Common Idiosyncratic Quantile Risk*

Jozef Baruník[a,b], and Matěj Nevrla[a,b†]

[a] Institute of Economic Studies, Charles University,
Opletalova 26, 110 00, Prague, Czech Republic

[b] The Czech Academy of Sciences, Institute of Information Theory and Automation
Pod Vodarenskou Vezi 4, 182 00, Prague, Czech Republic

August 30, 2022

Abstract

We propose a new model of asset returns with common factors that shift relevant parts of the stock return distributions. We show that shocks to such non-linear common movements in the panel of firm's idiosyncratic quantiles are priced in the cross-section of the US stock returns. Such risk premium is not subsumed by the common volatility, tail beta, downside beta, as well as other popular risk factors. Stocks with high loadings on past quantile risk in the left tail earn up to an annual five-factor alpha 7.4% higher than stocks with low tail risk loadings. Further, we show that quantile factors have predictive power for aggregate market returns.

Keywords: Cross-section of asset returns, factor structure of asset returns, idiosyncratic risk, quantiles, asymmetric risk
JEL: C21; C58; G12

*We appreciate helpful comments from participants at various seminars, workshops and conferences. Support from the Czech Science Foundation under the 19-28231X (EXPRO) project is gratefully acknowledged.

†Corresponding author, Tel. +420(734)593183, Email address: matej.nevrla@fsv.cuni.cz

1 Introduction

The question how relevant the information contained in various parts of the return distribution is for an investor has received considerable attention in the recent empirical asset pricing literature (Ang et al., 2006; Van Oordt and Zhou, 2016; Chabi-Yo et al., 2018; Lu and Murray, 2019) with few studies focusing on commonalities in tails or extremes in the cross-section of returns (Kelly and Jiang, 2014; Chabi-Yo et al., 2022). The goal of this paper is to explore the common, possibly non-linear movements in the panel of firm's idiosyncratic quantiles and investigate the effects of such time-varying quantile risk in asset markets. The major barrier to the investigation is that such common factors shifting relevant parts of return distributions are hard to be observed from data. The further we aim to explore the information in distribution away from the mean, the more infeasible the risk estimates are because of the infrequent nature of the events.

To estimate risks stemming from commonalities in the panel of return quantiles, we propose to use an approximate factor model being able to extract unobservable factors in different parts of observable panel of stock returns. In the spirit of popular principal component analysis that recovers the conditional mean, we work with more general quantile factor models (QFM) being flexible enough to capture quantile-dependent objects that standard tools are unable to retrieve. In contrast to the standard principal component analysis, the quantile factor models are able to capture hidden factors that shift characteristics of the distribution such as moments or quantiles. Moreover, these factors can vary across the distribution of each unit in the panel and allow to infer the factors properly when distributions of the idiosyncratic errors exhibit heavy tails.

Our main contribution is to investigate the pricing implications of the common non-linear factors, which are quantile specific, for the predictability of aggregate market return and the cross-section of stock returns. We are interested in factors for the whole distribution to identify the risk premium associated with both downside (or tail) risk and upside potential. To this end, we employ the quantile factor model of Chen et al. (2021) and investigate the pricing implications of quantile-dependent factors while, at the same time, controlling for various linear factors and exposures to them. Our goal is also motivated by the increasing evidence of the non-linearities present in the stock markets.[1] We aim to show that the common quantile risk present in the stock return data is not spanned by the common volatility risk and posses strong information for both cross-section of asset returns and time-series predictability of the equity premium.

[1] E.g., Amengual and Sentana (2020) report nonlinear dependence structure in short-term reversal and momentum. Ma et al. (2021) show that many firm-level characteristics posses complex relation to the returns with respect to quantiles.

We start by showing an extraordinary degree of comovement among idiosyncratic quantiles in the Center for Research in Security Prices (CRSP) stocks over a long sample spanning 1926 - 2015. A single factor explains up to 17% of the time variation in firm-level idiosyncratic quantile risk that is unrelated to common volatility structure. Further, we show that quantile factors have predictive power for aggregate market returns. Predictive regressions show that a one-standard-deviation increase in quantile risk predicts a statistically significant increase in annualized excess market returns of up to 7.676% in case of left tail. We also document predictive power of upper tail factor with smaller effect up to 3.985% increase in annualized returns hence the effect is asymmetric. These results hold out-of-sample, they are stronger for the left tail, and are robust to controlling for a broad set of popular predictors surveyed by Welch and Goyal (2007) as well as tail risk (Kelly and Jiang, 2014), common volatility risk (Herskovic et al., 2016) as well as variance risk premium (Bollerslev et al., 2009).

Further, we find that idiosyncratic quantile risk has substantial predictive power for the cross-section of average returns. We show that stocks with high loadings on past quantile risk in the left tail earn up to an annual five-factor alpha 7.405% higher than stocks with low tail risk loadings for 0.2 quantile. This risk premium is not subsumed by other common priced factors such as common volatility, tail, downside risk, as well as other popular risk factors. Investors thus possess a strong tail-risk aversion concerning the common movements of the idiosyncratic returns. On the other hand, the absence of the risk premium related to the factors for the upper quantiles suggests that investors are not upside-potential seekers. Both these results are consistent with the literature investigating the effect of asymmetric dependencies on asset prices.

Our work is connected to several strands of the literature. The first relates to the factor-based asset pricing models that are highly popular in the empirical asset pricing literature. In these models, only common return factors are valid candidate pricing factors, and sensitivities to those factors determine the risk premium associated with an asset (Ross, 1976). This strand of literature yields highly successful and popular results focusing on the parsimonious models (Fama and French, 1993), as well as exploration of statistically motivated latent factors.[2] Recently, Kelly et al. (2019) introduced instrumented principal component analysis, which enables to flexibly model the latent factors with time-varying loadings using the observable characteristics.[3]

[2]This approach dates back to Chamberlain and Rothschild (1983) and Connor and Korajczyk (1986). For a comprehensive overview of machine learning methods applied to asset pricing problems such as measuring expected returns, estimating factors, risk premia, or stochastic discount factor, model selection, and corresponding asymptotic theory, see Giglio et al. (2022).

[3]Other notable recent contributions to the factor literature are, e.g., Kozak et al. (2018) and Giglio

Our research agenda spans the non-linear factor models. Recently, Ma et al. (2021) introduced a semi-parametric quantile factor panel model that considers stock-specific characteristics, which may non-linearly affect stock returns in a time-varying manner. They find that many characteristics possess a non-linear effect on stock returns. In contrast to these authors, the approach used in our paper is more general since it allows not only loadings but also factors to be quantile-dependent. Moreover, our approach does not require the loadings to depend on observables and has direct relation of the approximate factor models that are ubiquitous in the finance literature.

The second strand of literature we contribute to is exploring the idiosyncratic risk and its pricing implications for the cross-section of asset returns. More specifically, we contribute to the literature interested in idiosyncratic risk that co-moves across assets and hence explores common trends not contained in the first-moment type factors. The bulk of this research is motivated by introducing the idiosyncratic volatility puzzle proposed by Ang et al. (2006). Unfortunately, all existing explanations of the anomaly via lottery preference-based, market frictions-based, or others account[4] only for 29-54% of the puzzle using individual stocks (Hou and Loh, 2016).

The third line of thought that we take into account deals with asymmetric properties of the systematic risk and how they are incorporated into asset prices. Interest in those kinds of models was reignited by Ang et al. (2006) and their introduction of downside beta, which captures covariance between asset and market return conditional on the market being below some threshold value. Bollerslev et al. (2021) further disentangle traditional market beta into semibetas characterized by the signed covariation between the market and asset return. They show that only the semibetas associated with the negative market and negative asset return predict significantly higher future returns. Bali et al. (2007) showed a significant cross-sectional relationship between hedge fund returns and value at risk. Similarly, Huang et al. (2012) discovered the cross-sectional effect of extreme downside risk, estimated using extreme value theory, on the returns of single stocks. Instead of using conditional mean linear models for predicting the equity premium, Meligkotsidou et al. (2014) embraced the quantile regression approach and,

et al. (2021). The recent availability of high-frequency return data also motivated the development of continuous-time factor models.Aït-Sahalia et al. (2020) proposed a generalization of the classical two-pass Fama-MacBeth regression from the classical discrete-time factor setting to a continuous-time factor model and enables uncovering complex dynamics such as jump risk and its role in the expected returns.

