Managing the risk of the "betting-against-beta" anomaly: does it pay to bet against beta?[1]

Pedro Barroso[2] Paulo Maio[3]

First version: November 2016. This version: December 2017.

[1]The present work has benefited from comments and suggestions from Andrew Detzel, Eser Arisoy, Li Yang and Paul Karehnke and numerous seminar participants in Paris-Dauphine, IESE, ESADE, Queensland University of Technology, the 2017 Australasian Banking and Finance Conference, and the annual meetings of the European Financial Management Association (EFMA) and the Spanish Finance Association. We thank in particular the discussants George Chalamandaris, Tomohiro Ando, and Yvonne Kreis. We are grateful to Kenneth French and Lasse Pedersen for providing stock market return data.

[2]University of New South Wales. E-mail: p.barroso@unsw.edu.au

[3]Hanken School of Economics, Department of Finance and Statistics. E-mail: paulofmaio@gmail.com

Abstract

We study the risk dynamics of the betting-against-beta anomaly. The strategy shows strong and predictable time variation in risk and no risk-return trade-off. A risk-managed strategy exploiting this achieves an annualized Sharpe ratio of 1.28 with a very high information ratio of 0.94 with respect to the original strategy. Similar strategies for the market, size, value, profitability, and investment factors achieve a much smaller information ratios of 0.15 on average. The large economic benefits of risk-scaling are similar to those of momentum and set these two anomalies apart from other equity factors. Decomposing risk into a market and a specific component we find the specific component drives our results. The performance of the strategy is also observed in international markets and is robust to transaction costs.

JEL classification: G11; G12; G17.

Keywords: Betting-against-beta, BAB, time-varying risk, realized volatility, momentum, risk factors, scaled factors, market anomalies.

1. Introduction

The capital asset pricing model (CAPM) of Sharpe (1964), Lintner (1965), and Treynor (1965) provides the first theoretically motivated measure of the riskiness of each asset and the expected return it should command in equilibrium. It is arguably the most taught asset pricing model and is widely used in corporate finance, portfolio performance measurement and investment valuation (see e.g. Fama and French (2004) and Damodaran (2012)). However, we know from early empirical tests of the model that low (high) beta stocks have consistently positive (negative) risk-adjusted returns, a result known as the beta anomaly (Black et al. (1972)). Recently, Frazzini and Pedersen (2014) propose an investment strategy ("betting-against-beta" (BAB)) that exploits this anomaly by buying low-beta stocks and shorting high-beta stocks. They report a Sharpe ratio for the strategy of 0.78, about double that of the US equity market.

Besides US equities, Frazzini and Pedersen (2014) show that BAB achieves abnormal returns in international equities, bonds, and currencies. Asness et al. (2014b) and Baker et al. (2014) find similar results examining industry portfolios and macro level country selection. Furthermore, in contrast to other anomalies in equities, the profits of exploiting the beta anomaly seem robust to transaction costs (Asness et al. (2014b)).

So the beta anomaly allows an impressive investment performance, at least from an unconditional perspective. But Cederburg and O'Doherty (2016) find that if one fully accounts for the time-varying systematic risk of the strategy its alpha already becomes insignificant. We propose an alternative approach to the conditional performance of the strategy and examine its time-varying volatility instead.

There is an extensive literature documenting the time varying risk of the stock market (e.g. Schwert (1989), Bollerslev (1987)) and the potential benefits of timing its volatility (Fleming et al. (2001)). Barroso and Santa-Clara (2015b) extend this discussion to the space of long-short equity factor investing by showing the benefits of managing the risk of momentum. More recently Moreira and Muir (2016) find similar benefits of volatility timing for other equity factors.

We find that volatility has an important economic value to condition exposure to the BAB strategy. In our sample, the Sharpe ratio of the BAB strategy increases from 0.91 to 1.28 with risk-management. The information ratio of the risk-managed strategy is 0.94 when compared to its original version, a result similar to that of momentum (0.93). The Fama-French 5-factor alpha of the strategy increases from an annualized 5.48% to 15.97%. Similar to momentum, the benefits come from strong predictability in risk and the absence of a risk-return trade-off. In fact, we find months with extreme high risk for the strategy are followed by lower returns on average.

Our results contrast with those of Cederburg and O'Doherty (2016). They find that conditional on time-varying beta BAB is less of a puzzle. We find the strategy also has strongly time-varying volatility and that using this to manage its risk makes BAB a much deeper puzzle.

We decompose the risk of the strategy into specific and market risk to assess the origins of the gains. We find that market risk plays a relatively minor role in our results. The specific component is the one showing interesting predictability.

Our study is closely related to the literature on the conditional behaviour of the beta anomaly. Cohen et al. (2005) find that consistent with the presence of money illusion in the stock market, the required real discount rate on low-beta stocks increases disproportionately with inflation. As a result the low-beta anomaly is concentrated in periods with moderate to high inflation. Antoniou et al. (2015) show that beta commands a reasonable risk premium in pessimistic periods and show that the anomaly is mainly present at times of optimism. They suggest that optimism attracts unsophisticated and overconfident investors to the market that result in the mispricing of beta. Huang et al. (2015) examine a measure of excess co-movement in the BAB portfolio as a proxy for arbitrage activity. They find that periods of high activity predict short-term returns positively but also more pronounced subsequent reversals. We add to this literature by examining the case for timing the volatility of the strategy.

Our paper is also related to the recent literature on timing the volatility of equity factors (Barroso and Santa-Clara (2015b),Moreira and Muir (2016), Barroso and Maio (2017)). Moreira and Muir (2016) in particular provide a prominent study of such benefits across a set of factors and show they do not seem to be explained by business cycles or analogous low-risk anomalies in the cross section. Comparatively, our study highlights the large economic benefits of managing the risk of BAB, shows its startling similarity with those found for momentum, identifies the component of risk driving the result, and examines potential implications for our understanding of the anomaly.

We test the robustness of our result along three dimensions: alternative methods to estimate volatility, incorporating frictions (leverage constraints and transaction costs), and international evidence. Overall, the results seem to be present in other samples; different methods to estimate volatility do not overturn the analysis; and the benefits of timing the volatility of BAB are not eliminated in constrained portfolios with plausible transaction costs.

This paper is structured as follows. Section 2. compares the performance of BAB with that of other equity factors. Section 3. shows the performance of risk-managed factor portfolios. Section 4. assesses the predictability of risk for the strategy and the existence of a risk-return trade-off. In section 5. we examine a decomposition of the risk of the strategy into its market and specific components. Section 6. examines robustness on three dimensions: i) alternative methods to estimate variance; ii) international evidence; and iii) portfolios with constrained leverage and incorporating transaction costs. Section 7. concludes.

2. Factor investing and betting-against-beta

In this section we describe the equity factors used in this study, provide descriptive statistics of their investment performance, and examine the risk-adjusted performance of betting-against-beta (BAB).

We use the BAB returns for US stocks obtained from AQR's data library ($https://www.aqr.com/library/data-sets$). The strategy consists on buying low-beta stocks and shorting high-beta stocks choosing weights such that the ex-ante beta of the strategy is zero. This implies having more than one dollar in the long leg and less than one dollar in the short leg. The difference is funded by a short position in the risk-free rate asset (T-bills) such that the overall portfolio has zero cost. Frazzini and Pedersen (2014) provide a complete description of the construction of the strategy's portfolio.

We compare the returns of BAB with those of the Fama and French (1993) three factors (FF3 model). The FF3 factors are: i) the excess return of the market over the risk-free rate (market factor or 'RM'); ii) the return of small firms in excess of large firms (size factor or 'SMB'); iii) the return of high book-to-market stocks in excess of low book-to-market stocks (value factor or 'HML'). For completeness we add to these the two newly proposed factors in Fama and French (2015) (FF5 model): operating profitability ('RMW' for robust-minus-weak) and investment ('CMA' for conservative-minus-aggressive). These two factors capture, respectively, the average positive excess returns of firms with high profitability and of those with low investments (as measured by asset growth). Taken together these factors reflect the sources of predictability for the cross section of equity returns found in the size (Banz (1981)), value (Basu (1983), Bondt and Thaler (1985), Rosenberg et al. (1985)), profitability (Novy-Marx (2013), Hou et al. (2014)), and investment (Hou et al. (2014)) variables.

We also use the Carhart (1997) 4-factor model (C4 model) that combines momentum with the FF3 factors. The return of momentum in a given month t is defined as the difference in value-weighted returns between the portfolio of previous winners and the portfolio of previous losers. The previous winners are the stocks in the top decile according to cumulative return from months $t-12$ to $t-2$ while the previous losers are those in the bottom decile. Only stocks listed in NYSE are used to compute the cut-off points of the deciles. This avoids having some of the deciles dominated by many small caps. The month $t-1$ is skipped to avoid confounding with the short-term reversal effect in monthly returns (Jegadeesh (1990), Jegadeesh and Titman (1995)). Momentum returns capture the tendency of recent winners, as defined by their returns in the previous 3 to 12 months, to continue outperforming recent losers (Levy (1967), Jegadeesh and Titman (1993)).

The returns of the momentum portfolios and the FF5 factors are from Kenneth

French's data library.[1] We have both daily and monthly returns available for all factors from July 1963 to December 2015. We use realized volatilities computed from one month and 6 months of data to perform the risk-scaling so our first risk-managed returns start in January 1964 and end in December 2015.

[Insert table 1 near here]

Table 1 shows the descriptive statistics of each factor-based investment strategy. Compared with the market, size, value, momentum, profitability, and investment factors, BAB has the highest Sharpe ratio. Its annualized Sharpe ratio is of 0.91, more than double the 0.38 of the market with is already a central puzzle in financial economics (Mehra and Prescott (1985)). It is also higher than the Sharpe ratio of momentum (0.67) which is often regarded has the major asset pricing anomaly. This illustrates the extent of the empirical failure of the CAPM. Black et al. (1972) find that the security market line (SML) is flatter than what should hold according to the CAPM. Frazzini and Pedersen (2014) show that a strategy exploiting this apparent mispricing has an economic performance even more impressive than momentum.

