^2$ in the $> 10$ sample is the highest and is monotonically declining in samples of younger funds, which is not surprising since a reasonable measure of optimal size ought to explain more of the variation in fund size in samples of older funds.

In columns 4 through 6, we run the multiple regression of $\log\left(q_{it}\right)$ on both $\log\left(\widehat{q}^*_i\right)$ and $\log\left(\widehat{q}^*_{iRD2}\right)$ in all three age-sorted samples. We consider two null hypotheses: that the slope coefficient on $\log\left(\widehat{q}^*_i\right)$ is zero, and that the slope on $\log\left(\widehat{q}^*_{iRD2}\right)$. We find that the slope on our main measure of optimal fund size is positive and significant in the $> 10$ sample, but its significance disappears in samples of younger funds. The slope on the alternative measure is positive and significant across all age-sorted samples.

The significantly positive coefficient on $\log\left(\widehat{q}^*_i\right)$ in the multiple regression reveals investors do recognize that there is heterogeneity in decreasing returns to scale, conditional on

[19] Using Vanguard index funds as benchmarks, the coefficient estimate is again statistically significant, indicating that an 1% increase in fund size is associated with a decrease in fund performance of 0.0013%, or 1.5 bp per year.

[20] To remove some implausible outliers, we winsorize these estimates at their 1st and 99th percentiles.

[21] Table 9 reports the double clustered (by fund and time) standard errors.

$\log(\widehat{q}^*_{iRD2})$. On the other hand, a significantly larger coefficient on $\log(\widehat{q}^*_{iRD2})$, and the much larger $R^2$ from the multiple regressions, suggests that this simpler version of optimal size better explains sizes in equilibrium. Our results offer the following narrative. Investors want to account for heterogeneity in decreasing returns to scale, but estimating $b$ fund by fund leads to imprecise estimates especially for young funds, which renders the estimation error in $q^*_i$ severe. To reduce the estimation error, investors seem to ignore fund-level variation in $b$ for young funds, which allows them to use cross-sectional information in quantifying fund-specific $q^*_i$. In particular, the investors only use the $\widehat{q}^*_i$ estimate (together with $\widehat{q}^*_{iRD2}$) in making their capital allocation decisions when a fund grows old enough such that the estimation error in its optimal size based on fund-specific $b$ is relatively modest.

Consistent with this idea, we find that the $\log(\widehat{q}^*_{iRD2})$ estimates are informative of $\log(\widehat{q}^*_i)$. Figure 5 plots the main measure of optimal fund size $\log(\widehat{q}^*_i)$ versus the alternative measure of optimal fund size $\log(\widehat{q}^*_{iRD2})$. The circles represent pairs of $(\log(\widehat{q}^*_{iRD2}), \log(\widehat{q}^*_i))$. The red line depicts the identity line. If the two measures of optimal fund size coincide, the red line would fit the data perfectly. We see that $\log(\widehat{q}^*_i)$ tends to move nearly one-for-one with $\log(\widehat{q}^*_{iRD2})$, so this estimator is a reasonable way to measure a fund's optimal size that also circumvents the need to address the estimation error in decreasing returns to scale. However, the two quantities generally do differ: the R-squared from a cross-sectional regression of $\log(\widehat{q}^*_i)$ on $\log(\widehat{q}^*_{iRD2})$ is 0.55 using the CAPM and only 0.14 if we use the Vanguard benchmark. Therefore, $\widehat{q}^*_{iRD2}$ alone does not suffice in capturing optimal size, which leads investors to directly estimate $\widehat{q}^*_i$ for funds with sufficiently long track records.

In short, the estimates of optimal size largely explains capital allocation to older funds. Both measures of optimal fund size matter, which is consistent with our narrative that investors account for not only the presence of decreasing returns to scale, but the heterogeneity of decreasing returns to scale.

# 6 Conclusion

The main contribution of this paper is to provide and verify predictions unique to a rational model for active management: the role of decreasing returns to scale in equilibrating the market for mutual funds. Not only do we find that steeper decreasing returns to scale attenuate flow sensitivity to performance, we also find that differences in decreasing returns to scale across funds are quantitatively important for explaining capital allocation in the market for mutual funds. Interestingly, the magnitude of empirical DRS-FSP relation estimates are much smaller than were our simple model to hold perfectly in the data. Bridging this gap by using more accurate measurements of a fund's returns to scale or flow sensitivity, or by considering new aspects of learning about the parameters governing fund returns, is an important area for future research. Overall, our results strongly support that, as a group, investors in the mutual fund market are sophisticated.

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Table 1: Summary Statistics

This table shows summary statistics for our sample of active equity mutual funds from 1979–2014. The unit of observation is the fund/month. All returns are in units of fraction per month. Net return is the return received by investors. Net alpha equals net return minus the return on benchmark portfolio, calculated using the CAPM or using a set of Vanguard index funds. Fund size is the fund's total AUM aggregated across share classes, adjusted by inflation. The numbers are reported in Y2000 $ millions per month. Flow is the monthly change in the fund's net assets not attributable to its return gains or losses. Turnover is in units of fraction per year. Volatility is the standard deviation of a fund's alphas, calculated over the prior 1 year. Fund age is the number of years since the fund's first offer date (from CRSP) or, if missing, since the fund's inception date (from Morningstar). # of managers is the number of managers managing the fund in a given month. $\widehat{DRS}_{it}$ is the fund's returns to scale estimated as of the previous month-end; $\widetilde{DRS}_{it}$ is the economically interpretable component of $\widehat{DRS}_{it}$ based on fund characteristics. $\widehat{FSP}_{it}$ is the fund's flow sensitivity to performance going forward.

