History Doesn’t Repeat, But It Rhymes.
- Cash Flow Risk and Expected Returns

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Abstract
Chava, Hsu, and Zeng (2019) find that investors don’t fully incorporate business cycle variation in cash flow growth and thus conditional Sharpe ratio can be informative for future industry returns. It suggests that cash flow risk at the idiosyncratic level is not fully incorporated into the prices by investors. I develop a stochastic volatility framework to evaluate the unexpected cash flow news through the variance decomposition perspective and apply the method to U.S. industry data. I find that i) The common cash flow volatility estimated from unexpected industry-level cash flow news is highly correlated to Uncertainty index constructed by Jurado, Ludvigson, and Ng (2015); ii) the idiosyncratic cash flow risk is robustly priced and the explanation power cannot be consumed by current well-known risk factors and firm characteristics; iii) stocks with high conditional Sharpe ratios tend to have higher idiosyncratic cash flow volatility and higher compensated returns, which is consistent with Chava, Hsu, and Zeng (2019)’s finding. A strategy that goes long the decile portfolio with the largest idiosyncratic cash flow volatility and short the decile portfolio with the smallest idiosyncratic cash flow volatility yields a Fama-French-Five-Factor alpha of 37 bps per month (t-stat: 6.90) in long sample (1931-2018) and 64 bps per month (t-stat: 12.28) in the modern sample (1963-2018).

JEL classification: G10.

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1. Introduction

The fundamental question in empirical asset pricing is the determinants of the cross-sectional stock returns. While a large body of recent research proposing new factors based on a host of empirically motivated economic or financial characteristics, we address this question from a new perspective, offering evidence that the idiosyncratic cash flow risk - the unexpected cash flow news at individual level - is important for understanding the cross-sectional stock returns.

In this paper, I argue that cash flow risk at the idiosyncratic level is not fully incorporated into the prices by investors. The cash flow risk and discount rates risk have been well defined in the pioneer work of Campbell (1991). Campbell and Vuolteenaho (2004a) apply the technique and use the market level unexpected news to explain the cross-section of stock returns and following their “Bad Beta, Good Beta” work, many papers have explored the role of the unexpected news risk in asset pricing. However, most of them focus on the cash flow risk at the market level and cash flow risk at individual level has been rarely explored. A recent paper by Chava, Hsu, and Zeng (2019) shows that there is significant variation in cash flow growth across industries over the business cycle and they find investors do not fully incorporate business cycle fluctuations into the industry level cash flows. If the business cycle information is not reflected in each industry’s cash flow, then conditional Sharpe ratio can be informative for future industry returns. In their paper, sector rotation strategy based on history-dependent Sharpe ratio can produce significant returns. It suggests that cash flow risk at the idiosyncratic level is not fully incorporated into the prices by investors. However, no theoretical model is provided to rationalize the documented Sharpe ratio premium and the role of idiosyncratic cash flows should be re-highlighted. In this paper, I develop a stochastic volatility framework to evaluate the unexpected cash flow news through the variance decomposition perspective, and I relate the conditional Sharpe ratio to the firm’s cash flow volatility - especially the idiosyncratic cash flow volatility - to justify the premium.
Campbell and Vuolteenaho (2004a) apply the technique in Campbell (1991) and use the market level unexpected news to explain the cross-section of stock returns. Following their “Bad Beta, Good Beta” work, many papers have explore the unexpected news risk like Da and Warachka (2009), Botshekan, Kraeussl, and Lucas (2012), Maio (2013), Chen, Da, and Zhao (2013), Campbell, Giglio, and Polk (2013), Cooper and Maio (2018), and Campbell, Giglio, Polk, and Turley (2018). Da and Warachka (2009) show that stock returns are partially driven by the unexpected cash flows by using data of analysts’ earnings forecast revisions on market earnings. Botshekan et al. (2012) construct a four-factor model to reflect the cash flow and discount rates risk under downside market and upside market. They find the downside cash flow risk is robust priced across different specifications and the downside cash flow risk premium is mainly attributable to small stocks. Maio (2013) extend the Campbell and Vuolteenaho (2004a)’s model and allow the price of aggregate cash flow to be time-varying by setting the conditional cash-flow beta to be linear in a state variable. Chen et al. (2013) show that cash flow news plays significant roles in determining stock returns and the importance increases with the investment horizon by using direct cash flow forecasts data. A most recent paper by Campbell et al. (2018) introduces the stochastic volatility into the initial homoskedastic ICAPM model and show that the volatility of future expected returns is negatively priced in the cross-sectional of stock returns. Different from their research, I find that the cash flow news and discount rates news at individual level tend to move together, which suggests the existence of common factors behind the big picture. Therefore I apply the stochastic volatility model to disentangle the common and idiosyncratic volatility from the individual-level news. To the best of my knowledge, however, no one has tried to disentangle the pricing properties of cash flow and discount rate news from the variance decomposition perspective. To motive the empirical results, I build up a cash flow model where each firm’s dividend growth is driven by two independent stochastic volatility processes - the common cash flow shock and the idiosyncratic cash flow shock - and the equilibrium solutions imply that the idiosyncratic and common cash flow risk are priced in
the cross-sectional stock returns.

My main intention is simple. I argue that the unexpected cash flow volatility could carry additional information besides current risk factors and firm characteristics. To verify my proposition, I apply the method to U.S. industry portfolios. In the main empirical results, I find that the common cash flow volatility estimated from unexpected industry-level cash flow news is highly correlated to Uncertainty index constructed by Jurado, Ludvigson, and Ng (2015). The idiosyncratic cash flow risk is robustly significant in explaining the cross-section of stock returns. The explanation power can not be consumed by current risk factors and firm characteristics. A strategy that goes long the decile portfolio with the largest idiosyncratic cash flow volatility and short the decile portfolio with the smallest idiosyncratic cash flow volatility can produce robust alpha across different specifications. The alpha significantly exists with respect to asset pricing models like Fama-French three factor model, Carhart four factor model and Fama-French five factor model. For example, the single-sorted strategy yields a Fama-French five factor alpha of 0.37% per month (t-stat: 6.90) in long sample (1931-2018) and 0.64% per month (t-stat: 12.28) in modern sample (1963-2018). By the double sorting, we find the abnormal alpha is mainly driven by the growth industries. I also build a theoretical connection between conditional Sharpe ratio and idiosyncratic cash flow volatility. I find that stocks with high conditional Sharpe ratios tend to have higher idiosyncratic cash flow volatility and higher compensated returns, which is consistent with Chava et al. (2019)’s finding.

One related literature is to study the role of the idiosyncratic and common stock return volatility in cross-sectional stock return literature. Their focus is the realized return volatility while my focus is the unexpected cash flow volatility. These two are closely connected and could help to understand the mechanism behind. In the realized return volatility literature, Ang, Hodrick, Xing, and Zhang (2006) document that high exposure to systematic return volatility or higher idiosyncratic return volatility corresponds to lower stock returns. The negative coefficients of common stock return volatility have been widely accepted while
the negative role of idiosyncratic return volatility is controversial. For the common stock volatility, the negative association can be justified by the leverage theory of Black (1976) and Christie (1982) and the risk premia theory of French, Schwert, and Stambaugh (1987). The leverage hypothesis argues that the firms become more levered when the stock prices fall which increase the aggregate volatility. The risk premium hypothesis argue investors demand higher risk premia when market volatility increase which depresses the firms’ value and results in the negative relationship. Both two explanation can justify the negative relationship among stock returns and aggregate return volatility. For the idiosyncratic return volatility, Ang et al. (2006) document that portfolios with high realized idiosyncratic volatility deliver low value-weighted average returns in the subsequent month while Bali and Cakici (2008) document no robustly significant relationship among stock returns and the idiosyncratic return volatility. Huang, Liu, Rhee, and Zhang (2009) find that the negative relationship is due to the short-term reversal and confirm the positive relationship among expected returns and the idiosyncratic volatility. Similar explanation is made by Fu (2009) where he uses the exponential GARCH models to estimate expected idiosyncratic volatility and find a significantly positive relation between the estimated conditional idiosyncratic volatility and expected returns. Fu (2009) argue that Ang et al. (2006)’s findings are largely explained by the return reversal of a subset of small stocks with high idiosyncratic volatility. These can go back the initial puzzle documented by Duffee (1995). Duffee (1995) documented the positive relationship among stock returns and the idiosyncratic volatility and argue that the positive contemporaneous relationship cannot be justified by the leverage hypothesis or the risk premium hypothesis. Grullon, Lyandres, and Zhdanov (2012) resolve this puzzle by showing that the positive relation between firm-level stock returns and firm-level return volatility is due to firms’ real options. Here the documented positive relationship among idiosyncratic cash flow volatility and stock returns which can also be backed up by the argument of Grullon et al. (2012). They take the firm’s future investment as potential growth options and the value of the growth options increase with the idiosyncratic return
volatility which justifies the positive relationship among volatility and stock returns. Our evidence on cash flow volatility support their argument on the amplified effect of good news on growth options.

