Do Hedge Funds Hedge?
New Evidence from Tail Risk Premia Embedded in Options *

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Abstract

This paper deciphers tail risk in hedge funds from option-based dynamic trading strategies. It demonstrates multiple and tradable tail risk premia strategies as measured by pricing discrepancies between real-world and risk-neutral distributions are instrumental determinants in hedge fund performance, in both time-series and cross-section. After controlling for Fung-Hsieh factors, a positive one-standard deviation shock to volatility risk premia is associated with a substantial decline in aggregate hedge fund returns of 25.2% annually. The results particularly evidence hedge funds that significantly load on volatility (kurtosis) risk premia subsequently outperform low-beta funds by nearly 11.7% (8.6%) per year. This finding suggests to what extent hedge fund alpha arises actually from selling crash insurance strategies. Hence, this paper paves the way for reverse engineering the performance of sophisticated hedge funds by replicating implied risk premia strategies.

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1 Introduction

The recent news[*] of the closure of Eton Park Capital Management, one of the most emblematic figure of the hedge fund industry, came as a shock to the financial community. It has brought the light to the most complicated periods hedge fund industry is experiencing, since liquidations strongly outpaced launches in 2016 according to Hedge Fund Research. In particular, the unexpected outcomes of the Brexit referendum and the U.S. elections have drawn doubts on their ability to manage tail risks. Indeed, hedge funds are often described as *"insurance companies selling earthquake insurance"* (Duarte, Longstaff, and Yu, 2007; Stulz, 2007), since they usually make penny-by-penny gains before incurring substantial losses. Hence, there is scarcely any doubt that hedge funds are particularly sensitive to market crashes, since they replicate short positions on equity index put options (Agarwal and Naik, 2004). Nevertheless, there is only limited literature on sophisticated option-based dynamic trading strategies that secretive hedge funds usually pursue, and how they explain hedge fund performance, risk, and compensation scheme. This research topic has become critical to remunerating hedge fund managers' skills and to understanding to what extent hedge fund alpha actually arises from beta, and specifically from alternative beta and alternative risk premia. Indeed, the highly entrepreneurial hedge fund industry has maintained a strong culture of secret and opaqueness to keep their investment process from fierce competition. Therefore, although U.S. institutional investment managers must report their portfolio holdings on Form 13F to the Securities and Exchange Commission (SEC), section 13(f) securities only concern equities and plain vanilla derivatives. In this way, SEC Form 13F doesn't reflect the highly exotic, out-of-the counter (OTC), and nonlinear payoffs usually hold by hedge fund managers. Hence, we test the following assumptions. First, does crash sensitivity of hedge funds arise from the tail risk premia strategies they usually trade? In particular, does tail risk premia investing explain the variation in hedge fund performance, in both the time-series and the cross-section of returns? Second, does crash sensitivity arise from a particular tail risk premia strategy? Specifically, at the hedge fund investment style level, which hedge funds are the most exposed to extreme events? Third, contrary to recent common beliefs, to what extent hedge funds can be simply considered as the *last insurers* against tail risk? In other words, to what extent does hedge fund alpha arise from selling crash insurance?

This paper is the first to explain the time-series and cross-sectional variation in hedge fund performance by tail risk premia. Although existing literature used tail risk measures and simple tail risk strategies, the specificity of the paper rests on: *i/* First, alternative risk premia since divergent swaps are more widely used

∗. Source: Bloomberg, on March 23, 2017.

by hedge fund managers because they fully reflect market price of risk; *ii/* Second, multiple tail risk premia since we decompose implied volatility smirks into three distinct tradable implied risk premia that fully reflect the market price of uncertainty associated to the realization of future extreme events. Tail risk premia usually designate tradable option-based payoffs pricing the market price of tail risk, as measured by the discrepancy between real-world and risk-neutral probability distributions. To that purpose, we derive from risk-neutral distributions and high-frequency data the tradable tail risk premia embedded in VIX options that are widely used by hedge funds, since they become the second most traded contracts at the Chicago Board Options Exchange (CBOE). As evidenced by Al Wakil (2016) in [6], tail risk premia embedded in options incorporate agents' risk attitudes and expectations about higher-order risks, and fully determine the market price of risk embedded in implied volatility surfaces. Therefore, this paper shows tail risk in hedge funds particularly arises from the distinct tradable volatility VRP, skewness SRP, and kurtosis KRP risk premia strategies that hedge funds usually pursue. Thus, they are instrumental determinants in the variation of hedge fund performance, both in the time-series and the cross-section.

This paper finds that exposures of hedge funds to tail risk premia are statistically significant across most investment strategies. Indeed, for the Global Hedge Fund Index, a four-factor model with our tail risk premia has the same explanatory power than the seven-factor model of Fung and Hsieh (2004) over the whole period. In particular, when considering tail events, adjusted R^2 associated to our augmented Fung- Hsieh model significantly increases across all investment styles. First, we exhibit to what extent hedge fund alpha actually arises from selling crash-insurance strategies. After controlling for loadings on Fung-Hsieh seven factors and forming quantile portfolios of cross-sectional hedge fund index returns sorted on the loadings of each of the tail risk premia, we evidence hedge funds that significantly load on volatility (kurtosis) risk premia substantially outperform low-beta funds by nearly 11.7% (8.6%) per year. In other words, when considering cross-sectional exposure to the volatility VRP (kurtosis KRP) risk premium, the high-minus-low portfolio realizes on average an annualized excess return of -11.7% (-8.6%). This finding particularly suggests hedge funds in quantile one are generally selling crash insurance, realizing on average annualized excess returns that compensate for bearing tail risks. Second, we evidence crash sensitivity of hedge funds substantially comes from volatility risk exposure. After controlling for loadings on Fung-Hsieh seven factors, a one-standard deviation increase in the volatility risk premium VRP is associated with a strong decline in aggregate hedge fund returns of 0.10% per day, or 25.20% per year over 2008-2013. Besides, over tail events, a one-standard deviation increase in the volatility risk premium VRP is associated with a substantial decline in aggregate hedge fund returns of 0.32%

per day, or 80.64% per year. In particular, at hedge fund investment style level, Relative Value and Equity Hedge are the most negatively exposed strategies to volatility risk, particularly during crises when volatility swap returns are the most profitable. This finding is consistent with literature, since Relative Value hedge funds are usually considered as *the last insurer* against tail risks, executing risk transfer from financial institutions, whereas Equity Hedge managers usually overlay hedge their long positions. Therefore, the associated payoff return profile is equivalent to buying a call option partially hedged by selling realized volatility. Third, at hedge fund investment style level, we evidence Relative Value and Directional hedge funds are the most positively exposed strategies to skewness risk. This result is consistent since they usually profit from underlying's volatility of volatility: Relative Value is commonly long gamma as described by Jaeger (2008) in [29], and trend-followers aim to buying optimally max lookback straddles according to Fung and Hsieh (2001) in [23]. In addition, we show Relative Value hedge funds are not simple insurance sellers, since they partially hedge their volatility risk exposure by buying skewness risk, whereas Global Macro hedge funds are usually negatively exposed to skewness risk. This last result is also consistent since Global Macro managers usually take contrarian bets on tail risks, i.e. selling realized skewness during crises, as their convergence trades are based on mid and long-term macroeconomic trends.

This paper extends the asset pricing literature associated to hedge fund performance for two reasons. First, it provides a new evidence for tail risk in hedge fund performance, showing it is an instrumental determinant in both the time-series and the cross-section of hedge fund returns, and to what extent hedge fund alpha actually arises from selling crash insurance strategies. Specifically, this paper deciphers hedge fund tail risk from multiple option-based dynamic trading strategies, defined as tradable tail risk premia, and decomposed into volatility, skewness, and kurtosis divergent swaps. Hence, we extend among others Asness, Krail, and Liu (2001) in [9], Geman and Kharoubi (2003) in [27], Agarwal and Naik (2004) in [3], Patton (2009) in [34], Agarwal, Ruenzi, and Weigert (2015) in [4], Agarwal, Arisoy and Naik (2017) in [1]. Nevertheless, this vast and recent literature usually deciphers tail risk in hedge funds from nontradable tail risk measures, or from standard and fragementary option-based strategies. Second, this paper contributes to the literature by providing a new evidence from multiple tail risk premia strategies that are widely traded by hedge funds, since divergent swap contracts fully incorporate the market price of risk. In particular, we show the volatility, skewness, and kurtosis risk premia, i.e. pricing discrepancies between risk-neutral and physical probability distributions, are distinguishable mimicking portfolios for insurance risk premia, usually harvested by hedge funds. Indeed, this paper evidences most hedge fund styles sell crash insurance, but tail risk exposures across hedge funds are distinct, since they depend on the specific trading strategies hedge fund managers use to arbitrate crash risks. In this sense, we extend the literature among others Aït-Sahalia, Wang, and Yared (2000) in [5], Alireza (2005) in [7], Chang, Zhang, and Zhao (2013) in [18], Bondarenko (2004) in [15], Schneider and Trojani (2015) in [36], and Al Wakil (2016) in [6]. Although this recent literature evidences new profitable divergence trading strategies to monetize compensation for higher-order risks, it generally doesn't explore the issue from hedge fund standpoint.

This paper arises practical implications especially within the industries of hedge funds, asset management, and smart indices. Since we find clear evidence that tradeable tail risk premia explain the variation in hedge fund returns, both in the time-series and the cross-section, this paper paves the way for reverse engineering sophisticated hedge funds by replicating the volatility VRP, skewness SRP, and kurtosis KRP risk premia strategies. Besides, this paper sheds light on the secretive drivers of hedge fund performance, since it disentangles it into real alpha and exotic beta like insurance-crash selling strategies.

The remainder of this paper is organized as follows. Section 1 describes the data used for this study, in particular the data from hedge funds, options, high-frequency trading, and futures. I document the methodology used to derive the tail risk premia embedded in options, and the Fung-Hsieh trend-following factors on a daily frequency. Section 2 investigates the time-varying exposure of various investment styles to tail risk, while Section 3 extends the analysis to the cross-section of investment styles. Robustness checks are provided in Section 4.

2 Literature

There is a vast literature about the instrumental contribution of tail risk in the pricing of hedge fund performance. More generally, this research question falls into the literature investigating the sources of hedge fund performance. In particular, it examines to what extent hedge fund alpha arises actually from market exposure, i.e. beta, and more recently from exotic beta, i.e. alternative beta, since hedge fund managers usually have recourse to sophisticated strategies.

Among others, Geman and Kharoubi (2003) in [27] show hedge funds are particularly sensitive to market distress. Jiang and Kelly (2012) in [28] exhibit a persistently exposure to the left-tail risk, both in the time-series and the cross-section of hedge fund returns. The differential asset pricing relations between dynamic tail and asset prices have been particularly well documented by Gabaix (2011) in [26], Wachter (2012) in [38], Drechsler and Yaron (2011) in [20], and Kelly (2012) in [31], among others. More recently, Agarwal, Ruenzi, and Weigert (2015) in [4] estimate a new tail risk measure from portfolio holdings to investigate the impact of tail risk on hedge fund performance. Specifically, they identify the sources of tail risk in the cross-section of hedge fund returns as tail-sensitive stocks and options.

The underlying assumption postulates hedge funds generally earn extra returns in good states for selling crash insurance, but suffer substantial losses during tail events episodes. Hence, a rich literature has suggested hedge funds are not really hedged, but rather exposed to risk factors, including Asness, Krail, and Liu (2001) in [9], Patton (2009) in [34], and Bali, Brown, and Caglayan (2012) in [10]. Indeed, a vast literature disentangles the sources of hedge fund performance, examining to what extent hedge fund alpha arises from beta. For illustration, the seminal paper of Jensen (1967) in [30] decomposes the mutual fund performance into the market risk exposure and the fund managing skills. This research topic has been particularly determinant since it puts into question the hedge fund compensation scheme, as market exposure (i.e. beta) is cheaper than active performance (i.e. alpha) and manager skills.

Furthermore, the reference papers of Fung and Hsieh (2001, 2004) in [23] and [24] that we extend here decomposes hedge fund performance into a seven-factor model that includes lookback straddles strategies to replicate the dynamics of trend-following hedge funds. Similarly, Mitchell and Pulvino (2001) in [32], and Fung, Hsieh, Naik, and Ramadorai (2008) in [25] evidence hedge fund strategies exhibit option-like payoffs, since systematic risk exposures can be replicated by option-based strategies. Indeed, recent literature examines to what extent hedge fund performance arises by now from complex and exotic beta, i.e. alternative beta, since hedge managers frequently have recourse to sophisticated strategies, using out-of-the counter derivatives and nonlinear payoffs. In particular, Agarwal and Naik (2004) in [3] clearly show left-tail risk in hedge funds arise from replicating short positions on equity index put options, especially for equity-oriented hedge fund styles that bear considerable crash risk. More recently, Agarwal, Arisoy and Naik (2017) in [1] find the uncertainty about equity market volatility is an instrumental determinant of hedge fund performance, both in the cross-section and over time. Specifically, they replicate the volatility of aggregate volatility with tradable lookback straddles on the VIX Index, and evidence a negative risk premium for uncertainty exposure in the cross-section of hedge fund returns.

