Is it Efficient to Buy the Index?

A Worldwide Tour with Stochastic Dominance*

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Abstract

The paper extends the model of Kuosmanen (2004) and develops an operational approach to test for stochastic dominance efficiency of a given portfolio at orders higher than two. Applying this approach to equity indices representing seventeen developed and developing markets across the globe, we find that all of these indices are inefficient, nearly always at order three and very often at order two, implying that all of the prudent and most of the risk averse investors would be better off not investing in these market indices. The indices are often dominated by individual industry sub-indices, with consumer goods, services, and utilities performing especially well. A simple trading rule based on past stochastic dominance information improves the average out-of-sample return of a global portfolio by 2% per year while simultaneously reducing the return standard deviation by 3% per year. It substantially limits global portfolio losses during the financial crises of 2007–2008. Portfolios of low-beta and low-volatility stocks consistently stochastically dominate the market indices and seem to be more desirable alternatives for prudent and risk-averse investors.

JEL Classification: D81, G11, G15

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1 Introduction

One of the most dynamic markets over the last decade has been the ETF market. By the end of June 2017, the total size of the assets under management (AUM) by ETFs reached USD 4 trillion, overcoming that of hedge funds.[1] ETFs provide an easy and cheap way for investors to track various market indices. The most heavily traded single ETF is SPDR S&P 500 ETF with the AUM being over USD 240 billion as of November 14, 2017.[2] The popularity of this ETF is not surprising, given that it tracks a well diversified stock portfolio, which often serves as a proxy for the "market" portfolio.

Starting from the seminal fork of Markowitz (1952), the diversification idea has been playing a decisive role in portfolio allocation theory and practice. Such concepts as the market portfolio and the security market line has become a natural benchmark for portfolio managers and academics. Despite its obvious merits, a problem with this approach is that it works only for jointly normally distributed asset returns or for mean-variance investors with quadratic utility function.

The optimality of the market portfolio has been challenged ever since it was introduced. For example, preference for skewness, behavioral biases, ambiguity with respect to underlying distribution and investor heterogeneity lead to optimal deviation from the market portfolio (Conine and Tamarkin 1981, Shefrin and Statman 2000, Uppal and Wang 2003, Mitton and Vorkink 2007).

Taking into account investor preference for skewness is an important step forward in portfolio theory. However, looking at a limited number of moments of return distribution is still restrictive, especially if a decision maker has a complex utility function, or an optimal portfolio should be constructed to satisfy preferences of multiple investors with heterogeneous utility functions (as, e.g., is the case of delegated portfolio management, including mutual and pension funds). A concept of stochastic dominance overcomes these limitations and provides an efficient tool for comparing complete distributions between each other, instead of focusing on a limited number of moments. First developed as a

[1] Financial Times, September 10, 2017, "Regulators descend on booming ETF market".
[2] `http://etfdb.com/etf/SPY/`.

statistical tool (Markowitz 1952, Lehmann 1955), the concept soon found its way into economics and finance (Hanoch and Levy 1969, Porter and Gaumnitz 1972, Tehranian 1980, Post 2003, Kuosmanen 2004, De Giorgi and Post 2008, Annaert et al. 2009, Constantinides et al. 2011, Hodder et al. 2015, Longarela 2016, Post et al. 2018, to name a few). One of the appealing features of the concept of stochastic dominance (and related stochastic dominance efficiency) is that it can be easily linked to the expected utility preference framework, and then applied to ranking potential portfolio return distributions. For example, if the return distribution X second-order stochastically dominates the return distribution Y, then all risk averse investors, regardless of the exact shape of their utility functions and the levels of risk aversion, would prefer X over Y. Similarly, third-order stochastic dominance leads to all risk averse and prudent investors choosing the dominating distribution.

Stochastic dominance efficiency is an even broader concept, which is applied to portfolios of assets. If a portfolio is stochastically efficient at a given order with respect to a set of underlying assets, then it is not possible to construct any other portfolio using the underlying set of assets that would dominate the portfolio in question at this order. Put differently, if a portfolio is not efficient, for example, at the second order, it is possible to construct a different portfolio using the same underlying assets that would be preferred by all risk averse investors.

To this end, the approach of evaluating portfolios from the stochastic dominance perspective is extremely appealing for delegated portfolio management industry, in which portfolio managers should cater for interests of multiple heterogeneous investors. Similarly, the rising in popularity ETFs should be assessed in terms of the stochastic dominance efficiency of their returns, or that of the indices they track. Previous empirical evidence worryingly suggest, however, that the market portfolio is inefficient relative even to the Fama and French benchmark size and book-to-market portfolios (Post 2003).

If the benchmark (market) portfolio is found not to be efficient, the follow up question naturally arises of whether it is possible to construct a dominating efficient portfolio. Kuosmanen (2004) has developed an operational test that, using standard linear programming algorithms, not only allows testing for the first and second order efficiency of a given portfolio relative to the underlying set of assets, but also provides optimal weights for an efficient portfolio. Applying this approach to twenty five Fama and French industry portfolios, Hodder et al. (2015) further show that efficient portfolios chosen in such a way perform reasonably well out of sample.

Post and Versijp (2007) develop a multivariate tests for second and third order stochastic dominance and show that the CRSP all-share index is not mean-variance efficient relative to the 10 beta-sorted portfolios, but the second order stochastic dominance efficiency cannot be rejected. Post and Kopa (2009) show that the U.S. market portfolio is not first-order stochastic dominance efficient in their sample relative to portfolios formed on book-to-market and size. Post (2017) develop a bootstrap empirical likelihood ratio test for stochastic dominance optimality, which jointly compares a given distribution with multiple possible alternatives, and show that the Fama and French small growth stock portfolio is not optimal for risk-averse investors. Post and Levy (2003) apply a wider range of stochastic dominance criteria including prospect stochastic dominance (that assumes an S-shape utility function) and Markowitz stochastic dominance (that assumes a reverse S-shaped utility function) to the market portfolio and show that the market portfolio is clearly inefficient using second order or prospect stochastic dominance criteria, but Markowitz stochastic dominance efficiency cannot be rejected. Post et al. (2018) combined the stochastic dominance decision criterion and the empirical likelihood optimization technique to improve the out-of-sample performance of portfolios relative to a set of benchmarks.

The majority of the existing empirical studies focus largely on second order stochastic dominance[3] and the U.S. market. There exists, however, growing evidence of substantial cross-county differences that result in different economic decisions of agents and pricing of assets. The important differences include, for example, observable legal rules that can impact ownership concentration (Porta et al. 1998), perception of risk that leads to variations in option pricing (Weber and Hsee 1998), cultural differences influencing

[3]Notable exceptions are Post and Kopa (2017) and Fang and Post (2017).

trade agreements (Guiso et al. 2009), investor portfolio choice (Grinblatt and Keloharju 2001), and takeover activities (Frijns et al. 2013). Cross-county differences also manifest themselves through a better performance of country-specific Fama-French three-factor model compared to its global version in explaining time-series variation in international stock returns (Griffin 2002). A related issue raised in Jorion and Goetzmann (1999) is that the U.S. market is one of the most successful markets in the world, and, consecutively, the estimates of the expected return on equity derived from this market are subject to a survivorship bias. The authors show substantial differences in the expected real return on assets across 39 countries, with the U.S. equity having the highest real return.

Our paper makes several contributions to the literature. First, on the theoretical front, we extend the operational approach of Kuosmanen (2004) to allow us to test for higher order stochastic dominance efficiency in majorization sense and derive efficient portfolios of orders higher than two.

Second, we use this methodology to test for the efficiency of well diversified stock indices across seventeen countries, spanning both developed and developing markets. We show that these indices are usually not efficient at order two, which also implies inefficiency at order three and beyond. In the cases where inefficiency at order two is not shown, then, in the vast majority of circumstances, stock indices are inefficient at order three and beyond. That is, all of the prudent investors and most of the risk averse investors would be better off not investing in those well-diversified indices but should instead hold more concentrated portfolios, focusing on several industries. The average potential improvement of a portfolio Sharpe ratio is 0.73 per year when an inefficient market index is substituted by an efficient portfolio.

Next, we perform a comparison of each well-diversified market index with its industry components and find striking difference across countries, which cannot be attributed only to the fact that a country is an emerging or developed economy. For example, the Japanese Nikkei 225 index is dominated by some of its sub-indices in 13 of 14 years in our sample (93% of years), whereas the Indian BSE SENSEX index is dominated only in 3 of 11 years (27%). Some counter-cyclical industries, such as consumer goods and services, health care, and utilities, are more likely to dominate the respective market indices in all countries in our sample.

Looking further into determinants of dominating industries, we find that the key factor is the relative volatility of the industry compared to the market, as well as past dominance of a sector index over the market index. Macroeconomic variables also help to forecast the years during which market indices are dominated and are not suitable for risk-averse and prudent investors. The information content of the aggregate indicators, however, is very distinct for developed and developing markets. For example, a higher GDP growth in developed markets indicates a relative homogeneous improvement in all sectors and, thus, predicts a lower likelihood of the market index to be dominated. A higher GDP growth in developing economies, that are usually tilted towards one or two main industries, signals a disproportional growth of one industries that makes the market index a relatively less desirable investment for a risk-averse investor as compared to the GDP-driving industry.

Motivated by the prediction results, we propose a simple trading rule based on past information on stochastic dominance. The rule allows improving the out-of-sample performance relative to a benchmark global portfolio with the mean return increasing on average by 1–2% annualized and return standard deviation declining by 2–3%. The improvement is consistent across time.

Last but not least, we contribute to the discussion on the exceptional performance of low-beta low-risk stocks. We show that the portfolios of low beta and low volatility stocks stochastically dominate the market indices in majority of years at order 3 and often at order 2. Thus, these portfolios are more suitable for risk-averse and prudent investors than the market indices across most of world economies considered in this paper.

2 Key Concepts and Theoretical Results

Let $F = F^{[1]}$ be a cumulative distribution function defined on real numbers. Define $F^{[n]}$ recursively as follows:

$$F^{[n]}(r) = \int_{-\infty}^{r} F^{[n-1]}(s)\; ds \tag{1}$$

Then, distribution F dominates distribution G at order n in the stochastic sense when $F^{[n]}(r) \leq G^{[n]}(r)$ for all $r \in \mathbb{R}$ and provided there exists $r_0 \in \mathbb{R}$ such that $F^{[n]}(r_0) < G^{[n]}(r_0)$.

The order of stochastic dominance is closely linked to an investor's preferences and to the shape of her utility function U. For example, "F dominates G at order 2" is equivalent to "all non-satiated ($U' \geq 0$) and risk-averse ($U'' \leq 0$) agents prefer F to G". The dominance of G by F at order 3 is equivalent to the preference of F over G for all non-satiated, risk-averse and prudent ($U^{(3)} \geq 0$) investors. As shown in Eeckhoudt and Schlesinger (2008), the positive third derivative of the utility function is associated with a higher degree of saving when distributions have a higher variance, keeping the mean constant. Finally, the dominance of G by F at order 4 is equivalent to the preference of F over G by all non-satiated, risk-averse, prudent, and temperant ($U^{(4)} \leq 0$) agents. The negative fourth derivative of the utility function is associated with a higher degree of savings when distribution have a smaller skewness, keeping the mean and the variance constant.

The concept of pairwise stochastic dominance can be extended to stochastic dominance efficiency (SD efficiency). A portfolio of assets is SD efficient of order K relative to a given span of the underlying assets when it is not possible to fund any other linear combination of the assets that dominates this portfolio at order K or higher. On the contrary, if a portfolio is not efficient, it is dominated by at least one other portfolio. Consider, for example, second order stochastic dominance. If a portfolio is not second order efficient, one can construct another portfolio using the same assets, such that the latter portfolio second order stochastically dominates the former portfolio. All risk averse investors favor this latter portfolio.

2.1 Pairwise comparisons of distribution

In this section, we extend the arguments of Marshall and Olkin (1979) used in Kuosmanen (2004) and show that pairwise SD comparison of distributions can be achieved at any order of stochastic dominance. We are not the first ones who attempt to tests for higher order SD. For example, Post and Kopa (2017) use a superconvex TSD (third order stochastic dominance) formulation – a more restrictive sufficient condition for TSD – to construct portfolios with enhanced out-of-sample performance relative to a benchmark. Fang and Post (2017) derive systems of equations that can exactly characterize portfolio SD efficiency up to the order 5. Davidson (2009) proposes a test of restricted stochastic dominance at any order that applies to theoretical cumulative distribution functions. This test considers stochastic dominance of distributions on a restrictive support, and it does not incorporate the most extreme information in the tails of the distributions. Accounting for tail events, however, is crucial for the analysis of financial returns, as has become apparent after the financial crisis of 2007-2008. The (extended) approach of Kuosmanen (2004) uses empirical cumulative distribution functions and accounts for all of the available information on the stock performance up to date. We believe it is the most appropriate for application to financial data.[4] Hodder et al. (2015) also show that the optimal second-order SD portfolios constructed using the Kuosmanen (2004) approach exhibit good out-of-sample performance.

To compare distributions F and G in terms of stochastic dominance, we consider T return observations $x_{t=1,\ldots,T}$ associated with F, and T return observations $y_{t=1,\ldots,T}$ associated with G. We denote the corresponding observations ranked in increasing order by $\tilde{x}_{t=1,\ldots,T}$ and $\tilde{y}_{t=1,\ldots,T}$.

Next, we define the cumulative sum of $\tilde{x}$ at order n as follows:

$$\forall t \leq T \quad \tilde{x}_t^{[n]} = \sum_{j_{n-1}=1}^{t} \sum_{j_{n-2}=1}^{j_{n-1}} \cdots \sum_{j_1=1}^{j_2} \tilde{x}_{j_1}. \tag{2}$$

These cumulative sums are discrete equivalents to the integrals of a cumulative distribu-

[4] Post and Kopa (2017)

tion function as defined in Equation (1).

For our practical purposes, we are interested in stochastic dominance up to order four. The corresponding cumulative sums for the first four orders are given by

$$\forall t \leq T \quad \tilde{x}_t^{[1]} = \tilde{x}_t \tag{3}$$

and

$$\forall t \leq T \quad \tilde{x}_t^{[2]} = \sum_{j_1=1}^{t} \tilde{x}_{j_1} \tag{4}$$

and

$$\forall t \leq T \quad \tilde{x}_t^{[3]} = \sum_{j_2=1}^{t} \sum_{j_1=1}^{j_2} \tilde{x}_{j_1} \tag{5}$$

and also

$$\forall t \leq T \quad \tilde{x}_t^{[4]} = \sum_{j_3=1}^{t} \sum_{j_2=1}^{j_3} \sum_{j_1=1}^{j_2} \tilde{x}_{j_1}. \tag{6}$$

Using the above definition of cumulative sums at order n, we now extend the concept of dominance in the majorization sense (see Marshall and Olkin 1979) to any order. We state that x dominates y at order n in the majorization sense, and we write $x \succ^{[n]} y$, when

$$\tilde{x}_t^{[n]} \geq \tilde{y}_t^{[n]} \quad \text{for all} \quad t \leq T. \tag{7}$$

We can now state the core theoretical result of this section, which extends Theorem 1 in Kuosmanen (2004) to any order.

Let $\hat{F}$ and $\hat{G}$ be the empirical cumulative distribution functions associated with distributions F and G. By observing that $\hat{F}^{[n]}$ and $\hat{G}^{[n]}$ are monotonically increasing piecewise linear functions with vertices located in $\tilde{x}_t^{[n]}$ and $\tilde{y}_t^{[n]}$, for $t = 1, ..., T$, we readily have:

Proposition 1. *Stochastic dominance of empirical distribution functions at order n is equivalent to dominance in the majorization sense at order n. In explicit terms:*

$$\hat{F}^{[n]}(r) \leq \hat{G}^{[n]}(r) \quad \forall r \in \mathbb{R} \quad \Leftrightarrow \quad \tilde{x}_t^{[n]} \geq \tilde{y}_t^{[n]} \quad \forall t \leq T. \tag{8}$$

Empirically, we apply the comparison of the distributions in the majorization sense, in order to compare them in the stochastic dominance sense. We use the ranked returns of the sub-indices and the corresponding indices to compute the cumulative sums of lower order return series (up to order four) as in Equations (3) to (6). These cumulative sums of subindex returns are then compared with the cumulative sums of index returns in the spirit of the majorization theorem, reflecting the corresponding order of stochastic dominance.

2.2 Portfolio dominating sets

For a given portfolio of assets, the dominating set includes all the portfolios that can be constructed using the same assets and that dominate this portfolio at a given order n.