Different from current discussions on the volatility of stock returns, my focus is the cash flow volatility estimated from the unexpected news. Since the basic economic theory tells us that prices should fully reflect the future cash flows and the future cash flow news should price today’s financial ratios, a direct approach to identify the role of cash flow can be helpful.

The aim of this paper is three-fold. First I build up a cash flow model where the firm’s cash flow is driven by a common factor and an idiosyncratic factor and I argue that the cash flow news will be priced in the cross-section stock returns. The model provides a clear closed-form solution to show the relationship among idiosyncratic and common cash flow risk, cross-section stock returns and the conditional Sharpe ratio. For the corresponding identification method, I propose a stochastic volatility econometric method to extract the common and idiosyncratic cash flow volatility from cross-section observed data. Second I apply the method to the U.S. industry portfolios and results suggest that the common cash flow volatility is closely to the whole economy uncertainty (see Jurado et al. (2015)) and the idiosyncratic cash flow volatility is not fully consumed by the current well-known risk factors and firm characteristic factors. The idiosyncratic cash flow volatility is positively related to the stock returns. Third, I relate the conditional Sharpe ratio to the idiosyncratic cash flow risk. Firms with higher idiosyncratic cash flow volatility tend to have higher Sharpe ratio and higher stock returns, which justifies the Sharpe ratio premium (see Chava et al. (2019)).

The rest of the paper is organized as follows. In the next section, I introduce the cash flow model that motives my empirical analysis and derive the equilibrium solution to show the relationship among idiosyncratic and common cash flow risk, cross-section stock returns and the conditional Sharpe ratio. Section 3 contains the estimation method to extract the volatility measures from the cash flow news. In section 4 I apply the method to US industry portfolio data and provide the main findings of this paper, namely that the common cash flow
volatility is closely to the whole economy uncertainty and the idiosyncratic cash flow news volatility cannot be fully explained by the well-known risk factors and firm characteristics. Strategy based on the idiosyncratic cash flow volatility can produce alpha in both long and modern samples. Section 5 concludes.

2. Theory

2.1. Motivation

In the influential “Bad Beta, Good Beta” paper, Campbell and Vuolteenaho (2004a) break the CAPM beta into two components: the bad one reflecting the future market cash flow news and the good one reflecting the future discount rates news. The economically motivated two-factor model is well applied to explain the size and value “anomalies”. They decompose unexpected market returns into the discount rate and cash flow components by using the return decomposition technique of Campbell and Shiller (1988) and Campbell (1991). The Campbell and Shiller’s technique is using a log-linear approximation of the present relation for stock prices that allows for time-varying discount rates. In Campbell and Vuolteenaho’s paper, the market return is decomposed into

\[
    r_{m,t+1} - E_t r_{m,t+1} \simeq (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \cdot \Delta d_{m,t+1+j} - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \cdot r_{m,t+1+j}
\]

\[
    = N_{m,CF,t+1} + N_{m,DR,t+1}
\]

where \( \rho = 1/(1 + exp(dp)) \) is a (log-linearization) discount coefficient that depends on the mean of log dividend-price ratio \( dp \), \( r_{m,t+j} \) is the log market return and \( \Delta d_{m,t+j} \) is the log market dividend growth. \( N_{CF} \) denotes news about future market cash flows and \( N_{DR} \) denotes news about future market expected returns.

The technique allows the unexpected market returns to be represented as the sum of cash flow news and discount rates news. By the construction, they can estimate each stock’s
beta by looking at the co-variance of the individual stock returns and market level news. The fitting two-beta ICAPM greatly improves the poor performance of the standard CAPM, which suggests that information is hidden in the unexpected cash flow and discount rates news.

Rather than look at market level news, I explore the information that might be hidden at individual level news. In this paper, the work is not limited to the market level decomposition since the return decomposition also works at the individual stock level. For example, the log-linearization formulation works at individual stock level, which is

\[
    r_{i,t+1} - E_t r_{i,t+1} \approx (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \cdot \Delta d_{i,t+1+j} - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \cdot r_{i,t+1+j}
\]

\[= \hat{N}_{i,CF,t+1} + \hat{N}_{i,DR,t+1}\]

(2)

where \(N_{i,CF}\) denotes news about future cash flows of stock \(i\) and \(N_{i,DR}\) denotes news about future expected returns of stock \(i\). If I bring this thought to real data, I find that cash flow news or discount rates news at individual level are driven by a common factor besides their idiosyncratic exogenous shocks. For example, I show the cash flow news and the discount rates news of each 30 industry (defined as Fama and French)\(^1\) in figure 1. I find that the individual news move together which is consistent with our argument.

\[\text{[Insert Figure 1 near here]}\]

The Campbell and Vuolteenaho (2004a)'s cash flow and discount rates decomposition at aggregate market level has became an important contribution to the ability of the CAPM model in explaining the cross-sectional differences in average returns. Following this framework, a large number of papers have shown that the cash flow and discount rates news are priced in the stock prices like Da and Warachka (2009), Botshekan et al. (2012), Maio (2013), Chen et al. (2013), Campbell et al. (2013), Cooper and Maio (2018), and Campbell et al. (2018). To the best of my knowledge, however, no one has tried to disentangle the

\(^1\)Results are robust when other industry definitions are applied. e.g. 48 industry.
pricing properties of cash flow and discount rate news from the variance decomposition - the idiosyncratic and common factor - perspective. To economically motive the empirical evidence, I provide a cash flow model where the individual firm’s dividend growth is driven by the common shock and its own idiosyncratic shock and I derive the proposition showing that the individual stock returns are determined by both two risk sources.

2.2. A Cash Flow Model

I start the theoretical framework from the pricing kernel as Constantinides (1992). In a no-arbitrage world, I always have the following condition holds as

\[
1 = E_t[M_{t+1}R_{t+1}]
\]

Here I assume the state pricing kernel at \( t + 1 \) in this economy follows

\[
m_{t+1} = -r_f - \frac{1}{2} \cdot \sigma^2_{m,t} + m_{t+1} \cdot \epsilon \quad m_{t+1} \sim N(0, \sigma^2_{m,t})
\]

where \( m_{t+1} \) is the state pricing density at time \( t + 1 \), \( r_f \) is the constant risk free rates and \( \sigma^2_{m,t} \) are exogenously determined. The pricing kernel form has been applied by previous researchers (e.g. Amin and Ng (1993), Wu (2001)) and here I adopt this functional form to easily derive the closed-form equilibrium solutions.