Subsequently, this paper particularly falls into the recent literature investigating the alternative risk premia strategies usually traded by hedge fund managers to arbitrate the implied volatility smirks. Bondarenko (2004) in [15] estimates the market price of variance risk and clearly evidences that variance swap return is a key determinant in explaining hedge fund performance. Furthermore, he shows hedge fund managers usually sell variance risk, since they are negatively exposed to the variance swap return. More generally, Schneider and Trojani (2015) in [36] propose swap trading strategies studied by Bondarenko (2014) in [16] to capture the isolated tradeable compensation for time-varying risks in higher-order moments. Inspired by a new class of divergence trading strategies in Alireza (2005) in [7], they exploit the inconsistency between the option-implied risk-neutral distribution, i.e. the fair price of moments, and the physical distribution of the underlying asset. Similarly, Chang, Zhang, and Zhao (2013) in [18] introduce new derivative contracts, such as skewness and kurtosis swaps, to trade the forward realized third and fourth cumulants. Using S&P 500 index options from 1996 to 2005, they shed light on persistent time-varying properties of higher-order risk premia, offering a justification for such swap strategies. Less recently, Aït-Sahalia, Wang, and Yared (2000) in [5], and Blaskowitz and Schmidt (2002) in [13] document the profitability of skewness and kurtosis trades, exploiting the discrepancies between risk-neutral densities implied by DAX option prices and the historical state-price densities. Recently, Al Wakil (2016) in [6] evidence implied volatility smirks can be analytically and empirically decomposed into a parsimonious combination of alternative risk premia, mimicking tradable portfolios of option-implied volatility, skewness, and kurtosis risk premia to take bets on the level, slope, and convexity associated to the volatility smirks. These three distinct tail risk premia strategies are usually traded by hedge fund managers to monetize pricing discrepancies reflected in the implied higher-order risks.

Nevertheless, there is limited literature about the detailed tail risk trading strategies usually executed within each hedge fund investment style. Subsequently, we provide thorough understanding from Jaeger (2008) in [29] that sheds light on tail risk strategies implemented by various hedge fund investment styles. In the following paragraphs, we describe three assumptions about hedge funds' exposures that we test in the empirical analysis, in both the time-series and the cross-section of hedge fund returns.

Over 2008-2013, major tail events occured including among others the US Subprime crisis and the Lehman Brothers bankruptcy in 2008, the European sovereign debt crisis in 2010, the US sovereign debt crisis in 2011, and the Taper Tantrum in 2013. Subsequently, the time period had been particularly favourable to insurance-selling strategies, just in the aftermath of tail risk events when central banks envisaged unprecedented bailouts to contain the Global Financial Crisis. Specifically, many hedge funds sold crash insurance when it was expensive in the aftermath of extreme events, earning extra returns over 2008-2013, but making themselves particularly crash sensitive. In particular, Volatility Arbitrage managers increased short positions on expensive realized volatility and went long on cheaper implied volatility, when volatility swap returns were the highest. More globally, hedge funds also usually sold the forward realized third and fourth cumulants, i.e. the skewness and kurtosis via divergent swap contracts. Consequently, we assume hedge funds that substantially loaded on tail risk premia over 2008-2013 should have subsequently outperformed low-beta funds, shedding light to what extent hedge fund alpha arises from selling crash insurance strategies.

At investment style level, although Equity Hedge and in particular Long/Short Equity managers are directional long biased, they partially overlay hedge long positions using short index futures, long OTM puts, and short covered calls. Subsequently, payoff return profile is equivalent to buying a call option hedged by selling realized volatility via volatility swaps. This is particularly true when considering: i/ Equity Market Neutral strategies that try to generate returns uncorrelated to market risk; ii/ Short Selling strategies that partially hedge the short sale bias with OTM call options for example. Similarly, Relative Value strategies are non-equity-directional and they are commonly called arbitrage, spread, or alternative risk premia strategies. In particular, Fixed Income Arbitrage monetizes pricing anomalies associated to global yield curves but fully neutralizes exposure to systematic risk factors. Nevertheless, they are usually considered as the last insurer against market tail risks, as they execute alternative risk transfer strategies from global financial institutions. Hence, payoff return profile is equivalent to shorting put options and realized volatility via volatility swaps. Since available risk premia are small, arbitrageurs have usually recourse to high leverage level, ranging from five to 15 times the asset base, exposing themselves to tail risks. It was particularly true when LTCM (Long Term Capital Management) increased leverage to 30:1 to keep returns targets when assets under management reached USD 4 billion. Consequently, we assume that Relative Value and Equity Hedge strategies are the most negatively exposed hedge fund styles to volatility risk, and we expect this is particularly true risk during crisis periods, when volatility swap returns are the highest.

Considering Relative Value strategies, Fixed Income managers exploit and monetize higher-order risks embedded in the curvature of global yield curves, but neutralize net exposure to yield-curve changes. Spread trades in fixed income usually consist in yield-curve arbitrage, especially butterflies along the yield curve (e.g. long cheap 3-year and 5-year, short expensive 4-year), and related to strong institutional demand. Concerning other Relative Value hedge funds, Convertible and Volatility Arbitrage strategies intensively execute gamma trading to exploit positive convexity of delta hedge ratio function. Specifically, Convertible arbitrageurs are long gamma, i.e. gamma designates delta variation with underlying, since strategies are especially profitable when delta strongly changes, whatever the direction of the move. Since relation between derivative price and underlying price is positively convex, Convertible and Volatility arbitrageurs capture positive gamma by dynamically hedging their delta. Hence, the payoff return profile is equivalent to buying realized skewness, commonly interpreted as an insurance-buying strategy. Concerning Directional strategies, Fung and Hsieh (2001) in [23] show the payoff return profile of Systematic Managed Futures strategies is equivalent to a long straddle position. Indeed, trend-following strategies optimally aim to *buy low and sell high*, corresponding ideally to buying max lookback straddles. Consequently, Directional strategies have usually recourse to buying realized skewness since they generate profit from underlying's volatility of volatility. Alternatively, Global Macro hedge funds identify mid and long-term macro-economic trends, and execute convergence trades to exploit mispricings when market prices substantially deviate from their fair values. Hence, they usually take contrarian bets, maintaining for example a negative exposure to market risk or selling crash risk in crisis periods. Consequently, we assume that Relative Value and Directional strategies are the most positively exposed hedge fund styles to skewness risk, since they are usually buyers of realized skewness via long straddles or positive gamma. Furthermore, contrary to common beliefs, our assumption suggests that Relative Value hedge funds are not completely insurance-sellers strategies, since they partially hedge volatility risk by buying skewness risk. In addition, we assume Global Macro hedge funds can be negatively exposed to skewness risk, since they usually take contrarian bets on tail risk realization in crisis periods.

3 Data

Data samples primarily consist in daily hedge fund return provided by HFR and classified into major investment styles; daily VIX options data provided by OptionMetrics, including closing bid-ask mid prices, expiration dates, strike prices, open interest, and trading volume for all the listed maturities - data sample has been filtered following the methodology documented by Al Wakil (2016) in [6]; high-frequency data related to tick-by-tick historical VIX index prices provided by Bloomberg; and a broad range of futures data associated to 15 markets and provided by Datastream and Bloomberg.

3.1 Hedge Fund Return Data

Since the time period of our study is restricted by the scarcity of our high-frequency data sample that we use to estimate accurately the tail risk premia, we have recourse to daily hedge fund return data obtained from the HFR Database over the period 2008-2013. HFR indices are constructed to measure the aggregate performance of a wide range of hedge funds grouped by a specific strategy criterion. The hedge fund strategy classification aims to capture pure strategies that reflect the evolution of major trends in the hedge fund industry.

Table 1 reports the summary statistics of the hedge fund data sample used for our study. Overall, the sample includes 1,650 daily hedge fund index returns associated to the 5 investment styles and the global index over the period 2008-2013. We restrict the hedge fund data sample to the availability of the tail risk premia that we estimate by using in particular short-length high-frequency data. The average daily hedge fund return is nearly 1 basis point and the daily standard deviation is 0.32%. When comparing the daily returns distribution accross the data sample years over 2008-2013, Panel A exhibits significant disparities between turbulent and calm years. In particular, the returns distribution in 2008 is the only one that shows a negative average daily return of -0.12%. In addition, it exhibits the highest returns dispersion over the time period, including very high and low returns, respectively equal to -2.31% and 2.58%. High returns dispersion is also reflected in the magnitude of the standard deviations: about 0.76% in 2008, when compared to less than 0.28% during 2009-2013. Interestingly, the data sample covers both highly turbulent and calm periods.

[Insert Table 1 here]

Panel B associated to Table 1 disentangles the descriptive statistics by investment style. We consider the following 5 investment styles: Directional, Equity Hedge, Macro, Merger Arbitrage, and Relative Value (see the Appendix A for further details). Although academic literature points out ambiguity in the hedge fund classification, this strategy classification is currently used by HFR, and by related research, e.g. Patton and Ramadorai (2013) in [35]. Nevertheless, Panel B exhibits some significant disparities between investment styles, what allows to testing the impact of tail risk on hedge fund performance. In particular, Merger Arbitrage and Relative Value exhibits by far the less volatile investment styles (respectively 0.24% and 0.26%) when compared to the Equity Hedge and the Directional strategies (respectively 0.42% and 0.41%). Intuitively, Merger Arbitrage is an event-driven strategy that invests both in long and short positions in the companies that are involved in mergers and acquisitions. Since risk arbitrageurs take risk on deals, Merger Arbitrage strategy typically makes profits when equity markets are up. Hence, it tends to be strongly delta-hedged and lowly volatile. Accordingly, Risk Arbitrage exhibits the highest minimum daily return (-1.25%) over 2008-2013, by contrast with the Directional style (-2.31%) that typically uses trend-following strategies.

[Insert Table 1 here]

Figure 1 plots the hedge fund investment style performance over the sample period. Overall, they all exhibit negative shocks to highly turbulent and volatile time periods as embodied by the VIX Index that represents the markets fear gauge. Over 2008-2013, major tail events include the US Subprime crisis and the Lehman collapse in 2008, the European sovereign debt crisis in 2010, the US sovereign debt crisis in 2011, and the Taper Tantrum in 2013. Nevertheless, hedge fund investment styles exhibit very distinct dynamics during extreme events. In particular, Merger Arbitrage, and Macro strategies show stronger resilience to the Lehman collapse, the European sovereign debt crisis, and the US sovereign debt crisis, when compared to the Directional and the Equity Hedge strategies. This suggests structural and time-varying tail risk exposures of hedge fund styles.

[Insert Figure 1 here]

3.2 Fung-Hsieh Factors

In accordance with the hedge fund literature, the paper also includes various factors that appeared to be important in the hedge fund performance, in particular Fung and Hsieh in [22], [23], and [24]. The seven risk factors considered are: MKT-RF and SMB of Fama and French in [21], the change in the term spread (the daily change in the 10-year treasury constant maturity yield), the change in the credit spread (the daily change in Moody's Baa yield less 10-year treasury constant maturity yield), and the Fung-Hsieh trend-following factors, i.e. PTFSBD (bonds), PTFSFX (currencies), and PTFSCOM (commodities). We calculate proxies of the trend-following factors on a daily basis as described by Fung and Hsieh in [24] when modelling the perfect trend-follower strategy (see the Appendix E for further details). When put together, the above seven factors are known in the hedge fund literature as the Fung-Hsieh seven-factor model.

Table 3 reports the time-series Pearson pairwise correlations of the Fung-Hsieh seven factors and the tail risk premia. The volatility risk premium VRP appears to be significantly correlated to all the other factors, at least at the 5% level of confidence, whereas the skewness risk premium SRP does not covary significantly with the changes in term spread and credit spread and with the trend-following factors in bonds and commodities. Interestingly, the factor most negatively correlated to both the volatility and the skewness risk premia is the market return, respectively at -0.27% and -0.31%. By construction of the risk premium, this strong negative correlation is consistent with the fact that the realization of tail risks generate stock market crashes. By contrast, the kurtosis risk premium KRP only exhibits significant correlations with the risk premia of volatility VRP and skewness SRP, respectively at 0.19% and 0.21%.

[Insert Table 3 here]

3.3 Tail Risk Premia

Tail risk premia usually designate disaster insurance that investors pay to hedge against tail events. Intuitively, investors have considerably high marginal utility in such bad states, and they are willing to pay a lot of money to insure extreme event risks. This implies that the market price of tail risk is negative, and thus, tail risk premia generate negative excess returns over long period, but they compensate for paying an insurance by generating income in bad times (see the Appendix D for further details). Since options data reflect agents' attitudes and beliefs towards risk, market option prices incorporate the market price of uncertainty about the realization of future tail risks. Henceforth, tail risks are fully captured by the risk-neutral probability distribution, as market option prices determine the fair price of moments. From an economic motivation, vanished volatility smirk's slope and curvature reduce the risk-neutral probability distribution to the Black-Scholes lognormal distribution, whereas positive slope and curvature make the risk-neutral density respectively more right-skewed and leptokurtic, i.e. more peaked and heavy tailed.