Consider a simple illustrative example: we construct the dominating set of a portfolio for which there are two return observations: $(1, 4)$. We look for all the pairs of returns (x_1, x_2) that satisfy $(x_1, x_2) \succ^{[n]} (1, 4)$, for $n = 1$ to 4. The cases $n = 1$ and $n = 2$ of first and second order stochastic dominance are studied in Kuosmanen (2004). For simplicity, we only consider the case where $x_1 < x_2$. For each order n, the case $x_1 > x_2$ is readily obtained by symmetry.

Using Equation (3), we see that a portfolio (x_1, x_2) dominates the portfolio $(1, 4)$ at order 1 in the majorization sense, and we write $(x_1, x_2) \succ^{[1]} (1, 4)$, when $x_1 > 1$ and $x_2 > 4$. Then, using Equation (4), we have that (x_1, x_2) dominates $(1, 4)$ at order 2 in the majorization sense, or $(x_1, x_2) \succ^{[2]} (1, 4)$, when $x_1 > 1$ and $x_1 + x_2 > 1 + 4$. The latter condition can be associated with the following limit segment: $x_2 = 5 - x_1$, which starts at $(1, 4)$ and stops on the straight line $x_2 = x_1$.

[Figure 1 around here]

The third and fourth order dominating sets can be constructed in a similar way, using Equations (5) and (6). Specifically, a portfolio (x_1, x_2) dominates $(1, 4)$ at order 3 in the majorization sense, or $(x_1, x_2) \succ^{[3]} (1, 4)$, when $x_1 > 1$ and $x_1 + x_1 + x_2 > 1 + 1 + 4$. The latter condition can be associated with the following limit segment: $x_2 = 6 - 2x_1$, which starts at $(1,4)$ and stops on the straight line $x_2 = x_1$. Finally, (x_1, x_2) dominates $(1,4)$ at order 4 in the majorization sense, or $(x_1, x_2) \succ^{[4]} (1,4)$, when $x_1 > 1$ and $x_1 + x_1 + x_1 + x_2 > 1+1+1+4$. The latter condition can be associated with the following limit segment: $x_2 = 7 - 3x_1$, which starts at $(1,4)$ and stops on the straight line $x_2 = x_1$.

Figure 1 summarizes these results. The sets that dominate $(1,4)$ are all convex except the dominating set at order 1. The figure confirms that dominating sets are increasing by inclusion: the dominating set at order m is included in the dominating set at order n, for $n \geq m$. This result has strong implications that are explored in the remainder of this text. If we cannot find empirical portfolios in a dominating set at order 3, for instance, then the dominating sets at order 1 and 2 are empty. Equivalently, empirical emptiness of dominating portfolios is a more powerful property when the order n increases.

2.3 SD efficiency of portfolios

From Hardy et al. (1934), a portfolio dominates another portfolio at order two if the former portfolio can be expressed as the product of a doubly stochastic matrix by the latter portfolio.[5] By extension, we have:

$$\forall t \leq T \quad \tilde{x}_t^{[2]} \quad \geq \quad \tilde{y}_t^{[2]} \quad \Leftrightarrow \quad \exists W \in \Xi \mid x \geq Wy, \tag{9}$$

where Ξ is the set of all doubly stochastic matrices and W is one element of this set.

Replacing x by $x^{[n-1]}$ and y by $y^{[n-1]}$ at any order n, we have the generalized result:

Proposition 2 (High Order Stochastic Dominance and Doubly Stochastic Matrices)**.** *A portfolio x dominates a portfolio y at order n if and only if the cumulative sum at order $n-1$ of the returns of portfolio x is larger than the product of a doubly stochastic matrix by the cumulative sum at order $n-1$ of the returns of portfolio y:*

$$\forall t \leq T \quad \tilde{x}_t^{[n]} \quad \geq \quad \tilde{y}_t^{[n]} \quad \Leftrightarrow \quad \exists W \in \Xi \mid x^{[n-1]} \geq Wy^{[n-1]}. \tag{10}$$

[5] A doubly stochastic matrix is a square matrix with all entries being non-negative real numbers and with the sums of the elements along each row and column being equal to one.

Specifically, for third order stochastic dominance:

$$\forall t \leq T \quad \tilde{x}_t^{[3]} \quad \geq \quad \tilde{y}_t^{[3]} \quad \Leftrightarrow \quad \exists W \in \Xi \mid x^{[2]} \geq W y^{[2]}, \tag{11}$$

and for fourth order stochastic dominance:

$$\forall t \leq T \quad \tilde{x}_t^{[4]} \quad \geq \quad \tilde{y}_t^{[4]} \quad \Leftrightarrow \quad \exists W \in \Xi \mid x^{[3]} \geq W y^{[3]}. \tag{12}$$

While stochastic dominance comparisons are conducted with permutation matrices[6] at order 1 and with doubly stochastic matrices at order 2, Proposition 2 shows that stochastic dominance comparisons at any higher order can also be achieved using doubly stochastic matrices as well.

Denote by y a portfolio being tested for n^{th} order stochastic dominance efficiency. We want to compare this portfolio to a market represented by N assets for which we have T observations. This market is represented by the database $(y^1, ..., y^N)$, where each element y^j is a vector of T observations. We also construct a broader database Y comprised of the market database completed by the portfolio being tested: $Y = (y, y^1, ..., y^N)$.

Using the generalization (10) of (9), we extend Theorem 5 in Kuosmanen (2004) to an arbitrary order n:

Proposition 3 (n^{th} Order SD Efficiency, Necessary Condition)**.** *Denote*

$$\theta_n^{nec}(y) = \frac{1}{T} \max_{\lambda, W} \left(\sum_{t=1}^{T} \sum_{i=1}^{N+1} Y_{i,t} \lambda_i - \sum_{t=1}^{T} y_t \right),$$

such that

$$(Y\lambda)^{[n-1]} \geq W y^{[n-1]},$$

where W is a doubly stochastic matrix and λ a vector of portfolio weights in the portfolio being tested and in the reference market. Then, $\theta_n^{nec}(y) = 0$ is a necessary condition for the portfolio y to be n^{th} order SD efficient given the market information $(y^1, ..., y^N)$.

[6] A permutation matrix is a square matrix with all entries being equal to zero or one and with the sums of the elements along each row and column being equal to one.

Similarly, using the generalization (10) of (9), we extend the sufficient condition of Theorem 6 in Kuosmanen (2004) to an arbitrary order n:

Proposition 4 (n^{th} order SD Efficiency, Sufficient Condition)**.** *Define*

$$\theta_n^{suf}(y) = \min_{W,\lambda,s^+,s^-} \sum_{j=1}^{T} \sum_{i=1}^{T} \left(s_{ij}^+ + s_{ij}^-\right),$$

such that

$$(Y\lambda)^{[n-1]} = Wy^{[n-1]},$$

where s_{ij}^+ and s_{ij}^- are non-negative numbers satisfying:

$$s_{ij}^+ - s_{ij}^- = W_{ij} - \frac{1}{2},$$

and where W is a doubly stochastic matrix and λ a vector of portfolio weights in the portfolio being tested and in the reference market.

Denote by d_t the number of occurrences where t values are identical in the portfolio being tested. Then, $\theta_n^{suf}(y) = \frac{T^2}{2} - \sum_{t=1}^{T} t d_t$ is a sufficient condition for this portfolio to be n^{th} order SD efficient given the market information $(y^1, ..., y^N)$.

Propositions 3 and 4 give necessary and sufficient conditions for a candidate portfolio to be efficient with respect to a given market. As a by-product of Proposition 3, one also obtains the optimal portfolio weights, defining the SD efficient portfolio. These weights are given by the optimal values of λ, obtained as the solution of the optimization problem defined in Proposition 3.

3 The road map of the empirical analysis

We conduct our analysis in several steps. First, for each market index i in our sample and each year t for which the information on the index and its constituents is available we construct time series of sub-index returns. We sort all year-beginning components of a benchmark index into sector groups according to their ICB codes. The return time series for sub-indices are calculated with weighting scheme consistent with that of the benchmark index (so, for value weighted indices, the sub-indices are also value weighted).

Next, we test if each index i is SD efficient with respect to any other portfolio constructed as a combination of its sub-indices. We use the extended approach of Kuosmanen (2004) for higher order stochastic dominance, as detailed in Section 2.1. For those cases in which the index is not efficient, we compute portfolio weights for sub-indices to construct a dominating efficient portfolio with the highest mean improvement relative to the index under consideration. Then, we compare the performance of the indices with that of the dominating efficient portfolios to assess the maximum possible gain for investors that optimize their portfolios using the stochastic dominance approach. The technical details of the efficiency tests and portfolio construction are provided in Section 2.3.

When implementing the above approach, we obtain the optimal portfolios from the necessary stochastic dominance condition. We start by testing for the first order SD efficiency of each index against all possible combinations of its sub-indices and itself. If the index turns efficient at a given order, we move on to test for a higher order efficiency. Conversely, because inefficiency at a given stochastic dominance order also implies inefficiency at higher orders, there is no need to conduct tests at higher orders for a portfolio that is inefficient at a given order.

The necessary test we implement is a linear programming optimization problem that maximizes the mean return difference of a potentially dominating portfolio over the portfolio in test. This maximization is subject to the constraint that the returns of the potentially dominating portfolio are larger than the returns of a mean preserving anti-spread of the portfolio in test. This mean preserving anti-spread is formed as the portfolio in test is permuted by a non-negative doubly stochastic matrix. If the mean return difference is 0, it is straightforward that the portfolio in test is necessarily efficient and we can further conduct necessary stochastic dominance test at higher order.

Although it cab be possible to construct a dominating portfolio relative to an index using realized returns, frequent re-balancing may lead to high transaction costs. Further, relying on in-sample optimality may lead to overfitting and not necessarily stellar out-of-sample performance.[7] Thus, in a second step, we refrain from the in-sample optimization and perform a pairwise stochastic dominance comparison of each index with the underlying sub-indices every year.

Next we analyze the determinants of dominance of sub-indices over the corresponding market indices using logistic regression framework. Last but not least, we use a simple trading strategy that equally weights the sub-indices that have dominated the index in the past, up to the fourth-order of stochastic dominance, and evaluate the performance of this portfolio.

4 Data

In this paper we use seventeen equity indices from Datastream spanning most representative regions of the world (including American, Asian, European developed and developing markets) and their constituent industry-based sub-indices. We use the total return approach to calculating daily return series for each index in our sample.

In order to construct industry sub-indices, we first group the constituents of each of the equity indices according to Industry Classification Benchmark (ICB), which assigns each company to one of 10 macro industries. Then the return on an industry sub-index is calculated as a (weighted) average of the returns on all stocks classified in this industry. The methodology for computing sub-indices is consistent with that of the corresponding benchmark index. If the benchmark index is price weighted (market capitalization weighted), its sub-indices are also price weighted (market capitalization weighted). In order to assure that the constructed sub-indices are investible and are not subject to a look-ahead bias, we determine the constituents of each index at the beginning of a year and keep them unchanged until the end of a year. If any of the firms are delisted during that year (due to various corporate events such as a bankruptcy or a merger), we use their return series until the last active trading day, and subsequently re-weight the sub-index after deletion of the concerned stocks without adding any new stock until the year end

[7]See, for example,Hodder et al. (2015). Post et al. (2018) suggest an approach to improve the out-of-sample performance of the portfolios chosen using SD criteria up to the order 3, by jointly utilizing an empirical likelihood estimation method for the multivariate return distribution.

is reached.

Our sample traces each equity index back to its earliest complete year covered in Datastream. A few components do not have complete series of total return or market capitalization in certain years. We exclude such firms from our sub-indices composition. Table 1 lists the indices used in our study together with the starting date of their histories. Table 2 reports the corresponding return descriptive statistics.

[Tables 1 and 2 around here]

5 Empirical results: Market indices vs. sector sub-indices

5.1 SD index efficiency

We start the section by presenting an example of application of our methodology to a single index, the S&P 100 index. Then, we proceed with a discussion of the complete set of empirical results based on all seventeen domestic indices.

5.1.1 Worked example: the S&P 100 index

Table 3 reports the results of the efficiency test of the S&P 100 index. Efficiency of the index is tested against all possible portfolios which can be constructed using the index and its industry-based sub-indices using daily returns for each year from 1990 to 2015. The first row of the table reports the lowest order of index inefficiency. Note that dominance at order 2 implies dominance at order 3 and beyond. However, the converse is not true. So, a number 2 in the table means that the index is not efficient of order 2, that is, one can construct a portfolio that dominates the index at orders 2, 3, 4, and beyond. A number 3 in the table indicates that it is possible to construct a portfolio that dominates the index at orders 3, 4, and beyond, but not at order 2.

The middle part of the table reports the optimal portfolio weights for each sub-index for every year. Then, the descriptive statistics of the index and the optimal portfolio are reported. The last row of the table reports the improvement of the mean return which could have been achieved should an investor have invested in the optimal portfolio and not the index. The last column of Table 3 summarises the average performance of the optimal portfolios relative to the index across the last quarter of a century. The order of inefficiency reflect the inefficiency of the index in majorization sense over the complete sample period.

Almost always (with the exception of years 1994, 2000, and 2005) the S&P 100 index is not efficient of order 2. This implies that most of time risk averse investors would be better off by not tracking the index, but investing in a different portfolio in which some industries are overweighed relative to the index. In years 1994, 2000, and 2005 the index is inefficient at order 3, implying that risk averse and prudent investors should optimally deviate from holding the index. Taken together as one time series, the S&P 100 index is still inefficient of order 3 over the complete sample.

Optimal portfolios show substantially higher mean returns than the index and deliver a higher Sharpe ratio every year. Overall, over the 25 years the potential mean improvement is 15% annualized. At the same time, such attractive gains may be difficult to achieve in practice, as the optimal portfolio weights are rather volatile. For example, the health care sub-index has a weight of 54% in 1990, 77% in 1991, and then three consecutive years of zero weights until the weight increases again to 36% in 1995. Such a volatility of optimal weights makes the out-of-sample construction of SD efficient portfolios rather difficult. This result confirms findings in Hodder et al. (2015) based on Fama-French industry portfolios that the construction of out-of-sample SD efficient portfolios is challenging.

[Table 3 around here]

5.1.2 Global indices results

Table 4 summarises the average results for the 17 global indices under study. For each index, we report the average values of the optimal portfolio weights[8], as well as the average descriptive statistics of the optimal portfolios and corresponding index across the years (similar to the structure of the last column of Table 3).[9] The SD inefficiency results for other indices are even more striking than those for the S&P100 index. Over the complete sample available for each index, all indices are not SD efficient at order two. Any risk-averse investor across the globe would be better off not investing in the well diversified index, but deviating from it. Table 5 further reports the orders of inefficiency of the market indices year by year. It indicates, that all indices have been consistently inefficient at least of order three over their histories.

[Tables 4 and 5 around here]

The potential average gains from deviating from indices are large, but vary across countries. The minimum average gain of 10% annualized is associated with the DJIA index, and the maximum of 44% annualized is achievable for the RTS index. Together with decreasing return standard deviations, this translates into substantial gains in terms of the Sharpe ratios. Figure 2 depicts the average improvements in annualized mean returns, standard deviations, and daily Sharpe ratios for our 17 indices.

[Figure 2 around here]

Following Kuosmanen (2004), the degree of portfolio inefficiency can be measures by the optimal parameter θ from Proposition (3). This parameter indicates the maximal possible improvement in the portfolio mean return that can be achieved by moving from an SD inefficient portfolio to an efficient one. Figure 3 plots the time series of the estimated θ-s for 17 market indices under study together with the average value of θ for each year. Overall, individual and average θ-s tend to spike during turbulent periods,

[8]In Appendix A we further report the average correlations between optimal portfolio weights.

[9]The detailed results for every year are available from the authors upon request.

such as year 2000 (the burst of Internet bubble) and 2008 (the pick of the financial crisis of 2007-2009), indicating serious SD inefficiency of the market indices during these periods. Individual θ-s also spike during market specific events. For example, in 2014 during Russian-Ukrainian geo-political crisis the θ of the RTS index reached a record level of 1.34, implying that during this year, investors who would optimally deviate from the Russian market index could generate 134% of return more per year, while also investing in an SD efficient portfolio. Importantly, however, one cannot identify any particularly trend that would suggest that market index SD efficiency improves over time. The degree of market SD inefficiency seems to be rather persistent, or even marginally increasing during the recent years.

[Figure 3 around here]

We next consider the SD efficiency of the market indices from the point of view of a long-term investor. This investor does not re-balance their portfolio each year, but instead has been holding the market indices for decades. We, thus, test for the SD efficiency of all the indices across the complete sample of monthly returns. We find that all but the S&P 100 and Nikkei 225 indices are not efficient at order 2 across their entire life, indicating, that most of market indicted across the globe have not been a good investment for any risk-averse individual.

