Expected Stock Returns and the Correlation Risk Premium*

Adrian Buss **Lorenzo Schönleber** **Grigory Vilkov**

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In general equilibrium settings with stochastic variance and correlation, the market return is driven by shocks to consumption, market variance and average correlation between stocks, and hence the equity risk premium is composed of compensations for variance, correlation and consumption risks. Model insights inspires a new empirical methodology of market return prediction, such that estimating variance and correlation betas from the joint dynamics of option-implied variables and index returns, we find significant *out-of-sample* R^2's of 10.4% and 7.0% for 3- and 12-months forecast horizons, respectively. While the predictability of the variance risk premium is strongest at the intermediate (quarterly) horizon, the correlation risk premium dominates at longer horizons. In line with a risk-based explanation for the existence of a correlation risk premium, we document that expected correlation predicts future diversification risks.

Keywords: correlation risk premium, out-of-sample return predictability, option-implied information, trading strategy, diversification, factor risk

JEL: G11, G12, G13, G17

*Adrian Buss is affiliated with INSEAD, France, adrian.buss@insead.edu; Lorenzo Schönleber and Grigory Vilkov are affiliated with Frankfurt School of Finance & Management, Germany, l.schoenleber@fs.de and vilkov@vilkov.net. We received helpful comments and suggestions from Turan Bali, Bruno Biais, Agostino Capponi (discussant), Peter Carr, Bernard Dumas, Thierry Foucault, Amit Goyal (discussant), Peter Kolm, Hugues Langlois, Philippe Müller, Viktor Todorov, and Andrea Vedolin. We also thank participants of the China International Conference in Finance 2017, the 2nd Annual Eastern Conference on Mathematical Finance (ECMF) 2017, and seminar participants at Erasmus University (Rotterdam), Frankfurt School of Finance & Management, HEC Paris, INSEAD, UNSW Sydney, University of Sydney, University of Technology Sydney, University of Balearic Islands, and University of Maastricht for useful comments; the remaining bugs and errors are ours.

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It is long recognized that the variance of the aggregate market return is stochastic, and that investors are ready to pay a premium to hedge against changes in variance—the variance risk premium.[1] Market index variance is is affected by individual variances and correlations between individual stocks, and the correlations are also time-varying. Moreover, by pricing index options using relatively higher expected variance than for individual options, investors are willing to pay a correlation risk premium to hedge against changes in correlation. [2] Empirically, both—aggregate index variance and average correlation—are co-moving negatively with the market return, that is, they tend to increase during bear markets, and, hence, should contribute to the equity risk premium.[3] While the relation between the variance risk premium and the equity risk premium has been studied extensively (see, among others, Bollerslev, Tauchen, and Zhou (2009), Carr and Wu (2016), and Bandi and Renò (2016)), the theoretical and empirical evidence for the correlation risk premium is scarce, with the theoretical model by Buraschi, Trojani, and Vedolin (2014) as a notable exception.

The focus of this paper is on *correlation risk*. In particular, we address the following questions: Are correlation and variance risks jointly priced in a theoretical model? Does the correlation risk premium provide non-redundant information, relative to the variance risk premium, in determining the market risk premium? Can the variance and correlation risk premiums predict the market excess return, especially *out-of-sample*? What is the economics behind the correlation risk premium?

We make four major contributions. First, using as motivation a general equilibrium model with stochastic variance and correlation, we decompose the equity risk premium into three components: (i) the market variance risk premium; (ii) the stock market correlation risk premium; and (iii) the standard risk premium due to consumption volatility. This representation gives us a theoretically founded prediction equation for the market excess return.

[1]See Carr and Wu (2009) and Bollerslev, Tauchen, and Zhou (2009) for evidence on the variance risk premium and Todorov (2009), Bollerslev and Todorov (2011) and Todorov and Tauchen (2011) for evidence on its composition.

[2]See Driessen, Maenhout, and Vilkov (2009), Buraschi, Kosowski, and Trojani (2014), Mueller, Stathopoulos, and Vedolin (2017), and Krishnan, Petkova, and Ritchken (2009).

[3]Christie (1982), Roll (1988), Bekaert and Wu (2000) and Longin and Solnik (2001) document a negative correlation between the market return and index variance (equal to -0.77 in our sample). For our sample period, we document a correlation of -0.61 between the market return and expected correlation.

Second, we propose a novel methodology for estimating the prediction equation parameters, where instead of running a standard regression of excess returns for a given horizon on past variance and correlation risk premiums, we estimate the variance and correlation betas directly from the market dynamics equation by regressing high-frequency returns on high-frequency shocks to variance and correlations. Moreover, we show how one can estimate these betas (under the physical measure) combining realized returns and increments of risk-neutral quantities, that is, implied variance and implied correlation. Compared to the standard predictive regression, the new methodology provides far more stable beta estimates in the presence of outliers, and it uses the most up-to-date information for estimation instead of having a lag equal to the return horizon. The proposed methodology is general in a sense that it can easily be adapted for the use with other predictors.

Third, we show, empirically, that the variance and correlation risk premium predict the market excess return *out-of-sample*, with out-of-sample R^2s of up to 10% at a quarterly, and up to 8% at an annual horizon. Most of this out-of-sample predictability can be attributed to our novel beta estimation methodology. While the predictability by the variance risk premium peaks at the quarterly horizon and declines after that, the predictive power of the correlation risk premium is strongest for longer horizons—up to one year. Thus, in line with our theoretical predictions, we provide strong empirical evidence for the existence of *two components in the equity premium* that can be estimated in an *ex ante* fashion using options data and contain non-redundant information. We demonstrate that these predictability results imply highly significant economic benefits for a representative investor.

Fourth, we empirically study the economic channels through which a correlation risk premium might arise. In particular, if the correlation risk commands a risk premium, it should be linked to the future investment opportunities in a sense of Merton (1973)'s Intertemporal CAPM (ICAPM); moreover, if the variance and correlation risk premiums are not redundant, the aggregate variance and correlation risks should be related to the future risks in a different way. We show that this risk-based foundation of the correlation and variance risk pricing is supported by the data. That is, expected correlation has a strong predictive power for future diversification benefits for horizons of up to one year, measured by the average future correlation or by the non-diversifiable portfolio risk. Similar to the market return predictability results, variance has a shorter predictability horizon for future risks. Note that we concentrate on return predictability by observable variables, and stay agnostic about the underlying economic forces creating stochastic consumption variance and dividend correlation in the first place; as a prominent example of a model that generates stochastic correlation from the structure of the economy we can refer to Buraschi, Trojani, and Vedolin (2014), who link correlation risk premium to disagreement.

Our paper is related to several strands of the literature. First, the work on models with priced market variance and correlation risks, analyzing variance and correlation risk premiums and its sources. Bollerslev, Tauchen, and Zhou (2009) introduce a model with priced variance risk using insights from the long-run risk literature. Buraschi, Trojani, and Vedolin (2014) propose a general equilibrium model with difference in beliefs, where higher uncertainty about future dividends leads agents to expect that stocks behave more like the market in the future. These beliefs increase the expected correlation under the pricing probability measure, and, hence, generates a correlation risk premium. Driessen, Maenhout, and Vilkov (2009) suggest a risk-based explanation of the correlation risk premium with the average correlation serving as a state variable that has predictive power for future market risks and, thus, is priced. However, they do not pin down the character of risks predicted by the correlation. Later, Buraschi, Kosowski, and Trojani (2014) empirically relate correlation risk to a "no-place-to-hide" state variable. Mueller, Stathopoulos, and Vedolin (2017) investigate the correlation risk premium using foreign exchange markets.

We contribute to this literature by developing a general equilibrium model, in which the stochastic variance of aggregate consumption is driven by the stochastic variance of each dividend tree and the stochastic correlation among them. Effectively, we are extending the model of Bollerslev, Tauchen, and Zhou (2009) to multiple dividend trees with stochastic correlation.

Such a setup leads to a version of two-component variance of the aggregate consumption process, similar to the model with short- and long-run volatility components in Zhou and Zhu (2015), where each component bears a risk premium, and the model provides a significant improvement in fitting the empirical data over models with a single variance component. We also solve the model in closed-form for correlation driven by a square-root process, and overall our model is similar in style to the model with two-component variance. Our major difference is that we are able to give an easy interpretation to the long-run component, linking it theoretically and empirically to a well-observed average correlation between stocks. In addition we show that variance and correlation risk premiums are not redundant, even though empirically the correlation risk premium is a part of the market variance risk premium. As state variables expected variance and correlation predict future risks differently, i.e., different types of risks, and at different horizons. We also contribute to this literature by studying the sources of the correlation risk premium, concentrating on risk-based and disagreement-based explanations. Our results suggest that the correlation risk premium should not serve as a proxy for uncertainty or disagreement because it is negatively related to uncertainty (measured by the economic policy uncertainty index) and disagreement (measured by the aggregate difference in beliefs proxy), contrary to the theoretical predictions. In contrast, we show that the risk-based explanation can rationalize the observed patters of return predictability, because expected correlation predicts future diversification risks.

Second, though we concentrate on option-implied predictors, we contribute to the literature on the predictability (especially, the out-of-sample one) of the aggregate market return. In an empirical application Bollerslev, Tauchen, and Zhou (2009) show for the U.S., and Bollerslev, Marrone, Xu, and Zhou (2014) repeat the analysis in international settings, that the variance risk premium is a strong and robust predictor of aggregate market returns for up to one quarter ahead. The evidence on market return predictability using the correlation risk premium is scarce. That is, while several existing studies document return predictability by correlations itself for a horizon of up to one year (for example, Driessen, Maenhout, and Vilkov (2005, 2012) and Faria, Kosowski, and Wang (2016) using implied correlations and Pollet and Wilson (2010) using the realized correlations), only Cosemans (2011) finds some in-sample return predictability by the correlation risk premium.

In addition to developing an equilibrium model that establishes a link between the market expected return and correlation risk premium, we confirm this relation empirically by testing the market return predictability—in-sample and out-of-sample for horizons of up to one year. Particularly, while borrowing the performance criteria to evaluate the out-of-sample return forecasts from influential studies like Goyal and Welch (2008), Campbell and Thompson (2008), and others, we develop a fundamentally new methodology for estimating the betas with respect to the option-based predictors in the forecasting equation. This new methodology is based on estimating "contemporaneous betas" from the joint dynamics of market returns and option-implied variables, and it substantially improves the out-of-sample predictability of both option-implied variables, compared to the traditionally used regressions of long-term returns on past predictors.

Last but not least, we contribute to a growing literature on using option-implied information in forecasting and asset pricing—an overwhelming overview of the recent research can be found in Christoffersen, Jacobs, and Chang (2013). The papers in this are can be roughly split into cross-sectional studies, where panel stock data are used, and into time-series studies, where aggregate quantities are predicted. We list just a few related papers: Bali and Zhou (2016) use the variance risk premium in the cross-sectional context to show how exposure to uncertainty is compensated in individual stocks. Bali and Hovakimian (2009), Xing, Zhang, and Zhao (2010), Cremers and Weinbaum (2010), Rehman and Vilkov (2010), Stilger, Kostakis, and Poon (2017) use different proxies of variance risk premium and forward-looking skewness to link them to the cross-section of future stock returns, and DeMiguel, Plyakha, Uppal, and Vilkov (2013) apply their results in portfolio selection exercise. Chang, Christoffersen, Jacobs, and Vainberg (2012) and Buss and Vilkov (2012) use option-implied correlations to measure market risk in the cross-section of stock returns. Kostakis, Panigirtzoglou, and Skiadopoulos (2011) use option-implied distributions to improve market-timing of the index investment.

We contribute to this literature by providing theoretical foundation and new empirical support for predictability of market returns using variance and correlation risk premiums; moreover, we develop a principally new methodology for estimating exposure to option-implied variables, and it can easily be extended to other time-series and cross-sectional studies. Judging by our experience, it can significantly boost the predictive qualities of option-based variables, and extending our results to the cross-section of stock returns is within our immediate agenda.

The remainder of the paper is organized as follows: Section I contains the derivations of the pricing equation linking the equity risk premium to the variance and correlation risk premiums, as well as a discussion of our novel estimation approach for contemporaneous variance and correlation betas. Section II.A discusses data preparation procedures. In Section III, we study market return predictability—in-sample and out-of-sample. Section IV analyzes the potential economic channels behind the correlation risk premium. Section V contains a number of robustness tests, and Section VI concludes. Appendix contains theoretical derivations, and Internet Appendix contains tables for robustness tests.

I. Equity Risk Premium Decomposition

In this section we introduce a model of a general-equilibrium economy that produces priced variance and correlation risks, and allows for a decomposition of the aggregate market index process into variance and correlation shocks. We use this decomposition later as motivation for developing a new estimation methodology for predicting market excess returns. Subsection I.A presents the setup and selective results from the model solution, subsection I.B links the equity risk premium to the risk premiums on market variance and average stock correlation, and subsection I.C presents our new estimation strategy for betas with respect to variance and correlation risks.

A. *Economic Framework*

Aggregate consumption is produced by a large number of individual Lucas (1978) trees (denoted by $i = 1, \ldots, I$) with fixed proportions w_i. In particular, we assume that its dynamics are described by the following Ito process with stochastic variance:

$$\begin{cases} \frac{dC_t}{C_t} & = \mu_c dt + \delta_c \sqrt{V_t} dB_{c,t} \\ dV_t & = \kappa_1(\bar{V} - V_t)dt + \sigma_1 \sqrt{V_t} dB_{V,t} + \sigma_\rho d\rho_t [= \kappa_2(\bar{\rho} - \rho_t)dt + \sigma_2 \nu(\rho_t) dB_{\rho,t}]. \end{cases} \tag{1}$$

The stochastic variance on the aggregate level arises from two *distinct* features of the underlying dividend trees. First, dividend trees are driven by a systematic source of risk $B_{c,t}$ and an idiosyncratic one $B_{i,t}$ with stochastic variance $V_{i,t}$, following a square-root process:

$$\begin{cases} \frac{dD_i}{D_i} & = \mu_{D,i} dt + \sigma_{D,i} \sqrt{V_{i,t}} dB_{i,t} + \sigma_{DC,i} \sqrt{V_t} dB_{c,t} \\ dV_{i,t} & = \kappa_{1,i}(\bar{V}_i - V_{i,t})dt + \varsigma_i \sqrt{V_{i,t}} dB_{V_i,t}, \end{cases} \tag{2}$$

with a constant "volatility of volatility" ς_i. Second, the pairwise correlations between dividends are stochastic and driven by a single state variable (following the approach in Driessen, Maenhout, and Vilkov (2009)). That is, the instantaneous correlation between trees i and j, $i \neq j$ is modeled as

$$\frac{dD_{i,t} \times dD_{j,t}}{\sqrt{(dD_{i,t})^2 \times (dD_{j,t})^2}} = \rho_{ij,t} dt = \rho_t dt. \tag{3}$$

In particular, Driessen, Maenhout, and Vilkov (2005) show that a similar fixed-weight aggregation of individual stocks with stochastic variance and stochastic correlation leads to an index with stochastic variance, which is driven by the weighted average shock to individual variances and by correlation state variable.

The correlation state variable ρ_t follows a mean-reverting process with long-run mean $\overline{\rho}$, speed of mean-reversion κ_2 and diffusion scaling parameter σ_ρ:

$$d\rho_t = \kappa_2 \left(\overline{\rho} - \rho_t\right) dt + \sigma_2 \nu(\rho_t) dB_{\rho,t}, \tag{4}$$

and it shows up with some scaling parameter in the aggregate variance dynamics (1) above. To obtain a closed-form solution we assume a square-root process for correlation with $\nu(\rho_t) = \sqrt{\rho_t}$.[4] With such a correlation process we are very close mathematically to the two-component variance model of Zhou and Zhu (2015), though instead of studying an effect of a latent variance process, we concentrate on the observable correlation.

We assume that there exists a representative investor with continuous-time, recursive preferences defined by Duffie and Epstein (1992b), with the relative risk aversion $\gamma > 0$, intertemporal elasticity of substitution $\psi > 0$, and rate of time preference β; the objective of the investor is to choose consumption process to maximize utility lifetime utility.[5] Solving for the equilibrium, we arrive at an expression for the pricing kernel with the risk premiums λ_1, λ_2, and λ_3 for all priced sources of risk in our economy—consumption, aggregate consumption variance, and correlation between dividends, respectively:

$$\frac{d\pi_t}{\pi_t} = -r_f dt - \lambda_1 dB_{c,t} - \lambda_2 dB_{V,t} - \lambda_3 dB_{\rho,t}, \tag{5}$$

where

$$\begin{cases} \lambda_1 &= \gamma \delta_c \sqrt{V_t} \\ \lambda_2 &= -\frac{1-\gamma\psi}{1-\gamma} A_1 \sigma_1 \sqrt{V_t} \\ \lambda_3 &= -\frac{1-\gamma\psi}{1-\gamma} \left(A_1 \sigma_\rho + A_2 \sigma_2\right) \sqrt{\rho_t}, \end{cases} \tag{6}$$

with $A_j, j = 1, 2$ being the coefficients by the state variables V_t and ρ_t in the optimal value function.

Having verified that in the specified economy both variance and correlation are priced, we derive now the equations that will motivate our empirical analysis later on. First, solving for the aggregate market process (i.e., wealth process), we obtain:

$$\frac{dW_t}{W_t} = \zeta_W dt + \delta_c \sqrt{V_t} dB_{c,t} - A_{1a} dV_t - A_{2a} d\rho_t, \tag{7}$$

[4]Thus, the correlation could end up being above 1, and in calibrations one needs to choose parameters such that the correlation stays effectively bounded.

[5]The details of the solution are collected in Appendix.

where $A_{ia} = \frac{1-\psi}{1-\gamma} A_i, i = 1, 2$ and the term ζ_W denotes a "partial" drift.[6] The aggregate market index is driven by (standard) consumption uncertainty, as well as by consumption variance and dividend correlation shocks.

Second, we are especially interested in the processes for market (index) variance and correlation between stocks, because unlike the latent consumption variance and dividend correlations, they are observable and can be estimated from either the historical data (i.e., under the true probability measure) or the option prices (i.e., under the risk-neutral measure).

The aggregate market variance is driven solely by the consumption variance and dividend correlation:

$$dV_{W,t} = (\delta_c^2 + A_{1a}^2 \sigma_1^2) dV_t + (A_{1a}\bar{\sigma}_\rho + A_{2a}\sigma_2)^2 d\rho_t. \tag{8}$$

Pricing individual dividend claims (i.e., stocks S_i with dividends $D_i,\ i = 1 \ldots N$), and computing the correlation process between them,[7] we obtain

$$d\rho_{S,t} = \zeta_{\rho_S} dt + \frac{V_S - Cov_S}{V_S^2} \left[(\sigma_{DC}^2 + A_{1m}^2 \sigma_1^2) dV_t + (A_{1m}\bar{\sigma}_\rho + A_{2m}\sigma_2)^2 d\rho_t \right] - \frac{1}{V_S^2} \sigma_{D,i}^2 dV_{i,t}, \tag{9}$$

where ζ_{ρ_S} is the partial drift, $A_{jm}, j = 1, 2$ are the coefficients in the equilibrium price-dividend ratio, V_S is the individual dividend claim variance, and Cov_S is the covariance between stocks—the expressions for the second moments are provided in the Appendix. Note that the *average correlation between stocks* is driven by the same sources of risk as the market variance—consumption variance and dividend correlation,—and by an idiosyncratic variance part. Only the first two—systematic sources of risk—are priced in the model.

B. The Equity Risk Premium and its Link to Correlation Risk

The equity risk premium for the aggregate market is determined by the covariance between the pricing kernel (5) and the market index (wealth) process. In particular, using the formu-

[6] We call it "partial," because both the dV_t and $d\rho_t$ contain deterministic terms. One can easily write the wealth process in terms of original sources of risk $dB_{V,t}$ and $dB_{\rho,t}$, but the given representation is more convenient for our interpretation.

