Betting Against Alpha

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Abstract. I sort stocks based on realized alphas estimated from the CAPM, Carhart (1997), and Fama-French Five Factor (FF5, 2015) models and find that realized alphas are negatively related with future stock returns, future alpha, and Sharpe Ratios. Thus, I construct a Betting Against Alpha (BAA) factor that buys a portfolio of low-alpha stocks and sells a portfolio of high-alpha stocks. Using rank estimation methods, I show that the BAA factor spans a dimension of stock returns different than Frazzini and Pedersen's (2014) Betting Against Beta (BAB) factor. Additionally, the BAA factor captures information about the cross-section of stock returns missed by the CAPM, Carhart, and FF5 models. The performance of the BAA factor further improves if the low alpha portfolio is calculated from low beta stocks and the high alpha portfolio from high beta stocks. I call this factor Betting Against Alpha and Beta (BAAB). I discuss several reasons that support the existence of this counter-intuitive low-alpha anomaly.

Keywords. betting against beta, betting against alpha, low-alpha anomaly, leverage, benchmarking.
JEL classification. G10, G12.

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1 Introduction

The primary purpose of this paper is to document a new anomaly and propose a new tradable factor based on it. I call this new anomaly the *low-alpha anomaly*: Portfolios constructed with assets having low realized alphas (overpriced according to the CAPM) have higher Sharpe Ratios, future alphas, and expected returns than portfolios constructed with assets having high-realized alphas (underpriced according to the CAPM). I show that the low-alpha anomaly captures different information about the cross-section of stock returns than the low-beta and low-volatility anomalies do. The new tradable factor I propose based on this new anomaly is a zero-net investment strategy with leverage that buys a portfolio of low-alpha stocks and sells a portfolio of high-alpha stocks. I call it *Betting Against Alpha* (BAA). The new factor generates Sharpe Ratios of similar magnitude to those of the Betting Against Beta (BAB) factor of Frazzini and Pedersen (FP, 2014) and is not priced by the CAPM,[1] Carhart (1997), or Fama-French Five Factor model (FF5, 2015). It generated monthly abnormal returns of around 1% between January 1972 and December 2015. Additionally, the performance of the BAA factor can be significantly improved by combining it with the insights of FP's BAB factor, by buying a portfolio with assets having low realized alphas from the set of assets with low realized betas, and selling a portfolio with assets having high realized alphas from the set of assets with high realized betas. Using this strategy, an investor can achieve Sharpe Ratios much higher than those achievable by either the BAA or BAB factors alone, and up to three times that of the market factor. I call this factor *Betting Against Alpha and Beta* (BAAB).

The BAA factor's fundamentals follow from already existing theoretical and empirical results. The discussion follows this paragraph. Importantly, the discussion assumes that the empirical Market factor used for testing the CAPM cannot capture all sources of risk, and thus, a multifactor asset pricing model is needed. This assumption is validated by decades of financial research.

- *Leverage constrained investors:* FP showed theoretically that leverage constrained investors bid up high Market-beta assets to augment the expected returns of their portfolios. Consequently, high Market-beta stocks are overpriced relative to low Market-beta stocks. The empirical implication of the FP model is that betting against beta should work. The BAB factor proposed by FP is a market-neutral strategy with leverage that buys a zero-net investment portfolio of low-beta stocks and sells a zero-net investment portfolio of high-beta stocks. The excess returns of the value-weighted portfolio over the one-month T-bill (Market factor) is used as a factor in most empirical asset pricing models, but it is well known that it fails to explain the cross-section of stock returns if it is not augmented with other empirical factors. For example, Harvey et al. (2016) categorize 314 factors from 311 different papers published in top-tier finance journals and working papers between 1967 and 2014. Therefore, leverage constrained

[1] The theoretical development of the CAPM using excess returns over the risk-free rate is attributed to Jack Treynor (1962), William Sharpe (1964), John Lintner (1965), and Jan Mossin (1966). Black (1972) extended the model to the case in which a risk-free asset is not available and a zero-beta portfolio is used to calculate excess returns.

investors can bid up on any of these non-Market factors' betas for the same reason they bid up on the Market beta in FP. Then, betting against any of the hundreds of factors proposed in the literature might also work. Additionally, buying assets with a high beta corresponding to a factor other than the CAPM's Market factor is equivalent to buying an asset with a positive CAPM's alpha. More precisely, if a certain multifactor model is the correct model to rule out arbitrage, then the estimated alpha by the CAPM – or any model that does not capture all sources of risk – includes the risk premium of the factors missed by the model times their betas. Thus, if leverage constrained investors bid up in assets with high non-Market betas too, a strategy of betting against alpha should work for the same reasons that a strategy of betting against beta does.

- *Non-Market betas interpreted as alpha:* Barber et al. (2016) showed that when evaluating a mutual fund's performance, investors pay attention mostly to the market factor and treat other risk factors as alpha. Their finding implies that an asset with a high (low) realized beta with respect to a factor missed by the CAPM is interpreted by investors as an asset having a high (low) realized CAPM alpha. Consequently, I argue that fund managers have incentives to tilt their portfolios towards assets with high non-Market betas too, since that signals superior performance and increases the flow of capital towards their fund. Therefore, betting against alpha should work in this case too, even if managers do not face leverage constraints. My conjecture about managers having incentives to buy assets with high realized non-Market betas – and thus high realized CAPM alphas – is also consistent with the empirical findings of Dasgupta et al. (2011). They show that stocks that are persistently sold by institutions outperform stocks that are bought by them. Therefore, if fund managers buy underperforming stocks with high realized non-Market betas – and thus high realized CAPM alphas – and sell overperforming stocks with low realized non-Market betas – and thus low realized CAPM alphas – the BAA factor should work, and the empirical results of Dasgupta et al. (2011) follow.

- *Benchmarking and the limits to arbitrage*: Related to the previous two points, the large body of empirical and theoretical literature on benchmarking attests to the fact that mutual fund managers have incentives to tilt their portfolios towards high beta stocks, even if these assets are overpriced.[2] Baker et al. (2011) stated that "among the many candidates for the greatest anomaly in finance, a particularly compelling one is the long-term success of low-volatility and low-beta stock portfolios." They hypothesized that the survival of this anomaly is due

[2]For example, Karceski (2002) developed a model where mutual fund investors present behavioral biases that encourage fund managers to tilt their portfolios towards high-beta stocks in bull markets. Buffa et al. (2017) showed that benchmarking causes managers to exacerbate price distortions because trading against overvaluation exposes managers to a greater risk of deviating from the index. Consistent with the previous paper, Christoffersen and Simutin (2017) showed that fund managers from large pension funds tilt their portfolios toward high-beta stocks in an effort to beat the index. They relate this behavior to the fact that pension plan sponsors often monitor a fund's performance relative to a benchmark.

to benchmarking and the limits to arbitrage. They also argued that "a typical contract for institutional equity management contains an implicit or explicit mandate to maximize the *information ratio* relative to a specific, fixed capitalization-weighted benchmark without using leverage. For example, if the benchmark is the S&P 500 Index, the numerator of the information ratio (IR) is the expected difference between the return earned by the investment manager and the return on the S&P 500. The denominator is the volatility of these returns' difference, also called the tracking error".[3] It follows that, for example, for assets with similar Market betas, those with higher non-Market betas, and thus higher CAPM alphas, will increase the numerator of the IR. If the differential impact on the tracking error is negligible, then fund managers will always prefer the asset with high non-Market betas even if they are overpriced when taking into account all sources of risk. Therefore, benchmarking provides incentives for fund managers to tilt their portfolios towards assets with high realized alphas, which reflects high non-Market factors' betas.

