Jump risk and pricing implications

Nancy Zambon
Department of Economics, University of Padua, Italy
(nancy.zambon@phd.unipd.it)

Massimiliano Caporin
Department of Statistics, University of Padua, Italy
(massimiliano.caporin@unipd.it)

Walter Distaso
Department of Economics, Imperial College, London
(w.distaso@imperial.ac.uk)

September 25, 2017

Abstract

This paper identifies a new common risk factor in stock returns related to the fear of future jumps: the Jump Factor. It is possible to include the factor in standard asset-pricing models leading to a five-factor model which is directed at capturing the size, value, profitability, momentum and jump expectation in stock returns. Standard analysis show that the jump component proxy, in stock returns, for sensitivity to a common risk factor and that the Jump Factor is able to explain much of the variation in returns both in time and cross-section. Moreover, the risk premia associated with the Jump Factor is negative, significant, and close to its factor portfolio mean excess returns.

Introduction

Finance literature has been focusing for decades on the patterns that asset returns follow. A cornerstone in the description of market returns is the asset-pricing model of Sharpe (1964), Lintner (1965), Mossin (1966) and Black (1972). According to the CAPM the market portfolio is mean-variance efficient in the sense of Markowitz (1959). The efficiency implies that only systematic market risk, measured by the beta of an asset, should be priced. Several studies, for example Reinganum (1981), Lakonishok and Shapiro (1986), and Fama and French (1992), challenge the ability of the CAPM to explain the cross-section of expected stock returns. Fama and French (1993) propose an extension of the one-beta CAPM starting from the observation that there is not a positive relation between average stock returns and market betas. They suggest that stock risks are multidimensional and include other two sources of risk: one proxied by *size* (or market value, or market equity price, or ME), and one proxied by *value* (or BE/ME), defined as the ratio of book value (or BE) to market value (or ME). Carhart (1997) showed that there are patterns in average returns related to the momentum factor (or MOM) based on Jegadeesh and Titman (1993).

In this paper we examine if perceptions of price uncertainty, the fear of sharp price movements, constitute a common risk factor in returns and how they impact on asset prices. Uncertainty plays a primary role in economics, finance, and decision sciences and may help explaining the empirical "fat tails" we observe in stock returns. A possible explanation for the leptokurtosis lies in the presence of discontinuous variations in the price process. There is evidence, Ball and Torous (1983), Jarrow and Rosenfeld (1984), and Jorion (1988) among others, that stock prices show sudden but infrequent movements of large magnitude, that are commonly known as jumps. The first models that incorporate jumps in the dynamic of stock prices are those of Press (1967) and Merton (1976), and several subsequent studies show that such a structure is necessary to fit the observed prices. There exist several studies that provide evidence of this for the option market, among others Ball and Torous (1985), Naik and Lee (1990), Bakshi et al. (1997), Duffie et al. (2000), Andersen et al. (2002), and Eraker et al. (2003). In particular Pan (2002) shows the presence of a priced aggregate jump risk in option prices. The Bates (2000) model, who extends the Heston (1993) stochastic volatility model by incorporating jumps, analysis the existence of the jump premium. More recently, Bollerslev et al. (2016) study the contribution of jump risk in explaining the cross-section of expected stock returns. They extend the CAPM by decomposing the market beta into three separate parts, continuous, discontinuous and overnight, and find that there are significant risk premiums, in the cross-section, for discontinuous and overnight movements and that their estimated betas are generally higher that the corresponding continuous betas.

Our work differs from these studies since it analyses the presence of a jump risk factor and relates to Yan (2011) and Cremers et al. (2015). Both studies use option prices because they contain forward-looking information that helps matching the time-varying nature of the jump risk. Focusing on the behaviour of put prices around the crash of October 1987, Bates (1991) finds that jump expectations in stock market returns change over time. Christoffersen et al. (2012) find that jump intensity is significantly time-varying and that discrete-time models have better performances when incorporating jumps. Yan (2011) proxies the average jump size using the slope of option implied volatility smile and finds that stocks with high positive (negative) slopes more probably will have large positive (negative) jumps in the future. He also highlights the existence of a negative relation between average jumps sizes and expected stock returns. Cremers et al. (2015) study the effect of jump risk using factor-mimicking portfolios that they build using straddles. Their results show that stocks with high sensitivity to jump risk present lower expected returns and that the aggregate stock market jump risk is significantly priced in the cross-section of returns.

While the precedent researchers justify their use of the option market recalling the synchronized information content of stock and option markets, we make a step further by focusing directly on the former. In order to maintain a forward-looking perspective, we focus on the market expectation of future jumps encoded in stock prices. To this end we follow Chan and Maheu (2002) who propose a model for stock returns with time-varying conditional jump intensity. After estimating the model, using past stock returns, it is possible to compute the expected jump component as a function of both the mean of the conditional jump size and the time-varying jump intensity. It follows that we can construct the factor-mimicking portfolio for the jump risk in the same way Fama and French (1993) build the portfolio mimicking the BE/ME risk source.

Our work is also related to the literature about rare disasters and tail risk. Relative to the former, it is important to notice that, even if jumps and disasters show various similarities, jumps happen more frequently than disasters. About the latter, Bollerslev and Todorov (2011) and Gabaix (2012), among others, show that jump tail risk may explain an important part of the aggregate equity risk premium and embedded temporal variation. According to Bollerslev and Todorov (2011) compensation for rare events accounts for a large fraction of the average equity and variance risk premia. By using high-frequency intraday data and short maturity out-of-the-money options they show that the market usually incorporates the possible occurrence of rare disasters in the way it prices risky payoffs. Furthermore, they discuss how the fear of these events account for a large part of the historically observed premia.

The main contribution of this paper is the construction of a jump factor (or JF) capturing investors fear of future sudden and sharp price movements, using a dataset of considerable dimension: 89 years of assets prices and more than 24,000 stocks. We model our factor, using all CRSP stocks over the 1925-2014 sample period, as the return differential between the high and low expected jump component quantile portfolios. The simple plot of JF time series, makes clear its ability of capturing jump forecasts changes over time. This is important since it confirms that our factor is able to reflect the jump probability evolution over time.

The factor average return is significantly different from zero, negative, and about |1.6|% per year. Since low and high expected jump portfolios are formed, respectively, by assets with negative and positive expected jump component, the observation of a negative mean monthly JF return (-0.13%) indicates that assets with negative expected jump present higher returns. It is possible to explain this result by considering the concepts of loss aversion and probability weighting in the field of prospect theory (see among others Kahneman and Tversky (1979), Barberis and Thaler (2003), and Barberis (2013)). Indeed, investors show greater sensitivity to large negative expected jumps and overweight low probabilities, pushing them to demand an insurance over large negative expected jumps.

To empirically test the relevance of the JF in explaining time-series and cross sectional return variations, we add the factor to the Carhart 4-factor model. The high mean and variance values and the low correlations with the other factors suggest that our new factor is relevant. We also test the usefulness of adding the JF to the Carhart (1997) asset-pricing model. Our empirical investigation makes use of the one-month abnormal returns from the Fama-French model which become the dependent variables in time-series regressions that investigate if we can consider JF and MOM missing factors in the 3-factor model. Estimated factor loadings show, especially when focusing on short time-windows, similar outcomes for the two factors. The highly positive significance slope results, both for JF and MOM, justify the extension of the Carhart (1997) model with the inclusion of our new factor.

Time series regressions results for our 5-factor model, both on the full sample period and on sub-periods, confirm that the JF captures common variation in stock returns. We compute factor loadings and coefficients of determination using as dependent variables two different sets of portfolios. In all cases we observe that the estimated jump factor loadings are statistically different from 0, at conventional levels, in a relevant number of regressions and that their values are also large. Specifically, the fraction of slopes on JF that are more than 1.645 standard errors from 0[1] are 52% and 56%. For all time-series regressions we also obtain important increases in the coefficient of determination by adding the jump factor to the asset-pricing model. Factor loadings and R^2 results suggest that the expected jump component proxy, in stock returns, for sensitivity to a common risk factor.

Lastly, we compute the risk premiums associated with the five factors of our 5-factor model using two approaches, Black, Jensen and Scholes (1972) and Fama and MacBeth (1973), and applying the Hou and Kimmel (2006) extrapolation correction. The risk premiums are not significantly different from their factor means, and, with the exception of the JF, they are always positive. JF risk premiums range from -0.22 to -0.01 and are significantly different from 0 (90% confidence level) in two out of four cases.

The paper proceeds as follows. Section 1 presents the models we employ to study the presence of a priced risk factor in stock returns. It also describes the construction of our jump risk factor-mimicking portfolio. Section 2 presents and describes summary statistics for the jump factor and other factors returns. It also introduces and describes some portfolios that will be useful for the subsequent analysis. Section 3 investigates if the jump factor is a missing factor in a standard asset-pricing model. Section 4 presents our main results on the ability of the jump factor to capture common variation in stock returns. It also describes how well a model including our jump factor explains average returns in the dependent portfolios. Section 5 presents the same results of Section 4 but for sub-periods. Section 6 presents the estimated risk premiums. Section 7 concludes.

1 Jump factor and relative asset-pricing model

1.1 Modelling returns with jumps

Our goal is the construction of a risk factor reflecting the jump impact on returns in a forward-looking perspective. This requires a preliminary step: we must recover a measure of jump sensitiveness. To this end, we refer to Chan and Maheu (2002) who propose a model for stock returns with GARCH volatility and time-varying conditional jump intensity. In the vast literature focusing on jumps, only few works analyse the presence of a jump risk factor. They recover a Jump Factor from a specific option database and

[1] Since we are testing using a 10% significance level, we consequently expect to obtain 10% false rejections even if the coefficient is null under the true model.

check if the jump component is priced by the market (see for example Yan (2011) and Cremers et al. (2015)). The model of Chan and Maheu (2002) differs from the previous ones by the possibility of recovering asset-specific jump expectations and, in turn, to use them for pricing, at the market level, the jump risk.

According to Chan and Maheu (2002) we model stock returns including a jump component, Z_t:

$$R_t = \mu + \sum_{i=1}^{l} \phi_i R_{t-i} + Z_t + \epsilon_t, \tag{1}$$

where R_t is the daily stock log return, $\Phi_t = \{R_t, ..., R_1\}$ is the information set at time t, $Z_t = \sum_{k=1}^{N_t} Y_{t,k}$ is the sum of the conditional jump sizes $Y_{t,k}$, and ϵ_t follows a conditionally normal density with GARCH error. The conditional jump size, given the information set Φ_{t-1}, is normally and independently distributed: $Y_{t,k}|\Phi_{t-1} \sim N(\Theta, \Delta)$ with constant mean and variance. It is possible to obtain the jump component by summing up the sizes of the jumps arriving between $t-1$ and t, where the number of jumps, N_t, is a Poisson random variable with parameter $\lambda_t > 0$. Recalling that the mean and variance of a Poisson random variable both equal its parameter, it is easy to compute the conditional mean of the counting process:

$$\lambda_t \equiv E[N_t|\Phi_{t-1}] \equiv Var[N_t|\Phi_{t-1}].$$

Moreover, λ_t, the conditional jump intensity, follows an approximate autoregressive moving average (ARMA) process:

$$\lambda_t = \lambda_0 + \sum_{i=1}^{r} \rho_i \lambda_{t-i} + \sum_{i=1}^{s} \gamma_i \xi_{t-i}, \tag{2}$$

where ξ_t, the innovation, equals:

$$\xi_t \equiv E[N_t|\Phi_t] - \lambda_t \equiv E[N_t|\Phi_t] - E[N_t|\Phi_{t-1}]. \tag{3}$$

In other words, the innovation results as the difference between the expectation of the Poisson random variable conditional to time t and its expectation conditional to time $t-1$.

In this framework, it is possible to show that, conditional to the information set, the expected jump component equals the mean of the conditional jump size, Θ, times the time-varying jump intensity, λ_t:

$$E[Z_t|\Phi_{t-1}] = \Theta\lambda_t. \tag{4}$$

We can not directly recover from market data the parameters of the model, but it is possible to estimate the elements of interest, Θ and λ_t. Chan and

Maheu (2002) model conditional jump intensity and size as function of observables and allow simple maximum likelihood estimation. We refer the reader to Chan and Maheu (2002) for further details about the model and the estimation approach. Differently from Chan and Maheu (2002) we set the parameters of $Y_{t,k}$ to be time invariant.

1.2 Jump factor construction

Our large dataset includes all the Center for Research in Security Prices (CRSP) assets with share code equal to 10 or 11, which covers NYSE and AMEX stocks until 1973 and adds NASDAQ stocks after that date. The sample includes a total of 24,122 equities over 89 years, from December 1925 until December 2014.[2] We obtain the parameters of the model using overlapping rolling windows with a size of one year each. Maximum likelihood estimations take place every year at the end of each month, in order to stick with the timing commonly in use to construct the mimicking portfolios for the momentum risk source. We run about a few millions estimations where each of them uses previous year simple daily returns as defined by CRSP.[3]

In order to capture the temporary presence of serial correlation, we consider an AR(2) process in equation 1 by imposing $l = 2$. The resulting equation does not change in time and across assets,

$$R_t = \mu + \phi_1 R_{t-1} + \phi_2 R_{t-2} + Z_t + \epsilon_t.$$

We restrict legs to one for the jump intensity process; this allows us to rewrite equation 2 as:

$$\lambda_t = \lambda_0 + \lambda_{t-1}(\rho_1 - \gamma_1) + \gamma_1 E[N_{t-1}|\phi_{t-1}].$$

Lastly, it is important to point out that we focus on the one-step-ahead values of the expected jump component in Equation 4. In this way we obtain the asset jump expectation for the following day.

Since each asset can show either positive or negative jumps, the range of values of the expected jump component (Equation 4) spans from negative to positive values. This comes from the sign of Θ that can be either positive or negative while λ_t is always positive. Figure I shows, for each estimation date, the number of assets with positive and negative expected jump component. The figure shows how the gap between the two groups tends to increase over time and that in almost 100% of the cases the number of positive expectations (expected jumps with positive sign) overcomes the number of negative expectations (expected jumps with negative sign).

---

[2] See Section A.1 of the Appendix for a detailed description of the monthly dataset dimension.

[3] CRSP daily return: $R_t = \frac{Price_t \times PriceAdjustmentFactor_t + CashAdjustment_t}{Price_{t-1}} - 1$.

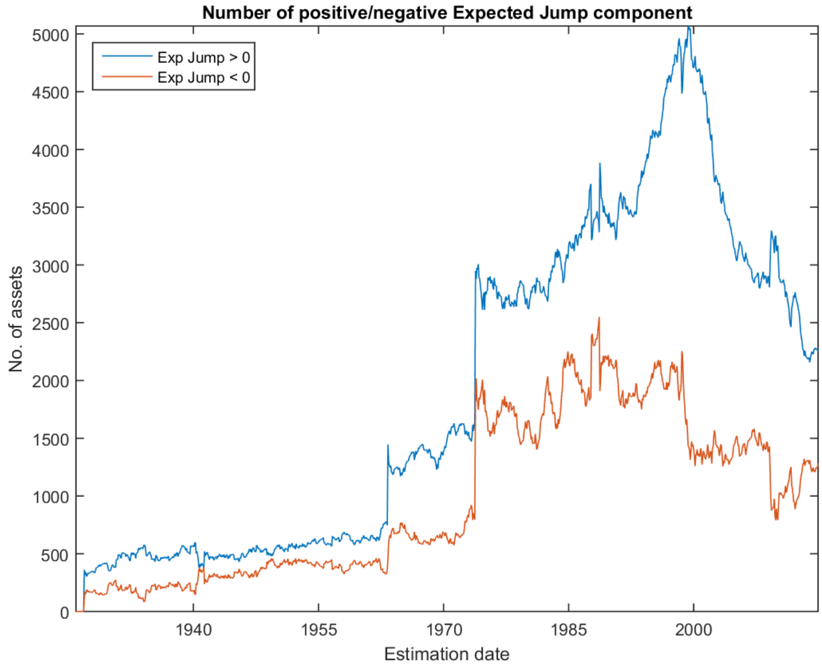

Figure I Expected Jump Factor sign. For each estimation date, from December 1925 till December 2014, it reports the number of assets for which the expected jump component ($\Theta\lambda_t$) is positive or negative. The estimation of the parameters uses overlapping rolling windows with a size of one year. Estimations take place every year at the end of each month and use previous year simple daily returns as defined by CRSP.

We then propose to recover a Jump Factor (or JF) as a factor-mimicking portfolio for the jump risk. For its construction we follow the approach that Fama and French (1993) use to build the size and value factors. In fact, both are portfolios mimicking risk sources, the size and BE/ME respectively. At the end of each month from January 1926 to December 2014, we sort all NYSE stocks on CRSP by *size* to determine the median breakpoint. The subsequent step is the allocation of all NYSE, Amex, and NASDAQ stocks to the two portfolios, Small and Big, according on the NYSE breakpoint. At the same dates we also split the NYSE, Amex, and NASDAQ stocks into three expected jump groups using $E[Z_t|\Phi_{t-1}]$: Low, Medium, and High. Using the ranked values of Expected jump component for NYSE stocks, we determine the breakpoints for the bottom 30%, the medium 40%, and the top 30%. From the intersection of the two size and the three expected jump groups we construct six portfolios: S/L, S/M, S/H, B/L, B/M, and B/H; adopting the same notation of Fama and French (1993). The re-balance of the portfolios takes place at the end of the each month, and for the time between two rebalances we calculate the monthly value-weighted returns for four of the six portfolios - S/L, S/H, B/L, and B/H. We then construct he Jump factor (JF) as the monthly difference between the average of the returns on the two low expected jump portfolios and the average of the returns on the two high expected jump portfolios:

$$JF = (1/2\, r_{S/H} + 1/2\, r_{B/H}) - (1/2\, r_{S/L} + 1/2\, r_{B/L}).$$

Coherently with Fama and French (1993), we believe that there exists a limited influence of the size, therefore, we focus only on the different return behaviours of low and high expected jump stocks.

To understand which is the proportion of assets that present a negative (positive) expectation with respect to the total number of assets, we report in Figure II the composition of the Low and High expected jump portfolios at each estimation date. It is clear that assets with negative expected jump

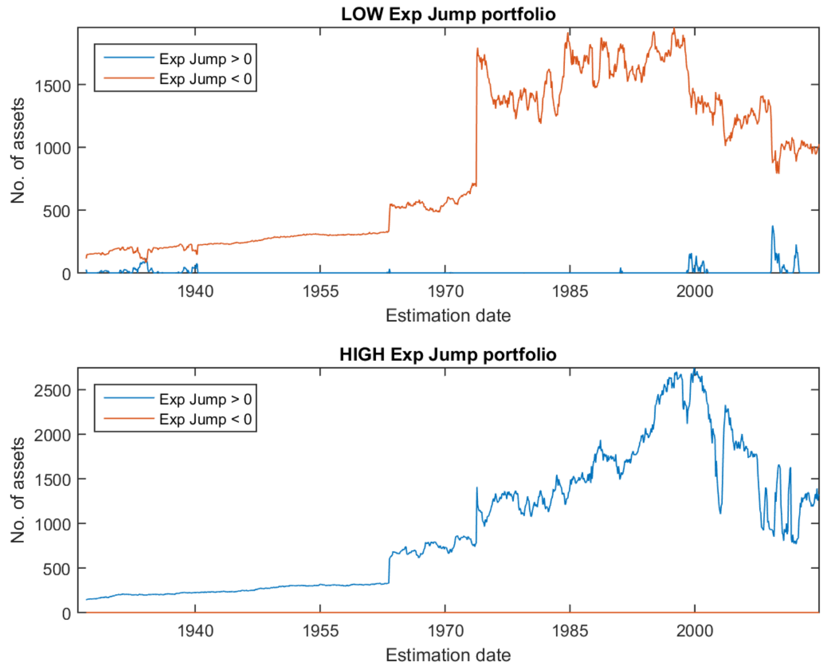

Figure II JF portfolios composition. For each estimation date, from December 1925 till December 2014, it shows the number of assets with positive and negative expected jump component ($\Theta\lambda_t$). The top panel plots the total number of assets with positive and negative expected jump component, considering only the stocks that are in the Low expected jump portfolio (expected jump component < bottom 30% breakpoint). Similarly, the bottom panel plots the same indicators but taking into account only the stocks that are in the High expected jump portfolio (expected jump component > top 30% breakpoint). The estimation of the parameters uses overlapping rolling windows with a size of one year. Estimations take place at the end of each month and use previous year simple daily returns as defined by CRSP.

component almost exclusively belong to the Low portfolio. Similarly, almost all assets with positive expected jump component flow into the High portfolio. The two portfolios, consequently, represent two opposite strategies: stocks with negative expected jumps versus stocks with positive expected jumps. From a risk-premium perspective, the sign of the expectation is useful in forecasting and understanding the sign of the factor loading. Investors demand a positive risk premium, measured as the extra return relative to the risk-free rate, for investing in risky assets. To understand the sign of the JF premium, we should focus on the signs of factor and corresponding factor loadings. If they coincide, we expect the JF premium to be positive. Since the JF covers negative and positive values, our expectation is that also the factor loading assumes positive and negative values.

