Understanding Alpha Decay

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Abstract

Received wisdom suggests that when investors correct a mispricing, the return for trading against the mispricing disappears. This paper shows that the return should instead *increase* as prices adjust to eliminate the mispricing. The decay signature is large; it corresponds to the prior alpha times a duration term that depends on the persistence of the anomaly. More generally, any changes in expected returns move returns in the opposite direction. When an anomaly attenuates over time, the average of past returns therefore overestimates the true ex-ante return, an important feature when testing anomalies. I document and quantify repricing returns in the data.

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Many anomalies see their alpha decay or disappear after publication. Received wisdom suggests that when investors eliminate a mispricing, the return for trading against the mispricing disappears after the discovery. To the contrary, this paper shows that the return should *increase* in the aftermath of the discovery. Otherwise, it means that the mispricing had been corrected before the presumed discovery date, or that there was no mispricing in the first place.

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The intuition underlying the above observation is best illustrated in the context of a mispriced bond, which earns an anomalous alpha. Suppose that this bond has a 5% yield but that its fair yield is 2%, so that the bond delivers a 3% alpha per year to its holders. When investors learn about the mispricing, for instance upon publication in an academic outlet, they require a 2% yield to hold that bond. For a sample period long enough, the researcher will observe a 3% alpha before discovery and a 0% alpha in the period after the discovery. But for the yield to decrease, the price of the bond must *rise*. This generates a positive return when the anomaly is revealed. As is well known, the sensitivity of a bond price to changes in its interest rate is its duration. The decay signature should thus equal the pre-discovery alpha times the duration of the bond.

This paper formalizes this intuition to the context of cross-sectional anomalies. Section I sets up a simple reduced-form model featuring cross-sectional predictability by some characteristic. Firm-level predictability resolves slowly over time and may reflect risk or mispricing. Because there is a large number of firms, exploiting this predictability generates a constant alpha, despite the fact that predictability tends to disappear at the firm level. For this alpha to change, for example after the discovery of a mispricing, prices must adjust in proportions equal to the change in alpha times the duration of the anomaly. This duration increases with the persistence of the anomaly, so that most persistent anomalies exhibit higher repricing returns.

More than forty years of academic research has studied the cross-section of stocks returns and other asset classes such as bonds, currencies, and commodities (see e.g. Cochrane (2011), Goyal (2012), and Nagel (2013) for recent surveys). Over the years, hundreds of predictive characteristics have been identified, yielding strategies that earn returns that are do not seem commensurate with their risk (Harvey et al., 2016). Because such anomalous returns may reflect unmodeled risk, limits to arbitrage, mispricings, or may simply be spurious, examining returns out-of-sample is crucial to understand their nature. When an anomaly reflects risk or limits to arbitrage, its "discovery" is no news to investors and alphas stay constant. When an anomaly reflects mispricings or a false positive, alphas disappear post-publication. This paper suggests that researchers can learn about the nature of an anomaly by examining returns around its presumed discovery. There cannot be any change in the alpha without a change in prices in the opposite direction. The absence of a repricing implies that there was no mispricing to be corrected, because, e.g., the anomaly is a false positive, or because it is dormant (e.g., the so-called "January effect" on a non-January month).

More generally, the main message of this paper is that realized returns differ from expected return when expected returns vary over time. Over the past decades, the size of the financial sector has grown substantially, and evidence indicates that prices have become more informative (Bai et al., 2016). Chordia et al. (2014), McLean and Pontiff (2016), and Green et al. (2016) have documented the attenuation of a host of anomalies over time. This persistent decline in anomaly returns is problematic for asset pricing tests that routinely assume constant, or at least stationary processes for expected returns.[1] Stationarity ensures that, asymptotically, the average of past realized returns converges to the true expected return. This paper argues that this approach is problematic whenever returns decay over time. Even when the true process is stationary, part of the returns earned on a factor reflects alpha decay, so that the average of past returns overestimates the true ex-ante return. The problem is even worse when the return decay is permanent, in which case the expected return is no longer well defined.

The cases discussed here are not just technical details. Section II provides some magnitudes. I first revisit results from a recent study by McLean and Pontiff (2016) to document and quantify repricing returns. McLean and Pontiff examine returns over 97 characteristics and find that on average anomaly returns decay out of sample, first in the sample period that is before publication, and then again in the period that follows publication. They attribute the first decrease to statistical biases and second decrease to investors learning from the publication. Interestingly, the authors observe two series of "outliers," in which returns increase when anomalies decay, the first at the very end of the in-sample period, and the second around the publication year. I show that a simple

[1] See DeMiguel et al. (2017); Feng et al. (2017); Freyberger et al. (2017); Kelly et al. (2017); Kozak et al. (2017) for recent examples.

trajectory for the true unobserved ex-ante return predicts decay signatures at the same time and magnitude as in the data. This result reinforces McLean and Pontiff's main point—that academic research contributes to anomaly decay. But this result also implies that trading against anomalies may start long before publications. In fact, the first repricings occur within the original sample, likely before researchers even identified the anomaly. This suggests that academics may sometimes learn from market participants as well, a new finding.[2] Using a smaller sample of 26 anomalies, I then replicate and extend the results in McLean and Pontiff (2016), and others, and show that the publication effect is significant and is strongest at the time of publication.

A duration of 3 and a learning phase of about two years seem to match the data very well. This result is useful in itself to guide future asset pricing tests, as discussed in Section III. Specifically, Chordia et al. (2014) find that the typical anomaly decays by about 30 bps per year. With a duration $\delta = 3$, the average of past returns overestimates the true ex-ante return by about 90 bps, under the conservative assumption that anomaly decay is only transitory. This pleads for a higher hurdle to assess anomalies, a point that is important to bring order to the "zoo of factors" that have been identified by the literature (Cochrane, 2011). Although the wedge between ex-ante and ex-post returns is well known (e.g., Elton 1999), quantifying the implications of anomaly decay on asset pricing tests is novel to the literature. In this dimension, this paper relates to recent papers that argue that many anomalies may be spurious (Harvey et al., 2016; McLean and Pontiff, 2016; Chen and Zimmermann, 2017; Chordia et al., 2017; Hou et al., 2017; Linnainmaa and Roberts, 2016).