[4]For comprehensive list of references belonging to each of the category, see Hou and Loh (2016). The only exception to this observation is the lottery-based explanation using highest realized return from the previous month proposed by Bali et al. (2011) and confirmed in the European markets by Annaert et al. (2013). But Hou and Loh (2016) argue that this explanation is not valid, as it is an almost perfect collinear range-based measure of idiosyncratic volatility.

using predictions of the whole distribution of the equity premium, robustly estimated the equity premium using the forecast combination methodology.

From a theoretical standpoint, there are many justifications for the deviation from the classical common factor pricing theory to the asymmetric forms of the utility function. Probably the most relevant to our work is the dynamic quantile decision maker of de Castro and Galvao (2019) who decides based on quantile dependent preferences. Barro (2006), building on Rietz (1988), introduced the rare disaster model and showed that tail events may possess significant ability to explain various asset pricing puzzles, such as the equity premium puzzle. The other prevalent model that considers asymmetric features of the risk is the generalized disappointment aversion model of Routledge and Zin (2010), which inherently assumes downside aversion of the investors. Based on these preferences, Farago and Tédongap (2018) introduced an intertemporal equilibrium asset pricing model and showed that the disappointment-related factors should be priced in the cross-section. Moreover, they prove that their model performs well empirically by jointly pricing various asset classes with significant prices of risk associated with the disappointment-related factors.

There are also attempts to combine the two or three of these research agendas. Herskovic et al. (2016) introduced a risk factor based on the common volatility of the idiosyncratic firm-level returns and showcased its pricing abilities for the cross-section of various asset classes. Kelly and Jiang (2014) show that a zero-cost portfolio sorted on exposure to the tail risk, which is built from the dynamic power law structure, earns significant 5.4% three-factor alpha. Similarly, Allen et al. (2012) proposed an aggregate tail risk measure constructed from the returns of financial sector firms capturing catastrophic risk exposure. Based on the conditional ICAPM framework, they argue that it should be priced and estimate a significantly positive market price of this systemic risk measure for both financial and non-financial firms. Bali et al. (2008) discovered time-series predictability of stock market returns using non-linear mean reversion using extreme daily returns. Jondeau et al. (2019) presented significant time-series predictability of average skewness for the market return. Finally, Renault et al. (2019) extended the arbitrage pricing theory (APT) to the case of pricing of squared returns.

Many research efforts that investigate common tail risk and its asset pricing implications rely on option data. They argue that the tail factor identifies additional information over the volatility factor. Andersen et al. (2020) show strong predictive power for future equity risk premiums in the U.S. and European equity-index derivatives. Bollerslev and Todorov (2011) combine high-frequency data and option data and use non-parametric approach to conclude that a large portion of the equity and variance risk premia is linked to the jump tail risk.

The rest of the paper is structured as follows. Section 2 motivates the study of the pricing implications of the common movements in idiosyncratic distribution of stock returns. Section 3 proposes the quantile factor model for asset returns, discusses the methodology of the estimation of quantile-specific factors, and data that we use. Section 4 presents the results regarding the time-series predictability of the aggregate market return using the common idiosyncratic quantile factors. Section 5 investigates the cross-sectional asset pricing implications of the proposed factors. Section 6 concludes.

2 The factor structure in the cross-section of return distributions

Researchers usually assume that time variation in equity returns can be captured by relatively small number of common factors with following structure[5]

$$r_{i,t} = \alpha_i + \beta_i^\top F_t + \epsilon_{i,t} \tag{1}$$

where $r_{i,t}$ is excess return of an asset $i = 1, \ldots, N$ at time $t = 1, \ldots, T$, F_t is a $k \times 1$ vector of common factors and β_i is a $k \times 1$ vector of the asset's i exposures to the common factors. Such cross-sectional regressions as the one in (1) yielding high R^2 are used to identify factors serving as good proxies for aggregate risks present in the economy. Exposures to the relevant factors captured by β_i coefficients should be compensated in the equilibrium and explain the risk premium of the assets

$$\mathbb{E}_t[r_{i,t+1}] = \beta_i^\top \lambda_t \tag{2}$$

where the λ_t is a $k \times 1$ vector of prices of risk associated with factor exposures. Importantly, while the arbitrage pricing theory (APT) of Ross (1976) suggests that any common return factors F_t are valid candidate asset pricing factors, the idiosyncratic return residuals $\epsilon_{i,t}$ are assumed not to be priced. This implication is due to many simplifying assumptions, such that an average investor can perfectly diversify her portfolio or that the linear model (1) is correctly specified.

While large literature have focused mainly on the diversification assumption, we aim to question linear nature of the factor model, and our focus is on exposure to parts of

[5]Recently, Lettau and Pelger (2020) introduce Risk-Premium Principal Component Analysis that allows for systematic time-series factors incorporating information from the first and second moment.

idiosyncratic return's distribution instead. Recently, Herskovic et al. (2016) documents strong comovement in idiosyncratic volatility that does not arise from omitted factors, and even after saturating the factor regression with up to ten principal components, residuals that are virtually uncorrelated display same co-movement seen in raw returns.

While the exposure to common movements in volatility seem to carry strong pricing implications, we ask if there exist additional structure insufficiently captured by volatilities especially in a non-linear and heavy tailed financial data. In other words, we ask if various parts of the return distributions may have pricing implications for the cross-section of stock returns.[6]

To motivate the discussion, we first look at the τth quantile of the US public firms' returns $r_{i,t}$ represented by its inverse probability distribution function

$$\mathbb{Q}_\tau\left(r_{i,t+1}\right) = \inf\{r_i : P(r_{i,t+1} < r_i) \geq \tau\}]. \tag{3}$$

We first estimate 0.1 quantile of firm-level monthly returns using data from CRSP from 1926 to 2015 as sample cross-sectional quantile $\widehat{\mathbb{Q}}_\tau\left(r_{i,t+1}\right)$. Specifically, we compute the sample 0.1 return quantile for each stock i in the sample within the calendar year. The average of the cross-sectional quantiles is depicted by black line in Figure 1. Next, we also compute average idiosyncratic return quantiles that are calculated as average of the sample quantiles of $\epsilon_{i,t}$ from the Equation 1 (we use the three factor model[7] of Fama and French (1993)) and are depicted in the Figure 1 by dashed line. Here we note that it is remarkable how close the sample average quantile of the returns and its idiosyncratic part are. As argued by Herskovic et al. (2016) in case of common volatility, the similarity could be attributed to some important factors being omitted in the regression. We argue that this is likely not the case since removing the factors almost perfectly eliminates all the linear dependencies between the assets. This is clear from the Figure 6 in the Appendix A, where we plot average yearly pairwise correlations for the raw and idiosyncratic returns and observe that there is essentially no linear dependence left after removing the factors.[8] This suggests that the common linear factors do not explain the extreme events.

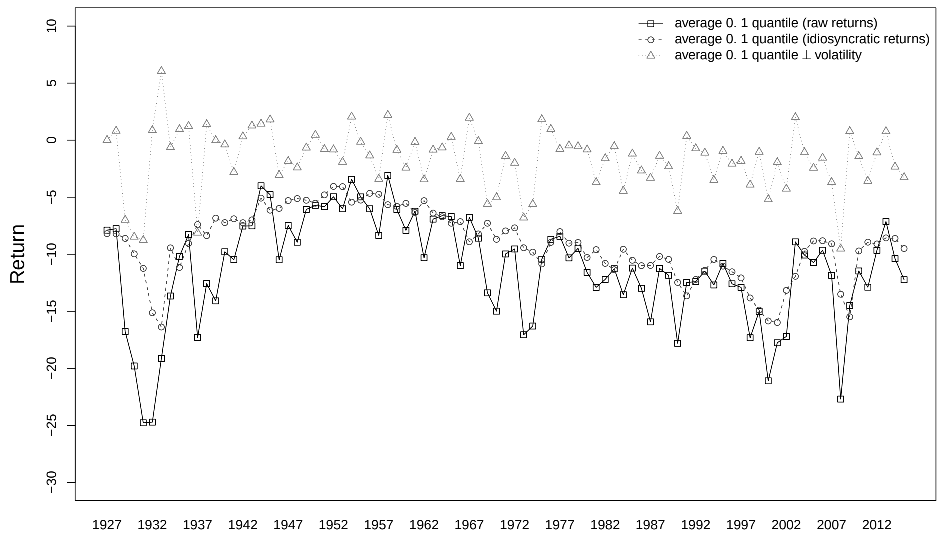
It is important to realize that this phenomenon may simply be driven by the common movements in volatility, especially if the first two moments satisfactorily describe the return distributions. Hence, we regress the time series of average idiosyncratic quantiles

[6]Ando and Bai (2020) document that the common factor structures explaining the upper and lower tails of the asset return distributions in global financial markets have become different since the subprime crisis.