Importantly, the previous explanations about why the BAA factor should work are also consistent with this factor summarizing the information about betting against beta strategies constructed using betas from factors other than the Market one. Consistent with this conjecture, I show that the BAB and BAA factors price the betting against beta strategies constructed using non-Market factors' betas. A relevant implication of this result is that in principle, these two strategies should subsume the information on most of the possible betting against beta strategies available.

The motivation for the second factor proposed in this paper, the BAAB factor, follows immediately from the previous discussion. If certain investors tilt their portfolios toward assets with high realized Market betas and also toward assets with high realized alphas, then a portfolio consisting of high realized alpha assets from the set of high realized beta ones should be even more overpriced. A similar logic follows for the long position used to construct the BAAB factor that comprises the portfolio of low realized alphas assets from the set of assets with low realized betas.

To construct the BAA factor presented in the main body of the paper, I use alphas estimated from the CAPM and data spanning January 1968 to Decemeber 2015. Then I extend the analysis in the Appendix twofold. First, I show that betting against the CAPM's realized alpha is not the only successful strategy. Using the alphas estimated from the Carhart and FF5 models works too. As is well documented in the literature, no factor model is capable of pricing the hundreds of anomalies that have been proposed in the empirical literature. Therefore, using alternative models to the CAPM to calculate alpha should, and does work. Importantly, the BAA factors constructed using alphas estimated from diferent models are all highly correlated among themselves, suggesting that they capture similar information. Second, I extend the period of analysis to data from January 1927 to December 2015 and all results hold.

[3] More specifically, suppose that R_A represents the returns on an active portfolio while R_M represents the returns on an index used as a benchmark. Then, the information ratio of the active portfolio is $IR_A = E(R_A - R_M)/\sigma_{(R_A - R_M)}$. Note that the numerator of IR_A in terms of the standard CAPM can be expressed as $\alpha_A + (\beta_A - 1)[E(R_M) - R_f]$.

Let me now move to discuss the construction of the factors and the paper's results in more detail. The BAA, BAB, and BAAB factors are constructed using leverage, although I show in the paper that the BAA factor works as a standard long-short strategy without leverage, too. More precisely, to compare across strategies, I developed a benchmark scenario in which I use the same weights for the low and high portfolios for every levered strategy. The common weights are calculated following the methodology developed in FP for the construction of their BAB factor.

In the benchmark scenario I rebalance all strategies once a year (12-month holding period). The monthly Sharpe Ratios of the BAA, BAB, and BAAB strategies are 0.22, 0.26, and 0.30, respectively.[4] These monthly Sharpe Ratios are quite high when compared to those of the Market (0.11), SMB (0.07), HML (0.12), RMW (0.11), CMA (0.17), and MOM (0.16) factors. The abnormal returns generated by the BAA and BAAB factors when controlling for the CAPM, Carhart, and FF5 models are around 1% per month, while that of the BAB is around 0.5% per month. The performance metrics of the BAA and BAAB factors increase with the holding period of the portfolios, reaching a maximum when rebalancing every 24 months. Again, this is consistent with our conjecture that institutional traders could be fueling the BAA and BAAB factors and the empirical results of Sias (2004) and Dasgupta et al. (2011) about trading persistence in mutual funds and institutional herding. In fact, Dasgupta et al. (2011) also find that their results about underperformance of stocks bought by institutional investors and outperformance of stocks sold by these investors are stronger using a 24-month period. Additionally, such a long period for rebalancing reduces the likelihood of facing data-snooping problems [see Novy-Marx and Velikov (2016)]. The monthly Sharpe Ratio of the BAAB factor using a 24-month holding period reaches 0.34, which is more than three times that of the Market factor (0.11).

According to the Arbitrage Pricing Theory (APT) developed by Ross (1977), a variable – with a positive risk premium – can be considered as a new risk factor only if it captures a source of comovement in the space of excess returns missed by other existing factors. Therefore, testing whether a long-short strategy captures new information is of primary concern. For this purpose, I estimate the rank of the beta matrix generated by several different sets of factors using the RBIC estimator of Ahn et al. (2017). I chose this estimator over others existing in the literature like Cragg and Donald (1997) because it is designed to handle data with a large number of cross-sectional units.[5] As Ahn et al. (2017) point out, "the rank of the beta matrix also provides important inferences on the number of common factors in risky asset returns. It increases whenever a factor having an identifiable risk premium or price (either zero or non-zero) is added to an asset pricing model." In other words, if a factor is added to a model and the rank of the beta matrix generated by the augmented model increases, then the added factor captures a source of comovement missed by the original model. My results suggest that the BAA and BAB strategies capture different information

[4] To avoid aggregation issues, I do not annualize the estimated monthly Sharpe Ratios [see Lo (2002)].

[5] Additionally, as argued by Ahn et al. (2017) and Burnside (2016), using rank tests can also control the proliferation of factors in the empirical asset pricing literature by requiring from a factor an additional proof of success beyond simply generating significant abnormal returns when regressed against some already existing empirical models.

among themselves. The results also suggest that the BAA strategy captures relevant information missed by the CAPM, Carhart, and FF5 models.

Finally, the low-beta anomaly has been related to the low-volatility anomaly [e.g. Baker et al. (2011)]. Consistent with this, I find that when I sort portfolios based on Market betas, the average total volatility and average idiosyncratic volatility of the assets in the sorted portfolios increase with beta. However, when I sort portfolios based on alphas, the average total volatility and average idiosyncratic volatility of the assets in the sorted portfolios present a U-shaped pattern, suggesting that the low-alpha anomaly is not related to the low-volatility one. Furthermore, in the Appendix I present results using two more strategies: a betting against total volatility strategy and a betting against idiosyncratic volatility strategy. The correlation coefficients of these two strategies with respect to the BAB factor are 0.56 and 0.51, respectively, while their correlation with respect to the BAA factor are -0.18 and -0.12, respectively.[6] The correlation between the BAA and BAB factors is just 0.21. Therefore, all my tests suggest that the low-alpha anomaly is different than the low-beta anomaly as well as the low-volatility anomaly.

The rest of the paper is organized as follows. Section 2 presents the data I will use and elaborates on the construction of the weights used to construct the factors with leverage. In Section 3, I present the main quantitative results. I conclude in Section 4 and present several robustness checks in the Appendix.

2 Data and construction of the levered strategies

2.1 Data

I use data on US individual stock returns from the Center for Research in Security Prices (CRSP) from January 1968 until December 2015. In the Appendix I corroborate the results using data from January 1927 until December 2015. The returns include dividends and correspond to common stocks traded on the NYSE, NASDAQ, and AMEX, excluding REITs and ADRs. Data on the Fama-French Five Factor model (FF5, 2015) as well as the Momentum factor for the Carhart (1997) model are from Kenneth French's website.[7] The FF5 factors are the excess market return (Market: the return on the CRSP value-weighted portfolio minus the return on the 1-month Treasury bill), Small Minus Big (SMB), High Minus Low (HML), Robust Minus Weak (RMW) and Conservative Minus Aggressive (CMA) factors. The Carhart factors are the Market, SMB and HML factors from

[6] The reason I focus in the paper on the low-alpha and low-beta anomalies but not on the low-volatility one is that the factors constructed based on the first two anomalies present much higher Sharpe Ratios and expected returns than those constructed based on the latter. Additionally, the factors based on the low-alpha and low-beta anomalies are not priced by any of the multifactor models used for controlling risks, while unreported results show that those based on the low-volatility are. These results do not mean that the low-volatility anomaly is not relevant. However, given the benchmark scenario used for comparison in this paper, the low-volatility factors definitely have less power to generate large performance metrics than the BAA, BAB, and BAAB factors.