5.2 Pairwise comparison with sub-indices

The precious sub-section discussed the *ex-post* analysis of SD efficiency of market indices. It provides insights into the maximum potential gains of deviating from an inefficient market portfolio to an efficient one. However, the relevant practical question remains of whether these gains can be realized by choosing portfolios *ex-ante*. This is especially challenging given that the optimal (in SD sense) portfolio weights are not persistent. Thus in this section we discuss a simple, but potentially more robust analysis – a pairwise comparison of the performance of each index and industry sub-indices. Again, we first present the case of the S&P 100 index in detail and then we summarize the results for the global indices.

5.2.1 Worked example: the S&P 100 index

Table 6 reports the results of the pairwise comparison between the S&P 100 index and its sub-indices for each of the years from 1990 to 2015. The numbers in Panel A indicate the order of dominance of the sub-index (reported in columns) over the S&P 100 index in a given year (reported in rows). The last column of the table reports the minimum order of stochastic dominance of any sub-index over the index. The last row of the table reports the percentage of years in which a given sub-index dominates the index.

Notably, there are several industries which never dominate the index. These include Oil and gas, Basic material, Financial and Technology sectors. At the same time, other sectors, such as Consumer goods, Consumer services and Health care (industries often described as countercyclical) often dominate the index. Consumer goods, for example, dominate the S&P 100 index in 42% of years. Generally, in 58% of years one can find a sub-index that dominates the index at order 3 or higher. This implies that all risk averse (and prudent) investors can still increase their utility functions by investing in sub-indices rather than in the index. Technically, this process is simpler and less costly than the frequent portfolio rebalancing of optimal portfolios implied by Table 3. Another emerging pattern is that the index efficiency decreases over the course of time, and the dominance by sub-indices is clustered after 2005. In particular, all risk averse investors would increase their utility function by investing in the Consumer goods sub-index after the year 2000, instead of tracking the diversified S&P 100 index.

[Table 6 around here]

We also conduct a reverse test to check if the index dominates its sub-indices. As reported in panel B of Table 6, the Consumer Goods sub-index is dominated by the S&P 100 index only in 31% of years. Thus, all risk averse (and prudent) investors would be better off investing only in Consumer goods, rather that in fully diversified portfolio in

42% of cases. They would be better off by sticking to the diversified index in 31% of cases, and in the remaining 17% of cases these portfolios lie in the same dominance class, implying that different types of investors can prefer one of them over another depending on the exact shape of their utility function.

5.2.2 Global indices results

Across the globe, the results for the diversified equity indices are also not extremely favorable (Panel A of Table 7). For most of the indices, it is possible to find a dominating sub-index in over 50% of years. Surprisingly, two relatively efficient indices are the Indian BSE SENSEX and Korean KOSPI 50 indices, which are dominated in 27% and 31% of years respectively. On the other extreme are the Russian RTS and Italian FTSE MIB indices that are always dominated by at least one industry sub-index.[10]

[Table 7 around here]

In terms of the dominating industries, there is considerable cross-country variation. For example, Consumer goods sub-index is often dominating in different markets. However, it never dominates the German DAX, Chinese SSE 50, Canadian TSX 60, and Korean KOSPI 50 indices.

Oil and gas relatively rarely dominates diversified indices, with the exception of Russian RTS, for which risk-averse investors would be better off investing just in oil and gas in 75% of years in our sample, which manifests a strong dependence of the Russian economy on oil and gas exports.

Remarkably, despite the booming financial industry before the crisis of 2007-2008, the Financial sector sub-index very rarely dominates diversified indices, and never does so for developed markets. It is also dominated by the diversified indices in most of cases as reported in Panel B of Table 7. These results suggest that only investing in the Financial sector has been an inferior strategy for any risk-averse investor even before the

[10]The results discussed here cover SD at orders 4, 3, and 2. In Appendix B we tabulate the results for SD orders 3 and 2 separately. The interpretation of the results does not change, since in majority of cases the indices are dominated at least at the 3rd order, and 4th order SD can be detected only in a handful of cases.

financial crisis. The only exception is the Chinese SSE 50 index, which is dominated by the Financial sub-index in 71% of cases and dominates it only in 14% of cases.

As far as the time variation in index efficiency is concerned, Figure 4 plots the share of the diversified indices that are dominated by at least one sub-index during each year starting from 2003. We choose the year 2003 as the starting point, as this is the first year in our sample that covers more than 10 indices across the world. The figure clearly reveals that in the rump up to the financial crises of 2007-2008, more indices have become SD inefficient, making them an unsuitable investment for risk-averse investors.

[Table 4 around here]

5.3 Determinants of dominating industries

In this section we take a closer look at the drivers of stochastic dominance, and check if it is possible to forecast if a market index will be stochastically dominated by any of the sector sub-indices.

SD inefficiency of market indices. We estimate a logit model for the probability that a sub-index dominates its parent equity index in a given year, and relate it to the past index and sub-index performance, volatility, as well as several key macroeconomic indicators, which potentially can explain SD inefficiency of equity markets.

As macroeconomic factors, we choose a wide range of indicators, including the GDP annual growth rate, annual consumer prices index, total unemployment rate, gross domestic savings, current account balance, and real effective exchange rate change. We obtain the data from the World Bank database. Apart from macroeconomic variable, we use several financial indicators, such as representative government bonds yield, and a central bank policy rate or main interest rate. These data are obtained from the IMF database and Datastream.

We also include several index and sub-index specific characteristics, such as index and sub-index annualized mean return and volatility over the previous year, and the volatility ratio defined as sector volatility over the index volatility, which measures a relative riskiness of the sub-index in normalized terms. We control for market liquidity and for each index we compute the Amihud (2002) illiquidity measure. Some indices, however, lack the trading volume data needed to calculate the measure. For example, Euro Stoxx 50 lacks volume data before 2005. Also, Argentina's government bonds yield data are unavailable before 2006. In our regression analysis we thus omit those index-years, for which we cannot construct all the required factors.

In addition to the aforementioned explanatory variables, we use index and sub-index fixed effects, and a dummy variable that takes a value of 1 if a given sub-index dominated the index during the previous year (or during any previous year) and zero otherwise.

There is substantial literature suggesting that there exist structural differences between advanced and developing economies, that impact firm productivity and, as a consequence, stock performance. These include, among others, differences in the infrastructure, regulations, human capital, financial constraints, adopted managerial practices and skills (Bloom et al. 2010, Bruhn et al. 2010). Thus, we split all the market indices under study into two groups – advanced and developing markets – and estimate logit models for them separately. The first group includes the market indices from the U.S., UK, Euro-zone, and Japan; all other countries are included in the second group. The results are reported in Table 8.

[Table 8 around here]

The model in general has a relatively food fit, with the Efron's pseudo R-squared being around 28–36%. The key significant variables are index and sub-index return volatilities, and the ration of the volatilities. More volatile indices are likely to be dominated by less volatile sub-indices in absolute and relative terms. The retaliations between index and sub-indices volatilities and the probability of future dominance over market indices are generally consistent for different types of economies, with individual index and sub-index measures having stronger statistical support for advanced economies, and the ratio of volatilities being statistically significant for the developing markets. Also, a market index is more likely to be dominated by a sub-index, which has been dominating in the past. A dummy for past dominance of a given sub-index over the market index is positive and highly significant for both groups of countries.

The results related to the macroeconomic factors, however, suggest that the actual information content of the aggregate economic indicator is quite different across developed and developing economies.

Three significant predictors of future dominance of the market index for the advanced economies are GDP growth, inflation, and current account balance. Higher values of these indicators reduce the probability that the market index will be dominated by any sector sub-indices. A higher GDP growth rate reflects overall growth of the economy; inflation usually increases on up marketers and goes hand in hand with higher growth; and higher current account balance reflects higher levels of export from the advanced economies. Overall, the results suggest that improving economic conditions in developed countries make diversified market indices more attractive options for risk averse and prudent investors.

For the developing countries, all these determinant change their signs. To begin with, the GDP growth rate and the current account balance are positively related to the probability of the market index to be dominated by sector sub-indices. A likely reason for such a sign flip is that the developed economies are more "homogeneous". A higher GDP growth rate reflects a balanced growth of all areas of a developed economy. In the developing markets, the growth is often driven by just a few key sectors, which also contribute to higher exports and higher resulting current account balance, and it does not translate into overall improved performance of other sectors (Koren and Tenreyro 2007). This makes the aggregate market index SD dominated by the fast growing, GDP driving industries, and less suitable for risk-averse investors. Also, the link between inflation and economic growth does not seem to be pronounced in the developing markets. Consequently, inflation is not a statistically significant predictor for the probability of the market index to be dominated by sector sub-indices.[11]

[11] As a robustness check we re-estimated the pooled logit regression jointly for all economies. Results reported in Table A.3 in Appendix C are consistent with the ones discussed in this section for return volatilities and past dominance, but none of the marco-factors are significant, due to the discussed differential impact on developed and developing markets.

6 An out-of-sample trading strategy

6.1 Market specific index-based trading strategy

So far we have established that diversified stock indices across the globe are often not SD efficient; past dominance of a sub-index over the corresponding market index is a strong and consistent predictor of future dominance. In this section we propose a genuinely simple trading rule that uses this past stochastic dominance information, and check if such a strategy allows us to outperform the indices. The trading rule is index specific and only relies on those sub-indices that are available for the index under study.

We conduct the analysis for advanced economies only, due to several major reasons. First and foremost, the developed market indices have longer histories. Thus, we have a sufficient number of training years during which we evaluate the indices from the SD perspective and a sufficient number of remaining years to perform out-of-sample performance assessments. Second, indices from the developed economies show a much better industrial coverage, and usually all 10 sub-indices are available for all years. Only the Utilities sub-index is absent in the DJIA 30 index due to historical reasons, and the Oil and gas sub-index is absent from the DAX 30 index. The long history, industry coverage, and continuity of the coverage across years are often missing for emerging economies, making forecasts difficult and at times pointless. Last but not least, stock indices in advanced economies are free from most of investment barriers, they are relatively easily investible, and they are also much more systemically important, in the sense of size, financial integration and worldwide influence. Therefore, we choose the S&P 100 index and DJIA 30 index of the U.S., the FTSE 100 index of the U.K., the CAC 40 index of France, the DAX 30 index of Germany, the Euro Stoxx 50 index of the Euro area, and the Nikkei 225 index of Japan for this exercise. The chosen economies cover on average 75% of the global GDP over the period 2009 to 2015 and around 81% of total world stock market capitalization, according to the Word Bank database, thus attracting most of business attention and investments around the world.

For each index, its sample period is divided into two parts. In the training sample (which we vary from 3 to 7 years), we record every sub-index that has dominated the respective index at any order of SD up to fourth. Then, for the first year of the prediction sub-sample, we choose those sub-indices that dominated the index at least twice in the past and construct an equally weighted portfolio from these dominating indices. We hold this portfolio for one year. Next, we roll over the training sample by one year forward and repeat the analysis. If we cannot find any sub-index that dominated at least twice the index in the past, the portfolio is 100% invested in the index itself for the next year. In fact, in our analysis it happened only once with the DJIA 30 index for the year of 2010.

Table 9 reports the annualized means and standard deviations of daily returns for total indices and our past-SD based portfolio strategy, and the corresponding Sharpe ratios. Table A.4 in the Appendix D reports more detailed results with first four moments of the return distribution for each available year and each index.

[Table 9 around here]

For all of the indices and estimation windows, the annualized volatility for the SD strategy is lower than the index volatility, implying a lower investment risk. The average returns are also often improved. For the American DJIA 30, British FTSE 100, and German DAX 30 indices, the SD strategies deliver consistently higher mean return than those of the corresponding indices for all estimation horizons. For the S&P 100 index the strategy performs best with a short estimation horizon of 3 years, and for the EURO STOXX 50 the 5-year estimation horizon is optimal.

The only notable exception is the Japanese Nikkei 225 index, for which our past-SD based strategy always fail to deliver higher or even comparable mean return. The detailed results of Table A.4 based on a 5-year estimation horizon reveal that this pattern is also consistent across years. In most of the years with some rare exceptions, SD-based strategies deliver higher returns with lower volatilities for the U.S., U.K, and continental Europe, but fail to do so for Japan. The reason for such a poor performance for Japan, seems to be the fact that the past stochastic dominance pattern is not quite consistent over the sample. Also, the past SD dominance of sub-indices over indices in Japan is mostly at order 3, and its effect on return improvement and risk reduction is not as remarkable as for lower order SD.

6.2 Global index-based trading strategy

We now make a step forward and consider if our simple past-SD allocation rule can improve the performance of a global equity portfolio. Specifically, we construct the index of indices using the developed economies' diversified equity indices, namely, S&P 100, DJIA 30, FTSE 100, CAC 40, DAX 30, Euro Stoxx 50 and Nikkei 225. We consider three weighting schemes while constructing the global index of indices: equally-weighted, GDP-weighted, and stock market capitalization weighted. Relevant data are from World Band database, Federal Reserve St. Louis, and knoema.com. We next apply our past-SD based rule and invest in those markets whose indices dominated the global index at least twice in the past. The allocation is rebalanced every year.

The results reported in Table 10 reveal that for our SD-based strategy the average return increases by about 2 percentage point annualized and return standard deviation declines by about 3 percentage point. These results are consistent across all three waiting schemes and hold for most of the years. Even during the pick of the financial crisis, the SD-based strategy helps to mitigate the losses of the global equity portfolio. In 2008, the equally weighted global index lost 38% of its value, and its GDP and market cap weighted counterparts lost 42% of their values. The SD-base alternatives limited the losses to about 25% while also decreasing return standard deviation by 8-9% annualized during this year.

[Table 10 around here]

6.3 Market specific individual stock-based trading strategy

Apart from past dominance of sub-index over the index, the most significant and consistent predictors of stochastic dominance are index and subindex volatilities and their ratio. We use this intuition and construct portfolios of individual stocks sorting them based on historical market beta. The market beta of each stock is proportional to the ratio the stock return volatility to the market return volatility. The results from Section 5.3 suggest that stocks having lower market beta are likely to stochastically dominate the index over the following year.

To implement this strategy, for each stock for each year end we download its historical beta from Datastream (code 897E). The beta is computed based on the previous five years of monthly data using a linear regression of the logarithmic adjusted returns onto the returns of the corresponding market index. For each sample year, we sort individual stocks within each index according to their previous year-end values of the market beta. The stocks are then sorted into three portfolios: a low beta portfolio, comprising 30% of stocks with the lowest betas, a medium beta portfolio, containing 40% of stocks with the medium beta, and high beta portfolio, that includes 30% of stocks with the highest historical beta. For each of the portoflios, the returns are calculated using the same methodology as that of the benchmark market index. That is, the portfolio returns market capitalization weighted average or price weighted average on the components, depending on the calculation methodology of the corresponding index. We then asses the SD relationship between these portfolios and the market index over the following year using the majorization theorem.

Table 11 reports the average descriptive statistics of the market betas for all indices. The average mean and median betas are all positive and close to 1 for all indices. The 30% and 70% thresholds used to construct portfolios are smaller and larger than 1, respectively. For example, for the S&P 100 index these thresholds are 0.81 and 1.23.

[Table 11 around here]

The results reported in Table 12 indicate that low-beta portfolios perform extremely well in terms of stochastic dominance over the corresponding market index. Almost always they dominate the corresponding market index in way more that 50% of years.

For example, the low-beta portfolio dominates the S&P 100 index in 73% of years.[12] There are only few exceptions when low-beta portfolios dominate the index in less than 50% of years. These include Argentinean MERVAL (21%), Italian MIB (17%), and South Korean KOSPI 50 (38%).

[Table 12 around here]

Looking at the actual performance of the low-beta portfolios (Table 13), we see the low-beta portfolio reduces out-of-sample volatility on all markets, but it often comes at a cost of also reducing the mean. The resulting Sharpe ratio depends on the country of interest. The low-beta portfolio works quite good in Europe, and less so in the U.S.

We repeat the analysis sorting portfolios based on historical total return volatility (as opposed to market beta). The historical volatility is estimated as a standard deviation of the returns over the past five years (Datastream code 400E). The results tabulated in Appendix E are qualitatively similar to the ones based on beta sorts, but are at times slightly weaker.[13]

Our stochastic dominance results complement a body of literature on exceptionally good performance of low-beta and low-volatility stocks (Blitz and van Vliet 2007, Ang et al. 2009, Baker et al. 2011, Frazzini and Pedersen 2014, Asness et al. 2014). We show that portfolios of these stocks not only perform well in the traditional mean-variance scenes, but often stochastically dominate the diversified equity market indices across the globe. Thus, these portfolios are more suitable for risk-averse and prudent investors as opposed to the market indices.

