Short Term Market Risks Implied by Weekly Options: An Alternative Perspective

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ABSTRACT

Andersen, Fusari and Todorov (2017) claim that in Weekly options there is variation in the negative tail risk that is not spanned by the option market. Using Stochastic Arbitrage (SA), we show that tail risk is adequately represented by the prices of out-of-the money options, but the at-the-money Weeklies are way above the prices justified by the dynamics of the S&P 500 index. We confirm the overpricing with out-of-sample tests that show high risk-adjusted profits from trading these options within the frictionless SA bounds. We attribute the AFT results to the assumptions and distortions imposed on their option data.

Anderson, Fusari and Todorov (AFT, 2017) use short term S&P 500 index options in order to study volatility and jump risks in option valuation. As they make it very clear (p. 1348), their theoretical framework is asset pricing theory in frictionless markets, according to which the option values are expectations of the payoffs with the transformed risk neutral jump diffusion dynamics. Such a transformation implies that the return distributions are multiplied by a pricing kernel, for which various forms are available. Nonetheless, AFT do not introduce an explicit pricing kernel, and in their empirical study they use only option market data and rely very little on the dynamics of the index return distribution. In their empirical analysis, option data are transformed to conform to the frictionless markets assumption, discarding the in-the-money (ITM) options and replacing them via put-call parity with the corresponding out-of-the money (OTM) options (p. 1345, footnote 8). They apparently use the midpoint of the observed bid-ask spread of the options (Appendix D, p. 1374) as representing the frictionless price, consistent with most empirical option market studies.

With these option data, AFT extract the parameters of the risk neutral frictionless process, which they fit to the entire option cross section. Their analysis is extremely flexible, insofar as it uses a semi-nonparametric approach that only imposes weak restriction on the jump distribution. It allows time-varying risk neutral jump amplitude distributions with daily calibration. In spite of this flexibility, however, they have trouble in identifying models that can adequately fit both right and left tails of this risk neutral distribution; see pp. 1354, 1356, 1358, 1364, etc. They conclude (p. 1370) that “there is a sharp separation between the dynamics of the actual jump risk and its pricing,” and mention the need to identify the economic forces that can rationalize their results.

In this paper, we show that the problems encountered by AFT stem from applying an option valuation model developed for frictionless markets to a market that is emphatically not frictionless, neither in trading index tracking funds or index futures nor, a fortiori, in trading options. We use an extended SPXW option dataset and demonstrate that even in the very liquid moneyness regions, put-call parity does not hold even approximately.[1] We also note that there are systematic differences in frictionless option valuation and observed option market data by degree of moneyness of the options. We use the stochastic arbitrage (SA) approach to map the regions in the option market where the frictionless option values consistent with the observed index dynamics should lie. We demonstrate that for large segments of the support of the index return distribution there is a complete disconnect between these mapped regions and the corresponding observed bid-ask spreads in our sample. This is probably the most important reason that led to the AFT conclusion about the shape of option skew across short-maturity options, for which they blame the scarcity of deep OTM call options. Last, we test rigorously whether this observed inconsistency of the option market data with the theoretically correct frictionless option bounds implied by the underlying return can lead to profitable option trading. Our tests demonstrate that the observed option data can produce large risk adjusted profits *provided* trading is frictionless, namely consistent with the SA bounds.

We evaluate the AFT results by using the stochastic arbitrage (SA) paradigm, formerly known as stochastic dominance. In that paradigm, we compare two generic identical risk averse investors. One of them holds the index and the riskless asset, the index trader (IT). The other one, the option trader (OT), adds a zero net cost option position to the IT holdings. OT should not dominate IT to the second degree. This absence of dominance creates a region within which the risk neutral option prices should lie. These bounds are the tightest intervals for the frictionless option prices to avoid risk-adjusted positive expected returns from the zero net cost portfolios. This paradigm was originally developed for frictionless markets in discrete time by Perrakis and Ryan (1984), Ritchken (1985), Levy (1985), Perrakis (1986), and Ritchken and Kuo (1988). It has been extended theoretically more recently and shown to converge in continuous time either to a single value in the cases of diffusion or stochastic volatility or to two distinct boundary distributions when the diffusions are mixed with independent jumps in Perrakis (2019, Ch. 2), Ghanbari, Oancea and Perrakis (2021), and Perrakis and Oancea (2022). The SA approach uses only the index return dynamics, without involving the option market. SA restricts the relevant universe of traders to those who are risk averse and hold the index and the riskless asset. This assumption implies that the index is identified with the market portfolio. Such index holders form a large and drastically increasing share of investors in the economy, as documented by Bogle (2005) and Charles (2017).

A frictionless equilibrium in the option market should not allow IT investors to realize positive risk adjusted extra profits by adding a zero net cost short or long option position to their holdings while trading in the frictionless market. A corollary of this condition is that frictionless option values can be extracted from the observed option market data bid-ask spread midpoint if and only if that midpoint is situated within the SA bounds for the entire support of the return distribution. This

[1] In our sample of end-of-week SPXW, the put volume is about 1.8 times that of the call volume, and this ratio is reasonably persistent from 9 days-to-maturity until 1 day-to-maturity. See also Constantinides and Lian (2021) on the partial segmentation of the markets for puta and calls.

happens seldom in our data. If frictionless option values cannot be extracted from a portion of the support of the return distribution, then there are frictionless trading strategies that generate risk adjusted profits for the IT investor. *Any* risk averse investor holding the index and the riskless asset will increase her expected utility by adopting such a strategy. In our data, this is confirmed rigorously by genuine out-of-sample and model-free tests. These tests compare the realized aggregate payoffs of the adopted strategies plus the returns of the IT portfolio, the OT time series of portfolio returns, to the returns of the IT portfolio. They allow inferences about the populations from which the two timeseries were drawn, thus confirming or rejecting the mispricing of the bounds violating option(s).[2]

The SA approach confirms the AFT-conjectured independence of the pricing of diffusive volatility and jump risk, represented by at-the-money (ATM) and OTM options respectively, by turning the AFT results upside down! We present empirical evidence that the bid-ask midpoint observed in the option market is an adequate representation of the risk neutral distribution supported by the index return dynamics *only* at the left tail of the distribution. By contrast, the option market overprices the highly liquid region around ATM for both calls and puts, with the admissible option prices lying way below the observed bid quotes for a large majority of the traded options. We verify that these ATM options are overpriced, by the out-of-sample SA tests in frictionless trading, with the profits disappearing if trading is done at the prevailing bid and ask prices or at their midpoint. In other words, the AFT results fail not because of "shifts in the pricing of negative tail risk" but because the underlying index dynamics are fatally inconsistent with observed option market prices for a key portion of the support of the risk neutral distribution. In fact, our results show that more than 50% of the observed bid-ask quotes for Weeklies are mispriced in frictionless markets with respect to the underlying index dynamics, while the profits disappear when trading in the presence of frictions. Further, the consistency of market data with the SA bounds is strongly dependent on the moneyness of the options, indicating clientele effects that attract separate types of investors for ATM and OTM options. This seems also to be implied by the comments of AFT (p. 1336), that ATM and OTM options are focused separately, respectively on the spot volatility and the jump component dynamics.

To our knowledge, this is the first empirical application of SA to frictionless markets, although it has been applied to markets with frictions. A major advantage of SA is the fact that it allows comparisons of the mapped option prices consistent with the index dynamics with the observed bid-ask spread at the level of each individual option, rather than an entire cross section as in conventional empirical option research. Our SPXW results are consistent with previous theoretical and empirical research that recognize market frictions, such as Constantinides and Perrakis (2002, 2007), Constantinides *et* al (2011), Post and Longarela (2021), and Arvanitis, Post and Topaloglou (2023). Constantinides, Czerwonko and Perrakis (CCP, 2020) show that the one week maturity option portfolios selected by their algorithms generated major risk adjusted expected excess returns in-sample, that were confirmed on the realized ex post series of returns.[3] If the options are mispriced

[2] See Constantinides *et al* (2011). The tests' hypotheses verify whether risk averse investors *unanimously* prefer the OT over the IT times series. Intuitively, the test examines the non-dominance of OT returns time series over that of IT returns.

[3] These excess returns in the market with frictions are not inconsistent with our own conclusions in Section 3, that the mispricing of the options in the frictionless markets disappears when trading at the appropriate bid and ask prices. Both CCP and PL used complex algorithms in order to identify mispriced option *portfolios*, not individual options.

in the market with frictions in a single period buy-and-hold model, then they are a fortiori mispriced in a continuous time frictionless model as in AFT, with the excess returns augmented by the bid-ask spread of the chosen portfolios. On the theoretical side, there are very few available studies that model transaction costs in the option market or consider the relation between the frictionless option values and the bid-ask spread in the market. A pioneering such study was Jouini and Kallal (1995), who showed (p. 188) that for the "correct" pricing of options in both the frictionless market and in the presence of frictions, the frictionless price should lie within the bid-ask spread. Similarly, Constantinides, Jackwerth, and Perrakis (2009) and Post and Longarela (PL, 2021) showed theoretically that their mispriced option portfolios were due to the fact that there was no monotone pricing kernel passing through the bid-ask spread in the option market.

Our results also provide some answers to the big question that AFT raise, about the identification of the economic forces that can rationalize their less than fully satisfactory results in fitting the risk neutral dynamics across moneyness domain. We fully agree with AFT that it is economic forces, not estimation technology, that are responsible for the impasse. Economic forces, however, manifest themselves through markets, where the equilibrium is established through supply and demand curves and possibly market power. The only market here is the option market, which is intermediated and universally assumed to be perfectly competitive. This assumption has never been tested empirically but it should be treated with suspicion, certainly for short maturity options and possibly for all index options. It is known that there is exactly one designated liquidity provider for the entire SPXW class, termed the Designated Primary Market Maker (DPM); see https://www.cboe.com/us/options/trading/liquidity_providers/. Since the trader population is diverse, there are issues of information asymmetry, which is a powerful barrier to entry. A systematic investigation of these issues requires specific datasets which are not available to the authors. However, we discuss our proposed line of inquiry briefly in the last section of this paper, and more extensively in our online appendix, and leave the empirical work for future studies.

The next section presents the SA derivation of the option trading bounds applicable to this paper, as well as the out-of-sample tests that verify whether the bounds are identifiers of mispriced options. Section II estimates the bounds for an extended data set of SPXW options and demonstrates via out-of-sample tests that the mispriced zero-net-cost option portfolios provide risk-adjusted excess returns to their holders in the frictionless market, but not in the presence of frictions. Section III concludes.

I. The Stochastic Arbitrage Option Bounds and their Violations

The SA bounds are derived by considering the risk averse IT investor holding the option and the riskless asset, who adds to her holdings a zero net cost portfolio containing a long or short option position and becomes the option trader or OT. The option prices should be such that OT returns would not dominate IT returns in the second degree at any time interval to option expiration, a condition that translates into upper and lower bound for the option. The bounds are consistent with a monotone decreasing pricing kernel. Any option price outside the bounds implies a stochastic arbitrage opportunity. The bounds are derived recursively, starting from the time $T-\Delta t$ before the option expiration at T, and applied to any time interval $(t, t+\Delta t)$ within the life of the option. The limiting form follows immediately for $\Delta t \to 0$. A key issue is the specification of the index return dynamics, to which we now turn.

Since we do need tradable entities, the dynamics of the index can be extracted from a tracking fund, as we assume in this section, or from a futures contract that matures with or after the option, as we illustrate in our online appendix when we introduce transaction costs. We follow AFT (eq. (1), p. 1348), who assume that the index returns are stochastic volatility mixed with jumps (SVJ), but also state that for such short maturities as in SPXW the volatility can be taken as constant. Hence, we use physical parameter estimates under jump diffusion and assume a total volatility that is time-varying between cross sections but constant for each cross section. That volatility is estimated from adjusting the observed VIX index for bias and maturity, as in Constantinides, Czerwonko and Perrakis (2020). As that paper showed, the volatility estimates from the adjusted VIX were excellent forecasts of the ex post observed realized volatility.[4] We verify them in our robustness checks.

For a tracking fund, let I_t denote the value of the index at *t*, μ the instantaneous mean assumed greater than the riskless return r, q the (assumed constant) ex-dividend rate, λ the jump intensity, κ the expected log jump amplitude minus 1, and σ_t the diffusion volatility, which is assumed to stay constant in each cross section. Let $[t,T]$ denote the interval to option expiration T and K the option strike price. The index dynamics then become

$$\frac{dI_t}{I_t} = (\mu - q - \lambda\kappa)dt + \sigma_t dW + (j-1)dN. \tag{1.1}$$

Hereafter we ignore the dividends, that do not enter into the risk neutral *Q*—dynamics, The discretized version of (1.1) is, setting the ex-dividend return $z_{t+\Delta t} = \dfrac{I_{t+\Delta t} - I_t}{I_t}$

$$z_{t+\Delta t} = \begin{cases} [\mu - \lambda\kappa]\Delta t + \sigma_t \varepsilon \sqrt{\Delta t} & \text{with probability } \ (1-\lambda\Delta t) \\ [\mu - \lambda\kappa]\Delta t + \sigma_t \varepsilon \sqrt{\Delta t} + (j-1) & \text{with probability } \ (\lambda\Delta t) \end{cases}, \tag{1.1'}$$

where $\varepsilon \sim N(0,1)$. We assume that the jump amplitude j is a *truncated* lognormal such that $j > j_{\min} \equiv \underline{j}$, which is common to all cross sections, consistent with the assumptions of the jump diffusion SA bounds. In such a case $\kappa = \exp\{E\left[\ln(j) \middle| j \geq \underline{j}\right]\} - 1$. With this specification the total variance of the ex-dividend index return till option expiration is given by the following expression, which is observable at every cross section and equal to the bias-adjusted VIX

$$Var\left[\ln\frac{I_T}{I_t} \middle| j \geq \underline{j}\right] = \left[\sigma_t^2 + \lambda\left\{\left(E\left[\ln(j)\middle| j \geq \underline{j}\right]\right)^2 + \left(\mathrm{var}\left[\ln(j)\middle| j \geq \underline{j}\right]\right)\right\}\right](T-t). \tag{1.2}$$

[4] An alternative approach would have been the full estimation of the *P*-distribution under SVJ, for which the risk neutralization under SD has been recently developed by Perrakis and Oancea (2022). As AFT imply (pp. 1349-1351), the weeklies may differ from longer maturity options, which makes our VIX approach more suitable.

The constant instantaneous mean μ can easily be varied or set proportional to variance or to volatility, with very little effect on the results. The derivation of the SA bounds stems from the Euler discretization of the diffusion component, in which the dynamics are as in (1.1)' but with the random term $\varepsilon \sim F(\varepsilon),\ \varepsilon \in [\underline{\varepsilon}, \overline{\varepsilon}],\ E[\varepsilon]=0,\ E[\varepsilon^2]=1,\ \underline{\varepsilon}<0<\overline{\varepsilon}<\infty$; as $\Delta t \to 0$ the discretized index dynamics (1.1)' tend to (1.1). This discretization is applied to the general expression for the frictionless SA bounds, which holds for any distribution of the random return $z_{t+\Delta t}$, with $Min\{z_{t+\Delta t}\} \equiv z_1,\ e^{r\Delta t} = R = 1 + r\Delta t + o(\Delta t)$ and $E_t[z_{t+\Delta t}] \equiv \hat{z}_t$,; note that z_1 corresponds to $j_{\min} = \underline{j}$ as $\Delta t \to 0$. This results in the following risk neutral dynamics $U_t(z_{t+\Delta t})$ for the upper bound[5]

$$U_t(z_{t+\Delta t}) = \begin{pmatrix} z_{t+\Delta t} \text{ with probability } \dfrac{R-(1+z_1)}{\hat{z}_t - z_1} \\ 1_{z_{t+\Delta t}=z_1} \quad \text{with probability } \dfrac{1+\hat{z}_t - R}{\hat{z}_t - z_1} \end{pmatrix}. \tag{1.3}$$

At the limit this expression yields, for $z_{t+\Delta t}$ given by (1.1)'

$$\begin{gathered} \frac{dI_t}{I_t} = \left(r - \left(\lambda + \lambda_{Ut}\right)k^U\right)dt + \sigma_t dW_t^Q + \left(j_t^U - 1\right)dN_t^Q,\ \lambda_t^U = \lambda + \lambda_{Ut},\ \lambda_{Ut} = -\frac{\mu - r}{\underline{j}-1}, \\ j_t^U = \begin{cases} j & \text{with probability } \frac{\lambda}{\lambda+\lambda_{Ut}} \\ \underline{j} & \text{with probability } \frac{\lambda_{Ut}}{\lambda+\lambda_{Ut}} \end{cases},\ E\left[j_t^U - 1\right] = \kappa^U = \frac{\lambda}{\lambda+\lambda_{Ut}}\kappa + \frac{\lambda_{Ut}}{\lambda+\lambda_{Ut}}(\underline{j}-1) \end{gathered}. \tag{1.4}$$

An SA lower bound also exists for this jump diffusion process, which is not shown since it is rarely violated in our empirical applications.