The strategy's high Sharpe ratio comes with a considerable higher order risk though with an excess kurtosis of 3.58 combined with a skewness of -0.62, both higher in absolute terms than the market that has 1.90 and -0.52 respectively. This higher order risk is small though when compared to the very high excess kurtosis of momentum (7.84) and respective negative skewness (-1.42).

In spite of the impressive long run performance of the strategy it is also exposed to substantial downside risk. Its maximum drawdown of -52.03 is close in magnitude

[1] $http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html$

to that of the market (-55.71) which has a much higher standard deviation (15.45% versus 11.21%). Still, both the higher order risk and the (closely related) maximum drawdown are smaller than those of momentum. So BAB offers a higher Sharpe ratio and it is not as exposed to crashes as momentum (see Daniel and Moskowitz (2016), Barroso and Santa-Clara (2015b) for a discussion of the crash risk of momentum).

Table 2 examines the ability of other risk factors to explain the returns of BAB. The second column shows the alpha of the strategy with respect to the different risk models. Columns 3 to 8 show the factor loadings of the strategy.

The market factor does not explain the returns of BAB. The strategy has a high annualized alpha of 10.54% with a t-statistic of 6.57. The beta of the strategy is very close to zero (-0.06). This shows that the strategy, which is constructed to have an ex-ante beta of zero, on average achieves this goal ex-post.

Jointly the size and value factors explain approximately 20% of the CAPM-alpha of the strategy. This is mainly due to the value factor. The BAB strategy in all specifications that include the value factor shows a significant exposure to that factor. The strategy is also exposed to the momentum, profitability and investment factors. Taken together we see that available stock market risk models explain up to 48% of the CAPM-alpha of the strategy ($1 - 5.48/10.54$). This happens because on average low beta firms also show some loading on value, profitability and (low) investment factors.[2]

This suggests that a substantial part of the empirical failure of the CAPM can

[2]In unreported results we also computed the α with the Hou et al. (2014) model. We find this model is the most successful explaining the returns of BAB, accounting for two-thirds of its α. The focus of this paper, the benefits of risk-management for BAB, are robust to using Hou et al. (2014) though.

be traced to failing to capture the multi-dimensional nature of risk that features in latter models.[3] But this ability of other factors to explain approximately half of the anomaly's risk-adjusted returns still leaves a considerable amount to be explained. The alpha of approximately 5.5 to 6 percentage points a year and respective t-statistics (3.49 in the C4 model and 3.48 in the FF5 model) are clearly economically and statistically significant.

[Insert table 2 near here]

Still, in a world with more than 300 factors or anomalies in the cross-section of stocks, it is legitimate to ask whether the beta anomaly is truly an anomaly or if it can rather be easily explained by some set of known risk factors. That is certainly plausible and some recent studies point in that direction (Hou et al. (2014), DeMiguel et al. (2017), Liu et al. (2016) for example). To this we observe that "age is rank" and, in a sense, the beta anomaly is the seminal anomaly in asset pricing. Nearly half a century after its discovery it can still be confirmed in the data and it remains quite strong. By contrast, many of the factors available now to explain it are of recent discovery and we don't know how robust they will look 50 years from now. That is a valid concern as McLean and Pontiff (2016) shows return predictability diminishes considerably after discovery. Therefore, the extensive post-discovery evidence supporting the beta anomaly should confer it some genuine status.

[3] It also suggests a possible reason for the particularly high Sharpe ratio of the strategy: it is analogous to a linear combination of weakly correlated stock market factors

3. Scaled factor strategies

Barroso and Santa-Clara (2015b) construct a risk-scaled version of momentum. We extend this to other factors and examine the resulting economic performance for an unconstrained investor. We compute the realized variance $RV_{F,t}$ from daily returns in the previous 21 sessions for each month and factor. Let $\{r_d\}_{d=1}^{D}$ be the daily returns and $\{d_t\}_{t=1}^{T}$ the time series of the dates of the last trading sessions of each month. Then the realized variance of factor F in month t is:

$$RV_{F,t} = \sum_{j=0}^{20} r_{F,d_t-j}^2. \tag{1}$$

Then we use the realized volatility $\hat{\sigma}_{F,t} = \sqrt{RV_{F,t}}$ to scale the returns in order to achieve a given target σ_{target}. Implicitly $\hat{\sigma}_{F,t}$ is used as the forecast of $\sigma_{F,t+1}$. All of the original factors used are zero-cost portfolios, so their scaled versions are still zero-cost and self financing strategies that we can scale without constraints. The scaled portfolio weight in the original factor at time t is:

$$W_t = \frac{\sigma_{target}}{\hat{\sigma}_{F,t}} \tag{2}$$

And so the risk-scaled factors are defined as $F_{t+1}^{*} = F_{t+1}W_t$. The choice of volatility target is arbitrary but influences directly the maximum, minimum, mean and the standard deviation of returns as well as the maximum drawdown of the scaled portfolio. However this choice is irrelevant for scale-invariant measures of portfolio performance such as the Sharpe ratio, (left) skewness, and excess kurtosis. As in Barroso and Santa-Clara (2015b) we pick a target corresponding to an annualized volatility of 12%. This choice of target has the desirable property of producing scaled factor portfolios with approximately the same ex-post volatility of the (unscaled) US stock market. Picking the same target for all portfolios facilitates comparison of those performance measures that are sensible to scaling.

Other volatility models could produce more accurate estimates of risk with increased potential for risk-scaling. We refrain from that pursuit in this paper and chose to focus instead on this somehow coarse measure of volatility. This also serves as an implicit control mechanism when assessing the existence of robust economic benefits in risk-scaling.[4]

[Insert table 3 near here]

Table 3 shows the performance of the scaled factors. Risk-scaling has economic gains for investors following (almost) all factor strategies. For the market, the risk-scaled factor has an information ratio of 0.20 with respect to the unscaled factor. This gain confirms the result of Fleming et al. (2001) who document the economic benefits of timing the volatility of the market. Their documented gains from market timing using volatility contrast with the difficulty of similar strategies trying to use predictability in returns (Goyal and Welch (2008)).

The benefits are not restricted to the market factor as 6 out of the 7 portfolios show positive information ratios. The exception is the size premium for which the scaled factor has a negative information ratio. Moskowitz (2003) show that the size premium increases with volatility and recessions in a way consistent with a risk-based explanation. Our results are consistent with theirs for the size factor.

By far the most impressive gains are found for the momentum and BAB factors. Comparing with table 1 the Sharpe ratio of momentum increases from 0.67 to

[4] As a robustness test we also use the usual workhorse of volatility modelling, the GARCH(1,1). We present those results in table 10.

1.08 (a 0.41 gain) and that of BAB increases from 0.91 to 1.28 (a 0.37 gain). The information ratio of the scaled strategies with respect to their original versions are very large for these two factors: 0.93 for MOM* and 0.94 for BAB*. These high information ratios reflect the fact that the risk-scaled portfolios are highly correlated with the original factors but much more profitable. For the FF5 factors the information ratio is on average 0.15, momentum and BAB have information ratios about 6 times larger. Our results for momentum confirm the findings of Barroso and Santa-Clara (2015b). Moreira and Muir (2016) recently examine the benefits of risk management in a set of factors that includes the FF5 but do not consider BAB. Therefore, to the best of our knowledge, we are the first to document that BAB shares this puzzling feature with momentum: the two strategies have particularly expressive economic benefits from risk management, gains that set them both apart from the FF5 factors.

[Insert table 4 near here]

Table 4 shows the risk-adjusted performance of BAB*. The strategy has a very high CAPM-alpha of 21.10% per year. The risk model with the best fit for the strategy is the C4 that explains 25% of its CAPM-alpha ((21.10%-15.83%)/21.10%). This is a smaller proportion than the almost half explained in table 2. So risk-managed BAB offers more diversification benefits than BAB for diversified investors exposed to other equity factors. The smallest annualized alpha is a very high 15.83% (with the C4 model) and the adjusted r-squares of the regressions are all smaller than in table 2. This confirms that BAB* is of value for diversified investors and more so than BAB.

[Insert figure 1 near here]

Figure 1 shows the cumulative returns of momentum, betting-against-beta and their respective risk-managed versions. As the strategies considered are zero-cost portfolios their returns are excess returns. In order to have gross returns we add to each strategy the gross return of an investment in the risk-free rate. So each moment in time the portfolio puts all wealth in the risk-free rate and combines this with a long-short portfolio. [5]

In a pure CAPM world none of the strategies in figure 1 should have any drift but empirically they have had an impressive economic performance. One US dollar invested in momentum in July 1963 grows to $9,990$ by the end of the sample. For the betting-against beta strategy the investment grows to a more modest amount of $1,675$ dollars but with much less risk than the momentum strategy. The scaled strategies have similar ex post standard deviation (17.37 for MOM* versus 16.84 for BAB*) but very different end results: the investment in risk-managed momentum grows to $87,843$ but in the beta anomaly to $364,121$. For comparison a similar investment in the US stock market grows to 139 US dollars by the end of the sample. This illustrates with an investment approach the extent of the puzzle in the performance of these strategies, particularly the benefits derived from managing their risk.

A related debate is whether investors could attain these performances in a realistic setting with transactions costs and other frictions (see, for example, Lesmond et al. (2004) and Asness et al. (2014a) for different interpretations of the evidence). If not, then one possible interpretation of our results is that the

[5] For example, for momentum the original strategy puts a notional amount in the long leg of $Wealth_t$ and in the short leg of $-Wealth_t$. For the risk-managed version the notional amount would be $W_tWealth_t$ and $-W_tWealth_t$ in the long and short legs respectively. For BAB the amounts in the long and short leg are different in order to target a beta of zero but they are offset by positions in the T-bills.

performance of these strategies, without frictions, is too puzzling to accept any explanation that does not incorporate them. While the argument is appealing, in section 6.2., we find the benefits of risk-management show interesting resilience to plausible trading costs and leverage constraints.