[7] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

FF5 plus the Momentum factor (MOM is selling losers and buying winners from the prior 6 to 12 months).

2.2 Construction of the Betting Against Alpha, Betting Against Beta, and Betting Against Alpha and Beta factors

The Betting Against Beta (BAB) factor developed by Frazzini and Pedersen (FP, 2014) consists of selling a portfolio made of high beta stocks' excess returns over the risk free asset and buying a portfolio made of low beta stocks' excess returns (note that the long and short portfolios are both zero-net investments). Additionally, both portfolios are scaled by the inverse of the risky assets' weighted betas, and thus, given that the average Market beta value fluctuates around one, the excess returns of the low beta portfolio are amplified, while the excess returns of the high beta portfolio are reduced. Therefore, the final long-short strategy when using leverage is a zero-net investment strategy, but it is not dollar-neutral on the risky portfolios.

Similarly, the Betting Against Alpha (BAA) strategy consists of selling a portfolio containing assets with realized alphas bigger then the median alpha and buying a portfolio of assets with realized alphas lower than the median alpha. The Betting Against Alpha and Beta (BAAB) strategy also sells a portfolio containing assets with high realized alphas and buys a portfolio containing assets with low realized alphas. The difference between the BAA factor and the BAAB factor is that for the latter, I first divide the sample into two groups: Assets with realized betas less than the median beta and those with realized betas higher than the median beta. Then, using the sample of assets with realized betas lower than the median beta, I create the long portfolio with assets having realized alphas lower than the median alpha within that sample. Similarly, from the sample of assets with realized betas higher than the median beta, I create the short portfolio with assets having realized alphas larger than the median alpha within the high beta sample.

The values of the estimated alphas and betas depend on the models used to calculate them. Following FP, the betas from the BAB factor always correspond to the sensitivity of the asset to the Market factor. In the main body of the paper I use the CAPM to estimate the realized alphas and realized market betas used to construct the long and short portfolios. The Appendix contains the results using the Carhart and FF5 models to estimate the realized alphas and the Market betas to construct the factors.

Modifying the weights used to augment and reduce the portfolios' excess returns affects the performance metrics of the factors. To simplify the comparison between the BAA, BAB, and BAAB factors, I use the same weights for the low and high portfolios in all of them. As shown in FP, further calibration of the weights applied to the long-short portfolio strategy can increase the resulting Sharpe Ratio. However, my goal is not to find the strategy with the largest Sharpe Ratio, but to check whether the BAA and BAAB strategies produce economically meaningful performance metrics and whether they capture different information than the BAB factor does. Additionally, I also check if they capture information about the comovement of stock returns missed by commonly used empirical asset pricing models.

FP suggests estimating the market betas differently than using traditional regression methods, but again, my goal is not to find the highest possible Sharpe Ratio but to construct a benchmark scenario that allows me to compare across strategies. Therefore, using the same estimation methods across strategies to calculate the parameters of interest seems appropriate for this paper.

A levered strategy consists of buying the low portfolio and selling the high portfolio with certain weights. As already mentioned, I will use the same weights for the BAA, BAB, and BAAB factors. I will construct those weights in a similar way as FP did. The next paragraphs explain how.

To calculate the common weight to use for the BAB, BAA and BAAB low and high portfolios, I follow closely the methodology developed in FP. For each asset i, $\beta_i = \rho_{iM}\,(\sigma_i/\sigma_M)$, where ρ_{iM} is the correlation between asset's i returns and the Market returns, while σ_i and σ_M are the asset i and Market estimated volatilities, respectively. As in FP, the ρ_{iM} is estimated using five year data while σ_i and σ_M are estimated using yearly data. The final market beta assigned to an asset i (β_i^M) is compressed towards one as in FP using the formula $\beta_i^M = 0.6\beta_i + 0.4$.

The low beta portfolios used to calculate the common weights contain all assets with β_i^M lower than the median, and the high beta portfolio contains the ones greater than or equal to the median. Securities β_i^M in the corresponding low or high portfolio are weighted using the same formula as in FP. More precisely, let *nl* be the number of assets in the low beta portfolio and *zl* be the $nl \times 1$ vector of beta ranks $zl_i = rank(\beta_i^M)$. The weight of an asset i in the low beta portfolio is given by $wl_i = (nl - zl_i + 1)/\sum zl_i$. Similarly, let *nh* be the number of assets in the high beta portfolio and *zh* be the $nh \times 1$ vector of beta ranks in this portfolio, where $zh_i = rank(\beta_i^M)$. The weight of an asset i in the high beta portfolio is given by $wh_i = zh_i/\sum zh_i$. Note that $\sum wl_i = \sum wh_i = 1$. The final weighted market beta of the low and high beta portfolios are $\beta^L = \sum wl_i\beta_i^M$ and $\beta^H = \sum wh_i\beta_i^M$ respectively. For every long-short strategy I use the same β^L and β^H. The returns of the low and high portfolio are $r^L = \sum wl_i r_i^L$ and $r^H = \sum wh_i r_i^H$, respectively. Importantly, remember that for the actual low and high beta portfolio I do not sort on β_i^M but on the CAPM's regression beta. The BAB factor's return rebalanced monthly and consisting of selling the high market beta portfolio and buying the low market beta portfolio is $r_{t+1}^{BAB} = \frac{1}{\beta_t^L}(r_{t+1}^L - r_{t+1}^f) - \frac{1}{\beta_t^H}(r_{t+1}^H - r_{t+1}^f)$. In the case of yearly rebalancing at the end of December, the BAB factor's return is $r_{t+s}^{BAB} = \frac{1}{\beta_t^L}(r_{t+s}^L - r_{t+s}^f) - \frac{1}{\beta_t^H}(r_{t+s}^H - r_{t+s}^f)$, where $s = 1, ..., 12$ and t corresponds to December.

The same weights β^L and β^H apply to the BAA and BAAB factors. The only difference is that for the BAA and BAAB factors $zl_i = rank(\hat{\alpha}_i^L)$ and $zh_i = rank(\hat{\alpha}_i^H)$, where $\hat{\alpha}_i^L$ and $\hat{\alpha}_i^H$ are alphas below and above the median alpha, respectively. The alphas are estimated using the standard CAPM regression.