For the cash flow model, I allow the heterogeneous cash flow shocks on individual stocks. In my model, the cash flow is driven by two independent stochastic volatility processes - the common cash flow shock \( \tilde{\varepsilon}_{d,t+1} \) and the idiosyncratic shock \( \tilde{i}_{d,t+1} \) - for each stock.

\[
\Delta d_{i,t+1} = \alpha_0 + \alpha_1 \cdot \Delta d_{i,t} + \varepsilon_{d,t+1} + i_{d,t+1};
\]

\[
(\sigma^2_{d,t+1})^2 = \beta_0^c + \beta_1^c \cdot (\sigma^2_{d,t})^2 + \sigma^2_{d,t} \cdot \varphi_{t+1}^c;
\]

\[
(\sigma^2_{d,t+1})^2 = \beta_0^i + \beta_1^i \cdot (\sigma^2_{d,t})^2 + \sigma^2_{d,t} \cdot \varphi_{t+1}^i;
\]
where
\[
\begin{align*}
    c_{d,t+1}|I_t &\sim N(0, (\sigma_{d,t}^c)^2), \\
    i_{d,t+1}|I_t &\sim N(0, (\sigma_{d,t}^i)^2); \\
    v_{t+1}^c &\sim N(0, (\eta_{t+1}^c)^2), \\
    v_{t+1}^i &\sim N(0, (\eta_{t+1}^i)^2);
\end{align*}
\]

I price the cash flow risk by the following way where \( \rho_m^{(i)} \) reflects the relationship among the cash flow growth and the value of dividends regarding different states. The positive sign of \( \rho_m \) implies the period of more valuable of dividend coincides with period of higher cash flow growth while the negative sign of \( \rho_m \) implies the period of more valuable of dividend coincides with period of lower cash flow growth. As long as \( \rho_m \) is not equal to zero, we have the cash flow risk being priced.

\[
\begin{align*}
    \text{cov}_t(c_{d,t+1}, \epsilon_{m,t+1}) &= \rho_m^c \cdot (\sigma_{d,t}^c)^2, \\
    \text{cov}_t(i_{d,t+1}, \epsilon_{m,t+1}) &= \rho_m^i \cdot (\sigma_{d,t}^i)^2;
\end{align*}
\]

I also allow the shock to the dividend and the shock to its volatility to be correlated which captures the leverage effect as argued by Black (1976) in explaining the asymmetric volatility of individual stock returns.

\[
\begin{align*}
    \text{corr}(c_{d,t+1}, v_{t+1}^c) &= \rho_{t+1}^c, \\
    \text{corr}(i_{d,t+1}, v_{t+1}^i) &= \rho_{t+1}^i;
\end{align*}
\]

I further assume that the two stochastic volatility processes are uncorrelated which allows us to derive a simple closed form solution.

\[
\begin{align*}
    \text{corr}(c_{d,t+1}, v_{t+1}^i) &= 0, \\
    \text{corr}(i_{d,t+1}, v_{t+1}^c) &= 0; \\
    \text{corr}(v_{t+1}^c, v_{t+1}^i) &= 0, \\
    \text{corr}(v_{m,t+1}, v_{t+1}^{c(i)}) &= 0;
\end{align*}
\]

I build up the house foundation step by step. The first three propositions show the formulations of the price-dividend ratio, stock returns and unexpected news. Then the fourth proposition shows how the cash flow volatility is related to the conditional Sharpe
ratios and the cross-section of stock returns.

**Proposition 1:** The log price-dividend ratio in the economy can be represented as

\[
(p_t - d_t) = c_0 + c_1 \cdot \Delta d_{t,t} + c_2 \cdot (\sigma^c_{d,t})^2 + c_3 \cdot (\sigma^i_{d,t})^2
\]

(7)

where

\[
c_0 = \frac{-r_f + \kappa + (\rho \cdot c_1 + 1)\alpha_0 + \rho \cdot c_2 \cdot \beta_0^c + \rho \cdot c_3 \cdot \beta_0^i}{1 - \rho}, \quad c_1 = \frac{\alpha_1}{1 - \rho \cdot \alpha_1};
\]

\[
c_2 = (1 - \rho \cdot \alpha_1) \cdot (1 - \rho \cdot \beta_1^c) - \rho \cdot \eta^e \cdot \rho^e_t \pm \sqrt{[(1 - \rho \cdot \alpha_1) \cdot (\rho \cdot \beta_1^c - 1) + \rho \cdot \eta^e \cdot \rho^e_t]^2 - \rho^2 \cdot (\eta^e)^2 \cdot [1 + 2 \cdot \rho^e_m \cdot (1 - \rho \cdot \alpha_1)]} \cdot \frac{(1 - \rho \cdot \alpha_1) \cdot \rho^2 \cdot (\eta^e)^2}{(1 - \rho \cdot \alpha_1) \cdot \rho^2 \cdot (\eta^e)^2}
\]

\[
c_3 = (1 - \rho \cdot \alpha_1) \cdot (1 - \rho \cdot \beta_1^i) - \rho \cdot \eta^i \cdot \rho^i_t \pm \sqrt{[(1 - \rho \cdot \alpha_1) \cdot (\rho \cdot \beta_1^i - 1) + \rho \cdot \eta^i \cdot \rho^i_t]^2 - \rho^2 \cdot (\eta^i)^2 \cdot [1 + 2 \cdot \rho^i_m \cdot (1 - \rho \cdot \alpha_1)]} \cdot \frac{(1 - \rho \cdot \alpha_1) \cdot \rho^2 \cdot (\eta^i)^2}{(1 - \rho \cdot \alpha_1) \cdot \rho^2 \cdot (\eta^i)^2}
\]

Proof: See Appendix.

Note for \(c_2\) and \(c_3\), each of them has two roots. The root selection actually depends on where does the volatility feedback come from. At aggregate level, the negative volatility feedback effect requires the sign of the volatility to be negative. However, I cannot conclude the signs at individual stock level.

**Proposition 2:** The realized return of each stock can be represented as

\[
r_{i,t+1} = \lambda_0 \cdot \Delta d_{i,t} + \lambda_1^c \cdot (\sigma^c_{d,t})^2 + \lambda_1^i \cdot (\sigma^i_{d,t})^2 + \lambda_2^c \cdot v_{d,t+1}^c + \lambda_2^i \cdot v_{d,t+1}^i + \lambda_3^c \cdot v_{d,t+1}^c + \lambda_3^i \cdot v_{d,t+1}^i
\]

(8)

where

\[
\lambda_0 = (\rho \cdot c_1) \cdot \alpha_1 - c_1; \quad \lambda_1^c = \rho \cdot c_2 \cdot \beta_1^c - c_2; \quad \lambda_1^i = \rho \cdot c_3 \cdot \beta_1^i - c_3;
\]

\[
\lambda_2^c = \frac{1}{1 - \rho \cdot \alpha_1}; \quad \lambda_3^c = \rho \cdot c_2 \cdot \sigma^c_{d,t}; \quad \lambda_3^i = \rho \cdot c_3 \cdot \sigma^i_{d,t}
\]

10
Proof: See Appendix.

Note that the return will be positively related to cash flow shock \( c_{d,t+1}^{(i)} \) and negatively related to the volatility shock \( v_{d,t+1}^{(i)} \).

The cash flow news framework is first proposed by Campbell and Hentschel (1992) that any unexpected returns can be decomposed into a cash flow news term and a discount rates news term. The derived shock to dividend level and to its volatility can be well fitted into the expected cash flow news and discount rates news framework (Campbell and Vuolteenaho (2004a), Campbell, Polk, and Vuolteenaho (2009), Botshekan et al. (2012)). By the model construction, I can represent the expected news term in the formulation of common and idiosyncratic shocks.

**Proposition 3:**

CF News:

\[
(E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \cdot \Delta d_{i,t+1+j} = \lambda_2^i \cdot c_{d,t+1}^{(i)} + \lambda_2^i \cdot d_{i,t+1}^{(i)} \tag{9}
\]

DR News:

\[
-(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \cdot r_{i,t+1+j} = \lambda_3^i \cdot v_{d,t+1}^{(i)} + \lambda_3^i \cdot v_{d,t+1}^{(i)} \tag{10}
\]

Proof: See Appendix.

Therefore I have the unexpected cash flow news are reflected by the shock to dividend and the unexpected discount rates news are reflected in the shock to the dividend volatility. The second derivation is a powerful justification of volatility feedback effect because it indicates the increase in volatility will decrease the expected returns which lead to drop in today’s stock prices.

**Proposition 4:**

Conditional Sharpe ratio increase with idiosyncratic cash flow volatility.

Proof: See Appendix.

Stocks with higher idiosyncratic cash flow risk tend to have higher conditional Sharpe ratio. This proposition relates the conditional Sharpe ratio to the cash flow risk, which
provide a risk-based explanation why stocks with high conditional Sharpe ratio have higher risk premia.

3. Estimation Methodology

3.1. Analytic Framework

In this section, I relate the economic dynamics to the cross-sectional asset pricing. I argue cash flow risk should be priced and the idiosyncratic cash flow risks can be priced in the cross section. The classical CAPM model may fail to reflect the role of idiosyncratic cash flow risk. In the traditional CAPM, the systematic market risks are considered. The systematic risks in standard CAPM are abstract and hard to interpret while the common cash flow risk corresponds to the market risk premia in our framework. The new perspective is to provide a risk framework where idiosyncratic cash flow risk determines the asset prices conditional on common cash flow risk. In sum, the CAPM model may fail to explain the idiosyncratic cash flow risk since the market betas only reflect the systematic risk.