[7] To obtain an "average" correlation between stocks, we just compute correlation between two claims on dividends with the same "average" parameter values. When dividend parameters are the same across all stocks, average correlation is equal to a pairwise correlation.

lation of the market process (7), we can express the market risk premium as a sum of three components (compare to two in Bollerslev, Tauchen, and Zhou (2009)):

$$E^{\mathbb{P}}\left[\frac{dW}{W}\right] - r_{f,t}dt = \lambda_1 \delta_c \sqrt{V_t}dt - A_{1a}(E^{\mathbb{P}}[dV_t] - E^{\mathbb{Q}}[dV_t]) - A_{2a}(E^{\mathbb{P}}[d\rho_t] - E^{\mathbb{Q}}[d\rho_t]), \quad (10)$$

The first component motivates the classic risk-return tradeoff relationship, whereas the second and the third components represent true compensation for aggregate consumption variance and dividend correlation risks. To be consistent with the recent literature and for ease of exposition, we define the variance (e.g., Carr and Wu (2009), Bollerslev, Tauchen, and Zhou (2009)) and correlation (e.g., Driessen, Maenhout, and Vilkov (2009)) risk premiums with the opposite sign, that is, as the expected process under $\mathbb{Q}$ measure minus the respective expectation under $\mathbb{P}$. Hence, the instantaneous market risk premium is given by

$$E^{\mathbb{P}}\left[\frac{dW}{W}\right] - r_{f,t}dt = \lambda_1 \delta_c \sqrt{V_t}dt + A_{1a}VRP_{C,t}dt + A_{2a}CRP_{C,t}dt. \quad (11)$$

Thus, knowing the variance and correlation risk premiums on the right, we would be able to predict the market excess return, however, aggregate *consumption* variance and dividend correlation risk premiums are not readily available from the data. To replace them with observable quantities, note that the risk premiums on *market variance* (8) and the *average stock correlation* (9) can be written as:

$$\begin{bmatrix} VRP \\ CRP \end{bmatrix} = \begin{bmatrix} (\delta_c^2 + A_{1a}^2\sigma_1^2) & (A_{1a}\bar{\sigma}_\rho + A_{2a}\sigma_2)^2 \\ \frac{V_S - Cov_S}{V_S^2}(\sigma_{DC}^2 + A_{1m}^2\sigma_1^2) & \frac{V_S - Cov_S}{V_S^2}(A_{1m}\bar{\sigma}_\rho + A_{2m}\sigma_2)^2 \end{bmatrix} \times \begin{bmatrix} VRP_C \\ CRP_C \end{bmatrix} \quad (12)$$

Idiosyncratic variance V_i is not priced, and hence does not enter the expression for CRP above. Now, because both market variance risk premium VRP and average stock correlation risk premium CRP are determined *exclusively* by the risk premiums for the aggregate consumption variance VRP_C and dividend correlation CRP_C, we can express two latter—latent—risk premiums in terms of the two observable premiums for the *market variance* and *stock (average) correlation* risks by solving the system (12) for latent variables. After substituting the solutions for VRP_C, CRP_C in (11) we can also write the equity risk premium as "instantaneous pricing equation":

$$E^{\mathbb{P}}\left[\frac{dW}{W}\right] - r_{f,t}dt = \lambda_1 \delta_c \sqrt{V_t}dt + A_{1z}VRP_t dt + A_{2z}CRP_t dt, \tag{13}$$

where A_{1z} and A_{2z} are the functions of a stock variance, covariance between average stocks, and other matrix elements in (12).[8]

Similar to Bollerslev, Tauchen, and Zhou (2009), who show that the variance and equity risk premiums share a common component due to stochastic vol-of-vol and thus provide theoretical foundation for using the variance risk premium to predict future market returns, we use both variance and correlation risk premiums for predicting excess market returns. To that end, we do not attempt to calibrate the model completely to identify the betas in the pricing equation (13), but instead develop a novel methodology to estimate the exposures from high-frequency observations of related variables.[9]

C. Estimation Framework

Substituting in (13) for the price of consumption risk λ_1 from the pricing kernel (5) and integrating over a desired period, yields the expected market excess return in the form of the following "finite horizon pricing equation":

$$E_t[r_{t+1}] - r_{f,t} = \gamma \delta_c^2 V_{t,t+1} + A_{1z}VRP_{t,t+1} + A_{2z}CRP_{t,t+1}. \tag{14}$$

The expected variance risk premium, $VRP_{t,t+1}$, and the expected correlation risk premium, $CRP_{t,t+1}$, can be estimated empirically. But, before one can use the pricing equation (14) to form a forecast for the market return, one first needs to estimate the coefficients (i.e., betas) for variance and correlation risks.

Traditionally, one would simply run a time-series regression as in equation (14), that is, regress realized market excess returns on lagged regressors using historical data. The estimated

[8]See Section VII.F in Appendix for details.

[9]We also calibrate the model to match a number macro and market indicators, and for some sensible parameter values we produce the equity premium of 4.5% to 5.5%, with the contribution of market VRP between 25% and 40% and the contribution of the CRP between 58% and 71%. Note that the model with the square-root process for the correlation is misspecified, and it should be interpreted as a qualitative exercise to obtain a decomposition of the equity risk premium.

betas could then, together with the variance and correlation risk premiums, be used to predict the future market excess return. This approach has been employed in a number of studies, but it relies heavily on *past information* and might therefore not lead to a strong out-of-sample performance. For example, Goyal and Welch (2008) demonstrate that many variables, which predict market excess returns in-sample, have a poor out-of-sample performance.

We propose an alternative approach that makes use of the fact that underlying the pricing equation (14) is the equation of aggregate wealth dynamics (7), in which we can carry out a procedure for substituting the market variance and stock pairwise correlation shocks dV_W and $d\rho_S$ for the consumption variance and dividend correlation shocks dV and $d\rho$, similar to the substitution of risk premiums in equation (13):

$$\frac{dW_t}{W_t} = \zeta'_W dt + \delta_c \sqrt{V_t} dB_{c,t} - A_{1z} dV_{W,t} - A_{2z} d\rho_{S,t} - A_{3z} dV_{i,t}, \tag{15}$$

where $A_{3z} = \frac{A_{2z}}{V_S^2}\sigma^2_{D,i}$, and the last term just compensates the additional idiosyncratic volatility term $dV_{i,t}$ introduced by $d\rho_{S,t}$ (as follows from (9)). Also note that the last term is not priced and hence does not affect the market risk premium.

Comparing the pricing equation (13) and dynamics equation (15), it turns out that the betas in pricing equation (14) essentially represent the integrated estimates of the diffusion coefficients in the dynamics dW/W, and, thus, can be obtained *directly* by regressing the return innovation, $dW/W - E[dW/W]$, on shocks to the index variance, dV_W, shocks to the pairwise correlation, $d\rho_S$, and shocks to the consumption component, dB_c.[10]

Under the actual measure, shocks to a predictor z are given by the difference between the realization and its conditional expectation: $z_{t+1} - E_t[z]$. Along these lines, Pyun (2016) uses high-frequency data to estimate contemporaneous variance betas, that is, he computes the exposure to innovations in daily realized variance. Unfortunately, using the same procedure to

[10]Note that in empirical implementation we will concentrate on out-of-sample return predictability, and concerned with potential overfitting we will include in the predictive regression at most two variables linked to variance and correlation risks. The consumption shock dB_c is not correlated with the other regressors, and its omission does not bias the estimated coefficients; however, omitting the "typical" idiosyncratic variance shock dV_i may lead to an omitted-variable bias depending on its correlation with the variance and correlation shocks. We do not expect it to be high, and neglect the effect of potential bias on the remaining betas.

obtain daily innovations in correlation is considerably more complicated, because one has to deal with a large number of stocks, so that data availability and micro-structural issues pose a problem. However, note that a change of measure—from the actual measure $\mathbb{P}$ to the risk-neutral measure $\mathbb{Q}$—only changes the drift of a process, but not the diffusion components (see, for example, (Karatzas and Shreve, 1991, page 190)):

$$\frac{dW_t^{\mathbb{Q}}}{W_t} = \zeta_W'' dt + \delta_c \sqrt{V_t} dB_{c,t}^{\mathbb{Q}} - A_{1z} dV_{W,t}^{\mathbb{Q}} - A_{2z} d\rho_{S,t}^{\mathbb{Q}} - A_{3z} dV_{i,t}^{\mathbb{Q}}, \tag{16}$$

where the actual-measure drift ζ_W' is adjusted by risk premiums to become ζ_W''. Thus, one can also estimate the slope coefficients—contemporaneous betas—using shocks to variables under either actual or the risk-neutral probability measure. Moreover, we are free to choose a non-matching probability measure for the dependent variable, because changing the measure affects only its drift (=mean), and hence only the intercept in the estimated regression.

Specifically for the independent variables under the risk-neutral measure, one can obtain, on each day, implied variances and correlations, which are the risk-neutral expected integrated variance and correlation until option maturity T:

$$IV(t,T) = E_t^{\mathbb{Q}} \left[\int_t^T V_W(s) ds \right], \qquad IC(t,T) = E_t^{\mathbb{Q}} \left[\int_t^T \rho_S(s) ds \right]. \tag{17}$$

Note that one can always decompose the implied variance, $IV(t,T)$, as follows

$$\begin{aligned} IV(t,T) &= E_t^{\mathbb{Q}} \left[E_{t+1}^{\mathbb{Q}} \left[\int_t^{t+1} V_W(s) ds + \int_{t+1}^T V_W(s) ds \right] \right] \\ &= E_t^{\mathbb{Q}} \left[\int_t^{t+1} V_W(s) ds \right] + E_t^{\mathbb{Q}} \left[IV(t+1,T) \right], \end{aligned} \tag{18}$$

so that its daily increments are given by

$$\begin{aligned} \Delta IV(t+1,T) &= IV(t+1,T) - IV(t,T) \\ &= IV(t+1,T) - E_t^{\mathbb{Q}} \left[IV(t+1,T) \right] - E_t^{\mathbb{Q}} \left[\int_t^{t+1} V_W(s) ds \right]. \end{aligned} \tag{19}$$

Similar computations for the implied correlation, $IC(t,T)$, imply that

$$\begin{aligned}\Delta IC(t+1,T) &= IC(t+1,T) - IC(t,T) \\ &= IC(t+1,T) - E_t^{\mathbb{Q}}\left[IC(t+1,T)\right] - E_t^{\mathbb{Q}}\left[\int_t^{t+1} \rho_S(s)ds\right].\end{aligned} \tag{20}$$

If the last term in equations (19) and (20)—expected integrated variance and correlation over a single day—is small, implied variance and correlation can be well approximated by a martingale. Accordingly, one can use the daily increments in implied variance and implied correlation as proxies for daily (or other short interval) shocks to variance and correlation.

Empirical evidence lends support to this approximation. For example, Filipović, Gourier, and Mancini (2016) find that a "martingale model provides relatively accurate forecasts for the one-day horizon" variance. Moreover, integrated expected variance and integrated expected correlation are highly persistent, with first-order autocorrelations in our data between 0.97 and 0.994 for variance and between 0.97 and 0.993 for correlations at various maturities. Moreover, average daily increments are statistically not different from zero.

Consequently, to obtain the contemporaneous betas for pricing equation (14), one can simply estimate the following discrete version of equation (16), based on the same-period (t to $t+1$) returns and the shocks to the risk-neutral quantities:

$$r_{t+1} - r_{f,t} = \alpha + \beta_{t,\Delta IV}\Delta IV(t+1,T) + \beta_{t,\Delta IC}\Delta IC(t+1,T) + \Xi_{t+1}, \tag{21}$$

where the error term Ξ_{t+1} captures the consumption and typical dividend idiosyncratic variance shocks.

Note that the betas, $\beta_{t,\Delta IV}$ and $\beta_{t,\Delta IC}$, estimated from the equation (21), need to be "normalized" before they can be used to form the return forecast (14). Specifically, one needs to adjust the betas for the difference in magnitudes of the regressors used for beta estimation and of the predictors in the pricing equation. A beta with respect to one of the implied variables can be decomposed into the correlation between the market excess return and the specific variable as well as the ratio of their volatilities. Consequently, one can simply adjust the variance beta by the ratio of the volatility of the right-hand side variable used in estimation (increments in implied variance) and the volatility of the predictor in the forecast equation (variance risk premium)[11]

$$
\begin{aligned}
\beta_{t,VRP} &= \mathrm{Cor}\left(r_{t+\tau}, VRP(t,t+\tau)\right) \times \frac{\mathrm{Vol}\left(r_{t+\tau}\right)}{\mathrm{Vol}\left(VRP(t,t+\tau)\right)} \\
&= \mathrm{Cor}\left(r_{t+\tau}, \Delta IV(t,t+\tau)\right) \times \frac{\mathrm{Vol}\left(r_{t+\tau}\right)}{\mathrm{Vol}\left(\Delta IV(t,t+\tau)\right)} \times \frac{\mathrm{Vol}\left(\Delta IV(t,t+\tau)\right)}{\mathrm{Vol}\left(VRP(t,t+\tau)\right)} \\
&= \beta_{t,\Delta IV} \times \frac{\mathrm{Vol}\left(\Delta IV(t,t+\tau)\right)}{\mathrm{Vol}\left(VRP(t,t+\tau)\right)}. \qquad (22)
\end{aligned}
$$

The transformation above uses the fact that correlation between the return process and shocks to the variance equals to the correlation between the return process and the variance risk premium, i.e., $\mathrm{Cor}\left(r_{t+\tau}, VRP(t,t+\tau)\right) = \mathrm{Cor}\left(r_{t+\tau}, \Delta IV(t,t+\tau)\right)$, and hence does not require extra adjustments. It can be observed from comparing the dynamics of the aggregate market return in equation (7) and the instantaneous market risk premium in equation (11): the difference in coefficients stemming from a given source of risk is just the scaling parameter λ, i.e., the unit risk premium, which is taken care of by the volatility adjustments above. Similar computations for the correlation risk premium yield

$$
\beta_{t,CRP} = \beta_{t,\Delta IC} \times \frac{\mathrm{Vol}\left(\Delta IC(t,t+\tau)\right)}{\mathrm{Vol}\left(CRP(t,t+\tau)\right)}. \qquad (23)
$$

II. Data and Preparation of Variables

To compute variance and correlation risk premiums, we rely on data for realized and implied variances for the market index and all its components, and on average realized and implied correlations among the index components. In Subsection II.A, we briefly introduce data sources and move to the estimation of variances and correlations in Subsection II.B. In Subsection II.C we also discuss the price of variance and correlation risk for various market index proxies as well as their constituents for various option maturities.

[11] Also, because the variance and correlation risk premium are defined as the risk-neutral quantity minus physical ones, and the expected excess return is the difference between the physical and risk-neutral measures, we need multiply the betas by -1.

A. Data Sources and Preparation

Our analysis focuses on three major U.S. indices, and their constituents, namely, the S&P500, the S&P100, and the DJ Industrial Average (DJ30) for a sample period from January 1996 to April 2016. For each index, we obtain its composition from Compustat and data on the constituents' daily returns and market capitalizations from CRSP.[12] We proxy for the index weights on each day using the constituents' relative market capitalization (for S&P500 and S&P100) or price (for DJ30) from the previous day.

For the option-based variables, we rely on the Surface File from OptionMetrics, selecting for each index and its constituents options with 30, 91, 182, 273, and 365 days to maturity and an (absolute) delta smaller or equal to 0.5.[13] While options data for the S&P500 and the S&P100 is available from January 1996, the data for the DJ30 starts only in October 1997. Typically, option data is available for about 98% of the stocks in the index. For example, for the S&P500, the median number of stocks for which option data is available is 491.

We also take into account a number of traditional predictors of the market return borrowed from Goyal and Welch (2008).[14] All these variables are used at monthly frequency.

B. Variances and Correlations

Option-implied variances (IV) are computed using simple variance swaps, as in Martin (2013, 2017), which capture the total quadratic variation due to diffusion and jump components—for each option maturity. For robustness, we also compute implied variances using log contracts, that is, model-free implied variance, as in Dumas (1995), Britten-Jones and Neuberger (2000), Bakshi, Kapadia, and Madan (2003), and others.[15] Realized variances (RV) are estimated as

[12] We merge the two datasets through the CCM Linking Table using GVKEY and IID to link to PERMNO, following the second best method from Dobelman, Kang, and Park (2014).

[13] Matching the historical data with options works through the historical CUSIP link provided by OptionMetrics. Particularly, while S&P500, S&P100, and DJ30 indices are directly used as underlying for options, PERMNO is used as the identifier for single stocks in our merged database.

[14] We are grateful to Amit Goyal for making the data available on his web-site www.hec.unil.ch/agoyal/.

[15] In earlier versions of the paper, Martin (2013) discussed the issue of estimating implied correlations, and suggested that implied correlations / correlation swaps should be estimated using simple variance swaps as opposed to model-free variances.

the sum of squared daily returns. The *ex ante* variance risk premium, $VRP(t, t+\Delta)$, for options with maturity Δ can then be computed as the implied variance at the end of day t minus the realized variance from $t - \Delta$ to t.

Consistent with our assumption that all pairwise correlations are driven by a single state variable, we constructed correlations as equicorrelations, that is, all pairwise correlations are set equal. This method yields a positive-definite covariance matrix, as long as the equicorrelation is non-negative,[16] which is always the case in our samples and, in general, holds for large baskets of stocks.

We identify the equicorrelations using the restriction that the variance of an index I has to be equal to the variance of the portfolio of its constituents

$$\sigma_I^2(t) = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i(t)w_j(t)\sigma_i(t)\sigma_j(t)\rho_{ij}(t),$$

which holds under both—objective and risk-neutral—measures. Particularly, given the variances of the index $\sigma_I^2(t)$ as well as its components $\sigma_i^2(t), i = 1 \ldots N$, and the index weights $w_i(t)$, the equicorrelation $\rho_{ij}(t) = \rho(t)$ is calculated as

$$\rho(t) = \frac{\sigma_I^2(t) - \sum\limits_{i=1}^{N} w_i(t)^2 \sigma_i^2(t)}{\sum\limits_{i=1}^{N} \sum_{j \neq i} w_i(t) w_j(t) \sigma_i(t) \sigma_j(t)}. \tag{24}$$

When using risk-neutral (implied) variances in equation (24), we arrive at the implied correlation (IC), whereas when using expected actual variances, we obtain the realized correlation (RC). The *ex ante* correlation risk premium, $CRP(t, t+\Delta)$, is then constructed as the difference between the implied correlation for options with maturity Δ observed at the end of day t and the corresponding realized correlation from $t - \Delta$ to t.

[16]See Proposition 1 in Appendix B of Driessen, Maenhout, and Vilkov (2012).

C. The Price of Variance and Correlation Risk

Tables I and II provide summary statistics for the variance risk premium of the three indices as well as their constituents for various option maturities. For an easier comparison across maturities, all quantities are annualized. Focusing on the S&P500 and the average variance risk premium reported in Table I, we can see that the variance risk premium for individual stocks is typically not significantly different from zero. With the exception of a maturity of 30 days, all point estimate are actually negative, that is, the realized variance is, on average, higher than the implied variance for individual stocks. In contrast, the variance risk premium for the S&P500 itself is always positive, and statistically significant.

Note, however, that, as shown in Table II, variance risk premiums for individual stocks in the S&P500 demonstrate a lot of heterogeneity. That is, while for a majority of the stocks we fail to reject the null hypothesis of an insignificant variance risk premium, there is still a sizeable fraction of stocks for which we can either reject the null of a positive or a negative variance risk premium.

The results shown in Table III demonstrate that the correlation risk premium for the S&P500 is positive for all maturities, that is, the implied correlation is always higher than the realized one. Particularly, the correlation risk premium is significant at all conventional levels and is monotonically increasing in option maturity. Focusing on the two components of the correlation risk premium, it is apparent that the increase in the correlation risk premium with option maturity is exclusively due to the increase of the implied correlation with maturity.

In summary, similar to Driessen, Maenhout, and Vilkov (2005), we find that index variance is priced predominantly due to a priced correlation component, though the dynamics of the individual variance risk premiums should not be neglected. Hence, both—correlation and index variance risk premiums—potentially *contain non-redundant information.*

The results for the other two indices—the S&P100 and the DJ30—confirm these findings. This is not very surprising because all considered variables (implied and realized correlations, as well as implied and realized variances) tend to be strongly correlated across indices, with the average correlation being about 0.97. Qualitatively, the magnitude and the statistical significance of the variance risk premium as well the correlation risk premium decrease with the number of index constituents, that is, are highest for the DJ30. In what follows, we concentrate on the S&P500, and provide results for the S&P100 and the DJ30 for completeness.

III. Return Predictability

We now proceed with testing market return predictability empirically—in-sample in Subsection III.A, and then out-of-sample in Subsection III.B, using the novel estimation strategy of variance and correlation betas developed in Section I.C and comparing its performance to the traditional prediction methods.

A. In-sample Tests

In a first step, we analyze the predictability of the market excess return *in-sample*, when using the variance and correlation risk premiums as regressors. Specifically, we run the following simple predictive regression

$$r_{s \to s+\tau_r} = a + b\,VRP(s, s+\tau_r) + c\,CRP(s, s+\tau_r) + \epsilon,$$

where $r_{s \to s+\tau_r}$ denotes the compounded market excess return from date s to $s + \tau_r$. We use returns from the end of each month in our sample period and Newey-West standard errors to correct for auto-correlation introduced by overlapping data.

The results are reported in Table IV—for regressions with a single explanatory variable as well as for the joint regression. When using the variance risk premium as the sole explanatory variable, it is highly statistically significant for horizons of up to one quarter, with a maximum (adjusted) R^2 of 6.90%. However, for longer horizons, the variance risk premium has no explanatory power and the coefficient even turns negative, that is, a high variance risk premium at time t would predict a low future market excess return—contrary to theory. These results are consistent with the findings in Bollerslev, Tauchen, and Zhou (2009) and Bollerslev, Marrone, Xu, and Zhou (2014), who demonstrate that the variance risk premium is able to predict market excess returns for a horizon of up to 3 months.

The correlation risk premium, when used alone, is statistically significant for horizons of up to 273 days. Its explanatory power is quite high even for long horizons of up to one year, and peaks at a horizon of 273 days, with an R^2 of 9.87%. These findings are comparable to Cosemans (2011) who uses the correlation risk premium in in-sample market return predictability tests. Interestingly, a vast majority of existing studies (for example, Driessen, Maenhout, and Vilkov (2005, 2012) and Faria, Kosowski, and Wang (2016)) documents return predictability for longer horizons of up to one year by implied correlation, and not by the correlation risk premium.