Equation (1.4) is sufficient for the estimation of the upper bounds of the options in a given cross section, which are expectations of the corresponding payoffs. These bounds can be applied individually to every option. If there is an option whose observed bid quote lies above the upper bound then this option is obviously mispriced in the frictionless world. The rest of this section is devoted to the design of strategies to exploit this overpricing for both calls and puts, which stem from the proof of the derivation of the risk neutral dynamics (1.3) in Perrakis (2019, pp. 23-27).

Suppose we observe at time t a call option in a cross section of a given maturity, whose bid price lies above the SD upper bound, or $C_{bt}(I_t, K, T) > \overline{C}(I_t, K, T)$. The strategy consists in shorting one call per unit index at the bid price and allocating $\beta_t C_{bt}$ and $(1-\beta_t)C_{bt}$ in the riskless bond and the index, respectively. This position is closed at time $t+\Delta t$ at a price equal to the call upper bound $\overline{C}(I_{t+\Delta t}, T)$, which is derived from the discretized Q-dynamics in (1.3) for all risk averse traders.

[5] See Perrakis, (2019, pp. 23-27), and Ghanbari, Oancea and Perrakis (2021, pp. 252-254). The latter study also shows that the width of the jump diffusion bounds increases with the degree of moneyness, a factor that will play a role in the following section's empirical results.

The allocation β_t is chosen so that at the lowest value of the return $z_{\min,t+\Delta t} = z_1$ that corresponds to the left tail of the jump amplitude the portfolio payoff will be zero at $t+\Delta t$, implying that

$$\beta_t = \frac{\bar{C}(I_t(1+z_1),K,T)-(1+z_1)C_{bt}(I_t,K,T)}{RC_{bt}(I_t,K,T)-(1+z_1)C_{bt}(I_t,K,T)} \equiv \beta_t^* . \tag{1.5}$$

If the portfolio is rebalanced at every point $\tau \in (t, T-1]$, with β_τ similarly chosen so that at the lowest value of the return z_1 the portfolio payoff at the upper bound $\bar{C}(I_{\tau+\Delta\tau},T)$ would be equal to 0, then at option maturity T the cumulated allocation would be equal to $C_{bt}\left[\prod_{\zeta=t}^{\zeta=T} R^{\Delta\zeta}\beta_\zeta^* + \prod_{\zeta=t}^{\zeta=T}(1-\beta_\zeta^*)(1+z_{\zeta+\Delta\zeta})\right]$, from which we need to subtract the proceeds of the closed short call $\bar{C}(I_T,K,T)=(I_T-K)^+$. In the empirical tests of the following section, the above strategy is applied to a portfolio of options violating the SA bounds at time *t*, and the position is closed at the beginning of the next day, for each option in the portfolio. It is closed at the upper bound $\bar{C}(I_{t+1},K,T)$ for frictionless trading and at the observed ask price $C_{a,t+1}(I_{t+1},K,T)$ if frictions are partially recognized.

For an overpriced put option, we write a put at its bid price P_{bt}, short $\beta_t I_t - P_{bt}$ of index with $\beta_t < 1$, and invest $\beta_t I_t$ in the riskless asset. The portfolio payoff at $t+\Delta t$ is $\beta_t I_t R - [\beta_t I_t - P_{bt}](1+z_{t+\Delta t}) - P(I_t(1+z_{t+\Delta t}),T)$, whose lowest value is when the put is at its upper bound at $t+\Delta t$, or $P(I_t(1+z_{t+\Delta t}),T) = \bar{P}(I_t(1+z_{t+\Delta t}),T)$. This payoff is clearly increasing in the put bid price P_{bt} for every β_t. At the lowest value of the index return support $z_{t+\Delta t} = z_1$ the payoff should be nonnegative, implying that the optimal allocation at time t would be

$$\beta_t \geq \frac{\bar{P}(I_t(1+z_1),K,T)-P_{bt}(1+z_1)}{I_t[R-(1+z_1)]} \equiv \beta_t^* , \tag{1.6}$$

Setting β_t at this value, at $t+\Delta t$ the expected portfolio payoff is $\beta_t^* I_t R - E_t[(\beta_t^* I_t - P_{bt})(1+z_{t+\Delta t}) - \bar{P}(I_t(1+z_{t+\Delta t}))]$. This expectation should be 0 at $P_{bt} = \bar{P}(I_t(1+z_{t+\Delta t}),T)$, which is verified using (1.3). Hence, it is positive for $P_{bt} > \bar{P}$, implying a positive put portfolio payoff when the put position is closed at $\bar{P}(I_{t+\Delta t},K,T)$. As with the overpriced calls, the excess returns at *T* would be

$P_{bt}\prod_{\zeta=t}^{\zeta=T-1}((1+z_{\zeta+\Delta\zeta}) + \beta_t^* I_t(R^{T-t} - \prod_{\zeta=\tau}^{\zeta=T-1}(1+z_{\zeta+\Delta\zeta})) - (K-I_T)^+$ As with the calls, in our empirical applications in the following section we close the position at the end of the first day at the value $\bar{P}(I_{t+1},K,T)$ for frictionless trading and $P_{a,t+1}(I_{t+1},K,T)$ in the presence of frictions. Hence, each overpriced call and put option will contribute the following quantity

$$C_{bt}\left[R^{T-t}\beta_t^* + (1-\beta_t^*)\prod_{\zeta=t}^{\zeta=T}(1+z_{\zeta+\Delta\zeta})\right] - R^{T-t+1)}\begin{pmatrix}\overline{C}(I_{t+1},K,T) \text{ for frictionless} \\ C_{a,t+1}(I_{t+1},K,T) \text{ with bid-ask spread}\end{pmatrix}$$

$$P_{bt}\prod_{\zeta=t}^{\zeta=T-1}((1+z_{\zeta+\Delta\zeta}) + \beta_t^* I_t(R^{T-t} - \prod_{\zeta=\tau}^{\zeta=T-1}(1+z_{\zeta+\Delta\zeta})) - R^{T-(t+1)}\begin{pmatrix}\overline{P}(I_{t+\Delta t},K,T) \text{ for frictionless} \\ P_{a,t+1}(I_{t+1},K,T) \text{ with bid-ask spread}\end{pmatrix} \tag{1.7}$$

For the empirical work verifying the SA model option portfolios are set at time t using options violating the SA bounds and then liquidated with option positions closed at $t+1$ at the option upper bound, $\overline{C}(I_{t+1},K,T)$ or $\overline{P}(I_{t+1},K,T)$, for frictionless trading. In the presence of frictions the option positions are closed at the prevailing ask prices, $C_a(I_{t+1},K,T)$ and $P_a(I_{t+1},K,T)$.[6] This work leaves open the determination of the unique equilibrium price for the frictionless options, for which the SA methodology only provides trading bounds. As we see in the following section, these bounds are quite wide at the tails of the distribution. A full determination of the equilibrium price requires further assumptions and a much longer treatment that transcends the purpose of this paper. We discuss these issues briefly in the following sections.

II. Data and Empirical Results

Weekly S&P 500 index options (SPXW) are similar to standard monthly options except they have a shorter life span, are PM-settled on their expiration date, and are expiring every day of the week. They are typically listed several weeks in advance. Since launch, Weekly options have grown to become one of CBOE' s most-actively traded products. A total of 345 million S&P 500 index option contracts were traded in 2021, with an average daily volume (ADV) of 1.4 million contracts. Among them, there were approximately 247 million SPXW contracts in 2021, with an ADV of more than 981,000 contracts, accounting for nearly 72 percent of total SPX trading volume.[7] In this study we only focus on the End-of-Week SPXW options, maturing every Friday. Although the sample data in the AFT study contains all short term options in their estimations, including both SPX and SPXW, we do not include the SPX in our sample, since they have different market structure, settlement mechanism and DPM.

For the empirical estimation of the S&P 500 index returns' P-dynamics we use ex-dividend daily returns over the period January 3, 1963, to December 31, 2010. The underlying return sample is chosen so that it has no overlap with the option sample.[8] During this period the average annualized return is 6.25%, the standard deviation of returns is 16.27%, and their skewness and kurtosis are -1.07 and 32.18 respectively. In robustness checks we also use daily returns from January 2, 1980, to December 31, 2010. For the intraday index price, we use the price provided by the CBOE

[6] Note, however, that frictions were not taken into account in establishing the option portfolios, implying that the resulting profits under frictions, if any, are upper bounds in exploiting the observed frictionless SA bounds' violations.

[7] https://ir.cboe.com/news-and-events/2022/04-13-2022/cboe-add-tuesday-and-thursday-expirations-spx-weeklys-options

[8] Although SPXW started trading in 2005, they were AM-settled until December 2010 and PM-settled thereafter. To prevent any bias due to the settlement mechanism, our option sample starts in in January 2011.

reporting system.[9] For the dividend yield we use daily cash payouts obtained from Standard and Poor. For the interest rate, we use the 3-month constant maturity T-bill rate obtained from the Federal Reserve Economic Data.

We use the adjusted VIX index as a proxy for total variance of the index returns. The adjustment is equal to the mean difference between the VIX and the realized volatility from 1986 to the observation date. The amount by which VIX exceeds the realized volatility is relatively stable in our sample, with an average annual premium of 5.27%, with the maximum value of 5.61% in September 2007 and minimum value of 4.96% in April 2020. Both the VIX and the realized volatility of daily returns are measured in one-week intervals without overlap, with the latter quantity defined as the square root of 252 times the mean squared daily return. In our robustness checks, we also consider alternative specifications of the VIX bias with very little impact on our results.

We estimate the jump diffusion parameters for the S&P 500 index using the generalized method of moments (GMM) framework. The moments of the jump-diffusion dynamics with a constant intensity and truncated log-normal amplitude are shown in Ghanbari, Oancea and Perrakis (2021, Appendix B). The mean return and diffusion standard deviation are 7.07% and 14.97% respectively. The jump intensity is 0.26 per year and the mean and standard deviation of the truncated jump size are -3.22% and 10.07% per year. The lognormal jump amplitude is truncated at a maximum 20% negative jump size. The statistical properties of our parameter estimates are in Table OA1 of the online appendix. From equation (1.2) we extract a diffusion volatility for each option cross section given the parameter estimates of the jump distribution and the adjusted VIX observations. The average diffusion volatility across all cross sections is 13.49% with the minimum (maximum) volatility of 2.55% (74.44%). From these *P*-parameters we estimate the upper and lower bounds on the option prices in each cross section.

Our option sample contains end-of-the week Weeklies with one week-to-maturity, expiring on Friday from March 17, 2011, until March 17, 2022, for a total of 477 expiring Fridays and 3127 observation days. We follow each SPXW option in its daily trading over 7 different maturities, 9, 8, 7 and 4 to 1 days to maturity, from Wednesday till the next Thursday. Unlike AFT that use end-of-day data from OptionMetrics, we collect option data from time-stamped intraday option quotes from CBOE. Our sample only contains options with nonzero trading volume and minimum bid price of 10 cents. Standard arbitrage violations filters across moneyness were also applied. The trading volume is obtained from the CBOE Market Data Express Open/Close database. This dataset shows daily buy volume, sell volume, and open interest, separately for different types of market participants. We confirm that the net daily buy volume across all market participants is zero. The full data summary is shown in Tables OA2-OA3 of our online appendix, together with the average estimated SA bounds, the overlap of quotes and bounds and the location of the bid-ask midpoint within the bounds, separately for calls and puts. The online appendix also shows option summary statistics across different trading days. Overall, the sample has 140,305 (194,470) contracts of calls (puts) with average implied volatility (IV) of 22.7% (23.4%).

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[9] In studies that used earlier data the intraday index price was found from the cost-of-carry relationship between the cash index and its futures price, due to poor reporting of the cash index. As of 2007 the quality of this reporting has significantly improved and there is no reason to use futures.

Several general conclusions can be drawn from this comprehensive data set, which to our knowledge is presented here for the first time. First, the market data for call and put options do not support put-call parity even approximately. For all seven days to maturity the total volume is more than 80% higher for puts than for calls across the entire sample. For the three observation days with 9, 7 and 4 days to maturity that are the focus of our empirical analysis the put volume is more than 77%, 120% and 108% respectively higher. Second, the market makers (MM) hold a large and relatively stable share of about 50% in the intermediated market for both calls and puts.[10] Such a level of volume certainly does not support the free entry of dealers into that market, assumed in the equilibrium model of Garleanu, Petersen and Poteshman (GPP, 2009, p. 4264).[11] Last, the DPM's order imbalance, which plays an important role in establishing equilibrium in the intermediated market in GPP and also in Fournier and Jacobs (2020), is not reliable as a key variable for the very simple reason that it is highly unstable during the life of the option. Although its averages start at the low levels of 5.35% for calls and 1.37% for puts 9 days before maturity, they rise dramatically to 18.72% for calls and 22.56% for puts at 7-day maturities, only to fall at 0.2% and 2.4% three days later.

The most striking result in Table 2, however, is the inconsistency of the intermediated market with the fundamental conditions that must exist to extract the frictionless option market prices. Panels B and D of Table 2 show the results of the comparisons of the SA option bounds with the observed bid-ask prices of the options. In all seven days-to-maturity (DTM) categories and for both puts and calls a very large majority of the cross sections are fundamentally inconsistent with the SA bounds, insofar as the observed bid-ask spreads lie entirely above the option bounds for more than 37% of the calls and 35% of the puts across all ranges of moneyness. Similarly, the bid-ask midpoint, the universally accepted proxy for the "correct" estimate of the equilibrium option price, lies within the SA bounds for only 24% of the observed calls and 39% of the puts. In what follows we focus on the end-of-the week SPXW options with 9-, 7- and 4 DTM, in order to explore in depth the economic reasons for these inconsistencies.

Tables 1-3 below show the detailed results for these three maturities, disaggregated by degree of moneyness categories for both put and call options. We set a minimum volume filter of 1000 contracts in these tables, which encompasses more than 95% of the total volume in all three DTM groups. In terms of degrees of moneyness $m_t \equiv \frac{K}{I_t}$ the liquid zone shown in the three tables is $m_t = [0.86, 1.02]$ for puts and $m_t = [0.98, >1.06]$ for calls for the two longer maturities, and $m_t = [0.90, 1.02]$ for puts and $m_t = [0.98, 1.06]$ for calls for the 4-day maturity. The tables also show the relative positions of the SA bounds and the option market bid-ask spread, as well as the DPM's fully hedged (matched) positions and exposures in terms of net buys across each moneyness category. The full data is shown in Tables OA4-OA5 of the online appendix.

[Table 1 about here]

[10] Although in later years there was one designated MM, the DPM, it may not have been so in earlier years.

