Low Risk Anomalies? *

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Abstract

This paper shows that stocks' CAPM alphas are negatively related to CAPM betas if investors demand compensation for negative skewness. Thus, high (low) beta stocks appear to underperform (outperform). This apparent anomaly merely reflects compensation for residual coskewness ignored by the CAPM. Empirically, we find that option-implied ex-ante skewness is strongly related to ex-post residual coskewness and alphas. Beta- and volatility-based low risk anomalies are largely driven by a single principal component, which is in turn largely explained by skewness. Controlling for skewness renders the alphas of betting-against-beta and -volatility insignificant.

Keywords: Low risk anomaly, skewness, risk premia, equity options.

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1 Introduction

Empirical findings that low-beta stocks outperform high beta stocks and that (idiosyncratic) volatility negatively predicts equity returns have spurred a large literature on 'low risk anomalies' (e.g., Haugen and Heins, 1975; Ang et al., 2006b; Baker et al., 2011; Frazzini and Pedersen, 2014). This paper shows that the returns to trading such anomalies can be rationalized when accounting for the skewness of equity returns, which is ignored by standard measures of market and idiosyncratic risk.

Our theoretical analysis starts from a generic arbitrage-free economy in which investors demand a premium for stocks that exhibit negative coskewness (e.g., Kraus and Litzenberger, 1976; Harvey and Siddique, 2000). We show that the standard CAPM systematically misestimates a firm's market risk because it does not explicitly account for return coskewness. The resulting bias is contained in the CAPM pricing errors and leads to predictable excess returns relative to the CAPM, i.e. to alphas, that are directly connected to the firm's coskewness. High-beta and high volatility stocks have more highly (i.e. less negatively) coskewed CAPM residuals than low-beta and low volatility stocks. Hence, the alphas of betting-against-risk strategies do not reflect anomalous returns but merely compensation for coskewness, and we document precisely the same pattern in empirical data on low risk anomalies (LRAs). We then employ an asset pricing model that uses the market as systematic risk factor, nests the standard CAPM as an approximation, but also allows for higher moments of the return distribution. With this model, we simulate (i) a world with a CAPM pricing kernel and find no LRAs, (ii) a skew-aware world and find that LRAs appear and are driven by coskewness, and (iii) a world where all moments higher than skewness are also accounted for, and we find that these higher moments contribute much less towards explaining LRAs. These simulation results suggest that skewness is the main driver of LRAs. Importantly, the model also demonstrates that there is a direct link between a firm's ex-ante skewness and its realized coskewness. This result motivates our empirical approach of using equity option-implied firm ex-ante skewness to construct skewness factors for our study of LRAs.

We establish our empirical results for a cross-section of approximately 5,000 US firms for the period from January 1996 to August 2014, covering all CRSP firms for which data on common stock and equity options are available. To comprehensively capture asymmetries in the return distribution, we compute three measures of ex-ante skewness from portfolios of out-of-the-money (OTM) options: upper skewness from OTM call options, covering the right part of the distribution; to account for the left part of the distribution, we compute lower skewness from a portfolio that is short in OTM put options, and hence by definition negative; total skewness, which is the sum of upper and lower skewness. Thus, total skewness becomes more negative, the more expensive put options are relative to call options, i.e. if investors are willing to pay high premia for protection against downside risk.

Our empirical analysis starts by showing that in the data ex-ante skewness is related to residual coskewness and alphas in the same way as in our simulated skew-aware world: The more extreme a firm's ex-ante skewness, the higher its residual coskewness and the lower its CAPM alpha. The results are almost identical when we compute alphas and residual coskewness relative to the Fama-French three factor model (FF3, Fama and French, 1993). When we additionally control for momentum (FF4, following Carhart, 1997), the results become quantitatively less pronounced but the qualitative patterns remain the same for lower and upper ex-ante skewness. These findings suggest that ex-ante skewness is linked to residual coskewness and alphas in a way that is consistent with skew-aware asset pricing and empirically not captured by standard risk factors.

Having established that ex-ante skewness is a forward-looking proxy for residual coskewness, we study the main prediction of our model: Controlling for skewness should eliminate positive alphas and negative residual coskewness of beta- and volatility-related LRAs. Since the different anomalies have been established as mostly unrelated asset pricing puzzles in the literature, we proceed in three steps. First, we show that the anomalies based on CAPM betas, idiosyncratic volatility, and options-implied variance all have a common driver, using principal component analyses of anomaly returns. We find that the first principal component explains more than 90% of the variation in anomaly excess returns and more than 70% of the variation in FF4 residual returns. Second, we show that the first principal component is related to the returns of skewness factors, constructed from decile portfolios sorted by different measures of firms' ex-ante skewness. When we regress the first principal component on skew factor returns, we find R^2s of up to 95% for excess returns and up to 80% for FF4 residual returns. These results provide strong evidence that LRAs have a common driver which is driven by skewness.

Third, we reconsider LRAs when controlling for skewness. Using several specifications of skewness factors, we find that anomaly alphas decrease substantially and are not statistically different from zero. For instance, using our most flexible specification, we find for betting-against-beta that the CAPM-alpha drops from 125 to 33 basis points per month, the FF3-alpha from 109 to 21 basis points, and the FF4-alpha from 73 to 21 basis points. Furthermore, all alphas become statistically insignificant. For all anomalies, the reduction in alphas goes in lockstep with a reduction in the strategies' negative coskewness. These results suggest that controlling for ex-ante skewness indeed renders alphas insignificant because it captures coskewness risk. This is confirmed by cross-sectional regressions of alphas on residual coskewness for 80 beta- and volatility-sorted portfolios. Without controlling for skewness, the regression R^2s are 73%, 73%, and 48% when using CAPM-, FF3-, and FF4-alphas and residual coskewness, respectively. Once we control for skewness, the R^2 for the CAPM-based regression drops to 29%, for the FF3- and FF4-based regressions the R^2s drop to zero.

In essence, our theoretical and empirical results imply that empirical patterns labeled as 'low risk anomalies' may not necessarily pose asset pricing puzzles. Taking into account that stock returns exhibit higher moments, our findings suggest that the CAPM beta may not be a sufficient metric to measure a firm's market risk, and that equity returns - reflecting the firm's true market risk - may only appear anomalous when benchmarked against the CAPM. These arguments also provide an understanding for the seemingly anomalous relations of (idiosyncratic) volatility to stock returns. Our results provide evidence that all these empirical patterns can be directly connected to skewness.

Various robustness checks confirm our findings and corroborate our conclusions. For instance, we show that controlling for skewness also leads to a substantial decrease in the alpha of the BaB-factor returns of Frazzini and Pedersen (2014), even though their factor is constructed from a broader cross-section that does not require options data. Other checks verify that our results are robust to variations in the portfolio-weighting schemes, including other control factors, and over subsample periods. We also provide preliminary evidence for a time-series prediction of our model, namely that alphas of betting-against-beta/volatility strategies should be related to the skewness of the market. Finally, we show that the model-implied relations between ex-ante skewness and credit spreads are strongly supported by empirical data using credit ratings and CDS spreads. Taken together with our equity results, our findings may also provide new insights on the 'distress puzzle'.

Related Literature. While the capital asset pricing model (CAPM, see Sharpe, 1964; Lintner, 1965; Mossin, 1966) postulates a positive relation between beta and return, there is a large body of research documenting that the empirical relation is flatter than implied by the CAPM or even negative. Early studies providing such evidence and attempting to explain the empirical failure of the CAPM include Brennan (1971), Black (1972), Black et al. (1972), and Haugen and Heins (1975). Recent research confirms these puzzling patterns. Ang et al. (2006b, 2009) show that (idiosyncratic) volatility negatively predicts equity returns and that stocks with high sensitivities to aggregate volatility risk earn low returns. While Fu (2009) finds that the sign of the relation between idiosyncratic risk and returns depends on the specific risk measure employed, other papers argue that a negative relation can be understood when accounting for leverage (e.g. Johnson, 2004) or differences in beliefs and short-selling constraints (e.g. Boehme et al., 2009). Related, Stambaugh et al. (2015) argue that the sign of the relation between idiosyncratic risk and returns depends on whether stocks are over- or underpriced and that arbitrage asymmetry explains why the overall relation is negative. Campbell et al. (2017) show that the low returns of stocks with high sensitivities to aggregate volatility risk are consistent with the intertemporal CAPM (Campbell, 1993) that allows for stochastic volatility.

To rationalize the profitability of betting against beta (BaB) strategies, Frazzini and Pedersen (2014) build on the idea of Black (1972) that restrictions to borrowing affect the shape of the security market line (SML). They present a model where leverage constrained investors bid up high-beta assets which in turn generate low risk-adjusted returns. Jylhä (2017) provides further evidence for the role of leverage constraints by showing that the SML-slope is connected to margin requirements. Baker et al. (2011) argue that institutional investors' mandates to beat a fixed benchmark discourages arbitrage activity and thereby contributes to the anomaly. Building on their insights, Wang (2014) claims that mandated investors with financial constraints contribute to the beta- but mitigate the idiosyncratic volatility-anomaly. Hong and Sraer (2016) present a model with short-sale constrained investors in which high beta assets are more prone to speculative overpricing because they are more sensitive to macro-disagreement. Bali et al. (2011) find that accounting for the lottery characteristics of stocks reverses the relation between idiosyncratic volatility and equity returns, and Bali et al. (2017) argue that the BaB anomaly is consistent with investors' preference for lottery stocks. Other papers study the properties of BaB returns, e.g., Baker et al. (2014) decompose these returns into micro and macro components. The results of Novy-Marx (2016) suggest that the performance of LRA-strategies is linked to firms' size, profitability, and book-to-market. Moreover, recent evidence suggests similar patterns in other asset classes (e.g. Frazzini and Pedersen, 2014) and in international markets (e.g. Walkshäusl, 2014). Huang et al. (2016) find that BaB activity itself affects the profitability of the strategy. Cederburg and O'Doherty (2016) study BaB returns through the lense of the conditional CAPM and argue that positive alphas reported in prior research are attributable to biases in unconditional performance measures.

This paper takes a different approach by directly linking LRAs to return skewness. Specifically, we build on the insight of Rubinstein (1973) and Kraus and Litzenberger (1976) that the empirical failure of the CAPM may be due to ignoring the effect of skewness on asset returns. Friend and Westerfield (1980) also find that coskewness with the market entails information for stock returns beyond covariance, Sears and Wei (1985) discuss the interaction of skewness and the market risk premium in asset pricing tests, and Harvey and Siddique (2000) show that conditional skewness helps to explain the cross-section of equity returns.[1] With the widespread availability of equity options data, recent papers explore the relation of option-implied ex-ante skewness on subsequent equity returns but provide mixed evidence (e.g. Bali and Hovakimian, 2009; Xing et al., 2010; Rehman and Vilkov, 2012; Bali and Murray, 2013; Conrad et al., 2013), with differences in results driven by differences in skew-measure construction.[2] Bali et al. (2015) provide complementary evidence by showing that ex-ante skewness is positively related to ex-ante stock returns estimated from analyst price targets. Other recent papers suggesting that skewness matters for the cross-section of equity returns are Amaya et al. (2015), who find a negative relation between realized skewness and subsequent equity returns, and Chang et al. (2013), who show that stocks that are most sensitive to changes in the market's ex-ante skewness, exhibit lowest returns. Buss and Vilkov (2012) apply the measure of Chang et al. (2013) to individual stocks, but do not find a pronounced relation to equity returns, whereas they do find that betas constructed from option-implied correlations exhibit a positive relation to subsequent stock returns. This paper differs from all of the literature above since our focus is on the relation between skewness and low risk anomalies. We show that options-implied ex-ante skewness of LRA portfolios is informative about their future residual coskewness and thus about CAPM mispricing.

The remainder of this paper is organized as follows. Section 2 presents the theoretical framework that guides our empirical analysis. We describe the data and construction of variables in Section 3 and present the empirical results in Section 4. Section 5 discusses additional results and robustness checks. Section 6 concludes. The Appendix describes technical details and the separate Internet Appendix reports additional results.

[1]Our approach, thus, builds on the idea that firms' skewness matters for asset prices through their coskewness component. This is conceptually very different from work that studies how idiosyncratic skewness can be priced in stock returns (e.g., Brunnermeier et al., 2007; Mitton and Vorkink, 2007; Barberis and Huang, 2008; Boyer et al., 2010). The pricing of coskewness is conceptually and empirically also different from the pricing of downside risk (e.g. Ang et al., 2006a; Lettau et al., 2014) and Ang et al. (2006a) elaborate in detail.

[2]For instance, Rehman and Vilkov (2012) and Conrad et al. (2013) both use the ex-ante skew measure of Bakshi et al. (2003) but find a positive and negative relation to subsequent returns, respectively. Apparently, this difference in results stems from Rehman and Vilkov (2012) measuring ex-ante skew from the latest option-data only whereas Conrad et al. (2013) compute ex-ante skew measures for every day over the past quarter and then take the average, thereby smoothing out recent changes in skewness. In preceding work, Bali and Hovakimian (2009) show that the spread between near-the-money call and put option implied volatility positively predicts stock returns. Similarly, Xing et al. (2010) find that stocks with steep implied volatility smirks, defined as the difference between OTM put minus ATM call implied volatility, underperform stocks with less pronounced smirks. Bali and Murray (2013) construct a skewness asset as a combination of positions in equity options and the underlying stock and and find that its returns are negatively related to the option-implied skewness measure of Bakshi et al. (2003).

2 Theoretical Framework

In this Section, we present the theory to guide our analysis of low risk anomalies (LRAs), such as the finding that high CAPM beta stocks underperform relative to low beta stocks. Kraus and Litzenberger (1976) are the first to note that the lack of empirical support for the CAPM may be due to the model ignoring the effect of skewness on asset prices. Building on this insight, Harvey and Siddique (2000) show that conditional skewness indeed helps to explain the cross-section of equity returns. Therefore, skewness appears to be a plausible candidate to provide insights for beta- and volatility-based LRAs that receive considerable attention in the recent literature (e.g., Ang et al., 2006b; Frazzini and Pedersen, 2014).

To motivate our analysis, Table 1 reports the alphas of LRAs, computed as Low-minus-High returns of decile portfolios sorted by CAPM beta, idiosyncratic volatility, and option-implied variance. These results suggest that the strategies' positive alphas come at the cost of their residual returns being negatively coskewed with the market. Figure 1 plots the alphas of the decile portfolios against their residual coskewness, illustrating a strong negative relation, with R^2s of 48% to 73% in regressions of alphas on residual coskewness across 80 portfolios. This negative relation is in line with Kraus and Litzenberger (1976), Harvey and Siddique (2000), and others who argue that investors demand compensation for accepting negative coskewness.

In what follows, we show that the empirical properties of LRAs are consistent with the implications of skew-aware asset pricing models. In particular, we show that alphas that appear 'anomalous' from the perspective of the CAPM indeed reflect compensation for skewness. Moreover, we discuss how betting-against-beta arises in the skew-aware CAPM and provide further evidence for the link between coskewness and LRA alphas in a simulation study of a skew-aware world populated by Merton-type firms, where (co)skewness arises endogenously from leverage and stochastic asset volatility. Within this framework, we also study the link between firms' coskewness and their implied skewness. These insights will later guide our empirical approach of constructing factors based on option-implied skewness.

2.1 Skew-Aware Asset Pricing

The results in Figure 1 are consistent with the interpretation that investors care about skewness and that excess returns appear 'abnormal' only because the factor-based risk adjustments fail to account for investors' aversion towards negative skewness. To study the effect of skewness on asset prices and LRAs in more depth, we draw on the ideas of Kraus and Litzenberger (1976, KL), Harvey and Siddique (2000, HS), Chabi-Yo et al. (2014) and

Schneider (2015). KL are the first to propose a three moment CAPM to account for skew preferences in asset pricing. HS provide a conditional version of their skew-aware CAPM. Schneider (2015) argues that any pricing kernel can be projected onto polynomials (or any other basis) of the market return and this projection implies an error which contains higher-order information by construction. Chabi-Yo et al. (2014) show that this projection error can have important asset pricing implications, and we argue along these lines. By construction, models with a linear approximation of the pricing kernel, such as the CAPM and standard linear factor models, ignore these implications.

For now, we assume that the world complies with a skew-aware CAPM in the spirit of KL and HS; as we show in Appendix A, starting from a general pricing kernel and projecting it on basis functions ultimately leads to the same result. In the skew-aware world that we assume here, the (true) pricing kernel is given by

$$\mathcal{M} := a_2 + b_2 R_T + c_2 R_T^2 \tag{1}$$

where R_T is the market excess return from time t to T, with $b_2 < 0$ and $c_2 > 0$. This implies that investors accept lower (demand higher) expected returns on assets with positive (negative) coskewness.[3] Building on the results of Schneider (2015), for details see Appendix A, we can express stock excess returns from time t to T for a generic firm i to follow

$$R_{i,T} = \beta_{i,t}^{skew} \cdot R_T + \varepsilon_{i,T}^{skew} \tag{2}$$

with

$$\beta_{i,t}^{skew} = \frac{b_2 \cdot \sigma_{i,m,t} + c_2 \cdot \sigma_{i,m^2,t}}{b_2 \cdot \sigma_{m,t}^2 + c_2 \cdot \sigma_{m,m^2,t}}, \tag{3}$$

where $\sigma_{i,m,t}$ is the covariance between stock i and market excess returns, $\sigma_{i,m^2,t}$ is the covariance between stock i and squared market excess returns, $\sigma_{m,t}^2$ is the variance of market excess returns, and $\sigma_{m,m^2,t}$ is the covariance between market and squared market excess returns (i.e. the skewness of the market). Finally, in a world that complies with this skew-aware CAPM, we have that $\mathbb{E}_t^{\mathbb{P}}\left[\varepsilon_{i,T}^{skew}\right] = 0$ and that the expected stock excess return is given by

$$\mathbb{E}_t^{\mathbb{P}}\left[R_{i,T}\right] = \beta_{i,t}^{skew} \cdot \mathbb{E}_t^{\mathbb{P}}\left[R_T\right]. \tag{4}$$

[3] While b_2 and c_2 may change over time, we omit time subscripts for the sake of simpler notation. We note that the signs of b_2 and c_2 could be different in incomplete markets but the empirical analysis of Schneider and Trojani (2017) suggests that this is not the case in the period from 1990 to 2016. Similarly, other recent evidence confirms that the market price of coskewness risk is negative, e.g. Christoffersen et al. (2016).

It is obvious that using the standard CAPM to assess market risk and to evaluate whether returns are 'abnormal' is inaccurate in such a skew-aware world. Ignoring the last term in equation (1), the CAPM postulates that

$$R_{i,T} = \beta_{i,t}^{capm} \cdot R_T + \varepsilon_{i,T}^{capm}, \tag{5}$$

where

$$\beta_{i,t}^{capm} = \frac{\sigma_{i,m,t}}{\sigma_{m,t}^2}, \tag{6}$$

thereby ignoring skewness-related pricing implications. As a consequence, $\mathbb{E}_t^{\mathbb{P}}\left[\varepsilon_{i,T}^{capm}\right]$ is generally not zero and the CAPM suffers from systematic pricing errors.[4] Following the convention to refer to returns unexplained by the CAPM as alphas, we write

$$\begin{aligned} \alpha_{i,T} :&= \varepsilon_{i,T}^{capm} \\ &= R_{i,T} - \beta_{i,t}^{capm} \cdot R_T \\ &= \left(\beta_{i,t}^{skew} - \beta_{i,t}^{capm}\right) \cdot R_T + \varepsilon_{i,T}^{skew}. \end{aligned} \tag{7}$$

Using the above equations, we can compute firm i's expected alpha and its residual coskewness, $\sigma_{\alpha_i,m^2,t}$, i.e. the covariance of excess returns not explained by the CAPM with squared market returns, as

$$\mathbb{E}_t^{\mathbb{P}}\left[\alpha_{i,T}\right] = \left(\beta_{i,t}^{skew} - \beta_{i,t}^{capm}\right) \cdot \mathbb{E}_t^{\mathbb{P}}\left[R_T\right], \tag{8}$$

$$\sigma_{\alpha_i,m^2,t} = \sigma_{i,m^2,t} - \beta_{i,t}^{capm} \cdot \sigma_{m,m^2,t}. \tag{9}$$

Combining equation (8) with the skew-aware beta from equation (3), we can rewrite the expected alpha to make its relation to CAPM beta more explicit:

$$\mathbb{E}_t^{\mathbb{P}}\left[\alpha_{i,T}\right] = \left(\sigma_{i,m^2,t} - \beta_{i,t}^{capm} \cdot \sigma_{m,m^2,t}\right) \cdot B_t \cdot \mathbb{E}_t^{\mathbb{P}}\left[R_T\right], \tag{10}$$

where

$$B_t = \frac{c_2}{b_2 \cdot \sigma_{m,t}^2 + c_2 \cdot \sigma_{m,m^2,t}}. \tag{11}$$

[4] The condition $\mathbb{E}_t^{\mathbb{P}}\left[\epsilon_{1,T}^{capm}\right] = \cdots = \mathbb{E}_t^{\mathbb{P}}\left[\epsilon_{n,T}^{capm}\right] = 0$ would imply that $\frac{\sigma_{1,m}}{\sigma_{1,m^2}} = \cdots = \frac{\sigma_{n,m}}{\sigma_{n,m^2}} = C$, where C depends on market moments. This follows from equating $\beta_{i,T}^{capm} = \beta_{i,T}^{skew}$.

Market skew, $\sigma_{m,m^2,t}$, is typically negative, i.e. times of high volatility are usually associated with market declines – in our sample, $\sigma_{m,m^2,t} = -0.000052$. Thus, equation (9) implies that, ceteris paribus, stocks with higher CAPM betas exhibit more positively (or less negatively) coskewed residual returns. Furthermore, in a skew-aware world $B_t < 0$.[5] Hence, equation (10) reveals that, ceteris paribus, stocks with higher CAPM betas exhibit less positive (or more negative) alphas.

Finally, to derive the relation between expected alpha and residual coskewness, we combine equation (10) with equation (9):

$$\mathbb{E}_t^{\mathbb{P}}\left[\alpha_{i,T}\right] = \sigma_{\alpha_i,m^2,t} \cdot B_t \cdot \mathbb{E}_t^{\mathbb{P}}\left[R_T\right]. \tag{12}$$

Given the negative sign of B_t, equation (12) shows that expected alphas are negatively related to residual coskewness. The negative relation between factor model alphas and residual coskewness in Figure 1 is indeed consistent with this implication from skew-preferences.[6]

With these results suggesting that the world behaves more in line with a skew-aware CAPM rather than the standard CAPM, we study the role of skewness specifically for LRAs in the next subsection. Our focus will be on betting-against-beta, starting from the observation that equation (9) implies a direct connection between CAPM betas and residual coskewness.

2.2 Implications for Cross-Sectional Trading Strategies

In a betting-against-beta (BaB) strategy, investors buy stocks with low CAPM betas (L) and sell stocks with high CAPM betas (H), generating a positive alpha when the CAPM underestimates expected excess returns on L and/or overestimates returns on H,[7]

$$\mathbb{E}_t^{\mathbb{P}}\left[\alpha_{BaB,T}\right] = \mathbb{E}_t^{\mathbb{P}}\left[\alpha_{L,T}\right] - \mathbb{E}_t^{\mathbb{P}}\left[\alpha_{H,T}\right]. \tag{13}$$

In a world that complies with the CAPM, the alpha of betting-against-beta is zero by definition. In a skew-aware world, the alpha can be nonzero and depends on the residual

[5] We have that $B_t < 0$ because $b_2 < 0$ and $c_2 > 0$, as discussed above for equation (1) and in Footnote 3, and, moreover, $\left|b_2 \cdot \sigma_{m,t}^2\right|$ is larger than $\left|c_2 \cdot \sigma_{m,m^2,t}\right|$ by several orders of magnitude except for pathological market distributions.

[6] The cross-sectional regression of alphas on residual coskewness, indicated by the solid lines in Figure 1, can be directly interpreted in terms of equations (12) and (11). The slope coefficient of this regression is the sample estimate for $B_t \cdot \mathbb{E}_t^{\mathbb{P}}\left[R_{t+1}\right]$ given by $\widehat{B}_t \cdot \overline{R}_{t+1}$.

[7] Positive alphas would also arise when the returns of L are less overestimated than returns of H or when returns of L are more underestimated than returns of H.

coskewness of low compared to high CAPM beta stocks. Using equation (12), we can write

$$\mathbb{E}_t^{\mathbb{P}}\left[\alpha_{BaB,T}\right] = \left(\sigma_{\alpha_L,m^2,t} - \sigma_{\alpha_H,m^2,t}\right) \cdot B_t \cdot \mathbb{E}_t^{\mathbb{P}}\left[R_T\right]. \tag{14}$$

From equation (9) we know that a firm's residual coskewness is directly related to its CAPM beta, and we can express the BaB-alpha as

$$\mathbb{E}_t^{\mathbb{P}}\left[\alpha_{BaB,T}\right] = \left(\left(\sigma_{L,m^2,t} - \sigma_{H,m^2,t}\right) - \left(\beta_{L,t}^{capm} - \beta_{H,t}^{capm}\right) \cdot \sigma_{m,m^2,t}\right) \cdot B_t \cdot \mathbb{E}_t^{\mathbb{P}}\left[R_T\right]. \tag{15}$$

In equation (15), the first term in the outer parentheses represents the difference in the return coskewness of low and high CAPM beta firms, whereas the second represents the difference in CAPM beta-related coskewness for low and high beta firms. The sign of this second term depends on market skewness, $\sigma_{m,m^2,t}$, which is typically negative, as discussed above. Hence, should high and low beta firms be equally coskewed (i.e. $\sigma_{H,m^2,t} = \sigma_{L,m^2,t}$), BaB delivers a positive expected alpha whenever market returns are negatively skewed. In other words, if risk is measured via the CAPM beta and the market is skewed, then this mechanically links a stock's implied coskewness to its beta. For a negative market skew, high-beta stocks inherit more negatively coskewed returns via the linear mapping than low-beta stocks, irrespective of the stocks' true coskewness. Thus, for the case in which low-beta portfolios are equally coskewed as high-beta portfolios, i.e. when $\sigma_{L,m^2,t} - \sigma_{H,m^2,t} = 0$, the measured betting against beta strategy alpha is positive.

In general, however, the term in the first inner bracket in equation (15) will not be zero, i.e. the coskewness of low and high beta portfolios will not be the same. In fact, Panel A of Figure 2 shows that, in our sample, firms with high CAPM beta are typically more negatively coskewed than low beta firms. In other words, stocks that appear most risky in terms of CAPM betas also exhibit the worst coskewness properties, delivering comparably low returns when market volatility is high. While this effect tends to reduce the alpha measured according to equation (15), the effect from the mismeasured coskewness due to beta-mapping in the second inner bracket in equation (15) dominates. This can be seen from the analysis of the coskewness of the residual returns defined in equation (9), i.e. the coskewness of returns after controlling for CAPM market risk. Panel B of Figure 2 shows that the firms with the highest CAPM betas have the highest residual coskewness. Thus, mapping stocks linearly on a negatively skewed market in a linear CAPM representation, prices in too much coskewness risk in high beta stocks and too little in low beta stocks. Therefore, the high beta stocks appear to underperform in a CAPM representation whereas low-beta stocks appear to outperform.[8]

The results in Figures 1 and 2 show that the empirical patterns for other volatility-based LRAs are in accordance with this intuition, i.e. the skewness implications for these LRAs resemble those for BaB. Sorting firms by volatility leads to portfolios that on average have similar CAPM betas as beta-sorted portfolios, and as a consequence the coskew implications through equation (15) apply accordingly.[9] Hence, our results suggest that excess returns of LRA strategies should have a common driver that is linked to coskewness and that controlling for coskewness, we should find alphas to become insignificant.

While these predictions for the empirical analysis appear straight-forward, an empirical challenge is to measure ex-ante coskewness as already stressed by, e.g., Harvey and Siddique (2000) and Christoffersen et al. (2016). Previous research has developed methods for measuring ex-ante moments for individual firms from equity options data, however, it is not possible to implement an analogous approach to measure co-moments due to the lack of derivatives having both the market and the stock as underlying. However, if a firm's skewness is related to its coskewness, then measures of ex-ante skewness are informative about the coskewness of the firm's future returns. In the next section, we present a simulation study on the link between skewness and LRAs and we also use this simulation to show that the link between measures of a firm's ex-ante skewness and its residual coskewness is strong in a skew-aware world. We will rely on these insights to guide our empirical approach of constructing skewness factors based on equity options data.

2.3 Simulation of a Skew-Aware World with Merton-Type Firms

To provide further support for the notion that skewness matters for LRAs, we show that the properties of LRAs in a simulated skew-aware world populated with Merton-type firms are qualitatively identical to those that we observe in the empirical data. In this framework, skewness and coskewness arise endogenously from firms' leverage, stochastic asset volatility, and the skewness of the market. For details on the setup of the simulation study, see

[8] As a concrete example, consider the equally-weighted BaB strategy in the first column of Panel A in Table 1 that delivers a CAPM alpha of around 92 basis points. Multiplying all coskewness quantities by 10,000, we have $\sigma_{L,m^2} = -0.39$, $\sigma_{H,m^2} = -0.69$, $\beta_H^{capm} = 2.24$, $\beta_L^{capm} = 0.56$, and $\sigma_{m,m^2} = -0.52$. Therefore, as discussed above, the second term in the brackets of equation (15) dominates the first one. Since $B_t \cdot \mathbb{E}_t^{\mathbb{P}}[r_m]$ is negative, the resulting $\alpha_{BaB,T}$ is positive.

[9] Moreover, note that empirical measures of idiosyncratic volatility are also directly linked to CAPM beta, as these measures are typically computed as the volatility of CAPM or factor model regression residuals. In our skew-aware world, volatility that is idiosyncratic from a CAPM perspective is given by

$$\sigma^2_{\alpha_{i,t}} = \sigma^2_{i,t} + (\beta^{capm}_{i,t})^2 \cdot \sigma^2_{m,t} - 2 \cdot \beta^{capm}_{i,t} \cdot \sigma_{i,m,t}.$$

Appendix B. In this Appendix, we also discuss alternative model specifications, such as including jumps in the asset process or allowing a firm to default before debt reaches its maturity, for which we present results in the Internet Appendix.