In joint regressions, the variance risk premium dominates at a short horizon of one month. For longer horizons the coefficient becomes negative again. In contrast, the correlation risk premium is still highly significant for longer horizons, indicating that there exist two components that provide non-redundant information. While the results for the variance risk premium are essentially same for all indices, the significance of the correlation risk premium is a bit stronger for the S&P100, but a bit weaker for the DJ30. The predictors survive a number of standard controls (for example, from Goyal and Welch (2008)), which we will discuss in more detail in Section V.

B. Out-of-sample Tests

While many variables have been shown to predict market returns in-sample, there is hardly any evidence for *out-of-sample predictability*, as shown convincingly by Goyal and Welch (2008). Accordingly, we now concentrate on the out-of-sample performance of the variance and correlation risk premium. We are particularly interested whether the two risk premium components provide different information and work at different horizons.

For our out-of-sample analysis, we deliberately deviate from the traditional approach of running, at each date, a time-series regression of past market excess returns on lagged regressors, whose coefficients are then used to form the out-of-sample forecast. Instead, we rely on the estimation strategy described in Section I.C.

That is, in the first step, we estimate, at the end of each month, the contemporaneous betas of the market excess return with respect to innovations in implied variance and correlation from equation (21). Specifically, we regress daily market excess returns on daily increments of implied variance and/or daily increments of implied correlation for options with a given maturity—using data from the past year. This results in initial variance and correlation betas $\beta_{t,\Delta IV}$ and $\beta_{t,\Delta IC}$. We then compute normalized betas $\beta_{t,VRP}$ and $\beta_{t,CRP}$ for the variance and correlation risk premium, as in equations (22) and (23) using the appropriate scaling factor estimated from the same backward window. In the second step, we then form the out-of-sample prediction for the market excess return, $\hat{r}_{t\rightarrow t+\tau_r}$, for horizon τ_r by combining the normalized betas with the time-t variance and correlation risk premiums

$$\hat{r}_{t\rightarrow t+\tau_r} = \beta_{t,VRP}\, VRP(t, t+\tau_r) + \beta_{t,CRP}\, CRP(t, t+\tau_r) \tag{25}$$

where $\beta_{t,VRP}$ and $\beta_{t,CRP}$ denote the normalized exposures of the market excess return with respect to innovations in implied variance and correlation, and $VRP(t, t+\tau_r)$ and $CRP(t, t+\tau_r)$ denote the current (date t) variance and correlation risk premium, respectively. Consistent with the theoretical prediction (14), we use implied variance and correlation as well the variance and correlation risk premium from options with a maturity matching the forecast horizon.

We consider four different forecast models, denoting the predicted returns by $\hat{r}_{j,t,\tau_r}, j \in \{1, ..., 4\}$. The first model simply uses the historical mean of the market excess return and serves as the natural benchmark. The second and third model use our out-of-sample methodology, but rely only on the variance risk premium ($j = 2$) or only on the correlation risk premium ($j = 3$) in forecasting the market excess return. Finally, the last model ($j = 4$) combines the variance and correlation risk premium to forecast market excess returns. For each model j, each point in time t, and each horizon τ_r, we define the forecast error as the difference between the predicted and the realized market excess return $e_{j,t,\tau_r} \equiv \hat{r}_{j,t,\tau_r} - r_{t,t+\tau_r}$. For ease of exposition, let $\hat{r}_{j,\tau_r}$ denote the vector of predicted returns for horizon τ_r, and e_{j,τ_r} denote the vector of rolling out-of-sample forecast errors—for model j.

Traditionally, one evaluates the time-series of out-of-sample forecast errors by a loss function that is either an economically meaningful criterion, such as utility or profits (for example, Leitch and Tanner (1991), West, Edison, and Cho (1993), Della Corte, Sarno, and Tsiakas (2009)), or using some statistical criterion (for instance, Diebold and Mariano (1995), McCracken (2007)). These approaches have recently been unified and extended by Giacomini and White (2006), who developed out-of-sample tests to compare the predictive ability of competing forecasts, given a general loss function under conditions of possibly mis-specified models.

In the following, we rely on three criteria. First, the out-of-sample R^2_{j,τ_r} relative to the forecasts from the (benchmark) historical average return model ($j = 1$)

$$R^2_{j,\tau_r} = 1 - \frac{MSE_{j,\tau_r}}{MSE_{1,\tau_r}}, \text{ with } MSE_{j,\tau_r} = \frac{1}{N}\left(e^{\top}_{j,\tau_r} \times e_{j,\tau_r}\right),$$

where N denotes the number of prediction errors. Second, the average square-error loss δ_{j,τ_r}, again defined relative to the prediction from the benchmark model

$$\delta_{j,\tau_r} = MSE_{j,\tau_r} - MSE_{1,\tau_r},$$

which is one of the loss functions underlying the Diebold-Mariano tests. Third, to measure the true economic benefit of a better return forecast, we compute the gain in the certainty equivalent return of a mean-variance investor (similar to Campbell and Thompson (2008)). Specifically, at the end of each month t, we derive, for each model and forecast horizon, the optimal portfolio consisting of the market portfolio and a risk-free investment for a myopic mean-variance investor with horizon τ_r and a risk aversion of 1.[17] Using the resulting time-series of realized portfolio returns $r^{MV}_{j,\tau}$, we compute the mean-variance certainty equivalent,

[17]The optimal weight in the market is given by $w_{t,\tau,j} = \frac{\hat{r}_{j,t,\tau_r}}{\sigma^2}$, where σ^2 denotes the one-year historical variance (same for all models). Following Campbell and Thompson (2008), we restrict the optimal weights to be in $[0, 1.5]$ range.

CE_{j,τ_r}, as well as the gain in the certainty equivalent return relative to the benchmark model

$$\Delta CE_j, \tau_r = CE_{j,\tau_r} - CE_{1,\tau_r}, \text{ where } CE_{j,\tau_r} = E[r^{MV}_{j,\tau_r}] - \frac{\gamma}{2}\sigma^2(r^{MV}_{j,\tau_r}).$$

For robustness, we also compute a certainty equivalent improvement relative to model using CRP as a predictor, i.e., $CE_{j,\tau_r} - CE_{3,\tau_r}$.

Note that a particular model, $j > 1$, out-performs the benchmark model that uses simply the average historical return if R^2_{j,τ_r} is significantly different from zero, if δ_{j,τ_r} is significantly negative and if $\Delta CE_{j,\tau_r}$ is significantly positive. Due to the short sample period of less than 20 years, the asymptotic standard errors may not be very accurate, so that we resort to bootstrapping. Specifically, we use the moving-block bootstrap procedure by Künsch (1989),[18] randomly resampling with replacement from the time-series of a model's forecasts,[19] to construct a bootstrapped distribution for the performance measures.

The results based on the "contemporaneous betas approach" are collected in Table V. Panel A, showing the out-of-sample R^2 and the square-error loss, demonstrates that using the variance risk premium alone generates significant return predictability that peaks at the quarterly horizon, but then declines monotonically. Similarly, Panel B shows that the variance risk premium significantly improves the certainty equivalent relative to the benchmark model, with a maximum gain of 3.5% at the quarterly horizon, but declining for longer horizons.

The correlation risk premium produces significantly better return predictability than the benchmark model *for all horizons*. For example, the out-of-sample R^2 reaches its maximum of 7.9% at the 9 months horizon and decreases only slightly to 7.0% for a one year forecast horizon. The Diebold-Mariano test statistic yields similar results and the correlation risk premium shows improvements in the certainty equivalent return of 3.9% for the monthly horizon that stays above 2% for up to 9 months, and is slightly less than one percent for one year. Starting from

[18]MBB is shown (e.g., in Lahiri (1999)) to be comparable in performance to other widely used methods like stationary bootstrap by Politis and Romano (1994) or circular block bootstrap from their 1992 paper, while constant block size leads to smaller mean-squared errors than with random block size as in stationary bootstrap.

[19]We draw 10,000 random samples of size equal to 200 blocks, with blocks of twelve observations (i.e., one-year blocks) to preserve the autocorrelation in the data, which is at maximum equal to eleven lags for annual prediction horizon due to overlapping observations each month.

a horizon of 6 months, the R^2 is always higher than for the variance risk premium. Moreover, comparing the certainty equivalent gain for the two models directly ($CE_{2,\tau_r} - CE_{3,\tau_r}$), confirms that the correlation risk premium performs better than variance risk premium for long horizons with gains of $0.5 - 1\%$. Finally, using the variance and correlation risk premium jointly only improves predictability for short horizons of to 3 months.

In summary, comparable to the in-sample analysis, the variance and correlation risk premium provide non-redundant information, with the the predictive power of the correlation risk premium being economically and statistically significant for a longer period compared to the variance risk premium.

To highlight the importance of our contemporaneous betas approach, that is, the timely update of the regression coefficients, we now also run the traditional predictive procedure. That is, we regress, at the end of each month t, historical market excess returns on lagged regressors:

$$r_{s \to s+\tau_r} = \beta_{VRP}\, VRP(s, s+\tau_r) + \beta_{CRP}\, CRP(s, s+\tau_r), \quad s+\tau_r \leq t, \tag{26}$$

and use the resulting betas together with the time-t observable variables $VRP(t, t+\tau_r)$ and $CRP(t, t+\tau_r)$ to form an market excess return forecast $\hat{r}_{t \to t+\tau_r}$ for the forecast horizon. We use a 3-year rolling window of past data at each time t to estimate the regression and apply the same evaluation criteria as we did before. The results are shown in Table VI.

Panel A shows that the out-of-sample R^2 and Diebold-Mariano loss function are substantially weaker than for the contemporaneous betas. For example, for the variance risk premium the R^2 is never significantly positive. The correlation risk premium implies some modest certainty equivalent gains. However, in general, the results are always considerably weaker than in Table V.

Recall that with the traditional approach, when estimating the betas in (26) at time t, the latest observation of the predictive variables comes from $t - \tau_r$, whereas in the contemporaneous betas approach one can use information up to time t. For example, for predicting annual returns using the traditional approach, one cannot use any (option-implied) information from the past year. To illustrate this difference between the contemporaneous beta approach and the traditional approach, Figure 1 contrasts the variance risk premium and correlation risk premium betas for the two approaches. The differences in betas can be quite large, particularly so for longer horizons. While the variance and correlation betas are quite volatile in general, the contemporaneous betas are considerably more stable than the standard ones, which adds to the stability of the return forecast. For example, visually, traditional betas seem to overreact to large "outliers" in returns and / or predictive variables. In contrast, using high-frequency (daily) returns and variance / correlation increments in the contemporaneous approach mitigates the effect of outliers on the estimated quadratic co-variation and the resulting betas.

In summary, three important messages emerge: (i) the variance and correlation risk premiums perform statistically significantly better than the simple average historical return, and the improvements in predictability have clear economic benefits; (ii) the performance of the variance risk premium peaks at the quarterly horizon, and then declines, while the correlation risk premium yields significant predictability for horizons of up to a year; and (iii) using the contemporaneous betas approach is important, especially for longer-term predictions.

IV. Source of Correlation Risk

While for our analysis of the market return predictability, the source of the positive price for correlation risk is secondary, understanding the economic mechanism that creates it, is of importance itself. In this section, we study potential explanations for priced correlation risk.

A. The Correlation Risk Premium and Uncertainty

Buraschi, Trojani, and Vedolin (2014) offer, in a general equilibrium setting, an explanation for the existence of a correlation risk premium that relies on economic uncertainty, measured as the aggregate difference in beliefs regarding future earnings. Intuitively, both—the correlation risk premium and uncertainty—are associated with a positive compensation for risk, and, hence, the correlation risk premium might potentially proxy for uncertainty. Buraschi, Trojani, and Vedolin (2014) also provide solid empirical support, showing that the ex-post correlation risk premium is positively related to differences in beliefs.

To proxy for economic uncertainty we rely on the measure of economic policy uncertainty (EPU) by Baker, Bloom, and Davis (2016),[20] and also construct the disagreement proxy DIB, as in Buraschi, Trojani, and Vedolin (2014). Specifically, following Diether, Malloy, and Scherbina (2002) we define a firm-specific disagreement proxy as the standard deviation of a firm's earnings-per-share forecasts for the next fiscal year (scaled by the absolute value of the forecasts), and compute it using the Unadjusted Summary History file for U.S. firms from I/B/E/S. The market-wide disagreement DIB is simplified defined as an equal-weighted average of the firm-specific disagreement proxies.[21]

For the sample period used in Buraschi, Trojani, and Vedolin (2014), that is, January 1996 to July 2007, we also find a positive correlation between aggregate DIB and the ex-post correlation risk premium, ranging from 0.11 for the 30-day correlation risk premium to 0.06 for the 365-day correlation risk premium. However, for our whole sample period until April 2016, the correlation with the 30-day correlation risk premium becomes literally zero (0.008), turning negative for longer maturities, and reaching -0.19 for the 365-day correlation risk premium.

While the theory about the link between the uncertainty (disagreement) and the correlation risk premium is very logical and appealing, the data in the last several years does not fully support it. One of the potential reasons for it is the change in the information processing technologies and market microstructure, happening in the last decade. We leave it as an open topic for future research to understand the exact reasons for such changes in empirical evidence and potentially to refine the existing model.

[20]We appreciate having an opportunity to download updated series of the Economic Policy Uncertainty Index from the web-site of the authors www.policyuncertainty.com/.

[21]Buraschi, Trojani, and Vedolin (2014) report a correlation of almost one with the corresponding variable based on market capitalization weights.

B. The Correlation Risk Premium and Future Market Risk

Another potential explanation for the existence of a correlation risk premium is the role of correlation as a state variable in the ICAPM, predicting future investment opportunities.

Future investment opportunities can be related to the future portfolio variance of a simple equal-weighted portfolio of N stocks

$$\sigma_p^2 = \left(\frac{1}{N}\right)\left(\frac{1}{N}\sum_{i=1}^{N}\sigma_i^2\right) + \left(\frac{N^2-N}{N^2}\right)\left(\frac{1}{N^2-N}\sum_{i=1}^{N}\sum_{j\neq i}^{N}\sigma_{ij}\right),$$

which, as N becomes large, is driven by the average covariance only. Accordingly, future investment opportunities are characterized by the future average correlation among stocks and future average variance, which define the lowest attainable bound of systematic risk in a well-diversified portfolio. Intuitively, an increase in correlations reduces diversification benefits, and, thus, increases the total portfolio risk.[22] Moreover, if one thinks in terms of a simple one-factor market model for returns, the covariances, given by $\sigma_{ij} = \beta_{M,i}\beta_{M,j}\sigma_M^2$, are decreasing in the cross-sectional dispersion of betas.[23] Thus, the portfolio's expected non-diversifiable risk relative to the market risk is decreasing in the dispersion of betas.

Table VII provides the regression results for predicting future risk measures (realized correlation, realized market variance, and dispersion of market betas) by expected correlation as well as expected variance. This allow us to disentangle the roles of variance and correlation as state variables defining future investment opportunities, and, thus, to understand why variance and correlation risk premiums predict returns over different horizons.

Expected correlation, and especially implied correlation, predict future realized correlation very well, delivering a high R^2 for all horizons. The strongest predictability arises for shorter-term realized correlation (with an R^2 of 35% for the monthly horizon), though the R^2 for the

[22] In a more practical sense, the average correlation between stocks also strongly affects the number of stocks needed to form a well-diversified portfolio.

[23] Assuming that market betas are distributed around mean one with the same variance, that is, $\beta_{M,k} = 1 + \epsilon_M \sim Dist(1, \sigma_\epsilon^2)$, it holds that $E[\beta_{M,i}\beta_{M,j}] = E[(1+\epsilon_{M,i})(1+\epsilon_{M,j})] = 1 + cov(\epsilon_{M,i}, \epsilon_{M,j}) = 1 - \sigma_\epsilon^2$, where the covariance between the deviation of betas from the mean is negative, because their mean does not change, and an increasing beta is necessarily compensated by a decreasing one.

annual horizon is still above 15%. Similarly, expected correlation predicts the future dispersion of market betas, with higher expected correlation predicting lower dispersion and, hence, more portfolio risk. At the monthly horizon, the R^2 is a modest (10.70%), but increases to around 30% for six months and longer horizons. Both expected correlations—implied and past realized—do a very poor job in predicting future market variance.

In contrast, expected variance, specifically so implied variance, predict future market variance, with an impressive explanatory power at short horizons. The R^2 is almost 50% for one-month predictions, but drops quickly and is only about 12% for one-year future variance. Expected variances can also predict future realized correlations, with a more modest R^2 of 15.9% for one month, and less than 5% for one year. Contrary to expectation, expected variances predict the dispersion of future market betas with a *positive* sign. Accordingly, a higher expected variance predicts better future diversification, that is, lower portfolio risk for well-diversified portfolios at longer horizons.

In summary, similar to our empirical results on market return predictability, variance and correlation seem to provide non-redundant information for future investment opportunities. Particularly, their predictive power varies with the horizon. While variance predicts shorter-term risk in the form of future realized market variance, correlation plays an important role in determining longer-term risks in the form of diversification benefits. This allows the link to Buraschi, Kosowski, and Trojani (2014), with correlation as a "no-place-to-hide" state variable, which predicts risks and returns at longer horizons compared to variance.

V. Robustness Tests

We carry out a number of additional tests to check the robustness of our results. The additional tables are provided in the Internet Appendix.

Instead of relying on variance and correlation risk premiums from options with maturities matching the forecasting horizon, the literature has often used options with a maturity of one month for all return forecast horizons (see, among others, Bollerslev, Tauchen, and Zhou (2009) and Bollerslev, Marrone, Xu, and Zhou (2014)). Accordingly, in the following, we reproduce the in-sample and out-of-sample predictability results, always using the implied variance from options with one month to maturity to estimate the beta, $\beta_{t,\Delta IV}$, and the variance risk premium with the same maturity for the return forecast (25).

Interestingly, the in-sample predictive power for these short-maturity options is typically slightly better (Table AI101), and the coefficients on the variance risk premium do not turn negative with the 30-day options. However, qualitatively, the pattern is the same, that is, the R^2 peaks at a quarterly frequency and then declines. A similar picture emerges for the out-of-sample return predictability (Table AI102). In summary, it seems that options with a 30-day maturity deliver slightly better results than options with a maturity matching the return horizon.

We also test how the option-based variables compare in predicting the future market excess return with a number of fundamental variables, that have been used in the literature. While there is a myriad of possible explanatory variables used in different studies (see, among others, Goyal and Welch (2008) and Ferreira and Santa-Clara (2011)), we limit our choice to five variables that encompass non-redundant economic information and have been show to be highly significant in-sample, constructed as in Goyal and Welch (2008). Specifically, we use the Earnings Price Ratio ($EP12$), defined as the difference between the log of earnings and the log of prices; the Term Spread (TMS), that is, the difference between the long term yield on government bonds and the Treasury-bill; the Default Yield Spread (DFY), computed as the difference between BAA and AAA-rated corporate bond yields; the Book-to-Market Ratio (BM), that is, the ratio of book value to market value for the Dow Jones Industrial Average; and the Net Equity Expansion ($NTIS$), defined the ratio of the 12-month moving sum of net issues by NYSE listed stocks divided by the total end-of-year market capitalization of NYSE stocks. While a number of fundamental variables successfully improve explaining future market returns, they typically do not change the sign or significance of the correlation risk premium (Table AI103). In some cases adding the term or default spread actually improves the significance of the correlation risk premium, for example, for 9- and 12-month predictions.

VI. Conclusion

Implied correlation uses forward-looking information from option markets, and is typically interpreted as an indicator of diversification risk in the future. In this paper, we show that the correlation risk premium, inferred from major U.S. stock indices, is able to predict market excess returns in-sample and *out-of-sample* at horizons of up to one year. In contrast, the predictability of the variance risk premium peaks already at the quarterly frequency.

We first derive, in a reduced-form model, a beta representation of the equity risk premium that links it to the variance and correlation risk premiums. Next, we develop a new methodology for estimating contemporaneous betas with respect to variance and correlation risk, using daily increments of option-implied variance and correlation. Our methodology substantially improves the out-of-sample predictability of market returns, and leads to out-of-sample R^2s of 7.9% even at the 9 month horizon, and 7% at the annual horizon. These predictability results imply considerable statistical and economic gains in portfolio optimization (such as the gain of 2% p.a. in the certainty equivalent return of a myopic mean-variance investor).

Analyzing the link between correlation and uncertainty as well as future risks, we document that the average correlation can be interpreted as a "no-place-to-hide" state variable, that predicts future diversification risks for horizons of up to one year. Particularly, expected correlation predicts future realized correlations and non-diversifiable market risk in equity portfolios in the form of dispersion of market betas. Expected variance performs better only in predicting shorter-term risks. This allows an interpretation of market variance and average correlation as state variables in the form of the ICAPM (or proxies of state variables) that predict future investment opportunities, and hence bear risk premiums as compensation. Intuitively, while correlations predict risks for a longer term compared to variances, they are able to predict returns for longer horizons as well.