[11] Note, however, that this may have been true in the 1996-2001 period of that study's data.

[Table 2 about here]
[Table 3 about here]

The results are stunning. The inconsistency of the bid-ask spread with the SA bounds is very definitely related to the moneyness, with the observed option prices in the ATM zone of 0.98-1.02 virtually entirely disconnected from the corresponding SA bounds, for both calls and puts and for all three DTM groups. A small minority of the bid-ask spread midpoints lie within the SA bounds, and for a large majority of the cross sections most option spreads have no overlap with the SA bounds. Exactly the opposite happens in the deep OTM put option zone of $m_t = [0.86, 0.93]$ for the 9- and 7-day DTM groups, and $m_t = [0.90, 0.96]$ for the 4-day DTM groups, where the bid-ask spread midpoint lies within the SA bounds for a majority of the options and across a majority of the cross sections. For the intermediate put option OTM zones $m_t = [0.93, 0.98]$ and $m_t = [0.96, 0.98]$ the consistency of bid-ask spread and bounds is in-between those of the deep OTM and ATM zones. Last for OTM and deep OTM calls there is overlap of spread and bounds for most cross sections, but the spread midpoint is not in it.

These results explain fully the inability of AFT to extract satisfactory risk neutral dynamics from the observed option market data, even after the daily structural calibration of a semi-nonparametric model combined with heavy data manipulation to transform it into a frictionless format. For a large segment of the support of the index return distribution there is simply no option market data even minimally consistent with the return dynamics that can be extracted from the S&P data. This fact, combined with the Jouini and Kallal (1995), Constantinides, Jackwerth and Perrakis (2009), Beare (2011, 2023) and Post and Longarela (2021) theory, implies that the frictionless option market data is mispriced.[12]

In the remaining part of this section, we focus our investigation on the two important results implied by our empirical work. The first one is the verification of the mispricing of the options with the most reliable tool for such an assessment, which is the ability of investors to profit from it. The relevant tests were described in equations (1.5)-(1.7) and are carried out below, for both frictionless trading and in the presence of an option bid-ask spread; see footnote 6.

The second result derived from the data in Tables 1-3 concerns the inconsistency of the observed option market prices with the frictionless equilibrium implied by the *P*-distribution, which need to be reconciled. As discussed in the introduction, on theoretical grounds this reconciliation can only come from those prices that are broadly consistent with the SA bounds. Ironically, in view of the difficulties experienced by AFT in fitting the tails of the *Q*-distribution to option market data, the only such "correctly" priced options are the deep OTM puts! By contrast, the market for call options cannot be reconciled with the SA bounds implied by the *P*-dynamics, since a large part of it lies almost entirely above them everywhere.

For the mispricing verification, we focus on the close to ATM zone $m_t = [0.98, 1.02]$ in Tables 1-3, where the SA bounds do not intersect with the bid-ask spread interval for both calls and puts for a

[12] This is also consistent with the SPXW option findings in the presence of frictions of Constantinides, Czerwonko and Perrakis (2020).

large majority of the observed data. Hence, with the given return distribution parameters a sustainable frictionless equilibrium is not consistent with the observed option market data in the ATM zone. This implies that there exist stochastic arbitrage strategies generating risk adjusted positive profits for any risk averse investor. In the remainder of this section, we illustrate these strategies for individual investors.

In our mispricing tests, for each call option cross section with *m* observed overpriced options such that $C_{bt}^{i}(I_t, K_i, T) > \overline{C}_i(I_t, K_i, T),\ \ i = 1,...,m$ we short the overpriced options and form a portfolio of the weighted sums of the differences $\sum_{1}^{m} w_i (C_{bt}^{i} - \overline{C}_i)$, with each short option position allocated as in (1.7)..The portfolio weights are proportional to the violations of the bid prices from the calls' upper bounds. We then consider SA strategies for profiting from mispriced options in the frictionless world that are available to any risk averse investor holding at least one index unit in her IT portfolio. Such strategies are available to all risk averse investors, but their application must be tailored to each individual investor. It will depend on the location of the equilibrium prices of the overpriced ATM options with respect to the particular investor's IT holdings. Such an equilibrium must also encompass the options that are consistent with the SA bounds, in our case the deep OTM put options. For these options the equilibrium is not uniquely defined from the overlap of the bid-ask spread with the interval between the SA bounds, both of which are quite wide.

As in conventional no arbitrage equilibrium (NAE) studies, we model this equilibrium based on the assumption of a representative constant relative risk aversion (CRRA) investor, defined uniquely from the RRA parameter. For our mispricing tests we fix the RRA parameter, define the equilibrium consistent with it *and* with the SA bounds, and verify the mispricing of the ATM zone given the equilibrium. With a CRRA utility and with the RRA denoted by φ the risk allocation of the jump diffusion dynamics takes the following form[13]

$$\mu - r = \varphi\sigma_t^2 + \lambda\kappa - \lambda^Q\kappa^Q,\ \ \lambda^Q = \lambda E_t(j^{-\varphi}),\ \kappa^Q = E_t[\frac{(j-1)j^{-\varphi}}{E_t(j^{-\varphi})}] = E_t[\frac{(j^{1-\varphi}}{E_t(j^{-\varphi})} - 1]. \tag{2.1}$$

This allocation stems from the IT investor, who maximizes successively in the discretized dynamics with $\Delta t = 1$ the allocation without any consumption till a horizon $T' > T$, for a return $R_{T'} = \alpha_t \prod_{\tau=t+1}^{\tau=T'} (1 + z_\tau) + (1 - \alpha_t) R^{T'-t}$:, yielding, for $W_{T'} = W_t R_{T'}$ and the maximization of $E_t[U(W_{T'})]$ the condition $\frac{E_t[U'(W_T)\prod_{\tau=t+1}^{\tau=T'}(1+z_\tau)]}{E_t[U']} = R^{T'-t}$, for any concave utility function $U(W_{T'})$. Setting for simplicity *T'*=*T* and denoting by R_τ^* the optimal ex-dividend index returns at τ, the first order

[13] See Ghanbari, Oancea and Perrakis (2021, p. 259), itself derived from several earlier NAE-based studies. The proof is also shown in our online appendix

conditions become $\frac{E_t[(\prod_{\tau=t+1}^{\tau=T} R_\tau^*)^{-\varphi}(1+z_{t+1})]}{E_t[(\prod_{\tau=t+1}^{\tau=T} R_\tau^*)^{-\varphi}]} = e^{r\Delta t}$. This yields a kernel $\frac{(\prod_{\tau=t+1}^{\tau=T} R_\tau^*)^{-\varphi}}{E_t[(\prod_{\tau=t+1}^{\tau=T} R_\tau^*)^{-\varphi}]}$, which is dependent on the RRA parameter φ and must be consistent with the SA bounds. This kernel is independent of wealth, but its application to the derivation of the SA bounds is not, since it must correspond to at least one index unit per valued option, in which case if takes the form $\frac{I_T^{-\varphi}}{E_t[I_T^{-\varphi}]}$.

From (2.1) it is clear that there is a one-to-one correspondence between the RRA coefficient φ and the risk neutralization of the jump component. On the other hand, setting in the upper bound (1.4) of the risk neutral dynamics $(\lambda^U, \kappa^U) = (\lambda^Q, \kappa^Q)$, it is easy to see that $\mu - r = \lambda\kappa - \lambda^Q\kappa^Q$, implying that the entire risk premium compensates for the jump risk at the SA upper bound. This automatically defines an SA-implied upper bound on φ in order to have the NAE option price and IT portfolio consistent with the SA option bound, provided the OT investor trades in one option per unit index in her portfolio. Consistent with (1.3)-(1.4), this upper bound is given by

$$\varphi_{t,T}^{\max} = Max\left(\varphi \middle| \lambda^Q = \lambda E_t(j^{-\varphi}),\ \kappa^Q = E_t[\frac{(j-1)j^{-\varphi}}{E_t(j^{-\varphi})}]\right) \quad . \tag{2.2}$$

For $\varphi \in (0, \varphi_{t.T}^{m\,ax})$, if there is an equilibrium corresponding to a "representative" CRRA investor with risk aversion within the bounds the relation (2.1) holds. The portfolio allocation of such an investor coincides with the NAE allocation of the premium to diffusion and jump risk.

From (2.1) and (2.2), a unique SA-implied RRA coefficient $\varphi_{t,T}^{\max}$ can be obtained for every cross section given the underlying return distribution parameters and riskless rate. The riskless rate, however, has fluctuated significantly over the period of our data, from almost 0 in 2011 to about 2.5% in early 2019 and again back to 0 in 2020. Consequently, the upper bound $\varphi_{t,T}^{\max}$ on RRA has fluctuated within relatively wide ranges, which are very similar in the three reference DTM groups of 9, 7 and 4 days. In decreasing order of DTM, we have $Min\{\varphi_{t,T}^{\max}\} = [6.20, 6.99, 6.79]$, $Max\{\varphi_{t,T}^{\max}\} = [14.1, 13.98, 13.90]$.[14] Within these ranges each cross section j has its own $\varphi_{j,t:T}^{\max}$, $j = 1,...,n$, where n is the number of the cross sections in our sample. Since the SA between the IT and OT trader portfolios that will exploit the mispriced options must have the same RRA coefficient φ for all cross sections, the OT zero net cost option portfolio must take this variation in $\varphi_{jt,T}^{\max}$ into account.

Since the payoffs of the zero-net-cost portfolio strategies are risky, our intuition tells us that the portfolio payoffs will be lower when the RRA increases, and this turns out to be the case. Any CRRA

[14] The full distributions of the implied RRA coefficients for all DTM from 1 to 9 are shown in Figure I of our online appendix.

investor who is more risk averse, with $\varphi > \varphi_{jt.T}^{max}$ will invest in only a fraction of an option contract in that cross section. In our online appendix we show that this fraction is equal to the ratio $\frac{\varphi_{jt,T}^{max}}{\varphi} \equiv \zeta_j < 1$, hereafter termed the risk preference multiplier (RPM).[15]

In the first set of mispricing tests shown in Table 4 below we consider RRA values with an RPM greater than 1 for all cross sections. In such a case the portfolio of the weighted sums of the differences $\sum_1^m w_i(C_{bt}^i - \overline{C}_i)$ has positive weights summing to one and maximizing the total mispricing. A similar procedure is applied to the violating ATM put options in every cross section. At option maturity each one of these violating options will contribute to the mispricing an amount equal to (1.7). At the end of the first day, we may close the position either at the SA upper bound as in (2.2) or at the equilibrium value of the option corresponding to the chosen RRA. This is a proxy for the mispricing in the frictionless world. Closing the position at the available ask prices yields an upper bound for the mispricing in the economy with frictions.

[Table 4 about here]

Table 4 presents the formal testing of the SA relationship with the Davidson-Duclos (DD, 2013) tests, to verify whether the series of OT returns stochastically dominates the corresponding series of IT in the second degree for the 9-, 7-, and 4 DTM groups. These tests were first used in Finance by Constantinides *et al* (2011) and are particularly convincing because the null is non-dominance of OT over IT, or $H_0 : OT \not\succ_2 IT$. In this table the DD tests in the upper and lower panels compare the two IT and OT series for all cross sections, respectively for calls and puts. The cross sections include those with overpricing of the corresponding options, as well as those in which there is no overpricing, which have identical IT and OT values in both series. In turn, each one of these panels contains Panels A to E, reflecting two different RRA values for the IT-OT comparisons. In Panels A to C the RRA is equal to 4 for all cross sections, significantly lower than $Min\{\varphi_{jt,T}^{max}\},\ j = 1,...,n$. For this investor Panel A shows the test results for the frictionless market, when the positions are closed at the SA upper bound. Panel B is identical to A with respect to the option strategies, except for the fact that the closing of the positions takes place at the prevailing equilibrium price of each option as in (2.1). Last, Panel C shows the results of the strategies when the positions are closed at the ask prices of the options. In all cases the short option proceeds are invested as in (1.7).

For each maturity and for both call and put panels and each closing strategy the table entries show the realized excess return of the OT minus IT series in annualized format, its volatility and the resulting information ratio. Consistent with the DD theory, the null hypothesis is tested under three different conditions with respect to handling the joint support of the paired sample. In all cases there is a 10% trimming of the left tail of the support, while the right tail has a 0, 5 and 10 percent

[15] When the jump risk is not priced as in Merton (1976) the entire premium compensates diffusive risk. This case is not relevant in our data.

trimming. The inference becomes progressively weaker as the right tail trimming increases. The *p*-values show the difference of OT and IT means for each right tail trimming.

The results of both call and put panels are clear and unequivocal for all three DTM groups and all three closing strategies, although there are some interesting and potentially informative differences between the groups. In all three groups and in the frictionless market the DD tests reject the non-dominance null everywhere and for both types of closing the positions, with *p*-values equal to zero or very low everywhere. As expected, the profits from the short option strategies are higher in Panel B, when the positions are closed at the equilibrium price, but the ex-post test results are identical. These tests are resounding confirmations of the Jouini and Kallal (1995) theoretical result, that frictionless option prices that do not lie inside the bid-ask spread are mispriced. Panel C also shows why these mispriced options persist, which is very simply the fact that they are not mispriced in the market with frictions, even the minimal ones that we have included! The DD tests are consistent with the non-dominance null in the 9- and 4-day DTM groups, since the excess OT returns are negative. Unreported results show that the 8-day DTM group is very similar to the 9-day group, and the frictionless DD tests also reject the non-dominance null for all other DTM groups.

The 7-day maturity is somewhat different in its Panel C results, insofar as the excess OT return is positive and significant at 5% and 10% right trimming, although not at the 0 trimming. It is also similar in its frictionless metrics to the 9-day maturity for both calls and puts, even though that maturity does not survive as profitable trading in the market with frictions. There are also major differences in the frictionless metrics between the 7- and 4-day maturities, with the latter having much lower excess OT return and information ratio. Since the SA bounds do not change very much because of the small difference in maturities, the change can only be attributed to heavy trading in the mispriced options that came closer to the SA bounds. This is something that should be examined in further research.

The results of Table 4 also illustrate the ranges of the RRA parameter consistent with a frictionless equilibrium within the SA bounds for each DTM group. An obvious exercise for anyone who wishes to extract the best fitting frictionless equilibrium based on the "correctly" priced deep OTM puts would be the extraction of a common RRA parameter from the bid-ask spread midpoints within the SA bounds in all cross sections. This can then be extended to all degrees of moneyness beyond the OTM puts. This will be left for further research.

In the last set of empirical results shown in Table 4 we demonstrate in Panels D and E the exploitation of the mispriced ATM options by more risk averse CRRA investors, whose equilibrium IT portfolios put them outside the SA bounds. These would need to modify the strategies, but they will still reap excess risk adjusted profits by exploiting option quotes violating the SA bounds. Since the SA-implied maximum RRA of (2.2) varies by cross section, we standardize the RRA by setting it to the maximum SA-implied RRA across all cross-sections, $\{\varphi_{t,T}^{\max}\}^* = Max_j[\{\varphi_{jt,T}^{\max}\}],\ j = 1,\ldots,n$, where n is the number of the cross sections in our sample. The investor with RRA equal to $\{\varphi_{t,T}^{\max}\}^*$ will trade optimally in order to exploit the mispricing in each cross section, by shorting one option at the cross section where $\{\varphi_{t,T}^{\max}\} = \{\varphi_{t,T}^{\max}\}^*$, but only shorting the RPM amount $\zeta_j = \dfrac{\varphi_{t,T}^{\max}}{\{\varphi_{t,T}^{\max}\}^*} < 1$

option in cross sections where $\{\varphi_{t,T}^{\max}\} < \{\varphi_{t,T}^{\max}\}^*$. We set the common RRA at the value of 14.16, the largest value observed in the three reference maturities.