[12] Over the past decade, the index was dominated by the low-beta portfolio in all years, except of 2013, at orders 3 or 2.

[13] There are potentially other fundamental factors, which could be related to stochastic dominance. Fama and French (1992), Fama and French (1993), McLean and Pontiff (2016), and Yan and Zheng (2017) provide a systematic overview on the predictability of various indicators, including value signals, of stock market returns. Following these research and given the data coverage we select additional fundamental indicators to perform portfolio sorts. They include 12 Month forward earnings per share (FEPS, Datastream I/B/E/S), 12 Month Forward Price/Earnings Ratio (Datastream I/B/E/S), and last available earnings per share (EPS). None of these indicators provides a strong signal for out-of-sample dominance. The dominance of portfolios sorted based on these fundamentals over the index happens rather rarely and randomly.

7 Conclusion

In this paper we extend the approach of Kuosmanen (2004) for testing for stochastic dominance efficiency of a given portfolio with respect to a set of underlying assets. The extended approach allows us to test for dominance efficiency of higher orders than two, and, similar to the original paper, to obtain the optimal weights for an efficient portfolio in case the test portfolio is proved to be inefficient.

We apply this approach to 17 stock market indices covering developed and developing markets across the globe, and find that in majority of years these indices are inefficient at least at order 3, and often at order 2. Thus, all prudent and most of risk averse investors should optimally deviate from the equity indices, investing instead in portfolios that overweight individual industries. The average mean return improvement that could be achieved by investing in an SD efficient portfolio is 23% annualized. Such a high return improvement is hard to achieve in practice, as this result stems from the in-sample optimization and knowledge of the realized return distribution. At the same time, the magnitude of the potential improvement suggests that even moderate deviations from the well diversified indices towards the optimal portfolio can result is substantial gains for investors.

Then, we conduct pairwise comparisons of the market equity indices with their sector sub-indices. Since here we use the ex-ante industry classification, this strategy is practically implementable. We find at on average in 67% of years not only the indices are inefficient but they are dominated by at least one sub-index. The percentage of dominated indices is especially high during the years 2008 – 2012. On the aggregate level, counter-cyclical industries such as Consumer Foods and Services, Health Care and Telecommunication are more likely to dominate their diversified equity indices. At the same time, the types of industries that are likely to dominate vary across the countries. For example, the Oil and Gas industry often dominates the Russian RTS index but not the other indices and the Financial sector often dominates the Chinese SSE 50 index but almost never the other equity markets.

Further, we estimate a Logit model for the determinants of the probability of a sub-index to dominate its index. Remarkably, macro factors contain different information with respect to future dominance for developed and developing markets. For more homogeneous and balanced economies, aggregate indicators of growth (such as GDP growth rate, inflation, and the current account balance) predict lower likelihood of the market index to be dominated. However, for the developing economies, which often rely on just one or several key industries, such aggregate indicators predict a higher likelihood that the market index will be dominated. The most significant and consistent predictors o dominance are index and sub-index volatilities, with the former being positively related to the probability of a sub-index to dominate the index, and the latter being negatively related to the probability, as well as the ratio of volatilities. Also, past dominance of a sector sub-index over the market index predicts higher likelihood for the future dominance.

Given that past stochastic dominance is a strong predictor of the future dominance of a sub-index over the index, we further suggest a simple trading rule based on the information on past dominance, that invests only in those sub-indices that dominate the index at least twice during a given number of previous years. Applying this strategy to the developed markets, we find that the rule results in consistent mean return improvement and volatility reduction in the U.S., the U.K., and Europe, but does not perform that well in Japan. Applying this strategy to a global portfolio results in about 1–2% annualized return improvement and 2–3% decline in the annualized standard deviation. Such improvements in the return distribution are consistent across time. Our past SD based approach also substantially limits the losses during market downturns like the financial crisis of 2007–2008.

Last but not least, we sort individual stocks into tercile portfolios based on their market betas and volatility and show that low-beta and low-volatility portfolios stochastically dominate the market indices in majority of years at order 3 and often at order 2 across most of world economies considered. These results contribute to the discission of a stellar performance of low-beta and low-risk stocks in the mean-variance sense, and suggest that these portfolios are likely to be preferred by risk-averse and prudent investors over diversified market indices.

Overall, our findings suggest that diversified equity indices across the globe are not SD efficient. Risk averse and prudent investors could benefit from switching between different industry sub-indices, by taking positions in those industries that were dominating in the past, or by investing in low-beta stocks. The sector-based strategies can rely on trading ETFs at low frequency, re-balancing portfolios once every year, thus, delivering improved return distributions with low transaction costs, which can be rather appealing for regulated long-term investors such as pension funds or insurance companies.

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8 Tables

Table 1: Equity indices

The table lists seventeen equity indices used in our study together with their country of origin. The source of the data is Datastream. "Start year" indicates the first full year when the index is available in our sample. The sample ends on December 31, 2015.

	Ticker	Short name	Country	Start year
S&P 100 Index	OEX	S&P 100	U.S.	1990
Dow Jones Industrial Average	DJI	DJIA	U.S.	2004
FTSE 100 Index	UKX	FTSE 100	U.K.	1996
CAC 40	PX1	CAC	France	2001
DAX Performance Index	DAX	DAX	Germany	2001
EURO STOXX 50	SX5E	Euro Stoxx 50	Eurozone	2001
Nikkei 225	NI225	Nikkei 225	Japan	2002
Andice Bovespa	IBOV	Indice Bovespa	Brazil	2007
RTS Index	RTSI	RTS Index	Russia	2008
S&P BSE SENSEX	SENSEX	BSE SENSEX	India	2005
Shanghai Stock Exchange 50 A Share Index	000016	SSE 50	China	2009
FTSE/JSE Top 40 Index	JTOPI	FTSE/JSE Top 40	South Africa	2003
S&P/ASX 50	AS31	S&P/ASX 50	Australia	2001
MERVAL Index	MERVAL	MERVAL Index	Argentina	2002
S&P/TSX 60	SPTSX60	S&P/TSX 60	Canada	2003
FTSE MIB	FTSEMIB	FTSE MIB	Italy	2010
KOSPI 50 Index	KOSPI50	KOSPI 50	South Korea	2003

Table 2: Equity indices: Descriptive statistics

The table reports the descriptive statistics of the daily returns of the seventeen equity indices used in our study. For each index, the sample starts when the complete data for index components becomes available and ends on December 31, 2015. The mean, standard deviation (Std), minimum (Min) return, maximum (Max) return, and Sharpe ratio are annualised, whereas skewness and kurtosis based on the original daily returns.

	Mean	Std	Median	Min	Max	Skewness	Kurtosis	Sharpe ratio
S&P 100	0.09	0.18	0.08	-23.97	27.81	-0.17	11.35	0.31
DJIA	0.07	0.18	0.09	-21.40	27.43	-0.07	14.64	0.31
FTSE 100	0.06	0.19	0.06	-24.18	24.49	-0.16	8.98	0.13
CAC	0.02	0.24	0.05	-24.72	27.65	0.03	8.09	-0.18
DAX	0.03	0.25	0.13	-23.16	28.18	-0.02	7.63	-0.10
Euro Stoxx 50	0.01	0.24	0.00	-21.37	27.24	0.01	7.57	-0.04
Nikkei 225	0.05	0.24	0.00	-31.61	34.54	-0.48	10.64	0.11
Indice Bovespa	0.00	0.29	0.00	-31.57	35.70	0.02	9.49	-0.40
RTS Index	-0.11	0.39	0.00	-55.33	52.73	-0.30	13.77	-0.44
BSE SENSEX	0.14	0.24	0.03	-30.29	41.73	0.08	11.88	0.36
SSE 50	0.10	0.27	0.00	-25.71	19.70	-0.34	7.16	0.31
FTSE/JSE Top 40	0.16	0.21	0.11	-20.46	20.22	-0.10	6.73	0.36
S&P/ASX 50	0.08	0.17	0.07	-22.52	15.84	-0.36	8.73	0.16
MERVAL Index	0.26	0.33	0.06	-33.80	32.88	-0.35	7.37	0.53
S&P/TSX 60	0.08	0.18	0.17	-26.90	25.65	-0.68	15.21	0.37
FTSE MIB	0.02	0.26	0.00	-18.39	27.89	-0.10	5.47	-0.04
KOSPI 50	0.09	0.23	0.00	-28.38	30.54	-0.30	8.88	0.28

Table 3: (In-)Efficiency of the S&P 100 index

The table reports the results of the SD efficiency tests for the S&P 100 index. The first row reports the lowest order of index inefficiency for each year from 1990 to 2015. The middle part of the table report the optimal weights of sub-indices in each year, and the descriptive statistics of the index returns and that of the optimal portfolio. The last row or the table reports the difference in means between the optimal portfolio and the index. The mean return, return standard deviation (Std), and Sharpe ratio are annualised, whereas skewness and kurtosis are based on the original daily returns.

SP100 Domi Weights	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	Average
Inefficiency order	2	2	2	2	3	2	2	2	2	2	3	2	2	2	2	3	2	2	2	2	2	2	2	2	2	2	2
S&P 100	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Oil and gas	0.15	0.00	0.07	0.00	0.00	0.18	0.27	0.00	0.00	0.14	0.00	0.00	0.00	0.00	0.08	1.00	0.24	0.46	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.10
Basic materials	0.00	0.00	0.00	0.00	0.13	0.00	0.20	0.00	0.00	0.23	0.00	0.39	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.32	0.00	0.00	0.00	0.00	0.00	0.05
Industrials	0.00	0.00	0.00	0.71	0.00	0.16	0.00	0.00	0.00	0.04	0.00	0.00	0.00	0.50	0.44	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.41	0.00	0.00	0.09
Consumer goods	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.15	0.62	0.00	0.08	0.00	0.30	0.26	0.00	0.03	0.67	0.00	0.00	0.00	0.00	0.00	0.08
Health care	0.54	0.77	0.00	0.00	0.00	0.36	0.08	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.28	0.04	0.05	0.00	0.00	0.00	0.21	0.06	0.00	0.09
Consumer Services	0.00	0.13	0.00	0.00	0.00	0.00	0.00	0.21	0.00	0.26	0.00	0.00	0.00	0.00	0.03	0.00	0.00	0.00	0.96	0.00	0.00	0.00	0.74	0.28	0.00	0.83	0.13
Telecommunications	0.00	0.00	0.50	0.00	0.00	0.00	0.00	0.58	0.19	0.00	0.00	0.46	0.00	0.00	0.00	0.00	0.35	0.00	0.00	0.00	0.01	0.00	0.00	0.04	0.00	0.17	0.09
Utilities	0.00	0.10	0.08	0.29	0.00	0.06	0.00	0.00	0.24	0.00	1.00	0.00	0.00	0.24	0.25	0.00	0.11	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.62	0.00	0.15
Financials	0.00	0.00	0.34	0.00	0.00	0.20	0.26	0.21	0.00	0.00	0.00	0.00	0.38	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.26	0.00	0.00	0.00	0.07
Technology	0.31	0.00	0.00	0.00	0.87	0.05	0.19	0.00	0.58	0.33	0.00	0.00	0.00	0.22	0.12	0.00	0.00	0.00	0.00	0.92	0.00	0.00	0.00	0.05	0.32	0.00	0.15
Index Mean	-0.04	0.25	0.06	0.11	0.03	0.34	0.23	0.26	0.29	0.28	-0.13	-0.15	-0.26	0.23	0.06	0.01	0.17	0.06	-0.43	0.20	0.12	0.03	0.15	0.27	0.12	0.03	0.09
Index Std	0.17	0.17	0.10	0.09	0.10	0.08	0.12	0.19	0.21	0.19	0.24	0.23	0.27	0.18	0.11	0.10	0.09	0.16	0.40	0.26	0.17	0.22	0.12	0.11	0.11	0.16	0.18
Index skewness	-0.16	0.52	0.04	0.00	-0.18	0.02	-0.59	-0.48	-0.69	-0.02	0.00	0.03	0.43	0.01	-0.08	0.03	0.02	-0.49	0.01	0.03	-0.23	-0.52	0.10	-0.24	-0.38	-0.16	-0.17
Index kurtosis	3.87	11.89	3.20	5.10	4.26	4.27	4.73	8.98	8.36	2.85	4.20	4.96	3.80	3.91	2.97	3.12	4.41	4.80	6.85	5.46	5.12	5.93	4.08	4.69	4.54	5.24	11.35
Sharpe ratio	-0.71	1.11	0.22	0.85	-0.17	3.43	1.42	1.10	1.14	1.24	-0.82	-0.79	-1.00	1.27	0.44	-0.20	1.29	0.10	-1.12	0.78	0.68	0.14	1.19	2.48	1.07	0.16	0.02
Opt. Port mean	0.08	0.41	0.19	0.19	0.15	0.40	0.28	0.34	0.41	0.35	0.47	-0.06	-0.12	0.28	0.15	0.17	0.22	0.16	-0.11	0.44	0.20	0.16	0.24	0.30	0.25	0.11	0.22
Opt. Port Std	0.17	0.17	0.10	0.09	0.19	0.08	0.13	0.18	0.21	0.19	0.29	0.23	0.22	0.17	0.11	0.23	0.10	0.15	0.35	0.26	0.18	0.15	0.13	0.11	0.11	0.15	0.18
Opt. Port skewness	-0.21	-0.12	0.12	-0.19	0.23	-0.04	-0.39	-0.72	-0.41	-0.06	-0.13	-0.03	0.30	0.03	-0.04	-0.36	0.00	-0.67	0.35	0.11	-0.08	-0.22	0.14	-0.21	0.08	-0.14	-0.03
Opt. Port kurtosis	3.48	4.90	2.83	3.39	5.43	3.99	3.76	7.39	5.68	2.77	3.25	4.29	4.35	3.58	3.11	3.38	3.68	4.16	8.21	5.31	4.79	6.05	3.52	4.43	3.19	4.99	8.82
Sharpe ratio	0.04	2.13	1.52	1.78	0.60	4.16	1.80	1.57	1.72	1.61	1.44	-0.42	-0.60	1.53	1.27	0.62	1.84	0.76	-0.37	1.69	1.11	1.05	1.92	2.85	2.16	0.73	0.06
Mean improvement	0.13	0.16	0.14	0.08	0.13	0.07	0.05	0.08	0.12	0.07	0.61	0.09	0.14	0.04	0.09	0.16	0.05	0.10	0.32	0.24	0.08	0.13	0.10	0.04	0.13	0.09	0.13
Std improvement	0.00	-0.01	0.00	0.00	0.09	0.00	0.00	-0.01	0.00	0.00	0.05	0.00	-0.05	0.00	0.00	0.13	0.00	0.00	-0.05	0.00	0.01	-0.07	0.00	0.00	0.00	-0.01	0.00
Skewness improvement	-0.05	-0.64	0.09	-0.19	0.40	-0.06	0.20	-0.24	0.28	-0.04	-0.13	-0.05	-0.13	0.02	0.04	-0.39	-0.02	-0.18	0.34	0.08	0.15	0.30	0.04	0.03	0.46	0.02	0.14
Kurtosis improvement	-0.38	-6.99	-0.37	-1.72	1.17	-0.28	-0.98	-1.59	-2.67	-0.08	-0.95	-0.67	0.55	-0.33	0.14	0.25	-0.73	-0.65	1.36	-0.15	-0.33	0.11	-0.56	-0.26	-1.35	-0.25	-2.53
Sharpe ratio improvement	0.75	1.02	1.30	0.93	0.76	0.74	0.39	0.47	0.59	0.37	2.26	0.37	0.39	0.26	0.84	0.82	0.55	0.66	0.75	0.91	0.44	0.92	0.73	0.37	1.09	0.57	0.04

Table 4: SD optimal portfolios

The table reports the results of the SD efficiency tests for 17 indices under study. The first row reports the minimum order of SD inefficiency of the index. Then, the average optimal portfolio weights and the descriptive statistics of the optimal portfolio returns are reported. The mean return, return standard deviation (Std), and Sharpe ratio are annualised, whereas skewness and kurtosis are based on the original daily returns. The last rows report the average improvement of the optimal portfolio over the index.