Formally, let $IV_{t,T}$ the implied volatility smirk computed at time t for maturity T associated to moneyness ξ. As specified by Zhang and Xiang (2008) in [39], assume the following three-dimensional representation of the smirk IV approximated by a second-order polynomial function in the log-moneyness ξ. Then:

$$IV_{t,T}(\xi) = \begin{cases} \gamma_{0,t,T} & \text{Black} - \text{Scholes}: \text{ flat smile} \\ \gamma_{0,t,T}\left[1+\gamma_{1,t,T}\xi\right] & \text{Skewed IV smile} \\ \gamma_{0,t,T}\left[1+\gamma_{1,t,T}\xi+\gamma_{2,t,T}\xi^2\right] & \text{Smirked IV smile} \end{cases} \tag{1}$$

where tail risks are contained in $\gamma_{0,t,T}, \gamma_{1,t,T}, \gamma_{2,t,T}$ that respectively designate the level, the slope, and the curvature effects associated to the shape of the volatility smirk. Subsequently, Zhang and Xiang (2008) in [39] derive asymptotic approximations to clearly evidence the level, slope, and curvature are fully determined by the risk-neutral probability distribution, particularly the risk-neutral volatility $RNVol_{t,T}$, skewness $RNSkew_{t,T}$, and kurtosis $RNKurt_{t,T}$.

$$\begin{aligned} \gamma_{0,t,T} &\approx \left[1-\frac{1}{24}\left(RNKurt_{t,T}+3\right)\right]RNVol_{t,T}, \\ \gamma_{1,t,T} &\approx \frac{1}{6}RNSkew_{t,T}, \\ \gamma_{2,t,T} &\approx \frac{1}{24}\left[RNKurt_{t,T}+3\right] \end{aligned} \tag{2}$$

Furthermore, hedge fund managers typically exploit the discrepancies between risk-neutral and real-world distributions, i.e. option-implied risk premia. In particular, Carr and Wu (2009) in [17], and Bollerslev, Tauchen, and Zhou (2009) in

[14] define the volatility risk premium as the difference between the realized and the risk- neutral volatilities, i.e. $VRP_{t,t+\tau,T}$ computed at time t over period τ as the difference between the ex post realized return volatility over $[t-\tau, t]$ time interval and the ex ante risk-neutral expectation of the future return volatility over $[t, t+\tau]$, associated to options and futures for the given maturity T.

$$VRP_{t,t+\tau,T} \equiv E_t^P\left[\sigma_{t,t-\tau,T}\right] - E_t^Q\left[\sigma_{t,t+\tau}\right] \tag{3}$$

where $E_t^Q[\cdot]$ and $E_t^P[\cdot]$ denote the time-t conditional expectation operator under respectively risk-neutral Q and physical measure P. Therefore, $E_t^P[\sigma_{t,t+\tau,T}]$, and $E_t^Q[\sigma_{t,t+\tau}]$ are the expected values conditional to time t of the volatility realized over time period τ under respectively physical and risk-neutral probability measures. Furthermore, the volatility risk premium $VRP_{t,t+\tau,T}$ multiplied by a notional dollar amount usually defines the payoff at maturity $t+\tau$ of a return volatility swap. Under the no-arbitrage condition, the constant volatility swap rate $SW_{t,t+\tau}$ determined at time t and paid at time $t+\tau$ equals the risk-neutral expectation of the future realized volatility.

In line with Bollerslev et al. (2009) in [14], we estimate $E_t^P[\sigma_{t,t+\tau}]$ in Equation (3) by the realized volatility $RDVol_t^{(TS)}$ over day t. For the sake of simplicity, we henceforth drop the subscript τ and we denote $VRP_{t,t+T}$ as the volatility risk premium computed at time t over the period $\tau = 1$ day, associated to options and futures for the given maturity T. Similarly, volatility swaps can be theoretically extended to forward contracts written on the third and fourth moments, i.e. swaps respectively associated to skewness risk premium $SRP_{t,T}$ and kurtosis risk premium $KRP_{t,T}$ as follows:

$$Tail\ Risk\ Premia \begin{cases} VRP_{t,T} = RDVol_t^{(TS)} - RNVol_{t,T} & Volatility \\ SRP_{t,T} = RDSkew_t^{(TS)} - RNSkew_{t,T} & Skewness \\ KRP_{t,T} = RDKurt_t^{(TS)} - RNKurt_{t,T} & Kurtosis \end{cases} \tag{4}$$

where risk-neutral moments $RNVol_{t,T}$, $RNSkew_{t,T}$, and $RNKurt_{t,T}$ are extracted from market option prices by using the model-free approach of Bakshi, Kapadia, and Madan (2003) in [11] (see the Appendix B for further details). Realized volatility $RDVol_t^{(TS)}$ designates the Aït-Sahalia, Mykland and Zhang (2005) Two-Scales Realized Volatility measure introduced in [40] that uses subsampling, averaging, and bias correction for the market microstructure noise. This bias-corrected realized measure is then extended to higher moments $RDSkew_t^{(TS)}$ and $RDKurt_t^{(TS)}$ (see the Appendix B for further details).

The three tail risk premia $VRP_{t,T}$, $SRP_{t,T}$, and $KRP_{t,T}$ have been broadly documented by recent literature as mimicking portfolios that harvest the tradeable compensation for time-varying risks in higher-order moments. By construction, these risk premia are generally negative because risk-neutral volatility, skewness, and kurtosis are generally higher than the associated realized moments, since it contains investor's expectations for future non-realized tail risks. Hence, $VRP_{t,T}$, $SRP_{t,T}$, and $KRP_{t,T}$ are similar to insurance-buying strategies that compensate for bearing tail risks by generating positive payoffs in crisis periods. Aït-Sahalia, Wang and Yared (2001) in [5], Blaskowitz and Schmidt (2002) in [13], Alireza (2005) in [7], Chang, Zhang and Zhao (2013) in [18], Bondarenko (2014) in [16], and Schneider and Trojani (2015) in [36] among others, investigate this new class of divergence trading strategies to exploit the discrepancy between risk-neutral and real-world distributions of the underlying asset.

Specifically, Blaskowitz and Schmidt (2002) in [13] and Alireza (2005) in [7] arbitrate implied volatility smile for higher-order moments. Let the option-implied risk-neutral distribution be more skewed to the left than the real distribution of the underlying asset. Then, OTM put options may be relatively overpriced with respect to the OTM call options, since the risk-neutral distribution should reflect the fair price of skewness. Subsequently, trading the skewness consists in selling the OTM put option $P(S_t, K_C)$ and buying the OTM call option $C(S_t, K_C)$, associated to the underlying asset price S_t and the strike price K_C. The skewness trade then equals to selling realized skewness. Therefore, the payoff value Π_{Skew} associated to the corresponding delta-vega-neutral portfolio is

$$\Pi_{Skew} = C(S_t, K_C) - \frac{\nu_C}{\nu_P} P(S_t, K_C) - \left[\Delta_C - \frac{\nu_C}{\nu_P}\Delta_P\right] S_t \tag{5}$$

where (Δ_C, Δ_P) and (ν_C, ν_P) respectively designate the delta and vega of call and put options. Furthermore, skewness trades are commonly interpreted as long risk reversals or long synthetic stocks. Similarly, let the risk-neutral distribution has a sharper peak and fatter tails than the real-world distribution of the underlying asset. Then, OTM options may be relatively overpriced with respect to ATM options. Subsequently, trading the kurtosis consists in selling the OTM call $C(S_t, K_3)$ and put options $P(S_t, K_1)$, and buying the ATM call $C(S_t, K_2)$ and put option $P(S_t, K_2)$ for strike prices $K_1 > K_2 > K_3$. The kurtosis trade then equals to selling realized kurtosis. Hence, the payoff value Π_{Kurt} associated to the corresponding delta-vega-neutral portfolio can be interpreted as a long modified butterfly

$$\begin{aligned}\Pi_{Kurt} \quad &= \quad C\left(S_t, K_2\right) + \frac{\nu_{C_2}}{\nu_{P_2}} P\left(S_t, K_2\right) \\ &\quad - C\left(S_t, K_3\right) - \frac{\nu_{C_3}}{\nu_{P_1}} P\left(S_t, K_1\right) \\ &\quad - \left[\Delta_{C_3} + \frac{\nu_{C_3}}{\nu_{P_1}} \Delta_{P_1} - \Delta_{C_2} - \frac{\nu_{C_2}}{\nu_{P_2}} \Delta_{P_2}\right] S_t\end{aligned} \tag{6}$$

Skewness and kurtosis trades are usually interpreted as insurance-selling strategies, whereas mirror trades are equivalent to buying respectively realized skewness and kurtosis. Typically, hedge fund managers widely trade insurance strategies like volatility, skewness, and kurtosis swaps to arbitrate higher-order risks. To that purpose, VIX options have been widely traded to trade portfolio insurance, since they are European options written on VIX futures. Figure 2 plots the trading volume of VIX options and its decomposition into call and put options. As observed, they have become strongly popular since the Lehman Brothers crisis, being by now the second most liquid option contracts listed on CBOE and CFE, as they provide a purer exposure to tail risks than S&P 500 options. More specifically, VIX call options are strongly more actively traded than put options since hedge fund managers usually traded OTM and deep OTM call options to pay tail risk insurance. As documented by Al Wakil (2016) in [6], we derive the tail risk premia $VRP_{t,T}$, $SRP_{t,T}$, and $KRP_{t,T}$ from VIX options on a daily frequency and for different maturities over 2008-2013. We use high-frequency data provided by Bloomberg to estimate the real-world distribution, whereas OptionMetrics provides options data to estimate the risk-neutral distribution.

[Insert Figure 2 here]

Figure 4 plots the time series of VIX tail risk premia respectively associated to volatility, skewness, and kurtosis, for 30-days time to maturities, over 2008-2013. Overall, the three tail risk premia are generally negative, since favourable states of nature correspond to extreme events. In particular, brief episodes of positive volatility risk premia consistently correspond to the US Subprime crisis and the Lehman collapse in 2008, the European sovereign debt crisis in 2010, the US sovereign debt crisis in 2011, and the Taper Tantrum in 2013.

[Insert Figure 4 here]

As observed in Figure 3, turmoil periods are associated to realized volatility peaks. Table 2 reports summary statistics for tail risk premia. Student t- stats indicate that average risk premia are clearly all significantly negative at the 1% confidence level across the 30, the 60, and the 120-days time to maturities.

[Insert Figure 3 here]

[Insert Table 2 here]

4 Hedge Fund Exposure to Tail Risk Across Time

We investigate the contribution of tail risk premia to the performance of hedge fund investment styles, after controlling for the loadings on the Fung and Hsieh seven factors. Specifically, we perform time-series Ordinary Least Squares (OLS) regressions over the whole 2008-2013 period to evaluate crash sensitivity at hedge fund investment style.

For each hedge fund investment style, Table 4 summarizes the results of two time-series OLS regressions: *i/* a first regression of style index returns on the market factor $MKT - RF$ and the three tail risk premia VRP, SRP, and KRP; and *ii/* a second regression while adding the Fung-Hsieh seven factors. Overall, t-stats and p-values suggest that Global index is significantly loaded on two of the three tail risk premia after controlling for the loadings on the Fung-Hsieh seven factors. Indeed, a one-standard deviation increase in the volatility risk premium VRP is associated with a strong decline in aggregate hedge fund returns of 0.1% per day, or 25.2% per year. This effect has a statistical significance at the 1% level of confidence (t-stat of -3.40). Comparing regression *i/* with *ii/* shows that the four factor model with tail risk factors has the same explanatory power than the Fung-Hsieh seven factor model (adjusted R^2 at 0.41).

[Insert Table 4 here]

When analyzing investment styles, we find that the exposure of hedge funds to tail risk is statistically significant across most investment strategies. More specifically, four of the five investment styles (Relative Value, Directional, Equity Hedge, Macro) present a significant loading on at least one of the three tail risk premia, and for at least one of the regression specifications. Only one investment style (Merger Arbitrage) exhibits a statistically insignificant tail risk loading. Precisely, the four styles exhibit all negative and significant loading on the volatility risk premium VRP, at least at the 10% level of confidence. In terms of magnitudes, returns associated to hedge funds that pursue long-short equity strategies, e.g. Equity Hedge (1% level of confidence, t-stat of -2.63) and Relative Value (5% level of confidence, t-stat of -2.25), are especially sensitive to tail risk shocks. A one-standard deviation increase in the volatility risk premium VRP is associated with a drop in returns of respectively 0.13% and 0.10% per day for hedge funds investing in Equity Hedge and Relative Value. This finding is consistent with literature since Relative Value hedge funds are usually considered as the last insurer against tail risks, executing risk transfer from financial institutions, whereas Equity Hedge hedge strategies usually overlay hedge long positions. Therefore, their payoff return profile is equivalent to buying a call option hedged by selling realized volatility.