This paper is also related to recent contributions that study anomalies in theoretical settings. van Binsbergen and Opp (2017) differentiate the aggregate mispricing wedge from alphas, and stress that persistent anomalies impose stronger inefficiencies than short-

[2] Relatedly, recent evidence argues that anomalies do not decay outside the United States (Jacobs and Müller, 2016; Lu et al., 2017). The results in this paper suggest that these conclusions may be premature. Because realized returns equal the ex-ante return plus the effect of alpha decay, a possibility—that remains to be tested—is that this non-attenuation in realized returns reflects late alpha decay. If alpha decays with a lag in international data, the effect of repricing returns may be large enough to hide anomaly attenuations.

lived anomalies. The present paper introduces the notion of characteristic duration and shows that the mispricing wedge simply equals the alpha multiplied by the duration. Jones and Pomorski (2016) study the investment problem of a Bayesian investor investing in a decaying anomaly. Liu et al. (2015) and Cho (2016) study the effect of arbitrage capital on anomalies returns, but not the effect of alpha decay on realized returns.

I. A Present-Value Model

This section presents a simple, reduced-form, economy featuring an anomaly associated with a characteristic. Sorting assets into portfolios based on the characteristic yields differences in average returns across portfolios, so that a portfolio that goes long the assets with a high average return and that shorts assets with a low average return generates a significant excess return. I next study the effect on returns of anomaly decay.

Consider an economy with a continuum of assets (firms) with mass normalized to one indexed by i. I assume that firms pay dividends $D_{i,t}$ in each time period. Denote $\alpha_{i,t}$ the expected conditional rate of return on a given asset in excess of a constant rate, $\alpha_{i,t} \equiv E_t(r_{i,t+1})$. The constant rate may be risk-free, or incorporate a compensation that investors require to hold the asset. For simplicity, I normalize this constant rate to 0, and use ex-ante return, or expected return, to refer to alphas. The firm-specific alpha can be positive or negative and is attached to some observable characteristic of the firm. It may reflect a risk premium, or limits to arbitrage, when agents are aware of the relation between the characteristic and future returns. Alternatively, $\alpha_{i,t}$ may reflect agents' biased beliefs regarding future dividends or regarding the riskiness of the asset; in that case it reflects a mispricing. In all instances, changes in alphas require prices to move, so for convenience I describe $\alpha_{i,t}$ as a mispricing.

We are interested in an economy where mispricings tend to resolve randomly over time. For simplicity, I assume that $\alpha_{i,t}$ follows a zero-mean AR(1):

$$\alpha_{i,t+1} = \phi\alpha_{i,t} + \varepsilon^{\alpha}_{i,t+1}, \tag{1}$$

where the mispricing shocks $\varepsilon^{\alpha}_{i,t1}$ are identically distributed normal shocks, and where for convenience firms share the same persistence parameter ϕ, which I assume positive and lower than 1. Because there is a large number of firms, there is no mispricing at the aggregate level, so that the expected excess return on the aggregate equally weighted portfolio is:

$$E(r^M_{t+1}) = \int \alpha_{i,t} dF(\alpha_{i,t}) = 0,$$

where F is the c.d.f. of the distribution of the firm-level mispricing; this distribution is Gaussian, following my normality assumption for the shocks $\varepsilon^{\alpha}_{i,t}$. The fact that the expected return is constant follows from the assumption of a large number of firms, and that the $\alpha_{i,t}$ are uncorrelated.

Ex-ante return Consider a portfolio with weights equal to 1 on the firms for which $\alpha_{i,t}$ is positive and weights equal to -1 on the remaining firms for which $\alpha_{i,t}$ is negative. The ex-ante return—the alpha—on this long-short portfolio is

$$\alpha = \int |\alpha_{i,t}| dF(\alpha_{i,t}). \tag{2}$$

Unlike the alpha on individual firms, the alpha on the long-short portfolio is positive by definition. Observe that alpha is constant, which follows from my assumption that both ϕ and the volatility of the mispricing shocks are constant, and that the individual ex-ante returns are uncorrelated. I make this assumption for simplicity and to emphasize the focus on cross-sectional return predictability.

Characteristic duration I express mispricing in terms of *expected returns.* Since we are interested in the impact of anomaly discoveries on realized returns, we need to express the effect of changes in expected returns on asset prices. To do so, I follow Campbell and Shiller (1988) and write log excess returns as

$$r_{i,t+1} = \kappa + \rho pd_{i,t+1} + \Delta d_{i,t+1} - pd_{i,t}, \tag{3}$$

where $pd_{i,t}$ is the log price-dividend ratio and $\Delta d_{i,t+1} = \log(D_{i,t+1}/D_{i,t})$ is log dividend growth. The constants κ and ρ are common to all firms and defined by $pd = E[pd_{i,t}]$, $\kappa = \log(1 + \exp(pd)) - \rho pd$, and $\rho = \frac{\exp(pd)}{1+\exp(pd)} \approx 1$. Iterating Equation (3) forward and taking expectations conditional upon time t yields

$$pd_{i,t} = \frac{\kappa}{1-\rho} + \sum_{j=1}^{\infty} \rho^{j-1} E_t[\Delta d_{i,t+j} - r_{i,t+j}] \tag{4}$$

where I assumed that $\lim_{j\to\infty} \rho^j pd_{i,t+j} = 0$. We can assume further that expected dividend growth is constant, with a long-term growth rate g common to all firms, so that dividend growth is given by:

$$\Delta d_{i,t+1} \quad = \quad \log\left(\frac{D_{i,t+1}}{D_{i,t}}\right) = g + \varepsilon^d_{i,t+1}. \tag{5}$$