The simulation evidence in Figure 3 illustrates the relation between alphas and residual coskewness for a population of 2,000 (levered) firms for three different pricing kernel specifications. Analogous to Figure 1, we also illustrate the long and short positions of an investor who bets against beta, idiosyncratic volatility, or total volatility. We first show that if the world were governed by a CAPM pricing kernel, there would be no alphas to betting against beta or volatility (see Panel A); investors do not care about coskewness and all CAPM alphas are zero. Second, we simulate a skew-aware CAPM world and Panel B illustrates the relation between alphas and residual coskewness discussed above. These patterns are qualitatively identical to those in the empirical data, i.e. low beta stocks generate higher alphas than high beta stocks and, hence, betting against beta delivers a positive alpha.[10] Finally, we simulate a world in which investors also care about moments higher than (co)skewness. The cross-sectional pattern in CAPM alphas and residual coskewness in Panel C is very similar to that in the skew-aware CAPM world (Panel B) which suggests that coskewness is the main driver of LRA alphas. To make this point more explicit, we also plot (in grey circles) skew-adjusted alphas, i.e. the expected excess returns of firms beyond the skew-beta; these are close to zero, indicating that moments beyond skewness matter much less. The results are essentially the same when we allow for jumps in firms' assets and for defaults prior to debt maturity (see Figures IA.1 and IA.2 in the Internet Appendix).

Moreover, we can use this simulation framework to study the relation between a firm's skewness to its coskewness. As mentioned above, ideally, we would like to control for the role of coskewness on LRAs by using forward-looking information on a stock's return co-moments. Due to lack of data that allows for measuring ex-ante coskewness we will rely on measures of firms' ex-ante skewness, i.e. the firm's implied skewness under the $\mathbb{Q}$-measure. In the simulation analysis, we can study how measures of implied skewness as well as measures of

[10]In the simulations, betting-against-beta is mostly driven by the short position in high beta stocks and less by the long position in low beta stocks. Empirically, the relative contribution of the long position can be somewhat higher. This can be seen from the fitted lines of regressing alphas on residual coskewness being on a higher level in the empirical data illustrated in Figure 1 compared to the simulated data in Figure 3. We have experimented with different parameter values for the stochastic volatility market model and found that we can reduce this level-difference when we make the market variance less persistent (explosive). Given that our main goal is to show the negative relation between LRA alphas and residual coskewness, we only show this relation for standard model parametrizations established in the literature (e.g. Aït-Sahalia and Kimmel, 2007) as we discuss in Appendix B.2. For future research, it will be interesting to explore the quantitative implications in more detail building on ongoing research efforts that aim for modeling a more realistic interplay between the cross-section of firms and the pricing kernel (e.g. Gourier, 2016; Boloorforoosh et al., 2017).

expected realized skewness (under the $\mathbb{P}$-measure) are related to each other as well as to residual coskewness and alphas.

We evaluate three dimensions of skewness to comprehensively capture asymmetries in the return distribution. First, we measure implied skewness that originates from the upper part of the distribution; this *upper skewness* is defined to be positive and can be measured from prices of out-of-the-money (OTM) call options. Second, we measure implied skewness from the lower part of the distribution, which is defined to be negative; this *lower skewness* can be measured as the price of a portfolio that is short in OTM put options. Third, we measure the firm's overall skewness, defined as the sum of upper and lower skewness, which can be positive or negative and quantifies the overall asymmetry of the distribution.[11]

Bearing in mind that our empirical objective will be to construct factors based on equity option-implied skew measures, Figure 4 reports the residual coskewness and the alphas of decile portfolios sorted by implied and expected realized lower skewness, upper skewness, and skewness; portfolios P1 and P10 contain the firms with the lowest and highest values of the conditioning variables, respectively. The left column shows that firms with low (i.e. most negative) lower skewness have highest residual coskewness and lowest alphas, and that residual coskewness decreases whereas alphas increase when moving towards portfolio P10 containing firms with the highest lower skewness (i.e. close to zero). Conversely, we see in the middle column that residual coskewness increases from firms with lowest (i.e. closest to zero) upper skewness in P1 to firms with highest (i.e. most positive) upper skewness in P10. Accordingly, the alphas decrease from P1 to P10. In other words, the higher implied lower or upper skewness in absolute terms, the higher the portfolio's residual coskewness and the more negative its alpha. In line with these patterns, we find that portfolios sorted by implied skewness (i.e. the sum of lower and upper semi-skew) exhibit a U-shaped relation towards coskewness and an inversely U-shaped relation to alphas. These findings remain unchanged when we allow for jumps in firms' assets and for default prior to debt maturity (Figures IA.3 and IA.4 in the Internet Appendix).

These results suggest a strong link of implied skewness measures to residual coskewness and alphas in our simulated world. In our empirical analysis, we find qualitatively very similar relations in the data.

[11] Considering upper and lower skewness separately besides total skewness can be useful, because identical values of total skewness can arise from different combinations of values for lower and upper skewness. This can be easily illustrated by two firms with total skewness close to zero. For one firm both upper and lower skewness may be close to zero, whereas for the other firm upper skewness takes high positive value (e.g. due to growth options) which is offset by a large negative value of lower skewness (e.g. due to leverage).

3 Setup of Empirical Analysis

This Section details the data used in the empirical analysis, describes the computation of ex-ante skewness and ex-ante variance from equity option data, and discusses the construction of beta-, volatility-, and coskewness-measures from stock and market returns.

3.1 Data

The data set for our empirical analysis of US firms is constructed as follows. The minimum requirement for firms to be included is that options data and equity data are available at a daily frequency, with sufficient historical data for stocks to construct the beta and volatility measures as described below. We start with volatility surface data from OptionMetrics and keep the firms for which we find corresponding stock returns in CRSP (common stocks with share code 10 or 11) and firm data in COMPUSTAT to compute market capitalizations and book-to-market ratios. The merged data set contains 400,449 monthly observations across 4,967 firms from January 1996 to August 2014. Additionally, we obtain daily return data for the CRSP value-weighted market index as well as daily and monthly market, size, book-to-market, momentum factor and riskfree returns from Ken French's data library. Other data we use in further robustness checks are described in the corresponding sections.

3.2 Measuring Ex-Ante Moments from Prices of Stock Options

Harvey and Siddique (2000) measure coskewness from historical stock returns, but also discuss the limitations of evaluating ex-ante moments using historical data. Recent research shows that model-free measures of a firm's higher equity moments implied by stock options are more accurate. While option-implied ex-ante moments can be measured on an individual firm level, options on the cross-moments of stock returns generally do not exist. However, our simulation study in Section 2.3 suggests that firms' ex-ante skewness is closely linked to firms' coskewness and we draw on this insight in our empirical analysis. We therefore use option-implied information (rather than historical data) in a model-free way (rather than assuming a parametric correlation framework) to explore how firms' ex-ante skewness relates to the joint distribution of their stock returns with the market and the prevalence of LRAs.

Building on the concepts of Breeden and Litzenberger (1978) and Neuberger (1994), recent research proposes to assess ex-ante moments of the equity return distribution based on equity option prices. The fundamental idea is that differential pricing of a firm's equity options across different strike prices reveals information about the shape of the risk-neutral return distribution (see, e.g., Bakshi and Madan, 2000). By now, a large literature discusses options-implied measures of ex-ante moments as well as corresponding realizations and associated risk premia (e.g., Bakshi et al., 2003; Carr and Wu, 2009; Todorov, 2010; Neuberger, 2013; Kozhan et al., 2013; Martin, 2013; Schneider and Trojani, 2014; Andersen et al., 2015).

The common theme across these papers is to measure ex-ante skewness as an options portfolio that takes long positions in out-of-the-money (OTM) calls and short positions in OTM puts. Differences in approaches arise from the associated portfolio weights and the behavior of moment measures when the underlying reaches a value of zero.[12] In these respects, the approach of Schneider and Trojani (2014) is most suitable for our objective of studying higher moments of individual firms. First, their portfolio weights specification complies with the notion of put-call symmetry developed by Carr and Lee (2009); this is important because this concept connects the observable slope of the implied volatility surface to the unobservable underlying distribution. Second, their measures are well-defined when the stock price reaches zero, a feature that is essential for individual firms which can default.

Schneider and Trojani (2014) suggest to measure skewness as follows.[13] Denote the price of a zero coupon bond with maturity at time T by $p_{t,T}$, the forward price of the stock (contracted at time t for delivery at time T) by $F_{t,T}$, and the prices of European put and call options with strike price K by $P_{t,T}(K)$ and $C_{t,T}(K)$, respectively. The portfolio of OTM put and OTM call options that measures option-implied skewness ($SKEW^{\mathbb{Q}}_{t,T}$) is given by

$$SKEW^{\mathbb{Q}}_{t,T} = \frac{6}{p_{t,T}} \left(\int_{F_{t,T}}^{\infty} \log\left(\frac{K}{F_{t,T}}\right) \frac{\sqrt{\frac{K}{F_{t,T}}}\, C_{t,T}(K)}{K^2} dK - \int_0^{F_{t,T}} \left(\log \frac{F_{t,T}}{K}\right) \frac{\sqrt{\frac{K}{F_{t,T}}}\, P_{t,T}(K)}{K^2} dK \right),$$

which is constructed precisely to measure deviations from put-call symmetry in the sense of Carr and Lee (2009). To comprehensively capture asymmetries in the return distribution, we decompose $SKEW^{\mathbb{Q}}_{t,T}$ into upper skewness and lower skewness, to separately account for

[12] With respect to the latter, see for instance the discussion in Martin (2013). The contract underlying the VIX implied volatility index, for example, becomes infinite as soon as the price of the underlying S&P 500 touches zero. Also OTC variance swaps which pay squared log returns have been reported to cause difficulties in particular in the single-name market.

[13] The exposition below rests on the assumption that options markets are complete, but only for notational convenience. In our empirical analysis we use the 'tradable' counterparts which are computed from available option data only; see Schneider and Trojani (2014). In other words, they account for market incompleteness and do not require interpolation or extrapolation schemes to satisfy an assumption that a continuum of option prices is available.

the left part and the right part of the distribution,

$$upperSKEW^{\mathbb{Q}}_{t,T} = \frac{6}{p_{t,T}} \left(\int_{F_{t,T}}^{\infty} \log\left(\frac{K}{F_{t,T}}\right) \frac{\sqrt{\frac{K}{F_{t,T}}} C_{t,T}(K)}{K^2} dK \right),$$

$$lowerSKEW^{\mathbb{Q}}_{t,T} = -\frac{6}{p_{t,T}} \left(\int_{0}^{F_{t,T}} \left(\log \frac{F_{t,T}}{K}\right) \frac{\sqrt{\frac{K}{F_{t,T}}} P_{t,T}(K)}{K^2} dK \right),$$

i.e. we have that $SKEW^{\mathbb{Q}}_{t,T} = upperSKEW^{\mathbb{Q}}_{t,T} + lowerSKEW^{\mathbb{Q}}_{t,T}$. While upper skewness is by definition positive and lower skewness is by definition negative, the sign of overall ex-ante skewness depends on the relative prices of OTM put and OTM call options. As already mentioned above (in footnote 11), considering upper and lower skewness separately besides total skewness can be useful, because identical values of total skewness can arise from different combinations of values for lower and upper skewness.

Moreover, we use options data to measure ex-ante variance. One of the low risk anomalies that we study in this paper is that options-implied variance negatively predicts stock returns (e.g., Conrad et al., 2013). We again follow Schneider and Trojani (2014) and compute

$$VAR^{\mathbb{Q}}_{t,T} = \frac{2}{p_{t,T}} \left(\int_{0}^{F_{t,T}} \frac{\sqrt{\frac{K}{F_{t,T}}} P_{t,T}(K)}{K^2} dK + \int_{F_{t,T}}^{\infty} \frac{\sqrt{\frac{K}{F_{t,T}}} C_{t,T}(K)}{K^2} dK \right).$$

In our empirical analysis, we measure ex-ante skewness and variance from equity options with a maturity of 30 days, thereby matching the monthly horizon of equity returns.

3.3 Construction of Variables from Stock Returns

In our empirical analysis of beta- and idiosyncratic volatility-related low risk anomalies, we apply exactly the same measures that were used in the studies originally documenting the anomalies; therefore, we delegate the details of how these measures are constructed to Appendix C. First, we estimate CAPM betas as described in Frazzini and Pedersen (2014), using the CRSP value-weighted market index. Second, we use the residuals of this CAPM estimation to construct our measure of idiosyncratic volatility relative to the CAPM (Ivol CAPM). Third, we estimate idiosyncratic volatility following Ang et al. (2006b) from the residuals of Fama French three factor model regressions (Ivol FF3). Using the CAPM residuals is conceptually closer to our theoretical setup as these residuals can be directly interpreted as pricing errors of the CAPM approximation to our asset pricing model in Section 2. In related research on the idiosyncratic volatility puzzle, Ivol FF3 is the measure commonly used and empirically the results are very similar using either estimate of idiosyncratic volatility.

To measure coskewness and residual coskewness, we compute the covariance of (portfolio) excess returns and residual returns with squared excess returns of the CRSP value-weighted market index.

4 Empirical Analysis

Our theoretical results in Section 2 suggest that the positive alphas of betting-against-beta and -volatility strategies may be driven by compensation required by skew-aware investors. In this case the excess returns of low risk anomalies (LRAs) should have a common driver that is related to skewness. Controlling for skewness should reduce the negative residual coskewness of LRA strategies and, as a consequence, render LRA alphas insignificant. In this section we provide strong support for these predictions.

4.1 Implied Skewness, Residual Coskewness, and Alphas

To study the importance of skewness for LRAs, we use stock options-implied measures of a firm's ex-ante skewness. In a skew-aware world in the spirit of Kraus and Litzenberger (1976) and Harvey and Siddique (2000) such measures of ex-ante skewness are directly related to the residual coskewness and the CAPM alpha of the firm's stock. Figure 5 shows that the empirical links between skewness, residual coskewness, and alpha are qualitatively identical to those in the simulations presented in Figure 4.

The empirical results in Figure 5 are based on decile portfolios sorted by three measures of options-implied skewness, as defined in Section 3.2: lower skewness (left column), upper skewness (middle column), and skewness computed as the sum of lower and upper skewness (right column). Panel A presents the portfolios' CAPM residual coskewness, i.e. the coskewness of excess returns after controlling for the market factor, and Panel B plots the corresponding CAPM alphas. In all plots, blue lines with bullets represent value-weighted portfolios and the green lines with diamonds depict results for equally-weighted portfolios. The results accord very well with those from the simulation study: firms with low (i.e. most negative) lower skewness have highest residual coskewness and lowest alphas. The residual coskewness decreases whereas alpha increases when moving towards portfolio P10 containing firms with the highest lower skewness (i.e. close to zero). Conversely, we see in the middle column that residual coskewness increases from firms with lowest (i.e. closest to zero) upper skewness in P1 to firms with highest (i.e. most positive) upper skewness in P10. Accordingly, alphas decrease from P1 to P10. In other words, the higher the implied lower skewness or implied upper skewness in absolute terms, the higher the portfolio's residual coskewness and the more negative its alpha. In line with these patterns, we find that portfolios sorted by implied skewness, computed as the sum of lower and upper skewness, exhibit a U-shaped relation towards residual coskewness and tend to show an inversely U-shaped relation to alphas. Descriptive statistics for the skewness measures, betas and volatilities, as well as size and book-to-market of the equally- and value-weighted portfolios are summarized in Tables IA.1 and IA.2 in the Internet Appendix.

Encouragingly, these empirical results show that in the data ex-ante skewness is indeed as informative for future CAPM residual coskewness and alphas as it is in our simulated world. The results are very similar when we also control for the small-minus-big (SMB) and high-minus-low (HML) factors of Fama and French (1993, FF3); see Figure IA.5 in the Internet Appendix. When we additionally control for momentum (FF4, following Carhart, 1997), we find the same patterns in the relation of residual coskewness and alphas to upper skewness and lower skewness (see Figure IA.6). We also find the U-shaped relation between skewness and residual coskewness whereas the link between skewness and alphas is mostly increasing and the inverse U-shape is less pronounced. As a robustness check, we repeat the analysis with the equity-option implied skewness measure of Kozhan et al. (2013) which gives very similar results (see Figure IA.7).[14] All these results are consistent with the notion that alphas reflect compensation for residual coskewness.

4.2 Skewness as a Common Driver of LRA Returns

With the above results suggesting that measures of ex-ante skewness are indeed informative for stocks' future residual coskewness and alphas, we now proceed to study whether LRAs have a common driver, and in turn, to what extent this common driver is related to skewness.

We start by performing a principal components analysis on the LRA excess returns as well as on their CAPM-, FF3-, and FF4-residual returns. In the main text, we present results for Low-minus-High returns of value-weighted decile portfolios in Table 2. Panel A shows that the first principal component (PC1) explains around 91% of the variation in LRA excess returns. After controlling for the market and other risk factors (size, value, and momentum),

[14]We choose to use the skewness measure of Kozhan et al. (2013) for the robustness check because similar to our measure it represents a measure of skewness which is not standardized by variance, whereas for instance the measure by Bakshi et al. (2003) is such a standardized measure. Since we explicitly study the interaction between (co)skewness and (co)variance, our analysis focuses on non-standardized measures of skewness. Additionally, we decompose the skewness measure of Kozhan et al. (2013) in separate upper skewness and lower skewness components and find results that very similar to the ones reported above.

PC1 still explains around 72% of variation in the LRA residual returns. In turn, when we regress the individual LRA's (residual) returns on PC1, we find that all LRAs load on PC1 with a positive sign with coefficients in the range from 0.37 to 0.58 (across all LRAs and all specifications). The explanatory power of PC1 for the individual LRA returns is high, with R^2s in the range of 85% to 95% for excess returns and in the range of 61% to 80% for four factor residual returns. For equally- and rank-weighted portfolios the same analysis suggests an even higher degree of comovement among LRAs and the results are very similar when additionally controlling for liquidity, profitability, and investment factors.[15]

Thus, LRAs have a common driver that explains a large part of their variation in returns. To explore whether this common driver is related to (co)skewness, we construct skew factors based on portfolios sorted by ex-ante skewness, i.e. using the portfolios presented in Figure 5. We start by computing the high-minus-low returns of portfolios sorted by lower skewness (LSK) as well as by upper skewness (USK). When we regress PC1 on these factor returns, we find that LSK and USK individually explain more than 93% of the variation in PC1 estimated from LRA excess returns and more than 75% when PC1 is estimated from LRA four factor residual returns. The signs of the regression coefficients are consistent with compensation for skewness. Across all specifications the coefficient on LSK is positive. This implies that positive alphas of LRAs, delivered through their positive loading on PC1, are associated with negative residual coskewness, because we know from Figure 5 that firms with high (low) values of lower skewness have low (high) residual coskewness. Accordingly, the regression coefficients on USK are always negative.

Next, we construct a skew factor from portfolios sorted by ex-ante skewness (shown in the right column of Figure 5). To capture the U-shaped relation between ex-ante skewness and residual coskewness, we compute the returns of going long the extreme portfolios (P1 and P10) and going short the middle portfolios (P5 and P6). Regressing the LRAs' first principal component on the returns of this factor ($\text{SK}_{1+10} - \text{SK}_{5+6}$), we find a regression-$R^2$ of around 83% when PC1 is estimated from excess returns and 52% when using FF4-residual returns, respectively. A potential advantage of this factor is that it embeds information from lower and upper skewness, however, the comparably lower, but still high, R^2 reflects that it is more difficult to construct factors from a U-shaped relation. To analyze whether

[15] In the Internet Appendix, Tables IA.3 and IA.4 report results for equally- and rank-weighted portfolios, respectively. Table IA.5 presents results when we augment the FF4 specification with the liquidity factor of Pástor and Stambaugh (2003, FF4 LIQU) and when computing residuals relative to the five factor model of Fama and French (2015, FF5), i.e. augmenting the FF3 specification with profitability and investment factors. Since all these results are qualitatively identical to the results reported in the paper and the LRA literature typically reports CAPM-, FF3, and FF4-alphas, we focus on these specifications in the remainder of our empirical analysis. Very similar to the results reported below, we find that controlling for skewness also reduces the FF4 LIQU- and FF5-alphas and the negative residual coskewness of LRAs, respectively.

incorporating information on both lower and upper skewness increases the explanatory power we regress PC1 on LSK and USK. We find that the R^2 increases to over 80% for the PCA based on FF4 residuals. Finally, we construct a factor as the difference between USK and LSK, which corresponds to a restricted version of the regression on both factors that imposes the coefficients to have the same magnitudes in absolute terms but with opposing signs. For this USK−LSK-factor the explanatory power is in the range of 95% for the PC1 estimated from excess returns to 80% for the PC1 estimated from FF4 residual returns.

Summarizing, 72% of the variation in the FF4 residual returns of LRAs, i.e. excess returns after controlling for the market, SMB, HML, and momentum, are driven by a single component. In turn, up to 80% of this common component can be explained by the returns of factors constructed from portfolios sorted by ex-ante skewness. The signs of the loadings of LRA returns on PC1 and the signs of the loadings of PC1 on skewness factor returns are consistent with predictions based on our theoretical analysis where investors require positive alpha as compensation for negative coskewness. Accordingly, we should find that once we control for skewness, LRA strategies should exhibit less negatively coskewed returns and that alphas become less significant. In the next section, we show that this is indeed the case.

4.3 Skew-Adjusted Returns of Betting-Against-Beta and -Volatility

We now show that controlling for skewness reduces, both, the alphas as well as the negative coskewness of LRAs, which is in line with the notion that investors require compensation for negative coskewness. We use the skewness factors constructed in the previous subsection based on information in lower and upper skewness and run factor model regressions that include either the $(\text{SK}_{1+10} - \text{SK}_{5+6})$-factor, the (USK−LSK)-factor, or both the LSK- and the USK-factor. From these regressions we compute the LRA strategies' skew-adjusted alphas and residual coskewness.

Figure 6 shows that any of the skew-adjustments leads to a substantial decrease in alphas compared to the alphas without skew-adjustments reported in Table 1 above. Figure 7 reveals that these reductions in alpha are associated with the residual coskewness of the LRA strategies becoming much less negative and closer to zero. These results are consistent with the predictions developed in Section 2: First, measures of ex-ante skewness contain information about future residual coskewness, which can be seen from the fact that skew-adjusted LRA have less negative residual coskewness. Second, controlling for coskewness largely eliminates the alphas. The reductions in alphas are economically large and in fact mostly render alphas of betting-against-beta and -volatility statistically insignificant, as we report in more detail in Table 3. The only alphas that remain borderline significant are the CAPM alphas of betting-against-beta when we use the ($SK_{1+10} - SK_{5+6}$)-factor or the (USK−LSK)-factor as control variable, with the t-statistics being 1.87 and 1.70, respectively. But even for those alphas there is a large reduction in the magnitude from an unadjusted 125 basis points (with t-statistic of 2.87) to 66 basis points and 56 basis points, respectively. All other alphas are insignificant, with all three skew-adjustments producing quite similar results; in most cases the skew-adjustment using both LSK and USK leads to the largest reduction in alphas and residual coskewness; the difference compared to the other skew-adjustments is not too big, though. All these results remain qualitatively unchanged when we use equally-weighted and rank-weighted portfolios (see Figures IA.8 to IA.11 in the Internet Appendix).

Finally, we revisit the link between alphas and residual coskewness at the portfolio level, as we did at the outset of the theory section in Figure 1. In Figure 8, we illustrate the relation between alphas and residual coskewness when controlling for skewness (using LSK and USK) at the portfolio level. The results show that after accounting for skewness, there is much less dispersion in the 80 portfolios' alphas and residual coskewness. Compared to the results without skew-adjustment in Figure 1, the R^2s of the cross-sectional regressions of alphas on coskewness decreases from 73% to 29% for CAPM residuals, from 73% to 0% for FF3-residuals, and from 48% to 0% for FF4-residuals.

All these results strongly support the view that the positive alphas of betting-against-beta and -volatility represent compensation for coskewness risk, rather than representing anomalous returns. In the next section, we summarize further robustness checks that support this conclusion. We show, for instance, that our results are not driven by any particular sub-sample period and that they also apply to broader cross-sections by directly using the returns of the BaB-factor of Frazzini and Pedersen (2014).

5 Additional Results and Further Robustness Checks

Throughout the paper, we have referred to several robustness results in the Internet Appendix, such as replicating the value-weighted portfolio analysis using equally- or rank-weighted portfolios. In this Section, we summarize additional results, for which we delegate Tables and Figures to the Internet Appendix.

5.1 Subsample Evidence: Cumulative Alphas

To show that our results are not driven by a particular subsample period, we plot the cumulative alphas and cumulative skew-adjusted alphas of the four LRA strategies in Figures IA.12 to IA.15. We compute the cumulative CAPM-, FF3-, and FF4-alphas by cumulating the intercept and the residuals of the corresponding regressions over time, i.e. the final values of the cumulative alpha series divided by the number of periods correspond to the unadjusted alpha values reported in Table 1 and the skew-adjusted alphas in Table 3, respectively.

5.2 Skewness and Frazzini and Pedersen (2014)'s BaB-factor

The construction of the skew factors restricts our analysis to the cross-section of firms for which options data are available, i.e. on average around 1,800 firms per month. Many studies of LRAs use broader cross-sections such as the CRSP universe.[16] To explore whether coskewness also plays a role for betting-against-beta in the broader CRSP sample, we apply our skew factors to the returns of the BaB-factor provided by Frazzini and Pedersen (2014). The overlapping sample period is from 01/1996 to 03/2012, and we use the most flexible skew-adjustment with LSK and USK, based on rank-weighted returns.[17]

While the cross-section underlying the BaB-factor is much broader than that used to construct the skewness factors, we still find that controlling for skewness leads to a substantial reduction in the BaB strategy's alpha and its negative coskewness. Figure IA.16 and Table IA.6 show that the skew-adjustment decreases the CAPM-alpha from 92 to 48 basis points, the FF3-alpha from 75 to 11 basis points, and the FF4-alpha from 61 to 12 basis points. The plots of cumulative alphas in Figure IA.17 suggest that our findings are not driven by particular subsample periods.

5.3 Market Skewness and LRA Alphas

Our theoretical framework in Section 2 also provides time-series predictions for the excess returns of LRAs. While we think that an in-depth study warrants a separate paper, we provide preliminary evidence for a prediction based on equation (15): other things equal, LRAs should generate higher alphas when market skew is low (more negative) compared to periods when market skew is high (less negative or positive). We split our sample's

[16] Typically anomalies are even more pronounced in cross-sections that also include the smallest firms (e.g. Novy-Marx, 2016).

[17] Frazzini and Pedersen (2014) also use rank-weighted portfolios. Furthermore, they rescale the long and short portfolios to have a beta of one at portfolio formation. We have verified that any such rescaling of our skew factors hardly affects the results below.

time-series of market skewness (simply measured as rolling covariances between market and squared market returns over the past 250 days) into low and high market skewness periods and find support for this prediction. Figure IA.18 shows that CAPM-, FF3-, and FF4-alphas of all four LRAs are substantially higher in times of low market skewness than in times of high market skewness; the only exception is the FF4-alpha of betting against idiosyncratic volatility based on the FF3 model.

5.4 Skewness and Corporate Credit Risk

While the main goal of this paper is to study the skewness implications for equity returns, we now also provide some evidence on the link between skewness and credit risk. In the conceptual framework of our simulation study, we follow Merton (1974) to model levered firms, which implies that credit risk and equity option-implied skewness are related (e.g. Geske, 1979; Hull et al., 2005).

First, we illustrate the link between skewness and credit risk in our simulated skew-aware world from Section 2.3; we compute the Merton-implied credit spreads as discussed in more detail in Appendix B.2. Panel A of Figure IA.19 shows that credit spreads increase as absolute lower and upper skewness increase, and we find a U-shaped relation to total skewness. The patterns are qualitatively very similar when we use empirical data for CDS spreads (Panel B) and credit ratings (Panel C).[18] We find that firms with high absolute values of lower and upper skewness are the firms with highest CDS spreads and worst ratings. The closer firms' lower and upper skewness are to zero, the lower their CDS spreads and the better their credit ratings. Accordingly, we find a U-shaped relation of CDS spreads and credit ratings to total skewness. Thus, our model does not only capture the relation between skewness and equity returns but also the link to credit spreads and ratings.

Taking together the credit and equity results, the findings of this paper may also shed light on another low risk anomaly, namely the distress puzzle. Previous research finds that firms with high distress risk underperform relative to firms with low distress risk. For instance, Campbell et al. (2008) show that distressed stocks have low returns, high loadings on risk factors, and negative alphas. Within our framework, these results appear consistent with the notion of skew-aware asset pricing. Our findings suggest that firms with high credit spreads

[18]The analyses involving CDS data and credit ratings are conducted on subsamples of our original data set due to data availability. For the analysis of CDS spreads, we use the dataset compiled by Friewald et al. (2014), which contains Markit CDS data for 573 firms from 01/2001 to 03/2010 with a total of 37,514 observations. For the analysis of credit ratings, we obtain the S&P long-term credit ratings vie Compustat whenever available for firms in our sample. This results in a subsample of 2,066 firms with a total of 179,816 observations over our full sample period.

(bad ratings) are firms with high residual coskewness and hence should earn negative alphas.

6 Conclusion

This paper provides a novel perspective on beta- and (idiosyncratic) volatility-based low risk anomalies established in previous research. We show that these apparently anomalous empirical patterns may not necessarily pose asset pricing puzzles when accounting for the skewness of the equity return distribution. In our theoretical framework of a skew-aware world in which investors demand compensation for negative coskewness, we show that the pricing errors of the Capital Asset Pricing Model (CAPM) exhibit residual coskewness. This residual coskewness is directly linked to a stock's CAPM beta and is compensated through excess returns relative to the CAPM, i.e. alpha. In the cross-section, a betting-against-beta strategy that buys low and sells high CAPM-beta stocks generates an alpha commensurate to the difference in their residual coskewness.

