Robust difference-in-differences analysis when there is a term structure[1]

Kjell G. Nyborg

University of Zurich,
Swiss Finance Institute,
and CEPR

Jiri Woschitz

BI Norwegian Business School

March 2023

[1]We gratefully acknowledge financial support from the Swiss National Science Foundation (project #100018_172679 "Trading and Financing during Market Stress"). We would also like to thank participants of the brownbag seminar at BI Norwegian Business School. Nyborg: Department of Banking and Finance, University of Zurich, Plattenstrasse 14, CH-8032 Zurich, Switzerland. email: kjell.nyborg@bf.uzh.ch. Woschitz: Department of Finance, BI Norwegian Business School, B4y, NO-0442 Oslo, Norway. email: jiri.woschitz@bi.no.

Abstract

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It is common practice in finance to use difference-in-differences analysis to examine fixed-income pricing data. In this paper, simulations show that difference-in-differences methodology applied to fixed-income pricing variables that exhibit a term structure, such as yields or credit spreads, produces false, systematically biased and mismeasured treatment effects. This is the case even under random assignment of the treatment. Both bias and mismeasurement are sample-induced and result from differential effects in different parts of the term structure in combination with the relative distributions of control and treated bond samples across maturity. Neither bond-fixed effects nor explicit yield-curve control in the specification resolve the issues. Acknowledging that a sensible estimation approach must control for term structure effects, we provide new methodology to overcome both bias and mismeasurement by combining difference-in-differences analysis with yield-curve modeling.

JEL classification: C20, G12, E43, E47

Keywords: Fixed-income pricing, yield curve, term structure of interest rates, yields, credit spreads, difference-in-differences analysis, treatment effect, bias and mismeasurement

1. Introduction

In finance, difference-in-differences (DiD) methodology is widely used to analyze fixed-income pricing data. Typically a security's price is expressed in terms of its yield (or credit spread) and DiD analysis is applied by running a classical DiD regression of the form

$$yield_{it} = \alpha_i + \delta_t + \beta_{Post}\ \mathbb{1}_{Post,t} + \beta_{DiD}\ \mathbb{1}_{Treated,i} \times \mathbb{1}_{Post,t} + \varepsilon_{it}, \tag{1}$$

where $yield_{it}$ is security i's yield-to-maturity on day t and the right side of the equation represents the typical DiD structure with α_i and δ_t corresponding to security- and time-fixed effects, respectively, $\mathbb{1}_{Treated,i}$ and $\mathbb{1}_{Post,t}$ treatment and post-event date indicator variables, β_{DiD} the treatment effect, and ε_{it} the error term. DiD methodology is designed to deal with endogeneity to measure the causal impact of a treatment on an outcome variable, yield in this case, by comparing treated to non-treated control units over the treatment date with the units being fixed-income securities in this case. Simultaneously, fixed-income securities data are characterized by two ever-present features. First, pricing variables such as yields or credit spreads exhibit a yield curve as these securities are priced against the term structure of interest rates, typically of the issuer, using its individual maturities. Thereby, the shapes of the issuer-specific term structures vary widely over time (even on a day-to-day basis). Second, it is hardly ever possible to match two securities of the same issuer on maturity because in practice individual issuers have only a few securities outstanding and these usually have wide ranges of maturities.

This paper uses simulations to show that the classical DiD specification in (1) applied to fixed-income pricing variables that exhibit a term structure such as yield or credit spread produces false, systematically biased and mismeasured treatment effects. This is the case even under random assignment of the treatment. Researchers may erroneously conclude that there is a statistically significant treatment effect if, in fact, a true underlying treatment effect is inexistent; or, conversely, they find no treatment effect if in reality there is one. It is even possible to measure a significant *negative* effect while the true underlying treatment impact is *positive*, or vice versa. The reason for bias and mismeasurement is the inability of Specification (1) to properly control for term structure effects, which we will elaborate on shortly. This is particularly unfavorable since DiD methodology is designed precisely for the purpose of dealing with endogeneity to elicit and quantify causal effects.

The paper demonstrates bias and mismeasurement in the most trivial setting with bond-level data, yield as dependent variable, and residual maturity (or bond fixed effects) as independent variable.[1] However, bias and mismeasurement survive (1) if the unit of analysis is an aggregation of the bond-level, such as the firm, the country, or the bank-firm relationship, (2) with other dependent pricing variables such as expected returns, loan rates or spreads, yield spreads, or logarithms of these variables, and (3) for any variable transformation of residual maturity or linear combination of transformations on the right side of the regression equation. As for the dependent pricing variable in Specification (1), using any variable that exhibits a time-varying term structure can lead to estimation bias and mismeasurement.[2] Table 1 provides a list of top finance publications that use DiD methodology in ways potentially affected, which therefore may benefit from an application of the methodology proposed in this paper.

Insert Table 1 here.

To show the bias we run Specification (1) for two types of true yield-curve effects, namely general and treatment effects. Specifically, first, we model general yield-curve effects, which are not related to the treatment. From pre to post treatment, they move yields of treated and control bonds irrespective of treatment and vary along maturity. The analysis shows that, even in absence of a true underlying treatment effect, the classical DiD specification in (1) produces false treatment effects. Second, we model treatment effects which move yields of treated bonds only but differentially so across maturity. In this case the classical DiD specification produces a potentially mismeasured, sample-specific average treatment effect, which may cause researchers to draw incorrect conclusions. Third, combining general and treatment effects shows that the classical DiD specification is genuinely not able to identify the true treatment effects and separate them from treatment-unrelated general effects.

The limitation of the classical DiD specification, which leads to the false and mismea-

[1] This setting is used, for example, in Todorov (2020), Macaire and Naef (2021), or Bremus, Schütze, and Zaklan (2021). The authors intend to measure the impact of quantitative easing or green labeling on bond yields. Both of these are very hot and important topics at the moment.

[2] Loan or yield spreads are typically calculated with maturity-matched interpolated LIBOR or treasury rates. Bao and Hou (2017), for example, show that an issuer's relatively longer-dated bonds have larger yield spreads and more co-movement with the issuer's equity and, hence, provide evidence for a term structure in yield spreads rather than yields. See also John, Lynch, and Puri (2003), Chava, Livdan, and Purnanandam (2009), Ayotte and Gaon (2011).

sured treatment effects, is its inability to capture treatment-unrelated general yield-curve effects as well as the treatment effects themselves if either of these effects vary along maturity. Both false and mismeasured treatment effects are the result of an endogeneity problem, misspecification, or omitted-variables problem. We show that the magnitude of both bias and mismeasurement is sample-induced and dependent on the combination of differential effects, both general and treatment effects, in different parts of the term structure and, simultaneously, the relative distributions of residual maturity in the treated and control bond samples.[3] Both bias and mismeasurement can be economically large, can go in either direction, and the larger they are in absolute terms the more likely they also are statistically significant.

To deal with differential treatment effects the literature typically either estimates heterogeneous treatment effects over the distribution of the dependent variable or uses fixed effects on the different discrete right-side units present in the data.[4] The former is not applicable with yield as dependent variable because yield curves can be flat or up- or downward sloping and the yield-curve shapes vary significantly over time. Applying the latter in a fixed-income setting, researchers sometimes measure DiD effects separately for individual maturity buckets (see, e.g., Bao, O'Hara, and Zhou, 2018; Todorov, 2020). We show that running the classical DiD specification individually by maturity bucket does not resolve the bias but merely shifts the issue to the maturity-bucket level and, simultaneously, leads to small samples in some of the regressions. This illustrates the main problem, namely that residual maturity in a fixed-income setting is a continuous habitat variable and, as such, demands a different approach.

Instead of Specification (1) researchers sometimes use DiD specifications that explicitly model the term structure. In that case the DiD specification exhibits one or several terms to parametrically control for the yield curve in the data. We proceed by running such a specification and show that explicit term structure control does neither resolve the bias nor the mismeasurement. In fact, due to the simplicity of our setting with only two time periods and no measurement error (the bonds always lie exactly on the yield curves), the DiD specification with explicit term structure control produces the exact same false and

[3]We use the terms "yield curve" and "term structure" interchangeably.

[4]To name a few, for the former see Heckman, Smith, and Clements (1997), Bitler, Gelbach, and Hoynes (2006), Callaway and Li (2019), the latter de Chaisemartin and D'Haultfœuille (2020), and both in one Callaway, Li, and Oka (2018).

mismeasured treatment effects as the classical DiD specification in (1). The problem with these specifications is that they impose, either explicitly through the parametric term structure or implicitly through the bond fixed effects, parallel yield-curve level-shifts between the involved groups (control and treated bonds before and after treatment) while the true effects are not limited to parallel level-shifts. In fact, the false and mismeasured treatment effects shown in this paper survive any DiD regression model that assumes parallel yield-curve level-shifts from pre to post treatment. A sensible approach must acknowledge that the underlying yield curve movements, both treatment-unrelated general as well as actual treatment effects, may vary over the maturity spectrum.

A simple solution to the problem would be to perfectly match each treated bond with a control bond. However, for fixed-income securities perfect matching on maturity is rarely feasible in practice. The main solution provided in this paper, which we call a "fully flexible yield-curve DiD specification," combines the DiD method with flexible yield-curve modeling and allows to separate general yield curve effects from actual treatment effects over the term structure. To provide intuition, the specification estimates the yield curve separately for treated and controls both pre- and post-treatment and measures the treatment effects as the incremental difference between the yield curves of treated and control bonds over the treatment. With zero-coupon yields as dependent variable the treatment effects can be estimated by running one single regression using standard software. The DiD estimator, in this case, does not only control for residual maturity but is a function of it. We show that the fully flexible yield-curve DiD specification resolves both bias and mismeasurement and provides precisely estimated treatment effects. Furthermore, since the specification uses the full panel structure of the data, it permits to cluster standard errors at the bond level as recommended by Bertrand, Duflo, and Mullainathan (2004). Our second solution, which we call "semi-matching," splits the first approach into its pieces and applies them step-by-step to the data. Semi-matching is more involved but at the same time also more broadly applicable, as it is not limited to matching on maturity only, but also on other potentially relevant features such as coupons, callability, etc.

Technically, the paper relates to the literature on latent confounding factor structures in fixed-effects settings. Bai (2009) develops a panel data estimator with interactive fixed effects. Correlation between an unobserved ignored factor structure and observed characteristics for the units leads to biases in the slope coefficients. DiD analysis is a special case of this. Xu (2017) shows that the treatment effect is biased if one ignores that the factors, the loadings, or both are correlated with the treatment. Our contribution is twofold. First we provide evidence that in a standard practical application in finance, i.e. using DiD methodology to analyze fixed-income pricing data, factor structures *naturally* confound the estimation of the treatment effects. To model effects we employ Diebold and Li (2006)'s factorization of the seminal yield-curve specification of Nelson and Siegel (1987). The term structure is given by three factors, level, slope, and curvature and the factor loadings, which are functions of a fourth parameter λ and bond maturity. Both loadings and factors are unobserved. To simulate treatment-unrelated general and treatment effects we manipulate the factors, and since we model differential effects at short and long ends via the factors they naturally are correlated with residual maturity. In this setting, in which the factor structure is correlated with residual maturity in a natural manner, we show that the classical DiD specification produces false treatment effects.[5]

Second, the proposed new approaches both overcome the bias and mismeasurement because they directly deal accurately with the confounding factor structure. They do so, however, in different ways. Semi-matching, our second approach, removes the confounding factor structure before using DiD analysis to examine the remaining variation. This is the same approach as emphasized in the synthetic control literature (see Abadie and Gardeazabal, 2003; Abadie, Diamond, and Hainmueller, 2010; Xu, 2017; Abadie, 2021), which is also concerned with correctly measuring treatment effects in presence of a factor structure, but applied to fixed-income data. The synthetic control method, however, typically needs a lot of pre-treatment periods to accurately estimate the factor structure. In our case, making use of a rich yield-curve fitting literature in finance (Nelson and Siegel, 1987; Svensson, 1994; Diebold and Li, 2006; Liu and Wu, 2021; to name a few), we can identify the factor structure with only one pre-treatment period. The fully flexible yield-curve DiD method, our first approach, illustrates an alternative to deal with the confounding factor structure. Instead of removing it, the approach models it as part of the DiD estimator. This enables one to not only control for the unobserved factor structure but, simultaneously, to also correctly estimate maturity-dependent treatment effects. As will be shown, the latter is relevant because, even after removing the confounding factor structure, the

[5] The DiD model allows for the presence of unobserved factors but their effects must be constant in time as otherwise taking the difference does not eliminate them (see Abadie, Diamond, and Hainmueller, 2010).

classical DiD specification still produces potentially misleading effects that may lead to incorrect conclusions.

The rest of the paper is structured as follows. Section 2 presents the term structure model and the data simulation. Section 3 shows how Specification (1) measures false treatment effects when there is no treatment effect. Section 4 illustrates how the specification mismeasures treatment effects. Section 5 provides results when false and mismeasured treatment effects are combined. Section 6 shows that any DiD specification that restricts yield-curve movements to parallel level-shifts is going to produce the same bias and mismeasurement. Section 7 provides methodology to overcome the omitted-variables problem and resolve the bias. Section 8 concludes.

2. Term structure modeling and data simulation

In the process of generating the data our focus lies on the two features that, combined, cause bias and mismeasurement. They are described and discussed in this section. As touched on in the Introduction, the first feature are differential general trends or treatment effects in different parts of the term structure. The second feature are differential relative distributions of residual maturity of control and treated sample bonds.

2.1 Modeling term structure effects

To model general and treatment effects in the term structure we employ Diebold and Li (2006)'s factorization of Nelson and Siegel (1987)'s term structure parameterization.[6] The spot rate, or *yield*, of a zero-coupon bond with maturity x at time t is

$$yield_t(x;\lambda_t) = \gamma_{0,t} + \gamma_{1,t}\left(\frac{1-e^{-\lambda_t x}}{\lambda_t x}\right) + \gamma_{2,t}\left(\frac{1-e^{-\lambda_t x}}{\lambda_t x} - e^{-\lambda_t x}\right), \qquad (2)$$

where $\gamma_{0,t}$ is a long-term or level factor, $\gamma_{1,t}$ a short-term or slope factor, $\gamma_{2,t}$ a medium-term or curvature factor, and λ_t the decay parameter. For simplicity we follow Diebold and Li (2006) and fix $\lambda_t = \lambda = 0.7308$.[7] To model effects in the term structure, we manipulate

[6]As explained by Diebold and Li (2006) their specification suffers less from multicollinearity between the parameters as compared to the original Nelson and Siegel (1987) specification.

[7]The authors explain that λ_t determines the point where the loading on the curvature factor, $\gamma_{2,t}$, obtains its maximum and pick this, based on practice, to be at a maturity of 30 months. If maturity is

the level, slope, and curvature parameters while holding the decay parameter fix.

Exhibit 1 gives an overview of the effects modeled in this paper. First, we model *general yield-curve effects*, which are unrelated to the treatment. One can think of general effects as trends in the term structure that are structural and the result of economic forces other than the treatment.[8] For a selected group of countries, Table 2 illustrates that in practice the shape of the term structure of interest rates varies widely over time. Using yield curve data from Bloomberg from January 2000 to December 2022, the table shows distributions of daily and monthly changes in the term spread in Panels A and B, respectively, computed as the end-of-day or end-of-month ten-year minus two-year zero-coupon spot yield. Across the twelve selected countries, daily term spreads vary from between −16 and +20 basis points (bps) in Japan to between −171 to +271 bps in Ireland.[9] In Spain, which is one of the two median countries in the list, daily (monthly) changes in term spreads vary between −58 and +60 (−114 and +172) bps.

Insert Table 2 here.

With the magnitudes motivated by these yield-curve movements observed in practice, we model two types of differential general yield-curve effects across maturity. Either the general effect pushes down the yield curve only at the short-end with −50 (+1) bps at the one-year (fifteen-year) maturity or only at the long-end with +4 (−50) bps, respectively. Previewing results, the analysis in Section 3 will show that the classical DiD specification produces potentially large, statistically significant treatment effects even if a true underlying treatment effect is entirely absent from the data.

Exhibit 1: Overview effects and sections discussing them

		Treatment effect	
		No	Yes
General effect	No	–	Section 4
	Yes	Section 3	Section 5

Second, following the same logic, we model differential *term-structure treatment effects* across maturity in two ways. Either the treatment affects treated bonds only at the short-end with −6 (0) bps at the one-year (ten-year) maturity or the treatment leads to a term-

measured in months $\lambda_t = \lambda = 0.0609$, which translates to $\lambda = 0.7308$ if maturity is measured in years.