4. Risk and return of BAB

We examine the predictability of the risk of BAB and its relation with expected returns.

First we examine the predictability of risk. Figure 2 shows the time series of the (annualized) monthly realized variances computed from daily returns.

[Insert figure 2 near here]

The plot illustrates the typical patterns of volatility known from Bollerslev (1987), Schwert (1989) and many others. Namely, volatility varies over time in a persistent manner. The realized volatility of the strategy varies significantly from a minimum of 1.75% in April 1965 to a maximum of 83% reached in May 2002. The series is also highly non-normal with a kurtosis of 35.44 and a very high positive skewness of 4.52 indicating that the sample contains extreme high risk outliers.

To examine the predictability of risk we run the regression:

$$\frac{1}{\hat{\sigma_{i,t+1}}} = \rho_0 + \rho_1 \frac{1}{\hat{\sigma_{i,t}}} + \varepsilon_{t+1}. \tag{3}$$

We focus on the inverse of realized volatility for two reasons. First, the risk-managed strategy puts a weight on BAB proportional to this quantity so it is the variable of direct interest to understand the source of the gains. Second, the sample of the inverse of volatilities is much closer to normal. It has a kurtosis of 2.99 and a skewness of -0.11, this allows for a much better inference reducing the large weight of outliers in the fit.

Table 5 shows the results of regression 3. Given the similar gains of risk managements for BAB and momentum we focus on these two strategies for the remainder of the paper.

[Insert table 5 near here]

The table shows that both series have similar levels of predictability. The slope coefficient is positive and it has a t-statistic of 23.88 for BAB and 25.31 for MOM. This shows that safe months tend to follow safe months. Most of the variation in the dependent variable is explained by its lagged value (R^2 of 60.79% for BAB and 56.11% for MOM).

But in sample predictability can be misleading and so we also compute the out-of-sample (OOS) R^2 of the regression. For that purpose we use an initial window S of 120 months ro run the first in-sample regression and use this to make a forecast for month $S+1$ that we compare with the ex-post realized value of the variable. The following month we use an expanding window to estimate the regression and re-iterate the procedure until the end of the sample. The OOS R^2 for a factor i is estimated as:

$$R^2_{i,OOS} = 1 - \frac{\sum_{t=S}^{T-1} (\widehat{\rho}_{0,t} + \widehat{\rho}_{1,t} \widehat{1/\sigma_{i,t}} - \widehat{1/\sigma_{i,t+1}}}{\sum_{t=S}^{T-1} (\overline{\widehat{1/\sigma_{i,t}}} - \widehat{1/\sigma_{i,t+1}})^2}, \tag{4}$$

where $\overline{\widehat{1/\sigma_{i,t}}}$ is the historical average up to time t and both $\widehat{\alpha}_t$ and $\widehat{\rho}_t$ are estimated with information available only up to time t. A positive OOS R^2 shows that on average the forecast from the regression outperforms that obtained from the historical average.

We find the OOS R^2 for BAB is not only positive but also very close to its in-sample counterpart. This shows that the regression shows strong robustness out-of-sample – a stark contrast with similar regressions for the equity premium that typically achieve negative OOS R^2's (Goyal and Welch (2008)). The predictability of the (inverse of) realized volatility for BAB is quite similar to the one documented for momentum.

[Insert table 6 near here]

We next examine if this predictability in risk is related with expected returns. For this purpose we regress the returns of each strategy on its lagged (inverse of) realized volatility.

$$r_{i,t+1} = \gamma_0 + \gamma_1(1/\hat{\sigma_{i,t}}) + \eta_{t+1}. \tag{5}$$

The existence of a risk-return trade-off should be captured by a statistically significant negative γ_1 meaning that months with less risk for the strategy also predict smaller returns. The results in table 6 show there is no evidence of any predictability of risk for the returns of BAB (or momentum). In fact the point estimates for both strategies are positive, although not statistically significant.

[Insert figure 3 near here]

In figure 3 we further examine the relation between risk and return for the strategies. For each strategy, we sort all months into quintiles according to realized volatility. Then, for each quintile, we take the subset of months subsequent to those in that quintile and compute the average return, standard deviation, and Sharpe ratio over that subset. The results for momentum confirm the pattern in Barroso and Santa-Clara (2015b) that higher risk predicts both lower returns and higher risk for the strategy, so the Sharpe ratio of the strategy is much higher following quintiles 1 to 3 than in the two top quintiles. The results for BAB show a similar pattern. In panel A the returns following a month in quintile 5 are lower (4.84%) on average than for other quintiles (11.53%). The relation is not monotonous and the underperformance of BAB in terms of expected returns is concentrated in months of particularly high risk. On the other hand, panel B shows that subsequent risk rises monotonically with lagged risk. The annualized volatility is 5.13% after a month in quintile 1 and it is 19.06% after a month in quintile 5. As a result the Sharpe ratio is much smaller after months in the top quintile (0.25) versus months after the lowest quintile (1.83). Both effects combine such that BAB has a quite weak performance as a factor subsequent to months of very high risk.

[Insert figure 4 near here]

Figure 4 shows the weights of BAB* and MOM*. On average BAB* has a higher weight than MOM*. This happens because both strategies target the same volatility but original BAB has a much lower standard deviation than MOM.[6] As the choice of target volatility is arbitrary this difference in the average weight is not very informative. We note though that the two series have a strong and statistically significant correlation of 0.63 (at the 1%) level. So an hypothetical unconstrained

[6] A version of BAB* re-scaled to have a similar average weight as MOM* would also show a very similar standard deviation in weights, so the apparent wilder swings of the weights in BAB are only due to a level effect.

arbitrageur following both strategies simultaneously would see speculative capital being absorbed and freed-up from these two uses at the same time. This suggests a possible limit to arbitrage for risk-management.

5. Anatomy of BAB risk

Grundy and Martin (2001) show that the beta of momentum with respect to the market varies substantially over time. Cederburg and O'Doherty (2016) argue that the conditional beta of BAB explains its premium. Motivated by this we examine if time-varying systematic risk can account for the similar gains of risk management for BAB and MOM and the performance of BAB in particular.

[Insert figure 5 near here]

Figure 5 shows the plot of the time series of monthly betas of BAB and MOM with respect to the market. The results for momentum confirm Grundy and Martin (2001). The beta of MOM ranges between -2.86 and 2.57. We find that BAB also has some time-varying market risk. In spite of being deliberately constructed to be market neutral, its beta ranges between -1.57 and 0.30 and it shows some persistence. Still the beta of BAB shows relatively little time-variation when compared to momentum.

We use the CAPM to decompose the risk of BAB and MOM into a market and a specific component:

$$RV_{F,t} = \beta_t^2 RV_{RM,t} + \sigma_{\varepsilon,t}^2. \tag{6}$$

The realized variances are estimated from daily returns in each month. [7] We find that on average 37% of the risk of BAB is systematic, so the market neutral strategy has a substantial market risk component. To assert the origin of the gains, we examine the performance of a risk-managed strategy using each of the risk components separately.

[Insert table 7 near here]

Table 7 shows the performance of risk-scaled BAB and momentum with total risk, specific risk, and market risk respectively. The Sharpe ratio of BAB scaled with specific risk is 1.21, close to the one using total risk (1.32). By contrast, the strategy using the systematic risk has a very low Sharpe ratio of 0.18 and a extreme value for standard deviation (3898.63), excess kurtosis (610.79), and skewness (24.68). The results for momentum are similar to those of BAB. They both show that the gains of risk-management come from using the informational content in the specific component of risk.

We also examine the predictability of the inverse of each component of risk. Table 8 shows the results of an auto-regression for each of those components (which correspond to the weights in the strategies examined in table 7).

[Insert table 8 near here]

For BAB the specific part of risk is highly persistent with a slope coefficient of 0.68 and a t-statistic of 20.30. The regression shows a good fit both in-sample and OOS with R-squares of 46.13% and 47.46% respectively. The predictability of

[7] To ensure the decomposition holds exactly, we use in this section the variance formula. This de-means returns each month instead of just summing the squares of returns as in equation 1. This does not change any of the results substantially and avoids the possibility of getting a negative specific risk.

the specific component is almost as high as that of total risk. For the systematic component there is no predictability. The slope coefficient is zero and so is the R-squared. The results for momentum are similar and confirm those of Barroso and Santa-Clara (2015b). For both strategies the predictable part of risk is the specific component.

6. Robustness tests

6.1. Alternative estimates of variance

We assess the robustness of our results with respect to the horizon used to compute the realized variance and using the GARCH(1,1) instead to estimate variance.

Table 9 shows the performance of scaled factors using 6 months to estimate variance instead of just one. First we note that for this particular window the result of the economic value of timing the volatility of the market is not robust.

[Insert table 9 near here]

The information ratio for risk-managed momentum is 0.98, so the benefits for this strategy are robust to this alternative period to compute variance. For BAB, with a Sharpe ratio of 1.17 and an information ratio of 0.76 with respect to the original strategy, we conclude that the benefits of risk-management are robust to using an alternative window.

We also use GARCH(1,1) from monthly returns of each strategy to compute risk-managed strategies. In order to assess the benefits of risk-management in a realistic OOS environment we only use volatility estimates obtained from a

GARCH(1,1) model estimated in real time. We use the initial 120 months of data to infer the parameters of the initial model and use them to make a forecast for month 121. The following month we expand the sample with a new observation and re-estimate the model. We keep re-iterating the procedure until the end of the sample. Table 10 shows the economic performance of the risk-managed portfolios. For comparison table 11 shows the performance of the original strategies for this different sample period.

[Insert tables 10 and 11 near here]

In this setting the benefits of risk-management for the market are not robust with a negative information ratio of -0.18. For momentum the benefits are robust with a gain in Sharpe ratio from 0.57 to 0.71 and a substantial reduction in the maximum drawdown from -80.36% to just -31.88%. For BAB the results are quite robust. The information ratio of BAB* is 0.65, even higher than the 0.43 of MOM*. So in this particular exercise the benefits of risk-management for BAB are stronger than for MOM.