Together with my assumptions regarding mispricing and dividend dynamics (1) and (5), this lets me write

$$pd_{i,t} = \frac{\kappa + g}{1-\rho} - \frac{1}{1-\rho\phi}\alpha_{i,t}. \tag{6}$$

Finally, inserting (6) in (3) gives

$$r_{i,t+1} = \alpha_{i,t} + \varepsilon^d_{it+1} - \delta\varepsilon^\alpha_{i,t+1},, \text{ where } \delta \equiv \frac{\rho}{1-\rho\phi}. \tag{7}$$

The ex-post return on a given asset $r_{i,t+1}$ equals the expected return plus a combination of a cash flow shock and a shock to expected returns. Because expected returns are persistent, their shocks are multiplied by an additional term δ, which can be interpreted as the *duration* of the anomaly. The intuition is related, but distinct, to the notions of bond and equity durations (e.g. Dechow et al. 2004), that describe the sensitivity of a bond or stock prices to a change in interest rate. The characteristic duration describes the sensitivity of characteristic-sorted portfolio to a change in the expected return.

Ex post return Aggregating at the portfolio level, the ex-post return equals

$$r_{t+1} = \alpha + \varepsilon^d_{t+1}. \tag{8}$$

In this simple model, the only source of risk is given by cash flow shocks $\varepsilon^d_{t+1} \equiv \int \varepsilon^d_{i,t+1} dF(\alpha_{i,t})$. Recall that I made no assumption regarding the covariance structure of cash flow shocks, while I assumed that ex-ante return (alpha) shocks are independently distributed. Hence cash flow shocks persist at the portfolio level, while alpha shocks are fully diversified away. This is only a restatement of my assumption that alpha is constant. Empirically, anomaly portfolio returns are quite volatile in the data, which points toward excess volatility beyond pure cash flow shocks (see e.g. Cohen et al. 2003; Lochstoer and Tetlock 2016). Some anomalies are also cyclical by nature, e.g. the January effect. This assumption is only here to simplify notations: the next section studies the effect of a change in the ex-ante return.

Anomaly decay I can now express the effect of anomaly decay on the anomaly portfolio return. I model anomaly decay as a decline in the ex-ante return. This decay can come from many sources, such as investors learning (from past dividends as in Lewellen and Shanken (2002) or from academic publication as in McLean and Pontiff 2016), changes in liquidity and transaction costs (Chordia et al., 2014), or changes in the stochastic discount factor(s), as in, e.g., Cho (2016). Also note that while empirical evidence points toward declines in alpha over time, the results hold symmetrically for an increase in alpha.

Suppose that at time $t+1$ mispricings $\alpha_{i,t}$ shrink toward zero at a constant decay rate θ. For example, investors learn about the mispricing from the publication in an academic journal and adjust their asset demands accordingly. If at time t a firm i earns an anomalous ex-ante return $\alpha_{i,t} > 0$, this return falls in $t+1$ to $(1-\theta)\alpha_{i,t}$, where $0 < \theta \leq 1$. This is equivalent to a mispricing shock $\varepsilon^\alpha_{i,t+1} = -\theta\alpha_{i,t}$ for all assets. If the required return to hold an asset falls, its price must increase; the same logic, in reverse, applies if $\alpha_{i,t} < 0$. For a typical firm, this generates an abnormal return equal to

$$-\delta\varepsilon^{\alpha}_{i,t+1} = \delta\theta\alpha_{i,t}.$$

This repricing aggregates at the level of the long-short portfolio, with an abnormal return given by

$$\int \delta \times \theta|\alpha_{i,t}|dF(\alpha_{i,t}) = \underbrace{\delta}_{\text{Duration}} \times \underbrace{\theta\alpha}_{\substack{\text{Anomaly}\\ \text{decay}}} \tag{9}$$

which is reminiscent of the familiar effect of a change in an interest rate on the price of a bond.

Another interpretation of the characteristic duration is that it gives a measure of the aggregate mispricing wedge in the economy. To see this, observe that the anomaly is entirely eliminated when $\theta = 1$. The mispricing wedge corresponds to the returns that eliminate the anomaly, i.e. $\delta\alpha$. The anomaly alpha hence only gives a partial measure of the economic importance of an anomaly, as it is a flow measure. Multiplying the alpha by the characteristic duration turns this flow into a stock—the mispricing wedge (see also van Binsbergen and Opp 2017).

Magnitudes Observe that the duration depends on the persistence of the anomaly ϕ and on the long-term value of the (average) price-dividend ratio (P/D):

$$\delta = \frac{1 + P/D}{1 + (1 - \phi)P/D} \tag{10}$$

The more persistent the anomaly (the closer is ϕ to 1), the larger the duration and thus the larger the repricing shocks. Suppose, to borrow the example from Cochrane (2008), that the anomaly is attached to a permanent characteristic of the stock, e.g. the first letter of its ticker symbol. In that case, $\phi = 1$ and δ will logically equal the duration of a perpetual bond. To fix magnitudes, for $P/D = 20$ the duration would be 21. Symmetrically, the duration is 1 for $\phi = 0$. Although crucial to the problem at hand, anomaly persistence is neither well documented nor well understood. Portfolio turnover is sometimes reported in the literature. Unfortunately, there is no direct mapping between portfolio turnover and anomaly persistence: turnover depends on both the persistence ϕ and the volatility of the mispricing shocks. In addition, portfolio turnover can often be reduced to minimize transaction costs, while maintaining a positive alpha (e.g. Frazzini et al. 2014; Novy-Marx and Velikov 2016). I return to this point in section II.