	S&P 100	DJIA	FTSE 100	CAC	DAX	Euro Stoxx 50	Nikkei 225	Indice Bovespa	RTS	BSE SENSEX	SSE 50	FTSE JSE Top 40	S&P/ ASX 50	MERVAL	S&P/ TSX 60	FTSE MIB	KOSPI 50	
Inefficiency order	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
								Optimal portfolio average weights										
Index	0.00	0.05	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.07	0.14	0.06	0.00	0.00	0.05	0.00	0.04	0.02
Oil and gas	0.10	0.11	0.01	0.06	-	0.08	0.14	0.04	0.22	0.05	0.00	-	0.07	0.07	0.11	0.17	0.15	0.09
Basic materials	0.05	0.02	0.11	0.10	0.18	0.09	0.04	0.03	0.13	0.01	0.00	0.04	0.15	0.24	0.04	0.00	0.08	0.08
Industrials	0.09	0.08	0.06	0.13	0.05	0.06	0.00	0.02	0.13	0.06	0.02	0.07	0.02	0.22	0.00	0.04	0.03	0.06
Consumer goods	0.08	0.02	0.23	0.19	0.10	0.08	0.02	0.22	0.10	0.16	0.27	0.28	0.10	0.19	0.14	0.39	0.23	0.16
Health care	0.09	0.25	0.14	0.15	0.35	0.20	0.11	0.00	0.03	0.23	0.00	0.19	0.19	0.00	0.07	0.21	-	0.14
Consumer Services	0.13	0.24	0.03	0.00	0.02	0.12	0.11	0.33	0.34	0.00	0.08	0.27	0.14	-	0.18	0.13	0.06	0.14
Telecommunications	0.09	0.08	0.09	0.13	0.06	0.15	0.20	0.21	0.03	0.04	0.29	0.10	0.12	0.17	0.17	0.12	0.18	0.13
Utilities	0.15	-	0.27	0.19	0.09	0.16	0.20	0.13	0.00	0.07	0.03	-	0.14	0.13	0.03	0.02	0.12	0.11
Financials	0.07	0.05	0.05	0.03	0.02	0.03	0.08	0.01	0.05	0.12	0.19	0.01	0.08	0.26	0.14	0.00	0.07	0.07
Technology	0.15	0.11	0.02	0.01	0.13	0.05	0.09	-	0.00	0.20	-	-	-	-	0.07	0.02	0.06	0.08
								Optimal portfolio descriptive statistics										
Opt. Port mean	0.22	0.17	0.20	0.21	0.22	0.16	0.21	0.24	0.34	0.31	0.27	0.33	0.25	0.51	0.25	0.26	0.26	0.26
Opt. Port Std	0.18	0.18	0.18	0.23	0.22	0.23	0.23	0.26	0.35	0.22	0.24	0.21	0.17	0.34	0.16	0.25	0.21	0.23
Opt. Port skewness	-0.03	0.12	-0.11	0.11	0.16	-0.02	-0.24	0.13	0.46	0.11	0.60	-0.08	0.88	0.57	0.02	-0.05	-0.14	0.15
Opt. Port kurtosis	8.82	10.75	7.10	6.60	8.96	6.95	8.65	7.77	23.20	9.32	9.44	6.22	24.51	13.36	6.49	4.45	5.33	9.88
Sharpe ratio	0.06	0.87	0.88	0.67	0.72	0.61	0.82	0.50	0.78	1.16	1.02	1.20	1.17	1.23	1.50	0.91	1.10	0.89
								Index portfolio descriptive statistics										
Index mean	0.09	0.07	0.06	0.02	0.03	0.01	0.05	0.00	-0.11	0.14	0.10	0.16	0.08	0.26	0.08	0.02	0.09	0.07
Index Std	0.18	0.18	0.19	0.24	0.25	0.24	0.24	0.29	0.39	0.24	0.27	0.21	0.17	0.33	0.18	0.26	0.23	0.24
Index skewness	-0.17	-0.07	-0.16	0.03	-0.02	0.01	-0.48	0.02	-0.30	0.08	-0.34	-0.10	-0.36	-0.35	-0.68	-0.10	-0.30	-0.19
Index kurtosis	11.35	14.64	8.98	8.09	7.63	7.57	10.64	9.49	13.77	11.88	7.16	6.73	8.73	7.37	15.21	5.47	8.88	9.62
Sharpe ratio	0.02	0.31	0.13	-0.20	-0.11	-0.04	0.12	-0.40	-0.44	0.36	0.31	0.36	0.17	0.53	0.37	-0.04	0.28	0.10
								Optimal portfolio improvement										
Mean	0.13	0.10	0.14	0.20	0.19	0.15	0.16	0.24	0.44	0.17	0.16	0.17	0.18	0.24	0.17	0.24	0.17	0.19
Std	0.00	0.00	-0.01	-0.01	-0.03	-0.02	-0.01	-0.03	-0.04	-0.02	-0.03	-0.01	0.01	0.01	-0.02	-0.01	-0.01	-0.01
Skewness	0.14	0.20	0.05	0.08	0.19	-0.03	0.24	0.12	0.76	0.03	0.94	0.02	1.24	0.92	0.70	0.05	0.16	0.34
Kurtosis	-2.53	-3.89	-1.87	-1.49	1.33	-0.62	-1.99	-1.71	9.43	-2.57	2.28	-0.51	15.78	5.99	-8.73	-1.02	-3.55	0.26
Sharpe ratio	0.04	0.57	0.75	0.86	0.83	0.65	0.70	0.90	1.22	0.80	0.71	0.84	1.00	0.70	1.13	0.96	0.81	0.79

Table 5: Time-series of orders of inefficiency of global indices

The tables reports the orders of inefficiency of each of the market indices during each of the years from 1990 to 2015. “-” indicates that the index is not available for this particular year in our sample.

	S&P 100	DJIA	FTSE 100	CAC	DAX	Euro Stoxx 50	Nikkei 225	Indice Bovespa	RTS	BSE SENSEX	SSE 50	FTSE JSE Top 40	S&P/ ASX 50	MERVAL	S&P/ TSX 60	FTSE MIB	KOSPI 50
1990	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
1991	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
1992	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
1993	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
1994	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
1995	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
1996	2	-	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-
1997	2	-	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-
1998	2	-	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-
1999	2	-	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2000	3	-	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2001	2	-	2	2	2	2	-	-	-	-	-	-	2	-	-	-	-
2002	2	-	2	2	2	2	2	-	-	-	-	-	2	2	-	-	-
2003	2	-	2	2	2	2	2	-	-	-	-	2	3	2	2	-	2
2004	2	2	2	2	2	2	2	-	-	-	-	2	2	2	2	-	2
2005	3	3	2	2	2	2	2	-	-	2	-	2	2	2	2	-	2
2006	2	2	2	2	2	2	2	-	-	2	-	2	2	2	2	-	2
2007	2	2	2	2	3	2	3	2	-	2	-	2	2	2	2	-	2
2008	2	2	2	2	2	2	2	2	2	2	-	2	2	2	2	-	2
2009	2	2	2	2	2	2	3	2	2	2	2	2	3	2	2	-	2
2010	2	2	2	3	2	2	2	2	2	2	2	2	3	2	2	2	2
2011	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
2012	2	2	2	2	2	2	2	2	2	2	3	2	2	2	2	2	2
2013	2	2	2	2	2	2	2	2	2	2	2	2	2	3	2	2	2
2014	2	2	3	2	2	2	2	2	2	2	3	2	2	2	2	2	2
2015	2	3	3	2	2	2	2	2	2	3	2	2	2	2	2	2	2

Table 6: Pairwise comparison of S&P 100 index and sub-indices

The table reports the results of the pairwise comparison of the S&P 100 index and its industry sub-indices for each year from 1990 to 2015. In Panel A, the numbers indicate the minimum order of dominance of the sub-index over S&P 100. The last row summarizes the percentage of years during which the index was dominated by a given sub-index. In Panel B, the numbers indicate the minimum order of dominance of the S&P 100 index over each sub-indices. The last row summarizes the percentage of years during which the index dominated a given sub-index.

	Oil &gas	Basic mat.	Ind.	Cons. goods	Health care	Cons.Services	Telecom.	Util.	Fin.	Tech.	Minimum SD order
					Panel A: Market index is dominated						
1990								2			2
1991								2			2
1992											
1993											
1994											
1995											
1996											
1997						3		3			3
1998								3			3
1999											
2000											
2001				2	3						2
2002											
2003				3				3			3
2004											
2005											
2006				3							3
2007			4	2	3						2
2008				2	2			2			2
2009				3	3	3		4			3
2010				2	3	2	2	3			2
2011				2	2	2	2	2			2
2012				3	2	2					2
2013											
2014				3							3
2015				2			3				2
Domi ratio	0	0	0.04	0.42	0.27	0.19	0.12	0.35	0	0	0.58
					Panel B: Market index is dominating						
1990		2	2	3	3	2	2		2	3	2
1991										2	2
1992	2	2	3	3	2	3	3	3	3	2	2
1993	2	2	4		2	2	3	3	3	2	2
1994	3	3		2	3	2	2	2	3	3	2
1995	2	2	3	2	3	2	2	2	3	3	2
1996	3		3	2	2	2	2	2	3	3	2
1997	3	2			2				3	2	2
1998				2	3	3				3	2
1999	2	3	2		2	3	2		2	3	2
2000		2		3		2	2		3	2	2
2001		3	3			3	3	2	3	2	2
2002	3	3	2				2	3	3	2	2
2003		3	3		2	2	2		3	3	2
2004	3	2	3		2	2	3	4	3	2	2
2005	3	2	3		2	2	2	3	3	2	2
2006	3	2	2		2		3	3	3	2	2
2007	3	2				3	3	3	2	3	2
2008	3	2							2		2
2009	2	3	2						2		2
2010	3	3	3						2	2	2
2011	3	2	2						2		2
2012	2	2	2				3		3	2	2
2013	2	3	3	2	3	3	2	2	3	2	2
2014	2	2	2		3	3	2	3	3	3	2
2015	2	2			3			2	2	3	2
Domi ratio	0.77	0.88	0.69	0.31	0.62	0.62	0.69	0.54	0.92	0.88	1

Table 7: Market indices dominated by/dominating sector sub-indices

Panel A of the table reports the fraction of years during which each of the 17 diversified equity indices is dominated by each of sector sub-indices at orders 4, 3 or 2. The last column reports the fraction of years during which the index is dominated by at least one sub-index. Panel B reports the fraction of years during which each of the 17 diversified equity indices dominated each of sector sub-indices at orders 4, 3 or 2. The last column reports the fraction of years during which the index dominates at least one sub-index.

	Oil&gas	Basic mat.	Ind.	Cons. goods	Health care	Cons. Services	Telecom.	Util.	Fin.	Tech.	Any
Panel A: Market indices are dominated											
S&P 100	0	0	0.04	0.42	0.27	0.19	0.12	0.35	0	0	0.58
DJIA	0	0	0	0.33	0.33	0.33	0.25	-	0	0	0.58
FTSE 100	0	0	0.1	0.55	0.2	0.3	0.05	0.45	0	0	0.75
CAC	0	0	0.07	0.4	0.13	0.33	0.27	0.07	0	0	0.67
DAX	-	0.07	0.07	0	0.33	0.07	0.07	0.2	0.07	0.07	0.53
Euro Stoxx 50	0.2	0	0	0.27	0.07	0.2	0.2	0.2	0	0.07	0.53
Nikkei 225	0.07	0.07	0	0.57	0.71	0.36	0.07	0.43	0	0	0.93
Indice Bovespa	0	0	0.22	0.89	0	0.33	0.33	0.56	0	-	0.89
RTS Index	0.75	0.38	0.13	0.25	0.13	0.5	0.75	0.13	0.13	0	1
BSE SENSEX	0	0	0	0.27	0	0	0	0	0	0	0.27
SSE 50	0.14	0	0.14	0	0	0	0.14	0.43	0.71	-	0.71
FTSE/JSE Top 40	-	0	0.31	0.38	0	0	0	-	0.31	-	0.69
S&P/ASX 50	0	0	0.13	0.2	0	0.4	0	0	0.13	-	0.6
MERVAL Index	0.21	0.36	0.07	0.14	-	-	0.07	0.07	0	-	0.50
S&P/TSX 60	0	0	0.08	0	0.08	0.77	0.23	0.08	0.38	0	0.92
FTSE MIB	0.5	0	0.83	0.83	0	0	0.17	0.83	0	0	1
KOSPI 50	0	0	0	0	-	0	0.23	0.15	0	0	0.31
Panel B: Market indices are dominating											
S&P 100	0.77	0.88	0.69	0.31	0.62	0.62	0.69	0.54	0.92	0.88	1.00
DJIA	1.00	0.92	0.83	0.33	0.50	0.33	0.67	-	0.92	0.75	1.00
FTSE 100	0.80	0.95	0.55	0.25	0.60	0.20	0.80	0.20	1.00	0.90	1.00
CAC	0.60	0.80	0.73	0.13	0.67	0.33	0.60	0.87	0.87	1.00	1.00
DAX	-	0.60	0.67	0.67	0.13	0.73	0.73	0.73	0.73	0.73	1.00
Euro Stoxx 50	0.53	0.73	0.87	0.27	0.73	0.33	0.40	0.47	1.00	0.73	1.00
Nikkei 225	0.71	0.57	0.79	0.00	0.07	0.14	0.57	0.36	0.79	0.57	1.00
Indice Bovespa	1.00	0.89	0.11	0.00	0.33	0.22	0.44	0.22	0.44	-	1.00
RTS Index	0.25	0.13	0.38	0.63	0.25	0.00	0.00	0.25	0.75	0.13	1.00
BSE SENSEX	1.00	0.91	0.73	0.27	0.73	0.09	0.91	0.73	0.91	0.73	1.00
SSE 50	0.14	0.86	0.71	0.14	0.00	0.71	0.43	0.14	0.14	-	0.86
FTSE/JSE Top 40	-	0.92	0.31	0.31	0.77	0.69	0.92	-	0.38	-	1.00
S&P/ASX 50	1.00	1.00	0.53	0.53	0.87	0.40	0.80	0.80	0.53	-	1.00
MERVAL Index	0.29	0.14	0.00	0.21	0.00	-	0.50	0.36	0.71	-	0.93
S&P/TSX 60	0.92	1.00	0.85	0.85	0.92	0.08	0.46	0.62	0.15	1.00	1.00
FTSE MIB	0.33	0.83	0.17	0.00	0.50	0.00	0.67	0.00	1.00	1.00	1.00
KOSPI 50	0.92	0.92	0.85	0.69	-	0.62	0.46	0.54	0.69	0.92	1.00

Table 8: Stochastic dominance determinants: advanced vs. developing economies

The table reports the estimation results for the Logit model for the probability of each sector sub-index to dominate its respective diversified equity index. The model is estimated for the advanced economies (the U.S., the U.K., Eurozone, and Japan) and the rest of the regions separately.

