

# Optimal Timing and Tilting of Equity Factors\*

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## Abstract

Given the pervasive low yield environment investors strive to allocate capital to alternative building blocks. Within equities this endeavor amounts to identifying equity factors that are typically associated with firm characteristics and have proved meaningful in explaining the cross-section of stock returns. A major challenge is to combine these factors into a coherent portfolio that is capable of optimally harvesting the associated factor premia. In this vein, we put forward an integrated framework to optimally exploit time-series and cross-sectional factor allocation signals. In particular, we consider a parametric portfolio policy that allows for both: Timing factors according to time-series predictors and tilting factors according to cross-sectional factor characteristics. While the time-series predictors are insignificant, the resulting factor allocation outperforms the equal-weighted benchmark. On the other hand, the cross-sectional evidence is strong in a multivariate framework also leading the resulting factor allocation to outperform.

*Keywords:* Asset allocation, factor investing, factor timing, parametric portfolio policy  
*JEL Classification:* G11, D81, D85

# 1 Introduction

The pervasive low yield environment in major developed markets severely challenges investors who strive for positive and stable investment performance. For this purpose, the bedrock of investment management was to diversify investments across asset classes, a concept which is often referred to as the only “free lunch” in investing, see Ilmanen and Kizer (2012) among others. Yet, the concept of diversification mostly failed during the financial crisis in 2007–08. Except for high-quality sovereign debt, virtually all asset classes suffered and were characterized by high volatility. In turn, many investors were not as diversified as they had thought and started to consider new sources to generate sufficient investment returns at a reasonable level of risk.

In a related vein, the recent academic literature synthesizes that all asset classes are subject to some common underlying factor exposures or risk premia. For instance, Ang, Goetzmann, and Schaefer (2009) argue that a high proportion of active fund returns can be explained by exposure to various factors. Based on the observation that factors are hardly correlated, Ilmanen and Kizer (2012) argue that the concept of diversification is not dead but that investors simply failed to diversify across these factors historically.

While this concept of factor investing has recently attracted considerable interest, the underlying factor theory is not new. The first approach known as the capital asset pricing model (CAPM) of Sharpe (1964) builds on the foundation of diversification and the mean-variance paradigm introduced by Markowitz (1952). The CAPM states that the expected return of an asset is proportional to its sensitivity to the market, i.e. its beta. In other words, the market premium is the sole risk premium available to investors and beta is the unique pricing factor. Unfortunately, the simplicity of the CAPM model comes at the cost of many assumptions and the CAPM has been challenged empirically. In that regard, a recurring theme are patterns in the cross-section of stock returns that cannot be explained by the exposure to the market factor and the associated market risk premium alone, see Martellini and Milhau (2015). Among the most prominent findings are the size, value, and momentum effects which describe a persistent link of future stock returns to the corresponding stock characteristic over a sustained time period and in several markets, see Banz (1981), Basu (1977), and Jegadeesh and Titman (1993), respectively.

Resurrecting the CAPM, models using multiple factors were introduced starting with the intertemporal CAPM by Merton (1973) and the Arbitrage Pricing Theory by Ross (1976). These approaches were followed by the three-factor model of Fama and French (1993) that builds on the empirical observation that size and value are complementary in explaining the cross-section of stock returns. Building on the work of Jegadeesh and Titman (1993), Carhart (1997) incorporated momentum into the three-factor model as a further priced factor.

To rationalize the explanatory power of equity factors one way is to relate them to the stochastic discount factor (SDF) denoted as  $m$ . In modern asset pricing theory the SDF acknowledges the notion of uncertainty and time-varying expected returns when defining the

price  $P_t^i$  of a given risky asset, see Cochrane (2009):

$$P_t^i = E_t[m_{t+1} x_{t+1}]s \quad (1)$$

where  $x_t$  is the cash flow for asset  $i$  in period  $t$ . The SDF in  $t$  can be interpreted as an index of “bad times”, i.e., times where the marginal utility of returns for investors is high: If period  $t+1$  turns out to be a bad state,  $m_{t+1}$  will be high, and vice versa. For further insights we consider the beta representation of expected returns, see Cochrane (2009):

$$E(R_{t+1}^i) - R_{t+1}^f = \underbrace{\frac{\text{cov}_t(R_{t+1}^i, m_{t+1})}{\text{var}_t(m_{t+1})}}_{=: \beta_t^{i, m_{t+1}}} \times \underbrace{\left( -\frac{\text{var}_t(m_{t+1})}{E_t(m_{t+1})} \right)}_{=: \lambda_t^{m_{t+1}}} \quad (2)$$

Representation (2) shows that the required return of asset  $i$  is linked to its covariation with bad times,  $\text{cov}_t(R_{t+1}^i, m_{t+1})$ : Assets that perform well in bad states are particularly valuable for investors leading them to accept lower expected returns or risk premia. Conversely, investors require a higher risk premium for assets that perform poorly in bad states. While a higher sensitivity to the market increases the assets’ expected return in the CAPM, it is now the sensitivity of the asset to the SDF ( $\beta_t^{i, m_{t+1}}$ ) that is multiplied by  $\lambda_t^{m_{t+1}}$ , i.e., the SDF risk premium. However, the SDF is unobservable and needs to be associated to observable variables as pursued in consumption-based models<sup>1</sup>. The unobservability issue prompts to use factor models as a proxy for the SDF. Given that  $K$  observable factors are represented by dollar-neutral excess returns, the SDF depends on a  $(K \times 1)$  vector of factor returns  $\mathbf{f}$ , a scalar  $a$  and a  $(K \times 1)$  vector  $\mathbf{b}$  of pricing factors (abstracting from time indices):

$$m = a + \mathbf{b}' \mathbf{f} \quad (3)$$

This formulation is equivalent to the following beta representation of expected returns, see Cochrane (2009):

$$E(R_i) = R_f + \beta_i' \mathbf{\Lambda} \quad (4)$$

where  $\beta_i$  is a  $(K \times 1)$  vector of multivariate regression coefficients of  $R_i$  on  $\mathbf{f}$  with a constant. The vector of factor risk premia,  $\mathbf{\Lambda}$ , contains the expected factor returns  $E(\mathbf{f})$ , see Cochrane (2009). Each of the  $K$  factors in  $\mathbf{f}$  (deemed to be relevant in explaining the cross-section of stock returns) make up the stochastic discount factor and each define a different set of *bad times*. In equilibrium, investors have to be compensated by factor risk premia for bearing these risks.

The present paper thoroughly investigates the idea of directly investing into factors instead of traditional asset classes. The challenge is to optimally combine factors which is “still

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<sup>1</sup>Obviously, such associations may raise new problems, e.g., aggregated macroeconomic consumption data is characterized by low frequencies, lagged releases and revisions, see Martellini and Milhau (2015).

unchartered territory” [Briere and Szafarz (2016)]. In the literature, most studies focus on the timing of factors by developing factor return prediction models, often restricted to single factors. Examples are momentum-based models, see Clare, Sapuric, and Todorovic (2010) or Chen and De Bondt (2004), and multivariate models using economy- or stock market-related predictors, see Copeland and Copeland (1999), Kao and Shumaker (1999), Levis and Liodakis (1999), Cooper, Gulen, and Vassalou (2001), Lucas, van Dijk, and Kloek (2002), and Bauer, Derwall, and Molenaar (2004). Conversely, Medvedev and Vaucher (2017) concentrate on optimal stock selection based on factor exposures but do not consider allocating between factors used as input for portfolio construction.

Our main contribution is to combine equity factors into a coherent portfolio that is capable of optimally harvesting the associated factor premia. In particular, we distinguish between *factor timing* that seeks to exploit time-series information and *factor tilting* that seeks to exploit cross-sectional information. Both approaches are couched into the parametric portfolio policy of Brandt and Santa-Clara (2006) and Brandt, Santa-Clara, and Valkanov (2009), respectively. This work therefore will contribute to the ongoing debate or “quantroversy” among quantitative investment managers and/or academic scholars regarding the degree to which it is possible to time equity factors, see Parker, Hayes, Ortega, and Naha (2016) for a quite positive view and Asness (2016) for a rather sceptical one. In a similiar vein, Hodges, Hogan, Peterson, and Ang (2017) use business cycle factors to time factors and building on work of Asness, Friedman, Krail, and Liew (2000) and Lewellen (2002) when incorporating cross-sectional information to tilt factors. Jointly investigating such information in a parametric portfolio policy sheds new insights into active factor allocation.

Our empirical analysis is driven by a broad set of some 20 global equity factors, that we compile from a broad sample of global companies. Ultimately, we thus can build our main analysis on two-decades of global equity factor returns ranging from 1997 to 2016. First taking an agnostic perspective regarding expected factor returns, we start by examining a quite diversified multi-factor portfolio based on equal factor weights. To improve this benchmark allocation, we strive for *optimal factor investing* in two ways: First, we try our hand at *factor timing* based on a variety of fundamental and technical indicators commonly used for predicting the equity risk premium, see for example Campbell and Thompson (2008) and Neely, Rapach, Tu, and Zhou (2014). These indicators are exploited in the parametric portfolio policy framework of Brandt and Santa-Clara (2006). Second, we engage in *factor tilting* according to cross-sectional factor characteristics. In particular, we consider spreads, valuation, the 1-month price momentum, see Avramov, Cheng, Schreiber, and Shemer (2017), volatility, and two centrality measures: Building on the work of Pozzi, Di Matteo, and Aste (2013) and Lohre, Papenbrock, and Poonia (2014) we use the factors’ centrality in the corresponding factor correlation network. A related but yet distinct characteristic is a factor’s distance to the market factor as revealed in the correlation network. These six characteristics are then couched into the parametric portfolio policy of Brandt, Santa-Clara, and Valkanov (2009). Both, *timing* and *tilting* of factors, are based on a benchmark-relative utility maximization of a mean-variance investor with a quadratic utility function. Given

strong signals in the data, the optimal strategy will actively deviate from the benchmark allocation. Conversely, it will embrace the benchmark when the informational content is weak. This framework allows to capitalize the diversification benefits embedded in the risk-based benchmark while exploiting the opportunity of active positioning. As a consequence, one might harvest utility gains when some factors enjoy their good times or avoid large drawdowns when some factors suffer from bad times.

While relying on time-series information, embedded in fundamental and technical predictor variables, the model is insignificant. Although the statistical evidence is weak, the resulting factor allocation outperforms the equal-weighted factor allocation benchmark. For factor tilting we use information embedded in the cross-section of the factor set. In a multivariate framework the resulting tilting-coefficients show a strong statistical evidence and slightly helps to improve performance.

## 2 Building global equity factors

To allow for a comprehensive and relevant analysis of factor timing and tilting we put together a representative set of global equity factors. These global equity factors derive from a global universe encompassing the constituents of MSCI, FTSE, S&P or STOXX global as well as regional indices throughout time. The source for company specific data such as financial statement data is the Worldscope database. The sample of monthly factor returns starts in January 1997 to allow for a reasonably broad universe, even in the regional subsets. The last month of the sample period is December 2016 giving us two decades of equity factor returns. The overall investable universe comprises roughly 4500 stocks in December 1996 and this number increases to 5000 companies in December 2016. As for the regional split, the universe on average includes 1700 European stocks and 1300 U.S. companies where both figures are quite constant throughout time. In December 1996 1000 companies belong to the Asia-Pacific region, mostly made up of Japan and Australia. The number of companies increases to 1400 at the end of our sample period. The remainder is recruited from a Rest of the World universe, including companies from Canada, New Zealand, Israel and Hong Kong. To not suffer from investability concerns we focus our analysis on this global Large/Mid cap universe, having a quite diversified universe region-wise.