Our empirical results confirm the model predictions for betting-against-beta and -volatility. We document that these strategies' positive alphas relative to the CAPM and other factor models are associated with negative residual coskewness. Once we control for skewness, using portfolio factors based on equity option-implied skewness, the low risk anomalies disappear. None of the strategies delivers a positive alpha and their residual coskewness becomes substantially less negative.

Appendix

A Skew-Aware Asset Pricing

The pricing kernel is likely to depend not only on the market, but also on additional risk factors. To relate such a multi-factor pricing kernel to the traditional asset pricing literature, one can work with its expectation conditional on R_T, the market return from time t to $T > t$,

$$\mathcal{M}(R_T) := \mathbb{E}^{\mathbb{P}}\left[\mathcal{M} \mid R_T\right]. \tag{A.1}$$

In the absence of arbitrage, the stochastic discount factor prices all risky asset payoffs in the economy. The t-expected return on asset i from time t to T, $\mathbb{E}_t^{\mathbb{P}}\left[R_{i,T}\right]$, is given by the expected excess return on the market, scaled by asset i's covariation with the pricing kernel relative to the market's covariation with $\mathcal{M}(R_T)$,[19]

$$\mathbb{E}_t^{\mathbb{P}}\left[R_{i,T}\right] = \underbrace{\frac{Cov_t^{\mathbb{P}}\big(\mathcal{M}, R_{i,T}\big)}{Cov_t^{\mathbb{P}}\big(\mathcal{M}(R_T), R_T\big)}}_{\text{'true beta'}} \mathbb{E}_t^{\mathbb{P}}\left[R_T\right], \tag{A.2}$$

where we refer to the ratio of pricing kernel covariances as the 'true beta'. To bring this no-arbitrage relation into the context of linear asset pricing models, we use in the following the first-order approximation $\mathcal{M}_1(R_T) := a_1 + b_1 R_T$, and the second-order approximation $\mathcal{M}_2(R_T) := a_2 + b_2 R_T + c_2 R_T^2$, for both $\mathcal{M}$ and $\mathcal{M}(R_T)$ in equation (A.2), respectively.[20] This yields

$$\begin{aligned} \mathbb{E}_t^{\mathbb{P}}\left[R_{i,T}\right] &\approx \frac{Cov_t^{\mathbb{P}}(a_1 + b_1 R_T, R_{i,T})}{Cov_t^{\mathbb{P}}(a_1 + b_1 R_T, R_T)} \mathbb{E}_0^{\mathbb{P}}\left[R_T\right] \\ &= \underbrace{\frac{Cov_t^{\mathbb{P}}(R_T, R_{i,T})}{V_t^{\mathbb{P}}\left[R_T\right]}}_{\text{CAPM beta } (\beta_i^{CAPM})} \mathbb{E}_t^{\mathbb{P}}\left[R_T\right], \end{aligned} \tag{A.3}$$

[19] Under mild technical conditions it can be shown that $\mathcal{M}(R_T) = \sum_{i=0}^{\infty} f_i R_T^i$, where the coefficients f depend on conditional $\mathbb{P}$ and $\mathbb{Q}$ moments.

[20] Here we are exposed to two sources of errors. The first is the error from using the finite-order projection $\mathcal{M}_j(R_T)$ rather than $\mathcal{M}_\infty(R_T) = \mathcal{M}(R_T)$. The second error arises from using the projection instead of the true kernel in the covariance in the numerator of (A.2).

and

$$
\begin{aligned}
\mathbb{E}_t^{\mathbb{P}}\left[R_{i,T}\right] &= \frac{Cov_t^{\mathbb{P}}(a_2 + b_2 R_T + c_2 R_T^2, R_{i,T})}{Cov_t^{\mathbb{P}}(a_2 + b_2 R_T + c_2 R_T^2, R_T)} \mathbb{E}_t^{\mathbb{P}}\left[R_T\right] \\
&= \underbrace{\frac{b_2 Cov_t^{\mathbb{P}}(R_T, R_{i,T}) + c_2 Cov_t^{\mathbb{P}}(R_T^2, R_{i,T})}{b_2 V_t^{\mathbb{P}}\left[R_T\right] + c_2 Cov_t^{\mathbb{P}}(R_T, R_T^2)}}_{\text{`skew-adjusted beta' } (\beta_i^{skew})} \mathbb{E}_0^{\mathbb{P}}\left[R_T\right].
\end{aligned} \tag{A.4}
$$

B Simulation study

This Section presents details on the simulation study for which we report results in Section 2.3 of the paper. We describe (i) how we model the market, (ii) how we model firms, and (iii) the general setup of the simulation study.

B.1 Market Model

To gauge how higher moments such as skewness matter for asset pricing, consider a representative power-utility investor who is exposed to stochastic volatility. We model the dynamics of the forward market price $M_{t,T}$, contracted at time t for delivery at T, as [21]

$$
\begin{aligned}
\frac{dM_{t,T}}{M_{t,T}} &= \eta_t\, dt + \kappa_t \,(\xi dW_t^{1\mathbb{P}} + \sqrt{1-\xi^2} dW_t^{2\mathbb{P}}), \\
d\kappa_t^2 &= (\nu_0 + \nu_1 \kappa_t^2) dt + \kappa_t \vartheta dW_t^{1\mathbb{P}}.
\end{aligned} \tag{B.5}
$$

With γ denoting the coefficient of constant relative risk aversion, $\eta_t = \gamma \kappa_t^2$ is the instantaneous market return in excess of the risk-free rate and κ_t is the associated market volatility. Campbell et al. (2017) develop an empirically successful asset pricing model with stochastic volatility in a similar way.[22] We define the discrete market excess return $R_T := \frac{M_{T,T}}{M_{t,T}} - 1$, where we suppress time subscripts here and subsequently for notational convenience, and set $M_{t,T} = 1$. Given the agent's local risk aversion γ, we obtain the forward pricing kernel as

$$
\mathcal{M} := \frac{(R_T + 1)^{-\gamma}}{e^{1/2(\gamma - \gamma^2)\int_0^T \kappa_s^2 ds}}. \tag{B.6}
$$

[21] We choose to specify the dynamics of the forward price (rather than the spot price) because this naturally accounts for interest rates and ensures that the forward price is a martingale under the forward measure ($\mathbb{Q}_T$) with the T-period zero coupon bond as numeraire.

[22] The less realistic but more parsimonious case of modeling the market by a geometric Brownian motion only leads to qualitatively the same asset pricing implications as the stochastic volatility dynamics in equation (B.5). In other words, higher moments of the return distribution matter for asset prices even if the market does not exhibit skewness; this point is also stressed by Kraus and Litzenberger (1976).

This kernel is one example with stochastic volatility generating the *true beta* in equation (A.2), such that the kernel is not measurable with respect to the market alone. Stochastic volatility greatly impacts the signs and magnitudes of the coefficients in the projections $\mathcal{M}_1(R_T)$ and $\mathcal{M}_2(R_T)$ in Section 2.1.

In the next Section we introduce a cross-section of firms into the economy that exhibits skewness in returns through both stochastic volatility and default risk.

B.2 Model for Levered Firms

Previous research shows that skewness of stock returns may originate from different sources such as credit risk (Merton, 1974), sentiment (e.g., Han, 2008), demand pressure in options markets (Gârleanu et al., 2009), or differences in beliefs (Buraschi et al., 2014); the latter also discuss the interaction of disagreement and credit risk. In contrast to the aggregate market, the skewness of individual firms' stock returns is often positive; recent studies providing evidence on the properties of skewness across firms and in the aggregate market include Albuquerque (2012) and Engle and Mistry (2014).

To parsimoniously model both positive and negative skewness, we specify a firm's asset process A to incorporate jumps and stochastic volatility,

$$\begin{aligned} d\log A_t &= \left(\mu - \frac{\sigma_t^2}{2}\right) dt + \sigma_t \left(\rho dW_t^{\mathbb{P}} + \sqrt{1-\rho^2} dB_t^{\mathbb{P}}\right) + \eta dJ_t^{\mathbb{P}}, \\ d\sigma_t^2 &= (\nu_2 + \nu_3\sigma_t^2 + \nu_4\kappa_t^2)dt + \psi\sigma_t dB_t^{\mathbb{P}}, \end{aligned} \tag{B.7}$$

where J is a pure jump process with intensity ω and η is a constant, $W_t^{\mathbb{P}} = \xi W_t^{1\mathbb{P}} + \sqrt{1-\xi^2} W_t^{2\mathbb{P}}$ and κ_t^2 is the stochastic market variance from equation (B.5). We consider two different default specifications for this setup for a level of debt $D_0 \leq A_0$. The first with a Merton (1974)-style default at maturity T, if $A_T < D_0$. The second allows for default prior to maturity if $A_t < D_0$, for at least one $t \in [0, T]$. Equity (E) represents the forward price of a European call option on the firm's assets with strike equal to D_0. The corresponding forward price $F_{0,T} := \mathbb{E}_0^{\mathbb{P}}[\mathcal{M}(A_T - D_0)^+]$, so that the forward gross return on equity is

$$\frac{(A_T - D_0)^+}{\mathbb{E}_0^{\mathbb{P}}[\mathcal{M}(A_T - D_0)^+]}.$$

The model-implied credit spreads for Merton-style default, i.e. when $A_T < D_0$, are given by

$$cs_T := \mathbb{E}_0^{\mathbb{P}}\left[\mathcal{M} \underbrace{\frac{D_0 - A_T}{D_0}}_{\text{loss rate conditional on default}} \underbrace{\mathbb{1}(D_0 \geq A_T)}_{\text{default probability}}\right], \tag{B.8}$$

and when we allow the firm to default before maturity, i.e. as soon as $A_t < D_0$, by

$$cs_T^{\text{early default}} := \mathbb{E}_0^{\mathbb{P}}\left[\mathcal{M} \underbrace{\frac{D_0 - A_T}{D_0}}_{\text{loss rate conditional on default}} \underbrace{\mathbb{1}(D_0 \geq A_t; 0 \leq t \leq T)}_{\text{default probability}}\right]. \tag{B.9}$$

With the asset value dynamics accounting for systematic and idiosyncratic shocks, we explore the impact of higher moments on expected equity returns within the market framework discussed above in Section B.1. In the paper, we report results for the baseline specification without jumps (i.e. we set the jump intensity $\omega = 0$) and for Merton-style default at maturity. In the Internet Appendix, we also present results for specifications that include jumps and early defaults.

B.3 Setup for the Simulation Study

Our simulation study is designed to generate data that matches the properties of our empirical data along several dimensions. In what follows, we sketch the most important steps of this procedure.

To simulate an economy according to the joint model for the market and asset prices from Sections B.1 and B.2, we first generate sets of parameters with plausible values. To model the dynamics of the market, we fix the coefficient of relative risk aversion γ at 2, the instantaneous correlation between forward market returns and stochastic variance ξ is set to the value of -0.85, the unconditional mean of index variance to $-\nu_0/\nu_1 = 0.048$ and the mean reversion of market variance $\nu_1 = -1$.[23] From these parameters we discretize the stochastic differential equation (B.5) and simulate a market time series of 320 months from daily increments.

In a second step, we generate 2,000 firms for which we draw the parameters from distributions reflecting the observed cross section. We draw $\rho \sim \mathcal{U}(0,1)$, a uniform distribution on the unit interval, leverage $D \sim \mathcal{B}(2,5)$, a Beta distribution, the asset drift $\mu \sim \Gamma(2, 0.01)$, from a Gamma distribution, same as the volatility of asset variance ψ, which is taken as

[23] These are parameter values similar to those in Aït-Sahalia and Kimmel (2007).

the square-root of a $\Gamma(2, 0.01)$ random variable. The parameters ν_2, ν_3, ν_4 are drawn from Gamma distributions $\Gamma(2, 0.01)$, $-\Gamma(2, 0.5)$, and $\Gamma(2, 0.25)$, respectively, so that the unconditional mean $\mathbb{E}^{\mathbb{P}}[\sigma_t^2]$ exists. To better reflect the cross section of US corporations we set additionally 25% of the population's leverage to zero (see, e.g., Strebulaev and Yang, 2013). When simulating the asset value processes, we keep the trajectory of the forward market fixed to ensure it is identical for all assets. Given these sample paths for firm assets, we then compute the sample paths of corporate equity values, expected equity returns, implied and expected realized skewness, CAPM betas, etc.

C Variables Constructed from Stock Returns

In our empirical analysis we explore whether accounting for skewness improves our understanding of low risk anomalies. This Section summarizes how we estimate CAPM betas and idiosyncratic volatility from past equity returns.

CAPM betas. We estimate ex-ante CAPM betas exactly as described in Frazzini and Pedersen (2014). For security i, the beta estimate is given by

$$\beta_i^{\hat{TS}} = \hat{\rho}_i \frac{\hat{\sigma_i}}{\hat{\sigma_m}} \tag{C.1}$$

where $\hat{\sigma_m}$ and $\hat{\sigma_i}$ denote the volatilities for stock i and the market excess returns, and ρ_i denotes their correlation with the market. We estimate volatilities as one-year rolling standard deviations of one-day log returns and correlations using a five-year rolling window of overlapping three-day log returns. As a minimum, we require 120 and 750 trading days of non-missing data, respectively. To reduce the influence of outliers, Frazzini and Pedersen (2014) follow previous research and shrink the time-series estimate $\beta_i^{\hat{TS}}$ to the cross-sectional beta mean ($\hat{\beta}^{XS}$),

$$\hat{\beta}_i = w \times \hat{\beta_i^{ts}} + (1 - w) \times \hat{\beta}^{XS}, \tag{C.2}$$

where they set $w = 0.6$ and $\hat{\beta}^{XS} = 1$. Following this procedure, we generate end-of-month estimates of CAPM betas for the period January 1996 to July 2014.

Idiosyncratic volatility. For our empirical analysis, we estimate two series of idiosyncratic volatility. First, we estimate idiosyncratic volatility following Ang et al. (2006b) as the square root of the residual variance from regressing daily equity excess returns of firm i on the daily returns of the three Fama French factors (market, size, and value) over the previous month. As a second estimate, we use the square root of the residual variance resulting from the CAPM beta estimation following Frazzini and Pedersen (2014) as described above.

References

Aït-Sahalia, Y. and Kimmel, R. (2007). Maximum likelihood estimation of stochastic volatility models. *Journal of Financial Economics*, 83:413–452.

Albuquerque, R. (2012). Skewness in stock returns: reconciling the evidence on firm versus aggregate returns. *Review of Financial Studies*, 25(5):1630–1673.

Amaya, D., Christoffersen, P., Jacobs, K., and Vasquez, A. (2015). Does realized skewness predict the cross-section of equity returns? *Journal of Financial Economics*, 118(1):135 – 167.

Andersen, T. G., Fusari, N., and Todorov, V. (2015). The risk premia embedded in index options. *Journal of Financial Economics*, 117(3):558 – 584.

Andrews, D. (1991). Heteroskedasticity and autocorrelation consistent covariance matrix estimation. *Econometrica*, 59:817–858.

Ang, A., Chen, J., and Xing, Y. (2006a). Downside risk. *The Review of Financial Studies*, 19:1191–1239.

Ang, A., Hodrick, R. J., Xing, Y., and Zhang, X. (2006b). The cross-section of volatility and expected returns. *The Journal of Finance*, 61:259–299.

Ang, A., Hodrick, R. J., Xing, Y., and Zhang, X. (2009). High idiosyncratic volatility and low returns: International and further u.s. evidence. *Journal of Financial Economics*, 91(1):1 – 23.

Baker, M., Bradley, B., and Taliaferro, R. (2014). The low-risk anomaly: A decomposition into micro and macro effects. *Financial Analysts Journal*, 70(2):43–58.

Baker, M., Bradley, B., and Wurgler, J. (2011). Benchmarks as limits to arbitrage: Understanding the low-volatility anomaly. *Financial Analysts Journal*, 67(1):40–54.

Bakshi, G., Kapadia, N., and Madan, D. (2003). Stock return characteristics, skew laws, and the differential pricing of individual equity options. *Review of Financial Studies*, 16:101–143.

Bakshi, G. and Madan, D. (2000). Spanning and derivative-security valuation. *Journal of Financial Economics*, 55(2):205–238.

Bali, T. G., Brown, S., Murray, S., and Tang, Y. (2017). A lottery demand-based explanation of the beta anomaly. *Journal of Financial & Quantitative Analysis*. forthcoming.

Bali, T. G., Cakici, N., and Whitelaw, R. F. (2011). Maxing out: Stocks as lotteries and the cross-section of expected returns. *Journal of Financial Economics*, 99(2):427–446.

Bali, T. G. and Hovakimian, A. (2009). Volatility spreads and expected stock returns. *Management Science*, 55(11):1797–1812.

Bali, T. G., Hu, J., and Murray, S. (2015). Option implied volatility, skewness, and kurtosis and the cross-section of expected stock returns. *Working paper.*

Bali, T. G. and Murray, S. (2013). Does risk-neutral skewness predict the cross-section of equity option portfolio returns? *Journal of Financial and Quantitative Analysis*, 48(04):1145–1171.

Barberis, N. and Huang, M. (2008). Stocks as lotteries: The implications of probability weighting for security prices. *American Economic Review*, 98(5):2066–2100.

Black, F. (1972). Capital market equilibrium with restricted borrowing. *The Journal of Business*, 45(3):pp. 444–455.

Black, F., Jensen, M. C., and Scholes, M. (1972). The capital asset pricing model: Some empirical tests. In Jensen, M., editor, *Studies in the Theory of Capital Markets*, pages 79–121. Praeger.

Boehme, R. D., Danielsen, B. R., Kumar, P., and Sorescu, S. M. (2009). Idiosyncratic risk and the cross-section of stock returns: Merton (1987) meets miller (1977). *Journal of Financial Markets*, 12:438 – 468.

Boloorforoosh, A., Christoffersen, P., Fournier, M., and Gourieroux, C. (2017). Beta risk in the cross-section of equities. Working Paper, Concordia University, University of Toronto, and HEC Montreal.

Boyer, B., Mitton, T., and Vorkink, K. (2010). Expected idiosyncratic skewness. *Review of Financial Studies*, 23(1):169–202.

Breeden, D. T. and Litzenberger, R. H. (1978). Prices of state-contigent claims implicit in option prices. *Journal of Business*, 51(4):621–651.

Brennan, M. J. (1971). Capital market equilibrium with divergent borrowing and lending rates. *Journal of Financial and Quantitative Analysis*, 6(05):1197–1205.

Brunnermeier, M. K., Gollier, C., and Parker, J. A. (2007). Optimal beliefs, asset prices, and the preference for skewed returns. *American Economic Review*, 97(2):159–165.

Buraschi, A., Trojani, F., and Vedolin, A. (2014). Economic uncertainty, disagreement, and credit markets. *Management Science*, 60(5):1281–1296.

Buss, A. and Vilkov, G. (2012). Measuring equity risk with option-implied correlations. *Review of Financial Studies*, 25(10):3113–3140.

Campbell, J. Y. (1993). Intertemporal asset pricing without consumption data. *The American Economic Review*, 83:487–512.

Campbell, J. Y., Giglio, S., Polk, C., and Turley, R. (2017). An intertemporal capm with stochastic volatility. *Journal of Financial Economics*. forthcoming.

Campbell, J. Y., Hilscher, J., and Szilagyi, J. (2008). In search of distress risk. *The Journal of Finance*, 63 (6):2899–2939.

Carhart, M. (1997). On persistence in mutual fund performance. *Journal of Finance*, 52:57–82.

Carr, P. and Lee, R. (2009). Put-call symmetry: Extensions and applications. *Mathematical Finance*, 19(4):523–560.

Carr, P. and Wu, L. (2009). Stock options and credit default swaps: A joint framework for valuation and estimation. *Journal of Financial Econometrics*, pages 1–41.

Cederburg, S. and O'Doherty, M. S. (2016). Does it pay to bet against beta? on the conditional performance of the beta anomaly. *The Journal of finance*, 71(2):737–774.

Chabi-Yo, F., Leisen, D. P., and Renault, E. (2014). Aggregation of preferences for skewed asset returns. *Journal of Economic Theory*, 154:453 – 489.

Chang, B. Y., Christoffersen, P., and Jacobs, K. (2013). Market skewness risk and the cross section of stock returns. *Journal of Financial Economics*, 107(1):46–68.

Christoffersen, P., Fournier, M., Jacobs, K., and Karoui, M. (2016). Option-based estimation of the price of co-skewness and co-kurtosis risk. *Working Paper*.

Conrad, J., Dittmar, R., and Ghysels, E. (2013). Ex ante skewness and expected stock returns. *Journal of Finance*, 68:85–124.

Engle, R. and Mistry, A. (2014). Priced risk and asymmetric volatility in the cross section of skewness. *Journal of Econometrics*, 182(1):135–144.

Fama, E. F. and French, K. R. (1993). Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics*, 33:3–56.

Fama, E. F. and French, K. R. (2015). A five-factor asset pricing model. *Journal of Financial Economics*, 116(1):1 – 22.

Frazzini, A. and Pedersen, L. H. (2014). Betting against beta. *Journal of Financial Economics*, 111(1):1–25.

Friend, I. and Westerfield, R. (1980). Co-skewness and capital asset pricing. *The Journal of Finance*, 35(4):897–913.

Friewald, N., Wagner, C., and Zechner, J. (2014). The cross-section of credit risk premia and equity returns. *Journal of Finance*, 69:2419–2469.

Fu, F. (2009). Idiosyncratic risk and the cross-section of expected stock returns. *Journal of Financial Economics*, 91:24 – 37.

Gârleanu, N., Pedersen, L. H., and Poteshman, A. M. (2009). Demand-Based Option Pricing. *Review of Financial Studies*, 22:4259–4299.

Geske, R. (1979). The valuation of compound options. *Journal of Financial Economics*, 7(1):63 – 81.

Gourier, E. (2016). Pricing of idiosyncratic equity and variance risks. Working Paper, Queen Mary University London.

Han, B. (2008). Investor sentiment and option prices. *Review of Financial Studies*, 21:387–414.

Harvey, C. and Siddique, A. (2000). Conditional skewness in asset pricing tests. *Journal of Finance*, 55:1263–1295.

Haugen, R. and Heins, A. (1975). Risk and the rate of return on financial assets: Some old wine in new bottles. *Journal of Financial and Quantitative Analysis*, 10:775–784.

Hong, H. and Sraer, D. A. (2016). Speculative betas. *The Journal of Finance*, 71(5):2095–2144.

Huang, S., Lou, D., and Polk, C. (2016). The booms and busts of beta arbitrage. Technical report, Working paper.

Hull, J. C., Nelken, I., and White, A. D. (2005). Merton's model, credit risk and volatility skews. *Journal of Credit Risk*, 1(1):3–26.

Johnson, T. C. (2004). Forecast dispersion and the cross section of expected returns. *The Journal of Finance*, 59(5):1957–1978.

Jylhä, P. (2017). Margin constraints and the security market line. *Journal of Finance*. forthcoming.

Kozhan, R., Neuberger, A., and Schneider, P. (2013). The skew risk premium in the equity index market. *Review of Financial Studies*, 26(9):2174–2203.

Kraus, A. and Litzenberger, R. H. (1976). Skewness preference and the valuation of risk assets. *The Journal of Finance*, 31:1085–1100.

Lettau, M., Maggiori, M., and Weber, M. (2014). Conditional risk premia in currency markets and other asset classes. *Journal of Financial Economics*, 114(2):197 – 225.

Lintner, J. (1965). The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets. *The review of economics and statistics*, pages 13–37.

Martin, I. (2013). Simple variance swaps. Working paper, Stanford University.

Merton, R. (1974). On the pricing of corporate debt: The risk structure of interest rates. *Journal of Finance*, 29:449–470.

Mitton, T. and Vorkink, K. (2007). Equilibrium underdiversification and the preference for skewness. *Review of Financial Studies*, 20(4):1255–1288.

Mossin, J. (1966). Equilibrium in a capital asset market. *Econometrica*, pages 768–783.

Neuberger, A. (1994). The log contract. *Journal of Portfolio Management*, 20(2):74–80.

Neuberger, A. (2013). Realized skewness. *Review of Financial Studies*, 25(11):3423–3455.

Newey, W. and West, K. (1987). A simple positive semi-definite, heteroscedasticity and autocorrelation consistent covariance matrix. *Econometrica*, 55:703–708.

Novy-Marx, R. (2016). Understanding defensive equity. Technical report, Working Paper.

Pástor, L. and Stambaugh, R. F. (2003). Liquidity risk and expected stock returns. *Journal of Political economy*, 111(3):642–685.

Rehman, Z. and Vilkov, G. (2012). Risk-neutral skewness: Return predictability and its sources. Working paper, Goethe University Frankfurt.

Rubinstein, M. E. (1973). The fundamental theorem of parameter-preference security valuation. *The Journal of Financial and Quantitative Analysis*, 8(1):pp. 61–69.

Schneider, P. (2015). Generalized risk premia. *Journal of Financial Economics*, 116(3):487 – 504.

Schneider, P. and Trojani, F. (2014). Fear trading. Working paper, University of Lugano and SFI.

Schneider, P. and Trojani, F. (2017). (almost) model-free recovery. *Journal of Finance*. forthcoming.

Sears, R. S. and Wei, K. C. J. (1985). Asset pricing, higher moments, and the market risk premium: A note. *The Journal of Finance*, 40(4):pp. 1251–1253.

Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *Journal of Finance*, 19(3):425–442.

Stambaugh, R. F., Yu, J., and Yuan, Y. (2015). Arbitrage asymmetry and the idiosyncratic volatility puzzle. *The Journal of Finance*, 70(5):1903–1948.

Strebulaev, I. A. and Yang, B. (2013). The Mystery of Zero-Leverage Firms. *Journal of Financial Economics*, 109(1):1–23.

Todorov, V. (2010). Variance risk-premium dynamics: The role of jumps. *Review of Financial Studies*, 23:345–383.

Walkshäusl, C. (2014). International low-risk investing. *The Journal of Portfolio Management*, 41(1):45–56.

Wang, C. (2014). Institutional holding, low beta and idiosyncratic volatility anomalies. *Working paper*.

Xing, Y., Zhang, X., and Zhao, R. (2010). What does the individual option volatility smirk tell us about future equity returns? *Journal of Financial and Quantitative Analysis*, 45:641–662.

Table 1: Low Risk Anomalies: Alphas and Residual Coskewness

This Table reports excess returns, factor model alphas, and residual coskewness of low risk anomalies (LRAs) using value- and equally-weighted decile portfolios (Panels A and B, respectively). At the end of every month, we sort firms into decile portfolios based one their CAPM beta, idiosyncratic volatility (measured from the residual variance of CAPM and Fama French three factor model regressions), or their equity option-implied ex-ante variance. From these portfolios, we compute Low-minus-High returns generated by betting-against-beta/volatility strategies, and report raw excess returns as well as alphas of CAPM-, Fama-French three- (FF3), and four-factor- (FF4) regressions. We also report the corresponding coskewness of residual returns, i.e. the covariance of residual returns with squared market excess returns. Values in square brackets are t-statistics based on standard errors following Newey and West (1987) where we choose the optimal truncation lag as suggested by Andrews (1991). The data covers 4,967 US firms, is sampled at a monthly frequency over the period January 1996 to August 2014, and contains a total of 400,449 observations.

Panel A. Low-minus-High-returns of value-weighted portfolios

	CAPM Beta	Ivol (CAPM)	IVol (FF3)	Ex-ante Var
Excess return	20.43	30.18	33.44	31.12
	[0.27]	[0.40]	[0.54]	[0.37]
Coskewness	0.49	0.34	0.28	0.44
CAPM alpha	125.06	123.79	109.02	133.17
	[2.87]	[2.52]	[2.43]	[2.49]
Coskewness	-0.39	-0.45	-0.35	-0.41
FF3 alpha	109.07	121.31	112.05	132.34
	[2.82]	[2.91]	[2.53]	[3.00]
Coskewness	-0.38	-0.47	-0.37	-0.44
FF4 alpha	72.98	60.61	52.04	68.11
	[1.58]	[1.50]	[1.78]	[1.69]
Coskewness	-0.28	-0.31	-0.21	-0.26

Panel B. Low-minus-High-returns of equally-weighted portfolios

	CAPM Beta	Ivol (CAPM)	IVol (FF3)	Ex-ante Var
Excess return	-7.58	32.21	28.65	5.83
	[-0.10]	[0.46]	[0.49]	[0.08]
Coskewness	0.30	0.47	0.32	0.38
CAPM alpha	91.80	121.56	102.27	96.97
	[1.87]	[2.47]	[2.27]	[1.79]
Coskewness	-0.54	-0.29	-0.30	-0.39
FF3 alpha	79.47	116.71	98.29	91.24
	[2.23]	[4.31]	[3.72]	[2.81]
Coskewness	-0.54	-0.31	-0.32	-0.41
FF4 alpha	39.04	78.55	56.77	38.91
	[0.87]	[2.33]	[2.26]	[1.09]
Coskewness	-0.43	-0.21	-0.20	-0.27

Table 2: Skewness as a Common Driver of Low Risk Anomalies

This Table presents evidence that skewness is a common driver of beta- and volatility-based low risk anomalies (LRAs). We compute LRA returns as Low-minus-High returns of value-weighted decile portfolios that we sort by CAPM beta, idiosyncratic volatility (measured from the residual variance of CAPM and Fama French three factor model regressions), or their equity option-implied ex-ante variance at the end of every month. We compute the LRAs' excess returns as well as alphas of CAPM-, Fama-French three- (FF3), and four-factor- (FF4) regressions and Panel A presents results for principal component analyses of the corresponding residual returns. The left part of Panel A reports the return variation explained by each of the four principal components (PCs). The right part reports the coefficients as well as the associated R^2s of regressing LRA returns on PC1. Panel B shows that the LRAs' PC1 is related to skewness by reporting results from regressing PC1 of excess returns as well as CAPM, FF3, and FF4 residual returns on skew factor returns. We compute skew factor returns as the High-minus-Low returns of value-weighted decile portfolios sorted by ex-ante skewness and consider the following specifications: we use lower skewness to construct the LSK-factor, upper skewness for the USK-factor, and total skewness to construct the $(SK_{1+10} - SK_{5+6})$-factor. We report results for regressions using these skew factors and additionally for regressions where we use the difference between USK and LSK as skew factor and a regression in which we include both USK and LSK returns. Values in square brackets are t-statistics based on standard errors following Newey and West (1987) where we choose the optimal truncation lag as suggested by Andrews (1991). The data covers 4,967 US firms, is sampled at a monthly frequency over the period January 1996 to August 2014, and contains a total of 400,449 observations.