[7]Robustness tests using purely statistical model based on principal components produce qualitatively similar results.

[8]For more details on this observation, see discussion in Herskovic et al. (2016)

Figure 1: *Average 0.1 sample quantile of stock returns.* The figure shows average 0.1 sample quantile obtained from the monthly data of returns of CRSP stocks during calendar year in black, average idiosyncratic 0.1 quantile obtained from the residuals of three factor Fama and French (1993) model as dashed line and average 0.1 idiosyncratic quantile orthogonalized by common volatility component in dotted line. The time span covers the period between January 1926 and December 2015.



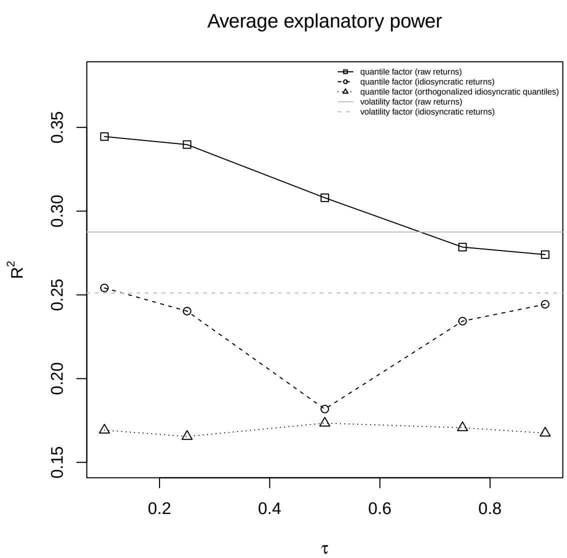
on the average volatility and plot the residuals by the dotted line in Figure 1. Although the common volatility captures substantial variation in the average idiosyncratic quantile, there is still some non-trivial variation left that we aim to explore.

We next estimate factor regression models for firm-level quantiles as well as volatility to see how strong the factor structure in the quantiles is, especially in comparison to factor structure in volatility. Figure 2 shows the average R^2 of time-series regressions of individual stock-level quantiles (volatilities) on its averaged value and Figure 7 in the Appendix A shows average coefficients from the predictive regressions.

High R^2 values of factor models on idiosyncratic returns show that removing the common linear factors does not erase factor structure. Consistent with Herskovic et al. (2016) we find close to 30% explanatory power of common volatility[9] while we report even higher explanatory power of common quantiles, especially in the left tail. Importantly, when we orthogonalize the average idiosyncratic quantiles by the average idiosyncratic volatility, more than half of the factor structure in both tails remains present. The strength of the quantile factor structure is comparable to the strength of the idiosyncratic volatility structure, which suggests that the idiosyncratic quantiles are not subsumed by

[9]The slight difference can be attributed to different sample span.

Figure 2: *Factor structure in quantiles.* The figure depicts average R^2 from time-series regression of CRSP stock-level quantiles (volatilities) on the averaged value of quantile (volatility) across all stocks. The quantile (or volatility) factor is defined as the equal-weighted cross-sectional average of firm quantiles (volatilities) within a year computed on monthly data. The estimated factor models take form: $q^{\tau}_{i,t} = \alpha_i(\tau) + \beta_i(\tau)\bar{q}^{\tau}_t(x) + v^{\tau}_{i,t}$ and $\sigma^2_{i,t} = \alpha_i + \beta_i \bar{\sigma}_t(x) + v_{i,t}$. We report R^2 for a factor model of (i) raw return quantile on average return quantile in black line (ii) idiosyncratic quantiles on average idiosyncratic quantiles in black dashed line (iii) idiosyncratic quantiles on average idiosyncratic quantiles orthogonalized by average volatility in black dotted line across all $\tau \in (0, 1)$ quantile levels. In addition, a factor model of (iv) raw volatility on average volatility in gray line, (v) idiosyncratic volatility on average idiosyncratic volatility in gray dashed line. Note that results for factor model on volatilities are constant across quantiles.



the common volatility a provide potentially valuable information for the asset prices.

The main observation from this illustratory exercise is that there is a strong structure present across the distributions of idiosyncratic returns that carries different information to structure in the volatility. This means that there is a common structure in tails that may be valuable for investors. In the rest of the paper, we will investigate the implications of the common idiosyncratic quantile factors for both predictability of the equity premia and the cross-section of stock returns.

3 Common Idiosyncratic Quantile Factors

The evidence presented in Section 2 indicates that firm-level idiosyncratic quantiles share high degree of comovement that can be described by a factor model. The preliminary discussion relies on the sample quantile estimates that suffer from small sample bias and are not flexible enough to investigate pricing implications due to lack of data to be able to characterize required parts of the distribution precisely.

Instead of the average sample quantiles of the panel returns, we propose an approximate factor model similar to principal component in means being able to extract unobservable factors that shift relevant parts of the distributions of observable returns. That is, in parallel to genuine factor structure in idiosyncratic volatility of a panel of returns recovered by cross-sectional averages (or PCA) once mean factors have been removed, we aim to recover genuine unobserved structure in idiosyncratic quantiles.

3.1 Quantile Factor Model

To formalize the discussion, we assume the returns from the time-series regression eliminating common factors

$$r_{i,t} = \alpha_i + \beta_i^\top F_t + \epsilon_{i,t} \tag{4}$$

to have τ-dependent structure $f_t(\tau)$ in idiosyncratic errors that we coin a common idiosyncratic quantile – CIQ(τ) – factors, $f_t(\tau)$

$$Q_{\epsilon_{i,t}}\Big[\tau | f_t(\tau)\Big] = \gamma_i^\top(\tau) f_t(\tau), \tag{5}$$

that implies

$$\epsilon_{i,t} = \gamma_i^\top(\tau) f_t(\tau) + u_{i,t}(\tau), \tag{6}$$

where $f_t(\tau)$ is an $r(\tau) \times 1$ vector of random common factors, and $\gamma_i(\tau)$ is $r(\tau) \times 1$ vector of non-random factor loadings with $r(\tau) \ll N$ and the quantile-dependent idiosyncratic error $u_{i,t}(\tau)$ satisfies the quantile restriction $P[u_{i,t}(\tau) < 0 | f_t(\tau)] = \tau$ almost surely for all $\tau \in (0, 1)$.

To estimate the common factors that capture co-movement of quantile-specific features of distributions of the idiosyncratic parts of the stock returns, we use Quantile Factor Analysis (QFA) introduced by Chen et al. (2021). In contrast to PCA, QFA allows to capture hidden factors that may shift more general characteristics such as moments or quantiles, of the distribution of returns other than mean. The methodology is also suitable for large panels.

The quantile-dependent factors and its loadings can be estimated as

$$\underset{(\gamma_1,\dots,\gamma_N, f_t,\dots,f_T)}{\operatorname{argmin}} \frac{1}{NT} \sum_{i=1}^{N} \sum_{t=1}^{T} \rho_\tau \left(\epsilon_{it} - \gamma_i^\top f_t \right) \tag{7}$$

where $\rho_\tau(u) = (\tau - \mathbf{1}\{u \leq 0\})u$ is the check function while imposing the following normalizations $\frac{1}{T}\sum_{t=1}^{T} f_t f_t^\top = \mathbb{I}_r$, and $\frac{1}{N}\sum_{i=1}^{N} \gamma_i\gamma_i^\top$ is diagonal with non-increasing diagonal elements.