Panel D of Table 4 shows the results of the DD test for an IT with an RRA equal to $\{\varphi_{t,T}^{\max}\}^* = 14.16$, with OT option portfolio positions equal to $\zeta_j \sum_{1}^{m_j} w_i (C_{jbt}^i - \bar{C}_j^i),\ j = 1,...,n,\ i = 1,...,m_j$, with m_j denoting the number of overpriced options in cross section j, for DTM equal to 9, 7 and 4 days for frictionless trading. In the presence of frictions $\bar{C}_j^i(I_{t+1}, K_j^i, T)$ is replaced by $C_{ja}^i(I_{t+1}, K_j^i, T)$ .Similar results without and with frictions also hold for overpriced puts. We first observe that in the frictionless option market the OT excess returns are uniformly lower, as expected, than in the corresponding Table 4 results, but the standard deviations are also lower, resulting in small changes in the information ratios. As for the DD tests, they are identical in both Panels A-C and Panels D and E, both in the frictionless world and in the presence of frictions.

Figure II of the online appendix shows the time series of the percentages of overpriced options, both calls and puts across each DTM group, for the entire cross section as well as the ATM zone. It shows that in the ATM zones for both calls and puts the overpricing covered between 60% and 100% of the options between 2011 and 2020. This percentage, however, decreased sharply to around 20% during the last two Covid years. We illustrate this observed mispricing in the frictionless world by showing in Figure 1 the cumulative OT portfolio returns from the mispriced portfolios separately for calls and puts over the period of our data.

[Figure 1 about here]

Figure III in the online appendix shows the contemporary cumulative IT returns. It is obvious that the enhancement from the overpriced option components is significant, which is also confirmed by statistical tests. A major advantage of the SA paradigm is that it allows the identification of such options. We explore this hypothesis by setting zero-net-cost OT portfolios of index and options using all but the previously identified overpriced options. The same Figure III shows the cumulative excess returns of these portfolios. It is obvious that these portfolios offer no advantage to investors and are if anything decreasing the cumulative returns of the IT investor. We conclude that our SA bounds' violations are excellent identifiers of overpriced options in the frictionless world, among which overpricing under frictions may also be sought.

III. Conclusions

The empirical evidence of the previous section showed clearly that attempting to extract the frictionless *Q*-dynamics from the observed bid-ask midpoint in the option market is a futile effort on both theoretical and empirical grounds. The observed option market data is fundamentally inconsistent with the frictionless option price format, insofar as it indicates a clear segmentation of the markets for call and put options and displays obvious moneyness effects. For the overwhelming majority of the cross sections not only the bid-ask midpoint but the entire width of the quotes for both calls and puts lay above their SA bounds in the highly liquid ATM region. This non-overlap was shown to be economically significant in the frictionless world, but its profitable exploitation disappeared when trading at the appropriate bid and ask prices. This frictionless mispricing of the options is consistent with the documented mispricing in the few studies that recognized frictions and the even fewer theoretical studies that compared option pricing without and with frictions.

Our empirical results also showed that the highly liquid deep OTM put options are the only ones that are broadly consistent with the index dynamics in a frictionless world. This important finding needs to be verified in longer maturities, in which the stochastic evolution of volatility during the life of the option must be taken into account. The inconsistency of the dynamics of the index with the risk neutral distribution extracted from option market data is also found in longer maturities that adopt the NAE approach and is not limited to AFT. It has given rise to a long debate about the shape of the pricing kernel and the alleged overpricing of OTM put options.[16] From the fragments of empirical work that have included frictions, it is conjectured that in standard index options SPX, the disconnect between the option market and the index dynamics will significantly be lower than the SPXW, although the verification will be left for future research.

As for the resolution of the twenty-year debate on the proper way to model the index option market, our results suggest that a starting point should be the recognition that there is an intermediate market where the bid and ask option prices are set. Our CBOE data file provides data on the option transactions of the market makers (MM), CBOE member firms and customers. In the case of SPXW, the first category consists of the designated monopolist liquidity provider or DPM, who obviously pays for this privilege.[17] The second category are firms licensed to trade on the CBOE platform, which we interpret as entities licensed to compete with the DPM. As for the 3rd category, they are the end users who place orders through their brokers based on the posted quotes.

To our knowledge, the only study that paid at least lip service to the intermediate option market was Garleanu, Pedersen and Poteshman (2009) which, however, assumed a free entry perfectly competitive frictionless market, in which there was no information asymmetry, no market power and no bid-ask spreads or any other types of frictions. In the online appendix we formulate models that relax several of these assumptions, first under competitive conditions and then taking into account the monopolistic liquidity provider, who sets the size and depth of the quotes and hedges her positions in the presence of transaction costs. These models can be taken only as templates for empirical work, which is a major undertaking and transcends the scope of this paper, but is nonetheless a highly worthwhile project.

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[16] See, for instance, Jackwerth (2000) for the shape of the kernel and Driessen and Maenhout (2007) for the overpriced put. The long list of references on the debate, which was still ongoing as of 2020, is surveyed in Perrakis (2022).

[17] See https://www.sec.gov/rules/sro/cboe/2022/34-93955-ex5.pdf.

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Table 1: Summary Statistics Weeklys - 4 Days-to-Maturity (Monday)

	<.90	.90-.93	.93-.96	.96-.98	.98-1.0	1.0-1.02	1.02-1.04	1.04-1.06	>1.06	All
Panel A: Summary Statistics Calls										
# Contracts	1,081	863	2,474	3,437	4,309	3,761	2,198	1,017	919	20,059
% Contracts	5.4%	4.3%	12.3%	17.1%	21.5%	18.7%	11.0%	5.1%	4.6%	100%
Avg IV	71.8%	34.2%	25.0%	19.1%	15.6%	13.8%	16.1%	21.0%	36.0%	21.2%
Avg Mid-Quotes	622.7	255.6	157.2	89.4	40.0	10.5	3.6	2.5	2.4	90.5
Relative BA Spread	1.4%	2.0%	2.9%	3.7%	3.3%	5.8%	22.4%	39.9%	56.0%	10.0%
Option Bounds Spread	0.1%	0.5%	1.0%	1.9%	4.9%	18.2%	29.7%	30.8%	23.1%	10.8%
Avg NetBuy MM	0	2	-5	-18	4	23	8	-26	-22	0.2
% Volume MM	54%	56%	49%	47%	52%	51%	49%	47%	50%	51%
Total Volume	12	13	103	269	2,400	7,351	2,177	666	337	13,327
Total Open Interest	93	208	1,493	3,932	11,599	14,696	4,974	1,657	1,072	39,723
Panel B: Calls Mispricing vs. SA Bounds										
Mid-Quotes Within Bounds	313	529	1,487	1,731	642	279	216	101	47	5,345
	29%	61%	60%	50%	15%	7%	10%	10%	5%	27%
Overpriced Contracts	37	54	234	776	2,470	1,873	426	163	301	6,334
	3%	6%	9%	23%	57%	50%	19%	16%	33%	32%
Cross-sections >1 Overpriced	11	14	53	153	335	307	88	22	14	345
	4%	5%	13%	36%	78%	72%	33%	18%	22%	81%
Cross-sections >50% Overpriced	6	14	35	90	272	261	66	19	11	152
	2%	5%	9%	21%	64%	61%	25%	15%	17%	36%
Cross-sections 100% Overpriced	3	14	21	72	158	230	61	16	11	4
	1%	5%	5%	17%	37%	54%	23%	13%	17%	1%
Panel C: Summary Statistics Puts										
# Contracts	1,123	4,590	6,405	4,319	4,340	3,991	1,941	891	1,472	29,072
% Contracts	3.9%	15.8%	22.0%	14.9%	14.9%	13.7%	6.7%	3.1%	5.1%	100%
Avg IV	44.7%	31.4%	24.3%	19.1%	15.5%	13.4%	17.6%	23.8%	49.5%	23.1%
Avg Mid-Quotes	2.3	1.2	2.0	4.7	12.1	37.6	86.3	144.2	402.0	38.9
Relative BA Spread	20.5%	35.6%	23.2%	10.3%	4.3%	4.9%	4.7%	3.7%	2.6%	14.9%
Option Bounds Spread	155.8%	147.2%	127.9%	94.8%	41.8%	2.5%	0.4%	0.2%	0.0%	78.1%
Avg NetBuy MM	3	-44	11	65	2	-4	-6	-8	-6	4
% Volume MM	50%	44%	48%	48%	52%	52%	50%	50%	50%	49%
Total Volume	387	2,338	6,878	7,737	8,007	1,973	253	81	70	27,724
Total Open Interest	1,575	12,456	22,548	17,188	14,504	5,505	1,913	945	1,291	77,925
Panel D: Puts Mispricing vs. SA Bounds										
Mid-Quotes Within Bounds	801	3,079	4,123	2,388	706	339	210	91	56	11,793
	71%	67%	64%	55%	16%	8%	11%	10%	4%	41%
Overpriced Contracts	296	627	1,197	1,582	2,763	1,502	193	67	117	8,344
	26%	14%	19%	37%	64%	38%	10%	8%	8%	29%
Cross-sections >1 Overpriced	24	52	105	231	350	292	35	11	5	365
	19%	14%	25%	54%	82%	68%	9%	5%	4%	85%
Cross-sections >50% Overpriced	23	46	74	155	310	168	17	8	1	66
	18%	12%	17%	36%	72%	39%	5%	4%	1%	15%
Cross-sections 100% Overpriced	21	43	57	104	204	51	12	5	1	1
	17%	11%	13%	24%	48%	12%	3%	2%	1%	0%

This table presents summary statistics for the merged CBOE time-stamped End-of-Week Weekly option quotes data (EoW Weeklys) with the trading volume data from Market Data Express Open/Close database. Option quotes observed on Monday at 3:00 PM from 20110317 to 20220317 with one week to maturity for the total of 428 expiring Fridays. The sample only contains options with non-zero trading volume and minimum bid price of 10 cents. Options are grouped across different moneyness bins. "Relative BA Spread" ("Option Bounds Spread") is the average bid-ask (upper and lower bounds) spreads in proportion of mid-quotes (mid-bounds). "Avg NB MM" is the average number of contracts purchased minus sold by market makers. "Total Volume" and "Total Open Interest" are in thousands. "Cross-sections >50% Overpriced" shows number of cross sections where more than 50% of contracts in the cross section are overpriced.

Table 2: Summary Statistics Weeklys - 7 Days-to-Maturity (Friday)

	<.90	.90-.93	.93-.96	.96-.98	.98-1.0	1.0-1.02	1.02-1.04	1.04-1.06	>1.06	All
Panel A: Summary Statistics Calls										
# Contracts	1,204	965	2,681	3,667	4,594	4,095	2,471	1,286	1,327	22,290
% Contracts	5.4%	4.3%	12.0%	16.5%	20.6%	18.4%	11.1%	5.8%	6.0%	100%
Avg IV	64.9%	33.9%	25.4%	19.8%	16.3%	14.3%	16.1%	19.8%	31.0%	21.4%
Avg Mid-Quotes	614.4	249.1	155.0	90.7	42.9	13.0	4.7	2.8	2.4	89.6
Relative BA Spread	1.6%	2.2%	3.2%	3.5%	3.1%	5.3%	16.5%	34.8%	64.4%	10.3%
Option Bounds Spread	0.2%	0.7%	1.2%	2.4%	6.0%	20.6%	32.3%	33.0%	25.9%	12.6%
Avg NetBuy MM	2	3	-8	-10	14	37	112	87	8	25
% Volume MM	60%	52%	52%	51%	51%	50%	50%	46%	50%	50%
Total Volume	16	17	107	259	2,562	7,871	2,790	926	509	15,058
Total Open Interest	89	254	1,382	3,953	11,787	13,818	4,976	1,996	1,477	39,732
Panel B: Calls Mispricing vs. SA Bounds										
Mid-Quotes Within Bounds	405	622	1,514	1,438	621	248	358	137	87	5,430
	34%	64%	56%	39%	14%	6%	14%	11%	7%	24%
Overpriced Contracts	49	104	483	1,411	3,202	2,526	634	172	183	8,764
	4%	11%	18%	38%	70%	62%	26%	13%	14%	39%
Cross-sections >1 Overpriced	15	28	106	255	404	373	136	27	13	415
	5%	9%	24%	55%	88%	81%	46%	18%	16%	90%
Cross-sections >50% Overpriced	9	25	70	177	360	338	115	21	8	217
	3%	8%	16%	38%	78%	73%	39%	14%	10%	47%
Cross-sections 100% Overpriced	6	24	54	122	249	298	95	19	7	5
	2%	8%	12%	27%	54%	65%	32%	13%	9%	1%
Panel C: Summary Statistics Puts										
# Contracts	2,343	5,944	6,952	4,652	4,640	4,311	2,176	976	1,467	33,461
% Contracts	7.0%	17.8%	20.8%	13.9%	13.9%	12.9%	6.5%	2.9%	4.4%	100%
Avg IV	43.0%	31.1%	24.8%	19.9%	16.3%	13.9%	16.9%	22.5%	39.0%	23.9%
Avg Mid-Quotes	2.7	1.8	3.2	7.0	15.3	39.8	86.1	145.1	350.4	34.6
Relative BA Spread	20.6%	24.0%	14.7%	7.3%	3.9%	4.6%	4.8%	3.8%	2.9%	11.4%
Option Bounds Spread	157.6%	150.1%	128.7%	96.0%	44.5%	3.2%	0.5%	0.2%	0.1%	84.4%
Avg NetBuy MM	35	-21	96	84	72	44	8	3	-10	46
% Volume MM	48%	43%	47%	49%	50%	50%	50%	50%	49%	48%
Total Volume	1,058	5,209	8,337	8,496	8,691	2,143	215	80	87	34,315
Total Open Interest	2,567	14,151	20,632	15,362	13,257	5,083	1,775	799	1,043	74,668
Panel D: Puts Mispricing vs. SA Bounds										
Mid-Quotes Within Bounds	1,637	4,387	4,642	2,104	699	274	233	90	56	14,122
	70%	74%	67%	45%	15%	6%	11%	9%	4%	42%
Overpriced Contracts	608	1,169	1,997	2,392	3,373	2,194	270	55	38	12,096
	26%	20%	29%	51%	73%	51%	12%	6%	3%	36%
Cross-sections >1 Overpriced	46	94	178	312	406	371	57	12	4	418
	20%	21%	39%	68%	88%	81%	14%	5%	3%	91%
Cross-sections >50% Overpriced	44	83	123	248	377	274	24	5	1	125
	19%	19%	27%	54%	82%	60%	6%	2%	1%	27%
Cross-sections 100% Overpriced	40	72	105	181	287	93	16	4	1	0
	17%	16%	23%	39%	62%	20%	4%	2%	1%	0%

This table presents summary statistics for the merged CBOE time-stamped End-of-Week Weekly option quotes data (EoW Weeklys) with the trading volume data from Market Data Express Open/Close database. Option quotes observed on Friday at 3:00 PM from 20110317 to 20220317 with one week to maturity for the total of 461 expiring Fridays. The sample only contains options with non-zero trading volume and minimum bid price of 10 cents. Options are grouped across different moneyness bins. “Relative BA Spread” (“Option Bounds Spread”) is the average bid-ask (upper and lower bounds) spreads in proportion of mid-quotes (mid-bounds). “Avg NB MM” is the average number of contracts purchased minus sold by market makers. “Total Volume,”“Avg Volume,” and “Total Open Interest” are in thousands. “Cross-sections >50% Overpriced" shows number of cross sections where more than 50% of contracts in the cross section are overpriced.

