Estimating Portfolio Risk for Tail Risk Protection Strategies

David Happersberger*
EMP, Lancaster University Management School

Harald Lohre†
Invesco
EMP, Lancaster University Management School

Ingmar Nolte‡
EMP, Lancaster University Management School

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Abstract

This paper investigates appropriate forecasting models to estimate portfolio risk for timely managing the investment exposure of dynamic portfolio insurance strategies. In particular, we employ sophisticated risk forecasting techniques based on extreme value theory, quantile regression and Copula-GARCH methods to forecast the overnight risk of a global asset allocation portfolio. We find that these more sophisticated models do dominate more naive approaches in modelling the tail of the portfolio return distribution. As a result, dynamic portfolio insurance strategies building on these models work effectively in protecting investors from downside risk. Given that the mechanics of the portfolio insurance strategy automatically reduce investment exposure when approaching a protection level, the weakness of a less sophisticated risk forecast can be alleviated by this second line of defense.

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*Centre for Financial Econometrics, Asset Markets and Macroeconomic Policy (EMP), Department of Accounting and Finance, Lancaster University Management School, Bailrigg, Lancaster LA1 4YX, UK; d.happersberger@lancaster.ac.uk

†Invesco Quantitative Strategies, An der Welle 5, 60322 Frankfurt am Main, Germany; harald.lohre@invesco.com; Centre for Financial Econometrics, Asset Markets and Macroeconomic Policy, Lancaster University Management School, Bailrigg, Lancaster LA1 4YX, United Kingdom

‡Centre for Financial Econometrics, Asset Markets and Macroeconomic Policy, Department of Accounting and Finance, Lancaster University Management School, Bailrigg, Lancaster LA1 4YX, UK; i.nolte@lancaster.ac.uk

I. Introduction

Investors' objectives are generally expressed as a combination of risk and return targets. Defining the return target is usually relatively simple – but the definition of risk targets is less straightforward. One conventional approach is to consider *volatility*. For many investors, however, *maximum drawdown* is a more relevant statistic, as it points to the maximum loss of portfolio value. To limit the maximum drawdown investors typically follow broadly diversified investment strategies that include a tactical asset allocation component designed to avoid losses as often as possible. However, without perfect foresight investors might fail in navigating any given market drawdown. Thus, to effectively limit maximum drawdown, a given investment strategy often include some form of portfolio insurance. Portfolio insurance strategies primarily aim to improve the downside risk profile of an investment strategy without jeopardizing its long-term return potential. Of these, the constant proportion portfolio insurance (CPPI) strategy (Perold, 1986; Black and Jones, 1987, 1988; Perold and Sharpe, 1988) is probably the most popular method among practitioners. It is based on a dynamic portfolio allocation between a risky and a risk-free asset, aiming to guarantee a minimum capital level at the end of the investment period. Key element to determine the investment exposure of the risky asset is the so-called multiplier. It represents the inverse of the maximum sudden loss of the risky asset so that a given risk capital (i.e., the spread between portfolio value and protection level) is not fully consumed and the portfolio value does not fall below the protection level.[1]

The academic literature provides various methods to determine the multiplier in a static unconditional way (e.g. Bertrand and Prigent, 2002; Balder, Brandl, and Mahayni, 2009; Cont and Tankov, 2009), but they all reduce the risk dimension of the strategy to the risky asset exposure (see Black and Perold, 1992). Hence, these traditional unconditional methods do not fully account for the risk of the underlying asset that is known to change with market conditions. In other words, the risky asset's risk is considered constant during the whole investment period. Given the empirical characteristics of financial assets concerning risk (see for instance Longin and Solnik, 1995; Andersen, Bollerslev, Christoffersen, and Diebold, 2006), the academic literature proposes to model the multiplier as time-varying and conditional.

We contribute to the existing literature on time-varying conditional multipliers by investigating which risk model is most suitable in timely managing the investment exposure of a dynamic portfolio insurance strategy.

Following Bertrand and Prigent (2002) who determine their unconditional multiplier according to quantile conditions Ameur and Prigent (2006, 2014) introduce the conditional time-varying multiplier depending on the market evolution. Specifically, they employ GARCH-type models including an EGARCH(1,1)-model and compare it to a constant multiplier specification. In this vein, further methods have been developed to determine the conditional multiplier. While Hamidi,

[1]For a comprehensive literature review on portfolio insurance and CPPI multipliers see for instance Benninga (1990); Black and Perold (1992); Basak (2002); Dichtl and Drobetz (2011); Hamidi, Maillet, and Prigent (2014).

Jurczenko, and Maillet (2009) and Hamidi, Maillet, and Prigent (2008) define the multiplier as a function of a dynamic autoregressive quantile model of the Value-at-Risk according to Engle and Manganelli (2004), Chen, Chang, Hou, and Lin (2008) propose a multiplier framework based on genetic programming. Closely related to our paper, Hamidi, Maillet, and Prigent (2014) employ a dynamic autoregressive expectile (DARE) model to estimate the conditional multiplier. In this framework, the multiplier is modeled as a function of the Expected Shortfall determined by a combination of quantile functions in order to reduce the potential model error. Specifically, they combine the historical simulation approach, three methods based on distributional assumptions, and four methods based on expectiles and conditional autoregressive specifications (CAViaR) into the DARE approach. All these papers evidence that the dynamic proportion portfolio insurance (DPPI) strategy with a multiplier based on a time-varying conditional risk estimate is more profitable than the traditional CPPI strategy.

We are particularly interested in comparing different ways to compute the conditional time-varying multiplier – by assessing various models to estimate portfolio risk. While the academic literature suggests a myriad of risk models – Andersen, Bollerslev, Christoffersen, and Diebold (2006) and Kuester, Mittnik, and Paolella (2006) provide good summaries on risk modelling – practitioners still only use a limited number of them. There may be various reasons for this discrepancy such as complexity, (computational) efficiency, or that the additional benefit of implementing a highly sophisticated model could be minor. We therefore examine both simple methods that are common among practitioners such as the historical simulation approach and more sophisticated methods based on quantile regressions or Copula-GARCH models to estimate portfolio risk.

In particular, we perform an empirical study using a global multi-asset data set consisting of stock, bond, commodity, and currency indices. When using international daily return data one faces the problem of different market closing times. Ignoring this fact will lead to distorted portfolio risk estimates given that the degree of co-movement is considerably underestimated (see Scholes and Williams, 1977; Lo and MacKinlay, 1990; Burns, Engle, and Mezrich, 1998; Audrino and Bühlmann, 2004; Scherer, 2013). Hence, we first synchronize daily returns by extrapolating asset prices of closed markets, based on information from markets that close later. Following Audrino and Bühlmann (2004) we thus employ a synchronization approach based on a VAR(1) model. Second, we estimate portfolio risk using the following models: historical simulation, RiskMetrics, Cornish-Fisher approximation, quantile regression, extreme value theory and Copula-GARCH. To quantify risk, we resort to the classic downside risk measure, the Value-at-Risk. Third, we employ the ensuing risk forecasts for the application of dynamic portfolio insurance strategies. We focus on the DPPI strategy but also investigate a dynamic variant of the time invariant portfolio protection (TIPP) strategy (Estep and Kritzman, 1988) and the synthetic put strategy (Rubinstein and Leland, 1981) for comparison purposes. Our findings indicate a superiority of sophisticated risk forecasts over simple approaches in terms of historical accuracy and statistical fit. Using the most prominent Value-at-Risk tests (Kupiec, 1995; Christoffersen, 1998; Christoffersen and Pelletier, 2004), we find that the quantile regression and the Copula-GARCH approach are the most suitable methods. When implementing the portfolio insurance framework, we find evidence that dynamic portfolio insurance strategies building on these sophisticated risk models are capable to protect investors from downside risk. However, more naive approaches are also able to provide downside protection. Given that the mechanics of the portfolio insurance strategy automatically reduce investment exposure when approaching the protection level, the less sophisticated methods seem to particularly profit from this second line of defense.

The remainder of the paper is structured as follows: Section II discusses the most prominent dynamic portfolio insurance strategies. Section III briefly presents the different models to estimate portfolio risk. In Section IV, we perform the empirical study using a global multi-asset data set to compare the performance of dynamic portfolio insurances strategies with conditional multipliers based on different risk models. Section V concludes.

II. Portfolio Insurance Techniques

We consider an highly risk-averse investor who primarily aims to limit the downside risk of an investment during period h. Typical choices are annual or semi-annual investment horizons. Further, let $t = 1, 2, ..., T$ be the time index of portfolio rebalancing during investment period h. It is common in the literature to rebalance on a daily basis implying that $T = 252$. At $t = 0$ the investor determines the minimum value, the so-called *floor* F_h, that should be preserved at $t = T$ in order to hedge his investment $W_{h,t}$.

Figure 1 illustrates the timing of investment decisions. The floor is chosen at the inception of each investment period.

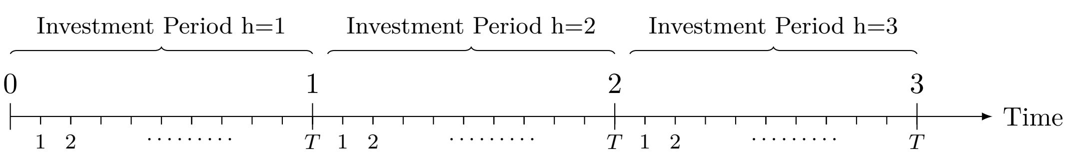

Figure 1. Graphical illustration of the timing of investment decisions in the porfolio insurance framework.

The academic literature suggests various investment strategies that incorporate loss protection. We will focus on the most prominent dynamic portfolio insurance strategies, which are briefly discussed in the following.[2]

A. Constant and Dynamic Proportion Portfolio Insurance

The basic idea of the constant proportion portfolio insurance (CPPI) strategy (see Perold, 1986; Black and Jones, 1987, 1988; Perold and Sharpe, 1988) is a portfolio that dynamically shifts between the risk-free and the risky investment according to a discrete trading rule. Each investment

[2] For a more detailed analysis see Benninga (1990) or Dichtl and Drobetz (2011), among others.

period the investor determines the minimum portfolio value, the so-called *floor*, that should be preserved at the end of the period. The corresponding risk capital, called the *cushion*, obtains as the difference of wealth, W_t, and discounted floor, i.e. the net present value, $\mathrm{NPV}(\cdot)$, of the floor F_h:

$$C_t = W_t - \mathrm{NPV}_t(F_h). \tag{1}$$

The cushion represents a certain amount of the portfolio value that allows to absorb potential market shocks before the manager can rebalance the portfolio. In order to avoid a breach of the discounted floor[3] the investment exposure of the risky investment, defined as E_t, should be set such that the cushion is at least as high as the maximum sudden loss of the investment, i.e.

$$C_t \geq W_t \ \mathrm{MaxLoss}(W_t), \tag{2}$$

where $\mathrm{MaxLoss}(\cdot)$ gives the maximum sudden loss of the investment in percentage terms. As the portfolio consists of two assets, a risky and a risk-free asset, we can write:

$$C_t \geq e_t W_t \ \mathrm{MaxLoss}(\text{risky investment}), \tag{3}$$

where e_t describes the exposure to the risky investment in percentage terms. Rearranging (3) then yields the (total) exposure of the risky investment[4] as

$$E_t \leq \frac{C_t}{\mathrm{MaxLoss}(\text{risky investment})} = m \cdot C_t, \tag{4}$$

where the multiplier

$$m \equiv \frac{1}{\mathrm{MaxLoss}(\text{risky investment})} \tag{5}$$

allows for a neat interpretation: it indicates how often a given cushion can be invested in the risky asset without breaching the floor. Thus, it reflects the investor's risk tolerance. The higher the multiplier, the more the investor will participate in upward market movements of the underlying. But, the higher the multiplier, the faster the portfolio will reach the floor when there is a sustained decrease in the underlying's price. In order to allow for the highest possible participation in the underling risky investment, it is common to set E_t such that equality holds in (4). The remainder of the investor's wealth is invested in the risk-free asset.

The classic CPPI strategy is based on a static unconditional multiplier – often reflecting a constant worst-case scenario. Although such a conservative stance would have meaningfully

[3] The risk of losing more than the cushion between two rebalancing dates and thus failing to ensure the protection at the end of the investment period is known as overnight or gap risk.