Panel A: Advanced economies								
	Coef.	t-stat.	Coef.	t-stat.	Coef.	t-stat.	Coef.	t-stat.
Intercept			3.03	1.48	2.56	1.19	0.67	0.31
GDP growth	-0.14*	-1.86	-0.14**	-2.05	-0.12*	-1.71	-0.12*	-1.72
Inflation	-0.45***	-3.66	-0.42***	-3.61	-0.39***	-3.12	-0.39***	-3.12
Unemployment	0.00	0.07	-0.01	-0.14	-0.01	-0.13	0.00	-0.03
Gross savings	-0.04	-0.77	-0.02	-0.36	-0.02	-0.43	-0.01	-0.25
Current account balance	-0.11*	-1.67	-0.15**	-2.17	-0.16**	-2.26	-0.16**	-2.25
Real effective exchange	-0.02	-0.87	-0.01	-0.54	0.00	-0.19	0.00	-0.10
Government bonds yield	-0.04	-0.26	-0.07	-0.45	-0.10	-0.59	-0.05	-0.28
Central bank policy rate	0.10	0.79	0.13	1.03	0.12	0.94	0.12	0.91
Liquidity	-2.97	-0.32	-2.83	-0.32	-5.31	-0.59	-1.94	-0.21
Index return	-1.04	-1.07	-1.05	-1.11	-0.70	-0.71	-0.56	-0.56
Index volatility	18.32**	2.17	21.78**	2.48	24.13***	2.67	23.68***	2.68
Sector return	0.26	0.33	0.29	0.36	0.30	0.38	0.25	0.31
Sector volatility	-19.91**	-2.46	-24.81***	-2.93	-26.18***	-3.00	-24.84***	-2.93
Volatility ratio	-1.30	-0.84	-2.31	-1.42	-1.97	-1.16	-1.17	-0.72
D Dominance, last year					-0.07	-0.24		
D Dominance, any year							0.67**	2.40
Sector dummies	yes		no		no		no	
Efron's pseudo R-sq	0.33		0.29		0.28		0.29	
N obs	1025		1025		967		967	
Panel B: Developing economies								
Intercept			6.84***	3.43	6.30***	3.01	4.57**	2.17
GDP growth	0.13**	2.27	0.13**	2.29	0.11*	1.89	0.13**	2.10
Inflation	0.05	1.01	0.05	1.00	0.05	0.91	0.04	0.64
Unemployment	-0.05**	-2.03	-0.05**	-1.99	-0.05*	-1.90	-0.06**	-2.15
Gross savings	-0.11***	-3.85	-0.11***	-3.87	-0.10***	-3.52	-0.10***	-3.45
Current account balance	0.09*	1.89	0.09*	1.79	0.08*	1.71	0.09*	1.80
Real effective exchange	0.03	1.53	0.03	1.55	0.02	0.86	0.03	1.10
Government bonds yield	-0.06	-1.04	-0.06	-1.02	-0.05	-0.87	-0.05	-0.80
Central bank policy rate	-0.02	-0.25	-0.02	-0.26	0.00	-0.01	0.01	0.08
Liquidity	-3.63	-1.57	-3.48	-1.54	-3.42	-1.46	-4.69*	-1.96
Index return	0.92	1.52	0.79	1.33	0.63	0.96	0.78	1.17
Index volatility	11.91	1.50	12.07	1.55	10.37	1.30	11.39	1.44
Sector return	-0.73	-1.19	-0.51	-0.86	-0.57	-0.87	-0.68	-1.04
Sector volatility	-11.74	-1.52	-12.10	-1.58	-11.88	-1.47	-11.74	-1.47
Volatility ratio	-4.69***	-2.60	-4.87***	-2.73	-4.30**	-2.32	-3.35*	-1.83
D Dominance, last year					0.24	0.70		
D Dominance, any year							0.98***	3.03
Sector dummies	yes		no		no		no	
Efron's pseudo R-sq	0.36		0.34		0.33		0.34	
N obs	798		798		721		721	

Table 9: Average out-of-sample performance of past-SD based trading strategy

The table reports the average annualized means and return standard deviations (in % per year) and the corresponding annualized Sharpe ratios of seven equity indices of the developed economies (Index) as well as the descriptive statistics of the corresponding portfolios that include only those sector sub-indices that have dominated the index in the past at least twice (SD).

	Mean		Std	
	Index	SD	Index	SD
3-year estimation window				
S&P 100	8.71	9.84	17.05	15.70
DJIA 30	6.37	8.22	18.01	17.18
FTSE 100	3.72	5.84	18.19	16.77
CAC 40	5.76	4.47	21.04	20.33
DAX 30	8.33	12.08	20.56	19.99
EURO STOXX 50	5.30	4.89	21.12	20.72
NIKKEI 225	6.26	2.58	22.94	20.68
5-year estimation window				
S&P 100	8.91	6.75	17.77	16.66
DJIA 30	12.45	14.14	15.67	14.29
FTSE 100	3.54	6.83	18.15	15.22
CAC 40	3.47	3.06	22.73	21.90
DAX 30	6.88	13.29	21.86	21.16
EURO STOXX 50	3.14	4.22	22.83	22.26
NIKKEI 225	2.89	0.67	24.35	20.68
7-year estimation window				
S&P 100	6.88	3.97	18.55	16.66
DJIA 30	10.72	13.07	13.87	11.94
FTSE 100	7.18	11.11	17.15	13.98
CAC 40	1.45	1.57	24.44	23.33
DAX 30	3.60	10.84	23.45	21.33
EURO STOXX 50	0.52	-1.15	24.72	24.70
NIKKEI 225	12.75	6.04	22.16	19.31
All-past-years estimation window				
S&P 100	7.79	9.76	17.40	11.97
DJIA 30	11.12	13.09	8.45	4.90
FTSE 100	4.73	8.10	15.73	12.56
CAC 40	9.01	9.98	13.63	13.38
DAX 30	11.47	1.49	14.54	9.38
EURO STOXX 50	8.30	2.99	13.59	10.58
NIKKEI 225	12.75	6.04	19.98	21.13

Table 10: Index of indices out-of-sample performance

The table reports the annualized mean returns (in % per year) of global index-of-indices for each year starting from 2003 to 2015 and the corresponding mean returns of SD-based index. The index is based on seven indices of the developed economies: S&P 100, DJIA 30, FTSE 100, CAC 40, DAX 30, Euro Stoxx 50, and Nikkei 225. The index-of-indices is constructed using three different weighting schemes: equally weighted (Equal), weighted by the GDP of the economy it represents (GDP), and weighted by the total market capitalization the index represents (Cap). The last two rows report the average annualized mean and return standard deviation over the entire sample.

	Equal				GDP				Market Cap			
	Index		SD-based		Index		SD-based		Index		SD-based	
	Mean	St. dev.	Mean	St. dev.	Mean	St. dev.	Mean	St. dev.	Mean	St. dev.	Mean	St. dev.
2003	0.23	0.18	0.23	0.17	0.23	0.18	0.23	0.17	0.23	0.18	0.23	0.17
2004	0.06	0.11	0.14	0.10	0.06	0.11	0.14	0.10	0.06	0.11	0.14	0.10
2005	0.01	0.10	0.08	0.11	0.01	0.10	0.08	0.11	0.01	0.10	0.08	0.11
2006	0.15	0.10	0.19	0.08	0.16	0.09	0.18	0.08	0.16	0.09	0.18	0.08
2007	0.07	0.15	0.13	0.13	0.06	0.15	0.16	0.14	0.06	0.15	0.16	0.14
2008	-0.38	0.34	-0.25	0.26	-0.42	0.37	-0.25	0.28	-0.42	0.37	-0.25	0.29
2009	0.22	0.22	0.11	0.15	0.22	0.22	0.08	0.16	0.21	0.22	0.08	0.15
2010	0.07	0.17	0.02	0.13	0.07	0.16	0.03	0.13	0.09	0.16	0.06	0.13
2011	-0.08	0.21	-0.04	0.17	-0.04	0.21	0.01	0.16	0.00	0.20	0.06	0.16
2012	0.17	0.14	0.09	0.10	0.16	0.13	0.10	0.10	0.14	0.12	0.10	0.09
2013	0.26	0.11	0.20	0.10	0.26	0.10	0.20	0.10	0.26	0.10	0.20	0.09
2014	0.06	0.11	0.11	0.10	0.08	0.11	0.11	0.10	0.09	0.10	0.12	0.09
2015	0.06	0.16	0.05	0.15	0.04	0.15	0.05	0.14	0.03	0.15	0.06	0.13
Average	0.07	0.16	0.08	0.13	0.07	0.16	0.08	0.14	0.07	0.16	0.09	0.13

Table 11: Beta descriptive statistics

The table reports the time series averages of the descriptive statistics of individual stock market betas for the stocks that are constituents of the 17 market indices used in this paper. "Effective stocks" reports the average percentage of stocks in each index that are alive at the end of a calendar year and are used for portfolio construction over the following year.

	Effective Stocks	Mean	Median	Std	Min	Max	30% quantile	70% quantile
S&P 100	98%	1.06	1.03	0.50	0.05	2.75	0.81	1.23
DJIA	100%	1.01	0.94	0.50	0.18	2.21	0.71	1.19
FTSE 100	98%	1.06	0.98	0.61	-0.34	3.28	0.74	1.27
CAC	100%	1.12	1.06	0.54	0.22	2.58	0.83	1.34
DAX	100%	0.99	0.99	0.44	0.23	2.06	0.77	1.15
Euro Stoxx 50	97%	1.05	1.03	0.47	0.25	2.54	0.76	1.25
Nikkei 225	100%	1.10	1.08	0.52	-0.42	3.33	0.87	1.30
Indice Bovespa	99%	0.92	0.92	0.47	-0.02	2.32	0.64	1.12
RTS Index	99%	1.11	1.10	0.41	-0.19	2.07	0.94	1.30
BSE SENSEX	100%	0.96	0.91	0.37	0.36	1.72	0.75	1.11
SSE 50	98%	1.11	1.13	0.35	0.38	1.95	0.93	1.27
FTSE/JSE Top 40	100%	0.86	0.78	0.44	-0.13	1.86	0.63	1.02
S&P/ASX 50	99%	0.93	0.86	0.56	-0.09	2.67	0.63	1.10
MERVAL Index	94%	0.87	0.90	0.31	0.30	1.47	0.69	1.04
S&P/TSX 60	100%	0.89	0.78	0.66	-0.13	3.41	0.52	1.06
FTSE MIB	97%	0.96	0.95	0.43	-0.06	1.86	0.79	1.21
KOSPI 50	96%	1.05	1.07	0.46	0.04	2.13	0.83	1.29

Table 12: Market indices vs. beta-sorted portfolios

The table reports the fraction of years during which each of the 17 diversified equity indices is dominated by beta-sorted portfolios of individual stocks at orders 4, 3 or 2.

	Low-beta	Medium-beta	High-beta
S&P 100	0.73	0.08	0.00
DJIA	0.50	0.00	0.00
FTSE 100	0.70	0.05	0.00
CAC	0.67	0.20	0.00
DAX	0.60	0.00	0.00
Euro Stoxx 50	0.73	0.20	0.00
Nikkei 225	0.79	0.00	0.00
Indice Bovespa	0.89	0.22	0.00
RTS Index	0.75	0.63	0.13
BSE SENSEX	0.64	0.00	0.00
SSE 50	0.86	0.14	0.00
FTSE/JSE Top 40	0.77	0.38	0.00
S&P/ASX 50	0.67	0.13	0.00
MERVAL Index	0.21	0.21	0.07
S&P/TSX 60	0.77	0.38	0.00
FTSE MIB	0.17	0.67	0.17
KOSPI 50	0.38	0.00	0.00

Table 13: Out-of-sample performance of low-beta portfolios

The table report the average descriptive statistics of the low-beta portfolios for 17 market indices under study. The mean return, return standard deviation (Std), and Sharpe ratio are annualised, whereas skewness and kurtosis are based on monthly returns. The last rows report the average improvement of the low-beta portfolio over the corresponding market index.

	S&P 100	DJIA	FTSE 100	CAC	DAX	Euro Stoxx 50	Nikkei 225	Indice Bovespa	RTS	BSE SENSEX	SSE 50	FTSE JSE Top 40	S&P/ ASX 50	MERVAL	S&P/ TSX 60	FTSE MIB	KOSPI 50
Low-beta portfolio																	
Mean	0.06	0.05	0.06	0.02	0.05	0.02	0.00	0.07	-0.04	0.09	0.05	0.14	0.05	0.25	0.09	0.05	0.10
St.D	0.15	0.16	0.16	0.22	0.21	0.21	0.24	0.25	0.36	0.24	0.22	0.19	0.14	0.37	0.18	0.20	0.21
Dkewness	-0.20	0.06	-0.35	0.04	-0.11	0.10	-0.55	-0.21	-0.11	-0.38	-0.53	-0.12	-0.38	-0.56	-0.52	-0.07	-0.25
Kurtosis	13.42	16.63	16.49	9.82	13.07	10.91	15.77	14.51	33.80	13.91	11.00	9.05	9.11	12.93	20.13	5.22	8.07
Sharpe ratio	0.20	0.25	0.15	-0.23	-0.07	0.00	-0.10	-0.16	-0.27	0.18	0.12	0.29	-0.01	0.43	0.41	0.07	0.36
Difference between low-beta portfolio and the index																	
Mean	-0.03	-0.01	0.00	0.00	0.01	0.01	-0.05	0.07	0.07	-0.04	-0.06	-0.02	-0.03	-0.02	0.01	0.02	0.01
St.D	-0.03	-0.02	-0.02	-0.02	-0.04	-0.03	0.00	-0.04	-0.02	0.00	-0.05	-0.02	-0.03	0.04	0.00	-0.06	-0.02
Skewness	-0.03	0.14	-0.19	0.02	-0.09	0.09	-0.07	-0.22	0.19	-0.46	-0.19	-0.01	-0.02	-0.21	0.16	0.03	0.04
Kurtosis	2.07	1.99	7.51	1.73	5.44	3.34	5.13	5.03	20.03	2.03	3.84	2.32	0.38	5.56	4.91	-0.25	-0.81
Sharpe ratio	-0.12	-0.05	0.01	-0.03	0.03	0.04	-0.22	0.22	0.17	-0.18	-0.19	-0.07	-0.18	-0.10	0.04	0.11	0.07

9 Figures

Figure 1: Dominating sets at order one to four: an illustration

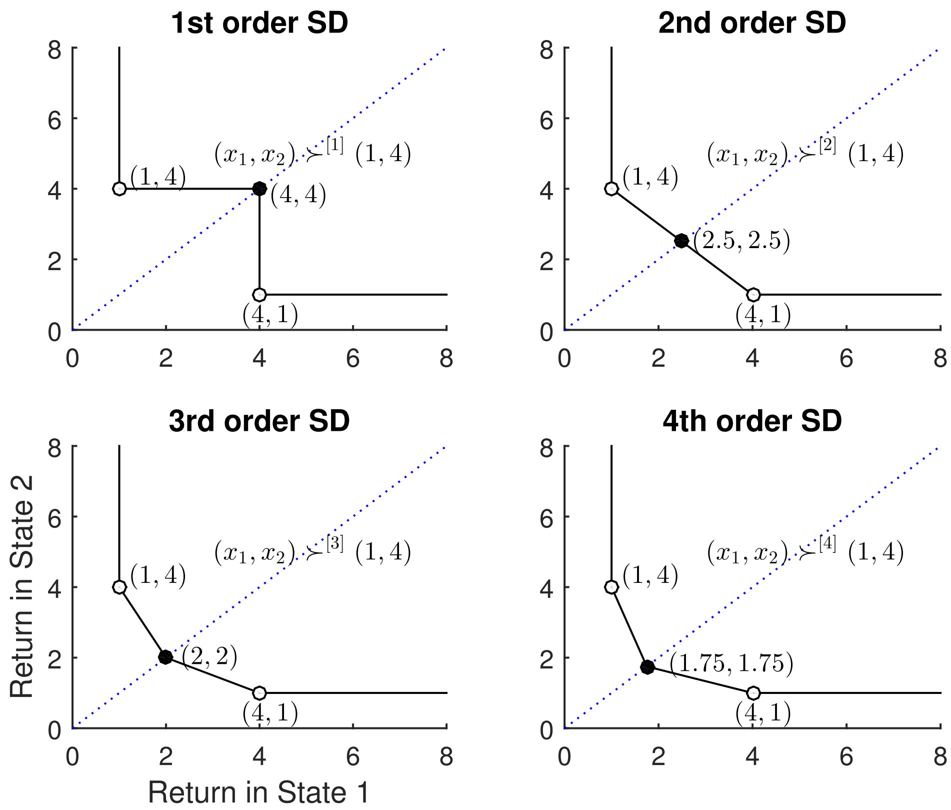
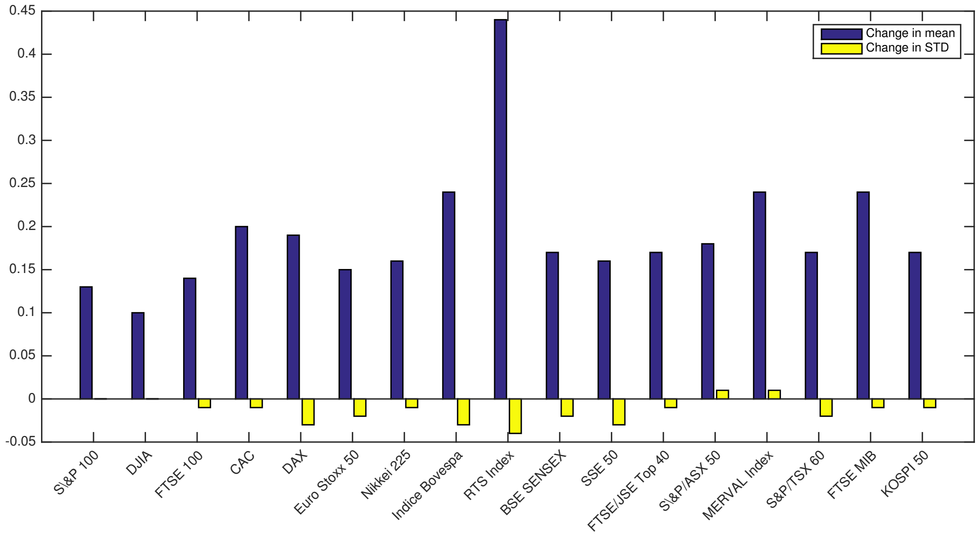
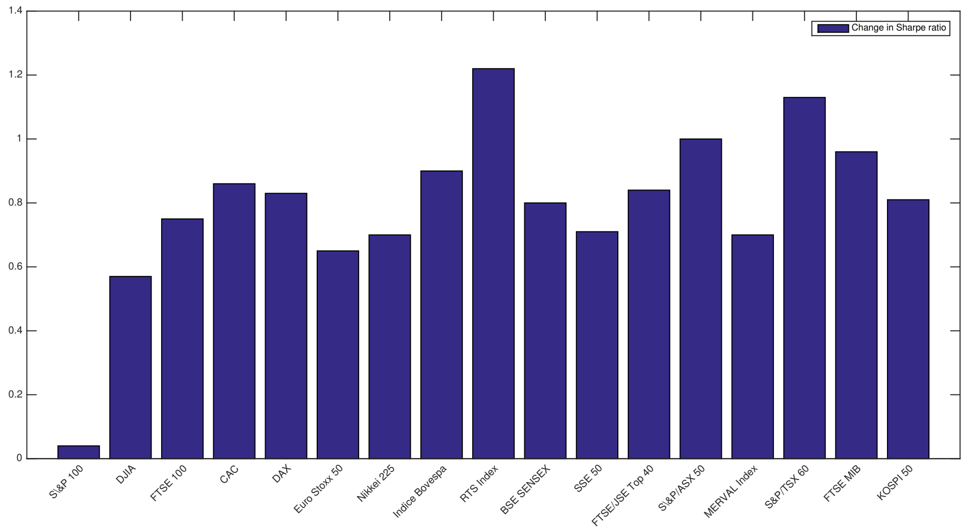


Figure 2: Average improvements in mean return, standard deviations, and Sharpe ratios

The figure plots the average changes in the annualized mean returns and return standard deviations (sub-plot (a)), and the average changes in the Sharpe ratios (sub-plot (b)) for the 17 indices under study, when the market index portfolio is swapped for an optimal SD efficient portfolio.