Table 3: Summary Statistics Weeklys - 9 Days-to-Maturity (Wednesday)

	<.90	.90-.93	.93-.96	.96-.98	.98-1.0	1.0-1.02	1.02-1.04	1.04-1.06	>1.06	All
Panel A: Summary Statistics Calls										
# Contracts	776	688	2,047	3,182	4,407	4,062	2,749	1,459	1,394	20,764
% Contracts	3.7%	3.3%	9.9%	15.3%	21.2%	19.6%	13.2%	7.0%	6.7%	100%
Avg IV	53.5%	30.8%	24.2%	18.8%	15.6%	13.6%	14.5%	17.5%	27.7%	19.0%
Avg Mid-Quotes	560.7	251.7	165.3	95.9	47.2	15.9	5.5	3.3	3.0	74.6
Relative BA Spread	1.6%	2.1%	2.6%	2.7%	2.4%	3.9%	11.0%	21.3%	38.9%	7.6%
Option Bounds Spread	0.2%	0.9%	1.7%	3.2%	7.5%	22.2%	33.3%	35.6%	31.4%	15.7%
Avg NetBuy MM	0	1	-6	-7	-3	7	24	-8	16	3
% Volume MM	53%	50%	51%	54%	51%	50%	50%	50%	51%	50%
Total Volume	9	13	55	133	1,565	4,070	1,515	361	249	7,970
Total Open Interest	81	194	1,105	3,410	10,283	13,045	5,491	2,185	1,851	37,646
Panel B: Calls Mispricing vs. SD Bounds										
Mid-Quotes Within Bounds	222	396	1,004	1,112	623	243	361	236	213	4,410
	29%	58%	49%	35%	14%	6%	13%	16%	15%	21%
Overpriced Contracts	77	125	624	1,529	3,140	2,411	745	147	112	8,910
	10%	18%	30%	48%	71%	59%	27%	10%	8%	43%
Cross-sections >1 Overpriced	19	35	121	278	369	334	162	24	13	392
	7%	14%	32%	66%	87%	79%	52%	14%	13%	92%
Cross-sections >50% Overpriced	10	29	81	187	343	311	137	19	7	242
	4%	12%	21%	44%	81%	73%	44%	11%	7%	57%
Cross-sections 100% Overpriced	4	26	57	142	253	277	112	17	4	7
	2%	11%	15%	34%	60%	65%	36%	10%	4%	2%
Panel C: Summary Statistics Puts										
# Contracts	3,479	5,874	6,613	4,502	4,489	3,995	1,745	731	1,027	32,455
% Contracts	10.7%	18.1%	20.4%	13.9%	13.8%	12.3%	5.4%	2.3%	3.2%	100%
Avg IV	37.1%	28.0%	22.7%	18.5%	15.5%	13.3%	15.7%	19.8%	37.6%	22.4%
Avg Mid-Quotes	2.5	2.2	4.4	9.0	18.9	43.9	90.9	147.2	358.0	30.4
Relative BA Spread	17.3%	18.0%	10.2%	5.5%	3.0%	3.5%	4.1%	3.3%	2.6%	9.2%
Option Bounds Spread	157.8%	148.1%	123.8%	89.1%	41.7%	4.1%	0.8%	0.3%	0.1%	87.6%
Avg NetBuy MM	-1	-8	-9	10	10	16	-12	15	-6	1
% Volume MM	44%	46%	48%	50%	51%	51%	49%	46%	45%	49%
Total Volume	855	1,829	3,268	3,211	3,749	974	134	62	61	14,142
Total Open Interest	5,882	13,732	17,906	12,975	10,584	4,123	1,510	666	862	68,240
Panel D: Puts Mispricing vs. SD Bounds										
Mid-Quotes Within Bounds	2,559	4,655	4,187	1,841	654	272	235	94	82	14,579
	74%	79%	63%	41%	15%	7%	13%	13%	8%	45%
Overpriced Contracts	878	1,072	2,277	2,518	3,271	2,075	205	49	32	12,377
	25%	18%	34%	56%	73%	52%	12%	7%	3%	38%
Cross-sections >1 Overpriced	54	86	191	313	368	330	51	7	4	396
	15%	20%	45%	74%	87%	78%	14%	3%	3%	93%
Cross-sections >50% Overpriced	49	64	132	223	344	280	19	5	0	109
	13%	15%	31%	53%	81%	66%	5%	2%	0%	26%
Cross-sections 100% Overpriced	48	55	88	173	277	104	14	4	0	1
	13%	13%	21%	41%	65%	25%	4%	2%	0%	0%

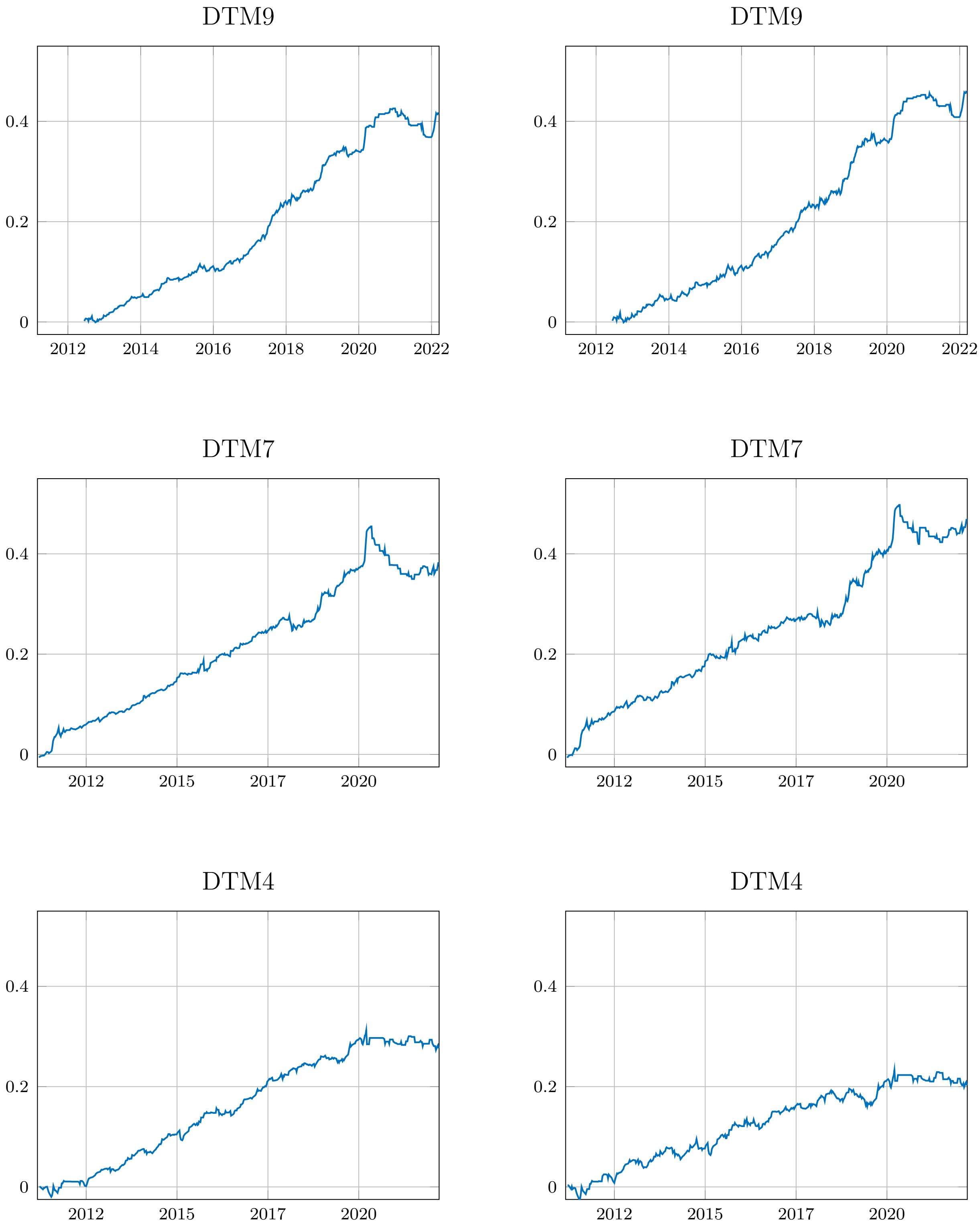
This table presents summary statistics for the merged CBOE time-stamped End-of-Week Weekly option quotes data (EoW Weeklys) with the trading volume data from Market Data Express Open/Close database. Option quotes observed on Wednesday at 3:00 PM from 20110317 to 20220317 with one week to maturity for the total of 424 expiring Fridays. The sample only contains options with non-zero trading volume and minimum bid price of 10 cents. Options are grouped across different moneyness bins. “Relative BA Spread” (“Option Bounds Spread”) is the average bid-ask (upper and lower bound) spreads in proportion of mid-quotes (mid-bounds). “Avg NB MM” is the average number of contracts purchased minus sold by market makers. “Total Volume” and “Total Open Interest” are in thousands. “Cross-sections >50% Overpriced" shows number of cross sections where more than 50% of contracts in the cross section are overpriced.

Table 4: Return Characteristics Option Trader Portfolios

	Panel A: Open @Bid - Close @UB			Panel B: Open @Bid - Close @NAE			Panel C: Open @Bid - Close @Ask			Panel D: Open @Bid - Close @UB			Panel E: Open @Bid - Close @Ask		
	DTM4	DTM7	DTM9	DTM4	DTM7	DTM9	DTM4	DTM7	DTM9	DTM4	DTM7	DTM9	DTM4	DTM7	DTM9
Calls:															
Avg Return	17.2%	23.7%	25.9%	17.7%	24.9%	29.3%	-10.0%	4.1%	-7.5%	13.6%	19.2%	20.5%	-8.5%	3.4%	-6.5%
St. Dev.	5.9%	6.8%	5.2%	5.9%	6.8%	5.2%	6.0%	6.4%	4.9%	5.0%	5.7%	4.3%	5.1%	5.4%	4.1%
Info Ratio	2.91	3.47	4.95	2.99	3.65	5.67	-1.66	0.64	-1.54	2.74	3.35	4.74	-1.65	0.63	-1.61
H_0: OT $\not\succ_2$ IT	0.001	0	0	0	0	0	1	0.206	1.000	0.002	0	0	1	0.210	1
H_0: OT $\not\succ_2$ IT (5% Trim)	0	0	0	0	0	0	1	0.011	1	0	0	0	1	0.009	1
H_0: OT $\not\succ_2$ IT (10% Trim)	0	0	0	0	0	0	1	0	1	0	0	0	1	0	1
% CS Ret>0	52%	60%	60%	53%	60%	62%	39%	50%	42%	52%	60%	60%	39%	50%	42%
No. CS	318	361	354	318	361	354	318	361	354	318	361	354	318	361	354
No. Contracts	3,698	4,944	5,170	3,698	4,944	5,170	3,698	4,944	5,170	3,698	4,944	5,170	3,698	4,944	5,170
Puts:															
Avg Return	12.7%	29.1%	28.6%	18.6%	37.7%	41.7%	-12.3%	6.1%	-7.6%	9.9%	23.6%	22.6%	-10.4%	5.1%	-6.7%
St. Dev.	7.2%	8.4%	6.2%	7.2%	8.5%	6.2%	6.7%	7.4%	5.2%	6.0%	7.0%	5.1%	5.6%	6.2%	4.3%
Info Ratio	1.76	3.45	4.62	2.57	4.45	6.75	-1.84	0.82	-1.45	1.66	3.38	4.44	-1.87	0.82	-1.55
H_0: OT $\not\succ_2$ IT	0.013	0	0	0.003	0	0	1	0.15	1	0.021	0	0	1	0.144	1
H_0: OT $\not\succ_2$ IT (5% Trim)	0.005	0	0	0	0	0	1	0.014	1	0.006	0	0	1	0.004	1
H_0: OT $\not\succ_2$ IT (10% Trim)	0	0	0	0	0	0	1	0	1	0	0	0	1	0	1
% CS Ret>0	47%	54%	55%	49%	56%	60%	40%	48%	42%	47%	54%	55%	40%	48%	42%
No. CS	332	362	353	332	362	353	332	362	353	332	362	353	332	362	353
No. Contracts	3,967	4,925	5,147	3,967	4,925	5,147	3,967	4,925	5,147	3,967	4,925	5,147	3,967	4,925	5,147

The table reports statistical properties of excess returns of option trader (OT) portfolio, including arithmetic average daily returns (annualized) and standard deviations. The row "% CS Ret>0" shows the percentage of weeks (trades) with positive portfolio returns. The returns in Panels A to C are computed when an OT writes an overpriced option at its bid quote and closes her position, respectively at the option upper bound, equilibrium price, and at the ask quote. Panels D and E reports similar results when OT is more risk averse. The trading strategy implement using 2% ATM options, end-of-week expiration, with non-zero volume in two consecutive days. Column DTM7 shows returns when the portfolio is set using options with four days-to-maturity on Monday and the positions are closed the next trading day. The OT portfolios are across all weeks in the sample and when there is no overpriced ATM option, OT=IT. The table also reports p-values for Davidson-Duclos (2007) second order stochastic dominance test for paired (correlated) outcomes.The row H_0: OT $\not\succ_2$ reports p-values for the null of non-dominance of time series of OT portfolio returns over time series of IT portfolio returns, with no trimming in the right tail (fourth row), 5% trimming in the right tail (fifth row), and 10% trimming in the right tail (six row).

Figure 1: Cumulative Excess Return OT Portfolio



This figure shows the cumulative return OT portfolio in excess of the index returns separately for calls (left panel) and puts (right panel) across all cross-sections. OT portfolio is set only with 2% ATM Weeklys, end-of-week expiration, with non-zero volume on any two consecutive trading days.

Short Term Market Risks Implied by Weekly Options: An Alternative Perspective

MICHAL CZERWONKO, HAMED GHANBARI, AND STYLIANOS PERRAKIS

Online Appendix

A. Proof of equation 2.1

The kernel $\frac{I_T^{-\varphi}}{E_t[I_T^{-\varphi}]}$ is obviously monotone decreasing in I_T, from which it follows that the implied upper boundary Q-distribution is given by (1.3). Since the return distribution is independent and identically distributed according to (1.1)' for every time partition Δt, we apply (1.3) recursively. Changing slightly the notation, we set $\frac{I_{t+\Delta t}}{I_t} = \exp(z_{D,t+\Delta t} + J)$, where $J = \ln(j)$ and $z_{D,t+\Delta t} \sim N[(\mu - \lambda\kappa)\Delta t, \sigma_t\sqrt{\Delta t}]$ is the diffusion component. Marginal analysis of borrowing \$1 and investing in the index should in equilibrium yield $\begin{pmatrix} E_t[I_{t+\Delta t}^{-\varphi}[\exp(z_{D,t+\Delta t} + J - r\Delta t)]] = 0 \Rightarrow \\ E_t\{\exp[-\varphi(z_{D,t+\Delta t} + J)]\exp(z_{D,t+\Delta t} + J)\} = e^{r\Delta t}E_t\{\exp[-\varphi(z_{D,t+\Delta t} + J)]\} \end{pmatrix}$. Since the two random terms are independent, the second line becomes

$$E_t\{\frac{\exp[(1-\varphi)z_{D,t+\Delta t}]}{E_t\{\exp(-\varphi z_{D,t+\Delta t})}\}E_t\{\frac{\exp[(1-\varphi)J)]}{E_t\{\exp(-\varphi J)\}}\} = e^{r\Delta t} =$$

$$\exp[(\mu - \lambda\kappa + \frac{\sigma_t^2}{2} - \varphi\sigma_t^2)\Delta t]E_t\{\frac{\exp[(1-\varphi)J)]}{E_t\{\exp(-\varphi J)\}}\}$$

from which we get (2.1) for $\Delta t \to 0$ after setting $\exp(-\varphi J) = j^{-\varphi}$, QED.