**Panel A: Fund-Level Variables**

| | | | | Percentiles | | |
|---|---|---|---|---|---|---|
| | # of obs. | Mean | Stdev. | 25% | 50% | 75% |
| Net return | 424,793 | 0.0079 | 0.0497 | −0.0193 | 0.0123 | 0.0387 |
| Net alpha (CAPM Risk Adj.) | 354,427 | 0.0001 | 0.0209 | −0.0105 | −0.0002 | 0.0104 |
| Net alpha (Vanguard Benchmark) | 420,163 | −0.0001 | 0.0155 | −0.0083 | −0.0001 | 0.0080 |
| Fund size (in 2000 $millions) | 421,701 | 995 | 4008 | 45 | 163 | 616 |
| Flows | 421,697 | 0.0049 | 0.0529 | −0.0143 | −0.0021 | 0.0145 |
| Turnover | 402,907 | 0.8317 | 0.7027 | 0.34 | 0.64 | 1.1 |
| Volatility (CAPM Risk Adj.) | 322,939 | 0.0188 | 0.0115 | 0.0106 | 0.0158 | 0.0239 |
| Volatility (Vanguard Benchmark) | 387,197 | 0.0142 | 0.0082 | 0.0086 | 0.0122 | 0.0177 |
| Fund age (years) | 423,871 | 13.44 | 13.38 | 4.58 | 9.28 | 16.77 |
| # of managers | 404,551 | 2.36 | 2.12 | 1 | 2 | 3 |

**Panel B: Estimated DRS and FSP**

| | | | | Percentiles | | |
|---|---|---|---|---|---|---|
| | # of obs. | Mean | Stdev. | 25% | 50% | 75% |
| $\widehat{DRS}$ (CAPM Risk Adj.) | 252,434 | 0.0084 | 0.0165 | −0.0002 | 0.0052 | 0.0137 |
| $\widetilde{DRS}$ (CAPM Risk Adj.) | 247,989 | 0.0084 | 0.0044 | 0.0053 | 0.0077 | 0.0106 |
| $\widehat{DRS}$ (Vanguard Benchmark) | 300,963 | 0.0044 | 0.0109 | −0.0008 | 0.0028 | 0.0081 |
| $\widetilde{DRS}$ (Vanguard Benchmark) | 294,815 | 0.0044 | 0.0024 | 0.0028 | 0.0043 | 0.0059 |
| $\widehat{FSP}$ (CAPM Risk Adj.) | 266,376 | 0.1045 | 0.1910 | 0.0140 | 0.0756 | 0.1694 |
| $\widehat{FSP}$ (Vanguard Benchmark) | 293,895 | 0.1487 | 0.2898 | 0.0171 | 0.1094 | 0.2499 |

Table 2: Relation Between DRS and FSP

The dependent variable in each regression model is $\widehat{FSP}_{it}$, the fund's flow sensitivity to performance going forward. $\widehat{DRS}_{it}$ is the fund's returns to scale estimated as of the previous month-end. The first three columns report the results using the raw estimates. The last two columns repeat the same analysis using percentile ranks for each variable across funds. In each version, from left to right, we gradually saturate with month and fund fixed effects to focus on variation coming from the market equilibrating mechanism beyond differences in sensitivity across funds and over time. Standard errors, two-way clustered by fund and by month, are in parentheses.

**Panel A: CAPM Risk Measure**

| | Dependent Variable: $\widehat{FSP}_{it}$ | | | | |
|---|---|---|---|---|---|
| $\widehat{DRS}_{it}$ | −0.375 | −0.296 | −0.279 | −0.0424 | −0.0769 |
| | (0.0938) | (0.0941) | (0.0898) | (0.0119) | (0.00984) |
| Month FE | No | Yes | Yes | No | No |
| Fund FE | No | No | Yes | No | Yes |
| Observations | 182691 | 182691 | 182676 | 182691 | 182676 |
| Scale | Raw est. | Raw est. | Raw est. | Pctl. rank | Pctl. rank |

**Panel B: Vanguard Benchmark**

| | Dependent Variable: $\widehat{FSP}_{it}$ | | | | |
|---|---|---|---|---|---|
| $\widehat{DRS}_{it}$ | −0.355 | −0.414 | −0.375 | −0.0264 | −0.0597 |
| | (0.219) | (0.223) | (0.167) | (0.0102) | (0.00863) |
| Month FE | No | Yes | Yes | No | No |
| Fund FE | No | No | Yes | No | Yes |
| Observations | 221749 | 221749 | 221743 | 221749 | 221743 |
| Scale | Raw est. | Raw est. | Raw est. | Pctl. rank | Pctl. rank |

Table 3: Relation Between DRS and Size

The dependent variable in each regression model is the fund's log real AUM in \$ millions (base year is 2000). $\widehat{DRS}_{it}$ is the fund's returns to scale estimated as of the previous month-end. Across the first three columns, we gradually saturate with month and fund fixed effects to focus on variation coming from the market equilibrating mechanism beyond differences in size across funds and over time. The last column repeats the same analysis by further including controls that are plausibly correlated with the fund size: family size, fund age, and turnover. Standard errors, two-way clustered by fund and by month, are in parentheses.

| | **Panel A: CAPM Risk Measure** | | | |
|---|---|---|---|---|
| | Dependent Variable: Log Real AUM | | | |
| $\widehat{DRS}_{it}$ | $-3.86$ | $-5.23$ | $-1.30$ | $-1.12$ |
| | (1.65) | (1.68) | (0.558) | (0.447) |
| Month FE | No | Yes | Yes | Yes |
| Fund FE | No | No | Yes | Yes |
| Controls | No | No | No | Yes |
| Observations | 252420 | 252420 | 252411 | 247054 |

| | **Panel B: Vanguard Benchmark** | | | |
|---|---|---|---|---|
| | Dependent Variable: Log Real AUM | | | |
| $\widehat{DRS}_{it}$ | $-3.97$ | $-5.36$ | $-2.34$ | $-2.66$ |
| | (2.20) | (2.22) | (0.755) | (0.611) |
| Month FE | No | Yes | Yes | Yes |
| Fund FE | No | No | Yes | Yes |
| Controls | No | No | No | Yes |
| Observations | 300947 | 300947 | 300936 | 294412 |