The main difference between this paper's calculation of the weights β^L and β^H and the ones used in FP is that they use daily data to estimate betas and this paper uses monthly data. Additionally, they allow for assets to have at least three years of data in their calculation, while this paper uses five years of data to estimate all parameters. On average, this paper's weights imply an investment of around \$1.67 in the long portfolio and \$0.63 in the short portfolio using yearly rebalanced strategies (end of December). For robustness, I also use monthly, 6-month, 24-month and 48-month rebalancing periods in the calculations.[8]

The effect of these methodological differences in the calculated weights does not change the results of this paper. Figure 1 below shows the weights, $\frac{1}{\beta_t^L}$ and $\frac{1}{\beta_t^H}$, calculated with my modified technique rebalancing every twelve months and the monthly weights calculated with the FP method (using daily data available for at least 36 months and rebalancing monthly) for the period of January 1973 - December 2015. It can be observed that both sets of weights show a similar pattern between 1973 and 2015. The main difference is that the modified technique seems to create a slightly larger spread between the low beta and high beta portfolios' weights. However, since I apply the same weights to all strategies, this difference in the weights' spread does not modify the qualitative results of this paper.

[Insert Figure 1 around here]

3 Results

3.1 Factors' performance metrics for the benchmark scenario

In a first pass to the data, I study the performance metrics of the BAA, BAB, and BAAB factors under the following scenario: (i) The holding period return for each strategy is 12 months, where betas and alphas are estimated at the end of December. Then, portfolios are formed on the first trading day of January and maintained for 12 months until the last trading day of December. (ii) Portfolios are formed using the entire universe of the CRSP database as explained in Section 2.1. (iii) The assets' alphas and betas are estimated using the standard CAPM.[9] (iv) The period of analysis is 1973-2015; thus, I use data from 1968-2015 since the first five years of data are needed for estimating the initial realized alphas and betas. Section 3.2 and Section 3.3 contain results for the factors' performance metrics using different holding periods and across different ranges of market cap values, respectively. In the Appendix, I present results for the factors constructed using alphas

[8]The correlation between this paper's BAB factor and that of FP is 0.73. The BAB factor differs from that of FP because I use CAPM's betas to sort assets into low and high beta portfolios, while they use the above formula for β_i, which I only use to estimate the weights of the low and high portfolios. I use 5 years of monthly data to estimate betas and they use 5 years of daily data for ρ_{iM} and one year of daily data for σ_i and σ_M. I rebalance the portfolios every twelve months while FP rebalance every month. However, when I replicate all the results of this paper using FP's BAB factor, there are no qualitative differences. The FP BAB factor is available at https://www.aqr.com/library/data-sets/.

[9]The betas used to construct the weights of the long-short strategies are estimated using FP's methodology. For more information please see Section 2.2.

and market betas estimated from the Carhart and FF5 models, and I extend the sample to the period 1927-2015.[10] It is important to clarify that this Section presents the benchmark results. As shown in the next Section, the highest values for the performance metrics using the BAA and BAAB factors are usually obtained using a 24-month holding period, while for the BAB factor the highest values for the metrics are obtained at the 1-month holding period.

Table 1 shows the summary statistics for the low portfolios, high portfolios, and levered BAA and BAB factors (weighted low portfolio returns minus weighted high portfolio returns). I report the monthly Sharpe Ratios[11], average monthly excess return, monthly CAPM alpha, monthly Carhart alpha, monthly FF5 alpha, and average market capitalization value of the weighted portfolios in thousands of USD (Size) at the time of rebalancing. Heteroskedastic robust t-statistics are in parenthesis below each model's estimated alphas.

[Insert Table 1 around here]

The first line of Table 1 shows that the Sharpe Ratios decrease for both strategies when we move from the low to the high portfolio. The BAB factor's monthly Sharpe Ratio is 0.26 while the BAA one is 0.22. Both factors have a higher monthly Sharpe Ratio than the Market (0.11), SMB (0.07), HML (0.12), RMW (0.11), CMA (0.17), and MOM (0.16) factors do. The excess return row shows that the risk premium of the BAA strategy doubles that of the BAB one.

Both factors produce sizeable CAPM, Carhart, and FF5 abnormal returns with t-statistics surpassing the hurdle of 3 suggested by Harvey et al. (2016). The abnormal returns of the BAA strategy are usually more than twice those of the BAB one. Finally, the BAA low portfolio is comprised on average of smaller market cap stocks than those of the BAB low portfolio, while the BAA high portfolio is comprised of larger market cap stocks than those of the BAB high portfolio. Therefore, size might affect the BAA factor more than the BAB one. We will come back to this issue later in Section 3.3.

The correlation between the BAA and BAB strategies is quite low, only 0.21, which implies that betting against alpha is probably not the same as betting against beta. This issue is studied in more depth in Section 3.4.

As I argue in the introduction, assets with high realized alphas from the sample of assets with high realized betas will be the most overpriced, while those with low alphas from the sample of low beta assets will be the most underpriced. Before moving to the results for the BAAB factor, I first check if betting against alpha works across different values of market betas. Table 2 presents the same performance metrics as before for the low alpha portfolio, high alpha portfolio, and levered

[10] I decided to present the shorter period 1972-2015 in the main body of the paper since the FF5 model seems to be the one that best captures some of the strategies' information. There is no available information for the RMW and CMA factors corresponding to the FF5 model prior to July 1963.

[11] I do not annualize the estimated monthly Sharpe ratios to avoid problems arising from aggregation [see Lo (2002)].

low-high strategy. However, in this table I calculate those metrics for the subset of assets with betas above the median beta (High Beta) and for the subset of assets with betas below the median beta (Low Beta) separately.

[Insert Table 2 around here]

Table 2 shows that Sharpe Ratios and CAPM, Carhart, and FF5 abnormal returns decrease as alpha increases for the low and high beta portfolios. Consistent with this paper's conjecture, the high alpha portfolio constructed from high beta assets has the lowest Sharpe Ratio, while the low alpha portfolio constructed from low beta assets has the highest one. As expected, Sharpe Ratios decrease as beta increases. Therefore, the BAAB factor that shorts the high alpha assets of the high beta portfolio and buys the low alpha assets of the low beta portfolio most probably improves over the single-sorted strategies.[12] Results are in Table 3.

[Insert Table 3 around here]

As expected, the monthly Sharpe Ratio of the combined strategy surpasses that of the BAA and BAB factors alone and almost triples that of the Market factor. Monthly abnormal returns for the combined strategy are around 1%.

I now analyze the correlation between the constructed factors with leverage and the factors used to control for risk in this paper (FF5 and MOM factors). Results are shown in Table 4.

[Insert Table 4 around here]

The correlation between the BAA factor and the FF5 factors is larger than that of the BAB factor and the FF5 factors, especially the one between the BAA factor and the Market factor (0.61). However, once the BAA strategy is combined with the BAB one into the BAAB factor, the correlation between the BAAB factor and the FF5 and MOM factors decreases. For example, the correlation between the BAAB factor and the Market factor is just 0.32, and no correlation with any other factor surpasses 0.4. Additionally, the BAAB factor still has a relatively high correlation with the BAA factor (0.82) and a relatively low correlation with the BAB factor (0.55).

Finally, Table 5 presents performance metrics for the dataset divided into decile portfolios sorted on alphas in Panel (a) and on betas in Panel (b). Assets within each portfolio are equally-weighted.

[Insert Table 5 around here]

[12] One important difference between the combined strategy with respect to the single-sorted one is that the combined uses around 50% of the CRSP cleaned database, while the single sorted one uses 100%.