6.2. Leverage constraints and transaction costs

Standard asset pricing research focuses on frictionless markets where portfolios combine assets freely in a linear space. As an illustration of this, Asness et al. (2013) explain in page 976 of their paper: "[...] Like most academic studies, we focus on gross returns, which are most suitable to illuminating the relation between risk and returns." We also believe analysis with gross returns are informative per se and the often used alternative of computing the costs of isolated strategies is not necessarily preferable.

In a world where investors can combine different signals to form portfolios, it is not straightforward to assess after-cost profitability of isolated signals. Barroso and Santa-Clara (2015a) show that the trading costs of a stand-alone strategy (e.g. momentum) can be substantially reduced if the same strategy is used in combination with other characteristics (such as the carry trade). DeMiguel et al. (2017) quantify precisely the reduction in trading costs of using several characteristics in combination and show that depends crucially on their interaction.

Still, from an asset pricing perspective, it is relevant to distinguish between correct prices driven by risk loadings from mispricings that are hard to arbitrage. Figure 4 shows that risk-managed BAB, as we define it, implies both substantial leverage and fast changes in weights. A typical investor is constrained on the leverage he can assume and incurs in costs to trade. So, it is important to ask if an hypothetical arbitrageur, as a hedge fund, still finds any benefit in timing the volatility of BAB in a realistic setting. As a result, we examine the conventional case of an investor exploiting a single source of predictable risk-adjusted returns - our strategy - while keeping in mind sophisticated arbitrageurs would probably want to combine several strategies to reduce costs.

We follow Moreira and Muir (2016) who estimate alphas net of costs under different assumptions for transaction costs: 1 basis point (bp) as in Fleming et al. (2003), 10bps as in Frazzini et al. (2012) and an additional scenario of 14bps reflecting the cost of trading in conditions of high volatility (VIX at 40%, its 98$^{\text{th}}$ percentile). Volatility management implies trading to reduce positions after increases in risk, when risk is high. This makes the 14 bps assumption potentially pertinent to our study.[8]

[8] On the other hand it should be kept in mind that BAB* also implies trading to increase

Besides trading costs we also consider a version of BAB* constrained to have maximum leverage of 50% (an exposure of 150% to the original factor) and no leverage (an exposure of 100% to the factor).

[Insert tables 12]

Table 12 shows the results. BAB* has an α w.r.t. original BAB of 8.86% but this comes with a very high turnover of 102% per month (2×0.51). Still, after transaction costs of 14bps, most of the α remains (7.15%). The break-even transaction cost that eliminates the α of volatility timing is 73bps, above typical estimates of transactions costs.

Nevertheless, much of the turnover of the original strategy can be reduced implementing leverage constraints. The strategy in panel A, with no leverage has a turnover of just 6% a month, less than a tenth of BAB* (most of the time the strategy would just keep an exposure of 100% to the factor). The break-even cost of this strategy would be very high (220 bps) but is α is also much smaller (1.55% a year). This is understandable as the strategy is very close to the original.

As mentioned above, changing the volatility target is not relevant to assess Sharpe ratios and information ratios. It is just akin to a dislocation along the CAL and not a shift in the curve. But the same does not apply to the after-cost alphas of different volatility managed strategies. Panels B and C show that it is possible to achieve higher risk-adjusted returns for given transaction costs if the volatility target is smaller. Intuitively, strategies with high volatility will find the leverage constraint binding most of the time - as a result they are almost identical to original BAB and their α's are small. For the same reasoning, strategies with

positions after decreases in risk (when risk is low) and, generally, much larger positions in low-risk environments.

small volatility targets imply little leverage on average and produce interesting risk-adjusted returns.

Generally the results in table 12 suggest transaction costs and leverage constraints do not eliminate the benefits of volatility-management. In fact, versions of BAB* with combinations of tight leverage constraints and modest volatility targets show very high cut-off transactions costs between 91 and 220bps. So we conclude the performance of BAB* has an interesting resilience to practical considerations.

6.3. International evidence

In this subsection we show the performance of BAB* in five major international markets available in AQR's website: Global market, Global without the US, Europe, North America, and Pacific. These samples are different from the ones in the rest of our study both geographically and temporally.

[Insert tables 13]

The results in table 13 show that BAB* has information ratios between 0.68 and 1.07, all of them positive and statistically significant at the 1% level. While there is some overlap of the Global and the North American market with our main sample, the results for Europe, Pacific, and Global Ex-USA provide external validation. Our initial version of the paper only had US data. So we see this as out-of-sample confirmation of our main results.

7. Conclusion

Approximately half of the risk-adjusted returns of betting-against-beta with respect to the market are explained by the Carhart (1997) or the Fama and French (2015) factor models. While the remainder risk-adjusted performance is still a puzzle in its own right, managing the risk of the strategy makes it much larger. The risk-managed strategy has a high information ratio of 0.94, similar to that of momentum. The scaled BAB has a higher alpha with respect to the market and less of it (only 25%) is explained by equity risk factors. So managing the risk of BAB creates and economic value orthogonal to those equity factors and even the original BAB itself.

Risk is highly predictable for BAB, similar to that of momentum. There is also no evidence of a risk-return trade-off for the strategy. If anything there seems to be the opposite of a trade-off: BAB generally performs worse after months of high risk in the strategy. This bad performance is concentrated in periods of extreme risk for the strategy suggesting a regime switch in high volatility states.

Time-varying systematic risk has been proposed as an explanation of the premium of BAB (Cederburg and O'Doherty (2016)). We find evidence confirming the existence of this time-varying systematic risk but also find that it conveys relatively little information about the conditional performance of the strategy. Decomposing the risk of BAB into market and specific risk we find that the predictable part is the specific one. Also, the performance of a strategy scaled by specific risk only is very similar to one using total risk. So we conclude that specific risk is causing the gains of risk management.

Generally, our results show that risk-managed BAB is a deeper puzzle than its original version. The informational content of its own lagged volatility to condition exposure to the strategy is very large, similar to momentum and much higher than that found for the other equity factors examined.

The results are robust to using a different window to compute volatilities or a GARCH(1,1) model to estimate it.

References

Antoniou, C., Doukas, J. A., and Subrahmanyam, A. (2015). Investor sentiment, beta, and the cost of equity capital. *Management Science*, 62(2):347–367.

Asness, C. S., Frazzini, A., Israel, R., and Moskowitz, T. J. (2014a). Fact, fiction and momentum investing. *Journal of Portfolio Management, Fall*.

Asness, C. S., Frazzini, A., and Pedersen, L. H. (2014b). Low-risk investing without industry bets. *Financial Analysts Journal*, 70(4):24–41.

Asness, C. S., Moskowitz, T. J., and Pedersen, L. H. (2013). Value and momentum everywhere. *The Journal of Finance*, 68(3):929–985.

Baker, M., Bradley, B., and Taliaferro, R. (2014). The low-risk anomaly: A decomposition into micro and macro effects. *Financial Analysts Journal*, 70(2):43–58.

Banz, R. W. (1981). The relationship between return and market value of common stocks. *Journal of financial economics*, 9(1):3–18.

Barroso, P. and Maio, P. F. (2017). The risk-return tradeoff among equity factors.

Barroso, P. and Santa-Clara, P. (2015a). Beyond the carry trade: Optimal currency portfolios. *Journal of Financial and Quantitative Analysis*, 50(5):1037–1056.

Barroso, P. and Santa-Clara, P. (2015b). Momentum has its moments. *Journal of Financial Economics*, 116(1):111–120.

Basu, S. (1983). The relationship between earnings' yield, market value and return for nyse common stocks: Further evidence. *Journal of financial economics*, 12(1):129–156.

Black, F., Jensen, M. C., and Scholes, M. S. (1972). The capital asset pricing model: Some empirical tests. In Jensen, M., editor, *Studies in the Theory of Capital Markets*. Praeger Publishers Inc.

Bollerslev, T. (1987). A conditionally heteroskedastic time series model for speculative prices and rates of return. *The review of economics and statistics*, pages 542–547.

Bondt, W. F. and Thaler, R. (1985). Does the stock market overreact? *The Journal of finance*, 40(3):793–805.

Carhart, M. M. (1997). On persistence in mutual fund performance. *The Journal of finance*, 52(1):57–82.

Cederburg, S. and O'Doherty, M. S. (2016). Does it pay to bet against beta? on the conditional performance of the beta anomaly. *The Journal of finance*, 71(2):737–774.

Cohen, R. B., Polk, C., and Vuolteenaho, T. (2005). Money illusion in the stock market: The modigliani-cohn hypothesis. *The Quarterly journal of economics*, 120(2):639–668.

Damodaran, A. (2012). *Investment valuation: Tools and techniques for determining the value of any asset*, volume 666. John Wiley & Sons.

Daniel, K. D. and Moskowitz, T. J. (2016). Momentum crashes. *Journal of Financial Economics*, 122(2):221–247.

DeMiguel, V., Martin-Utrera, A., Nogales, F. J., and Uppal, R. (2017). A portfolio perspective on the multitude of firm characteristics.

Fama, E. F. and French, K. R. (1993). Common risk factors in the returns on stocks and bonds. *Journal of financial economics*, 33(1):3–56.

Fama, E. F. and French, K. R. (2004). The capital asset pricing model: Theory and evidence. *The Journal of Economic Perspectives*, 18(3):25–46.

Fama, E. F. and French, K. R. (2015). A five-factor asset pricing model. *Journal of Financial Economics*, 116(1):1–22.

Fleming, J., Kirby, C., and Ostdiek, B. (2001). The economic value of volatility timing. *The Journal of Finance*, 56(1):329–352.

Fleming, J., Kirby, C., and Ostdiek, B. (2003). The economic value of volatility timing using realized volatility. *Journal of Financial Economics*, 67(3):473–509.

Frazzini, A., Israel, R., and Moskowitz, T. J. (2012). Trading costs of asset pricing anomalies.

Frazzini, A. and Pedersen, L. H. (2014). Betting against beta. *Journal of Financial Economics*, 111(1):1–25.