Understanding anomaly discoveries It should be apparent that there cannot be any change in ex-ante return if there is no change in price. This is important to interpret anomaly discoveries. If realized return decline after the discovery of an anomaly but one does not observe high returns, it had to be that the anomaly did not exist to begin with, or that the mispricing wedge was exactly zero at the time of the discovery. This latter case is actually likely for cyclical anomalies, where ex-ante returns may be zero at some point in time (e.g., the discovery of the January effect in June).

Relatedly, failing to account for repricing returns may lead the econometrician to wrongly conclude that an anomaly is robust out of sample. Given the relatively small post-discovery sample sizes, these returns have a similar order of magnitude as the anomaly itself. For example, for an out-of-sample period of 10 years and a duration of 5, the returns corresponding to the elimination of the anomaly weight as large as half the effect of the anomaly itself over the out-of-sample period. As noted earlier, this could help explain why recent research does not find evidence of anomaly decay in international data (Jacobs and Müller, 2016; Lu et al., 2017). In principle, the econometrician could use discovery dates to build more powerful tests of changes in ex-ante returns. However, this requires that the researcher can precisely date when arbitrageurs have started trading against the anomaly. In practice, various patterns of post-discovery return could emerge depending on how fast arbitrageurs learn about the mispricing, and crucially on the duration of the anomaly, complicating inference. The next section provides empirical guidance regarding when and how fast markets learn about mispricings around academic publications. Section III discusses to the question of inference in more detail.

II. Empirical Evidence

Earlier evidence indicates that some of the most prominent anomalies saw their return decay post-publication. For instance, Horowitz et al. (2000) finds that the size factor had very low returns in the fifteen years that followed the publication of the effect in academic journals (Banz, 1981; Reinganum, 1981). Schwert (2003) notes that the discovery coincided with the emergence of small cap-based funds that sought to exploit the anomaly, a prominent example being Dimensional Fund Advisors (DFA), which started offering a fund trading against the anomaly in 1981. Schwert also finds that the value premium disappeared after 1993. While the value premium was readily known in the academic community in the nineties, it is plausible that it gathered attention around the publication of the Fama and French (1993) model. In principle, researchers could detect the elimination of an anomaly by examining the history of returns around the time of publication (or suspected arbitrage activity). Size and value returns were in fact large during these periods. Small firms outperformance persisted the two years after 1981 in the US market, and it is only in 1983 that the size effect presumably vanished (Hirshleifer, 2001).[3] Similarly, HML returns were high over the 1992-1993 years. Anomaly-level estimates are unfortunately very noisy, which motivates pooling anomalies together. This section looks for more systematic evidence of decay signature in the data. I begin by discussing results in a recent paper by McLean and Pontiff, and then provides further evidence on a smaller subset of anomalies.

A. Decay Signature in McLean and Pontiff (2016)

McLean and Pontiff (2016, hereafter MP) propose a clever design to decompose alpha decay into components due to mispricing and false positives. Their design compares returns in three periods: (i) in sample, (ii) out-of-sample but before publication, and (iii) after publication. Comparing returns between (i) and (ii) gives a measure of statistical

[3] Dimson and Marsh (1999) notice a similar two-year "honeymoon period" after the introduction of an index trading against the size anomaly in the UK market.

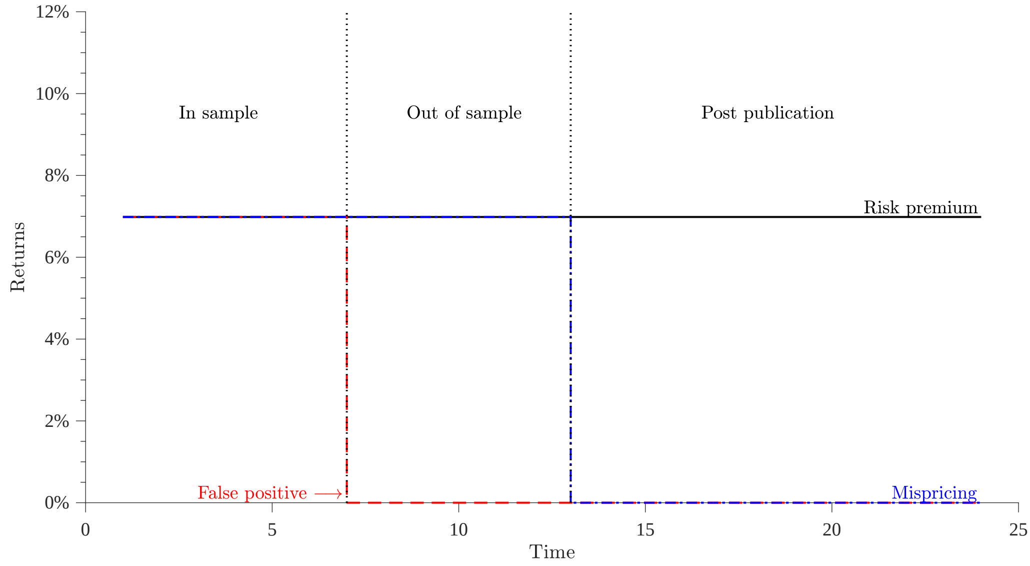
bias due to false positives. If the null of a zero return was rejected in-sample while being true, there is no reason for the return to be significant in the sample period that is not covered in the published paper. If the return declines between periods (ii) and (iii), it has to be that arbitrageurs learned from the academic research. Alternatively, if returns never decay, then it means that the anomaly reflects some risk. This empirical design is illustrated in Panel (a) of Figure 1, which plots the anomaly's ex-ante return as measured by the econometrician under the three alternatives. The point of the present paper is that the econometrician measures *ex post* returns, which differ from ex-ante returns whenever ex-ante returns change over time. (I return to this point below).