As for the nature of the global equity factors build on factors which are widely used and well documented in academic research. Specifically, the factor set includes:

- *Profitability (PROF)*: This factor is long stocks with robust operating profitability and short stocks with weak profitability. Profitability is calculated as annual revenues less cost of goods sold and interest and other expenses, divided by book value for the last fiscal year-end. The factor is based on academic research of Haugen and Baker (1996), Cohen, Gompers, and Vuolteenaho (2002), Novy-Marx (2013) and Fama and French (2006, 2016)
- *Cashflow yield (CFY)*: The cashflow yield factor captures the excess return of going long stocks with a high cashflow-to-price ratio and short those with a low one, see

Sloan (1996), Da and Warachka (2009) and Hou, Karolyi, and Kho (2011). Cashflows are measured as the sum of funds from operations, extraordinary items and funds from other operating activities.

- *Accruals (ACC)*: The accruals factor is long stocks with low accruals and short those with high accruals, where accruals are measured as the change in working capital per share, divided by the book value per share, see Sloan (1996).
- *Dividend yield (DY)*: The dividend yield factor is long stocks with a high dividend-to-price ratio and short those with a low dividend-to-price ratio, see Litzenberger and Ramaswamy (1979), Blume (1980), Fama and French (1988) and Campbell and Shiller (1988). Dividends include all extra dividends declared during the year.
- *Asset turnover (AT)*: Asset turnover measures asset utilization and efficiency. Following Soliman (2008) companies with high asset turnover are associated with future positive returns as those firms manage their inventory more efficiently. The factor is defined as sales, divided by the average net operating assets.
- *Book to Market (BTM)*: The factor is constructed by going long stocks with a high book-to-market ratio and short stocks with a small book-to-market ratio. The factor builds on the findings of Basu (1977), Rosenberg, Reid, and Lanstein (1985), Jaffe, Keim, and Westerfield (1989), Chan, Hamao, and Lakonishok (1991) and Fama and French (1992) that value stocks outperform growth stocks in the long-run.
- *12-month momentum (MOM12) and 6-month momentum (MOM6)*: 12-month momentum as well as 6-month momentum capture a medium-term continuation effect in returns by buying recent winners and selling recent losers. We control for the short-term reversal effect (see below) by excluding the most recent month ( $t - 1$ ) at time  $t$ . Jegadeesh (1990) and Jegadeesh and Titman (1993) were the first that documented the momentum effect in the cross-section of stock returns.
- *Short-term reversal (STR)*: Jegadeesh (1990) and Lehmann (1990) documented a short-term reversal effect in the cross-section of stock returns. The factor is going long stocks with a weak previous month performance and short stocks with a high performance in the previous month.
- *Long-term reversal (LTR)*: De Bondt and Thaler (1985) documented reversal patterns in the long-term past performance. Following DeMiguel, Martin-Utrera, Nogales, and Uppal (2017) we choose the horizon to be 36 months. To control for the momentum effect we exclude the most recent year from our three year horizon of past performance. The factors goes long in stocks with a weak long-term past performance and short in stocks with a strong long-term past performance.
- *Change in long-term debt (DLTD)*: An increase in long-term debt could hint at empire-building behaviour of the company which is associated with negative future returns,

see Richardson, Sloan, Soliman, and Tuna (2005). The factor builds on year-on-year changes, divided by the long-term debt in  $t - 2$ .

- *Change in shares outstanding (DSO)*: Ritter (1991) and Loughran and Ritter (1995) were the first to document that firms with a high change in shares outstanding underperform relative to non-issuing firms, see also Daniel and Titman (2006) and Pontiff and Woodgate (2008). Change in shares outstanding is measured by the year-on-year change in shares outstanding, divided by outstanding shares in  $t - 2$ .
- *Size*: The size factor builds on the observation that stocks with a larger market capitalization tend to underperform stocks with smaller market capitalizations, see Banz (1981). The factor is going long stocks with the smallest market capitalization and short stocks with the highest market capitalizations, see Fama and French (1992).
- *Asset growth (AG)*: This factor is based on research by Fairfield, Whisenant, and Yohn (2003), Richardson, Sloan, Soliman, and Tuna (2005), Titman, Wei, and Xie (2004), Fama and French (2006) and Cooper, Gulen, and Schill (2008), all documenting a negative relation between investment activity and returns. The factor is long stocks with a low asset growth ratio and short stocks with a high asset growth ratio. Asset growth is measured by the year-on-year change in total assets, divided by the total assets in  $t - 2$ .
- *Cash productivity (CP)*: Chandrashekar and Rao (2009) find the productivity of cash to be a strong and robust negative predictor of returns. Firms with high cash productivity have low subsequent stock returns, and low cash productivity firms have high future returns. The factor is defined as market value plus long-term debt minus total assets, divided by cash.
- *Profit margin (PMA)*: Soliman (2008) stated that firms which are able to ensure a high profit margin are often associated with a first mover advantage or a high brand recognition which translates into a high pricing power. Profit margin is defined as operating income divided by sales. The factor goes long stocks with a high profit margin and goes short firms with a lower profit margin.
- *Earnings yield (EY)*: The earnings yield factor is long stocks with a high earnings-to-price ratio and short those with a low earnings-to-price ratio, see Basu (1977).
- *Leverage (LEV)*: The leverage factor of Bhandari (1988) is defined as total liabilities, divided by the market value of the company.
- *Return on Assets (ROA)*: High return on assets indicates a successful firm described by Balakrishnan, Bartov, and Faurel (2010). The factor is therefore long in firms with a high return on assets ratio and short those with a low return on assets ratio.
- *Sales to cash (STC)*: The factor is based on research of Ou and Penman (1989) who show a positive relationship between a high sales to cash ratio and future returns. The



factor is long stocks with a high sales to cash ratio and short those with a low sales to cash ratio.

- *Sales to inventory (STI)*: Sales to Inventory measures the effective use of the firms assets. Ou and Penman (1989) stated that a high sales-to-inventory ratio indicates firm effectiveness and is associated with higher future returns.

To compute all equity factors we sort the global universe of companies according to the factor characteristic on a monthly basis and compute the mean of the subsequent 1-month local return of the respective quintiles. The ultimate long-short factor return results from taking the spread between top and bottom quintiles.

[Figure 1 about here.]

The correlation chart in Figure 1 shows that we have constructed a quite heterogenous factor set. Yet, factors looking to harvest a “value” premium do have a higher correlation. The correlation of *CFY*, *DY*, *BTM* or *EY* ranges from 0.8 to 0.9. Also, the momentum factors *MOM12* and *MOM6* do have a high positive correlation of 0.9. *Size* and *ACC* are factors adding diversification potential to our factor set, having negative correlation to the most other factors in the range from  $-0.1$  to  $-0.4$  for *Size* and from  $-0.4$  to  $-0.8$  for *ACC*.

[Table 1 about here.]

The best performing factors are the momentum factors, *MOM12* and *MOM6*, with two digit annualized returns of 12.1% and 10.2%, respectively. *ACC*, *STR* and *STI* have rather modest returns with 0.3%, 1.9% and 2.5% p.a., respectively. Thus, all factors have a positive premium in the sample period. All equity factors used throughout this study have a strong economic rationale and are widely accepted in academic research. However, Schwert (2003), Chordia, Roll, and Subrahmanyam (2011) and McLean and Pontiff (2016) show that factors tend to weaken after their publication. As the interest of this paper is to optimally combine factors and not provide the most stable and robust ones we refrain from cherry picking and include all factors in the analysis. Especially, time-variation in factor returns is calling for actively managing factor exposures to navigate potential factor cyclicity. From a volatility perspective the momentum factors are the most volatile, with 20.2% for *MOM12* and 19.2% for *MOM6*, and *ACC*, *AT* and *STI* are the least volatile. The ensuing Sharpe Ratios range from 0.05 for *ACC* to 1.15 for *PROF*.

### 3 Factor timing

To improve an equal-weighted benchmark factor allocation we consider factor *timing* by relating factor returns to a variety of fundamental variables and technical indicators commonly used for predicting the equity risk premium. The identification of *good* and *bad times* of a given factor should help to improve the overall risk-return profile of the equal-weighted benchmark strategy. In particular, we operationalize the potential predictive content of predictor variables in the parametric portfolio policy framework of Brandt and Santa-Clara (2006).

### 3.1 Predictor variables

#### 3.1.1 Fundamental variables

We use fundamental variables to track macroeconomic conditions which could inform about the future state of the economy and therefore about different *good* or *bad times*, see Neely, Rapach, Tu, and Zhou (2014). In particular, we employ the following variables as deployed in Welch and Goyal (2008) and publicly available from July 1926 to December 2016 on Amit Goyal’s web page<sup>2</sup>: Dividend Price Ratio (*dp*), Dividend Yield (*dy*), Earnings Price Ratio (*ep*), Dividend Payout Ratio (*de*), Stock Variance (*svar*), Book to Market Ratio (*bm*), Net Equity Expansion (*ntis*), Treasury Bills (*tbl*), Long Term Yield (*lty*), Long Term Rate of Return (*ltr*), Term Spread (*tms*), Default Yield Spread (*dfy*), Default Return Spread (*dfr*) and Inflation (*infl*). See Appendix A.1 for a definition of the variables. Following Rapach, Strauss, and Zhou (2013) using U.S. based fundamental variables have a good predictive ability for other developed non-U.S. countries.

To avoid spurious findings resulting from high autocorrelations it is useful to detrend the variables, see Ferson, Sarkissian, and Simin (2003). We thus standardize any predictor variable  $X$  at time  $t$  by subtracting its arithmetic mean and dividing by its standard deviation. For the calculation of the mean and standard deviation we use a rolling window covering the 12 months preceding (and thus excluding)  $t$ . Hence, the current observation of  $X$  is not included which allows for stronger innovations:

$$X_t^{std} = \frac{X_t - \frac{1}{N} \sum_{i=t-N}^{t-1} X_i}{\sqrt{\frac{1}{N-1} \sum_{i=t-N}^{t-1} (X_i - \bar{X})^2}} \quad (5)$$

with  $N = 12$ . Furthermore, as few standardized fundamental variables might attain extreme values, we truncate the variables at  $\pm 5$ :

$$X_t^{std} = \begin{cases} 5 & \text{if } X_t^{std} > 5 \\ -5 & \text{if } X_t^{std} < -5 \\ X_t^{std} & \text{otherwise} \end{cases} \quad (6)$$

#### 3.1.2 Technical indicators

Besides variables capturing the state of the economy, we follow Neely, Rapach, Tu, and Zhou (2014) in using technical indicators or trading rules using past factor returns. Similar to Hammerschmid and Lohre (2017), we include 16 technical indicators based on three sets of trading rules related to the general concepts of momentum ( $MOM_m$ ), moving averages ( $MA_{s-l}$ ) and stochastic oscillator ( $KDS_m$ ). These technical trading rules reasonably capture the trend-following idea of technical analysis and are representative of typical rules analyzed

<sup>2</sup>The dataset is available on <http://www.hec.unil.ch/agoyal/>. For a more detailed description of the variables please refer to Welch and Goyal (2008).

in the literature, see for example Brock, Lakonishok, and LeBaron (1992) and Sullivan, Timmermann, and White (1999).