Goyal, A. and Welch, I. (2008). A comprehensive look at the empirical performance of equity premium prediction. *Review of Financial Studies*, 21(4):1455–1508.

Grundy, B. D. and Martin, J. S. M. (2001). Understanding the nature of the risks and the source of the rewards to momentum investing. *Review of Financial studies*, 14(1):29–78.

Hou, K., Xue, C., and Zhang, L. (2014). Digesting anomalies: An investment approach. *Review of Financial Studies*.

Huang, S., Lou, D., and Polk, C. (2015). The booms and busts of beta arbitrage. *Available at SSRN 2666910*.

Jegadeesh, N. (1990). Evidence of predictable behavior of security returns. *The Journal of Finance*, 45(3):881–898.

Jegadeesh, N. and Titman, S. (1993). Returns to buying winners and selling losers: Implications for stock market efficiency. *The Journal of finance*, 48(1):65–91.

Jegadeesh, N. and Titman, S. (1995). Short-horizon return reversals and the bid-ask spread. *Journal of Financial Intermediation*, 4(2):116–132.

Lesmond, D. A., Schill, M. J., and Zhou, C. (2004). The illusory nature of momentum profits. *Journal of financial economics*, 71(2):349–380.

Levy, R. A. (1967). Relative strength as a criterion for investment selection. *The Journal of Finance*, 22(4):595–610.

Lintner, J. (1965). The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets. *The review of economics and statistics*, pages 13–37.

Liu, J., Stambaugh, R. F., and Yuan, Y. (2016). Absolving beta of volatility's effects.

McLean, R. D. and Pontiff, J. (2016). Does academic research destroy stock return predictability? *The Journal of Finance*, 71(1):5–32.

Mehra, R. and Prescott, E. C. (1985). The equity premium: A puzzle. *Journal of monetary Economics*, 15(2):145–161.

Moreira, A. and Muir, T. (2016). Volatility managed portfolios. *The journal of finance*, forthcomming.

Moskowitz, T. J. (2003). An analysis of covariance risk and pricing anomalies. *Review of Financial Studies*, 16(2):417–457.

Novy-Marx, R. (2013). The other side of value: The gross profitability premium. *Journal of Financial Economics*, 108(1):1–28.

Rosenberg, B., Reid, K., and Lanstein, R. (1985). Persuasive evidence of market inefficiency. *The Journal of Portfolio Management*, 11(3):9–16.

Schwert, G. W. (1989). Why does stock market volatility change over time? *The journal of finance*, 44(5):1115–1153.

Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *The journal of finance*, 19(3):425–442.

Treynor, J. L. (1965). How to rate management of investment funds. *Harvard business review*, 43(1):63–75.

Table 1

Performance of factor-based strategies

The performance of the betting-against-beta factor (BAB) is compared with the Fama-French-Carhart risk factors: market (RMRF), size (SMB), value (HML), momentum (MOM), profitability (RMW), and investment (CMA). All statistics are computed with monthly returns. Reported are the maximum and minimum one-month returns observed in the sample, the mean average excess return (annualized), the (annualized) standard deviation of each factor, excess kurtosis, skewness, (annualized) Sharpe ratio, and the maximum drawdown (MDD). The sample returns are from 1964:01 to 2015:12. The max, min, mean, standard deviation and maximum drawdown are in percentage points.

Factor	Max.	Min.	Mean	S.D.	Kurt.	Skew.	Sharpe	MDD
RMRF	16.10	-23.24	5.92	15.45	1.90	-0.52	0.38	-55.71
SMB	22.32	-16.70	2.89	10.79	5.64	0.53	0.27	-52.82
HML	13.91	-13.11	4.04	9.94	2.61	0.01	0.41	-45.21
MOM	26.16	-45.79	16.23	24.26	7.84	-1.42	0.67	-80.36
RMW	12.19	-17.57	2.94	7.38	11.55	-0.40	0.40	-39.17
CMA	9.51	-6.81	3.69	6.98	1.65	0.29	0.53	-17.62
BAB	12.89	-15.22	10.19	11.21	3.58	-0.62	0.91	-52.03

Table 2
Alphas for BAB

This table presents the alphas and factor loadings from time-series regressions of the betting-against-beta factor (BAB). The factor models are the CAPM, Fama-French three-factor model (FF3), Carhart's four-factor model (C4), and the Fama-French five-factor model (FF5). The factors are the market (RM), size (SMB), value (HML), momentum (MOM), profitability (RMW), and investment (CMA). The sample is 1964:01 to 2015:12. For each regression, the first row presents the coefficient estimates and the second row reports Heteroskedasticity-adjusted t-ratios (in parentheses). α is the annualized intercept. R^2 denotes the adjusted coefficient of determination. Bold t-ratios indicate statistical significance at the 5% level.

Model	α(%)	β_{RM}	β_{SMB}	β_{HML}	β_{MOM}	β_{RMW}	β_{CMA}	R^2(%)
CAPM	10.54	−0.06						0.51
	(**6.57**)	(−1.25)						
FF3	8.26	0.03	−0.01	0.44				14.31
	(**5.27**)	(0.56)	(−0.10)	(**6.46**)				
C4	5.97	0.07	0.00	0.50	0.11			19.64
	(**3.49**)	(1.40)	(0.01)	(**7.06**)	(**4.10**)			
FF5	5.48	0.08	0.11	0.28		0.46	0.38	22.00
	(**3.48**)	(1.76)	(**2.17**)	(**3.06**)		(**5.90**)	(**3.21**)	

Table 3

Performance of scaled factor strategies

The performance of the scaled betting-against-beta factor (BAB*) is compared with the Fama-French-Carhart scaled risk factors: market (RMRF*), size (SMB*), value (HML*), momentum (MOM*), profitability (RMW*), and investment (CMA*). Each scaled factor uses the realized variance in the previous month to scale the exposure to the original factor. All statistics are computed with monthly returns. Reported are the maximum and minimum one-month returns observed in the sample, the mean average excess return (annualized), the (annualized) standard deviation of each factor, excess kurtosis, skewness, (annualized) Sharpe ratio, and the maximum drawdown (MDD). IR denotes the (annualized) information ratio of the scaled factor with respect to the unscaled benchmark. The sample returns are from 1964:01 to 2015:12. The max, min, mean, standard deviation and maximum drawdown are in percentage points.

Factor	Max.	Min.	Mean	S.D.	Kurt.	Skew.	Sharpe	IR	MDD
RMRF*	13.94	-18.66	6.30	14.63	1.14	-0.27	0.43	0.20	-52.82
SMB*	23.60	-22.94	4.21	18.86	1.85	-0.10	0.22	-0.04	-84.68
HML*	22.84	-22.78	8.41	18.67	1.19	0.15	0.45	0.20	-57.42
MOM*	20.74	-15.43	18.79	17.37	0.82	-0.17	1.08	0.93	-37.33
RMW*	18.26	-17.15	8.65	17.14	0.49	0.03	0.50	0.31	-57.97
CMA*	18.22	-13.84	8.15	16.24	0.07	0.17	0.50	0.07	-44.02
BAB*	17.59	-28.54	21.48	16.84	2.70	-0.41	1.28	0.94	-56.75

Table 4

Alphas for scaled BAB

This table presents the alphas and factor loadings from time-series regressions of the scaled betting-against-beta factor (BAB*). BAB* uses the realized variance in the previous month to scale the exposure to the original factor. The factor models are the CAPM, Fama-French three-factor model (FF3), Carhart's four-factor model (C4), and the Fama-French five-factor model (FF5). The factors are the market (RM), size (SMB), value (HML), momentum (MOM), profitability (RMW), and investment (CMA). The sample is 1964:01 to 2015:12. For each regression, the first row presents the coefficient estimates and the second row reports Heteroskedasticity-adjusted t-ratios (in parentheses). α is the annualized intercept. R^2 denotes the adjusted coefficient of determination. Bold t-ratios indicate statistical significance at the 5% level.

Model	α(%)	β_{RM}	β_{SMB}	β_{HML}	β_{MOM}	β_{RMW}	β_{CMA}	R^2(%)
CAPM	21.10	0.06						0.19
	(**8.75**)	(0.90)						
FF3	18.22	0.16	0.03	0.55				9.22
	(**7.68**)	(**2.33**)	(0.45)	(**7.66**)				
C4	15.83	0.20	0.04	0.61	0.12			11.75
	(**6.42**)	(**2.87**)	(0.54)	(**8.39**)	(**4.04**)			
FF5	15.97	0.22	0.10	0.35		0.30	0.44	11.14
	(**6.34**)	(**2.93**)	(1.33)	(**3.66**)		(**2.96**)	(**2.83**)	

Table 5

Autoregression of monthly realized volatilities

The realized variances of each factor represent the sum of factor squared daily returns in each month. The factors are betting-against-beta (BAB) and momentum (MOM). The AR(1) process regresses the (non-overlapping inverse of the) factor monthly realized volatility on its own lagged value. Heteroskedasticity-adjusted t-ratios are presented in parentheses. OOS R^2(%) denotes the out-of-sample (OOS) R^2, which relies on an initial (in-sample) window of 120 months. The sample period is from 1963:12 to 2015:12. Bold t-ratios indicate statistical significance at the 5% level.