[8]For example, Foley-Fisher, Ramcharan, and Yu (2016) show how the Fed's maturity extension program depresses yields of long-term but not short-term bonds.

[9]In Greece changes in daily term spreads vary even between −906 and 1,822 bps. However, Greece was hit exceptionally hard during the European sovereign debt crisis.

structure twist, which pushes up (down) the yields of treated bonds at the one-year (ten-year) maturity by +6 (−6) bps. By choosing small treatment effects, we acknowledge the fact that in reality treatment effects are typically small and, in particular, smaller than the structural general yield-curve effects. In these cases, as shown in Section 4, the classical DiD specification produces a sample-specific average treatment effect, which may lead a researcher to draw incorrect conclusions.

Third, Section 5 combines the treatment-unrelated general with the actual treatment effects. Differential yield-curve movements across maturity both in the form of general and treatment effects are the first critical feature that drives the estimation bias.

2.2 Simulation of residual maturity

In practice control and treated bonds are often distributed differentially over maturity. This property arises naturally in fixed-income data as individual issuers inherently issue only a limited number of securities but with wide maturity ranges. For the twelve countries used previously, Table 3 provides the number of securities and the percentage of debt per maturity bucket as of the country's total debt in January 2023 (Panel A) and 2011 (Panel B). First, across panels, the number of securities is small and lies between 16 in Ireland and 559 in Japan. In 2023, seven of the twelve countries have less than 100 securities outstanding. Second, issuers have relatively more debt outstanding at short maturities but the exact maturity structure is issuer- and time-specific. For example, while the US' maturity structure is tilted towards the short-end, the UK's is tilted rather towards the (5-10]-year bucket. Or, the Netherlands has short maturities in 2011 but rather uniformly distributed maturities in 2023. Non-governmental issuers typically issue even fewer securities but with more dispersed maturities. The small number of securities issued per issuer and the wide maturity ranges inherently lead to unequal maturity distributions in the samples of treated and control bonds. This is the second feature that causes to the estimation bias.

Insert Table 3 here.

Based on these properties of fixed-income data in practice, we make use of simulation technique to generate bond maturity, which will be the only source of randomness. A bond's residual maturity, x, is drawn from a triangular probability density function, $P(x)$, that ranges from zero to twenty years, $x \in [0, 20]$, and has mode m. The mode determines the location of the triangular probability density function's peak.[10] Hence,

$$P(x) = \begin{cases} 0, & \text{if } x < 0 \text{ or } x > 20, \\ \frac{x}{10m}, & \text{if } 0 \leq x \leq m, \\ \frac{20-x}{10(20-m)}, & \text{if } m < x \leq 20. \end{cases} \quad (3)$$

To analyze the impact of differential residual maturity distributions in the treated and control groups we build families of samples and sample couplets. Family F_j consists of five samples, namely one control sample and four treated samples, with fifty bonds each. Residual maturity of the fifty control bonds is drawn randomly from $P(x)$ with $m = 0.25$ years and denoted by $S_{j,C,0.25}$. Residual maturities of the four treated bond samples are drawn, respectively, from $P(x)$ with $m = \{0.25, 1, 3, 10\}$ years and denoted by $\{S_{j,T,m}\}_{m=0.25,1,3,10}$. Together this gives the total of five samples per family $\{F_j\} = \{S_{j,C,0.25}, \{S_{j,T,m}\}_{m=0.25,1,3,10}\}$ with $P(x)$ provided in Figure 1. For family F_j we pair each sample of treated with the sample of control bonds, which gives the family of sample couplets, F_j^{SC}. Hence, $\{F_j^{SC}\} = \{\{S_{j,C,0.25}, S_{j,T,m}\}_{m=0.25,1,3,10}\} = \{\{F_{j,m}^{SC}\}_{m=0.25,1,3,10}\}$. Each family consists of four sample couplets. $F_{j,m=0.25}^{SC} = \{S_{j,C,0.25}, S_{j,T,0.25}\}$ is the first sample couplet with residual maturity drawn from $P(x)$ with $m = 0.25$ years for both the fifty control and the fifty treated bonds. The other three sample couplets, $\{F_{j,m}^{SC}\}_{m=1,3,10} = \{\{S_{j,C,0.25}, S_{j,T,m}\}_{m=1,3,10}\}$, are comprised of the control bond sample with $m = 0.25$ years and the three treated bond samples with $m = \{1, 3, 10\}$ years. The analysis is carried out on 1,000 randomly drawn families of samples and deduced sample couplets.

Insert Figure 1 here.

An important artifact of the simulation is the large variation in the relative distributions of residual maturity of treated and control bonds across sample couplets. In Table 4 we slice the data by family, treatment group, and triangular distribution mode and compute the average of residual maturity across the fifty bonds in each sample. For each sample couplet $\{F_{j,m}^{SC}\}_{m=0.25,1,3,10}$, we calculate the ratio of average residual maturity as average of treated divided by average of control bonds. Panels A and B show populations means as well as distributions, respectively, of the sample averages and the average-maturity ratio across the 1,000 families by treatment group and mode m.

[10]The triangular distribution is a continuous distribution function. We use the terms "triangular distribution" and "triangular probability density function" interchangeably.

Insert Table 4 here.

With mode $m = 0.25$, the population mean in Panel A is 6.75 years and, by definition, the ratio in Panel B is one. With increasing mode the population mean increases and its ratio rises above one. Across the 1,000 families, means and medians of the samples in Panel A and the sample couplets in Panel B are very similar to the population means. Crucially, however, minimums and maximums of average residual maturity in Panel A and the average-maturity ratio in Panel B exhibit large dispersion. For example, if mode $m = 0.25$ for both control and treated bonds, the average sample maturity in Panel A ranges, respectively, from 4.33 to 8.81 and from 4.32 to 9.17 years. As a result, the average-maturity ratio in Panel B ranges from 0.59 to 1.52 and can, therefore, be very different from the population mean even when residual maturity of treated and control bonds is drawn from the *same* underlying distribution. Thus, as artifact resulting from fixed-income data properties observed in practice, namely small security samples per issuer but wide issuer- and time-specific maturity ranges (Table 3), the simulation naturally leads to unequal maturity distributions in the treated and control bond samples. This artifact is illustrated by the large variation in the average-maturity ratio across the 1,000 families within and across modes m. This is the second critical feature that drives the estimation bias.[11]

The following sections make use of the simulated maturity data and the modeled term structures to analyze the classical DiD specification's performance in fixed-income settings.

3. General effects: Measuring false treatment effects

In this section we run the classical DiD specification in (1) on the simulated maturity data when the term structure exhibits differential general, or structural, effects across maturity. These effects are not related to the treatment whatsoever and, therefore, treatment effects are entirely absent from the data. From pre to post treatment, the general effect pushes down the yield curve either only at the short-end or only at the long-end. To model the scenarios, we use Diebold and Li (2006)'s yield curve specification given in (2) and two

[11] Given the maturity structures observed in practice, the triangular distribution function seems a reasonable benchmark to draw residual maturity from. However, notice that, if anything, this is probably a rather conservative benchmark as the dispersion in the average-maturity ratio would increase if we were to draw residual maturity, for example, from a uniform distribution.

periods, with different yield curves for pre- and post-treatment.[12] Figure 2 illustrates the two scenarios graphically together with the underlying yield curve parameter values used to build the curves and the resulting yield levels and differences.[13]

Insert Figure 2 here.

With the chosen parameter values the general short-end effect corresponds to a yield reduction of -50 bps at a residual maturity of one year and an effect close to zero (+1 bps) at fifteen years. The long-end effect amounts to a yield decline of -50 bps at fifteen years and an effect close to zero (+4 bps) at one year.

3.1 Main result: The treatment effect bias

We estimate treatment effects by running the classical DiD specification in (1) using ordinary least squares methodology (OLS). Since Bertrand, Duflo, and Mullainathan (2004) show that the persistence of the treatment indicator in DiD settings induces serial correlation in the error term and that clustering at the level of the treated unit helps to diminish this issue, standard errors are clustered at the individual bond-level.

Figure 3 provides a first set of estimation results. Each graph plots the 1,000 estimated DiD effects against the corresponding t-statistics. The first (second) row of graphs covers the general yield-curve short-end (long-end) effect and graphs on the left (right) the case when $m = 0.25$ ($m = 10$) years.

Insert Figure 3 here.

The figure illustrates that the classical DiD specification always generates false treatment effects if there are differential trends, or general effects, in the term structure at short- compared to long-end. Figures 3a and 3c show that even if residual maturity of the control and treated bonds is drawn from the *same* underlying maturity distribution with $m = 0.25$ years the treatment effects are biased. The effects can go in either direction and the larger in absolute value, the more likely they are statistically different from zero. Across the 1,000

[12] We think about it as follows: Prior to treatment, control and treated bonds share the same term structure, denoted as the pre-treatment curve. Since there is no treatment effect, control and treated bonds also share the same term structure after the treatment but due to other, undetermined economic forces the post-treatment curve has shifted to a different location compared to the pre-treatment curve.

[13] We chose parameter values so that the curves are upward sloping but the argument is independent of the shape of the yield curve. The same bias can be shown for downward sloping or flat yield curves.

families and in case of a general yield-curve short-end (long-end) effect, the bias ranges from -11.59 to 10.85 (-11.34 to 12.01) bps and 91 (88) of them are statistically significant at the 10%-level. When residual maturity is drawn from a triangular distribution function with $m = 10$ years instead and in case of a general yield-curve short-end (long-end) effect, the distribution of estimated treatment effects is shifted upward (downward) with all the 1,000 estimated treatment effects becoming positive (negative) and ranging from 2.30 to 22.99 (-24.06 to -2.59) bps, and 992 (991) of the 1,000 coefficients are statistically significant at the 10%-level, as illustrated in Figure 3b (3d).

3.2 The main driver of the bias

A key driver of the estimation bias is the average-maturity ratio of treated sample bonds relative to controls. For illustration purposes, in Table 5 we sort the 1,000 families of sample couplets on this ratio, index the sample couplets in ascending order, and show the estimated treatment effects for a selection of eleven sample couplets.[14] For each $m \in \{0.25, 1, 3, 10\}$, the first and the last sample couplets in the distribution, $F^{SC}_{1,m}$ and $F^{SC}_{1000,m}$, are the sample couplets with the minimum and maximum average-maturity ratio, respectively. In between, with order indices $j = 10, 50, 100, 250, 501, 751, 901, 951, 991$, $F^{SC}_{j,m}$ represents the sample couplet with average-maturity ratio approximately at percentile $j/10$.

Insert Table 5 here.

Panel A shows the average-maturity ratio for the eleven selected families of sample couplets by mode m. From the upper left corner (order index 1 and $m = 0.25$) to the lower right corner (order index 1,000 and $m = 10$) this ratio tends to go up.[15] Panels B and C show the DiD coefficients and, underneath in parentheses, the associated p-values for each of the eleven selected sample couplets by mode m for a general term-structure short-end (long-end) effect, respectively. In case of a general yield-curve short-end (long-end) effect, the estimated DiD effects tend to increase (decrease) with the average-maturity ratio both within and across modes. This shows that the bias results from the relative distributions of residual maturity in the samples of treated and control bonds. For the same sample couplet the false treatment effect is biased away from the true zero-effect in opposite direction for

[14] Therefore, for each $m \in \{0.25, 1, 3, 10\}$, we have 1,000 *ordered* families of samples couplets with different orders for each m.

[15] Panel A of Table 5 is essentially just a more granular depiction of Panel B in Table 4.

general short- and long-end effects. For example, the estimated treatment effects of the sample couplet with order index 1 and mode $m = 0.25$ years are -11.59 and $+12.01$ bps, respectively.

Furthermore, the table highlights that larger, more extremely biased coefficients are more likely to be statistically different from zero. Coefficients that are statistically significant at the 10% level are marked in bold and the letters a, b, and c indicate significance at the 1%, 5%, and 10% levels, respectively. The larger is the average-maturity ratio both within and across modes the more likely the false treatment effect is statistically significant.

The analysis so far illustrates that the classical DiD specification applied to bond data with bond yield as dependent variable produces biased regression slope coefficients. Even in absence of a true treatment effect the specification generates potentially economically large and statistically significant but false treatment effects. The bias can go in either direction, which is driven by the combination of differential general effects in the term structure at short- and long-end and unequal distributions of residual maturity in the samples of control and treated bonds. Furthermore, the larger is the bias in absolute terms, the more likely it also is statistically different from zero.

3.3 Estimation separately by individual maturity buckets

To deal with differential treatment effects the literature typically uses fixed effects on the discrete units present in the data. Applied to fixed-income securities with yield on the left side of the regression equation, researchers sometimes measure the DiD effects separately by individual maturity buckets (for example Bao, O'Hara, and Zhou, 2018; Todorov, 2020).

Table 6 shows the results if we run the classical DiD specification in (1) on four buckets with residual maturities in the ranges $[0, 2]$, $(2, 5]$, $(5, 10]$, and $(10, 20]$ years. Panel A (B) covers the case when residual maturity of the treated bonds is drawn from a triangular distribution function with mode $m = 0.25$ ($m = 10$) years. Each panel shows the distributions of estimated treatment effects by the different yield-curve effects and the maturity buckets as well as separately for when the coefficients are statistically significant or not at the two-sided 10%-significance level.

Insert Table 6 here.

The results show that running the classical DiD specification separately for individual maturity buckets diminishes the bias but is by no means immune against it. In our case, the bias is the strongest in the $(2,5]$-year maturity bucket. For example, if there is a general yield-curve short-end effect and the mode $m = 0.25$ years in Panel A, across the 1,000 estimated treatment effects $53 + 47 = 100$ range from -7.49 to -3.31 and from $+3.55$ to $+8.47$ bps and are statistically different from zero at the 10%-significance level. The remaining 900 coefficients are biased and range from -4.69 to $+4.73$ bps but are not statistically significant. If $m = 10$ years in Panel B, in 2 cases it is not possible to estimate effects because there are no observations in the treated group. Out of the remaining 998 coefficients $25 + 211 = 236$ are statistically significant and they range more extremely from -10.51 to -3.32 and from $+3.26$ to $+11.89$ bps. In case of a general yield-curve long-end effect the results are similar but, if anything, even slightly more extreme. Importantly, a bias-immune specification would measure effects of zero in all of these cases.

The analysis illustrates that maturity-bucket controls do not resolve the bias but merely shift the issue to the maturity-bucket level. Moreover, taking this approach to the extreme and making the maturity buckets shorter and shorter, in the extreme case we are left with many infinitesimal short maturity buckets and, therefore, no or only very few bonds remaining in each bucket. Hence, because residual maturity is a continuous habitat variable we need a different approach.

Next, we turn to the case when there is no treatment-unrelated general effect but only a yield-curve treatment effect.

4. Mismeasured treatment effects

This section models treatment effects and examines whether the classical DiD specification in (1) is able to identify them. It focuses exclusively on the case when only the yield curve of treated bonds – through the treatment effect – experiences a shift. Hence, there are no treatment-unrelated general effects.[16] The treatment either affects the yield curve only at the short-end or its impact leads to a yield-curve twist. In terms of preview, the analysis will show that the estimated DiD effects represent *sample-specific average* treatment effects, which can cause researchers to draw incorrect conclusions.

[16] We think of it as follows: Control and treated bonds share the same term structure prior to treatment. Since there is no treatment-unrelated general effect, control bonds stay on the pre-treatment curve when the treatment takes place. Only the yield curve of treated bonds moves to a different location.

To model the underlying term structures we continue to employ Diebold and Li (2006)'s yield-curve specification in (2). Figure 4 provides the underlying parameter values and shows the resulting yield curves graphically together with the numbers for yield levels and differences. The short-end treatment effect corresponds to a yield reduction for treated bonds of −6 bps at a maturity of one year and no effect (0 bps) at ten years relative to the pre-curve. The treatment yield-curve twist leads to a yield-curve increase for treated bonds of +6 bps at a maturity of one year and a reduction of −6 bps at ten years. In either case control bonds stay on the pre-treatment curve.

Insert Figure 4 here.

Figure 5 provides the estimated treatment effects using the classical DiD specification in (1) run with OLS. Standard errors are clustered at the bond-level. Figures 5a and 5b show the results, respectively, for a treatment short-end effect and a treatment yield-curve twist. From left to right, the graphs show 1) the true underlying treatment effects as a function of maturity, 2) the distributions of estimated treatment effects when residual maturity of the treated bonds is drawn from triangular distributions with modes at 0.25 and 10 years, and 3) the DiD effects against the corresponding t-statistics for these two modes.

Insert Figure 5 here.