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Table I Individual and Index Variances, and Variance Risk Premiums

The table reports the time-series averages of realized (RV) and model-free implied variances (IV), expressed in volatility terms, and the difference between them ($VRP = IV - RV$), expressed as a difference in variances, for three samples of stocks—components of S&P500, S&P100, and DJ30 indices, for the sample period from 1996 to 04/2016, and from 10/1997 to 04/2016, respectively, and for three different maturities—30, 91, 182, 273, and 365 (calendar) days. For individual stocks the variances are equal-weighted cross-sectional averages across all constituent stocks. Implied variance (IV) is computed as simple variance swap (Martin (2013)) on each day using out-of-the money options with the respective maturity, and realized variance RV is calculated on each day from daily returns over a respective window, corresponding to the maturity of IV. All numbers are expressed in annual terms. The p-value is for the null hypothesis that implied and realized variances are on average equal; the p-values are computed from standard errors with Newey and West (1987) adjustments for autocorrelation.

	Individual Stocks				Indices			
Days	$\sqrt{IV}$	$\sqrt{RV}$	VRP	$p-val$	$\sqrt{IV}$	$\sqrt{RV}$	VRP	$p-val$
SP500 Sample								
30	0.398	0.397	0.001	0.807	0.210	0.185	0.005	0.007
91	0.381	0.395	-0.011	0.125	0.210	0.184	0.006	0.049
182	0.371	0.393	-0.017	0.115	0.211	0.184	0.007	0.087
273	0.368	0.392	-0.019	0.154	0.213	0.184	0.007	0.090
365	0.365	0.392	-0.020	0.171	0.215	0.185	0.008	0.078
SP100 Sample								
30	0.361	0.368	-0.005	0.309	0.210	0.186	0.005	0.007
91	0.348	0.366	-0.012	0.095	0.211	0.185	0.006	0.034
182	0.342	0.363	-0.015	0.115	0.212	0.185	0.007	0.055
273	0.340	0.361	-0.015	0.175	0.214	0.185	0.008	0.067
365	0.339	0.361	-0.016	0.217	0.217	0.186	0.009	0.053
DJ30 Sample								
30	0.320	0.325	-0.003	0.314	0.206	0.175	0.006	0.000
91	0.308	0.323	-0.009	0.121	0.206	0.175	0.007	0.007
182	0.302	0.320	-0.011	0.231	0.208	0.175	0.008	0.023
273	0.303	0.317	-0.009	0.349	0.210	0.175	0.009	0.032
365	0.304	0.316	-0.007	0.476	0.212	0.175	0.010	0.032

Table II Individual Variance Risk Premiums

The table reports the results of individual tests of variance risk premiums, for three samples of stocks–components of S&P500, S&P100, and DJ30 indices, for the sample period from 01/1996 to 04/2016 for S&P500 and S&P100, and from 10/1997 to 04/2016 for DJ30, and five three different maturities – 30, 91, 182, 273 and 365 (calendar) days. The table shows the number of stocks, for which the respective hypothesis is either rejected ($IV - RV \geq 0$ and $IV - RV \leq 0$), or failed to be rejected ($IV = RV$). Implied variance (IV) is computed on each day using out-of-the money options with the respective maturity, and realized variance (RV) is calculated on each day from daily returns over a respective window, corresponding to the maturity of IV. The test statistics for each stock are based on Newey-West (1987) autocorrelation consistent standard errors with lags equal to the number of overlapping observations (20, 62, 125, 188 or 251, respectively).

Days	$IV - RV \geq 0$	$IV = RV$	$IV - RV \leq 0$
SP500 Sample			
30	54	669	344
91	70	824	171
182	83	839	143
273	86	810	168
365	95	765	197
SP100 Sample			
30	12	150	51
91	9	173	25
182	16	176	25
273	13	166	30
365	12	159	40
DJ30 Sample			
30	3	38	7
91	4	42	4
182	2	41	4
273	1	39	10
365	0	37	10

Table III Index Implied and Realized Correlations: Summary

The table reports summary statistics (time-series mean, p-value for the mean, median, and the standard deviation) for the implied correlation (IC), realized correlation (RC), and for the correlation risk premium ($CRP = IC - RC$), for three samples of stocks—components of S&P500, S&P100 and DJ30 indices, for the sample period from 1996 to 04/2016, and from 10/1997 to 04/2016, respectively, and for five different maturities—30, 91, 182, 273, and 365 (calendar) days. $IC(t)$ ($RC(t)$) are calculated from daily observations of implied (realized) variances for the index and for all index components. Implied variances are computed as simple variance swaps (Martin (2013)). The p-values for significance of the means are computed with Newey and West (1987) adjustments for autocorrelation.

	IC					*RC*					*IC-RC*				
	30	91	182	273	365	30	91	182	273	365	30	91	182	273	365
SP500															
Mean	0.387	0.423	0.446	0.454	0.459	0.327	0.326	0.327	0.328	0.327	0.060	0.097	0.123	0.130	0.133
p-val	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Median	0.375	0.423	0.454	0.462	0.464	0.298	0.308	0.310	0.307	0.308	0.060	0.094	0.126	0.140	0.142
StDev	0.126	0.113	0.106	0.104	0.099	0.145	0.125	0.119	0.116	0.115	0.103	0.084	0.081	0.080	0.076
SP100															
Mean	0.423	0.463	0.485	0.494	0.498	0.356	0.357	0.359	0.359	0.358	0.067	0.106	0.126	0.135	0.140
p-val	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Median	0.412	0.466	0.496	0.506	0.509	0.331	0.344	0.339	0.342	0.341	0.066	0.103	0.125	0.144	0.144
StDev	0.130	0.114	0.106	0.103	0.101	0.152	0.129	0.122	0.119	0.116	0.114	0.090	0.090	0.094	0.093
DJ30															
Mean	0.464	0.497	0.523	0.529	0.528	0.371	0.373	0.376	0.378	0.377	0.082	0.112	0.138	0.140	0.137
p-val	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Median	0.456	0.503	0.535	0.541	0.539	0.352	0.363	0.368	0.359	0.359	0.078	0.102	0.134	0.144	0.141
StDev	0.148	0.129	0.118	0.113	0.105	0.169	0.148	0.143	0.142	0.141	0.130	0.102	0.095	0.094	0.090

Table IV In-sample Market Return Predictability: Correlation and Variance Risk Premiums

The table shows the coefficients (and corresponding p-values) and the R^2 of the market predictive regressions, for the sample period from 01/1996 to 04/2016 for S&P500 and S&P100, and from 10/1997 to 04/2016 for DJ30-based variables. We regress overlapping excess market returns compounded over a specified horizon (30, 91, 182, 273, and 365 calendar days) on a constant and a given set of explanatory variables, which are the correlation risk premium (CRP) for 30, 91, 182, 273, and 365 calendar days, and the variance risk premium (VRP), which equals to the difference between implied variance and lagged realized variance computed over the matching period of 30, 91, 182, 273, and 365 calendar days. Implied variances are computed as simple variance swaps (Martin (2013)). The p-values (under the coefficients) for the null hypothesis that the coefficients are equal zero are computed using Newey and West (1987) standard errors. The adjusted R^2 are given as percentages.

	Return, 30 days			Return, 91 days			Return, 181 days			Return, 273 days			Return, 365 days		
SP500 Sample															
CRP	0.076	-	0.027	0.254	-	0.195	0.381	-	0.729	0.588	-	1.108	0.559	-	1.071
	0.027	-	0.362	0.002	-	0.027	0.051	-	0.002	0.067	-	0.002	0.150	-	0.031
VRP	-	0.322	0.289	-	0.562	0.270	-	-0.304	-1.606	-	-0.599	-2.554	-	-0.740	-2.501
	-	0.004	0.007	-	0.002	0.175	-	0.553	0.000	-	0.380	0.000	-	0.253	0.028
R^2	2.48	6.90	6.81	7.26	5.08	7.73	6.90	0.15	16.20	9.87	0.81	23.90	5.43	0.85	14.77
SP100 Sample															
CRP	0.051	-	0.011	0.234	-	0.161	0.363	-	0.647	0.561	-	1.029	0.527	-	0.994
	0.076	-	0.678	0.004	-	0.062	0.047	-	0.003	0.042	-	0.001	0.082	-	0.017
VRP	-	0.333	0.319	-	0.652	0.400	-	-0.270	-1.567	-	-0.437	-2.642	-	-0.467	-2.645
	-	0.004	0.006	-	0.001	0.042	-	0.683	0.006	-	0.592	0.001	-	0.518	0.031
R^2	1.27	6.68	6.35	6.74	6.07	8.10	7.24	-0.03	15.10	12.18	0.21	26.16	7.60	0.08	17.28
DJ30 Sample															
CRP	0.040	-	0.010	0.205	-	0.133	0.273	-	0.545	0.330	-	0.741	0.150	-	0.581
	0.117	-	0.675	0.007	-	0.120	0.123	-	0.018	0.243	-	0.017	0.642	-	0.198
VRP	-	0.292	0.277	-	0.679	0.420	-	-0.436	-1.718	-	-0.942	-2.690	-	-1.323	-2.628
	-	0.005	0.005	-	0.000	0.044	-	0.475	0.008	-	0.236	0.004	-	0.079	0.072
R^2	0.90	4.53	4.16	6.27	6.06	7.51	4.47	0.40	12.33	3.93	1.86	15.64	0.11	2.81	7.85

Table V Out of Sample Predictability - Contemporaneous Beta Approach

The table reports the Out-of-Sample R^2_{j,τ_r} and the Diebold-Mariano test statistic δ_{j,τ_r} in Panel A, and the improvement of the certainty equivalent of a mean-variance investor optimizing her portfolio using forecasts by a specific model instead of either using the past mean market return (Model 0), or CRP as a predictor, in Panel B. The variance and correlation risk premiums are computed as the difference between implied and lagged realized variances ($VRP = IV - RV$), and as the difference between implied and lagged realized correlations ($CRP = IC - RC$), for three samples of stocks—components of S&P500, S&P100, and DJ30 indices, for the sample period from 1996 to 04/2016 for S&P500, S&P100, and from 10/1997 to 04/2016 for DJ30, and for five different maturities—30, 91, 182, 273, and 365 (calendar) days. Implied variance (IV) is computed as the simple variance swap (Martin (2013)) on each day using out-of-the money options with the respective maturity, and realized variance is calculated on each day from daily returns over the corresponding historical window. The betas are computed on each day using daily increments over a 12-month rolling window. The p-values are obtained from a bootstrapped distribution using moving-block bootstrap by Künsch (1989) with 10,000 samples and block length of 12 months.

Panel A: OOS R^2 and δ

	R^2_{j,τ_r}			δ_{j,τ_r}		
Days	VRP	CRP	VRP+CRP	VRP	CRP	VRP+CRP
SP500 Sample						
30	0.094	0.025	0.096	-0.000	-0.000	-0.000
	0.000	0.000	0.000	0.000	0.000	0.000
91	0.103	0.081	0.104	-0.001	-0.001	-0.001
	0.000	0.000	0.000	0.000	0.000	0.000
182	0.062	0.067	0.063	-0.001	-0.001	-0.001
	0.000	0.000	0.001	0.000	0.000	0.001
273	0.039	0.079	0.032	-0.001	-0.002	-0.001
	0.064	0.000	0.144	0.065	0.000	0.145
365	0.037	0.070	0.024	-0.001	-0.003	-0.001
	0.091	0.001	0.210	0.091	0.001	0.213
SP100 Sample						
30	0.091	0.012	0.087	-0.000	-0.000	-0.000
	0.000	0.015	0.000	0.000	0.015	0.000
91	0.111	0.072	0.107	-0.001	-0.000	-0.001
	0.000	0.000	0.000	0.000	0.000	0.000
182	0.029	0.055	0.020	-0.000	-0.001	-0.000
	0.086	0.000	0.215	0.088	0.000	0.218
273	0.019	0.066	0.000	-0.001	-0.002	-0.000
	0.239	0.000	0.498	0.241	0.000	0.498
365	0.031	0.070	0.016	-0.001	-0.003	-0.001
	0.134	0.001	0.298	0.136	0.001	0.301
DJ30 Sample						
30	0.065	-0.004	0.066	-0.000	0.000	-0.000
	0.000	0.283	0.000	0.000	0.283	0.000
91	0.103	0.064	0.094	-0.001	-0.000	-0.001
	0.000	0.000	0.000	0.000	0.000	0.000
182	0.041	0.049	0.025	-0.001	-0.001	-0.000
	0.006	0.001	0.079	0.007	0.001	0.079
273	-0.001	0.028	-0.017	0.000	-0.001	0.000
	0.462	0.092	0.193	0.460	0.094	0.193
365	-0.005	0.008	-0.023	0.000	-0.000	0.001
	0.396	0.387	0.100	0.394	0.390	0.098

...Table V continued

Panel B: Certainty Equivalent Improvement

Days	$CE_{j,\tau_r} - CE_{0,\tau_r}$ VRP	CRP	VRP+CRP	$CE_{j,\tau_r} - CE_{CRP,\tau_r}$ VRP	CRP	VRP+CRP
SP500 Sample						
30	0.023	0.039	0.023	-0.016	-	-0.016
	0.003	0.000	0.003	0.000	-	0.002
91	0.035	0.022	0.032	0.013	-	0.010
	0.000	0.001	0.000	0.001	-	0.038
182	0.030	0.021	0.030	0.009	-	0.009
	0.000	0.000	0.000	0.038	-	0.061
273	0.016	0.020	0.012	-0.004	-	-0.008
	0.008	0.000	0.061	0.237	-	0.105
365	-0.001	0.007	-0.007	-0.008	-	-0.015
	0.456	0.086	0.159	0.040	-	0.002
SP100 Sample						
30	0.021	0.024	0.017	-0.004	-	-0.007
	0.008	0.000	0.022	0.264	-	0.158
91	0.034	0.027	0.031	0.008	-	0.005
	0.000	0.000	0.000	0.018	-	0.196
182	0.024	0.021	0.017	0.004	-	-0.004
	0.000	0.000	0.015	0.218	-	0.242
273	0.010	0.017	-0.001	-0.007	-	-0.018
	0.071	0.002	0.455	0.085	-	0.001
365	0.002	0.011	-0.008	-0.009	-	-0.019
	0.404	0.039	0.137	0.014	-	0.000
DJ30 Sample						
30	0.005	-0.015	0.014	0.020	-	0.029
	0.315	0.087	0.094	0.000	-	0.000
91	0.023	0.013	0.021	0.010	-	0.008
	0.000	0.034	0.003	0.002	-	0.036
182	0.028	0.017	0.025	0.011	-	0.008
	0.000	0.013	0.001	0.020	-	0.102
273	0.008	0.013	0.003	-0.005	-	-0.011
	0.169	0.015	0.369	0.202	-	0.071
365	-0.016	-0.008	-0.019	-0.008	-	-0.012
	0.027	0.144	0.010	0.090	-	0.055

Table VI Out of Sample Predictability - Traditional Beta Approach

The table reports the Out-of-Sample R^2_{j,τ_r} and the Diebold-Mariano test statistic δ_{j,τ_r} in Panel A, and the improvement of the certainty equivalent of a mean-variance investor optimizing her portfolio using forecasts by a specific model instead of either using past mean market return (Model 0), or CRP as a predictor, in Panel B. The VRP and CRP betas are computed by the traditional predictive approach using a 36-month historical rolling window for estimation. The variance and correlation risk premiums are computed as the difference between implied and lagged realized variances ($VRP = IV - RV$), and as a difference between implied and lagged realized correlations ($CRP = IC - RC$), for three samples of stocks—components of S&P500, S&P100, and DJ30 indices, for the sample period from 1996 to 04/2016 for S&P500, S&P100, and from 10/1997 to 04/2016 for DJ30, and for five different maturities—30, 91, 182, 273, and 365 (calendar) days. Implied variance (IV) is computed as the simple variance swap (Martin (2013)) on each day using out-of-the money options with the respective maturity, and realized variance is calculated on each day from daily returns over the corresponding historical window. Betas are computed using monthly-sampled variables over a 60-month rolling window. The p-values are obtained from a bootstrapped distribution using moving-block bootstrap by Künsch (1989) with 10,000 samples with the block size of 60 months.

Panel A: OOS R^2 and δ

	R^2_{j,τ_r}			δ_{j,τ_r}		
Days	VRP	CRP	VRP+CRP	VRP	CRP	VRP+CRP
SP500 Sample						
30	-0.438	-0.014	-0.679	0.001	0.000	0.001
	0.000	0.000	0.000	0.000	0.000	0.000
91	-0.938	-0.022	-0.841	0.008	0.000	0.007
	0.000	0.000	0.000	0.000	0.000	0.000
182	-1.667	-0.177	-0.674	0.037	0.004	0.015
	0.000	0.000	0.000	0.000	0.000	0.000
273	-0.453	-0.325	-1.223	0.017	0.012	0.045
	0.000	0.000	0.000	0.000	0.000	0.000
365	-1.628	-0.226	-3.687	0.085	0.012	0.192
	0.000	0.000	0.000	0.000	0.000	0.000
SP100 Sample						
30	-0.333	-0.023	-0.555	0.001	0.000	0.001
	0.000	0.000	0.000	0.000	0.000	0.000
91	-1.232	-0.022	-1.088	0.010	0.000	0.009
	0.000	0.000	0.000	0.000	0.000	0.000
182	-6.846	-0.208	-2.973	0.160	0.005	0.069
	0.000	0.000	0.000	0.000	0.000	0.000
273	-1.067	-0.496	-0.116	0.041	0.019	0.004
	0.000	0.000	0.000	0.000	0.000	0.000
365	-1.607	-0.320	-2.311	0.084	0.017	0.121
	0.000	0.000	0.000	0.000	0.000	0.000
DJ30 Sample						
30	-0.775	-0.012	-1.104	0.002	0.000	0.003
	0.000	0.000	0.000	0.000	0.000	0.000
91	-1.317	0.009	-0.747	0.012	-0.000	0.007
	0.000	0.086	0.000	0.000	0.087	0.000
182	-0.809	-0.136	-0.441	0.020	0.003	0.011
	0.000	0.000	0.000	0.000	0.000	0.000
273	-0.376	-0.358	-0.627	0.015	0.014	0.025
	0.000	0.000	0.000	0.000	0.000	0.000
365	-1.172	-0.295	-1.960	0.067	0.017	0.112
	0.000	0.000	0.000	0.000	0.000	0.000

...Table VI continued

Panel B: Certainty Equivalent Improvement

Days	$CE_{j,\tau_r} - CE_{0,\tau_r}$ VRP	CRP	VRP+CRP	$CE_{j,\tau_r} - CE_{CRP,\tau_r}$ VRP	CRP	VRP+CRP
SP500 Sample						
30	0.021	0.009	0.027	0.012	-	0.018
	0.000	0.000	0.000	0.003	-	0.000
91	0.030	0.043	0.034	-0.012	-	-0.008
	0.000	0.000	0.000	0.000	-	0.000
182	-0.017	0.016	0.035	-0.033	-	0.019
	0.000	0.000	0.000	0.000	-	0.000
273	0.057	0.017	0.111	0.041	-	0.095
	0.000	0.000	0.000	0.000	-	0.000
365	0.057	0.013	0.050	0.044	-	0.037
	0.000	0.000	0.000	0.000	-	0.000
SP100 Sample						
30	0.013	0.006	0.017	0.006	-	0.011
	0.001	0.000	0.000	0.096	-	0.001
91	0.037	0.041	0.035	-0.004	-	-0.006
	0.000	0.000	0.000	0.000	-	0.000
182	-0.023	0.040	0.022	-0.063	-	-0.019
	0.000	0.000	0.000	0.000	-	0.000
273	0.021	0.022	0.105	-0.001	-	0.083
	0.000	0.000	0.000	0.383	-	0.000
365	0.030	0.007	0.100	0.022	-	0.093
	0.000	0.000	0.000	0.000	-	0.000
DJ30 Sample						
30	0.017	0.008	0.006	0.009	-	-0.002
	0.000	0.000	0.072	0.019	-	0.343
91	0.036	0.051	0.049	-0.015	-	-0.002
	0.000	0.000	0.000	0.000	-	0.000
182	-0.017	0.020	0.031	-0.037	-	0.011
	0.000	0.000	0.000	0.000	-	0.000
273	0.036	0.013	0.091	0.023	-	0.078
	0.000	0.000	0.000	0.000	-	0.000
365	0.030	0.012	0.038	0.018	-	0.026
	0.000	0.000	0.000	0.000	-	0.000

Table VII Risk Predictability

The table shows the coefficients (with corresponding p-values) and the R^2 of the risk predictive regressions, for the sample period from 01/1996 to 04/2016 for S&P500 and S&P100, and from 10/1997 to 04/2016 for DJ30-based variables. We regress risk measures for a specified future horizon of 30, 91, 181, 273, and 365 calendar days on a constant and one of the explanatory variables, which are the lagged realized and current implied variances (RV and IV), and lagged realized and current implied correlations (RC and IC) for 30, 91, 181, 273, and 365 calendar days. The risk measures are the cross-sectional variance of market betas $\sigma^2(\beta_M)$ for all stocks in an index in Panel A, realized equicorrelation (RC) in Panel B, and realized market variance (RV) for a given index in Panel C. Implied variances are computed as simple variance swaps Martin (2013). The p-values (under the coefficients) for the null hypothesis that the coefficients are equal zero are computed using Newey and West (1987) standard errors.