Factor	ρ_0 (t-stat)	ρ_1 (t-stat)	R^2(%)	OOS R^2(%)
BAB	12.96	0.78	60.79	63.36
	(**7.49**)	(**23.88**)		
MOM	7.16	0.75	56.11	60.41
	(**9.12**)	(**25.31**)		

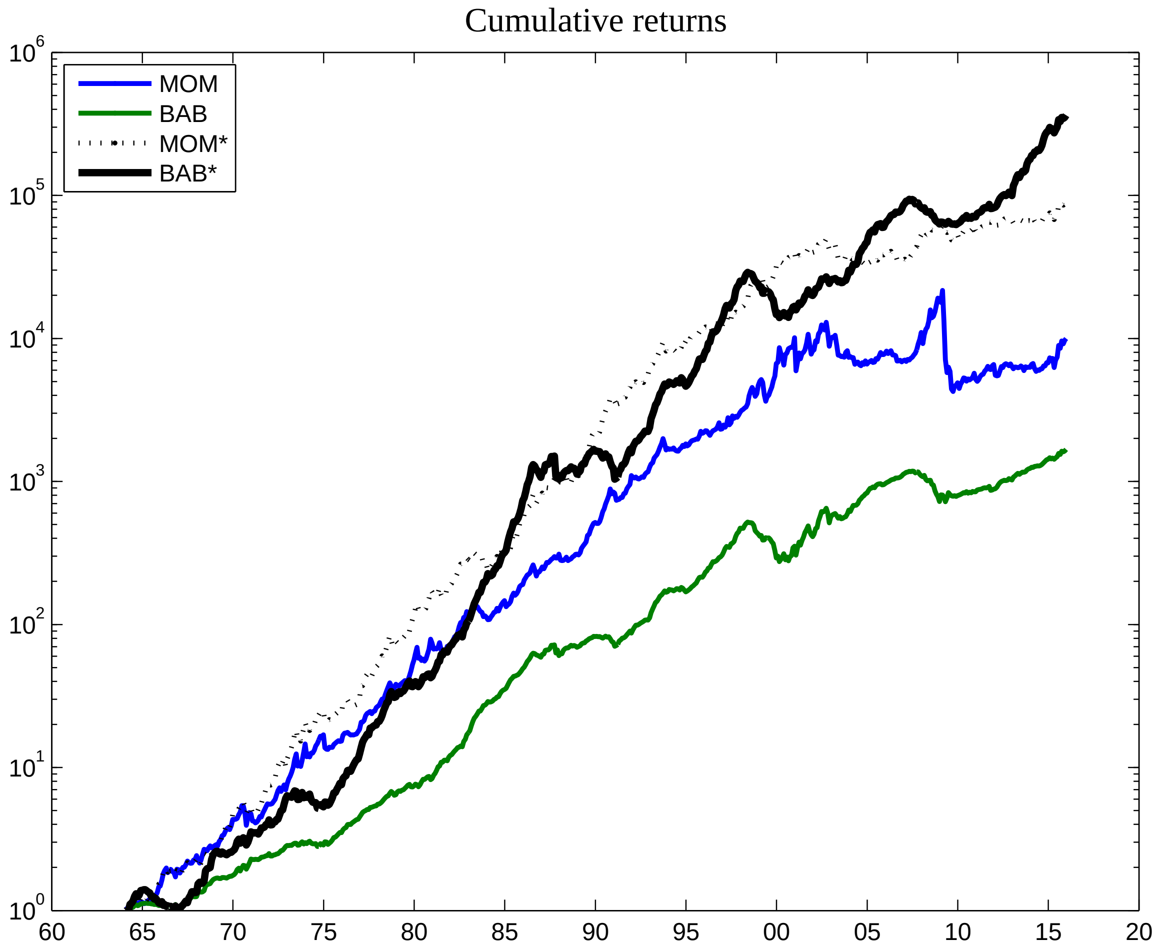

Fig. 1. The cumulative returns of BAB, MOM and its risk-managed versions using the previous month realized volatility. The returns are from 1964:01 to 2015:12 and include the return of the risk-free rate asset plus the excess return of the respective zero-cost portfolio.

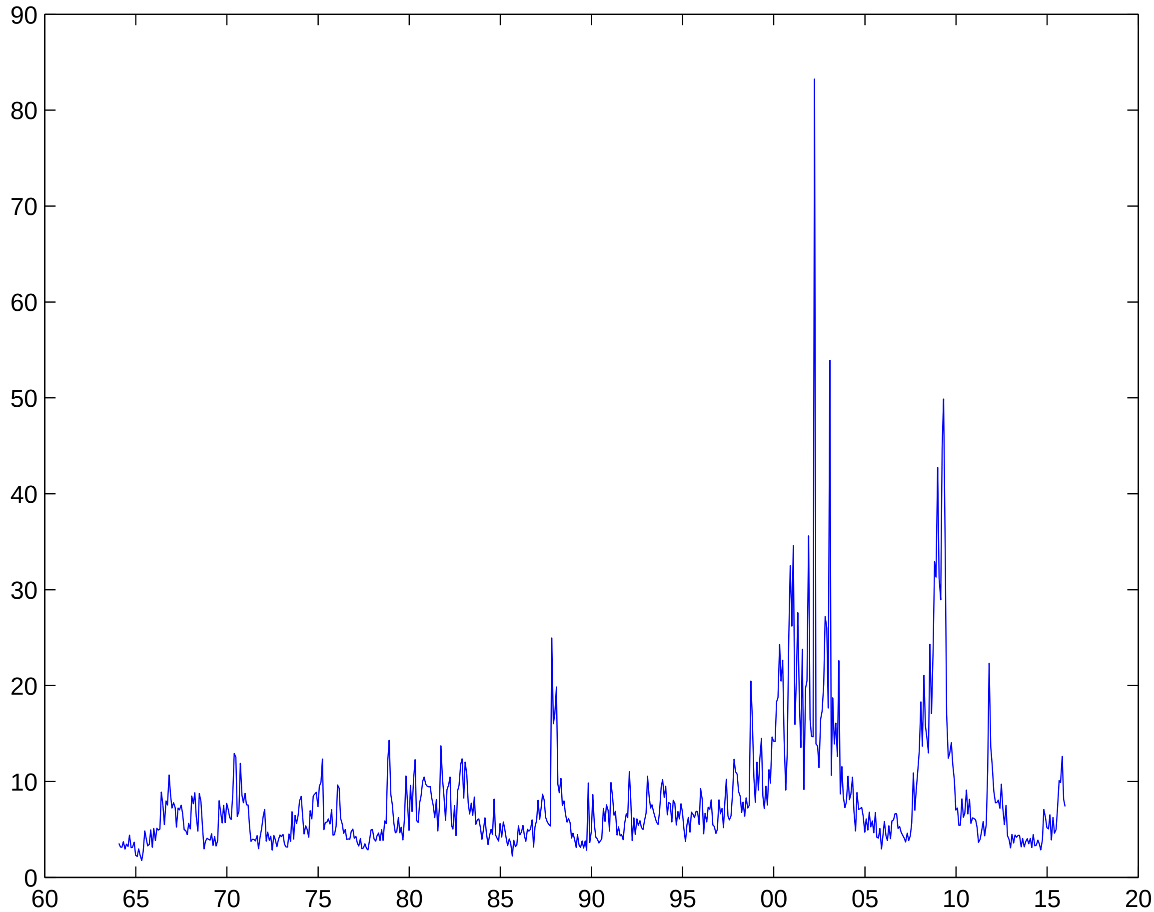

Fig. 2. Realized volatility of BAB
This figure plots the time-series of the annualized realized volatility (in %) associated with the BAB factor computed from daily returns each month. The sample is 1964:01 to 2015:12.

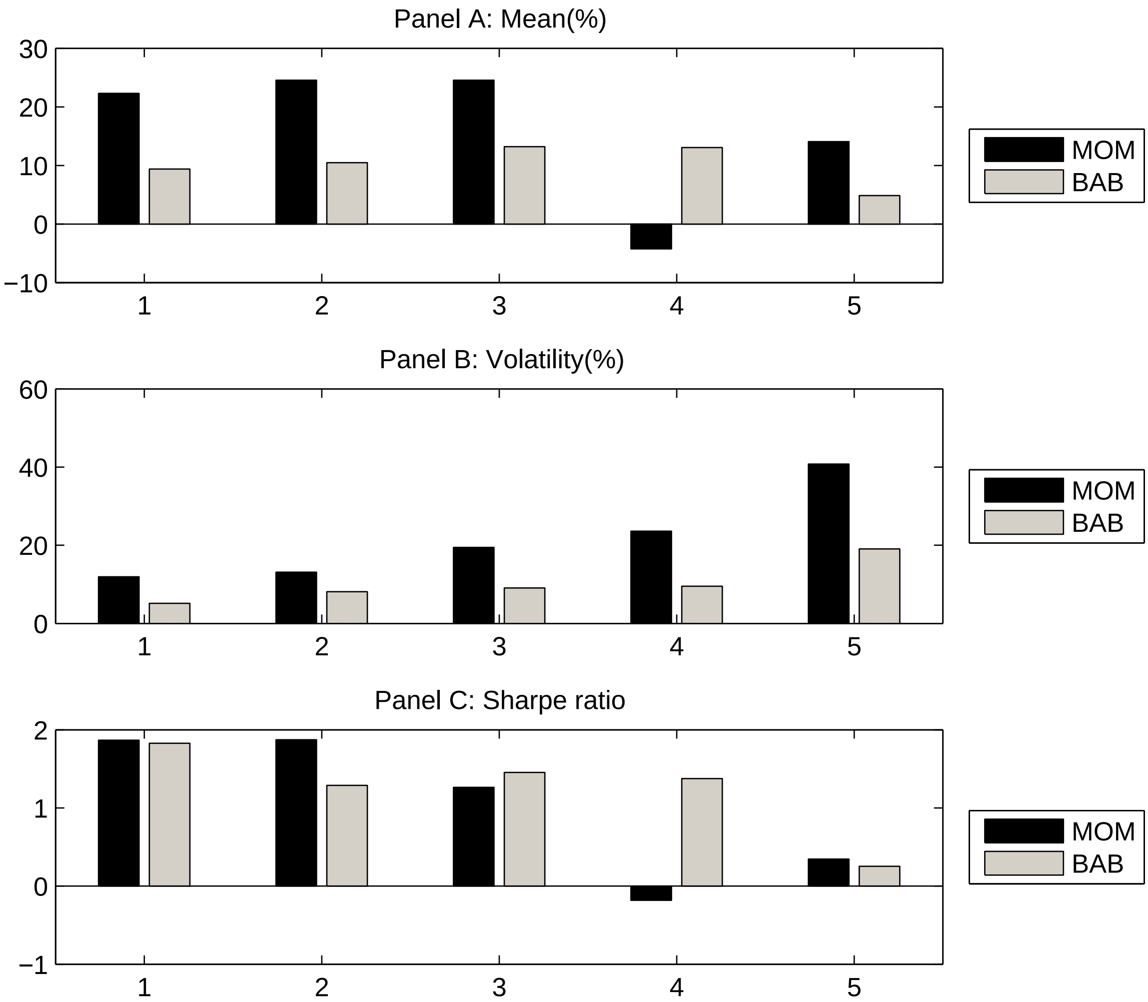

Fig. 3. Conditional performance of BAB and MOM
The returns of each factor are sorted on quintiles based on the realized volatility in the previous month. The figure presents the following annualized average return, volatility, and Sharpe ratio. The sample is 1964:01 to 2015:12.