The cash flow and discount rates risks are first explored by Campbell and Vuolteenaho (2004a)’s paper. Campbell and Vuolteenaho (2004a) estimate the unexpected market-level news and show aggregate level risks are priced in the cross-section stock returns. By the novel cash flow setting, we manage to show that idiosyncratic cash flow risk is also priced in the cross section. For the economic dynamics, the cash flow framework is actually inspired by Wu (2001)’s earlier work. However, his paper focus on the market level cash flow and provide no insights on heterogeneous cash flow risks while our interests mainly lie in the cross-sectional stock pricing. My framework allows us to take one step further to study the determinants of cross sectional returns.
3.2. **Stochastic Volatility Model Estimation**

The priced volatility terms are estimated from the stochastic volatility model as below. Let \( X_{i,t} \) be the individual cash flow news \( \tilde{N}_{i,CF,t} \) and we can estimate the common factor from all individual news term, which can lead to the estimated common volatility and idiosyncratic volatility.

\[
X_{i,t} = B_i^c \cdot F_t^c + e_{i,t}; \quad (11)
\]

\[
F_t^c = \alpha + \sum_{j=1}^{p} \rho_j^c \cdot F_{t-j}^c + \Omega^{0.5} \cdot v_t; \quad (12)
\]

\[
\Omega^{0.5} = A_t^{-1} \cdot \text{diag}(\gamma_t) \cdot A_t^{-1}; \quad (13)
\]

\[
e_{i,t} = \sum_{j=1}^{p} \rho_j^i \cdot e_{t-j} + h_{i,t}^{0.5} \cdot \epsilon_t; \quad (14)
\]

Therefore we have the variance decomposition of the unexpected news term \( X_{i,t} \).

\[
\text{var}(X_{i,t}) = \text{var}(B_i^c F_t^c) + \text{var}(e_{i,t}) \quad (15)
\]

by which we have the cash flow news variance decomposition as follows where the total variations are equal to the sum of common and idiosyncratic volatility.

\[
\text{var}(\tilde{N}_{i,CF}) = (\sigma_{CF}^c)^2 + (\sigma_{CF}^i)^2 \quad (16)
\]

4. **Application**

In this section, I mainly study the asset pricing property at U.S. industry level. Evidence suggests that the idiosyncratic cash flow risk is robust priced at different specifications.
4.1. Data

I choose the U.S. industry portfolio data to test our framework where Fama-French Industry 30 data are explored here. I choose the industry specification (30 industries) due to the long documented data history than other industry specifications. The sample spans from 1926m6 to 2018m12 at monthly frequency.

The cash flow news and discount rates news are estimated as Campbell and Vuolteenaho (2004b) where the state variables are chosen as term spread, default spread and the adjusted PE ratios.

The term spread (TS) is defined as the difference between the ten-year yield and the three-month yield. The default spread (DS) is defined as the difference between Moody’s Seasoned Aaa and Baa bond yields. The CAPE is cyclically adjusted Price Earnings ratio downloaded from Robert Shiller’s website.

The \[-(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \cdot r_{i,t+1+j} \] is estimated from the VAR system while the \[ (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \cdot \Delta d_{i,t+1+j} \] is backed from the unexpected returns \[ r_{i,t+1} - E_t[r_{i,t+1}] \]. I present the calculated news term in Figure 1. A fact that can be documented here is that either cash flow news or discount rates news are driven by a common factor and they tended to move in the same direction. Therefore it is consistent with our argument that each news term is driven by a common shock sources and their volatility can be decomposed into two parts - a common part and an idiosyncratic part.

4.1.1. Volatility

I apply the estimation framework discussed in the methodology part. I estimate the cash flow volatility where the cash flow volatility follows an AR(1) process. I also run robust check letting the cash flow volatility follows an stationary AR(p) process and the main conclusion holds in our U.S industry portfolio application.

For cash flow volatility, I estimate it by letting the volatility term follows an AR(1) process. The AR(1) framework is consistent with the economic model and reflects the stationary
property of volatility updating process.

\[
\begin{align*}
\ln(\gamma_t) &= \bar{a} + \bar{b} \cdot \ln(\gamma_{t-1}) + Q^{0.5} \hat{\eta}_t; \\
\ln(h_{i,t}) &= a + b \cdot \ln(h_{i,t-1}) + q^{0.5} \eta_{i,t};
\end{align*}
\]  

(17)  
(18)

The stochastic volatility model is estimated via Gibbs sampling. Detailed procedures to carry out the estimation are introduced in the technical appendix. In the benchmark specifications, we use 20,000 replications and base our inference on the last 5000 replications. The lag in cash flow estimation is equal to four. Detailed processes are introduced in the technical appendix. I find that the idiosyncratic volatility varies across different industries. Each industry has its idiosyncratic cash flow volatility evolving pattern. It could be attributed to its industry’s life cycle and other industry characteristics.

[Insert Figure 2 near here]

[Insert Figure 3 near here]

4.2. Common Cash Flow Volatility

The common cash flow volatility is estimated from the U.S. whole industry’s cross-sectional cash flows. It is the common source that drive each industry’s dividend growth. Compared to the economic uncertainty index constructed by Jurado et al. (2015), I find that the common cash flow volatility is highly correlated to both financial uncertainty and macroeconomic uncertainty at 82% and 73%, respectively.

[Insert Figure 4 near here]

In Jurado, Ludvigson, and Ng (2015)’s construction, it takes 132 macro series to construct the macroeconomic uncertainty \(\text{UNC}^{\text{macro}}\) and it takes 147 financial time series to
construct the financial uncertainty $UNC^{fin}$. The macro data represents broad categories of macroeconomic time series including real output and income, employment and hours, different economic sector orders, inventories, and sales, consumer spending, compensation and labor costs, capacity utilization measures, price indexes, bond and stock market indexes, and foreign exchange measures while the financial data-set includes valuation ratios such as the dividend-price ratio and earnings-price ratio, growth rates of aggregate dividends and prices, default and term spreads, yields spreads of private and public bond, and a broad cross-section of portfolio equity returns. Here I simply use the unexpected cash flow news from thirty industry portfolios and the estimated common cash flow volatility is tightly co-moves with Jurado et al. (2015)’s macroeconomic and financial uncertainty.

4.3. Idiosyncratic Cash Flow Volatility

4.3.1. ICFV in the Cross-Section

I first investigate how the idiosyncratic cash flow volatility (ICFV) is related to the industry characteristics. Monthly cross-sectional regressions are run for the modern sample (1963-2018):

$$Y_t = \alpha + \gamma \cdot F_t + \epsilon$$

where $Y_t = \{ICFV_i\}$, $ICFV_i$ is the idiosyncratic cash flow volatility and $F_t$ are firm characteristics including the operating profitability $ROE$, the book-to-market ratio $BM$, the average firm size $Size$, leverage $LEV$ as Johnson (2004), idiosyncratic stock volatility $IVOL$ constructed as Ang, Hodrick, Xing, and Zhang (2009) and risk factors such as the economic uncertainty $UNC$ from Jurado, Ludvigson, and Ng (2015), lottery demand factor $FMAX$ from Bali, Brown, Murray, and Tang (2017) and liquidity factor $ILLIQ$ from Pastor and Stambaugh (2003). In order to control for the potential economic explanation of the estimated volatility measures, we include these industry characteristics and risk factors in the cross-sectional regressions. In table 1, the intercepts $Cons$ remain significant across
different specifications and the explained $R^2$ are less than 5% except specification (5) and (7). Results suggest that the idiosyncratic cash low volatility can not be fully explained by firm’s characteristics. We find that $IVOL$, $LEV$ and $UNC$ factors can increase the explanatory power $R^2$ a lot. For industry characteristics, evidence suggests that value firms and large firms tend to have larger idiosyncratic cash flow volatility. High firm leverage corresponds to high idiosyncratic cash flow volatility which is consistent with findings of Ang, Hodrick, Xing, and Zhang (2009). The interesting finding is that high past idiosyncratic stock return volatility $IVOL$ corresponds to high idiosyncratic cash low volatility and it has the largest explanatory power on the idiosyncratic cash flow risk. For the risk factors, the high economic uncertainty $UNC$ indicates high idiosyncratic cash flow volatility. Both $ILLIQ$ and $FMAX$ factors are significant but the explanatory power is trivial.