Finally, Figure III shows the time series of the JF. The top panel compares the JF and its rolling mean, where the latter makes use of the last twelve monthly values of the JF. The bottom panel, instead, reports JF values that are preceded by a Jump Factor value of the same sign, that is we focus on the JF runs. The goal is to obtain a clear image of the JF clusters.

The plots not only make clear the tendency of the jump factor values to be clustered by sign, but also show some peculiar behaviours associated with the JF levels.[4] The top panel, in particular, shows that the factor and its mean assume values far from zero in periods of market turmoils as the 1929-1932 crises and the dot-com bubble crash around 2000. It is reasonable to forecast that the fear of future jumps increases and is more relevant in periods of greater market uncertainty. The empirical findings of Chan and Maheu (2002) suggest an explanation to this behaviour: autocorrelation in the conditional jump intensity is positive and persistent, which means that high probability of few (many) jumps today is generally followed by a high probability of few (many) jumps tomorrow. The JF historical behaviour suggests that its use could be relevant especially in periods of market turmoils.

1.3 A model including Jump Factor

We now briefly describe and evaluate traditional asset pricing models with a double objective: first of all, our interest is to verify the postulated impact of the JF on returns and the sign of its factor loading; secondly, we want to detect potential changes on other more traditional factors, both in terms of their loadings as well as for their significance. We empirically assess this goal by starting from a model that can work as benchmark due to its recognized performances: the Carhart (1997) 4-factor model. We will then extend the

[4] We identify the processes in charge of them by focusing on the behaviour of the jump probability. The Press (1967) hypothesis of a constant Poisson distribution has been undermined by Bates (1991) who verified that jump probability changes over time. Following this intuition Chan and Maheu (2002) assumes that the conditional jump intensity follows an ARMA process, thus we expect the transmission of this property to the factor via the time-varying jump intensity, λ_t.

Carhart (1997) model by adding the JF returns and thus putting forward a 5-factor model. We thus follow the existing standard in the literature, with results comparable to the reference works in this field of study (see for example Fama and French (1993) and Fama and French (2015)).

In line with the common interpretation of factor asset-pricing models, we can consider our 5-factor model as a performance-attribution model where coefficients and premia, on the factor-mimicking portfolios, represent the proportion of mean return due to five elementary strategies. For the 4-factor model these strategies cover stocks with high or low betas, stocks with large or small market capitalization, value or growth stocks, and one-year return momentum or contrarian stocks. Our new factor represents, instead, the elementary strategy of high versus low expected jump stocks.

It is now clear that our contribution to the existing models centers on the inclusion of a measure of jump sensitiveness, JF_t, that is the difference between the returns on portfolios of stocks with high and low jump expectations. Our 5-factor model time-series regression representation is:

$$\begin{aligned} R_{i,t} - R_{F,t} = \\ &\alpha_i + \beta_i MKT_t + \beta_{SMB,i} SMB_t + \beta_{HML,i} HML_t \\ &+ \beta_{MOM,i} MOM_t + \beta_{JF,i} JF_t + e_{i,t}, \end{aligned} \tag{5}$$

where $R_{i,t}$ is the return on a security or portfolio i, $R_{F,t}$ is the risk-free return, $MKT_t = (R_{M,t} - R_{F,t})$ is the excess return on a value weighted market portfolio, and e_{it} is a zero-mean residual. SMB_t (Small [market capitalization] Minus Big), HML_t (High [book-to-market ratio] Minus Low), and MOM_t are the returns on value-weighted factor-mimicking portfolios for, respectively, $Size$, book-to-market equity, and one-year momentum. For $\beta_{JF,i} = 0$ we fall back to the benchmark case: the Carhart (1997) 4-factor model.

If the exposure to the five factors, β_i, $\beta_{SMB,i}$, $\beta_{HML,i}$, $\beta_{MOM,i}$ and $\beta_{JF,i}$ capture all variation in expected returns, the intercept α_i in Equation 5 is zero for all securities and portfolios i.

2 Jumps factors and other factors returns

2.1 JF and other risk factors

Tables 1 and 2 show the summary statistics for the monthly returns of the market portfolio of stocks (MKT)and the mimicking portfolios for $size$ (SMB), book-to-market equity (HML), momentum (MOM), and expected jump (JF).

Volatilities and correlations give a perspective about the model capability of explaining time-series variations. For the former we can observe that

Table 1 Factors summary statistics. Summary statistics for the monthly factor returns in percent. RF is the one-month Treasury bill return. MKT is the market proxy. SMB and HML are Fama and French's factor-mimicking portfolios for size and book-to-market equity. MOM and JF are the factor-mimicking portfolio respectively for one-year momentum and expected jump component.

Factor portfolio	Mean monthly return %	Standard deviation	t-statistic for Mean=0
RF	0.28	0.25	36.38
MKT	0.65	5.40	3.94
SMB	0.22	3.23	2.22
HML	0.40	3.54	3.65
MOM	0.67	4.74	4.62
JF	-0.13	1.87	-2.32

the JF variance is the smaller among the factor-mimicking portfolio variances but it is still relatively high. For the latter, instead, factor-mimicking portfolio correlations, both among each other and with the market proxy, are low. In addition, all the correlations among factors and Risk-Free rate are not significantly different from zero at the 99% confidence level. Focusing on the JF for all but one correlations we reject the null hypothesis (zero correlation) at 1% significance level, and their values run from a minimum (in absolute value) 0.00 with HML to a maximum 0.58 with the market factor. The combined observation of high variances and low correlations suggests that the 5-factor model, and our new factor, can describe sizeable time-series variation. In addition, the low values of the cross-correlations indicate that multicollinearity should not affect the estimation of the factor loadings.

Turning now our attention to the first moment, we observe a range of values that goes from $|0.67|$% per month for the momentum to a still considerably high $|0.13|$% per month for the JF. In a time-series regression approach, they correspond to the average premiums per unit of risk and, from a statistical point of view, they are all significantly different from zero (5% significance level). Mean values reach the minimum with the JF but, from an investment perspective, it is still large (about $|1.6|$% per year). The high value of the JF mean also suggests that it explains a considerable part of the mean return variation on stock portfolios, at the cross-sectional level.

Lastly, it is interesting to notice that the mean monthly JF return is negative. As previously discussed, we construct the JF as the monthly return difference between High and Low expected jump portfolios. We also learnt from Figure II that Low and High expected jump portfolios are formed, respectively, by assets with negative and positive expected jump component. Thus, a negative mean value signals that assets with negative expected jump components are associated with higher returns than assets with positive expectations. This result is in line with the findings in the field of prospect theory (see among others Barberis and Thaler (2003) and Barberis (2013)).

Table 2 Factors correlations. RF is the one-month Treasury bill return. MKT is the market proxy. SMB and HML are Fama and French's factor-mimicking portfolios for size and book-to-market equity. MOM and JF are the factor-mimicking portfolio respectively for one-year momentum and expected jump component.

Factor	Cross correlations					
	RF	MKT	SMB	HML	MOM	JF
RF	1.00					
MKT	-0.07	1.00				
SMB	-0.05	0.33	1.00			
HML	0.02	0.23	0.11	1.00		
MOM	0.05	-0.34	-0.15	-0.40	1.00	
JF	-0.05	0.58	0.42	0.00	-0.18	1.00
	P-value for corr=0					
	RF	MKT	SMB	HML	MOM	JF
RF	0.00					
MKT	0.03	0.00				
SMB	0.08	0.00	0.00			
HML	0.62	0.00	0.00	0.00		
MOM	0.08	0.00	0.00	0.00	0.00	
JF	0.09	0.00	0.00	0.97	0.00	0.00

In particular, Kahneman and Tversky (1979) propose a model for gambles with two non-zero outcomes, which incorporates, among others, the concepts of loss aversion, diminishing sensitivity, and probability weighting. Some features of the model are useful in explaining the observed negative mean value. First, the model accounts for the greater sensitivity of people to losses (even small losses) than to gains with the same magnitude. In our context, the concept of loss aversion suggests that investors are more sensitive to large negative expected jumps than large positive expected jumps, thus leading to higher returns. Second, Kahneman and Tversky design the evaluation process of the agents to reflect the tendency of people to overweight (underweight) low (high) probabilities. In our framework this means that individuals overweight the negative tail of the expected jump distribution, thus putting to much weight on unlikely extreme outcomes.

Moreover, Kahneman and Tversky (1979) model helps explaining preferences for insurance. In this field Barberis and Thaler (2003) underline that, while according to the Kahneman and Tversky agents are risk-seeking over losses, the overweight of small probabilities leads to risk aversion for gambles that can cause large losses even if with a small probability. A similar reasoning can be applied to our case: agents require an insurance over large negative expected jumps.

2.2 JF and sorted portfolios

It is possible to use the five risk sources of our 5-factor model, alone or in combination, to form portfolios of stocks. Among all the possible combinations of two risk sources, we focus on those that are more relevant for our case study: stock portfolios formed according to size and book-to-market equity (or Size-BE/ME portfolios), and stock portfolios formed on the basis of size and expected jump (or size-expected jump portfolios).

The size-BE/ME portfolios were first introduced by Fama and French in 1993 who also proposed their use as dependent variables in time-series regressions. They not only produce a wide range of average excess returns, but also allow to study if SMB and HML capture common factors in stock returns. Similar reasons drive the choice of the second group of portfolios.

When used as dependent variables, our portfolios values define the range of returns that competing sets of risk factors must explain. In our case, these competing factors, are those forming the 4-factor and the 5-factor models since our interest is in understanding the importance and relevance of the JF.

2.2.1 Size-BE/ME portfolios

In June of each year we allocate NYSE, Amex, and NASDAQ stocks into five size quintiles and five book-to-market quintiles, using NYSE breakpoints. We then calculate value-weighted monthly excess returns from July of year t to June year $t+1$, when portfolios are reformed.

Table 3 shows the average monthly excess returns for the 25 size-BE/ME portfolios. The 25 stock portfolios produce a wide range of average excess returns, from 0.58% to 1.38% per month. The patterns in average returns confirm the presence of size and value effects: controlling for book-to-market, returns tend to decrease from small to big stocks, and controlling for $size$ average returns tend to increase with BE/ME. The only exceptions for the size effect are in the first two columns: the first does not show a clear relation between $size$ and average return, while in the second the only outlier is the low average return for the microcap. All but one average returns are more than two standard errors away from 0. The low-small portfolio shows a high standard deviation (12.36% per month) that makes its average return not significantly different from 0. This is a well-known problem already underlined by Merton (1980). Lastly, note that in each column volatility falls from small to big stocks, with the only exception of the High BE/ME portfolios (last column). For them we observe, in all but the small portfolio case, stable standard deviations with a value of about 8.6% per month.

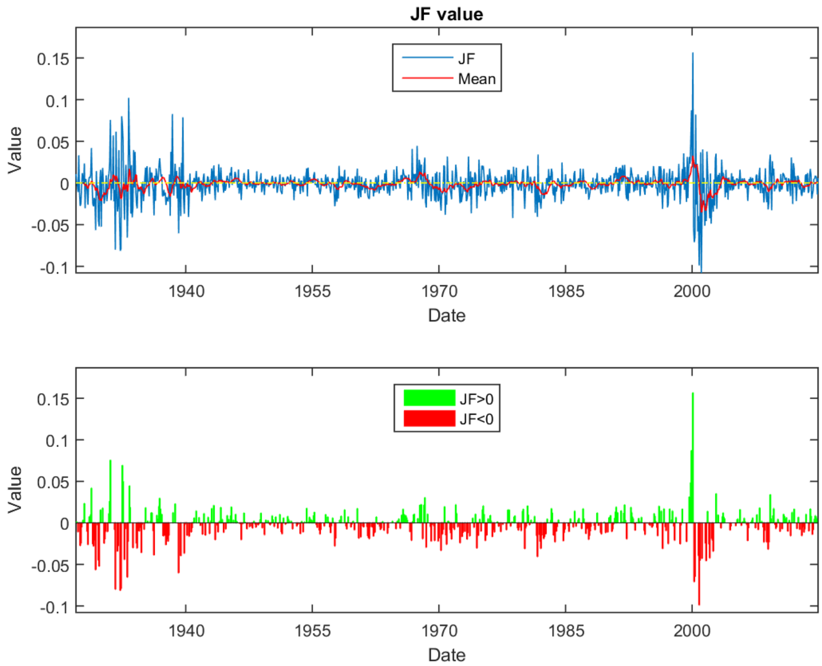

Figure III JF historical values. At the end of each month from December 1926 to December 2014, all NYSE, Amex, and NASDAQ stocks are allocated to the two size portfolios (Small and Big) and the three expected jump portfolios (Low, Medium, and High) according on the NYSE breakpoints. From their intersection we construct six portfolios: S/L, S/M, S/H, B/L, B/M, and B/H. The re-balance of the portfolios takes place at the end of each month, and for the time between two rebalances we calculate the monthly value-weighted returns for four of the six portfolios - S/L, S/H, B/L, and B/H. We then construct the Jump factor (JF) as the monthly difference between the average of the returns on the two low expected jump portfolios and the average of the returns on the two high expected jump portfolios. Top panel plots the time series of the JF and its mean. For the JF mean we use overlapping rolling windows with a size of one year. The calculation of the mean takes place at the end of every month and uses previous year monthly JF values. The bottom panel plots the monthly Jump Factors that are preceded by a Jump Factor of the same sign.

Table 3 Excess returns size-BE/ME portfolios. Value-weighted monthly percent excess returns of 25 stock portfolios formed on size and book-to-market. Portfolios are formed in June of year t, from June 1925 until June 2014, by the intersection of size and BE/ME quintiles. The allocation of NYSE, Amex, and NASDAQ stocks into five size quintiles and five book-to-market quintiles, makes use of the NYSE breakpoints.

	Book-to-market equity (BE/ME) quintiles										
Size quintile	Low	2	3	4	High		Low	2	3	4	High
	Means						Standard Deviations				
Small	0.58	0.71	1.02	1.18	1.38		12.36	9.91	9.08	8.40	9.40
2	0.63	0.91	1.02	1.08	1.26		8.03	7.55	7.32	7.51	8.76
3	0.71	0.90	0.96	1.01	1.17		7.47	6.52	6.57	6.96	8.53
4	0.72	0.75	0.87	0.96	1.05		6.29	6.15	6.51	6.87	8.72
Big	0.62	0.64	0.68	0.69	0.94		5.40	5.34	5.71	6.55	8.50
	t-statistic for Mean=0										
Small	1.53	2.33	3.68	4.59	4.79						
2	2.54	3.94	4.53	4.70	4.67						
3	3.10	4.50	4.74	4.75	4.45						
4	3.74	3.95	4.36	4.56	3.93						
Big	3.71	3.89	3.85	3.44	3.59						

2.2.2 Size-expected jump portfolios

The modelling of the size-expected jump portfolios is much like the six expected jump portfolios discussed in subsection 1.2. Using only NYSE stocks, we compute the breakpoints for *size* and *expected jump* that we then use to allocate NYSE, Amex, and NASDAQ stocks into five *size* groups and five *expected jump* groups at the end of each month. From the intersection of the groups we construct the 25 portfolios for which we then compute the value weighted excess returns monthly, from the end of one month to the end of the following month.

Table 4 reports average monthly excess returns for the 25 size-expected jump portfolios. The range of average excess returns covered by the stock portfolios goes from 0.58% to 3.61% per month. Similarly to Table 3, there is a negative relation between size and average return, when controlling for expected jump. When controlling for size there is no evidence of a clear pattern between expected jump and average excess returns. All average excess returns are more than two standard errors away from 0, and correspondent standard deviations fall from small to big stocks when considering a single column. Focusing on the first three rows, we observe very high values for the standard deviations, and in particular for the microcaps. High volatility is a characteristic of the small stocks already observed in Table 3. In this case, however, standard deviations for small portfolios are particularly high: from 14.43 to 25.66.

3 Missing factor

This section evaluates the appropriateness of adding the JF to the asset-pricing model by measuring its marginal effect on the abnormal performance.

Each month we estimate the 3-factor model loadings. Estimations cover the prior three years excess returns for all the 25 dependent portfolios, using a minimum of 30 observations. We report here the corresponding time-series regression equation:

$$R_{i,t} - R_{F,t} = \alpha_i + \beta_i MKT_t + \beta_{SMB,i} SMB_t + \beta_{HML,i} HML_t + e_{i,t},$$

where $R_{i,t}$ is the return on a security or portfolio i, $R_{F,t}$ is the risk-free return, $MKT_t = (R_{M,t} - R_{F,t})$ is the excess return on a value weighted market portfolio, SMB_t is the return on a value-weighted factor-mimicking portfolio for $Size$, HML_t is the return on a value-weighted factor-mimicking portfolio for book-to-market equity, and $e_{i,t}$ is a zero-mean residual.

We then use the results to compute the one-month abnormal return from the 3-factor model:

$$\breve{\alpha}_{i,t} = (R_{i,t} - R_{F,t}) - \hat{\beta}_i MKT_t - \hat{\beta}_{SMB,i} SMB_t - \hat{\beta}_{HML,i} HML_t. \qquad (6)$$

Table 4 Excess returns size-expected jump portfolios. Value-weighted monthly percent excess returns of 25 stock portfolios formed on size and expected jump component. Portfolios formation takes place at the end of each month from January 1926 until December 2014, by the intersection of size and expected jump quintiles. The allocation of NYSE, Amex, and NASDAQ stocks into five size quintiles and five expected jump quintiles, makes use of the NYSE breakpoints.

	expected jump (JF) quintiles									
Size quintile	Low	2	3	4	High	Low	2	3	4	High
	Means					Standard Deviations				
Small	3.61	2.05	2.02	2.93	2.10	18.49	14.43	15.22	17.11	25.66
2	1.86	1.79	1.49	1.62	1.56	12.32	11.49	11.40	11.91	14.37
3	1.28	1.38	1.22	1.35	0.80	10.10	10.03	9.20	10.50	10.75
4	1.06	0.93	1.13	1.00	0.79	8.19	7.28	8.08	8.68	9.68
Big	0.70	0.71	0.61	0.71	0.58	5.56	5.11	5.61	5.65	6.23
	t-statistic for Mean=0									
Small	5.89	4.22	3.89	5.12	2.54					
2	4.88	5.00	4.18	4.41	3.51					
3	4.13	4.48	4.30	4.16	2.43					
4	4.21	4.15	4.55	3.73	2.64					
Big	4.11	4.50	3.50	4.08	3.02					

We consider two different sets of dependent portfolios: (i) 25 size-BE/ME portfolios, and (ii) 25 size-expected jump portfolios. For each kind of dependent variable, we estimate three different time series regressions:

$$\breve{\alpha}_{i,t} = a_i + \beta_{M,i} MOM_t + \xi_{i,t}, \tag{7}$$

$$\breve{\alpha}_{i,t} = a_i + \beta_{J,i} JF_t + \xi_{i,t}, \tag{8}$$

$$\breve{\alpha}_{i,t} = a_i + \beta_{M,i} MOM_t + \beta_{J,i} JF_t + \xi_{i,t}. \tag{9}$$

Equations 7, 8, and 9 use data from either the previous three years and the full sample.