The classical DiD specification estimates a sample-specific average treatment effect. In case of a yield-curve treatment short-end effect (Figure 5a) and residual maturity of the treated and control bonds drawn from a triangular distribution with mode $m = 0.25$ years, this average effect ranges from −2.94 to −0.48 bps. The researcher will draw the correct conclusion in terms of the sign of the coefficient but the magnitude of the effect is not identified. If, however, the residual-maturity distribution of treated bonds is tilted sufficiently to the long-end ($m = 10$ years), the estimated effects range from −0.72 to +0.06 bps. Across the 1,000 estimates the researcher concludes in 699 cases that there is no treatment effect because the DiD estimate is not statistically significant at the 10%-level even if, in fact, the true underlying treatment effect at the one-year maturity is a negative −6 bps.[17]

In case of a treatment yield-curve twist (Figure 5b) and residual maturity of treated

[17] In 294 cases the researcher finds a statistically significant negative effect and in 7 cases even a small positive effect because the true treatment effect is slightly positive at the very long-end of the yield curve.

and control bonds drawn from the *same* triangular distribution with mode $m = 0.25$ years, across the 1,000 estimates the researcher concludes in 121 cases that there is no treatment effect because the DiD estimate is not statistically significant at the 10%-level and in 879 cases that the treatment effect is statistically significantly *negative* even if, in fact, the true underlying treatment effect at the one-year maturity is a *positive* +6 bps. If the residual maturity distribution of the treated bonds is shifted towards the long-end ($m = 10$ years), this becomes more extreme and the researcher concludes in all the 1,000 cases that the treatment effect is negative despite the positive true treatment effect of +6 bps at the one-year maturity.

The analysis shows that the classical DiD specification may lead a researcher to draw incorrect conclusions. This is the case even in absence of treatment-unrelated general yield-curve effects if the treatment effects themselves vary over the term structure. The reason is that the classical DiD specification produces a sample-specific average treatment effect that can be tilted in either direction depending on the relative distributions of residual maturity of the control and treated bond samples. The classical DiD specification is not designed to identify the true underlying treatment effects because the specification is not able to capture differential treatment effects across the maturity spectrum.

5. Combine false and mismeasured treatment effects

This section combines treatment-unrelated general and treatment effects. We continue to employ Diebold and Li (2006)'s specification given in (2) to build the underlying term structures. By choosing the yield-curve parameter values we generate a general short-end (long-end) effect, which reduces yields at a residual maturity of one year (fifteen years) by −50 bps and is close to zero at a maturity of fifteen years (one year). On top of the general effects, we add a treatment yield-curve short-end effect (twist), which pushes yields of treated bonds down (up) by 6 bps at a maturity of one year and has a zero effect (pushes yields down by 6 bps) at a maturity of ten years relative to the general effects. Hence, for the total of four combinations of general and treatment effects, the true effects in this section equal the sum of the individual true effects in Sections 3 and 4. The yield curve parameter values and resulting yield levels and differences are provided in Table 7.

Insert Table 7 here.

When both treatment-unrelated general and actual treatment effects are present in the data at the same time, it turns out that the estimated treatment effect for each sample couplet and combination of true effects is identical to adding up the corresponding bias from Section 3 and the mismeasured effect from Section 4.[18] For illustration purposes, we use OLS to estimate the classical DiD specification in (1) with standard errors clustered at the bond-level using the data with both effects present simultaneously and compare the estimated treatment effect to the sum of the individual components, namely the bias from Section 3 and the mismeasured effect from Section 4, using the same sample couplet and yield data with the corresponding true effects. We run a total of 16,000 different regressions, namely for the four combinations of general and treatment effects, the four modes $m \in \{0.25, 1, 3, 10\}$, and the 1,000 families. The difference between the coefficient estimated in the data with both effects present simultaneously and the sum of the individual coefficients estimated separately in the data when only one of the effects is present across the 16,000 regressions ranges from -0.003 to 0.003 bps and is, therefore, virtually zero.

Inherently in Specification (1) the treatment effect is assumed to be the same for all treated bonds and the treatment-unrelated structural effect the same for all sample bonds. The specification, therefore, does neither allow for treatment-unrelated structural habitats nor for treatment habitats in the yield curve. If the treatment-unrelated general effects or the treatment effects themselves are different in different parts of the yield curve, the classical DiD specification in (1) produces biased and mismeasured effects. Both bias and mismeasurement depend on the true underlying habitat effects and the relative distributions of residual maturity in the samples of treated and control bonds.

6. Model with explicit term structure control

Instead of using the classical DiD specification in (1) researchers sometimes use DiD specifications that explicitly control for the term structure or maturity and higher orders of it when dealing with bond yield data (e.g. Qiu and Yu, 2009; Ayotte and Gaon, 2011; Fuhrer, Müller, and Steiner, 2017; Gao, Lee, and Murphy, 2020). This section studies whether explicit parametric control for the term structure enables a DiD specification to identify the

[18]This results from the simplicity of having just two time periods and no bond-individual noise (the bonds always lie on the term structure).

true underlying effects or, at least, diminish the estimation bias. Specifically, we run

$$yield_{it} = \mathbf{B}' \mathbf{L}_{it} + \alpha \, \mathbb{1}_{Treated,i} + \delta \, \mathbb{1}_{Post,t} + \beta_{DiD} \, \mathbb{1}_{Treated,i} \times \mathbb{1}_{Post,t} + \varepsilon_{it}, \tag{4}$$

where $\mathbb{1}_{Treated,i}$ and $\mathbb{1}_{Post,t}$ are defined as above, α and δ are the corresponding parameters and β_{DiD} is the DiD estimator. The term $\mathbf{B}'\mathbf{L}_{it}$ parametrically controls for the term structure of interest rates. We continue to employ Diebold and Li (2006)'s term structure specification in (2) where $\mathbf{L}_{it}$ is a three-dimensional vector with elements 1, $l_1(x_{it}; \lambda)$, and $l_2(x_{it}; \lambda)$, with the latter two terms defined as

$$l_{1,t}(x; \lambda) = \left(\frac{1 - e^{-\lambda x}}{\lambda x} \right) \quad \text{and} \quad l_{2,t}(x; \lambda) = \left(\frac{1 - e^{-\lambda x}}{\lambda x} - e^{-\lambda x} \right), \tag{5}$$

$\mathbf{B}$ the corresponding vector of coefficients with individual elements β_k, $k = 0, \ldots, 2$, and the decay parameter λ assumed to be independent of time t.

In terms of estimation, since we know the underlying parameter value of lambda, which is $\lambda = 0.7308$, we could simply plug it into the expressions in (5) and use OLS to run Specification (4). Instead, we use nonlinear least squares methodology (NLS) to estimate λ in-sample together with the other parameters. As start value we use $\lambda_{Seed} = 1$.[19] Standard errors are clustered at the bond level.

It turns out that the specification with explicit term structure control in (4) produces the exact same bias and mismeasurement as the classical DiD specification in (1). For illustration purposes, we compute the difference between the coefficients estimated with Specifications (1) and (4) for all possible combinations of modeled yield-curve effects (eight in total) and all sample couplets (4,000 in total, namely four modes and 1,000 families), which gives a total of 32,000 different regressions run with each specification. Across the 32,000 differences in DiD coefficients between Specifications (1) and (4) even minimum and maximum amount to 0.0000 and 0.0001 bps, respectively. This shows that the two specifications produce the exact same biased and mismeasured treatment effects.[20]

The specification with explicit term structure control in (4) controls for the yield curve

[19] Alternatively, Nyborg and Woschitz (2021) use NLS to first estimate the yield curves separately for treated and control bonds pre- and post-treatment and, then, average across those within-group $\widehat{\lambda}$s, plug the average into (5) and use OLS to estimate Specification (4).

[20] We have also calculated the differences in p-values across the 32,000 different cases. The difference in p-values ranges from -0.0056 to 0.0000 showing that Specification (4) is slightly more conservative than Specification (1) but the difference is tiny.

parametrically. However, $\mathbf{B}'\mathbf{L}_{it}$ just removes the average term structure in the pooled sample data of the different groups (treated, controls, pre-, post-treatment). The specification therefore explicitly restricts yield-curve movements between the different groups to parallel yield-curve level-shifts. While this feature is explicit with $\mathbf{B}'\mathbf{L}_{it}$, the classical DiD specification imposes the same parallel level-shift restrictions more implicitly through the bond fixed effects as they capture, among other things, bond maturity. If, however, treatment-unrelated general effects or the treatment effects themselves come as movements other than parallel level-shifts, both of these specifications produce false and mismeasured treatment effects. In fact, any specification that restricts yield-curve movements to parallel level-shifts will produce the same false and mismeasured treatment effects. The results illustrate that specifications with explicit term structure control can suffer from the same inability to capture treatment-unrelated general habitat as well as treatment habitat effects as the classical DiD specification because these specifications are not designed to deal with differential effects in different parts of the yield curve.

The next section provides methodology acknowledging that both general and treatment effects can vary along maturity and, therefore, eliminates both bias and mismeasurement.

7. Resolution of the bias and mismeasurement

As touched on in the Introduction, a simple solution to the problem is perfect matching. However, in practice perfect maturity matching is rarely feasible. This section acknowledges this feature of fixed-income securities data and provides an alternative solution to resolving the bias and mismeasurement of treatment effects. The main approach is to combine DiD methodology with flexible yield-curve modeling. In particular, two approaches are discussed, namely 1) the fully flexible yield-curve DiD specification and 2) semi-matching. Both approaches acknowledge that the underlying movements in yield curves, the general as well as the treatment effects, may vary across maturity and both approaches are able to separate treatment-unrelated general yield-curve effects from true treatment effects.

7.1 The fully flexible yield-curve DiD specification

The fully flexible yield-curve DiD specification was first used by Nyborg and Woschitz (2021). It does not impose any particular relation between pre- and post-curves of treated and control bonds. On the contrary, the specification uses the estimated yield curves of the different groups and measures the DiD in yield curves. It treats maturity as a continuous habitat variable. Specifically, we estimate the specification

$$yield_{it} = \mathbf{B}_1' \ \mathbf{L}_{it} + \mathbf{B}_2' \ \mathbf{L}_{it} \ \mathbb{1}_{Treated,i} + \mathbf{B}_3' \ \mathbf{L}_{it} \ \mathbb{1}_{Post,t} + \mathbf{B}_4' \ \mathbf{L}_{it} \ \mathbb{1}_{Treated,i} \times \mathbb{1}_{Post,t} \ + \ \varepsilon_{it}, \quad (6)$$

where notation is as above except that each of the four indicators (constant, $\mathbb{1}_{Treated,i}$, $\mathbb{1}_{Post,t}$, and $\mathbb{1}_{Treated,i} \times \mathbb{1}_{Post,t}$) has its own Diebold-Li curve, $\mathbf{B}_j'\mathbf{L}_{it}$, $j = 1, \ldots, 4$, with three individual coefficients each, $\beta_{k,j}$, $k = 0, \ldots, 2$. For simplicity, the fourth parameter, λ, is assumed to be time-invariant and the same for treated and control bonds.[21]

The first curve, when $j = 1$, represents the spot curve of control bonds pre treatment and is given by

$$s^{dl}(x; \widehat{\lambda}) = \widehat{\beta}_{0,1} + \widehat{\beta}_{1,1} \ l_1(x; \widehat{\lambda}) + \widehat{\beta}_{2,1} \ l_2(x; \widehat{\lambda}), \quad (7)$$

where $\{\widehat{\beta}_{k,1}\}_{k=0}^2$ are the estimated regression coefficients, x is residual maturity, and l_1 and l_2 are as in (5) with λ replaced by $\widehat{\lambda}$. The other three curves, when $j = 2, \ldots, 4$, are what we call the delta curves, namely

$$\Delta_j^{dl}(x; \widehat{\lambda}) = \widehat{\beta}_{0,j} + \widehat{\beta}_{1,j} \ l_1(x; \widehat{\lambda}) + \widehat{\beta}_{2,j} \ l_2(x; \widehat{\lambda}), \quad (8)$$

where $\{\widehat{\beta}_{k,j}\}_{k=0}^2$ are the estimated regression coefficients. The delta curves with $j = 2, \ldots, 4$ capture, respectively, the incremental differences for i) treated bonds, ii) the post-treatment period, and iii) treated bonds over the post-treatment period. Adding them to the spot curve for control bonds pre treatment, $s^{dl}(x; \widehat{\lambda})$, returns the spot curve, respectively, for i) treated bonds pre treatment, $s^{dl}(x; \widehat{\lambda}) + \Delta_2^{dl}(x; \widehat{\lambda})$, ii) control bonds post treatment, $s^{dl}(x; \widehat{\lambda}) + \Delta_3^{dl}(x; \widehat{\lambda})$, and iii) treated bonds post treatment, $s^{dl}(x; \widehat{\lambda}) + \sum_{j=2}^4 \Delta_j^{dl}(x; \widehat{\lambda})$.

$\Delta_4^{dl}(x)$, the delta curve when $j = 4$, measures the DiD in yield curves of treated relative to control bonds from pre- to post-treatment. $\Delta_4^{dl}(x)$ is a function of residual maturity x and, therefore, captures the treatment effect at different maturities. Treatment-unrelated general effects from pre- to post-treatment will be captured by the delta curve on the indicator variable for the post-treatment period ($j = 3$). Hence, the specification is able to separate treatment-unrelated general yield-curve effects from actual treatment effects and

[21] This assumption can easily be relaxed.

allows for differential general and treatment effects in different parts of the yield curve.

To analyze its performance, we estimate Specification (6) with NLS and cluster standard errors at the bond level. We therefore continue to estimate lambda in-sample and use the start value $\lambda_{Seed} = 1$. For each regression, the estimation gives twelve coefficients and one estimate for lambda. Because it is difficult to grasp the economics from these coefficients, however, they are used to predict the treatment effects at selected maturities and the corresponding standard errors are calculated with the delta method.[22] As an illustration, separately for each of the eight combinations of modeled yield-curve movements[23], Table 8 shows the distributions of the estimated treatment effects at the selected maturities across the 1,000 families of sample couplets when $m = 0.25$ years. The first column in each block provides the true underlying treatment effect at the selected maturities.

Insert Table 8 here.

The measured quantity is intuitive. For example, the second row of blocks shows results for all combinations of effects that include a yield-curve treatment short-end effect. In that case the true underlying treatment effect corresponds to −6.23, −2.97, −1.39, −0.26, 0.00, 0.08, and 0.09 bps at the maturities, respectively, of 1, 2, 3, 5, 7, 10, and 15 years. The table shows that the procedure generates highly accurate coefficients. Across the eight combinations of modeled yield-curve effects and the 1,000 sample couplets and, hence, a total of 8,000 different regressions, even minimums and maximums of the estimated DiD coefficients exhibit a measurement error compared to the true treatment effects of maximally ±0.01 bps. This is a large improvement compared to Figure 3 where Specification (1) generated a bias ranging from −11.59 to 12.01 bps.

To test for the performance more generally, Table 9 calculates the difference between estimated and true treatment effects at the selected maturities and shows minimum and maximum across the 8,000 regression results not only for $m = 0.25$ but also for $m = \{1, 3, 10\}$ years (which is a total of 32,000 regressions).

Insert Table 9 here.

Panel A, if $m = 0.25$ years, shows the result from above: Across the 8,000 regressions,

[22] The Internet Appendix illustrates the procedure in detail step-by-step.

[23] Across the combinations of general yield-curve effects (no effect, short-end effect, long-end effect) and yield-curve treatment effects (no effect, short-end effect, yield-curve twist) we leave out the no effect/no effect combination, which gives a total of eight combinations.

the difference between estimated and true effects ranges between ±0.01 bps. If, however, m increases to 1, 3, and 10 years, the tails of the distributions exhibit relatively more measurement error at the short-end of the yield curve. The largest error is measured when $m = 10$ years at the one-year maturity. Across the 8,000 regressions, the difference ranges between −0.21 and +0.18 bps. Notice, however, this is still a large improvement compared to the estimated bias of between −24.06 and 22.99 bps produced with Specification (1) in Figure 3. Furthermore, Panel B shows that the increasing measurement error with increasing m results from a lack of data. In Panel B, as a crude test, we repeat the analysis in Panel A but restrict the underlying sample data to "good sample couplets," namely to those which contain at least one treated and one control bond in the [0,1]-year maturity bucket. Using the remaining 21,872 "good sample couplets" (from the total of 32,000) only, the measurement error at the short-end disappears. The difference between estimated and true treatment effects lies between ±0.01 bps across the whole maturity spectrum including the short-end when $m = 10$ years. This shows that the estimation error at the short-end in Panel A results from a lack of data at the short-end in some samples.