[Insert Table 1 near here]

Results in table 2 suggest the coefficients of idiosyncratic cash flow volatility are positive across all specifications while the magnitudes range from 0.075 to 0.150. We find that the idiosyncratic cash flow volatility can explain 15% in the first column. The positive magnitudes implies that a portfolio buying stocks with the highest idiosyncratic cash flow volatility and short-selling stocks with the lowest cash flow volatility can generate returns in the following month controlling for all else. The $\beta_{mkt}$ coefficients are positive and insignificant. We find that the coefficients of $BM$ are positive across all specifications which is consistent with the value effect. The coefficients of $SIZE$ are negative but insignificant. The leverage and the lagged idiosyncratic stock volatility are negatively priced which is consistent with Ang et al. (2009). The economic uncertainty is significantly priced as documented by Bali, Brown, and Tang (2017) and investors get compensated by economic uncertainty exposure. The liquidity factor by Stambaugh (1999) is negatively priced in the industry cross section. As shown in Column 5, 6 and 7, including $UNC$, $ILLIQ$ and $FMAX$ do not affect the power of the idiosyncratic cash flow volatility and other firm characteristic variables.
Here we study the role of idiosyncratic cash flow volatility which is related to the idiosyncratic and common stock return volatility covered in previous cross-sectional stock return literature. Ang et al. (2006) document that high exposure to systematic return volatility or higher idiosyncratic return volatility corresponds to lower stock returns. The negative coefficients of common return volatility have been widely accepted while the negative role of idiosyncratic return volatility is controversial. For the common return volatility, the negative association can be justified by the leverage theory of Black (1976) and Christie (1982) and the risk premia theory of French et al. (1987). The leverage hypothesis argues that the firms become more levered when the stock prices fall which increase the aggregate volatility. The risk premium hypothesis argue investors demand higher risk premia when market volatility increase which depresses the firms’ value and results in the negative relationship. Both two explanation can justify the negative relationship among stock returns and aggregate volatility. For the idiosyncratic volatility, Ang et al. (2006) document that portfolios with high realized idiosyncratic volatility deliver low value-weighted average returns in the subsequent month while Bali and Cakici (2008) document no robustly significant relationship among stock returns and idiosyncratic volatility. Huang et al. (2009) find that the negative relationship is due to the short-term reversal and confirm the positive relationship among expected returns and idiosyncratic volatility. Similar explanation is made by Fu (2009) where he use the exponential GARCH models to estimate expected idiosyncratic volatility and find a significantly positive relation between the estimated conditional idiosyncratic volatility and expected returns. Fu (2009) argue that Ang et al. (2006)’s findings are largely explained by the return reversal of a subset of small stocks with high idiosyncratic volatility. These can go back the initial puzzle documented by Duffee (1995). Duffee (1995) documented the positive relationship among stock returns and idiosyncratic volatility and argue that the positive contemporaneous relationship cannot be justified by the leverage hypothesis or the risk premium hypothesis. Grullon et al. (2012) resolve this puzzle by showing that the positive relation between firm-level stock returns and firm-level return volatility is due to
firms’ real options. Here the documented positive relationship among idiosyncratic cash flow volatility and stock returns which can also be backed up by the argument of Grullon et al. (2012). They take the firm’s future investment as potential growth options and the value of the growth options increase with the idiosyncratic volatility which justifies the positive relationship among volatility and stock returns. Here the amplified effect of good news on growth options is closely related to the cash flow volatility we estimated from each stock’s unexpected cash flows. Later we double sort the industry stocks by the idiosyncratic cash flow uncertainty and the book-to-market ratios. Results suggest the pricing of idiosyncratic cash flow risk is mainly driven by the growth industry. There are other hypothesis to explain the relationship among idiosyncratic volatility and stock returns. Stambaugh, Yu, and Yuan (2015) argue the negative relationship of some stocks is due to the relatively higher constraint on short selling.

[Insert Table 2 near here]

Due to extensive data mining in research on cross-sectional expected returns, Harvey, Liu, and Zhu (2016) argue that we should raise the threshold for accepting empirical results as evident of true economic phenomena. Their results suggests that today a newly discovered factor needs to clear a much higher hurdle, with a $t$ statistics greater than 3.0. As shown in table 2, the Fama-MacBeth cross-sectional regression indicates that the industry level cash flow volatility passes this test with a $t$ statistic above the threshold 3.0 when firm’s characteristics are considered.

4.3.2. Sorted Portfolios

Uni-variate Sorted Portfolios: At the end of each month, I sort all stocks into five groups based on the estimated idiosyncratic cash flow volatility. A strategy that goes long the decile portfolio with the largest idiosyncratic cash flow volatility and short the decile portfolio with the smallest idiosyncratic cash flow volatility can produce robust alpha across different
specifications. The alpha significantly exists with respect to asset pricing models like Fama-French three factor model, Carhart four factor model and Fama-French five factor model. For example, the single-sorted strategy yields a Fama-French five factor alpha of 0.37% per month (t-stat: 6.90) in long sample (1931-2018) and 0.64% per month (t-stat: 12.28) in modern sample (1963-2018).

[Insert Table 3 near here]

Double-Sorted Portfolios: I show that the abnormal returns can be obtained by sorting stocks into different idiosyncratic cash flow volatility groups. Here I proceed to evaluate the role of idiosyncratic cash flow volatility by further sorting the stocks into different industry characteristic groups. I consider the well-known characteristics like the book-to market ratio $BM$, the debt-to-asset ratio $LEV$ and the average market capitalization $Size$. At the end of each month, we sort all stocks into three groups based on the estimated idiosyncratic volatility and sort stocks in each volatility group into two groups based on an ascending sort of the industry characteristics. The intersections of the two industry characteristics groups and the three volatility groups generate six portfolios. Therefore we obtain the cash flow volatility premium by taking difference of high volatility and low volatility. Panel A of Table 4 shows that the equally-weighted volatility factor generates an average monthly return of 0.50% with a Newey-West t-statistic of 2.88 in Growth group and an insignificant average monthly return of 0.21% in Value group. It suggest the industry cash flow volatility is more likely priced in the growth industry which is supposed to have high cash flow volatility. The finding here is consistent with Grullon et al. (2012)’s argument that the value of firms’ growth options increases with the idiosyncratic volatility which results in the positive relationship among stock returns and idiosyncratic volatility. Panel B of Table 4 shows that the equally-weighted uncertainty factor generates an average monthly return of 0.45% with a Newey-West t-statistic of 2.65 in High leverage group and an average monthly return of 0.26% with a Newey-West t-statistic of 1.82 in Low leverage group. Panel C of Table 4 shows that the
equally-weighted uncertainty factor generates an average monthly return of 0.32% with a Newey-West t-statistic of 1.90 in Small firm group and an average monthly return of 0.40% with a Newey-West t-statistic of 2.84 in Large firm group. These results indicate that the idiosyncratic cash flow volatility is more likely to be priced in the growth industries.

[Insert Table 4 near here]

4.4. Conditional Sharpe Ratio

As argued by Chava et al. (2019), investors fail to incorporate the business cycle information into the cash flow growth and it affect the cross-sectional returns. If the pattern holds, then the price ratio during the similar history regime should predict the future returns. In their paper, they showed that firms with higher conditional (regime-dependent) Sharpe ratios correspond to higher stock returns and they find those firms have stronger fundamentals and more upward analyst forecast revisions. Here I argue that higher idiosyncratic cash flow volatility leads to higher conditional Sharpe ratio and brings higher risk compensation as shown in proposition 4.

[Insert Table 5 near here]

Table 5 shows that portfolio with higher idiosyncratic cash flow risk has higher conditional Sharpe ratio and higher average stock returns. The result provides empirical support for the previous proposition. Figure 5 shows how the conditional Sharpe ratios of top quintile and bottom quintile evolve during 1963 to 2018. The conditional Sharpe ratio of top quintile is larger than the bottom Sharpe ratio for most of the time.

[Insert Figure 5 near here]
4.5. Further Discussions

I apply the method to the US industry portfolios. Results suggest that the common cash flow volatility represents the economic uncertainty while the idiosyncratic cash flow volatility is persistent priced in the cross-sectional stock returns. Investors are compensated by holding a diversified portfolio. Results suggest that the volatility measure estimated from the unexpected stock returns are not fully explained by the current risk factors and the firm characteristics. My argument here is that there are information embedded in the unexpected return news at individual level and we can extract new factors from the individual cash flow news. It can also help to better understand the role of cash flows in pricing the current stocks.