Panel A. Principal Components of Low Risk Anomalies

	Variation explained by PCs				Anomaly return loadings on PC1			
	PC1	PC2	PC3	PC4	CAPM Beta	Ivol (CAPM)	Ivol (FF3)	Ex-ante Var
Excess returns	91.05	4.67	2.24	2.04	0.47	0.54	0.43	0.56
					[36.70]	[40.67]	[22.88]	[46.74]
R^2 (%)					84.60	94.08	88.76	94.68
CAPM residual returns	85.41	6.94	3.96	3.69	0.37	0.57	0.45	0.58
					[20.05]	[31.62]	[17.96]	[36.68]
R^2 (%)					66.94	91.55	83.09	91.29
FF3 residual returns	81.34	8.80	5.08	4.77	0.39	0.55	0.49	0.56
					[16.69]	[27.76]	[19.92]	[34.17]
R^2 (%)					64.06	87.34	81.50	87.21
FF4 residual returns	72.43	12.64	7.71	7.21	0.46	0.55	0.44	0.54
					[14.01]	[23.62]	[17.26]	[25.00]
R^2 (%)					61.09	80.46	67.54	78.42

Panel B. Skew Factor Returns and the First Principal Component of Low Risk Anomalies

	Excess returns	CAPM residuals	FF3 residuals	FF4 residuals
Lower skewness (LSK)	1.74	1.65	1.61	1.48
	[44.09]	[37.57]	[23.54]	[16.59]
R^2 (%)	94.66	90.76	86.61	77.77
Upper skewness (USK)	-1.65	-1.52	-1.45	-1.32
	[-42.57]	[-38.85]	[-24.92]	[-18.26]
R^2 (%)	93.13	89.35	85.18	75.35
Skewness $(SK_{1+10} - SK_{5+6})$	-1.24	-1.06	-0.96	-0.89
	[-16.87]	[-15.58]	[-22.76]	[-12.43]
R^2 (%)	82.82	71.92	71.40	51.83
Upper minus lower skewness (USK − LSK)	-0.86	-0.81	-0.78	-0.73
	[-47.74]	[-44.30]	[-27.88]	[-21.34]
R^2 (%)	94.98	91.85	88.25	79.96
Lower skewness (LSK)	1.18	1.00	0.95	0.90
	[8.48]	[7.20]	[6.36]	[6.95]
Upperskewness (USK)	-0.55	-0.63	-0.64	-0.58
	[-4.19]	[-5.11]	[-5.19]	[-4.94]
R^2 (%)	95.13	91.95	88.34	80.14

Table 3: Low Risk Anomalies: Controlling for Skewness

This Table reports skew-adjusted factor model alphas and residual coskewness of low risk anomalies (LRAs) using value-weighted decile portfolios. At the end of every month, we sort firms into decile portfolios based one their CAPM beta, idiosyncratic volatility (measured from the residual variance of CAPM and Fama French three factor model regressions), or their equity option-implied ex-ante variance. From these portfolios, we compute Low-minus-High returns generated by betting-against-beta/volatility strategies, and report alphas of CAPM-, Fama-French three-, and four-factor regressions that additionally include controls for skewness. To adjust for skewness, we use either the ($SK_{1+10} - SK_{5+6}$)-factor (Panel A), the (USK−LSK)-factor (Panel B), or both the LSK- and the USK-factor (Panel C). We also report the corresponding coskewness of residual returns, i.e. the covariance of residual returns with squared market excess returns. Values in square brackets are t-statistics based on standard errors following Newey and West (1987) where we choose the optimal truncation lag as suggested by Andrews (1991). The data covers 4,967 US firms, is sampled at a monthly frequency over the period January 1996 to August 2014, and contains a total of 400,449 observations.

Panel A. Controlling for Skewness using the ($SK_{1+10} - SK_{5+6}$)-factor

	CAPM Beta	Ivol (CAPM)	IVol (FF3)	Ex-ante Var
CAPM alpha	66.45	30.80	32.70	34.01
	[1.87]	[0.85]	[1.08]	[1.23]
Coskewness	-0.15	-0.06	-0.03	0.00
FF3 alpha	43.70	29.17	29.91	31.37
	[1.34]	[0.82]	[0.92]	[1.15]
Coskewness	-0.12	-0.11	-0.05	-0.04
FF4 alpha	44.85	27.41	26.32	30.23
	[1.41]	[0.78]	[0.96]	[1.15]
Coskewness	-0.12	-0.11	-0.06	-0.04

Panel B. Controlling for Skewness using the (USK−LSK)-factor

	CAPM Beta	Ivol (CAPM)	IVol (FF3)	Ex-ante Var
CAPM alpha	56.47	11.60	19.18	7.53
	[1.70]	[0.39]	[0.66]	[0.80]
Coskewness	-0.18	-0.10	-0.07	-0.02
FF3 alpha	40.35	19.29	19.38	10.16
	[1.26]	[0.74]	[0.68]	[1.09]
Coskewness	-0.15	-0.14	-0.06	-0.03
FF4 alpha	40.16	16.06	14.42	9.80
	[1.29]	[0.59]	[0.56]	[1.04]
Coskewness	-0.15	-0.13	-0.06	-0.03

Panel C. Controlling for Skewness using the LSK- and the USK-factor

	CAPM Beta	Ivol (CAPM)	IVol (FF3)	Ex-ante Var
CAPM alpha	33.02	12.33	10.93	3.72
	[1.06]	[0.37]	[0.39]	[0.31]
Coskewness	-0.16	-0.10	-0.06	-0.02
FF3 alpha	21.27	24.21	8.80	7.43
	[0.64]	[0.73]	[0.31]	[0.61]
Coskewness	-0.14	-0.14	-0.06	-0.03
FF4 alpha	21.11	21.06	3.99	7.09
	[0.60]	[0.61]	[0.16]	[0.62]
Coskewness	-0.14	-0.14	-0.05	-0.03

Figure 1: Low Risk Anomalies: Alphas and Residual Coskewness

This Figure reports results for the equally-weighted and value-weighted decile portfolios used to compute the low risk anomaly (LRA) returns in Table 1. At the end of every month, we sort firms into decile portfolios based one their CAPM beta, idiosyncratic volatility (measured from the residual variance of CAPM and Fama French three factor model regressions), or their equity option-implied ex-ante variance. In total, we have 80 portfolios: 10 portfolios for each of the four LRAs, both, equally- and value-weighted. We plot CAPM-, Fama-French three-, and four-factor-alphas against their corresponding residual coskewness. Blue circles mark the low risk portfolios that a betting-against-beta/volatility strategy goes long. Red diamonds mark the high risk portfolios that a betting-against-beta/volatlity strategy goes short. Each figure header reports the R^2 of a cross-sectional regression of alphas on coskewness and the dashed lines represent the regression-fitted values. The data covers 4,967 US firms, is sampled at a monthly frequency over the period January 1996 to August 2014, and contains a total of 400,449 observations.

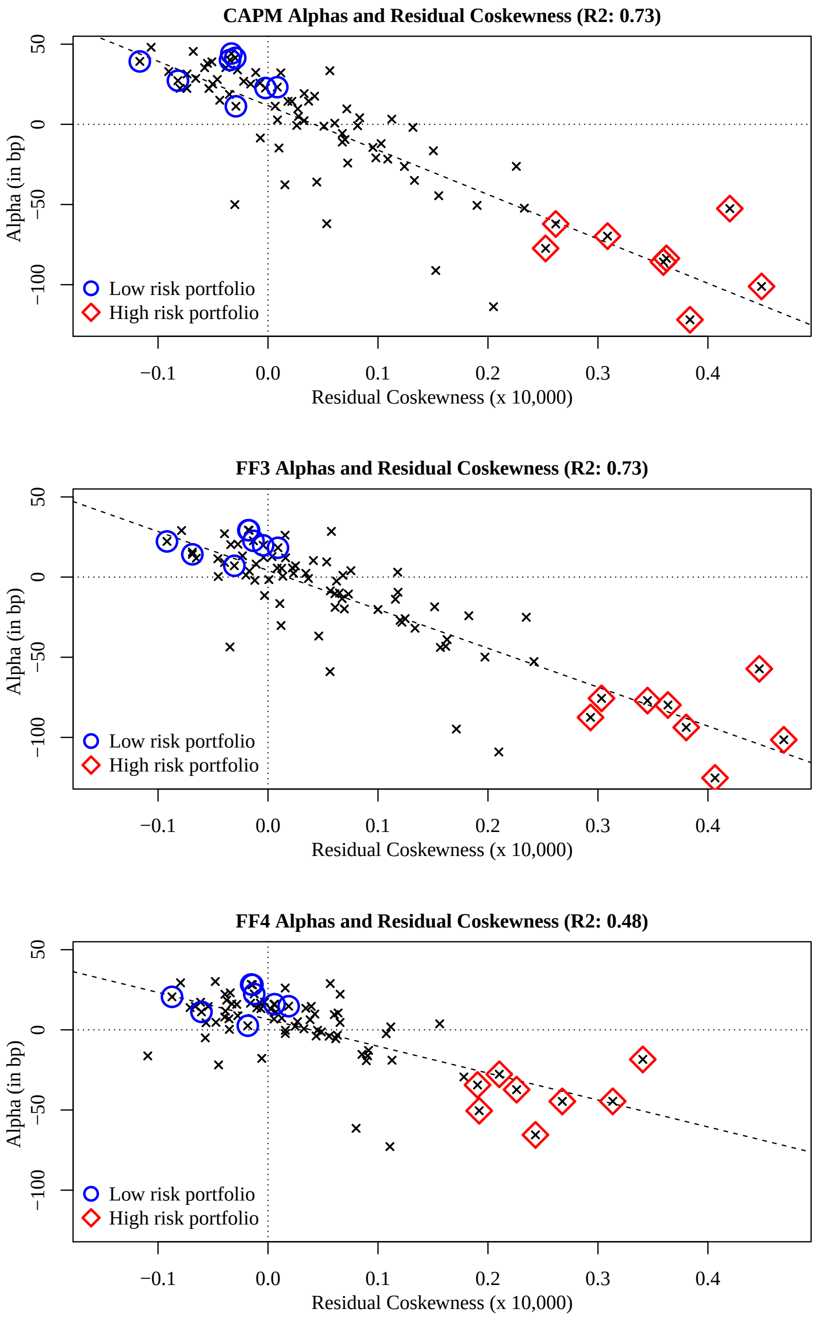
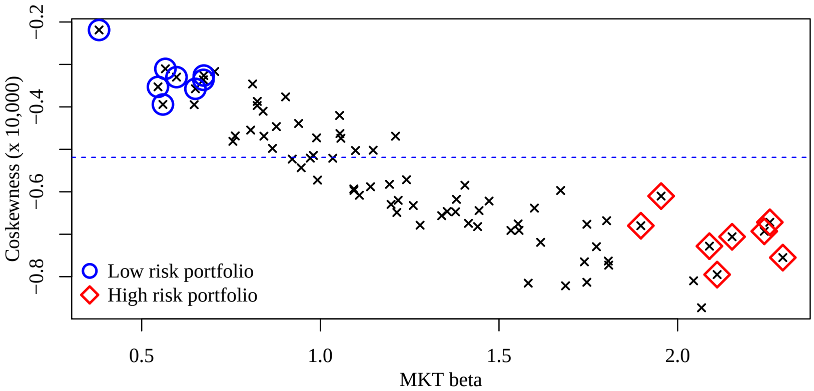


Figure 2: Low Risk Anomalies: Beta, Coskewness, and Residual Coskewness

This Figure reports results for the equally-weighted and value-weighted decile portfolios used to compute the low risk anomaly (LRA) returns in Table 1. At the end of every month, we sort firms into decile portfolios based one their CAPM beta, idiosyncratic volatility (measured from the residual variance of CAPM and Fama French three factor model regressions), or their equity option-implied ex-ante variance. In total, we have 80 portfolios: 10 portfolios for each of the four LRAs, both, equally- and value-weighted. We plot the portfolios' market beta (x-axis) against the portfolios' return coskewness (y-axis) in Panel A and against the portfolios' residual return coskewness computed from excess returns after controlling for the market factor in Panel B. Blue circles mark the low risk portfolios that a betting-against-beta/volatility strategy goes long. Red diamonds mark the high risk portfolios that a betting-against-beta/volatlity strategy goes short. The blue dashed line (in Panel A) plots the average market skewness (σ_{m,m^2}) over our sample period. The data covers 4,967 US firms, is sampled at a monthly frequency over the period January 1996 to August 2014, and contains a total of 400,449 observations.

Panel A: Beta and Coskewness



Panel B: Beta and Residual Coskewness

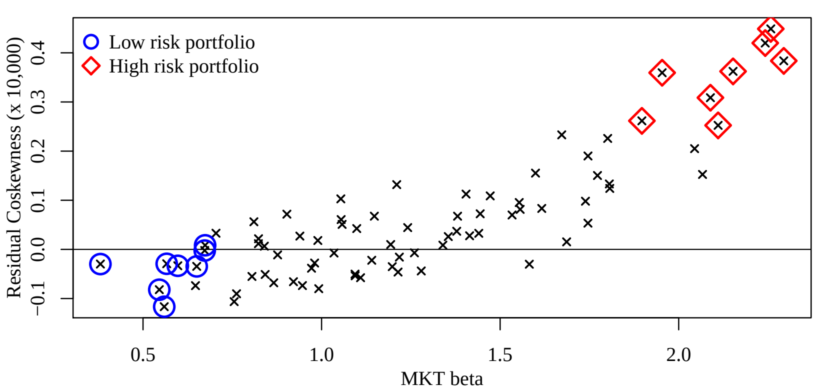
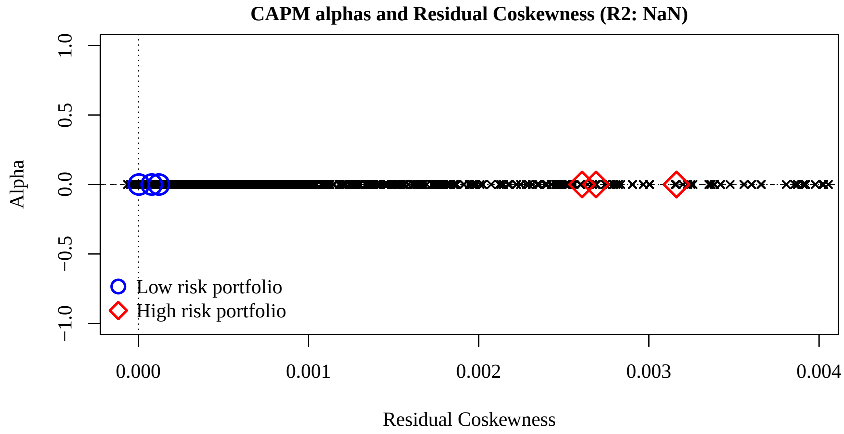


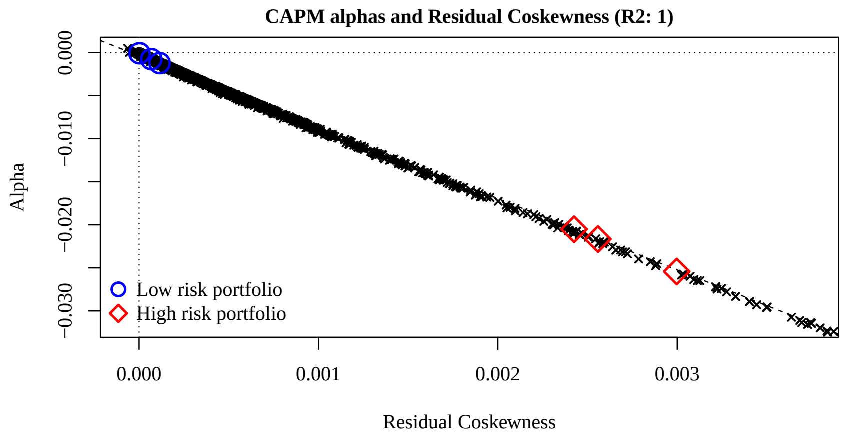
Figure 3: Alphas and Residual Coskewness: Simulation Evidence

This Figure presents results for the relation between CAPM alphas and residual coskewness in simulated worlds with 2,000 Merton-type firms. The three plots are based on different pricing kernel specifications: a CAPM world (Panel A), a skew-aware world (Panel B), and a world where also moments higher than skewness are accounted for (Panel C). The figure also reports results for LRA strategies based on equally-weighted decile portfolios, betting against CAPM beta, idiosyncratic volatility, and implied volatility. Blue circles mark the low risk portfolios that a betting-against-risk strategy goes long. Red diamonds mark the high risk portfolios that a betting-against-risk strategy goes long. Each figure header reports the R^2 of a cross-sectional regression of alphas on coskewness. In Panel C, the grey circles mark the alphas after additionally controlling for skewness.

Panel A. Simulation of a CAPM world



Panel B. Simulation of a skew-aware CAPM world



Panel C. Simulation of true pricing kernel world

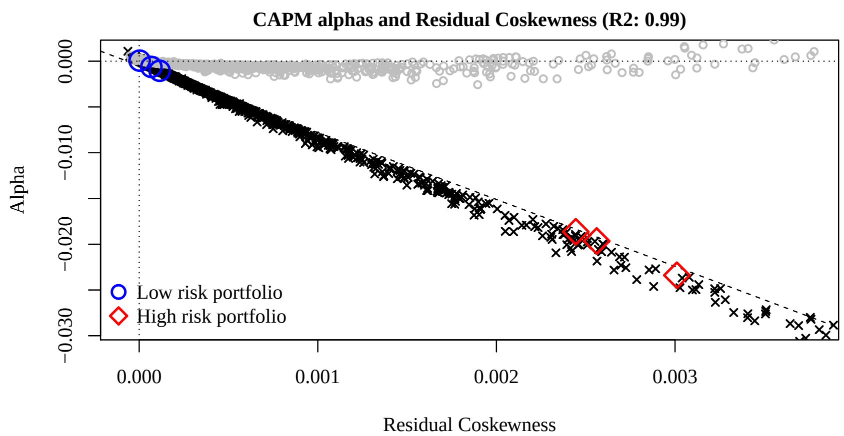
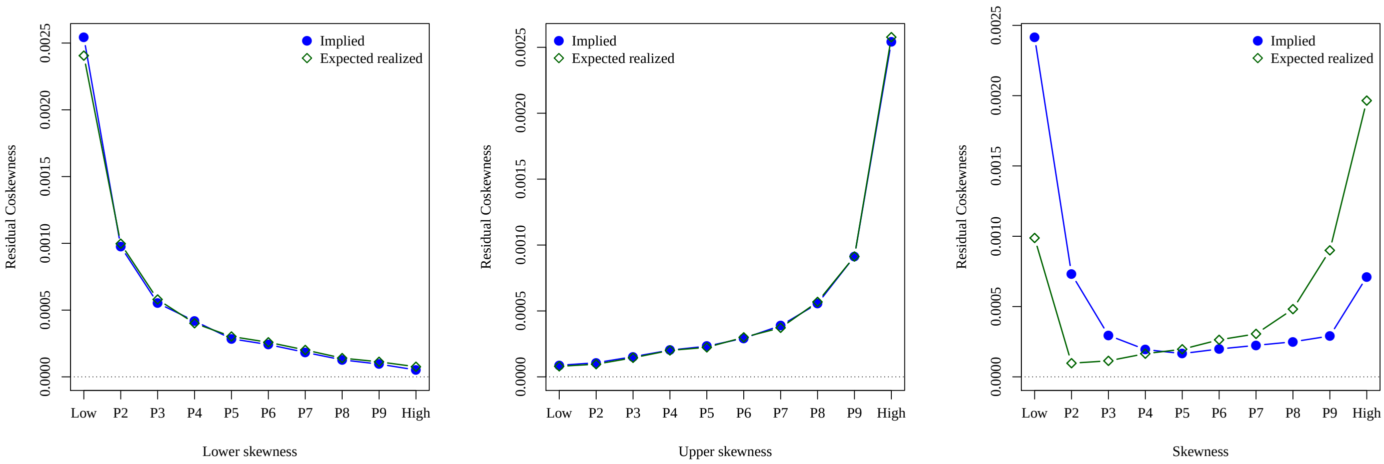


Figure 4: Implied Skewness, Residual Coskewness, and Alphas: Simulation Evidence

This Figure presents results for the relation between firms' skewness, CAPM residual coskewness, and CAPM alphas in a simulated skew-aware world with 2,000 Merton-type firms. For each firm, we compute measures of implied skewness (under the Q-measure) and expected realized skewness (under the P-measure). We compute lower skewness (by definition always negative), upper skewness (by definition always positive), and skewness which is the sum of lower and upper skewness. We sort firms into decile portfolios based on the three Q-, and on the three P-skew measures, and compute the portfolios' CAPM alphas and residual coskewness. Portfolios P1 and P10 contain the firms with the lowest and highest values of the sort variables, respectively. Panel A reports the portfolios' CAPM residual coskewness when using measures of Q-skew (blue line with bullets) and P-skew (green line with diamonds) using lower skewness (left), upper skewness (middle), and skewness (right). Panel B plots the portfolios' corresponding CAPM alphas.

Panel A. CAPM Residual Coskewness



Panel B. CAPM Alphas

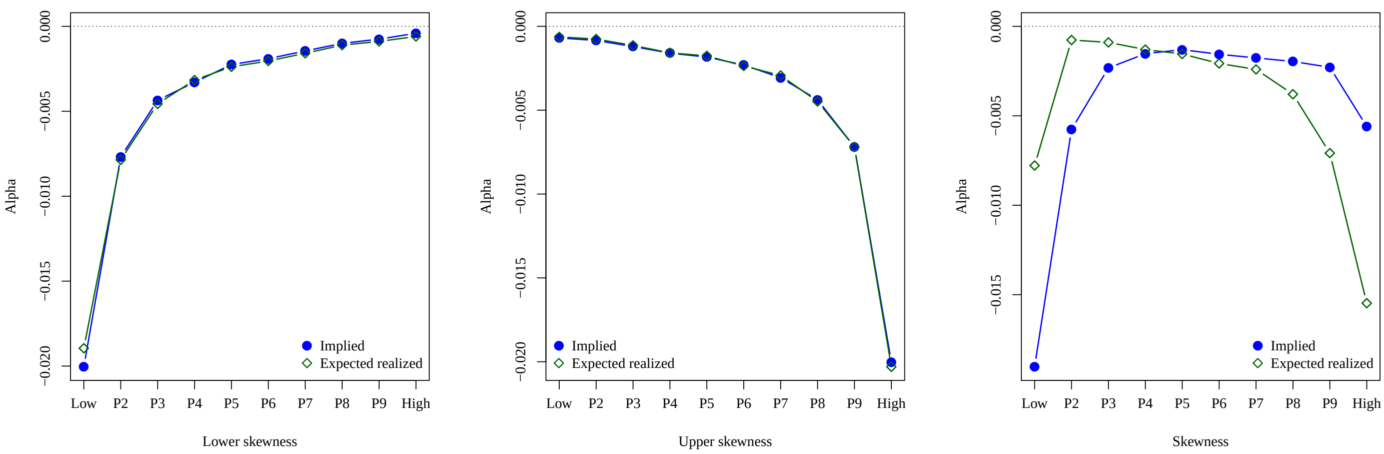
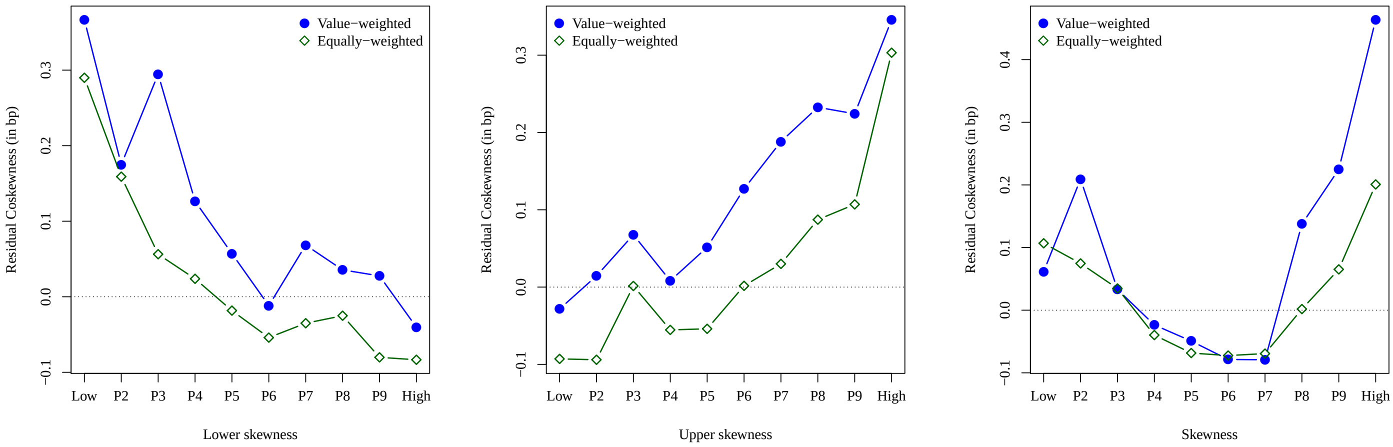


Figure 5: Option-Implied Ex-Ante Skewness, Residual Coskewness, and Alphas

This Figure presents results for the relation between firms' equity-option implied ex-ante skewness, CAPM residual coskewness, and CAPM alphas. For each firm, we compute option-implied lower skewness (by definition always negative), upper skewness (by definition always positive), and skewness which is the sum of upper and lower skewness. We sort firms into decile portfolios based on the three option-implied skew measures and compute the portfolios' CAPM alphas and residual coskewness. Portfolios P1 and P10 contain the firms with the lowest and highest values of the sort variables, respectively. Panel A reports the portfolios' CAPM residual coskewness for equally-weighted portfolios (blue line with bullets) and value-weighted portfolios (green line with diamonds) using lower skewness (left), upper skewness (middle), and skewness (right). Panel B plots the portfolios' corresponding CAPM alphas. The data covers 4,967 US firms, is sampled at a monthly frequency over the period January 1996 to August 2014, and contains a total of 400,449 observations.

Panel A. CAPM Residual Coskewness



Panel B. CAPM Alphas

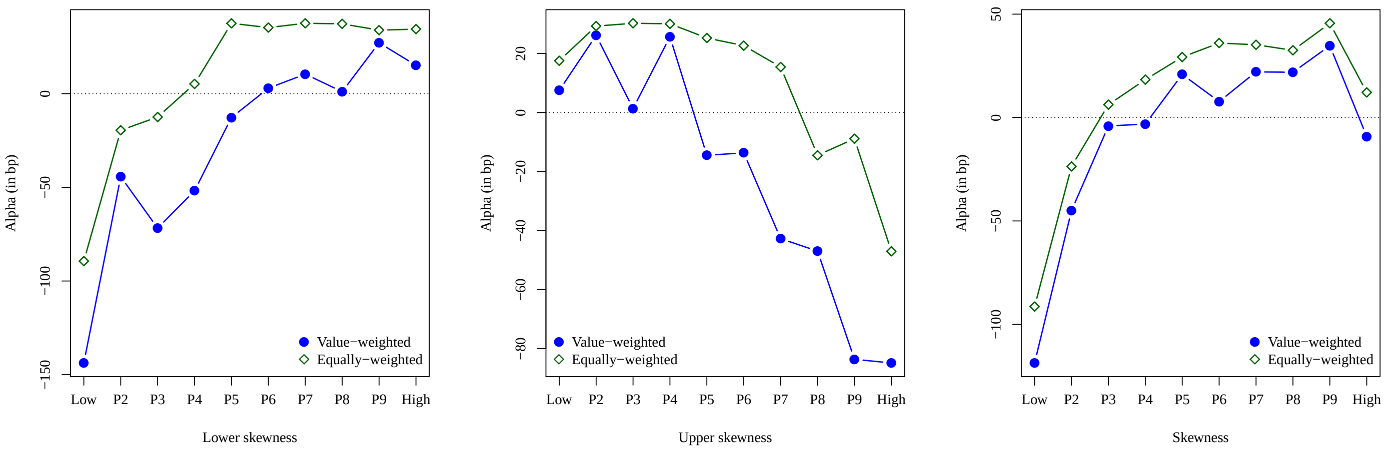
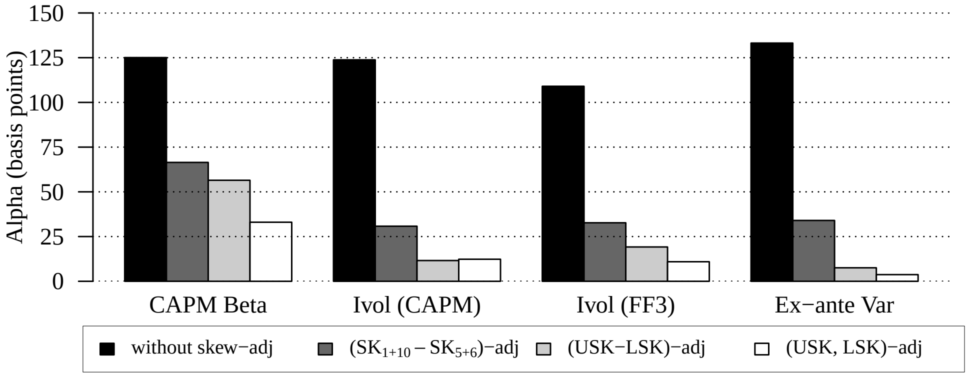


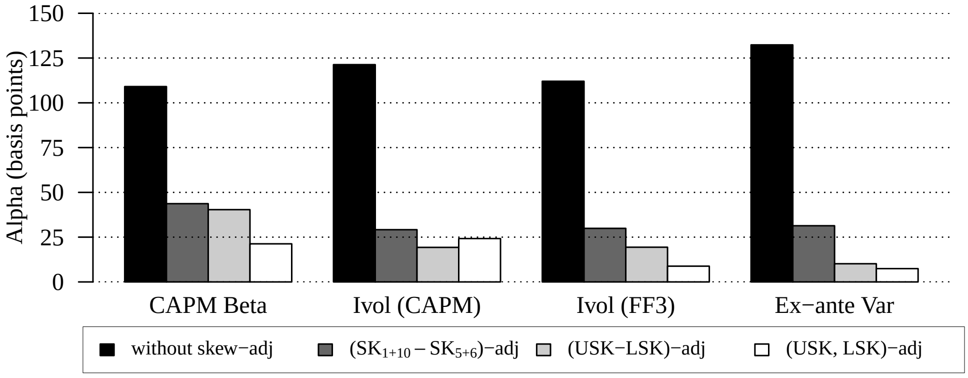
Figure 6: Alphas and Skew-Adjusted Alphas

This Figure reports alphas and skew-adjusted alphas of low risk anomalies (LRAs). At the end of every month, we sort firms into value-weighted decile portfolios based on their CAPM beta, idiosyncratic volatility (measured from the residual variance of CAPM and Fama French three factor model regressions), or their equity option-implied ex-ante variance. From these portfolios, we compute Low-minus-High returns generated by betting-against-beta/volatility strategies, and report alphas of CAPM-, Fama-French three-, and four-factor regressions (black bars in Panels A, B, and C, respectively) as well as alphas that additionally include controls for skewness. To adjust for skewness, we use either the ($SK_{1+10} - SK_{5+6}$)-factor (dark grey bars), the (USK−LSK)-factor (light grey bars), or both the LSK- and the USK-factor (white bars). The data covers 4,967 US firms, is sampled at a monthly frequency over the period January 1996 to August 2014, and contains a total of 400,449 observations.