Table 4: Determinants of Returns to Scale

The dependent variable in each regression model is $\widehat{DRS}_{it}$, the fund's scale effect computed using only its observations prior to the month. $Std\,(Alpha)$ is the standard deviation of the fund's alphas, which we calculate over the prior 1 year. $1\,(MultiMgr)$ is a dummy variable that is equal to one if the fund is managed by many managers. $1\,(Outflows)$ is an indicator for whether the fund experienced outflows over the prior 1 year. $LogFundAge$ is the natural logarithm of years since the fund's first offer date (from CRSP) or, if missing, since the fund's inception date (from Morningstar). All models are estimated by OLS. Standard errors, two-way clustered by fund and time, are in parentheses.

| | **Panel A: CAPM Risk Measure** | | | | |
|---|---|---|---|---|---|
| | Dependent Variable: $\widehat{DRS}_{it}$ | | | | |
| $Std\,(Alpha)$ | 0.267 | | | | 0.265 |
| | (0.0239) | | | | (0.0244) |
| $1\,(MultiMgr)$ | | −0.00164 | | | −0.000998 |
| | | (0.000364) | | | (0.000354) |
| $1\,(Outflows)$ | | | 0.00217 | | 0.00214 |
| | | | (0.000398) | | (0.000388) |
| $LogFundAge$ | | | | 0.000602 | 0.000785 |
| | | | | (0.000369) | (0.000349) |
| Controls | Yes | Yes | Yes | Yes | Yes |
| Observations | 252362 | 248077 | 252434 | 252200 | 247989 |

| | **Panel B: Vanguard Benchmark** | | | | |
|---|---|---|---|---|---|
| | Dependent Variable: $\widehat{DRS}_{it}$ | | | | |
| $Std\,(Alpha)$ | 0.154 | | | | 0.147 |
| | (0.0185) | | | | (0.0185) |
| $1\,(MultiMgr)$ | | −0.00102 | | | −0.000717 |
| | | (0.000234) | | | (0.000225) |
| $1\,(Outflows)$ | | | 0.00179 | | 0.00152 |
| | | | (0.000242) | | (0.000225) |
| $LogFundAge$ | | | | 0.00146 | 0.00136 |
| | | | | (0.000225) | (0.000210) |
| Controls | Yes | Yes | Yes | Yes | Yes |
| Observations | 300883 | 294922 | 300963 | 300666 | 294815 |

Table 5: Relation Between DRS and FSP

This table is the same as Table 2 but replaces $\widehat{DRS}_{it}$ by their predicted values based on fund characteristics, $\widetilde{DRS}_{it}$.

| | Panel A: CAPM Risk Measure | | | | |
|---|---|---|---|---|---|
| | Dependent Variable: $\widehat{FSP}_{it}$ | | | | |
| $\widetilde{DRS}_{it}$ | −2.95 | −2.28 | −2.37 | −0.0536 | −0.0854 |
| | (0.568) | (0.657) | (0.596) | (0.0152) | (0.0148) |
| Month FE | No | Yes | Yes | No | No |
| Fund FE | No | No | Yes | No | Yes |
| Observations | 178624 | 178624 | 178612 | 178624 | 178612 |
| Scale | Raw est. | Raw est. | Raw est. | Pctl. rank | Pctl. rank |

| | Panel B: Vanguard Benchmark | | | | |
|---|---|---|---|---|---|
| | Dependent Variable: $\widehat{FSP}_{it}$ | | | | |
| $\widetilde{DRS}_{it}$ | −7.83 | −6.88 | −3.39 | −0.0546 | −0.0404 |
| | (1.84) | (1.89) | (1.99) | (0.0133) | (0.0134) |
| Month FE | No | Yes | Yes | No | No |
| Fund FE | No | No | Yes | No | Yes |
| Observations | 216044 | 216044 | 216040 | 216044 | 216040 |
| Scale | Raw est. | Raw est. | Raw est. | Pctl. rank | Pctl. rank |

Table 6: Relation Between DRS and Size

This table is the same as Table 3 but replaces $\widehat{DRS}_{it}$ by their predicted values based on fund characteristics, $\widetilde{DRS}_{it}$.

| | **Panel A: CAPM Risk Measure** | | | |
|---|---|---|---|---|
| | Dependent Variable: Log Real AUM | | | |
| $\widehat{DRS}_{it}$ | $-44.1$ | $-63.3$ | $-49.0$ | $-36.4$ |
| | (8.10) | (9.08) | (3.83) | (3.29) |
| Month FE | No | Yes | Yes | Yes |
| Fund FE | No | No | Yes | Yes |
| Controls | No | No | No | Yes |
| Observations | 247975 | 247975 | 247965 | 243079 |

| | **Panel B: Vanguard Benchmark** | | | |
|---|---|---|---|---|
| | Dependent Variable: Log Real AUM | | | |
| $\widehat{DRS}_{it}$ | $-16.7$ | $-16.1$ | $-88.7$ | $-75.5$ |
| | (14.9) | (15.7) | (7.10) | (5.91) |
| Month FE | No | Yes | Yes | Yes |
| Fund FE | No | No | Yes | Yes |
| Controls | No | No | No | Yes |
| Observations | 294,799 | 294,799 | 294,790 | 288,867 |