The first line of both panels shows that Sharpe Ratios are decreasing in both alphas and betas. However, the second line shows that excess returns are decreasing in alphas while they slightly increase in betas. The last column of the table shows the results from using a low minus high strategy without leverage.[13] The first five lines of the Low-High column in Panel (a) show that for the BAA strategy, in principle, one does not need to use leverage as with the BAB one. However, since the BAAB factor requires combining the BAA strategy with the BAB, I will keep using in our main results all strategies with leverage. As expected, the sixth line in Panel (a) shows that average realized alphas are increasing for portfolios sorted by alpha, while the sixth line of Panel (b) shows that realized betas are increasing for portfolios sorted by betas. The prediction of FP still holds: A low realized beta implies a future high alpha, while a high realized beta implies a future low alpha. My prediction also holds: A low realized alpha implies a future high alpha, while a high realized alpha implies a future low alpha. This can be seen in lines three to five of Panel (a) and Panel (b), where I present the abnormal returns for the CAPM, Carhart, and FF5 models. For both sets of portfolios, the FF5 model seems to price most decile portfolios but the lowest ones. As expected, lines seven and eight in Panel (b) show that the Average Total Volatility and the Average Idiosyncratic Volatility of the assets in the portfolios increase with the average Market beta [see for example Baker et al. (2011)]. Importantly, Panel (a) shows that the relationship between realized alphas and volatility present a U-shape, suggesting that the low-alpha anomaly presented in this paper is not related to the low-volatility anomaly. To further study the relationship between the low-beta, low-alpha, and low-volatility anomalies, I created two more strategies: A betting against total volatility strategy and a betting against idiosyncratic volatility strategy. Details about the performance metrics of these strategies are in the Appendix. The correlation coefficients of these two strategies with respect to the BAB factor are 0.56 and 0.51, respectively, while their correlations with respect to the BAA factor are -0.18 and -0.12, respectively. Overall, the low-volatility anomaly does not seem related to the low-alpha anomaly presented in this paper.

Finally, the last line of Panel (a) shows that the relationship between market capitalization (Size) and Average Alpha has an inverted U-shape. The last line of Panel (b) shows that this inverted U-shape is even more pronounced for the relationship between Size and Average Market Beta. Thus, as confirmed in Section 3.3 below, removing small stocks from the sample negatively impacts the performance metrics of the factors since it is equivalent to removing assets from both extreme decile portfolios. Additionally, this inverted U-shape between market capitalization and alpha (and beta) implies that using value-weighted portfolios to construct these strategies should have a negative impact on their performance metrics too: While the BAA, BAB, and BAAB factors are constructed overweighting the assets in the extreme range of the alpha and beta values, a value-weighted portfolio will overweight the assets in the middle of the range. Thus, a strategy constructed using value-

[13]Note that here the strategies use only the largest and smallest decile portfolios, while the results presented in the previous tables use much more data.

weighted portfolios will overweight alphas close to zero and betas close to one. Overweighting such assets is exactly the opposite of what these strategies require.[14]

3.2 BAA, BAB, and BAAB factors for different holding periods

In this Section, I analyze the performance of the different strategies using holding period returns from 1 month to 48 months. As in the previous Section, I use the same weights for every strategy, where I recalculate the weights at the end of the holding period using the formulas presented in Section 2.2.

Table 6 presents the performance metrics across holding period returns for the BAA, BAB, and BAAB factors.

[Insert Table 6 around here]

For the BAA factor, the Sharpe Ratio increases in the holding period return. The Sharpe Ratio of the 48 month holding period is slightly larger than that of the 24 month holding period. However, average and abnormal returns for the 48 month holding period are below those observed for the 12 month holding period. The highest monthly abnormal returns are obtained when rebalancing the portfolios every 24 months.[15] For the BAB factor, Sharpe Ratios and abnormal returns decrease in holding period returns. For this strategy, the highest Sharpe Ratio, average returns, and abnormal returns are observed when the portfolios are re-balanced monthly.

Results for the BAAB factor are in the last three columns of Table 6. Like the BAA factor, the BAAB factor obtains the highest values for the performance metrics when re-balancing portfolios every 24 months. The monthly Sharpe Ratio of this strategy rebalancing every 24 months is 0.34. This is three times the monthly Sharpe Ratio of the Market factor and significantly higher than the highest Sharpe Ratio that either the BAA factor (0.25) or the BAB factor (0.28) can achieve.

Finally, the abnormal returns generated by these three factors across holding period returns are statistically significant for the three models at a less than 1% level of significance. The only exceptions are the FF5's alpha for the BAA factor using a 1-month holding period and the Carhart's alpha for the BAB factor using a 48-month holding period.

[14] As explained in this paragraph, using value-weighted or any other type of weights violates the strategies' premise of overweighting low alpha (beta) assets in the low alpha (beta) portfolio, while underweighting this type of asset in the high alpha (beta) portfolio. Thus, results with equally-weighted (or value-weighted) portfolios should be considered of second order importance. Studying the impact of size across strategies is of paramount importance, but should be performed taking into account the strategies' premises. This can be done by studying the performance of the strategies across sets of assets belonging to different market capitalization ranges. The latter is the objective of Section 3.3 below.

[15] In the Appendix, I obtain similar results for the BAA factors based on alphas from the Carhart or FF5 models. In that case, both the highest Sharpe Ratios and highest abnormal returns are obtained using a 24 month holding period.

3.3 BAA, BAB, and BAAB factors across size

Most long-short strategies that produce abnormal returns show decreasing performance as companies with low market capitalization values are removed from the sample used to construct them [e.g. Fama and French (2008)]. In fact, the positive risk premiums generated by many factors disappear once the small companies (or even micro cap companies) are removed from the sample. Therefore, it is important to study the performance of the BAA, BAB, and BAAB factors for different levels of market cap.

Before performing the analysis clustering companies by market capitalization, it is important to remember that the relationship between size and alpha, as well as that between size and beta, has an inverted U-shape form.[16] This mean that there are small stocks at both extremes of the alpha and beta ranges. Thus, removing small stocks will affect the BAA, BAB, and BAAB factors' performance metrics.

Using the NYSE 30th and 70th percentile for market capitalization cutoff values, every December, I divide the dataset into three categories: (i) *30% Small*, which contains all firms whose market cap is equal to or below the 30th percentile; (ii) *40% Medium*, which contains all firms whose market cap is greater than the 30th percentile and lower than or equal to the 70th percentile; and (iii) *30% Big*, which contains those firms with a market cap value greater than the 70th percentile. For each group, I construct the BAA, BAB, and BAAB factors and run the same performance metrics as before. As in the benchmark scenario, all strategies use a 12-month holding period.[17] Results are presented in Table 7.

[Insert Table 7 around here]

First of all, and as expected, Sharpe Ratios decrease as the market capitalization value of the companies used to construct the strategies increases. However, an important desirable property is maintained: for the three strategies, the Sharpe Ratio of the low portfolio surpasses that of the large one across size groups. When looking at average returns, all strategies produce positive risk premiums across size groups too.

The CAPM cannot price the factors constructed with any set of market capitalization clusters, generating abnormal returns at the 1% level or less across all groups. The only exception is the BAA factor, which generates abnormal returns at the 5% level or less when large stocks are used. When controlling for the Carhart model, the results for the BAA factor are quite similar to those obtained when controlling for the CAPM. The BAB factor presents a different scenario: It only generates statistically significant abnormal returns when small stocks are used. The BAAB factor loses some

[16] See Table 5 in Section 3.1 and the corresponding discussion in the last paragraph of that section.