[4] We follow Benninga (1990) and Dichtl and Drobetz (2011) and restrict the participation ratio to vary between 0% and 100% of the risky investment in order to rule out short positions. This approach leads to a slightly different exposure definition: $E_t = \max[\min(mC_t, W_t), 0]$.

addressed catastrophic drawdowns during crisis periods, it would also have capped upside potential over the long term. Dynamic proportion portfolio insurance (DPPI) may be a more suitable choice. Instead of using a static multiplier, the risk budget adapts dynamically to changes in a dynamic risk forecast such as the Value-at-Risk. Hence, the investment exposure is adjusted by introducing a time component t:

$$E_t \leq \frac{C_t}{\text{MaxLoss}_t(\text{risky investment})} = m_t \cdot C_t. \tag{6}$$

The dynamic multiplier is then defined as

$$m_t \equiv \frac{1}{\text{MaxLoss}_t(\text{risky investment})}. \tag{7}$$

In this way, the portfolio's exposure to the risky investment reacts to changes in the risk forecast – ensuring that it does not remain artificially low as a result of a constant conservative risk assumption. For this to work in practice, the risk model must be capable of quickly homing in on volatility spikes, and just as quickly readjusting to a normalization of market volatility.

The advantage of the CPPI and DPPI strategy, respectively, is the simple practical implementation that does not call for forecasting the returns of the risky investment. Major drawbacks are the strategies' path dependencies as well as the lock-in effect. Essential to the strategy's resulting return profile is the course of the risky investment during the investment period. Depending on the path the CPPI/DPPI strategy can deliver considerably different results. In general, the more volatile the risky investment the lower is the average participation ratio. While the CPPI strategy is fully exposed to the problem of path dependency, the DPPI strategy can mitigate this problem at least to some extent due to its capability to quickly react to market changes. The lock-in effect occurs when the cushion is entirely consumed by losses. The CPPI/DPPI strategy is then fully invested in the risk-free asset until the end of the investment period and no participation in subsequent upward movements is possible.

B. Time Invariant Portfolio Protection

The time invariant portfolio protection (TIPP) strategy suggested by Estep and Kritzman (1988) is a modification of the CPPI strategy: Instead of using a fixed floor the TIPP strategy ratchets up the floor if the value of the portfolio increases. The main argument of the authors is that investors do not only care about a protection of their initial wealth but also about any interim capital gain. However, Choie and Seff (1989) argue that the continuous adjustments of the floor to the highest portfolio value increase the likelihood of breaching the prevailing floor. Therefore, the TIPP strategy will generally participate less in upward movements than the classic CPPI strategy. In our analysis we do not use the classic TIPP strategy but also resort to a dynamic variant based on a dynamic risk forecast – the dynamic TIPP (dTIPP).

C. Synthetic Put Portfolio Insurance

An obvious way to protect a risky portfolio against losses is the purchase of a European put option. The so-called protective put strategy ensures that the portfolio value will not breach the targeted floor at expiry. However, such a strategy can be expensive, since the option premium is payable each investment period, although the portfolio insurance could prove unnecessary in the majority of periods. Moreover, it may be difficult to find option contracts that fit the needs of the portfolio – particularly when it comes to complex investment vehicles like a multi-asset portfolio (Figlewski, Chidambaran, and Kaplan, 1993). Yet, both of these problems can be addressed by synthetically replicating the necessary European put option.

Using the Black and Scholes (1973) option pricing formula the synthetic put portfolio insurance strategy (Rubinstein and Leland, 1981) creates a continuously adjusted synthetic European put option on the risky investment. Combining the purchase of a risky asset with the purchase of a put option on this asset is equivalent to buying a continuously-adjusted portfolio, which is a combination of the risky and the risk-free asset. Pricing the put option with the Black and Scholes (1973) option pricing formula, the value of a portfolio that consists of a stock S plus a put P can be calculated as:

$$\begin{aligned} S_t + P_t &= S_t - S_t N(-d_{1,t}) + K_h e^{-r_t(T-t)} N(-d_{2,t}) \\ &= S_t N(d_{1,t}) + K_h e^{-r_t(T-t)} N(-d_{2,t}), \end{aligned} \tag{8}$$

where K_h denotes the strike price determined at the beginning of each investment period and r_t the risk-free rate. Moreover, $N(\cdot)$ is the standard normal cumulative distribution function with d_1 and d_2 defined as:

$$d_{1,t} = \frac{\ln\left(S_t/K_h\right) + \left(r_t + 0.5\sigma_t^2\right)(T-t)}{\sigma_t\sqrt{T-t}}, \quad d_{2,t} = d_{1,t} - \sigma_t\sqrt{T-t}, \tag{9}$$

where σ_t is the standard deviation of risky asset returns. The investment exposure of the replicating portfolio is then calculated using the delta[5] of the portfolio in (8):

$$w_t^{\text{risky}} = \frac{S_t N(d_{1,t})}{S N(d_{1,t}) + K_h e^{-r_t(T-t)} N(-d_{2,t})}, \tag{10}$$

i.e., an investor needs to increase the proportion of the risky asset in the portfolio if the price of the risky asset increases, and vice versa. In order to maintain the targeted floor, the strike price K_h must be set such that the following relationship holds:

$$K_h = \frac{F_h}{W_h}\left(S_{t_0} + P_{t_0}(K_h)\right), \tag{11}$$

[5] The delta measures the sensitivity of the synthetic put option to changes in the underlying. In this context, it defines how much of the risky asset must be purchased in order to replicate the portfolio consisting of the risky asset and the put option.

where the ratio (F_h/W_h) is the percentage floor of investment period h. As the value of the put option $P_{t_0}(K_h)$ depends on the strike price itself, the solution of (11) must be determined iteratively.

Two drawbacks lead to the limited use of this strategy in practice. First, a synthetic put strategy is based on the assumptions of the Black and Scholes (1973) option pricing model. Among other assumptions, the model suggests that stock returns are normally distributed following a geometric Brownian motion. Second, the synthetic put strategy entails the disadvantage of frequent portfolio reallocation resulting in high transaction costs.

The most crucial issue in the synthetic put portfolio insurance strategy is, however, to estimate the volatility of the asset whose value needs to be secured.[6] Accordingly, the quality of the protection strongly depends on the precision of this estimate (Zhu and Kavee, 1988; Bird, Cunningham, Dennis, and Tippett, 1990; Rendleman Jr and O'Brien, 1990). In this context, we investigate appropriate risk models generating volatility forecasts that are suitable in the synthetic put portfolio insurance strategy.

III. Estimating Portfolio Risk

When estimating portfolio risk for the use in dynamic portfolio insurance strategies we first need to define how to measure risk. As we are particularly interested in the left tail of the distribution, we resort to the classic downside risk measure, the Value-at-Risk (VaR). Conditional on the information given up to time t, the VaR for $t+1$ is the negative p-quantile of the conditional return distribution, that is,

$$\mathrm{VaR}_{t+1}^{p} \equiv -Q_p(r_{t+1}|\mathcal{F}_t) = -\inf_x\{x \in \mathbb{R} : P\left(r_{t+1} \leq x|\mathcal{F}_t\right) \geq p\}, \tag{12}$$

where $Q_p(\cdot)$ denotes the quantile function, r_t reflects the portfolio return in period t, and $\mathcal{F}_t$ represents the information available at date t.

Having defined the risk measure, the obvious question is which risk model is the most suitable in timely managing the investment exposure of a dynamic portfolio insurance strategy. Given the vast amount of available models, we focus on a few, yet distinct approaches, ranging from rather simple to quite advanced approaches. In particular, we examine a non-parametric historical simulation method, a semi-parametric RiskMetrics approach, a fully parametric Cornish-Fisher approximation approach, and a more sophisticated quantile regression approach, an extreme value

[6] We follow Dichtl and Drobetz (2011) and incorporate the so-called transaction costs effect when estimating volatility (see Leland, 1985). The latter author suggests the use of a modified volatility estimator:

$$\sigma_t = \sigma_t\sqrt{1+\sqrt{\frac{2}{\pi}}\frac{k}{\sigma\sqrt{\Delta t}}},$$

where k captures the round-trip transaction costs, and Δt denotes the length of the rebalancing interval.

theory approach and a Copula-GARCH approach. We briefly summarize these methods in the following.[7]

A. Historical Simulation

The simplest way to estimate the Value-at-Risk is to employ the sample quantile estimate based on historical pseudo portfolio returns constructed using historical asset prices but todays' portfolio weights. Known as historical simulation (HS), the VaR estimate for $t+1$ can be computed as

$$\widehat{\mathrm{VaR}}_{t+1}^{p} = -\hat{Q}_p(r_t, r_{t-1}, ..., r_{t-n+1}), \tag{13}$$

where $\hat{Q}_p(\cdot)$ is the empirical p-quantile and n the number of observations. For example, with a moving window of length 1000 observations, the 99% VaR estimate is simply the negative of the 10^{th} sample order statistic.

The main advantage of the HS approach is its computational simplicity and non-parametric dimension, i.e. the VaR does not rely on any distributional assumptions. In contrast, the most pertinent disadvantage[8] is its incapability to properly incorporate conditionality (see Andersen, Bollerslev, Christoffersen, and Diebold, 2006). This deficiency of the conventional HS approach is forcefully highlighted by clustering in time of the corresponding VaR violations, reflecting a failure to properly account for persistent changes in market volatility. The only source of dynamics in the HS-VaR is the evolving window used to construct historical pseudo portfolio returns. Though, the choice of the window size is crucial: too few observations will lead to sampling error, whereas too many observations will slow down estimates when reacting to changes in the true distribution of financial returns. Moreover, VaR estimates can exhibit jumps when large negative returns either enter or exit the estimation window.

B. RiskMetrics Approach

Given the shortcomings of the HS-VaR approach, a simple alternative can be found in the RiskMetrics (RM) approach. While the HS-VaR methodology makes no explicit assumptions about the distributional model generating the returns, the RM model implicitly assumes a very tight parametric specification by incorporating conditionality via exponential smoothing of squared portfolio returns. More precisely, in the RM model, the portfolio returns are assumed to follow a normal distribution $r_{t+1}|\mathcal{F}_t \sim \mathcal{N}(0, \sigma_{t+1}^2)$, where the conditional variance σ_{t+1}^2 is defined as

$$\sigma_{t+1}^2 = (1-\lambda) r_t^2 + \lambda \sigma_t^2, \tag{14}$$

[7]For a rigorous discussion of the analyzed risk models, see Kuester, Mittnik, and Paolella (2006), Andersen, Bollerslev, Christoffersen, and Diebold (2006) or Andersen, Bollerslev, Christoffersen, and Diebold (2013).

[8]For a rigorous discussion of several serious issues of the HS approach see Pritsker (2006).

which corresponds to an Integrated GARCH(1,1) model with $\alpha + \beta = 1$ and $\omega = 0$. λ is a smoothing parameter.[9] Using repeated substitution (and an initial $\hat{\sigma}^2$) the variance can also be formulated as an exponentially weighted moving average of past squared returns:

$$\sigma^2_{t+1} = \sum_j (1-\lambda)\lambda^j r^2_{t-j}. \tag{15}$$

The VaR is then simply obtained as

$$\widehat{\text{VaR}}^p_{t+1} = -\hat{\sigma}_{t+1}\Phi^{-1}_p, \tag{16}$$

where Φ^{-1}_p is the p-th quantile of the standard normal distribution.

The RM model is appealing because no parameters need to be estimated – which is due to the implicit assumption of zero mean returns, a fixed smoothing parameter, and conditional normality. However, as the normality assumption is not consistent with the behaviour of financial returns, the RM model tends to underestimate the Value-at-Risk. As the variance scales with the return horizon the RM model can be thought of as a random walk model in the conditional variance. The historical record of volatility across numerous asset classes, however, suggests that volatilities are unlikely to follow random walks, and hence that the flat forecast function associated with exponential smoothing is inappropriate for modeling volatility. In particular, the lack of mean-reversion in the RM variance calculations implies that the term structure of volatility is always flat, which violates both intuition and empirical evidence (see Andersen, Bollerslev, Christoffersen, and Diebold, 2013).