Table 7: Calibration

The top panel summarizes the model parameters that we set directly and their parameter values. Then, for each value of $b$, we construct 2,500 samples of simulated panel data for 100 funds over 100 months. In each sample, we estimate the flow-performance sensitivity $\gamma$. Given $b$, we set $\sigma_0$ so that the median of the $\gamma$ estimates across simulated samples matches the flow-performance relation implied by the regression model in column 3 of Panel A of Table 5 (i.e., $\widehat{FSP} = 0.117 - 2.40 \times DRS$). Panel B contains the values of $\sigma_0$ that resulted from this process for five plausible values of $b$: 0.00357, 0.00531, 0.00770, 0.0106, and 0.0140. These values correspond to the 10th, 25th, 50th, 75th, and 90th percentiles of fund-specific $b$ estimates, respectively. Column 3 shows the target flow-performance sensitivities computed using $\widehat{FSP} = 0.117 - 2.40 \times DRS$, while the resulting median of the $\gamma$ estimates across simulated samples are reported in the last column. Panel C repeats the same analysis for each $b$, but using the counterfactual value of $\sigma_0 = 0.162\%$ per month.

**Panel A: Parameter Values**

| Variable | Symbol | Value |
|---|---|---|
| Return standard deviation | $\sigma$ | 5.77% |
| Exit mean | $\overline{\theta}$ | 0% |

**Panel B: Calibration**

| Parameter set | Calibrated parameter | $FSP$ | |
|---|---|---|---|
| Decreasing returns to scale ($b$) | Prior standard deviation ($\sigma_0$) | Target | Model |
| 0.00357 | 0.00116 | 0.109 | 0.110 |
| 0.00531 | 0.00139 | 0.105 | 0.107 |
| 0.00770 | 0.00162 | 0.099 | 0.099 |
| 0.01065 | 0.00186 | 0.092 | 0.093 |
| 0.01396 | 0.00204 | 0.084 | 0.084 |

**Panel C: Counterfactual**

| Parameter set | Counterfactual parameter | $FSP$ | |
|---|---|---|---|
| Decreasing returns to scale ($b$) | Prior standard deviation ($\sigma_0$) | Target | Model |
| 0.00357 | 0.00162 | 0.109 | 0.213 |
| 0.00531 | 0.00162 | 0.105 | 0.143 |
| 0.00770 | 0.00162 | 0.099 | 0.099 |
| 0.01065 | 0.00162 | 0.092 | 0.071 |
| 0.01396 | 0.00162 | 0.084 | 0.054 |

Table 8: Simulation

We construct 2,500 samples of simulated panel data for 10,000 funds over 100 months. To simulate a given sample, we first draw each fund's DRS $b_i$ randomly from a geometric distribution consistent with the distribution of fund-specific $b$ estimates, while we draw the fund's skill $\theta_i$ from a normal distribution with mean $\theta_0(b_i)$ and standard deviation $\sigma_0(b_i)$. Next, we draw the random values of $\varepsilon_{it}$, building up the panel data of $r_{it}$ and $q_{it}$. For every $i$ and $t$, we compute the fund's size under the counterfactual, $q_{it}^C$, as detailed in Section 5.4.1. Finally, for each sample, we calculate the R-squared from a regression of $\log(q_{it})$ on $\log\left(q_{it}^C\right)$ to check the goodness of fit by counterfactuals. Panel A reports the results from our first counterfactual; Panel B focuses on the second counterfactual. Panel C shows summary statistics of the DRS-FSP relations in both the fully calibrated model and the uncalibrated model (i.e., using the counterfactual value of $\sigma_0 = 0.162\%$ per month).

**Panel A: First Counterfactual**

$\log(q_{it}) = \kappa + \lambda \log\left(q_{it}^C\right) + \xi_{it}$

| | | | Percentiles | | | | |
|---|---|---|---|---|---|---|---|
| | Mean | Stdev. | 1% | 25% | 50% | 75% | 99% |
| $\widehat{\kappa}$ | 10.175 | 0.0023 | 10.169 | 10.173 | 10.175 | 10.176 | 10.180 |
| $\widehat{\lambda}$ | −1.033 | 0.0005 | −1.034 | −1.033 | −1.033 | −1.033 | −1.032 |
| $R^2$ | 0.5067 | 0.0006 | 0.5053 | 0.5063 | 0.5068 | 0.5072 | 0.5081 |

**Panel B: Second Counterfactual**

$\log(q_{it}) = \kappa + \lambda \log\left(q_{it}^C\right) + \xi_{it}$

| | | | Percentiles | | | | |
|---|---|---|---|---|---|---|---|
| | Mean | Stdev. | 1% | 25% | 50% | 75% | 99% |
| $\widehat{\kappa}$ | 0.0006 | 0.0055 | −0.012 | −0.003 | 0.0005 | 0.0042 | 0.0138 |
| $\widehat{\lambda}$ | 0.9999 | 0.0011 | 0.9973 | 0.9992 | 0.9999 | 1.0006 | 1.0024 |
| $R^2$ | 0.9999 | 0.0000 | 0.9998 | 0.9998 | 0.9999 | 0.9999 | 0.9999 |

**Panel C: Estimated DRS-FSP Relation**

| | | | Percentiles | | |
|---|---|---|---|---|---|
| Data | −2.367 | | | | |
| | Mean | Stdev. | 25% | 50% | 75% |
| Calibrated Model | −2.643 | 0.0001 | −2.644 | −2.643 | −2.643 |
| Uncalibrated Model | −11.17 | 0.0001 | −11.17 | −11.17 | −11.17 |