In their sample of 97 long-short characteristic-sorted portfolios, MP find that the average return is 6.9% annually in-sample, then declines to 4.8% out-of-sample, and finally to about 3.2% post-publication. They conclude that academic research in finance helped to eliminate mispricings, and that a large fraction of in sample returns reflect false positives as well. Interestingly, the authors document several "outliers", reproduced in Panel (b) in Figure 1 together with their main findings (with black dots). Returns are "too large" at the end of the in-sample period and the first year out of sample. In the former, they should not be different from the typical in-sample alpha, but are 9% larger on average. MP argue that this should not necessarily come as a surprise, if researchers strategically time the end of their sample period to maximize the in-sample alpha. However, they note that returns are also large in the first year out of sample, when they equal the typical in-sample alpha, while the true alpha should be zero. A similar pattern emerges again around publication date, with returns increasing the first two years post-publication.

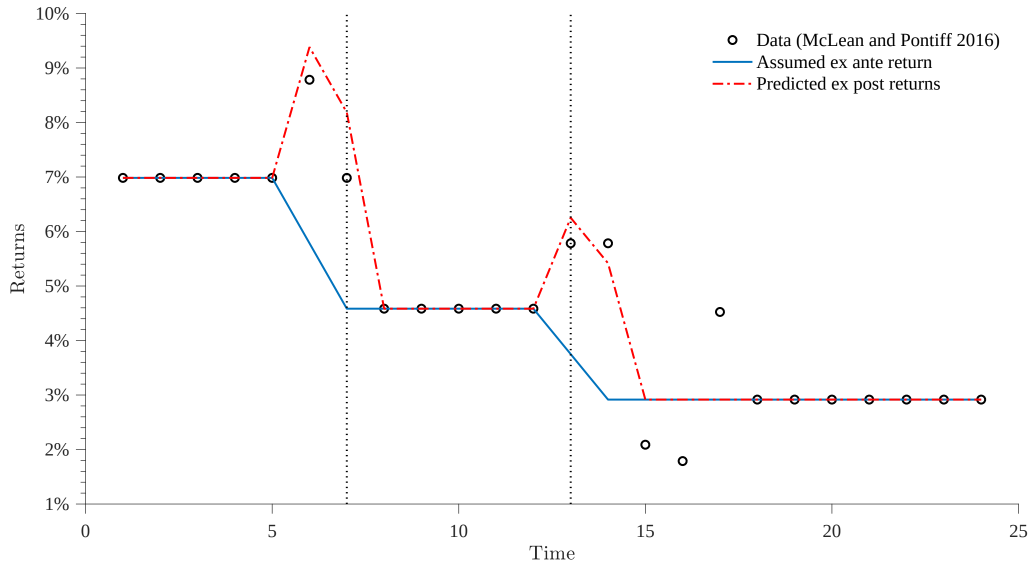
A natural explanation is that these observations are not outliers and reflect the price adjustments required to eliminate mispricings. In fact, the absence of a decay signature would be puzzling, given that McLean and Pontiff's design nicely controls for false positives. Of course, new anomalies are often discussed and made available before publication, and we may expect the repricing to unfold over long periods of time. Yet Figure 1 shows that the average decline in returns is sharp after publication. Given that many anomalies

Figure 1: Return dynamics around publication

(a) Empirical design



(b) Model-implied ex-ante returns and predicted ex-post returns



Panel (a) illustrates the research design employed in McLean and Pontiff (2016). Panel (b) compares the returns observed in the data, reproduced from Figure 2 of McLean and Pontiff (2016), together with the model-implied ex-ante returns and predicted returns obtained assuming a duration $\delta = 3$. The implied trajectory for expected return assumes three plateaus during which the true ex-ante returns equal measured returns in the three sub-periods (in-sample, out-of-sample, post-publication), and assumes that the declines in expected return take two years each. The start of each decline is chosen to match the return patterns in the data.

in MP's sample are persistent, prices should adjust precisely at these times. To illustrate this point, Panel (b) in Figure 1 shows average returns (black dots), together with the predicted returns accounting for the repricing (dashed red line), under an assumed trajectory for the true expected return implied by the model (solid blue line). For clarity, I make simple assumptions regarding this trajectory, namely that the true expected return has three plateaus during which it equals the measured returns in the three sub-periods, and that it takes two periods for the expected return to drop from one plateau to the next. I then "reverse engineer" the dates at which the expected return starts to decline, in order to match the return patterns around then end-of-sample and publication dates. Figure 1 shows that one obtains a good fit of the data, assuming a duration $\delta = 3$. This suggests that ex-ante returns do move after publication—the main message of MP—but also earlier. The first drop occurs two years before the end of the in-sample period, and can therefore explain why returns persist the first year out-of-sample. The second drop occurs the year before publication, and can explain the second bump in returns around the publication date.[4]

How different is this interpretation from MP's? First and foremost, the existence of decay signature strengthens the case that arbitrageurs eliminate mispricings. This confirms and reinforces the main point of MP. However, Figure 1 suggests a possibly different timeline. MP's assumptions imply that average ex-post returns should decline monotonically at the end of the in-sample period (due to false positives), and then exhibit a cycle around publication (due to arbitrageurs eliminating mispricings). Instead we observe two cycles, suggesting two distinct waves of repricings, one before the end of the in-sample period and one slightly before publication. The second one is identified in MP, that market participants learn from academic research. The first one—if significant—is novel and suggests that, in many cases, arbitrageurs have started to trade against anomalies before researchers identified them. Of course, noise could explain this first cycle. MP emphasize that their estimates are imprecise; I actually observe a more modest

[4]Observe that returns seem to overshoot in the data in the years 3-5 post-publication, which can be accounted for by letting the expected return fluctuate later on.

effect in my smaller sample of anomalies. Nevertheless this seems unlikely, given that the increase in return coincides with the subsequent decline in returns in the post-sample period. A perhaps more likely explanation is that academics "hear from Wall Street" about these profitable strategies, and popularize them to a larger audience. In these cases, academics are in fact the ones who learn from markets, rather than the opposite. Finally, while this interpretation leaves little room for false positives, I emphasize that MP's results describe average effects. That we observe large repricing returns in the data in spite of false positives indicates that mispricings are likely to be large for some anomalies.