Panel A: Dispersion of Market Betas – $\sigma^2(\beta_M)$

	$\sigma^2(\beta_M)$, 30			$\sigma^2(\beta_M)$, 91			$\sigma^2(\beta_M)$, 181			$\sigma^2(\beta_M)$, 273			$\sigma^2(\beta_M)$, 365		
	β	$p-val$	R^2	β	$p-val$	R^2	β	$p-val$	R^2	β	$p-val$	R^2	β	$p-val$	R^2
SP500 Sample															
RV	-0.117	0.573	0.05	0.761	0.011	6.80	1.390	0.000	14.64	1.214	0.004	12.90	1.004	0.026	9.97
IV	-0.148	0.651	0.02	1.328	0.005	7.62	1.818	0.000	11.26	2.105	0.000	15.63	2.334	0.000	20.93
RC	-0.531	0.000	6.59	-0.226	0.097	3.04	-0.255	0.188	4.61	-0.251	0.276	5.06	-0.231	0.351	4.91
IC	-0.783	0.000	10.70	-0.487	0.002	11.59	-0.677	0.001	28.54	-0.684	0.004	32.30	-0.643	0.020	28.58
SP100 Sample															
RV	0.224	0.425	0.23	0.987	0.006	11.08	1.473	0.000	26.81	1.417	0.000	24.53	1.210	0.005	17.08
IV	0.672	0.120	0.97	1.765	0.001	14.27	2.161	0.000	23.71	2.460	0.000	29.36	2.421	0.000	30.74
RC	-0.315	0.000	2.77	-0.097	0.421	0.61	-0.017	0.929	0.00	-0.020	0.928	0.02	-0.055	0.811	0.33
IC	-0.422	0.000	3.64	-0.296	0.046	4.55	-0.399	0.062	10.84	-0.454	0.061	16.30	-0.414	0.130	15.02
DJ30 Sample															
RV	0.294	0.258	0.46	1.219	0.000	22.26	1.779	0.000	35.26	1.520	0.000	30.70	1.305	0.001	24.57
IV	0.696	0.064	1.30	1.781	0.000	21.71	2.263	0.000	32.67	2.406	0.000	41.02	2.501	0.000	46.27
RC	-0.089	0.368	0.35	0.044	0.706	0.24	0.029	0.827	0.14	0.020	0.898	0.07	0.022	0.899	0.10
IC	-0.153	0.175	0.78	-0.073	0.551	0.54	-0.205	0.165	6.14	-0.213	0.188	7.50	-0.179	0.331	4.98

Panel B: Realized Correlation – RC

	RC, 30			RC, 91			RC, 181			RC, 273			RC, 365		
	β	$p-val$	R^2	β	$p-val$	R^2	β	$p-val$	R^2	β	$p-val$	R^2	β	$p-val$	R^2
SP500 Sample															
RV	0.768	0.000	12.09	0.650	0.000	8.43	0.529	0.014	2.80	0.485	0.023	2.50	0.637	0.002	4.70
IV	1.359	0.000	15.93	1.200	0.000	10.58	0.854	0.052	3.30	0.646	0.256	1.78	0.558	0.398	1.38
RC	0.510	0.000	26.03	0.544	0.000	29.97	0.493	0.000	23.12	0.529	0.000	27.58	0.514	0.000	28.60
IC	0.688	0.000	35.44	0.548	0.000	25.05	0.451	0.000	16.88	0.422	0.001	15.01	0.440	0.003	15.67
SP100 Sample															
RV	0.757	0.000	9.97	0.610	0.000	6.40	0.268	0.169	1.02	0.216	0.318	0.57	0.392	0.134	1.72
IV	1.278	0.000	12.80	1.065	0.000	7.87	0.526	0.150	1.62	0.216	0.676	0.21	0.112	0.853	0.04
RC	0.470	0.000	22.10	0.523	0.000	27.74	0.425	0.000	18.55	0.447	0.000	21.10	0.440	0.001	21.62
IC	0.647	0.000	30.64	0.512	0.000	20.69	0.386	0.001	11.90	0.297	0.026	7.20	0.267	0.075	6.04
DJ30 Sample															
RV	0.861	0.000	8.89	0.703	0.000	5.38	0.471	0.155	1.14	0.389	0.204	0.77	0.601	0.120	1.93
IV	1.245	0.000	9.13	0.879	0.016	3.83	0.156	0.833	0.05	-0.281	0.763	0.20	-0.475	0.660	0.60
RC	0.522	0.000	27.28	0.609	0.000	37.30	0.560	0.000	28.65	0.593	0.000	31.97	0.577	0.000	31.91
IC	0.671	0.000	33.79	0.558	0.000	23.85	0.454	0.000	14.11	0.380	0.015	9.41	0.354	0.069	7.27

...Table VII continued

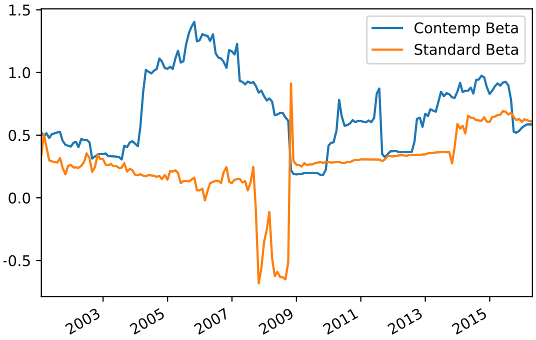
Panel C: Realized Variance – RV

	RV, 30			RV, 91			RV, 181			RV, 273			RV, 365		
	β	$p-val$	R^2	β	$p-val$	R^2	β	$p-val$	R^2	β	$p-val$	R^2	β	$p-val$	R^2
SP500 Sample															
RV	0.694	0.000	48.09	0.464	0.000	21.61	0.401	0.002	10.03	0.256	0.058	5.00	0.167	0.107	2.62
IV	1.074	0.000	48.54	0.847	0.000	26.44	0.754	0.000	15.91	0.645	0.000	12.83	0.569	0.000	11.87
RC	0.150	0.002	10.97	0.099	0.038	4.93	0.059	0.221	2.01	0.059	0.264	2.48	0.050	0.333	2.22
IC	0.143	0.001	7.43	0.044	0.201	0.80	-0.029	0.319	0.41	-0.040	0.162	0.97	-0.025	0.348	0.39
SP100 Sample															
RV	0.681	0.000	46.29	0.458	0.000	20.97	0.311	0.005	9.72	0.219	0.081	4.84	0.162	0.156	2.65
IV	1.023	0.000	47.09	0.796	0.000	25.60	0.639	0.000	16.89	0.539	0.000	11.68	0.484	0.000	10.70
RC	0.122	0.003	8.60	0.081	0.060	3.85	0.056	0.220	2.24	0.054	0.296	2.47	0.044	0.413	1.96
IC	0.126	0.001	6.67	0.033	0.342	0.47	-0.021	0.518	0.23	-0.061	0.073	2.46	-0.048	0.120	1.74
DJ30 Sample															
RV	0.660	0.000	43.56	0.436	0.000	19.07	0.385	0.002	9.23	0.237	0.073	4.21	0.136	0.212	1.68
IV	0.960	0.000	45.34	0.723	0.000	23.92	0.626	0.000	14.01	0.513	0.000	10.59	0.442	0.000	9.18
RC	0.102	0.005	8.61	0.069	0.061	4.32	0.038	0.337	1.52	0.034	0.408	1.45	0.023	0.556	0.82
IC	0.100	0.001	6.29	0.036	0.193	0.86	-0.033	0.203	0.88	-0.048	0.094	2.14	-0.042	0.144	1.72

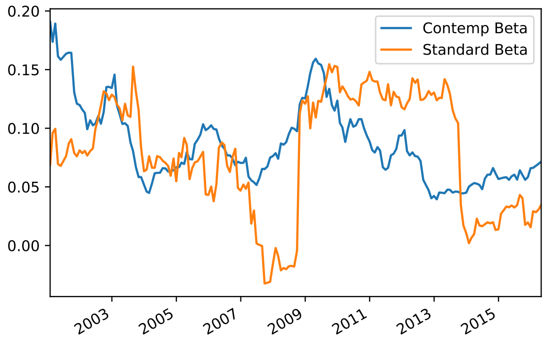
Figure 1. Beta Comparison: Contemporaneous vs. Traditional Approach

The figure shows the time series of the variance (VRP) and correlation betas (CRP), estimated using our novel, contemporaneous approach (Contemp Beta) as well as the traditional, predictive approach (Standard Beta). The contemporaneous approach uses a 12-months historical window of daily returns, and the standard approach uses a historical rolling window of 60 months. We depict betas for 30, 91, 182, and 365-day variance and correlation risks.

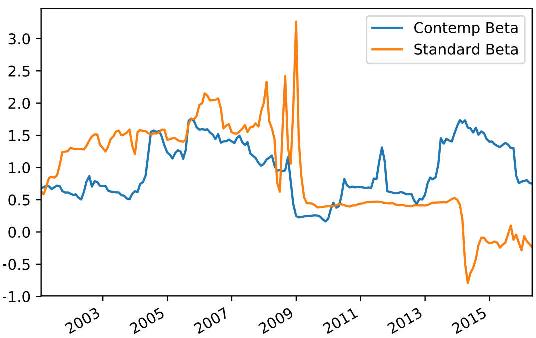
(a) 30-day VRP Betas



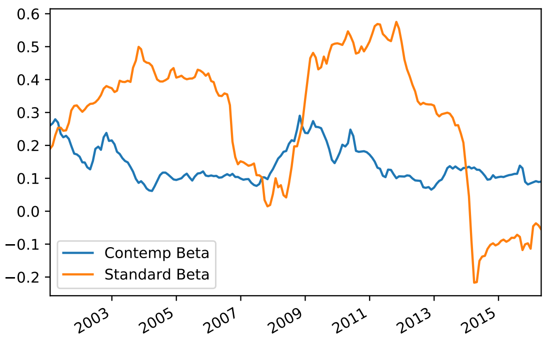
(b) 30-day CRP Betas



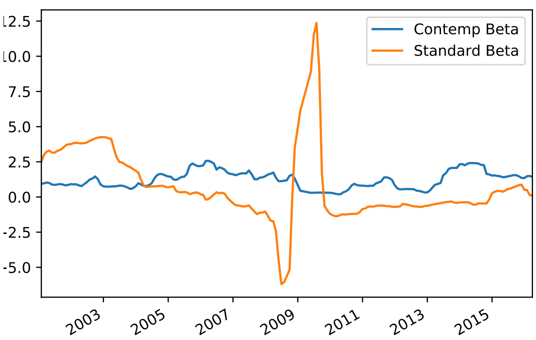
(c) 91-day VRP Betas



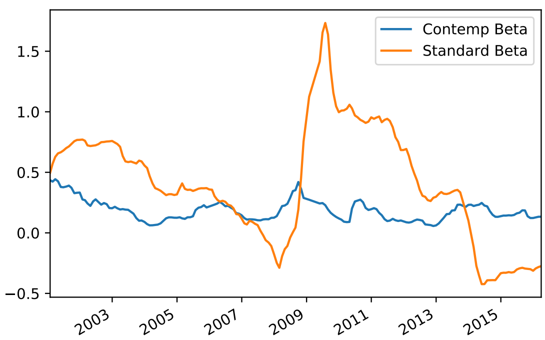
(d) 91-day CRP Betas



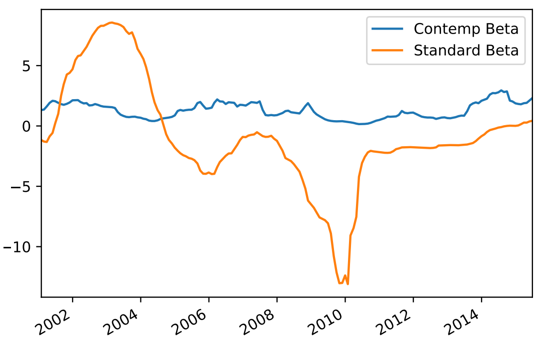
(e) 182-day VRP Betas



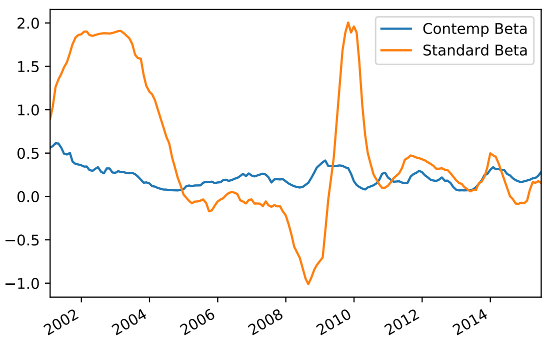
(f) 182-day CRP Betas



(g) 365-day VRP Betas



(h) 365-day CRP Betas



VII. Appendix

A. Setup

The consumption dynamics is given by

$$\begin{cases} \frac{dC_t}{C_t} & = \mu_c dt + \delta_c \sqrt{V_t} dB_{c,t} \\ dV_t & = \kappa_1(\bar{V} - V_t)dt + \sigma_1 \sqrt{V_t} dB_{V,t} + \sigma_\rho d\rho_t \\ d\rho_t & = \kappa_2(\bar{\rho} - \rho_t)dt + \sigma_2 \sqrt{\rho_t} dB_{\rho,t}, \end{cases} \tag{A1}$$

where $\mu_c, \kappa_1, \kappa_2, \sigma_1, \sigma_\rho, \sigma_2, \bar{V}, \bar{\rho} \in \mathbb{R}$ and B_c, B_V and B_ρ are standard Brownian motions. Plugging in the expression for $d\rho$ into the second equation above, we obtain the variance process:

$$dV_t = [\kappa_1(\bar{V} - V_t) + \bar{\kappa}_2(\bar{\rho} - \rho_t)]dt + \sigma_1 \sqrt{V_t} dB_{V,t} + \bar{\sigma}_\rho \sqrt{\rho_t} dB_{\rho,t}, \tag{A2}$$

where $\bar{\kappa}_2 = \sigma_\rho \kappa_2$, and $\bar{\sigma}_\rho = \sigma_\rho \sigma_2$.

The representative agent in the economy has Epstein-Zin recursive preferences. The intertemporal value function is defined recursively (see Duffie and Epstein (1992b)) by

$$J_t = \mathrm{E}_t \left[\int_t^T f(C_s, J_s) ds \right] \tag{A3}$$

Thus the representative investor chooses consumption C in order to maximize the value function:

$$J_t = \max_{C_s} \mathrm{E}_t \left[\int_t^T f(C_s, J_s) ds \right], \tag{A4}$$

where the normalized aggregator $f(C_t, J_t)$ is given by

$$f(C_t, J_t) = \frac{\beta}{1 - \frac{1}{\psi}} (1 - \gamma) J_t \left[\left(\frac{C_t}{((1 - \gamma)J_t)^{\frac{1}{1-\gamma}}} \right)^{1 - \frac{1}{\psi}} - 1 \right] \tag{A5}$$

with relative risk aversion γ and elasticity of intertemporal substitution ψ. To simplify notation restate f as follows:

$$f(C_t, J_t) = \frac{\beta}{1 - \frac{1}{\psi}} (1 - \gamma) J_t [G_t - 1] = \theta J [\beta G_t - \beta], \tag{A6}$$

where

$$G_t := \left(\frac{C_t}{((1-\gamma)J_t)^{\frac{1}{1-\gamma}}} \right)^{1-\frac{1}{\psi}} \quad \text{and } \theta := \frac{1-\gamma}{1-\frac{1}{\psi}} \tag{A7}$$

The value function follows the Hamilton-Jacobi-Bellman equation

$$\max_C \{f(C,J) + \mathcal{A}J\} = 0, \tag{A8}$$

where $\mathcal{A}$ denotes the infinitesimal generator.

B. *Solving for Equilibrium*

Conjecture. The conjectured solution for J has the following form

$$J(W_t, V_t, \rho_t) = \exp(A_0 + A_1 V_t + A_2 \rho_t) \frac{W_t^{1-\gamma}}{1-\gamma} \tag{A9}$$

Approximation. To substitute consumption for wealth in the value function we use the continuous-time log-linear approximation (Chacko and Viceira (2005)). With g_1 denoting the long-run mean of the consumption-wealth ratio (lower case variables denotes log variables) $g_1 = \exp(\mathrm{E}[c_t - w_t])$ we can write

$$\frac{C_t}{W_t} = \exp(\log(\frac{C_t}{W_t})) = \exp(c_t - w_t) \approx g_1 - g_1 \log(g_1) + g_1 \log(\frac{C_t}{W_t}) \tag{A10}$$

Compute the partial derivatives of J and f:

$$\begin{aligned} J_W &:= \frac{\partial J(W,V,\rho)}{\partial W} \\ &= \frac{\partial}{\partial W} \left[\exp(A_0 + A_1 V_t + A_2 \rho_t) \frac{W_t^{1-\gamma}}{1-\gamma} \right] \\ &= \exp(A_0 + A_1 V_t + A_2 \rho_t) W_t^{-\gamma} \end{aligned} \tag{A11}$$

and

$$
\begin{aligned}
f_C &:= \frac{\partial f(C,J)}{\partial C} \\
&= \frac{\partial}{\partial C}\left[\frac{\beta}{1-\frac{1}{\psi}}(1-\gamma)J_t\left[\left(\frac{C_t}{((1-\gamma)J_t)^{\frac{1}{1-\gamma}}}\right)^{1-\frac{1}{\psi}}-1\right]\right] \\
&= \frac{\partial}{\partial C}\left[\frac{\beta}{1-\frac{1}{\psi}}\frac{(1-\gamma)J_t}{((1-\gamma)J_t)^{\frac{1}{1-\gamma}(1-\frac{1}{\psi})}}C_t^{1-\frac{1}{\psi}}\right] \\
&= \beta\frac{(1-\gamma)J_t}{((1-\gamma)J_t)^{\frac{\psi-1}{(1-\gamma)\psi}}}C_t^{-\frac{1}{\psi}}
\end{aligned}
\tag{A12}
$$

and hence

$$
\begin{aligned}
f_C\frac{((1-\gamma)J_t)^{\frac{\psi-1}{(1-\gamma)\psi}}}{(1-\gamma)J_t} &= \beta C_t^{-\frac{1}{\psi}} \\
\Rightarrow \quad f_C((1-\gamma)J_t)^{\frac{\psi\gamma-1}{(1-\gamma)\psi}} &= \beta C_t^{-\frac{1}{\psi}} \\
\Rightarrow \quad f_C^{-\psi}\beta^{\psi}((1-\gamma)J_t)^{\frac{1-\psi\gamma}{(1-\gamma)}} &= C_t
\end{aligned}
\tag{A13}
$$

Use the envelope condition $f_C = J_W$ to rewrite the last:

$$
J_W^{-\psi}\beta^{\psi}((1-\gamma)J_t)^{\frac{1-\psi\gamma}{(1-\gamma)}} = C_t \tag{A14}
$$

and plug in J_t and J_W from equations (A9) and (A11), respectively, to obtain

$$
\begin{aligned}
C_t &= (\exp(A_0 + A_1V_t + A_2\rho_t)^{-\psi}W_t^{\gamma\psi})\beta^{\psi}\exp(A_0 + A_1V_t + A_2\rho_t)^{\frac{1-\psi\gamma}{(1-\gamma)}}W_t^{1-\psi\gamma} \\
&= \beta^{\psi}\exp(A_0 + A_1V_t + A_2\rho_t)^{\frac{1-\psi}{(1-\gamma)}}W_t
\end{aligned}
\tag{A15}
$$

Thus, we obtain the wealth-consumption ratio:

$$
\begin{aligned}
\frac{C_t}{W_t} &= \beta^{\psi}\exp(A_0 + A_1V_t + A_2\rho_t)^{\frac{1-\psi}{1-\gamma}} \\
&= \beta^{\psi}\exp(A_{0a} + A_{1a}V_t + A_{2a}\rho_t)
\end{aligned}
\tag{A16}
$$

with $A_{ia} = A_i \frac{1-\psi}{1-\gamma}$.