C. Cornish-Fisher Approximation

Another simple approach is the Cornish-Fisher approximation (Cornish and Fisher, 1938) method that consists of a Taylor-series type expansion of the VaR around the VaR of a normal distribution. In detail, the Cornish-Fisher VaR is an extension of the normal quantile function by accounting for skewness γ and kurtosis δ leading to

$$\widehat{\text{VaR}}^p_{t+1} = -\mu_{t+1} - \sigma_{t+1} F^{-1}_{CF}(p), \tag{17}$$

where

$$F^{-1}_{CF}(p) \equiv \Phi^{-1}_p + \left([\Phi^{-1}_p]^2 - 1\right)\frac{\gamma}{6} + \left([\Phi^{-1}_p]^3 - 3\Phi^{-1}_p\right)\frac{\delta-3}{24} - \left(2[\Phi^{-1}_p]^3 - 5\Phi^{-1}_p\right)\frac{\gamma^2}{36}. \tag{18}$$

The main advantage of the Cornish-Fisher approximation is the ability to account for fat tails. However, an issue is that the CFA-VaR is not necessarily monotone, i.e. the 99% VaR might be smaller than the 95% VaR.

[9] In practice, λ is typically fixed at a preset value of 0.94 for daily returns.

D. Quantile Regression

The determination of the Value-at-Risk naturally lends itself to the concept of quantile regression. The basic idea is to directly model the conditional quantile, rather than the whole distribution of portfolio returns. More precisely, the conditional p-quantile, $Q_p(r_t|X_t) = -\text{VaR}_t$, is modeled as a function of the information $\mathbf{X}_t \in \mathcal{F}_{t-1}$:

$$\text{VaR}_t^p \equiv -g_p\left(\mathbf{X}_t; \boldsymbol{\beta}_p\right), \tag{19}$$

where $g(\cdot,\cdot)$ and the parameter vector $\boldsymbol{\beta}$ explicitly depend on p. Following Koenker and Bassett (1978), the conditional sample p-quantile can be found as the solution to

$$\min_{\boldsymbol{\beta}_p \in \mathbb{R}^k} \left\{ p \sum_{r_t \geq \text{VaR}_t^p} |r_t + \text{VaR}_t^p| + (1-p) \sum_{r_t < -\text{VaR}_t^p} |r_t + \text{VaR}_t^p| \right\}, \tag{20}$$

where we determine VaR_t^p by the CAViaR specification of Engle and Manganelli (2004). In particular, we adopt their asymmetric slope CAViaR model[10] that is given by

$$\text{VaR}_t^p = \beta_0 + \beta_1 \text{VaR}_{t-1}^p + \beta_2 \max\left[r_{t-1}, 0\right] + \beta_3 \max\left[-r_{t-1}, 0\right]. \tag{21}$$

The attractiveness of the quantile regression approach is that no explicit distributional assumption for the time series behavior of returns is needed, thus reducing the risk of model misspecification. The main drawback of the CAViaR modeling strategy is similar to the CFA method. It might generate out-of-order quantiles. Also, estimation of model parameters is challenging.

E. Extreme Value Theory

As we are primarily interested in the tails of the distributions a natural way is to resort to extreme value theory (EVT) which allows for meaningfully estimating the tails based on extrapolating from available observations. McNeil and Frey (2000) propose a semi-parametric framework based on extreme value theory to describe the tail of the conditional distribution. In a first step, the authors employ pseudo-maximum-likelihood fitting of AR(1)-GARCH(1,1) models to estimate conditional volatility forecasts $\hat{\sigma}_{t+1}$ and conditional mean forecasts $\hat{\mu}_{t+1}$. In a second step, they resort to EVT for estimating the tail of the innovation distribution of the AR(1)-GARCH(1,1) model. In particular, they use the peak-over-threshold method where a Generalized Pareto distribution (GPD) is fitted to losses over a specified threshold. The quantile $\hat{z}_p$ can then be estimated as

$$\hat{z}_p = u + \frac{\hat{\beta}}{\hat{\xi}} \left[\left(\frac{1-q}{n_u/n} \right)^{-\hat{\xi}} - 1 \right], \tag{22}$$

[10] For the sake of simplicity, we focus on one CAViaR model. Particularly, we choose the asymmetric slope specification because of its ability to accommodate the leverage effect.

where $\hat{\beta}$ and $\hat{\xi}$ are the GPD estimates and n_u is the number of observations above threshold u. Consequently, the VaR forecast can be computed as

$$\widehat{\text{VaR}}_{t+1}^{p} = -\hat{\mu}_{t+1} - \hat{\sigma}_{t+1}\hat{z}_p. \tag{23}$$

The ARMA-GARCH fitting in the first step allows to capture certain stylized facts such as time-varying volatility, fat tails, and volatility clustering. Then, EVT is particularly suitable to estimate the tails of the distribution. The crucial assumption of EVT is, however, that we are in the tails of the distribution. Hence, the difficulty is the determination of the threshold. If the threshold is too low, then the approximation can hardly be justified and the associated risk estimates may be biased. Vice versa, if the threshold is too high, there are too few observations over the threshold resulting in highly volatile estimates.

F. The Copula-GARCH Approach

To reasonably capture both the risk dynamics of the univariate return series and the whole dependence structure between the portfolios' assets Jondeau and Rockinger (2006) and Patton (2006) propose a Copula-GARCH approach. This framework is based on the concept of inference from margins, i.e. dependencies between the marginal distributions are captured by a copula.

In a first step, univariate GARCH(1,1)-models are fitted to the underlying return series. Assuming a return process $(r_{it})_{i\in\mathbb{N},t\in\mathbb{Z}}$ the mean and variance equations are given by

$$r_{it} = \mu_i + \varepsilon_{it} \tag{24}$$

$$\varepsilon_{it} = z_{it}\sqrt{\sigma_{it}^2} \tag{25}$$

$$z_{it} \sim \mathcal{D}_i(0,1,\xi_i,\nu_i) \tag{26}$$

$$\sigma_{it}^2 = \omega_i + \alpha_i\varepsilon_{it-1}^2 + \beta_i\sigma_{it-1}^2, \tag{27}$$

where $\omega_i > 0, \alpha_i \geq 0$ and $\beta_i \geq 0$, $i = 1, ..., n$. Moreover, r_i are the returns of the i-th portfolio asset and $\mathcal{D}_i$ reflects the skewed standardized-t distribution with skewness parameter ξ_i and shape parameter ν_i.

In a second step, we use a time-varying copula to estimate the marginal distributions of the asset returns together with the dependence structure. In particular, the joint distribution of the n GARCH return processes can be expressed depending on an n-dimensional copula C:

$$F_t\left(\mathbf{r}_t|\boldsymbol{\mu}_t,\boldsymbol{\sigma}_t\right) = C_t\left(F_{1t}(r_{1t}|\mu_{1t},\sigma_{1t}), ..., F_{nt}(r_{nt}|\mu_{nt},\sigma_{nt})|\mathcal{F}_{t-1}\right), \tag{28}$$

where $F_1(\cdot), ..., F_n(\cdot)$ are the conditional marginal distributions of the return processes. The dependence structure of the margins is assumed to follow a Student-t copula with conditional correlation $\mathbf{R}_t$ and constant shape parameter η. We resort to the Student-t copula for modelling the dependence of financial assets since the normal copula cannot account for tail dependence.

The conditional density of the Student-t copula at time t is given by:

$$c_t\left(u_{it},...,u_{nt}|\mathbf{R}_t,\eta\right)=\frac{f_t\left(F_{it}^{-1}(u_{it}|\eta),...,F_{it}^{-1}(u_{nt}|\eta)|\mathbf{R}_t,\eta\right)}{\prod_{i=1}^{n} f_i\left(F_{it}^{-1}(u_{it}|\eta)|\eta\right)}, \tag{29}$$

where $u_{it}=F_{it}(r_{it}|\mu_{it},\sigma_{it},\xi_i,\nu_i)$ is the probability integral transformation of each series by its conditional distribution F_{it} estimated via the first stage GARCH process, $F_{it}^{-1}(u_{it}|\eta)$ represents the quantile transformation of the uniform margins subject to the common shape parameter of the multivariate density, $f_t(\cdot|\mathbf{R}_t,\eta)$ is the multivariate density of the Student-t distribution with conditional correlation $\mathbf{R}_t$ and shape parameter η and $f_i(\cdot|\eta)$ is the univariate margins of the multivariate Student-t distribution with common shape parameter η. Furthermore, we allow the parameters of the conditional copula to vary through time in a manner analogous to a GARCH model for conditional variance (e.g. Patton, 2006). Specifically, we assume the dynamics of $\mathbf{R}_t$ to follow an Asymmetric Generalized Dynamic Conditional Correlation (AGDCC) model according to Cappiello, Engle, and Sheppard (2006).

The Copula-GARCH model has several advantages over more simplistic approaches. The GARCH models with skewed standardized-t distribution applied in the first stage allow to capture the main empirical characteristics of financial asset returns. Moreover, as correlation coefficients can capture the dependency between random variables correctly only for elliptical distributions and asset returns hardly follow elliptic distributions, dependencies between the portfolios' assets cannot be measured correctly in most cases. Using the concept of copulas enables us to separate the marginal distributions and the dependence structure so that dependencies between the portfolios' assets can be incorporated in the VaR estimation. Given the associated computational effort and complexity, however, most practitioners rather resort to more simple methods.

IV. Empirical Analysis

A. Global Asset Allocation Data

In the empirical analysis we use a global multi-asset data set consisting of the stock market indices *Nikkei 225, Euro Stoxx 50, FTSE 100, S&P 500, MSCI EM*, the government bonds *JGB 10Y, Euro Bund, UK Gilt, U.S. 10Y*, the commodity indices *Oil, Gold, Copper*, and the foreign exchange rates *JPY/USD, EUR/USD, GBP/USD*. The money market investment is based on the *3-month U.S. Treasury Bill*. We retrieve all data from Bloomberg.[11] The sample spans from January 2nd, 1991 to March 31st, 2017, giving rise to 6,847 daily return observations for each series.

When calculating portfolio returns, we assume a given static strategic allocation of portfolio

[11] In particular, we use future series of the described indices. All asset returns are in local currency. Portfolio returns and values are computed from the perspective of an U.S. investor who is hedging any currency exposure.

weights (see Table V). Alternatively, one could consider a dynamic weight structure driven by a tactical asset allocation component. However, one could no longer determine whether an increase in performance is due to superior risk forecasts or due to predictability of the tactical component. Thus, to clearly identify the impact of different risk models we build on a static strategic allocation. Moreover, from a practitioner's perspective it is also reasonable to choose such a static allocation since, in practice, it is common to have a strategic allocation (long-term focus) and a tactical component (medium-term focus) in the first and second place. If these two components fail to navigate drawdowns, portfolio insurance is the ultima ratio to avoid breaching the floor (daily focus).

[Table I about here]

Table I reports descriptive statistics of the portfolio returns and the risk-free returns based on the 3-month U.S. treasury bill.[12] We see that the multi-asset portfolio is conservative with a mean return of 4.6% and a volatility of 7.5%.

For estimating the risk models we split the sample in two periods: we use a rolling window of 1,000 daily returns for dynamically estimating the parameters in-sample and an out-of-sample period from 3/11/1994 to 31/3/2017 consisting of 5,846 daily returns for out-of-sample testing the various risk models.

B. Synchronizing Returns

When modelling risk using international daily return data one has to properly account for the fact that markets have different closing times.[13] Even worse, for some markets trading times do not overlap at all, as is the case for the U.S. and Japan. Obviously, these differences will render equity markets to appear less (cor)related than they actually are. As a result, portfolio risk estimates will overstate the diversification benefit attached to investing across these assets (see Scholes and Williams, 1977; Lo and MacKinlay, 1990; Burns, Engle, and Mezrich, 1998). Ideally, daily returns can be computed for all series using the same time stamp. This approach, however, is hardly feasible, even when using high-frequency data. Instead, the academic literature suggests to synchronize daily returns by extrapolating asset prices for those markets that close earlier, based on information from markets that close latest. While Burns, Engle, and Mezrich (1998) use a first-order vector moving average model with a multivariate GARCH covariance matrix to estimate synchronized returns, Audrino and Bühlmann (2004) employ a simple first-order vector autoregressive model. We follow the latter approach due to its computational efficiency and performance advantages.