Table 9: Relation Between Optimal Size and Fund Size

The dependent variable in each regression model is the fund's log real AUM in $ millions (base year is 2000). $\widehat{q}_i^*$ is an estimate for the optimal fund size; $\widehat{q}_{iRD2}^*$ is an alternative measure of optimal fund size, assuming that fund size has the same effect on performance for all funds. We assign funds to one of three samples based on fund age: $[0,5]$, $(5,10]$, and $> 10$ years. Columns 1–3 show the results from running panel regressions of log real AUM on the fund's log optimal fund size estimate in each age-sorted sample. The last three columns show the results from running multiple regressions of $\log(q_{it})$ on both $\log(\widehat{q}_i^*)$ and $\log(\widehat{q}_{iRD2}^*)$ in all three age-sorted samples. The double clustered (by fund and time) standard errors are in parentheses.

| | **Panel A: CAPM Risk Measure** | | | | | |
|---|---|---|---|---|---|---|
| | Dependent Variable: Log Real AUM | | | | | |
| $\log(\widehat{q}_i^*)$ | 0.218 | 0.364 | 0.430 | −0.0189 | 0.00599 | 0.0474 |
| | (0.0158) | (0.0208) | (0.0330) | (0.0148) | (0.0110) | (0.00942) |
| $\log(\widehat{q}_{iRD2}^*)$ | | | | 0.497 | 0.762 | 0.916 |
| | | | | (0.0230) | (0.0166) | (0.0143) |
| $R^2$ | 0.146 | 0.354 | 0.428 | 0.308 | 0.665 | 0.824 |
| Observations | 64718 | 105288 | 196233 | 64718 | 105288 | 196233 |
| Fund ages | $[0,5]$ yr. | $(5,10]$ yr. | $> 10$ yr. | $[0,5]$ yr. | $(5,10]$ yr. | $> 10$ yr. |

| | **Panel B: Vanguard Benchmark** | | | | | |
|---|---|---|---|---|---|---|
| | Dependent Variable: Log Real AUM | | | | | |
| $\log(\widehat{q}_i^*)$ | 0.0546 | 0.0745 | 0.132 | 0.0147 | 0.0126 | 0.0251 |
| | (0.00655) | (0.00906) | (0.0166) | (0.00472) | (0.00423) | (0.00514) |
| $\log(\widehat{q}_{iRD2}^*)$ | | | | 0.273 | 0.484 | 0.692 |
| | | | | (0.0111) | (0.0113) | (0.0128) |
| $R^2$ | 0.0497 | 0.0721 | 0.134 | 0.248 | 0.510 | 0.699 |
| Observations | 75870 | 110319 | 196467 | 75870 | 110319 | 196467 |
| Fund ages | $[0,5]$ yr. | $(5,10]$ yr. | $> 10$ yr. | $[0,5]$ yr. | $(5,10]$ yr. | $> 10$ yr. |