The method can also be applied to other situations. For example, we can study the cross-country stock returns to evaluate the role of idiosyncratic and common cash flows, the analysis which may complement our understanding in global investment. It is also possible to extend the sample to the individual stocks in a larger sample and to evaluate the role of current risk factors and the well-known firm characteristics by the newly estimated volatility measures.

5. Conclusion

The fundamental question in empirical asset pricing is the determinants of the cross-sectional stock returns. While a large body of recent research proposing new factors based on a host of empirically motivated economic or financial characteristics, I address this question from a new perspective, offering evidence that idiosyncratic and common cash flow volatility is important for understanding stock returns. My main intention is simple. I argue that the unexpected cash flow news should carry additional information besides current risk factors and firm characteristics. In particular, drawing on classic work of Campbell and Vuolteenaho (2004a) and the insightful framework of Wu (2001), I link uncertainty to cross-sectional stock
returns through the common and idiosyncratic volatility perspective.

A recent paper by Chava et al. (2019) shows that there is significant variation in cash flow growth across industries over the business cycle and they find investors do not fully incorporate business cycle fluctuations into the industry level cash flows. If the business cycle information is not reflected in each industry’s cash flow, then conditional Sharpe ratio can be informative for future industry returns. In their paper, sector rotation strategy based on history-dependent Sharpe ratio can produce significant returns. It suggests that cash flow risk at the idiosyncratic level is not fully incorporated into the prices by investors. However, no theoretical model is provided to rationalize the documented Sharpe ratio premium and the role of idiosyncratic cash flows should be re-highlighted. In this paper, I develop a stochastic volatility framework to evaluate the unexpected cash flow news through the variance decomposition perspective, and I relate the conditional Sharpe ratio to the firm’s cash flow volatility - especially the idiosyncratic cash flow volatility - to justify the premium.

I propose a method to estimate common and idiosyncratic cash flow volatility from Campbell and Vuolteenaho (2004a)’s cs cash flow news. Papers have been developed based on the aggregate unexpected news but the individual dimension has been less explored. Moreover, the pure news shock has less been connected to the macroeconomic business cycles. I am inspired by a previous work of Wu (2001) where they explored the cash flow model and connected the unexpected stock returns to the model implied shock on cash flow and on its volatility term. I extend the aggregate level cash flow model by allowing a common factor and an idiosyncratic factor driving each firm’s cash flow growth. The setting allows us to have a new perspective and able to study the cross-sectional pricing from the volatility perspective and to provide a theoretical justification for Chava et al. (2019)’s findings on Sharpe ratios.

I apply the method to the U.S. industry portfolios and to study the role of newly estimated volatility measure. I find that the common cash flow volatility estimated from unexpected industry-level cash flow news is highly correlated to Uncertainty index constructed
by Jurado, Ludvigson, and Ng (2015). I also documented that the idiosyncratic cash flow volatility is positively priced in the cross-sectional stock returns. I control for well-known risk factors and firm characteristics to see the economic mechanism behind and results suggest the idiosyncratic cash flow volatility is not consumed by the current factors. I do the double sorting by the book-to-market ratio, the industry leverage and the average capitalization and find that the abnormal alphas are main driven by the growth industries. A strategy that goes long the decile portfolio with the largest idiosyncratic cash flow volatility and short the decile portfolio with the smallest idiosyncratic cash flow volatility yields a Fama-French-Five-Factor alpha of 37 bps per month (t-stat: 6.90) in long sample (1931-2018) and 64 bps per month (t-stat: 12.28) in the modern sample (1963-2018). The results suggest the idiosyncratic cash flow risk is not fully reflected by current risk factors. The results may not be limited to U.S. industry portfolios. Our method can also be applied to other situations, for example the cross-country asset returns and the cross-section individual firm returns.
References


Table 1: Comparison with Firm Characteristics: $ICFV_i$

This table shows results from regressing the idiosyncratic cash flow volatility on firm characteristics. The variables are economic uncertainty factor $UNC$ from Jurado, Ludvigson, and Ng (2015), lottery demand factor $FMAX$ from Bali, Brown, Murray, and Tang (2017), liquidity factor $ILLIQ$ from Pastor and Stambaugh (2003), operating profitability $ROE$, book-to-market ratio $BM$, average firm size $SIZE$, leverage $LEV$ as Johnson (2004) and idiosyncratic stock volatility $IVOL$ constructed as Ang, Hodrick, Xing, and Zhang (2009). Newey-West adjusted t statistics are reported in brackets. The sample period is from 1963 to 2018.

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The sample period is from 1963 to 2018.
Table 2: Fama-MacBeth Cross-Section Regressions

This table reports the time-series averages of the slope coefficients obtained from regressing monthly excess returns (in percentage) on the cash flow volatility and a set of factors. The control variables are the $\beta_{mkt}$ of market risk factor ($MktRf$) from Fama and French (1993 & 2015), economic uncertainty factor $UNC$ from Jurado, Ludvigson, and Ng (2015), lottery demand factor $FMAX$ from Bali, Brown, Murray, and Tang (2017), liquidity factor $ILLIQ$ from Pastor and Stambaugh (2003), operating profitability $ROE$, book-to-market ratio $BM$, average firm size $Size$, leverage $LEV$ as Johnson (2004) and idiosyncratic stock volatility $IVOL$ constructed as Ang, Hodrick, Xing, and Zhang (2009). Newey-West adjusted t-statistics are reported in brackets. The sample period is from 1963 to 2018.

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Table 3: Uni-variate Sorted Portfolios

This table shows results of real equally-weighted returns of industry portfolios sorted according to their industry-level cash flow volatility. Return data are monthly over the long sample from 1931 to 2018 and over the modern sample from 1963 to 2018. Industry definitions are from Kenneth French’s website. CAPM (FF3, Carhart4, and FF5) denotes average excess returns unexplained by the CAPM (Fama-French three-factor model, Carhart four-factor model and Fama-French five-factor model). The numbers in parentheses are t statistics according to Newey and West (1987). One, two, and three asterisks denote significance at the 10%, 5%, and 1% levels, respectively.

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<td>(8.08)</td>
<td>(10.57)</td>
<td>(11.12)</td>
<td>(7.54)</td>
</tr>
<tr>
<td>Carhart4Alpha</td>
<td>0.60</td>
<td>0.59</td>
<td>0.58</td>
<td>0.79</td>
<td>0.96</td>
<td>0.37***</td>
</tr>
<tr>
<td></td>
<td>(6.96)</td>
<td>(7.19)</td>
<td>(7.19)</td>
<td>(9.60)</td>
<td>(10.18)</td>
<td>(6.90)</td>
</tr>
<tr>
<td>Modern Sample</td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td>$H − L$</td>
</tr>
<tr>
<td>CAPMAlpha</td>
<td>0.09</td>
<td>0.11</td>
<td>0.09</td>
<td>0.30</td>
<td>0.44</td>
<td>0.36***</td>
</tr>
<tr>
<td></td>
<td>(1.86)</td>
<td>(2.55)</td>
<td>(2.23)</td>
<td>(7.20)</td>
<td>(7.79)</td>
<td>(6.95)</td>
</tr>
<tr>
<td>FF3Alpha</td>
<td>-0.18</td>
<td>-0.17</td>
<td>-0.16</td>
<td>0.05</td>
<td>0.28</td>
<td>0.46***</td>
</tr>
<tr>
<td></td>
<td>(-6.15)</td>
<td>(-6.08)</td>
<td>(-5.98)</td>
<td>(1.76)</td>
<td>(5.73)</td>
<td>(8.91)</td>
</tr>
<tr>
<td>Carhart4Alpha</td>
<td>-0.02</td>
<td>-0.01</td>
<td>-0.02</td>
<td>0.17</td>
<td>0.42</td>
<td>0.44***</td>
</tr>
<tr>
<td></td>
<td>(-0.72)</td>
<td>(-0.20)</td>
<td>(-0.69)</td>
<td>(5.49)</td>
<td>(7.69)</td>
<td>(8.13)</td>
</tr>
<tr>
<td>FF5Alpha</td>
<td>-0.23</td>
<td>-0.25</td>
<td>-0.19</td>
<td>0.00</td>
<td>0.42</td>
<td>0.64***</td>
</tr>
<tr>
<td></td>
<td>(-6.76)</td>
<td>(-8.58)</td>
<td>(-6.04)</td>
<td>(0.14)</td>
<td>(7.94)</td>
<td>(12.28)</td>
</tr>
</tbody>
</table>
### Table 4: Double-Sorted Portfolios

This table shows results of real equally-weighted returns of industry portfolios sorted according to their industry-level cash flow volatility and their industry characteristics. Return data are monthly over the modern sample from 1970 to 2018. Industry definitions are from Kenneth French’s website. Industry characteristics include the book-to-market ratio $BM$, the industry leverage $LEV$ and the average firm size factor $Size$. The numbers in parentheses are t statistics according to Newey and West (1987). One, two, and three asterisks denote significance at the 10%, 5%, and 1% levels, respectively.