B. Proof that $\frac{\varphi_{t,T}^{\max}}{\varphi}$ is the highest fraction of option that can be shorted when $\varphi > \varphi_{t.T}^{\max}$

Let ζC denote the maximum quantity of a given overpriced option per unit index held in the IT portfolio that can be shorted by the OT investor at *t* to achieve SA. The return of this short option position, equal to $\zeta(1 + z_{t+\Delta t})$, is invested in proportions β_t^* and $1 - \beta_t^*$ respectively in the index and the riskless asset. As shown in the model free derivation of the SA bounds,[1] the overpriced option position must exceed the value of the expected return of the position with discretized risk neutral dynamics given by (1.3), with $\zeta(1 + z_{t+\Delta t})$ replacing $(1 + z_{t+\Delta t})$. At the continuous time limit of the jump diffusion, however, these risk neutral dynamics tend to (λ^Q, κ^Q) such that $\mu - r = \lambda\kappa - \lambda^Q\kappa^Q$. Since for $\zeta = 1$ and $\varphi > \varphi_{t.T}^{\max}$ this equation defines a higher SA upper bound than (2.2), the only way it can hold is if $\zeta\varphi = \varphi_{t.T}^{\max}$, QED.

[1] See Perrakis (2019, pp. 23-26).

C. Intermediate Market Structure

Under competitive conditions assume that an IT investor holds a portfolio with x_t in the riskless asset and a starting long position y_t in a futures contract, corresponding to at least one unit of the index at maturity *T'*. The SPXW option matures at *T* and the futures at some value *T'>T*, the nearest futures maturity time. At *T'* the IT liquidates her portfolio by maximizing a concave utility function. As an OT she adds an appropriate position in one call option at *t*, which is closed at *T*. The IT portfolio is being revised along the path from *t* to *T'*, with transaction costs 1+*k* for additions to the long and 1-*k* to the short futures positions, with no costs for the riskless asset.

In the presence of frictions continuous time IT portfolio revisions are infeasible. As Constantinides showed in his seminal 1979 article, there exists a no trade (NT) zone for the IT investor, with trades occurring only when the risky asset dynamics bring the value of the risky asset holdings outside the NT zone. Analytical expressions for the derivation of this NT zone and the corresponding optimal IT portfolio policy for a finite horizon *T'* are available only for CRRA utilities and diffusion or jump diffusion asset dynamics, with a numerical algorithm presented in Czerwonko and Perrakis (2016). If $V(x_t, y_t, t)$ denotes the IT value function and $W_{T'}$ the terminal wealth then $V(x_{T'}, y_{T'}, T') = U(w_{T'}) = U(x_{T'} + (1-k)y_{T'})$, which is equal to $w_{T'}^{1-\alpha}/(1-\alpha)$ under CRRA.

Hedging the IT position with futures is equivalent to setting $\frac{dI_t}{I_t} = \frac{dF_t}{F_t} + \alpha_t dt$ plus a random shock equal to the basis risk. To see this denote by I_t and F_t the underlying and futures prices at *t* and by Z_{t+1} the ex dividend return $Z_{t+1} = \frac{I_{t+1}e^{-q_t}}{I_t} = \frac{F_{t+1}e^{\psi(t,T')}}{F_t} \Rightarrow \ln(\frac{I_{t+1}e^{-q_t}}{I_t}) = \ln(\frac{F_{t+1}}{F_t}) + \psi(t,T')$, where q_t is the (assumed constant) dividend yield and $\psi(t,T')$ the basis risk, a zero mean independent error term that varies with the distance from futures maturity. At T' we have $I_{T'} = F_{T'}$ without error. It is clear from the above formulation that the dynamics of the index return can be very closely approximated by the dynamics of the futures return in our case, provided the diffusive volatility is increased to represent the basis risk. Hereafter the basis risk will be ignored in the expressions.

The derivation of the NT zone was done numerically for the CRRA class of utility functions and shown in Czerwonko and Perrakis (2016, equations A.8-A.9) to converge to a continuous time limit when the return $\ln(\frac{F_{t+1}}{F_t}) \equiv z_{t+1}$ converged for both diffusion and jump diffusion. Although we do not need the CRRA assumption for most of our results, we shall assume in what follows that the IT stays in the NT zone for the entire period to option expiration. This is probably a very good approximation for SPXW options, given that the costs for restructuring to the nearest NT boundary are very low in the CRRA case. In such a case at option expiration *T* the value function at *t* is $V(x_t, y_t, t) = E_t[V(x_t R^{T-t}, y_t z_T, T)]$.

Suppose there are $i=1,...,n$ IT investors, each one of whom adopts a long position in one call option with price C and set $m_t \equiv \frac{K}{F_t}$. Under competitive conditions that eliminate profits we have, if $J^i(x_t, y_t, t)$ denotes the corresponding OT value function for the ith IT investor that

$$V^i(x_t, y_t, t) = J^i(x_t - C, y_t, t) = E_t[V^i(x_t - C)R + (F_T - K)^+, y_T, T)]. \tag{3.1}$$

Maximizing, we get $F_t \frac{E_t[V_x^i(z_T - m_t)^+]}{RE_t[V_x^i]} = C$. Since we know that $V_x(T) \in [\frac{V_y(T)}{1+k}, \frac{V_y(T)}{1-k}]$ and that IT traders are marginal in the option market, marginal analysis shows that to a very close approximation we have for every trader $\frac{V_x^i(T)}{E_t[V_x^i(T)]} \approx \frac{V_y^i(T)}{E_t[V_y^i(T)]}$, which is clearly monotone decreasing in z_T. Hence, the term $\sum_i \frac{V_y^i(T)}{E_t[V_y^i(T)]}$ has the interpretation of a market pricing kernel, which is monotone decreasing in the return and is equal in equilibrium to the competitive ask price. An equivalent result holds for the short option. A monotone pricing kernel implies automatically that the risk neutralization of the discretized jump diffusion index futures dynamics in (1.1)' converges to an option value that lies within the two SA bounds and depends on the kernel aggregation. This value can be determined only for CRRA investors and with specific assumptions about their holdings in riskless asset and index futures. In such a case each point within the bounds corresponds to a state-varying partition of the risk premium into volatility and jump risk components, as shown in the previous section. We have thus shown that the SA bounds are consistent with perfect competition in the option market even in the presence of frictions in trading the underlying.

Such a competitive environment is also consistent with equal prices for long and short options and with a DPM constrained to be a passive liquidity provider. Let the market consist of competitive end users and a monopolistic liquidity provider. Assume that the total end user demand for a given put option is $N_s = D_s(P_b, P_a, t)$ for short and $N_l = D_l(P_b, P_a, t)$ for long, where the first is increasing in P_b and the second is decreasing in P_a. Assume also, as is reasonable, that $\frac{\partial D_S}{\partial P_a} \geq 0$ and $\frac{\partial D_l}{\partial P_b} \leq 0$. The total net position is $N_l - N_e \equiv N_e > 0$, where N_e is the put market's net exposure, which is on average positive in the OTM region in our data. Then the passive monopolistic market maker is fully hedged in $N_s = D_s(P_b, P_a, t)$ but must cover the net long end user demand with a short position in N_e, which is increasing in both P_b and P_a, but which she must cover at a price of P_b. Exactly the opposite occurs if the end users are net short and the DPM must cover it by purchasing at P_a.

Since this is clearly a suboptimal decision, we now allow the DPM to set freely the prices of the long and short options. Assume as above that the DPM faces a total demand that is net long, with a fully hedged component $N_s = D_s(P_b, P_a, t)$, and residual exposure $N_e(P_b, P_a, t)$, increasing in both arguments. We then have a t time t $\hat{x}_t = x_t + N_s(P_a - P_b) + N_e P_b$ and the following maximization problem $Max_{P_a,P_b} J_b\left(\hat{x}_t, y_t, t\right)$, where instead of (3.1) we have

$$\begin{aligned} &J_b\left(\hat{x}_t, y_t, t\right) = E_t[V(\hat{x}_T - N_e F_t(m_t - z_T)^+, y_T, T)] = \\ &E_t[V(R(x_t + N_s(P_a - P_b) + N_e P_b) - N_e F_t(m_t - z_T)^+, y_T, T)] \end{aligned}. \tag{3.2}$$

Maximizing $J_b\left(\hat{x}_t, y_t, t\right)$ with respect to P_a and P_b we get after simplification

$$\begin{aligned} &F_t \frac{E_t[V_x(m_t - z_T)^+]}{RE_t[V_x]} = \frac{1}{\frac{\partial N_e}{\partial P_a}}[N_s + \frac{\partial N_s}{\partial P_a}(P_a - P_b) + P_b \frac{\partial N_e}{\partial P_a}] \\ &F_t \frac{E_t[V_x(m_t - z_T)^+]}{RE_t[V_x]} = \frac{1}{\frac{\partial N_e}{\partial P_b}}[-N_s + \frac{\partial N_s}{\partial P_b}(P_a - P_b) + P_b \frac{\partial N_e}{\partial P_b}] \end{aligned}. \tag{3.3}$$

If P_a and P_b are constrained to be equal to P then it can be shown that the right-hand-side of both expressions in (3.3) is equal to $P(1+e_N^{-1})$, where $e_N > 0$ is the elasticity of residual demand $N_e(P,t)$. This appears perverse, since the DPM charges less than P for the short position, but on the other hand it reduces the fully hedged position, from which she receives no income. It also justifies partially the observed consistency of the option market data with the SA bounds at the left tail. Conversely, the elasticity is negative when the DPM exposure is net long, as it is in the ATM zone. In both cases the bid-ask spread raises the prices. In either case the equilibrium prices depend on the demand elasticity, varying inversely with $|e_N|$. They do not depend on the asset dynamics and are specific to the degree of moneyness. Hence, they can accommodate the differing distances between the SA bounds and the observed bid-ask midpoint in our results since there is no reason for the demand forces to be the same along the cross section.[2]

[2] In fact, AFT point out in several places (e.g., p. 1336) that the pricing rules differ between OTM and ATM options, as shown also in our data.

Table OA1: Jump Diffusion Parameter Estimates

Parameter	Estimate	Std Err	t Value	Approx Pr > |t|
μ	0.0707	0.0234	3.03	0.0025
σ	0.1497	0.0031	48.40	<.0001
μ_j	-0.0322	0.0164	-1.96	0.0497
σ_j	0.1007	0.0107	9.42	<.0001
λ	0.2625	0.1572	1.67	0.095

This table presents parameter estimates for the the jump diffusion model obtained from the GMM procedure using daily returns of the S&P 500 index from 1963 to 2010.

Table OA2: Summary Statistics Weeklys - All Cross Sections

	All	DTM1	DTM2	DTM3	DTM4	DTM7	DTM8	DTM9
Panel A: Summary Statistics Calls								
# Contracts	140,305	17,170	18,769	19,601	20,059	22,290	21,652	20,764
Avg IV	22.7%	32.0%	25.7%	22.6%	21.2%	21.4%	20.1%	19.0%
Avg Moneyness (K/S)	0.98	0.96	0.97	0.98	0.98	0.99	0.99	0.99
Avg Mid-Quotes	94.5	128.8	115.5	94.3	90.5	89.6	77.2	74.6
Relative BA Spread	8.4%	6.3%	7.6%	8.8%	10.0%	10.3%	7.9%	7.6%
Option Bounds Spread	10.6%	3.5%	6.2%	8.8%	10.8%	12.6%	14.5%	15.7%
Avg NetBuy MM	4	-13	-1	-0.2	0.2	25	9	3
% Volume MM	51%	51%	52%	52%	51%	50%	51%	50%
Total Volume	101,788	24,412	16,716	13,562	13,327	15,058	10,744	7,970
Total Open Interest	265,293	33,565	36,738	40,007	39,723	39,732	37,882	37,646
Panel B: Calls Mispricing vs. SA Bounds								
Mid-Quotes Within Bounds	33,128	3,687	4,495	4,985	5,345	5,430	4,776	4,410
	24%	21%	24%	25%	27%	24%	22%	21%
Overpriced Contracts	51,880	4,866	6,397	6,947	6,334	8,764	9,662	8,910
	37%	28%	34%	35%	32%	39%	45%	43%
Cross-sections >1 Overpriced	2,807	412	404	411	345	415	428	392
	90%	93%	90%	90%	81%	90%	93%	92%
Cross-sections >25% Overpriced	2,274	251	305	330	285	369	385	349
	73%	56%	68%	72%	67%	80%	83%	82%
Cross-sections >50% Overpriced	1,279	88	153	167	152	217	260	242
	41%	20%	34%	37%	36%	47%	56%	57%
Panel C: Summary Statistics Puts								
# Contracts	194,470	15,423	23,284	27,455	29,072	33,461	33,320	32,455
Avg IV	23.4%	24.5%	23.8%	23.1%	23.1%	23.9%	23.4%	22.4%
Avg Moneyness	0.98	1.01	0.99	0.98	0.98	0.97	0.97	0.96
Avg Mid-Quotes	37.2	60.2	45.2	34.4	38.9	34.6	30.8	30.4
Relative BA Spread	12.4%	10.1%	16.6%	16.6%	14.9%	11.4%	9.2%	9.2%
Option Bounds Spread	77.2%	39.4%	65.4%	75.4%	78.1%	84.4%	86.5%	87.6%
Avg NetBuy MM	5	-15	-13	-8	4	46	-2	1
% Volume MM	50%	51%	51%	50%	49%	48%	49%	49%
Total Volume	188,261	34,302	31,262	26,684	27,724	34,315	19,832	14,142
Total Open Interest	482,479	38,406	71,039	82,927	77,925	74,668	69,274	68,240
Panel D: Puts Mispricing vs. SA Bounds								
Mid-Quotes Within Bounds	75,122	2,774	7,404	10,314	11,793	14,122	14,136	14,579
	39%	18%	32%	38%	41%	42%	42%	45%
Overpriced Contracts	68,378	5,642	7,693	8,746	8,344	12,096	13,480	12,377
	35%	37%	33%	32%	29%	36%	40%	38%
Cross-sections >1 Overpriced	2,853	419	409	413	365	418	433	396
	91%	94%	91%	90%	85%	91%	94%	93%
Cross-sections >25% Overpriced	2,250	389	329	317	231	313	353	318
	72%	87%	73%	69%	54%	68%	76%	75%
Cross-sections >50% Overpriced	785	138	111	92	66	125	144	109
	25%	31%	25%	20%	15%	27%	31%	26%

This table presents summary statistics for the merged CBOE time-stamped End-of-Week Weekly option quotes data (EoW Weeklys) with the trading volume data from Market Data Express Open/Close database. Option quotes are at 3:00 PM from 20110317 to 20220317 for the total of 477 expiring Fridays and 3,127 observation days. The sample only contains options with non-zero trading volume and minimum bid price of 10 cents. Options are grouped based on the number of days-to-maturity (DTM), from 9 days to 1 day. "Relative BA Spread" ("Option Bounds Spread") is the average bid-ask (upper and lower bounds) spreads in proportion of mid-quotes (mid-bounds). "Avg NB MM" is the average number of contracts purchased minus sold by market makers. "Total Volume" and "Total Open Interest" are in thousands. "Cross-sections >25% Overpriced" shows number of cross sections where more than 25% of contracts in the cross section are overpriced.