As discussed in Chen et al. (2021), this estimator is related to the principal component analysis (PCA) estimator studied in Bai and Ng (2002) and Bai (2003) similarly as quantile regression is related to classical least-square regression. Unlike the PCA estimator of Bai (2003), the estimator does not yield an analytical closed form solution. To solve for the stationary points of the objective function, Chen et al. (2021) proposed a computational algorithm called iterative quantile regression. Moreover, they show that the estimator possess same convergence rate as the PCA estimators for AFM. We follow their approach when estimating the quantile factors.[10]

3.2 Common Idiosyncratic Quantile Factor and the US firms

To estimate the CIQ(τ) factors, we use returns on common stocks from the Center for Research in Securities Prices (CRSP) database sampled between January 1963 and December 2015. We include all stocks with codes 10 and 11 in estimating the CIQ(τ) factors. When forming the portfolios, we follow the standard practice in the literature and exclude all "penny stocks" with prices less than one dollar to avoid biases related to these stocks.[11] We performed the analysis using all the stocks, and the results did not qualitatively change. When not stated otherwise, we use monthly data for both factor estimation and beta calculations.

In the process of the factor estimation, we proceed in a few steps. First, we use a moving window of 60 months of monthly sampled observations. We select the stocks that have all the observations in this window. For all these stocks, we run time-series regression to eliminate the influence of the common (linear) factors

$$\forall i : r_{i,t} = \alpha_i + \beta_i^\top F_t + e_{i,t}, \quad t = 1, \ldots, T \tag{8}$$

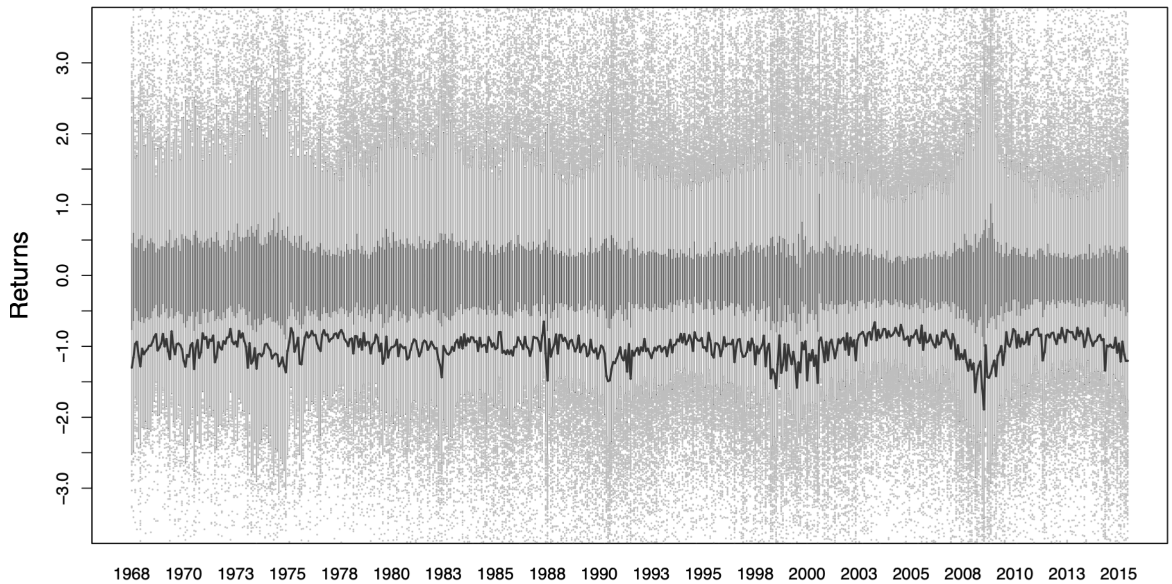
and save the residuals $e_{i,t}$. For the common factors F_t, which we eliminate from the stock returns, we resort to the three factors of Fama and French (1993).[12] Second, we use the residuals from the first step and, for every τ, estimate common idiosyncratic quantile

[10] We employ the authors' Matlab codes provided on the Econometrica webpage.

[11] See, e.g., Amihud (2002).

[12] As discussed in Herskovic et al. (2016), there is a little difference between the results obtained using factors of Fama and French (1993) and purely statistically motivated ones estimated using the PCA framework.

Figure 3: *CIQ(0.1) factor* The figure depicts 0.1 common idiosyncratic quantile – CIQ(τ) – factor estimated from the 60-month rolling window using CRSP stocks in black line. Gray boxplots show cross-sectional distributions of CRSP stock returns. Note the returns are standardized.



factors, $f_t(\tau)$

$$\forall \tau : e_{i,t} = \gamma_i(\tau) f_t(\tau) + u_{i,t}(\tau) \tag{9}$$

where the quantile-dependent idiosyncratic error $u_{i,t}(\tau)$ satisfies the quantile restriction following the methodology discussed in the previous subsection. We use only the first – the most informative – estimated factor for our purposes. In the overwhelming majority of the cases, the algorithms proposed in Chen et al. (2021) select exactly one factor to be the correct number of factors that explain the panels of idiosyncratic returns.[13]

Third, consistent with common volatility factor literature, we focus on the changes in the CIQ(τ), and we work with ΔCIQ(τ) factors.[14] In the case of the aggregate market prediction, we use the last estimated value from a given window to predict market return at time $T + 1$. Fourth, to assess the pricing implications of the exposures to the CIQ(τ) factors in the cross-section, for every stock, we estimate their betas with respect to the

[13]Robustness tests using more factors produce qualitatively similar results.

[14]If not stated otherwise, in the rest of the paper, we perform all the analyses using ΔCIQ(τ) factors.

changes in CIQ(τ) factors

$$r_{i,t} = \alpha_i + \beta_i^{\Delta}(\tau)\Delta f_t(\tau) + v_{i,t} \tag{10}$$

by employing classical linear regression based on the least-square estimator. And finally, we use these betas to infer the predictive implications regarding the next period returns $r_{i,t+1}$, which are either subsequent month or year returns.

Figure 3 illustrates estimated levels of the CIQ(τ) factor. We chose CIQ(0.1) level estimated using a 60-month rolling window, and the last estimated value from each window is plotted in black line. In addition, whole cross-sectional distribution of CRSP stock returns is depicted in gray boxplots. We can see that CIQ(0.1) recovers the unobservable quantile factor well.

Having variety of quantile factors at hand, it is tempting to explore pricing implications of common quantiles with different levels and shift focus of the analysis from tails to other parts of distribution. Table 6 provides correlations between CIQ(τ) factors at different quantiles. Correlation between CIQ(τ) in levels for the upper and lower part of the distribution are far from perfect, e.g., the correlation between the lower tail factor CIQ(0.1) and upper tail CIQ(0.9) is -0.554. This observation suggests that the factors do not simply duplicate information and are hence not likely to be rescaled information contained in common volatility factor. Moreover, this dependence almost perfectly disappears if we look at the increments of the CIQ(τ) factors – dependence between lower and upper tail factors reduces to -0.053. These results suggest that there is a potential for different pricing information across quantiles and that this information does not simply mirror information contained in the common volatility.

4 Time-series Predictability of Market Return

We start examining the information content of CIQ(τ) factors for subsequent short-term market returns. Here we aim to predict monthly excess return on the market that we approximate by the value-weighted return of all CRSP firms. In the regressions, we also control for popular predictive variables used in Welch and Goyal (2007) as well as three closely related factors – the tail risk (TR) factor of Kelly and Jiang (2014), the innovations of common idiosyncratic volatility (ΔCIV) factor of Herskovic et al. (2016), and the variance risk premium (VRP) factor of Bollerslev et al. (2009).[15] Because the

[15]We replicated tail risk factor construction of Kelly and Jiang (2014) by ourself; we acquired data of Herskovic et al. (2016) from Bernard Herskovic's webpage and data of Bollerslev et al. (2009) from Hao Zhou's webpage.

Table 1: *Predictive power of the $\Delta CIQ(\tau)$ factors.* The table reports results from the univariate predictive regressions of the value-weighted return of all CRSP firms on ΔCIQ(τ) factors for various $\tau \in (0, 1)$. Coefficients are scaled to capture the effect of one standard deviation increase in the factor on the annualized market return in percent. The corresponding t-statistics are computed using the Newey-West robust standard errors using six lags. We report both in-sample (IS) and out-of-sample (OOS) R^2s. We also truncate the predictions at zero following Campbell and Thompson (2007) (CT) and report corresponding IS and OOS R^2s.