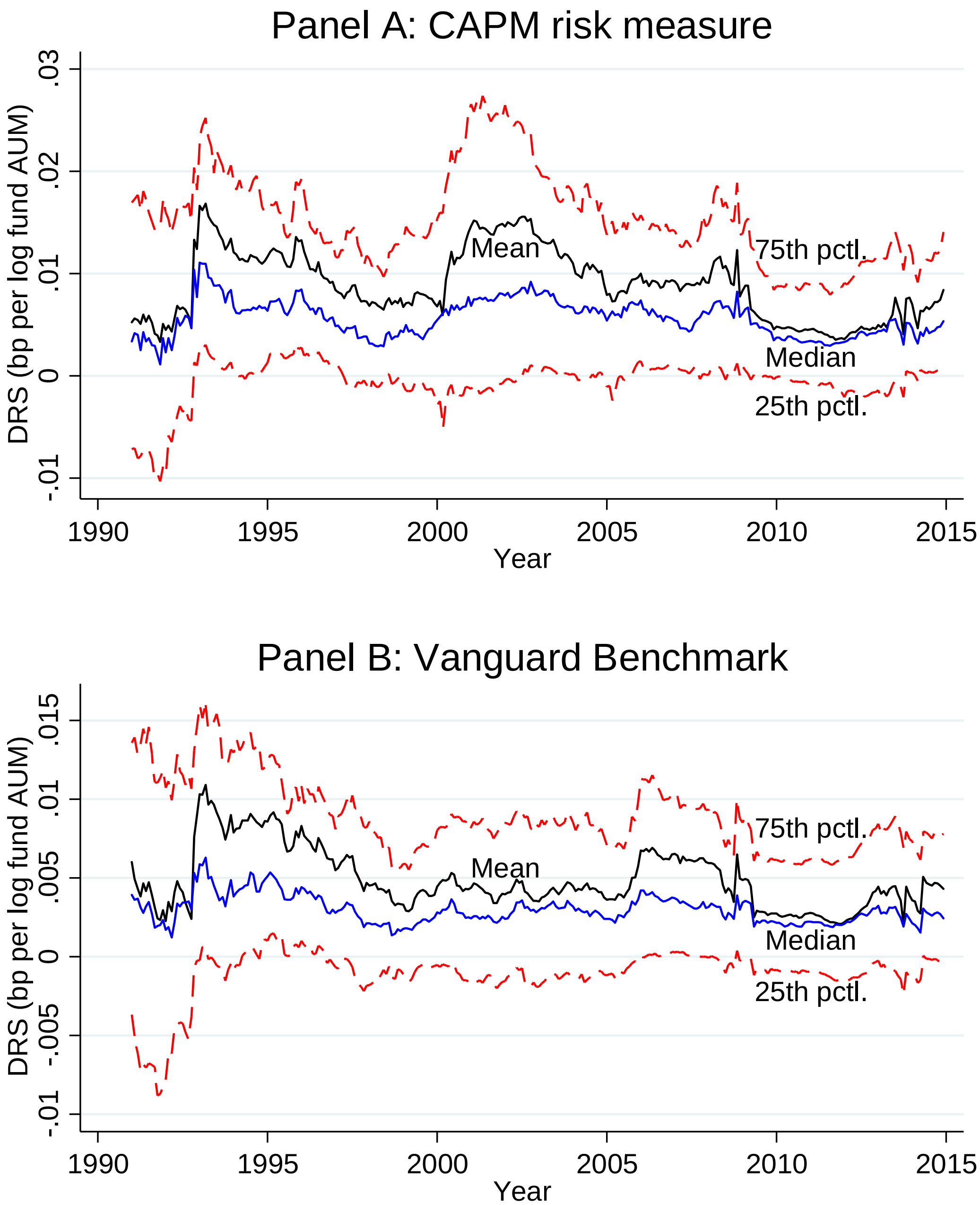



Figure 1: **Distribution of fund-specific decreasing returns to scale over time:** The figure plots each month's mean and percentiles of estimated size effect on performance across all funds operating during that month. Panel A estimates DRS using the CAPM alpha to measure fund performance. Panel B estimates DRS when we take Vanguard index funds as the alternative investment opportunity set.

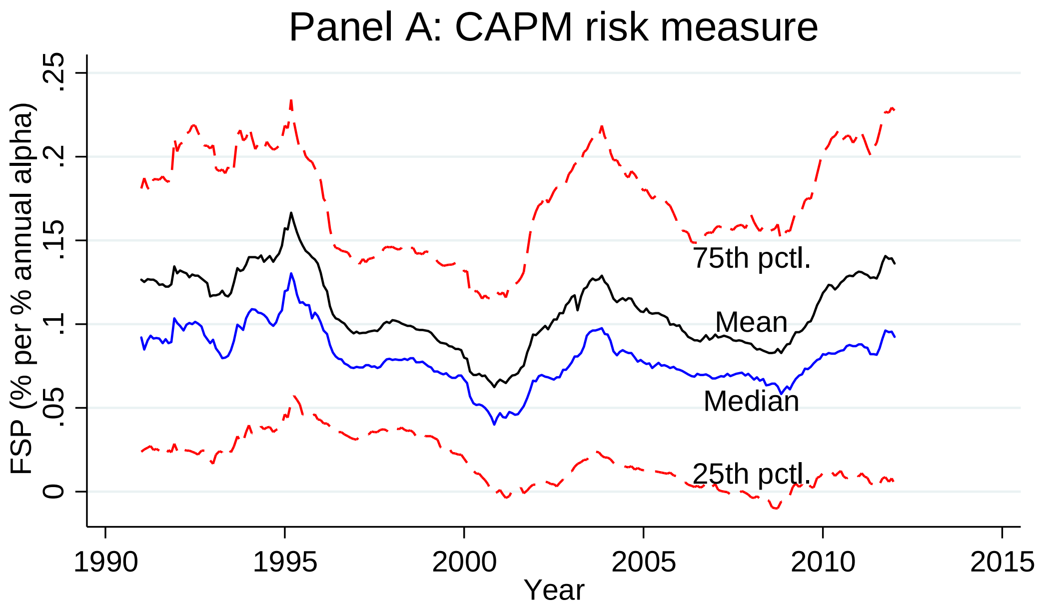



Figure 2: **Distribution of fund-specific flow sensitivity to performance over time:** The figure plots each month's mean and percentiles of estimated flow sensitivity to performance across all funds operating during that month. Panel A estimates FSP using the CAPM alpha to measure fund performance. Panel B estimates FSP when net alpha is computed using Vanguard index funds as benchmark portfolios.