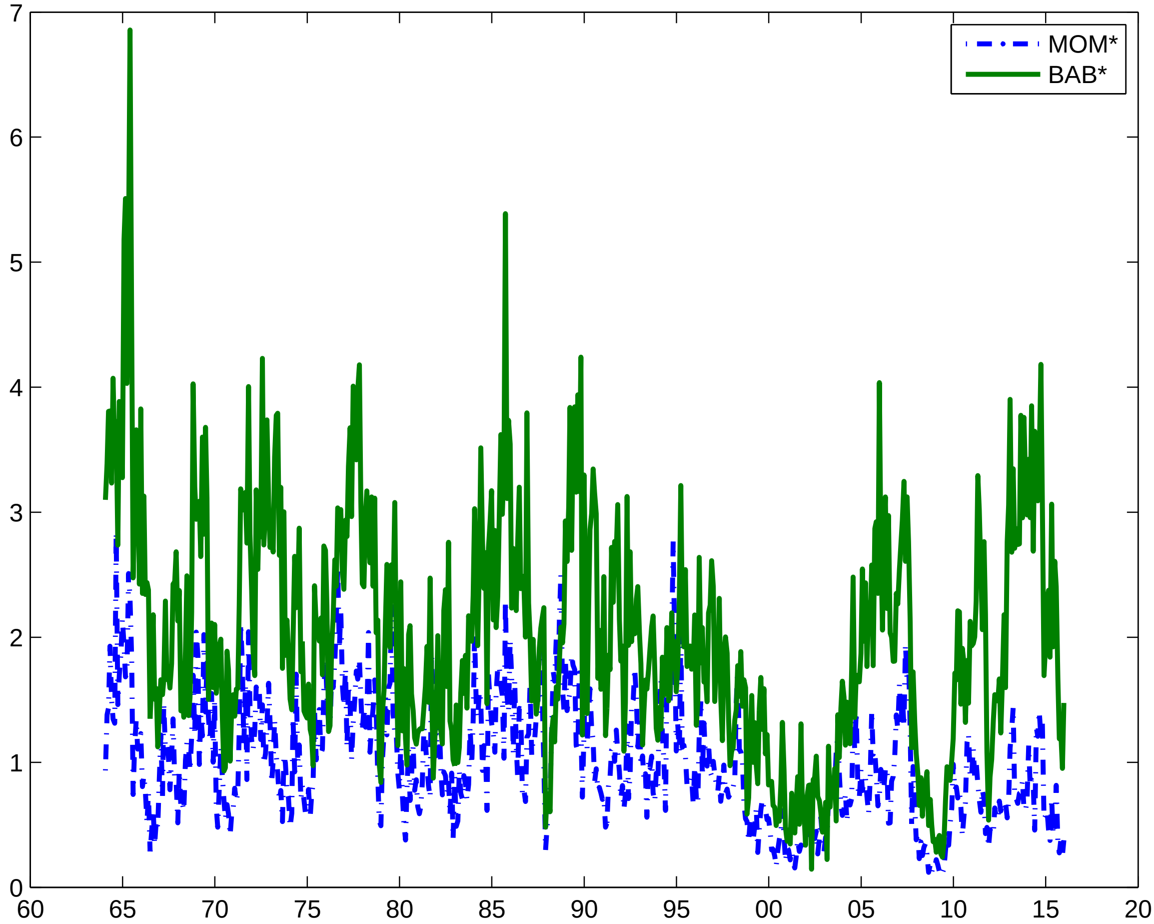

Fig. 4. Weights of scaled BAB and MOM
This figure plots the time-series of the weights associated with the scaled BAB and MOM factors. The sample is 1964:01 to 2015:12.

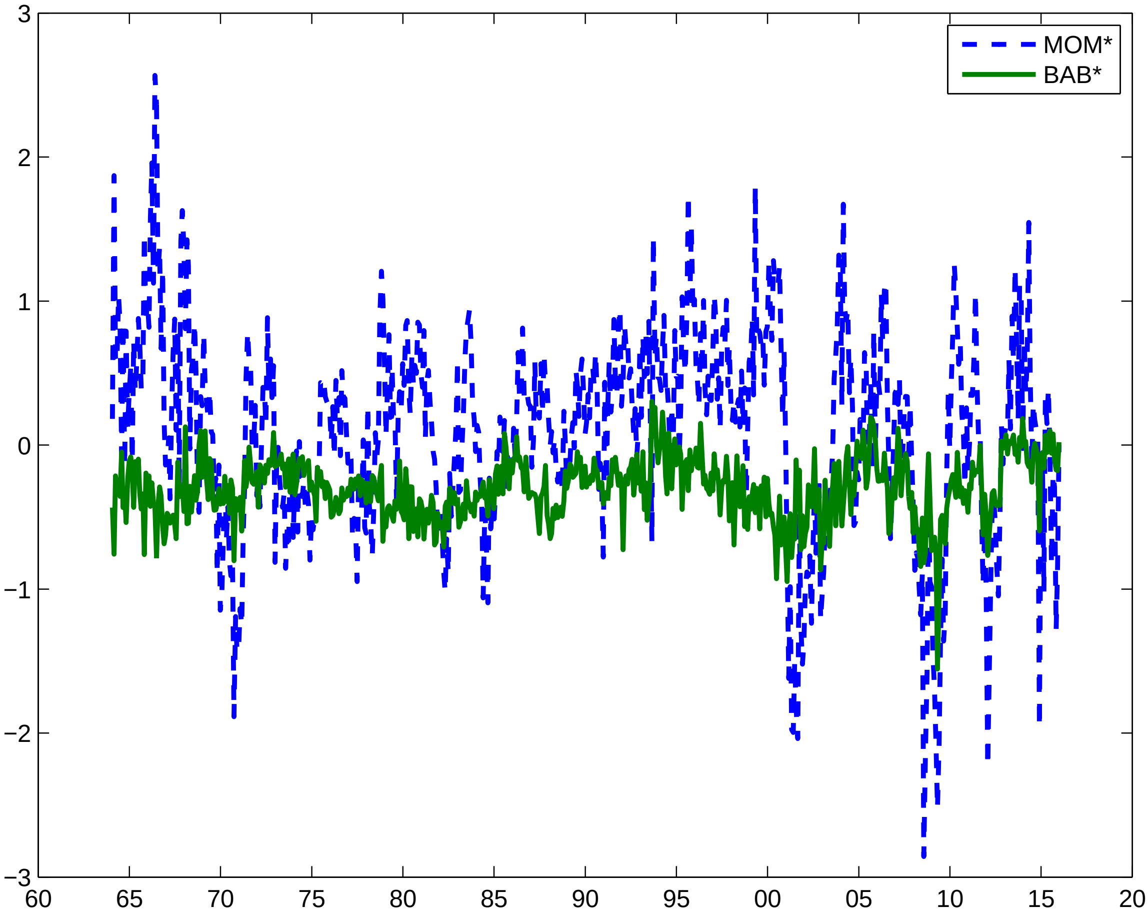

Fig. 5. Betas for BAB and MOM
This figure plots the time-series for the realized market beta associated with the BAB and MOM factors. The sample is 1964:01 to 2015:12.

Table 6

Volatility-return tradeoff

The realized variances of each factor represent the sum of factor squared daily returns in each month. The factors are betting-against-beta (BAB) and momentum (MOM). The monthly return of each factor is regressed on the lagged (non-overlapping inverse of the) factor monthly realized volatility. Heteroskedasticity-adjusted t-ratios are presented in parentheses. OOS $R^2(\%)$ denotes the out-of-sample (OOS) R^2, which relies on an initial (in-sample) window of 120 months. The sample period is from 1963:12 to 2015:12. Bold t-ratios indicate statistical significance at the 5% level.

Factor	$\gamma_0 \times 100$ (t-stat)	$\gamma_1 \times 100$ (t-stat)	$R^2(\%)$	OOS $R^2(\%)$
BAB	0.72	0.00	0.03	−0.45
	(1.67)	(0.38)		
MOM	0.43	0.03	0.41	0.00
	(0.47)	(1.36)		

Table 7

Performance of scaled factor strategies: components of variance

The performance of the scaled betting-against-beta factor (BAB*) is compared with momentum (MOM*). Each scaled factor uses the realized variance in the previous month, and its specific and systematic components, to scale the exposure to the original factor. The total variance is decomposed on its specific and systematic components based on the CAPM pricing equation. All statistics are computed with monthly returns. Reported are the maximum and minimum one-month returns observed in the sample, the mean average excess return (annualized), the (annualized) standard deviation of each factor, excess kurtosis, skewness, (annualized) Sharpe ratio, and the maximum drawdown (MDD). IR denotes the (annualized) information ratio of the scaled factor with respect to the unscaled benchmark. The sample returns are from 1964:01 to 2015:12. The max, min, mean, standard deviation and maximum drawdown are in percentage points.