[12] See Table V for descriptive statistics of the individual components of the multi-asset portfolio and the allocation of portfolio weights.

[13] The opening times of the markets in our sample are as follows: Japanese markets are open from 19:00(-1) to 1:00 ET, EU/UK markets from 3:00 to 11:30 ET, and U.S. markets open from 09:30 to 16:15 ET.

Let $S_{t_j,j}$ denote the continous time price of asset j $(j = 1, ..., M)$, where time t_j is the closing time of market j measured in local time of the base market, i.e. the market where to synchronize to. The corresponding synchronized price $S^s_{t,j}$ is then defined as

$$\log\left(S^s_{t,j}\right) = \mathbb{E}\left[\log\left(S_{t,j}\right)|\mathcal{F}_t\right] = \mathbb{E}\left[\log\left(S_{t_j+1,j}\right)|\mathcal{F}_t\right], \quad t_j \leq t \leq t_j + 1 \ (t \in \mathbb{N}), \tag{30}$$

where $t = t_1$ and $\mathcal{F}_t$ is the complete information of all recorded prices up to time t. The logarithms are used to be consistent with continuously compounded returns. Clearly, if the closing price S is observed at time $t \in \mathbb{N}$, then its conditional expectation given $\mathcal{F}_t$ is the observed price. This is the case for the assets from the base market. If the market closes before t, then its past prices and all the other markets may be useful in predicting S at time t. As a simplifying approximation, the authors therefore assume that, given the information $\mathcal{F}_t$, the best predicted log-prices at t and at the nearest succeeding closing time $t_j + 1$ remain the same, saying that future changes up to $t_j + 1$ are unpredictable.

Then, we denote r_t as the vector of log-returns in different markets using the multi index $t = (t_1, t_2, \cdots, t_M)$ and define the synchronized returns r^s_t as the change in the logarithms of the synchronized prices:

$$r_t = \begin{bmatrix} \log\left(\dfrac{S_{t_1,1}}{S_{t_1-1,1}}\right) \\ \vdots \\ \log\left(\dfrac{S_{t_M,M}}{S_{t_M-1,M}}\right) \end{bmatrix} \qquad\qquad r^s_t = \begin{bmatrix} \log\left(\dfrac{S^s_{t,1}}{S^s_{t-1,1}}\right) \\ \vdots \\ \log\left(\dfrac{S^s_{t,M}}{S^s_{t-1,M}}\right) \end{bmatrix}. \tag{31}$$

In order to estimate the relationship between the individual asset markets, the authors employ a simple „auxiliary“ VAR(1) model:

$$r_t = A \cdot r_{t-1} + \varepsilon_t, \tag{32}$$

where the innovations ε_t are i.i.d.$\sim \mathcal{N}(\mathbf{0}, \Sigma)$, independent from $\{r_s; s < t\}$ and A is the matrix of VAR coefficients. We can then derive the synchronized returns as follows

$$\begin{aligned} r^s_t &= \log\left(\mathbf{S}^s_t\right) - \log\left(\mathbf{S}^s_{t-1}\right) \\ &= \mathbb{E}\left[\log\left(\mathbf{S}_{t+1}\right)|\mathcal{F}_t\right] - \mathbb{E}\left[\log\left(\mathbf{S}_t\right)|\mathcal{F}_{t-1}\right] \\ &= \mathbb{E}\left[\log\left(\mathbf{S}_{t+1}\right) - \log\left(\mathbf{S}_t\right)|\mathcal{F}_t\right] - \mathbb{E}\left[\log\left(\mathbf{S}_t\right) - \log\left(\mathbf{S}_{t-1}\right)|\mathcal{F}_{t-1}\right] + \log\left(\frac{\mathbf{S}_t}{\mathbf{S}_{t-1}}\right) \\ &= \mathbb{E}\left[r_{t+1}|\mathcal{F}_t\right] - \mathbb{E}\left[r_t|\mathcal{F}_{t-1}\right] + r_t \\ &= Ar_t - Ar_{t-1} + r_t \\ &= r_t + A(r_t - r_{t-1}). \end{aligned} \tag{33}$$

That is, any synchronized return r^s_t is still anchored in the actual realized return r_t plus an anticipated innovation according to the estimated VAR-relation as captured in matrix A. The „missing“ dynamics of markets closing early in the day are thus proxied according to the short-term relationship with respect to those markets closing later that day.

Sorting markets according to their closing times allows to readily formulate a restriction matrix for the VAR model such that markets are explained only by those markets with a later closing time. Given that U.S. markets are the last to close in our sample, we anchor our synchronization of daily returns in U.S. markets. Thus, the U.S. time series remain unchanged but are still included in the VAR-model to serve as explanatory markets, i.e. the final set of synchronized daily returns does not build on forecasted time series for the U.S. but uses their original daily returns. Non-U.S. data is, however, forecasted to the closing time of the US market by the VAR(1) (see Table VI for median t-statistics of the obtaining coefficients).

Based on our sample, we compare the synchronized daily returns to the original ones. Table II shows the descriptive statistics of the original and synchronized daily returns. Looking at the mean, we observe that differences are only marginal. Volatilities, however, are slightly higher when synchronizing. Thus, the return characteristics of the original data are maintained, which can be confirmed by analyzing the plots in Figure 6.

[Table II about here]

To check the effectiveness of synchronization Figure 2 shows the correlation matrices of both return types. For the synchronized daily returns, the chosen VAR(1) model is successfully re-correlating the within-asset class correlations. While equity correlations are no longer underestimated, equity-bond correlations tend to be more negative when using synchronized returns. Hence, the improved equity-bond diversification could mitigate the equity risk pick-up. However, we learn that the latter effect dominates and unreported results evidence an increase of portfolio risk figures that average to 15% for the conservative multi-asset portfolio. These findings are in line with Scholes and Williams (1977) and Lo and MacKinlay (1990).

[Figure 2 about here]

C. Validity of Risk Forecasts

In order to test the validity of the risk forecasts based on the estimated models we resort to various Value-at-Risk tests (see Kuester, Mittnik, and Paolella, 2006; Christoffersen, 2012). In principle, backtesting VaR forecasts boils down to evaluating the distribution of VaR violations. That is, counting the number of realized returns that fall below the predicted VaR-level for a given estimation period. Hence, for example, in a set of 252 forecasts of daily 99%-VaRs per year, there should be 2.52 violations in theory. The most prominent VaR test, the test for unconditional coverage of Kupiec (1995), assesses whether the frequency of violations is consistent with the quantile of loss a VaR measure is intended to reflect. However, this test does not account for serial independence of the number of violations. Christoffersen (1998) offers a remedy by jointly testing the frequency as well as the independence of violations, assuming that VaR violations are modeled with a first order Markov chain. Thus, this test for conditional coverage is able to reject a VaR model that generates too many clustered violations. An alternative way to account for clustering of extremes is the duration test of Christoffersen and Pelletier (2004), which examines the duration between violations by testing the null hypothesis that the duration between violations is exponentially distributed against a Weibull alternative.

Figure 3 presents the forecasted VaR figures of the Historical Simulation approach (HS), RiskMetrics (RM), Cornish-Fisher approximation (CFA), Quantile Regression (QR), Extreme Value Theory (EVT) and Copula-GARCH (CG) model over a rolling estimation window of 1,000 days leading to 5,846 out-of-sample forecasts. Panel (a) shows the HS-VaR forecast. As expected, the majority of realized returns were higher than the forecasted VaR. In the sample period from November 1994 to March 2017, there are only 60 violations (red dots) – which is very close to the number of expected violations (=1% of 5,846). An analysis of VaR violations throughout time, however, calls into doubt the validity of the HS-VaR – given a latent underestimation of risk with most violations occuring during the 2008 Financial Market Crisis. Subsequently, the HS-VaR forecast was overly conservative, and there were no violations in the following five years. Thus, it seems that a portfolio insurance strategy on this basis would have held investment exposure much too low over time. This conclusion is confirmed by rigorous statistical testing. Using the unconditional coverage test, the HS-VaR does indeed deliver a conclusive number of violations over the entire period. But, based on the test for correct coverage and independence and the duration test, it is clear that the violations are not occurring independently, but rather appear in clusters (cf. Table III).

Regarding the Cornish-Fisher VaR in Panel (c) we can draw similar conclusions. Although the Cornish-Fisher approximation approach accounts for fat tails by incorporating skewness and kurtosis it still remains sluggish over time. Like the HS-VaR, the CFA-VaR passes the test for unconditional coverage but fails when accounting for clustering of returns (cf. Table III).

[Figure 3 about here]

The remaining risk models are much more sensitive and quicker to react to the prevailing risk environment (see Panel (b), (d), (e), (f)). Moreover, the occurrence of violations is markedly less clustered – as confirmed by the statistical tests. All four risk methods pass the duration test. The RM-VaR, however, fails the unconditional and conditional coverage test which is mostly due to the high deviation of the realized to the expected number of violations (96 vs. 58). The EVT-VaR also fails the test for unconditional coverage but passes the tests that account for independence. In contrast, the QR-VaR and the CG-VaR pass all tests. Although the QR approach has the higher p-values one would presumably prefer the Copula-GARCH forecasts since the risk estimates of the QR approach are much more volatile than the ones of the Copula-GARCH approach.

[Table III about here]

D. The Economic Relevance of Risk Forecasting for Dynamic Portfolio Insurance Strategies

In this section, we analyze the performance of the different risk models when fed into dynamic portfolio insurance strategies. We first discuss the DPPI strategy with respect to the different multipliers. Subsequently, we investigate the dTIPP and synthetic put strategy as robustness checks.

D.1 *Historical Path*

To examine how a dynamic proportion portfolio insurance strategy based on different risk models would have performed over the period from 1994 to 2017 we calculate the historical path using the multi-asset portfolio as risky and the 3-month U.S. Treasury Bill as risk-free investment. Further, we choose a floor level of 95%. An investment horizon of one calendar year – a typical choice of institutional and private investors alike (see Benartzi and Thaler, 1995) – implies that the floor is adjusted to the current portfolio value at the start of each year to initialize the cushion. This procedure helps to mitigate the lock-in effect.

Figure 4 illustrates how the mechanism of a DPPI strategy generally works. The chart shows the performance of the conservative multi-asset portfolio using the DPPI strategy (red line) in relation to the floor (green line) over time. The investment exposure is mainly driven by two components: the floor and the multiplier. If the portfolio value of the underlying approaches the floor line, i.e. the cushion shrinks, the exposure is reduced (see grey background of Figure 4) and parts of the investment are shifted into the risk-free asset. Similarly, the exposure is reduced if risk estimates predict too high (overnight) risk, i.e. the multiplier decreases given that the distance to the floor is not excessive. In this example, the conditional multiplier is based on the Copula-GARCH-VaR at 99% confidence.[14] For comparison, we also include the performance of the underlying multi-asset portfolio (blue line) and a money market investment (black line).

[Figure 4 about here]

Examining the whole sample period, we learn that the DPPI strategy indeed prevented catastrophic drawdowns. With the onset of the financial market crisis, investment exposure drops to zero, so that the portfolio value at the end of 2008 is equal to the floor. Then, even with the V-shaped market evolution (steep decline followed by a rapid recovery) in early 2009 – a major impediment for portfolio insurance – the DPPI portfolio does not end up in a „cash lock“. It participates in at least part of the subsequent recovery. On the whole, the DPPI strategy has an average investment exposure of approximately 90% and delivers an excess return of 23bp per year

[14]In order to reflect the preferences of highly risk-averse investors, we follow Soupé, Heckel, and de Carvalho (2014) and scale the risk forecast by a term consisting of an investor's risk aversion parameter and the expected Sharpe ratio given a Constant Relative Risk Aversion (CRRA) utility function of the investor. Assuming a highly risk-averse investor, we choose the risk aversion parameter to be 0.05 and an expected Sharpe ratio of 0.3.

compared to the pure multi-asset portfolio (see column PF and CG in Table IV). This advantage remains when considering risk-adjusted returns. The Sharpe ratio of the DPPI strategy (0.35) is significantly higher than the one for the underlying (0.26). The lower maximum drawdown of the DPPI strategy (11pp) evidences that downside protection is indeed provided.