<table>
<thead>
<tr>
<th>BM/$ICFV_i$</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>$H - L$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Growth</strong></td>
<td>0.58</td>
<td>0.66</td>
<td>1.08</td>
<td>0.50***</td>
</tr>
<tr>
<td></td>
<td>(2.02)</td>
<td>(2.46)</td>
<td>(3.50)</td>
<td>(2.88)</td>
</tr>
<tr>
<td><strong>Value</strong></td>
<td>0.66</td>
<td>0.62</td>
<td>0.88</td>
<td>0.21</td>
</tr>
<tr>
<td></td>
<td>(2.32)</td>
<td>(2.30)</td>
<td>(2.95)</td>
<td>(1.33)</td>
</tr>
<tr>
<td>LEV/$ICFV_i$</td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>$H - L$</td>
</tr>
<tr>
<td><strong>High</strong></td>
<td>0.64</td>
<td>0.65</td>
<td>1.10</td>
<td>0.45***</td>
</tr>
<tr>
<td></td>
<td>(2.26)</td>
<td>(2.35)</td>
<td>(3.47)</td>
<td>(2.65)</td>
</tr>
<tr>
<td><strong>Low</strong></td>
<td>0.60</td>
<td>0.63</td>
<td>0.86</td>
<td>0.26*</td>
</tr>
<tr>
<td></td>
<td>(2.10)</td>
<td>(2.43)</td>
<td>(3.18)</td>
<td>(1.82)</td>
</tr>
<tr>
<td>SIZE/$ICFV_i$</td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>$H - L$</td>
</tr>
<tr>
<td><strong>Small</strong></td>
<td>0.61</td>
<td>0.63</td>
<td>0.92</td>
<td>0.32*</td>
</tr>
<tr>
<td></td>
<td>(2.04)</td>
<td>(2.15)</td>
<td>(2.81)</td>
<td>(1.90)</td>
</tr>
<tr>
<td><strong>Large</strong></td>
<td>0.63</td>
<td>0.65</td>
<td>1.03</td>
<td>0.40***</td>
</tr>
<tr>
<td></td>
<td>(2.32)</td>
<td>(2.67)</td>
<td>(3.99)</td>
<td>(2.84)</td>
</tr>
</tbody>
</table>

### Table 5: Conditional Sharpe Ratio

This table shows results of real equally-weighted returns of industry portfolios sorted according to their industry-level cash flow volatility. Return data are monthly over the modern sample from 1963 to 2018. Industry definitions are from Kenneth French’s website. The numbers in parentheses are t statistics according to Newey and West (1987). One, two, and three asterisks denote significance at the 10%, 5%, and 1% levels, respectively.

<table>
<thead>
<tr>
<th>Modern Sample</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>$H - L$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Average Ret</strong></td>
<td>0.67</td>
<td>0.69</td>
<td>0.66</td>
<td>0.87</td>
<td>1.09</td>
<td>0.42***</td>
</tr>
<tr>
<td></td>
<td>(8.27)</td>
<td>(8.69)</td>
<td>(8.60)</td>
<td>(11.27)</td>
<td>(11.71)</td>
<td>(7.96)</td>
</tr>
<tr>
<td><strong>Sharpe Ratio</strong></td>
<td>0.423</td>
<td>0.433</td>
<td>0.438</td>
<td>0.447</td>
<td>0.469</td>
<td>0.046***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(4.92)</td>
</tr>
</tbody>
</table>
Fig. 1. Cash Flow News - Industry Portfolios

Fig. 2. Common Cash Flow Volatility
Fig. 3. Idiosyncratic Cash Flow Volatility
Fig. 4. Common Cash Flow Volatility and Uncertainty Index of Jurado, Ludvigson, and Ng (2015)

![Fig. 4](image-url)

Fig. 5. Conditional Sharpe Ratios Sorted by Idiosyncratic Cash Flow Volatility

![Fig. 5](image-url)
Appendix A. A Cash Flow Model

\[ \Delta d_{i,t+1} = \alpha_0 + \alpha_1 \cdot \Delta d_{i,t} + \frac{c_{d,t+1}}{d_{t+1}} + \frac{i_{d,t+1}}{c_{d,t+1}} \]

\[ (\sigma_{d,t+1}^c)^2 = \beta_0^c + \beta_1^c \cdot (\sigma_{d,t}^c)^2 + \sigma_{d,t}^c \cdot v_{t+1}^c \]

\[ (\sigma_{d,t+1}^i)^2 = \beta_0^i + \beta_1^i \cdot (\sigma_{d,t}^i)^2 + \sigma_{d,t}^i \cdot v_{t+1}^i \]

A.1. Proposition 1

\[ (p_t - d_t)_i = c_0 + c_1 \cdot \Delta d_{i,t} + c_2 \cdot (\sigma_{d,t}^c)^2 + c_3 \cdot (\sigma_{d,t}^i)^2 \]

Proof:

\[ 1 = E_t(M_{t+1}R_{t+1}) = E_t(exp(-r_{f,t+1} - \frac{1}{2} \sigma_{m,t}^2 + m_{t+1} + r_{t+1})) \]

where

\[ r_{t+1} = \kappa + \rho \cdot (p_{t+1} - d_{t+1}) + \Delta d_{t+1} - (p_t - d_t) \]

Let \( A(\cdot) = -r_{f,t+1} - \frac{1}{2} \sigma_{m,t}^2 + m_{t+1} + r_{t+1} \), we have

\[ E[A(\cdot)] + \frac{1}{2} Var(A(\cdot)) = 0 \]

By the educated guess,

\[ (p_t - d_t)_i = c_0 + c_1 \cdot \Delta d_{i,t} + c_2 \cdot (\sigma_{d,t}^c)^2 + c_3 \cdot (\sigma_{d,t}^i)^2 \]

Substitute the guess into \( A(\cdot) \) and the corresponding equation:

- For the constant term:

\[ -r_f + \kappa + \rho \cdot c_0 + (\rho \cdot c_1 + 1)\alpha_0 + \rho \cdot c_2 \cdot \beta_0^c + \rho \cdot c_3 \cdot \beta_0^i - c_0 = 0 \]
\[ c_0 = \frac{-r_f + \kappa + (\rho \cdot c_1 + 1)\alpha_0 + \rho \cdot c_2 \cdot \beta_0^c + \rho \cdot c_3 \cdot \beta_0^i}{1 - \rho} \]

- For the \( \Delta d \) corresponding term:

\[ (\rho \cdot c_1) \cdot \alpha_1 - c_1 = 0 \]

\[ \Rightarrow c_1 = \frac{\alpha_1}{1 - \rho \cdot \alpha_1} \]

- For the \( (\sigma_{d,t}^c)^2 \) corresponding term:

\[ \frac{1}{2} \rho^2 \cdot c_2^2 \cdot (\eta_i^c)^2 + (\rho \cdot \beta_1^c - 1) \cdot c_2 + \frac{1}{2} (\rho \cdot c_1 + 1)^2 + (\rho \cdot c_1 + 1) \cdot \rho^c + (\rho \cdot c_1 + 1) \cdot \rho^i \cdot \eta_i^c \cdot \rho_i^c = 0 \]

\[ \Rightarrow c_2 = \frac{(1 - \rho \alpha_1) \cdot (1 - \rho \beta_1^c) - \rho \cdot \eta_i^c \cdot \rho_i^c \pm \sqrt{(1 - \rho \alpha_1) \cdot (\rho \beta_1^c - 1) + \rho \cdot \eta_i^c \cdot \rho_i^c} - \rho^2 \cdot \eta_i^c \cdot \rho_i^c \cdot [1 + 2 \cdot \rho \cdot \rho_m \cdot (1 - \rho \alpha_1)]}{(1 - \rho \alpha_1) \cdot \rho^2 \cdot \eta_i^c \cdot \rho_i^c} \]