Therefore, concerning the skewness risk premium SRP, Relative Value (t-stat of 3.47) and Directional (t-stat of 2.68) styles exhibit both the most positive and significant loadings at the 1% level of confidence, whereas Global Macro (t-stat of -2.5) presents rather a negative and significant loading. In other words, Relative Value and Directional hedge funds are the most positively exposed to skewness risk, whereas Global Macro hedge funds are usually negatively exposed to skewness risk. Furthermore, our results show Relative Value hedge funds are not completely insurance sellers, since they partially hedge volatility risk exposure by buying skewness risk. From the economic intuition, these findings generally make sense, since Relative Value and Directional strategies usually profit from underlying's volatility of volatility. As described by Jaeger (2008) in [29], Relative Value style is commonly long gamma, i.e. trading gamma to adjust the delta hedge ratio, whereas trend-followers aim to optimally buying max lookback straddles as documented by Fung and Hsieh (2001) in [23]. Therefore, their payoff return profile is equivalent to buying realized skewness. Alternatively, Global Macro managers usually take contrarian bets, especially on tail risks by selling realized skewness during crises, since they base their convergence trades on mid and long-term macro-trends. For illustration, the loading on the market excess returns $MKT-RF$ associated to Global Macro style is negative (but then nonsignificant in the ii/-regression), whereas all the other investment strategies exhibit significant and positive loadings on the market excess returns $MKT-RF$. Finally, none of the hedge fund styles present a significant loading on the kurtosis risk premium KRP.

The findings are robust[†] to the expiration time used to calculate the tail risk premia VRP, SRP, and KRP. Findings clearly show that tail risk loading varies both across investment styles and time, and henceforth, tail risk premia are an instrumental pricing factor in the universe of hedge funds. More precisely, our findings clearly evidence that crash sensitivity associated to hedge funds mainly arises from volatility risk exposure. Indeed, Relative Value and Equity Hedge are the most negatively exposed strategies to volatility risk, even if Relative Value hedge funds partially hedge their volatility risk exposure by buying skewness risk. Conversely, Global Macro managers are the only hedge funds significantly negatively exposed to skewness risk.

†. Additional tests have been also performed with tail risk premia estimated for the 60, 90, and 120-days to maturity and can be provided on demand.

5 Tail Risk in the Cross-Section of Hedge Funds

In the previous section, we investigate embedded tail risk across time at the hedge fund investment style level. Consistent with the hedge fund literature, the evidence shows that hedge fund returns, in particular for equity-oriented investment strategies, are generally sensitive to tail risk shocks across times, after controlling for commonly used hedge fund risk factors. This finding suggests tail risk premia are instrumental determinants of hedge fund performance. In what follows, we provide now cross-sectional evidence that supports this theory.

Each day, three hedge fund portfolios are formed by sorting the five hedge fund investment styles on their exposures to tail risk. Specifically, at the end of each day, I perform time-series monthly rolling regressions of excess returns associated to hedge fund investment strategies on the market return and on respectively each of the three tail risk premia VRP, SRP, and KRP. In the daily estimation window, the tail risk loading of each investment style is calculated with at least 18 days of data. Nevertheless, the results are robust to running longer rolling windows,and to using longer maturities for the tail risk premia. Therefore, the five investment strategies are sorted into three quantile portfolios based on their tail risk factor loadings. The Fung-Hsieh alpha then designates the intercept associated to the regression of the daily tail-risk beta portfolio excess returns on the seven Fung-Hsieh hedge fund factors. This methodology is consistent with the hedge fund literature about tail risk, including Jiang and Kelly (2012) in [28] among many others, but it investigates tail risk in the cross-section of returns of investment styles rather than of individual hedge funds.

Table 5 reports the performance of the three quantile portfolios sorted on hedge fund tail betas, respectively associated to the three tail risk premia VRP, SRP, and KRP. For each of the tail risk factors, it summarizes the average daily tail risk betas, the average annualized excess returns, and the Fung-Hsieh seven factor alpha associated to the three quantile portfolios and to the high minus low portfolio, defined as the return spread between the high-tail-loading and the low-tail-loading portfolios of hedge funds. Results show significant returns dispersion in the investment styles captured by their betas on the tail risk premia.

[Insert Table 5 here]

Considering cross-sectional exposure to the volatility (Panel A) risk premia VRP, Table 5 shows the low-tail-loading portfolio (average beta of -0.216%) of hedge funds has the highest average annualized excess return (0.57%). Inversely, the high-tail-loading portfolio (average beta of 0.091%) of hedge funds has the lowest average daily excess return (-11.13%). Precisely, hedge funds in quantiles one and two have negative tail risk loadings, respectively of -0.216% and -0.026%.

Intuitively, these hedge funds have on average negative returns when tail risk is high, and therefore they are particularly sensitive to tail risk shocks. This suggests that funds in quantiles one and two are generally selling crash insurance, realizing on average higher annualized excess returns of respectively 0.57% and 0.51% that compensate for bearing volatility risk. Inversely, hedge funds in the last quantile have high and positive volatility risk loadings, on average of 0.091%, and are thus generally buyers of crash insurance. Hence, they earn on average significantly lower excess returns of -11.13%. The high-minus-low portfolio realizes on average an annualized return spread of -11.7%, that is significant at the 5% level of confidence, with a t-statistic of -2.38.

Similarly, considering cross-sectional exposure to the kurtosis (Panel C) risk premia KRP, Table 5 shows that the low-tail-loading portfolio (average beta of -0.003%) of hedge funds has the highest average annualized excess return (0.28%). Inversely, the high-tail-loading portfolio (average beta of 0.009%) of hedge funds has the lowest average daily excess return (-8.32%). Precisely, hedge funds in quantile one has negative tail risk loadings (-0.003%), and they tend to realize on average negative returns when tail risk is high, and therefore they are particularly sensitive to tail risk shocks. This suggests that funds in quantile one are generally selling crash insurance, realizing on average higher annualized excess returns of 0.28% that compensate for bearing kurtosis risk. Inversely, hedge funds in the second and last quantile have high and positive volatility risk loadings, on average of respectively 0.005% and 0.009%, and are thus generally buyers of crash insurance. Hence, they earn on average significantly lower excess returns of respectively -1.85% and -8.32%. The high-minus-low portfolio realizes on average an annualized return spread of -8.6%, significant at the 5% level of confidence, with a t-statistic of -1.86.

The findings are robust[‡] to the expiration time used to calculate the tail risk premia VRP and KRP. Nevertheless, when considering cross-sectional exposure to the skewness (Panel B) risk premia SRP, Table 5 shows non-significant extra returns for the high-minus-low portfolio. The average annualized excess return of 0.8% (with a t-statistic of 0.15) suggests the return spread between insurance hedgers and sellers is negligible over the post-Lehman period 2008-2013, and when considering the cross-section of hedge funds at the investment styles level.

Finally, we find clear evidence that hedge funds that significantly load on volatility (kurtosis) risk premia substantially outperform low-beta funds by nearly 11.7% (8.6%) per year. This result sheds light to what extent hedge fund alpha arises actually from selling crash insurance strategies, in particular selling realized volatility and kurtosis via divergent swaps.

‡. Additional tests have been also performed with tail risk premia estimated for the 60, 90, and 120-days to maturity and can be provided on demand.

6 Robustness Checks

In this section, we perform various robustness checks to ensure our results are consistent. Specifically, we investigate the contribution of tail risk premia to the performance of hedge fund investment styles, after controlling for the loadings on the Fung and Hsieh seven factors, over tail events to disentangle the time-varying and structural exposures of hedge fund styles.

6.1 Tail Risk Periods

Since crises episodes are violent but rare, scarcity implies that above results could be dominated by long non-crisis periods. Hence, we investigate the time-varying and structural exposure of hedge funds to tail risks by considering only financial turmoil. Specifically, we use the VIX *fear gauge* to time-slice the initial data sample by identifying peaks and bottoms associated to realized tail risks. Then, time-sliced sample period includes the US Subprime crisis and the Lehman collapse in 2008, the European sovereign debt crisis in 2010, the US sovereign debt crisis in 2011, and the Taper Tantrum in 2013, providing 71 observation points for data analysis. As expected, Table 6 exhibits much stronger adjusted R^2, suggesting that tail risk premia are particularly instrumental in pricing time-varying hedge fund performance. Considering the first regression of style index returns on the market factor $MKT - RF$ and the three tail risk premia VRP, SRP, and KRP, adjusted R^2 increases for all the investment styles, especially for Macro (from 0.03 to 0.19), Relative Value (from 0.10 to 0.18), and Merger Arbitrage (from 0.35 to 0.54); and to a lesser extent, for Equity Hedge (from 0.53 to 0.60) and Directional (from 0.49 to 0.58). Since the loadings on intercept and market factor remain generally unchanged from regression $i/$ to $ii/$, adding tail risk factors appears instrumental to pricing the hedge fund performance.

[Insert Table 6 here]

Overall, t-stats and p-values suggest that Global index is significantly loaded on one tail risk premia after controlling for the loadings on the Fung-Hsieh seven factors. Indeed, a one-standard deviation increase in the volatility risk premium VRP is associated with a considerable drop in aggregate hedge fund returns of 0.32% per day, or 80.64% per year. This effect has a statistical significance at the 1% level of confidence (t-stat of -3.01). Comparing regression $i/$ with $ii/$, the four factor model with tail risk factors has again the same explanatory power than the Fung-Hsieh seven factor model (adjusted R^2 at 0.46). More precisely, t-stats and p-values indicate that now all of the five style indexes present a significant loading on at least one of the three tail risk premia, and for at least one of the regression specifications. Precisely, three investment styles exhibit negative and significant loading on the volatility risk premium VRP, at least at the 10% level of confidence. In terms of magnitudes, it is especially true for Equity Hedge (5% level of confidence, t-stat of -2.46) and Relative Value (5% level of confidence, t-stat of -2.52) that significantly load on tail risk over 2008-2013 period. Precisely, a one-standard deviation increase in the volatility risk premium VRP is associated with a considerable drop in returns of respectively 0.39% and 0.42% per day for hedge funds investing in Equity Hedge and Relative Value. This finding particularly exhibit that Equity Hedge and Relative Value are the most negatively exposed strategies to volatility risk, especially during crises when volatility swap returns are the highest. In other words, they are particularly crash sensitive since they profit from selling realized volatility when it is considered as expensive during crises.

Then, considering the skewness risk premium SRP, Relative Value (t-stat of 2.02) and Directional (t-stat of 1.91) styles exhibit again both a positive and significant loading. This finding exhibits they usually buy realized skewness since they profit from underlying's volatility of volatility. Indeed, Relative Value managers are long gamma, and trend-followers aim to optimally buying max lookback straddles. Their associated payoff return profile is then equivalent to buying realized skewness. Global Macro (t-stat of -1.89) presents again the only negative and now significant loading on skewness risk. This result particularly validates that Global Macro hedge funds are usually selling realized skewness during crises since they base their convergence trades on long-term macro-trends. Interestingly, the Merger Arbitrage investment style presents by now a positive and significant loading on the kurtosis risk premium KRP (t-stat of 2.28). Figure 5 illustrates the sensitivities of hedge fund investment styles to the Fung and Hsieh seven factors (without market factor) and the three tail risk premia, where sensitivities are estimated by absolute values of t-statistics after controlling for the market factor loading.

[Insert Figure 5 here]

Intuitively, the tail risk embedded in the five hedge fund investment strategies makes sense over crisis periods. All the five investment strategies exhibit generally significant negative loadings on the volatility risk premium VRP, and significant positive loadings on the market excess returns $MKT - RF$.[§] These findings are consistent with Agarwal and Naik (2004) that evidence equity-oriented

§. Similarly, the loading on the market excess returns $MKT-RF$ associated to Global Macro style is negative but then nonsignificant in the $ii/$-regression. In additional tests with tail risk premia estimated for the 60, 90, and 120-days to maturity, the loading becomes significantly positive.

hedge fund styles generally bear considerable left-tail risk, incurring considerable losses in equity market downward moves. Furthermore, they are also consistent with our previous results: *i/* Relative Value and Equity Hedge are the most negatively exposed strategies to volatility risk, since they are usually volatility sellers; *ii/* Relative Value and Directional are the most positively exposed strategies to skewness risk, and Relative Value managers partially hedge their volatility risk exposure by buying realized skewness; And *iii/* Global Macro hedge funds are usually negatively exposed to skewness risk to take contrarian bets.