Comparing the performance of the DPPI strategy across risk models yields less clear-cut results. Panel A of Table IV shows the corresponding results. Analyzing returns we find a 19bp-difference between the best performing risk model, the Copula-GARCH, and the weakest model, the HS approach (4.58 vs. 4.39). However, when risk-adjusting returns, this spread diminishes so that we observe marginal differences among the different models. In particular, the Sharpe ratio ranges from 0.33 to 0.36. The same conclusions can be drawn in terms of maximum drawdown. In addition, we apply the Calmar and the Sortino ratio suggested by the literature to measure downside risk.[15] While the Calmar ratio is defined as the ratio of annualized return over the absolute value of the maximum drawdown, the Sortino Ratio is calculated as the difference of mean return and minimum acceptable return divided by downside deviation.[16] Evaluating the risk models on the basis of Calmar and Sortino ratio shows only marginal differences. The range from 0.29 to 0.32 and from 0.14 to 0.15, respectively, indicates that not only sophisticated but also more naive approaches are able to provide downside protection in the context of DPPI. Among the best models are the CFA (Calmar ratio of 0.32) and the Copula-GARCH approach (Calmar ratio of 0.31). This finding can be rationalized as follows: In general, few allocation changes are necessary to protect from downside risks if the DPPI strategy is reasonably calibrated. In particular, the investment exposure will be decreased when one is approaching the floor, irrespective of the underlying risk forecast. This embedded line of defense is most likely preventing less accurate risk forecasts from inhibiting overall performance. As a result, we see that all risk models likewise dominate the underlying risky portfolio when evaluating Calmar and Sortino ratio.

Similar to several studies (Bertrand and Prigent, 2002; Ameur and Prigent, 2006; Hamidi, Jurczenko, and Maillet, 2009; Ameur and Prigent, 2014; Hamidi, Maillet, and Prigent, 2014), we also benchmark the DPPI performance with multipliers based on the different risk models against the CPPI performance based on static unconditional multipliers. In particular, we employ two static multipliers representing different levels of loss aversion. While the fixed multiplier of 8 (FM8) is calculated as the maximum loss of the underlying indicating high loss aversion, the fixed multiplier of 32 (FM32), determined as the average over the HS-VaR, reflects an average attitude towards downside risk. Even with the average risk estimator (FM32) the maximum drawdown is halved – similar to the DPPI strategies. However, there is a performance drag relative to these strategies. It reveals that this static multiplier underperforms in terms of Sharpe ratio (0.27 vs. approx. 0.33) and Calmar ratio (0.23 vs. approx. 0.30) while exhibiting a similar participation

[15] See Bertrand and Prigent (2011) for general definitions of the Calmar and the Sortino ratio.

[16] In our context, we set the *minimum acceptable return* (MAR) to the corresponding floor return, i.e. MAR $= -(1 - \text{FloorPercentage})$. Furthermore, *downside deviation* measures the variability of underperformance below a minimum target rate. More precisely, it eliminates positive returns when calculating risk and uses the minimum acceptable return instead of the mean return.

in the risky asset (approx. 88%). This performance drag is exacerbated when plugging a worst-case assumption into the constant multiplier (FM8). In terms of downside measures, however, FM8 shows slightly better results than the competing DPPI strategies owing to a more defensive investment exposure (approx. 60%).

D.2 *Historical Simulation*

Due to the problem of path dependency that arises when analyzing the historical path, we also conduct a historical simulation. In particular, we use 250 subsequently following daily portfolio and risk-free returns on a rolling window basis and implement the DPPI strategy based on the different risk models. Intuitively, this approach enables us to assess the DPPI strategy's robustness with respect to alternative entry dates. Moreover, it allows to use the available data in the most efficient way (5,597 overlapping yearly performance data) while preserving all dependency effects in the series, such as autocorrelation and conditional heteroskedasticity (see Dichtl and Drobetz, 2011).

Figure 5 shows the distribution of the simulated yearly returns of the DPPI strategy (red shade). The black dashed line indicates the floor level of 95%. For comparison, we also include the return distribution of a pure buy-and-hold portfolio investment strategy (blue shade). The chart clearly highlights the effect of portfolio insurance. The left tail of the return distribution is shifted towards the floor level such that downside risk is reduced – however, at the expense of some return potential in the right tail.

[Figure 5 about here]

Panel B in Table IV reports the median performance data of the historical simulation. Compared to the historical path, we find slightly different results. Concerning the performance of the underlying, we learn that its Sharpe ratio increases (from 0.26 to 0.48) and its maximum drawdown decreases (from -21% to -6%) such that we have similar figures to the DPPI strategy. Moreover, we observe that the Calmar Ratio increases substantially for all strategies (from approx. 0.3 to approx. 0.95). Both findings can be explained by the fact that the massive drawdown year 2008 loses weight when performing historical simulations. In other words, the crisis year 2008 is „averaged out". In terms of Sortino ratio we still find no significant differences across risk models. Given the amount of simulations we can resort to another downside risk measure, the cumulative prospect value. It is based on cumulative prospect theory (CPT) according to Tversky and Kahneman (1992) that especially accounts for the finding that many investors are more sensitive to losses than to gains (loss aversion) and have the tendency to overweight extreme, but unlikely events. Dichtl and Drobetz (2011) were the first to apply CPT in the context of portfolio insurance.[17] In our analysis, evaluating in terms of the cumulative prospect value shows that the DPPI strategy

[17] To compute the cumulative prospect value we define the involved parameters according to Abdellaoui (2000), i.e. $\alpha \approx \beta \approx 0.88, \lambda \approx 2.25$, $\delta^+ = 0.65$, $\delta^- = 0.84$, $\gamma^+ = 0.65$, $\gamma^- = 0.84$.

based on different risk models outperforms both the underlying and the money market investment (approx. 5.9 to 3.8 and 5.5). On the other hand, we find only slight differences across the risk models' performances. While the Copula-GARCH based multiplier dominates among sophisticated models, it shows a slightly weaker performance compared to the HS and CFA approach as well as to the static FM8.

[Table IV about here]

In essence, the results support the conclusion drawn from the historical path analysis. Dynamic proportion portfolio insurance strategies building on sophisticated risk models do a good job in protecting investors from downside risk. Given that the mechanics of the portfolio insurance strategy automatically reduce investment exposure when approaching the protection level, a less sophisticated risk forecast is mostly profiting from this second line of defense.

D.3 *dTIPP and Synthetic Put*

In order to verify the robustness of the findings of the historical path and historical simulation analysis we investigate the dTIPP and the synthetic put portfolio insurance strategy. The more conservative dTIPP strategy differs from the DPPI strategy by locking in any intermediate gain.

Panel (a) of Figure 7 shows the evolution of the dTIPP strategy, based on the identical risk forecast as the DPPI strategy. Exposure over the entire period is roughly 20 percentage points lower than that of the DPPI strategy – a consequence of the floor being always closer to the portfolio value so that no additional cushion can be built up. This feature implies a clear reduction of returns compared to the DPPI strategy – but one that is less dramatic in risk-adjusted terms. Panel (b) of Figure 7 shows that the dTIPP strategy is the most efficient approach to protect the initial floor. Compared to the other strategies, the smallest mass of the return distribution is to the left of the floor line. However, it can also be observed that this protection comes at the high cost of reduced upside return potential. The dTIPP has considerably less mass in the right tail than the DPPI.

Figure 8 charts the evolution of the synthetic put strategy over time. We note that the rate of investment (exposure) varies significantly, depending on the expected volatility and the difference between the portfolio value and the strike price. This finding demonstrates one weakness of a synthetic put strategy: frequent portfolio reallocations. Nonetheless, we see that the strategy is able to prevent investors from massive drawdowns – the exposure would have been reduced early enough in 2008 to avoid a massive drawdown – and benefit from upward market movements.

In essence, when comparing the different portfolio insurance strategies we find similar results to Dichtl and Drobetz (2011). The DPPI strategy dominates the dTIPP strategy as well as the synthetic put strategy and the buy-and-hold portfolio investment strategy. Table VII allows to compare the estimation results of the historical path and the historical simulation across the different risk models for both the dTIPP and the synthetic put strategy. We see that, no matter which performance measure is used, there are only marginal differences. This finding confirms the results of the analysis of the DPPI strategy.

V. Conclusion

This paper investigates a number of forecasting models to generate portfolio risk estimates that are suitable in timely managing the investment exposure of dynamic portfolio insurance strategies. In particular, we focus on the dynamic proportion portfolio insurance (DPPI) strategy that works with a conditional multiplier based on a dynamic risk forecast. Therefore, it adapts much more readily to the market environment than the CPPI approach with its constant unconditional multiplier. To this end, we analyze risk models both prominent in the academic literature and popular among practitioners – from simple historical simulation, the RiskMetrics approach, and the Cornish-Fisher approximation, to quantile regression, extreme value theory and the Copula-GARCH approach. Empirically, we build our analysis on a global multi-asset return data set including stocks, bonds, commodities, and foreign exchange rates. To take account of different market closing times we apply a return synchronization technique by extrapolating prices of closed markets, based on information from markets which close later. It turns out that the risk forecasts of the quantile regression approach and the Copula-GARCH method dominate the more naive approaches in terms of statistical fit. When feeding these risk forecasts into dynamic portfolio insurance strategies we find less clear-cut results. We evidence that dynamic portfolio insurance strategies building on sophisticated risk models are capable to protect investors from downside risk. However, more naive approaches are also able to provide downside protection. Given that portfolio insurance only leads to few allocation changes simple risk models might have simply been lucky. Thus, forward looking a more accurate quantile regression or Copula-GARCH approach appear to be more likely to help mitigating the next downturn.

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Table I

Descriptive Statistics of the Multi-Asset Portfolio and Money Market Investment

This table reports annualized return and annualized volatility of the multi-asset portfolio of 5 stock indices, 4 bond indices, 3 commodities, 3 foreign exchange rates and a money market investment based on the 3-month U.S. treasury bill rate over the period from from 02/01/1991 to 31/03/2017.

	Multi-asset portfolio	Money market investment
Annualized mean return (in %)	4.63	2.40
Annualized volatility (in %)	7.51	0.14

Table II

Synchronized vs. Original Daily Returns: Descriptive Statistics

This table reports the annualized return and annualized volatility of the synchronized and original daily returns based on the international multi-asset data set.

	Nikkei	JGB10Y	Bund	Gilt	EuroSTOXX	FTSE
Mean original returns (in %)	-0.0035	0.0136	0.0165	0.0149	0.0305	0.0178
Mean synchronized returns (in %)	-0.0034	0.0136	0.0165	0.0149	0.0307	0.0179
Volatility original returns (in %)	1.5109	0.2486	0.3341	0.4098	1.3875	1.1306
Volatility synchronized returns (in %)	1.6338	0.2561	0.3581	0.4386	1.5829	1.2916

Table III
Results of VaR Testing

This table shows the results of the unconditional coverage test, the conditional coverage test and the duration test to evaluate the risk forecasts of the different risk models – Historical Simulation (HS), RiskMetrics (RM), Cornish-Fisher Approximation (CFA), Quantile regression (QR), Extreme Value Theory (EVT) and Copula-GARCH (CG). Reported are the number of realized VaR violations, the p-value and the test decision using the synchronized multi-asset portfolio over the period from 3/11/1994 to 31/3/2017. We calculate the Value-at-Risk at 99% confidence level and expect 58 violations over the whole sample period.