1. *Momentum* ( $MOM_m$ ): The momentum indicator gives a buy signal if the price at time  $t$ ,  $P_t$ , is higher than the price at time  $(t - m)$ ,  $P_{t-m}$ , and a sell signal otherwise:

$$MOM_m = \begin{cases} 1 & \text{if } P_t > P_{t-m} \\ 0 & \text{if } P_t \leq P_{t-m} \end{cases} \quad (7)$$

We compute five momentum indicators for different look-back periods using  $m = 1, 3, 6, 9, 12$  months. The conjecture is that factor returns are trending such that recent positive returns are followed by subsequent positive returns.

2. *Moving Average* ( $MA_{s-l}$ ): The moving average indicator is based on the comparison of a short-term and a long-term moving average which are calculated as:

$$MA_{j,t} = \frac{1}{j} \sum_{i=0}^{j-1} P_{t-i} \quad \text{for } j = s, l \quad (8)$$

where  $s$  and  $l$  are the length of the lookback period for the short- and long-term moving averages in months using  $s = 1, 2, 3$  and  $l = 9, 12$ . The indicator gives a buy signal if the short-term moving average is greater than the long-term moving average

$$MA_{s-l} = \begin{cases} 1 & \text{if } MA_{s,t} > MA_{l,t} \\ 0 & \text{if } MA_{s,t} \leq MA_{l,t} \end{cases} \quad (9)$$

providing us with six moving average indicators. The conjecture is that a crossing of the long-term moving average from below by the short-term moving average signals an upshift in the trend while smoothing out noise from the price data.

3. *Stochastic Oscillator* ( $KDS_m$ ): The stochastic oscillator was introduced by George C. Lane in the 1950s, see Murphy (1999), and tracks the speed or momentum of price movements. It builds on the idea that momentum changes often precede price changes. To compute the indicator, we first calculate  $K_t^{fast}$  which gives the position of the price relative to the high-low price range over a specific period ranging from  $(t - m)$  to  $t$  (denoted  $t - m, t$ ). We create five stochastic oscillators based on five look-back periods with  $m = 12, 24, 36, 48, 60$  months.

$$K_t^{fast} = \left[ \frac{P_t - L_{t-m,t}}{H_{t-m,t} - L_{t-m,t}} * 100 \right] \quad (10)$$

where  $L_{t-m,t}$  and  $H_{t-m,t}$  are the lowest and the highest price respectively within the last  $m$  months. Equation (10) yields a value between 0 and 100 with a high figure indicating that the factor trades close to its high (measured over the  $m$ -months period). The smoothed version using a 3-months moving average of (10) constitutes  $K_t^{slow}$  which

is also denoted as  $D_t^{fast}$

$$D_t^{fast} = K_t^{slow} = \text{MA}_{3,t} \left( K_t^{fast} \right) \quad (11)$$

We further decelerate  $D_t^{fast}$  to get a less choppy indicator by taking another 3-month average

$$D_t^{slow} = \text{MA}_{3,t} \left( D_t^{fast} \right) \quad (12)$$

The final stochastic oscillator gives a buy signal if the shorter moving average ( $D_t^{fast}$ ) is greater than the longer moving average ( $D_t^{slow}$ ):

$$KDS_m = \begin{cases} 1 & \text{if } D_t^{fast} > D_t^{slow} \\ 0 & \text{if } D_t^{fast} \leq D_t^{slow} \end{cases} \quad (13)$$

Intuitively, the stochastic oscillator follows the speed or momentum of price changes: If  $D_t^{fast}$  is bigger than  $D_t^{slow}$ , the factor's price increased strongly more recently (relative to its trading range) and gained momentum compared to the realization of the longer term average of this number. Hence, the stochastic oscillator pictures an upward trend in the factor's price. Conversely, if  $D_t^{fast}$  is smaller than  $D_t^{slow}$ , the price increase slowed down and could possibly reverse displaying a downward trend.

As we “lose” observations for technical indicators that are based on longer periods (for instance, for  $KDS_{24}$  we do not have observations in the first 23 months) we fill missing values with indicator values from shorter periods. In the  $KDS$  example, the missing observations for month 13 to 23 for  $KDS_{24}$  are filled with the values of  $KDS_{12}$ . Still, the first 12 months are “lost” such that all technical predictors start in January 1998.

To check for multicollinearity of predictors figure 2 gives the correlation matrix for the fundamental variables and technical indicators that obtain for the equity factor  $MOM12$  over the whole sample from January 1998 to December 2016.<sup>3</sup> As expected, the technical indicators are highly correlated with correlations up to 0.9 especially within the three trading rule sets. Only  $MOM_1$  does exhibit rather small correlations to the other technical predictors ranging from 0.2 to 0.4. Among the fundamental variables, the correlation structure is more heterogenous. Yet, the valuation ratios  $dp$ ,  $dy$ ,  $ep$ , and  $bm$  appear to be highly correlated. Moreover,  $ltr$  and  $tms$  are perfectly collinear. The most negative correlation is found for  $dp$  and  $de$  with  $-0.7$ . As can be seen, technical and fundamental predictors are rather uncorrelated in general, suggesting complimentary predictive content (if any).

[Figure 2 about here.]

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<sup>3</sup>Note that the fundamental variables used to predict  $MOM12$  are also used to predict other equity factors, whereas the technical indicators are factor-specific and therefore the correlation structure does differ across factors. Still, the general notion of highly correlated technical indicators prevails across all equity factors.

### 3.1.3 Reducing the number of predictor variables

We reduce the number of independent variables while preserving the information embedded in them. For this purpose, we resort to principal component analysis (PCA) that is separately applied to the fundamental variables and technical indicators in the spirit of Neely, Rapach, Tu, and Zhou (2014), see also Ludvigson and Ng (2007, 2009) and Hammerschmid and Lohre (2017). The aim is to come up with a reduced number of predictive factors that synthesize the heterogeneous information contained in the 30 predictor variables and to get rid of the noise within the predictors. Also, the PCA gives orthogonal predictors such that multicollinearity problems are avoided. In our main analysis we use the first<sup>4</sup> principal component for fundamental variables (denoted as *FUN1*) and the first principal component for technical indicator (denoted as *TECH1*). Both capture a significant proportion of variation in the underlying variables and indicators (27% and 86%, respectively).

## 3.2 Optimal factor timing

We ultimately want to examine whether a risk averse investor may profit from timing equity factors with respect to fundamental variables and technical indicators. To this end, we use the parametric portfolio policy (PPP) of Brandt and Santa-Clara (2006) that ties predictive variables and investor utility in a portfolio-theoretic framework. Their approach translates the predictive power embedded in the above PCA factors into optimal portfolio weights. To do so, one augments the set of 21 equity factors by synthetic assets that invest into the equity factors in proportion to the conditioning variables. In our case the conditioning variables are the PCA factors. Optimal portfolio weights then derive from a classic Markowitz mean-variance optimization over this augmented space of equity factors.

### 3.2.1 Methodology of Brandt and Santa-Clara (2006)

Brandt and Santa-Clara (2006) consider the maximization problem of a mean-variance investor who is risk averse according to her risk aversion parameter  $\gamma$  and is thus solving:

$$\max_{w_t} E \left[ w' r_{t+1} - \frac{\gamma}{2} w' r_{t+1} r'_{t+1} w_t \right] \quad (14)$$

where  $r_{t+1}$  is the vector of future excess return of the  $N$  equity factors and  $w_t$  denotes the vector of equity factor portfolio weights. The use of excess returns implies that the remainder is invested into the risk-free asset with return  $r_f$  if the PPP is not fully invested. The crucial ingredient of Brandt and Santa-Clara (2006) is to assume the optimal portfolio strategy  $w_t$  to be linear in the vector  $z_t$  of the  $K$  conditioning variables (of which the first element is simply a constant):

$$w_t = \theta z_t \quad (15)$$

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<sup>4</sup>As a robustness check we have also analyzed the use of 3 PCAs (jointly capturing 56% of variation) which gives rise to similar conclusions. Moreover, a smaller number of predictors allows to have a longer out-of-sample backtesting window.

and  $\theta$  is an  $N \times K$  matrix of parameters. Plugging the linear portfolio policy from representation (15) in Equation (14), the problem becomes

$$\max_{\theta} E_t \left[ (\theta z_t)' r_{t+1} - \frac{\gamma}{2} (\theta z_t)' r_{t+1} r_{t+1}' (\theta z_t) \right] \quad (16)$$

Using some linear algebra<sup>5</sup> to rearrange the following term

$$(\theta z_t)' r_{t+1} = z_t' \theta' r_{t+1} = \underbrace{vec(\theta)'}_{=: \tilde{w}} \underbrace{(z_t \otimes r_{t+1})}_{=: \tilde{r}_{t+1}} \quad (17)$$

one can write Equation (16) as

$$\max_{\tilde{w}} E_t \left[ \tilde{w}' \tilde{r}_{t+1} - \frac{\gamma}{2} \tilde{w}' \tilde{r}_{t+1} \tilde{r}_{t+1}' \tilde{w} \right] \quad (18)$$

As the same  $\tilde{w}$  maximizes the conditional expected utility at all  $t$ , it also maximizes the unconditional expected utility, hence optimization problem (18) is equivalent to

$$\max_{\tilde{w}} E \left[ \tilde{w}' \tilde{r}_{t+1} - \frac{\gamma}{2} \tilde{w}' \tilde{r}_{t+1} \tilde{r}_{t+1}' \tilde{w} \right] \quad (19)$$

Thus, the original dynamic optimization problem can be restated as a static Markowitz optimization applied to an augmented set of assets that does not only include the basis assets (i.e., the equity factors) but also “managed” portfolios. Each of these managed portfolios invests in a single basis asset according to the realization of one of the conditioning variables. To illustrate the augmented asset space, consider a simple two-factor example using Book-to-Market (*BTM*) and 12-month momentum (*MOM12*) with just one conditioning variable:

$$\tilde{r}_T = \begin{bmatrix} f_{t_1}^{BTM} & f_{t_1}^{MOM12} & f_{t_1}^{BTM} * z_{t_0} & f_{t_1}^{MOM12} * z_{t_0} \\ f_{t_2}^{BTM} & f_{t_2}^{MOM12} & f_{t_2}^{BTM} * z_{t_1} & f_{t_2}^{MOM12} * z_{t_1} \\ \vdots & \vdots & \vdots & \vdots \\ f_{t_T}^{BTM} & f_{t_T}^{MOM12} & f_{t_T}^{BTM} * z_{t_{T-1}} & f_{t_T}^{MOM12} * z_{t_{T-1}} \end{bmatrix} \quad (20)$$

Instead of directly solving for portfolio weights as in the classical Markowitz problem, one optimizes over the  $\theta$  parameters that govern the linear portfolio policy. Having obtained an optimal parameter for each managed portfolio, one can simply multiply these with the current realizations of the conditioning variables to arrive at the optimal weight  $w$ . That is, by adding up the corresponding weight components that are related to a specific factor, we can infer the optimal weights for any single factor. In the above example, one thus adds up the optimal weights for  $f^{MOM12}$  and  $f^{MOM12} * z$  to obtain the total momentum factor weight.