[17] Results improve for the BAA and BAAB factors when using a 24-month holding period and for the BAB strategy when using a 1-month holding period.

power when controlling for the Carhart model instead of the CAPM, generating statistically significant abnormal returns at the 1% level or less only for the small and medium group, while for the group of large stocks the abnormal returns are significant at the 5% level. Finally, when I control for the FF5 model, the strategies only generate abnormal returns when small cap assets are used. This result should not be surprising: The factors included in the FF5 model are constructed with the entire universe of the CRSP database, including small cap stocks (which, in addition, present more volatility than the average). Therefore, it is not surprising that they can project a space of factors constructed using the subset of very large stocks (which present less volatility than the average).

Overall, I find that the analyzed strategies maintain desirable properties across all size groups, like decreasing Sharpe Ratios across portfolios and positive risk premiums. As I include more factors in the empirical asset pricing model used to test the strategies, abnormal returns for strategies using only large stocks diminish and for the FF5 model disappear. However, Sharpe Ratios and risk premiums are still large. For example, when using the 30% biggest stocks, the BAAB strategy generates a monthly risk premium of almost 1% and a Sharpe Ratio 0.18, which is 65% larger than the Market factor and surpasses that of any of the FF5 or MOM factors.[18]

3.4 BAA factor as a new source of stock returns' comovement

In the previous sections I showed that the BAA and BAB factors are not priced by either the CAPM, Carhart, or FF5 models. Additionally, and not shown in this paper, I found that the BAA factor cannot price the BAB factor and vice-versa.[19] However, that a factor generates significant pricing errors when regressed against other factors is not sufficient evidence about that factor capturing a missing dimension in the space of stock returns. For example, using rank estimation methods, Ahn et al. (2017) found that 26 commonly used factors capture at most five independent vectors in the space of stock returns.

Therefore, an important question that remains to be answered is whether the BAA and BAB strategies capture different information, or even information missed by the FF5 and momentum factors. A natural way to answer this question is to estimate the rank of the beta matrix generated by these strategies when they are used as regressors. As Ahn et al. (2017) point out, "the rank of the beta matrix corresponding to a set of factors equals the number of factors whose prices are identifiable." In other words, the rank of the beta matrix will tell us the number of different sources of stock returns' comovement captured by a set of factors.

First I will test whether the BAA and BAB factors increase the rank of the beta matrix using them separately and together. This will allow me to assess whether they are capturing different or similar information.[20] Then, I will use these factors to augment the CAPM, Carhart, and FF5

[18] For the 1972-2015 period, the monthly Sharpe Ratios of the other factors are 0.11 for the Market Portfolio, 0.07 for SMB, 0.12 for HML, 0.11 for RMW, 0.17 for CMA, and 0.16 for MOM.

[19] Results are available upon request.

[20] The results of this Section remain unchanged if I directly use Frazzini and Pedersen's BAB factor. Their factor

models to analyze if the BAA and BAB factors contain information missed by any of these empirical models.

As response variables in my tests I will use two sets of portfolio returns.[21] First, following the suggestion of Lewellen et al. (2010), I consider the combined set of the 25 Size and Book to Market portfolios together with the 30 Industrial portfolios. The second set of response variables consists of the 25 Size and Book to Market, 25 Size and Investment, 25 Size and Operating Profitability, 25 Book to Market and Investment, 25 Book to Market and Operating Profitability, and 25 Investment and Operating Profitability portfolios.[22] The rationale for using the second set of portfolios is to have a scenario in which the FF5 model produces a full rank beta matrix. As shown in Ahn et al. (2017), the FF5 model fails to produce a full rank beta matrix when using the first set of portfolios. Thus, if the BAA or BAB factors increase the rank of the beta matrix when used in tandem with the FF5 factors using the second set of portfolios as response variables, where the FF5 factors generate a full rank beta matrix, then we have strong evidence that the the BAA or BAB factors capture information about the cross-section of stock returns missed by FF5.

While many alternative rank estimators are available in the literature, they are designed for the analysis of data with a small number of cross section units (N). Consequently, they may not be appropriate for the estimation of the beta matrix with large N. Ahn et al. (2017), however, found that a restricted version of the BIC (RBIC) rank estimator of Cragg and Donald (1997) has good finite-sample properties if the return data used contains the time series observations of at least 240 months ($T \geq 240$) over individual portfolios whose number does not exceed one half of the time series observations ($N \leq T/2$). Our data fits the desirable properties for the RBIC rank estimator since our time span is January 1973 - September 2015 ($T = 516$) and the number of cross-sectional units is $N = 55$ and $N = 150$ for the first and second set of response variables, respectively.

Table 8 presents the rank estimations' results. The first column shows the results using the first set of 55 portfolios as response variables, while the second column corresponds to the results using the larger set of 150 portfolios as response variables. Each row corresponds to a set of k factors used as regressors to generate the estimated beta matrix (or matrix of factor loadings). Thus, k corresponds to the maximum rank attainable by the beta matrix.

[Insert Table 8 around here]

The results in the first three lines correspond to using only the BAA and BAB factors.[23] Both

is available at https://www.aqr.com/library/data-sets/.

[21] I use portfolio returns as response variables since they contain a stronger factor structure than individual stock returns. Ahn et al. (2017) show that a higher signal to noise ratio of the factors with respect to the response variables increases the accuracy of their rank estimator.

[22] All the portfolio data comes from Kenneth French's website.

[23] I do not add to the analysis the BAAB factor since the main objective is to assess whether the BAA factor captures different information than the BAB factor. Rank estimation results with the BAAB factor are very similar

strategies capture a relevant source of comovement according to the RBIC estimator when using the first set of response variables (the rank estimator equals 1 when the response variables are regressed against the BAA and BAB factors separately). When the BAA and BAB factors are used together, they generate a full rank beta matrix (the RBIC rank estimator equals 2), indicating that the information they capture is different. With the second set of response variables, only the BAA factor seems to capture a source of comovement. Overall, both strategies seem to contain different information. I now move to analyze if the factors capture a different source of comovement than the CAPM, Carhart, and FF5 models.

Focusing on the first column for now, we observe that both factors augment the rank generated by the CAPM. Thus, both strategies capture information missed by the Market factor. When both factors are added to the CAPM (k=3), the estimated rank of the beta matrix is 3, which confirms our result that BAA and BAB capture different information. Similar results are obtained when augmenting the Carhart model with the BAA and BAB strategies. By itself, the Carhart model generates a rank deficient beta matrix (k=4 but the estimated rank is 3). The rank augments to 4 when adding either the BAA or BAB factor and augments to 5 when adding both factors. This is further evidence about the BAA and BAB factors capturing different information. Finally, the FF5 model also produces a rank-deficient beta matrix using the first set of response variables [as already showed in Ahn et al. (2017)]. However, in this case only the BAA factor increases the rank of the beta matrix. Therefore, it seems that the BAA factor captures a source of comovement missed by FF5.