B. Further Evidence on the Publication Effect

The results in the prior section show that publication decay is associated with large returns, strengthening McLean and Pontiff (2016)'s point that arbitrageurs eliminate anomalies over time. This section examines this publication effect further. I first replicate alpha decay results from MP and others in a smaller sample of 26 and document that returns are significantly higher within months of the official publication date. I also examine whether this effect differs across anomalies.

My dataset combines long-short value-weighted portfolios from Stambaugh et al. (2012) and Novy-Marx and Velikov (2016), that the authors generously share on their webpages. Stambaugh et al. (2012) provide data for 11 anomalies, spanning January 1963 to December 2016. Novy-Marx and Velikov (2016) uses only 23 portfolios, but Robert Novy-Marx provides a larger dataset of 31 anomalies on his website, covering the period July 1963 to December 2012. I collect the publication dates (year and month) for all published articles (or the first time the article was officially online, when this information is available), as well as the end-of-sample dates. To facilitate comparison, I focus only on published papers and drop portfolios for unpublished works as well as combinations of several strategies discussed in Novy-Marx and Velikov (2016). Whenever an anomalies is available in two datasets, I retain Stambaugh et al. (2012)'s, which has a slightly longer coverage. Overall, my sample contains 26 anomalies; the average publication year is 2001 and the average sample end is 1997. While this is a rather small sample, it includes the most well known and more robust strategies that have been proposed in the literature. Also, interestingly, Novy-Marx and Velikov (2016) classifies anomalies in term of turnover, which is useful to compare repricing effects between anomalies that differ in persistence.

My design largely borrows from the recent literature. I mostly follow McLean and Pontiff (2016) in regressing portfolio returns on dummies equal to one after the end of the sample period considered in each paper, as well as after the publication date. The first dummy captures the drop in returns between the in-sample period and the immediate period before publication. Under the assumption that the anomaly is still unknown during this period, this captures the effect of false positives on average returns.[5] The second dummy captures returns after publication, and therefore measures out-of-sample ex-ante returns as well as unexpected returns due to alpha decay. To separate the two effects, I introduce a third dummy equal to one if the date is within 6 months before and after publication. (I next consider variants of this specification). I also test for slow-moving declines in returns by including a time trend (Chordia et al., 2014) and a dummy equal to one after 2003. Green et al. (2016) find a significant decline in anomaly returns after 2003, which they attribute to the changes occurred in information and trading environment during the period 2002-2003. I also consider a specification with time fixed effects. Finally, all regressions include predictor fixed effects.

Before turning to the results, I must emphasize that this design does not test whether the model presented in Section I is an accurate representation of traders' beliefs, or whether anomalies have a non-zero durations. As argued earlier, as long as anomalies represent some inefficiency, there must be a mispricing wedge and this wedge can be summarized as the strategy's alpha times a duration. Failure to detect a publication effect indicates that either the repricing is small and the design has low power (e.g. for non-persistent anomalies), or that there is no repricing. Inversely, a positive publication

[5] As noted earlier, in this sample, the first return cycle at the end of the in-sample period is less pronounced than in MP.

effect does not necessarily imply that an anomaly is eliminated, only that prices adjust in that direction. Prices could still overshoot, e.g., if to many arbitrageurs start trading against the anomaly at the time of publication.

The main results are presented in Table I. In all configurations, the coefficients associated with post-sample and post-publication dummies are negative, indicating that returns decay after the end of the in-sample period and after publication; the magnitudes are very similar to the ones reported in McLean and Pontiff (2016). For instance, in column (1) the post-publication coefficient is -0.224, which indicates that returns decay by about 22.4 bps per month post-publication; McLean and Pontiff find a 33.7 bps decay in a similar setup. Also in line with Chordia et al. (2014) and Green et al. (2016) there is an overall downward trend in anomaly returns over time. In all instances the publication dummy is strongly positive, indicating that returns are larger around publication than in the other sub-periods. In the baseline specification (column 2), the publication dummy indicates that returns are 72.7 bps higher within six months of the publication dates. This suggests that publication has indeed a causal effect on prices. The result in unaffected by time trends and fixed effects (columns 5-6).

Another implication of the results in this paper is that, everything else equal, the decay signature should be stronger for more persistent anomalies. While persistence is unobserved, I use the fact that Novy-Marx and Velikov (2016) categorize their anomalies in terms of turnover. Regression (7) interacts the publication dummy with dummies equal to one when turnover is low (high). To the extent anomaly returns all reflect quantitatively similar mispricings, we should expect a muted publication effect for high turnover anomalies and a stronger effect for more persistent (low turnover) characteristics. Column (7) indicates that the publication effect is indeed smaller for high-turnover portfolios, but surprisingly it is also smaller for low-turnover portfolios. However, the estimates are imprecise and the interaction dummies are insignificant.[6]

Finally, Table II distinguish the publication effect that corresponds to the period before publication, to the period immediately after the publication. Separating the two

[6] MP also mention they considered this specification and report insignificant estimates.

periods amounts to asking whether exploiting the publication effect is a potentially profitable strategy for non-informed (or less-informed) investors. Formally, I split the publication dummy in two. In the baseline case in column (2), a first dummy equals one in the six months before publications and a second equals one if the month is within the six months after publication. I also consider time periods from one month to one year around publication. Table II suggests that most of the repricing occurs before publication, indicating that information about potential mispricings (or more generally new strategies) diffuses quickly. The effect is strongest the month of publication, with a 208.8 bps additional return, on average; it weakens in wider intervals of time. Overall, these results indicate that most of the repricing occurs in the few months before publication, so that less-informed investors are unlikely to profit from the repricing effect.