The conditioning variables used for the equity factors are the first fundamental PCA factor and the first technical PCA factor. Note that any given equity factor can be joined

<sup>5</sup>Note that *vec* is a linear transformation which converts the matrix into a column vector and  $\otimes$  denotes the Kronecker product of two matrices.

with a factor-specific set of conditioning variables, i.e., one could focus on selecting factor-conditioning variables that are deemed meaningful. Yet, we refrain from pursuing such a cherry picking exercise, but rather aim for including the maximal amount of information as represented by the 2 PCA factors. We compute the first optimal portfolio weights over a 66 months window which is expanded going through time such that we obtain the first portfolio for June 2003<sup>6</sup>. As for the risk aversion parameter  $\gamma$  governing the quadratic utility function, we choose a quite conservative value of 10 implying a relatively high risk aversion, see Ang (2014). The parameters in  $\theta$  pertaining to the original basis equity factor are constrained such that they equal the weight of the equal-weighted benchmark portfolio. Hence, we perform benchmark-relative portfolio allocation where deviations from the equal-weighted benchmark only result from changes in the conditioning variables<sup>7</sup> (i.e., in the above example for  $t = 1$ :  $f_{t_1}^{BTM} * z_{t_0}$  and  $f_{t_1}^{MOM12} * z_{t_0}$ ). To avoid extreme allocations, we allow a predictor to change the factor weight by twice its benchmark weight at most. In addition, we rescale the timing portfolio weights such that they obey a maximum ex-ante annualized tracking error of 2.5%. In particular, this rescaling ensures that we do not unnecessarily force the strategy into more extreme allocations when the signals from the PCA factors are deemed weak. In the latter case the strategy will naturally resort to the equal-weighted benchmark. The (annualized) ex-ante tracking error at time  $t$  ( $TE_t$ ) is calculated as

$$TE_t = \sqrt{12 (w_s - w_b)' \Sigma (w_s - w_b)} \quad (21)$$

where  $w_s$  is the weights vector of the strategy and  $w_b$  is the weights vector of the benchmark such that the difference gives the active weights of the factor timing strategy.  $\Sigma$  denotes the covariance matrix based on the factor returns that are available upon estimation.

As the PPP expresses the portfolio problem in an estimation context, it is possible to compute standard errors for the  $\theta$  coefficients and to evaluate the significance of a given predictor. According to Brandt and Santa-Clara (2006), we calculate standard errors from the covariance matrix of  $\tilde{w}$  which is calculated as

$$\frac{1}{\gamma^2 T - N \times K} (\iota_T - \tilde{r}\tilde{w})' (\iota_T - \tilde{r}\tilde{w}) (\tilde{r}'\tilde{r})^{-1} \quad (22)$$

where  $\iota_T$  denotes a  $T \times 1$  vector of ones.

### 3.2.2 Empirical results

Table 2 gives the estimates of the  $\theta$ -coefficients and their significance. Significance is assessed in terms of the corresponding confidence bands calculated as  $\hat{\theta}_i \pm 1.96 \times SE_i$  with  $i$  ranging from 1 to  $21 \times 2 = 42$ . Note that all the  $\theta$ -coefficients representing the factor timing strategy

<sup>6</sup>Additionally, we need another 12 months to calibrate the technical indicators.

<sup>7</sup>The approach of Brandt and Santa-Clara (2006) is designed to also allow the policy to fully deinvest if deemed necessary: When equity factors are expected to disappoint, the PPP resorts to a risk-free investment instead. Imposing a full-investment constraint might thus prevent the strategy from fully exploiting the information content embedded in the PCA factors. In unreported tests we observe that relaxing the full investment constraint hardly leads the PPP to deinvest.

are insignificant at the 5% level. Nevertheless, simply looking at the signs, a positive coefficient for *TECH1* would indicate that factor buy signals lead to even higher weightings in the factor allocation.

[Table 2 about here.]

While the statistical evidence of the  $\theta$ -coefficients is weak, following the approach of Leitch and Tanner (1991), we wonder whether the economic performance is likewise weak. In a first step we inspect the ensuing factor allocations over time. In particular, Figure 3 depicts the transfer of  $\theta$ -coefficients into the optimal portfolio weights using the examples of Book-to-Market (*BTM*) and 12-month momentum (*MOM12*). Specifically, we decompose the optimal weights by contributions of each conditioning variable. While *TECH1* favors *MOM12* in nearly every period throughout our sample - except for a few months in 2009, *FUN1* has a more cyclical contribution to the weights decomposition. Especially during the crisis of 2007 to 2009, in 2012 to 2013 and in 2016 it is reducing the weight on *MOM12*. As a result, one normally overweights the factor, except for the above mentioned periods. For the *BTM* factor the picture is different. We underweight and even short this factor most of the time, driven by *TECH1* as well as *FUN1* predictors.

Considering the remaining factors the average weight of *PROF*, *CFY*, *AT*, *MOM12*, *CP*, *EY* and *ROA* is increased compared to the benchmark case. The strongest increase obtains for *PROF* with 2.5% per month on average. For the remaining factors, the PCA factors lead to a decrease in the weight. This decrease is most pronounced for *DY* with  $-2.5$  percentage points and *STR* with  $-2.2$  percentage points. The general overweight in *PROF* and *MOM12* as well as the underweight in *ACC* should help active performance while an underweight in *MOM6* and an overweight in *CP* should be detracting active performance. As a result, the biggest average weight over the time period is assigned to *PROF* (7.3%), *MOM12* (6.9%), and *CFY* (6.7%), while *MOM6* (3.4%), *DY* (2.3%), and *STR* (2.3%) account for the factors with the lowest weight. Thus, the factor allocation ensuing from the PPP is quite active and might be rewarded performance-wise, despite the insignificance of the  $\theta$ -coefficients.

[Figure 3 about here.]

In fact, the factor timing strategy is outperforming the equal-weighted benchmark by 1.23% p.a. Given an ex-post tracking error of 1.12% this results in an information ratio of 1.10, see Table 3. The absolute performance is 4.60% excess return at 2.89% volatility p.a. which responds to a sharpe ratio of 1.59 and compares to a equal-weighted factor allocation benchmark sharpe ratio of 1.21. Yet, the maximum drawdown of the factor timing strategy is slightly more severe (-6.11% vs. -5.31%).

[Table 3 about here.]



## 4 Factor tilting

A potentially complementary way of optimal equity factor investing could exploit differences in cross-sectional factor characteristics. For this purpose, we couch suitable characteristics into the cross-sectional parametric portfolio policy developed by Brandt, Santa-Clara, and Valkanov (2009). We employ six cross-sectional factor characteristics that can be computed from the factors' return time series: valuation, spread, price momentum, volatility, and the factors' position in the factor correlation network.

### 4.1 Cross-sectional factor characteristics

#### 4.1.1 Valuation (*VAL*)

The underlying rationale of any value strategy is to invest in relatively cheap assets while avoiding (or shorting) securities that are relatively expensive. Translating this idea to equity factors one could consider overweighting factors which are cheap and underweighting those that are expensive. To operationalize this rationale we focus on valuation levels. Following the idea of Basu (1977), value, as measured by fundamental factor metrics (such as price-to-book or price-to-earnings), is a good predictor of future stock returns. Translating this rationale to the factor level one can determine the relative cheapness of a given factor by comparing the average valuation of the factor's top quintile to the one of the bottom quintile. The academic literature provides several explanations for the value premium ranging from risk considerations to behavioural arguments (cf. Fama and French (1993, 1995) and Lakonishok, Shleifer, and Vishny (1994)). Therefore, one would expect a positive prognostic ability of future performance on a factor level as well (cf. Arnott, Beck, Kalesnik, and West (2016)). Still, we have to keep in mind, that any factor will trade on its own norm. Value factors will be cheap by definition compared to growth factors. Using *Book-to-Market* ratios as a proxy for valuation in our research, we especially have to address if there is any additional benefit to using a valuation indicator for factor timing when there is already a value factor in the overall factor allocation model.

#### 4.1.2 Spread

A factor spread measures the distance in its defining characteristic from top quintile to bottom quintile. Thus, stocks are sorted in descending order according to the factor-defining characteristic. If the mean difference between top and bottom quintile is large, the factor is relatively cheap in terms of the factor-defining characteristic as one can easily distinguish between attractive and unattractive stocks. Asness (1997) showed that realized performance over the short term is heavily driven by the dispersion of returns. Consequently, it is difficult to show superior skill if returns act homogeneously. Following this rationale, we use factor spreads to proxy for their potential future return dispersion: If the factor spread is wide the

factor’s return opportunity is expected to be largest (cf. Huang, Liu, Ma, and Osiol (2010)). As we have a diverse set of factors included in our factor set, we standardize their spreads<sup>8</sup>.

#### 4.1.3 1-month price momentum ( $PM$ )

We use 1-month price momentum ( $PM$ ), to capture short-term factor momentum. Avramov, Cheng, Schreiber, and Shemer (2017) document a naïve active factor momentum strategy applied to a set of 15 equity factors to consistently outperform a  $1/N$  benchmark. The momentum measure for equity factor  $i$  at time  $t$ ,  $PM_{i,t}$ , is simply calculated as the return of the respective equity factor in the previous month,  $r_{i,t-1}$ .

#### 4.1.4 Volatility ( $VOL$ )

As early as in the 1970s low volatility stocks have been documented to outperform high volatility stocks, see Jensen, Black, and Scholes (1972) and Haugen and Baker (1991). In this vein, we test whether there is also a volatility effect amongst equity factors. We calculate  $VOL_{i,t}$  from the variance-covariance matrix of equity factors using observations up to  $t$  which are weighted according to an exponentially weighted moving average (EWMA). We use an initial window of 36 months. Hence, the first factor volatilities are available as of December 1999.

#### 4.1.5 Factor position in correlation networks

We consider two factor centrality measures that build on the work of Mantegna (1999), Pozzi, Di Matteo, and Aste (2013) and Lohre, Papenbrock, and Poonia (2014). In a similar vein to Montagu, Krause, Jalan, Murray, Chew, and Yusuf (2016), we investigate the centrality of an equity factor in the factor correlation network as given by the factor’s node betweenness. This reasoning is in line with empirical findings of Pozzi, Di Matteo, and Aste (2013) who show that portfolios of peripheral U.S. stocks provide a superior risk-adjusted performance compared to central stocks. In a similar vein, Lohre, Papenbrock, and Poonia (2014) use the node betweenness in a parametric portfolio policy of Brandt, Santa-Clara, and Valkanov (2009) and demonstrate a notable outperformance of peripheral against central equity sectors. A related and yet distinct characteristic is the factor’s distance to the market portfolio as revealed in the correlation network.