Factor	Max.	Min.	Mean	S.D.	Kurt.	Skew.	Sharpe	IR	MDD
Panel A: BAB*									
Total	18.36	-28.14	23.60	17.89	2.08	-0.26	1.32	1.01	-57.90
Specific	23.47	-45.73	30.13	24.93	3.83	-0.67	1.21	0.82	-75.07
Systematic	28032.09	-147.31	684.20	3898.63	610.79	24.68	0.18	0.15	-29202.68
Panel B: MOM*									
Total	24.86	-17.47	20.55	18.87	1.17	-0.02	1.09	0.93	-40.51
Specific	26.70	-21.16	24.49	22.83	1.26	-0.12	1.07	0.92	-54.16
Systematic	413.05	-22164.70	-299.27	3080.05	615.07	-24.80	-0.10	-0.29	-38579.63

Table 8

Autoregression of the components of realized volatilities

The realized variances of each factor represent the sum of factor squared daily returns in each month. The total variance is decomposed on its specific and systematic components based on the CAPM pricing equation. The factors are betting-against-beta (BAB) and momentum (MOM). The AR(1) process regresses the (non-overlapping inverse of the) factor monthly realized volatility (or each of its components) on its own lagged value. Heteroskedasticity-adjusted t-ratios are presented in parentheses. OOS $R^2(\%)$ denotes the out-of-sample (OOS) R^2, which relies on an initial (in-sample) window of 120 months. The sample period is from 1963:12 to 2015:12. Bold t-ratios indicate statistical significance at the 5% level.

Volatility	ρ_0 (t-stat)	ρ_1 (t-stat)	$R^2(\%)$	OOS $R^2(\%)$
Panel A: BAB				
Total	16.72	0.74	54.82	57.06
	(**8.09**)	(**20.89**)		
Spec.	26.96	0.68	46.13	47.46
	(**10.17**)	(**20.30**)		
Syst.	930.83	−0.00	0.00	−0.15
	(1.54)	(−0.01)		
Panel B: MOM				
Total	9.38	0.70	49.05	55.19
	(**9.75**)	(**21.48**)		
Spec.	11.81	0.68	46.11	50.42
	(**10.81**)	(**22.45**)		
Syst.	210.73	−0.00	0.00	−0.50
	(**6.62**)	(−0.11)		

Table 9
Performance of scaled factor strategies: alternative variance measures

The performance of the scaled betting-against-beta factor (BAB*) is compared with the Fama-French-Carhart scaled risk factors: market (RMRF*), size (SMB*), value (HML*), momentum (MOM*), profitability (RMW*), and investment (CMA*). Each scaled factor uses the realized variance in the previous six months to scale the exposure to the original factor. All statistics are computed with monthly returns. Reported are the maximum and minimum one-month returns observed in the sample, the mean average excess return (annualized), the (annualized) standard deviation of each factor, excess kurtosis, skewness, (annualized) Sharpe ratio, and the maximum drawdown (MDD). IR denotes the (annualized) information ratio of the scaled factor with respect to the unscaled benchmark. The sample returns are from 1964:01 to 2015:12. The max, min, mean, standard deviation and maximum drawdown are in percentage points.

Factor	Max.	Min.	Mean	S.D.	Kurt.	Skew.	Sharpe	IR	MDD
RMRF*	11.39	-20.35	4.99	13.99	1.64	-0.57	0.36	0.00	-57.36
SMB*	21.79	-21.52	4.34	17.96	1.50	0.11	0.24	-0.02	-82.76
HML*	23.13	-17.50	7.41	17.78	0.92	0.12	0.42	0.12	-55.81
MOM*	21.74	-17.06	18.67	17.04	1.18	-0.15	1.10	0.98	-28.28
RMW*	15.82	-24.01	7.63	16.65	1.10	-0.19	0.46	0.23	-62.33
CMA*	13.77	-14.23	8.38	15.63	-0.07	0.16	0.54	0.14	-37.14
BAB*	18.25	-22.79	19.16	16.38	1.63	-0.43	1.17	0.76	-57.14

Table 10

Performance of scaled factor strategies: GARCH(1,1)

The performance of the scaled betting-against-beta factor (BAB*) is compared with the Fama-French-Carhart scaled risk factors: market (RMRF*), size (SMB*), value (HML*), momentum (MOM*), profitability (RMW*), and investment (CMA*). Each scaled factor uses the volatility forecast obtained from a GARCH(1,1) model to scale the exposure to the original factor. All statistics are computed with monthly returns. Reported are the maximum and minimum one-month returns observed in the sample, the mean average excess return (annualized), the (annualized) standard deviation of each factor, excess kurtosis, skewness, (annualized) Sharpe ratio, and the maximum drawdown (MDD). IR denotes the (annualized) information ratio of the scaled factor with respect to the unscaled benchmark. The sample returns are from 1963:07 to 2015:12, while the first volatility forecast and scaled factor observation occurs for 1973:07. The max, min, mean, standard deviation and maximum drawdown are in percentage points.

Factor	Max.	Min.	Mean	S.D.	Kurt.	Skew.	Sharpe	IR	MDD
RMRF*	9.14	-18.52	4.49	12.24	2.25	-0.75	0.37	-0.18	-42.77
SMB*	16.83	-13.35	3.01	11.96	1.75	0.14	0.25	0.02	-62.68
HML*	12.87	-11.67	3.79	12.23	0.73	-0.13	0.31	-0.18	-48.07
MOM*	12.94	-27.62	9.99	13.99	6.31	-1.22	0.71	0.43	-31.88
RMW*	9.54	-18.18	5.91	12.64	1.78	-0.45	0.47	0.19	-57.12
CMA*	9.53	-9.80	6.76	11.75	-0.13	0.04	0.58	0.01	-31.88
BAB*	14.49	-18.18	14.58	13.18	2.37	-0.71	1.11	0.65	-54.01

Table 11

Performance of factor-based strategies: 1973:07–2015:12

The performance of the betting-against-beta factor (BAB) is compared with the Fama-French-Carhart risk factors: market (RMRF), size (SMB), value (HML), momentum (MOM), profitability (RMW), and investment (CMA). All statistics are computed with monthly returns. Reported are the maximum and minimum one-month returns observed in the sample, the mean average excess return (annualized), the (annualized) standard deviation of each factor, excess kurtosis, skewness, (annualized) Sharpe ratio, and the maximum drawdown (MDD). The sample returns are from 1973:07 to 2015:12. The max, min, mean, standard deviation and maximum drawdown are in percentage points.

Factor	Max.	Min.	Mean	S.D.	Kurt.	Skew.	Sharpe	MDD
RMRF	16.10	-23.24	6.71	15.94	2.00	-0.56	0.42	-54.36
SMB	22.32	-16.70	2.94	10.78	6.96	0.59	0.27	-52.82
HML	13.91	-13.11	3.94	10.36	2.59	0.00	0.38	-45.21
MOM	26.16	-45.79	14.53	25.39	7.69	-1.44	0.57	-80.36
RMW	12.19	-17.57	3.33	7.76	11.45	-0.45	0.43	-39.17
CMA	9.51	-6.81	4.11	6.87	1.83	0.38	0.60	-17.30
BAB	12.89	-15.22	10.90	11.92	3.22	-0.67	0.91	-52.03

Table 12

Transaction costs and leverage constraints

Estimates of the turnover and after-cost of three strategies timing exposure to the BAB equity factor for different volatility targets. The strategies are: i) the BAB* strategy targeting constant volatility using recent one-month realized volatility; ii) a version of BAB* constrained to have no leverage w.r.t. the original factor; iii) a strategy allowing up to 50% leverage. Panel A, B and C show results for a target volatility of 12%, 9%, and 6%, respectively. The columns show the average absolute change in weights of the strategy; the average (annualized) excess return; the alpha of the dynamic strategy w.r.t. to plain BAB assuming transaction costs of, respectively, zero, one, ten and fourteen basis points. The last column shows the transaction costs that would eliminate the alpha of each dynamic strategy (in basis points). The returns are from 1964:01 to 2015:12.

Strategy	$\lvert\Delta w\rvert$	ER	α	1bps	10bps	14bps	Break even
Panel A: volatility target of 12%							
BAB*	0.51	21.31	8.86	8.73	7.64	7.15	73
No leverage	0.03	9.54	1.66	1.65	1.58	1.55	220
50% leverage	0.08	13.92	3.89	3.87	3.69	3.61	194
Panel B: volatility target of 9%							
BAB*	0.38	15.98	6.64	6.55	5.73	5.36	73
No leverage	0.04	9.42	2.36	2.35	2.25	2.21	213
50% leverage	0.13	12.72	4.23	4.19	3.90	3.78	131
Panel C: volatility target of 6%							
BAB*	0.25	10.65	4.43	4.37	3.82	3.57	72
No leverage	0.09	8.48	2.82	2.80	2.60	2.52	130
50% leverage	0.18	10.07	3.93	3.89	3.50	3.33	91

Table 13
International evidence

Descriptive statistics of BAB* in five international regions. Reported are the start and end months for each series of returns, the maximum and minimum one-month returns observed in the sample, the mean average excess return (annualized), the (annualized) standard deviation of the scaled factor in each market, its excess kurtosis, skewness, (annualized) Sharpe ratio, the (annualized) information ratio of BAB* w.r.t. its original version, the respective t-statistic, and the maximum drawdown (MDD). The max, min, mean, standard deviation and maximum drawdown are in percentage points. Bold t-ratios indicate statistical significance at the 5% level.

Market	Global	Global Ex-USA	Europe	North America	Pacific
Start	1987:04	1987:04	1989:04	1987:04	1989:01
End	2016:12	2016:12	2016:12	2016:12	2016:12
Max	10.45	11.59	14.42	13.34	14.83
Min	-12.50	-7.92	-11.74	-14.29	-9.23
Mean	8.90	9.95	9.74	9.10	8.60
STD	10.07	9.88	11.58	12.53	11.87
Kurt	2.49	0.88	1.87	2.96	1.18
Skew	-0.55	0.03	-0.05	-0.60	0.17
SR	0.88	1.01	0.84	0.73	0.72
IR	1.07	0.86	0.88	0.91	0.68
t-stat	**5.82**	**4.70**	**4.63**	**4.95**	**3.61**
MDD	-0.36	-0.42	-0.23	-0.57	-0.44