III. Implications for Asset Pricing Tests

A standard assumption in tests of asset pricing anomalies is that the ex-ante return is constant, or at least follows a stationary process. Yet the prior section shows that this assumption is strongly rejected in the data. This section discusses the consequences of a slow-moving or permanent decline in ex-ante returns for asset pricing tests.

An anomaly arises whenever the high average excess return on a given characteristic-sorted portfolio persists when controlling for a set of factors, for instance the Fama and French (1993) model. Specifically, since an asset pricing model should price any asset, the researcher can project the returns of the anomaly portfolio on the set of model-implied factors f_t:

$$r_t = \alpha + \beta' f_t + \varepsilon_{j,t}, \tag{11}$$

where r_t is again a log excess return. (I work with log-returns to maintain notational consistency, although such test is usually performed in levels). Suppose, for simplicity, that the portfolio returns are uncorrelated from the factor returns, so that β is a vector of zeros. Then the anomaly's alpha coincides with the ex-ante return, and the estimate of $\mathrm{E}r_t = \alpha$ is simply the average of ex-post returns:

$$\hat{\alpha} = \frac{1}{T}\sum_{t=1}^{T} r_t. \tag{12}$$

If $\hat{\alpha}$ is statistically positive, then the portfolio constitutes an anomaly with respect to the proposed asset pricing model. Of course, inference requires α to follow a stationary process (or be constant). I next discuss the implication of α decay over the sample period, either in a stationary fashion (so that the alpha should revert to the mean in the long run), or non-stationarily.

Stationary case Suppose that over the sample period ex-ante returns decay over time but remain stationary (i.e., the decay is transitory). The sample average of past returns (12) remains unbiased but is not efficient. Results in Section I imply that the average of ex-ante returns will differ from the average of ex-post returns (see also Avdis and Wachter 2017). Assume for simplicity that each period the ex-ante return decays by $\theta\alpha_t$. Form Equations (8) and (9), it is clear that ex-post returns equal

$$r_{t+1} = \alpha_t + \varepsilon_{t+1}^d + \delta\theta\alpha_t. \tag{13}$$

Inserting (13) in (12) gives

$$\hat{\alpha} = \frac{1}{T}\sum_{t=1}^{T}\alpha_{t-1} + \frac{1}{T}\sum_{t=1}^{T}\varepsilon_t^d + \frac{1}{T}\sum_{t=1}^{T}\delta\theta\alpha_{t-1}. \tag{14}$$

The average of ex-post returns thus equals the average of the true ex-ante returns, plus a noise term equal to the average of cash flow shocks, and an additional term reflecting anomaly decay. When the sample size T goes to infinity, our stationarity assumption implies that the last two terms disappear. Still, in finite samples, anomaly decay introduces a wedge between the measurement of ex-ante and ex-post returns. How big is this wedge? Chordia et al. (2014) find that anomaly returns decay on average by 3.9% a year (i.e. they find that it takes 12.8 years for an anomaly return to decline by half), or

30 bps expressed in returns units (the average anomaly return is 7.7% annually in their sample). With a duration $\delta = 3$—a plausible number as Section II argues—this suggests that returns are overestimated by about 90 bps, or 11.7%, a non-negligible number.

Non-stationary case Observe that non-stationarity implies that the unconditional ex-ante return $\mathrm{E}r_t$ non longer exists. Naturally, strict non-stationarity seems an odd assumption, and ex-ante returns are likely to be stationary in subsamples (e.g. before and after publication). Non-stationarity requires the econometrician to spell out over which period the expectation is defined. Suppose for example that the ex-ante return breaks a time τ; the true ex-ante return equals

$$\alpha_t = \begin{cases} \alpha^0 & \text{when } t < \tau \\ \alpha^1 & \text{otherwise.} \end{cases}$$

Presumably, the researcher is interested in the ex-ante return after the break α^1. It is clear that the sample average of past return (12) cannot deliver an unbiased estimate of α^1. In this example, the econometrician should test for a structural break in ex-ante returns. The best estimate of α^1 is then the average of realized returns after the break. The argument is similar, albeit less trivial, if ex-ante returns trend downward, as argued in Chordia et al. (2014). In that case, Equation (12) should account for such a trend, but the econometrician needs to choose a specification where the trend disappears at sample size T grows, so that the forward-looking ex-ante return remains finite when T goes to infinity.

To summarize, asset pricing tests should account for the evidence that anomaly returns have decayed over time. The econometric implications differ according to whether the possible decay is permanent or transitory, but in both cases, ignoring the decay leads the econometrician to overestimate the ex-ante return.

IV. Summary and Implications

Changes in ex-ante returns are discount rate shocks, a property that has seemingly been overlooked in the literature on cross-sectional return predictability. This paper formalizes this property and introduces the notion of characteristic duration. Like a bond duration, the characteristic duration is the sensitivity of an asset's value to a changes in its underlying rate. In the context of anomalies, the duration is the sensitivity of the value of the anomaly portfolio to changes in its ex-ante return. The duration is important to quantify the total mispricing wedge, which corresponds to the change in value of the anomaly portfolio if the anomaly is discovered and eliminated.