7 Conclusion

This paper has been motivated by filling the gap in the hedge funds and the asset pricing literature. Although there is scarcely any doubt that hedge funds are particularly sensitive to market crashes, there is limited literature on sophisticated option-based dynamic trading strategies that hedge funds usually pursue, and how they explain hedge fund performance. In particular, I address the following assumptions. First, does tail risk in hedge funds come from their tail risk premia strategies? Second, does tail risk premia investing explain the variation in hedge fund performance, in both the time-series and the cross-section? Finally, to what extent does hedge fund alpha arise from managerial skill or from actually selling crash insurance? In particular, do hedge funds can actively time tail risk before market crashes? To our knowledge, this paper is the first to explain the time-series and cross-sectional variation in hedge fund performance by tail risk premia dynamic strategies. Therefore, this paper shows tail risk in hedge funds particularly arises from the tradeable volatility VRP, skewness SRP, and kurtosis KRP risk premia strategies that hedge funds pursue. Thus, they are instrumental determinants in the variation of hedge fund performance, both in the time-series and the cross-section.

This paper finds that exposures of hedge funds to tail risk premia are statistically significant across most investment strategies. Indeed, for the Global Hedge Fund Index, a four-factor model with our tail risk premia has the same explanatory power than the seven-factor model of Fung and Hsieh (2004) over the whole period. In particular, when considering tail events, adjusted R^2 associated to our augmented Fung- Hsieh model significantly increases across all investment styles. First, we exhibit to what extent hedge fund alpha actually arises from selling crash-insurance strategies. After controlling for loadings on Fung-Hsieh seven factors and forming quantile portfolios of cross-sectional hedge fund returns sorted on tail risk loadings, we evidence hedge funds that significantly load on volatility (kurtosis) risk premia substantially outperform low-beta funds by nearly 11.7% (8.6%) per year. In other words, when considering cross-sectional exposure to the volatility VRP (kurtosis KRP) risk premium, the high-minus-low portfolio realizes on average an annualized return spread of -11.7% (-8.6%). This finding particularly suggests hedge funds in quantile one are generally selling crash insurance, realizing on average annualized excess returns that compensate for bearing tail risks. Second, we show crash sensitivity of hedge funds mainly comes from volatility risk exposure. After controlling for loadings on Fung-Hsieh seven factors, a one-standard deviation increase in the volatility risk premium VRP is associated with a strong decline in aggregate hedge fund returns of 0.10% per day, or 25.20% per year over 2008-2013. Over tail events, a one-standard deviation increase in the volatility risk premium VRP is associated with a substantial decline in aggregate hedge fund returns of 0.32% per day, or 80.64% per year. In particular, at HF investment style level, Relative Value and Equity Hedge are the most negatively exposed strategies to volatility risk, particularly during crises when volatility swap returns are the highest. This finding is consistent with literature, since Relative Value hedge funds are usually considered as the last insurer against tail risks, executing risk transfer from financial institutions, whereas Equity Hedge hedge funds usually overlay hedge long positions. Therefore, payoff return profile is equivalent to buying a call option partially hedged by selling realized volatility. Third, at hedge fund investment style level, we evidence Relative Value and Directional hedge funds are the most positively exposed strategies to skewness risk. This result is consistent since they usually profit from underlying's volatility of volatility: Relative Value is commonly long gamma as described by Jaeger (2008) in [29], and trend-followers aim to buying optimally max lookback straddles according Fung and Hsieh (2001) in [23]. Therefore, we show Relative Value hedge funds are not completely insurance sellers, since they partially hedge volatility risk by buying skewness risk, whereas Global Macro hedge funds are usually negatively exposed to skewness risk. This result is also consistent since Global Macro managers usually take contrarian bets on tail risks, i.e. selling realized skewness during crises, as their convergence trades are based on long-term macroeconomic trends.

This paper extends the asset pricing literature of hedge fund performance for two reasons. First, it extends Agarwal and Naik (2004) in [3], and Agarwal, Ruenzi, and Weigert (2015) in [4] that evidence tail risk in hedge funds arise from dynamic strategies replicating short positions in equity index put options. Second, this paper sheds light on Agarwal, Bakshi, and Huij (2010) in [2] that evidence hedge funds are particularly sensitive to market crashes through their exposures to the S&P 500 risk-neutral volatility, skewness, and kurtosis. To that extent, this paper clearly shows tail risk premia strategies that trade the higher-order risks embedded in options are an instrumental determinant in the performance of hedge funds.

This paper arises practical implications especially within the industries of hedge funds, asset management, and smart indices. Since we find clear evidence that tradeable tail risk premia explain the variation in hedge fund returns, both in the time-series and the cross-section, this paper paves the way for reverse engineering sophisticated hedge funds by replicating the volatility VRP, skewness SRP, and kurtosis KRP risk premia strategies. Besides, this paper sheds light on the secretive drivers of hedge fund performance, since it disentangles it into real alpha and alternative beta like insurance-crash selling strategies.

Nevertheless, due to the lack of data about hedge funds, we investigated tail risk sensitivity only at the hedge fund investment style on a daily basis, leaving for future research examination at the individual hedge fund level. Besides, my further research will focus on estimating a new statistical measure to evaluate the timing ability and managerial skills of hedge fund managers to mitigate tail risk exposure.

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Appendix

A. Investment Styles

HFR indices are constructed to track the aggregate performance of a wide range of hedge fund managers grouped by a specific strategy criterion. The hedge fund strategy classification aims to capture pure strategies that reflect the evolution of major trends in the hedge fund industry. The 5 major investment styles used in HFR are based as follows on the definitions provided below by HFR.

- Directional:
  This investment strategy employs quantitative techniques to forecast future price movements and relations between securities. They include in particular Factor-based and Statistical Arbitrage/Trading strategies. Factor-based strategies are based on the systematic analysis of common relationships between securities, while Statistical Arbitrage/Trading strategies consist in exploiting pricing anomalies inherent in security prices. Directional strategies typically maintain time-varying levels of long and short equity market exposure over distinct market cycles.
- Equity Hedge:
  This strategy consists in maintaining positions both long and short in equity stocks and equity derivative instruments. Equity Hedge managers can be either broadly diversified or narrowly concentrated on specific sectors, and they can adjust their net exposure, leverage, holding period, and concentrations. They typically maintain at least 50% exposure to equity, and can be completely invested in, both long and short.
- Macro:
  This investment style covers a broad range of strategies in which investment process is based on the movements in economic variables and their impact these have on various asset classes. Managers use various techniques, both systematic and discretionary, both fundamental and quantitative, and both bottom-up and top-down approaches. Macro strategies usually depart from relative value strategies since they are based on the movements in macroeconomic variables rather than on the discrepancy between securities.
- Merger Arbitrage:
  Merger Arbitrageurs focus on companies that are primarily involved in announced corporate transactions, typically with restricted or no exposure to situations that don't include formal announcement. Since investment process consists typically in going long the stock of the acquired company and going short the stock of the acquirer, deal-failure risk designates the major

risk arbitrage risk. These investment strategies typically maintain at least 75% exposure to announced transactions over a given market cycle.

- Relative Value:

  Relative Value arbitrageurs take profit from the realization of a valluation discrepancy between various securities. They use both quantitative and fundamental techniques and a broad range of securities among asset classes to identify attractive risk-adjusted spreads. This investment style can be also involved in corporate transactions, but they depart from Merger Arbitrage since they are based on pricing anomalies between securities, rather than on the outcome of a transaction.

B. Risk-Neutral Distribution

Following the model-free approach of Bakshi, Kapadia, and Madan (2003) in [11], we extract risk-neutral moments from the market option prices. Let $R(t,T) \equiv \ln[S(t+T)] - \ln[S(t)]$ the log return at time t over the time period T. We define the risk- neutral mean of returns $\mu(t,T)$, volatility $RNVol(t,T)$, skewness $RNSkew(t,T)$, and kurtosis $RNKurt(t,T)$ measured at time t over period T by

$$\mu(t,T) \equiv E_t^Q[R(t,T)] \tag{B.1}$$

$$RNVol(t,T) \equiv \left[E_t^Q\left[R(t,T)^2\right] - \mu(t,T)^2\right]^{\frac{1}{2}} \tag{B.2}$$

$$RNSkew(t,T) \equiv \frac{E_t^Q\left[\left(R(t,T) - E_t^Q[R(t,T)]\right)^3\right]}{\left(E_t^Q\left[\left(R(t,\tau) - E_t^Q[R(t,T)]\right)^2\right]\right)^{\frac{3}{2}}} \tag{B.3}$$

$$RNKurt(t,T) \equiv \frac{E_t^Q\left[\left(R(t,T) - E_t^Q[R(t,T)]\right)^4\right]}{\left(E_t^Q\left[\left(R(t,T) - E_t^Q[R(t,T)]\right)^2\right]\right)^2} \tag{B.4}$$

From Bakshi and Madan (2000) in [12], any payoff function $H[S]$ can be spanned algebraically by a continuum of OTM European call and put options. Therefore, let r the risk-free rate, $C(t,T;K)$ $(P(t,T;K))$ the price of a European call (put) option at time t, with time to expiration T, and strike price K. Let the volatility $V(t,T)$, the cubic $W(t,T)$, and the quartic $X(t,T)$ contracts associated to the payoff function $H[S]$. As below, Equations (B.1), (B.2), (B.3), and (B.4) can be expressed in terms of the volatility, cubic, and quartic contracts' fair values under the risk-neutral expectation operator conditional on information at time t :

$$\mu(t,T) = \exp(rT) - 1 - \frac{\exp(rT)}{2} V(t,T) - \frac{\exp(rT)}{6} W(t,T) - \frac{\exp(rT)}{24} X(t,T) \tag{B.5}$$

$$RNVol(t,T) = \left[V(t,T)\exp(rT) - \mu(t,T)^2 \right]^{\frac{1}{2}} \tag{B.6}$$

$$RNSkew(t,T) = \frac{\exp(rT)\,W(t,T) - 3\mu(t,T)\exp(rT)\,V(t,T) + 2\mu(t,T)^3}{\left[\exp(rT)\,V(t,T) - \mu(t,T)^2\right]^{\frac{3}{2}}} \tag{B.7}$$

$$RNKurt(t,T) = \frac{\exp(rT)\,X(t,T) - 4\mu(t,T)\exp(rT)\,W(t,T) + 6\exp(rT)\,\mu(t,T)^2\,V(t,T) - 3\mu(t,T)^4}{\left[\exp(rT)\,V(t,T) - \mu(t,T)^2\right]^2} \tag{B.8}$$

Furthermore, in Equations (B.5), (B.6), (B.7), and (B.8), contracts' fair values $V(t,T)$, $W(t,T)$, and $X(t,T)$ can be spanned by a linear combination of OTM European call and put options, the stock and the risk-free asset, requiring a large continuum of traded OTM options. However, since we observe in practice only few option market prices for discretely spaced strike prices, we apply the non-parametric approach of Völkert (2014) in [37] to adress discreteness by applying a cubic smoothing spline to interpolate implied volatilities amongst strike prices. Therefore, we approximate numerically the integral functions of volatility, cubic, and quartic contracts by using trapezoidal approximations.

C. Real-World Distribution

Recent literature about high-frequency econometrics, including Bollerslev, Tauchen, and Zhou (2009) in [14], and Neumann and Skiadopoulos (2013) in [33] among others, usually estimates the daily realized variance under a nonparametric approach by summing frequently sampled squared returns. Similarly, Amaya, Christoffersen, Jacobs, and Vasquez (2013) in [8] derive the daily realized skewness and kurtosis from intradaily returns. Nevertheless, since this standard econometric approach is widely biased by the market microstructure noise on volatility estimation, a naive practice consists in throwing away a lot of available data by sampling less frequently the intradaily underlying asset prices. In this way, we rather use the model-free approach proposed by Aït-Sahalia, Mykland and Zhang (2005) in [40] to fully exploit the tick-by-tick data, to correct for the bias of market microstructure noise; and furthermore, to estimate similarly the higher-order realized moments.