	HS	RM	CFA	QR	EVT	CG
Realized Violations	60	96	46	66	77	70
a) Test for unconditional coverage (H_0: Correct Violations)						
p-Value	0.84	0.00	0.09	0.33	0.02	0.14
Test Decision	✓	x	✓	✓	x	✓
b) Test for conditional coverage (H_0: Correct & Independent Violations)						
p-Value	0.00	0.00	0.00	0.30	0.07	0.33
Test Decision	x	x	x	✓	✓	✓
c) Duration test (H_0: Duration between Violations have no memory)						
p-Value	0.00	0.92	0.00	0.92	0.86	0.57
Test Decision	x	✓	x	✓	✓	✓

Table IV
Estimation Results of DPPI

This table shows the estimation results of the DPPI strategies with conditional multipliers based on different risk forecasts for the historical path and historical simulations over the sample period from 1994 to 2017 using a synchronized international multi-asset portfolio and a money market investment. The analyzed risk models are the Historical Simulation approach (HS), RiskMetrics (RM), Cornish-Fisher approximation (CFA), Quantile Regression (QR), Extreme Value Theory (EVT) and Copula-GARCH (CG). For comparison, we include the performance of the underlying multi-asset portfolio (PF), the money market investment (Cash), and two fixed multipliers based on the maximum loss of the portfolio returns (FM8) and the average risk measured by the average over the HS-VaR (FM32), respectively. As performance measure, we employ the annualized mean return, annualized volatility, maximum drawdown, Sharpe Ratio, Calmar and Sortino ratio, and the cumulative prospect value. Moreover, we show the average participation of the strategy in the risky investment. For the historical simulation, we report the median of these performance measures and add the cumulative prospect (CP) value. In each calendar year, a floor of 95% of the initial portfolio value is targeted.

			CPPI		DPPI					
	PF	Cash	FM8	FM32	HS	RM	CFA	QR	EVT	CG
Panel A: Historical Path										
Return p.a. (in %)	4.35	2.40	3.73	4.04	4.39	4.52	4.48	4.50	4.43	4.58
Volatility p.a. (in %)	7.51	0.14	4.05	6.14	5.94	6.18	5.83	6.12	6.13	6.16
Sharpe Ratio	0.26	0.00	0.33	0.27	0.33	0.34	0.36	0.34	0.33	0.35
Max Drawdown (in %)	-20.86	0.00	-5.73	-10.04	-9.51	-10.07	-9.56	-9.72	-9.86	-9.90
Calmar Ratio	0.13	-	0.38	0.23	0.29	0.30	0.32	0.30	0.30	0.31
Sortino Ratio	0.11	-	0.21	0.14	0.15	0.14	0.15	0.14	0.14	0.14
Participation (in %)	100	0	59.1	88.4	88.0	90.2	86.1	89.9	89.9	90.3
Panel B: Historical Simulation										
Return p.a. (in %)	5.93	1.70	4.60	5.61	5.45	5.49	5.40	5.43	5.49	5.47
Volatility p.a. (in %)	6.71	0.01	4.73	6.17	6.11	6.15	6.10	6.09	6.14	6.13
Sharpe Ratio	0.48	0	0.43	0.44	0.41	0.43	0.41	0.42	0.43	0.43
Max Drawdown (in %)	-6.09	0.00	-4.23	-6.14	-6.05	-6.13	-5.92	-6.12	-6.13	-6.11
Calmar Ratio	1.02	-	1.03	0.94	0.93	0.94	0.94	0.93	0.93	0.95
Sortino Ratio	0.15	-	0.18	0.15	0.15	0.15	0.15	0.15	0.15	0.15
CP Value	3.80	5.46	6.56	5.95	5.94	5.88	5.99	5.84	5.87	5.89

Table V

Descriptive Statistics and Portfolio Weight Structure of the Multi-Asset Data Set

This table reports the annualized mean return and annualized volatility of the components of the multi-asset portfolio over the period from January 2nd, 1991 to March 31st, 2017 (including 6,847 daily returns). In addition, the static portfolio weights are given in the third column.

	Mean return (in %)	Volatility (in %)	Portfolio Weights
A. Stocks			
Nikkei 225	-0.88	23.98	5%
Euro Stoxx 50	7.69	22.03	5%
MSCI EM	6.08	18.09	5%
FTSE 100	4.50	17.95	5%
S&P 500	6.24	17.87	15%
B. Bonds			
JGB 10Y	3.44	3.95	10%
Euro Bund	4.16	5.30	10%
UK Gilt	3.75	6.15	10%
U.S. 10Y	3.80	5.88	10%
C. Commodities			
Oil	0.79	34.26	5%
Gold	4.18	16.04	5%
Copper	6.15	25.49	5%
D. Foreign Exchange Rates			
EUR/USD	-0.99	9.84	15%
UK/USD	-1.61	9.47	15%
JPY/USD	0.70	10.82	15%

Table VI

Return Synchronization: VAR Estimation Results

This table reports median t-statistics of the estimated VAR matrix. To estimate the VAR relationship we use a rolling window of 1,000 days over the period from January 2nd, 1991 to March 31st, 2017 and employ a parsimonious specification of the VAR restriction matrix in which the return series to be synchronized are only predicted by U.S. equities and U.S. 10Y.

	Nikkei 225	JGB 10Y	Euro Bund	UK Gilt	Euro Stoxx 50	FTSE 100
S&P 500	2.93	-5.22	-3.99	-4.43	0.25	3.55
U.S. 10Y	-5.62	2.31	1.21	1.33	-2.96	-3.95

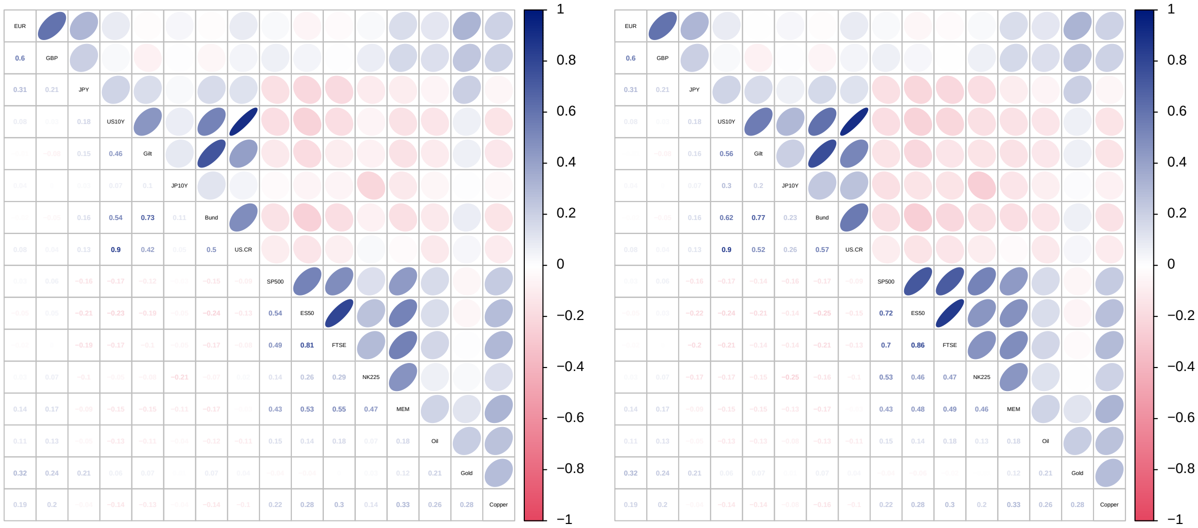
Table VII
Estimation Results of dTIPP and Synthetic Put Insurance

This table shows the estimation results of the dTIPP and the synthetic put strategy based on different risk forecasts for the historical path and historical simulations over the sample period from 1994 to 2017 using a synchronized international multi-asset portfolio and a money market investment. The analyzed risk models are the Historical Simulation approach (HS), RiskMetrics (RM), Cornish-Fisher approximation (CFA), Quantile Regression (QR), Extreme Value Theory (EVT) and Copula-GARCH (CG). For comparison, we include the performance of the underlying multi-asset portfolio (PF), the money market investment (Cash), and two fixed multipliers based on the maximum loss of the portfolio returns (FM8) and the average risk measured by the average over the HS-VaR (FM32), respectively. As performance measure, we employ the annualized mean return, annualized volatility, maximum drawdown, Sharpe Ratio, Calmar and Sortino ratio. Moreover, we show the average participation of the strategy in the risky investment. For the historical simulation, we report the median of these performance measures and add the cumulative prospect (CP) value. In each calendar year, a floor of 95% of the initial portfolio value is targeted.

			CPPI		DPPI					
	PF	Cash	FM8	FM32	HS	RM	CFA	QR	EVT	CG
dTIPP										
Panel A: Historical Path										
Return p.a. (in %)	4.35	2.40	3.17	3.38	3.79	3.88	3.82	3.77	3.75	3.92
Volatility p.a. (in %)	7.51	0.14	2.30	4.87	4.60	4.91	4.49	4.82	4.85	4.90
Sharpe Ratio	0.26	0.00	0.33	0.20	0.30	0.30	0.32	0.28	0.28	0.31
Max Drawdown (in %)	-20.9	0.00	-4.66	-6.42	-6.18	-6.37	-5.70	-6.13	-6.19	-6.02
Calmar Ratio	0.13	-	0.43	0.31	0.34	0.33	0.37	0.34	0.33	0.36
Sortino Ratio	0.11	-	0.36	0.16	0.18	0.17	0.19	0.17	0.17	0.17
Participation (in %)	100	0.0	34.4	71.5	69.4	72.0	67.6	70.6	71.2	72.2
Panel B: Historical Simulation										
Return p.a. (in %)	5.93	1.70	5.55	4.56	4.57	4.35	4.61	4.38	4.39	4.41
Volatility p.a. (in %)	6.71	0.01	2.35	5.27	4.78	5.22	4.65	5.03	5.16	5.17
Sharpe Ratio	0.48	0.00	1.20	0.39	0.49	0.37	0.52	0.41	0.38	0.39
Max Drawdown (in %)	-6.09	0.00	-1.59	-4.32	-3.90	-4.15	-3.75	-3.97	-4.11	-4.06
Calmar Ratio	1.02	-	3.32	1.10	1.26	1.07	1.31	1.16	1.11	1.11
Sortino Ratio	0.15	-	0.46	0.17	0.19	0.17	0.20	0.17	0.17	0.17
CP Value	3.80	5.46	7.15	6.49	6.55	6.41	6.59	6.45	6.44	6.47
Synthetic Put										
Panel C: Historical Path										
Return p.a. (in %)	4.35	2.40	3.80	4.34	4.49	4.29	-	-	4.32	4.33
Volatility p.a. (in %)	7.51	0.14	4.13	5.17	5.83	5.87	-	-	5.87	5.86
Sharpe Ratio	0.26	0.00	0.34	0.37	0.36	0.32	-	-	0.33	0.33
Maximum Drawdown (in %)	-20.9	0.0	-10.9	-8.58	-10.9	-11.1	-	-	-11.0	-11.0
Calmar Ratio	0.13	-	0.22	0.29	0.30	0.25	-	-	0.25	0.26
Sortino Ratio	0.11	-	0.20	0.17	0.15	0.15	-	-	0.15	0.15
Participation (in %)	100	0.0	59.4	75.2	85.3	86.7	-	-	86.5	86.3
Panel D: Historical Simulation										
Return p.a. (in %)	5.93	1.70	5.37	5.30	5.43	5.33	-	-	5.38	5.36
Volatility p.a. (in %)	6.71	0.01	4.02	5.20	5.96	5.98	-	-	5.97	5.93
Sharpe Ratio	0.48	0.00	0.68	0.49	0.46	0.41	-	-	0.43	0.43
Maximum Drawdown (in %)	-6.09	0.00	-3.45	-4.84	-5.65	-5.83	-	-	-5.80	-5.73
Calmar Ratio	1.02	-	1.50	1.07	0.99	0.95	-	-	0.96	0.97
Sortino Ratio	0.15	-	0.24	0.17	0.15	0.15	-	-	0.15	0.15
CP Value	3.80	5.46	6.06	6.07	5.84	5.37	-	-	5.41	5.46

Figure 2. The Effects of Return Synchronization

This figure shows the correlation matrices of the original and synchronized returns based on the international multi-asset data set including stocks, bonds, commodities, and foreign exchange rates. Blue shade indicates positive correlation, red shade negative correlations. The more straight the circles, the higher is the correlation.