Appendix A.2 details the computation of a correlation network as given by a minimum-spanning tree. To foster intuition, Figure 4 displays the MST that obtains using equity factor data from January 1997 to December 2016 together with the market ( $S\&P$  500) with equal weight on all observations.

[Figure 4 about here.]

Note that many factors are generally quite different from the market as the  $S\&P$  500 is found at the periphery. This observation is expected as the long-short construction of factors

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<sup>8</sup>We use an expanding window to standardize the spread according to the factors own history.

should remove most of the market risk. The factors, among others, *PROF*, *LTR*, *LEV* and *BTM* turn out to be rather peripheral to the correlation network similar to *S&P 500* and *Size*, while *MOM6* and *DY* are central.

#### 4.1.5.1 Node betweenness (*NB*)

Specifically, we compute the betweenness centrality which is indicative of a factor's centrality in the network. The node betweenness of factor  $i$  is equal to the number of shortest lines from all nodes to all others that pass through that node. Based on the minimum spanning tree (*MST*) in Figure 4, *Accruals* has a node betweenness of 19, as all shortest paths from 19 factors to the *S&P 500* cross *ACC*. Only the one for *Size* does not pass *ACC*. In contrast, the node betweenness for some factors, including *PROF*, *EY*, *STR*, *AT* and *S&P 500*, is zero, as there are no shortest paths from one factor to another passing through these. Note that we rely on a correlation network consisting of the 21 equity factors alone (excluding the market factor) when calculating the factors' node betweenness. Tilting towards peripheral factors can be interpreted as an additional layer of risk control that implicitly provides a higher degree of diversification.

#### 4.1.5.2 Distance-to-market (*DTM*)

As a novel contribution to the literature we also include a characteristic that measures an equity factor's distance to the market. The conjecture is that factors more distant from the market outperform those closest to the market. Specifically, tilting towards factors most different from the general market allows for putting emphasis on those factors that appear most genuine relative to the market return. The distance-to-market (*DTM*) characteristic is calculated as the length from the shortest paths from each factor to the *S&P 500* in the *MST*. For the total period and an equally weighted covariance matrix, Figure 4 shows that *ACC* and *Size* have the shortest *DTM* on average whereas for example *AG* has the longest.

## 4.2 Methodology of Brandt, Santa-Clara, and Valkanov (2009)

We couch the above characteristics into the parametric portfolio policy of Brandt, Santa-Clara, and Valkanov (2009) which allows for utility-driven portfolio optimization to exploit cross-sectional characteristics. While an application of the mean-variance approach of Markowitz (1952) would require to estimate first and second moments of all assets, the authors propose a more parsimonious optimization problem that leads to a tremendous reduction in dimensionality. In particular, they suggest to parameterize the weight of an asset as a function of its characteristics. The associated coefficients are estimated by maximizing investor utility. Specifically, the authors consider an investor seeking to maximize her conditional expected utility of her portfolio return  $r_{p,t+1}$ :

$$\max_{\{w_{i,t}\}_{i=1}^{N_t}} E_t [u(r_{p,t+1})] = E_t \left[ u \left( \sum_{i=1}^{N_t} w_{i,t} r_{i,t+1} \right) \right] \quad (23)$$

where  $w_{i,t}$  denotes the portfolio weight for asset  $i$  and  $N_t$  the number of assets at time  $t$ . Brandt, Santa-Clara, and Valkanov (2009) propose to model the portfolio weight as a linear function of its characteristics  $x_{i,t}$ :

$$w_{i,t} = f(x_{i,t}; \phi) = \overline{w_{i,t}} + \frac{1}{N_t} \phi' \hat{x}_{i,t} \quad (24)$$

where  $\overline{w_{i,t}}$  denotes the benchmark weights,  $\phi$  is the vector of coefficients to be estimated through utility maximization and  $\hat{x}_{i,t}$  are standardized factor characteristics. Parameterization (24) implicitly assumes that the characteristics fully capture the joint distribution of asset returns that are relevant for portfolio optimization. The characteristics are cross-sectionally standardized at time  $t$  across all factors

$$\hat{X}_{i,t} = \frac{x_{i,t} - \overbrace{\frac{1}{N} \sum_{i=1}^{12} x_{i,t}}^{=: \bar{x}}}{\sqrt{\frac{1}{N-1} \sum_{i=1}^{12} (x_{i,t} - \bar{x})^2}} \quad (25)$$

As a consequence, the cross-sectional distribution of the standardized characteristics is stationary through time and the cross-sectional mean for each standardized characteristic is zero such that deviations from the benchmark are equivalent to a zero-investment portfolio. The weights of the resulting portfolio thus always add up to 100 %. The optimization problem is further simplified by noting that the coefficients that maximize the conditional expected utility of the investor at a specific time  $t$  are constant through time and across assets such that the optimization problem can be written in terms of the  $\phi$ -coefficients:

$$\max_{\phi} E[u(r_{p,t+1})] = E \left[ u \left( \sum_{i=1}^{N_t} f(x_{i,t}; \phi) r_{i,t+1} \right) \right] \quad (26)$$

To estimate the  $\phi$ -coefficients we rely on the corresponding sample moments:

$$\max_{\phi} \frac{1}{T-1} \sum_{t=0}^T u(r_{p,t+1}) = \frac{1}{T} \sum_{t=0}^{T-1} u \left( \sum_{i=1}^{N_t} \left( \overline{w_{i,t}} + \frac{1}{N_t} \phi' \hat{x}_{i,t} \right) r_{i,t+1} \right) \quad (27)$$

As the PPP expresses the portfolio problem as a statistical estimation problem, it is possible to obtain standard errors for the  $\phi$  coefficients and to evaluate whether a characteristic is a significant determinant of the portfolio policy. The optimization problem in Equation (27) satisfies the first order conditions, see Brandt, Santa-Clara, and Valkanov (2009):

$$\frac{1}{T} \sum_{t=0}^{T-1} h(r_{t+1}, x_t; \phi) \equiv \frac{1}{T} \sum_{t=0}^{T-1} u'(r_{p,t+1}) \left( \frac{1}{N_t} \hat{x}_t' r_{t+1} \right) = 0 \quad (28)$$

where  $u'(r_{p,t+1})$  denotes the first derivative of the utility function and  $\hat{x}_t'$  the transpose of the factor characteristics vector. Thus, the optimization problem can be interpreted as a method

of moments estimator and the asymptotic covariance matrix estimator  $AVar[\hat{\phi}]$  is given by, see Hansen (1982):

$$\Sigma_{\phi} \equiv AVar[\hat{\phi}] = \frac{1}{T}[G'V^{-1}G]^{-1} \quad (29)$$

where

$$G \equiv \frac{1}{T} \sum_{t=0}^{T-1} \frac{\delta h(r_{t+1}, x_t; \phi)}{\delta \phi} = \frac{1}{T} \sum_{t=0}^{T-1} u''(r_{p,t+1}) \left( \frac{1}{N_t} \hat{x}'_t r_{t+1} \right) \left( \frac{1}{N_t} \hat{x}'_t r_{t+1} \right)' \quad (30)$$

and  $V$  is a consistent estimator of the covariance matrix of  $h(r, x; \phi)$ .

### 4.3 Empirical results

Table 4 gives estimation results and performance statistics for six univariate parametric portfolio policies. Across the univariate models, the only significant coefficients obtain for *PM*, while the coefficients for *VOL*, *NB*, *DTM*, *VAL*, and *Spread* are insignificant. We observe a significant positive sign for *PM* suggesting a short-term price momentum effect among equity factors. Hence, factors with positive price momentum are overweighted relative to the benchmark while factors with negative price momentum are underweighted. The annualized return of the parametric portfolio policy using *PM* is 0.62 percentage points higher than the one for the equal-weighted benchmark, the volatility increases by 0.31 percentage points. This results in an information ratio of 0.88. While *VOL*, *NB* and *VAL* display negative information ratios, *DTM* and *Spread* have a positive one (0.30 and 0.06, respectively).

[Table 4 about here.]

[Figure 5 about here.]

Instead of relying on *PM* only, we include six characteristics in a multivariate parametric portfolio policy as the interactions of characteristics in a multivariate setting might alter the evidence. Panel C in Table 4 gives the estimation results for the parametric portfolio policy based on six characteristics. the estimation coefficients for *VAL* and *NB* are insignificant and detract from performance. Focussing on factors whose estimation coefficients are significant in a multivariate framework and which economically foster performance, we reduce our characteristic set to four factors. Panel D in Table 4 and Figure 5 show the  $\phi$ -estimates and the corresponding confidence bands calculated as  $\hat{\phi}_i \pm 1.96 \times SE_i$ . Similar to the univariate case we find *PM* still having a significant  $\phi$ -coefficient. Using a multivariate approach result in significant coefficients for *DTM*, *Spread* and *VOL* in the sample period. *Spread* and *VOL* have a negative  $\phi$ -estimator meaning that the model favors less volatile factors with wide spreads. On the other hand, *DTM* shows a positive  $\phi$ -coefficient indicating that the model favors factors which are more distant to the market.

[Figure 6 about here.]

Figure 6 illustrates the ensuing optimal linear portfolio weight over time using the two factors Book-to-Market (*BTM*) and 12-month momentum (*MOM12*). As for *BTM*, there is an overweight relative to the equal-weighted benchmark due to *Spread* and *DTM*. As this changed in the course of the financial crisis in 2009 the PPP shys away from *BTM* to the extent that this factor is underweighted. The two other characteristics hardly influence the *BTM* factor. As the factor is a middle-ground factor according to its volatility, the *VOL* characteristic does not significantly add or detract weight from the equal-weighted benchmark anchor. The evidence is different for the *MOM12* factor. The overall weight is much more volatile going through time. *MOM12* does have the highest volatility in the factor set, thus the PPP reduced its weight due to the negative  $\phi$ -coefficients for *VOL*. *DTM* and *VOL* lead to an underweight until the beginning of 2009. Afterwards *DTM* and *Spread* lead to an overweight due to the changing  $\phi$ -coefficients for *DTM* over time.

In Table 3 we present the resulting strategy performance. The factor tilting approach delivers an excess return of 0.51 percentage points p.a. over the benchmark, whereas the volatility is increased by 0.32 percentage points. As a result, the SR (1.25) is comparable to the the equal-weighted benchmark (1.21). The return distribution is also more extreme, as the minimum and maximum returns attain higher absolute values. However, the maximum drawdown is slightly reduced from  $-5.3\%$  to  $-4.6\%$ , indicating that the strategy helps to navigate some of the *bad times*. The strategy's IR is 0.61.

We infer that the chosen characteristics are useful to tilt equity factors. By couching the cross-sectional characteristics into the parametric portfolio policy of Brandt, Santa-Clara, and Valkanov (2009) we are able to construct a portfolio which slightly increase on a risk-return basis compared to an equal-weighted benchmark and is able to limit the maximum drawdown.