Panel A. CAPM alphas



Panel B. FF3 alphas



Panel C. FF4 alphas

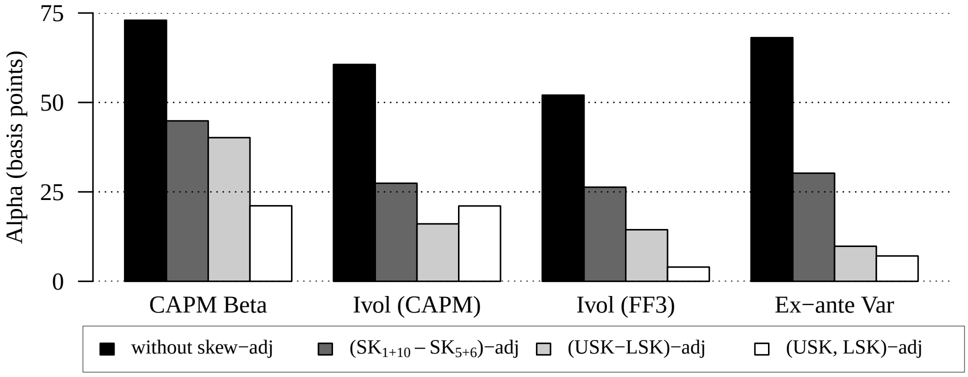


Figure 7: Residual Coskewness associated with Alphas and Skew-Adjusted Alphas

This Figure reports the residual coskewness associated with alphas and skew-adjusted alphas of low risk anomalies (LRAs). At the end of every month, we sort firms into value-weighted decile portfolios based on their CAPM beta, idiosyncratic volatility (measured from the residual variance of CAPM and Fama French three factor model regressions), or their equity option-implied ex-ante variance. From these portfolios, we compute Low-minus-High returns generated by betting-against-beta/volatility strategies, and report the coskewness associated with residual returns of CAPM-, Fama-French three-, and four-factor regressions (black bars in Panels A, B, and C, respectively) as well as the residual coskewness when additionally including controls for skewness. To adjust for skewness, we use either the ($SK_{1+10} - SK_{5+6}$)-factor (dark grey bars), the (USK−LSK)-factor (light grey bars), or both the LSK- and the USK-factor (white bars). The data covers 4,967 US firms, is sampled at a monthly frequency over the period January 1996 to August 2014, and contains a total of 400,449 observations.

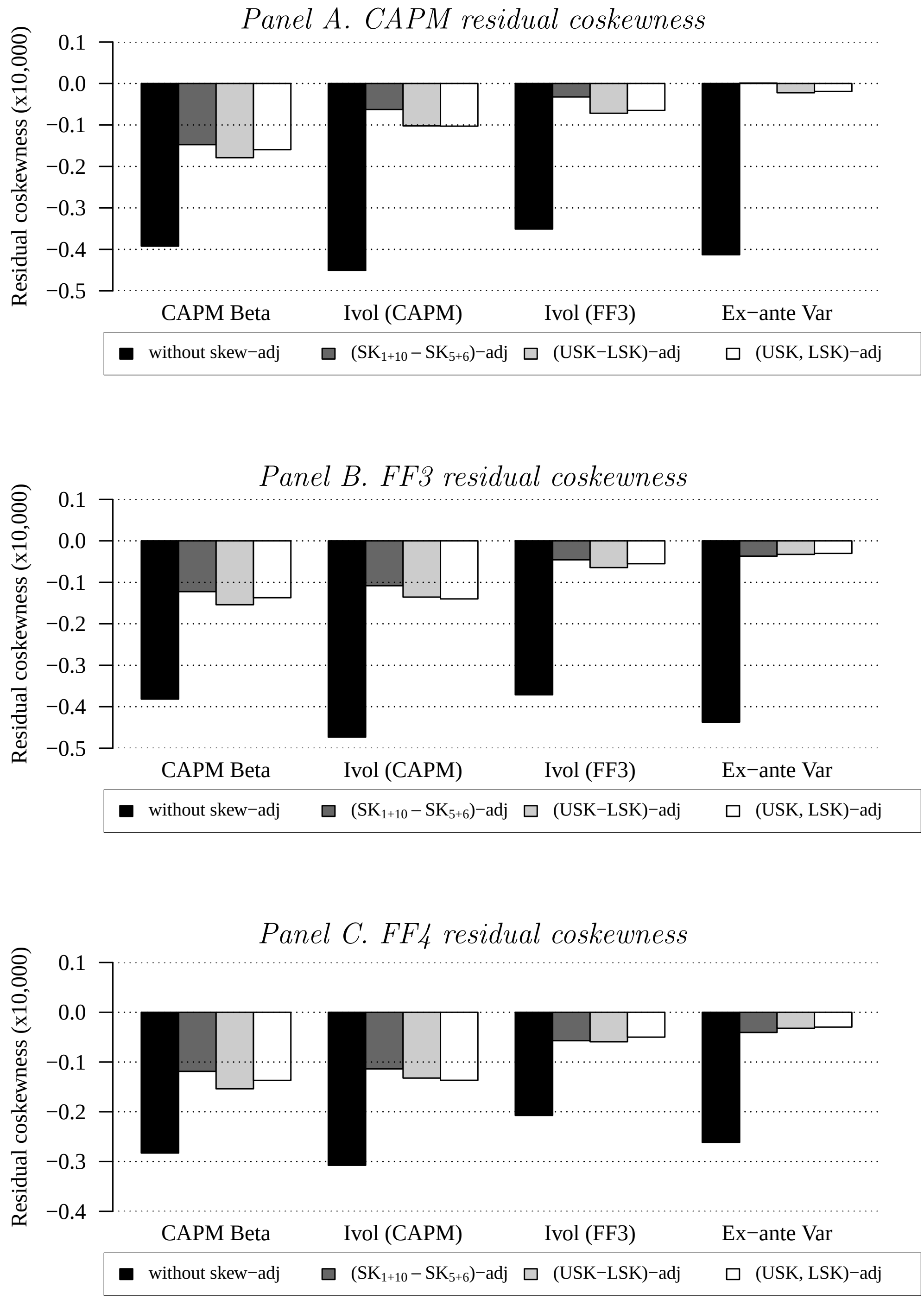
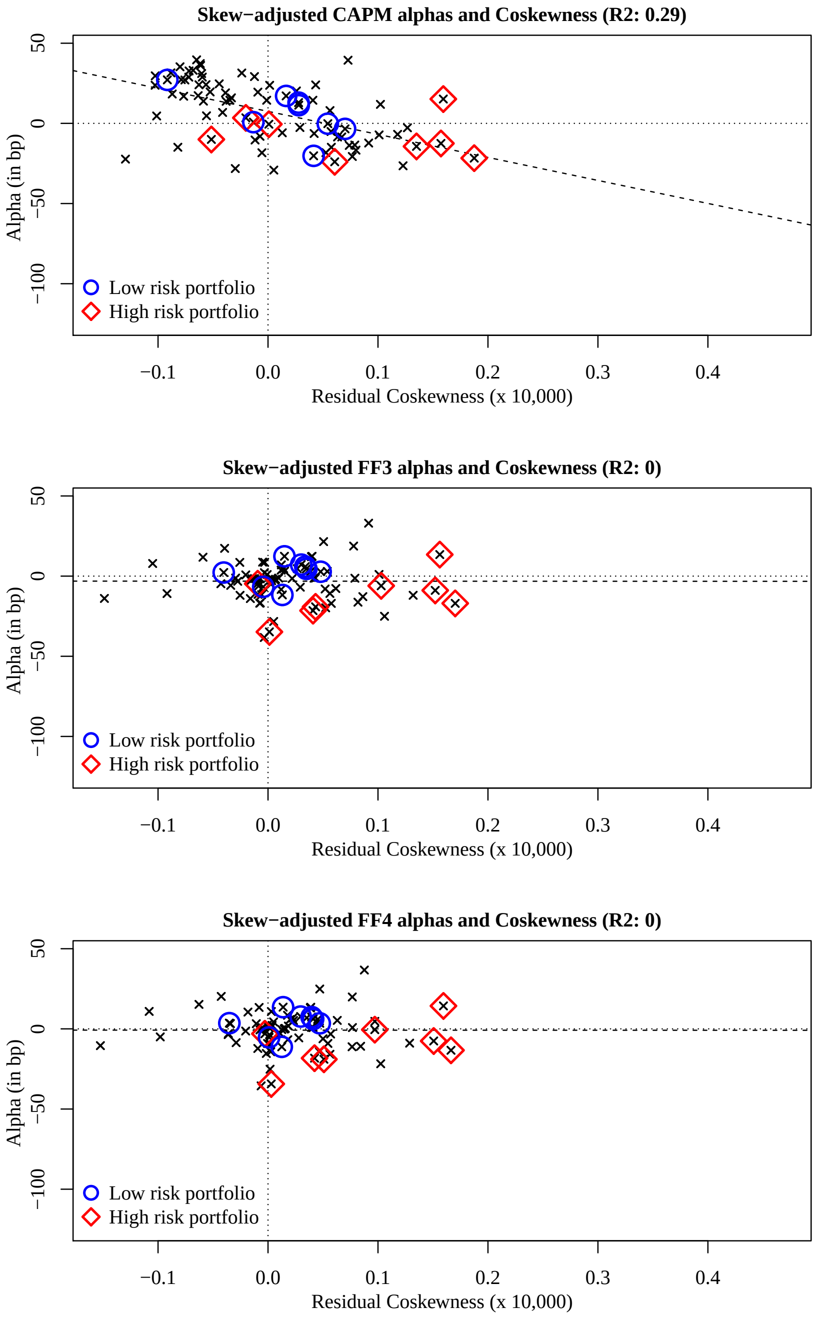


Figure 8: Low Risk Anomalies: Skew-Adjusted Alphas and Residual Coskewness

This Figure reports results for the equally-weighted and value-weighted decile portfolios used to compute the skew-adjusted low risk anomaly (LRA) returns in Table 3. At the end of every month, we sort firms into decile portfolios based one their CAPM beta, idiosyncratic volatility (measured from the residual variance of CAPM and Fama French three factor model regressions), or their equity option-implied ex-ante variance. In total, we have 80 portfolios: 10 portfolios for each of the four LRAs, both, equally- and value-weighted. We plot skew-adjusted CAPM-, Fama-French three-, and four-factor-alphas against their corresponding residual coskewness. To adjust for skewness, we add the LSK- and the USK-factor to the factor model regressions. Blue circles mark the low risk portfolios that a betting-against-beta/volatility strategy goes long. Red diamonds mark the high risk portfolios that a betting-against-beta/volatlity strategy goes short. Each figure header reports the R^2 of a cross-sectional regression of alphas on coskewness and the dashed lines represent the regression-fitted values. The data covers 4,967 US firms, is sampled at a monthly frequency over the period January 1996 to August 2014, and contains a total of 400,449 observations.



Internet Appendix for

Low Risk Anomalies?

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This Appendix contains additional results referred to throughout the paper as well as for the empirical analyses discussed in Section 5.

Table IA.1: Descriptives Statistics for Skew-Sorted Portfolios: EW Means

This Table presents descriptives statistcis for the equally-weighted decile portfolios sorted by measures of ex-ante skewness, for which we report CAPM alphas and residual coskewness in Figure 5. For each firm, we compute option-implied lower skewness (by definition always negative), upper skewness (by definition always positive), and skewness which is the sum of upper and lower skewness. We sort firms into decile portfolios based on the three option-implied skew measures and compute equally-weighted portfolio means of the variables indicated in the rownames. Portfolios P1 and P10 contain the firms with the lowest and highest values of the sort variables, respectively. We report results for portfolios sorted by lower skewness (Panel A), upper skewness (Panel B), and skewness (Panel C). The data covers 4,967 US firms, is sampled at a monthly frequency over the period January 1996 to August 2014, and contains a total of 400,449 observations.

Panel A. Portfolios sorted by lower skewness

	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
Lower skewness (LSK) [%]	-13.63	-5.05	-3.34	-2.39	-1.75	-1.30	-0.97	-0.71	-0.49	-0.26
Upper skewness (USK) [%]	17.36	6.35	3.95	2.79	1.97	1.40	1.05	0.75	0.53	0.33
Skewness (SK) [%]	3.73	1.30	0.61	0.40	0.22	0.10	0.08	0.04	0.05	0.07
CAPM Beta	1.31	1.27	1.21	1.15	1.08	1.04	0.99	0.95	0.89	0.80
Ivol (CAPM) [%]	4.62	3.85	3.39	3.00	2.65	2.34	2.09	1.83	1.60	1.35
Ivol (FF3) [%]	3.98	3.12	2.71	2.40	2.13	1.89	1.69	1.49	1.29	1.08
Ex-ante Var [%]	82.69	43.96	32.65	25.68	20.50	16.50	13.40	10.69	8.26	5.48
Market cap [in logs]	20.83	21.08	21.30	21.54	21.78	22.12	22.44	22.83	23.25	23.83
Book-to-market	0.47	0.50	0.51	0.51	0.50	0.49	0.48	0.47	0.45	0.43

Panel B. Portfolios sorted by upper skewness

	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
Lower skewness (LSK) [%]	-0.37	-0.60	-0.83	-1.09	-1.45	-1.93	-2.62	-3.59	-5.33	-12.08
Upper skewness (USK) [%]	0.19	0.38	0.58	0.84	1.18	1.68	2.43	3.60	5.84	19.77
Skewness (SK) [%]	-0.18	-0.22	-0.25	-0.25	-0.27	-0.25	-0.19	0.01	0.51	7.69
CAPM Beta	0.81	0.90	0.95	0.99	1.04	1.09	1.14	1.21	1.26	1.31
Ivol (CAPM) [%]	1.36	1.60	1.83	2.08	2.35	2.65	3.00	3.40	3.86	4.62
Ivol (FF3) [%]	1.08	1.29	1.48	1.68	1.88	2.12	2.40	2.71	3.13	4.00
Ex-ante Var [%]	5.42	8.23	10.66	13.33	16.47	20.40	25.61	32.67	43.95	83.12
Market cap [in logs]	23.90	23.26	22.81	22.39	22.05	21.76	21.45	21.17	20.90	20.62
Book-to-market	0.42	0.44	0.47	0.48	0.49	0.50	0.51	0.51	0.52	0.48

Panel C. Portfolios sorted by skewness

	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
Lower skewness (LSK) [%]	-7.32	-2.34	-1.57	-1.19	-1.02	-1.04	-1.38	-2.18	-3.53	-8.29
Upper skewness (USK) [%]	4.03	1.76	1.24	0.98	0.90	1.02	1.50	2.60	4.73	17.70
Skewness (SK) [%]	-3.30	-0.59	-0.33	-0.21	-0.12	-0.02	0.12	0.42	1.21	9.42
CAPM Beta	1.17	1.11	1.06	1.01	0.97	0.94	0.97	1.06	1.15	1.24
Ivol (CAPM) [%]	3.34	2.66	2.33	2.10	1.97	1.97	2.19	2.69	3.32	4.18
Ivol (FF3) [%]	2.70	2.09	1.85	1.67	1.58	1.60	1.79	2.19	2.70	3.58
Ex-ante Var [%]	40.95	21.51	16.53	13.70	12.42	12.73	15.91	23.03	34.11	68.85
Market cap [in logs]	21.75	22.27	22.67	22.91	23.11	23.25	22.99	21.99	21.03	20.47
Book-to-market	0.52	0.49	0.46	0.45	0.45	0.44	0.46	0.50	0.53	0.52

Table IA.2: Descriptives Statistics for Skew-Sorted Portfolios: VW Means

This Table presents descriptives statistcis for the value-weighted decile portfolios sorted by measures of ex-ante skewness, for which we report CAPM alphas and residual coskewness in Figure 5. For each firm, we compute option-implied lower skewness (by definition always negative), upper skewness (by definition always positive), and skewness which is the sum of upper and lower skewness. We sort firms into decile portfolios based on the three option-implied skew measures and compute value-weighted portfolio means of the variables indicated in the rownames. Portfolios P1 and P10 contain the firms with the lowest and highest values of the sort variables, respectively. We report results for portfolios sorted by lower skewness (Panel A), upper skewness (Panel B), and skewness (Panel C). The data covers 4,967 US firms, is sampled at a monthly frequency over the period January 1996 to August 2014, and contains a total of 400,449 observations.

Panel A. Portfolios sorted by lower skewness

	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
Lower skewness (LSK) [%]	-13.00	-4.98	-3.30	-2.37	-1.74	-1.29	-0.96	-0.70	-0.48	-0.26
Upper skewness (USK) [%]	14.05	5.20	3.28	2.21	1.54	1.09	0.79	0.55	0.36	0.28
Skewness (SK) [%]	1.05	0.22	-0.03	-0.16	-0.20	-0.20	-0.16	-0.15	-0.11	0.03
CAPM Beta	1.42	1.38	1.34	1.27	1.21	1.15	1.08	1.03	0.95	0.81
Ivol (CAPM) [%]	4.24	3.58	3.17	2.80	2.48	2.20	1.97	1.74	1.54	1.26
Ivol (FF3) [%]	3.77	2.98	2.56	2.27	2.03	1.78	1.60	1.43	1.25	1.00
Ex-ante Var [%]	75.54	41.18	30.84	24.18	19.37	15.62	12.64	10.08	7.69	5.02
Market cap [in logs]	22.88	23.44	23.63	23.90	24.09	24.41	24.62	24.86	25.00	25.35
Book-to-market	0.41	0.43	0.41	0.41	0.40	0.39	0.40	0.38	0.36	0.33

Panel B. Portfolios sorted by upper skewness

	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
Lower skewness (LSK) [%]	-0.30	-0.55	-0.79	-1.09	-1.47	-1.98	-2.69	-3.71	-5.51	-12.07
Upper skewness (USK) [%]	0.18	0.37	0.57	0.83	1.17	1.67	2.41	3.56	5.75	19.04
Skewness (SK) [%]	-0.12	-0.18	-0.22	-0.26	-0.29	-0.31	-0.28	-0.15	0.23	6.97
CAPM Beta	0.82	0.97	1.03	1.09	1.16	1.22	1.28	1.35	1.39	1.41
Ivol (CAPM) [%]	1.27	1.56	1.78	2.02	2.27	2.55	2.86	3.27	3.68	4.34
Ivol (FF3) [%]	1.01	1.27	1.47	1.65	1.85	2.08	2.36	2.68	3.09	3.89
Ex-ante Var [%]	5.10	8.12	10.63	13.40	16.62	20.63	25.85	32.92	44.03	80.92
Market cap [in logs]	25.37	25.02	24.85	24.54	24.23	24.09	23.75	23.41	23.17	22.76
Book-to-market	0.33	0.37	0.39	0.40	0.40	0.40	0.41	0.42	0.44	0.40

Panel C. Portfolios sorted by skewness

	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
Lower skewness (LSK) [%]	-5.28	-1.76	-1.10	-0.78	-0.63	-0.70	-0.98	-1.77	-3.25	-7.76
Upper skewness (USK) [%]	2.83	1.19	0.78	0.58	0.52	0.67	1.09	2.16	4.41	16.78
Skewness (SK) [%]	-2.46	-0.57	-0.33	-0.20	-0.12	-0.02	0.11	0.40	1.16	9.02
CAPM Beta	1.21	1.17	1.10	1.02	0.96	0.92	0.93	1.05	1.19	1.32
Ivol (CAPM) [%]	2.76	2.27	1.96	1.76	1.62	1.60	1.77	2.26	2.97	3.83
Ivol (FF3) [%]	2.29	1.79	1.55	1.41	1.32	1.32	1.50	1.90	2.50	3.40
Ex-ante Var [%]	32.10	17.23	12.75	10.24	8.91	9.36	11.97	19.12	31.01	64.24
Market cap [in logs]	24.07	24.46	24.67	24.77	24.95	25.06	24.96	24.21	23.25	22.79
Book-to-market	0.46	0.41	0.39	0.38	0.37	0.35	0.35	0.41	0.46	0.44

Table IA.3: Skewness as a Common Driver of Low Risk Anomalies: EW Portfolios

This Table presents evidence that skewness is a common driver of beta- and volatility-based low risk anomalies (LRAs). We compute LRA returns as Low-minus-High returns of equally-weighted decile portfolios that we sort by CAPM beta, idiosyncratic volatility (measured from the residual variance of CAPM and Fama French three factor model regressions), or their equity option-implied ex-ante variance at the end of every month. We compute the LRAs' excess returns as well as alphas of CAPM-, Fama-French three- (FF3), and four-factor- (FF4) regressions and Panel A presents results for principal component analyses of the corresponding residual returns. The left part of Panel A reports the return variation explained by each of the four principal components (PCs). The right part reports the coefficients as well as the associated R^2s of regressing LRA returns on PC1. Panel B shows that the LRAs' PC1 is related to skewness by reporting results from regressing PC1 of excess returns as well as CAPM, FF3, and FF4 residual returns on skew factor returns. We compute skew factor returns as the High-minus-Low returns of equally-weighted decile portfolios sorted by ex-ante skewness and consider the following specifications: we use lower skewness to construct the LSK-factor, upper skewness for the USK-factor, and total skewness to construct the $(SK_{1+10} - SK_{5+6})$-factor. We report results for regressions using these skew factors and additionally for regressions where we use the difference between USK and LSK as skew factor and a regression in which we include both USK and LSK returns. Values in square brackets are t-statistics based on standard errors following Newey and West (1987) where we choose the optimal truncation lag as suggested by Andrews (1991). The data covers 4,967 US firms, is sampled at a monthly frequency over the period January 1996 to August 2014, and contains a total of 400,449 observations.

Panel A. Principal components of Low Risk Anomalies

	Variation explained by PCs				Anomaly return loadings on PC1			
	PC1	PC2	PC3	PC4	CAPM Beta	Ivol (CAPM)	Ivol (FF3)	Ex-ante Var
Excess returns	96.28	2.74	0.57	0.42	0.49	0.52	0.44	0.54
					[25.97]	[38.00]	[67.89]	[71.02]
R^2 (%)					91.94	97.90	97.13	98.07
CAPM residual returns	94.16	4.07	1.02	0.75	0.43	0.53	0.46	0.56
					[19.15]	[31.37]	[79.01]	[56.88]
R^2 (%)					84.39	96.77	95.87	97.39
FF3 residual returns	89.62	6.87	1.93	1.58	0.52	0.48	0.43	0.55
					[18.90]	[37.29]	[31.26]	[32.05]
R^2 (%)					83.22	92.50	89.99	93.58
FF4 residual returns	84.98	10.04	2.58	2.40	0.56	0.51	0.40	0.52
					[25.49]	[33.74]	[31.25]	[32.29]
R^2 (%)					79.63	89.79	82.85	88.77

Panel B. Skew Factor Returns and the First Principal Component of Low Risk Anomalies

	Excess returns	CAPM residuals	FF3 residuals	FF4 residuals
Lower skewness (LSK)	1.88	1.80	1.77	1.77
	[77.80]	[67.46]	[33.35]	[30.32]
R^2 (%)	98.03	97.20	93.15	88.14
Upper skewness (USK)	-1.85	-1.77	-1.69	-1.67
	[-64.24]	[-46.37]	[-26.71]	[-22.82]
R^2 (%)	97.74	96.72	92.32	86.77
Skewness $(SK_{1+10} - SK_{5+6})$	-1.71	-1.55	-1.30	-1.26
	[-29.79]	[-19.51]	[-18.06]	[-17.16]
R^2 (%)	91.15	86.53	78.08	63.94
Upper minus lower skewness (USK − LSK)	-0.94	-0.90	-0.88	-0.88
	[-72.91]	[-55.66]	[-30.25]	[-26.53]
R^2 (%)	98.18	97.46	93.76	89.10
Lower skewness (LSK)	1.17	1.12	1.07	1.08
	[8.57]	[9.82]	[9.62]	[9.80]
Upperskewness (USK)	-0.71	-0.68	-0.69	-0.68
	[-5.16]	[-5.97]	[-6.18]	[-5.68]
R^2 (%)	98.20	97.49	93.81	89.18

Table IA.4: Skewness as a Common Driver of Low Risk Anomalies: RW Portfolios

This Table presents evidence that skewness is a common driver of beta- and volatility-based low risk anomalies (LRAs). We compute LRA returns as Low-minus-High returns of rank-weighted decile portfolios that we sort by CAPM beta, idiosyncratic volatility (measured from the residual variance of CAPM and Fama French three factor model regressions), or their equity option-implied ex-ante variance at the end of every month. We compute the LRAs' excess returns as well as alphas of CAPM-, Fama-French three- (FF3), and four-factor- (FF4) regressions and Panel A presents results for principal component analyses of the corresponding residual returns. The left part of Panel A reports the return variation explained by each of the four principal components (PCs). The right part reports the coefficients as well as the associated R^2s of regressing LRA returns on PC1. Panel B shows that the LRAs' PC1 is related to skewness by reporting results from regressing PC1 of excess returns as well as CAPM, FF3, and FF4 residual returns on skew factor returns. We compute skew factor returns as the High-minus-Low returns of rank-weighted decile portfolios sorted by ex-ante skewness and consider the following specifications: we use lower skewness to construct the LSK-factor, upper skewness for the USK-factor, and total skewness to construct the ($SK_{1+10} - SK_{5+6}$)-factor. We report results for regressions using these skew factors and additionally for regressions where we use the difference between USK and LSK as skew factor and a regression in which we include both USK and LSK returns. Values in square brackets are t-statistics based on standard errors following Newey and West (1987) where we choose the optimal truncation lag as suggested by Andrews (1991). The data covers 4,967 US firms, is sampled at a monthly frequency over the period January 1996 to August 2014, and contains a total of 400,449 observations.

Panel A. Principal components of Low Risk Anomalies

	Variation explained by PCs				Anomaly return loadings on PC1			
	PC1	PC2	PC3	PC4	CAPM Beta	Ivol (CAPM)	Ivol (FF3)	Ex-ante Var
Excess returns	97.57	2.07	0.19	0.17	0.48	0.52	0.45	0.54
					[21.69]	[41.15]	[53.04]	[196.78]
R^2 (%)					93.40	98.75	98.56	99.24
CAPM residual returns	96.47	2.90	0.33	0.29	0.43	0.54	0.47	0.55
					[19.17]	[38.90]	[53.20]	[129.79]
R^2 (%)					88.29	98.41	98.15	98.93
FF3 residual returns	93.68	4.86	0.76	0.71	0.54	0.49	0.43	0.54
					[37.74]	[49.38]	[37.40]	[59.79]
R^2 (%)					88.65	95.76	94.55	96.85
FF4 residual returns	90.86	7.18	1.05	0.91	0.55	0.51	0.42	0.51
					[33.24]	[57.28]	[39.87]	[55.25]
R^2 (%)					84.58	94.31	91.46	95.11

Panel B. Skew Factor Returns and the First Principal Component of Low Risk Anomalies

	Excess returns	CAPM residuals	FF3 residuals	FF4 residuals
Lower skewness (LSK)	1.87	1.83	1.86	1.89
	[151.03]	[166.66]	[63.68]	[56.49]
R^2 (%)	99.27	99.00	97.07	95.39
Upper skewness (USK)	-1.87	-1.82	-1.78	-1.82
	[-117.12]	[-64.11]	[-44.04]	[-37.41]
R^2 (%)	98.82	98.29	95.22	92.56
Skewness ($SK_{1+10} - SK_{5+6}$)	-0.94	-0.91	-0.92	-0.94
	[-166.83]	[-105.36]	[-52.92]	[-45.08]
R^2 (%)	99.19	98.88	96.69	94.83
Upper minus lower skewness (USK − LSK)	-0.94	-0.91	-0.92	-0.94
	[-166.83]	[-105.36]	[-52.92]	[-45.08]
R^2 (%)	99.19	98.88	96.69	94.83
Lower skewness (LSK)	1.69	1.61	1.71	1.71
	[9.61]	[11.23]	[9.60]	[9.99]
Upperskewness (USK)	-0.18	-0.22	-0.15	-0.18
	[-1.04]	[-1.52]	[-0.86]	[-1.02]
R^2 (%)	99.28	99.01	97.09	95.42

Table IA.5: Skewness as a Common Driver of Low Risk Anomalies: Additional Controls

This Table presents evidence that skewness is a common driver of beta- and volatility-based low risk anomalies (LRAs). We compute LRA returns as Low-minus-High returns of value-weighted decile portfolios that we sort by CAPM beta, idiosyncratic volatility (measured from the residual variance of CAPM and Fama French three factor model regressions), or their equity option-implied ex-ante variance at the end of every month. From the LRAs' excess returns, we compute alphas relative to the Fama-French five factor model as well as relative to the Fama-French four factor model extended with the liquidity factor of Pástor and Stambaugh (2003). Panel A presents results for principal component analyses of the corresponding residual returns. The left part of Panel A reports the residual return variation explained by each of the four principal components (PCs). The right part reports the coefficients as well as the associated R^2s of regressing LRA residual returns on PC1. Panel B shows that the LRAs' PC1 is related to skewness by reporting results from regressing PC1 of FF5 and FF4 LIQU residual returns on skew factor returns. We compute skew factor returns as the High-minus-Low returns of value-weighted decile portfolios sorted by ex-ante skewness and consider the following specifications: we use lower skewness to construct the LSK-factor, upper skewness for the USK-factor, and total skewness to construct the $(\text{SK}_{1+10} - \text{SK}_{5+6})$-factor. We report results for regressions using these skew factors and additionally for regressions where we use the difference between USK and LSK as skew factor and a regression in which we include both USK and LSK returns. Values in square brackets are t-statistics based on standard errors following Newey and West (1987) where we choose the optimal truncation lag as suggested by Andrews (1991). The data covers 4,967 US firms, is sampled at a monthly frequency over the period January 1996 to August 2014, and contains a total of 400,449 observations.