τ	Coeff.	t-stat	R^2 IS	R^2 OOS	R^2 IS CT	R^2 OOS CT
0.1	-7.332	-2.881	1.789	1.334	1.360	1.747
0.15	-7.676	-2.934	1.961	1.461	1.588	1.856
0.2	-7.572	-2.896	1.908	1.383	1.421	1.533
0.3	-7.567	-2.969	1.906	1.109	1.090	1.083
0.4	-7.019	-2.980	1.640	0.496	0.876	0.748
0.5	-3.401	-1.604	0.385	-0.057	0.381	0.173
0.6	-0.983	-0.420	0.032	-0.392	0.032	-0.303
0.7	-0.908	-0.466	0.027	-0.604	0.027	-0.471
0.8	1.388	0.763	0.064	-0.434	0.006	-0.354
0.85	2.429	1.404	0.196	-0.264	0.081	-0.151
0.9	3.985	2.067	0.529	0.044	0.264	-0.030

CIQ(τ) factors are estimated using a rolling window, we use the last value of the factors estimated from each rolling window to construct a single series of the CIQ(τ) factors.

First, we report the results from the univariate regressions of the market return on the differences of the CIQ(τ) factors at various τ quantile levels in Table 1. We report estimated scaled coefficients to capture the effect of one standard deviation increase of the independent variable on the subsequent annualized market return. The corresponding t-statistics are computed using Newey-West robust standard errors using six lags.

The results in Table 1 document strong predictive power using the ΔCIQ(τ) factors for the left part of the distribution, with the peak for τ being between 0.15 and 0.2, where the increase (decrease) of one standard deviation in the factors predict subsequent decrease (increase) of 7.676 and 7.572 percents in annualized market return.[16] There is also some predictive power for the upper tail factor when CIQ(0.9), but the effect is much smaller with only 3.985 percent increase in annualized market return accompanied with only one-fourth of the R^2 from the lower tail. From a perspective of an investor, in times of high risk – captured by large negative increments of the left-tail CIQ(τ) factor, she requires a premium for investing. And thus, these risky periods correlate with the high marginal utility states of the investors.

Together with in-sample (IS) R^2, we also report the out-of-sample (OOS) R^2 from expanding window scheme. We use data up to time t to estimate the prediction model and then forecast the $t + 1$ return (the first window contains 120 monthly periods to obtain

[16]Note that the lower tail factors are on average negative. Increase (decrease) of these factors corresponds to the decrease (increase) of risk, which leads to a decrease (increase) of the required risk premium.

sufficiently reasonable estimates). Then, the window is extended by one observation, the prediction model is re-estimated and a new forecast is obtained. We repeat this procedure until the whole sample is exhausted. The corresponding R^2 is computed by comparing conditional forecast and historical mean computed using the available data up to time t, i.e., $1-\sum_t(r_{m,t+1}-\widehat{r}_{m,t+1|t})^2/\sum_t(r_{m,t+1}-\bar{r}_{m,t})^2$ where $\widehat{r}_{m,t+1|t}$ is out-of-sample forecast of the $t+1$ return using data up to time t, and $\bar{r}_{m,t}$ is the historical mean of the market return computed up to date t. Unlike the case of the IS R^2, the OOS R^2 can attain negative values if the conditional forecasts perform worse than the historical mean forecast. The positive values of the OOS R^2 for τ between 0.1 and 0.3 provide strong evidence for the benefits of the ΔCIQ(τ) factors for predicting the market return in the real-world setting. On the other hand, the predictability vanishes for the higher values of τ.

To assess the economic usefulness for the investors, we further follow suggestions from Campbell and Thompson (2007) (hence CT). They propose to truncate the predictions from the estimated model at 0, as the investor would not have used a model to predict a negative premium. This non-linear modification of the model should introduce caution into the models. Based on this modification, we report both IS and OOS R^2s. Naturally, using this transformation, the IS R^2 does not improve for any of the models, but the performance rises for the OOS analysis. Results suggest that the common fluctuations in the lower part of the excess returns distributions robustly predict the subsequent market movement.

Next, we run bivariate regressions to assess whether the proposed quantile factors contain additional information not included in the relevant previously proposed variables. We separately control for variables that may contain duplicate information. First, in Table 2, we report coefficients and their t-statistics while controlling for the TR factor of Kelly and Jiang (2014), the ΔCIV of Herskovic et al. (2016), and the VRP factor of Bollerslev et al. (2009), respectively. In the first case, ΔCIQ(τ) factors mirror the results from the univariate regressions in terms of coefficients and their significance. TR factor is significant across all the specifications, although its effect is smaller and less significant than in the case of ΔCIQ(τ) for the lower tail values of τ. In the second case, while controlling for the ΔCIV, the results regarding the ΔCIQ(τ) factors remain the same, and ΔCIV proves not to predict future market returns. In the third case, the VRP factor appears to be the most closely related in terms of predictability to the ΔCIQ(τ) factors. The VRP is highly significant, and at the same time, it diminishes the effect of the ΔCIQ(τ) factors – the scaled coefficients decreases around 1.5 percentage points, and the corresponding t-statistics are now approximately 1.6.

Table 2: *Bivariate predictive regressions.* The table reports results from the bivariate predictive regressions of the value-weighted return of all CRSP firms on ΔCIQ(τ) factors for various $\tau \in (0,1)$ and other control variables. We employ the TR factor of Kelly and Jiang (2014), the ΔCIV of Herskovic et al. (2016), and the VRP factor of Bollerslev et al. (2009), respectively. Coefficients are scaled to capture the effect of one standard deviation increase in the factor on the annualized market return in percent. The corresponding t-statistics are computed using the Newey-West robust standard errors using six lags.

	τ	0.1	0.15	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.85	0.9
Kelly and Jiang (2014)	ΔCIQ(τ)	-7.296	-7.617	-7.568	-7.497	-6.961	-3.171	-0.771	-0.751	1.519	2.500	4.064
	t-stat	-2.875	-2.922	-2.909	-2.944	-2.960	-1.502	-0.329	-0.386	0.836	1.449	2.112
	TR	4.982	4.943	5.027	4.928	4.954	4.885	5.002	5.011	5.074	5.070	5.098
	t-stat	2.254	2.247	2.301	2.220	2.250	2.174	2.245	2.255	2.276	2.279	2.290
Herskovic et al. (2016)	ΔCIQ(τ)	-7.863	-8.126	-7.918	-7.831	-7.204	-3.470	-1.003	-0.902	1.420	2.464	3.995
	t-stat	-3.042	-3.063	-2.987	-3.022	-2.961	-1.630	-0.426	-0.461	0.771	1.393	2.046
	ΔCIV	-2.454	-2.325	-2.074	-1.865	-1.594	-0.995	-0.781	-0.748	-0.811	-0.855	-0.806
	t-stat	-0.718	-0.683	-0.607	-0.541	-0.452	-0.273	-0.210	-0.201	-0.217	-0.228	-0.216
Bollerslev et al. (2009)	ΔCIQ(τ)	-5.649	-6.019	-5.774	-5.513	-5.114	-2.271	1.814	-0.749	1.002	1.658	2.910
	t-stat	-1.494	-1.573	-1.604	-1.606	-1.562	-0.747	0.516	-0.290	0.389	0.716	1.235
	VRP	12.643	12.621	12.403	12.348	12.308	12.311	12.536	12.399	12.518	12.566	12.616
	t-stat	5.580	5.500	5.281	5.208	5.214	5.229	5.466	5.322	5.420	5.487	5.567

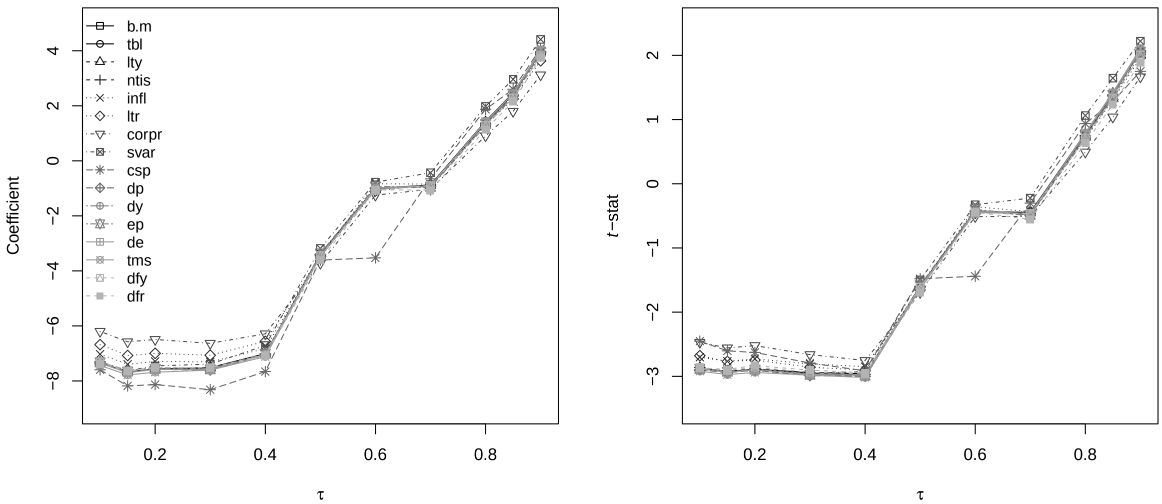
Second, we control for variables discussed in Welch and Goyal (2007).[17] Instead of large Table through all variables and quantiles, we summarize the results in the Figure 4, in which we plot the coefficients of the ΔCIQ(τ) factors while controlling for said variables. We observe that none of the variables drives out the significance of the ΔCIQ(τ) factors. Moreover, the magnitude of the effect remains very close to the ones from the univariate regressions.