In short, the fully flexible yield-curve DiD specification in (6) eliminates the estimation bias, is able to separate actual treatment effects from treatment-unrelated general yield-curve effects, and the measured quantity reflects that treatment effects depend on maturity.

7.2 Semi-matching

Perfect matching would solve the problem but in practice is rarely feasible. Researchers thus sometimes apply imperfect matching procedures (see, e.g., Ang, Bhansali, and Xing, 2010; Eser and Schwaab, 2016; Choi, Hoseinzade, Shin, and Tehranian, 2020).[24] This section seeks to perfect the imperfect-matching approach by combining DiD analysis with yield-curve modeling, and uses what we call "semi-matching." As perfect matching is not feasible, each treated bond is matched with a synthetic control bond whose yield is inferred from a contemporaneous yield curve of control bonds. We apply semi-matching as follows:

1. Separately for pre- and post-treatment periods, use Diebold and Li (2006)'s specification in (2) to estimate the yield curve of control bonds;[25]

[24] Eser and Schwaab (2016) use a matching approach indirectly by restricting their DiD analysis to five-year benchmark bonds.

[25] As before, we use NLS to estimate lambda in-sample and use a start value $\lambda_{Seed} = 1$.

2. Apply semi-matching: Separately for each period, subtract the spot yield of a maturity-matched synthetic control bond from the yield of each treated sample bond;[26]

3. For each maturity-matched pair of treated and synthetic control bond, calculate the difference in the pair's yield-difference from pre- to post treatment, which represents the DiD in yields for each bond pair i, $yield_{it}^{DiD}$.

Semi-matching is illustrated in Figure 6 using one random sample couplet with $m = 0.25$ years. The figure plots the difference between the estimated Diebold-Li control-bond curves from pre- to post-treatment for a general yield-curve short-end (long-end) effect on the left (right). The (red) diamonds and (green) crosses show the difference of the treated bonds' yields from pre- to post-treatment in the cases, respectively, of a yield-curve treatment short-end effect and a treatment yield-curve twist.

Insert Figure 6 here.

In the following the analysis shows in three steps that 1) semi-matching applied on the entire sample couplet will eliminate the bias related to general yield-curve effects but not the mismeasurement of treatment effects and may, therefore, still lead researchers to draw incorrect conclusions; 2) the analysis by maturity buckets resolves the bias and reduces the likelihood of mismeasurement but does not eliminate the latter; 3) the analysis bond-by-bond resolves both bias and mismeasurement of treatment effects. As will be shown, estimating a yield curve through the individual bond-level DiDs after having applied semi-matching will result in the DiD delta curve, $\Delta_4^{dl}(x)$, estimated with Specification (6).

Potentially misleading average treatment effects

Semi-matching eliminates the part of the bias that derives from general, treatment-unrelated yield-curve effects. For illustration purposes, we proceed as follows: First, we run a regression of the bond-level DiDs in yields on a constant C,

$$yield_{it}^{DiD} = \beta_{DiD} \times C + \varepsilon_{it}, \tag{9}$$

[26]The term "semi-matching" reflects that exact matching is not possible. Only semi-matching on a synthetic control bond whose yield is inferred from the surrounding bonds via yield-curve modeling is feasible.

to estimate the average treatment effect, β_{DiD}, for each sample couplet. We run Specification (9) on the simulated data from Section 5, which exhibits both true underlying general and treatment effects simultaneously. This is a total of 16,000 different regressions (two general effects, two treatment effects, four modes $m \in \{0.25, 1, 3, 10\}$, and 1,000 families of sample couplets). Importantly, general effects are present in that data.

Second, for each sample couplet, we compute the difference between the semi-matched DiD coefficient (using Specification (9)), when true underlying general effects are present in the data, and the DiD coefficient from the classical DiD specification in (1), but the latter run on the data that exhibits no general (but only treatment) effects (as in Section 4). Hence, the latter coefficients from the classical DiD specification exhibit mismeasurement only but are not biased because there are no true general yield-curve effects in the data.

Across the 16,000 different combinations, namely the two general effects, the two treatment effects, the four modes, and the 1,000 families of sample couplets, the differences in coefficients ranges from -0.003 to 0.004 bps.[27] This illustrates that semi-matching applied on the entire sample couplet produces the same mismeasurement as in Section 4, however, in the data where treatment-unrelated general effects are present. Hence, semi-matching eliminates the bias that stems from treatment-unrelated structural effects but may still lead to incorrect conclusions because of potentially mismeasured treatment effects.

Semi-matching separately by individual maturity buckets

This subsection shows that semi-matching applied by maturity buckets eliminates the bias related to general yield-curve effects and reduces the likelihood to draw incorrect conclusions based on mismeasured treatment effects but does not entirely eliminate the latter. For illustration, we run Specification (9), as in the previous subsection, on the data that exhibit both true general and treatment effects simultaneously but separately for subsets of bonds with residual maturity in the buckets $[0, 2]$, $(2, 5]$, $(5, 10]$, and $(10, 20]$ years.

Table 10 shows the results. Separately by maturity bucket and for the treatment yield-curve short-end effect and the twist in Panels A and B, respectively, each panel shows the number of estimated treatment effects, minimum and maximum number of involved bonds, the true treatment effects at maturity-range start and end, as well as minimum and

[27] Notice that this extreme similarity is not generic. It is the result of the simplicity of the setting with only two time periods and no noise of yields around the term structures.

maximum of the distributions of the estimated treatment effects for both types of general effects. Per panel and maturity bucket this involves all four modes and the 1,000 families of sample couplets and, therefore, represents at total of 4,000 different regressions.

Insert Table 10 here.

Across panels, the number of estimated coefficients is below 4,000 for the $[0, 2]$- and the $(2, 5]$-year buckets showing that, for some sample couplets and maturity buckets, there are no treated bonds. The remaining 3,614 and 3,998 coefficients for these two buckets are estimated with a minimum of one and a maximum of twenty and twenty-four treated bonds, respectively. Hence, the more maturity buckets, the shorter its maturity ranges, and the smaller the number of bonds per range.

Comparing true and estimated treatment effects shows that, for each maturity bucket, the estimated coefficients cover, with one exception, the range of true treatment effects.[28] Semi-matching applied by maturity bucket produces, per maturity bucket, a sample-specific average treatment effect, which is less likely to misguide researchers as, on the maturity bucket level, the sign of the coefficient is more likely to be correct. Notice, however, to draw incorrect conclusions regarding the sign of the coefficient is still possible, namely in our case with maturity bucket $(5, 10]$ in Panel A and $(2, 5]$ in Panel B.

Overall, this illustrates the trade-off between the accuracy of measured treatment effects and the power of the test. With an increasing number of maturity buckets and, thus, buckets of relatively shorter length, the precision of the estimated effects increases but is based on less bonds per bucket. Semi-matching applied by individual maturity buckets eliminates the bias due to treatment-unrelated general yield-curve effects and reduces the likelihood to draw incorrect conclusions but the latter is not entirely ruled out.

Semi-matching bond-by-bond

The extreme case is what is emphasized in the synthetic control literature, namely to analyze each sample couplet and the effects bond-by-bond (see, e.g., Abadie, 2021). A glance at Figure 6 is sufficient to detect the small treatment effects relative to the large treatment-unrelated general yield-curve effects and to see that both types of effects vary

[28]The one exception relates to the $(2, 5]$-year bucket in Panel A, where the maximum of -0.29 bps in case of a general short-end effect is outside the range of true effects of $[-2.97, -0.26]$ bps.

along maturity. Applying semi-matching via yield-curve modeling eliminates treatment-unrelated general yield-curve effects and is, as no averaging takes place, simultaneously immune against incorrect conclusions based on mismeasured treatment effects. However, as discussed in detail in the synthetic control literature (see, e.g., Xu, 2017), the estimation of adequate standard errors to draw inference is more laborious.[29]

One way to express the results of bond-by-bond semi-matching is to estimate and describe a curve through the bond-level DiDs in yields. As an illustration, separately for each of the four combinations of general and treatment effects and the same sample couplet as in Figure 6, we use NLS and $\lambda_{Seed} = 1$ to fit a Diebold-Li curve through the bond-level DiDs in yields in Figure 6. Exhibit 2 shows the true as well as the estimated treatment effects predicted at selected residual maturities of 1, 2, 3, 5, 7, 10, and 15 years.

In short, Exhibit 2 shows the exact same treatment effects as produced with the fully flexible yield-curve DiD specification given in (6) and the results provided in Table 8. In other words, estimating a curve through the bond-level DiDs in yields after having applied semi-matching results in the delta curve, $\Delta_4^{dl}(x;\widehat{\lambda})$, provided in (8), which builds the bridge between bond-level semi-matching and the fully flexible yield-curve DiD specification from Subsection 7.1.

Exhibit 2: Estimated treatment effects (in bps) with semi-matching

Residual maturity (in years)	True treatment effects		General and treatment effects			
			short-end		long-end	
	short-end	twist	short-end	twist	short-end	twist
1	-6.23	5.87	-6.23	5.88	-6.23	5.87
2	-2.97	3.75	-2.97	3.75	-2.97	3.74
3	-1.39	1.58	-1.39	1.58	-1.39	1.58
5	-0.26	-1.87	-0.26	-1.87	-0.26	-1.87
7	0.00	-4.11	0.00	-4.11	0.00	-4.11
10	0.08	-6.09	0.08	-6.09	0.08	-6.09
15	0.09	-7.72	0.09	-7.71	0.09	-7.71

Overall, the advantage of the fully flexible yield-curve DiD specification over semi-matching is its simple and fast implementation. In our case, with plain zero-coupon bond yields, it accurately estimates the DiD in yield curves with one single regression. Furthermore, it is possible to cluster standard errors at the individual bond-level (Bertrand, Duflo, and Mullainathan, 2004). However, it is less flexible with respect to including other

[29] Standard errors are typically based on bootstrapping methods.

bond features such as coupons or callability features. Semi-matching allows to match also on other relevant bond characteristics. However, semi-matching is laborious not only but especially also when it comes to the estimation of standard errors (see, e.g., Xu, 2017).

8. Concluding remarks

Difference-in-differences (DiD) methodology is frequently used in finance to assess the causal impact of a treatment on yields of fixed-income securities. Simultaneously, fixed-income securities are priced against a term structure of interest rates and perfect matching is rarely feasible in practice.

Using simulations, this paper shows that the classical DiD specification with yield as dependent variable produces false, systematically biased as well as mismeasured treatment effects. This holds even under random assignment of the treatment. To illustrate the bias we simulate residual maturity of treated and control bond samples and two types of yield-curve effects which both vary along maturity. First, general yield-curve effects are not related to the treatment whatsoever and affect treated and control bonds the exact same way. The analysis shows, however, that differential general effects across maturity lead to false, systematically biased coefficients even in absence of true underlying treatment effects. Second, yield-curve treatment effects influence only the treated bonds. However, differential yield-curve treatment effects across maturity lead to mismeasured treatment effects that may mislead to draw incorrect conclusions even without treatment-unrelated general effects present in the data. Both bias and mismeasurement can be economically large, can go in either direction, and statistical significance increases with the coefficients' absolute size. As shown, neither explicit term structure control in the specification nor regressions for individual maturity buckets resolve bias and mismeasurement.

The limitation of these specifications is the inability to capture general and treatment effects if these effects vary over maturity. The magnitudes of bias and mismeasurement are sample-induced and depend on 1) differential general and treatment effects in different parts of the term structure and 2) the relative distributions of treated and control bonds across maturity. The root of bias and mismeasurement lies in the specifications' implicit restrictions of movements in the underlying yield curves to parallel level-shifts between the involved groups (treated, controls, both pre- and post-treatment). In fact, bias and mismeasurement survive any DiD specification that implicitly assumes parallel yield-curve level shifts.

The paper provides new methodology to overcome both bias and mismeasurement by combining DiD analysis with yield-curve modeling. First, the fully flexible yield-curve DiD specification takes parametric yield curves explicitly into the DiD estimator. Instead of measuring the DiD in yields, this specification measures the DiD in yield curves between the involved groups and thereby tackles both bias and mismeasurement. Second, semi-matching tells apart the different pieces of the fully flexible yield-curve DiD specification and provides an approach that step-by-step teases out the true underlying treatment effects.

Bias and mismeasurement are shown in the trivial setting with bond yield on the left side of the regression equation and bond-fixed effects on its right side. The importance of this paper is owed, however, to the survival of both bias and mismeasurement when the unit of analysis is an aggregation of the bond level, with other dependent variables, and for other right-side controls of maturity. Overall, this shows that DiD methodology must be applied with great caution in fixed-income settings, especially with respect to residual maturity.

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Table 1: Recent top finance publications potentially affected.

This tables shows a collection of recent top finance publications potentially affected by the bias discussed in the present paper. The list was created by a manual search of The Journal of Finance (JF), The Journal of Financial Economics (JFE), and The Review of Financial Studies (RFS) over the period July 1 to September 10, 2021, using relevant combinations of key words. “Treas.” is short for Treasury and “mat.” is short maturity.

Authors	**Publ. year**	**Jour-nal**	**Analysis level**	**Dependent variable**	**Independent variable(s) to capture maturity**[1]
Chava, Livdan, Purnanandam	2009	RFS	Loan, or firm	Log changes in loan spread over LIBOR	–
Qiu, Yu	2009	JFE	Firm	Credit spread over mat.-matched Treas.	Duration and convexity[2]
Titman, Tsyplakov	2010	RFS	Mortgage	Credit spread over mat.-matched Treas.	Mortgage resid. time-to-mat.
Ayotte, Gaon	2011	RFS	ABS issuance	ABS spread over mat.-matched swap rates	Average life and its quadratic term
Hasan, Hoi, Wu, Zhang	2014	JFE	Loan facility	Log loan spread over LIBOR	Log resid. time-to-mat.
Rodano, Serrano-Velarde, Tarantino	2016	JFE	Bank-firm	Loan interest rate	Bank-firm fixed effects, loan mat.[3]
Adelino, Ferreira	2016	RFS	Loan facility	Loan spread over LIBOR	–
Cornaggia, Cornaggia, Israelsen	2018	RFS	Bond	Yield, credit spread over dur.-matched Treas.[4]	Duration
Dannhauser	2017	JFE	Bond	Yield spread over mat.-matched swap rates[5]	Bond fixed effects
Bao, O’Hara, Zhou	2018	JFE	Bond	Yield spread over mat.-matched Treas.	Log resid. time-to-mat.
Todorov	2020	JFE	Bond	Yield	Bond fixed effects
Gao, Lee, Murphy	2020	JFE	Bond	Yield spread over coupon-equiv. Treas. yield[6]	Resid. time-to-mat. and its inverse
Painter	2020	JFE	Bond	Annualized issuance cost[7], yield	Log resid. time-to-mat.[8]
Benetton, Fantino	2021	JFE	Bank-firm	Loan rate	Bank-firm fixed effects
Ding, Xiong, and Zhang	2021	JFE	Issuance	Issuance spread over Chinese Treas.	Resid. time-to-mat.

(1) Dashes mean that there are no independent variables to capture maturity

(2) Bond-level characteristics are converted into firm-level measures (value-weighted). Authors also run bond level regressions: Independent variables are not aggregated

(3) Loan maturity is measured with indicator variables for $<$ 1 year, 1-5 years, and $>$ 5 years

(4) Either using each bond’s time-to-maturity or with the callable bonds’ call dates in lieu of their maturity dates

(5) Monthly volume-weighted yield of a bond over the maturity-matched swap rate

(6) See Footnote 7 in Gao, Lee, and Murphy (2020) for details

(7) For details see Painter (2020), page 470

(8) Sample split at maturity of 25 years throughout the paper

Table 2: Time-variation in the term spread in practice.

This table shows the distribution of changes in the term spread for a selected group of countries over the period from January 3, 2000 to December 14, 2022. The term spread is measured in basis points (bps) and calculated as ten-year minus two-year zero-coupon spot yield. Panel A shows daily changes in the term spread (using end-of-day pricing data) and Panel B monthly changes (using end-of-month data). Data source: Bloomberg.