Table OA3: Summary Statistics Weeklys - All Options

	<.90	.90-.93	.93-.96	.96-.98	.98-1.0	1.0-1.02	1.02-1.04	1.04-1.06	>1.06	All
Panel A: Summary Statistics Calls										
# Contracts	8,170	6,415	17,427	24,783	31,476	25,786	13,847	6,480	5,921	140,305
% Contracts	5.8%	4.6%	12.4%	17.7%	22.4%	18.4%	9.9%	4.6%	4.2%	100%
Avg IV	80.3%	38.8%	28.1%	20.9%	16.3%	14.6%	16.7%	20.7%	33.0%	22.7%
Avg Mid-Quotes	609.6	255.5	158.0	90.1	40.3	11.3	4.3	2.9	2.6	94.5
Relative BA Spread	1.5%	2.0%	2.9%	3.7%	3.6%	6.0%	17.9%	32.1%	49.8%	8.4%
Option Bounds Spread	0.1%	0.5%	0.9%	1.9%	5.0%	18.6%	30.3%	32.4%	26.0%	10.6%
Avg NetBuy MM	-0.1	1	-5	-14	4	24	26	1	-13	4
% Volume MM	62%	51%	51%	49%	51%	51%	50%	49%	51%	51%
Total Volume	127	123	609	1,759	22,052	57,979	13,826	3,455	1,859	101,788
Total Open Interest	724	1,842	10,029	28,984	85,911	91,711	29,024	9,844	7,224	265,293
Panel B: Calls Mispricing vs. SA Bounds										
Mid-Quotes Within Bounds	2,011	3,335	8,929	9,826	4,403	1,533	1,763	774	554	33,128
	25%	52%	51%	40%	14%	6%	13%	12%	9%	24%
Overpriced Contracts	334	542	2,372	7,350	20,298	15,149	3,668	1,029	1,138	51,880
	4%	8%	14%	30%	64%	59%	26%	16%	19%	37%
Cross-sections >1 Overpriced	83	134	478	1,340	2,733	2,456	769	156	76	2,807
	4%	7%	17%	43%	87%	80%	45%	20%	18%	90%
Cross-sections >50% Overpriced	44	117	327	858	2,146	2,236	644	134	55	1,279
	2%	6%	11%	28%	69%	73%	38%	17%	13%	41%
Cross-sections 100% Overpriced	22	106	234	593	1,330	2,015	550	117	47	26
	1%	5%	8%	19%	43%	66%	32%	15%	11%	1%
Panel C: Summary Statistics Puts										
# Contracts	10,262	26,944	38,921	29,219	31,201	28,978	13,793	6,103	9,049	194,470
% Contracts	5.3%	13.9%	20.0%	15.0%	16.0%	14.9%	7.1%	3.1%	4.7%	100%
Avg IV	41.4%	31.1%	25.2%	20.3%	16.3%	14.0%	18.4%	25.3%	47.9%	23.4%
Avg Mid-Quotes	2.7	1.7	2.7	5.5	12.5	37.6	86.5	145.2	368.0	37.2
Relative BA Spread	18.3%	25.6%	20.6%	11.2%	5.0%	5.1%	4.7%	3.6%	2.5%	12.4%
Option Bounds Spread	156.1%	148.8%	128.0%	98.2%	47.3%	2.6%	0.4%	0.2%	0.0%	77.2%
Avg NetBuy MM	3	-19	-4	45	0	12	-2	-3	-7	5
% Volume MM	47%	45%	48%	49%	52%	51%	49%	50%	48%	50%
Total Volume	3,214	13,533	33,585	47,364	71,141	16,902	1,556	534	433	188,261
Total Open Interest	14,630	64,515	125,974	108,641	102,448	40,460	12,743	5,704	7,364	482,479
Panel D: Puts Mispricing vs. SA Bounds										
Mid-Quotes Within Bounds	7,168	19,968	24,935	14,046	4,953	1,839	1,339	501	373	75,122
	70%	74%	64%	48%	16%	6%	10%	8%	4%	39%
Overpriced Contracts	2,865	4,667	9,869	13,550	22,405	13,045	1,383	327	267	68,378
	28%	17%	25%	46%	72%	45%	10%	5%	3%	35%
Cross-sections >1 Overpriced	204	388	902	1,832	2,765	2,468	301	55	25	2,853
	18%	18%	34%	63%	88%	79%	11%	4%	3%	91%
Cross-sections >50% Overpriced	189	319	628	1,332	2,505	1,512	118	34	3	785
	17%	15%	23%	46%	80%	48%	4%	2%	0%	25%
Cross-sections 100% Overpriced	178	289	470	979	1,864	497	75	25	3	4
	16%	13%	18%	34%	60%	16%	3%	2%	0%	0%

This table presents summary statistics for the merged CBOE time-stamped End-of-Week Weekly option quotes data (EoW Weeklys) with the trading volume data from Market Data Express Open/Close database. Option quotes observed at 3:00 PM from 20110317 to 20220317, expiring on Fridays. The sample only contains options with non-zero trading volume and minimum bid price of 10 cents. Options are grouped across different moneyness bins. “Relative BA Spread” (“Option Bounds Spread”) is the average bid-ask (upper and lower bounds) spreads in proportion of mid-quotes (mid-bounds). “Avg NB MM” is the average number of contracts purchased minus sold by market makers. “Total Volume” and “Total Open Interest” are in thousands. “Cross-sections >50% Overpriced" shows number of cross sections where more than 50% of contracts in the cross section are overpriced.

Table OA4: Bid-Ask Quotes vs Option Bounds - All Cross Sections

	<.90	.90-.93	.93-.96	.96-.98	.98-1.0	1.0-1.02	1.02-1.04	1.04-1.06	>1.06	All
Panel A: Call Mid-Quotes Within Option Bounds										
DTM1-Thu	193	348	1029	1318	626	128	38	6	1	3687
	13%	32%	36%	35%	14%	5%	6%	5%	1%	21%
DTM2-Wed	327	495	1243	1461	665	135	151	16	2	4495
	21%	45%	47%	40%	15%	4%	12%	4%	1%	24%
DTM3-Tue	268	514	1471	1602	616	200	210	53	51	4985
	24%	55%	58%	44%	13%	6%	12%	7%	9%	25%
DTM4-Mon	313	529	1487	1731	642	279	216	101	47	5345
	29%	61%	60%	50%	15%	7%	10%	10%	5%	27%
DTM7-Fri	405	622	1514	1438	621	248	358	137	87	5430
	34%	64%	56%	39%	14%	6%	14%	11%	7%	24%
DTM8-Thu	283	431	1181	1164	610	300	429	225	153	4776
	31%	57%	54%	34%	13%	7%	15%	16%	11%	22%
DTM9-Wed	222	396	1004	1112	623	243	361	236	213	4410
	29%	58%	49%	35%	14%	6%	13%	16%	15%	21%
Panel B: Call Overpricing Given Option Bounds										
DTM1-Thu	13	15	78	384	2,353	1,665	246	45	67	4,866
	1%	1%	3%	10%	52%	61%	41%	41%	87%	28%
DTM2-Wed	38	61	195	737	2,720	1,936	389	145	176	6,397
	2%	6%	7%	20%	61%	58%	30%	35%	66%	34%
DTM3-Tue	38	56	213	867	3,034	2,092	404	130	113	6,947
	3%	6%	8%	24%	66%	58%	23%	17%	19%	35%
DTM4-Mon	37	54	234	776	2,470	1,873	426	163	301	6,334
	3%	6%	9%	23%	57%	50%	19%	16%	33%	32%
DTM7-Fri	49	104	483	1,411	3,202	2,526	634	172	183	8,764
	4%	11%	18%	38%	70%	62%	26%	13%	14%	39%
DTM8-Thu	82	127	545	1,646	3,379	2,646	824	227	186	9,662
	9%	17%	25%	48%	74%	63%	30%	16%	14%	45%
DTM9-Wed	77	125	624	1,529	3,140	2,411	745	147	112	8,910
	10%	18%	30%	48%	71%	59%	27%	10%	8%	43%
Panel C: Put Mid-Quotes Within Option Bounds										
DTM1-Thu	0	0	624	1,160	668	172	109	23	18	2,774
	0%	0%	68%	53%	17%	4%	5%	2%	1%	18%
DTM2-Wed	0	973	2,866	2,346	786	201	141	39	52	7,404
	0%	76%	61%	55%	17%	5%	7%	4%	4%	32%
DTM3-Tue	236	2,255	4,110	2,403	760	271	191	58	30	10,314
	62%	70%	64%	52%	16%	6%	10%	7%	2%	38%
DTM4-Mon	801	3,079	4,123	2,388	706	339	210	91	56	11,793
	71%	67%	64%	55%	16%	8%	11%	10%	4%	41%
DTM7-Fri	1,637	4,387	4,642	2,104	699	274	233	90	56	14,122
	70%	74%	67%	45%	15%	6%	11%	9%	4%	42%
DTM8-Thu	1,935	4,619	4,383	1,804	680	310	220	106	79	14,136
	67%	77%	63%	39%	15%	7%	11%	13%	7%	42%
DTM9-Wed	2,559	4,655	4,187	1,841	654	272	235	94	82	14,579
	74%	79%	63%	41%	15%	7%	13%	13%	8%	45%
Panel D: Put Overpricing Given Option Bounds										
DTM1-Thu	9	35	222	905	2,883	1,460	114	14	0	5,642
	100%	100%	24%	41%	74%	36%	6%	1%	0%	37%
DTM2-Wed	50	148	817	1,592	3,206	1,649	157	41	33	7,693
	100%	12%	17%	37%	71%	40%	8%	5%	2%	33%
DTM3-Tue	136	403	988	1,853	3,354	1,810	151	32	19	8,746
	36%	13%	15%	40%	72%	43%	8%	4%	2%	32%
DTM4-Mon	296	627	1,197	1,582	2,763	1,502	193	67	117	8,344
	26%	14%	19%	37%	64%	38%	10%	8%	8%	29%
DTM7-Fri	608	1,169	1,997	2,392	3,373	2,194	270	55	38	12,096
	26%	20%	29%	51%	73%	51%	12%	6%	3%	36%
DTM8-Thu	888	1,213	2,371	2,708	3,555	2,355	293	69	28	13,480
	31%	20%	34%	58%	76%	56%	15%	8%	3%	40%
DTM9-Wed	878	1,072	2,277	2,518	3,271	2,075	205	49	32	12,377
	25%	18%	34%	56%	73%	52%	12%	7%	3%	38%

Panels A and C report number and percentage of option contracts with the bid-ask quotes between option bounds. Panels B and D report number and percentage of overpriced option contracts with the ask price above option upper bounds. Statistics are grouped based on the number of days-to-maturity (DTM), from 9 days to 1 day.

Table OA5: Volume and Net Buy Market Makers

	<.90	.90-.93	.93-.96	.96-.98	.98-1.0	1.0-1.02	1.02-1.04	1.04-1.06	>1.06	All
Panel A: Call Percentage Volume MM and Total Volume (x1000)										
DTM1-Thu	77%	48%	52%	48%	51%	52%	51%	49%	53%	51%
	39	24	104	365	6,512	15,352	1,782	163	71	24,412
DTM2-Wed	51%	52%	48%	50%	51%	52%	51%	50%	50%	52%
	15	16	96	286	3,949	9,944	1,931	371	108	16,716
DTM3-Tue	58%	48%	52%	49%	52%	52%	50%	51%	51%	52%
	18	18	84	258	3,048	7,739	1,766	410	221	13,562
DTM4-Mon	54%	56%	49%	47%	52%	51%	49%	47%	50%	51%
	12	13	103	269	2,400	7,351	2,177	666	337	13,327
DTM7-Fri	60%	52%	52%	51%	51%	50%	50%	46%	50%	50%
	16	17	107	259	2,562	7,871	2,790	926	509	15,058
DTM8-Thu	57%	51%	53%	50%	52%	50%	51%	51%	54%	51%
	19	22	59	189	2,016	5,653	1,864	559	364	10,744
DTM9-Wed	53%	50%	51%	54%	51%	50%	50%	50%	51%	50%
	9	13	55	133	1,565	4,070	1,515	361	249	7,970
Panel B: Call Net Buy MM										
DTM1-Thu	-2	0	-2	-27	-21	-4	17	-122	2	-13
DTM2-Wed	0	2	-4	-12	14	30	-55	-77	-80	-1
DTM3-Tue	1	0	-2	-19	9	42	-3	-68	-108	0
DTM4-Mon	0	2	-5	-18	4	23	8	-26	-22	0
DTM7-Fri	2	3	-8	-10	14	37	112	87	8	25
DTM8-Thu	0	2	-9	-6	7	29	22	18	-5	9
DTM9-Wed	0	1	-6	-7	-3	7	24	-8	16	3
Panel C: Put Percentage Volume MM and Total Volume (x1000)										
DTM1-Thu	52%	53%	52%	51%	52%	52%	48%	49%	47%	51%
	10	60	1,078	6,000	21,566	5,070	361	107	50	34,302
DTM2-Wed	55%	50%	49%	50%	52%	52%	49%	51%	47%	51%
	18	563	4,367	9,264	13,738	2,947	253	51	62	31,262
DTM3-Tue	54%	47%	48%	49%	52%	52%	48%	49%	44%	50%
	87	1,021	5,288	7,738	9,834	2,406	177	79	56	26,684
DTM4-Mon	50%	44%	48%	48%	52%	52%	50%	50%	50%	49%
	387	2,338	6,878	7,737	8,007	1,973	253	81	70	27,724
DTM7-Fri	48%	43%	47%	49%	50%	50%	50%	50%	49%	48%
	1,058	5,209	8,337	8,496	8,691	2,143	215	80	87	34,315
DTM8-Thu	45%	44%	49%	48%	52%	51%	52%	53%	52%	49%
	799	2,513	4,370	4,919	5,556	1,389	164	74	47	19,832
DTM9-Wed	44%	46%	48%	50%	51%	51%	49%	46%	45%	49%
	855	1,829	3,268	3,211	3,749	974	134	62	61	14,142
Panel D: Put Net Buy MM										
DTM1-Thu	-393	-126	-26	104	-66	-25	-15	-10	-15	-15
DTM2-Wed	36	-90	-102	35	16	18	1	-8	-4	-13
DTM3-Tue	-34	-26	-51	78	-28	2	-3	-8	-8	-8
DTM4-Mon	3	-44	11	65	2	-4	-6	-8	-6	4
DTM7-Fri	35	-21	96	84	72	44	8	3	-10	46
DTM8-Thu	-13	9	0	-27	-17	28	11	-3	-3	-2
DTM9-Wed	-1	-8	-9	10	10	16	-12	15	-6	1