To understand how well MOM and JF help in explaining the abnormal return, we focus on estimated factor loadings resulting from equations 7, 8, and 9. The significance of the corresponding betas, using standard errors consistent to heteroscedasticity and autocorrelation, signals that the factors explain shared variation in stock returns that MKT, SMB, and HML are not able to capture, thus suggesting that MOM and JF are missing factors in the 3-factor model.

3.1 25 size-BE/ME portfolios

Full sample results in Table 5, highlight the importance of including the momentum factor in the 3-factor model. The absolute t-statistics on β_M greater than 1.645 are six in the single factor regressions, and nine in the multi-factor regressions, out of 25. The average value of the MOM slope, considering just those significantly different from 0, is in both cases about -0.04. Differently from the MOM, the relevance of the JF is less clear: in the single and in the multi-factor regressions, we observe respectively two and six portfolios for which β_J is more than 1.65 standard errors from 0 and with values from $|0.05|$ to $|0.29|$. The weaker performance of the JF may be due to the long time windows that we employ in the second regressions.

To further investigate this point it is useful to consider the three-years regressions. We present the results by focusing on portfolios, Table 6, and estimation dates, Figure IV. Table 6 shows, for each portfolio, the fraction of significant β_M and β_J, when considering a 90% confidence level. Even if significance percentages are generally greater for momentum, results for MOM and JF are not too different and suggest that both factors are relevant. β_M, from Equation 7, is on average significant in 25.18% of the regressions with a minimum of 13.9% and a maximum of 36.3%. Correspondent values for β_J, resulting from Equation 8, are slightly different: from 14.8% to 39.3%, with an average value of 21.84%. When we include both factors in the regression, Equation 9, we observe a small variation in average, 24.95%, minimum, 13.3%, and maximum, 36.4%, significance of β_M. For the JF, instead, only

Table 5 Size-BE/ME portfolios, full sample missing factors. Regressions of excess stock returns of 25 size-BE/ME portfolios on the excess market return (MKT) and the mimicking returns for the size (SMB), and book-to-market equity (HML) factors: December 1925 to December 2014. Using a rolling window of three years, it is possible to compute the correspondent monthly abnormal returns that we then use to test if MOM and JF are missing factors. To estimate the parameters of equations 7, 8, and 9 we use the full sample.

	Book-to-market equity (BE/ME) quintiles									
Size quintile	Low	2	3	4	High	Low	2	3	4	High
	$\alpha_{it} = a_i + \beta_{Mi} MOM_t + \xi_{it}$									
	β_M					$t(\beta_M)$				
Small	-0.07	-0.01	-0.06	0.00	-0.04	-1.10	-0.39	-1.61	-0.25	-1.65
2	-0.02	-0.03	-0.02	0.00	0.00	-0.88	-1.47	-1.35	-0.03	-0.30
3	-0.03	0.00	-0.01	-0.02	-0.04	-2.41	-0.22	-0.70	-1.15	-2.32
4	0.01	-0.02	-0.04	-0.02	-0.07	1.18	-1.35	-2.39	-1.00	-2.85
Big	-0.02	-0.01	-0.02	-0.03	0.03	-1.88	-1.43	-0.98	-1.23	0.39
	$\breve{\alpha}_{i,t} = a_i + \beta_{J,i} JF_t + \xi_{i,t}$									
	β_J					$t(\beta_J)$				
Small	0.29	-0.02	0.01	-0.04	-0.04	2.04	-0.28	0.17	-0.62	-0.65
2	-0.02	-0.06	0.03	0.01	-0.08	-0.31	-1.44	0.53	0.22	-1.62
3	0.08	-0.06	0.04	-0.01	0.05	1.46	-1.23	0.62	-0.16	0.86
4	-0.03	-0.02	-0.04	-0.05	0.11	-0.82	-0.41	-1.21	-1.39	1.64
Big	-0.01	-0.05	-0.02	0.06	0.01	-0.52	-2.30	-0.41	1.00	0.08
	$\breve{\alpha}_{i,t} = a_i + \beta_{M,i} MOM_t + \beta_{J,i} JF_t + \xi_{i,t}$									
	β_M					$t(\beta_M)$				
Small	-0.05	-0.01	-0.06	-0.01	-0.04	-0.80	-0.43	-1.71	-0.45	-1.93
2	-0.02	-0.03	-0.02	0.00	-0.01	-1.03	-1.77	-1.35	0.02	-0.80
3	-0.03	-0.01	-0.01	-0.02	-0.04	-1.85	-0.46	-0.53	-1.27	-2.28
4	0.01	-0.02	-0.05	-0.02	-0.06	0.98	-1.53	-2.62	-1.33	-2.70
Big	-0.02	-0.02	-0.02	-0.03	0.03	-2.02	-1.85	-1.10	-1.09	0.42
	β_J					$t(\beta_J)$				
Small	0.27	-0.03	-0.01	-0.04	-0.06	2.01	-0.35	-0.18	-0.68	-0.92
2	-0.03	-0.08	0.02	0.01	-0.08	-0.52	-2.01	0.39	0.22	-1.77
3	0.07	-0.07	0.04	-0.02	0.03	1.29	-1.25	0.57	-0.41	0.59
4	-0.02	-0.03	-0.06	-0.06	0.08	-0.63	-0.66	-1.99	-1.68	1.33
Big	-0.02	-0.06	-0.03	0.04	0.02	-0.91	-2.71	-0.61	0.80	0.27

Table 6 Size-BE/ME portfolios, three years window missing factors. Regressions of excess stock returns of 25 size-BE/ME portfolios on the excess market return (MKT) and the mimicking returns for the size (SMB), and book-to-market equity (HML) factors: December 1925 to December 2014. Using a rolling window of three years, it is possible to compute the correspondent monthly abnormal returns that we then use to test if MOM and JF are missing factors. To estimate the parameters of equations 7, 8, and 9 we use the prior three years of monthly data. The table reports, for each portfolio, the fraction of regressions with absolute t-statistics on β greater than 1.645.

	Book-to-market equity (BE/ME) quintiles									
Size quintile	Low	2	3	4	High	Low	2	3	4	High
	$\alpha_{it} = a_i + \beta_{Mi} MOM_t + \xi_{it}$					$\alpha_{it} = a_i + \beta_{Ji} JF_t + \xi_{it}$				
	β_M					β_J				
Small	35.30	23.90	33.70	21.80	24.40	34.90	15.60	16.40	15.50	15.70
2	27.10	27.80	21.40	19.00	13.90	20.60	24.80	23.70	18.30	15.50
3	24.30	19.30	24.90	30.00	18.10	14.80	18.20	39.30	27.30	17.90
4	18.50	20.70	27.50	17.00	33.70	21.70	27.40	31.00	24.10	24.80
Big	22.70	36.30	24.40	32.90	31.00	21.40	17.60	17.00	24.90	17.70
	$\alpha_{it} = a_i + \beta_{Mi} MOM_t + \beta_{Ji} JF_t + \xi_{it}$									
	β_M					β_J				
Small	34.10	25.80	27.60	23.80	26.40	28.80	15.70	16.50	20.50	19.70
2	28.80	29.00	23.20	17.10	13.30	16.00	22.20	28.80	16.60	18.60
3	23.40	24.60	30.50	22.50	14.90	18.10	17.60	41.40	29.40	14.60
4	20.20	19.20	29.00	20.40	25.90	21.20	27.70	32.50	21.90	21.90
Big	21.00	36.40	24.30	31.80	30.50	26.90	22.40	20.70	22.50	15.80

the minimum decreases to 14.6% while the maximum and the average show a small increase, respectively to 41.4% and 22.32%.

From Figure IV we obtain, for each estimation date, information on the fraction of portfolios for which the slopes are significant at the 90% confidence level. In the top panel there are the results for β_M of Equation 7

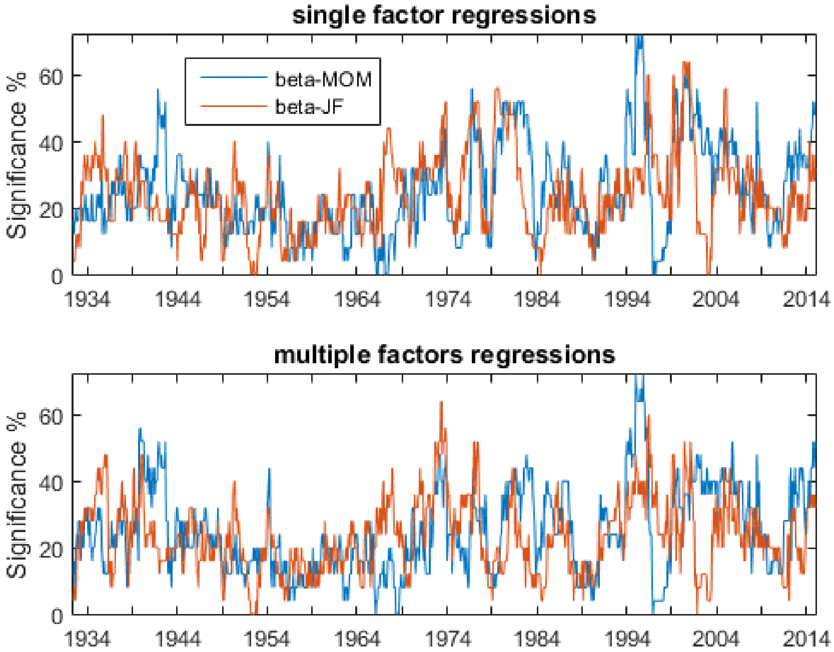

Figure IV Size-BE/ME portfolios, three years window missing factors. Regressions of excess stock returns of 25 size-BE/ME portfolios on the excess market return (MKT) and the mimicking returns for the size (SMB), and book-to-market equity (HML) factors: December 1925 to December 2014. Using a rolling window of three years, it is possible to compute the correspondent monthly abnormal returns that we then use to test if MOM and JF are missing factors. To estimate the parameters of equations 7, 8, and 9 we use the prior three years of monthly data. For each estimation date, the graph shows the fraction of portfolios with significant slopes (10% significance level). The top panel plots results for β_M of Equation 7 and β_J of Equation 8, while the bottom panel plots results for the slopes of Equation 9.

and β_J of Equation 8, while in the bottom panel the results for the slopes of Equation 9. Not only the two factors show similar behaviours, but in 41.43% and 40.42% of the dates in, respectively, the single- and multi-factor regressions, the fraction of significant β_J is greater than the corresponding fraction of β_M.

A global interpretation of the results requires a preliminary consideration: since jumps are short-time phenomena, regressions that use short time windows can better capture and describe their behaviour. In line with this statement, the use of the full time window leads to poor results for the JF. It is, however, more relevant to consider the results on shorter time-windows. In this last case performances for MOM and JF are similar and suggest that they are missing factors in the 3-factor model.

3.2 25 size-expected jump portfolios

Table 7 shows stronger results in favour of the inclusion in the 3-factor model for the JF with respect to the MOM. Considering a 90% confidence level, significant β_M are twenty in the single factor regressions and fourteen in the multi-factor regressions. The average absolute values of the MOM slope, considering just those significantly different from 0, are respectively 0.14 and 0.50. For the JF, instead, the number of portfolios for which β_J is more than 1.645 standard errors from 0 is six in both cases, with an average absolute slope (for significant portfolios) of 0.15.

As discussed before, since long time-windows impact on JF performance, we expect better results when using time-windows of three years. Results in Table 8 and Figure V confirm the importance of the JF.

In Table 8, the fraction of regressions with significant β_J is usually smaller than the corresponding fraction with significant β_M (10% significance level). Relative to the single-factor case, significance values on β_M, from Equation 7, show greater values with respect to the 25 size-BE/ME case: average, minimum and maximum values are 39.82%, 20.8%, and 53.8%. β_J, from Equation 8, shows a wider range of significance percentages, from 15.2% to 84.4%, and a lower average value, 30.07%.

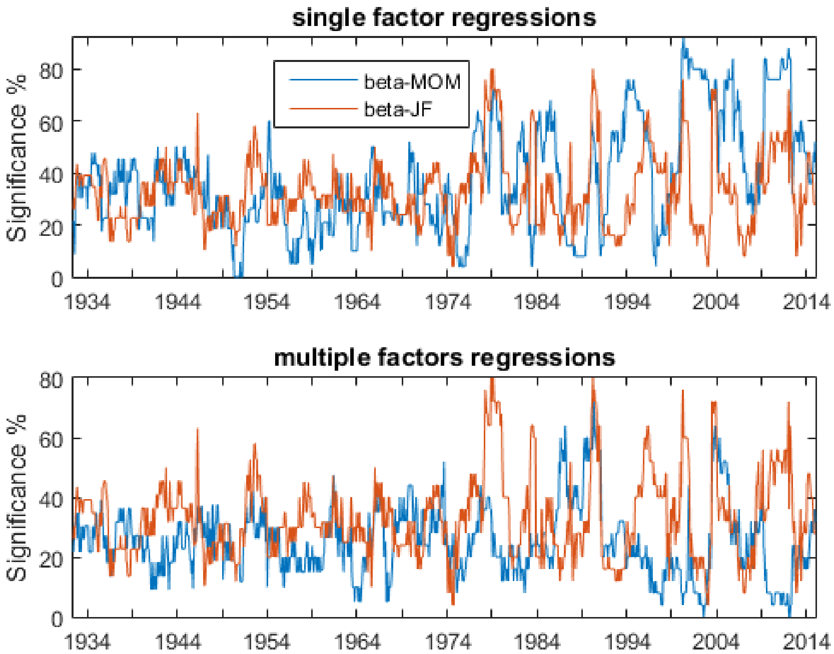

Figure V Size-expected jump portfolios, three years window missing factors. Regressions of excess stock returns of 25 size-expected jump portfolios on the excess market return (MKT) and the mimicking returns for the size (SMB), and book-to-market equity (HML) factors: December 1925 to December 2014. Using a rolling window of three years, it is possible to compute the correspondent monthly abnormal returns that we then use to test if MOM and JF are missing factors. To estimate the parameters of equations 7, 8, and 9 we use the prior three years of monthly data. For each estimation date, the graph shows the fraction of portfolios with significant slopes (10% significance level). The top panel plots results for β_M of Equation 7 and β_J of Equation 8, while the bottom panel plots results for the slopes of Equation 9.

Table 7 Size-expected jump portfolios, full sample missing factors. Regressions of excess stock returns of 25 size-expected jump portfolios on the excess market return (MKT) and the mimicking returns for the size (SMB), and book-to-market equity (HML) factors: December 1925 to December 2014. Using a rolling window of three years, it is possible to compute the correspondent monthly abnormal returns that we then use to test if MOM and JF are missing factors. To estimate the parameters of equations 7, 8, and 9 we use the full sample.

	Expected jump (JF) quintiles									
Size quintile	Low	2	3	4	High	Low	2	3	4	High
				$\alpha_{it} = a_i + \beta_{Mi} MOM_t + \xi_{it}$						
			β_M					$t(\beta_M)$		
Small	-0.06	-0.16	-0.11	-0.28	-0.75	-0.52	-2.30	-0.86	-3.00	-2.15
2	-0.19	-0.08	-0.08	-0.15	-0.23	-3.39	-1.65	-1.31	-1.84	-1.67
3	-0.17	-0.13	-0.07	-0.10	-0.10	-4.75	-3.43	-2.40	-3.12	-1.95
4	-0.06	-0.02	-0.07	-0.08	-0.04	-2.11	-0.75	-2.19	-3.13	-1.80
Big	-0.03	0.02	0.00	0.03	-0.02	-1.91	1.67	-0.21	2.37	-1.84
				$\alpha_{it} = a_i + \beta_{Ji} JF_t + \xi_{it}$						
			β_J					$t(\beta_J)$		
Small	-0.47	0.05	-0.17	-0.06	1.55	-1.24	0.25	-0.64	-0.31	1.16
2	0.01	-0.06	0.00	-0.05	0.10	0.10	-0.53	0.00	-0.29	0.49
3	-0.15	-0.06	-0.06	-0.06	0.07	-1.37	-0.60	-0.79	-0.66	0.51
4	-0.14	-0.12	-0.04	0.09	0.16	-2.07	-1.86	-0.49	1.29	1.96
Big	-0.14	-0.11	-0.01	0.02	0.23	-3.07	-5.08	-0.18	0.44	6.22
				$\alpha_{it} = a_i + \beta_{Mi} MOM_t + \beta_{Ji} JF_t + \xi_{it}$						
			β_M					$t(\beta_M)$		
Small	2.26	0.16	1.27	0.66	0.75	3.32	0.38	2.79	2.01	0.68
2	0.34	0.61	0.55	0.24	0.18	1.66	2.59	2.04	1.25	0.66
3	0.00	0.07	0.18	0.10	-0.42	-0.02	0.63	1.85	1.04	-3.55
4	-0.05	-0.01	0.14	-0.03	-0.28	-0.93	-0.22	2.34	-0.59	-4.24
Big	-0.03	0.06	-0.08	0.08	-0.06	-0.98	2.06	-1.98	2.10	-1.68
			β_J					$t(\beta_J)$		
Small	-0.47	0.05	-0.17	-0.06	1.55	-1.24	0.25	-0.64	-0.31	1.16
2	0.01	-0.06	0.00	-0.05	0.10	0.10	-0.53	0.00	-0.29	0.49
3	-0.15	-0.06	-0.06	-0.06	0.07	-1.37	-0.60	-0.79	-0.66	0.51
4	-0.14	-0.12	-0.04	0.09	0.16	-2.07	-1.86	-0.49	1.29	1.96
Big	-0.14	-0.11	-0.01	0.02	0.23	-3.07	-5.08	-0.18	0.44	6.22

Table 8 Size-expected jump portfolios, three years window missing factors. Regressions of excess stock returns of 25 size-expected jump portfolios on the excess market return (MKT) and the mimicking returns for the size (SMB), and book-to-market equity (HML) factors: December 1925 to December 2014. Using a rolling window of three years, it is possible to compute the correspondent monthly abnormal returns that we then use to test if MOM and JF are missing factors. To estimate the parameters of equations 7, 8, and 9 we use the prior three years of monthly data. The table reports, for each portfolio, the fraction of regressions with absolute t-statistics on β greater than 1.645.

	Expected jump (JF) quintiles									
Size quintile	Low	2	3	4	High	Low	2	3	4	High
	$\alpha_{it} = a_i + \beta_{Mi} MOM_t + \xi_{it}$					$\alpha_{it} = a_i + \beta_{Ji} JF_t + \xi_{it}$				
	β_M					β_J				
Small	46.60	47.00	53.80	48.80	49.80	21.90	18.10	25.70	32.60	28.40
2	44.10	29.30	47.60	42.80	42.30	15.20	20.50	17.90	21.60	26.50
3	48.50	38.80	33.20	42.90	43.10	25.20	21.20	17.10	17.90	33.60
4	42.60	28.30	47.50	41.20	33.70	36.70	32.60	29.20	30.00	39.40
Big	46.50	20.80	27.70	26.00	22.60	59.10	47.10	21.50	28.40	84.40
	$\alpha_{it} = a_i + \beta_{Mi} MOM_t + \beta_{Ji} JF_t + \xi_{it}$									
	β_M					β_J				
Small	26.00	33.90	32.00	20.30	21.10	21.90	18.10	25.70	32.60	28.40
2	27.30	20.70	25.20	23.70	36.50	15.20	20.50	17.90	21.60	26.50
3	25.70	16.10	17.40	19.10	51.00	25.20	21.20	17.10	17.90	33.60
4	12.30	21.50	16.50	24.90	45.50	36.70	32.60	29.20	30.00	39.40
Big	11.60	23.60	24.30	14.60	19.00	59.10	47.10	21.50	28.40	84.40

Results for the JF do not change when we include both factors in the regressions, Equation 9. For the MOM, instead, we observe a decrease in average, 24.39%, minimum, 11.6%, and maximum, 51.0%, significance with respect to the single-factor regressions.