Country	Mean	SD	Min	P1	P5	Med	P95	P99	Max	N
Panel A: Distribution of daily changes in the term spread (10y-2y) [in bps]										
Japan	0	2	-16	-5	-3	0	3	5	20	5,985
Netherlands	0	3	-19	-8	-4	0	4	8	19	5,986
Germany	0	3	-20	-8	-4	0	5	8	25	5,987
France	0	3	-20	-8	-4	0	5	8	18	5,217
United States	0	4	-28	-10	-6	0	6	11	25	5,987
United Kingdom	0	3	-37	-9	-5	0	5	9	25	5,985
Spain	0	4	-58	-11	-6	0	6	11	60	5,959
Italy	0	5	-137	-13	-5	0	6	13	81	5,987
China	0	9	-139	-25	-8	0	8	24	176	4,766
Portugal	0	10	-145	-26	-8	0	8	29	140	5,985
Ireland	0	9	-171	-19	-6	0	6	18	271	5,982
Greece	0	46	-906	-100	-22	0	17	93	1,822	5,976
Panel B: Distribution of monthly changes in the term spread (10y-2y) [in bps]										
Japan	0	8	-23	-18	-11	-1	12	23	39	219
Netherlands	-1	17	-51	-37	-22	-1	27	48	87	219
Germany	-1	15	-41	-37	-22	-3	26	48	82	219
France	-1	16	-37	-35	-24	-2	26	76	79	193
United States	0	18	-44	-40	-29	-2	34	49	59	219
United Kingdom	0	17	-46	-38	-26	0	24	47	98	219
Spain	0	25	-114	-47	-33	-3	33	91	172	216
Italy	0	25	-104	-52	-30	-3	29	95	184	219
China	0	21	-74	-69	-32	-1	36	62	106	179
Portugal	0	55	-412	-211	-43	-1	71	170	351	219
Ireland	-2	35	-177	-147	-42	-2	45	122	166	219
Greece	4	191	-802	-483	-219	0	126	761	1,745	219

Table 3: The maturity structure of outstanding debt in practice.

This table provides the number of outstanding securities as well as outstanding debt by maturity buckets for the same selection of countries as in Table 2 at the beginning of 2023 (Panel A) and at the beginning of 2011 (Panel B). For each country, outstanding debt by maturity bucket is provided as percentage of the total outstanding debt by that country. Data source: Bloomberg.

Country	# of sec.	[0-2]	(2-5]	(5-10]	(10-15]	(15-20]	(20-30]	>30y
Panel A: At the beginning of 2023								
Netherlands	31	23	21	26	9	11	9	1
Portugal	32	17	28	35	14	2	5	0
Ireland	59	10	20	38	11	3	13	4
Spain	82	23	25	30	8	5	7	2
Greece	82	21	18	27	18	6	9	1
Germany	84	30	24	26	6	4	10	0
France	97	21	25	31	6	7	7	4
United Kingdom	122	14	17	19	10	10	17	13
Italy	205	28	25	25	9	5	7	1
United States	444	42	24	17	0	6	10	0
China	493	33	29	23	1	2	9	4
Japan	559	31	20	21	8	8	9	2
Panel B: At the beginning of 2011								
Netherlands	41	35	25	22	6	4	4	3
Portugal	45	26	27	30	11	0	5	0
Ireland	16	12	20	59	9	0	0	0
Spain	63	33	24	23	7	4	8	2
Greece	105	22	28	28	10	5	7	1
Germany	274	33	26	26	3	4	8	1
France	92	30	24	25	8	4	6	3
United Kingdom	100	14	19	21	9	8	16	13
Italy	178	30	23	22	9	5	9	1
United States	305	41	26	23	3	3	5	0
China	285	32	24	21	12	5	5	1
Japan	466	36	25	22	6	7	3	0

Table 4: Overview on simulation of residual maturity.

This table provides an overview of the simulated families of samples and sample couplets. One family of samples comprises five simulated residual maturity samples, one for control bonds (with $m = 0.25$) and four for treated bonds (with $m = \{0.25, 1, 3, 10\}$). m is the mode of the triangular distribution from which residual maturity is drawn. Each sample is comprised of fifty bonds. In total, we simulate 1,000 families of samples. Panel A shows the distributions of average-maturity across the 1,000 families separately for each treatment group and mode. The four sample couplets within each family are built by pairing each sample of treated bonds with $m = \{0.25, 1, 3, 10\}$ with the sample of control bonds with $m = 0.25$. Each sample couplet contains fifty control and fifty treated bonds. Panel B provides the distributions of average-maturity ratios across the families of sample couplets separately for each mode m. The ratio is calculated as average residual maturity of the fifty treated bonds for each mode $m \in \{0.25, 1, 3, 10\}$ divided by average residual maturity of the fifty control bonds with $m = 0.25$.

Panel A: Average-maturity across families of samples by treatment group and mode

Samples	Group	m	Population mean	Sample distributions: No. of families	Mean	SD	Med	Min	Max
$S_{C,0.25}$	Control	0.25	6.75	1,000	6.76	0.67	6.77	4.33	8.81
$S_{T,0.25}$	Treated	0.25	6.75	1,000	6.73	0.67	6.71	4.32	9.17
$S_{T,1}$		1	7.00	1,000	7.03	0.61	7.03	5.36	9.01
$S_{T,3}$		3	7.67	1,000	7.65	0.60	7.63	5.82	9.56
$S_{T,10}$		10	10.00	1,000	9.98	0.59	10.02	8.04	11.98

Panel B: Average-maturity ratios across families of sample couplets by mode

Sample couplets	m treated bonds	Population mean	Sample distributions: No. of families	Mean	SD	Med	Min	Max
$\{S_{C,0.25}, S_{T,0.25}\}$	0.25	1.00	1,000	1.01	0.14	0.99	0.59	1.52
$\{S_{C,0.25}, S_{T,1}\}$	1	1.04	1,000	1.05	0.14	1.04	0.71	1.60
$\{S_{C,0.25}, S_{T,3}\}$	3	1.14	1,000	1.14	0.15	1.14	0.69	1.81
$\{S_{C,0.25}, S_{T,10}\}$	10	1.48	1,000	1.49	0.18	1.48	1.10	2.23

Table 5: False treatment effects.

This table shows estimated treatment effects using the simulated data with differential general yield-curve effects across maturity as indicated but without true treatment effects. The estimation uses OLS and the classical DiD specification $yield_{it} = \alpha_i + \delta_t + \beta_{Post}\,\mathbb{1}_{Post,t} + \beta_{DiD}\,\mathbb{1}_{Treated,i} \times \mathbb{1}_{Post,t} + \varepsilon_{it}$, where $yield_{it}$ is the yield-to-maturity of bond i on day t, the α_i's (δ_t's) are bond (time) fixed effects, $\mathbb{1}_{Treated,i}$ ($\mathbb{1}_{Post,t}$) is an indicator variable being one for bond i if bond i is treated (being one for event and post-event dates), β_{DiD} is the treatment effect, and ε_{it} the error term. The results are presented as follows: For each mode $m \in \{0.25, 1, 3, 10\}$ of the treated bonds the 1,000 families of sample couplets are ordered according to the average-maturity ratio (treated divided by control bonds) in ascending order. For each m, the selected sample couplets are the ones with order index as indicated in the table and with its average-maturity ratio provided in Panel A. For the corresponding sample couplet in Panel A, Panels B and C show the estimated DiD effects and, underneath in parentheses, p-values based on standard errors clustered at the bond-level, respectively, for a general term-structure short-end (long-end) effect. a, b, and c denote significance (two-sided) at the levels of 1%, 5%, and 10%, respectively. Coefficients statistically significant at the 10%-level or better are marked in bold.

m treated	Order index of families of sample couplets										
bonds	1	10	50	100	250	501	751	901	951	991	1000
Panel A: Ratio of average residual maturity of treated over control bonds											
0.25	0.592	0.742	0.799	0.843	0.904	0.994	1.086	1.184	1.259	1.379	1.524
1	0.710	0.766	0.838	0.877	0.948	1.039	1.142	1.244	1.304	1.419	1.598
3	0.691	0.838	0.913	0.963	1.044	1.135	1.228	1.334	1.400	1.527	1.805
10	1.095	1.144	1.232	1.277	1.362	1.482	1.594	1.715	1.790	2.027	2.231
Panel B: General yield-curve short-end effect											
0.25	**-11.59**[a]	**-7.22**[b]	-3.29	-4.57	-3.89	0.89	-0.58	4.10	4.16	**9.32**[a]	**10.79**[a]
	(0.00)	(0.04)	(0.35)	(0.22)	(0.28)	(0.81)	(0.86)	(0.24)	(0.26)	(0.01)	(0.00)
1	**-8.67**[a]	**-7.79**[b]	-4.82	-1.19	0.54	1.32	2.19	**6.82**[c]	**6.01**[c]	**7.84**[b]	**11.32**[a]
	(0.01)	(0.02)	(0.14)	(0.73)	(0.89)	(0.70)	(0.55)	(0.07)	(0.09)	(0.03)	(0.00)
3	**-8.73**[a]	-2.01	-0.82	1.93	2.30	4.24	**5.88**[c]	**7.98**[b]	**8.63**[b]	**11.35**[a]	**14.24**[a]
	(0.01)	(0.55)	(0.82)	(0.58)	(0.46)	(0.21)	(0.09)	(0.01)	(0.01)	(0.00)	(0.00)
10	**6.30**[c]	4.26	**8.61**[b]	**9.42**[a]	**9.88**[a]	**10.80**[a]	**14.14**[a]	**15.35**[a]	**15.34**[a]	**19.47**[a]	**22.99**[a]
	(0.06)	(0.14)	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Panel C: General yield-curve long-end effect											
0.25	**12.01**[a]	**7.67**[b]	3.86	4.72	4.17	-1.06	0.25	-4.15	-4.35	**-9.94**[a]	**-11.34**[a]
	(0.00)	(0.04)	(0.30)	(0.23)	(0.27)	(0.78)	(0.94)	(0.26)	(0.26)	(0.01)	(0.00)
1	**9.31**[a]	**8.31**[b]	5.32	1.56	-0.21	-1.51	-2.08	**-7.00**[c]	**-6.45**[c]	**-8.28**[b]	**-11.84**[a]
	(0.01)	(0.02)	(0.13)	(0.68)	(0.96)	(0.68)	(0.60)	(0.08)	(0.08)	(0.03)	(0.00)
3	**9.72**[a]	2.59	1.16	-1.69	-2.15	-4.40	-6.12	**-8.47**[b]	**-8.82**[b]	**-11.50**[a]	**-14.95**[a]
	(0.00)	(0.47)	(0.77)	(0.65)	(0.52)	(0.22)	(0.10)	(0.02)	(0.02)	(0.00)	(0.00)
10	**-6.56**[c]	-4.54	**-8.60**[b]	**-9.92**[a]	**-10.53**[a]	**-11.79**[a]	**-15.32**[a]	**-16.56**[a]	**-16.51**[a]	**-20.63**[a]	**-24.06**[a]
	(0.07)	(0.14)	(0.02)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)

Table 6: False treatment effects measured individually by maturity buckets.

This table provides the distributions of estimated treatment effects using OLS on the same data, the same modeled general yield-curve effects, and using the same classical DiD specification as in Table 5 but separately by four individual buckets with residual maturity in the ranges $[0, 2]$, $(2, 5]$, $(5, 10]$, and $(10, 20]$ years. Panel A (B) covers the case when residual maturity of the treated bonds is drawn from a triangular distribution function with mode $m = 0.25$ ($m = 10$) years. Each panel shows the estimated treatment effects by the general effects (short- or long-end) and the maturity buckets as well as separately for the cases when $t < -t_{cv}$, $-t_{cv} \leq t \leq t_{cv}$, and $t_{cv} < t$, where t_{cv} is the critical value of a two-sided t-test at the significance level of 10% (which is 1.645 in case of a z-test) with standard errors clustered at the bond-level.

General effect	Maturity bucket (in years)	Number of meas. effects, N	$t < -t_{cv}$ N	$t < -t_{cv}$ Min (in bps)	$t < -t_{cv}$ Max (in bps)	$-t_{cv} \leq t \leq t_{cv}$ N	$-t_{cv} \leq t \leq t_{cv}$ Min (in bps)	$-t_{cv} \leq t \leq t_{cv}$ Max (in bps)	$t_{cv} < t$ N	$t_{cv} < t$ Min (in bps)	$t_{cv} < t$ Max (in bps)
Panel A: m = 0.25 years											
Short-end	[0 - 2]	1,000	60	-4.24	-1.49	883	-2.77	2.51	57	1.54	4.17
	(2 - 5]	1,000	53	-7.49	-3.31	900	-4.69	4.73	47	3.55	8.47
	(5 - 10]	1,000	63	-6.14	-2.36	889	-3.43	3.46	48	2.39	5.54
	(10 - 20]	1,000	46	-3.44	-1.15	900	-2.18	2.11	54	1.31	3.10
Long-end	[0 - 2]	1,000	61	-2.85	-0.75	868	-1.97	1.92	71	0.74	3.17
	(2 - 5]	1,000	47	-8.94	-3.74	900	-5.03	4.97	53	3.50	7.90
	(5 - 10]	1,000	48	-6.24	-2.70	888	-3.87	3.66	64	2.65	6.90
	(10 - 20]	1,000	54	-3.53	-1.49	900	-2.39	2.48	46	1.30	3.92
Panel B: m = 10 years											
Short-end	[0 - 2]	640	94	-5.12	-1.14	308	-2.45	3.90	238	0.58	8.01
	(2 - 5]	998	25	-10.51	-3.32	762	-6.29	6.15	211	3.26	11.89
	(5 - 10]	1,000	11	-4.14	-2.08	787	-2.61	3.11	202	2.03	6.70
	(10 - 20]	1,000	49	-2.98	-1.18	888	-1.81	1.64	63	1.28	3.15
Long-end	[0 - 2]	640	204	-4.82	-0.43	288	-2.24	1.58	148	0.46	2.88
	(2 - 5]	998	210	-12.70	-3.35	762	-6.60	6.60	26	3.58	11.01
	(5 - 10]	1,000	202	-7.56	-2.28	787	-3.50	2.94	11	2.35	4.67
	(10 - 20]	1,000	62	-3.58	-1.46	889	-1.87	2.06	49	1.34	3.39

Table 7: Modeling general term-structure effects combined with treatment effects.

To model the term structure we employ Diebold and Li (2006)'s yield curve specification. This table shows the parameter values to create the true underlying term structures as well as the resulting yield levels and yield differences at selected maturities. Panels A and C cover the cases of a term-structure treatment effect only at the short-end and a term-structure treatment twist in case of a general short-end effect and Panels B and D, respectively, the same in case of a general long-end effect from pre- to post-treatment.