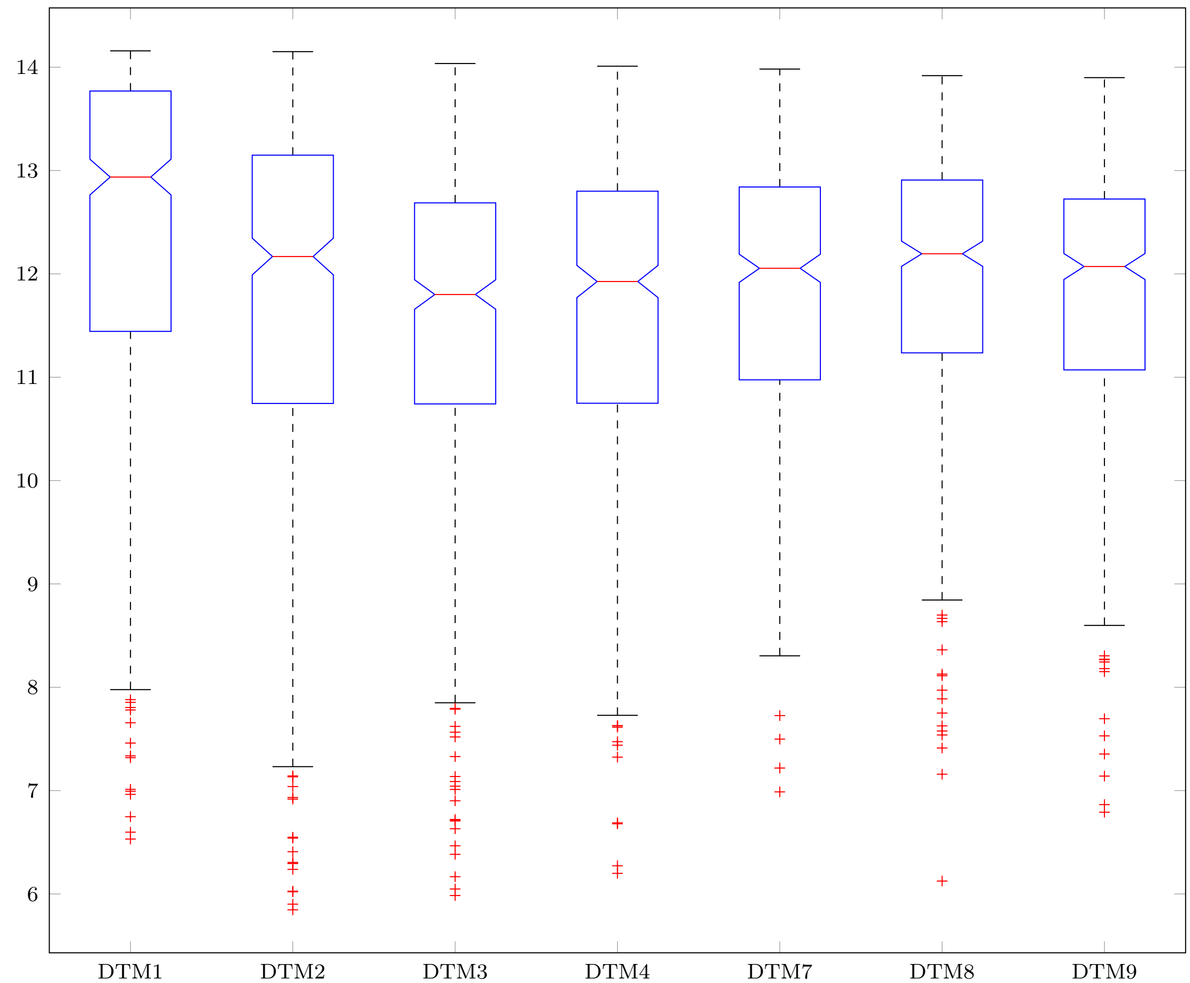
Panels A and C report percentage of volume by market markers and the total trading volume for calls and puts. Panels B and D reports the average net buy by market makers. Options are grouped based on the number of days-to-maturity (DTM), from 9 days to 1 day and statistics are provided across moneyness bins. The option data are based on the merged CBOE time-stamped End-of-Week Weekly option quotes data (EoW Weeklys) with the trading volume data from Market Data Express Open/Close database. Option quotes are at 3:00 PM from 20110317 to 20220317 for the total of 477 expiring Fridays and 3,127 observation days. The sample only contains options with non-zero trading volume and minimum bid price of 10 cents.

Table OA6: Return Characteristics Option Trader Portfolios

	Panel A: Open @Bid - Close @UB						Panel B: Open @Bid - Close @Ask						Panel C: Open @Midpoint - Close @Midpoint					
	DTM2	DTM3	DTM4	DTM7	DTM8	DTM9	DTM2	DTM3	DTM4	DTM7	DTM8	DTM9	DTM2	DTM3	DTM4	DTM7	DTM8	DTM9
Calls:																		
Avg Return	28.7%	16.6%	17.2%	23.7%	29.4%	25.9%	7.1%	-8.7%	-10.0%	4.1%	-2.4%	-7.5%	13.2%	-3.0%	-4.6%	9.9%	3.8%	-2.5%
St. Dev.	6.8%	7.0%	5.9%	6.8%	6.2%	5.2%	6.7%	6.9%	6.0%	6.4%	5.8%	4.9%	6.5%	6.7%	5.9%	6.4%	5.7%	4.8%
Info Ratio	4.20	2.36	2.91	3.47	4.78	4.95	1.06	-1.26	-1.66	0.64	-0.41	-1.54	2.02	-0.45	-0.78	1.55	0.66	-0.53
H_0: OT $\not\succ_2$ IT	0	0.004	0.001	0	0	0	0.082	1	1	0.206	1	1	0.004	1	1	0.044	0.189	1
H_0: OT $\not\succ_2$ IT (5%)	0	0	0	0	0	0	0.010	1	1	0.011	1	1	0.001	1	1	0	0.009	1
H_0: OT $\not\succ_2$ IT (10%)	0	0	0	0	0	0	0	1	1	0	1	1	0	1	1	0	0	1
% CS Ret>0	62%	62%	52%	60%	64%	60%	52%	48%	39%	50%	50%	42%	56%	51%	41%	53%	53%	44%
No. CS	382	387	318	361	399	354	382	387	318	361	399	354	382	387	318	361	399	354
No. Contracts	3,868	4,451	3,698	4,944	5,615	5,170	3,868	4,451	3,698	4,944	5,615	5,170	3,868	4,451	3,698	4,944	5,615	5,170
Puts:																		
Avg Return	22.6%	13.9%	12.7%	29.1%	30.3%	28.6%	2.7%	-9.3%	-12.3%	6.1%	-4.7%	-7.6%	7.2%	-5.3%	-8.5%	11.4%	0.7%	-3.2%
St. Dev.	7.7%	8.3%	7.2%	8.4%	7.7%	6.2%	7.0%	7.5%	6.7%	7.4%	6.6%	5.2%	7.1%	7.6%	6.7%	7.6%	6.7%	5.3%
Info Ratio	2.95	1.68	1.76	3.45	3.94	4.62	0.38	-1.23	-1.84	0.82	-0.71	-1.45	1.02	-0.70	-1.26	1.50	0.10	-0.61
H_0: OT $\not\succ_2$ IT	0	0.019	0.013	0	0	0	0.308	1	1	0.149	1	1	0.108	1	1	0.027	0.445	1
H_0: OT $\not\succ_2$ IT (5%)	0	0	0.005	0	0	0	0.133	1	1	0.014	1	1	0.024	1	1	0	0.099	1
H_0: OT $\not\succ_2$ IT (10%)	0	0	0	0	0	0	0.040	1	1	0	1	1	0.001	1	1	0	0.006	1
% CS Ret>0	55%	54%	47%	54%	59%	55%	49%	47%	40%	48%	49%	42%	51%	47%	41%	50%	51%	43%
No. CS	383	387	332	362	404	353	383	387	332	362	404	353	383	387	332	362	404	353
No. Contracts	4,158	4,879	3,967	4,925	5,712	5,147	4,158	4,879	3,967	4,925	5,712	5,147	4,158	4,879	3,967	4,925	5,712	5,147

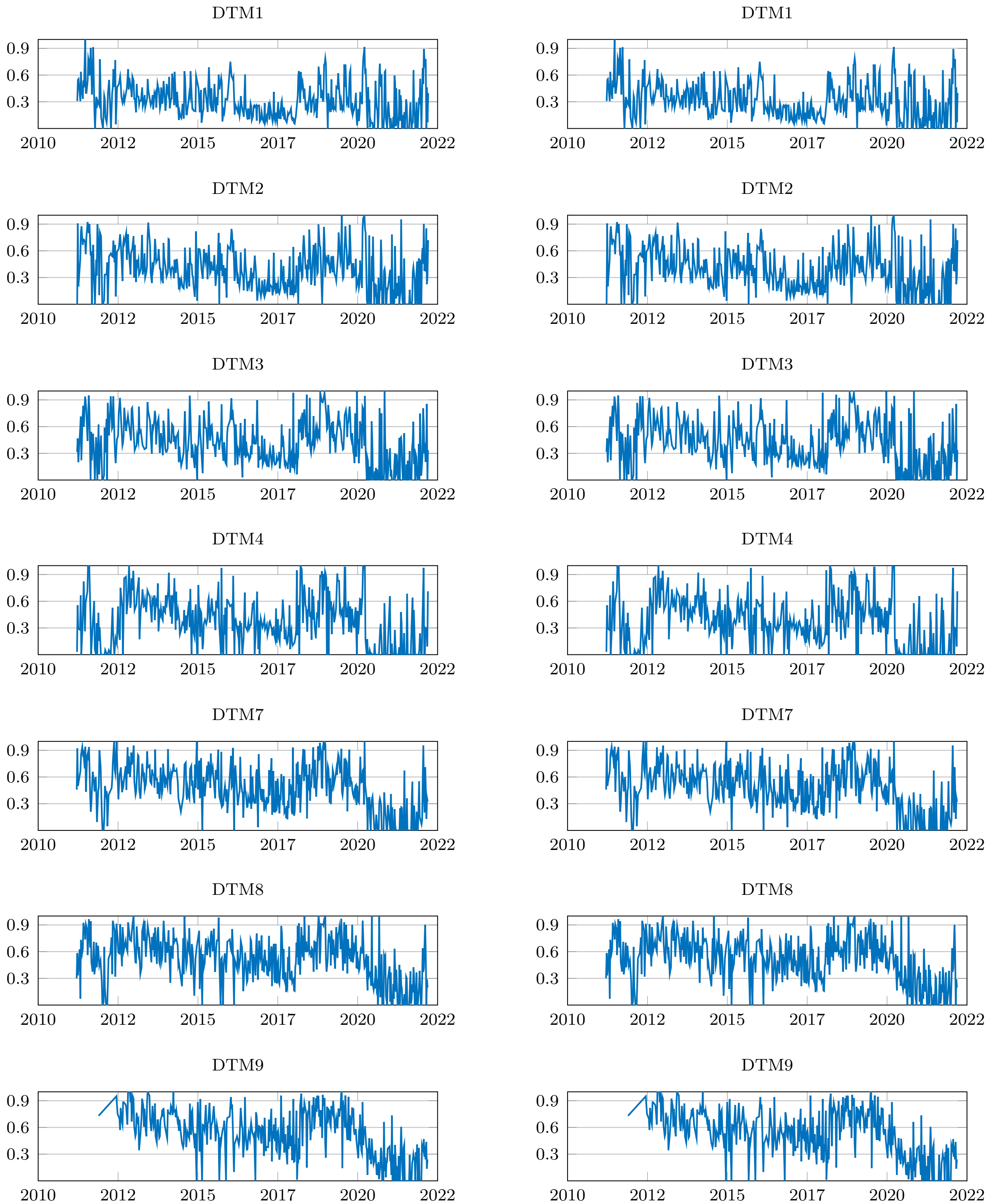
The table reports statistical properties of excess returns of option trader (OT) portfolios, including arithmetic average daily returns (annualized) and standard deviations. The row "% CS Ret>0" shows the percentage of weeks (trades) with positive portfolio returns. The returns in Panels A to B are computed when OT writes an overpriced option at its bid quote and closes her position at the option upper bound and at the ask quote. Panels C reports similar results when OT open and close her position at the midpoint of bid and ask quotes. The trading strategy is implemented by using 2% ATM options, end-of-week expiration, with non-zero volume in two consecutive days. Column DTM2 shows statistics when the portfolio is set using options with two days-to-maturity on Wednesday and the positions are closed at the next trading day. The OT portfolios are across all weeks in the sample and when there is no overpriced ATM option, OT=IT. The table also reports p-values for Davidson-Duclos (2007) second order stochastic dominance test for paired (correlated) outcomes.The row H_0: OT $\not\succ_2$ reports p-values for the null of non-dominance of time series of OT portfolio returns over time series of IT portfolio returns, with no trimming in the right tail (fourth row), 5% trimming in the right tail (fifth row), and 10% trimming in the right tail (six row).

Figure OA1: SA Implied Upper Bound RRA



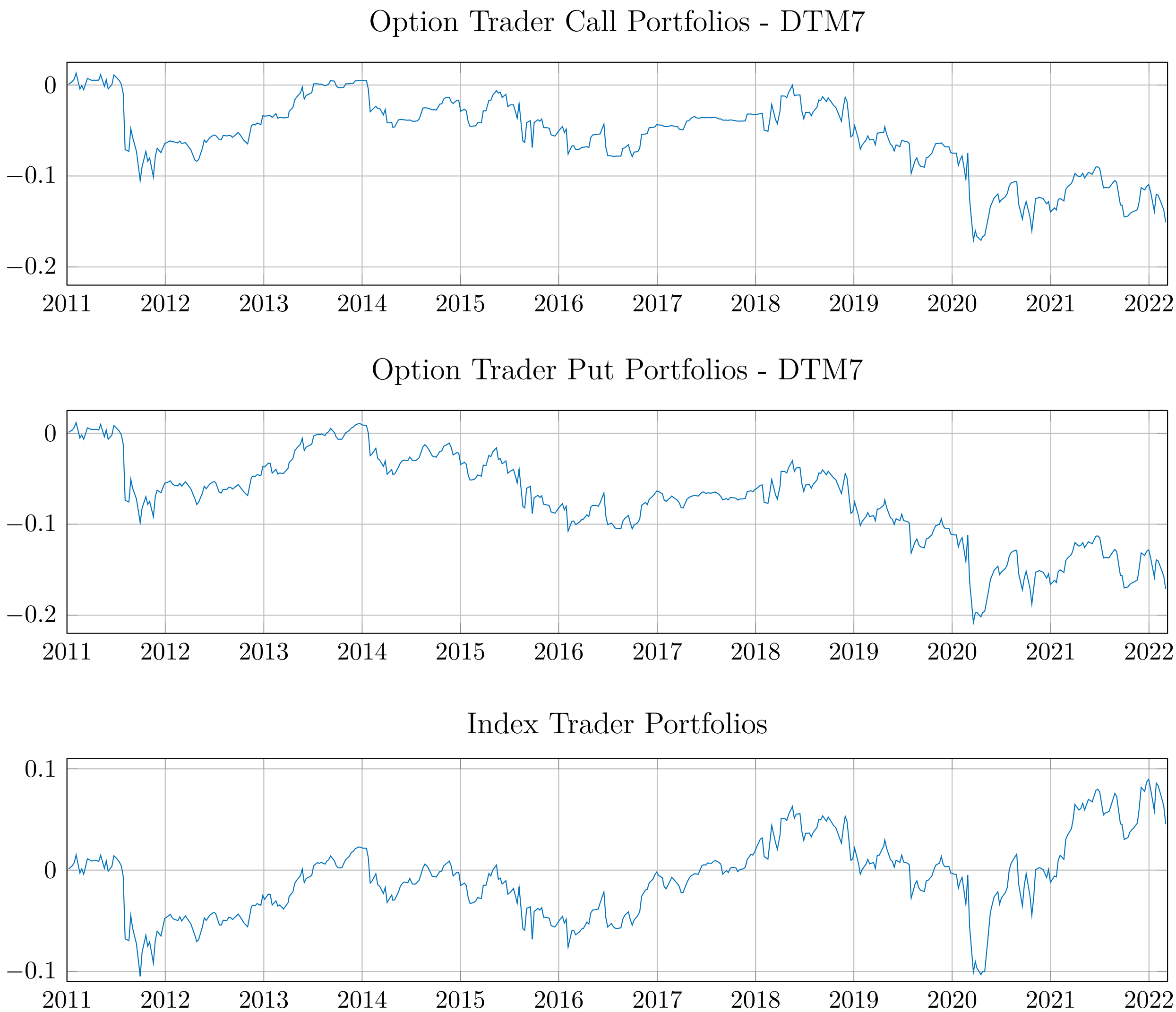
This figure shows the distribution of relative risk aversions implied by stochastic arbitrage upper bounds on option prices over the entire sample. The implied RRAs are reported in groups based on number of day-to-maturity. The results are obtained for weekly options, end-of-week expiration, with nonzero volume in two consecutive trading days.

Figure OA2: Percentage Overpriced Options



This figure shows the percentage overpriced call (left panel) and put (right panel) contracts with respect to the stochastic arbitrage upper bounds on option prices. The statistics are obtained for weekly options, end-of-week expiration, with nonzero trading volume in two consecutive trading days.

Figure OA3: 1-day Log-Return - All but Overpriced Options



The figure shows cumulative one-day returns for an option trader portfolio when OT trades all but overpriced options identified by the SA approach. The trading strategy is implemented by using 2% ATM options, end-of-week expiration, with non-zero volume in two consecutive days. The bottom panel plots the cumulative return of an index trader.