5 Pricing the CIQ(τ) Risks in the Cross-Section

In this section, we investigate the pricing implications of the presented common idiosyncratic quantile factors for the cross-section of stock returns. We hypothesize that the stochastic discount factor increases in the CIQ(τ) risk, as the risk-averse investor's marginal utility is high in the states of high CIQ(τ) risk. Based on that hypothesis, we assume that the assets that perform poorly in the states of high CIQ(τ) risk will require a higher risk premium for holding by the investors. On the other hand, assets that perform well during these states serve as a hedging tool and will be traded with higher prices and thus lower expected returns. The stocks sensitivities to the factors capture betas estimated by the linear regression of stocks returns on the factors. The betas are calculated following the notion discussed in Subsection 3.2, i.e., using a 60-month rolling window of monthly data up to time t, and are used to predict return at time $t+1$. If not explicitly stated otherwise, we use as our predicted variable monthly out-of-sample returns following the estimation window. We also try to predict one-year returns using

[17]For the information regarding the specification of the variables, see Welch and Goyal (2007). We obtained the data from the Iwo Welch's webpage.

Figure 4: *Predictive power of the $\Delta CIQ(\tau)$ factors with Welch and Goyal (2007) variables.* The figure depicts coefficients and corresponding t-statistics associated with the ΔCIQ(τ) factors from bivariate regressions when controlling for variables discussed in Welch and Goyal (2007). The dependent variable is the value-weighted return of all CRSP firms. Coefficients are scaled to capture the effect of one standard deviation increase in the factor on the annualized market return in percent. The t-statistics are computed using the Newey-West robust standard errors using six lags.



portfolios to assess the persistence of the CIQ(τ) betas and thus indirectly investigate the transaction costs related to the trading of these factors. The betas for the control variables that we employ in various parts of the following analysis are estimated similarly to the CIQ(τ) betas, i.e., using a 60-month rolling window for the monthly data up to time t to forecast the returns at time $t+1$.

5.1 Portfolio Sorts

Here we look at performance of the portfolios sorted on the CIQ(τ) betas. Every month, we estimate CIQ(τ) betas for all stocks that possess all the observations during the last 60 months using data up to time t. We sort the stocks into ten portfolios based on their betas for every τ separately. We then record the portfolios' performances at time $t+1$ using either an equal-weighted or value-weighted scheme. Then we move one month ahead, re-estimate all the betas, and create new portfolios. We expect that, for $\tau < 0.5$, there will be an increasing pattern of returns from the low exposure to the high exposure portfolios, and vice versa for $\tau > 0.5$. The results for sorts based on ten portfolios summarizes Panel A of Table 3. We observe an increasing return pattern for the portfolios with τ up to 0.4 for both equal-weighted and value-weighted schemes.

This pattern practically disappears when we look at the portfolios formed on higher τ CIQ(τ) betas. This observation suggests that only the exposure to the lower tail common movements is priced in the cross-section.

Table 3: *Ten univariate sorted portfolios using monthly data.* The table contains annualized out-of-sample excess returns of ten portfolios sorted on the exposure to the ΔCIQ(τ) factors computed from the monthly data. Panel A reports results based on the one-month returns following the formation period, Panel B results for the twelve-month period. We use all the CRSP stocks that have all 60 monthly observations in each window. We exclude penny stocks with prices less than 1$. We report returns of the high minus low (H - L) portfolios, their t-statistics, and annualized 5-factor alphas with respect to the four factors of Carhart (1997) and CIV shocks of Herskovic et al. (2016). We also report t-statistics for these alphas. Data contain the period between January 1963 and December 2015.

τ	Low	2	3	4	5	6	7	8	9	High	H - L	t-stat	α	t-stat
Panel A: One-month returns														
							Equal-weighted							
0.1	7.300	8.732	9.304	9.630	9.747	9.599	9.295	10.180	11.414	11.835	4.535	2.555	6.164	4.054
0.15	6.810	9.127	9.238	9.801	9.574	9.551	9.965	10.512	10.749	11.705	4.895	2.800	6.791	4.552
0.2	6.623	9.366	9.241	9.729	9.830	9.973	9.586	10.250	10.474	11.958	5.335	3.052	7.405	4.949
0.3	7.206	8.560	9.547	10.239	9.638	9.916	9.512	10.136	10.469	11.815	4.609	2.915	6.344	4.524
0.4	7.275	8.999	10.078	9.609	9.684	9.659	9.462	10.263	10.122	11.873	4.599	2.973	5.866	3.973
0.5	7.079	9.615	9.739	9.347	9.383	9.944	9.945	10.689	10.289	10.962	3.882	1.698	3.950	1.653
0.6	8.169	9.351	9.929	9.731	9.225	9.846	9.876	10.000	10.471	10.384	2.214	1.081	1.205	0.633
0.7	8.295	9.948	9.197	9.048	9.937	9.742	9.340	9.400	10.838	11.233	2.937	1.500	1.609	0.926
0.8	9.247	9.892	9.109	8.632	9.873	9.579	9.340	10.426	9.917	10.957	1.710	0.727	0.044	0.023
0.85	9.492	9.437	9.299	9.180	9.384	9.513	10.054	9.887	10.265	10.452	0.961	0.381	-0.945	-0.471
0.9	9.531	9.459	9.337	9.356	9.543	9.942	9.407	10.407	9.943	10.035	0.504	0.190	-1.371	-0.677
							Value-weighted							
0.1	5.031	5.357	6.086	6.722	6.546	6.282	5.977	6.694	9.139	8.329	3.298	1.263	2.686	1.223
0.15	4.607	5.309	6.775	6.852	6.345	6.617	6.451	6.380	8.468	8.400	3.793	1.505	3.689	1.805
0.2	4.114	6.110	6.817	6.884	6.443	5.649	7.329	6.698	7.766	8.918	4.804	1.957	4.905	2.543
0.3	4.737	6.356	6.177	6.859	5.983	6.716	6.535	6.509	5.874	9.419	4.682	1.970	5.267	2.593
0.4	5.116	5.775	6.639	6.081	7.470	6.244	6.203	5.972	4.886	10.012	4.896	1.914	6.329	2.497
0.5	5.062	6.233	5.604	5.664	5.802	6.610	6.187	7.293	5.297	7.855	2.793	0.999	3.105	1.097
0.6	5.099	6.521	6.098	5.917	6.061	5.825	6.708	6.413	5.530	6.857	1.758	0.658	1.838	0.719
0.7	5.756	6.656	5.952	6.767	6.394	5.776	6.228	5.277	6.328	8.184	2.428	0.957	1.803	0.760
0.8	5.430	6.464	5.941	6.008	6.659	6.398	5.638	6.937	4.361	8.848	3.418	1.197	2.892	1.164
0.85	5.149	6.412	5.302	6.275	6.524	6.660	6.596	5.654	6.699	6.872	1.723	0.577	1.061	0.437
0.9	4.711	5.961	6.109	6.053	6.733	6.509	7.003	6.472	5.960	6.283	1.572	0.502	0.509	0.214
Panel B: Twelve-month returns														
							Equal-weighted							
.1	10.191	10.722	11.122	10.925	11.134	11.030	10.997	11.501	12.421	14.352	4.161	3.394	5.672	4.671
0.15	10.051	11.081	11.056	11.245	10.942	10.963	11.008	11.476	12.128	14.446	4.396	3.666	5.809	4.921
0.2	10.221	11.099	10.974	11.281	10.997	10.769	11.334	11.243	11.861	14.610	4.389	3.772	5.548	4.703
0.3	10.702	10.861	10.902	11.277	10.890	11.036	11.156	11.056	11.928	14.577	3.875	3.681	4.589	3.833
0.4	11.004	11.039	11.132	10.703	10.745	11.023	10.793	11.289	12.219	14.432	3.428	3.561	3.730	2.920
0.5	10.264	11.028	10.961	10.916	10.969	10.777	11.170	11.713	12.723	13.856	3.592	3.162	4.118	3.436
0.6	10.154	10.850	11.319	10.906	10.649	11.006	11.094	11.788	12.550	14.060	3.906	3.443	2.921	1.954
0.7	11.277	11.451	11.059	10.792	10.783	10.905	10.865	11.098	12.149	13.995	2.717	2.130	0.289	0.193
0.8	11.911	11.513	10.549	10.657	10.761	11.224	10.946	11.319	12.025	13.470	1.558	1.001	-1.385	-0.768
0.85	12.125	11.120	10.646	10.615	10.897	11.197	11.095	11.477	12.028	13.175	1.050	0.623	-2.186	-1.095
0.9	11.867	11.040	10.741	10.813	10.976	11.158	11.031	11.744	12.132	12.877	1.010	0.564	-1.444	-0.729
							Value-weighted							
0.1	6.286	6.355	7.356	6.976	6.873	6.641	6.544	7.667	9.138	9.083	2.797	1.670	3.022	1.914
0.15	6.293	6.620	7.627	7.372	6.897	6.642	6.578	7.255	8.653	8.910	2.617	1.598	3.308	2.080
0.2	6.244	7.112	7.365	7.446	6.983	6.598	6.994	6.874	8.049	9.019	2.775	1.718	3.166	1.974
0.3	6.543	7.162	7.571	7.220	6.670	6.809	7.229	6.788	6.616	9.289	2.746	1.838	3.045	1.884
0.4	7.033	7.091	7.283	7.036	6.737	7.051	6.961	6.328	6.097	9.557	2.524	1.793	3.335	1.874
0.5	5.681	6.418	7.007	6.411	6.436	7.002	7.265	6.999	6.628	8.444	2.763	1.790	4.196	2.824
0.6	5.126	5.988	6.641	6.233	6.575	7.155	7.209	6.997	6.639	8.883	3.757	2.543	4.933	3.388
0.7	6.415	6.874	6.107	6.834	7.327	7.059	6.564	6.851	6.718	8.830	2.415	1.461	1.016	0.633
0.8	7.152	6.374	6.447	6.879	6.930	7.234	7.037	6.466	6.866	8.191	1.038	0.526	-0.434	-0.202
0.85	6.735	6.323	6.480	6.403	7.084	7.709	7.014	6.307	7.223	7.532	0.797	0.392	-1.043	-0.484
0.9	6.606	6.369	6.273	6.794	7.076	7.580	7.206	6.612	7.020	7.393	0.787	0.378	-1.585	-0.757