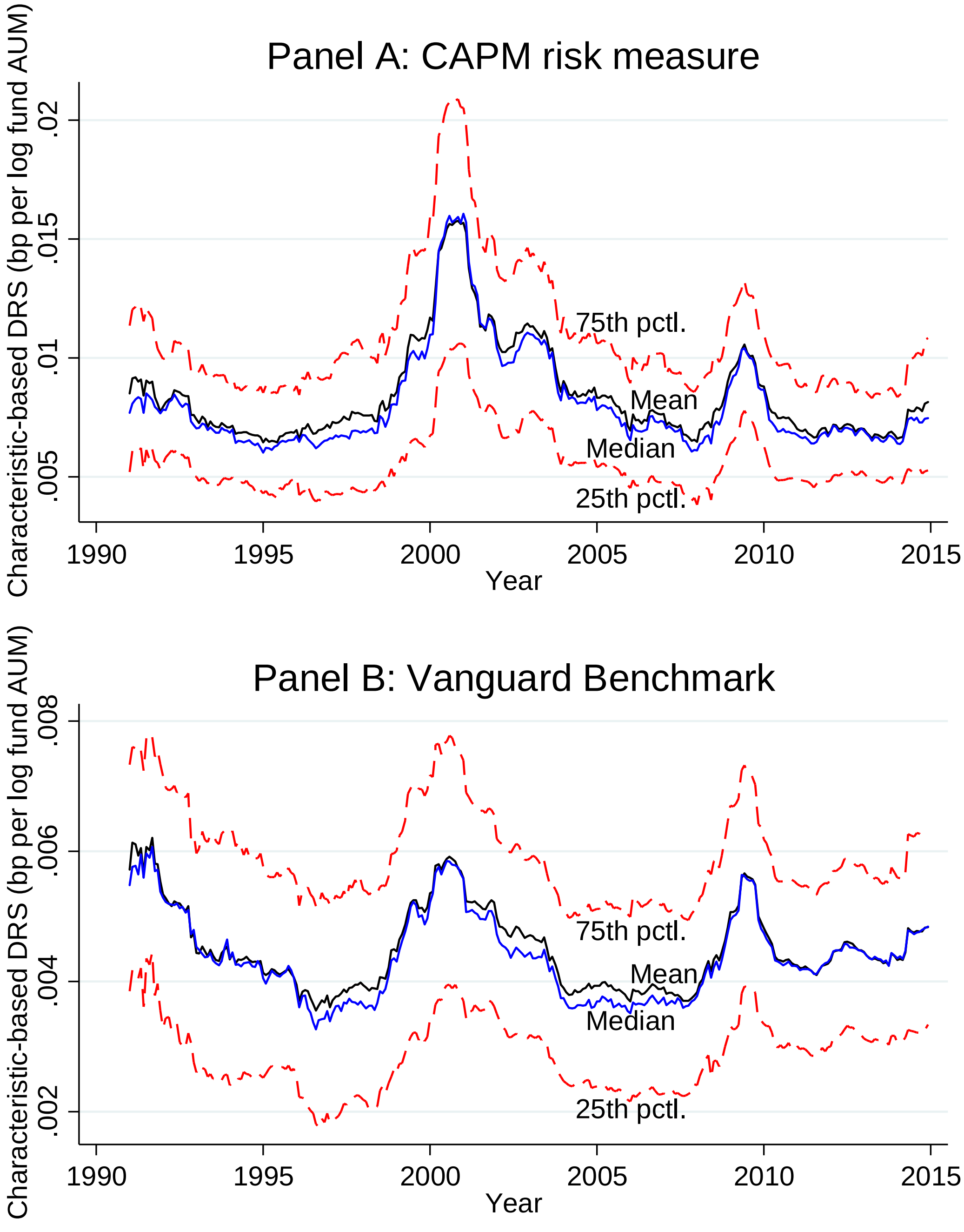



Figure 3: **Cross-sectional distribution of characteristic-based DRS over time:** This figure is the same as Figure 1 but measures size effect on performance not by $\widehat{DRS}_{it}$, but by their predicted values based on fund characteristics, $\widetilde{DRS}_{it}$.

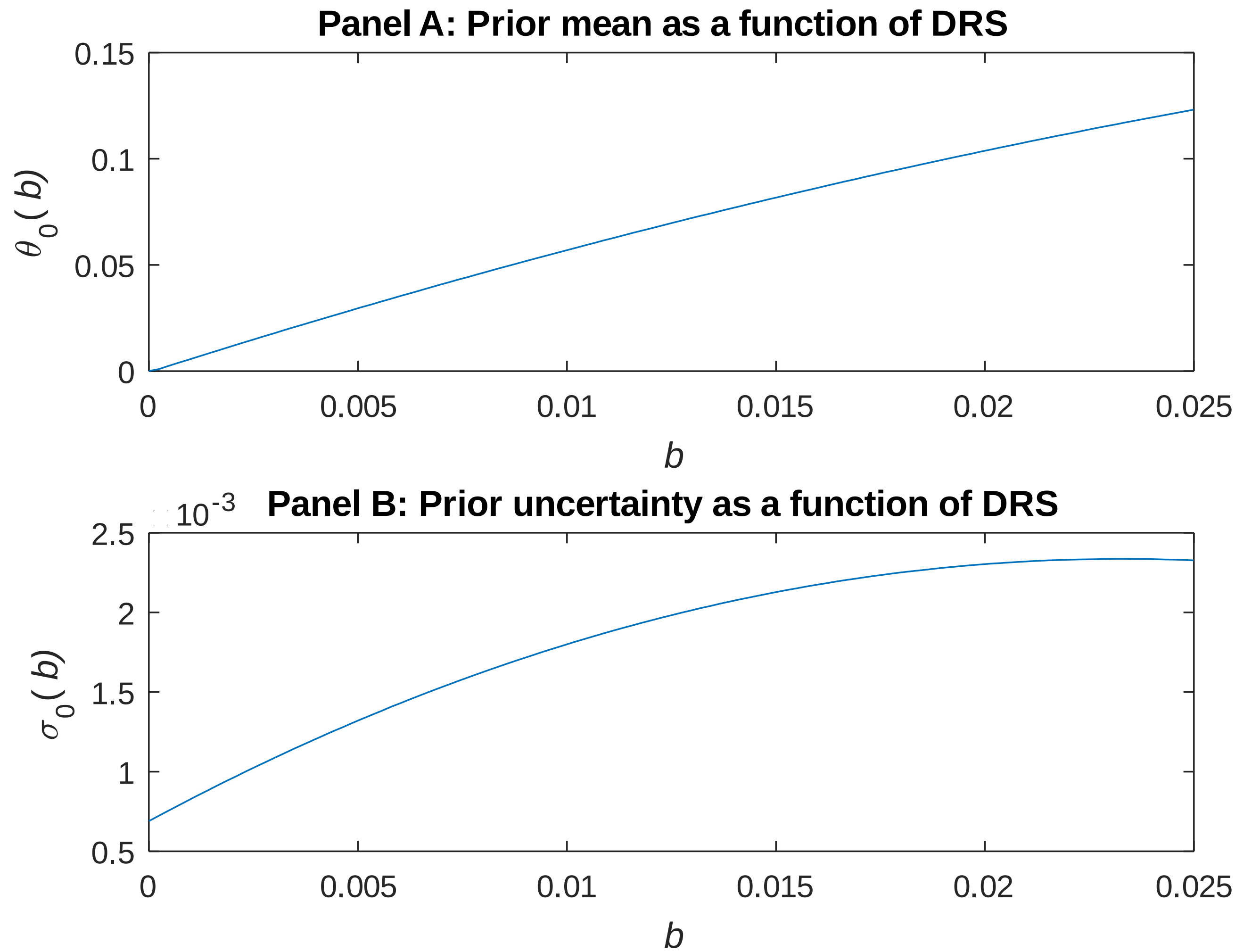
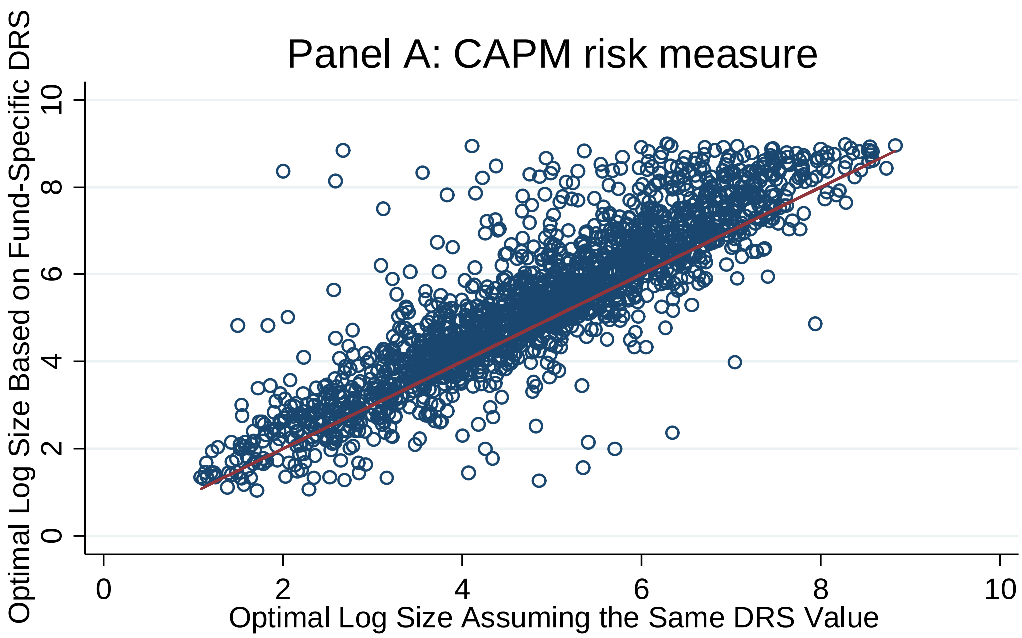