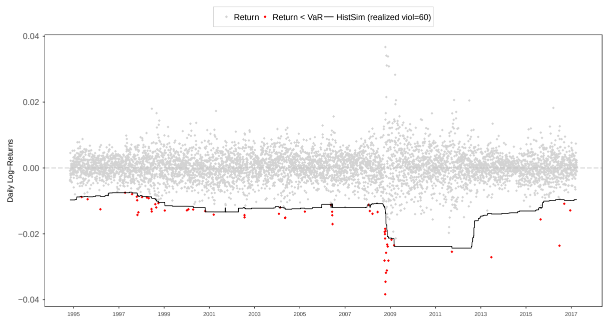


(a) Original Returns

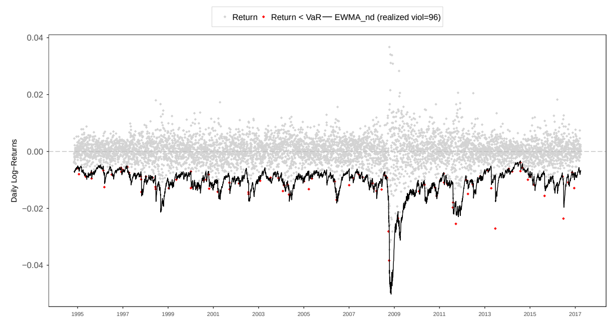
(b) Synchronized Returns

Figure 3. VaR Forecasts over Time

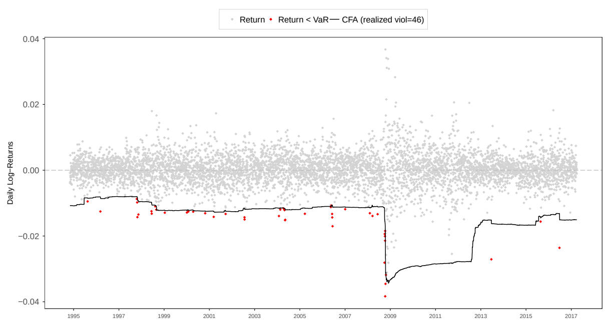
This figure shows the (negative) daily 99% VaR forecasts of the different risk models and the realized returns of the multi-asset portfolio (grey dots) over the period from 3/11/1994 to 31/3/2017. VaR violations are marked in red. At a confidence level of 99%, a total of 58 violations are expected over the model period.



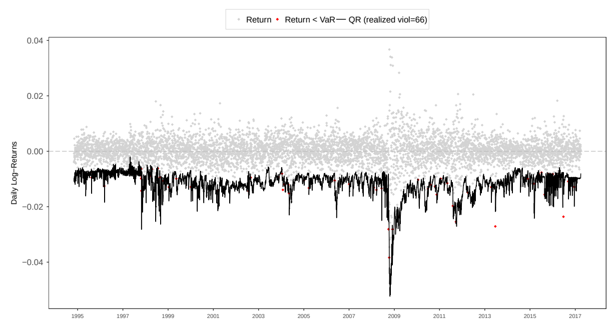
(a) HS-VaR



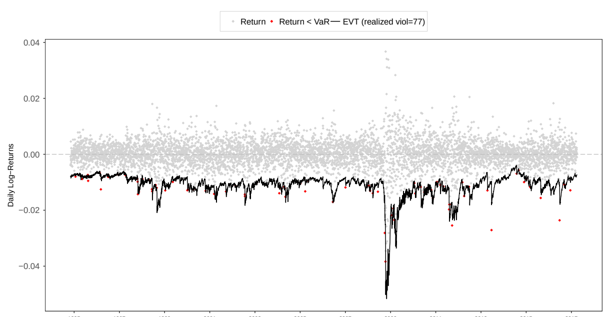
(b) RM-VaR



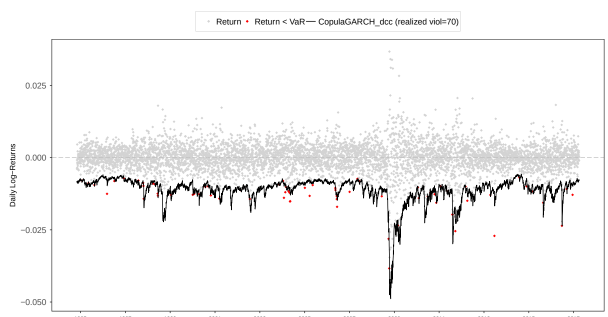
(c) CFA-VaR



(d) QR-VaR



(e) EVT-VaR



(f) Copula-GARCH-VaR

Figure 4. Historical Path of DPPI

This chart shows the performance of the conservative multi-asset portfolio (35% equities, 40% fixed income, 15% commodities, 45% currencies) using a DPPI strategy (red line) in relation to the floor (green line) over the sample period from 1994 to 2017. Exposure is calculated using the cushion (difference between the portfolio value and the floor; here: 95% of the initial annual portfolio value) and the conditional multiplier (based on the Copula-GARCH 99%-VaR). For comparison, we have included the performance of the underlying multi-asset strategy (blue line) and a money market investment (black line).

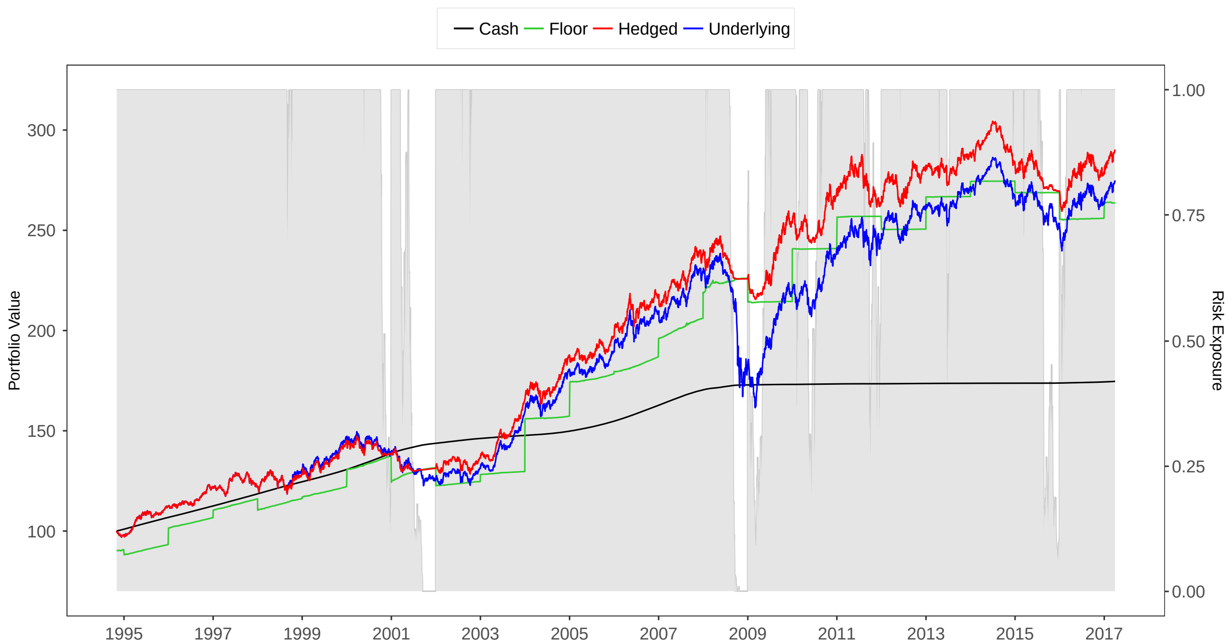


Figure 5. Historical Simulation of DPPI

This chart shows the distribution of historically simulated yearly returns of the DPPI strategy (red shade) and a pure buy-and-hold portfolio investment strategy (blue shade). The multiplier is based on the Copula-GARCH 99% VaR. The black dashed line indicates the floor level.

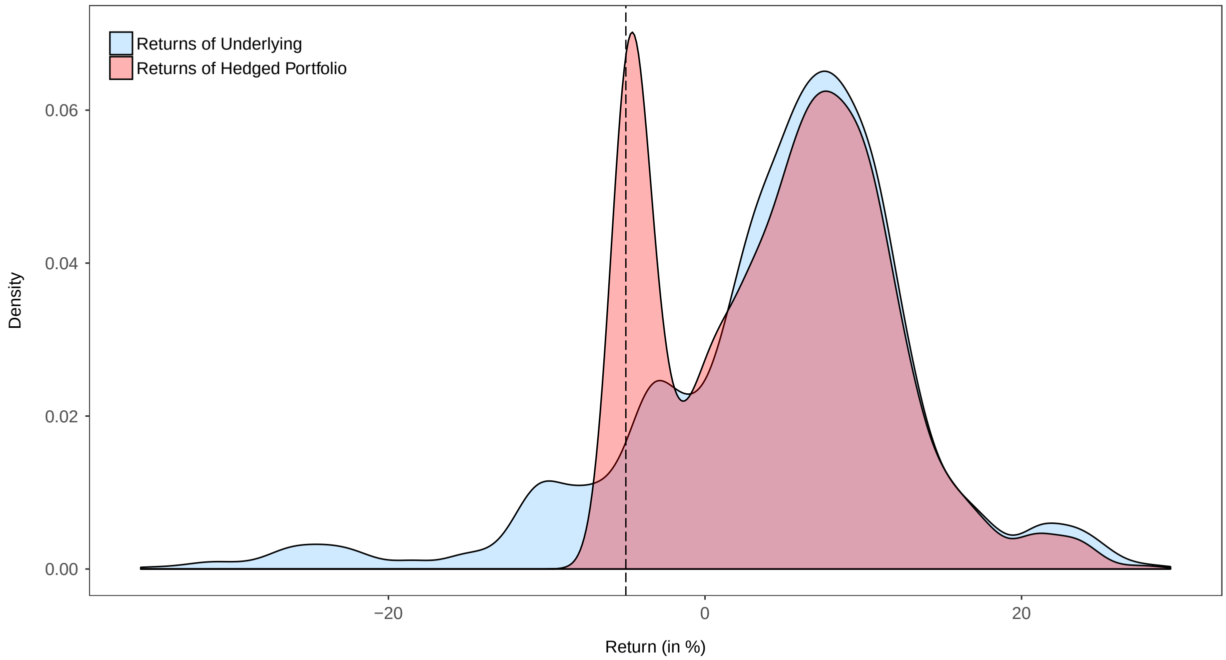
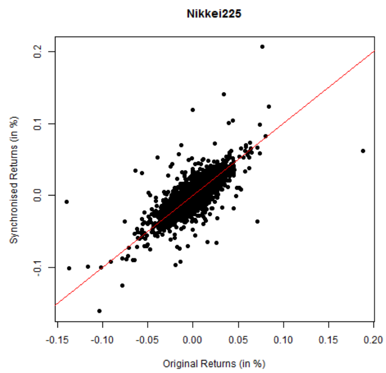
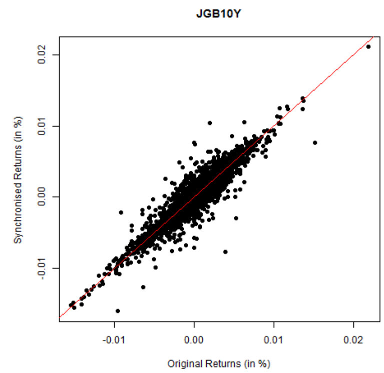


Figure 6. Synchronized vs. Original Daily Returns: QQ-Plots

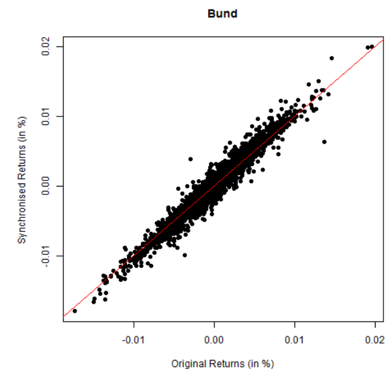
This figure presents QQ-plots of synchronized vs. original returns for those series that close before the U.S. markets, i.e. need to be synchronized.