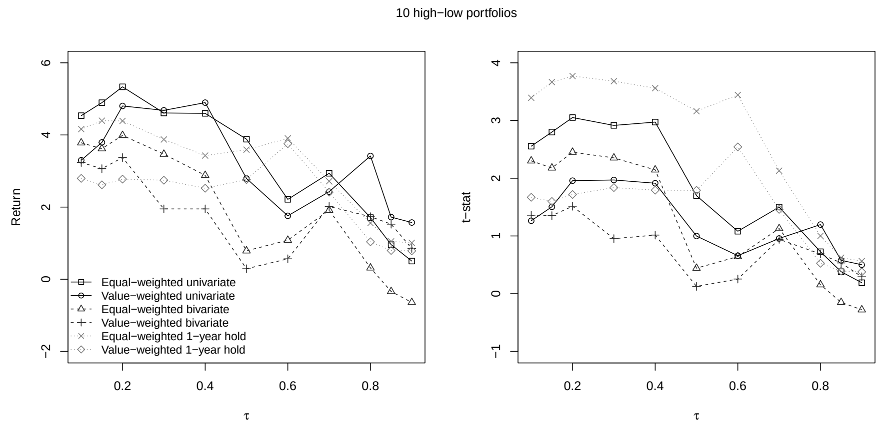
Moreover, to formally assess whether there is a compensation for bearing a risk of high exposure to the common movements in various parts of distributions of idiosyncratic returns, we present returns of high minus low portfolios. We obtain these returns as a difference between returns of portfolios with the highest CIQ(τ) betas and portfolios with the lowest CIQ(τ) betas. These portfolios are zero-cost portfolios and capture the risk premium associated with specific τ joint movements of idiosyncratic returns. Results of this analysis are also summarized in Panel A of Figure 7. As expected, we observe a significant positive premium for the difference portfolios only for τ being less or equal to 0.4. These premiums are both economically and statistically significant. In the case of the equal-weighted portfolios, the premium for CIQ(0.2) factors is 5.335% on the annual basis with a t-statistic of 3.052. The premiums are slightly lower in the case of the value-weighted portfolios – e.g., for $\tau = 0.2$ the premium is 4.804 with t-statistic of 1.957. This lower significance may be partially caused by the fact that the value-weighted portfolios possess a higher concentration, which leads to more volatile returns.

To make sure that the estimated premiums cannot be explained by exposure to other risks previously proposed in the literature, we regress the returns of the high minus low portfolios on four factors of Carhart (1997) and CIV shocks of Herskovic et al. (2016) and report corresponding annualized 5-factor alphas. From the results, we can see that the proposed factors do not capture the positive premium associated with both zero-cost portfolios. For the equal-weighted portfolio with $\tau = 0.2$, the estimated annualized alpha is 7.405% with t-statistic of 4.949, for value-weighted portfolios it is 4.904% premium with t-statistics being equal to 2.543.

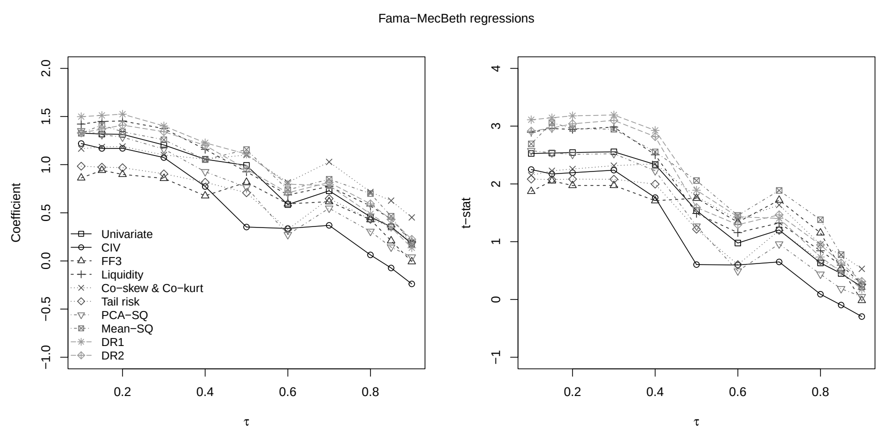
Next, in Panel B of Table 3, we look at the performance of the CIQ(τ) sorted portfolios captured by the following twelve-month returns. Each month, we construct portfolios as in the previous case. Instead of saving the one-month return of the sorted portfolios, we record a twelve-month return, which follows after the formation period. We observe slightly smaller returns but still consistent with the results obtained using one-month returns. The high minus low portfolios with $\tau = 0.2$ yield 4.389% ($t = 3.772$) and 2.775% ($t = 1.718$) for the equal- and value-weighted schemes, respectively. The other risk factors cannot explain these premiums as the 5-factor alphas stay economically and statistically significant.

Due to the fact that only the exposures to the lower tail common movements are priced, the previous results suggest that the CIQ(τ) risks are not driven by the effect of the common volatility. If it were the case that the volatility is the main driver of the obtained results, we would observe that both exposures to the lower and upper parts of the joint movements are priced, which is not the case. But to explicitly control for

Figure 5: *Cross-sectional results of the monthly data* Δ*CIQ*(τ) *factors.* The figure summarizes the cross-sectional asset pricing results of the ΔCIQ(τ) factors using monthly data. Panel (a) captures annualized returns of ten high minus low portfolios based on sorting stocks into ten portfolios. Panel (b) presents coefficients corresponding to the ΔCIQ(τ) betas from the Fama-MacBeth regressions while controlling for various competing risk measures. The results are based on data sampled between January 1963 and December 2015.