Panel A: General short-end effect, treatment short-end effect

i) Parameter values	γ_0	γ_1	γ_2	λ
Pre-curve	4.000	-2.000	0.000	0.7308
Post-curve controls	4.140	-2.650	-0.800	0.7308
Post-curve treated	4.141	-2.781	-0.670	0.7308

ii) Yields (in %) and differences (in pps) at selected maturities:

	1y	2y	3y	5y	7y	10y	15y
Pre-curve	2.58	2.95	3.19	3.47	3.61	3.73	3.82
Post-curve controls	2.08	2.51	2.83	3.24	3.47	3.67	3.83
Difference	-0.50	-0.44	-0.36	-0.23	-0.14	-0.06	0.01
Post-curve treated	2.02	2.48	2.82	3.24	3.47	3.67	3.83
Difference	-0.06	-0.03	-0.01	0.00	0.00	0.00	0.00

Panel B: General long-end effect, treatment short-end effect

i) Parameter values	γ_0	γ_1	γ_2	λ
Pre-curve	4.000	-2.000	0.000	0.7308
Post-curve controls	3.350	-1.350	1.000	0.7308
Post-curve treated	3.351	-1.481	1.130	0.7308

ii) Yields (in %) and differences (in pps) at selected maturities:

	1y	2y	3y	5y	7y	10y	15y
Pre-curve	2.58	2.95	3.19	3.47	3.61	3.73	3.82
Post-curve controls	2.62	2.93	3.10	3.23	3.28	3.30	3.32
Difference	0.04	-0.01	-0.09	-0.24	-0.34	-0.43	-0.50
Post-curve treated	2.56	2.90	3.08	3.23	3.28	3.30	3.32
Difference	-0.06	-0.03	-0.01	0.00	0.00	0.00	0.00

Panel C: General short-end effect, treatment yield-curve twist

i) Parameter values	γ_0	γ_1	γ_2	λ
Pre-curve	4.000	-2.000	0.000	0.7308
Post-curve controls	4.140	-2.650	-0.800	0.7308
Post-curve treated	4.030	-2.470	-0.620	0.7308

ii) Yields (in %) and differences (in pps) at selected maturities:

	1y	2y	3y	5y	7y	10y	15y
Pre-curve	2.58	2.95	3.19	3.47	3.61	3.73	3.82
Post-curve controls	2.08	2.51	2.83	3.24	3.47	3.67	3.83
Difference	-0.50	-0.44	-0.36	-0.23	-0.14	-0.06	0.01
Post-curve treated	2.14	2.55	2.85	3.22	3.43	3.61	3.75
Difference	0.06	0.04	0.02	-0.02	-0.04	-0.06	-0.08

Panel D: General long-end effect, treatment yield-curve twist

i) Parameter values	γ_0	γ_1	γ_2	λ
Pre-curve	4.000	-2.000	0.000	0.7308
Post-curve controls	3.350	-1.350	1.000	0.7308
Post-curve treated	3.240	-1.170	1.180	0.7308

ii) Yields (in %) and differences (in pps) at selected maturities:

	1y	2y	3y	5y	7y	10y	15y
Pre-curve	2.58	2.95	3.19	3.47	3.61	3.73	3.82
Post-curve controls	2.62	2.93	3.10	3.23	3.28	3.30	3.32
Difference	0.04	-0.01	-0.09	-0.24	-0.34	-0.43	-0.50
Post-curve treated	2.68	2.97	3.11	3.21	3.23	3.24	3.24
Difference	0.06	0.04	0.02	-0.02	-0.04	-0.06	-0.08

Table 8: Treatment effects using the fully flexible yield-curve DiD specification. This table shows treatment effects estimated with the fully flexible yield-curve DiD specification $yield_{it} = \mathbf{B}_1' \mathbf{L}_{it} + \mathbf{B}_2' \mathbf{L}_{it} \mathbb{1}_{Treated,i} + \mathbf{B}_3' \mathbf{L}_{it} \mathbb{1}_{Post,t} + \mathbf{B}_4' \mathbf{L}_{it} \mathbb{1}_{Treated,i} \times \mathbb{1}_{Post,t} + \varepsilon_{it}$ with notation as in Table 5, $\mathbf{L}_{it}$ a three-dimensional vector of regressors with elements 1, $l_1(x_{it}; \lambda)$, and $l_2(x_{it}; \lambda)$, the latter two terms defined as in (5), and $\mathbf{B}_j$ the corresponding three-dimensional vectors of coefficients with individual elements $\beta_{k,j}$, $k = 0, \ldots, 2$. The latter measure level, slope, and curvature of the baseline curve for control bonds pre treatment ($j = 1$) and the incremental differences of (i) treated bonds pre treatment ($j = 2$), (ii) control bonds post treatment ($j = 3$), and (iii) treated bonds post treatment ($j = 4$). $\mathbf{B}_4$ captures level, slope, and curvature of the DiD delta curve, $\Delta_4^{dl}(x)$, which provides the treatment effects at maturity x. The specification is estimated with NLS, $\lambda_{Seed} = 1$, and λ is assumed to be time-invariant and the same for treated and control bonds. The results are shown for the eight combinations of modeled true yield-curve movements as indicated in the table. Each block represents one of the combinations of true yield curve effects and shows minimum and maximum of the distribution of estimated treatment effects at selected maturities of $x \in \{1, 2, 3, 5, 7, 10, 15\}$ years using the DiD delta curve, $\Delta_4^{dl}(x)$, across the 1,000 families of sample couplets when $m = 0.25$ years. Standard errors are clustered at the bond-level and calculated using the delta method.

	Residual		General effects					
Treatment	maturity	True	None		Short-end		Long-end	
effect (TE)	(in years)	TE	Min	Max	Min	Max	Min	Max
None	1	0.00			-0.01	0.01	-0.01	0.01
	2	0.00			0.00	0.00	0.00	0.01
	3	0.00			-0.01	0.00	0.00	0.00
	5	0.00			0.00	0.00	0.00	0.00
	7	0.00			0.00	0.00	0.00	0.00
	10	0.00			0.00	0.00	0.00	0.00
	15	0.00			0.00	0.00	0.00	0.00
	R^2_{adj}				1.0000	1.0000	1.0000	1.0000
	$\widehat{\lambda}$				0.7306	0.7310	0.7306	0.7310
Short-end	1	-6.23	-6.24	-6.23	-6.24	-6.22	-6.24	-6.22
	2	-2.97	-2.97	-2.96	-2.97	-2.96	-2.97	-2.96
	3	-1.39	-1.40	-1.39	-1.40	-1.39	-1.40	-1.39
	5	-0.26	-0.27	-0.26	-0.27	-0.26	-0.27	-0.26
	7	0.00	0.00	0.01	0.00	0.00	0.00	0.01
	10	0.08	0.07	0.08	0.07	0.08	0.07	0.08
	15	0.09	0.09	0.09	0.09	0.10	0.09	0.09
	R^2_{adj}		1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	$\widehat{\lambda}$		0.7282	0.7337	0.7306	0.7310	0.7306	0.7310
Yield-curve twist	1	5.87	5.87	5.88	5.87	5.88	5.86	5.88
	2	3.75	3.74	3.75	3.74	3.75	3.74	3.75
	3	1.58	1.57	1.58	1.57	1.58	1.57	1.58
	5	-1.87	-1.87	-1.87	-1.87	-1.87	-1.87	-1.87
	7	-4.11	-4.12	-4.11	-4.12	-4.11	-4.12	-4.11
	10	-6.09	-6.09	-6.09	-6.09	-6.09	-6.09	-6.09
	15	-7.72	-7.72	-7.71	-7.72	-7.71	-7.72	-7.71
	R^2_{adj}		1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	$\widehat{\lambda}$		0.7295	0.7325	0.7305	0.7311	0.7306	0.7310

Table 9: Treatment effects from fully flexible yield-curve DiD specification by mode. This table shows minimum and maximum of the difference between estimated and true treatment effects using the fully flexible yield-curve DiD specification, as in Table 8, across the eight combinations of modeled yield-curve movements and the 1,000 families of sample couplets separately for each mode $m \in \{0.25, 1, 3, 10\}$ years. The table shows the estimated treatment effects at selected residual maturities $x \in \{1, 2, 3, 5, 7, 10, 15\}$ years. Standard errors are clustered at the bond-level and calculated using the delta method.

Residual	Mode m of triangular prob. density function of treated bonds							
maturity	$m = 0.25$		$m = 1$		$m = 3$		$m = 10$	
(in years)	Min	Max	Min	Max	Min	Max	Min	Max
Panel A: All sample couplets								
No. of SC	8,000		8,000		8,000		8,000	
1	-0.01	0.01	-0.01	0.02	-0.05	0.03	-0.21	0.18
2	0.00	0.01	-0.01	0.01	-0.01	0.01	-0.08	0.06
3	-0.01	0.00	-0.01	0.00	-0.01	0.01	-0.03	0.02
5	0.00	0.00	-0.01	0.00	0.00	0.00	-0.01	0.01
7	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00
15	-0.01	0.00	-0.01	0.01	-0.01	0.01	-0.01	0.01
*Panel B: Good sample couplets**								
No. of SC	7,816		7,272		4,840		1,944	
1	-0.01	0.01	-0.01	0.01	-0.01	0.01	-0.01	0.01
2	0.00	0.01	-0.01	0.01	-0.01	0.01	-0.01	0.01
3	-0.01	0.00	-0.01	0.00	-0.01	0.00	-0.01	0.01
5	0.00	0.00	-0.01	0.00	0.00	0.00	0.00	0.00
7	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00
15	0.00	0.00	-0.01	0.01	-0.01	0.01	0.00	0.00

* At least one treated and one control bond in one-year maturity bucket.

Table 10: Semi-matching separately for individual maturity buckets.

This table shows the results from applying semi-matching separately by maturity buckets using the data that exhibit both general and treatment effects simultaneously (as in Section 5), NLS with $\lambda_{Seed} = 1$ to estimate the Diebold-Li yield curve given in (2), and the specification $yield_{it}^{DiD} = \beta_{DiD} \times C + \varepsilon_{it}$ with $yield_{it}^{DiD}$ the bond-level DiDs in yields, C a constant, and β_{DiD} the treatment effect. Panel A (B) covers the case of a yield-curve treatment short-end effect (twist). Separately for each maturity bucket and general yield-curve short- and long-end effects, each panel shows the number of estimated treatment effects, minimum and maximum number of bonds involved in the estimations, the true treatment effects at maturity-range start and end, and minimum and maximum of the estimated treatment effects across the four $m \in \{0.25, 1, 3, 10\}$ and the 1,000 families of sample couplets (which is a total of 4,000 regressions per maturity bucket).

Maturity	Number of			True effects*		Distributions of $\widehat{\beta}_{DiD}$ (in bps)			
range	estim.	treat. bonds		(in bps)		GE: Short-end		GE: Long-end	
(in years)	coeff.	Min	Max	start	end	Min	Max	Min	Max
Panel A: Treatment yield-curve short-end effect									
$[0, 2]$	3,614	1	20	-13.00	-2.97	-12.40	-2.97	-12.40	-2.97
$(2, 5]$	3,998	1	24	-2.97	-0.26	-2.60	-0.29	-2.60	-0.28
$(5, 10]$	4,000	6	31	-0.26	0.08	-0.10	0.05	-0.10	0.05
$(10, 20]$	4,000	4	36	0.08	0.09	0.08	0.09	0.08	0.09
Panel B: Treatment yield-curve twist									
$[0, 2]$	3,614	1	20	7.00	3.75	3.75	7.00	3.75	7.00
$(2, 5]$	3,998	1	24	3.75	-1.87	-1.74	3.34	-1.73	3.34
$(5, 10]$	4,000	6	31	-1.87	-6.09	-5.30	-3.14	-5.30	-3.13
$(10, 20]$	4,000	4	36	-6.09	-8.54	-7.98	-6.61	-7.98	-6.61

* True treatment effects are given for start and end of maturity range in first column.

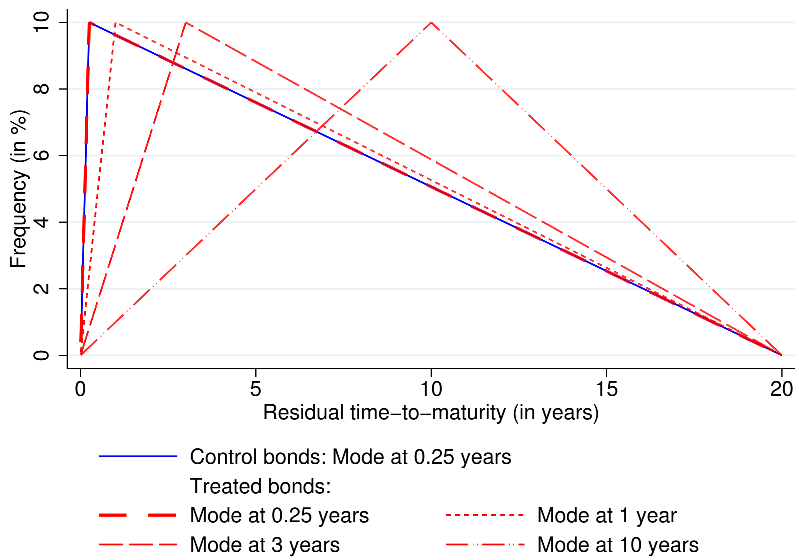
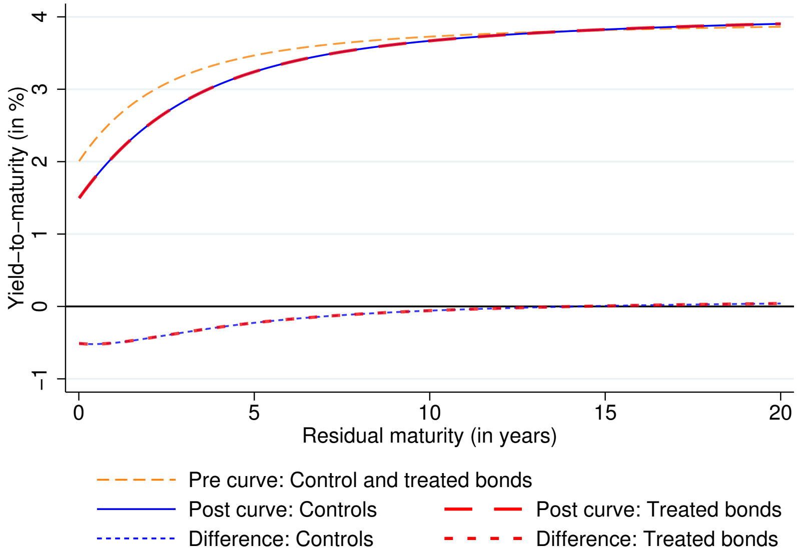
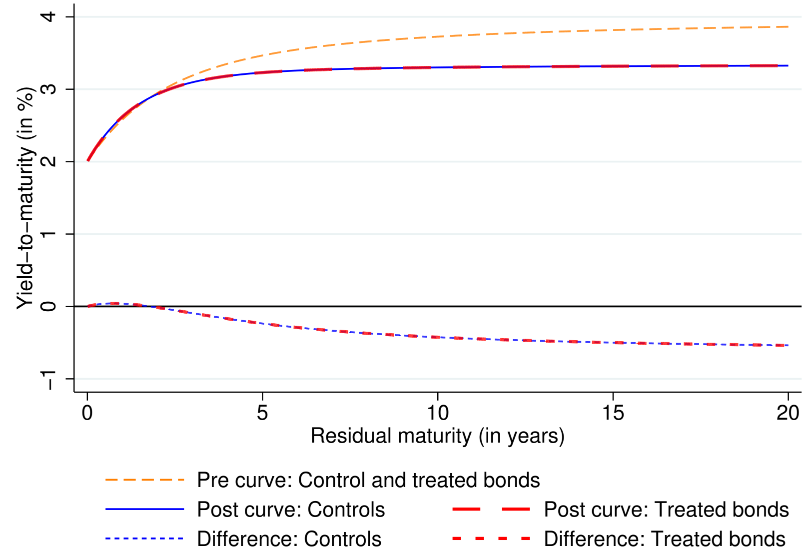


Figure 1: Triangular probability density functions with different modes m.
This figure shows the triangular probability density functions used to simulate residual maturity of the one control bond sample with mode $m = 0.25$ years and the four samples of treated bonds with modes $m = \{0.25, 1, 3, 10\}$ years while residual maturity x ranges from zero to twenty years ($x \in [0, 20]$) for either sample.





a) General yield-curve short-end effect:

i) Parameter values	γ_0	γ_1	γ_2	λ
Pre-curve	4.000	-2.000	0.000	0.7308
Post-curve	4.140	-2.650	-0.800	0.7308

ii) Yields (in %) and differences (in pps) at selected maturities:

	1y	2y	3y	5y	7y	10y	15y
Pre-curve	2.58	2.95	3.19	3.47	3.61	3.73	3.82
Post-curve	2.08	2.51	2.83	3.24	3.47	3.67	3.83
Difference	-0.50	-0.44	-0.36	-0.23	-0.14	-0.06	0.01

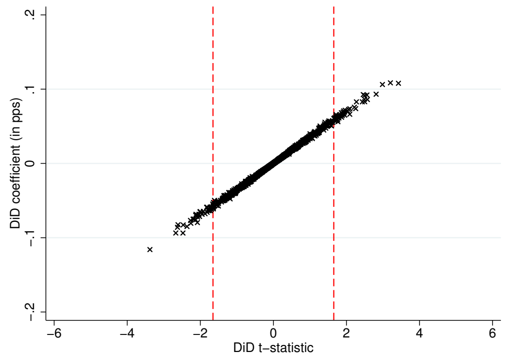
b) General yield-curve long-end effect:

i) Parameter values	γ_0	γ_1	γ_2	λ
Pre-curve	4.000	-2.000	0.000	0.7308
Post-curve	3.350	-1.350	1.000	0.7308

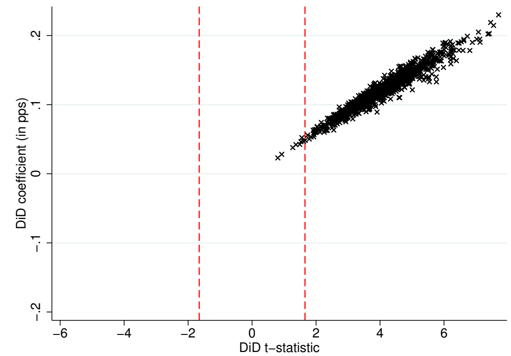
ii) Yields (in %) and differences (in pps) at selected maturities:

	1y	2y	3y	5y	7y	10y	15y
Pre-curve	2.58	2.95	3.19	3.47	3.61	3.73	3.82
Post-curve	2.62	2.93	3.10	3.23	3.28	3.30	3.32
Difference	0.04	-0.01	-0.09	-0.24	-0.34	-0.43	-0.50

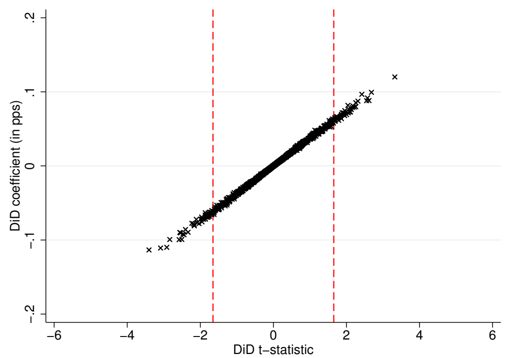
Figure 2: Modeling general effects in the term structure of interest rates.
To model the term structure we employ Diebold and Li (2006)'s yield curve specification. The minitable underneath each plot shows the parameter values to create the true underlying term structures as well as the resulting yield levels and yield differences at selected maturities. Figures 2a and 2b provide graphical illustrations of the resulting yield and differential curves when there is a general short-end or a long-end effect, respectively, from pre- to post-treatment.