(a) Mean and standard deviation



(b) Sharpe ratio

Figure 3: Time series of maximal mean improvement of inefficient market indices

The figure plots the time series of the estimated θ-s from Proposition 3 for 17 indices under study. The pentacles indicate individual θ estimates. The solid line plots the average θ-s for each year. Each θ reflects the maximum annualized mean return improvement that can be achieved by moving from an SD inefficient portfolio to an efficient one.

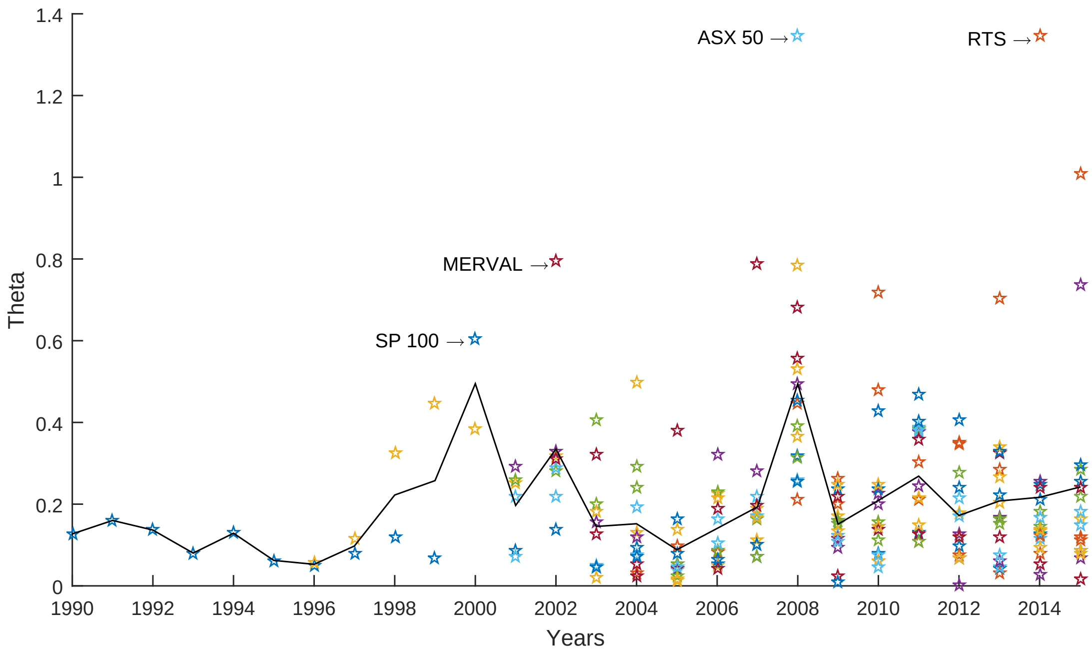
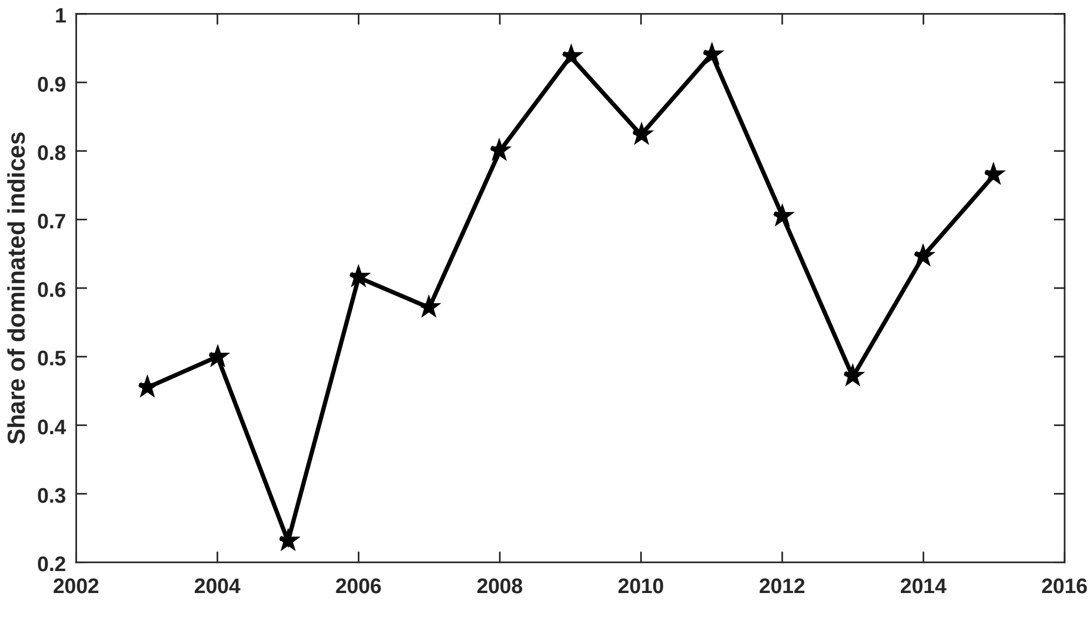


Figure 4: A share of of dominated indices

The figure plots a share of the market indices which are dominated at the order 3 or lower by at least one sector sub-index for each year from 2003 to 2015.



Appendices

A Optimal portfolio weights's correlations

The correlation coefficients of the optimal portfolio weights across the indices are reported in Table A.1. The average correlation across all the indices is 21%. It ranges between 62% for the Argentinean MERVAL and Chinese SSE 50 indices and −19% for the DJIA and SSE 50 indices.

Table A.1: Optimal weight correlation

The table reports the average correlation coefficients between optimal portfolio weights across 17 stock market indices used in this paper.

		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
(1)	S&P 100	1.00	0.26	0.34	0.12	0.08	0.24	0.17	0.15	0.21	0.18	0.22	0.19	-0.02	0.14	0.10	0.12	-0.04
(2)	DJIA		1.00	0.00	-0.04	0.15	-0.05	0.38	-0.10	0.14	0.26	-0.19	0.18	0.11	-0.03	0.18	0.12	0.00
(3)	FTSE 100			1.00	0.39	0.34	0.35	0.12	0.18	-0.01	0.14	0.47	0.18	-0.01	0.24	-0.08	0.19	0.16
(4)	CAC				1.00	0.20	0.39	-0.04	-0.02	0.12	0.06	0.14	0.12	0.05	0.17	0.11	0.46	0.09
(5)	DAX					1.00	0.43	0.21	0.29	0.08	0.08	0.18	0.23	0.34	-0.11	0.02	0.22	0.01
(6)	Euro Stoxx 50						1.00	0.21	0.15	0.04	-0.01	0.18	0.02	0.14	-0.02	0.07	0.05	0.13
(7)	Nikkei 225							1.00	0.41	0.10	0.18	0.24	0.18	0.25	0.22	0.09	-0.19	-0.02
(8)	Indice Bovespa								1.00	0.29	0.14	0.31	0.20	0.17	0.00	0.11	0.18	0.08
(9)	RTS Index									1.00	-0.13	0.29	0.24	0.11	0.22	0.01	0.46	-0.16
(10)	BSE SENSEX										1.00	0.25	0.06	0.00	0.31	0.14	0.02	0.24
(11)	SSE 50											1.00	0.10	0.18	0.62	0.14	0.08	0.49
(12)	FTSE/JSE Top 40												1.00	0.03	0.11	-0.02	0.34	0.19
(13)	S&P/ASX 50													1.00	-0.11	0.21	0.49	-0.03
(14)	MERVAL Index														1.00	-0.11	-0.14	0.11
(15)	S&P/TSX 60															1.00	0.20	0.17
(16)	FTSE MIB																1.00	0.19
(17)	KOSPI 50																	1.00

B Pairwise comparison of market indices and sector sub-indices, SD orders 3 and 2

In this appendix we report the results of the pairwise comparison of each index and corresponding sector sub-indices with respect to the 3rd order and 2nd order SD separately.

Table A.2: Percentage of indices being dominated by sub-indices at orders 2 and 3

The table reports the percentage of years during which each of the 17 diversified equity indices is dominated by each of sector sub-indices at order 3 (Panel A) or 2 (Panel B). The last column reports the percentage of years during which the index is dominated by at least one sub-index.

	Oil&gas	Basic mat.	Ind.	Cons. goods	Health care	Cons. services	Telecom.	Util.	Fin.	Tech.	Any
Panel A: 3rd order stochastic dominance											
S&P 100	0.00	0.00	0.00	0.42	0.27	0.19	0.12	0.31	0.00	0.00	0.58
DJIA	0.00	0.00	0.00	0.33	0.33	0.33	0.25	-	0.00	0.00	0.58
FTSE 100	0.00	0.00	0.05	0.55	0.20	0.20	0.05	0.45	0.00	0.00	0.65
CAC	0.00	0.00	0.07	0.33	0.13	0.27	0.27	0.07	0.00	0.00	0.67
DAX	-	0.07	0.00	0.00	0.33	0.07	0.07	0.20	0.07	0.07	0.53
Euro Stoxx 50	0.20	0.00	0.00	0.27	0.07	0.20	0.20	0.20	0.00	0.07	0.53
Nikkei 225	0.00	0.07	0.00	0.50	0.64	0.36	0.07	0.43	0.00	0.00	0.93
Indice Bovespa	0.00	0.00	0.22	0.89	0.00	0.33	0.33	0.56	0.00	-	0.89
RTS Index	0.75	0.38	0.13	0.25	0.13	0.50	0.63	0.13	0.13	0.00	1.00
BSE SENSEX	0.00	0.00	0.00	0.27	0.00	0.00	0.00	0.00	0.00	0.00	0.27
SSE 50	0.14	0.00	0.14	0.00	0.00	0.00	0.14	0.43	0.71	-	0.71
FTSE/JSE Top 40	-	0.00	0.31	0.31	0.00	0.00	0.00	-	0.31	-	0.62
S&P/ASX 50	0.00	0.00	0.13	0.20	0.00	0.40	0.00	0.00	0.13	-	0.60
MERVAL Index	0.21	0.29	0.07	0.07	-	-	0.00	0.07	0.00	-	0.50
S&P/TSX 60	0.00	0.00	0.08	0.00	0.08	0.77	0.23	0.08	0.38	0.00	0.92
FTSE MIB	0.50	0.00	0.83	0.83	0.00	0.00	0.17	0.67	0.00	0.00	1.00
KOSPI 50	0.00	0.00	0.00	0.00	-	0.00	0.23	0.15	0.00	0.00	0.31
Panel B: 2rd order stochastic dominance											
S&P 100	0.00	0.00	0.00	0.23	0.12	0.12	0.08	0.15	0.00	0.00	0.35
DJIA	0.00	0.00	0.00	0.08	0.25	0.25	0.25	-	0.00	0.00	0.42
FTSE 100	0.00	0.00	0.00	0.50	0.10	0.10	0.05	0.35	0.00	0.00	0.55
CAC	0.00	0.00	0.07	0.27	0.07	0.07	0.07	0.07	0.00	0.00	0.53
DAX	-	0.00	0.00	0.00	0.13	0.00	0.00	0.13	0.07	0.07	0.27
Euro Stoxx 50	0.07	0.00	0.00	0.27	0.00	0.07	0.07	0.07	0.00	0.00	0.27
Nikkei 225	0.00	0.00	0.00	0.14	0.29	0.07	0.07	0.21	0.00	0.00	0.43
Indice Bovespa	0.00	0.00	0.22	0.78	0.00	0.33	0.22	0.44	0.00	-	0.89
RTS Index	0.50	0.25	0.13	0.25	0.13	0.38	0.38	0.13	0.13	0.00	0.75
BSE SENSEX	0.00	0.00	0.00	0.18	0.00	0.00	0.00	0.00	0.00	0.00	0.18
SSE 50	0.14	0.00	0.14	0.00	0.00	0.00	0.00	0.14	0.29	-	0.43
FTSE/JSE Top 40	-	0.00	0.23	0.23	0.00	0.00	0.00	-	0.15	-	0.54
S&P/ASX 50	0.00	0.00	0.07	0.13	0.00	0.20	0.00	0.00	0.07	-	0.33
MERVAL Index	0.14	0.00	0.07	0.07	-	-	0.00	0.00	0.00	-	0.21
S&P/TSX 60	0.00	0.00	0.00	0.00	0.00	0.38	0.15	0.00	0.15	0.00	0.54
FTSE MIB	0.50	0.00	0.67	0.83	0.00	0.00	0.17	0.33	0.00	0.00	1.00
KOSPI 50	0.00	0.00	0.00	0.00	-	0.00	0.08	0.08	0.00	0.00	0.15

C Pooled logit regression for probability of stochastic dominance of a sub-index over the index

Table A.3 reports the estimation results of a pooled logit regression for dominance of sub-indices over the index base on all markets under study. The key predictors of the dominance are market and index volatilities, as well as their ratio. It also emerges that several sectors, namely, Consumer goods, Health care, Consumer services, Telecommunications, and Utilities are more likely to dominate their respective diversified indices. Their significance is suppressed, however, when we include a broader factor indicating sub-index dominance in the past. We do not find a single macro- or financial variable that can consistently explain the propensity of a market index to be dominated by any of its sub-indices. The absence of any link between the performance of the real economy and financial markets is explained by substantially different information content of the aggregate market indicators for different types of the economies, as explained in the main body of the paper.

Table A.3: Stochastic dominance determinants

The table reports the estimation results for the Logit model for the probability of each sector sub-index to dominate its respective diversified equity index.