We now move to analyze the results of the second column in more depth. Since the BAB factor alone does not capture a source of comovement using this larger set of assets as response variables, it is not surprising that it does not increase the rank of the beta matrix when it is used to augment the CAPM, Carhart, or FF5 models. This should not be interpreted as a robust negative result for the BAB factor. It might be simply that the factor structure generated by these portfolios has very low correlation with the BAB factor. What is important from the second column of this table is that the factor structure generated by these portfolios is highly correlated with the FF5 factors, which generate a full rank beta matrix in this case (k=5 and the estimated rank is 5). Additionally, when I augment the FF5 factors with the BAA factor, the estimated rank of the beta matrix increases to 6, suggesting that BAA captures a source of comovement missed by the FF5 the factors. Note that this happens even when using as response variables a set of assets that is highly correlated with the factor structure of the FF5 model. In fact, the BAA factor increases the rank of the beta matrix in all the cases I study.

In summary, this Section has two important results. First, the BAA and BAB factors capture different information. Second, the BAA strategy captures information missed by the CAPM, Carhart, and FF5 models, even when I use as response variables a set of portfolios that is highly correlated with the factor structure of these models.

to those obtained using just the BAA factor and are available from the author upon request.

3.5 Betting against other factors' betas

Previous research shows that, for example, liquidity constraints [Frazzini and Pedersen (2014)] and benchmarking [Baker et al. (2011)] generate incentives for mutual fund managers to tilt their portfolios toward high non-Market beta assets. Additionally, in the introduction I argued that investors' liquidity constraints, benchmarking, and investors' inattention to non-Market factors [Barber et al. (2016)] also generate incentives for mutual fund managers to tilt their portfolios toward high non-Market beta assets. These non-Market beta assets times their corresponding factors' risk premiums are reflected in the estimated CAPM's alpha. Therefore, as I also argued in the Introduction, the BAA and BAB factors should suffice to price the *betting against non-Market factors beta* strategies. Thus, in this Section I analyze whether the BAA and BAB factors can price betting against beta strategies constructed using the betas of the FF5 non-Market factors (SMB, HML, CMA, and RMW) and the MOM factor. Each factor's betas are estimated from a single-factor regressions.

First, Table 9 shows the summary statistics of these new strategies' performance metrics. I use the same weights for the long and short portfolios as I did for the BAB and BAA factors before. The long and short portfolios are rebalanced each December.

[Insert Table 9 around here]

Panel (a) and Panel (e) show that betting against the SMB beta and betting against the MOM beta strategies produce high monthly Sharpe Ratios (0.20 and 0.19, respectively) and significant abnormal returns. Betting against the betas generated by HML, CMA, and RMW does not produce high Sharpe Ratios. In fact, the Sharpe Ratios do not even decrease as we move from the low to the high portfolio. This result suggests that, for example, managers might be using Size and MOM to increase their expected returns more than the other non-Market factors.

Let me now move to the main objective of this Section, which is to assess whether the BAA and BAB factors suffice to price the levered strategies constructed with the non-Market factors, especially those using the betas corresponding to SMB and MOM, which produce non-negligible performance metrics. To answer this question I use an insight from Barillas and Shanken (2017). They showed that "it turns out that test assets tell us nothing about model comparison, beyond what we learn by examining the extent to which each model prices the factors in the other models." In other words, to compare factor models based on estimated pricing errors, we can simply regress one factor against another set of factors and see if the pricing error is statistically significant. If it is not, then the factor used as a dependent variable cannot improve over a model containing the factors used as regressors. For this purpose, I run regressions using the betting against non-market factor beta strategies presented in Table 9 as dependent variables and the BAA and BAB factors as regressors. Results are presented in Table 10.

[Insert Table 10 around here]

Looking at the intercepts, observe that the BAA and BAB factors price all the other strategies of interest. Therefore, my conjecture about the BAA and BAB factors subsuming the pricing information of other betting against beta strategies seems supported by the data.

4 Concluding remarks

I find that assets' realized alphas are negatively related to their future returns, future alphas, and Sharpe Ratios. I call this finding the low-alpha anomaly. Therefore, I propose a new factor called Betting Against Alpha (BAA) that produces sizable Sharpe Ratios, risk premiums, and abnormal returns. Following Frazzini and Pedersen's (2014) construction of their Betting Against Beta (BAB) factor, the new BAA factor is a zero-net investment strategy with leverage, where the long portfolio's excess returns are magnified and the short portfolio's excess returns are diminished. Furthermore, the performance of this new factor improves substantially when the assets with low alphas are selected from the set of assets with low betas, while the assets with high alphas are selected from the set of assets with high betas. I call this new factor Betting Against Alpha and Beta (BAAB). The BAAB factor produces a Sharpe Ratio three times larger than that of the Market factor when rebalanced every 24 months. The Sharpe Ratio of the BAAB factor also exceeds those of the BAA and BAB factors, as well as those of the Fama-French Five Factor model (FF5) and the Momentum factor.

The theoretical and empirical foundations for the BAA and BAAB factors follow from at least three well-known results. First, Frazzini and Pedersen (2014) show theoretically that leverage constrained investors overweight their portfolios with stocks having high realized Market betas and develop their BAB factor to take advantage of this friction. Following the same reasoning, leverage constrained investors pursuing higher expected returns also have incentives to tilt their portfolios towards assets with high non-Market betas for the same reasons they tilt their portfolios towards assets with high Market betas. Then, given that in a multifactor asset-pricing model the missing factor(s) will be reflected in the pricing error, and sustained on a large body of empirical literature about CAPM anomalies showing that the empirical Market portfolio does not capture all sources of risk, I am able to justify why betting against alpha works.

Second, Barber et al. (2016) show that investors are mostly concerned about Market risk and treat non-Market risk factors as alphas. Consequently, fund flows respond positively to the size of the CAPM alpha. I argue that this should create incentives for fund managers to tilt their portfolios toward assets with high non-Market factor betas, since that will be interpreted as superior performance by investors caring mostly about Market risk, increasing the flow of capital toward their fund. This second justification for the BAA factor suggests that it works even if fund managers do not face liquidity constraints, although this can magnify the fund managers' impact on mispricing.

Third, and related to the previous discussion, the literature on benchmarking and the limits to arbitrage show that fund managers with an explicit mandate to maximize the information ratio (IR) relative to a specific index have incentives to tilt their portfolios towards high beta assets, as argued by Baker et al. (2011) and many others.[24] The numerator of the IR depends on the alpha generated by a portfolio relative to an index, usually the S&P 500. Then, for assets with similar Market betas and similar impact on the tracking error, for example, those with higher non-Market betas, and thus higher CAPM alphas, will increase the numerator of the IR while keeping its denominator relatively constant. For this reason, benchmarked fund managers also have incentives to tilt their portfolios towards assets with high realized alphas.

A growing body of literature raised concern about the increasing quantity of factors available [e.g. Harvey et al (2016)]. I show that the BAA and BAB factors subsume the information contained on betting against non-Market factor betas, which should prevent the proliferation of betting against non-Market factors' betas. Furthermore, this result is consistent with my conjecture that the BAA factor works because agents have incentives to tilt their portfolios toward assets with high non-Market betas, whether it is because they face leverage constraints, managers try to show superior performance with respect to the CAPM, or are benchmarked against an index.

Finally, to show the relevance of the BAA factor, I not only rely on the usual tests based on pricing errors, but I also require the factors to pass the rigor imposed by rank estimation methods [e.g. Cragg and Donald (1997), Ahn et al. (2017)]. Using the RBIC rank estimator of Ahn et al. (2017), I check whether the BAA and BAB factors capture different information about the cross-section of stock returns. I find that the BAA factor captures not only different information than the BAB one, but also information missed by the FF5 and Momentum factors.