## 5 Conclusion

To summarize, this paper contributes to the ongoing debate of whether it is possible to time factors. In contrast to most existing studies, we use a multi-factor approach. Moreover, taking a portfolio-theoretic factor allocation view that considers time-series as well as cross-sectional based predictors also adds to the academic literature. In this context, we contrast our performance against an equal-weighted factor allocation benchmark. Using a well documented yet diversified factor set, we couch time-series and cross-sectional signals into the parametric portfolio policy by Brandt and Santa-Clara (2006) and Brandt, Santa-Clara, and Valkanov (2009) to improve the equal-weighted benchmark. For factor timing using time-series information we rely on fundamental and technical indicators as predictors. To avoid cherry picking and overcome multicollinearity problems we use the first PCA of both datasets. We reduce the number of variables to synthesize information embedded in all predictor variables and also reduce their noise. Although the factor timing coefficients are insignificant we are able to improve our benchmark results.

For factor tilting we rely on six different characteristics embedded in the cross-section of the factor set: valuation, spread, momentum, volatility and two factors based on the correlation network (note betweenness and distance-to-market). Couching this in the parametric portfolio policy on an univariate basis only momentum coefficients were significant. In a multivariate model only node betweenness and valuation stay insignificant. Therefore, we focus on a model including the remaining four significant characteristics to exploit their information in a factor tilting framework. Using this cross-sectional information the tilting-coefficients help to slightly improve the risk-return profile of the resulting factor allocation relative to the equal-weighted factor allocation benchmark and are able to limit the maximum drawdown remarkably.

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## A Appendices

### A.1 Definition of fundamental predictors

- *Dividend Price Ratio (dp)*: Difference between the log of 12-month moving sums of dividends paid on the *S&P* 500 index and the log of *S&P* 500 index prices.
- *Dividend Yield (dy)*: Difference between the log of 12-month moving sums of dividends paid on the *S&P* 500 index and the log of 1-month lagged *S&P* 500 index prices.
- *Earnings Price Ratio (ep)*: Difference between 12-month moving sums of earnings on the *S&P* 500 index and log of *S&P* 500 index prices.
- *Dividend Payout Ratio (de)*: Difference between 12-month moving sums of dividends on the *S&P* 500 index and log of 12-month moving sums of earnings on the *S&P* 500.
- *Stock Variance (svar)*: Realized variance calculated as the monthly sum of squared daily returns on the *S&P* 500.
- *Book to Market Ratio (bm)*: Ratio of book value to market value for the Dow Jones Industrial Average.
- *Net Equity Expansion (ntis)*: Ratio of 12-month moving sums of net issues by NYSE listed stocks divided by the total market capitalization of NYSE stocks.
- *Treasury Bills (tbl)*: Interest rate on a 3-month treasury bill traded on the secondary market.
- *Long Term Yield (lty)*: Yield on long-term government bonds.
- *Long Term Rate of Return (ltr)*: Return on long-term government bonds.
- *Term Spread (tms)*: Difference between the long-term yield on government bonds and the T-bill rate.
- *Default Yield Spread (dfy)*: Difference between BAA- and AAA-rated corporate bond yields.
- *Default Return Spread (dfr)*: Difference between the return on long-term corporate bonds and returns on the long-term government bonds.
- *Inflation (infl)*: Consumer Price Index (all urban consumers).

## A.2 Network Theory

Network theory deals with the representation of relations between elements in graphs. In a graph, the elements are represented by nodes which are connected by lines. A subgraph of a connected graph is the minimum spanning tree (MST) where the term tree refers to a graph in which any two nodes are connected by exactly one line and a *spanning* tree connects all nodes of the graph. The MST is the tree in which the sum of the length of all lines is minimized within the subclass of spanning trees without cycles.

In our case, the elements are equity factors and their relations are described in terms of correlations. As is common in the literature, we define the distance of factors by transforming the correlations as follows:

$$D_{i,j}^t = \sqrt{2(1 - \rho_{i,j}^t)} \quad (31)$$

This transformation ensures that all distances are positive and it yields a matrix with values in the range of  $[0,2]$ , in which smaller distances between factors represent higher correlations, and vice versa. We can then construct the MST by using Prim's (1957) algorithm: We start to initialize the MST with an arbitrarily chosen single factor  $A$ . Then, we examine all nodes that are not yet included in the graph and attach the node with the shortest distance to  $A$  (as measured by  $D_{i,j,t}$ ) to the graph. The algorithm of Prim then progressively adds further nodes which have not already been linked. This is done by repeatedly adding those which have the shortest distance to one of the nodes in the tree until all nodes are included. Identical to the computation of  $VOL$ , we weight the return observations to estimate the variance-covariance matrix using a decay parameter of  $\lambda = 0.97$ .

**Table 1: Stylized facts of equity factors.** The table shows stylized facts of the employed equity factors. Annualized excess returns are calculated using the arithmetic average of simple returns. The standard deviation and Sharpe ratio are annualized by multiplication with  $\sqrt{12}$ . Min and Max denotes the lowest and highest monthly excess return in the sample period. MaxDD describes the maximum drawdown the factor realized. Return, Volatility, Min, Max and MaxDD are in percentage terms. The time period is from 01/1997 to 12/2016.

	Return	Volatility	Min	Max	MaxDD	Sharpe Ratio	t-stat
<i>PROF</i>	7.70	6.66	-7.93	9.91	15.80	1.15	5.17
<i>CFY</i>	9.69	12.61	-14.00	17.14	46.01	0.76	3.43
<i>ACC</i>	0.29	5.70	-8.80	6.43	30.85	0.05	0.23
<i>DY</i>	5.63	14.08	-14.82	19.25	47.13	0.39	1.79
<i>AT</i>	4.55	5.25	-3.66	4.62	12.51	0.86	3.87
<i>BTM</i>	3.58	11.66	-14.26	16.95	46.36	0.30	1.37
<i>MOM12</i>	12.05	20.21	-33.12	22.45	56.04	0.59	2.67
<i>MOM6</i>	10.24	19.16	-30.25	27.83	43.93	0.53	2.39
<i>STR</i>	1.94	14.56	-16.26	15.87	37.14	0.13	0.60
<i>LTR</i>	3.20	12.72	-11.08	16.08	38.41	0.25	1.13
<i>DLTD</i>	4.96	7.22	-7.34	11.71	17.23	0.68	3.07
<i>DSO</i>	7.28	9.02	-8.61	12.55	20.83	0.80	3.61
<i>Size</i>	2.97	13.56	-11.45	12.84	45.27	0.21	0.98
<i>AG</i>	5.96	10.04	-10.51	15.71	25.09	0.59	2.66
<i>CP</i>	4.09	8.19	-7.30	12.18	22.82	0.50	2.24
<i>PMA</i>	3.99	8.63	-8.55	9.22	29.81	0.46	2.07
<i>EY</i>	8.33	11.35	-9.28	14.84	35.38	0.73	3.28
<i>LEV</i>	3.75	13.75	-18.49	18.20	51.84	0.27	1.22
<i>ROA</i>	5.07	7.12	-6.87	5.49	20.52	0.71	3.19
<i>STC</i>	5.28	11.84	-15.07	15.15	51.29	0.44	1.99
<i>STI</i>	2.48	5.68	-4.20	7.64	23.92	0.43	1.95



**Table 2: Factor timing.** The table shows the  $\theta$  coefficients for the fundamental and technical PCA factors that obtain in the parametric portfolio policy for factor timing. The coefficients are marked by \* if significant at the 5 %-level. The sample period is from 06/2003 to 12/2016.

	<i>FUN1</i>	<i>S.E.</i>	<i>TECH1</i>	<i>S.E.</i>
<i>PROF</i>	0.10	0.28	0.21	0.52
<i>CFY</i>	-0.10	0.39	0.03	0.38
<i>ACC</i>	0.10	0.41	0.13	0.62
<i>DY</i>	-0.10	0.27	-0.05	0.30
<i>AT</i>	0.10	0.49	0.08	0.45
<i>BTM</i>	-0.04	0.46	-0.03	0.50
<i>MOM12</i>	-0.10	0.18	0.20	0.15
<i>MOM6</i>	-0.08	0.18	-0.03	0.19
<i>STR</i>	0.02	0.08	-0.18	0.17
<i>LTR</i>	-0.02	0.21	0.00	0.20
<i>DLTD</i>	0.10	0.66	0.04	0.76
<i>DSO</i>	0.10	0.43	0.07	0.54
<i>Size</i>	0.01	0.16	0.00	0.17
<i>AG</i>	0.10	0.61	0.02	0.59
<i>CP</i>	0.07	0.48	0.02	0.62
<i>PMA</i>	0.10	0.45	-0.04	0.43
<i>EY</i>	-0.10	0.51	0.20	0.43
<i>LEV</i>	-0.10	0.35	0.01	0.30
<i>ROA</i>	0.05	0.64	0.13	0.44
<i>STC</i>	0.05	0.40	-0.09	0.44
<i>STI</i>	0.10	0.31	0.14	0.49

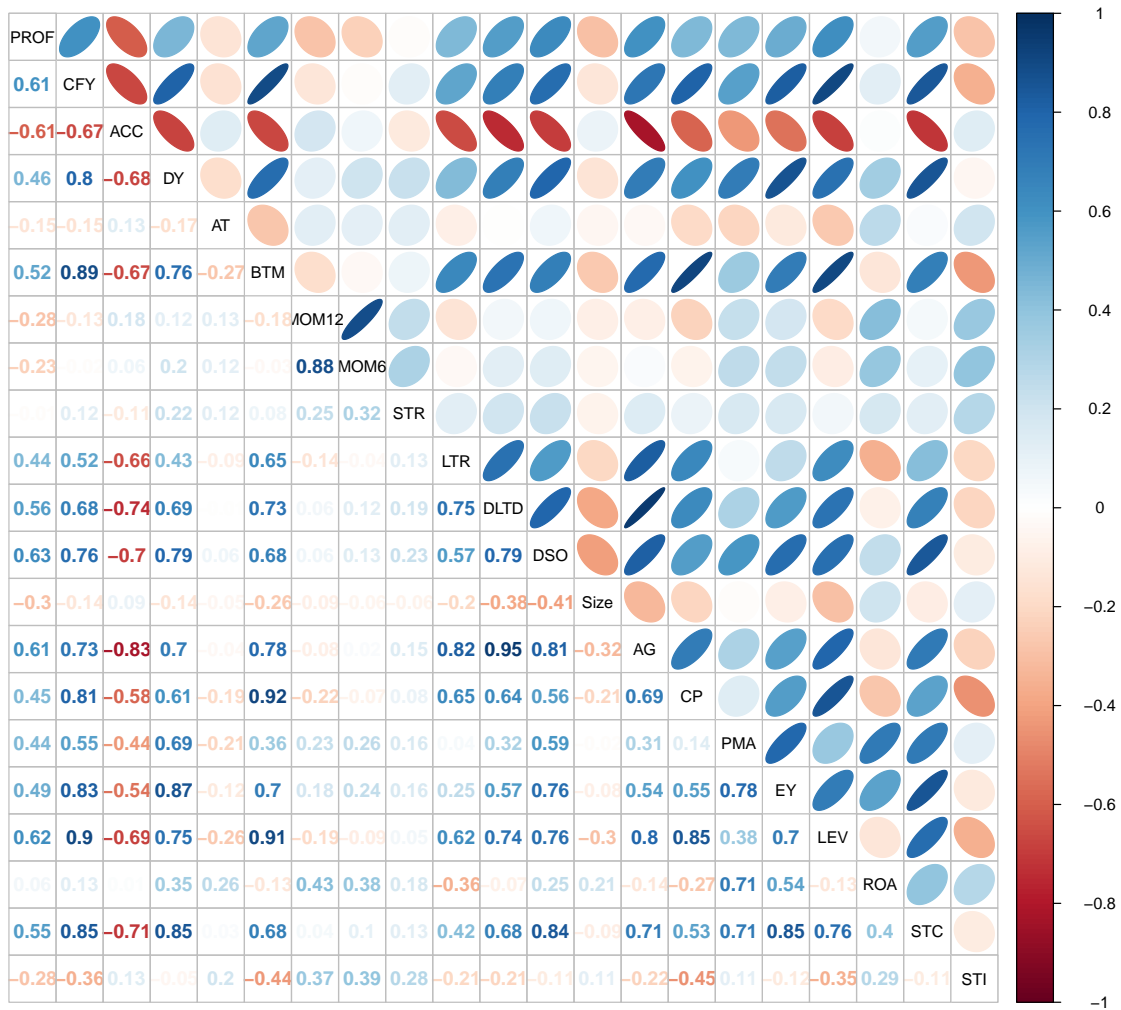
**Table 3: Performance statistics: Factor timing and factor tilting.** Excess return, standard deviation, minimum, maximum, maximum drawdown, and (ex-post) tracking error are in percentage points. Annualized excess returns are calculated using the arithmetic average of simple returns. The standard deviation and Sharpe ratio are annualized by multiplication with  $\sqrt{12}$ . The OOS period spans from 06/2003 to 12/2016.