Figure V shows, for each estimation date, the fraction of portfolios slopes different from 0 using a 10% significance level. Both in the top panel, which considers the single-factor regressions (Equation 7 and Equation 8), and in the bottom panel, that focuses on the multi-factor regressions (Equation 9) we observe a general higher level of significance with respect to the size-BE/ME case (Figure IV). We obtain similar results for MOM and JF when using single-factor regressions, and in 43.04% of the dates the fraction of significant β_J is greater than the corresponding fraction of β_M. Results are even more in favour of the JF when considering multi-factor regressions: levels of significance differs often substantially and in 60.79% of the dates the percentage of significant β_J is larger than the percentage of significant β_M.

Lastly, Table 9 reports the values, and corresponding P-values, of a Wald

Table 9 3-Factor regression, missing factor slope test. Regressions of excess stock returns of 25 size-BE/ME portfolios and 25 size-expected jump portfolios on the excess market return (MKT) and the mimicking returns for the size (SMB), and book-to-market equity (HML) factors: December 1925 to December 2014. Using a rolling window of three years, it is possible to compute the correspondent monthly abnormal returns that we then use to test if MOM and JF are missing factors. The table reports the test values, $\boldsymbol{\beta}'\boldsymbol{\Omega}^{-1}\boldsymbol{\beta}$, checking $H0 : \boldsymbol{\beta} = \mathbf{0}$, and the corresponding P-values. We substitute the unknown quantities $\boldsymbol{\beta}$, and $\boldsymbol{\Omega}$ with their estimated correspondents $\hat{\boldsymbol{\beta}}$, and $\hat{\boldsymbol{\Omega}} = \frac{1}{T}\sum_{t=1}^{T} \hat{\boldsymbol{\xi}}_t \hat{\boldsymbol{\xi}}_t'$. The test statistic, under H_0, has asymptotic distribution χ^2_N, where $N = 25$.

	P-value			
Portfolios	Single regression		Multiple regression	
	β_M	β_J	β_M	β_J
size-BE/ME	1.00	1.00	1.00	1.00
size-exp Jump	1.00	1.00	1.00	1.00
	Test-value			
Portfolios	Single regression		Multiple regression	
	β_M	β_J	β_M	β_J
size-BE/ME	0.0067	0.0247	0.0074	0.0283
size-exp Jump	0.0630	0.4754	0.6893	0.4767

test statistics that verifies that the betas (βs) from full sample regressions are simultaneously equal to zero: $\boldsymbol{\beta} = \mathbf{0}$. The correspondent test, $\boldsymbol{\beta}'\boldsymbol{\Omega}^{-1}\boldsymbol{\beta}$ is distributed as a χ^2_N where N equals the number of dependent portfolios, so in our case $N = 25$. To empirically compute the test it is necessary to substitute the unknown quantities $\boldsymbol{\beta}$, and $\boldsymbol{\Omega}$ with their estimated correspondents $\hat{\boldsymbol{\beta}}$, and $\hat{\boldsymbol{\Omega}} = \frac{1}{T}\sum_{t=1}^{T} \hat{\boldsymbol{\xi}}_t \hat{\boldsymbol{\xi}}_t'$. $\hat{\boldsymbol{\beta}}$ is the vector of estimated betas, $\hat{\boldsymbol{\Omega}}$ is the variance-covariance matrix of the regression residuals, and T is the length of the portfolios time series. Each row focus on a different set of dependent portfolios and reports, respectively, the P-values for β_M of Equation 7, β_J of Equation 8, and β_M and β_J of Equation 9. It is clear from the results that it is not possible to reject the null hypothesis in all the cases in analysis. A possible motivation for the negative results lies in the use of such a long time-window. The test assumes greater values in the size-expected jump cases but β_M and β_J test values are closer in the size-BE/ME case.

The analysis in this section support the inclusion of the JF, as well the MOM, in the 3-factor model. Results for the JF are stronger when focusing on short time-windows, reflecting the short-term peculiarity of the jumps, and when using 25 size-expected jump portfolios. This last evidence, suggests that the inclusion of the JF is relevant, in particular, for portfolios with strong size and expected jump tilts.

4 Common variation in stock returns

We turn now to the asset-pricing tests: we use time series regressions to analyse if the JF captures common variation in stock returns. In a time-series regressions framework, variables related to average returns must proxy for sensitivity to common (shared and thus undiversifiable) risk factors in returns, when assets are priced rationally. Slopes and R^2_{adj} values give evidence if the JF captures shared variation in stock returns that other factors are not able to explain. To judge the improvements that it is possible to obtain using our new factor we employ three different sets of dependent variables: 25 size-BE/ME portfolios, 25 size-expected jump portfolios, and all CRSP single assets with share code 10 or 11.[5]

4.1 25 size-BE/ME portfolios

To analyse the role of the JF we follow a process in two steps. We examine (a) regressions that use the four-factor model ($\beta_{JF,i} = 0$) and (b) regressions that use the five-factor model (Equation 5). Table 10 shows the results using model (a) while Table 11 summarizes the results for model (b).

The tables make clear the importance of the standard three Fama and French factors: market, size and value. For β, β_{SMB}, and β_{HML} values we observe minor changes when moving from the 4-factor to the 5-factor model. Their statistical behaviour is also very similar; market βs are always more than three standard errors from 0; and, with few exceptions, the absolute t-statistics on the SMB and HML slopes are greater than 1.645 in both tables. Our results confirm previous Fama and French findings: the three factors capture strong common variation in stock returns.

[5]We report single assets regression results in Section A.2 of the Appendix.

Table 10 Size-BE/ME portfolios, 4-Factor regression results. Regressions of excess stock returns of 25 size-BE/ME portfolios on the excess market return (MKT) and the mimicking returns for the size (SMB), book-to-market equity (HML), and momentum (MOM) factors: December 1925 to December 2014. t-statistics make use of Newey-West heteroscedasticity and autocorrelation consistent (or HAC) standard errors.

	Book-to-market equity (BE/ME) quintiles									
Size quintile	Low	2	3	4	High	Low	2	3	4	High
			β					$t(\beta)$		
Small	1.25	1.07	1.02	0.94	0.98	14.66	36.79	26.30	46.43	36.41
2	1.07	1.00	0.99	0.97	1.06	41.68	61.15	41.60	52.96	53.58
3	1.11	1.02	0.99	0.99	1.11	63.30	67.46	38.96	53.23	49.63
4	1.08	1.02	1.02	1.02	1.19	82.81	45.73	48.98	51.50	56.21
Big	1.02	0.98	0.97	1.02	1.14	98.73	59.38	42.55	44.53	35.24
			β_{SMB}					$t(\beta_{SMB})$		
Small	1.44	1.53	1.25	1.21	1.31	7.59	9.07	24.40	12.12	17.31
2	1.12	0.97	0.84	0.83	0.90	14.93	13.18	12.63	12.17	17.93
3	0.81	0.50	0.44	0.46	0.58	20.45	9.88	8.76	8.11	8.86
4	0.32	0.24	0.20	0.21	0.28	8.48	4.87	3.96	6.25	6.18
Big	-0.15	-0.21	-0.24	-0.17	-0.14	-5.31	-7.95	-8.99	-5.08	-1.72
			β_{HML}					$t(\beta_{HML})$		
Small	0.36	0.23	0.46	0.57	0.89	1.72	3.15	12.26	14.04	18.73
2	-0.23	0.13	0.36	0.57	0.88	-6.17	2.47	7.15	12.72	24.14
3	-0.25	0.05	0.32	0.56	0.85	-5.55	1.21	7.08	11.23	19.11
4	-0.35	0.08	0.33	0.55	0.93	-14.12	1.80	7.39	11.47	20.97
Big	-0.27	0.02	0.31	0.63	0.96	-14.50	0.56	10.56	18.77	18.21
			β_{MOM}					$t(\beta_{MOM})$		
Small	-0.17	-0.01	-0.13	-0.03	-0.06	-1.77	-0.27	-3.26	-0.92	-1.84
2	-0.03	-0.05	0.01	0.02	-0.03	-1.05	-1.92	0.30	0.79	-1.41
3	-0.06	0.00	0.01	-0.01	-0.06	-2.45	-0.03	0.25	-0.17	-1.92
4	0.01	-0.01	-0.03	-0.05	-0.07	0.33	-0.48	-1.09	-1.81	-2.38
Big	-0.02	-0.01	-0.02	-0.04	-0.12	-1.25	-0.71	-0.96	-1.73	-2.32

Table 11 Size-BE/ME portfolios, 5-Factor regression results. Regressions of excess stock returns of 25 size-BE/ME portfolios on the excess market return (MKT) and the mimicking returns for the size (SMB), book-to-market equity (HML), momentum (MOM), and expected jump (JF) factors: December 1925 to December 2014. t-statistics make use of Newey-West heteroscedasticity and autocorrelation consistent (or HAC) standard errors.

	Book-to-market equity (BE/ME) quintiles										
Size quintile	Low	2	3	4	High		Low	2	3	4	High
			β						$t(\beta)$		
Small	1.07	1.09	1.01	0.96	0.99		14.90	23.44	31.86	44.63	36.58
2	1.08	1.06	1.03	1.01	1.07		50.92	63.07	41.73	43.92	52.74
3	1.08	1.07	1.03	1.04	1.10		62.23	54.66	38.33	45.73	42.51
4	1.06	1.08	1.09	1.04	1.17		62.28	47.79	46.40	51.45	49.69
Big	1.03	1.02	0.99	1.02	1.07		83.51	57.47	42.69	48.24	30.02
			β_{SMB}						$t(\beta_{SMB})$		
Small	1.29	1.55	1.24	1.22	1.32		6.45	8.50	21.41	11.48	16.24
2	1.13	1.02	0.87	0.85	0.91		13.73	15.48	15.92	13.44	17.60
3	0.79	0.54	0.47	0.50	0.58		18.94	13.46	11.20	11.43	9.67
4	0.30	0.29	0.26	0.22	0.26		9.85	8.49	6.54	5.76	5.44
Big	-0.14	-0.17	-0.22	-0.17	-0.20		-5.30	-9.29	-7.27	-4.65	-2.48
			β_{HML}						$t(\beta_{HML})$		
Small	0.44	0.22	0.46	0.57	0.88		2.02	3.07	12.45	14.41	17.67
2	-0.24	0.10	0.34	0.55	0.87		-6.87	2.30	7.57	13.12	22.92
3	-0.24	0.03	0.31	0.53	0.85		-5.24	0.96	8.29	12.09	18.82
4	-0.34	0.06	0.30	0.54	0.94		-14.60	1.63	8.71	10.94	20.90
Big	-0.27	0.00	0.30	0.63	0.99		-15.39	-0.10	9.68	20.40	18.78
			β_{MOM}						$t(\beta_{MOM})$		
Small	-0.16	-0.02	-0.13	-0.03	-0.06		-1.84	-0.29	-3.29	-0.98	-1.86
2	-0.03	-0.05	0.01	0.02	-0.03		-1.08	-2.43	0.24	0.77	-1.45
3	-0.06	0.00	0.01	-0.01	-0.06		-2.37	-0.13	0.20	-0.30	-1.91
4	0.01	-0.02	-0.04	-0.05	-0.07		0.42	-0.76	-1.40	-1.91	-2.29
Big	-0.02	-0.02	-0.02	-0.04	-0.11		-1.28	-0.97	-1.04	-1.74	-2.31
			β_{JF}						$t(\beta_{JF})$		
Small	0.97	-0.11	0.06	-0.11	-0.04		2.82	-0.67	0.45	-1.13	-0.31
2	-0.04	-0.33	-0.23	-0.18	-0.07		-0.42	-4.89	-2.73	-2.07	-1.30
3	0.17	-0.25	-0.19	-0.28	0.05		2.81	-2.74	-1.66	-3.14	0.50
4	0.12	-0.34	-0.39	-0.10	0.13		1.74	-3.88	-5.13	-1.13	1.55
Big	-0.03	-0.22	-0.09	-0.01	0.38		-0.81	-4.05	-1.29	-0.06	3.02

When focusing, instead, on the momentum factor we still do not notice relevant variations in the significance and values of β_{MOM}. The slopes on MOM that are significant (10% significance level) are 40% of the portfolios in both cases.

The most interesting results are those about the JF: β_{JF} assumes values from -0.39 to the maximum of 0.97, and in 52% of the cases its absolute t-statistic is greater than 1.645 (in eleven cases even greater than 2). Considering also Table 12, it is clear that the JF captures shared variation in stock returns that MKT, SMB, HML, and MOM are not able to explain. The values of the coefficient of determination, do not decrease 80% of the times when we include the JF in the regressions. The lower values of R^2_{adj} are, in the 4-factor case, in correspondence of the small portfolios (first row) where R^2_{adj} spans from 65.7% to 94.1%. This apparent lower ability of the model, that is a consequence of the high volatility of microcaps (see Table 3), seems only limitedly captured by the JF.[6]

Table 12 Size-BE/ME portfolios, R^2_{adj}. Regressions of excess stock returns of 25 size-BE/ME portfolios on the returns of the 4-factor model (excess market return (MKT) and the mimicking returns for the size (SMB), book-to-market equity (HML), and momentum (MOM) factors), and on the returns of the 5-factor model (excess market return (MKT) and the mimicking returns for the size (SMB), book-to-market equity (HML), momentum (MOM), and expected jump (JF) factors). December 1925 to December 2014.

	Book-to-market equity quintiles				
Size quintile	Low	2	3	4	High
			R^2_{adj} 4-factor model		
Small	0.657	0.820	0.892	0.927	0.941
2	0.909	0.931	0.938	0.951	0.951
3	0.932	0.926	0.926	0.932	0.928
4	0.934	0.922	0.914	0.921	0.915
Big	0.955	0.933	0.908	0.925	0.840
			R^2_{adj} 5-factor model		
Small	0.669	0.820	0.892	0.928	0.941
2	0.909	0.935	0.940	0.952	0.951
3	0.933	0.929	0.928	0.936	0.927
4	0.935	0.928	0.921	0.921	0.915
Big	0.955	0.936	0.909	0.925	0.844

4.2 25 size-expected jump portfolios

Similarly to the previous case, we consider (a) regressions that use the four-factor model, Table 13, and (b) regressions that use the five-factor model,

[6]We present a comparison of the models in terms of residuals correlation and heteroskedasticity in section A.3 of the Appendix.

Table 14.

The slopes of the standard three Fama and French factors show minor changes when moving from the 4-factor to the 5-factor model. We observe

Table 13 Size-expected jump portfolios, 4-Factor regression results. Regressions of excess stock returns of 25 size-expected jump portfolios on the excess market return (MKT) and the mimicking returns for the size (SMB), book-to-market equity (HML), and momentum (MOM) factors: December 1925 to December 2014. t-statistics make use of Newey-West heteroscedasticity and autocorrelation consistent (or HAC) standard errors.

	Expected jump quintiles									
Size quintile	Low	2	3	4	High	Low	2	3	4	High
			β					$t(\beta)$		
Small	0.72	0.80	0.58	0.85	1.56	5.07	9.32	4.77	6.01	2.86
2	0.85	0.85	0.87	0.99	1.10	14.56	16.65	20.83	15.72	13.72
3	0.94	0.91	0.94	1.12	1.19	25.04	23.75	36.72	45.63	38.88
4	1.02	0.94	0.96	1.12	1.24	57.20	39.93	30.35	70.03	47.62
Big	0.98	0.95	1.00	1.02	1.11	53.58	86.81	75.16	76.64	82.03
			β_{SMB}					$t(\beta_{SMB})$		
Small	2.00	1.35	1.77	1.64	2.05	8.49	7.41	5.54	7.18	6.91
2	1.64	1.41	1.40	1.60	1.57	11.79	6.78	9.58	13.00	6.02
3	1.40	1.36	1.19	1.38	1.45	10.72	7.24	8.28	9.84	19.81
4	0.98	0.83	0.99	0.98	1.20	13.10	11.43	7.06	21.48	23.10
Big	-0.02	-0.09	-0.05	0.01	0.05	-0.42	-4.34	-2.09	0.65	2.48
			β_{HML}					$t(\beta_{HML})$		
Small	1.16	0.95	0.97	0.76	1.68	5.19	3.84	3.48	3.11	3.36
2	0.94	0.88	0.78	0.67	0.91	7.66	7.69	8.93	6.34	4.38
3	0.63	0.75	0.60	0.57	0.36	8.91	8.01	8.24	8.44	4.44
4	0.39	0.37	0.45	0.40	0.24	9.90	7.53	8.42	11.20	3.93
Big	0.08	0.05	0.09	0.05	-0.04	2.83	2.46	3.62	2.13	-1.43
			β_{MOM}					$t(\beta_{MOM})$		
Small	-0.48	-0.17	-0.54	-0.25	-0.58	-2.70	-1.54	-2.61	-1.64	-2.20
2	-0.37	-0.10	-0.37	-0.31	-0.45	-4.25	-1.35	-5.85	-4.58	-3.31
3	-0.25	-0.16	-0.15	-0.26	-0.23	-4.15	-2.75	-3.35	-4.03	-3.65
4	-0.15	-0.04	-0.14	-0.18	-0.17	-6.30	-1.31	-3.74	-8.64	-3.29
Big	-0.02	0.04	-0.01	0.05	-0.03	-0.82	2.67	-0.41	3.17	-1.71

Table 14 Size-expected jump portfolios, 5-Factor regression results. Regressions of excess stock returns of 25 size-expected jump portfolios on the excess market return (MKT) and the mimicking returns for the size (SMB), book-to-market equity (HML), momentum (MOM), and expected jump (JF) factors: December 1925 to December 2014. t-statistics make use of Newey-West heteroscedasticity and autocorrelation consistent (or HAC) standard errors.

	Expected jump quintiles									
Size quintile	Low	2	3	4	High	Low	2	3	4	High
			β					$t(\beta)$		
Small	0.79	0.82	0.58	0.85	1.35	5.90	8.55	3.88	5.79	3.42
2	0.87	0.92	0.89	1.02	0.95	12.44	12.21	16.94	14.01	12.21
3	1.06	1.01	1.02	1.14	1.10	22.14	18.60	29.79	29.95	27.98
4	1.10	1.01	1.01	1.08	1.12	55.24	43.41	38.54	48.55	47.27
Big	1.06	1.00	1.00	1.00	1.02	70.18	107.41	84.16	68.01	73.78
			β_{SMB}					$t(\beta_{SMB})$		
Small	2.06	1.37	1.76	1.64	1.86	8.00	7.33	6.13	6.34	4.53
2	1.66	1.46	1.43	1.62	1.44	12.94	7.19	9.04	12.42	5.37
3	1.50	1.44	1.25	1.40	1.38	13.16	8.17	9.44	9.97	17.00
4	1.05	0.90	1.04	0.95	1.10	18.37	15.92	7.23	22.32	28.26
Big	0.05	-0.05	-0.05	-0.01	-0.02	1.80	-3.04	-1.96	-0.32	-0.78
			β_{HML}					$t(\beta_{HML})$		
Small	1.13	0.94	0.97	0.76	1.77	5.02	3.68	3.19	2.98	3.23
2	0.93	0.85	0.77	0.66	0.97	7.18	7.75	9.52	5.75	4.85
3	0.57	0.71	0.56	0.56	0.40	8.14	8.13	8.76	8.60	5.46
4	0.35	0.33	0.43	0.41	0.30	11.25	8.99	9.84	11.85	6.31
Big	0.04	0.03	0.09	0.06	0.00	1.56	1.89	3.37	2.53	0.16
			β_{MOM}					$t(\beta_{MOM})$		
Small	-0.48	-0.18	-0.54	-0.25	-0.58	-2.70	-1.55	-2.65	-1.68	-2.19
2	-0.37	-0.10	-0.37	-0.31	-0.44	-4.27	-1.44	-5.99	-4.64	-3.42
3	-0.26	-0.17	-0.16	-0.26	-0.23	-4.42	-3.05	-3.65	-4.06	-3.90
4	-0.16	-0.04	-0.14	-0.18	-0.16	-7.63	-1.70	-4.04	-8.12	-3.49
Big	-0.02	0.04	-0.01	0.05	-0.03	-1.05	3.33	-0.40	3.07	-1.65
			β_{JF}					$t(\beta_{JF})$		
Small	-0.37	-0.11	0.05	-0.03	1.17	-0.86	-0.43	0.12	-0.07	1.16
2	-0.12	-0.34	-0.15	-0.12	0.82	-0.54	-1.35	-0.72	-0.51	2.71
3	-0.66	-0.53	-0.40	-0.12	0.50	-3.41	-3.08	-2.55	-0.87	2.84
4	-0.46	-0.42	-0.29	0.19	0.67	-5.19	-3.62	-1.88	1.92	9.90
Big	-0.43	-0.30	0.03	0.13	0.49	-11.77	-9.58	0.43	2.34	7.15

greater changes for β, β_{SMB} and β_{HML} when using as dependent variables size-expected jump portfolios with respect to the size-BE/ME portfolios. Similarly to the size-BE/ME case, the absolute t-statistics on the MKT slopes are always greater than 3, and the βs of SMB and HML are, with few exceptions, significant at the 10% significance level.