To obtain the value function as function of consumption, we rewrite W_t as follows:

$$\begin{aligned} W_t &= C_t \beta^{-\psi} \exp(A_0 + A_1 V_t + A_2 \rho_t)^{-\frac{1-\psi}{1-\gamma}} \\ \Rightarrow \quad W_t^{1-\gamma} &= C_t^{1-\gamma} \beta^{-\psi(1-\gamma)} \exp(A_0 + A_1 V_t + A_2 \rho_t)^{-(1-\psi)} \end{aligned} \tag{A17}$$

and substitute for $W^{1-\gamma}$ in $J(W_t, V_t, \rho_t) = \exp(A_0 + A_1 V_t + A_2 \rho_t) \frac{W_t^{1-\gamma}}{1-\gamma}$:

$$\begin{aligned} J(C_t, V, \rho) &= \exp(A_0 + A_1 V_t + A_2 \rho_t) \frac{C_t^{1-\gamma} \beta^{-\psi(1-\gamma)} \exp(A_0 + A_1 V_t + A_2 \rho_t)^{-(1-\psi)}}{1-\gamma} \\ &= \exp[\psi(A_0 + A_1 V_t + A_2 \rho_t)] \beta^{-\psi(1-\gamma)} \frac{C_t^{1-\gamma}}{1-\gamma} \end{aligned} \tag{A18}$$

Using the expression (A16) for $\frac{C_t}{W_t}$ and the conjecture (A9) for J we get

$$\begin{aligned} \beta G &= \beta \left(\frac{C_t}{((1-\gamma) J_t)^{\frac{1}{1-\gamma}}} \right)^{1-\frac{1}{\psi}} = \beta \left(\frac{W_t \beta^{\psi} \exp(A_0 + A_1 V_t + A_2 \rho_t)^{\frac{1-\psi}{(1-\gamma)}}}{((1-\gamma) \exp(A_0 + A_1 V_t + A_2 \rho_t) \frac{W_t^{1-\gamma}}{1-\gamma})^{\frac{1}{1-\gamma}}} \right)^{1-\frac{1}{\psi}} \\ &= \beta^{\psi} \left(\exp[(A_0 + A_1 V_t + A_2 \rho_t) \frac{1-\psi}{1-\gamma}] \right) = \frac{C_t}{W_t}, \end{aligned} \tag{A19}$$

and the $\frac{C_t}{W_t}$ approximation as in (A10) then gives

$$\beta G = \frac{C_t}{W_t} \approx g_1 - g_1 \log(g_1) + g_1 \log(\beta G), \tag{A20}$$

where from above we know that

$$\begin{aligned} \log(\beta G) &= \log(\beta^{\psi} (\exp[(A_0 + A_1 V_t + A_2 \rho_t) \frac{1-\psi}{1-\gamma}]) \\ &= \psi \log(\beta) + (A_0 + A_1 V_t + A_2 \rho_t) \frac{1-\psi}{1-\gamma}. \end{aligned} \tag{A21}$$

Thus, we arrive at the useable form of the aggregator function through approximation:

$$\begin{aligned} f &= \theta J[\beta G - \beta] \approx \theta J[g_1 - g_1 \log(g_1) + g_1 \log(\beta G) - \beta] \\ &= \theta J \left[g_1 - \beta - g_1 \log(g_1) + g_1 (\psi \log(\beta) + (A_0 + A_1 V_t + A_2 \rho_t) \frac{1-\psi}{1-\gamma}) \right] \\ &= \theta J \left[\xi + g_1 (A_0 + A_1 V_t + A_2 \rho_t) \frac{1-\psi}{1-\gamma} \right], \end{aligned} \tag{A22}$$

where $\xi = g_1 - \beta - g_1 \log(g_1) + g_1 \psi \log(\beta)$.

HJB Equation. Substituting consumption for wealth in the value function J_t and plugging it in the Hamilton-Jacobi-Bellman equation (A8), results in a PDE that needs to be solved for obtaining the value function parameters A_0, A_1, and A_2:

$$f(C, J) + \mathcal{A}J(C, V, \rho) = 0, \tag{A23}$$

where the state variables are given by:

$$\begin{cases} \frac{dC_t}{C_t} &= \mu_c dt + \delta_c \sqrt{V_t} dB_{c,t} \\ dV_t &= [\kappa_1(\bar{V} - V_t) + \bar{\kappa}_2(\bar{\rho} - \rho_t)]dt + \sigma_1 \sqrt{V_t} dB_{V,t} + \bar{\sigma}_\rho \sqrt{\rho_t} dB_{\rho,t} \\ d\rho_t &= \kappa_2(\bar{\rho} - \rho_t)dt + \sigma_2 \sqrt{\rho_t} dB_{\rho,t}, \end{cases} \tag{A24}$$

and $\mu_c, \kappa_1, \kappa_2, \sigma_1, \sigma_\rho, \sigma_2, \bar{V}, \bar{\rho} \in \mathbb{R}$ and B_c, B_V and B_ρ are standard Brownian motions.

$$\begin{aligned} \mathcal{A}J(C, V, \rho) = &\, C\mu_c J_C + [\kappa_1(\bar{V} - V_t) + \bar{\kappa}_2(\bar{\rho} - \rho_t)]J_V + \kappa_2(\bar{\rho} - \rho)J_\rho \\ &+ \frac{1}{2}[C_t^2 V_t \delta_c^2 J_{CC} + [\sigma_1^2 V_t + \bar{\sigma}_\rho^2 \rho_t]J_{VV} + 2\bar{\sigma}_\rho \sigma_2 \rho_t J_{V\rho} + \sigma_2^2 \rho_t J_{\rho\rho}], \end{aligned} \tag{A25}$$

where the respective derivatives of the value function are as follows:

$$\begin{aligned} J_C &= \frac{\partial}{\partial C}\left[\exp[\psi(A_0 + A_1 V_t + A_2 \rho_t)]\beta^{-\psi(1-\gamma)} \frac{C_t^{1-\gamma}}{1-\gamma}\right] = \frac{J(1-\gamma)}{C} \\ J_V &= A_1 \psi J \\ J_\rho &= A_2 \psi J, \end{aligned} \tag{A26}$$

and

$$\begin{aligned} J_{CC} &= -\frac{J(1-\gamma)\gamma}{C^2} \\ J_{V\rho} &= J_{\rho V} = \frac{\partial}{\partial \rho} J_V = A_1 \psi \frac{\partial}{\partial \rho} J = A_2 A_1 \psi^2 J \\ J_{VV} &= A_1^2 \psi^2 J \\ J_{\rho\rho} &= A_2^2 \psi^2 J \end{aligned} \tag{A27}$$

Using the approximation of $f(C,J)$ from (A22) and the expansion $\mathcal{A}J$ above, we get

$$\begin{aligned} f(C,J)+\mathcal{A}J = \theta J\left[\xi + g_1(A_0 + A_1V_t + A_2\rho_t)\frac{1-\psi}{1-\gamma}\right] \\ + C\mu_c J_C + [\kappa_1(\bar{V}-V_t) + \bar{\kappa}_2(\bar{\rho}-\rho_t)]J_V + \kappa_2(\bar{\rho}-\rho)J_\rho \\ + \frac{1}{2}[C_t^2 V_t \delta_c^2 J_{CC} + [\sigma_1^2 V_t + \bar{\sigma}_\rho^2\rho_t]J_{VV} + 2\bar{\sigma}_\rho\sigma_2\rho_t J_{V\rho} + \sigma_2^2\rho_t J_{\rho\rho}]. \end{aligned} \tag{A28}$$

Using

$$\theta\frac{1-\psi}{1-\gamma} = \frac{1-\gamma}{1-\frac{1}{\psi}}\frac{1-\psi}{1-\gamma} = -\frac{1-\psi}{\frac{1-\psi}{\psi}} = -\psi \tag{A29}$$

and plugging in the respective partial derivatives, we end up with the PDE to solve:

$$\begin{aligned} 0 = J[\theta\xi - g_1\psi A_0 - g_1\psi A_1 V_t - g_1\psi A_2\rho_t] \\ + J[\mu_c(1-\gamma) + A_1\psi\kappa_1\bar{V} + A_1\psi\bar{\kappa}_2\bar{\rho} + \kappa_2\bar{\rho}A_2\psi + V_t(-A_1\psi\kappa_1) \\ + \rho_t(-A_1\psi\bar{\kappa}_2 - \kappa_2A_2\psi)] \\ + \frac{J}{2}[V_t(-\delta_c^2(1-\gamma)\gamma + \sigma_1^2A_1^2\psi^2) + \rho_t(\bar{\sigma}_\rho^2A_1^2\psi^2 + 2\bar{\sigma}_\rho\sigma_2A_2A_1\psi^2 + \sigma_2^2A_2^2\psi^2)]. \end{aligned} \tag{A30}$$

or after collecting respective coefficients:

$$\begin{aligned} 0 = J[\theta\xi - g_1\psi A_0 + \mu_c(1-\gamma) + A_1\psi\kappa_1\bar{V} + A_1\psi\bar{\kappa}_2\bar{\rho} + \kappa_2\bar{\rho}A_2\psi \\ + V_t(-A_1\psi\kappa_1 - g_1\psi A_1 + \frac{1}{2}(-\delta_c^2(1-\gamma)\gamma + \sigma_1^2A_1^2\psi^2) \\ + \rho_t(-A_1\psi\bar{\kappa}_2 - \kappa_2A_2\psi - g_1\psi A_2 + \frac{1}{2}(\bar{\sigma}_\rho^2A_1^2\psi^2 + 2\bar{\sigma}_\rho\sigma_2A_2A_1\psi^2 + \sigma_2^2A_2^2\psi^2))]. \end{aligned} \tag{A31}$$

Setting the expressions next to V_t, ρ_t and the free terms to zero, we end up with the system:

$$\begin{cases} 0 &= \theta\xi - g_1\psi A_0 + \mu_c(1-\gamma) + A_1\psi\kappa_1\bar{V} + A_1\psi\bar{\kappa}_2\bar{\rho} + \kappa_2\bar{\rho}A_2\psi \\ 0 &= -A_1\psi\kappa_1 - g_1\psi A_1 + \frac{1}{2}(-\delta_c^2(1-\gamma)\gamma + \sigma_1^2A_1^2\psi^2) \\ 0 &= -A_1\psi\bar{\kappa}_2 - \kappa_2A_2\psi - g_1\psi A_2 + \frac{1}{2}(\bar{\sigma}_\rho^2A_1^2\psi^2 + 2\bar{\sigma}_\rho\sigma_2A_2A_1\psi^2 + \sigma_2^2A_2^2\psi^2). \end{cases} \tag{A32}$$

Solving for A_1:

$$0 = \sigma_1^2\psi^2A_1^2 + 2(-g_1\psi - \psi\kappa_1)A_1 + (-\delta_c^2(1-\gamma)\gamma). \tag{A33}$$

The solution is given by

$$A_1 = \frac{-b_1 \pm \sqrt{b_1^2 - a_1 c_1}}{a_1}, \tag{A34}$$

where

$$\begin{aligned} a_1 &= \sigma_1^2 \psi^2 \\ b_1 &= -(g_1 + \kappa_1)\psi \\ c_1 &= -\delta_c^2 (1-\gamma)\gamma. \end{aligned} \tag{A35}$$

Solving for A_2:

$$0 = A_2^2 \sigma_2^2 \psi + A_2 2[(-\kappa_2 - g_1) + \bar{\sigma}_\rho \sigma_2 A_1 \psi] + (\bar{\sigma}_\rho^2 A_1^2 \psi - 2A_1 \bar{\kappa}_2). \tag{A36}$$

The solution is given by

$$A_2 = \frac{-b_2 \pm \sqrt{b_2^2 - a_2 c_2}}{a_2}, \tag{A37}$$

where

$$\begin{aligned} a_2 &= \sigma_2^2 \psi \\ b_2 &= [-(\kappa_2 + g_1) + \bar{\sigma}_\rho \sigma_2 A_1 \psi] \\ c_2 &= \bar{\sigma}_\rho^2 A_1^2 \psi - 2A_1 \bar{\kappa}_2. \end{aligned} \tag{A38}$$

A_0 follows from the first equation of the system (A32).

C. *The Pricing Kernel and the Riskfree Rate*

The pricing kernel for the recursive utility (see Duffie and Epstein (1992a)) is defined as

$$\pi_t = \exp\left[\int_0^t f_J(C_s, J_s)ds\right] f_C(C_t, J_t). \tag{A39}$$

Differentiating both sides of the above equation we obtain the stochastic differential equation:

$$\frac{d\pi_t}{\pi_t} = f_J(C, J)dt + \frac{df_C(C, J)}{f_C(C, J)}. \tag{A40}$$

Applying Ito Lemma to expand $df_C(C, J)$, and setting the drift of the pricing kernel to minus the short rate $-rdt$ (which we can also compute, bit do not need to for now), we obtain the expression of the pricing kernel with the risk premiums λ_1, λ_2, and λ_3 for all priced sources of risk in our economy—consumption, aggregate variance, and correlation risks, respectively.

Combining the definition of G from (A7), and the expression for f_C from (A12) we get

$$f_C = \beta \frac{G}{C}(1-\gamma)J \tag{A41}$$

and then using formulation of J as a function of consumption given by (A18), the $\beta G = \beta^{\psi} \exp(A_0 + A_1 V_t + A_2 \rho_t)^{\frac{1-\psi}{(1-\gamma)}}$ from (A19) we end up with:

$$f_C = \beta^{\psi\gamma} \exp[(A_0 + A_1 V_t + A_2 \rho_t)\frac{1-\gamma\psi}{1-\gamma}]C_t^{-\gamma}. \tag{A42}$$

The first derivatives of f_C then can be computed as

$$\begin{aligned} f_{CC} &= -\gamma \beta^{\psi\gamma} \exp[(A_0 + A_1 V_t + A_2 \rho_t)\frac{1-\gamma\psi}{1-\gamma}]C_t^{-\gamma-1} = -\gamma \frac{f_C}{C} \\ f_{CV} &= A_1 \frac{1-\gamma\psi}{1-\gamma} \beta^{\psi\gamma} \exp[(A_0 + A_1 V_t + A_2 \rho_t)\frac{1-\gamma\psi}{1-\gamma}]C_t^{-\gamma} = A_1 \frac{1-\gamma\psi}{1-\gamma} f_C \\ f_{C\rho} &= A_2 \frac{1-\gamma\psi}{1-\gamma} \beta^{\psi\gamma} \exp[(A_0 + A_1 V_t + A_2 \rho_t)\frac{1-\gamma\psi}{1-\gamma}]C_t^{-\gamma} = A_2 \frac{1-\gamma\psi}{1-\gamma} f_C. \end{aligned} \tag{A43}$$

Plugging in these partials in the (A40), collecting deterministic terms in the expanded SDE for the pricing kernel into $-rdt$ and grouping the random terms, we get the process for our pricing kernel with the well-defined risk premiums:

$$\frac{d\pi_t}{\pi_t} = -rdt - \lambda_1 dB_{c,t} - \lambda_2 dB_{V,t} - \lambda_3 dB_{\rho,t}, \tag{A44}$$

where

$$\begin{cases} \lambda_1 &= \gamma \delta_c \sqrt{V_t} \\ \lambda_2 &= -\frac{1-\gamma\psi}{1-\gamma} A_1 \sigma_1 \sqrt{V_t} \\ \lambda_3 &= -\frac{1-\gamma\psi}{1-\gamma} \left(A_1 \sigma_\rho + A_2 \sigma_2\right) \sqrt{\rho_t} \end{cases} \tag{A45}$$

To determine the riskfree rate r, note that it is formed by all deterministic terms in (A40), i.e,

$$-rdt = f_J dt + E\left[\frac{df_C(C,J)}{f_C(C,J)}\right]. \tag{A46}$$

The approximate expression for f_J is as follows:

$$\begin{aligned} f_J &= (\theta-1)\beta G - \beta\theta \\ &\approx (\theta-1)[g_1 - g_1\log(g_1) + g_1\log(\beta G)] - \beta\theta \\ &= \xi_1 - g_1(A_1V_t + A_2\rho_t)\frac{1-\psi\gamma}{1-\gamma}, \end{aligned} \tag{A47}$$

where $\xi_1 = (\theta-1)\xi - \beta - g_1A_0\frac{1-\psi\gamma}{1-\gamma}$.

To get the $E\left[\frac{df_C(C,J)}{f_C(C,J)}\right]$, we also need the second-order partials for the $df_C(C,J)$ expansion, and they can be computed as

$$f_{CCC} = \frac{\partial}{\partial C}f_{CC} = -\gamma\frac{\partial}{\partial C}\frac{f_C}{C} = \gamma(\gamma+1)\frac{f_C}{C^2} \tag{A48}$$

$$f_{CVV} = \frac{\partial}{\partial V}f_{CV} = A_1^2(\frac{1-\gamma\psi}{(1-\gamma)})^2 f_C \tag{A49}$$

and

$$f_{C\rho\rho} = \frac{\partial}{\partial\rho}f_{C\rho} = A_2^2(\frac{1-\gamma\psi}{(1-\gamma)})^2 f_C \tag{A50}$$

$$f_{CV\rho} = \frac{\partial}{\partial\rho}f_{CV} = A_1A_2(\frac{1-\gamma\psi}{(1-\gamma)})^2 f_C \tag{A51}$$

Plugging in all the necessary terms into (A46), and dividing by dt, we get

$$\begin{aligned} -r_f &= \xi_1 - \gamma\mu_c + \frac{1-\gamma\psi}{1-\gamma}[A_1\kappa_1\bar{V} + A_1\bar{\kappa}_2\bar{\rho} + A_2\kappa_2\bar{\rho}] \\ &+ \frac{1-\gamma\psi}{1-\gamma}[-\kappa_1A_1 - g_1A_1 + \frac{1}{2}\gamma(\gamma+1)\delta_c^2 + \frac{1}{2}A_1^2\frac{1-\gamma\psi}{(1-\gamma)}\sigma_1^2]V_t \\ &+ \frac{1-\gamma\psi}{1-\gamma}[-A_2\kappa_2 - A_1\bar{\kappa}_2 - g_1A_2 + \frac{1-\gamma\psi}{2(1-\gamma)}(A_2^2\sigma_2^2 + 2A_1A_2\bar{\sigma}_\rho\sigma_2 + A_1^2\bar{\sigma}_\rho)]\rho_t \end{aligned} \tag{A52}$$

so that we can write the riskfree rate as $r = r_0 + r_1V_t + r_2\rho_t$ with

$$\begin{cases} r_0 &= -(\xi_1 - \gamma\mu_c + \frac{1-\gamma\psi}{1-\gamma}[A_1\kappa_1\bar{V} + A_1\bar{\kappa}_2\bar{\rho} + A_2\kappa_2\bar{\rho}]) \\ r_1 &= -\frac{1-\gamma\psi}{1-\gamma}[-\kappa_1A_1 - g_1A_1 + \frac{1}{2}\gamma(\gamma+1)\delta_c^2 + \frac{1}{2}A_1^2\frac{1-\gamma\psi}{(1-\gamma)}\sigma_1^2] \\ r_2 &= -\frac{1-\gamma\psi}{1-\gamma}[-A_2\kappa_2 - A_1\bar{\kappa}_2 - g_1A_2 + \frac{1-\gamma\psi}{2(1-\gamma)}(A_2^2\sigma_2^2 + 2A_1A_2\bar{\sigma}_\rho\sigma_2 + A_1^2\bar{\sigma}_\rho)] \end{cases} \tag{A53}$$

D. Aggregate Market (or Consumption) Claim

Use the wealth-consumption ratio from the equation (A16):

$$\frac{C_t}{W_t} = \beta^{\psi} \exp(A_{0a} + A_{1a}V_t + A_{2a}\rho_t). \tag{A54}$$

Rewrite it as

$$W_t = C_t\beta^{-\psi} \exp(-A_{0a} - A_{1a}V_t - A_{2a}\rho_t). \tag{A55}$$

and apply Ito Lemma to both sides:

$$\begin{aligned} dW_t &= dC_t\beta^{-\psi} \exp(-A_{0a} - A_{1a}V_t - A_{2a}\rho_t) + C_t d\beta^{-\psi} \exp(-A_{0a} - A_{1a}V_t - A_{2a}\rho_t) \\ &\quad + dC_t d\beta^{-\psi} \exp(-A_{0a} - A_{1a}V_t - A_{2a}\rho_t). \\ dW_t &= dC_t\frac{W_t}{C_t} + C_t\frac{W_t}{C_t}[-A_{1a}dV_t - A_{2a}d\rho_t + \frac{1}{2}A_{1a}^2(dV_t)^2 + \frac{1}{2}A_{2a}^2(d\rho_t)^2 + A_{1a}A_{2a}dV_t d\rho_t] \\ &\quad + dC_t\frac{W_t}{C_t}[-A_{1a}dV_t - A_{2a}d\rho_t]. \end{aligned} \tag{A56}$$

By assumption $dC_t dV_t = 0$ and $dC_t d\rho_t = 0$, and dividing both sides by W_t, and cancelling the terms we get:

$$\frac{dW_t}{W_t} = \frac{dC_t}{C_t} - A_{1a}dV_t - A_{2a}d\rho_t + \frac{1}{2}A_{1a}^2(dV_t)^2 + \frac{1}{2}A_{2a}^2(d\rho_t)^2 + A_{1a}A_{2a}dV_t d\rho_t. \tag{A57}$$

Collecting deterministic parts on the other side into a generic dt term (either all in ζ'_W, or partially in ζ_W) and expanding the state variables, we get two useful expressions for the aggregate market:

$$\begin{aligned} \frac{dW_t}{W_t} &= \zeta_W dt + \delta_c\sqrt{V_t}dB_{c,t} - A_{1a}dV_t - A_{2a}d\rho_t \\ &= \zeta'_W dt + \delta_c\sqrt{V_t}dB_{c,t} - A_{1a}\sigma_1\sqrt{V_t}dB_{V,t} - [A_{1a}\bar{\sigma}_\rho\sqrt{\rho_t} + A_{2a}\sigma_2\sqrt{\rho_t}]dB_{\rho,t} \end{aligned} \tag{A58}$$

The instantaneous variance of the aggregate market $\frac{dW_t}{W_t}$ can be calculated as follows

$$V_{W,t} = \left(\frac{dW_t}{W_t}\right)^2 /dt = (\delta_c^2 + A_{1a}^2\sigma_1^2)V_t + (A_{1a}\bar{\sigma}_\rho + A_{2a}\sigma_2)^2\rho_t \tag{A59}$$

And with Ito's Lemma it follows that

$$dV_{W,t} = (\delta_c^2 + A_{1a}^2\sigma_1^2)dV_t + (A_{1a}\bar{\sigma}_\rho + A_{2a}\sigma_2)^2 d\rho_t \tag{A60}$$

E. Individual Dividend Claims

The individual dividend process for asset i is given by

$$\frac{dD_{i,t}}{D_{i,t}} = \mu_D dt + \sigma_D\sqrt{V_{i,t}}dB_{D_i,t} + \sigma_{DC}\sqrt{V_t}dB_{c,t} \tag{A61}$$

thereby we assume that $dB_{D_i,t}$ and $dB_{c,t}$ are not correlated.