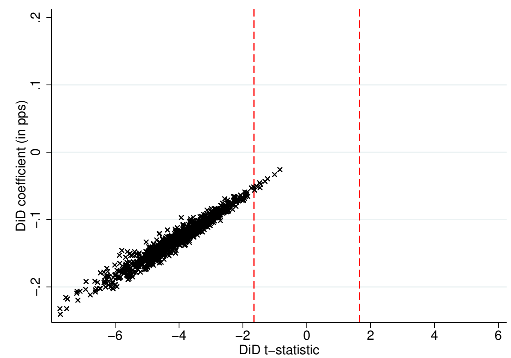
a) General short-end effect: $m = 0.25$ years
$\widehat{\beta}_{DiD} \in [-11.59, 10.85]$, $|t| > 1.653$: 91



b) General short-end effect: $m = 10$ years
$\widehat{\beta}_{DiD} \in [2.30, 22.99]$, $|t| > 1.653$: 992

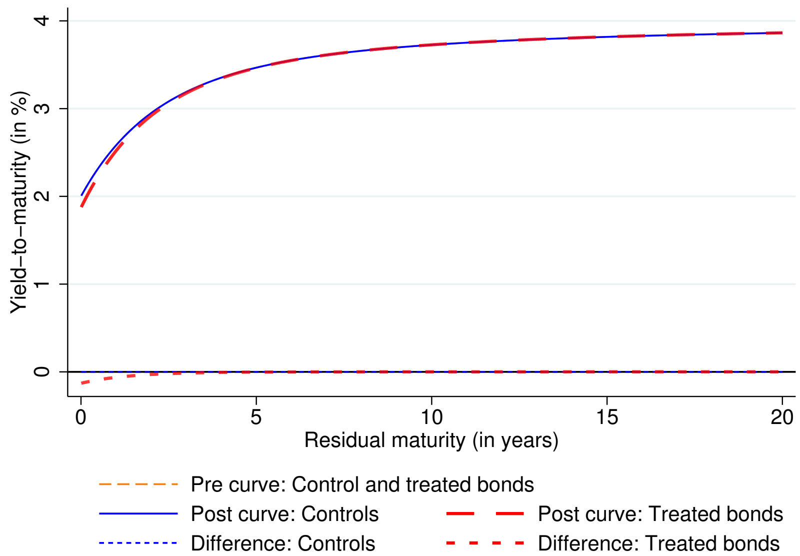


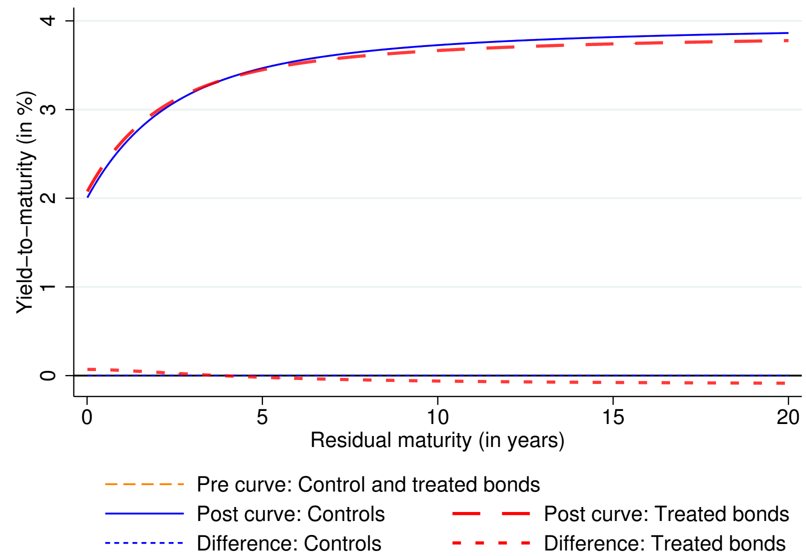
c) General long-end effect: $m = 0.25$ years
$\widehat{\beta}_{DiD} \in [-11.34, 12.01]$, $|t| > 1.653$: 88



d) General long-end effect: $m = 10$ years
$\widehat{\beta}_{DiD} \in [-24.06, -2.59]$, $|t| > 1.653$: 991

Figure 3: False treatment effects graphically.
This figure shows estimated treatment effects based on the 1,000 families of sample couplets when the modeled term structure exhibits differential general effects across maturity but a true treatment effect is entirely absent from the data. The specification is the same as in Table 5 and estimated with OLS. The (black) crosses in each plot show the 1,000 estimated DiD coefficients against the corresponding t-statistics. The vertical dashed (red) lines mark the values of ± 1.653, which correspond to two-sided confidence bands using a significance level of 10%. Subplots on the left (right) show the estimates when maturity of treated bonds is drawn from the triangular distribution with $m = 0.25$ ($m = 10$) years. The first (second) row of plots covers the general short-end (long-end) effect. The t-statistics are based on standard errors clustered on the bond level.





a) Yield-curve treatment short-end effect:

i) Parameter values	γ_0	γ_1	γ_2	λ
Pre-curve treated	4.000	-2.000	0.000	0.7308
Post-curve treated	4.001	-2.131	0.130	0.7308

ii) Yields (in %) and differences (in pps) at selected maturities:

	1y	2y	3y	5y	7y	10y	15y
Pre-curve treated	2.58	2.95	3.19	3.47	3.61	3.73	3.82
Post-curve treated	2.52	2.92	3.18	3.46	3.61	3.73	3.82
Difference treated	-0.06	-0.03	-0.01	0.00	0.00	0.00	0.00

b) Yield-curve treatment twist:

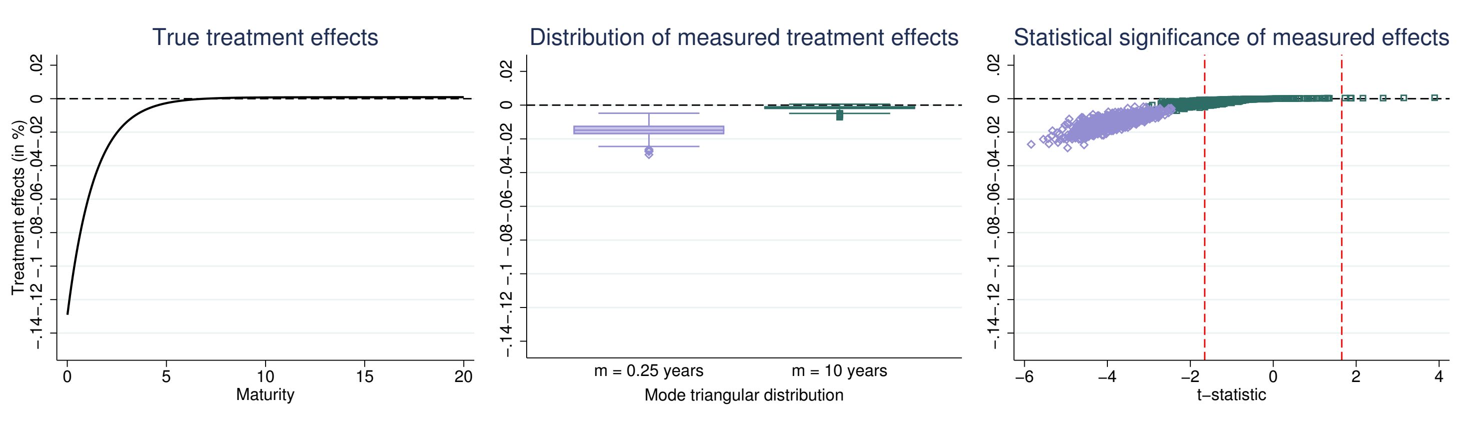
i) Parameter values	γ_0	γ_1	γ_2	λ
Pre-curve treated	4.000	-2.000	0.000	0.7308
Post-curve treated	3.890	-1.820	0.180	0.7308

ii) Yields (in %) and differences (in pps) at selected maturities:

	1y	2y	3y	5y	7y	10y	15y
Pre-curve treated	2.58	2.95	3.19	3.47	3.61	3.73	3.82
Post-curve treated	2.64	2.99	3.21	3.45	3.57	3.67	3.74
Difference treated	0.06	0.04	0.02	-0.02	-0.04	-0.06	-0.08

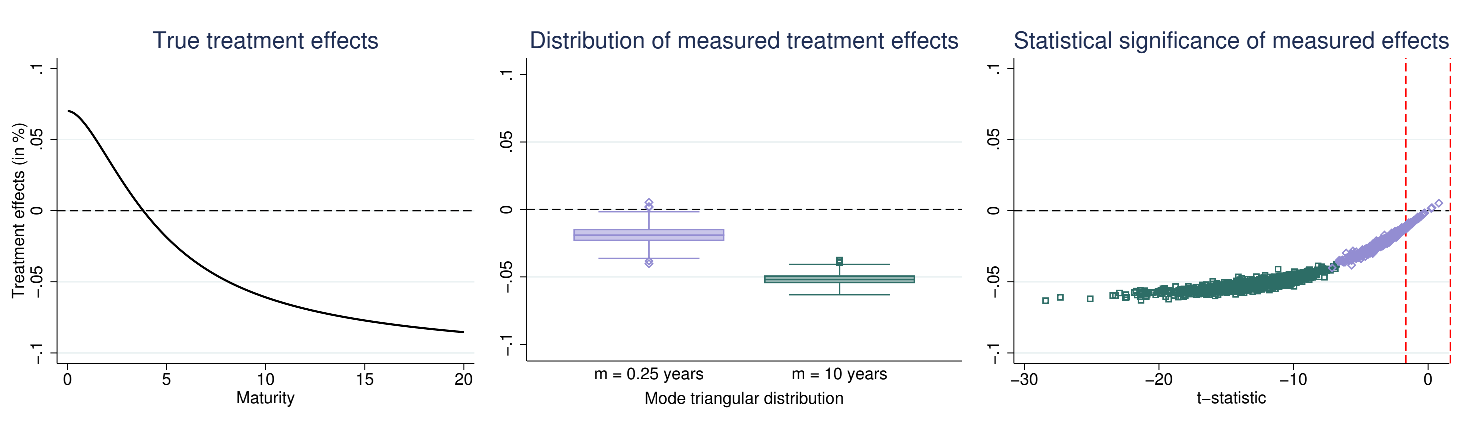
Figure 4: Modeling term-structure treatment effects.

To model the term structure we employ Diebold and Li (2006)'s specification. The minitable underneath each plot shows the parameter values to create the true underlying term structures as well as the resulting yield levels and yield differences at selected maturities. Figures 4a and 4b provide graphical illustrations of the resulting yield and differential curves when there is a yield-curve treatment short-end effect and a yield-curve treatment twist, respectively, from pre- to post-treatment.



a) Treatment short-end effect

$m = 0.25$ years: $\widehat{\beta}_{DiD} \in [-2.94, -0.48]$ bps, $|t| > 1.653$: 1000 $m = 10$ years: $\widehat{\beta}_{DiD} \in [-0.72, +0.06]$ bps, $|t| > 1.653$: 301

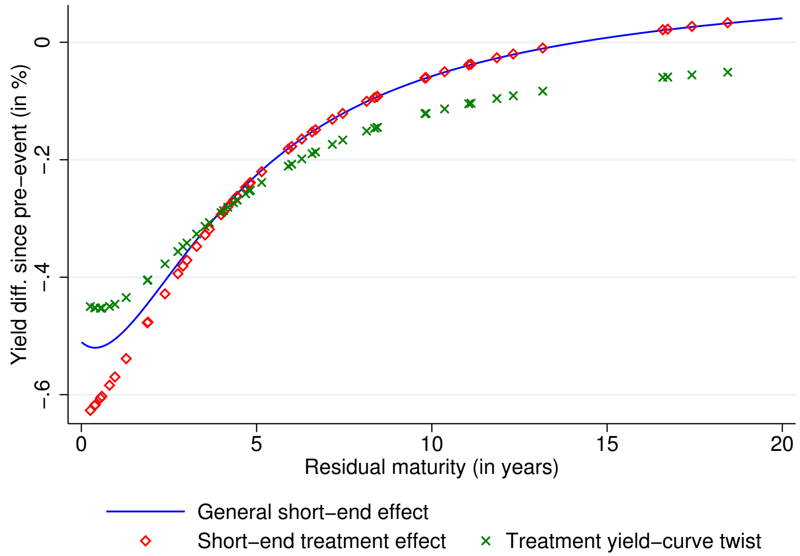


b) Treatment yield-curve twist

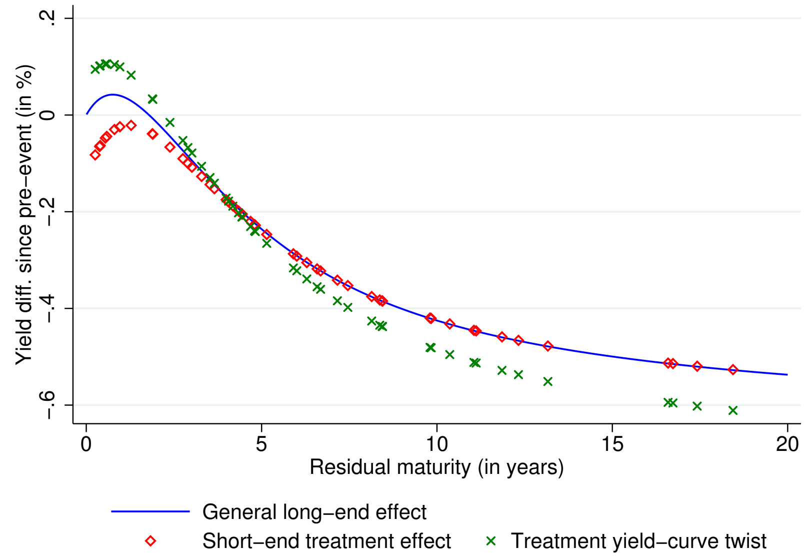
$m = 0.25$ years: $\widehat{\beta}_{DiD} \in [-4.02, +0.52]$ bps, $|t| > 1.653$: 879 $m = 10$ years: $\widehat{\beta}_{DiD} \in [-6.32, -3.75]$ bps, $|t| > 1.653$: 1000

Figure 5: Mismeasured treatment effects graphically.

Figures 5a and 5b show true and measured treatment effects on the 1,000 families of sample couplets for treatment short-end effect and yield-curve treatment twist, respectively, using OLS to estimate the same specification as in Table 5. From left to right, the graphs plot the true treatment effect over maturity, the distributions (box plots) if maturity of the treated bonds is drawn from triangular distributions with $m = 0.25$ years (purple diamonds) or $m = 10$ years (green squares), and the estimated treatment effects against the t-statistics. The vertical dashed (red) lines in the plots to the far right mark the values of ± 1.653 (two-sided confidence bands using 10%-significance level). The t-statistics are based on standard errors clustered at the bond-level.



a) General short-end effect



b) General long-end effect

Figure 6: Illustration of semi-matching.

This illustration is based on a random sample couplet when $m = 0.25$ years for both control and treated bonds. Figures 6a and 6b provide graphical illustrations for semi-matching when there is a general yield-curve effect only at the short-end or only at the long-end, respectively. In each plot, given the general yield-curve effects there is either an additional yield-curve treatment effect at the short-end or a yield-curve treatment twist.

Internet Appendix

Robust difference-in-differences analysis when there is a term structure[1]

Kjell G. Nyborg

University of Zurich,
Swiss Finance Institute,
and CEPR

Jiri Woschitz

BI Norwegian Business School

March 2023

[1]Nyborg: Department of Banking and Finance, University of Zurich, Plattenstrasse 14, CH-8032 Zurich, Switzerland. email: kjell.nyborg@bf.uzh.ch.
Woschitz: Department of Finance, BI Norwegian Business School, B4y, NO-0442 Oslo, Norway. email: jiri.woschitz@bi.no.