Anomaly discoveries are also useful on their own: they bring information regarding how fast markets learn about anomalies, and also about the magnitude of the typical characteristic duration. I use results in McLean and Pontiff (2016) to show that a duration of 3 is a reasonable estimate, and document two waves of investors learning about anomalies that last about two years each. Interestingly, these results suggest that academics learn as much from financial markets as financial markets learn from academic publications. I summarize below the main implications of the results in this paper, and discuss directions for further research.

Implication 1. *Prices must adjust when arbitrageurs eliminate mispricings. The more persistent the anomaly, the larger the price correction around the elimination of an anomaly.*

This implies that information on the nature of an anomaly—whether it reflects unmodeled risk, mispricing, or statistical bias—is revealed around its discovery.

Implication 2. *The out-of-sample performance of an anomaly does not imply that the anomaly will persist further.*

This is because post-publication returns may include repricing returns associated with the elimination of a prior mispricing. Similarly, evidence in Section II suggests that repricings may occur in the sample studied by the researcher, specifically at the end of the in-sample period. This casts suspicion on estimates of in-sample returns. In fact, not only anomaly discoveries, but also slow decay in limits to arbitrage (e.g. decline in transaction costs) cause anomaly to decay:

Implication 3. *Anomaly decay implies that the average of past returns overestimates the true ex-ante return.*

Section III shows that even if such decay is transitory, sample averages of past realized returns will overestimate the true ex-ante return. The problem is worse when returns decay permanently, because in this case the unconditional ex-ante return is no longer well defined.

These points are naturally related and call for a joint modeling of both ex-ante and ex-post returns. This task is crucial to understand which anomaly is priced as of today, and therefore to organize the "zoo of factors" that have been documented in the literature. This is complicated by the potential non-stationarity in anomaly ex-ante returns and the large volatility of the typical anomaly portfolio. This large volatility suggests that, beyond cash flow shocks, shocks to expected returns are an important component of anomaly returns. There is, unfortunately, little research on the time series behavior of expected anomaly returns. Beyond examining time series of returns and return predictors, researchers may also want to exploit trading volume and shorting interest to extract information about arbitrage activities. This task is beyond the scope of this manuscript, but will be pursued in future work.

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Table I: Publication Effect on Portfolio Returns

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Post-Sample	-0.186	-0.272			-0.257	-0.214	-0.271
t-stat	(-1.35)	(-1.94)			(-1.61)	(-1.37)	(-1.93)
Post-Publication	-0.224	-0.167			-0.133	-0.263	-0.167
t-stat	(-1.49)	(-1.07)			(-0.88)	(-1.67)	(-1.07)
Publication		0.727			0.733	0.710	
t-stat		(2.26)			(2.27)	(2.35)	
Time			-0.057		0.081		
t-stat			(-1.91)		(1.48)		
Post-2003				-0.439	-0.433		
t-stat				(-3.17)	(-1.81)		
Publication							1.130
t-stat							(2.24)
Publication x Low turn.							-0.697
t-stat							(-1.09)
Publication x High turn.							-0.897
t-stat							(-0.91)
Predictor FE?	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Time FE?	No	No	No	No	No	Yes	No
# obs.	15,702	15,702	15,702	15,702	15,702	15,702	15,702
# pred.	26	26	26	26	26	26	26

This table shows panel regression results regressing portfolio returns around publication time. The design tests for changes in realized returns in sub-samples. The left-hand side variable is the monthly return to 26 long-short portfolios. *Post-Sample* equals one when the month is after the sample period used in the original study and zero otherwise. *Post-Publication* equals one if the month is after the research publication date and zero otherwise. *Publication* is equal to one if the month within six months before or after the official publication date. *Time* is the number of months since January 1963, divided by 100. *Post-2003* equals one if the month is after January 2003. *Low (High) turn.* equals one if the strategy is classified as low (high) turnover in Novy-Marx and Velikov (2016). The sample spans January 1963 to December 2016. *t*-statistics (in parentheses) are computed under the assumption of contemporaneous cross-sectional correlation between panel portfolio residuals.

Table II: Timing the Publication Effect

	(1)	(2)	(3)	(4)	(5)	(6)
Post-Sample	-0.272	-0.278	-0.258	-0.276	-0.251	-0.239
t-stat	(-1.94)	(-1.97)	(-1.67)	(-1.99)	(-1.81)	(-1.75)
Post-Publication	-0.167	-0.159	-0.178	-0.150	-0.171	-0.168
t-stat	(-1.07)	(-1.02)	(-1.06)	(-0.98)	(-1.13)	(-1.12)
Publication	0.727					
t-stat	(2.26)					
Publication (6 mth. before)		0.778				
t-stat		(1.83)				
Publication (6 mth. after)		0.682				
t-stat		(1.53)				
Publication (1 yr. before)			0.286			
t-stat			(0.86)			
Publication (1 yr. after)			0.329			
t-stat			(1.00)			
Publication (3 mth. before)				1.725		
t-stat				(2.60)		
Publication (3 mth. after)				0.763		
t-stat				(1.37)		
Publication (2 mth. before)					1.583	
t-stat					(2.18)	
Publication (2 mth. after)					0.811	
t-stat					(1.26)	
Publication month						2.088
t-stat						(2.43)
Publication (1 mth. after)						-0.512
t-stat						(-0.71)
Predictor FE?	Yes	Yes	Yes	Yes	Yes	Yes
Time FE?	No	No	No	No	No	No
# obs.	15,702	15,702	15,702	15,702	15,702	15,702
# pred.	26	26	26	26	26	26

This table examines the effect of changing the window over which the publication effect is measured. *Publication* is equal to one if the month within six months before or after the official publication date. Column (1) reproduces Column (2) in Table I (see the table's legend for details). The next column decompose the publication effect between the month before and including the publication month, and the months after publication.

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