Statistic	Benchmark $1/N$	Factor Timing $PPP_{FI}^{time}$	Factor Tilting $PPP^{tilt}$
Excess return	3.37	4.60	3.88
Standard deviation	2.79	2.89	3.11
Minimum	-2.78	-2.07	-3.43
Maximum	3.34	3.50	3.50
Maximum drawdown	-5.31	-6.11	-4.66
Sharpe ratio	1.21	1.59	1.25
Tracking error		1.12	0.83
Information ratio		1.10	0.61
$t$ -statistic	4.46	5.87	4.60

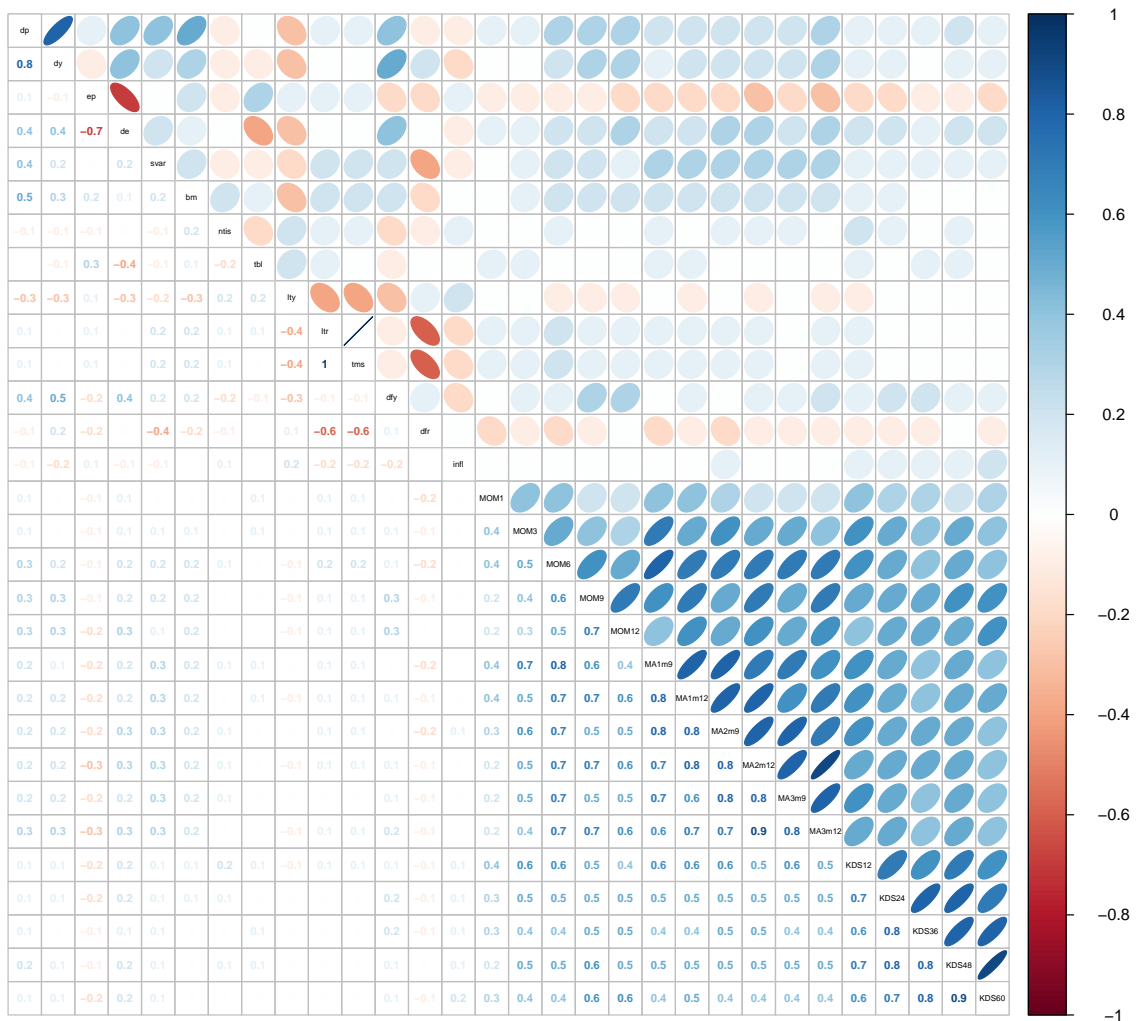
**Table 4: Factor tilting-coefficients.** The table gives estimation results and performance statistics of parametric portfolio policies for factor tilting based on cross-sectional characteristics. As for the estimation of the parametric portfolio policy, the first column reports the  $\hat{\phi}$ -coefficients, and the second column reports the associated standard errors (S.E.). The sample period is from 11/2002 to 12/2016. Panel B gives PPPs based on single characteristics, Panel C gives the PPP based on six characteristics, Panel D gives the PPP based on four characteristics. Panel A gives the benchmark model.

Characteristic	$\hat{\phi}$	S.E.	Return p.a.	Vola p.a.	Sharpe ratio	Tracking error	Information ratio
<i>Panel A: Benchmark model</i>							
1/N			3.37	2.79	1.21	-	-
<i>Panel B: Univariate model</i>							
PM	0.754*	0.261	3.99	3.10	1.29	0.71	0.88
VOL	-0.304	0.566	3.11	2.72	1.14	0.55	-0.48
NB	-0.209	1.215	3.04	2.63	1.15	0.78	-0.43
DTM	0.542	0.457	3.60	3.01	1.20	0.75	0.30
Val	-0.012	0.465	3.15	2.95	1.07	0.55	-0.40
Spread	-0.608	0.814	3.41	3.03	1.13	0.70	0.06
<i>Panel C: Multivariate model</i>							
PM	1.164*	0.256	3.69	3.15	1.17	0.84	0.38
VOL	-1.403*	0.496					
NB	1.857	1.045					
DTM	0.958*	0.403					
VAL	-0.536	0.356					
Spread	-1.214*	0.624					
<i>Panel D: Multivariate model</i>							
PM	0.946*	0.226	3.88	3.11	1.25	0.83	0.61
VOL	-0.854*	0.429					
DTM	0.822*	0.414					
Spread	-1.217*	0.638					

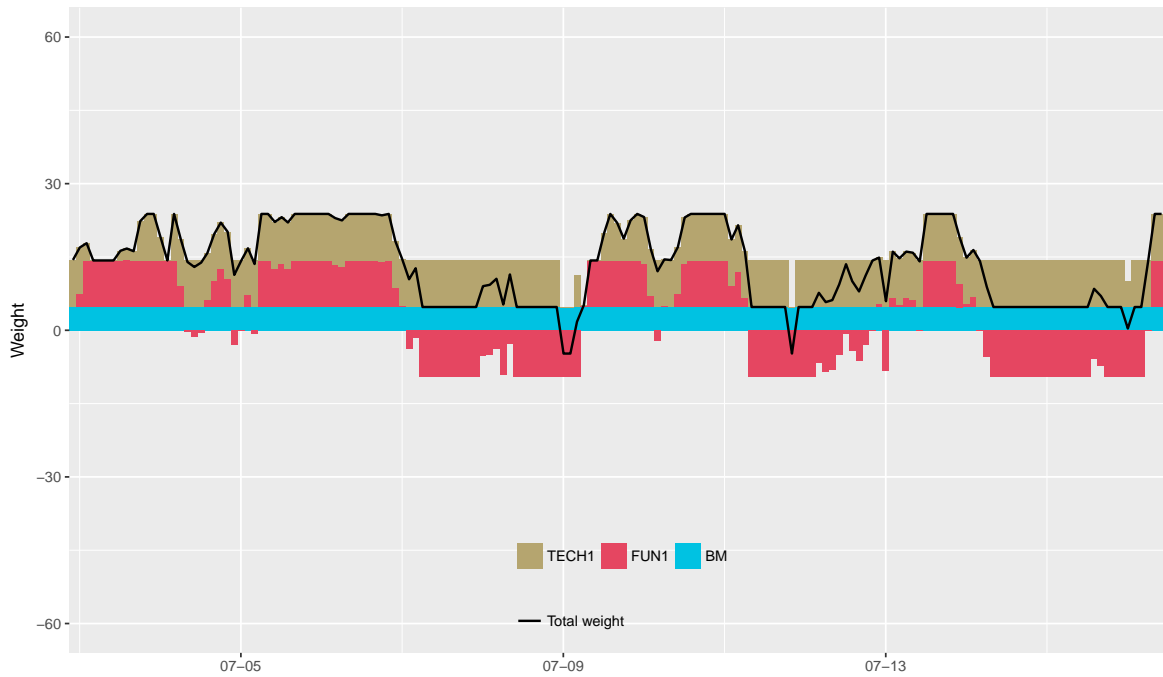
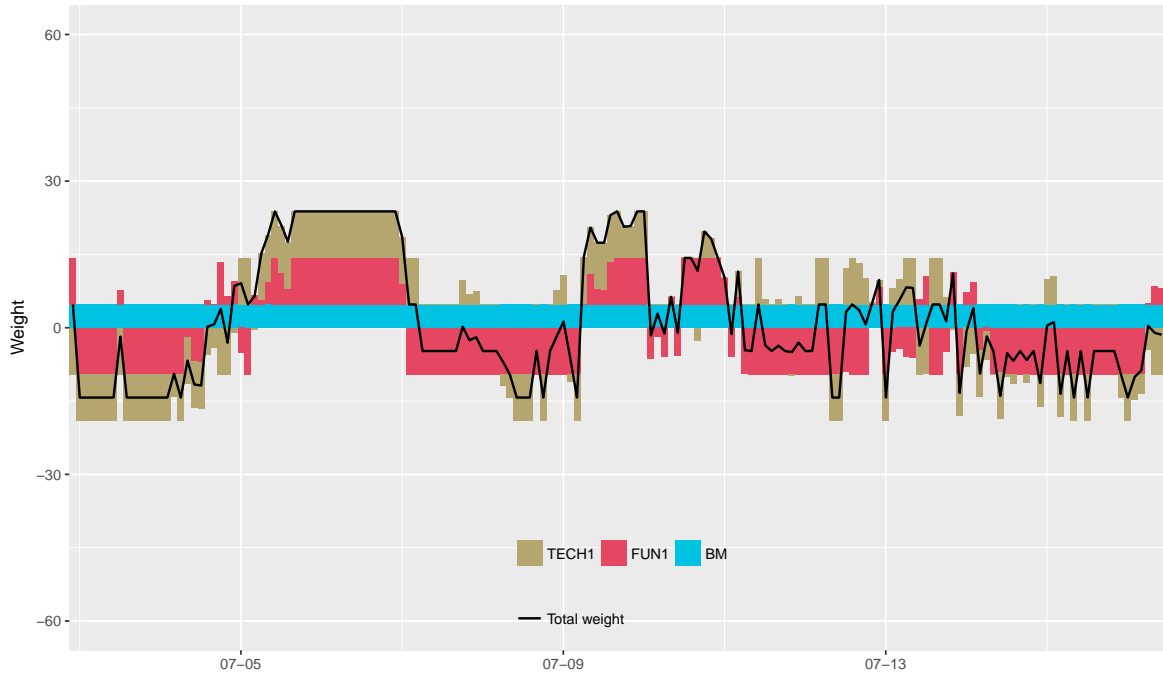
**Figure 1: Equity factor correlation matrix.** The figure gives the correlation matrix for the equity factors included in the factor set for the time period 01/1997 to 12/2016.