First, derive the individual dividend-price ratio. Denote the dividend claim price by $S_{i,t}$. For simplicity we drop the index i, hence $D_{i,t} = D_t$ and $S_{i,t} = S_t$.

$$\frac{D_t}{S_t} = \exp(A_{0m} + A_{1m}V_t + A_{2m}\rho_t) \tag{A62}$$

or equivalently

$$S_t = D_t\exp(-A_{0m} - A_{1m}V_t - A_{2m}\rho_t) \tag{A63}$$

Applying the Ito Lemma in the same way as for the consumption claim:

$$\begin{aligned} \frac{dS_t}{S_t} &= \frac{d\exp(-A_{0m} - A_{1m}V_t - A_{2m}\rho_t)}{\exp(-A_{0m} - A_{1m}V_t - A_{2m}\rho_t)} + \frac{dD_t}{D_t} \\ &= -A_{1m}dV_t - A_{2m}d\rho + \frac{1}{2}[A_{1m}^2(dV_t)^2 + A_{2m}^2(d\rho)^2] + A_{1m}A_{2m}dV_t d\rho + \frac{dD_t}{D_t} \end{aligned} \tag{A64}$$

Plugging in the equations for $dV_t, d\rho_t$ and with

$$\begin{aligned} (dV_t)^2 &= \sigma_1^2 V_t dt + \bar{\sigma}_\rho\rho_t dt \\ (d\rho_t)^2 &= \sigma_2^2\rho_t dt \\ dV_t d\rho_t &= \bar{\sigma}_\rho\sigma_2\rho_t dt \end{aligned} \tag{A65}$$

where we assumed that $dB_{V,t},\ dB_{\rho,t}$ are uncorrelated we get

$$\begin{aligned}\frac{dS_t}{S_t} = \frac{dD_t}{D_t} &- [A_{1m}(\kappa_1(\bar{V}-V_t)+\bar{\kappa}_2(\bar{\rho}-\rho_t))+A_{2m}\kappa_2(\bar{\rho}-\rho_t) \\ &- \frac{1}{2}(A_{1m}^2[\sigma_1^2 V_t+\bar{\sigma}_\rho\rho_t]+A_{2m}^2\sigma_2^2\rho_t)-A_{1m}A_{2m}\bar{\sigma}_\rho\sigma_2\rho_t]dt \\ &- [A_{1m}\sigma_1\sqrt{V_t}]dB_{V,t}-[A_{1m}\bar{\sigma}_\rho\sqrt{\rho_t}+A_{2m}\sigma_2\sqrt{\rho_t}]dB_{\rho,t}.\end{aligned} \tag{A66}$$

And hence

$$\begin{aligned}\mathrm{E}_t[\frac{dS_t}{S_t}]/dt = \mu_D &- A_{1m}(\kappa_1(\bar{V}-V_t)+\bar{\kappa}_2(\bar{\rho}-\rho_t))-A_{2m}\kappa_2(\bar{\rho}-\rho_t) \\ &+ \frac{1}{2}(A_{1m}^2[\sigma_1^2 V_t+\bar{\sigma}_\rho\rho_t]+A_{2m}^2\sigma_2^2\rho_t)+A_{1m}A_{2m}\bar{\sigma}_\rho\sigma_2\rho_t\end{aligned} \tag{A67}$$

Next compute the risk premium:

$$\frac{d\pi_t}{\pi_t}\frac{dS_t}{S_t} = -\lambda_1\sigma_{DC}\sqrt{V_t}dt+\lambda_2[A_{1m}\sigma_1\sqrt{V_t}]dt+\lambda_3[A_{1m}\bar{\sigma}_\rho\sqrt{\rho_t}+A_{2m}\sigma_2\sqrt{\rho_t}]dt \tag{A68}$$

so that

$$-\mathrm{E}_t[\frac{d\pi_t}{\pi_t}\frac{dS_t}{S_t}]/dt = \lambda_1\sigma_{DC}\sqrt{V_t}-\lambda_2[A_{1m}\sigma_1\sqrt{V_t}]-\lambda_3[A_{1m}\bar{\sigma}_\rho\sqrt{\rho_t}+A_{2m}\sigma_2\sqrt{\rho_t}] \tag{A69}$$

Next approximate $\frac{D_t}{S_t} = \exp(A_{0m}+A_{1m}V_t+A_{2m}\rho_t)$ following the same procedure as for the wealth-consumption ratio:

$$\begin{aligned}\frac{D_t}{S_t} &\approx g_{1m}-g_{1m}\log(g_{1m})+g_{1m}\log(\frac{D_t}{S_t}) \\ &= g_{0m}+g_{1m}(A_{0m}+A_{1m}V_t+A_{2m}\rho_t),\end{aligned} \tag{A70}$$

where $g_{0m} = g_{1m}-g_{1m}\log(g_{1m})$

In order to obtain expressions for A_{im} we use the following pricing relation:

$$\begin{aligned}\mathrm{E}_t[\frac{dS_t}{S_t}]+\frac{D_t}{S_t}dt &= r_f dt-\mathrm{E}[\frac{d\pi}{\pi}\frac{dS_t}{S_t}] \\ \Rightarrow \quad 0 &= r_f-\mathrm{E}[\frac{d\pi}{\pi}\frac{dS_t}{S_t}]/dt-\mathrm{E}_t[\frac{dS_t}{S_t}]/dt-\frac{D_t}{S_t}\end{aligned} \tag{A71}$$

Inserting the equations step by step

$$
\begin{aligned}
\mathrm{E}_t[\frac{dS_t}{S_t}]/dt + \frac{D_t}{S_t} &= \mu_D - A_{1m}(\kappa_1(\bar{V} - V_t) + \bar{\kappa}_2(\bar{\rho} - \rho_t)) - A_{2m}\kappa_2(\bar{\rho} - \rho_t) \\
&+ \frac{1}{2}(A_{1m}^2[\sigma_1^2 V_t + \bar{\sigma}_\rho \rho_t] + A_{2m}^2\sigma_2^2\rho_t) + A_{1m}A_{2m}\bar{\sigma}_\rho\sigma_2\rho_t \\
&+ g_{0m} + g_{1m}(A_{0m} + A_{1m}V_t + A_{2m}\rho_t) \\
&= \mu_D + g_{0m} + g_{1m}A_{0m} - A_{1m}(\kappa_1\bar{V} + \bar{\kappa}_2\bar{\rho}) - A_{2m}\kappa_2\bar{\rho} \\
&+ V_t(A_{1m}\kappa_1 + \frac{1}{2}A_{1m}^2\sigma_1^2 + g_{1m}A_{1m}) \\
&+ \rho_t[A_{1m}\bar{\kappa}_2 + A_{2m}\kappa_2 + \frac{1}{2}(A_{1m}^2\bar{\sigma}_\rho + A_{2m}^2\sigma_2^2) + A_{1m}A_{2m}\bar{\sigma}_\rho\sigma_2 + g_{1m}A_{2m}]
\end{aligned} \tag{A72}
$$

And

$$
\begin{aligned}
r_f - \mathrm{E}_t[\frac{d\pi_t}{\pi_t}\frac{dS_t}{S_t}]/dt &= r_0 + r_1V_t + r_2\rho_t + \lambda_1\sigma_{DC}\sqrt{V_t} - \lambda_2[A_{1m}\sigma_1\sqrt{V_t}] \\
&- \lambda_3[A_{1m}\bar{\sigma}_\rho\sqrt{\rho_t} + A_{2m}\sigma_2\sqrt{\rho_t}] \\
&= r_0 + V_t(r_1 + \gamma\delta_c\sigma_{DC} + \frac{1-\gamma\psi}{1-\gamma}A_1\sigma_1^2A_{1m}) \\
&\rho_t[r_2 + \frac{1-\gamma\psi}{1-\gamma}(A_1\sigma_\rho A_{1m}\bar{\sigma}_\rho + A_1\sigma_\rho A_{2m}\sigma_2 + A_2\sigma_2A_{1m}\bar{\sigma}_\rho + A_2\sigma_2^2A_{2m})]
\end{aligned} \tag{A73}
$$

Next, sorting and grouping in terms of the $0 = r_f - \mathrm{E}[\frac{d\pi}{\pi}\frac{dS_t}{S_t}]/dt - \mathrm{E}_t[\frac{dS_t}{S_t}]/dt - \frac{D_t}{S_t}$ leads to the following system

$$
\begin{cases}
0 &= r_0 - \mu_D - g_{0m} - g_{1m}A_{0m} + A_{1m}(\kappa_1\bar{V} + \bar{\kappa}_2\bar{\rho}) + A_{2m}\kappa_2\bar{\rho} \\
0 &= V_t[r_1 + \gamma\delta_c\sigma_{DC} + \frac{1-\gamma\psi}{1-\gamma}A_1\sigma_1^2A_{1m} - A_{1m}\kappa_1 - \frac{1}{2}A_{1m}^2\sigma_1^2 - g_{1m}A_{1m}] \\
0 &= \rho_t[r_2 + \frac{1-\gamma\psi}{1-\gamma}(A_1\sigma_\rho A_{1m}\bar{\sigma}_\rho + A_1\sigma_\rho A_{2m}\sigma_2 + A_2\sigma_2A_{1m}\bar{\sigma}_\rho + A_2\sigma_2^2A_{2m}) \\
& \quad -A_{1m}\bar{\kappa}_2 - A_{2m}\kappa_2 - \frac{1}{2}(A_{1m}^2\bar{\sigma}_\rho + A_{2m}^2\sigma_2^2) - A_{1m}A_{2m}\bar{\sigma}_\rho\sigma_2 - g_{1m}A_{2m}]
\end{cases} \tag{A74}
$$

Solving for A_{1m}:

$$
\begin{aligned}
0 &= 2r_1 + 2\gamma\delta_c\sigma_{DC} + 2\frac{1-\gamma\psi}{1-\gamma}A_1\sigma_1^2A_{1m} - 2A_{1m}\kappa_1 - A_{1m}^2\sigma_1^2 - 2g_{1m}A_{1m} \\
&= A_{1m}^2\sigma_1^2 + 2[-\frac{1-\gamma\psi}{1-\gamma}A_1\sigma_1^2 + \kappa_1 + g_{1m}]A_{1m} - 2r_1 - 2\gamma\delta_c\sigma_{DC}
\end{aligned} \tag{A75}
$$

And therefore the solution is given by

$$A_{1m} = \frac{-b_{1m} \pm \sqrt{b_{1m}^2 - a_{1m}c_{1m}}}{a_{1m}}, \tag{A76}$$

where

$$\begin{aligned} a_{1m} &= \sigma_1^2 \\ b_{1m} &= [-\frac{1-\gamma\psi}{1-\gamma}A_1\sigma_1^2 + \kappa_1 + g_{1m}] \\ c_{1m} &= -2r_1 - 2\gamma\delta_c\sigma_{DC} \end{aligned} \tag{A77}$$

Solving for A_{2m}:

$$\begin{aligned} 0 &= r_2 + \frac{1-\gamma\psi}{1-\gamma}(A_1\sigma_\rho A_{1m}\bar{\sigma}_\rho + A_1\sigma_\rho A_{2m}\sigma_2 + A_2\sigma_2 A_{1m}\bar{\sigma}_\rho + A_2\sigma_2^2 A_{2m}) \\ &- A_{1m}\bar{\kappa}_2 - A_{2m}\kappa_2 - \frac{1}{2}(A_{1m}^2\bar{\sigma}_\rho + A_{2m}^2\sigma_2^2) - A_{1m}A_{2m}\bar{\sigma}_\rho\sigma_2 - g_{1m}A_{2m} \\ &= -A_{2m}^2\sigma_2^2 + A_{2m}2[-\kappa_2 - A_{1m}\bar{\sigma}_\rho\sigma_2 - g_{1m} + \frac{1-\gamma\psi}{1-\gamma}(A_1\sigma_\rho\sigma_2 + A_2\sigma_2^2)] \\ &+ 2[r_2 - A_{1m}\bar{\kappa}_2 + \frac{1-\gamma\psi}{1-\gamma}(A_1\sigma_\rho A_{1m}\bar{\sigma}_\rho + A_2\sigma_2 A_{1m}\bar{\sigma}_\rho)] - A_{1m}^2\bar{\sigma}_\rho \end{aligned} \tag{A78}$$

and hence

$$A_{2m} = \frac{-b_{2m} \pm \sqrt{b_{2m}^2 - a_{2m}c_{2m}}}{a_{2m}}, \tag{A79}$$

where

$$\begin{aligned} a_{2m} &= \sigma_2^2 \\ b_{2m} &= -[-\kappa_2 - A_{1m}\bar{\sigma}_\rho\sigma_2 - g_{1m} + \frac{1-\gamma\psi}{1-\gamma}(A_1\sigma_\rho\sigma_2 + A_2\sigma_2^2)] \\ c_{2m} &= -2[r_2 - A_{1m}\bar{\kappa}_2 + \frac{1-\gamma\psi}{1-\gamma}(A_1\sigma_\rho A_{1m}\bar{\sigma}_\rho + A_2\sigma_2 A_{1m}\bar{\sigma}_\rho)] + A_{1m}^2\bar{\sigma}_\rho \end{aligned} \tag{A80}$$

The last coefficient A_{0m} follows from the first equation

$$A_{0m} = \frac{1}{g_{1m}}(r_0 - \mu_D - g_{0m} + A_{1m}(\kappa_1\bar{V} + \bar{\kappa}_2\bar{\rho}) + A_{2m}\kappa_2\bar{\rho}). \tag{A81}$$

Assuming that all dividend trees are homogenous, i.e., they have the same parameters, the average correlation among dividend claims is equal to the correlation between any two trees. Using the process for two dividend claims, compute the instantaneous covariance between them:

$$Cov_S := \frac{dS_{i,t}}{S_{i,t}} \frac{dS_{j,t}}{S_{j,t}} / dt = (\sigma_{DC}^2 + A_{1m}^2 \sigma_1^2) V_t + (A_{1m} \bar{\sigma}_\rho + A_{2m} \sigma_2)^2 \rho_t, \tag{A82}$$

and hence the process for the covariance between dividend claims can be written as:

$$dCov_S = (\sigma_{DC}^2 + A_{1m}^2 \sigma_1^2) dV_t + (A_{1m} \bar{\sigma}_\rho + A_{2m} \sigma_2)^2 d\rho_t, \tag{A83}$$

Denote by $V_{S,i}$ the total variance of a stock:

$$V_{S,i} = \sigma_{D,i}^2 V_{i,t} + (\sigma_{DC,i}^2 + A_{1m}^2 \sigma_1^2) V_t + (A_{1m} \bar{\sigma}_\rho + A_{2m} \sigma_2)^2 \rho_t. \tag{A84}$$

Its process is given by joint process of dividend variance and correlations:

$$dV_{S,i} = \sigma_{D,i}^2 dV_{i,t} + (\sigma_{DC,i}^2 + A_{1m}^2 \sigma_1^2) dV_t + (A_{1m} \bar{\sigma}_\rho + A_{2m} \sigma_2)^2 d\rho_t. \tag{A85}$$

To compute the instantaneous correlation we need to normalize the covariance by the product of volatilities:

$$\begin{aligned} \rho_S &:= \frac{Cov_S}{V_S} = \\ &= \frac{(\sigma_{DC}^2 + A_{1m}^2 \sigma_1^2) V_t + (A_{1m} \bar{\sigma}_\rho + A_{2m} \sigma_2)^2 \rho_t}{\sigma_D^2 V_{i,t} + (\sigma_{DC}^2 + A_{1m}^2 \sigma_1^2) V_t + (A_{1m} \bar{\sigma}_\rho + A_{2m} \sigma_2)^2 \rho_t}, \end{aligned} \tag{A86}$$

where we denote the idiosyncratic variance by V_i do differentiate it from the systematic volatility V_t, and again assume that the individual dividend trees are homogenous, i.e., have the same parameters, and hence the stocks have the same instantaneous variance. The correlation basically is given by the ratio of the systematic variance to total variance.

The process for correlation can be derived by applying Ito's Lemma:

$$
\begin{aligned}
d\rho_S &= \frac{1}{V_S} dCov_S + Cov_S d\frac{1}{V_S} + dCov_S d\frac{1}{V_S} \\
&= \frac{1}{V_S} dCov_S + Cov_S \left(-\frac{1}{V_S^2} dV_S + \frac{1}{V_S^3} (dV_S)^2 \right) - \frac{1}{V_S^2} dCov_S dV_S \\
&= \zeta_{\rho_S} dt - \frac{1}{V_S^2} \sigma_{D,i}^2 dV_{i,t} \\
&+ \left(\frac{V_S - Cov_S}{V_S^2} \right) (\sigma_{DC}^2 + A_{1m}^2 \sigma_1^2) dV_t + \left(\frac{V_S - Cov_S}{V_S^2} \right) (A_{1m}\bar{\sigma}_\rho + A_{2m}\sigma_2)^2 d\rho_t,
\end{aligned}
\tag{A87}
$$

where ζ_{ρ_S} is the partial drift.

F. *The Equity, Variance, and Correlation Risk Premiums*

The risk premiums are computed from the covariance of the corresponding process with the pricing kernel, and according to Girsanov theorem, they represent the change in drift due to the change from actual to the risk-neutral measure.

We will collect below some equations from previous derivations, so that we have them conveniently in one place. Our pricing kernel in equation (A44) contains premiums for the aggregate consumption, aggregate variance, and correlation risks:

$$\frac{d\pi_t}{\pi_t} = -r_f dt - \lambda_1 dB_{c,t} - \lambda_2 dB_{V,t} - \lambda_3 dB_{\rho,t},$$

where

$$
\begin{cases}
\lambda_1 &= \gamma \delta_c \sqrt{V_t} \\
\lambda_2 &= -\frac{1-\gamma\psi}{1-\gamma} A_1 \sigma_1 \sqrt{V_t} \\
\lambda_3 &= -\frac{1-\gamma\psi}{1-\gamma} \left(A_1 \sigma_\rho + A_2 \sigma_2 \right) \sqrt{\rho_t}.
\end{cases}
$$

Variance and Correlation Risk Premiums

We will compute and use several types of variance and correlation risk premiums, where following recent practice in academic papers we define the variance and correlation risk premiums as the difference between the risk-neutral and actual quantities:

1. Variance risk premium for the aggregate consumption variance and the correlation risk premium for the correlation between dividend trees. We call them VRP_C, and CRP_C, respectively. These two quantities are latent, and cannot be directly observed/ extracted from data.

$$VRP_C = (E^{\mathbb{Q}}[dV] - E^{\mathbb{P}}[dV])/dt = \frac{d\pi_t}{\pi_t} dV_t = -\lambda_2 \sigma_1 \sqrt{V_t} - \lambda_3 \bar{\sigma}_\rho \sqrt{\rho_t} \tag{A88}$$

$$CRP_C = (E^{\mathbb{Q}}[d\rho] - E^{\mathbb{P}}[d\rho])/dt = \frac{d\pi_t}{\pi_t} d\rho_t = -\lambda_3 \sigma_2 \sqrt{\rho_t}. \tag{A89}$$

2. Variance risk premium for the aggregate wealth (i.e., market) process variance, and correlation risk premium for the correlation between dividend trees. We call them VRP and CRP, respectively. These two variables can be estimated from data, and play a special role in our empirical analysis.