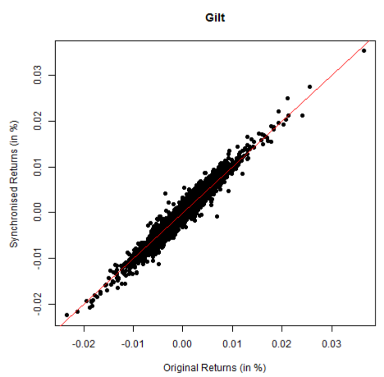
(a) Nikkei 225



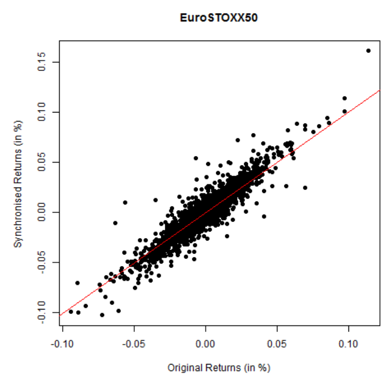
(b) JGB 10Y



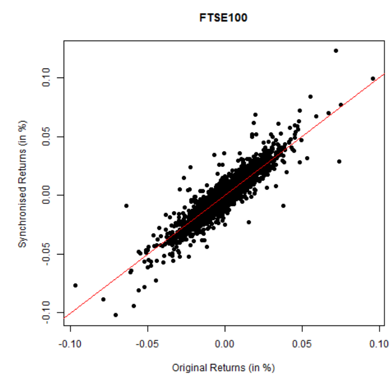
(c) Euro Bund



(d) UK Gilt



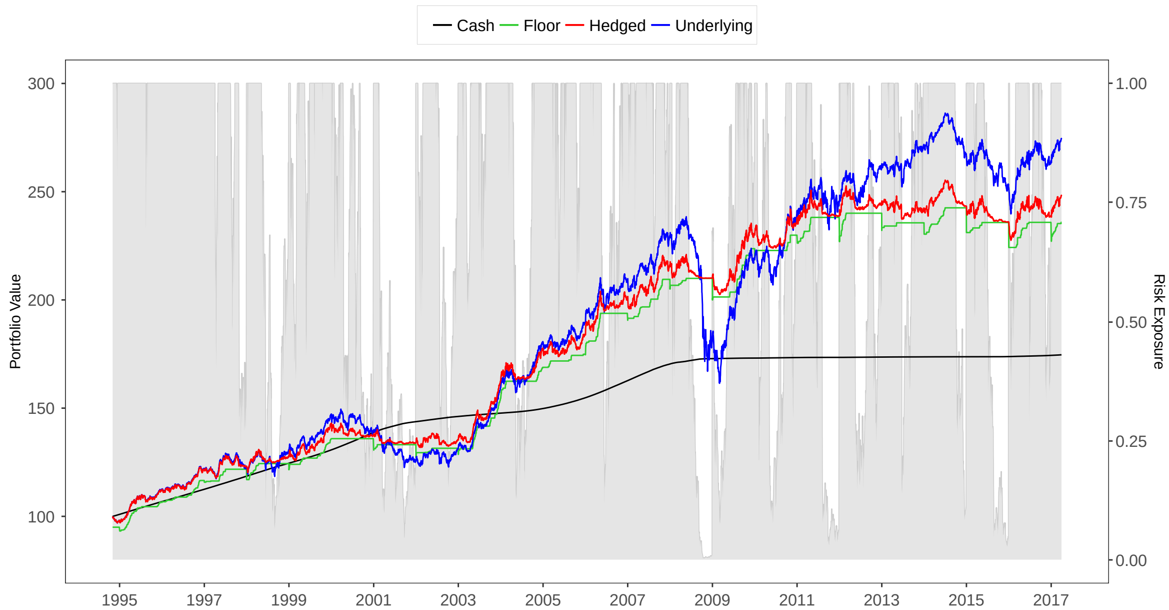
(e) Euro STOXX 50



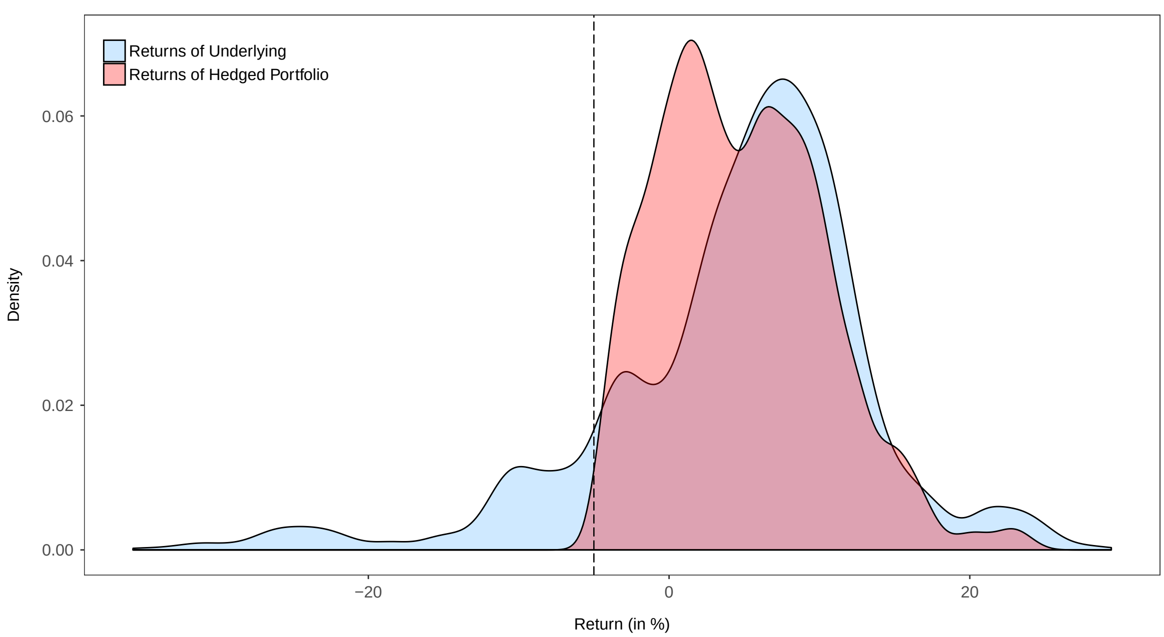
(f) FTSE 100

Figure 7. Historical Path and Historical Simulation of dTIPP

This chart shows the performance of the multi-asset portfolio (35% equities, 40% fixed income, 15% commodities, 45% currencies) using a dTIPP strategy for historical path and historical simulation. Panel (a) shows the historical path of the dTIPP portfolio (red line) in relation to the floor (green line) over the sample period from 1994 to 2017. Exposure is calculated using the cushion and the conditional multiplier (based on the Copula-GARCH 99%-VaR). The key characteristic of the dTIPP strategy lies in the „ratcheting-up“ of the floor (95%) once a new high is achieved. For comparison, we have included the performance of the underlying multi-asset strategy (blue line) and a money market investment (black line). Panel (b) shows the distribution of historically simulated yearly returns (red shade) and a pure buy-and-hold portfolio investment strategy (blue shade). The black dashed line indicates the floor level.



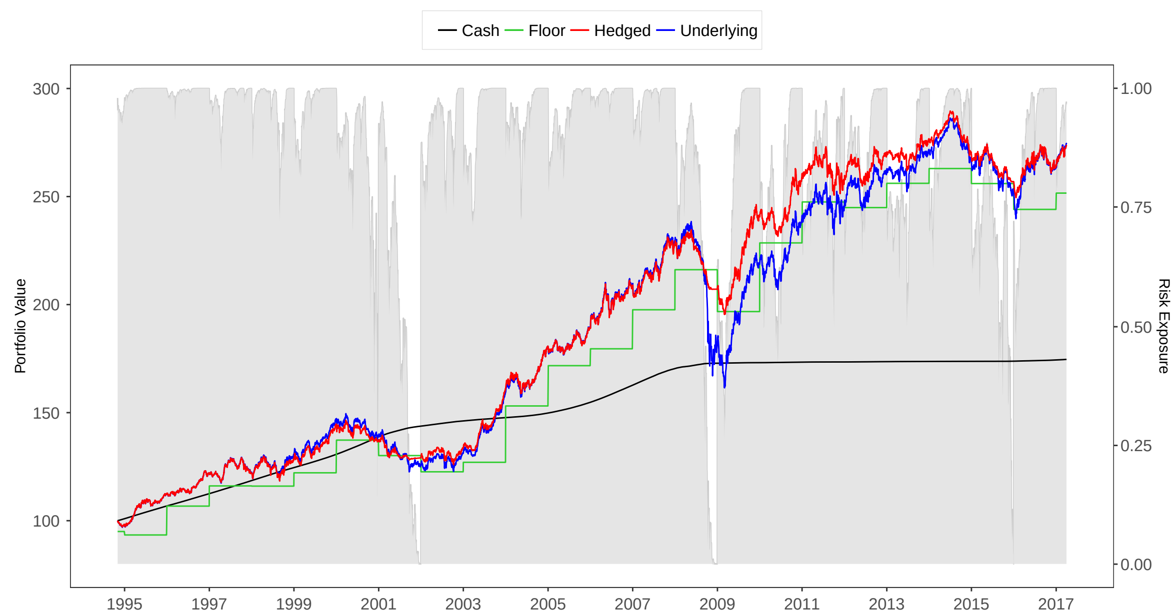
(a) Historical Path



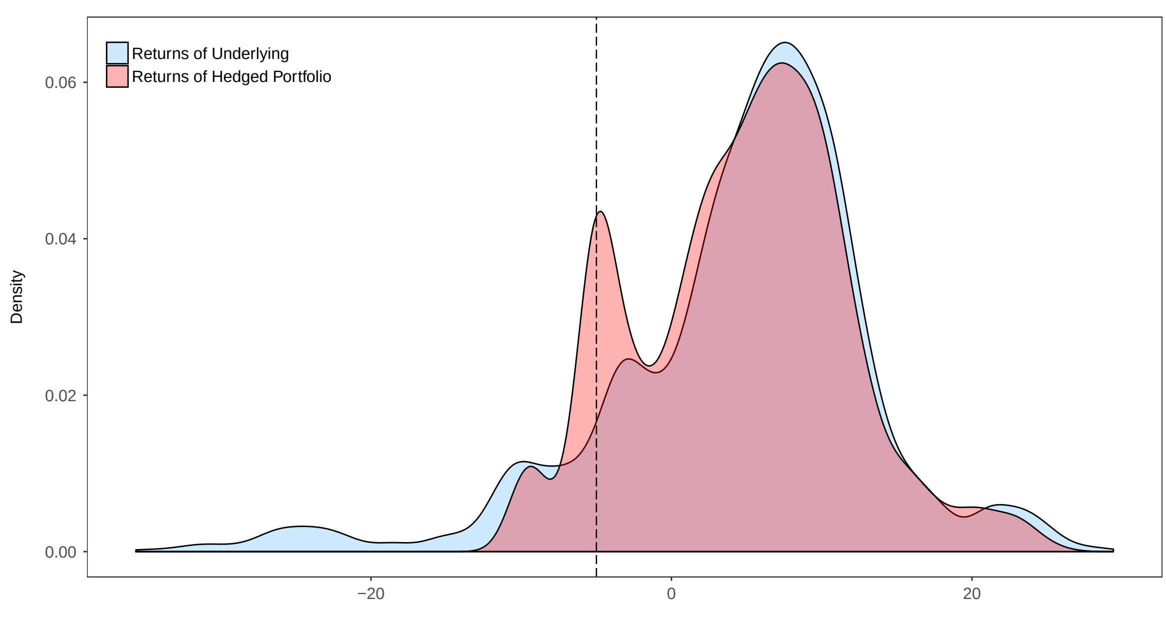
(b) Historical Simulation

Figure 8. Historical Path and Historical Simulation of Synthetic Put Insurance

This chart shows the performance of the multi-asset portfolio (35% equities, 40% fixed income, 15% commodities, 45% currencies) using a synthetic put strategy for historical path and historical simulation. Panel (a) shows the historical path of the synthetic put portfolio (red line) in relation to the floor (green line) over the sample period from 1994 to 2017. Participation in the risky asset's performance is calculated using a classic Black-Scholes formula measuring the sensitivity of a synthetic put with a strike price matching the floor value (here: 95% of the initial annual portfolio value). The volatility of the risky asset is forecasted based on the Copula-GARCH approach. For comparison, we have included the performance of the underlying multi-asset strategy (blue line) and a money market investment (black line). Panel (b) shows the distribution of historically simulated yearly returns (red shade) and a pure buy-and-hold portfolio investment strategy (blue shade). The black dashed line indicates the floor level.



(a) Historical Path



(b) Historical Simulation