The values of β_{MOM} never change considerably, while the slopes on MOM that are significant considering a 90% confidence interval increase from 76% to 88% of the portfolios.

In Section 1.2 we underlined how in order to get a positive JF premium the sign of the factor loading must coincide with the sign of the factor. As shown in Figure II, assets with negative (positive) expected jump principally belong to lower (higher) JF portfolios. To read JF slope results in Table 14, it is however important to remind that the mean monthly JF return is negative, thus leading to forecast a negative JF premium.

Focusing on the fourteen (56% of the portfolios) significant β_{JF}s (10% significance level), it is possible to note that the value of β_{JF} increases with the expected jump, controlling for the size. Moreover, in correspondence of lower JF portfolios (left columns) JF slopes are < 0, while for higher JF portfolios (right columns) the slopes are > 0. Thus, for portfolios composed by assets with negative expected jump (left columns) the exposition to the JF has a positive impact on return since the signs of factor and loadings are both negative. The opposite holds for portfolios formed by assets with positive expected jump (right columns), with divergent signs of JF premium and JF slopes. Results are coherent with the observations in Section 2, where we underlined how investors seek an insurance against large losses related to large negative expected jumps, even if they present a small probability.

Adding the information content of Table 15, it is possible to infer that the JF captures common variation that the other factors miss. When we add the JF in the regressions, the values of the R^2_{adj} do not decrease 76% of the times.[7]

In Section 2 we observed how the 25 size-expected jump portfolios present peculiar high volatility. By comparing Table 4 and Table 15 it is evident a correspondence between higher volatilities and lower R^2_{adj}. The first three rows show volatilities from 9.20 to 25.66 and R^2_{adj} from 0.326 to 0.869 in the 5-factor model. For the microcaps R^2_{adj} assumes values from a minimum of 32.6% (5-factor model) to a maximum of 42.0% (4-factor model). The microcaps R^2_{adj} average increase when moving from the 4-factor to the 5-factor is of 0.04 basis points; it was 0.25 basis points in the $size - BE/ME$ case. Anyway, the table shows major improvements for the big portfolios. The R^2_{adj} performance is good and we observe comparatively more significance for the β_{JF} than in the $size - BE/ME$ case. The bottom line is that, also in

[7] We also compare the models in terms of residuals correlation and heteroskedasticity, results are available in Section A.3 of the Appendix.

Table 15 Size-expected jump portfolios, R^2_{adj}. Regressions of excess stock returns of 25 size-expected jump portfolios on the returns of the 4-factor model (excess market return (MKT) and the mimicking returns for the size (SMB), book-to-market equity (HML), and momentum (MOM) factors), and on the returns of the 5-factor model (excess market return (MKT) and the mimicking returns for the size (SMB), book-to-market equity (HML), momentum (MOM), and expected jump (JF) factors). December 1925 to December 2014.

	Expected jump quintiles				
Size quintile	Low	2	3	4	High
			R^2_{adj} 4-factor model		
Small	0.404	0.347	0.420	0.327	0.353
2	0.680	0.605	0.668	0.709	0.605
3	0.817	0.791	0.830	0.869	0.852
4	0.937	0.913	0.917	0.955	0.932
Big	0.946	0.961	0.951	0.950	0.952
			R^2_{adj} 5-factor model		
Small	0.404	0.346	0.419	0.326	0.357
2	0.680	0.606	0.668	0.709	0.611
3	0.826	0.796	0.833	0.869	0.856
4	0.944	0.920	0.919	0.956	0.942
Big	0.958	0.968	0.951	0.951	0.965

this case, the JF is able to capture the variation in the dependent variables left unexplained by the 4-factor model.

4.3 Model performance

The regression slopes and R^2_{adj} values in Tables 10 to 15 establish that the JF proxy for a common risk factor in stock returns. We now study how well the 4-factor and the 5-factor models explain average excess returns on the portfolios of Table 3 and Table 4. The focus is on their relative performances since they allow to judge the improvements that the JF brings. The time-series regressions in this section use excess returns, on portfolios and single assets, as dependent variables. The explanatory variables are, instead, either excess returns ($MKT = RM - RF$) or returns on zero-investment portfolios (SMB, HML, MOM, and JF). In these regressions if the asset-pricing model completely captures expected returns, the intercept must be indistinguishable from 0 (Merton (1973)). We use the estimated intercepts to test whether the average premiums for the common risk factors in returns explain the cross-section of average returns. In addition to the simple comparison of the estimated values, we present three indicators, first introduced by Fama and French (2015), and one test on the intercepts. Table 16 reports a detailed description of both the indicators and the test.

The first indicator (A1) considers the average absolute intercepts. The interpretation is straightforward: the model that better describes the cross-

Table 16 Intercept indicators and test. a_i is the estimated intercept for portfolio i. $\bar{r}_i$ is the deviation of portfolio i from the cross-sectional average: $\bar{r}_i = \bar{R}_i - \bar{R}$ where $\bar{R}_i$ is the time-series mean for portfolio i and $\bar{R} = \frac{1}{n}\sum_{i=1}^{n} \bar{R}_i$. $\hat{\alpha}_i^2 = A(a_i^2) - SE_{a_i}^2$ and $\hat{\mu}_i^2 = A(\bar{r}_i^2) - SE_{\bar{r}_i}^2$ where SE are the standard errors. T is the length of the portfolios time series, N is the number of portfolios, and K is the number of factors. $\bar{\boldsymbol{f}}$ is the vector of mean value of the factors. $\hat{\boldsymbol{\Sigma}}_{\boldsymbol{f}}$ is the estimated variance-covariance matrix of the factors. $\hat{\boldsymbol{\alpha}}$ is the vector of estimated intercepts for the 25 portfolios. $\hat{\boldsymbol{\Omega}}$ is the estimated variance-covariance matrix of the regression residuals.

A1	$\frac{1}{n}\sum_{i=1}^{n} \lvert a_i \rvert$
A2	$\frac{\frac{1}{n}\sum_{i=1}^{n} \lvert a_i \rvert}{\frac{1}{n}\sum_{i=1}^{n} \lvert \bar{r}_i \rvert}$
A3	$\frac{\frac{1}{n}\sum_{i=1}^{n} \lvert \hat{\alpha}_i^2 \rvert}{\frac{1}{n}\sum_{i=1}^{n} \lvert \hat{\mu}_i^2 \rvert}$
A4	$\frac{T-N-K}{N}\left(1 + \bar{\boldsymbol{f}}\hat{\boldsymbol{\Sigma}}_{\boldsymbol{f}}^{-1}\bar{\boldsymbol{f}}\right)^{-1}\hat{\boldsymbol{\alpha}}'\hat{\boldsymbol{\Omega}}^{-1}\hat{\boldsymbol{\alpha}}$

section of returns, has intercepts that, on average, are closer to zero. The other two ratios allow to compare the models in terms of proportion of cross-section of expected returns left unexplained. The numerators measure the dispersion of the estimated intercepts produced by a given model (4-factor or 5-factor model) for a set of dependent variables. The denominators, instead, measure the dispersion of the excess returns in the dependent portfolios (25 size-BE/ME portfolios or 25 size-expected jump portfolios).

The second ratio (A2) has as numerator the average absolute intercept and as denominator the average absolute deviation. We obtain $\bar{r}_i$, the deviation of portfolio i from the cross-sectional average, as the difference between its time-series average excess return, $\bar{R}_i$, and the cross-sectional average of all the 25 $\bar{R}_i$: $\bar{r}_i = \bar{R}_i - \bar{R}$. It is important to notice that we are using estimated values and not true values. As a consequence estimation errors inflate both the numerator and the denominator: the true intercept is just the difference between the estimated intercept and the estimation error, $\alpha_i = a_i - e_i$, and the expected deviation results by subtracting from the estimated deviation the estimation error, $\mu_i = \bar{r}_i - \epsilon_i$.

The last ratio (A3), has a design that should help to correct the measurement errors affecting (A2). Consider first the denominator where μ_i is the deviation of portfolio i from the mean: $\mu_i = x_i - \bar{x}$. Its average value is zero, $E(\mu_i) = E(x_i - \bar{x}) = E(x_i) - E(\bar{x}) = \bar{x} - \bar{x} = 0$, and its variance is $Var(\mu_i) = E[\mu_i^2] - [E(\mu_i)]^2 = E[\mu_i^2]$. It is now clear that the average value of μ_i^2, $A(\mu_i^2)$, is the cross-section variance of expected portfolio returns. Focus now on the numerator: α_i is a constant ($E(\alpha_i) = \alpha_i$ and $Var(\alpha_i) = 0$) and, consequently, $E(a_i^2) = E[(\alpha_i + e_i)^2] = E[\alpha_i^2 + e_i^2 + 2\alpha_i e_i] = \alpha_i^2 + E(e_i^2)$. The estimates of α_i^2 and μ_i^2 are respectively $\hat{\alpha}_i^2 = A(a_i^2) - SE_{a_i}^2$ and $\hat{\mu}_i^2 = A(\bar{r}_i^2) - SE_{\bar{r}_i}^2$ where SE are the standard errors. Summing up it is possible to rewrite (A2) in terms of squared intercepts and deviations, the proportion of portfolios variance left unexplained by a model.

Lastly, test (A4) is the Gibbons, Ross and Shanken (1989) or GRS test statistic that allows to investigate if the intercepts (α) are simultaneously equal to 0.[8] The test statistic, under the null hypothesis, has distribution $F_{N,T-N-K}$, where N equals the number of dependent portfolios, so in our case $N = 25$, T is the length of the portfolios time series and K is the number of factors, consequently $K = 4$ or $K = 5$.

Table 17 shows the results using the 4-factor and the 5-factor models when considering 25 size-BE/ME portfolios.

Table 18 reports the results for the 25 size-expected jump portfolios both for the 4-factor model and the 5-factor model.

The first thing to notice is that the number of intercepts significantly different from 0 (90% confidence level), employing HAC standard errors, never increase when the 5-factor model is in use. In the 25 $Size-B/M$ case they are 7 in both cases while in the size-expected jump case they move from 18 to 10.

Tables 17 and 18 show that the average absolute intercept (A1) is always smaller for the five-factor model. A1 decreases of 18% and 6% respectively for the 25 $Size-B/M$ portfolios and the 25 size-expected jump portfolios. These results suggest that applying the 4-factor model to portfolios with strong size and value inclinations or to portfolios with strong size and expected jump tilts may lead to poor results.

Also for A2 and A3 we observe positive results for both sets of portfolios. A2 and A3 decrease respectively of 18% and 47% in the $Size - B/M$ case and of 6% and 5% in the size-expected jump case.

Lastly, the P-values of test A4, reject the null hypothesis ($H_0 : \boldsymbol{\alpha} = \mathbf{0}$) for both the 4-factor and the 5-factor regressions when using size-BE/ME dependent portfolios as well as using size-expected jump portfolios. The clear results of the tests on the intercepts in favour of the 5-factor model, together with the increase in 80% ($Size - B/M$) and 76% (size-expected jump) of the portfolios R_{adj}^2 suggest that the 5-factor model outperforms the 4-factor model.

[8] For more details on the GRS test statistic, we refer the reader to Section A.4 of the Appendix.

5 Sub periods

From section 4, where we obtain the results using the full sample, we learned that the JF proxies for common risk factors in stock returns, and that the 5-factor model outperforms the 4-factor model in explaining average excess returns on the dependent portfolios. In this section we repeat the tests to judge how the relevance of the JF changes in different periods. To this end, we consider four equally spaced sub-periods of 267 months: from December 1925 till March 1948, from April 1948 till June 1970, from July 1970 till September 1992, and from October 1992 till December 2014.

We present the results separately for regressions that use as dependent portfolios: 25 size-BE/ME portfolios, and 25 size-expected jump portfolios. For each dependent variable we compare results for (a) the four-factor model ($\beta_{JF,i} = 0$) and (b) the five-factor model (equation 5). We refer the reader to Section A.5 of the Appendix for full results about estimated parameters, coefficients of determination, and intercept tests.

5.1 25 size-BE/ME portfolios

Estimated coefficient results for market, size, and value are in line with full-sample regressions: no important changes when moving from the 4-factor to the 5-factor model in all sub-periods. While market betas are always significant (90% confidence level), β_{SMB} and β_{HML} show little variation in time and a minimum significance level in the first sub-period for both models.

Relative to the momentum we observe an increase in the significance level for the second sub-period when adding the JF. It is however in the last sub-period that the momentum factor seems to have greater importance, with a percentage of β_{MOM} significantly different from 0, considering a 10% significance level, of 36%.

The most interesting results are for the JF, which slope average value spans from $|0.012|$ in the third sub-period to $|0.185|$ in the fourth sub-period. Moreover, JF slopes show high levels of significance in all the sub-periods with the minimum in the 1925-1948 window, 16%, and the maximum in the 1970-1992 and 1992-2014 windows, 64%.

Considering also the coefficients of determination, it is clear that the JF captures shared variation in stock returns that MKT, SMB, HML, and MOM miss. The R^2_{adj}s, in each sub-period, are higher when using the 5-factor model with respect to the 4-factor model for 52%, 64%, 88%, and 92% of the 25 dependent portfolios.

Information on JF slopes and coefficients of determination suggest that the JF ability to capture common variation in stock returns is stronger in the two last sub-periods.

To study how well the 5-factor model explains average excess returns, we investigate the behaviour of estimated αs and intercept tests (A1, A2, A3 , and A4). The fraction of intercepts different from 0 (90% confidence level) decreases in the first, second, and last sub-periods when we use the 5-factor model. Intercept tests reinforce the idea of a superiority of model (b), as the values of the tests A1, A2, and A3 do not increase when we add the JF to the model in all but the first sub-period. Nonetheless, in three out of four sub-periods, the P-values of test A4 does not allow to reject the null hypothesis that $H_0 : \boldsymbol{\alpha} = \mathbf{0}$, at the 1% significance level, when using the 5-factor model. This means that the model is a complete description of expected returns.

5.2 25 size-expected jump portfolios

Similarly to the full-sample case, market, size, and value show minor changes when moving from the 4-factor to the 5-factor model. Both β_{SMB} and β_{HML} experience little variation in time and high percentages of significance.

For the momentum factor, instead, we report a decrease in significance for the second sub-period and an increase in the third sub-period. Model (a) leads to significance percentages of 48% (1948-1970) and 40% (1970-1992) that become 44% and 44% with model (b).

Relative to the JF, we not only obtain large average slope values, from -0.282 to 0.288, but also high levels of significance (90% confidence level). In the two last sub-periods β_{JF} is more than 1.645 standard errors from 0 for more than 60% of the portfolios (76% in the 1970-1992 window and 64% in the 1992-2014 window). Stunning significance percentages, suggest that the JF captures shared variation in stock returns that is missed by MKT, SMB, HML, and MOM. A comparison of the coefficients of determinations for the 4-factor and the 5-factor models, reinforce this finding. In each sub-period, the R^2_{adj}s from the 5-factor model are higher than the correspondent values in the 4-factor case for 52%, 64%, 88%, and 72% of the 25 dependent portfolios. As in the $size - BE/ME$ case, the ability of the JF to capture common variation in stock returns seems to be stronger in the last two sub-periods.

As previously discussed, to judge the ability of the model of explaining the cross-section of average returns we focus on estimated αs and tests on the intercepts. The inclusion of the JF in the model has a positive effect on α for which the fraction of intercepts different from 0 (90% confidence level) decreases in the first, second and last sub-periods respectively from 60% to 44%, from 32% to 28% and from 36% to 32%. Tests on the intercept give clear evidence of a superiority of the 5-factor model. A1-A3 test values decrease or do not vary in the great majority of the cases when the model includes the JF. The P-values of test A4 are, instead, clearly in favour of the 5-factor model: when we add the JF to the model the test never allows to reject the null hypothesis that $H_0 : \boldsymbol{\alpha} = \mathbf{0}$.

6 Risk Premium

In section 1.3 we introduced our 5-factor model, designed such as the assets excess returns obey a linear relationship with their exposures to various sources of risk, equation 5. Our interest is now in understanding what are the risk premia associated with those factors.

The risk premium, γ, measures the extra return an investor demands for investing in the asset relative to the risk-free rate. So, in our case, the total risk premium is the sum of different premiums:

$$\gamma_{tot} = \gamma_{MKT} + \gamma_{SMB} + \gamma_{HML} + \gamma_{MOM} + \gamma_{JF}.$$

To empirically compute the risk premiums we employ two similar approaches: Black, Jensen and Scholes (1972) and Fama and MacBeth (1973). Both approaches use a two pass technique.
In the first step, it is necessary to regress each asset's return on the time series of the factor realizations, to obtain the estimated beta coefficients of the portfolios on the factors:

$$\begin{aligned} R_{i,t} - R_{F,t} = & \\ & \alpha_i + \beta_i MKT_t + \beta_{SMB,i} SMB_t + \beta_{HML,i} HML_t \\ & + \beta_{MOM,i} MOM_t + \beta_{JF,i} JF_t + e_{i,t}. \end{aligned} \quad (10)$$

The second pass, instead, requires to regress, at each time t, the cross-section of assets returns against their beta coefficients:

$$\begin{aligned} R_{i,t} - R_{F,t} = & \\ & \hat{\beta}_i \gamma_{MKT,t} + \hat{\beta}_{SMB,i} \gamma_{SMB,t} + \hat{\beta}_{HML,i} \gamma_{HML,t} \\ & + \hat{\beta}_{MOM,i} \gamma_{MOM,t} + \hat{\beta}_{JF,i} \gamma_{JF,t} + \epsilon_{i,t}. \end{aligned} \quad (11)$$

In this way we obtain five time series of risk premia coefficients, $\hat{\gamma}$, each of length T.

It is important to notice that the regressors we use in the second step are not the real betas, which are unknown, but the estimated betas. This introduces the error-in-the-variables problem. As suggested among the others by Fama and MacBeth (1973), we attenuate this problem by using portfolios, 25 size-BE/ME portfolios and 25 size-expected jump portfolios, instead of assets as dependent variables. The $\hat{\beta}$'s of portfolios are more precise estimates of true β's than $\hat{\beta}$'s for single assets.

The difference between the Black et al. (1972) approach and the Fama and MacBeth (1973) approach lies in the choice of the explanatory variables in the second pass, $\hat{\beta}$s. According to the former we use full sample β estimates, while for the latter rolling β estimates.

Table 19 shows the estimated monthly percentage risk premia using Black et al. (1972) approach, $\breve{\gamma}$, and Fama and MacBeth (1973) approach, $\tilde{\gamma}$, and the correspondent t-statistics for $H_0 : \breve{\gamma} = 0$ and $H_0 : \tilde{\gamma} = 0$.[9]

Focusing on $\breve{\gamma}$, we observe that in the size-BE/ME case, with the exception of the JF factor portfolio for which we observe a negative, even if not statistically significant, risk premium, all other estimates are positive and fairly close to their factor portfolio mean monthly percentage excess return. Moreover, the estimated risk premiums for MKT and HML are more than two standard errors from 0. For the size-expected jump case, instead, we observe a negative risk premium for the SMB, MOM, and JF factor portfolios. In this case all absolute t-statistics on the risk premiums are greater than 2.