	Coef.	t-stat.	Coef.	t-stat.	Coef.	t-stat.	Coef.	t-stat.	Coef.	t-stat.	Coef.	t-stat.
S&P 100	-0.67	-0.31			-1.70	-0.75			-2.00	-0.88		
DJIA	-0.34	-0.16			-1.01	-0.44			-1.13	-0.49		
FTSE 100	-0.38	-0.16			-1.42	-0.55			-1.83	-0.70		
CAC	-0.50	-0.26			-1.30	-0.64			-1.60	-0.78		
DAX	-0.65	-0.33			-1.29	-0.61			-1.61	-0.76		
Euro Stoxx 50	-0.45	-0.24			-1.13	-0.58			-1.44	-0.74		
Nikkei 225	-0.01	-0.01			-1.07	-0.49			-1.43	-0.66		
Indice Bovespa	0.38	0.16			-0.30	-0.12			-0.77	-0.31		
RTS Index	1.24	0.64			0.52	0.25			0.11	0.05		
BSE SENSEX	-1.40	-1.18			-1.77	-1.44			-1.84	-1.47		
SSE 50	0.00	–			0.00	–			0.00	–		
FTSE/JSE Top 40	-0.80	-0.36			-0.92	-0.40			-1.21	-0.52		
S&P/ASX 50	-0.64	-0.36			-1.31	-0.69			-1.48	-0.77		
MERVAL Index	2.48	1.01			3.75	1.41			3.44	1.28		
S&P/TSX 60	0.77	0.40			0.54	0.27			0.36	0.18		
FTSE MIB	0.31	0.14			-0.59	-0.26			-0.96	-0.42		
KOSPI 50	-0.92	-0.65			-1.73	-1.13			-1.87	-1.22		
Oil and gas	1.61	0.38	2.46*	1.85	3.26	0.73	2.44*	1.74	2.74	0.61	1.12	0.80
Basic materials	1.32	0.31	2.12	1.56	2.82	0.63	2.04	1.42	2.28	0.51	0.68	0.47
Industrials	1.69	0.40	2.36*	1.80	3.43	0.76	2.41*	1.74	2.86	0.64	1.04	0.75
Consumer goods	2.97	0.71	3.52***	2.73	4.59	1.02	3.44**	2.51	3.97	0.89	2.00	1.46
Health care	2.54	0.61	3.07**	2.36	4.20	0.94	3.07**	2.23	3.58	0.80	1.64	1.19
Consumer Services	2.61	0.62	3.25**	2.54	4.30	0.96	3.25**	2.40	3.64	0.81	1.78	1.31
Telecommunications	2.40	0.57	3.06**	2.32	3.97	0.89	2.95**	2.12	3.38	0.76	1.56	1.12
Utilities	2.28	0.54	2.95**	2.29	3.80	0.85	2.79**	2.04	3.13	0.70	1.27	0.92
Financials	1.99	0.48	2.73**	2.08	3.47	0.78	2.56*	1.84	2.92	0.65	1.20	0.87
Technology	0.96	0.23	1.47	0.99	2.62	0.58	1.47	0.95	2.25	0.50	0.32	0.21
GDP growth	0.02	0.43	0.00	0.06	0.02	0.30	-0.02	-0.38	0.03	0.54	-0.01	-0.22
Inflation	-0.06	-1.06	0.00	-0.10	-0.07	-1.22	0.01	0.15	-0.09	-1.50	-0.01	-0.14
Unemployment	0.01	0.07	-0.01	-0.57	-0.05	-0.54	-0.02	-0.71	-0.06	-0.61	-0.02	-0.85
Gross savings	-0.02	-0.23	-0.03*	-1.82	-0.05	-0.58	-0.03*	-1.66	-0.06	-0.74	-0.03	-1.46
Current account balance	-0.09	-1.17	0.00	-0.14	-0.10	-1.15	-0.01	-0.44	-0.08	-0.88	-0.02	-0.56
Real effective exchange	0.02	1.44	0.02	1.56	0.02	1.41	0.02	1.27	0.03	1.60	0.02	1.54
Government bonds yield	-0.03	-0.50	-0.05	-1.30	0.00	0.02	-0.04	-1.15	0.01	0.24	-0.04	-0.97
Central bank policy rate	0.02	0.44	0.05	1.21	-0.01	-0.13	0.05	1.14	0.01	0.14	0.06	1.28
Liquidity	-8.21	-1.48	0.50	0.26	-16.12**	-2.49	0.30	0.15	-17.71***	-2.73	-0.53	-0.26
Index return	0.93*	1.80	0.57	1.18	0.93	1.64	0.43	0.83	1.01*	1.78	0.54	1.02
Index volatility	17.1***	3.06	15.52***	2.79	18.76***	3.23	15.3***	2.68	18.66***	3.28	15.63***	2.78
Sector return	-0.82*	-1.69	-0.59	-1.24	-0.67	-1.30	-0.52	-1.02	-0.77	-1.51	-0.60	-1.18
Sector volatility	-16.46***	-3.02	-15.28***	-2.82	-18.05***	-3.18	-16.02***	-2.86	-17.14***	-3.09	-15.58***	-2.83
Volatility ratio	-2.79**	-2.44	-3.3***	-2.87	-2.47**	-2.08	-3.07***	-2.58	-1.85	-1.61	-2.34**	-2.01
D dominance, last year					-0.07	-0.29	-0.04	-0.16				
D dominance, any year									0.59***	2.67	0.63***	2.95
Efron's pseudo R-sq	0.32		0.30		0.31		0.29		0.32		0.29	
N obs	1823		1823		1688		1688		1688		1688	

D Out-of-sample trading strategy: details

This appendix reports detailed results of the out-of-sample performance of our past-SD based trading strategy. The descriptive statistics of the resulting returns are reported for each index year by year.

Table A.4: Out-of-sample index performance: detailed

The table reports the descriptive statistics mean (Mean), standard deviation (Std), skewness (Sk), and kurtosis (kr) of market indices (Index) and corresponding index that includes those industries that dominated the index in the past at least two times (SD). Mean returns and standard deviations are annualized.

	Mean (Index)	Mean (SD)	Std (Index)	Std (SD)	Sk (Index)	Sk (SD)	Kr (Index)	Kr (SD)
				S&P 100				
2003	23.31%	23.06%	17.52%	16.67%	0.01	0.05	3.91	4.54
2004	6.21%	13.66%	11.10%	9.80%	-0.08	0.10	2.97	4.77
2005	1.17%	8.09%	9.85%	11.23%	0.03	-0.22	3.12	3.23
2006	17.01%	17.26%	9.47%	8.71%	0.02	-0.17	4.41	2.97
2007	5.94%	17.15%	15.74%	15.44%	-0.49	-0.53	4.80	4.65
2008	-43.39%	-25.68%	40.12%	30.75%	0.01	0.72	6.85	10.32
2009	20.12%	8.33%	25.54%	16.93%	0.03	-0.29	5.46	4.01
2010	11.78%	6.97%	17.24%	12.75%	-0.23	-0.33	5.12	5.64
2011	3.14%	12.41%	22.35%	15.87%	-0.52	-0.55	5.93	6.09
2012	14.89%	10.68%	12.47%	8.82%	0.10	0.14	4.08	3.99
2013	26.54%	18.91%	10.67%	10.43%	-0.24	-0.26	4.69	4.63
2014	11.99%	12.98%	11.18%	9.43%	-0.38	-0.19	4.54	4.34
2015	2.60%	3.06%	15.89%	13.63%	-0.16	-0.15	5.24	5.21
				DJIA 30				
2010	13.16%	13.16%	16.16%	16.16%	-0.17	-0.17	5.29	5.29
2011	8.08%	14.95%	21.12%	16.79%	-0.53	-0.35	5.61	5.66
2012	9.75%	11.09%	11.72%	9.11%	0.05	0.09	4.02	3.83
2013	25.97%	21.26%	10.15%	10.42%	-0.19	-0.35	4.48	4.73
2014	9.57%	11.58%	10.91%	9.53%	-0.35	-0.35	4.27	4.86
2015	0.21%	6.50%	15.45%	13.85%	-0.14	-0.05	4.58	5.13
				FTSE 100				
2006	13.53%	20.57%	12.61%	10.09%	-0.40	-0.34	4.59	3.89
2007	7.11%	9.61%	17.49%	14.32%	-0.36	-0.31	4.67	4.39
2008	-33.18%	-24.11%	37.50%	29.84%	0.12	0.10	6.59	7.04
2009	24.16%	14.67%	23.50%	15.55%	-0.21	-0.05	4.53	5.18
2010	11.89%	11.84%	17.42%	13.40%	0.05	-0.16	5.11	5.37
2011	-2.21%	7.75%	21.29%	16.22%	-0.24	-0.21	4.35	4.15
2012	9.51%	11.37%	13.96%	10.87%	-0.01	0.05	3.67	3.48
2013	17.11%	19.70%	12.12%	11.26%	-0.26	-0.42	4.78	6.21
2014	0.73%	5.27%	11.39%	11.24%	-0.35	-0.42	5.03	5.70
2015	-1.33%	4.32%	17.34%	15.78%	-0.28	-0.05	5.01	4.26
				CAC 40				
2009	24.36%	29.41%	26.73%	26.18%	-0.01	0.06	3.89	3.71
2010	0.55%	9.08%	23.57%	19.34%	0.57	0.41	9.13	7.84
2011	-14.43%	-14.82%	28.84%	24.32%	-0.13	-0.08	4.57	4.24
2012	18.54%	5.19%	20.73%	19.26%	0.09	0.16	3.98	4.12
2013	20.06%	14.24%	16.19%	17.09%	-0.18	-0.35	4.40	4.15
2014	2.67%	16.28%	16.23%	15.92%	-0.21	-0.13	4.28	3.96
2015	11.28%	10.50%	22.56%	24.11%	-0.25	0.05	4.22	3.97
				DAX 30				
2009	21.39%	9.67%	28.43%	20.59%	-0.07	-0.22	3.89	3.09
2010	14.89%	-3.38%	18.42%	13.90%	-0.02	0.00	4.73	4.10
2011	-15.95%	-8.69%	28.97%	25.33%	-0.15	-0.57	4.37	5.18
2012	25.51%	6.06%	18.91%	15.62%	-0.12	-0.09	4.29	4.04
2013	22.70%	6.80%	14.64%	14.10%	-0.30	-0.45	4.17	5.89
2014	2.62%	11.80%	16.76%	16.47%	-0.16	-0.46	3.86	4.43
2015	9.13%	-11.84%	23.59%	25.09%	-0.15	-0.11	3.57	3.23
				EURO STOXX 50				
2009	23.96%	3.65%	28.14%	26.98%	-0.08	-0.14	3.92	4.53
2010	-1.92%	-11.77%	23.70%	20.65%	0.77	-0.04	10.66	5.66
2011	-14.16%	-11.05%	28.94%	25.37%	-0.16	-0.06	4.38	4.24
2012	17.87%	8.73%	20.76%	18.92%	0.19	0.26	4.30	4.40
2013	20.49%	16.48%	16.39%	15.81%	-0.15	-0.17	4.27	3.63
2014	4.82%	8.15%	17.08%	16.51%	-0.16	-0.13	4.20	4.33
2015	7.03%	6.76%	23.30%	23.28%	-0.24	-0.17	4.22	4.62
				NIKKEI 225				
2009	19.17%	-0.74%	27.41%	18.93%	-0.04	0.05	3.75	3.33
2010	-1.33%	-8.73%	20.64%	14.29%	-0.24	-0.41	3.26	3.62
2011	-17.01%	-26.75%	23.54%	22.01%	-1.72	-4.05	16.57	40.68
2012	22.83%	9.92%	16.03%	13.82%	-0.09	0.12	3.00	3.48
2013	46.59%	39.85%	26.70%	25.43%	-0.76	-0.87	5.50	6.28
2014	8.57%	9.80%	20.08%	19.11%	-0.06	-0.19	4.49	4.61
2015	10.43%	18.93%	20.74%	21.61%	0.00	-0.14	7.98	8.11

E Market specific individual-stock based trading strategy: historical volatility

This appendix reports the results for SD tests and the average performance of portfolios sorted on stock historical volatility. Overall, the portfolios including 30% of stocks with the lowest historical volatility perform well in SD sense out-of-sample, consistent with findings of Hodder et al. (2015) on good performance of a global minimum variance portfolio.

Table A.5: Historical stock return volatility descriptive statistics

The table reports the time series averages of the descriptive statistics of individual stock return historical volatilities for stocks that are constituents of the 17 market indices used in this paper. "Effective stocks" reports the average percentage of stocks in each index that are alive at the end of a calendar year and are used for portfolio construction over the following year.

	Effective Stocks	Mean	Median	Std	Min	Max	30% quantile	70% quantile
S&P 100	98%	0.31	0.29	0.11	0.15	0.71	0.25	0.33
DJIA	100%	0.27	0.26	0.08	0.15	0.51	0.23	0.29
FTSE 100	99%	0.30	0.28	0.10	0.15	0.73	0.25	0.32
CAC	100%	0.34	0.32	0.12	0.18	0.71	0.27	0.38
DAX	100%	0.35	0.33	0.11	0.20	0.70	0.29	0.38
Euro Stoxx 50	100%	0.32	0.30	0.11	0.18	0.67	0.26	0.35
Nikkei 225	100%	0.37	0.36	0.10	0.16	0.79	0.32	0.41
Indice Bovespa	99%	0.41	0.37	0.14	0.22	0.91	0.33	0.44
RTS Index	99%	0.57	0.54	0.17	0.30	1.15	0.48	0.61
BSE SENSEX	100%	0.39	0.37	0.10	0.25	0.63	0.33	0.42
SSE 50	100%	0.48	0.46	0.13	0.20	0.76	0.40	0.54
FTSE/JSE Top 40	100%	0.31	0.29	0.09	0.17	0.59	0.26	0.34
S&P/ASX 50	99%	0.28	0.25	0.10	0.13	0.65	0.22	0.29
MERVAL Index	94%	0.54	0.53	0.14	0.31	0.84	0.46	0.60
S&P/TSX 60	100%	0.32	0.29	0.14	0.15	0.83	0.24	0.36
FTSE MIB	98%	0.36	0.36	0.11	0.13	0.66	0.30	0.42
KOSPI 50	96%	0.44	0.43	0.12	0.22	0.83	0.38	0.48

Table A.6: Market indices vs. historical volatility-sorted portfolios

The table reports the fraction of years during which each of the 17 diversified equity indices is dominated by historical volatility-sorted portfolios of individual stocks at orders 4, 3 or 2.

	Low-vol	Medium-vol	High-vol
S&P 100	0.62	0.04	0.00
DJIA	0.50	0.00	0.00
FTSE 100	0.50	0.05	0.10
CAC	0.73	0.13	0.00
DAX	0.73	0.07	0.00
Euro Stoxx 50	0.87	0.20	0.00
Nikkei 225	0.86	0.00	0.00
Indice Bovespa	0.78	0.11	0.00
RTS Index	0.75	0.38	0.38
BSE SENSEX	0.64	0.00	0.00
SSE 50	0.71	0.43	0.00
FTSE/JSE Top 40	0.62	0.00	0.00
S&P/ASX 50	0.67	0.00	0.00
MERVAL Index	0.36	0.21	0.36
S&P/TSX 60	0.92	0.15	0.00
FTSE MIB	0.83	0.33	0.00
KOSPI 50	0.38	0.00	0.00

Table A.7: Out-of-sample performance of low-historical volatility portfolios

The table report the average descriptive statistics of the low-historical volatility portfolios for 17 market indices under study. The mean return, return standard deviation (Std), and Sharpe ratio are annualised, whereas skewness and kurtosis are based on monthly returns. The last rows report the average improvement of the low-historical volatility portfolio over the corresponding market index.

	S&P 100	DJIA	FTSE 100	CAC	DAX	Euro Stoxx 50	Nikkei 225	Indice Bovespa	RTS	BSE SENSEX	SSE 50	FTSE JSE Top 40	S&P/ ASX 50	MERVAL	S&P/ TSX 60	FTSE MIB	KOSPI 50
Low-volatility portfolio																	
Mean	0.06	0.04	0.05	0.03	0.04	0.02	0.03	0.07	-0.02	0.11	0.06	0.17	0.08	0.09	0.07	0.04	0.07
St.D	0.16	0.16	0.17	0.21	0.21	0.21	0.20	0.25	0.37	0.23	0.23	0.19	0.16	0.35	0.17	0.22	0.21
Skewness	-0.19	0.03	-0.61	0.10	-0.18	0.06	-0.51	0.10	-0.14	-0.27	-0.47	-0.08	-0.19	0.12	-0.60	-0.05	-0.14
Kurtosis	12.07	18.18	15.16	9.48	12.78	10.44	13.47	13.28	28.21	13.85	10.68	8.28	9.32	8.40	21.91	5.27	7.58
Sharpe ratio	0.21	0.17	0.09	-0.18	-0.10	0.02	0.03	-0.14	-0.22	0.26	0.17	0.48	0.17	0.00	0.35	0.04	0.23
Difference between low-volatility portfolio and the index																	
Mean	-0.02	-0.03	-0.01	0.01	0.01	0.01	-0.02	0.07	0.09	-0.03	-0.05	0.02	0.00	-0.18	-0.01	0.02	-0.02
St.D	-0.02	-0.02	-0.02	-0.03	-0.04	-0.04	-0.04	-0.04	-0.02	-0.01	-0.04	-0.02	-0.01	0.01	-0.01	-0.05	-0.02
Skewness	-0.02	0.11	-0.45	0.07	-0.16	0.05	-0.03	0.08	0.16	-0.35	-0.13	0.02	0.17	0.47	0.08	0.05	0.15
Kurtosis	0.72	3.54	6.18	1.39	5.15	2.87	2.83	3.79	14.44	1.97	3.52	1.55	0.59	1.03	6.70	-0.20	-1.31
Sharpe ratio	-0.10	-0.14	-0.05	0.01	0.01	0.06	-0.09	0.23	0.22	-0.10	-0.14	0.12	0.01	-0.53	-0.02	0.08	-0.05