   The process for the aggregate market variance is given in equation (A60):

$$dV_{W,t} = (\delta_c^2 + A_{1a}^2 \sigma_1^2) dV_t + (A_{1a} \bar{\sigma}_\rho + A_{2a} \sigma_2)^2 d\rho_t$$

   and the variance risk premium is then directly related to the VRP_C and CRP_C:

$$VRP = (E^{\mathbb{Q}}[dV_W] - E^{\mathbb{P}}[dV_W])/dt = (\delta_c^2 + A_{1a}^2 \sigma_1^2) VRP_C + (A_{1a} \bar{\sigma}_\rho + A_{2a} \sigma_2)^2 CRP_C \tag{A90}$$

   The process for the correlation between individual stocks (dividend claims) is given in (A91):

$$\begin{aligned} d\rho_S = {} & \zeta_{\rho S} dt - \frac{1}{V_S^2} \sigma_{D,i}^2 dV_{i,t} + \\ & + \frac{V_S - Cov_S}{V_S^2} (\sigma_{DC}^2 + A_{1m}^2 \sigma_1^2) dV_t + \frac{V_S - Cov_S}{V_S^2} (A_{1m} \bar{\sigma}_\rho + A_{2m} \sigma_2)^2 d\rho_t, \end{aligned} \tag{A91}$$

   where V_S denotes the total variance of a dividend claim given in equation (A84):

$$V_{S,i} = \sigma_{D,i}^2 V_{i,t} + (\sigma_{DC,i}^2 + A_{1m}^2 \sigma_1^2) V_t + (A_{1m} \bar{\sigma}_\rho + A_{2m} \sigma_2)^2 \rho_t.$$

   The idiosyncratic variance $V_{i,t}$ it not priced in our economy (zero price of risk in the pricing kernel), and hence the correlation risk premium is then again directly related to the VRP_C and CRP_C:

$$CRP = (E^{\mathbb{Q}}[d\rho_S] - E^{\mathbb{P}}[d\rho_S])/dt = \\ = \frac{V_S - Cov_S}{V_S^2}\left[(\sigma_{DC}^2 + A_{1m}^2\sigma_1^2)VRP_C + (A_{1m}\bar{\sigma}_\rho + A_{2m}\sigma_2)^2 CRP_C\right]. \tag{A92}$$

Equity Risk Premium

The dynamics of aggregate market is given in (A58):

$$\frac{dW_t}{W_t} = \zeta_W dt + \delta_c\sqrt{V_t}dB_{c,t} - A_{1a}dV_t - A_{2a}d\rho_t \\ = \zeta_W' dt + \delta_c\sqrt{V_t}dB_{c,t} - A_{1a}\sigma_1\sqrt{V_t}dB_{V,t} - [A_{1a}\bar{\sigma}_\rho\sqrt{\rho_t} + A_{2a}\sigma_2\sqrt{\rho_t}]dB_{\rho,t}. \tag{A93}$$

Thus, we can formulate the risk premium for the aggregate market either using the prices for underlying risks $dB_{c,t}, dB_{V,t}$, and $dB_{\rho,t}$, or using the risk premiums for the state variables ρ and V directly:

$$E^{\mathbb{P}}\left[\frac{dW}{W}\right]/dt - r_{f,t} = \lambda_1\delta_c\sqrt{V_t} + A_{1a}VRP_C + A_{2a}CRP_C.$$

Note that because both market variance risk premium VRP and average stock correlation risk premium CRP are determined *exclusively* by the risk premiums for the aggregate consumption variance VRP_C and dividend correlation CRP_C, we can express two latter—latent—risk premiums in terms of the two observable risk premiums to the market variance and pairwise (average) correlation:

$$\begin{pmatrix} VRP_C \\ CRP_C \end{pmatrix} = \begin{pmatrix} (\delta_c^2 + A_{1a}^2\sigma_1^2) & (A_{1a}\bar{\sigma}_\rho + A_{2a}\sigma_2)^2 \\ \frac{V_S - Cov_S}{V_S^2}(\sigma_{DC}^2 + A_{1m}^2\sigma_1^2) & \frac{V_S - 1}{V_S^2}(A_{1m}\bar{\sigma}_\rho + A_{2m}\sigma_2)^2 \end{pmatrix}^{-1} \times \begin{pmatrix} VRP \\ CRP \end{pmatrix} \tag{A94}$$

The solution looks like

$$\begin{pmatrix} VRP_C \\ CRP_C \end{pmatrix} = Z \times \begin{pmatrix} \frac{V_S - Cov_S}{V_S^2}(A_{1m}\bar{\sigma}_\rho + A_{2m}\sigma_2)^2 & -(A_{1a}\bar{\sigma}_\rho + A_{2a}\sigma_2)^2 \\ -\frac{V_S - Cov_S}{V_S^2}(\sigma_{DC}^2 + A_{1m}^2\sigma_1^2) & (\delta_c^2 + A_{1a}^2\sigma_1^2) \end{pmatrix} \times \begin{pmatrix} VRP \\ CRP \end{pmatrix} \tag{A95}$$

where $Z = \frac{V_S^2}{V_S - Cov_S} \frac{1}{(\delta_c^2 + A_{1a}^2\sigma_1^2)(A_{1m}\bar{\sigma}_\rho + A_{2m}\sigma_2)^2 - (A_{1a}\bar{\sigma}_\rho + A_{2a}\sigma_2)^2(\sigma_{DC}^2 + A_{1m}^2\sigma_1^2)}$, and after substituting the solutions from (A95) for VRP_C, CRP_C we can also write the equity risk premium as:

$$E^{\mathbb{P}}\left[\frac{dW}{W}\right]/dt - r_{f,t} = \lambda_1\delta_c\sqrt{V_t} + A_{1z}VRP + A_{2z}CRP,$$

where A_{1z} and A_{2z} are the functions of the total variance of a dividend claim, matrix elements in (A95), and coefficients A_{1a}, A_{2a}:

$$\begin{cases} A_{1z} &= Z\frac{V_S - Cov_S}{V_S^2}\left(A_{1a}(A_{1m}\bar{\sigma}_\rho + A_{2m}\sigma_2)^2 - A_{2a}(\sigma_{DC}^2 + A_{1m}^2\sigma_1^2)\right) \\ A_{2z} &= Z\left(A_{2a}(\delta_c^2 + A_{1a}^2\sigma_1^2) - A_{1a}(A_{1a}\bar{\sigma}_\rho + A_{2a}\sigma_2)^2\right). \end{cases} \tag{A96}$$

As a result we have a nice decomposition of the market risk premium into the risk premium for the consumption volatility (standard risk premium), market variance risk premium, and the stock-based correlation risk premium.

VIII. Internet Appendix: Tables for Robustness Tests

Table AI101 In-sample Market Return Predictability: Correlation and Variance Risk Premiums

The table shows the coefficients (and corresponding p-values) and the R^2 of the market predictive regressions, for the sample period from 01/1996 to 04/2016 for S&P500 and S&P100, and from 10/1997 to 04/2016 for DJ30-based variables. We regress overlapping excess market returns compounded over a specified horizon (30, 91, 182, 273, and 365 calendar days) on a constant and a given set of explanatory variables, which are the correlation risk premium (CRP) for 30, 91, 182, 273, and 365 calendar days, and the variance risk premium (VRP), which equals to the difference between the 30-day implied variance and lagged realized variance computed over the historical period of 30 calendar days. Implied variances are computed as simple variance swaps (Martin (2013)). The p-values (under the coefficients) for the null hypothesis that the coefficients are equal zero are computed using Newey and West (1987) standard errors. The adjusted R^2 are given as percentages.

	Return, 30 days			Return, 91 days			Return, 181 days			Return, 273 days			Return, 365 days		
SP500 Sample															
CRP	0.076	-	0.027	0.254	-	0.165	0.381	-	0.343	0.588	-	0.575	0.559	-	0.559
	0.027	-	0.362	0.002	-	0.031	0.051	-	0.095	0.067	-	0.095	0.150	-	0.166
VRP	-	0.322	0.289	-	0.663	0.514	-	0.473	0.220	-	0.421	0.101	-	0.230	0.000
	-	0.004	0.007	-	0.000	0.000	-	0.007	0.178	-	0.060	0.705	-	0.430	0.999
R^2	2.48	6.90	6.81	7.26	9.60	11.99	6.90	2.24	7.02	9.87	0.74	9.55	5.43	-0.18	5.02
SP100 Sample															
CRP	0.051	-	0.011	0.234	-	0.154	0.363	-	0.313	0.561	-	0.551	0.527	-	0.517
	0.076	-	0.678	0.004	-	0.042	0.047	-	0.089	0.042	-	0.063	0.082	-	0.098
VRP	-	0.333	0.319	-	0.701	0.563	-	0.718	0.528	-	0.500	0.103	-	0.403	0.124
	-	0.004	0.006	-	0.000	0.000	-	0.000	0.002	-	0.052	0.725	-	0.219	0.696
R^2	1.27	6.68	6.35	6.74	9.75	12.08	7.24	4.24	9.23	12.18	1.01	11.87	7.60	0.25	7.27
DJ30 Sample															
CRP	0.040	-	0.010	0.205	-	0.128	0.273	-	0.227	0.330	-	0.306	0.150	-	0.132
	0.117	-	0.675	0.007	-	0.068	0.123	-	0.230	0.243	-	0.313	0.642	-	0.689
VRP	-	0.292	0.277	-	0.727	0.582	-	0.555	0.330	-	0.468	0.249	-	0.285	0.212
	-	0.005	0.005	-	0.000	0.000	-	0.007	0.105	-	0.085	0.463	-	0.405	0.536
R^2	0.90	4.53	4.16	6.27	9.42	11.25	4.47	2.39	4.91	3.93	0.73	3.80	0.11	-0.15	-0.19

Table AI102 Out of Sample Predictability - Continuous Beta Approach

The table reports the Out-of-Sample R^2_{j,τ_r} and the Diebold-Mariano test statistic δ_{j,τ_r} in Panel A, and the improvement of the certainty equivalent of a mean-variance investor optimizing her portfolio using forecasts by a specific model instead of either using past mean market return (Model 0), or CRP as a predictor, in Panel B. The variance and correlation risk premiums are computed as the difference between implied and lagged realized variances ($VRP = IV - RV$), and as the difference between implied and lagged realized correlations ($CRP = IC - RC$), for three samples of stocks—components of S&P500, S&P100, and DJ30 indices, for the sample period from 1996 to 04/2016 for S&P500, S&P100, and from 10/1997 to 04/2016 for DJ30, and for five different maturities—30, 91, 182, 273, and 365 (calendar) days. Implied variance (IV) is computed as the simple variance swap (Martin (2013)) on each day using out-of-the money options with 30 days maturity, and realized variance (RV) is calculated on each day from daily returns over a 30-day historical window. The betas are computed from daily increments of the variables over a 12-month rolling window. The p-values are obtained from a bootstrapped distribution using moving-block bootstrap by Künsch (1989) with 10,000 samples and block size of 12 months.

Panel A: OOS R^2 and δ

	R^2_{j,τ_r}			δ_{j,τ_r}		
Days	VRP	CRP	VRP+CRP	VRP	CRP	VRP+CRP
SP500 Sample						
30	0.094	0.025	0.096	-0.000	-0.000	-0.000
	0.000	0.000	0.000	0.000	0.000	0.000
91	0.081	0.081	0.119	-0.001	-0.001	-0.001
	0.000	0.000	0.000	0.000	0.000	0.000
182	-0.113	0.067	-0.031	0.002	-0.001	0.000
	0.000	0.000	0.100	0.000	0.000	0.099
273	-0.373	0.079	-0.275	0.010	-0.002	0.007
	0.000	0.000	0.000	0.000	0.000	0.000
365	-0.654	0.070	-0.493	0.024	-0.003	0.018
	0.000	0.001	0.000	0.000	0.001	0.000
SP100 Sample						
30	0.091	0.012	0.087	-0.000	-0.000	-0.000
	0.000	0.015	0.000	0.000	0.015	0.000
91	0.086	0.072	0.118	-0.001	-0.000	-0.001
	0.001	0.000	0.000	0.001	0.000	0.000
182	-0.086	0.055	-0.028	0.001	-0.001	0.000
	0.000	0.000	0.084	0.000	0.000	0.084
273	-0.344	0.066	-0.290	0.009	-0.002	0.008
	0.000	0.000	0.000	0.000	0.000	0.000
365	-0.576	0.070	-0.491	0.021	-0.003	0.018
	0.000	0.001	0.000	0.000	0.001	0.000
DJ30 Sample						
30	0.065	-0.004	0.066	-0.000	0.000	-0.000
	0.000	0.283	0.000	0.000	0.283	0.000
91	0.052	0.064	0.075	-0.000	-0.000	-0.001
	0.046	0.000	0.003	0.046	0.000	0.003
182	-0.159	0.049	-0.123	0.003	-0.001	0.002
	0.000	0.001	0.000	0.000	0.001	0.000
273	-0.476	0.028	-0.436	0.012	-0.001	0.011
	0.000	0.092	0.000	0.000	0.094	0.000
365	-0.745	0.008	-0.680	0.027	-0.000	0.025
	0.000	0.387	0.000	0.000	0.390	0.000

...Table AI102 continued

Panel B: Certainty Equivalent Improvement

Days	$CE_{j,\tau_r} - CE_{0,\tau_r}$ VRP	CRP	VRP+CRP	$CE_{j,\tau_r} - CE_{CRP,\tau_r}$ VRP	CRP	VRP+CRP
SP500 Sample						
30	0.023	0.039	0.023	-0.016	-	-0.016
	0.003	0.000	0.003	0.000	-	0.002
91	0.025	0.022	0.022	0.003	-	0.000
	0.000	0.001	0.000	0.344	-	0.489
182	0.011	0.021	0.008	-0.010	-	-0.012
	0.064	0.000	0.110	0.029	-	0.008
273	-0.002	0.020	-0.002	-0.022	-	-0.022
	0.403	0.000	0.417	0.001	-	0.001
365	0.002	0.007	0.002	-0.005	-	-0.005
	0.352	0.086	0.368	0.211	-	0.196
SP100 Sample						
30	0.021	0.024	0.017	-0.004	-	-0.007
	0.008	0.000	0.022	0.264	-	0.158
91	0.019	0.027	0.021	-0.008	-	-0.005
	0.000	0.000	0.000	0.069	-	0.153
182	0.004	0.021	0.005	-0.017	-	-0.016
	0.275	0.000	0.228	0.005	-	0.007
273	-0.005	0.017	-0.004	-0.022	-	-0.021
	0.287	0.002	0.312	0.003	-	0.004
365	0.001	0.011	0.002	-0.010	-	-0.009
	0.410	0.039	0.354	0.103	-	0.121
DJ30 Sample						
30	0.005	-0.015	0.014	0.020	-	0.029
	0.315	0.087	0.094	0.000	-	0.000
91	0.012	0.013	0.010	-0.000	-	-0.003
	0.056	0.034	0.118	0.458	-	0.247
182	0.006	0.017	0.005	-0.011	-	-0.013
	0.286	0.013	0.324	0.063	-	0.041
273	-0.013	0.013	-0.013	-0.027	-	-0.027
	0.138	0.015	0.141	0.001	-	0.001
365	-0.015	-0.008	-0.015	-0.007	-	-0.008
	0.091	0.144	0.082	0.149	-	0.128

Table AI103 In-sample Market Return Predictability with Controls: Correlation and Variance Risk Premiums

The table shows the coefficients (and corresponding p-values) and the R^2 of the market predictive regressions, for the sample period from 01/1996 to 04/2016 for S&P500 and S&P100, and from 10/1997 to 04/2016 for DJ30-based variables. We regress excess market return compounded over a specified horizon (30, 91, 182, 273, and 365 calendar days) and observed at the end of each month (i.e., overlapping by its horizon in months-1) on a constant and a given set of explanatory variables, which are the ex ante correlation risk premium (CRP) for 30, 91, 182, 273, and 365 calendar days, the variance risk premium (VRP), which equals the difference between implied variance and lagged realized variance computed using out-of-the money options with the respective maturity, and a number of control variables as defined and used in the study by Goyal and Welch (2008). The p-values (under the coefficients) for the null hypothesis that the coefficients are equal zero are computed using Newey and West (1987) standard errors.

Days	CRP	VRP	EP12	TMS	DFY	BM	NTIS	R^2
SP500 Sample								
30	0.027	0.289	-	-	-	-	-	6.81
-	0.362	0.007	-	-	-	-	-	-
30	0.027	0.314	0.011	0.048	0.639	-	-	6.27
-	0.361	0.009	0.246	0.826	0.604	-	-	-
30	0.030	0.267	-0.004	-0.217	-	0.109	0.223	7.39
-	0.310	0.009	0.739	0.374	-	0.105	0.265	-
91	0.195	0.270	-	-	-	-	-	7.73
-	0.027	0.175	-	-	-	-	-	-
91	0.205	0.532	0.015	-0.061	3.755	-	-	8.36
-	0.015	0.075	0.502	0.904	0.260	-	-	-
91	0.172	0.370	-0.058	-1.297	-	0.534	1.305	20.95
-	0.026	0.037	0.041	0.036	-	0.000	0.038	-
182	0.729	-1.606	-	-	-	-	-	16.20
-	0.002	0.000	-	-	-	-	-	-
182	0.752	-2.070	0.056	0.455	-0.419	-	-	17.59
-	0.000	0.014	0.170	0.599	0.933	-	-	-
182	0.498	-1.377	-0.055	-2.160	-	0.878	3.181	40.30
-	0.004	0.014	0.278	0.034	-	0.000	0.001	-
273	1.108	-2.554	-	-	-	-	-	23.90
-	0.002	0.000	-	-	-	-	-	-
273	1.264	-3.117	0.128	0.900	7.771	-	-	31.60
-	0.000	0.002	0.009	0.441	0.094	-	-	-
273	0.774	-2.829	-0.014	-1.943	-	1.081	4.226	52.63
-	0.000	0.001	0.853	0.116	-	0.000	0.000	-
365	1.071	-2.501	-	-	-	-	-	14.77
-	0.031	0.028	-	-	-	-	-	-
365	1.520	-3.161	0.177	2.260	15.905	-	-	30.91
-	0.001	0.011	0.024	0.160	0.004	-	-	-
365	0.682	-2.363	-0.042	-1.191	-	1.403	4.708	46.85
-	0.039	0.072	0.701	0.410	-	0.000	0.002	-

...Table AI103 continued

Days	CRP	VRP	EP12	TMS	DFY	BM	NTIS	R^2
SP100 Sample								
30	0.011	0.319	-	-	-	-	-	6.35
-	0.678	0.006	-	-	-	-	-	-
30	0.012	0.340	0.011	0.040	0.545	-	-	5.81
-	0.645	0.008	0.244	0.855	0.658	-	-	-
30	0.014	0.294	-0.003	-0.220	-	0.105	0.221	6.86
-	0.586	0.008	0.802	0.374	-	0.120	0.271	-
91	0.161	0.400	-	-	-	-	-	8.10
-	0.062	0.042	-	-	-	-	-	-
91	0.185	0.702	0.021	-0.161	4.496	-	-	9.42
-	0.025	0.016	0.352	0.755	0.150	-	-	-
91	0.157	0.471	-0.057	-1.350	-	0.546	1.228	21.38
-	0.045	0.007	0.042	0.031	-	0.000	0.051	-
182	0.647	-1.567	-	-	-	-	-	15.10
-	0.003	0.006	-	-	-	-	-	-
182	0.731	-1.887	0.068	0.239	2.748	-	-	17.37
-	0.000	0.034	0.113	0.777	0.597	-	-	-
182	0.481	-1.187	-0.074	-2.444	-	1.017	2.937	40.47
-	0.003	0.028	0.124	0.018	-	0.000	0.003	-
273	1.029	-2.642	-	-	-	-	-	26.16
-	0.001	0.001	-	-	-	-	-	-
273	1.348	-3.481	0.171	0.665	12.360	-	-	39.75
-	0.000	0.001	0.001	0.511	0.001	-	-	-
273	0.836	-2.949	-0.016	-1.983	-	1.178	3.648	55.34
-	0.000	0.000	0.799	0.093	-	0.000	0.000	-
365	0.994	-2.645	-	-	-	-	-	17.28
-	0.017	0.031	-	-	-	-	-	-
365	1.612	-3.999	0.239	1.898	21.149	-	-	40.53
-	0.000	0.002	0.003	0.189	0.000	-	-	-
365	0.813	-2.715	-0.036	-1.071	-	1.466	4.028	49.93
-	0.001	0.028	0.712	0.431	-	0.000	0.007	-

Days	CRP	VRP	EP12	TMS	DFY	BM	NTIS	R^2
DJ30 Sample								
30	0.010	0.277	-	-	-	-	-	4.16
-	0.675	0.005	-	-	-	-	-	-
30	0.012	0.302	0.011	0.099	0.436	-	-	3.52
-	0.624	0.013	0.280	0.648	0.738	-	-	-
30	0.012	0.258	-0.005	-0.216	-	0.122	0.231	5.16
-	0.616	0.009	0.716	0.379	-	0.087	0.292	-
91	0.133	0.420	-	-	-	-	-	7.51
-	0.120	0.044	-	-	-	-	-	-
91	0.148	0.730	0.023	0.196	4.271	-	-	9.21
-	0.080	0.026	0.318	0.694	0.181	-	-	-
91	0.120	0.558	-0.059	-1.128	-	0.579	1.126	22.55
-	0.088	0.009	0.027	0.067	-	0.000	0.091	-
182	0.545	-1.718	-	-	-	-	-	12.33
-	0.018	0.008	-	-	-	-	-	-
182	0.633	-1.965	0.065	0.652	2.912	-	-	14.85
-	0.003	0.057	0.119	0.450	0.538	-	-	-
182	0.394	-1.103	-0.087	-2.263	-	1.092	2.906	41.37
-	0.011	0.101	0.062	0.025	-	0.000	0.004	-
273	0.741	-2.690	-	-	-	-	-	15.64
-	0.017	0.004	-	-	-	-	-	-
273	1.126	-3.206	0.153	1.490	13.874	-	-	30.21
-	0.000	0.005	0.001	0.157	0.000	-	-	-
273	0.600	-2.133	-0.090	-2.027	-	1.486	3.676	53.60
-	0.001	0.020	0.188	0.097	-	0.000	0.002	-
365	0.581	-2.628	-	-	-	-	-	7.85
-	0.198	0.072	-	-	-	-	-	-
365	1.577	-4.226	0.245	3.148	24.742	-	-	35.47
-	0.000	0.003	0.001	0.037	0.000	-	-	-
365	0.594	-2.059	-0.103	-0.956	-	1.774	3.916	50.68
-	0.027	0.118	0.249	0.486	-	0.000	0.010	-