Panel A. Principal components of Low Risk Anomalies

	Variation explained by PCs				Anomaly return loadings on PC1			
	PC1	PC2	PC3	PC4	CAPM Beta	Ivol (CAPM)	Ivol (FF3)	Ex-ante Var
FF5 residual returns	78.55	10.25	5.78	5.41	0.39	0.55	0.49	0.55
					[16.04]	[25.24]	[17.91]	[32.46]
R^2 (%)					58.89	85.68	78.94	85.06
FF4 LIQU residual returns	72.22	12.82	7.82	7.15	0.46	0.55	0.45	0.54
					[13.21]	[22.38]	[17.58]	[25.74]
R^2 (%)					60.81	80.09	68.18	77.96

Panel B. Skew Factor Returns and the First Principal Component of Low Risk Anomalies

	FF5 residuals	FF4 LIQU residuals
Lower skewness (LSK)	1.59	1.48
	[23.23]	[17.26]
R^2 (%)	83.90	77.31
Upper skewness (USK)	-1.41	-1.32
	[-22.63]	[-18.51]
R^2 (%)	82.87	74.87
Skewness $(\text{SK}_{1+10} - \text{SK}_{5+6})$	-0.90	-0.89
	[-27.35]	[-12.45]
R^2 (%)	68.04	50.97
Upper minus lower skewness (USK − LSK)	-0.77	-0.73
	[-28.00]	[-22.11]
R^2 (%)	86.08	79.53
Lower skewness (LSK)	0.90	0.90
	[6.18]	[6.90]
Upperskewness (USK)	-0.66	-0.57
	[-5.57]	[-4.93]
R^2 (%)	86.14	79.71

Table IA.6: Alphas and Skew-Adjusted Alphas of the BaB-factor

This Table reports alphas and skew-adjusted alphas for the BaB factor of Frazzini and Pedersen (2014). We report alphas of CAPM-, Fama-French three-, and four-factor regressions as well as alphas of the same regressions when we additionally include controls for skewness. To adjust for skewness, we use both the LSK- and the USK-factor. We also report the corresponding coskewness of residual returns, i.e. the covariance of residual returns with squared market excess returns. Values in square brackets are t-statistics based on standard errors following Newey and West (1987) where we choose the optimal truncation lag as suggested by Andrews (1991). The overlapping period of our sample and the data of Frazzini and Pedersen (2014) is from 01/1996 to 03/2012.

	Without skew-adjustment	With skew-adjustment
CAPM alpha	91.69	47.89
	[2.34]	[1.19]
Coskewness	-0.44	-0.33
FF3 alpha	74.66	10.85
	[2.17]	[0.29]
Coskewness	-0.41	-0.23
FF4 alpha	60.68	12.32
	[1.69]	[0.42]
Coskewness	-0.36	-0.23

Figure IA.1: Alphas and Residual Coskewness: Simulation Evidence with Asset Jumps

This Figure presents results for the relation between CAPM alphas and residual coskewness in a simulated world with 2,000 Merton-type firms where we additionally allow for jumps in the firms' asset processes. The pricing kernel is specified to account for skewness and moments higher than skewness. The black crosses mark CAPM alphas and the grey circles mark alphas after additionally controlling for skewness. The figure also reports results for LRA strategies based on equally-weighted decile portfolios, betting against CAPM beta, idiosyncratic volatility, and implied volatility. Blue circles mark the low risk portfolios that a betting-against-risk strategy goes long. Red diamonds mark the high risk portfolios that a betting-against-risk strategy goes long. The figure header reports the R^2 of a cross-sectional regression of alphas on coskewness.

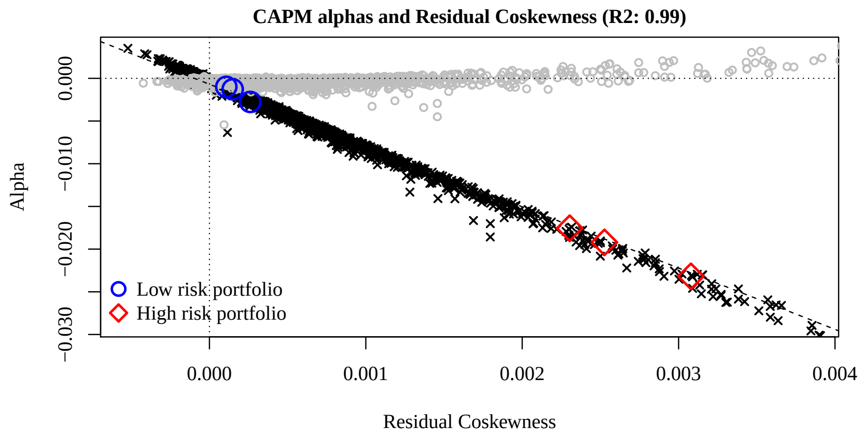


Figure IA.2: Alphas and Residual Coskewness: Simulation Evidence with Early Defaults

This Figure presents results for the relation between CAPM alphas and residual coskewness in a simulated world with 2,000 Merton-type firms where we additionally allow for early defaults, i.e. before debt maturity. The pricing kernel is specified to account for skewness and moments higher than skewness. The black crosses mark CAPM alphas and the grey circles mark alphas after additionally controlling for skewness. The figure also reports results for LRA strategies based on equally-weighted decile portfolios, betting against CAPM beta, idiosyncratic volatility, and implied volatility. Blue circles mark the low risk portfolios that a betting-against-risk strategy goes long. Red diamonds mark the high risk portfolios that a betting-against-risk strategy goes long. The figure header reports the R^2 of a cross-sectional regression of alphas on coskewness.

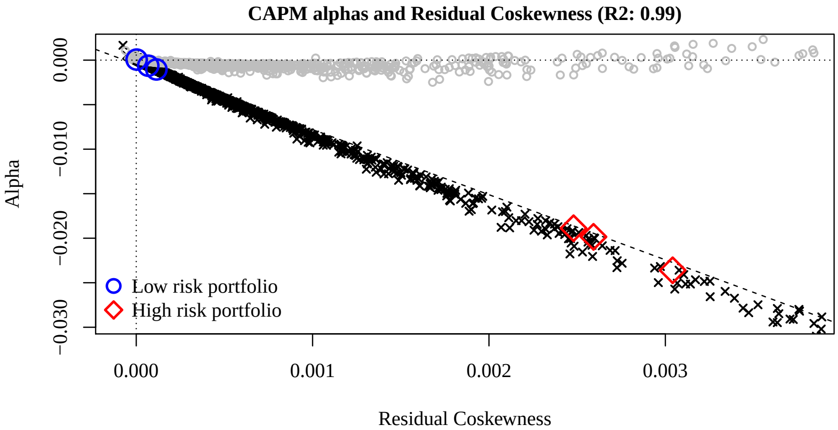
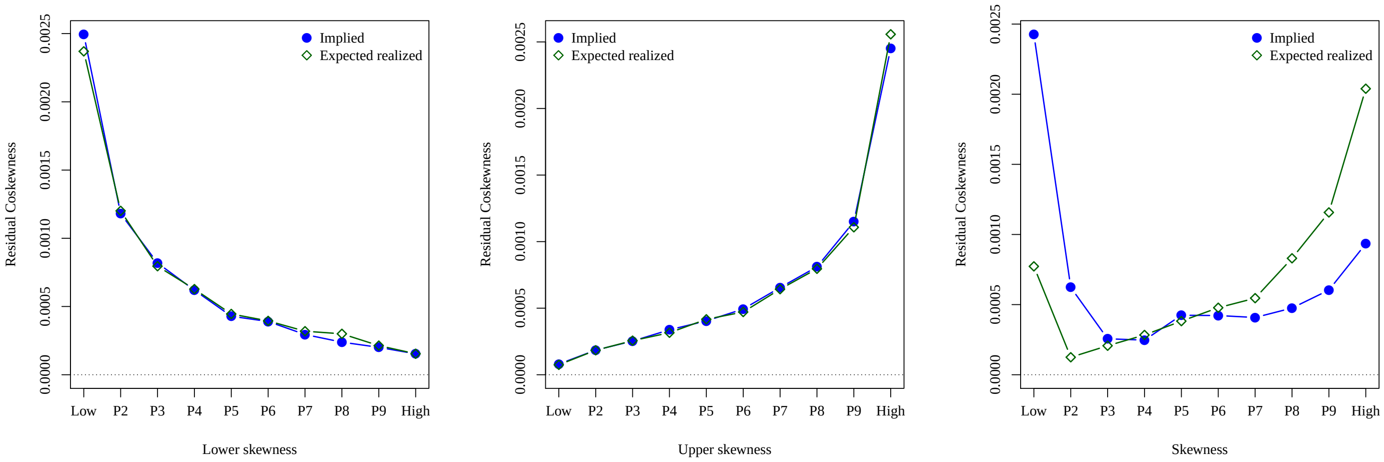


Figure IA.3: Implied Skewness, Residual Coskewness, and Alphas: Simulation Evidence with Asset Jumps

This Figure presents results for the relation between firms' skewness, CAPM residual coskewness, and CAPM alphas in a simulated skew-aware world with 2,000 Merton-type firms where we additionally allow for jumps in the firms' asset processes. For each firm, we compute measures of implied skewness (under the Q-measure) and expected realized skewness (under the P-measure). We compute lower skewness (by definition always negative), upper skewness (by definition always positive), and skewness which is the sum of lower and upper skewness. We sort firms into decile portfolios based on the three Q-, and on the three P-skew measures, and compute the portfolios' CAPM alphas and residual coskewness. Portfolios P1 and P10 contain the firms with the lowest and highest values of the sort variables, respectively. Panel A reports the portfolios' CAPM residual coskewness when using measures of Q-skew (blue line with bullets) and P-skew (green line with diamonds) using lower skewness (left), upper skewness (middle), and skewness (right). Panel B plots the portfolios' corresponding CAPM alphas.

Panel A. CAPM Residual Coskewness



Panel B. CAPM Alphas

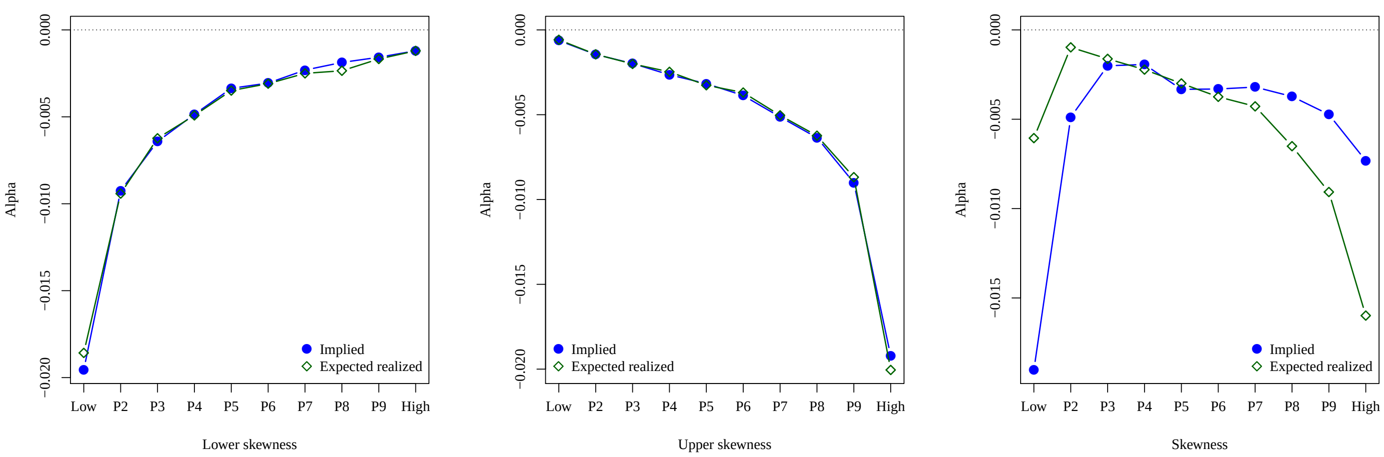
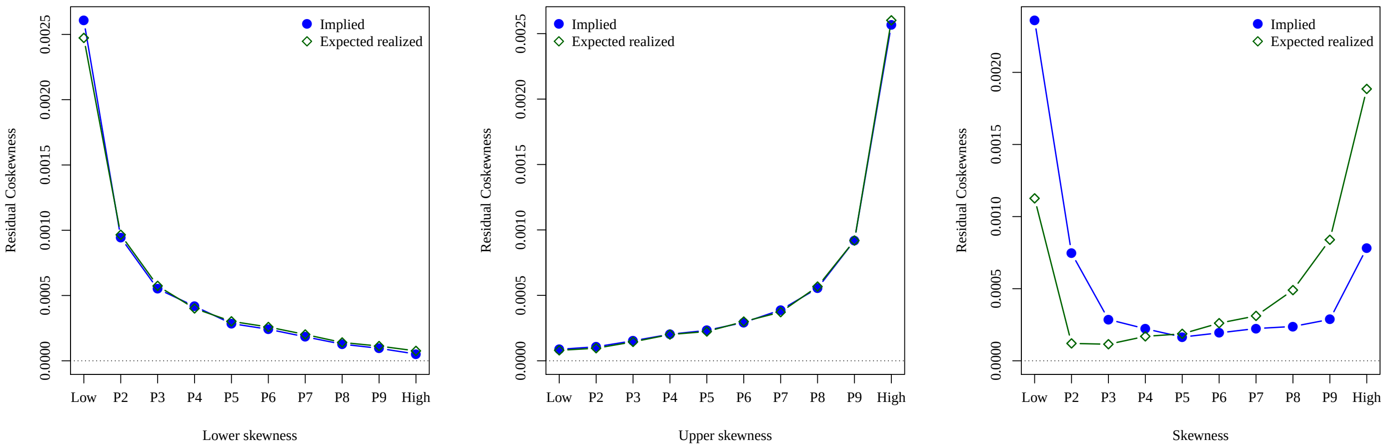


Figure IA.4: Implied Skewness, Residual Coskewness, and Alphas: Simulation Evidence with Early Defaults

This Figure presents results for the relation between firms' skewness, CAPM residual coskewness, and CAPM alphas in a simulated skew-aware world with 2,000 Merton-type firms where we additionally allow for early defaults, i.e. before debt maturity. For each firm, we compute measures of implied skewness (under the Q-measure) and expected realized skewness (under the P-measure). We compute lower skewness (by definition always negative), upper skewness (by definition always positive), and skewness which is the sum of lower and upper skewness. We sort firms into decile portfolios based on the three Q-, and on the three P-skew measures, and compute the portfolios' CAPM alphas and residual coskewness. Portfolios P1 and P10 contain the firms with the lowest and highest values of the sort variables, respectively. Panel A reports the portfolios' CAPM residual coskewness when using measures of Q-skew (blue line with bullets) and P-skew (green line with diamonds) using lower skewness (left), upper skewness (middle), and skewness (right). Panel B plots the portfolios' corresponding CAPM alphas.

Panel A. CAPM Residual Coskewness



Panel B. CAPM Alphas

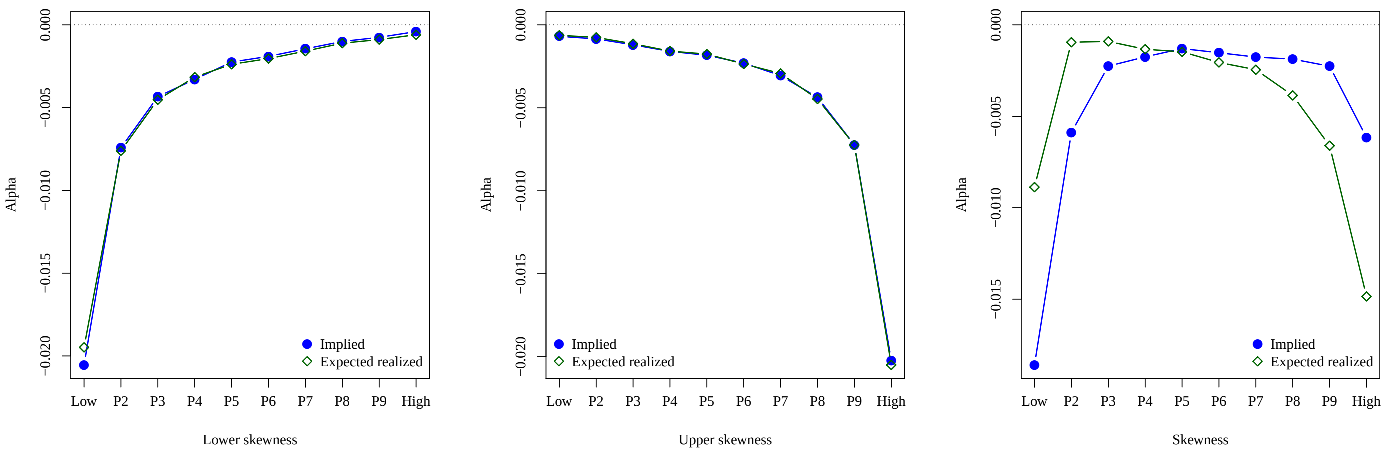
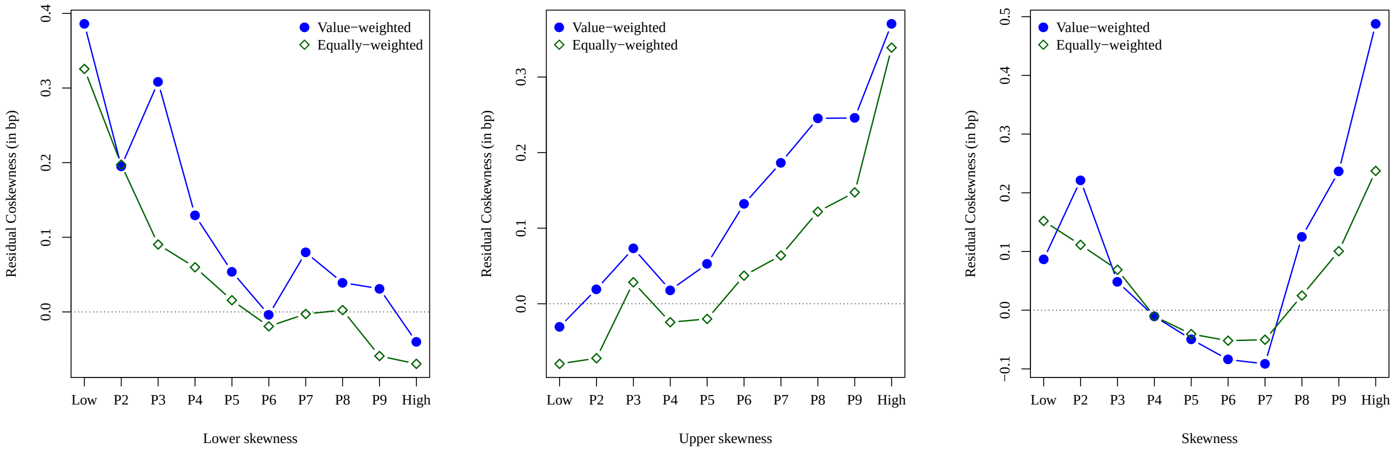


Figure IA.5: Option-Implied Ex-Ante Skewness, FF3 Residual Coskewness, and FF3 Alphas

This Figure presents results for the relation between firms' equity-option implied ex-ante skewness, FF3 residual coskewness, and FF3 alphas. For each firm, we compute option-implied lower skewness (by definition always negative), upper skewness (by definition always positive), and skewness which is the sum of upper and lower skewness. We sort firms into decile portfolios based on the three option-implied skew measures and compute the portfolios' FF3 alphas and residual coskewness. Portfolios P1 and P10 contain the firms with the lowest and highest values of the sort variables, respectively. Panel A reports the portfolios' FF3 residual coskewness for equally-weighted portfolios (blue line with bullets) and value-weighted portfolios (green line with diamonds) using lower skewness (left), upper skewness (middle), and skewness (right). Panel B plots the portfolios' corresponding FF3 alphas. The data covers 4,967 US firms, is sampled at a monthly frequency over the period January 1996 to August 2014, and contains a total of 400,449 observations.

Panel A. FF3 Residual Coskewness



Panel B. FF3 Alphas

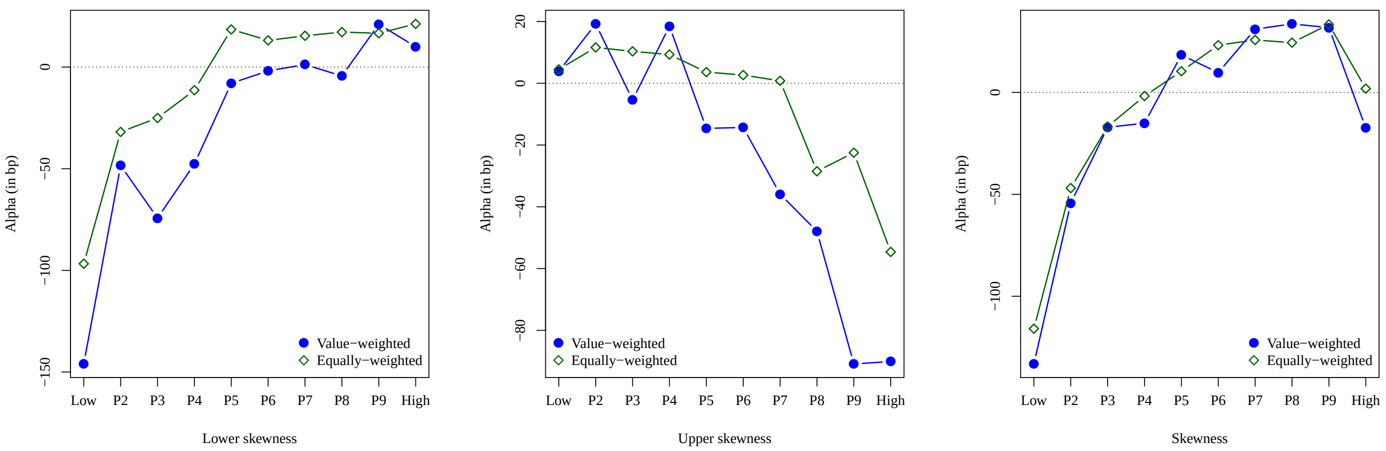
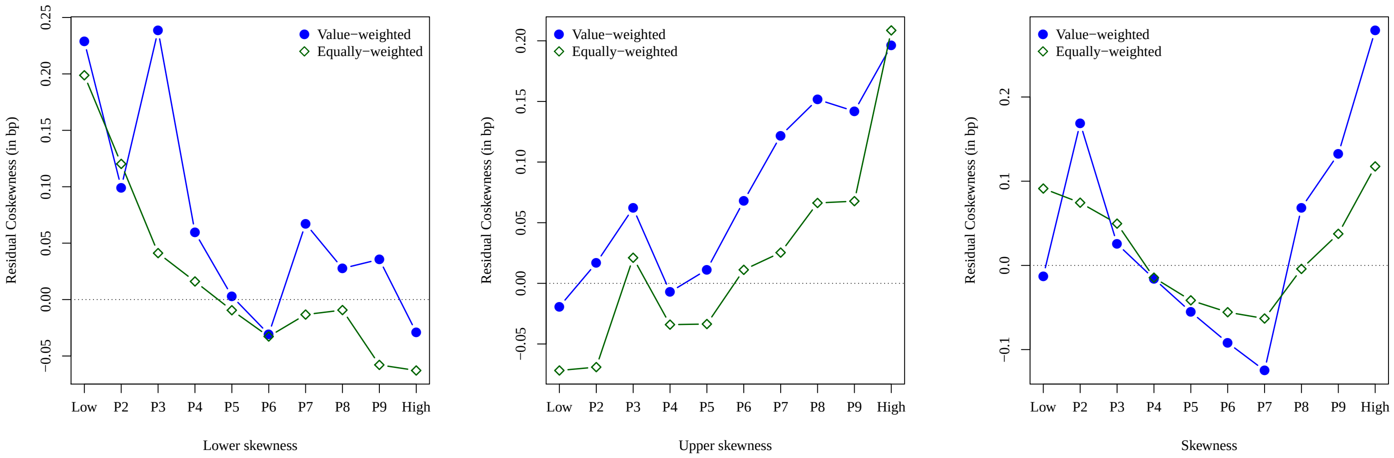


Figure IA.6: Option-Implied Ex-Ante Skewness, FF4 Residual Coskewness, and FF4 Alphas

This Figure presents results for the relation between firms' equity-option implied ex-ante skewness, FF4 residual coskewness, and FF4 alphas. For each firm, we compute option-implied lower skewness (by definition always negative), upper skewness (by definition always positive),and skewness which is the sum of upper and lower skewness. We sort firms into decile portfolios based on the three option-implied skew measures and compute the portfolios' FF4 alphas and residual coskewness. Portfolios P1 and P10 contain the firms with the lowest and highest values of the sort variables, respectively. Panel A reports the portfolios' FF4 residual coskewness for equally-weighted portfolios (blue line with bullets) and value-weighted portfolios (green line with diamonds) using lower skewness (left), upper skewness (middle), and skewness (right). Panel B plots the portfolios' corresponding FF4 alphas. The data covers 4,967 US firms, is sampled at a monthly frequency over the period January 1996 to August 2014, and contains a total of 400,449 observations.

Panel A. FF4 Residual Coskewness



Panel B. FF4 Alphas

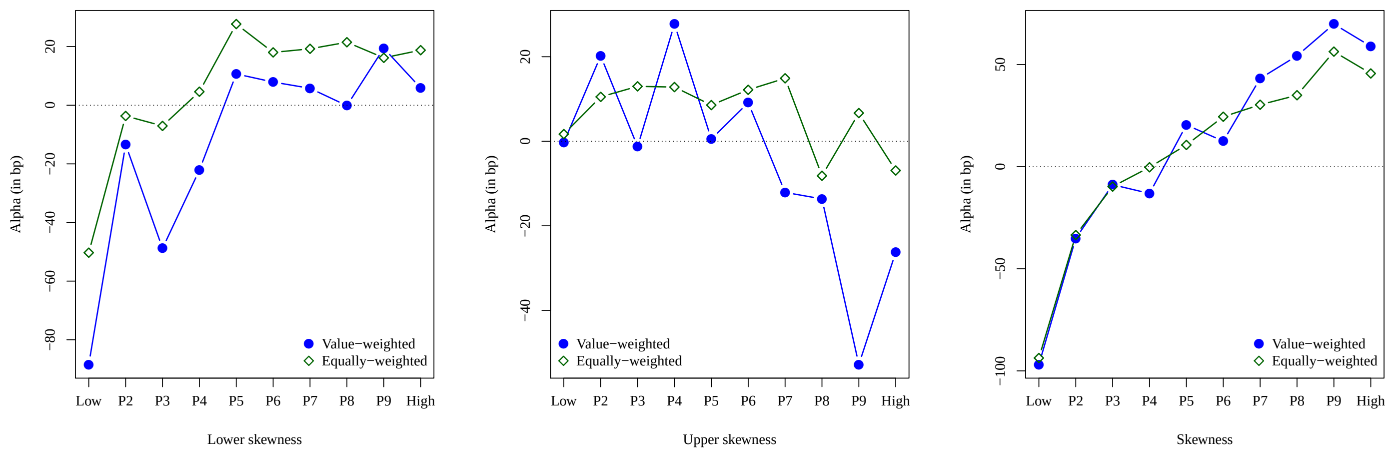
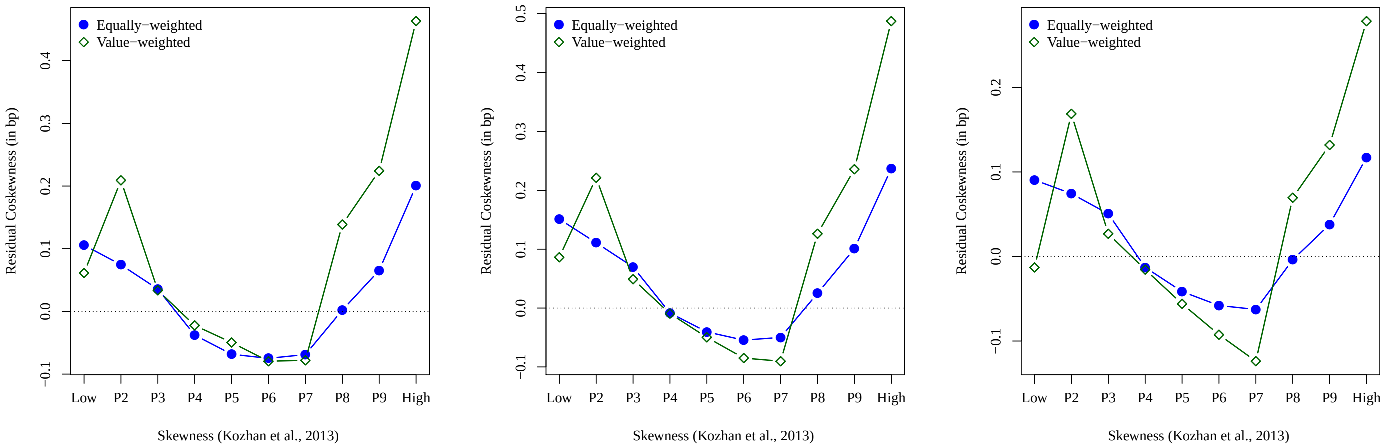


Figure IA.7: Option-Implied Ex-Ante Skewness, Residual Coskewness, and Alphas: Alternative Ex-Ante Skew Measure

This Figure presents results for the relation between firms' equity-option implied ex-ante skewness, factor model residual coskewness, and factor model alphas. For each firm, we compute the option-implied skewness measure of Kozhan et al. (2013). We sort firms into decile portfolios based on option-implied skewness and compute the portfolios alphas and residual coskewness. Portfolios P1 and P10 contain the firms with the lowest and highest values of the sort variables, respectively. Panel A reports the portfolios' CAPM (left), FF3 (middle), and FF4 (right) residual coskewness for equally-weighted portfolios (blue line with bullets) and value-weighted portfolios (green line with diamonds). Panel B plots the portfolios' corresponding CAPM, FF3, and FF4 alphas. The data covers 4,967 US firms, is sampled at a monthly frequency over the period January 1996 to August 2014, and contains a total of 400,449 observations.