Appendix: Robustness checks

A BAA and BAAB strategies using Carhart and FF5 models' alphas and market betas

In this Section I check that the main results still hold when the alphas for the BAA strategies are calculated using either the Carhart or FF5 model. For the levered low minus high strategies, I maintain the same weights used in the main body of the paper. Table A1.1 and Table A1.2 replicate Table 1 in the paper for the Carhart and FF5 models respectively. Overall, results are quite similar. Using the Carhart or FF5 models to estimate alphas slightly increases the Sharpe Ratios of the BAA strategies compared to using the CAPM's alphas. The magnitude of the abnormal returns generated by the BAA strategies slightly decreases when using the Carhart of FF5 models' alphas.

[Insert Tables A1.1 and A1.2 around here]

[24] See footnote 2.

Tables A2.1 and A2.2 replicate Table 2 using realized alphas estimated with the Carhart and FF5 models, respectively. Results are similar to those presented in the main body of the paper.

[Insert Tables A2.1 and A2.2 around here]

Tables A3.1 and A3.2 replicate Table 3 using the Carhart and FF5 models, respectively, to estimate the parameters used to create the low and high portfolios for the BAAB factor. Results are quite similar to those presented in the main body of the paper using the CAPM for the estimation of the parameters.

[Insert Tables A3.1 and A3.2 around here]

Finally, Table A4 augments Table 4 by adding the BAA and BAAB factors calculated with the Carhart and FF5 models' alphas.

[Insert Table A4 around here]

The BAA strategies calculated using the alphas from the Carhart and FF5 models are highly correlated with the one calculated using the CAPM's alphas (the correlation coefficients are 0.95 and 0.96, respectively). The BAAB factor constructed using the parameters estimated from the Carhart and FF5 models is also highly correlated with the BAAB factor constructed using parameters estimated from the CAPM (0.92 and 0.91, respectively).

B BAA and BAB strategies using data since 1927

In this Section I extend the time period of the data used to calculate the main results. The extended dataset starts in 1927, which means that the strategies estimated span from January 1932 to December 2015 (since we need 5 years of data to initialize them). Table B1 replicates Table 1. The BAA and BAB strategies still show high monthly Sharpe Ratios and significant abnormal returns when controlling for the CAPM and Carhart models. However, the performance metrics are slightly lower in Table B1 than those presented in Table 1 for both strategies, suggesting that these strategies were more profitable in the second half of the analyzed period.

[Insert Table B1 around here]

Tables B2 and B3 replicate Table 2 and Table 3, respectively, showing that the BAAB factor also improves over using the BAA and BAB separately when using the extended dataset.

[Insert Tables B2 and B3 around here]

C Betting against idiosyncratic and total volatility

In this Section I present the performance metrics of two more "betting against" strategies mentioned in the Introduction: (i) Betting Against Total Volatility (BATV) and (ii) Betting Against Idiosyncratic Volatility (BAIV). These strategies are constructed using the same weight for the long and short portfolios already used for the BAA, BAB, and BAAB factors. In the BATV strategy assets are sorted according to their realized variance during the 60 months prior to the sorting date. In the BAIV strategy assets are sorted according to their residual variance calculated from the CAPM regression using data from 60 months prior to the sorting date. Results are presented in Table C1 below.

[Insert Table C1 around here]

Finally, Table C2 presents the correlation coefficients between the BAA, BAB, BAAB, BATV, and BAIV strategies.

[Insert Table C2 around here]

References

[1] Ahn S.C., A. Horenstein, and N. Wang, 2017, Beta matrix and common factors in stock returns, forthcoming *Journal of Financial and Quantitative Analysis*.

[2] Baker M., B. Bradley, and J. Wurgler, 2011, Benchmarks as limits to arbitrage: Understanding the low-volatility anomaly, *Financial Analysts Journal* 67, 40-54.

[3] Barber B., X. Huang, and T. Odean, 2016, Which factors matter to investors? Evidence from mutual fund flows, *Review of Financial Studies* 29, 2600-2642.

[4] Barillas F. and J. Shanken, 2017, Which alpha?, *Review of Financial Studies* 30, 1316-1338.

[5] BLACK F., 1972, Capital market equilibrium with restricted borrowing, *Journal of Business* 45, 444-455.

[6] BUFFA A., D. VAYANOS, AND P. WOOLLEY, 2017, Asset management contracts and equilibrium prices, *working paper.*

[7] BURNSIDE C., 2016, Identification and inference in linear stochastic discount factor models with excess returns, *Journal of Financial Econometrics* 14, 295–330.

[8] CARHART M., 1997, On Persistence in Mutual Fund Performance, *Journal of Finance* 52, 57-82.

[9] CRAGG J. AND S. DONALD, 1997, Inferring the rank of a matrix, *Journal of Econometrics* 76, 223–250.

[10] CHRISTOFFERSEN S. AND M. SIMUTIN, 2017, On the demand for high-beta stocks: Evidence from mutual funds, forthcoming *Review of Financial Studies.*

[11] DASGUPTA A., A. PRAT, AND M. VERARDO, 2011, Institutional trade persistence and long-term equity returns. *Journal of Finance* 66, 635–653.

[12] FAMA E. AND K. FRENCH, 1992, The cross-section of expected stock returns, *Journal of Finance* 47, 427-465.

[13] FAMA E. AND K. FRENCH, 2008, Dissecting anomalies, *Journal of Finance* 63, 1653-1678.

[14] FAMA E. AND K. FRENCH, 2015, A five-factor asset pricing model, *Journal of Financial Economics* 116, 1-22.

[15] FRAZZINI A. AND L. H. PEDERSEN, 2014, Betting against beta, *Journal of Financial Economics*, 111, 1-25.

[16] HARVEY C., Y. LIU, AND H. ZHU, 2016, ... and the cross-section of expected returns, *Review of Financial Studies* 29, 5-68.

[17] KARCESKI J., 2002, Returns-chasing behavior, mutual funds, and beta's death, *Journal of Financial and Quantitative Analysis* 37, 559-594.

[18] LEWELLEN, J., S. NAGEL, AND J. SHANKEN, 2010, A Skeptical Appraisal of Asset Pricing Tests, *Journal of Financial Economics* 96, 175–194.

[19] LINTNER J., 1965, The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets, *Review of Economic and Statistics* 47, 13-37.

[20] LO A., 2002, The statistics of Sharpe Ratios, *Financial Analysts Journal* 58, 36-52.

[21] MOSSIN J., 1966, Equilibrium in a capital asset market, *Econometrica* 35, 768-783.

[22] Novy-Marx R. and M. Velikov, 2016, A taxonomy of anomalies and their trading costs, *Review of Financial Studies* 29, 104-147.

[23] Ross S., 1976, The arbitrage theory of capital asset pricing, *Journal of Economic Theory* 13, 341–360.

[24] Sias R., 2004, Institutional herding, *Review of Financial Studies* 17, 165–206.

[25] Sharpe W., 1964, Capital asset prices: A theory of market equilibrium under conditions of risk, *Journal of Finance* 19, 425-442.

[26] Treynor J. Toward a theory of market value and risky assets. Unpublished manuscript. Final version in Asset Pricing and Portfolio Performance, 1999, Robert A. Korajczyk ed., London: Risk Books, 15-22.