When considering, instead, $\tilde{\gamma}$ we obtain in the size-BE/ME case positive estimated values, which are also fairly close to their factor portfolio mean monthly percentage excess return and often statistically different from zero. In particular, the absolute t-statistics on risk premiums for MKT, SMB, and HML are greater than 2. In the size-expected jump case, instead, we observe a negative and statistically significant risk premium for MOM and JF.

We further test if the differences between estimated parameters and factor means (or $\bar{\boldsymbol{f}}$) are statistically simultaneously different from 0.[10] Table 20 reports the P-values of the tests for the 25 size-BE/ME portfolios and 25 size-expected jump portfolios. It is possible to observe that, using the Black et al. (1972) approach as well as the Fama and MacBeth (1973) approach, we can reject the null hypothesis at the 10% significance level only for the second group of portfolios.

The presence of factors not spanned by the assets might explain the unsatisfactory results. The following section presents the variation to the standard two pass technique, the Hou and Kimmel (2006) correction, we use to attenuate this problem and the corresponding corrected results.

6.1 Hou and Kimmel extrapolation correction

Hou and Kimmel (2006) define extrapolation as the phenomenon that arises when the factors of a linear factor model are not spanned by assets. In this case, in fact, the risk premium of a factor presents two components: the risk premium of the factor mimicking portfolio and an extrapolation of the risk premiums of the factors spanned components to the unspanned components. By purchasing the appropriate securities we can realize only the former. We can consider the latter a real risk premium only if additional assets that complete the market exist and the model is able to price them correctly.

Using 25 size-BE/ME portfolios, and 25 size-expected jump portfolios

[9] Sections A.6.1 and A.6.2 of the Appendix report detailed descriptions of the processes in place to compute risk premiums and correspondent t-statistics, both in the Black et al. (1972) case and in the Fama and MacBeth (1973) case.

[10] For further details about the tests we refer the reader to Section A.6.3 of the Appendix.

in the Black et al. (1972) and the Fama and MacBeth (1973) approaches, leads to compute risk premiums that are affected by extrapolation. In fact, we treat the factors as if they are unspanned even if they are traded assets that investors can buy. An investor who can only trade in the 25 portfolios (25 size-BE/ME portfolios or 25 size-expected jump portfolios) is not able to perfectly replicate the returns on the 5 factor portfolios. Hou and Kimmel (2006) suggest to augment the 25 dependent portfolios with the five factor portfolios, thus making the factors spanned.

To understand the change in the investment opportunities that we would introduce by the Hou and Kimmel (2006) correction, we regress the monthly returns of the factor portfolios on the monthly returns of the 25 other portfolios:

$$\begin{aligned} MKT_t &= \alpha_{MKT} + \beta_{MKT,1} R_{1,t} + \cdots + \beta_{MKT,25} R_{25,t} + \varepsilon_{MKT,t}, \\ SMB_t &= \alpha_{SMB} + \beta_{SMB,1} R_{1,t} + \cdots + \beta_{SMB,25} R_{25,t} + \varepsilon_{SMB,t}, \\ HML_t &= \alpha_{HML} + \beta_{HML,1} R_{1,t} + \cdots + \beta_{HML,25} R_{25,t} + \varepsilon_{HML,t}, \\ MOM_t &= \alpha_{MOM} + \beta_{MOM,1} R_{1,t} + \cdots + \beta_{MOM,25} R_{25,t} + \varepsilon_{MOM,t}, \\ JF_t &= \alpha_{JF} + \beta_{JF,1} R_{1,t} + \cdots + \beta_{JF,25} R_{25,t} + \varepsilon_{JF,t}. \end{aligned}$$

The corresponding R^2 values in Table 21 give us an idea of how well, using the 25 portfolios, we can replicate the returns of the factor portfolios. In the size-BE/ME case the high R^2 values for MKT, SMB, and HML suggest that the part not spanned by the 25 portfolios of the three Fama and French factors is small. The same conclusion does not hold for MOM and JF. Their low R^2 values, respectively 0.30 and 0.53, tell us that an investor who can only purchase the 25 size-BE/ME portfolios is not able to replicate the returns on MOM and JF. In the size-expected jump case, instead, we observe large R^2 values for MKT, SMB, and JF. The R^2 values for HML and MOM are both very low: 0.40 for the former and 0.28 for the latter.

The addition of the five factor portfolios significantly changes the investment opportunity set both in the size-BE/ME case and in the size-expected jump case. These results justify the inclusion of the factor portfolios in the set of dependent assets.

We repeat the regressions of the previous subsections but applying the Hou and Kimmel (2006) extrapolation correction. Tables 22 and 23 present the results for both the Black et al. (1972) approach and the Fama and MacBeth (1973) approach.

Relative to the Black et al. (1972) approach, we observe that in the size-BE/ME case all but the JF estimates are positive and close to their factor portfolio mean monthly percentage excess returns. In addition, results show that not only the absolute t-statistics on the risk premiums for MKT and HML, and MOM are greater than 3, but also that the estimated JF risk premium is significantly different from 0, at the 5% significance level. For the size-expected jump case, the correction resolves the anomaly we observed in the uncorrected case of negative risk premiums for SMB and

MOM. Estimated premiums are also, with the exception of the JF, more than two standard errors from 0.

In the Fama and MacBeth (1973) case, we observe positive estimated values, both using the size-BE/ME and the size-expected jump portfolios, for all but the JF. Also in this case, the negative estimated risk premium observed in the uncorrected case is no longer present when we apply the Hou and Kimmel (2006) extrapolation correction. In the size-BE/ME case we obtain four significant risk premiums: 5% significant level for SMB, and 1% significant level for MKT, HML, and MOM. In the size-expected jump case, instead, only for SMB and MOM we cannot reject the null, while the other risk premiums, and in particular the JF, are significant at the 95% confidence level.

Lastly, comparing Table 20 and Table 23 we do not observe large differences in the P-values. Therefore, also using Hou and Kimmel (2006) corrected regressions, we can reject the null hypothesis that the differences between estimated risk premiums and factor means are simultaneously equal to 0 only in the size-expected jump case.

7 Conclusions

This paper investigates the presence of a new common jump risk factor in stock returns and tests whether it captures the cross-section of average returns. We construct our factor, the Jump Factor (or JF), starting from the observation that it is possible to infer market fear of future jumps from observed returns using a model for stock returns with time-varying conditional jump intensity: the model of Chan and Maheu (2002). The large values of JF mean (-0.13% monthly return) and volatility (1.87) and its low correlations with the other factors (minimum and maximum correlations are respectively -0.18 and 0.58), show that the JF can explain much of the variation in returns both in time and cross-section.

Missing factor analysis provides supporting results about the usefulness of adding the JF to the Carhart (1997) asset-pricing model and justifies its inclusion. JF and MOM slopes, resulting from correspondent regressions, show similar behaviours thus suggesting that both factors are relevant and that it is possible to consider them missing factors in the Fama and French (1993) 3-factor model.

Empirical evidence also supports the hypothesis that the expected jump component proxy, in stock returns, for sensitivity to a common risk factor. We empirically investigate an extended CAPM model, our 5-factor model, and find that the new factor captures shared variation in stock returns that the four factors of the Carhart (1997) model (MKT, SMB, HML, and MOM) are not able to explain. The slopes on JF (or β_{JF}), resulting from 5-factor time series regressions, range respectively from -0.39 to 0.97 using 25 size-

BE/ME dependent portfolios, and from -0.66 to 1.17 for 25 size-expected jump dependent portfolios. They are not only large, in absolute value, but also often statistically different from 0. Indeed, considering a 90% confidence level, we obtain respectively 52% and 56% significant β_{JF}. A confirm to the power of our factor in capturing common variation comes also from the values of the coefficients of determination: the inclusion of the JF in the asset-pricing model, does not decrease R^2_{adj} values at least 76% of the times.

Lastly, we compute the risk premiums associated with the five factors of our new model (γ_{MKT}, γ_{SMB}, γ_{HML}, γ_{MOM}, and γ_{JF}), and find that they are close to their factor portfolio mean monthly percentage excess returns. Risk premiums signs and values are in line with our expectations since the premiums reflect the extra return that an investor demands for investing in the asset relative to the risk-free rate. Note that only for the JF we obtain a negative risk premium, reflecting investors desire to be insured against large negative expected jumps. We also observe that there is no statistical difference between estimated premiums and factor means and that risk premiums are, in most of the cases, statistically different from 0 at standard confidence levels.

References

Andersen, Torben G, Luca Benzoni and Jesper Lund (2002), 'An empirical investigation of continuous-time equity return models', *The Journal of Finance* **57**(3), 1239–1284.

Bakshi, Gurdip, Charles Cao and Zhiwu Chen (1997), 'Empirical performance of alternative option pricing models', *The Journal of finance* **52**(5), 2003–2049.

Ball, Clifford A and Walter N Torous (1983), 'A simplified jump process for common stock returns', *Journal of Financial and Quantitative analysis* **18**(01), 53–65.

Ball, Clifford A and Walter N Torous (1985), 'On jumps in common stock prices and their impact on call option pricing', *The Journal of Finance* **40**(1), 155–173.

Barberis, Nicholas C (2013), 'Thirty years of prospect theory in economics: A review and assessment', *The Journal of Economic Perspectives* **27**(1), 173–195.

Barberis, Nicholas and Richard Thaler (2003), 'A survey of behavioral finance', *Handbook of the Economics of Finance* **1**, 1053–1128.

Bates, David S (1991), 'The crash of'87: Was it expected? the evidence from options markets', *The journal of finance* **46**(3), 1009–1044.

Bates, David S (2000), 'Post-'87 crash fears in the s&p 500 futures option market', *Journal of Econometrics* **94**(1), 181–238.

Black, Fischer (1972), 'Capital market equilibrium with restricted borrowing', *The Journal of Business* **45**(3), 444–455.

Black, Fischer, Michael C. Jensen and Myron Scholes (1972), The capital asset pricing model: some empirical tests, *in* 'Studies in Theory of Capital Markets, Michael C. Jensen', ed. New York: Praeger Publishers Inc, pp. 79–121.

Bollerslev, Tim, Sophia Zhengzi Li and Viktor Todorov (2016), 'Roughing up beta: Continuous versus discontinuous betas and the cross section of expected stock returns', *Journal of Financial Economics* **120**(3), 464–490.

Bollerslev, Tim and Viktor Todorov (2011), 'Tails, fears, and risk premia', *The Journal of Finance* **66**(6), 2165–2211.

Breusch, Trevor Stanley (1978), 'Testing for autocorrelation in dynamic linear models', *Australian Economic Papers* **17**(31), 334–355.

Carhart, Mark M (1997), 'On persistence in mutual fund performance', *The Journal of finance* **52**(1), 57–82.

Chan, Wing H and John M Maheu (2002), 'Conditional jump dynamics in stock market returns', *Journal of Business & Economic Statistics* **20**(3), 377–389.

Christoffersen, Peter, Kris Jacobs and Chayawat Ornthanalai (2012), 'Dynamic jump intensities and risk premiums: evidence from s&p500 returns and options', *Journal of Financial Economics* **106**(3), 447–472.

Cremers, Martijn, Michael Halling and David Weinbaum (2015), 'Aggregate jump and volatility risk in the cross-section of stock returns', *The Journal of Finance* **70**(2), 577–614.

Duffie, Darrell, Jun Pan and Kenneth Singleton (2000), 'Transform analysis and asset pricing for affine jump-diffusions', *Econometrica* **68**(6), 1343–1376.

Engle, Robert F (1982), 'Autoregressive conditional heteroscedasticity with estimates of the variance of united kingdom inflation', *Econometrica: Journal of the Econometric Society* pp. 987–1007.

Eraker, Bjorn, Michael Johannes and Nicholas G Polson (2003), 'The impact of jumps in returns and volatility', *Journal of Finance* **53**(3), 1269–1300.

Fama, Eugene F and James D MacBeth (1973), 'Risk, return, and equilibrium: Empirical tests', *The journal of political economy* pp. 607–636.

Fama, Eugene F and Kenneth R French (1992), 'The cross-section of expected stock returns', *the Journal of Finance* **47**(2), 427–465.

Fama, Eugene F and Kenneth R French (1993), 'Common risk factors in the returns on stocks and bonds', *Journal of financial economics* **33**(1), 3–56.

Fama, Eugene F and Kenneth R French (2015), 'A five-factor asset pricing model', *Journal of Financial Economics* **116**, 1–22.

Gabaix, Xavier (2012), 'Variable rare disasters: An exactly solved framework for ten puzzles in macro-finance', *The Quarterly Journal of Economics, 127(2)* pp. 645–700.

Gibbons, Michael R, Stephen A Ross and Jay Shanken (1989), 'A test of the efficiency of a given portfolio', *Econometrica: Journal of the Econometric Society* pp. 1121–1152.

Godfrey, Leslie G (1978), 'Testing against general autoregressive and moving average error models when the regressors include lagged dependent variables', *Econometrica: Journal of the Econometric Society* pp. 1293–1301.

Heston, Steven L (1993), 'A closed-form solution for options with stochastic volatility with applications to bond and currency options', *Review of financial studies* **6**(2), 327–343.

Hou, Kewei and Robert Kimmel (2006), On the estimation of risk premia in linear factor models, Technical report, Working Paper, Ohio State University.

Jarrow, Robert A and Eric R Rosenfeld (1984), 'Jump risks and the intertemporal capital asset pricing model', *Journal of Business* pp. 337–351.

Jegadeesh, Narasimhan and Sheridan Titman (1993), 'Returns to buying winners and selling losers: Implications for stock market efficiency', *The Journal of finance* **48**(1), 65–91.

Jorion, Philippe (1988), 'On jump processes in the foreign exchange and stock markets', *Review of Financial Studies* **1**(4), 427–445.

Kahneman, Daniel and Amos Tversky (1979), 'Prospect theory: An analysis of decision under risk', *Econometrica: Journal of the econometric society* pp. 263–291.

Lakonishok, Josef and Alan C Shapiro (1986), 'Systematic risk, total risk and size as determinants of stock market returns', *Journal of Banking & Finance* **10**(1), 115–132.

Lintner, John (1965), 'The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets', *The review of economics and statistics* pp. 13–37.

Markowitz, Harry (1959), 'Portfolio selection: Efficient diversification of investments', *Wiley, New York* .

Merton, Robert C (1973), 'An intertemporal capital asset pricing model', *Econometrica: Journal of the Econometric Society* pp. 867–887.

Merton, Robert C (1976), 'Option pricing when underlying stock returns are discontinuous', *Journal of financial economics* **3**(1-2), 125–144.

Merton, Robert C (1980), 'On estimating the expected return on the market: An exploratory investigation', *Journal of financial economics* **8**(4), 323–361.

Mossin, Jan (1966), 'Equilibrium in a capital asset market', *Econometrica: Journal of the econometric society* pp. 768–783.

Naik, Vasanttilak and Moon Lee (1990), 'General equilibrium pricing of options on the market portfolio with discontinuous returns', *Review of financial studies* **3**(4), 493–521.

Pan, Jun (2002), 'The jump-risk premia implicit in options: Evidence from an integrated time-series study', *Journal of financial economics* **63**(1), 3–50.

Press, S James (1967), 'A compound events model for security prices', *Journal of business* pp. 317–335.

Reinganum, Marc R (1981), 'A new empirical perspective on the capm', *Journal of financial and quantitative analysis* **16**(04), 439–462.

Shanken, Jay (1992), 'On the estimation of beta-pricing models', *Review of Financial studies* **5**(1), 1–33.

Sharpe, William F (1964), 'Capital asset prices: A theory of market equilibrium under conditions of risk', *The journal of finance* **19**(3), 425–442.

Yan, Shu (2011), 'Jump risk, stock returns, and slope of implied volatility smile', *Journal of Financial Economics* **99**(1), 216–233.

Table 17 Size-BE/ME portfolios, regression intercepts. Regressions of excess stock returns of 25 size-BE/ME portfolios on 4-Factor model factors (excess market return (MKT) and mimicking returns for size (SMB), book-to-market equity (HML), and momentum (MOM)) or 5-Factor model factors (MKT, SMB, HML, MOM, and expected jump (JF)): December 1925 to December 2014. A1, A2, A3, and A4 are intercept indicators and tests and their formulas are available in Table 16. Test statistic A4, under H_0, has distribution $F_{N,T-N-K}$, where N equals the number of dependent portfolios, so in our case $N = 25$, T is the length of the portfolios time series and K is the number of factors, consequently $K = 4$. The table reports the P-value for test A4 (P(A4)).

	Book-to-market equity (BE/ME) quintiles									
Size quintile	Low	2	3	4	High	Low	2	3	4	High
4-Factor model										
			α					$t(\alpha)$		
Small	-0.59	-0.40	0.00	0.09	0.14	-3.48	-3.67	-0.05	1.43	2.05
2	-0.21	0.03	0.04	0.04	0.04	-3.11	0.47	0.84	0.75	0.60
3	-0.04	0.10	0.09	0.04	0.02	-0.87	1.84	1.60	0.80	0.31
4	0.08	0.00	0.05	0.07	-0.11	1.52	0.04	0.86	1.14	-1.37
Big	0.10	0.04	-0.01	-0.16	-0.07	2.62	0.89	-0.19	-2.99	-0.65
5-Factor model										
			α					$t(\alpha)$		
Small	-0.35	-0.43	0.01	0.06	0.13	-1.66	-3.37	0.11	0.90	1.78
2	-0.22	-0.05	-0.01	-0.01	0.02	-2.99	-0.73	-0.21	-0.14	0.31
3	0.00	0.04	0.04	-0.02	0.03	-0.08	0.76	0.72	-0.42	0.45
4	0.11	-0.08	-0.04	0.04	-0.07	1.86	-1.48	-0.72	0.65	-0.93
Big	0.09	-0.02	-0.03	-0.16	0.02	2.35	-0.36	-0.54	-3.05	0.18

	4-Factor	5-Factor
(A1)	0.1029	0.0845
(A2)	0.5629	0.4623
(A3)	0.6209	0.3291
P(A4)	0.0001	0.0010

Table 18 Size-expected jump portfolios, regression intercepts. Regressions of excess stock returns of 25 size-expected jump portfolios on 4-Factor model factors (excess market return (MKT) and mimicking returns for size (SMB), book-to-market equity (HML), and momentum (MOM)) or 5-Factor model factors (MKT, SMB, HML, MOM, and expected jump (JF)): December 1925 to December 2014. A1, A2, A3, and A4 are intercept indicators and tests and their formulas are available in Table 16. Test statistic A4, under H_0, has distribution $F_{N,T-N-K}$, where N equals the number of dependent portfolios, so in our case $N = 25$, T is the length of the portfolios time series and K is the number of factors, consequently $K = 4$. The table reports the P-value for test A4 (P(A4)).