Panel A. Residual Coskewness



Panel B. Alphas

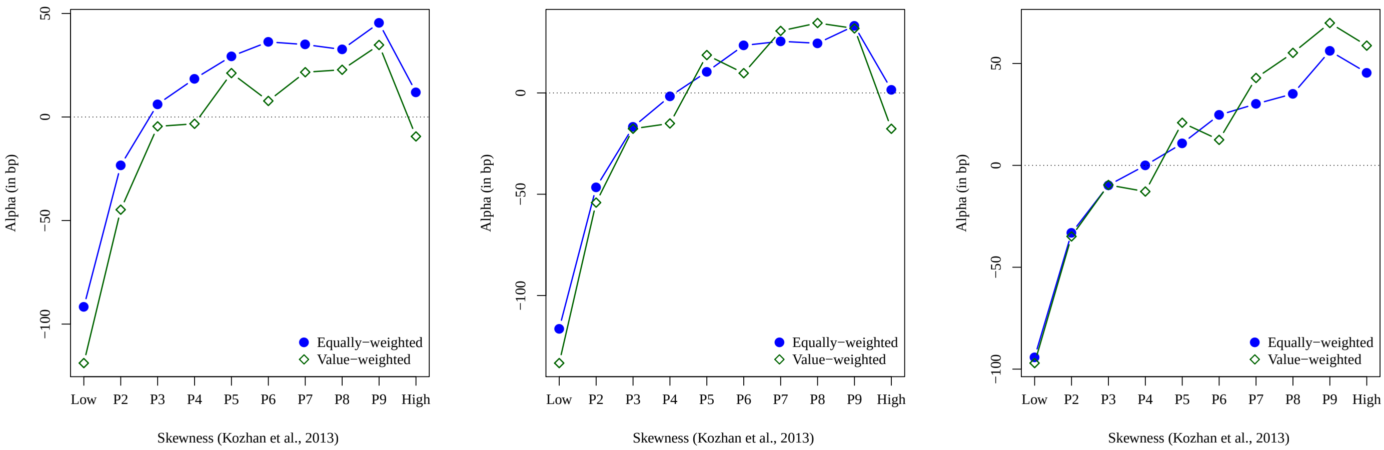
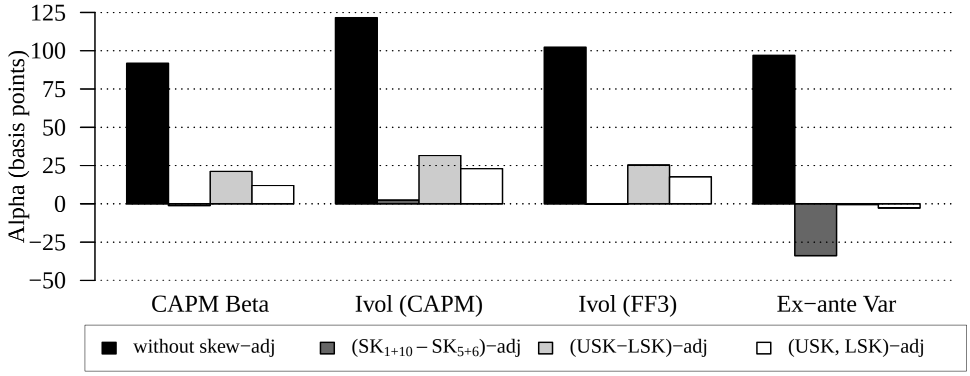


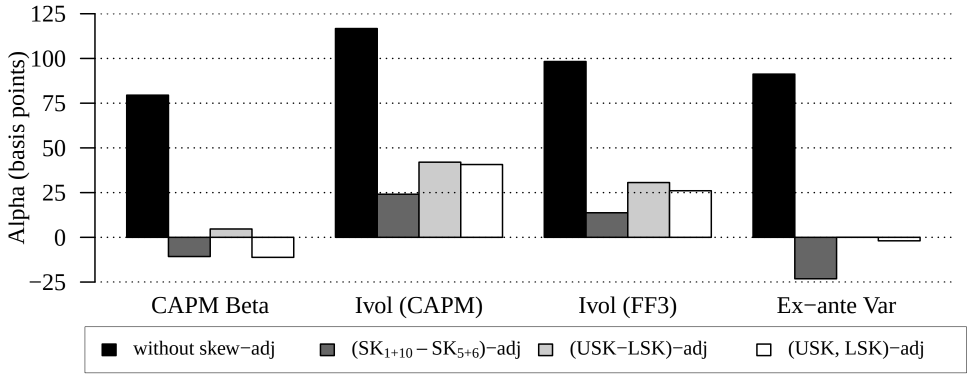
Figure IA.8: Alphas and Skew-Adjusted Alphas: EW Portfolios

This Figure reports alphas and skew-adjusted alphas of low risk anomalies (LRAs). At the end of every month, we sort firms into equally-weighted decile portfolios based on their CAPM beta, idiosyncratic volatility (measured from the residual variance of CAPM and Fama French three factor model regressions), or their equity option-implied ex-ante variance. From these portfolios, we compute Low-minus-High returns generated by betting-against-beta/volatility strategies, and report alphas of CAPM-, Fama-French three-, and four-factor regressions (black bars in Panels A, B, and C, respectively) as well as alphas that additionally include controls for skewness. To adjust for skewness, we use either the $(SK_{1+10} - SK_{5+6})$-factor (dark grey bars), the (USK−LSK)-factor (light grey bars), or both the LSK- and the USK-factor (white bars). The data covers 4,967 US firms, is sampled at a monthly frequency over the period January 1996 to August 2014, and contains a total of 400,449 observations.

Panel A. CAPM alphas



Panel B. FF3 alphas



Panel C. FF4 alphas

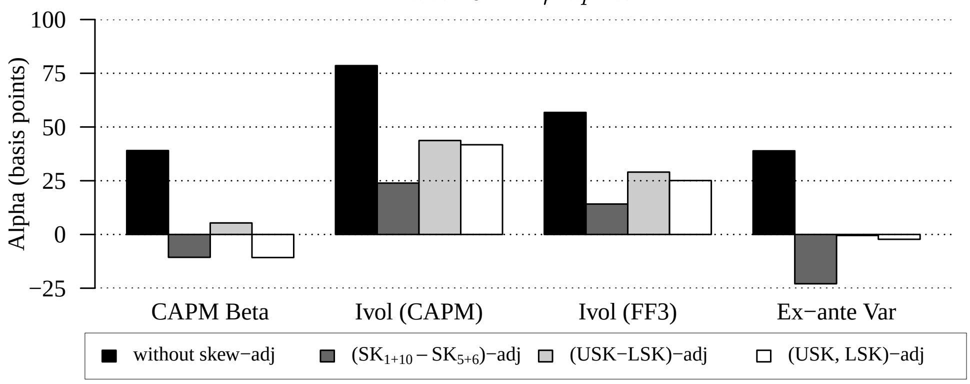
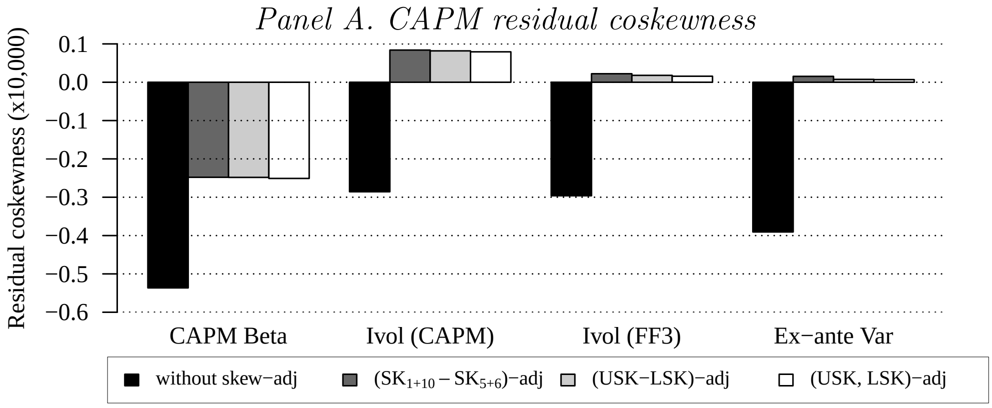
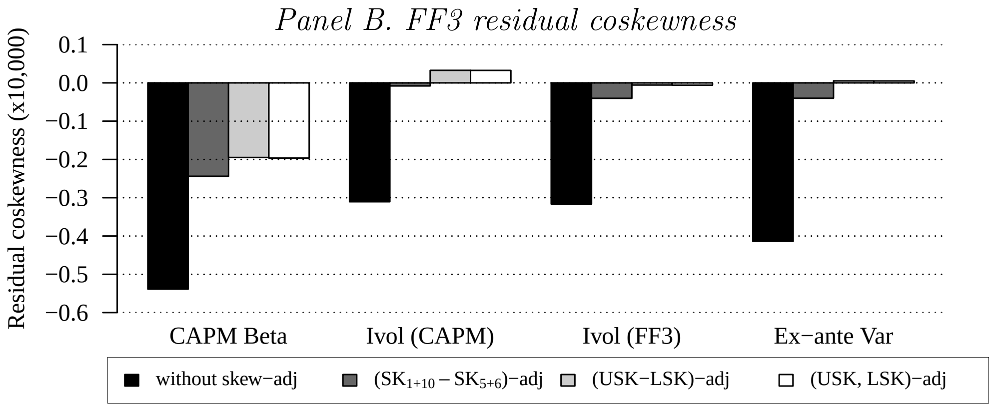


Figure IA.9: Residual Coskewness associated with Alphas and Skew-Adjusted Alphas: EW Portfolios

This Figure reports the residual coskewness associated with alphas and skew-adjusted alphas of low risk anomalies (LRAs). At the end of every month, we sort firms into equally-weighted decile portfolios based on their CAPM beta, idiosyncratic volatility (measured from the residual variance of CAPM and Fama French three factor model regressions), or their equity option-implied ex-ante variance. From these portfolios, we compute Low-minus-High returns generated by betting-against-beta/volatility strategies, and report the coskewness associated with residual returns of CAPM-, Fama-French three-, and four-factor regressions (black bars in Panels A, B, and C, respectively) as well as the residual coskewness when additionally including controls for skewness. To adjust for skewness, we use either the ($SK_{1+10} - SK_{5+6}$)-factor (dark grey bars), the (USK−LSK)-factor (light grey bars), or both the LSK- and the USK-factor (white bars). The data covers 4,967 US firms, is sampled at a monthly frequency over the period January 1996 to August 2014, and contains a total of 400,449 observations.





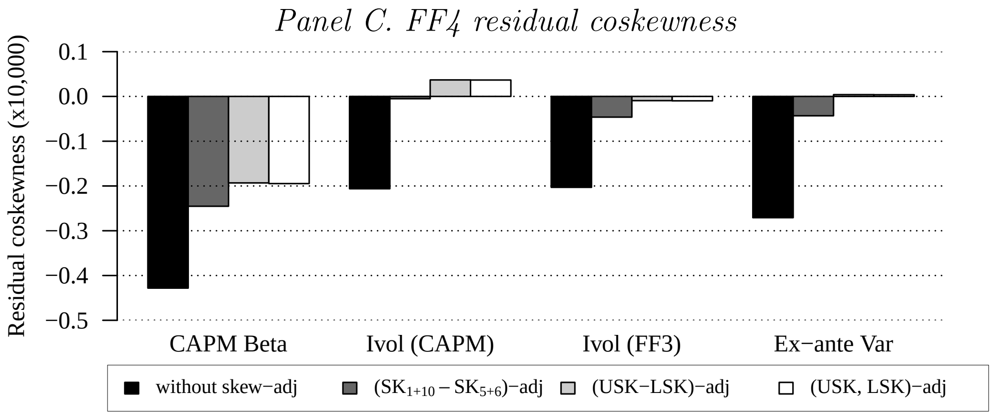


Figure IA.10: Alphas and Skew-Adjusted Alphas: RW Portfolios

This Figure reports alphas and skew-adjusted alphas of low risk anomalies (LRAs). At the end of every month, we sort firms into rank-weighted decile portfolios based on their CAPM beta, idiosyncratic volatility (measured from the residual variance of CAPM and Fama French three factor model regressions), or their equity option-implied ex-ante variance. From these portfolios, we compute Low-minus-High returns generated by betting-against-beta/volatility strategies, and report alphas of CAPM-, Fama-French three-, and four-factor regressions (black bars in Panels A, B, and C, respectively) as well as alphas that additionally include controls for skewness. To adjust for skewness, we use either the $(SK_{1+10} - SK_{5+6})$-factor (dark grey bars), the (USK−LSK)-factor (light grey bars), or both the LSK- and the USK-factor (white bars). The data covers 4,967 US firms, is sampled at a monthly frequency over the period January 1996 to August 2014, and contains a total of 400,449 observations.

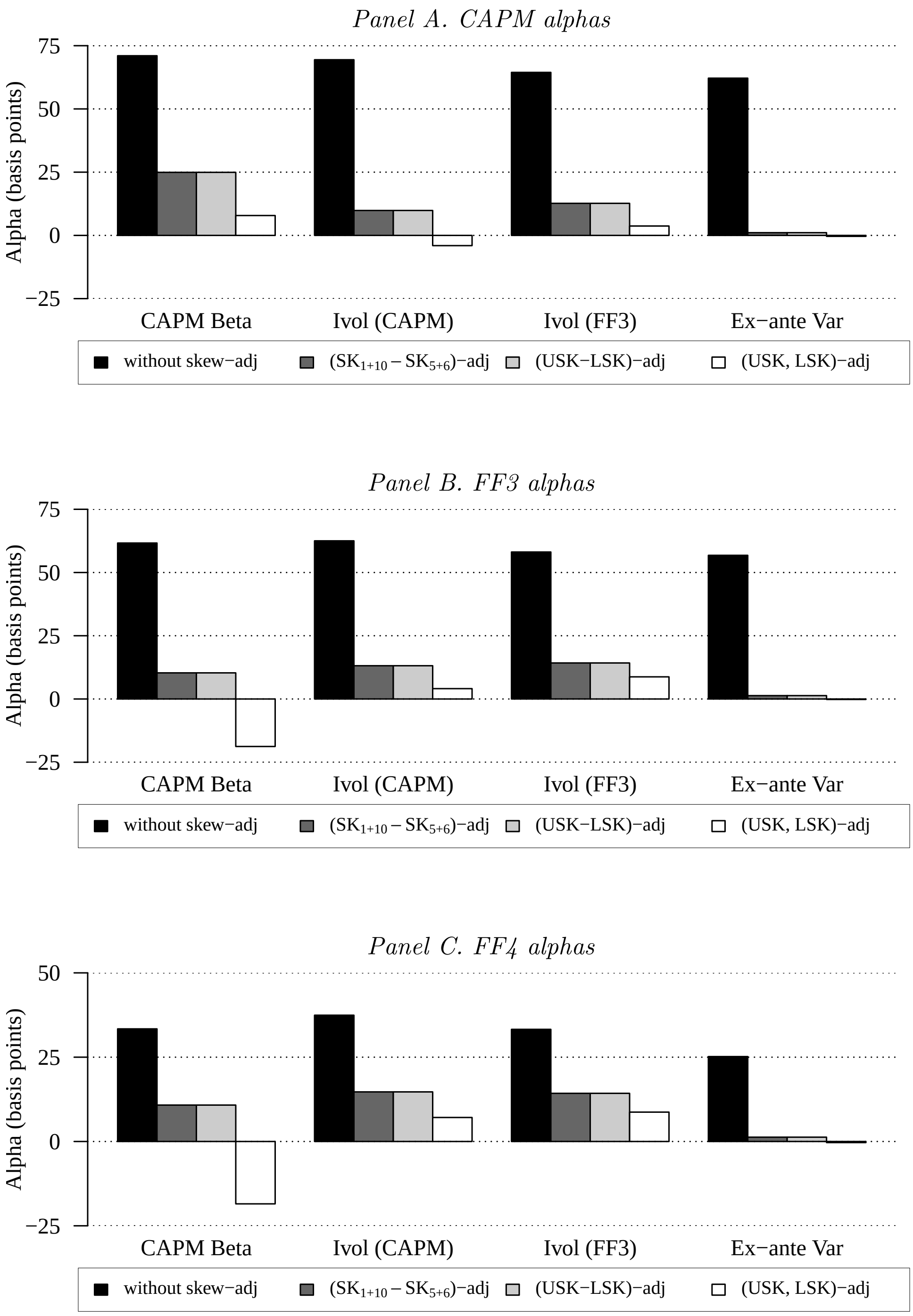


Figure IA.11: Residual Coskewness associated with Alphas and Skew-Adjusted Alphas: RW Portfolios

This Figure reports the residual coskewness associated with alphas and skew-adjusted alphas of low risk anomalies (LRAs). At the end of every month, we sort firms into rank-weighted decile portfolios based on their CAPM beta, idiosyncratic volatility (measured from the residual variance of CAPM and Fama French three factor model regressions), or their equity option-implied ex-ante variance. From these portfolios, we compute Low-minus-High returns generated by betting-against-beta/volatility strategies, and report the coskewness associated with residual returns of CAPM-, Fama-French three-, and four-factor regressions (black bars in Panels A, B, and C, respectively) as well as the residual coskewness when additionally including controls for skewness. To adjust for skewness, we use either the ($SK_{1+10} - SK_{5+6}$)-factor (dark grey bars), the (USK−LSK)-factor (light grey bars), or both the LSK- and the USK-factor (white bars). The data covers 4,967 US firms, is sampled at a monthly frequency over the period January 1996 to August 2014, and contains a total of 400,449 observations.

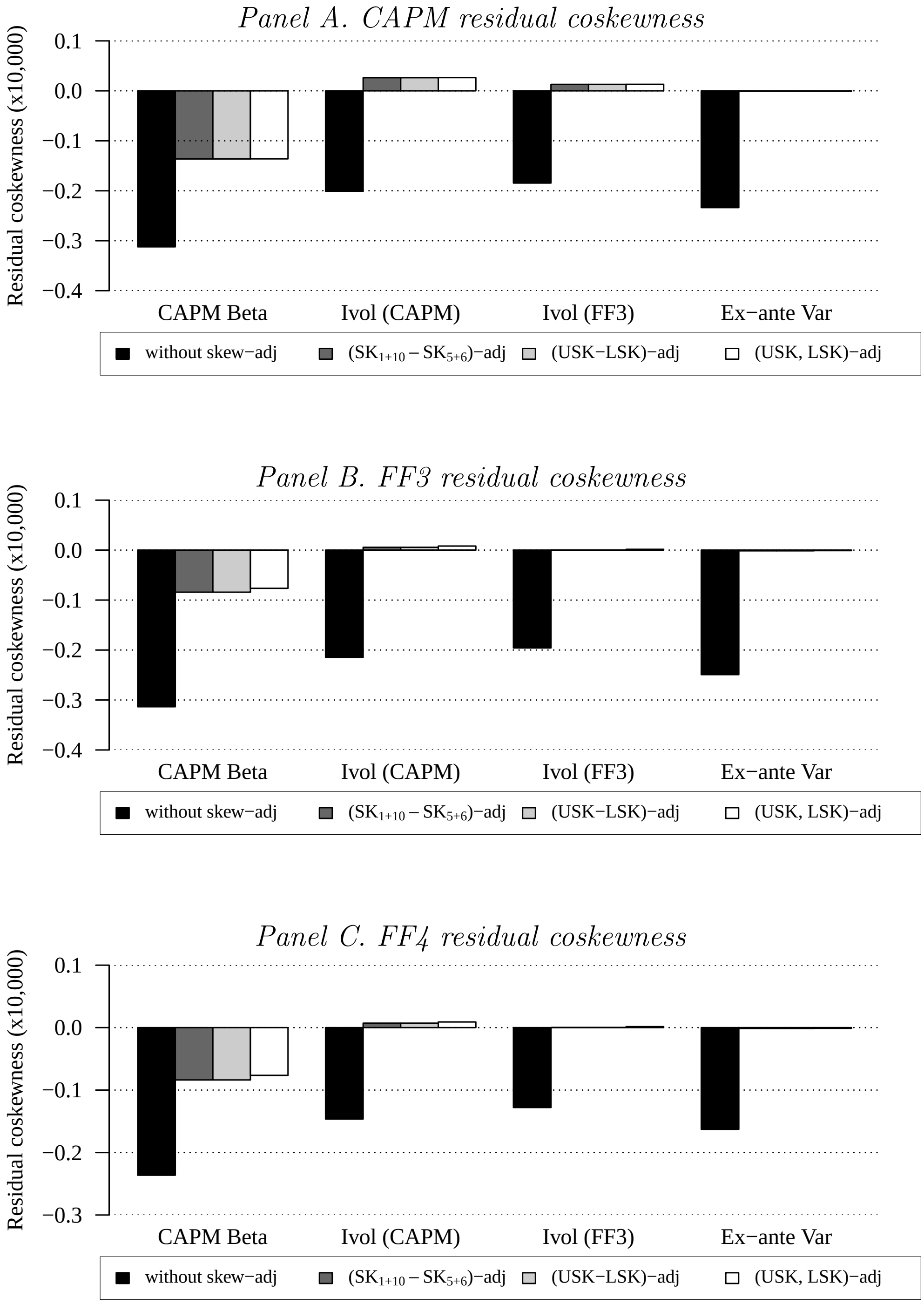
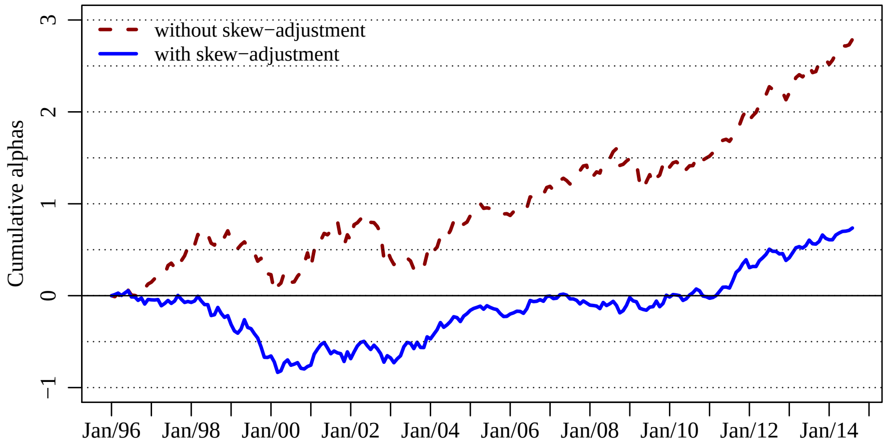


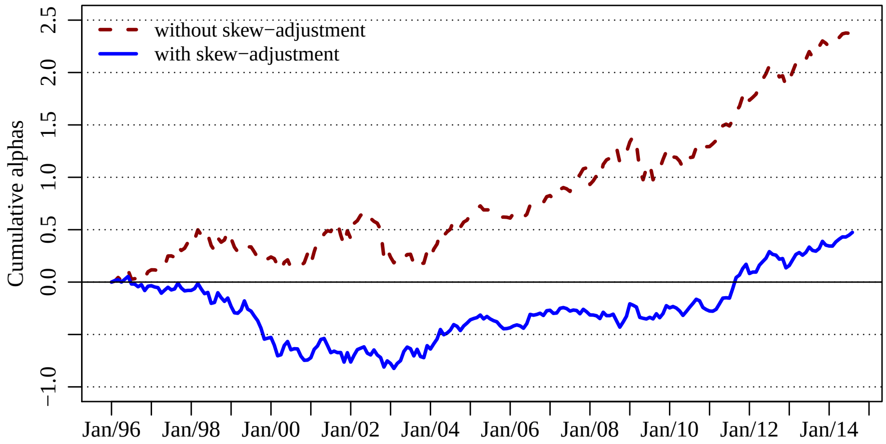
Figure IA.12: Cumulative Alphas of Betting-against-Beta

This Figure reports cumulative alphas and cumulative skew-adjusted alphas for betting-against-beta. At the end of every month, we sort firms into value-weighted decile portfolios based on their CAPM beta. From these portfolios, we compute Low-minus-High returns and report cumulative alphas and skew-adjusted of CAPM (Panel A), FF3 (Panel B), and FF4 (Panel C) regressions. To adjust for skewness, we use both the LSK- and the USK-factor. We compute cumulative alphas by cumulating the intercept and the residuals of the corresponding regressions over time, i.e. the final values of the cumulative alpha series divided by the number of periods correspond to the unadjusted alpha values reported in Table 1 and the skew-adjusted alpha values in Table 3, respectively. The data covers 4,967 US firms, is sampled at a monthly frequency over the period January 1996 to August 2014, and contains a total of 400,449 observations.

Panel A. CAPM alphas



Panel B. FF3 alphas



Panel C. FF4 alphas

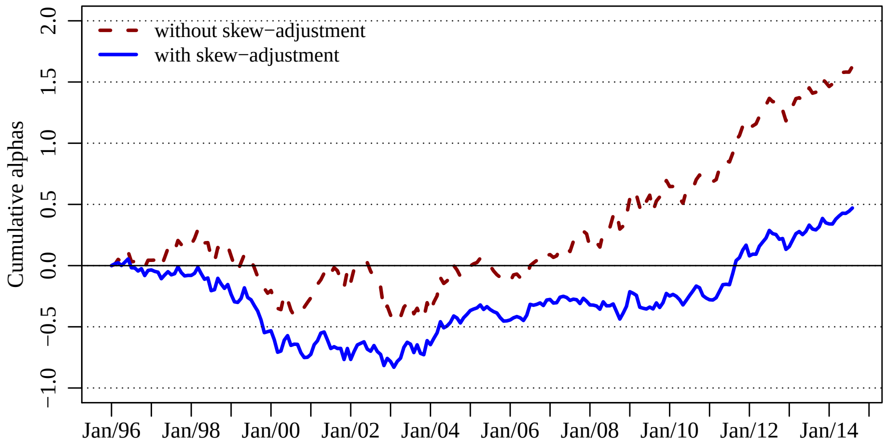
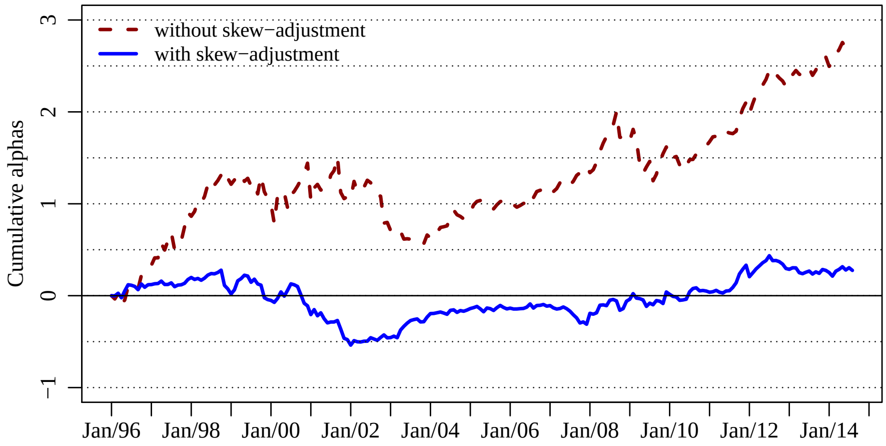


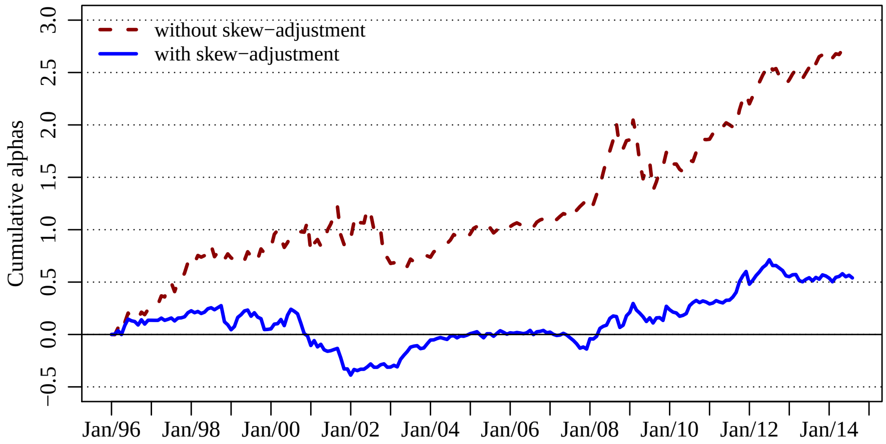
Figure IA.13: Cumulative Alphas of Betting-against-CAPM Idiosyncratic Volatility

This Figure reports cumulative alphas and cumulative skew-adjusted alphas for betting-against-idiosyncratic volatility. At the end of every month, we sort firms into value-weighted decile portfolios based on their CAPM idiosyncratic volatility. From these portfolios, we compute Low-minus-High returns and report cumulative alphas and skew-adjusted of CAPM (Panel A), FF3 (Panel B), and FF4 (Panel C) regressions. To adjust for skewness, we use both the LSK- and the USK-factor. We compute cumulative alphas by cumulating the intercept and the residuals of the corresponding regressions over time, i.e. the final values of the cumulative alpha series divided by the number of periods correspond to the unadjusted alpha values reported in Table 1 and the skew-adjusted alpha values in Table 3, respectively. The data covers 4,967 US firms, is sampled at a monthly frequency over the period January 1996 to August 2014, and contains a total of 400,449 observations.