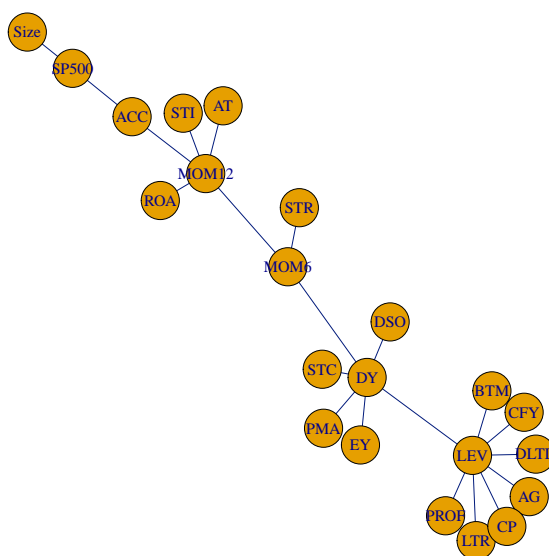
**Figure 2: Correlation matrix of timing predictors.** The figure shows the correlation matrix of the standardized fundamental variables and technical indicators for the 12-month momentum ( $MOM12$ ) factor. The correlation structure of fundamental variables is in the top left corner, the one of technical indicators in the bottom right corner. The time period is from 01/1997 to 12/2016



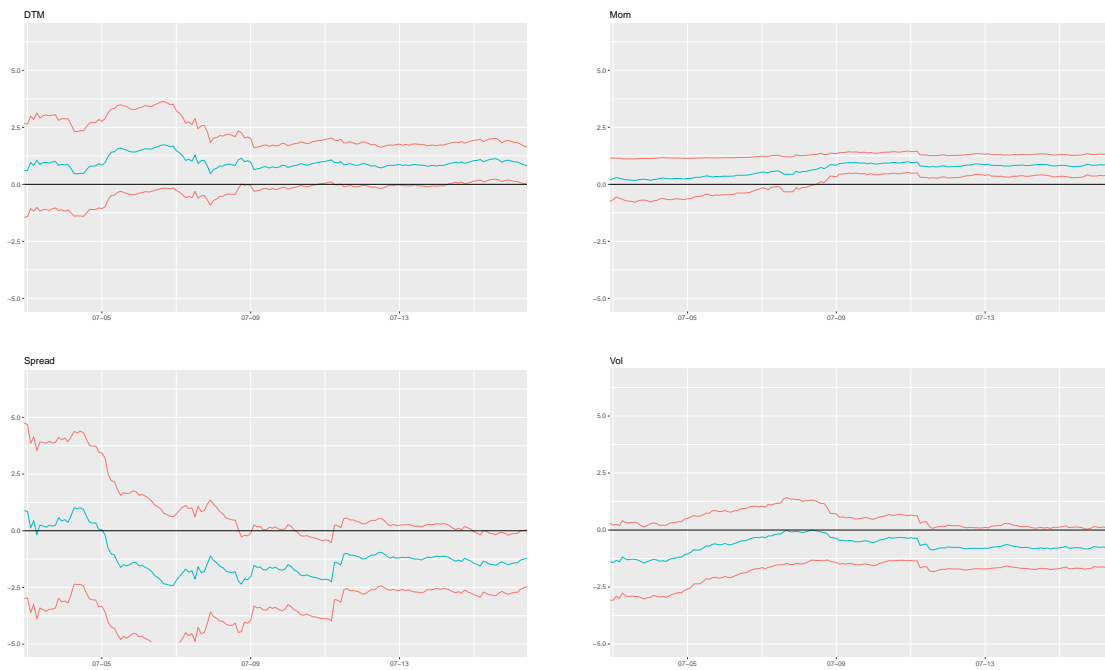
**Figure 3: Decomposition of optimal factor timing weights.** The figures decompose the factor timing weight for two factors (*BTM* at the top and *MOM12* at the bottom) into the contributions of the conditioning variables. *BM* depicts the benchmark weight. The black line shows the total weight of a respective factor. The time period is from 06/2003 to 12/2006.



**Figure 4: Minimum spanning tree.** The figure visualizes the correlation network in terms of a minimum spanning tree of equity factors plus the market. The plot builds on the equal-weighted variance-covariance matrix using monthly data from 01/1997 to 12/2016.

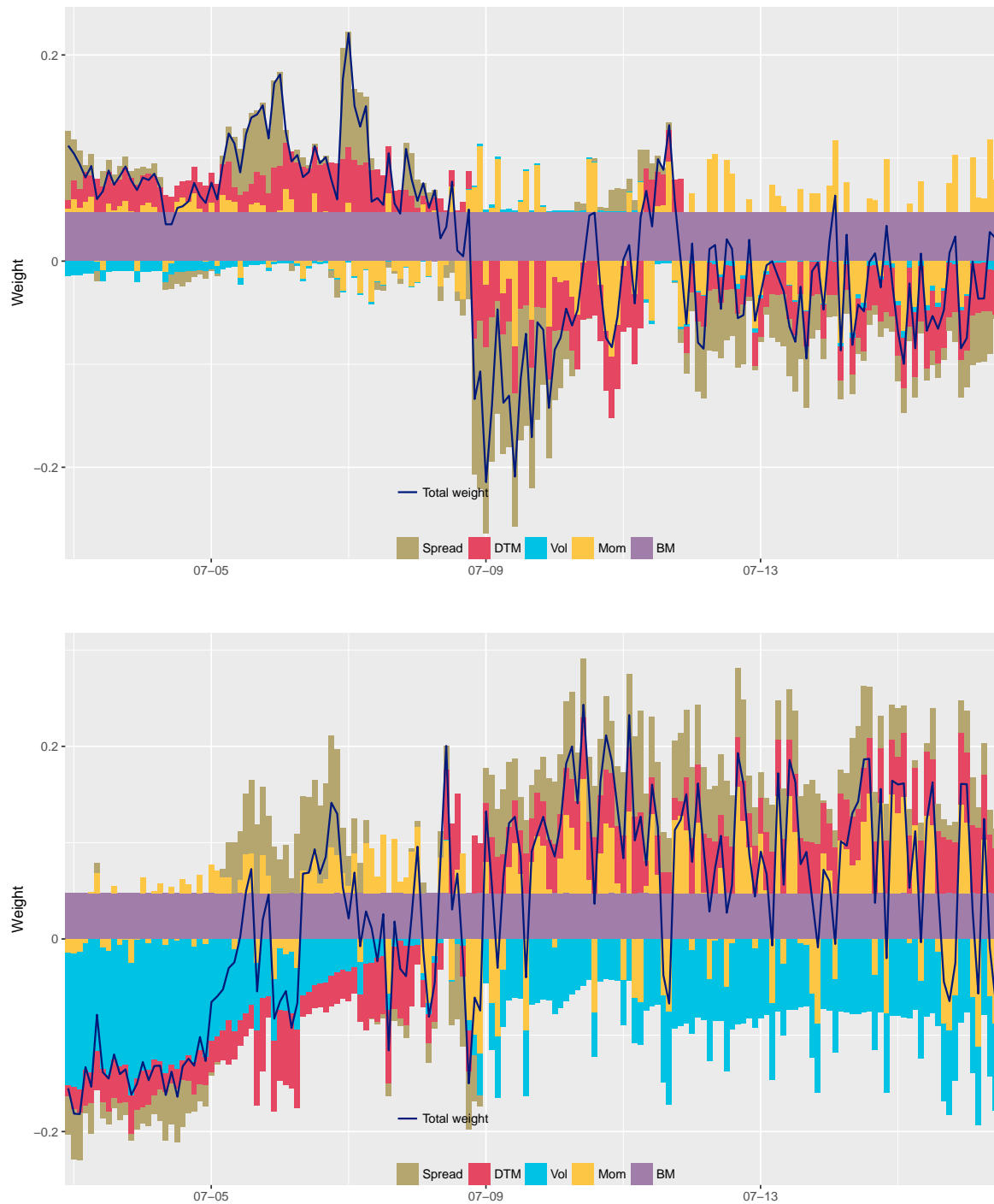


**Figure 5: Multivariate  $\phi$ -coefficients over time.** The figure depicts  $\phi$ -coefficients for the cross-sectional characteristics used in the parametric portfolio policy for factor tilting. The solid line depicts the coefficients while the dashed lines give the corresponding 95% confidence interval.





**Figure 6: Decomposition of optimal factor tilting weights.** The figure decomposes the weight in the parametric portfolio policy for factor tilting for a specific factor into the contributions of the characteristics. The solid line gives the overall weight. The upper chart is for *BTM* and the lower chart for *MOM12*. The time period is from 06/2003 to 12/2006.



**Figure 7: Univariate  $\phi$ -coefficients over time.** The figure shows  $\phi$ -coefficients for the cross-sectional characteristics used in the parametric portfolio policy for factor tilting. The solid line depicts the coefficients while the dashed lines give the corresponding 95% confidence interval.