	Expected jump quintiles									
Size quintile	Low	2	3	4	High	Low	2	3	4	High
	4-Factor model									
	α					$t(\alpha)$				
Small	2.46	1.12	1.27	1.73	0.50	4.56	3.17	3.01	4.13	1.12
2	0.82	0.63	0.59	0.53	0.37	3.49	3.11	2.61	2.84	1.39
3	0.28	0.30	0.20	0.25	-0.28	2.09	2.11	1.94	2.15	-2.11
4	0.13	0.01	0.20	0.01	-0.28	2.03	0.18	2.99	0.25	-3.39
Big	0.05	0.07	-0.07	-0.01	-0.11	0.99	2.15	-1.54	-0.33	-2.18
	5-Factor model									
	α					$t(\alpha)$				
Small	2.36	1.09	1.28	1.72	0.78	4.57	3.03	2.83	3.82	1.42
2	0.79	0.55	0.56	0.50	0.58	3.36	2.72	2.60	2.73	1.93
3	0.11	0.17	0.10	0.22	-0.16	0.89	1.22	0.91	1.87	-1.13
4	0.01	-0.09	0.13	0.06	-0.11	0.23	-1.52	1.61	0.98	-1.45
Big	-0.06	-0.01	-0.06	0.02	0.01	-1.24	-0.19	-1.24	0.45	0.11

	4-Factor	5-Factor
(A1)	0.4909	0.4614
(A2)	0.8871	0.8337
(A3)	1.0634	1.0106
P(A4)	0.0000	0.0000

Table 19 5-factor risk premiums. Two-pass estimated risk premia of the 5-factor model, using monthly excess stock returns of 25 size-BE/ME portfolios and 25 size-expected jump portfolios: December 1925 to December 2014. The table reports the mean monthly percentage excess return of each factor portfolio, Mean return %, the Black et al. (1972) monthly percentage risk premia, $\breve{\gamma}$, and the corresponding t-statistic for $\breve{\gamma} = 0$, the Fama and MacBeth (1973) monthly percentage risk premia, $\tilde{\gamma}$, and the corresponding t-statistic for $\tilde{\gamma} = 0$.

Factor portfolio	Mean return %	25 size-BE/ME portfolios $\breve{\gamma}$	t-statistic	$\tilde{\gamma}$	t-statistic
γ_{MKT}	0.65	0.63	3.70	0.71	4.27
γ_{SMB}	0.22	0.16	1.47	0.28	2.57
γ_{HML}	0.40	0.45	3.97	0.43	3.68
γ_{MOM}	0.67	0.63	1.04	0.27	1.26
γ_{JF}	-0.13	-0.28	-1.78	0.04	0.41
		25 size-Expected jump portfolios			
		$\breve{\gamma}$	t-statistic	$\hat{\gamma}$	t-statistic
γ_{MKT}	0.65	0.42	2.31	0.45	2.30
γ_{SMB}	0.22	-0.40	-2.35	-0.05	-0.37
γ_{HML}	0.40	1.85	3.91	0.43	2.12
γ_{MOM}	0.67	-1.48	-2.99	-0.88	-3.02
γ_{JF}	-0.13	-0.23	-2.42	-0.29	-3.14

Table 20 Risk premiums/factor means divergence tests. Two-pass regressions of the 5-factor model using monthly excess stock returns of 25 size-BE/ME portfolios and 25 size-expected jump portfolios: December 1925 to December 2014. The table reports the P-values of the tests that check if the differences between estimated parameters and factor means are statistically simultaneously equal to 0. The null hypothesis for the Black et al. (1972) and the Fama and MacBeth (1973) approaches are respectively $H_0 : \breve{\boldsymbol{\gamma}} - \bar{\boldsymbol{f}} = \mathbf{0}$ and $H_0 : \tilde{\boldsymbol{\gamma}} - \bar{\boldsymbol{f}} = \mathbf{0}$, where $\bar{\boldsymbol{f}}$ is the vector of the factor means.

Dependent portfolio	P-value $H_0 : \breve{\boldsymbol{\gamma}} - \bar{\boldsymbol{f}} = \mathbf{0}$	P-value $H_0 : \tilde{\boldsymbol{\gamma}} - \bar{\boldsymbol{f}} = \mathbf{0}$
25 size-BE/ME	1.000	0.996
25 size-Expected jump	0.010	0.002

Table 21 Factor portfolios, R^2. Table reports the R^2 values resulting from regressions of excess returns of 5 factor portfolios (MKT, SMB, HML, MOM, and JF) on the returns of 25 size-BE/ME portfolios or 25 size-expected jump portfolios: December 1925 to December 2014.

	R^2 for the dependent portfolio				
Explanatory portfolios	MKT	SMB	HML	MOM	JF
25 size-BE/ME	0.99	0.97	0.96	0.30	0.53
25 size-Expected jump	1.00	0.88	0.40	0.28	0.93

Table 22 5-factor corrected risk premiums. Two-pass estimated risk premia of the 5-factor model, using monthly excess stock returns of 25 size-BE/ME portfolios and 25 size-expected jump portfolios augmented by the 5 factor portfolios: December 1925 to December 2014. The table reports the mean monthly percentage excess return of each factor portfolio, Mean return %, the Black, Jensen and Scholes (1972) monthly percentage risk premia, $\check{\gamma}$, and the corresponding t-statistic for $\check{\gamma} = 0$, the Fama and MacBeth (1973) monthly percentage risk premia, $\tilde{\gamma}$, and the corresponding t-statistic for $\tilde{\gamma} = 0$.

Factor	Mean	25 size-BE/ME portfolios + 5 factor portfolios			
portfolio	return %	$\check{\gamma}$	t-statistic	$\tilde{\gamma}$	t-statistic
γ_{MKT}	0.65	0.63	3.77	0.71	4.32
γ_{SMB}	0.22	0.16	1.55	0.26	2.44
γ_{HML}	0.40	0.43	3.88	0.44	3.84
γ_{MOM}	0.67	0.68	4.65	0.52	3.46
γ_{JF}	-0.13	-0.22	-2.04	-0.01	-0.13
		25 size-Expected jump portfolios + 5 factor portfolios			
		$\check{\gamma}$	t-statistic	$\hat{\gamma}$	t-statistic
γ_{MKT}	0.65	0.41	2.30	0.44	2.26
γ_{SMB}	0.22	0.36	2.63	0.18	1.33
γ_{HML}	0.40	0.65	4.81	0.36	2.56
γ_{MOM}	0.67	0.45	2.57	0.26	1.36
γ_{JF}	-0.13	-0.12	-1.32	-0.20	-2.32

Table 23 Corrected risk premiums/factor means divergence tests. Two-pass regressions of the 5-factor model using monthly excess stock returns of 25 size-BE/ME portfolios and 25 size-expected jump portfolios augmented by the 5 factor portfolios: December 1925 to December 2014. The table reports the P-values of the tests that check if the differences between estimated parameters and factor means are statistically simultaneously equal to 0. The null hypothesis for the Black et al. (1972) and the Fama and MacBeth (1973) approaches are respectively $H_0 : \check{\boldsymbol{\gamma}} - \bar{\boldsymbol{f}} = \mathbf{0}$ and $H_0 : \tilde{\boldsymbol{\gamma}} - \bar{\boldsymbol{f}} = \mathbf{0}$, where $\bar{\boldsymbol{f}}$ is the vector of the factor means.

Dependent portfolio	P-value $H_0 : \check{\boldsymbol{\gamma}} - \bar{\boldsymbol{f}} = \mathbf{0}$	P-value $H_0 : \tilde{\boldsymbol{\gamma}} - \bar{\boldsymbol{f}} = \mathbf{0}$
25 size-BE/ME + 5 factor	1.000	1.000
25 size-Expected jump + 5 factor	0.044	0.003

A Appendix

A.1 Dataset

The dataset includes all CRSP stocks, with share code equal to 10 or 11, from December 1925 until December 2014. Of the overall 24,122 assets, figure VI reports for each month the dimension of the dataset in terms of number of quoted stocks. We consider as quoted those assets with at least one available return in the window of interest.

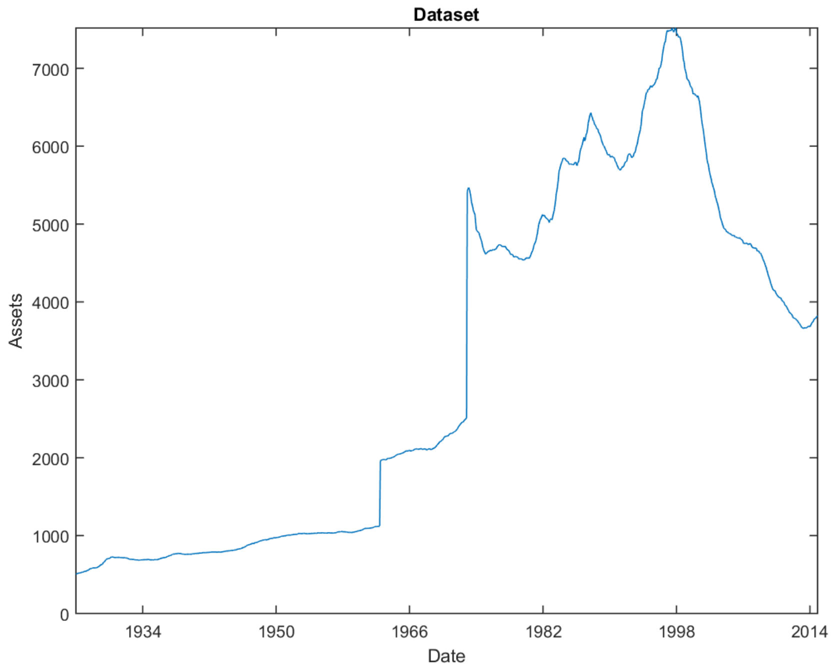

Figure VI Monthly dataset. For each month from January 1926 till December 2014, it shows the number quoted assets. Quoted stocks are those for which at least one return exist in the window of interest.

A.2 Single assets regression results

We compare factor slopes and coefficients of determination resulting from regressing single asset excess returns against the 4-factor and the 5-factor models. For the latter we observe that adding the JF in the regression increases the coefficient of determination 42.42% of the times.

Relative to the βs, instead, with the 5-factor model we obtain slopes for MKT, SMB, HML, and MOM that are significantly different from 0 (10% significance level) respectively in 62%, 50%, 28%, and 22% of the cases. More importantly, β_{JF} is significant 21% of the times.

Single asset results reinforce the conclusions, of section 4, from regressions that use portfolios as dependent variables: JF captures strong common variation in returns.

A.3 Residuals correlation and heteroskedasticity

This section presents a comparison of the 4-factor and the 5-factor models in terms of residuals correlation and heteroskedasticity. Table 24 shows the P-values of the Breusch (1978)-Godfrey (1978) autocorrelation test, or AR, while Table 25 reports the P-values of the Engle (1982) heteroskedasticity test, or ET, both with 3 lags.

In the size-BE/ME case it is possible to observe that regression errors are affected by heteroskedasticity and, slightly, by autocorrelation. While for the former we do not register great differences between the models, see left panels of Table 25, autocorrelation results improve by including the JF. In particular, the lower values of average residual correlations and average absolute residual correlations in Table 24, for the 5-factor model with respect to the 4-factor model, witness the superiority of the former.

Using 25 size-expected jump portfolios, instead, we obtain errors that are similarly affected by heteroskedasticity in the 4-factor and in the 5-factor models, but slightly more affected by autocorrelation in the 5-factor case with respect to the 4-factor case. In the right panels of Table 24 it is possible to observe a small increase, when using the 5-factor model, of the average residual correlation, from 0.212 to 0.226, and of the average absolute residual correlation, from 0.250 to 0.258.

A.4 GRS test statistic

The Gibbons, Ross and Shanken (1989), or GRS, statistic tests if the intercepts, α, are jointly zero: $H_0 : \boldsymbol{\alpha} = \mathbf{0}$: It requires that the errors are normal, uncorrelated, and homoskedastic.

Under the hypothesis of normal excess returns, the distribution of the estimated αs, or $\hat{\boldsymbol{\alpha}}$, conditional to the factors, or $\boldsymbol{F}$, is:

$$\hat{\boldsymbol{\alpha}}|\boldsymbol{F} \sim \mathcal{N}\Big[\boldsymbol{\alpha}, \frac{1}{T}\Big(1 + \bar{\boldsymbol{f}}'\hat{\boldsymbol{\Sigma}}_{\boldsymbol{f}}^{-1}\bar{\boldsymbol{f}}\Big)\boldsymbol{\Omega}\Big],$$

where $\bar{\boldsymbol{f}}$ is the vector of the mean value of the factors, $\boldsymbol{\Omega}$ is the variance-covariance matrix of the regression residuals, T is the length of the time series, and $\hat{\boldsymbol{\Sigma}}_{\boldsymbol{f}}$ is the estimated variance-covariance matrix of the factors: $\hat{\boldsymbol{\Sigma}}_{\boldsymbol{f}} = \frac{1}{T}\sum_{t=1}^{T}[f_t - \bar{\boldsymbol{f}}][f_t - \bar{\boldsymbol{f}}]'$.

It follows that the test statistic takes the following form:

$$\frac{T-N-K}{N}\Big(1 + \bar{\boldsymbol{f}}\hat{\boldsymbol{\Sigma}}_{\boldsymbol{f}}^{-1}\bar{\boldsymbol{f}}\Big)^{-1}\hat{\boldsymbol{\alpha}}'\hat{\boldsymbol{\Omega}}^{-1}\hat{\boldsymbol{\alpha}}.$$

Table 24 Residual correlations. Regressions of excess stock returns of 25 size-BE/ME portfolios and 25 size-expected jump portfolios on the returns of the 4-factor model (excess market return (MKT) and the mimicking returns for the size (SMB), book-to-market equity (HML), and momentum (MOM) factors), and on the returns of the 5-factor model (excess market return (MKT) and the mimicking returns for the size (SMB), book-to-market equity (HML), momentum (MOM), and expected jump (JF) factors). December 1925 to December 2014. Panels report the P-values of the Breusch (1978)-Godfrey (1978) autocorrelation test with 3 lags. The table also shows statistics for residual correlations between portfolios: $\rho_{i,j}$ with $i \neq j$. In detail, $\bar{\rho} = \frac{1}{N(N-1)} \sum_{i=1}^{N} \sum_{j=1}^{N} \rho_{i,j}, i \neq j$ is the average residual correlation, $\ddot{\rho} = \frac{1}{N(N-1)} \sum_{i=1}^{N} \sum_{j=1}^{N} |\rho_{i,j}|, i \neq j$ is the average absolute residual correlation, while $\min(\rho)$ and $\max(\rho)$ are the minimum and maximum values of the residual correlations.

	Book-to-market quintiles					Expected jump quintiles				
Size quintile	Low	2	3	4	High	Low	2	3	4	High
	AR 4-factor model					AR 4-factor model				
Small	0.00	0.06	0.00	0.01	0.00	0.94	0.60	0.48	0.53	0.71
2	0.00	0.10	0.79	0.01	0.01	0.75	0.13	0.13	0.11	0.66
3	0.16	0.92	0.57	0.10	0.00	0.03	0.94	0.07	0.00	0.03
4	0.54	0.00	0.07	0.19	0.35	0.93	0.03	0.18	0.26	0.24
Big	0.22	0.75	0.00	0.03	0.15	0.60	0.28	0.09	0.07	0.10
Size quintile	Low	2	3	4	High	Low	2	3	4	High
	AR 5-factor model					AR 5-factor model				
Small	0.00	0.04	0.00	0.01	0.00	0.94	0.64	0.46	0.57	0.99
2	0.00	0.03	0.77	0.03	0.01	0.75	0.15	0.14	0.10	0.70
3	0.19	0.55	0.53	0.03	0.00	0.26	0.73	0.20	0.00	0.00
4	0.60	0.02	0.39	0.20	0.38	0.75	0.31	0.99	0.45	0.74
Big	0.20	0.71	0.00	0.02	0.33	0.87	0.25	0.12	0.08	0.17
	Book-to-market equity					Expected jump				
Model	$\bar{\rho}$	$\ddot{\rho}$	$\min(\rho)$	$\max(\rho)$		$\bar{\rho}$	$\ddot{\rho}$	$\min(\rho)$	$\max(\rho)$	
4-factor	0.050	0.120	-0.431	0.473		0.212	0.250	-0.404	0.757	
5-factor	0.048	0.116	-0.436	0.427		0.226	0.258	-0.405	0.758	

Table 25 Residual heteroskedasticity. Regressions of excess stock returns of 25 size-BE/ME portfolios and 25 size-expected jump portfolios on the returns of the 4-factor model (excess market return (MKT) and the mimicking returns for the size (SMB), book-to-market equity (HML), and momentum (MOM) factors), and on the returns of the 5-factor model (excess market return (MKT) and the mimicking returns for the size (SMB), book-to-market equity (HML), momentum (MOM), and expected jump (JF) factors). December 1925 to December 2014. Tables report the P-values of the Engle (1982) heteroskedasticity test with 3 lags.

	Book-to-market quintiles					Expected jump quintiles				
Size quintile	Low	2	3	4	High	Low	2	3	4	High
	ET 4-factor model					ET 4-factor model				
Small	0.00	0.00	0.00	0.00	0.00	0.51	0.02	0.02	0.00	0.00
2	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00
3	0.24	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Big	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03
	ET 5-factor model					ET 5-factor model				
Small	0.00	0.00	0.00	0.00	0.00	0.50	0.02	0.01	0.00	0.00
2	0.00	0.00	0.00	0.00	0.00	0.04	0.00	0.00	0.00	0.00
3	0.30	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Big	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Under the null hypothesis, it has distribution $F_{N,T-N-K}$, where N equals the number of dependent portfolios, T is the length of the portfolios time series and K is the number of factors. $\hat{\boldsymbol{\alpha}}$ is the vector of estimated intercepts for the 25 portfolios and $\hat{\boldsymbol{\Omega}}$ is the estimated variance-covariance matrix of the regression residuals: $\hat{\boldsymbol{\Omega}} = \frac{1}{T}\sum_{t=1}^{T} \hat{\boldsymbol{e}}_t \hat{\boldsymbol{e}}_t^{'}$.

A.5 Subperiods full results

This section reports the results from regressing 25 size-BE/ME portfolios and 25 size-expected jump portfolios against (a) the four-factor model ($\beta_{JF,i} = 0$) and (b) the five-factor model (equation 5). Tables 27, and 28 report the results for the 25 size-BE/ME portfolios, while Tables 29, and 30 the results in the size-expected jump case.

In particular, we present the estimated coefficient results for model (a) in Tables 27 and 29, and for model (b) in Tables 28 and 30. The tables not only show the mean, the average standard error, the maximum and minimum values of the estimated parameter across the 25 portfolios, but also the percentage of portfolios for which the parameter of interest is significantly different from 0 using a 90% confidence level.

Tables 26 and 31, instead, compare the models and the dependent portfolios, respectively, in terms of variations in the coefficients of determinations and estimated intercepts.

Table 26 Sub-periods: coefficient of determination. Regressions of excess stock returns of 25 size-BE/ME portfolios and 25 size-expected jump portfolios on the returns of the 4-factor model, and on the returns of the 5-factor model: December 1925 to December 2014. R^2_{adj} increase reports the percentage of portfolios for which R^2_{adj} increases when moving from the 4-factor to the 5-factor model. R^2_{adj} max and R^2_{adj} min are respectively the maximum and minimum R^2_{adj} obtained using the 5-factor model.

Book-to-market equity (BE/ME) portfolios			
Period	R^2_{adj} increase	R^2_{adj} max	R^2_{adj} min
Dec1925-Mar1948	52%	0.979	0.607
Apr1948-Jun1970	64%	0.957	0.732
Jul1970-Sep1992	88%	0.966	0.805
Oct1992-Dec2014	92%	0.954	0.787
Expected jump (JF) portfolios			
Period	R^2_{adj} increase	R^2_{adj} max	R^2_{adj} min
Dec1925-Mar1948	52%	0.974	0.281
Apr1948-Jun1970	64%	0.962	0.241
Jul1970-Sep1992	88%	0.979	0.703
Oct1992-Dec2014	72%	0.964	0.457

Table 27 Size-BE/ME portfolios, 4-Factor sub-periods coefficient results. Regressions of excess stock returns of 25 size-BE/ME portfolios on the excess market return (MKT) and the mimicking returns for the size (SMB), book-to-market equity (HML), and momentum (MOM) factors. Period identifies the sub-period used for the regressions. Mean, A(SE), Max, and Min represents, respectively, the mean, the average Standard Error, the maximum value and the minimum value of the estimated parameter across the 25 portfolios. Significance reports the percentage of portfolios with $|t\text{-statistic}| > 1.645$.

