Frontiers of Factor Investing – Extended Abstract

Volatility Forecasting for Low-Volatility Investing

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Previous research has demonstrated that portfolios based on low-volatility stocks tend to outperform portfolios based on high-volatility stocks (Ang et al., 2006; Bali and Cakici, 2008). In our paper, we propose a new model selection criterion for low-volatility investing. Our procedure is based on the decomposition of any Bregman loss function into a weighted integral of elementary scores (Ehm et al., 2016) for defining forecast errors that are directly related to the problem of choosing the future 20%-quintile of low-volatility stocks.

Until now, high-frequency data based times series models are not widely used for constructing low-volatility portfolios even though they are considered to issue better predictions than models based on daily data. One explanation may be that it is not clear which model to choose for this particular problem. When comparing several models at hand, superior models are typically selected by evaluating past forecast performance with respect to some loss function, e.g. squared error or QLIKE loss. Thereafter, these losses are used to conduct tests on whether differences in forecast accuracy are statistically significant or due to chance. However, those loss functions evaluate overall forecast performance instead of the binary decision problem of choosing stocks for which next period's volatility is (relatively) low.

With our proposed loss function we closely match the portfolio choice problem at hand, which enables us to choose among volatility models in a problem-specific way. Creating portfolios of individual stocks from a large cross-section of 987 S&P500 stocks for the period 1998 to 2017 results in higher returns for both the HAR model and our elementary score compared to a widely used benchmark based on daily returns. These gains are even more pronounced when considering low-high portfolios.

First, the main contribution of our paper is to use recent advances in forecast evaluation theory for constructing a loss function that has a direct translation into the economic decision of financial

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investors. As mentioned above, a problem for portfolio sorts based on time series models is the inherent problem of choosing the "best" model. A natural choice is to evaluate out-of-sample forecasts y and corresponding realizations x using a loss function that is *consistent* for the mean functional, meaning that the expected loss should be minimized by the conditional mean forecast. However, the class of loss functions that is consistent for the mean, the so-called class of Bregman loss functions, consists of every function of the form

$$S(x, y) = \phi(y) - \phi(x) - \phi'(x)(y - x),$$

where $\phi(x)$ is convex with subgradient ϕ'. Hence, the widely used forecast evaluation measures squared error (SE), $L(x, y) = (y - x)^2$, and QLIKE loss, $L(x, y) = y/x - \log(x/y) - 1$, are only two examples of infinitely many. Additionally, Patton (2016) showed that the ranking of conditional mean models differ across consistent loss functions if models are misspecified, which is the case in almost all applications. Therefore, the choice of loss function matters in this regard.

Ehm et al. (2016) showed recently that any Bregman loss function $L(x, y)$ can be expressed as an integral of elementary scores $L_\theta(x, y)$:

$$L(x, y) = \int_{\theta=0}^{\infty} L_\theta(x, y) dH(\theta)$$

with

$$L_\theta(x, y) = \begin{cases} |y - \theta| & \text{if } \min(x, y) \leq \theta < \max(x, y) \\ 0 & \text{else} \end{cases}$$

and $H(\theta)$ being a positive weighting function on $\mathbb{R}^+$.[1] In words, $L_\theta(x, y)$ assigns a penalty if and only if θ lies in between x and y. Regarding portfolio sorts, the choice of H can be motivated economically: For example, consider the simple problem of whether or not to buy one unit of a risky asset. In this problem, it may matter whether the volatility is 5 or 10 (this may be the difference between buying versus not buying), whereas the distinction between volatilities of 200 and 300 may be irrelevant because the risky asset would be unattractive in either of these cases.

Second, in our empirical application we use a data set containing every stock that has been part of the S&P500 in between January 1998 and July 2017. The one-minute stock market prices are provided by QuantQuote[2] and are aggregated to five-minute log-returns. We use this data for calculating various high-frequency realized volatility measures (Andersen et al., 2012; Patton and Sheppard, 2015) on a daily frequency. This enables us to estimate a wide variety of volatility models for dynamically selecting the best model for each stock.

As an ideal forecast, we define the "oracle" low-volatility portfolio to consist of those stocks for which the realized variance is below the empirical 20%-quantile of the cross-section. This portfolio has an average annualized return of 13.28 compared to 9.49 when sorting by the sum of last month's

[1] SE is obtained by $H(\theta) = 1$ and QLIKE by $H(\theta) = 1/\theta^2$.

[2] https://www.quantquote.com

squared returns, which is often used by low-volatility funds. This indicates that there actually is a margin for improvement when using more accurate volatility forecasts. Our preliminary results for equally-weighted monthly portfolio sorts indicate an increase of average annualized returns for the HAR model by 0.17 and for the elementary score by 0.10. Moreover, the average difference of the lowest- and highest-quintile portfolio returns increases from 7.80 when sorting by last month's squared returns to 10.01 when sorting by the elementary score, making it a good candidate for constructing long-short portfolios. The improvements rely on including the volatility of overnight-returns in our HAR models, even though these are less accurately measured than intraday volatility. This is not typically done in HAR model applications and may be an explanation for other authors not finding significantly higher returns with model-based portfolio sorts - combining high-frequency data with overnight returns seems to be crucial.

Until your conference "Frontiers of Factor Investing", we intend to incorporate transaction costs, weekly portfolio sorts, forecasting idiosyncratic volatility instead of volatility itself, value-weighted portfolios, and robustness checks relative to the Fama, French and Carhart four-factor model (Fama and French, 1993; Carhart, 1997). Summing up, our paper is a novel approach for exploiting recent advances in forecast evaluation to a decision-theoretical question in financial markets.

References

Andersen, T. G., Dobrev, D., and Schaumburg, E. (2012). Jump-robust volatility estimation using nearest neighbor truncation. *Journal of Econometrics*, 169(1):75–93.

Ang, A., Hodrick, R. J., Xing, Y., and Zhang, X. (2006). The cross-section of volatility and expected returns. *Journal of Finance*, 61(1):259–299.

Bali, T. G. and Cakici, N. (2008). Idiosyncratic volatility and the cross section of expected returns. *Journal of Financial and Quantitative Analysis*, 43(1):29–58.

Carhart, M. M. (1997). On persistence in mutual fund performance. *Journal of Finance*, 52(1):57–82.

Ehm, W., Gneiting, T., Jordan, A., and Krüger, F. (2016). Of quantiles and expectiles: Consistent scoring functions, Choquet representations and forecast rankings. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, 78(3):505–562.

Fama, E. and French, K. (1993). Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics*, 33(1):3–56.

Patton, A. J. (2016). Comparing possibly misspecified forecasts. *Working Paper*.

Patton, A. J. and Sheppard, K. (2015). Good volatility, bad volatility: Signed jumps and the persistence of volatility. *The Review of Economics and Statistics*, 97(3):683–697.