According Bollerslev, Tauchen, and Zhou (2009) in [14], the daily realized variance is usually estimated by summing the intradaily returns of the underlying asset. Let $R_{t,i}$ the i-intraday log return calculated on day t and associated to the price index $P_{t,i}$. Then $R_{t,i,T} = \ln\left(P_{t,\frac{i}{N}}\right) - \ln\left(P_{t,\frac{(i-1)}{N}}\right)$, where N denotes the total number of observed intraday log returns in the trading day t. Therefore, the daily realized volatility $RDVol_t^{(all)}$ is usually estimated by summing naively all the n squares of intradaily log returns $R_{t,i}$:

$$RDVol_t^{(all)} = \left(\sum_{i=1}^{N} R_{t,i}^2\right)^{\frac{1}{2}} \tag{C.1}$$

Similarly, following Amaya et al. (2013) in [8], the ex-post realized daily skewness $RDSkew_t$ and kurtosis $RDKurt_{t,T}$ can be expressed as follows, respectively scaled by $N^{\frac{1}{2}}$ and N to ensure they correspond to the daily realized measures:

$$\begin{aligned} RDSkew_t^{(all)} &= \frac{N^{\frac{1}{2}} \sum\limits_{i=1}^{N} R_{t,i}^3}{RDVol_t^3}, \\ RDKurt_t^{(all)} &= \frac{n \sum\limits_{i=1}^{N} R_{t,i}^4}{RDVol_t^4} \end{aligned} \tag{C.2}$$

Nevertheless, Aït-Sahalia, Mykland and Zhang (2005) in [40] argue that using naively all the tick-by-tick data makes the market microstructure noise totally swamp the estimated realized volatility under the nonparametric case. Suppose the log price process X_t follows a continuous semi-martingale. Then, it is modeled by the stochastic differential equation $dX_t = \mu_t + \sigma_t dW_t$, where μ_t, σ_t, and W_t denote respectively the drift and the volatility of the log return process of X_t at time t, and a standard Brownian motion process. Therefore, the object of interest primarily consists in estimating the integrated variance, i.e. the quadratic variation $\langle X, X \rangle_T = \int\limits_0^T \sigma_t^2 dt$ over the time period $[0, T]$. Indeed, Zhang et al. (2005) show that $RDVol_T^{(all)}$ in the (C.1) converges in law to

$$RDVol_T^{(all)} \overset{L}{\simeq} \langle X, X\rangle_T + 2nE\left[\varepsilon^2\right] + \left[4nE\left[\varepsilon^4\right] + \frac{2T}{n}\int_0^T \sigma_t^4 dt\right]^{\frac{1}{2}} Z_{total} \tag{C.3}$$

where $RDVol_T^{(all)}$ is even more positively biased by the market microstructure noise $2nE\left[\varepsilon^2\right]$ when the sample size n of observed intradaily prices increases. Consequently, sampling sparsely either at an arbitrary frequency or even at an optimal frequency by decreasing n are tantamount to ignoring the microstructure noise and to throwing out a large fraction of the available intradaily data. In contrast, Aït-Sahalia, Mykland and Zhang (2005) propose the following Two-Scales Realized Volatility estimator $RDVol_T^{(TS)}$ that uses all the available tick-by-tick data but that incorporates subsampling, averaging, and bias correction for the market microstructure noise:

$$RDVol_T^{(TS)} = \frac{1}{K}\sum_{k=1}^{K} RDVol_T^{(k)} - \frac{\overline{n}}{n} RDVol_T^{(all)} \tag{C.4}$$

where the original grid $G = \{t_k, ..., t_n\}$ of observation times of log prices in a given trading day is partitioned into K non-overlapping and equal subsamples $G^{(k)}$ for $k = \{1, ..., K\}$. The k-th sub-grid is written as $G^{(k)} = \{t_{k-1}, t_{k-1+K} ..., t_{k-1+n_k K}\}$. Therefore, Zhang et al. (2005) average the estimators $RDVol_T^{(k)}$ obtained on each of the K grids of average size $\overline{n} = \frac{n-K+1}{K}$, giving rise to the estimator $RDVol_T^{(avg)} = \frac{1}{K}\sum_{k=1}^{K} RDVol_T^{(k)}$. Then, bias correction is determined by $K = n^{\frac{2}{3}}\left[12\widehat{E\left[\varepsilon^2\right]}^2 \Big/ \frac{Tn}{3}\int_0^T \sigma_t^4 dt\right]^{\frac{1}{3}}$. Finally, $RDVol_T^{(TS)}$ corrects for the bias $2\overline{n}E\left[\varepsilon^2\right]$ due to the microstructure noise of $RDVol_T^{(avg)}$, since it now increases with the average subsamples size $\overline{n}$.

Similarly, we derive to the higher-order moments the Aït-Sahalia, Mykland and Zhang (2005) methodology of subsampling, averaging, and bias correction for the market microstructure noise:

$$\begin{aligned} RDSkew_T^{(TS)} &= \frac{1}{K}\sum_{k=1}^{K} RDSkew_T^{(k)} - \frac{\overline{n}}{n} RDSkew_T^{(all)}, \\ RDKurt_T^{(TS)} &= \frac{1}{K}\sum_{k=1}^{K} RDKurt_T^{(k)} - \frac{\overline{n}}{n} RDKurt_T^{(all)} \end{aligned} \tag{C.5}$$

where $RDSkew_T^{(TS)}$ and $RDKurt_T^{(TS)}$ denote respectively the two-scales realized skewness and kurtosis.

D. Theory of Tail Risk Premia

From Cochrane (2005) in [19], I demonstrate that the expected excess return over long period on tail risk premia VRP, SRP, and KRP is negative, since it is negatively correlated with the negative covariance between factors and stochastic discount factor (SDF).

Let the representative agent modelled by utility function U defined for consumption $\tilde{c}_t$ and $\tilde{c}_{t+1}$:

$$U\left(\tilde{c}_t, \tilde{c}_{t+1}\right) = u\left(\tilde{c}_t\right) + \beta E_t\left[u\left(\tilde{c}_{t+1}\right)\right]$$

where β denotes the subjective discount factor. The intuition underlying tail risk premia makes sense since representative agent feels poorer in bad times, decreasing then their consumption. Hence, he consents to pay a positive risk premium over long period that is compensated by generating positive excess returns in adverse times. Therefore, allocation problem consists in a trade-off at time t over $[t, t+1]$ between consumption and investment in an amount ξ of the factor payoff $x_{t+1} = p_{t+1} + d_{t+1}$, where p_{t+1} and d_{t+1} are respectively price and dividend of the risk factor. Henceforth, agent's problem is to find the optimal amount of wealth ξ that maximizes the utility $U\left(\tilde{c}_t, \tilde{c}_{t+1}\right)$:

$$\max_{\{\xi\}} \left\{u\left(\tilde{c}_t\right) + \beta E_t\left[u\left(\tilde{c}_{t+1}\right)\right]\right\} \quad \text{s.t.} \left\{\begin{array}{l} \tilde{c}_t = c_t - \xi p_t \\ \tilde{c}_{t+1} = c_{t+1} + \xi x_{t+1} \end{array}\right. \tag{D.1}$$

By Lagrangean technique, the first-order condition (FOC) for an optimal consumption and portfolio choice gives the pricing equation of the factor at time t

$$p_t = E_t\left[\beta \frac{u'\left(c_{t+1}\right)}{u'\left(c_t\right)} x_{t+1}\right] \tag{D.2}$$

where $m_{t+1} \equiv \beta \frac{u'(c_{t+1})}{u'(c_t)}$ is the SDF, i.e. the intertemporal marginal rate of substitution between consumption and investment, written as a function of marginal utility u'. Assuming risk-free asset pays with certainty the payoff $x_{t+1} = 1$, then

$$E_t\left[m_{t+1}\right] = \frac{1}{r_{t+1}^f}$$

where r_{t+1}^f is the risk-free rate discounting the payoff x_{t+1} to give the risk-free asset price at t. Exhibiting in (D.2) the covariance term between SDF m_{t+1} and factor payoff x_{t+1}, factor price at t can now be written as the expected cashflow discounted at risk-free rate plus a risk premium:

$$p_t = \frac{E_t\left[x_{t+1}\right]}{r^f_{t+1}} + \mathrm{cov}_t\left[m_{t+1}, x_{t+1}\right] \tag{D.3}$$

Rearranging the expression below where $R_{t+1} = \frac{x_{t+1}}{p_t}$ is the gross return of the factor over $[t, t+1]$ and $r_{t+1} = R_{t+1} - r^f_{t+1}$ is the factor return in excess of the risk-free rate, we obtain the expected excess return on the tail risk factors, i.e. the tail risk premium:

$$E_t\left[r_{t+1}\right] = -r^f_{t+1} \cdot \mathrm{cov}_t\left[m_{t+1}, r_{t+1}\right] \tag{D.4}$$

$$= -\frac{\mathrm{cov}_t\left[m_{t+1}, r_{t+1}\right]}{E_t\left[m_{t+1}\right]} \tag{D.5}$$

Considering the theoretical underpinnings of a risk premium, Equation (D.4) provides very straightforward conclusions:

- If the factor excess return r_{t+1} and the SDF m_{t+1} are independent, there is no risk premium for bearing additional risk;

- If the factor excess return r_{t+1} covaries negatively with the SDF m_{t+1}, the risk premium compensates the agent for paying a positive risk premium over long period;

- If the factor excess return r_{t+1} covaries positively with the SDF m_{t+1}, the risk premium generates negative returns over long period, but pays a reward in bad times that compensates the agent for paying an insurance. This is exactly what happens when an investor pays the tail risk premia VRP, SRP, and KRP.

E. Trend-Following Factors

Since our study is restricted by the size of our tail risk premia data sample, we calculate proxies on a daily basis for the trend-following factors of Fung-Hsieh (2001, 2004) in [23] and [24].

For that purpose, we consider their special case of the perfect trend follower who captures systematically the largest asset price movement over the trading day. Hence, the Primitive Trend-Following Strategy (PTFS) captures the optimal payout $S_{\max} - S_{\min}$, where $S_{\max}$ and $S_{\min}$ respectively designate the maximum and the minimum price of an asset over a trading day. This special case assumes the trend follower can perfectly anticipate asset price movements without incurring trading costs, since he can formally *buy breakouts and sell breakdowns*.

Therefore, we empirically construct returns of the PTFS for each of the following 15 markets by using the futures data provided by Datastream and Bloomberg:

- Bonds: 30-year US bond, 10-year Gilts, 10-year Bund, 10-year French bond, 10-year Australian bond.
- Currencies: British pound, Deutschemark, Japanese yen, Swiss franc.
- Commodities: Corn, wheat, soybean, crude oil, gold, silver.

Finally, returns of the PTFSBD (bonds), PTFSFX (currencies), and PTFSCOM (commodities) used in the Fung-Hsieh seven-factor model are then calculated by equally-weighting the returns of the 15 PTFS associated to each of the 3 asset classes.

Table 1: Summary Statistics of Hedge Fund Investment Styles

	N	Min	Pct1	Pct25	Pct50	Pct75	Pct99	Max	Std
Panel A: All investment styles, per year									
2008	20	-0,0231	-0,0229	-0,0042	-0,0012	0,0020	0,0257	0,0258	0,0076
2009	52	-0,0099	-0,0089	-0,0009	0,0004	0,0018	0,0083	0,0118	0,0028
2010	51	-0,0138	-0,0088	-0,0014	0,0000	0,0012	0,0056	0,0105	0,0028
2011	52	-0,0196	-0,0090	-0,0017	0,0000	0,0013	0,0058	0,0072	0,0029
2012	55	-0,0088	-0,0059	-0,0010	0,0001	0,0011	0,0061	0,0071	0,0020
2013	45	-0,0123	-0,0078	-0,0010	0,0000	0,0013	0,0045	0,0068	0,0023
Panel B: Full sample, by investment style									
Aggregate	275	-0,0109	-0,0077	-0,0008	0,0001	0,0011	0,0048	0,0149	0,0022
Directional	275	-0,0231	-0,0184	-0,0017	0,0001	0,0017	0,0073	0,0244	0,0041
Equity Hedge	275	-0,0173	-0,0125	-0,0019	0,0000	0,0018	0,0099	0,0256	0,0042
Macro	275	-0,0160	-0,0088	-0,0022	-0,0003	0,0014	0,0081	0,0118	0,0033
Merger Arbitrage	275	-0,0125	-0,0070	-0,0009	-0,0001	0,0012	0,0087	0,0127	0,0024
Relative Value	275	-0,0188	-0,0071	-0,0007	0,0003	0,0012	0,0045	0,0258	0,0026
Panel C: All investment styles, full sample									
	1650	-0,0231	-0,0097	-0,0013	0,0001	0,0013	0,0077	0,0258	0,0032

This table reports summary statistics for the data of hedge fund investment styles from HFR Database over the period 2008-2013. The data sample that we constructed is restricted to the estimation points associated to the tail risk premia. Statistic N designates either the number of daily returns associated to all the hedge fund investment styles each year (Panel A), or for each investment style over the entire sample period (Panel B). Other statistics consist in the minimum, 1, 25, 50, 75, and 99 percentiles, maximum and standard deviation. Panel C summarizes the total number of daily hedge fund returns in the entire data sample and other statistics.