Book-to-market equity (BE/ME) portfolios					
Period	Mean	A(SE)	Max	Min	Significance
		β			
Dec1925-Mar1948	1.049	0.050	1.223	0.895	100%
Apr1948-Jun1970	1.010	0.027	1.111	0.875	100%
Jul1970-Sep1992	1.015	0.026	1.163	0.882	100%
Oct1992-Dec2014	0.992	0.035	1.082	0.858	100%
		β_{SMB}			
Dec1925-Mar1948	0.635	0.115	1.929	-0.369	92%
Apr1948-Jun1970	0.608	0.048	1.625	-0.209	96%
Jul1970-Sep1992	0.547	0.039	1.391	-0.247	96%
Oct1992-Dec2014	0.512	0.052	1.323	-0.270	96%
		β_{HML}			
Dec1925-Mar1948	0.368	0.080	1.147	-0.276	76%
Apr1948-Jun1970	0.274	0.046	0.883	-0.380	84%
Jul1970-Sep1992	0.216	0.047	0.819	-0.475	80%
Oct1992-Dec2014	0.337	0.060	0.904	-0.447	96%
		β_{MOM}			
Dec1925-Mar1948	-0.044	0.058	0.064	-0.304	16%
Apr1948-Jun1970	-0.021	0.040	0.059	-0.189	16%
Jul1970-Sep1992	-0.017	0.035	0.087	-0.105	20%
Oct1992-Dec2014	-0.034	0.035	0.028	-0.119	36%

Table 28 Size-BE/ME portfolios, 5-Factor sub-periods coefficient results. Regressions of excess stock returns of 25 size-BE/ME portfolios on the excess market return (MKT) and the mimicking returns for the size (SMB), book-to-market equity (HML), momentum (MOM), and expected jump (JF) factors. Period identifies the sub-period used for the regressions. Mean, A(SE), Max, and Min represents, respectively, the mean, the average Standard Error, the maximum value and the minimum value of the estimated parameter across the 25 portfolios. Significance reports the percentage of portfolios with $|t\text{-statistic}| > 1.645$.

Book-to-market equity (BE/ME) portfolios					
Period	Mean	A(SE)	Max	Min	Significance
		β			
Dec1925-Mar1948	1.045	0.058	1.264	0.931	100%
Apr1948-Jun1970	1.008	0.029	1.093	0.886	100%
Jul1970-Sep1992	1.016	0.026	1.140	0.876	100%
Oct1992-Dec2014	1.023	0.034	1.143	0.863	100%
		β_{SMB}			
Dec1925-Mar1948	0.635	0.114	1.932	-0.372	92%
Apr1948-Jun1970	0.605	0.050	1.515	-0.202	96%
Jul1970-Sep1992	0.549	0.041	1.307	-0.237	96%
Oct1992-Dec2014	0.551	0.047	1.249	-0.245	96%
		β_{HML}			
Dec1925-Mar1948	0.367	0.079	1.125	-0.275	76%
Apr1948-Jun1970	0.275	0.046	0.898	-0.371	88%
Jul1970-Sep1992	0.214	0.047	0.850	-0.455	84%
Oct1992-Dec2014	0.278	0.051	0.875	-0.470	92%
		β_{MOM}			
Dec1925-Mar1948	-0.044	0.057	0.061	-0.304	16%
Apr1948-Jun1970	-0.022	0.040	0.058	-0.210	20%
Jul1970-Sep1992	-0.017	0.035	0.091	-0.102	20%
Oct1992-Dec2014	-0.034	0.031	0.028	-0.120	36%
		β_{JF}			
Dec1925-Mar1948	0.018	0.164	0.801	-0.637	16%
Apr1948-Jun1970	0.016	0.110	0.566	-0.371	32%
Jul1970-Sep1992	-0.012	0.094	0.601	-0.386	64%
Oct1992-Dec2014	-0.185	0.089	0.406	-0.596	64%

Table 29 Size-expected jump portfolios, 4-Factor sub-periods coefficient results. Regressions of excess stock returns of 25 size-expected jump portfolios on the excess market return (MKT) and the mimicking returns for the size (SMB), book-to-market equity (HML), and momentum (MOM) factors. Period identifies the sub-period used for the regressions. Mean, A(SE), Max, and Min represents, respectively, the mean, the average Standard Error, the maximum value and the minimum value of the estimated parameter across the 25 portfolios. Significance reports the percentage of portfolios with $|t\text{-statistic}| > 1.645$.

Expected jump (JF) portfolios					
Period	Mean	A(SE)	Max	Min	Significance
		β			
Dec1925-Mar1948	0.881	0.153	1.656	-0.092	84%
Apr1948-Jun1970	1.025	0.099	1.333	0.867	100%
Jul1970-Sep1992	0.909	0.033	1.143	0.648	100%
Oct1992-Dec2014	0.896	0.048	1.214	0.490	100%
		β_{SMB}			
Dec1925-Mar1948	1.416	0.274	2.858	-0.120	88%
Apr1948-Jun1970	1.165	0.144	2.333	-0.069	92%
Jul1970-Sep1992	1.057	0.063	1.791	-0.111	96%
Oct1992-Dec2014	0.676	0.073	1.349	-0.124	84%
		β_{HML}			
Dec1925-Mar1948	0.756	0.221	2.244	-0.021	88%
Apr1948-Jun1970	0.563	0.152	1.484	-0.052	84%
Jul1970-Sep1992	0.208	0.065	0.563	-0.078	80%
Oct1992-Dec2014	0.217	0.077	0.565	-0.165	76%
		β_{MOM}			
Dec1925-Mar1948	-0.221	0.164	0.121	-1.199	44%
Apr1948-Jun1970	-0.169	0.147	0.072	-1.279	48%
Jul1970-Sep1992	-0.063	0.050	0.057	-0.175	40%
Oct1992-Dec2014	-0.234	0.056	0.073	-0.499	96%

Table 30 Size-expected jump portfolios, 5-Factor sub-periods coefficient results. Regressions of excess stock returns of 25 size-expected jump portfolios on the excess market return (MKT) and the mimicking returns for the size (SMB), book-to-market equity (HML), momentum (MOM), and expected jump (JF) factors. Period identifies the sub-period used for the regressions. Mean, A(SE), Max, and Min represents, respectively, the mean, the average Standard Error, the maximum value and the minimum value of the estimated parameter across the 25 portfolios. Significance reports the percentage of portfolios with $|t\text{-statistic}| > 1.645$.

Expected jump (JF) portfolios					
Period	Mean	A(SE)	Max	Min	Significance
		β			
Dec1925-Mar1948	0.939	0.165	1.688	-0.024	92%
Apr1948-Jun1970	1.015	0.107	1.348	0.864	100%
Jul1970-Sep1992	0.884	0.037	1.092	0.582	100%
Oct1992-Dec2014	0.901	0.049	1.122	0.462	100%
		β_{SMB}			
Dec1925-Mar1948	1.418	0.272	2.856	-0.120	88%
Apr1948-Jun1970	1.143	0.162	2.287	-0.091	88%
Jul1970-Sep1992	1.016	0.061	1.647	-0.138	92%
Oct1992-Dec2014	0.682	0.068	1.184	-0.151	92%
		β_{HML}			
Dec1925-Mar1948	0.771	0.222	2.252	-0.044	92%
Apr1948-Jun1970	0.571	0.156	1.510	-0.036	80%
Jul1970-Sep1992	0.237	0.065	0.665	-0.024	80%
Oct1992-Dec2014	0.208	0.075	0.416	-0.025	68%
		β_{MOM}			
Dec1925-Mar1948	-0.218	0.164	0.128	-1.201	44%
Apr1948-Jun1970	-0.174	0.147	0.061	-1.293	44%
Jul1970-Sep1992	-0.066	0.050	0.062	-0.184	44%
Oct1992-Dec2014	-0.234	0.052	0.073	-0.500	96%
		β_{JF}			
Dec1925-Mar1948	-0.282	0.447	0.459	-1.365	36%
Apr1948-Jun1970	0.103	0.326	0.898	-0.749	44%
Jul1970-Sep1992	0.288	0.146	1.027	-0.506	76%
Oct1992-Dec2014	-0.029	0.123	0.982	-0.612	64%

Table 31 Sub-periods: intercept results. Regressions of excess stock returns of 25 size-BE/ME portfolios and 25 size-expected jump portfolios on the returns of the 4-factor model, and on the returns of the 5-factor model: December 1925 to December 2014. Period identifies the sub-period used for the regressions. Mean, A(SE), Max, and Min represents, respectively, the mean, the average Standard Error, the maximum value and the minimum value of the estimated parameter across the 25 portfolios. Sig (Significance) reports the percentage of portfolios with $|t\text{-statistic}| > 1.645$. A1, A2, A3, and A4 are intercept tests and their formulas are specified in Table 16. P is the P-value for test A4 that, under H_0, has distribution $F_{N,T-N-K}$, where N equals the number of dependent portfolios, so in our case $N = 25$, T is the length of the portfolios time series and K is the number of factors, consequently $K = 4$ or $K = 5$.

	Book-to-market equity (BE/ME) portfolios					Expected jump (JF) portfolios				
Period	Mean	A(SE)	Max	Min	Sig	Mean	A(SE)	Max	Min	Sig
			$\alpha_{4-factor}$					$\alpha_{4-factor}$		
Dec1925-Mar1948	-0.011	0.189	0.336	-1.307	12%	1.545	0.622	7.858	-0.232	60%
Apr1948-Jun1970	-0.019	0.089	0.256	-0.349	36%	0.349	0.302	2.526	-0.541	32%
Jul1970-Sep1992	0.017	0.091	0.218	-0.544	20%	-0.051	0.133	0.430	-0.813	24%
Oct1992-Dec2014	0.031	0.114	0.278	-0.555	24%	0.243	0.179	0.951	-0.261	36%
			$\alpha_{5-factor}$					$\alpha_{5-factor}$		
Dec1925-Mar1948	-0.004	0.206	0.491	-1.531	8%	1.444	0.664	7.335	-0.252	44%
Apr1948-Jun1970	-0.016	0.091	0.235	-0.332	28%	0.365	0.307	2.559	-0.447	28%
Jul1970-Sep1992	0.014	0.095	0.241	-0.420	24%	0.009	0.139	0.577	-0.672	40%
Oct1992-Dec2014	0.000	0.108	0.283	-0.487	16%	0.238	0.175	1.000	-0.127	32%

	Intercept tests				Intercept tests			
Period	A1 (4F)	A1 (5F)	A2 (4F)	A2 (5F)	A1 (4F)	A1 (5F)	A2 (4F)	A2 (5F)
Dec1925-Mar1948	0.19	0.20	0.59	0.62	1.61	1.50	0.87	0.81
Apr1948-Jun1970	0.12	0.11	0.82	0.72	0.45	0.45	0.89	0.89
Jul1970-Sep1992	0.11	0.10	0.61	0.52	0.21	0.20	1.08	1.01
Oct1992-Dec2014	0.13	0.12	0.81	0.78	0.29	0.26	1.53	1.36

Period	A3 (4F)	A3 (5F)	P(A4) (4F)	P(A4) (5F) A3 (4F)	A3 (5F)	P(A4) (4F)	P(A4) (5F)			
Dec1925-Mar1948	0.41	0.51	0.17	0.11			1.02	0.87	-	-
Apr1948-Jun1970	0.44	0.25	0.01	0.02			0.72	0.73	0.07	1.00
Jul1970-Sep1992	0.31	0.18	0.07	0.18			1.11	0.97	0.00	1.00
Oct1992-Dec2014	0.42	0.36	0.00	0.00			2.61	2.54	0.00	1.00

A.6 Two pass technique

The two pass technique requires as a first step to regress each asset's return on the time series of the factor realizations (equation 10), to obtain the estimated betas, and in the second step to regress, at each time t, the cross-section of assets returns against the beta coefficients (equation 11).

A.6.1 Black, Jensen and Scholes (1972) approach

Following the Black, Jensen and Scholes (1972) approach, we run the first step using the full 1925-2014 sample of monthly percentage returns. From the T regressions of step two, we obtain the time series of risk premia coefficients, $\breve{\boldsymbol{\gamma}}_t$, and residuals, $\breve{\boldsymbol{\epsilon}}_t$, that we then use to estimate $\boldsymbol{\gamma}$ and $\boldsymbol{\epsilon}$ as the average of the cross-sectional regressions:

$$\breve{\boldsymbol{\gamma}} = \frac{1}{T}\sum_{t=1}^{T}\breve{\boldsymbol{\gamma}}_t \text{ and } \breve{\boldsymbol{\epsilon}} = \frac{1}{T}\sum_{t=1}^{T}\breve{\boldsymbol{\epsilon}}_t.$$

In order to test the statistical significance of the estimated risk premiums, $\breve{\boldsymbol{\gamma}}$s, it is necessary to consider their asymptotic distribution: $\sqrt{T}(\breve{\boldsymbol{\gamma}} - \boldsymbol{\gamma}) \xrightarrow{d} \mathcal{N}[\mathbf{0}, (1 + \boldsymbol{\mu}_f'\boldsymbol{\Sigma}_f^{-1}\boldsymbol{\mu}_f)(\boldsymbol{B}'\boldsymbol{B})^{-1}\boldsymbol{B}'\boldsymbol{\Omega}\boldsymbol{B}(\boldsymbol{B}'\boldsymbol{B})^{-1} + \boldsymbol{\Sigma}_f]$. Where T is the length of the portfolios time series, $\boldsymbol{B}$ is the matrix of estimated coefficients for the 25 portfolios, $\boldsymbol{\mu_f}$ is the vector of the expected value of the factors, $\boldsymbol{\Sigma_f}$ is the variance-covariance matrix of the factors, and $\boldsymbol{\Omega}$ is the variance-covariance matrix of step one regression residuals. Moreover, the multiplicative term $(1 + \boldsymbol{\mu}_f'\boldsymbol{\Sigma}_f^{-1}\boldsymbol{\mu}_f)$ is due to the Shanken (1992) correction for the fact that $\hat{\beta}$ are generated regressors. To empirically test if a risk premium is equal to 0, $H_0 : \breve{\gamma} = 0$, we need to substitute the unknown quantities $\boldsymbol{\mu_f}$, $\boldsymbol{\Sigma_f}$, and $\boldsymbol{\Omega}$ with their estimated correspondents $\bar{\boldsymbol{f}}$, $\hat{\boldsymbol{\Sigma}}_{\boldsymbol{f}}$, and $\hat{\boldsymbol{\Omega}} = \frac{1}{T}\sum_{t=1}^{T}\hat{\boldsymbol{e}}_{\boldsymbol{t}}\hat{\boldsymbol{e}}_{\boldsymbol{t}}'$.

A.6.2 Fama and MacBeth (1973) approach

The Fama and MacBeth (1973) approach requires the use of five-years rolling (overlapping) windows of monthly percentage returns in the first step. We then estimate the risk premia, γ, and the residuals, ϵ, as the average of the T cross-sectional regressions of step two:

$$\hat{\boldsymbol{\gamma}} = \frac{1}{T}\sum_{t=1}^{T}\hat{\boldsymbol{\gamma}}_t \text{ and } \hat{\boldsymbol{\epsilon}} = \frac{1}{T}\sum_{t=1}^{T}\hat{\boldsymbol{\epsilon}}_t.$$

We compute he sampling errors for the estimates following Fama and MacBeth (1973) who suggest the use of the standard deviations of the cross-sectional regression estimates: $\sigma^2(\hat{\gamma}) = \frac{1}{T}\sum_{t=1}^{T}(\hat{\gamma}_t - \hat{\gamma})^2$. We slightly diverge from their recommendation by using Newey-West HAC standard errors. Note, since it does not correct for the fact that $\hat{\beta}$ are generated regressors, it is necessary to check that the Shanken (1992) correction factor,

$(1+\boldsymbol{\mu}_f'\boldsymbol{\Sigma}_f^{-1}\boldsymbol{\mu}_f)$, is not too large. To empirically compute the correction factor we need to substitute the unknown quantities, $\boldsymbol{\mu_f}$ and $\boldsymbol{\Sigma_f}$, with their estimated correspondents, $\bar{\boldsymbol{f}}$ and $\hat{\boldsymbol{\Sigma}}_{\boldsymbol{f}}$. In our case for a monthly interval $\bar{\boldsymbol{f}}\hat{\boldsymbol{\Sigma}}_f^{-1}\bar{\boldsymbol{f}} \approx 0.09$. Since it is quite small, ignoring the multiplicative term does not make big difference.

A.6.3 Test for the differences between estimated parameters and factor means

Our goal is to understand if the differences between estimated parameters and factor means (or $\bar{\boldsymbol{f}}$) are statistically simultaneously different from 0.

In the Black et al. (1972) case we can use the time series of risk premia estimates, $\breve{\boldsymbol{\gamma}}_t$, and errors, $\breve{\boldsymbol{\epsilon}}_t$, to approximate the variance-covariance matrix of the difference between estimated risk premiums and factor mean values: $\breve{\boldsymbol{\gamma}} - \bar{\boldsymbol{f}}$. Renaming the differences as $\breve{\boldsymbol{\phi}}_t = \breve{\boldsymbol{\gamma}}_t - \boldsymbol{f}_t$ and $\breve{\boldsymbol{\phi}} = \breve{\boldsymbol{\gamma}} - \bar{\boldsymbol{f}}$, we can define the variance-covariance matrix as:

$$\breve{Var}(\breve{\boldsymbol{\gamma}} - \bar{\boldsymbol{f}}) \equiv \breve{Var}(\breve{\boldsymbol{\phi}}) = \frac{1}{T}\breve{Var}(\breve{\boldsymbol{\phi}}_t),$$

where $\breve{Var}(\breve{\boldsymbol{\phi}}_t) = \frac{1}{T}\sum_{t=1}^{T}(\breve{\boldsymbol{\phi}}_t - \breve{\boldsymbol{\phi}})(\breve{\boldsymbol{\phi}}_t - \breve{\boldsymbol{\phi}})'$. It is possible to use this estimates to test if the differences between estimated parameters and factor means are statistically simultaneously different from 0: $H_0 : \breve{\boldsymbol{\gamma}} - \bar{\boldsymbol{f}} = \boldsymbol{0}$. The correspondent test statistic $\breve{\boldsymbol{\phi}}'[\breve{Var}(\breve{\boldsymbol{\phi}})]^{-1}\breve{\boldsymbol{\phi}}$, under H_0, has distribution χ^2_N, where N equals the number of dependent portfolios, so in our case $N = 25$.

Similarly to the Black et al. (1972) case, also for the Fama and MacBeth (1973) approach we can use the time series of risk premia estimates, $\hat{\boldsymbol{\gamma}}_t$, and errors, $\hat{\boldsymbol{\epsilon}}_t$ to approximate the variance-covariance matrix of the difference between estimated risk premiums and factor mean values: $\hat{\boldsymbol{\gamma}} - \bar{\boldsymbol{f}}$. Calling the differences $\hat{\boldsymbol{\phi}}_t = \hat{\boldsymbol{\gamma}}_t - \boldsymbol{f}_t$ and $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\gamma}} - \bar{\boldsymbol{f}}$, we define the variance-covariance matrix as:

$$\hat{Var}(\hat{\boldsymbol{\gamma}} - \bar{\boldsymbol{f}}) \equiv \hat{Var}(\hat{\boldsymbol{\phi}}) = \frac{1}{T}\hat{Var}(\hat{\boldsymbol{\phi}}_t),$$

where $\hat{Var}(\hat{\boldsymbol{\phi}}_t) = \frac{1}{T}\sum_{t=1}^{T}(\hat{\boldsymbol{\phi}}_t - \hat{\boldsymbol{\phi}})(\hat{\boldsymbol{\phi}}_t - \hat{\boldsymbol{\phi}})'$. The test $\hat{\boldsymbol{\phi}}'[\hat{Var}(\hat{\boldsymbol{\phi}})]^{-1}\hat{\boldsymbol{\phi}}$ for $H_0 : \hat{\boldsymbol{\gamma}} - \bar{\boldsymbol{f}} = \boldsymbol{0}$, under H_0, has distribution χ^2_N, where N equals the number of dependent portfolios.