A.1. Appendix

A.1.1 Fully flexible yield-curve DiD: Example

To illustrate the functioning of the fully flexible yield-curve DiD specification, in this appendix we apply it to one sample couplet j of the simulated data for the eight combinations of general effects (no effect, short-end effect, long-end effect) and treatment effects (no effect, short-end effect, twist) leaving out the no effect/no effect combination.

To generate the true underlying yield curves in the simulated data of the paper's main body, we have chosen values for the parameters γ_0, γ_1, γ_2, and λ and have plugged them into Diebold and Li (2006)'s spot curve in Specification (2). Table A.1, Panel A, collects these parameter values of the spot curves from Figures 2 and 4 as well as Table 7 for the eight combinations of yield-curve movements. However, while the spot curve parameters of the control bonds prior to the treatment, $\{\beta_{k,1}\}_{k=0}^{2}$, measure the same quantity as the gammas, γ_0, γ_1, and γ_2, in Specification (6), the $\{\beta_{k,j}\}_{k=0}^{2}$ for $j = 2, \ldots, 4$ (for treated pre, control post, and treated post) represent *differential*, or *delta*, *curves* and are therefore quantities that differ from the corresponding gammas in Specification (2). In Table A.1, Panel A, since we want to compare estimated values to the true underlying parameter values, the γ-representation from Specification (2) is transformed into the β-representation in Specification (6).

Insert Table A.1 here.

In Panel A, except for λ, parameter values that are not zero are highlighted in bold. Panel B shows the result of estimating Specification (6) using NLS for the eight combinations of yield-curve effects using family couplet j of the simulated data. In Panel B, coefficients that are statistically significantly different from zero at a significance level of at least 1% are also marked in bold. Comparing Panel A, which provides the true underlying values for the βs in Specification (6), with Panel B, providing the estimated coefficients for sample couplet j, shows that the bold non-zero values in Panel A form the same pattern as the bold significant coefficients in Panel B. Measured in percentage points, the parameter values and coefficients in bold in the two panels are the same (up to at least the third decimal digit after the comma). These results show the feasibility of identifying the true underlying parameter values using a simple but well specified regression model.

Interesting are a few exceptions, where the parameter estimates seem to be slightly different from the true parameter values. These incidences appear, on the one hand, with the curvature factor of the control bonds prior to treatment, $\widehat{\beta}_{2,1}$, and, on the other hand, with the decay parameter, $\widehat{\lambda}$, at the bottom of the panel. As explained by Diebold and Li (2006), the decay parameter determines the point where the loading of the curvature factor obtains its maximum. Hence, these two parameters have more multicollinearity with each other than each of them has with the other parameters, level and slope. This relationship can, for example, be seen by looking at the case of a short-end treatment effect. The more *downward* and away-biased the estimated lambda, $\widehat{\lambda} = 0.7302$, from the true value, $\lambda = 0.7308$, the more *upward* and away-biased is the pre-treatment control-bond curvature estimate, $\widehat{\beta}_{2,1} = 0.001$, is from the true parameter value, $\beta_{2,1} = 0.000$. This shows that the estimation might be exposed to multicollinearity between the yield-curve parameters and estimates might, therefore, be confounded to a certain extent. We will discuss this further shortly below.

A different question, however, is whether this is the right quantity to consider. By looking, for example, at the case of a general yield-curve short-end effect and a yield-curve treatment twist in the sixth regression in Panels A and B of Table A.1, a researcher learns: First, the differential curve of treated compared to control bonds prior to the treatment is zero. Second, level, slope, and curvature factors of the control bonds change by 0.140, -0.650, and -0.800, respectively, from pre to post treatment (which represents a short-end effect). Third, compared to the post-curve of control bonds, the level factor of the curve of treated bonds is -0.110 smaller and the slope and curvature factors each 0.180 larger (the additional yield-curve treatment twist). Clearly it is very difficult to grasp what this information economically means. Panels C and D in Table A.1 provide an alternative to presenting the same results in a more readable and intuitive manner.

Table A.1, Panel C, shows the true underlying treatment effects of treated bonds from pre to post treatment, controlling for movements in the yield curve of treated compared to control bonds and movements in the yield curve from pre to post treatment. The DiD is a function of maturity and, hence, varies across different maturities as long as the DiD is not a pure level-shift. Panel C shows that the DiD at selected maturities, 1, 2, 3, 5, 7, 10, and 15 years, are the same across the three cases of no general effect, a general short-end, and a general long-end effect. That is how the true underlying effects are modeled and is therefore correct.

In Panel D we present the results if we estimate the DiD by using the estimation results from Panel B and predicting the DiD at the same selected maturities. We use the delta method to calculate standard errors, which are also clustered on the bond level. Marginal effects that are statistically significantly different from zero (all at the significance level of at least 1%) are marked in bold. To help visualize the similarities between Panels C and D, the true underlying non-zero marginal effects in Panel C are as well highlighted in bold. The results show that measuring the DiD in percentage points, the true and estimated numbers are the same up to the third decimal digit after the comma. With respect to multicollinearity between the regressors as touched upon above, this shows that presenting the results this way is not impacted by multicollinearity anymore.

Furthermore, the measured quantity is intuitive to understand. For example, the sixth regression in Table A.1, Panel D, the one for which we tried to describe the results already above, shows that in case of a yield-curve short-end effect of the control bonds and an additional yield-curve treatment twist of the treated bonds, the treatment effect corresponds to +5.87, +3.75, +1.58, −1.87, −4.11, −6.09, and −7.71 bps at maturities of 1, 2, 3, 5, 7, 10, and 15 years, respectively.

While this appendix section illustrated how to estimate a meaningful quantity using a random sample couplet j of simulated data, the paper applies the method to all simulated sample couplets and shows its power to eliminate both the bias and mismeasurement of treatment effects in fixed-income settings.

Table A.1: Illustration of the fully flexible yield-curve DiD specification.

This table illustrates the fully flexible yield-curve DiD specification by applying it to one sample draw of simulated data. This is done for the eight combinations of general (no, short-end, long-end effect) and treatment effects (no, short-end effect, twist), leaving out the no effect/no effect combination. The specification is $yield_{it} = \mathbf{B}_1' \, \mathbf{L}_{it} + \mathbf{B}_2' \, \mathbf{L}_{it} \, \mathbb{1}_{Treated,i} + \mathbf{B}_3' \, \mathbf{L}_{it} \, \mathbb{1}_{Post,t} + \mathbf{B}_4' \, \mathbf{L}_{it} \, \mathbb{1}_{Treated,i} \times \mathbb{1}_{Post,t} + \varepsilon_{it}$, where notation is as in Table 5 and, additionally, $\mathbf{B}_j$ are three-dimensional vectors of coefficients, with individual elements $\beta_{k,j}$, $k = 0, \ldots, 2$. They measure the baseline curve for control bonds pre-treatment ($j = 1$), and the incremental differences of (i) treated bonds pre-treatment ($j = 2$), (ii) control bonds post-treatment ($j = 3$) and, (iii) treated bonds post-treatment ($j = 4$). $\mathbf{B}_4$ is the DiD estimator and depends on maturity x. Panel A collects the parameter values of γ_k, $k = 0, ..., 2$, and λ of the spot curves from Figures 2 and 4 and Table 7 and transforms the γ-representation in those tables to the β-representation. In Panel A, except for λ, parameter values that are not zero are highlighted in bold. Panel B shows the result of estimating the specification using NLS for the eight combinations of yield-curve developments using the one sample draw of simulated data. The decay parameter λ is assumed to be time invariant and the same for treated and control bonds. In Panel B, coefficients that are statistically significantly different from zero (all at the significance level of at least 1%) are marked in bold. Panel C shows $\mathbf{B}_4$, the true underlying DiD of treated bonds post-treatment, for selected maturities. In Panel C, parameter values that are not zero are highlighted in bold. Panel D shows $\widehat{\mathbf{B}}_4$, estimated using the estimation results from Panel B and predicting the DiD at the same selected maturities as in Panel C. Standard errors are clustered at the bond level and calculated using the delta method. Coefficients that are statistically significantly different from zero (all at the significance level of at least 1%) are marked in bold. In Panels B and D, a, b, and c denote significance (two-sided) at the levels of 1%, 5%, and 10%, respectively.

Panel A: True parameter values

General effect	Short-end	Long-end	–		Short-end		Long-end	
Treatment effect	–	–	Short-end	Twist	Short-end	Twist	Short-end	Twist
$\beta_{0,1}$	**4.000**	**4.000**	**4.000**	**4.000**	**4.000**	**4.000**	**4.000**	**4.000**
$\beta_{1,1}$	**-2.000**	**-2.000**	**-2.000**	**-2.000**	**-2.000**	**-2.000**	**-2.000**	**-2.000**
$\beta_{2,1}$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$\beta_{0,2}$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$\beta_{1,2}$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$\beta_{2,2}$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$\beta_{0,3}$	**0.140**	**-0.650**	0.000	0.000	**0.140**	**0.140**	**-0.650**	**-0.650**
$\beta_{1,3}$	**-0.650**	**0.650**	0.000	0.000	**-0.650**	**-0.650**	**0.650**	**0.650**
$\beta_{2,3}$	**-0.800**	**1.000**	0.000	0.000	**-0.800**	**-0.800**	**1.000**	**1.000**
$\beta_{0,4}$	0.000	0.000	**0.001**	**-0.110**	**0.001**	**-0.110**	**0.001**	**-0.110**
$\beta_{1,4}$	0.000	0.000	**-0.131**	**0.180**	**-0.131**	**0.180**	**-0.131**	**0.180**
$\beta_{2,4}$	0.000	0.000	**0.130**	**0.180**	**0.130**	**0.180**	**0.130**	**0.180**
λ	0.7308	0.7308	0.7308	0.7308	0.7308	0.7308	0.7308	0.7308

Table A.1 – *continued*

Panel B: Estimated parameter values for family j of ordered sample couplets

General effect	Short-end	Long-end	–		Short-end		Long-end	
Treatment effect	–	–	Short-end	Twist	Short-end	Twist	Short-end	Twist
$\widehat{\beta}_{0,1}$	**4.000**^{a}	**4.000**^{a}	**4.000**^{a}	**4.000**^{a}	**4.000**^{a}	**4.000**^{a}	**4.000**^{a}	**4.000**^{a}
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
$\widehat{\beta}_{1,1}$	**-2.000**^{a}	**-2.000**^{a}	**-2.000**^{a}	**-2.000**^{a}	**-2.000**^{a}	**-2.000**^{a}	**-2.000**^{a}	**-2.000**^{a}
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
$\widehat{\beta}_{2,1}$	-0.000	-0.000	0.001	-0.000	-0.000^b	0.000	-0.000^c	-0.000
	(0.317)	(0.148)	(0.333)	(0.889)	(0.036)	(0.908)	(0.068)	(0.384)
$\widehat{\beta}_{0,2}$	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000
	(0.170)	(0.170)	(0.170)	(0.171)	(0.170)	(0.170)	(0.170)	(0.170)
$\widehat{\beta}_{1,2}$	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000
	(0.466)	(0.466)	(0.463)	(0.467)	(0.466)	(0.466)	(0.466)	(0.466)
$\widehat{\beta}_{2,2}$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	(0.117)	(0.117)	(0.116)	(0.117)	(0.117)	(0.117)	(0.117)	(0.117)
$\widehat{\beta}_{0,3}$	**0.140**^{a}	**-0.650**^{a}	0.000	0.000	**0.140**^{a}	**0.140**^{a}	**-0.650**^{a}	**-0.650**^{a}
	(0.000)	(0.000)	(0.980)	(0.858)	(0.000)	(0.000)	(0.000)	(0.000)
$\widehat{\beta}_{1,3}$	**-0.650**^{a}	**0.650**^{a}	-0.000	0.000	**-0.650**^{a}	**-0.650**^{a}	**0.650**^{a}	**0.650**^{a}
	(0.000)	(0.000)	(0.845)	(0.953)	(0.000)	(0.000)	(0.000)	(0.000)
$\widehat{\beta}_{2,3}$	**-0.800**^{a}	**1.000**^{a}	0.000	-0.000	**-0.800**^{a}	**-0.800**^{a}	**1.000**^{a}	**1.000**^{a}
	(0.000)	(0.000)	(0.972)	(0.877)	(0.000)	(0.000)	(0.000)	(0.000)
$\widehat{\beta}_{0,4}$	0.000^c	0.000	**0.001**^{a}	**-0.110**^{a}	**0.001**^{a}	**-0.110**^{a}	**0.001**^{a}	**-0.110**^{a}
	(0.094)	(0.227)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
$\widehat{\beta}_{1,4}$	0.000	-0.000	**-0.131**^{a}	**0.180**^{a}	**-0.131**^{a}	**0.180**^{a}	**-0.131**^{a}	**0.180**^{a}
	(0.576)	(0.941)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
$\widehat{\beta}_{2,4}$	-0.000^c	-0.000	**0.130**^{a}	**0.180**^{a}	**0.130**^{a}	**0.180**^{a}	**0.130**^{a}	**0.180**^{a}
	(0.050)	(0.411)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
$\widehat{\lambda}$	0.7308^a	0.7308^a	0.7302^a	0.7308^a	0.7309^a	0.7307^a	0.7309^a	0.7308^a
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Num. of obs.	200	200	200	200	200	200	200	200
Adjusted R^2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
RMSE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table A.1 – *continued*

Panel C: True difference-in-differences at selected maturities									
General effect		Short-end	Long-end	–		Short-end		Long-end	
Treatment effect		–	–	Short-end	Twist	Short-end	Twist	Short-end	Twist
Maturity	1	0.0000	0.0000	**-0.0623**	**0.0587**	**-0.0623**	**0.0587**	**-0.0623**	**0.0587**
(in years)	2	0.0000	0.0000	**-0.0297**	**0.0375**	**-0.0297**	**0.0375**	**-0.0297**	**0.0375**
	3	0.0000	0.0000	**-0.0139**	**0.0158**	**-0.0139**	**0.0158**	**-0.0139**	**0.0158**
	5	0.0000	0.0000	**-0.0026**	**-0.0187**	**-0.0026**	**-0.0187**	**-0.0026**	**-0.0187**
	7	0.0000	0.0000	**0.0000**	**-0.0411**	**0.0000**	**-0.0411**	**0.0000**	**-0.0411**
	10	0.0000	0.0000	**0.0008**	**-0.0609**	**0.0008**	**-0.0609**	**0.0008**	**-0.0609**
	15	0.0000	0.0000	**0.0009**	**-0.0772**	**0.0009**	**-0.0772**	**0.0009**	**-0.0772**
Panel D: Estimated difference-in-differences at selected maturities for family j of ordered sample couplets									
Maturity	1	0.0000	0.0000	**-0.0623**^{a}	**0.0587**^{a}	**-0.0623**^{a}	**0.0587**^{a}	**-0.0623**^{a}	**0.0587**^{a}
(in years)		(0.895)	(0.706)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
	2	-0.0000	-0.0000	**-0.0297**^{a}	**0.0375**^{a}	**-0.0297**^{a}	**0.0375**^{a}	**-0.0297**^{a}	**0.0374**^{a}
		(0.231)	(0.958)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
	3	-0.0000	-0.0000	**-0.0139**^{a}	**0.0158**^{a}	**-0.0139**^{a}	**0.0158**^{a}	**-0.0139**^{a}	**0.0158**^{a}
		(0.139)	(0.974)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
	5	-0.0000	0.0000	**-0.0026**^{a}	**-0.0187**^{a}	**-0.0026**^{a}	**-0.0187**^{a}	**-0.0026**^{a}	**-0.0187**^{a}
		(0.280)	(0.607)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
	7	-0.0000	0.0000	**0.0000**^{a}	**-0.0411**^{a}	**0.0000**^{a}	**-0.0411**^{a}	**0.0000**^{a}	**-0.0411**^{a}
		(1.000)	(0.231)	(0.002)	(0.000)	(0.000)	(0.000)	(0.001)	(0.000)
	10	0.0000	0.0000	**0.0008**^{a}	**-0.0609**^{a}	**0.0008**^{a}	**-0.0609**^{a}	**0.0008**^{a}	**-0.0609**^{a}
		(0.323)	(0.157)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
	15	0.0000	0.0000	**0.0009**^{a}	**-0.0772**^{a}	**0.0009**^{a}	**-0.0771**^{a}	**0.0009**^{a}	**-0.0771**^{a}
		(0.163)	(0.174)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Num. of obs.		200	200	200	200	200	200	200	200
Adjusted R^2		1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
RMSE		0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000