- For the \( (\sigma_{d,t}^i)^2 \) corresponding term:

\[ \frac{1}{2} \rho^2 \cdot c_3^2 \cdot (\eta_i^i)^2 + (\rho \cdot \beta_1^i - 1) \cdot c_3 + \frac{1}{2} (\rho \cdot c_1 + 1)^2 + (\rho \cdot c_1 + 1) \cdot \rho^i + (\rho \cdot c_1 + 1) \cdot \rho^c \cdot \eta_i^i \cdot \rho_i^i = 0 \]

\[ \Rightarrow c_3 = \frac{(1 - \rho \alpha_1) \cdot (1 - \rho \beta_1^i) - \rho \cdot \eta_i^i \cdot \rho_i^i \pm \sqrt{(1 - \rho \alpha_1) \cdot (\rho \beta_1^i - 1) + \rho \cdot \eta_i^i \cdot \rho_i^i} - \rho^2 \cdot \eta_i^i \cdot \rho_i^i \cdot [1 + 2 \cdot \rho \cdot \rho_m \cdot (1 - \rho \alpha_1)]}{(1 - \rho \alpha_1) \cdot \rho^2 \cdot \eta_i^i \cdot \rho_i^i} \]

Q.E.D.

### A.2. Proposition 2

\[ r_{i,t+1} = \lambda_0 \cdot \Delta d_{i,t} + \lambda_1^c \cdot (\sigma_{d,t}^c)^2 + \lambda_1^i \cdot (\sigma_{d,t}^i)^2 + \lambda_2^c \cdot \sigma_{d,t+1}^c + \lambda_2^i \cdot \sigma_{d,t+1}^i + \lambda_3^c \cdot \nu_{d,t+1}^c + \lambda_3^i \cdot \nu_{d,t+1}^i \]

Proof:

\[ r_{t+1} = \kappa + \rho \cdot (p_{t+1} - d_{t+1}) + \Delta d_{t+1} - (p_t - d_t) \]

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By proposition 1,

\[(p_t - d_t)_i = c_0 + c_1 \cdot \Delta d_{i,t} + c_2 \cdot (\sigma_{d,t}^c)^2 + c_3 \cdot (\sigma_{d,t}^i)^2\]

We have

\[r_{i,t+1} = \lambda_0 \cdot \Delta d_{i,t} + \lambda_1^c \cdot (\sigma_{d,t}^c)^2 + \lambda_1^i \cdot (\sigma_{d,t}^i)^2 + \lambda_2^c \cdot \sigma_{d,t+1}^c + \lambda_2^i \cdot \sigma_{d,t+1}^i + \lambda_3 \cdot v_{d,t+1}^c + \lambda_3 \cdot v_{d,t+1}^i\]

where

\[\lambda_0 = (\rho \cdot c_1) \cdot \alpha_1 - c_1;\]
\[\lambda_1^c = \rho \cdot c_2 \cdot \beta_1^c - c_2;\]
\[\lambda_1^i = \rho \cdot c_3 \cdot \beta_1^i - c_3;\]
\[\lambda_2^c = \lambda_2^i = \frac{1}{1 - \rho \cdot \alpha_1};\]
\[\lambda_3^c = \rho \cdot c_2 \cdot \sigma_{d,t}^c;\]
\[\lambda_3^i = \rho \cdot c_3 \cdot \sigma_{d,t}^i\]

Q.E.D.

A.3. Proposition 3

CF News:

\[\sum_{j=0}^{\infty} \rho^j \cdot \Delta d_{i,t+1+j} = \lambda_2^c \cdot \sigma_{d,t+1}^c + \lambda_2^i \cdot \sigma_{d,t+1}^i\]

DR News:

\[\sum_{j=1}^{\infty} \rho^j \cdot r_{i,t+1+j} = \lambda_3^c \cdot v_{d,t+1}^c + \lambda_3^i \cdot v_{d,t+1}^i\]

Proof:
CF News:

\[(E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho_j \cdot \Delta d_{i,t+1+j} = \sum_{j=0}^{\infty} \rho^j \cdot [E_{t+1}[\Delta d_{i,t+1+j}] - E_t[\Delta d_{i,t+1+j}]] \]

\[= \sum_{j=0}^{\infty} \rho^j \cdot \alpha_i \left( c_{d,t+1} + i_{d,t+1} \right) \]

\[= \frac{1}{1 - \rho \cdot \alpha_i} \left( c_{d,t+1} + i_{d,t+1} \right) \]

\[= \lambda^c_2 \cdot c_{d,t+1} + \lambda^i_2 \cdot i_{d,t+1} \]

DR News:

\[-(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho_i \cdot r_{i,t+1+j} = -\rho \sum_{j=1}^{\infty} \rho^{j-1} \cdot [E_{t+1}[r_{i,t+1+j}] - E_t[r_{i,t+1+j}]] \]

\[= -\rho \sum_{j=1}^{\infty} \rho^{j-1} \cdot [E_{t+1}[\lambda^c_i \sigma_{d,t+1+j} + \lambda^i_1 \sigma_{d,t+1+j}] - E_t[\lambda^c_i \sigma_{d,t+1+j} + \lambda^i_1 \sigma_{d,t+1+j}]] \]

\[= -\rho \sum_{j=0}^{\infty} (\rho \beta_1^c)^j \cdot (\lambda^c_1 \sigma_{d,t+1} v_{i,t+1}^c) \cdot (\lambda^i_1 \sigma_{d,t+1} v_{i,t+1}^i) \]

\[= \rho \cdot \frac{-\lambda^c_1}{1 - \rho \cdot \beta_1^c} \sigma_{d,t+1} v_{i,t+1}^c + \rho \cdot \frac{-\lambda^i_1}{1 - \rho \cdot \beta_1^i} \sigma_{d,t+1} v_{i,t+1}^i \]

\[= \rho \cdot c_2 \cdot \sigma_{d,t+1} v_{i,t+1}^c + \rho \cdot c_3 \cdot \sigma_{d,t+1} v_{i,t+1}^i \]

\[= \lambda^c_3 \cdot v_{d,t+1}^c + \lambda^i_3 \cdot v_{d,t+1}^i \]

Q.E.D.

A.4. Proposition 4

Proof:

In proposition 2, we have

\[r_{i,t+1} = \lambda_0 \cdot \Delta d_{i,t} + \lambda^c_1 \cdot (\sigma_{d,t}^c)^2 + \lambda^i_1 \cdot (\sigma_{d,t}^i)^2 + \lambda^c_2 \cdot c_{d,t+1} + \lambda^i_2 \cdot i_{d,t+1} + \lambda^c_3 \cdot v_{d,t+1}^c + \lambda^i_3 \cdot v_{d,t+1}^i \]
The conditional Sharpe ratio using log returns can be represented as

\[ SR_t = \frac{E_t[r_{i,t+1}] + \frac{1}{2} Var_t[r_{i,t+1}]}{\sqrt{Var_t[r_{i,t+1}]}} \]

\[ = \frac{\lambda_0 \cdot \Delta d_{i,t} + \lambda_1^i \cdot (\sigma_{d,t}^i)^2 + \lambda_2^i \cdot (\sigma_{d,t}^i)^2 + \frac{1}{2}((\lambda_3^i \cdot \sigma_{e,t}^i)^2 + (\lambda_3^i \cdot \eta_{e,t}^i)^2 + (\lambda_3^i \cdot \eta_{e,t}^i)^2)}{\sqrt{(\lambda_5^i \cdot \sigma_{d,t}^i)^2 + (\lambda_2^i \cdot \sigma_{d,t}^i)^2 + (\lambda_3^i \cdot \eta_{e,t}^i)^2 + (\lambda_3^i \cdot \eta_{e,t}^i)^2}} \]

Conditional Sharpe Ratio increases with idiosyncratic cash flow volatility \( \sigma_{d,t}^i \).

Q.E.D.