Panel A. CAPM alphas



Panel B. FF3 alphas



Panel C. FF4 alphas

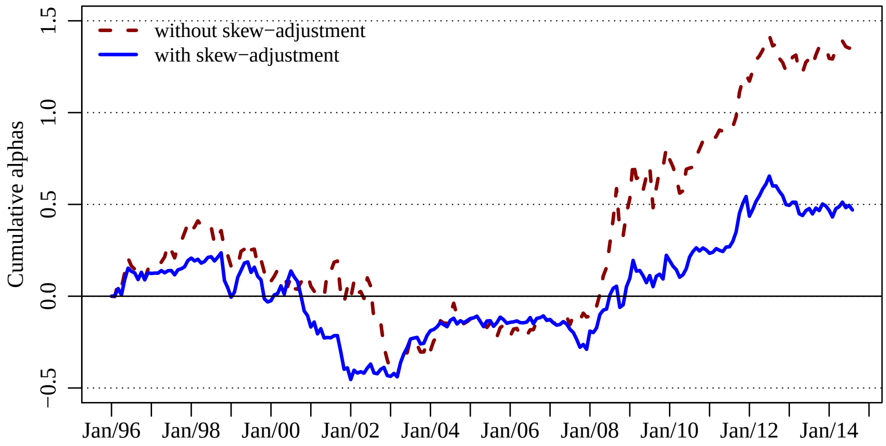
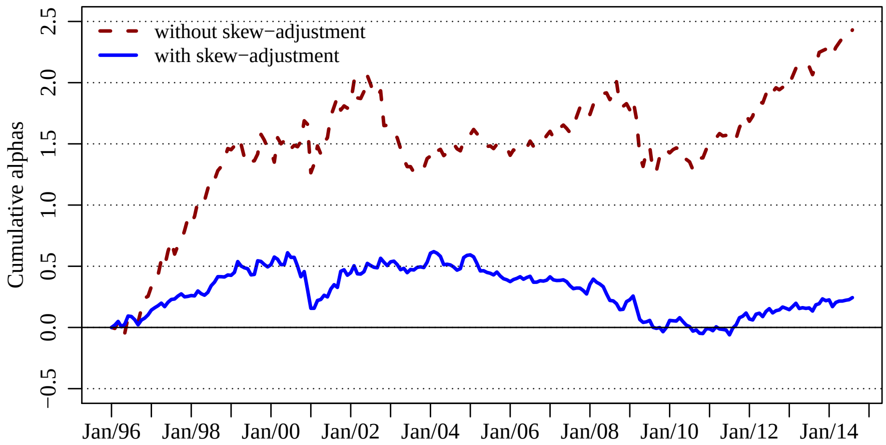


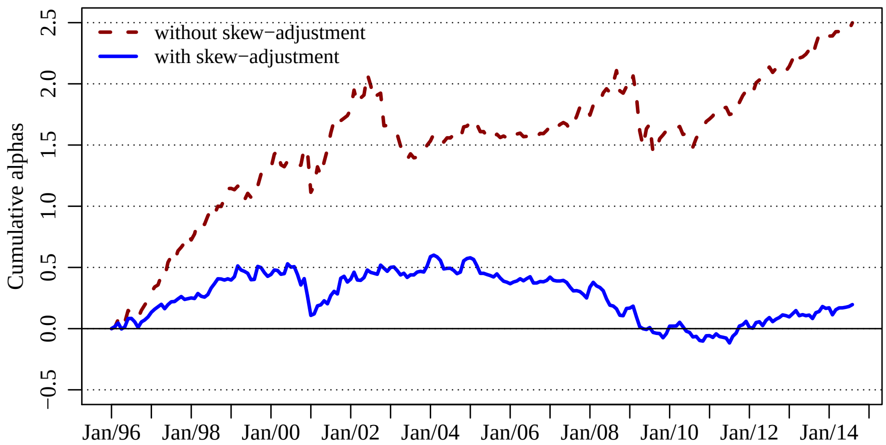
Figure IA.14: Cumulative Alphas of Betting-against-FF3 Idiosyncratic Volatility

This Figure reports cumulative alphas and cumulative skew-adjusted alphas for betting-against-idiosyncratic volatility. At the end of every month, we sort firms into value-weighted decile portfolios based on their FF3 idiosyncratic volatility. From these portfolios, we compute Low-minus-High returns and report cumulative alphas and skew-adjusted of CAPM (Panel A), FF3 (Panel B), and FF4 (Panel C) regressions. To adjust for skewness, we use both the LSK- and the USK-factor. We compute cumulative alphas by cumulating the intercept and the residuals of the corresponding regressions over time, i.e. the final values of the cumulative alpha series divided by the number of periods correspond to the unadjusted alpha values reported in Table 1 and the skew-adjusted alpha values in Table 3, respectively. The data covers 4,967 US firms, is sampled at a monthly frequency over the period January 1996 to August 2014, and contains a total of 400,449 observations.

Panel A. CAPM alphas



Panel B. FF3 alphas



Panel C. FF4 alphas

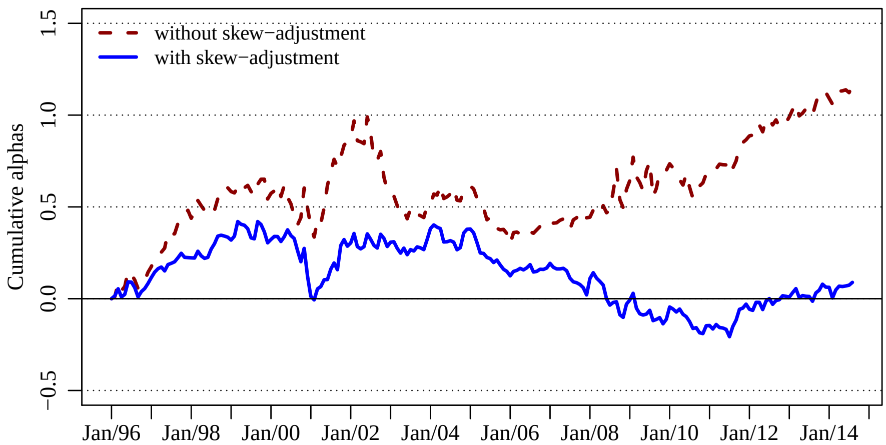
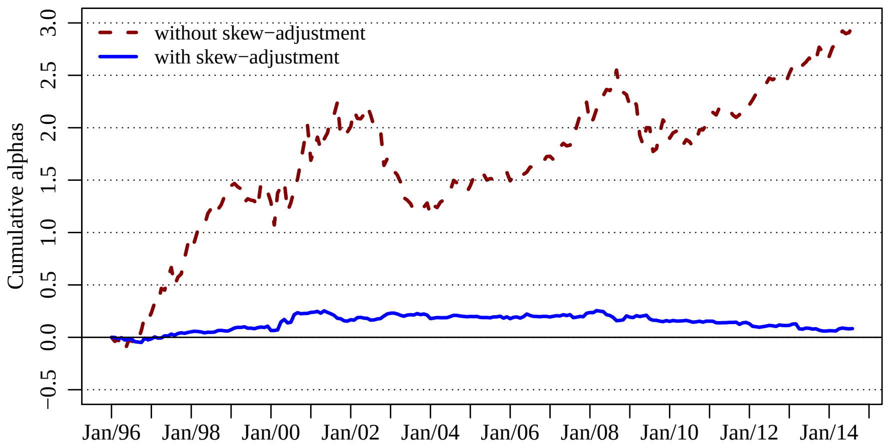


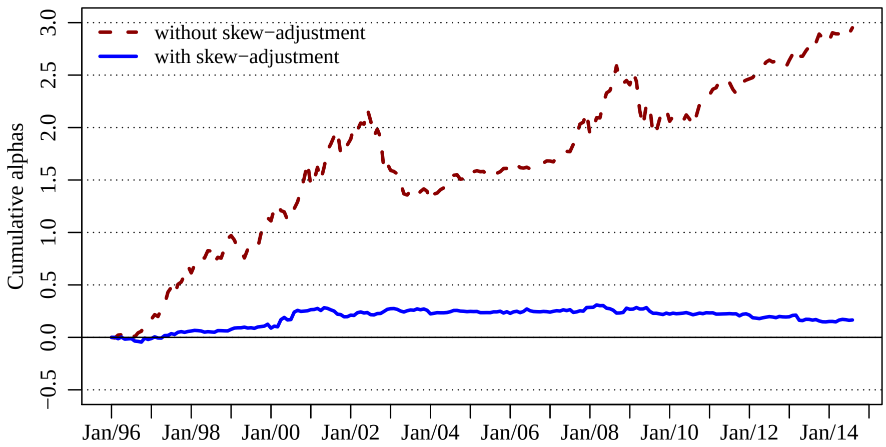
Figure IA.15: Cumulative Alphas of Betting-against-Ex-Ante Variance

This Figure reports cumulative alphas and cumulative skew-adjusted alphas for betting-against-ex-ante variance. At the end of every month, we sort firms into value-weighted decile portfolios based on their equity option-implied ex-ante variance. From these portfolios, we compute Low-minus-High returns and report cumulative alphas and skew-adjusted of CAPM (Panel A), FF3 (Panel B), and FF4 (Panel C) regressions. To adjust for skewness, we use both the LSK- and the USK-factor. We compute cumulative alphas by cumulating the intercept and the residuals of the corresponding regressions over time, i.e. the final values of the cumulative alpha series divided by the number of periods correspond to the unadjusted alpha values reported in Table 1 and the skew-adjusted alpha values in Table 3, respectively. The data covers 4,967 US firms, is sampled at a monthly frequency over the period January 1996 to August 2014, and contains a total of 400,449 observations.

Panel A. CAPM alphas



Panel B. FF3 alphas



Panel C. FF4 alphas

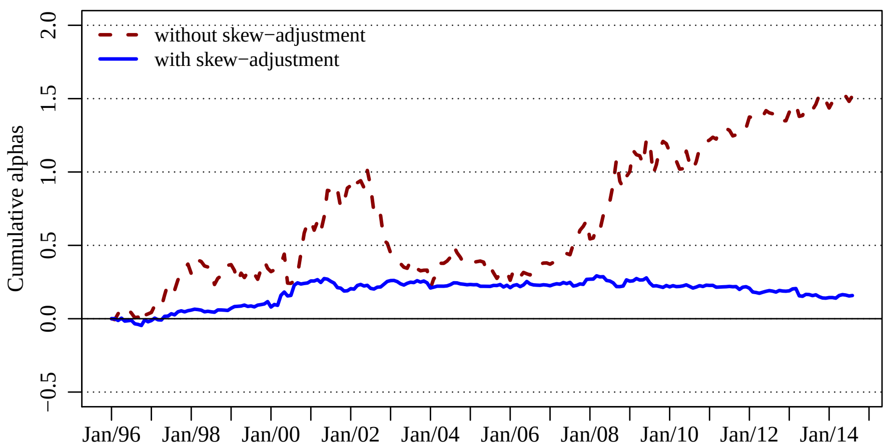
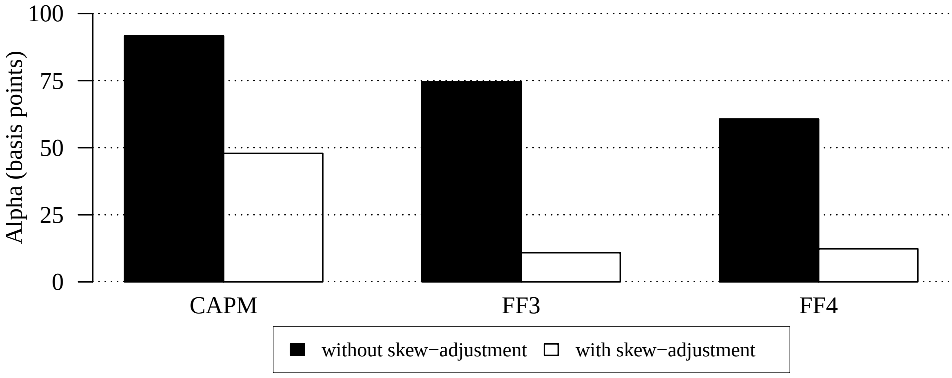


Figure IA.16: Alphas, Skew-Adjusted Alphas, and Residual Coskewness of the BaB-factor

This Figure reports alphas and skew-adjusted alphas of the BaB-factor returns of Frazzini and Pedersen (2014) and the associated residual coskewness. We report alphas of CAPM-, Fama-French three-, and four-factor regressions (black bars) as well as alphas that additionally include controls for skewness (white bars). To adjust for skewness, we use both the LSK- and the USK-factor. Panel B reports the associated residual coskewness. The overlapping period of our sample and the data of Frazzini and Pedersen (2014) is from 01/1996 to 03/2012.

Panel A. Alphas and Skew-Adjusted Alphas



Panel B. Residual Coskewness

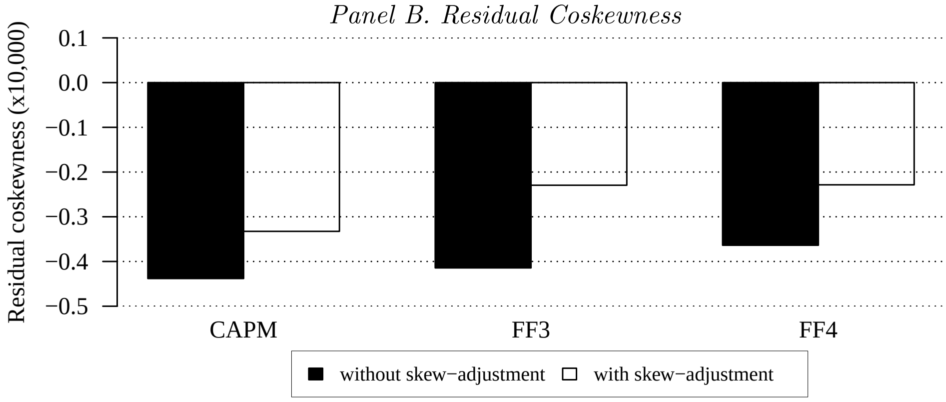
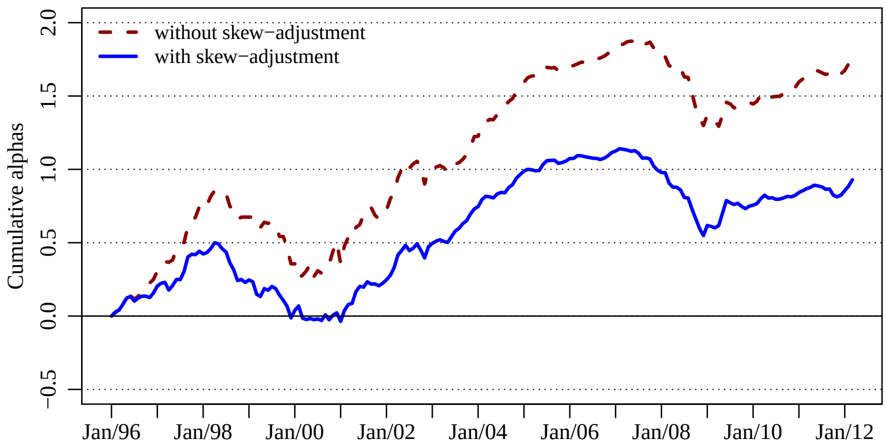


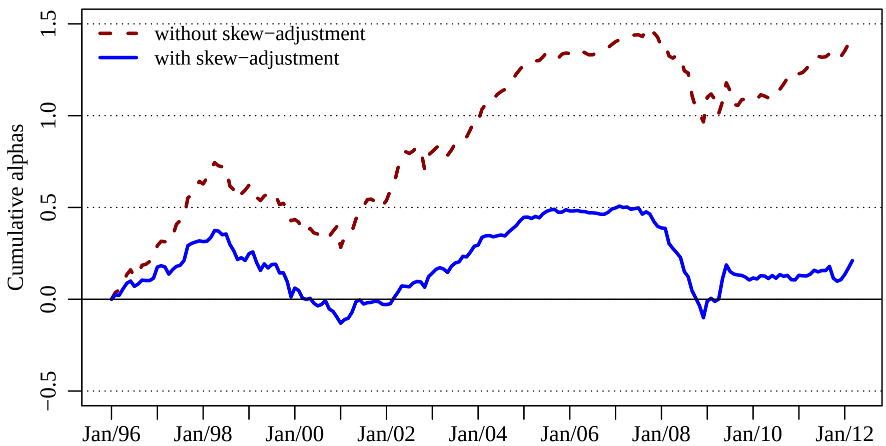
Figure IA.17: Cumulative Alphas of BaB-Factor Returns

This Figure reports cumulative alphas and cumulative skew-adjusted alphas for the BaB-factor of Frazzini and Pedersen (2014). We report cumulative alphas and skew-adjusted of CAPM (Panel A), FF3 (Panel B), and FF4 (Panel C) regressions. To adjust for skewness, we use both the LSK- and the USK-factor. We compute cumulative alphas by cumulating the intercept and the residuals of the corresponding regressions over time, i.e. the final values of the cumulative alpha series divided by the number of periods correspond to the unadjusted and skew-adjusted alpha values reported in Table IA.6. The overlapping period of our sample and the data of Frazzini and Pedersen (2014) is from 01/1996 to 03/2012.

Panel A. CAPM alphas



Panel B. FF3 alphas



Panel C. FF4 alphas

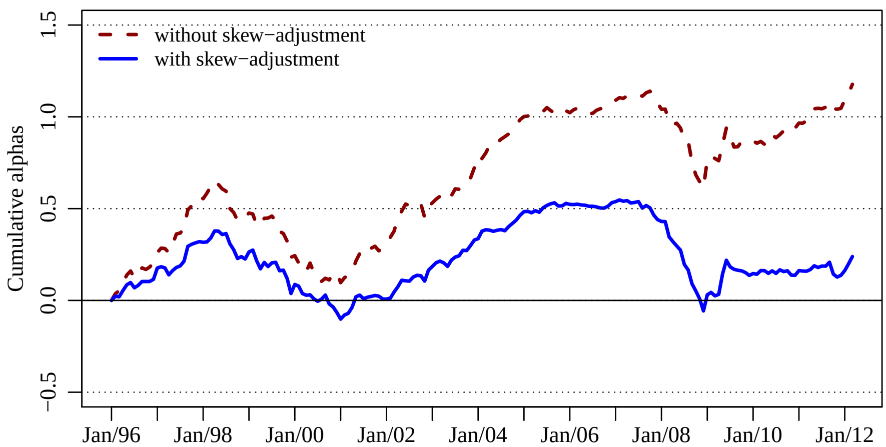
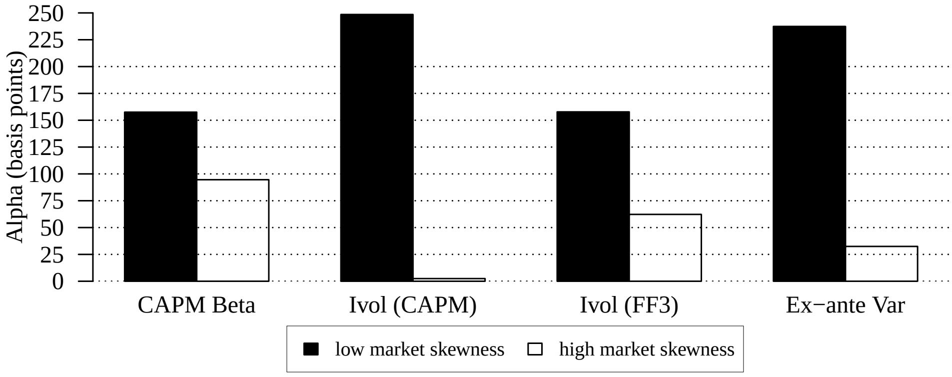


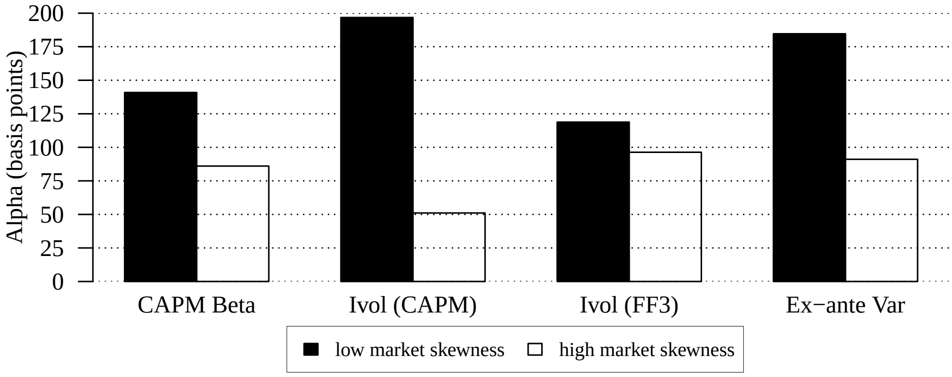
Figure IA.18: Low Risk Anomalies and Market Skewness

This Figure reports alphas and skew-adjusted alphas of low risk anomalies (LRAs) for periods of low compared to high (i.e. less negative) market skewness, measured from the rolling 250-day covariance of market excess returns with squared market excess returns. At the end of every month, we sort firms into value-weighted decile portfolios based on their CAPM beta, idiosyncratic volatility (measured from the residual variance of CAPM and Fama French three factor model regressions), or their equity option-implied ex-ante variance. From these portfolios, we compute Low-minus-High returns generated by betting-against-beta/volatility strategies, and alphas of CAPM-, Fama-French three-, and four-factor regressions. We distinguish between periods of low market skewness (black bars) and high market skewness (white bars) based on the rolling 250-day covariance of market excess returns with squared market excess returns. The data covers 4,967 US firms, is sampled at a monthly frequency over the period January 1996 to August 2014, and contains a total of 400,449 observations.

Panel A. CAPM alphas



Panel B. FF3 alphas



Panel C. FF4 alphas

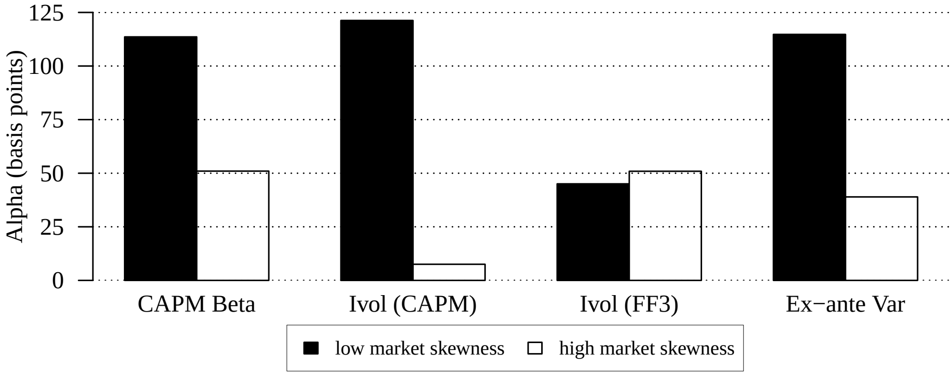
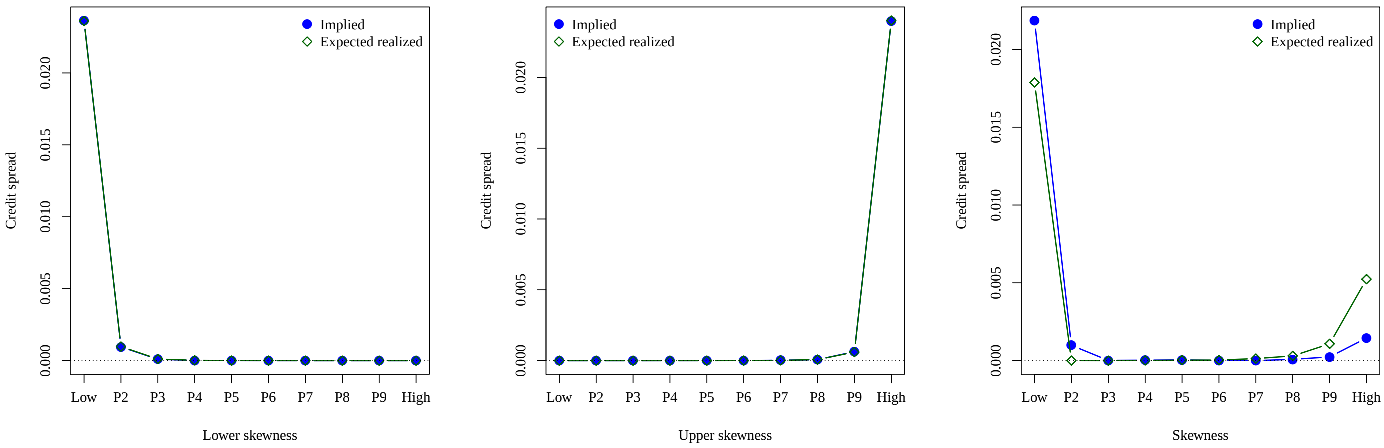


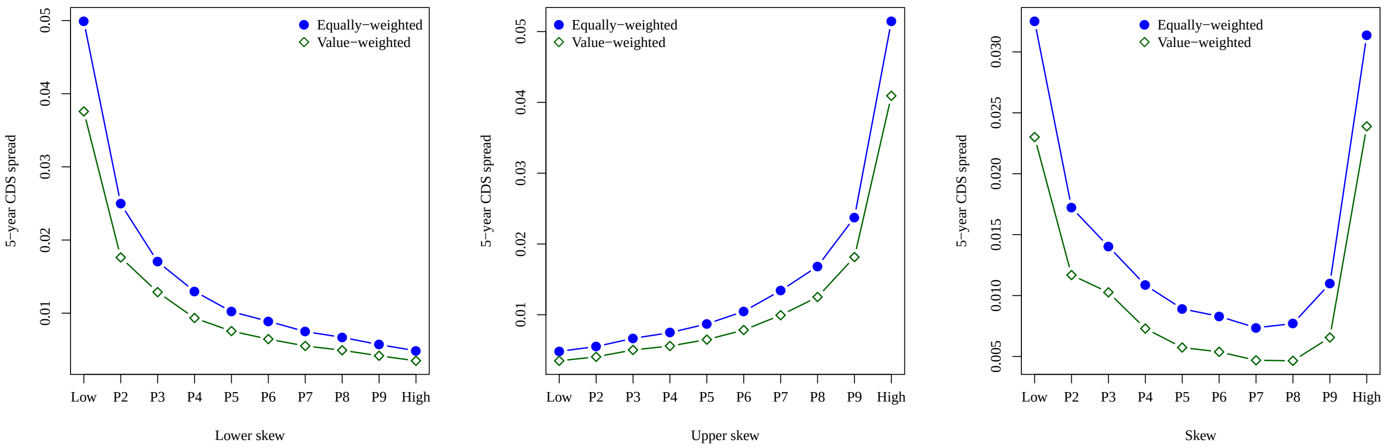
Figure IA.19: Implied Skewness and Credit Risk

This Figure presents results for the relation between firms' implied skewness and measures of credit risk. Panel A presents simulation evidence from a skew-aware world with 2,000 Merton firms and their model-implied credit spreads. Panels B and C present empirical evidence using option-implied measures of ex-ante skewness and CDS spreads or credit ratings respectively. We compute lower skewness (by definition always negative), upper skewness (by definition always positive), and skewness which is the sum of upper and lower skewness. We sort firms into decile portfolios based on the three option-implied skew measures and compute the portfolios average measure of credit risk, where high (low) values generally imply high (low) credit risk. Portfolios P1 and P10 contain the firms with the lowest and highest values of the sort variables, respectively. In the simulated data in Panel A we report results for using measures of Q-skew (blue line with bullets) and P-skew (green line with diamonds) using lower skewness (left), upper skewness (middle), and skewness (right). In the empirical data in Panels B and C, we present results for equally-weighted portfolios (blue line with bullets) and value-weighted portfolios (green line with diamonds). For the analysis of CDS spreads, our dataset contains Markit CDS data for 573 firms from 01/2001 to 03/2010 with a total of 37,514 observations. For the analysis of credit ratings, we obtain the S&P long-term credit ratings via Compustat whenever available for firms in our sample. We convert ratings into numerical data, with lower numbers indicating better ratings, i.e. we asign a value of 1 to AAA-rated firms, 2 to firms with a rating of AA+, 3 to firms with a rating of AA, etc. This results in a subsample of 2,066 firms with a total of 179,816 observations from January 1996 to August 2014.

Panel A. Model simulation: credit spreads



Panel B. Empirical: 5-year CDS spreads



Panel C. Empirical: Credit Ratings