Figure 4: **Priors, conditional on the fund's decreasing returns to scale parameter *b*:** Panel A plots the prior mean ($\theta_0$) as a function of $b$ that we use in our simulation analysis. Panel B plots the prior uncertainty ($\sigma_0$) as a function of $b$ that we use in our simulation analysis. Given $b$, $\sigma_0$ is calibrated so that the median of the FSP (flow sensitivity to performance) estimates across simulated samples matches the flow-performance relation implied by the regression model in column 3 of Panel A of Table 5. Our approach is to calibrate the prior uncertainty for 100 disperse values of $b$, fitting a quadratic polynomial to the data, $(b, \sigma_0)$, that resulted from this process to extrapolate the prior uncertainty, conditional on other values of $b$, that we use in our simulation analysis.



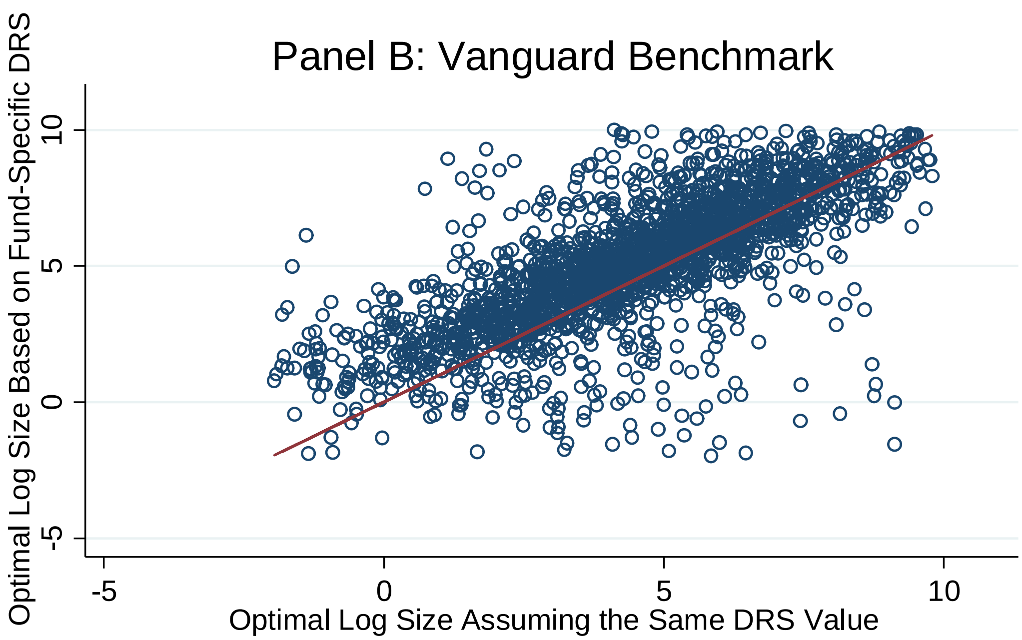




Figure 5: **Relation between two measures of optimal fund size:** The figure plots the main measure of optimal fund size, $\log(\widehat{q}_i^*)$, versus the alternative measure of optimal fund size, $\log(\widehat{q}_{iRD2}^*)$. We compute the optimal fund size, $\widehat{q}_i^*$, by estimating fund-specific $a$ and $b$ parameters. We construct an alternative measure of optimal log fund size, ignoring the fact that there is individual heterogeneity in decreasing returns to scale. The circles represent pairs of $(\log(\widehat{q}_{iRD2}^*), \log(\widehat{q}_i^*))$. The red line depicts the identity line.