Table 2: Descriptive Statistics of VIX Option-Implied Risk Premia

	30 Days Maturity			60 Days Maturity			90 Days Maturity		
	VRP	**SRP**	**KRP**	**VRP**	**SRP**	**KRP**	**VRP**	**SRP**	**KRP**
Panel A: Levels of Option-Implied Risk Premia									
Nb. Observations	300	300	300	267	267	267	237	237	237
Mean	-0,193	-0,718	-6,027	-0,411	-0,747	-5,818	-0,487	-0,652	-5,707
Median	-0,242	-0,849	-2,680	-0,447	-0,774	-2,264	-0,515	-0,766	-1,852
Max	1,361	3,031	4,855	1,033	2,645	5,358	0,971	2,507	4,723
Min	-1,280	-4,555	-35,560	-1,205	-5,522	-33,474	-1,258	-4,382	-33,029
Standard Dev.	0,397	1,437	8,073	0,354	1,502	8,056	0,388	1,450	8,420
Skewness	0,736	0,047	-1,437	1,011	-0,157	-1,392	0,662	-0,038	-1,467
Kurtosis	4,017	2,457	4,322	5,091	2,673	4,273	3,559	2,541	4,435
LBQ Test	79,14*	0,01	0,12	40,11*	0,05	0,57	37,21*	0	0
ADF Test	-8,57**	-14**	-12,07**	-6,33**	-13,16**	-10,98**	-5,52**	-12,87**	-11,01**
Student Test	-8,41**	-8,66**	-12,93**	-18,94**	-8,13**	-11,8**	-19,35**	-6,92**	-10,44**
Panel B: First Differences of Option-Implied Risk Premia									
Nb. Observations	299	299	299	266	266	266	236	236	236
Mean	0,000	-0,004	0,025	-0,216	-0,100	0,006	-0,312	-0,013	0,308
Median	-0,016	-0,032	0,055	-0,224	-0,103	0,359	-0,330	-0,223	0,091
Max	1,166	5,826	30,593	1,432	5,615	28,863	1,400	4,908	36,231
Min	-1,443	-5,247	-35,141	-2,003	-6,249	-33,166	-1,966	-5,965	-33,478
Standard Dev.	0,393	2,029	11,543	0,523	2,013	11,314	0,519	1,915	11,922
Skewness	0,051	-0,017	-0,059	-0,030	-0,126	-0,211	0,058	-0,030	-0,062
Kurtosis	3,687	2,687	3,528	3,784	3,179	3,638	3,593	2,930	3,716
LBQ Test	49,88*	61,45*	96,49*	52,91*	0,77	0,58	42,15*	0,89	0,9
ADF Test	-26,52**	-28,05**	-32,7**	-9,03**	-17,1**	-15,53**	-7,95**	-16,3**	-16,26**
Student Test	0	-0,03	0,04	-6,75**	-0,81	0,01	-9,24**	-0,1	0,4

Descriptive statistics associated to VIX option-implied risk premia for different maturities, from January 24, 2008 to August 29, 2014. Option-implied risk premia are computed as the difference between the physical measure and the risk-neutral measure of the expected value of realized moments. Moments estimated under the physical expectation are based on intradaily prices associated to the VIX spots. Moments estimated under the risk-neutral expectation are based on daily VIX option prices for different maturities. Stars *, ** denote statistical significance at respectively 5% and 1% level of confidence.

Table 3: Pairwise Correlations of Fung-Hsieh Factors and Tail Risk Premia

	MKT-RF	**SMB**	**dTERM**	**dCREDIT**	**PTFSBD**	**PTFSFX**	**PTFSCOM**	**VRP**	**SRP**
SMB	0,10								
	[0,08]								
dTERM	0,37	0,04							
	[0]	[0,49]							
dCREDIT	-0,33	-0,05	-0,95						
	[0]	[0,39]	[0]						
PTFSBD	-0,13	-0,06	-0,09	0,11					
	[0,04]	[0,3]	[0,12]	[0,06]					
PTFSFX	-0,18	-0,08	0,10	-0,07	0,08				
	[0]	[0,21]	[0,09]	[0,24]	[0,16]				
PTFSCOM	-0,12	-0,13	-0,08	0,17	0,31	0,12			
	[0,05]	[0,03]	[0,16]	[0,01]	[0]	[0,04]			
VRP	-0,27	-0,21	-0,13	0,15	0,18	0,25	0,34		
	[0]	[0]	[0,04]	[0,01]	[0]	[0]	[0]		
SRP	-0,31	-0,16	0,00	-0,02	0,10	0,20	0,04	0,14	
	[0]	[0,01]	[0,95]	[0,75]	[0,1]	[0]	[0,47]	[0,02]	
KRP	-0,04	0,00	-0,05	0,04	-0,02	0,00	-0,06	0,19	0,21
	[0,52]	[0,95]	[0,39]	[0,48]	[0,72]	[0,98]	[0,29]	[0]	[0]

This table summarizes the time-series Pearson pairwise correlations of the Fung-Hsieh (2001) seven factors and the tail risk premia in Al Wakil (2016). Sample period is August 2008 to October 2013. The Fung-Hsieh seven factors consist in the market portfolio in excess of the risk-free rate, and the SMB of Fama and French (1993), the term spread change, the credit spread change, and the factors associated to the best trend-following strategies, i.e. PTFSBD (bonds), PTFSFX (currencies), PTFSCOM (commodities). p-values are reported in square brackets.

Table 4: Multivariate Regressions Results of Hedge Fund Styles Returns on Fung-Hsieh Factors and Tail Risk Premia over 2008-2013

Investment Style	Nb. Obs.	Intercept	MKT-RF	SMB	dTERM	dCREDIT	PTFSBD	PTFSFX	PTFSCOM	VRP	SRP	KRP	R-Square/ Adj. R-Square
Global	275	0,0000	0,088***							-0,0012***	0,0002**	0,0000	0,41
		[-0,19]	[11,48]							[-4,26]	[2,09]	[0,51]	0,41
		-0,0231	0,0872***	0,0064	-0,0052	-0,0114	0,0024	4,3693	-0,0001**	-0,001***	0,0001*	0,0000	0,43
		[-0,63]	[10,45]	[0,37]	[-0,4]	[-0,63]	[1,6]	[0,63]	[-2]	[-3,4]	[1,8]	[0,36]	0,41
Directionnal	275	0,0003	0,1979***							-0,0009*	0,0004***	0,0000	0,50
		[1,22]	[15,11]							[-1,85]	[3,34]	[1,5]	0,49
		-0,0407	0,1908***	-0,0623**	-0,0027	-0,0206	0,0027	1,2206	-0,0002**	-0,0007	0,0004***	0,0000	0,52
		[-0,66]	[13,52]	[-2,13]	[-0,12]	[-0,67]	[1,06]	[0,1]	[-2,24]	[-1,36]	[2,68]	[1,48]	0,50
Equity Hedge	275	-0,0001	0,1973***							-0,0013***	0,0001	0,0000	0,54
		[-0,22]	[15,54]							[-2,96]	[1,06]	[0,49]	0,53
		0,0096	0,1906***	0,0084	0,0153	0,0051	0,0044*	16,4733	-0,0001*	-0,0013***	0,0001	0,0000	0,55
		[0,16]	[13,88]	[0,29]	[0,71]	[0,17]	[1,79]	[1,43]	[-1,66]	[-2,63]	[0,52]	[0,57]	0,54
Macro	275	-0,0009***	-0,0321**							-0,0011**	-0,0004**	0,0000	0,05
		[-3,2]	[-2,23]							[-2,23]	[-2,5]	[-0,01]	0,03
		-0,0991	-0,0059	0,0866***	-0,0691***	-0,0495	0,0018	-17,9721	-0,0001	-0,0005	-0,0002	0,0000	0,18
		[-1,54]	[-0,4]	[2,83]	[-3]	[-1,54]	[0,69]	[-1,46]	[-1,27]	[-0,95]	[-1,35]	[-0,82]	0,15
Merger Arbitrage	275	0,0002	0,0979***							-0,0001	0,0001	0,0000	0,36
		[1,21]	[11,3]							[-0,47]	[0,62]	[0,88]	0,35
		0,1272***	0,0901***	-0,0272	0,0497***	0,0633***	-0,0001	-8,6250	0,0000	-0,0002	0,0000	0,0000	0,39
		[3,12]	[9,68]	[-1,41]	[3,42]	[3,11]	[-0,06]	[-1,11]	[-0,7]	[-0,53]	[0,5]	[0,98]	0,37
Relative Value	275	0,0004**	0,0424***							-0,0009**	0,0005***	0,0000	0,12
		[2,01]	[3,88]							[-2,4]	[4,14]	[0,06]	0,10
		0,0338	0,0314***	-0,0229	0,0267	0,0168	0,0021	2,7385	-0,0001	-0,001**	0,0004***	0,0000	0,15
		[0,65]	[2,65]	[-0,93]	[1,44]	[0,65]	[1]	[0,28]	[-0,96]	[-2,25]	[3,47]	[0,27]	0,11

This table summarizes the results of time-series OLS regressions of major hedge fund style returns on the Fung-Hsieh (2001) seven factors and the tail risk premia in Al Wakil (2016). Sample period is August 2008 to October 2013. The Fung-Hsieh seven factors consist in the market portfolio in excess of the risk-free rate, and the SMB of Fama and French (1993), the term spread change, the credit spread change, and the factors associated to the best trend-following strategies, i.e. PTFSBD (bonds), PTFSFX (currencies), PTFSCOM (commodities). T-statistics are reported in square brackets and stars *, ** denote statistical significance at respectively 5% and 1% level of confidence.

Table 5: Quantile Portfolios Sorted on Hedge Fund Tail Loadings and Return Spread of the High-Minus-Low Portfolio

	Factor Beta Quantiles			
	1 [Low]	2	3 [High]	High - Low
Panel A: Volatility Risk Premia in the Cross-Section				
Average Tail Risk Beta	-0,216%	-0,026%	0,091%	0,307%
	[-18,35]	[-3,37]	[11,47]	[28,88]
Average Excess Return	0,0057	0,0051	-0,1113	-0,117***
	[0,16]	[0,15]	[-2,4]	[-2,38]
Fung-Hsieh Alpha	-11,01%	-3,61%	-1,59%	9,42%
	[-1,97]	[-0,71]	[-0,23]	[1,18]
Panel B: Skewness Risk Premia in the Cross-Section				
Average Tail Risk Beta	-0,019%	0,035%	0,059%	0,078%
	[-6,49]	[14,33]	[18,28]	[16,7]
Average Excess Return	0,001	-0,024	0,008	0,008
	[0,01]	[-0,6]	[0,18]	[0,15]
Fung-Hsieh Alpha	-0,87%	-10,24%	-7,56%	-6,69%
	[-0,15]	[-1,74]	[-1,12]	[-0,84]
Panel C: Kurtosis Risk Premia in the Cross-Section				
Average Tail Risk Beta	-0,003%	0,005%	0,009%	0,012%
	[-9,1]	[17,3]	[21,91]	[31,74]
Average Excess Return	0,0028	-0,0185	-0,0832	-0,086***
	[0,07]	[-0,52]	[-1,79]	[-1,86]
Fung-Hsieh Alpha	-8,91%	-2,50%	-4,01%	4,90%
	[-1,58]	[-0,48]	[-0,59]	[0,68]

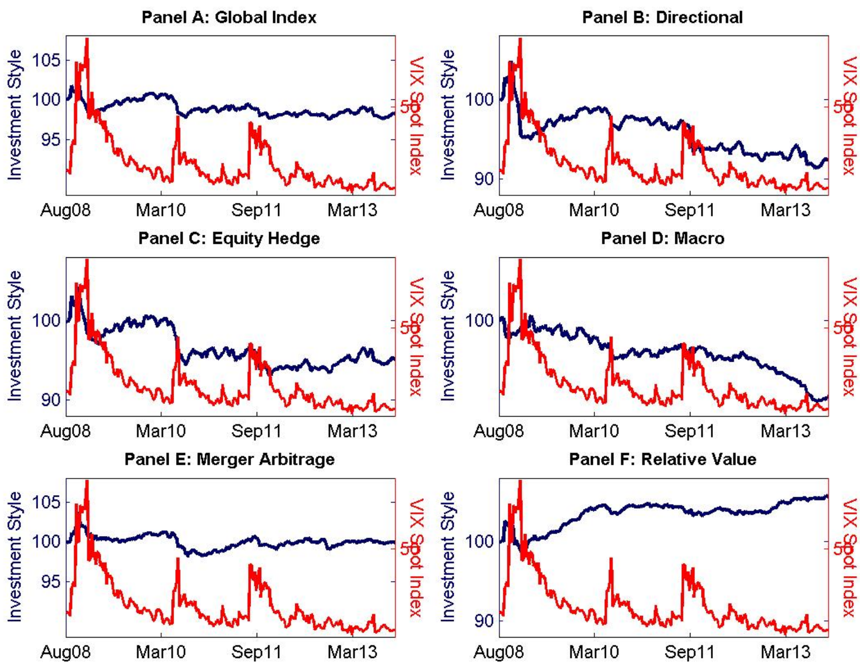
This table reports the average daily tail risk betas, the average annualized excess returns, and the Fung-Hsieh seven factor alpha for hedge fund portfolios sorted on the basis of their loadings on the tail risk premia, respectively associated to the volatility VRP, skewness SRP, and kurtosis KRP. Each day, three quantile portfolios are formed based on the investment styles loadings on the tail risk premia in a regression of returns on the market excess return and the tail risk factor in the past 22 days. Newey-West (1987) t-statistics are reported in square brackets and stars *** denote statistical significance at respectively 5% level of confidence for the average excess returns, and the Fung-Hsieh seven factor alpha.

Table 6: Multivariate Regressions Results of Hedge Fund Styles Returns on Fung-Hsieh Factors and Tail Risk Premia over Tail Events

Investment Style	Nb. Obs.	Intercept	MKT-RF	SMB	dTERM	dCREDIT	PTFSBD	PTFSFX	PTFSCOM	VRP	SRP	KRP	R-Square/ Adj. R-Square
Global	71	0,0007	0,0939***							-0,003***	0,0004	0,0000	0,49
		[1,51]	[6,54]							[-3,41]	[1,65]	[0,59]	0,46
		0,0571	0,1044***	-0,0100	0,0136	0,0288	0,0116*	1,6329	-0,0002	-0,0032***	0,0004	0,0000	0,53
		[0,74]	[5,3]	[-0,24]	[0,46]	[0,75]	[1,91]	[0,08]	[-1,58]	[-3,01]	[1,46]	[0,47]	0,46
Directionnal	71	0,0012	0,2318***							-0,0020	0,0011**	0,0000	0,60
		[1,53]	[9,49]							[-1,33]	[2,47]	[0,73]	0,58
		0,1378	0,207***	-0,0845	0,0725	0,0684	0,0146	-32,9534	-0,0004**	-0,0015	0,0009*	0,0000	0,66
		[1,08]	[6,35]	[-1,24]	[1,49]	[1,07]	[1,45]	[-0,97]	[-2,23]	[-0,84]	[1,91]	[0,4]	0,60
Equity Hedge	71	0,0010	0,207***							-0,0034**	0,0005	0,0000	0,62
		[1,4]	[9,47]							[-2,59]	[1,26]	[0,68]	0,60
		0,1467	0,2203***	-0,0033	0,0464	0,0741	0,0182*	5,4085	-0,0002	-0,0039**	0,0005	0,0000	0,66
		[1,26]	[7,4]	[-0,05]	[1,05]	[1,27]	[1,98]	[0,17]	[-1,26]	[-2,46]	[1,08]	[0,69]	0,60
Macro	71	-0,0005	-0,0637***							-0,0028**	-0,0011***	0,0001	0,24
		[-0,73]	[-3,12]							[-2,26]	[-3]	[1,1]	0,19
		-0,0696	-0,0372	-0,0290	-0,0646*	-0,0359	-0,0027	-40,9889	-0,0002	-0,0017	-0,0007*	0,0000	0,41
		[-0,69]	[-1,45]	[-0,54]	[-1,69]	[-0,72]	[-0,34]	[-1,53]	[-1,45]	[-1,22]	[-1,89]	[0,14]	0,32
Merger Arbitrage	71	0,0012**	0,1244***							-0,0009	-0,0002	0,0001**	0,57
		[2,64]	[8,39]							[-0,95]	[-0,66]	[2,3]	0,54
		0,2653***	0,1139***	-0,0527	0,0978***	0,1321***	0,0058	-19,6823	-0,0001	-0,0011	-0,0002	0,0001**	0,67
		[3,62]	[6,1]	[-1,35]	[3,52]	[3,62]	[1,01]	[-1,01]	[-0,78]	[-1,11]	[-0,81]	[2,28]	0,61
Relative Value	71	0,0012*	0,064***							-0,0033**	0,0011***	0,0000	0,23
		[1,68]	[2,8]							[-2,36]	[2,75]	[-0,11]	0,18
		0,0937	0,0662**	0,0199	0,0477	0,0481	0,0191**	31,5392	-0,0001	-0,0042**	0,0009**	0,0000	0,32
		[0,78]	[2,16]	[0,31]	[1,04]	[0,8]	[2,02]	[0,98]	[-0,82]	[-2,52]	[2,02]	[0,32]	0,21

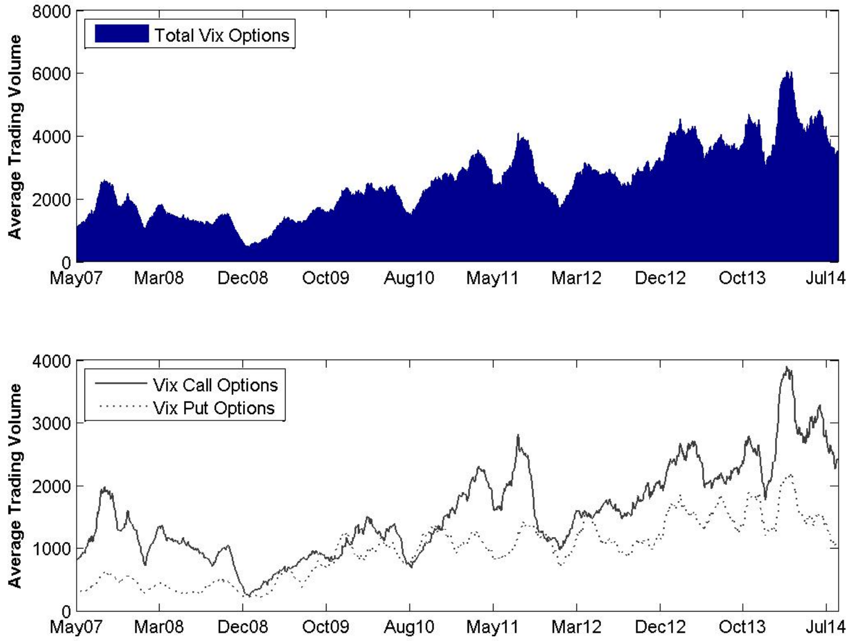
This table summarizes the results of time-series OLS regressions of major hedge fund style returns on the Fung-Hsieh (2001) seven factors and the tail risk premia in Al Wakil (2016). Sample period consists in time-slicing the initial data sample over 2008-2013 to consider major extreme events, including the US Subprime crisis and the Lehman collapse in 2008, the European sovereign debt crisis in 2010, the US sovereign debt crisis in 2011, and the Taper Tantrum in 2013. T-statistics are reported in square brackets and stars *, ** denote statistical significance at respectively 5% and 1% level of confidence.

Figure 1: Performances of Hedge Fund Investment Styles



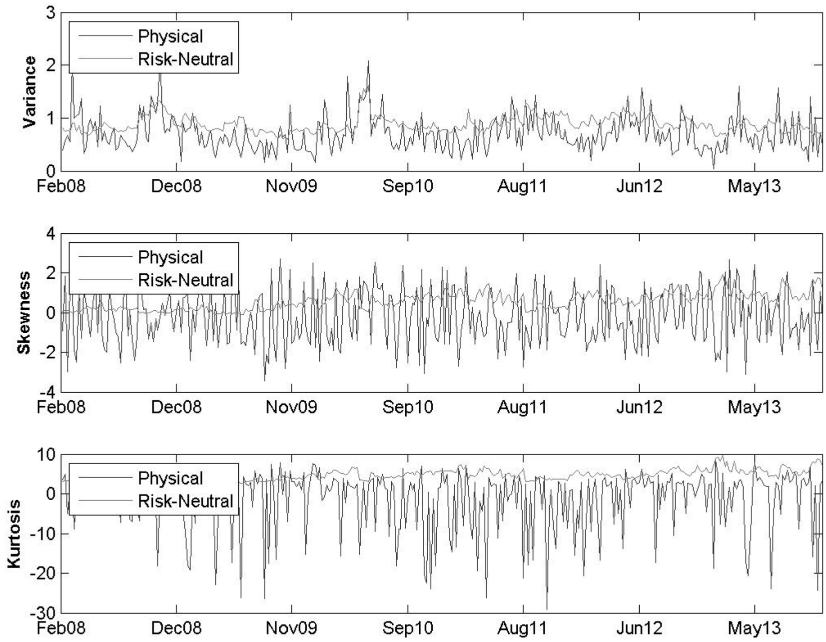
This figure displays the rebased performances of the Aggregate Index, Directional, Equity Hedge, Macro, Merger Arbitrage, and Relative Value strategies and the VIX over 2008-2013. The data sample covers some highly turbulent and volatile periods, including the US Subprime crisis and the Lehman collapse in 2008, the European sovereign debt crisis in 2010, the US sovereign debt crisis in 2011, and the Taper Tantrum in 2013.

Figure 2: Average Trading Volume of VIX Options



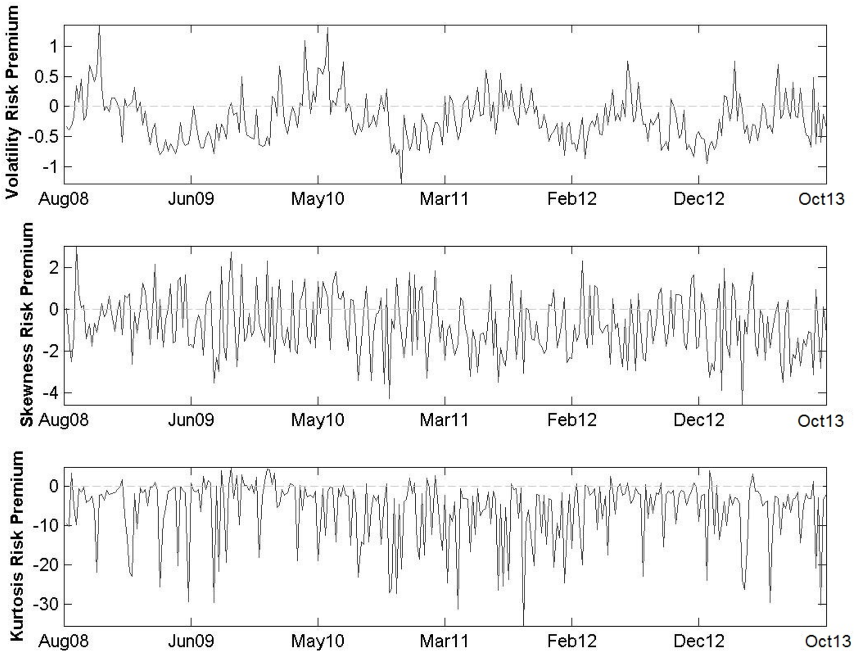
This figure displays the average daily trading volume of VIX options over 2007-2014. The lower panel represents the compared average daily trading volume of VIX call and put options. For clearness, computations are based on the 2-month moving average trading volume.

Figure 3: Higher-Order Moments under Real-World and Risk-Neutral Probability Measures



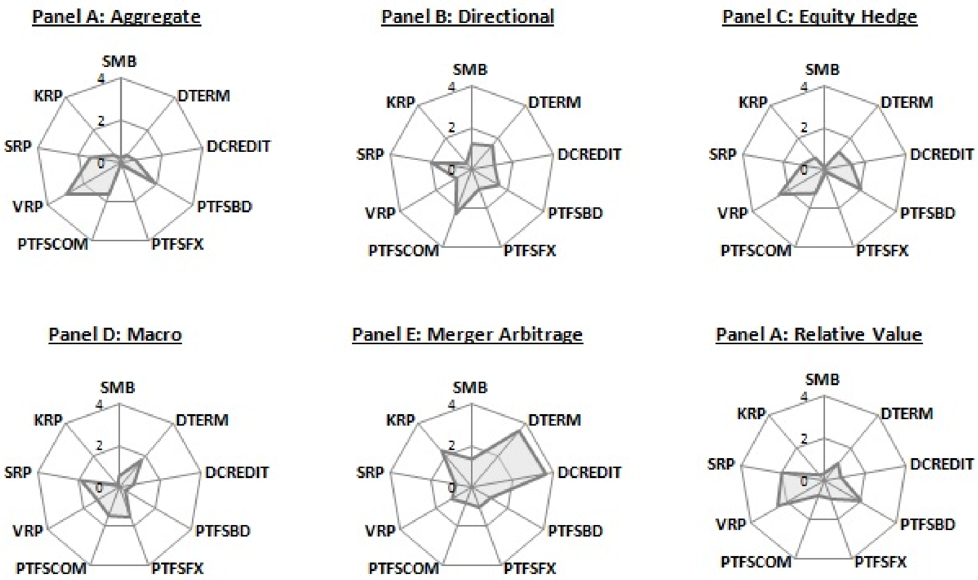
This figure displays the estimates of higher-order moments under real-world and risk-neutral probability measures associated to VIX markets over 2008-2013. Intradaily VIX spots are used to estimate physical moments, and daily VIX options and futures are used to estimate risk-neutral moments. The figures plot respectively on a daily basis the levels of the physical and risk-neutral volatility, skewness, and kurtosis, for 30 days time to expiration.

Figure 4: Option-Implied Risk Premia associated to VIX Options



This figure displays the VIX option-implied risk premia for 30 days time to maturity over 2008-2013. On a daily basis, the levels of the risk premia are associated to the volatility, the skewness, and the kurtosis, for 30 days time to expiration.

Figure 5: Sensitivity of Hedge Fund Investment Styles to Fung-Hsieh Factors and Tail Risk Premia



This figure displays the sensitivity of hedge fund investment styles to Fung-Hsieh seven factors (without market factor) and tail risk premia in crisis periods over 2008-2013. Option-implied risk premia associated to the volatility, the skewness, and the kurtosis are calculated for VIX options and 30 days time to maturity. Sensitivities are measured by the absolute values of t-statistics.