(a)



(b)

the effect of the common idiosyncratic volatility, we perform dependent bivariate sorts by double sorting on betas for increments of CIV factor and betas for increments of the CIQ(τ) factors. Every month, we first sort the stocks into ten portfolios based on their CIV betas. Then, within each of the CIV-sorted portfolios, we sort the stocks into ten portfolios based on their CIQ(τ) betas. Finally, for each CIQ(τ) portfolio, we collapse all the corresponding CIV portfolios into one CIQ(τ) portfolio. This procedure yields single-sorted portfolios which vary in their CIQ(τ) betas but possess approximately equal CIV betas. The obtained results summarizes Table 4. For the equal-weighted portfolios, we see that the risk premium captured by the returns of the high minus low portfolios

Table 4: *Ten bivariate sorted portfolios.* The table contains annualized out-of-sample excess returns of ten portfolios double sorted on the exposure to the ΔCIQ(τ) factors and ΔCIV. First, we perform sorts based on the ΔCIV betas, then, within each ΔCIV portfolio, we sort on the ΔCIQ betas, and then we collapse all the ΔCIV portfolios for a given ΔCIQ(τ) portfolio into one. The obtained portfolios vary in their ΔCIQ(τ) betas but not in their ΔCIV betas. We report returns of the high minus low (H-L) portfolios, their t-statistics, and annualized 5-factor alphas with respect to the four factors of Carhart (1997) and CIV shocks of Herskovic et al. (2016). We also report t-statistics for these alphas. Data contain the period between January 1963 and December 2015.

τ	Low	2	3	4	5	6	7	8	9	High	H - L	t-stat	α	t-stat
							Equal-weighted							
0.1	7.264	8.944	9.942	9.540	9.699	9.699	10.310	10.174	10.438	11.046	3.782	2.301	4.588	3.571
0.15	7.099	8.875	9.726	10.286	9.762	9.674	10.434	10.045	10.457	10.718	3.619	2.178	4.693	3.657
0.2	6.822	9.241	9.887	9.867	10.408	9.780	9.870	10.306	10.093	10.810	3.988	2.452	5.199	3.953
0.3	7.125	9.105	9.726	9.906	10.036	10.317	10.123	9.972	10.157	10.593	3.468	2.353	4.592	3.658
0.4	7.700	9.488	9.981	9.928	9.750	10.542	9.513	9.398	10.134	10.581	2.881	2.145	4.094	3.419
0.5	8.288	10.246	9.693	10.345	9.924	10.323	10.292	9.368	9.482	9.075	0.787	0.441	0.967	0.564
0.6	8.282	9.491	10.112	10.672	9.657	9.814	10.380	10.139	9.084	9.365	1.083	0.643	0.754	0.488
0.7	8.574	10.115	9.470	9.613	9.450	10.198	9.875	9.166	10.021	10.489	1.914	1.128	1.089	0.733
0.8	9.462	9.518	9.855	9.577	9.698	9.921	9.680	8.946	10.494	9.777	0.315	0.155	-0.783	-0.480
0.85	9.639	9.373	10.280	9.611	9.554	9.718	9.699	9.626	10.161	9.301	-0.339	-0.151	-1.598	-0.901
0.9	9.743	9.725	9.743	10.103	9.913	9.479	9.581	9.590	10.013	9.100	-0.643	-0.279	-1.924	-1.032
							Value-weighted							
0.1	5.646	5.925	7.337	7.587	7.920	8.382	8.202	7.360	7.804	8.884	3.238	1.359	2.388	1.453
0.15	5.579	5.992	7.343	8.036	8.326	7.903	8.120	7.830	7.628	8.644	3.065	1.352	2.847	1.764
0.2	5.395	6.293	8.066	7.861	8.202	8.138	7.946	8.900	7.039	8.768	3.373	1.515	3.319	1.957
0.3	5.825	6.455	7.074	8.304	8.098	8.239	8.777	7.239	7.390	7.775	1.950	0.953	2.439	1.538
0.4	5.984	6.937	8.634	8.248	7.154	9.210	7.521	5.922	7.280	7.933	1.949	1.015	3.212	1.836
0.5	5.693	7.552	6.719	7.706	7.825	8.531	7.263	8.062	7.119	5.984	0.291	0.126	0.730	0.316
0.6	5.358	7.278	7.434	7.359	8.010	8.618	8.256	7.927	6.593	5.924	0.566	0.254	0.441	0.213
0.7	6.281	7.595	8.176	7.137	7.429	8.611	7.173	7.527	7.595	8.298	2.018	0.942	1.268	0.631
0.8	6.110	7.064	7.408	7.963	8.139	7.410	7.350	6.675	8.098	7.844	1.734	0.689	1.054	0.504
0.85	5.863	7.445	7.483	7.584	7.843	7.693	7.345	7.679	7.043	7.390	1.527	0.532	0.750	0.319
0.9	6.509	7.363	7.426	8.327	7.858	7.738	6.832	7.626	7.407	7.363	0.855	0.293	0.025	0.011

for $\tau \leq 0.4$ remains significant with an annualized return of 3.988% ($t = 2.452$). In case of the value-weighted portfolios, the return decreases to 3.373% for $\tau = 0.2$ ($t = 1.515$). This observation suggests that the CIQ(τ) risk premium partly captures the interaction between size and CIV premium.

In Appendix A in Tables 7 and 8, we provide results of the same analysis using five portfolios instead of ten. The results are qualitatively very similar to the results from the above, confirming the robustness of our claim that the exposure to the common left tail events is priced in the cross-section of returns.

5.2 Pricing CIQ(τ) risk

Next, we perform a two-stage Fama and MacBeth predictive cross-sectional regressions to explore the ability of CIQ(τ) factors to explain the abnormal returns associated with CIQ(τ)-beta sorted portfolios. This type of asset pricing test moreover conveniently allows for simultaneous estimation of many risk premiums associated with various risk measures. That means that we can estimate the risk premium associated with the CIQ(τ) risks while controlling for other risk measures previously proposed in the literature. More specifically, for each time $t = 1, \ldots, T-1$ using all of the stocks $i = 1, \ldots, N$ available at time t and $t+1$,[18] we cross-sectionally regress all the returns at time $t+1$ on the betas estimated using only the information available up to time t. This procedure yields estimates of prices of risk $\lambda_{t+1}(\tau)$ while controlling for the most widely used competing measure of risk

$$r_{i,t+1} = \alpha + \beta_{i,t}^{CIQ(\tau)}(\tau)\lambda_{t+1}^{CIQ(\tau)}(\tau) + \beta_{i,t}^{\top Control}\lambda_{t+1}^{Control} + e_{i,t+1} \tag{11}$$

where $\beta_{i,t}^{Control}$ is vector of control betas and $\lambda_{t+1}^{Control}$ is vector of corresponding prices of risk. Using $T-1$ cross-sectional estimates of the prices of risk, we compute the average price of risk associated with each $\lambda^{CIQ}(\tau)$ as

$$\widehat{\lambda}^{CIQ(\tau)}(\tau) = \frac{1}{T-1}\sum_{t=2}^{T}\widehat{\lambda}_t^{CIQ(\tau)}(\tau) \tag{12}$$

and report their annualized values along with their t-statistics based on the Newey-West robust standard errors using 6 lags. For better comparability, each time t that we estimate the price of risk for time $t+1$, we multiply the estimated coefficient by the cross-sectional standard deviation of the corresponding betas from time t. We follow the same logic when reporting results for the control variables. Doing that enables us to compare the effect on the expected returns across τ and various controls. We report the results from these regressions in Table 5 and Panel (b) of Figure 7.

First, we report results from the univariate regressions on CIQ(τ) betas. We observe very similar results to those obtained from the portfolio sorts – the exposure to the common idiosyncratic left tail events is significantly compensated in the cross-section of stock returns. For example, ΔCIQ(τ) for $\tau = 0.2$ posses a coefficient of 1.315 (t-stat = 2.543), on the other hand, for $\tau = 0.8$, the estimated coefficient is equal to 0.458 (t-stat = 0.632).

Second, we report results from the multivariate regressions in which we include as a control volatility betas computed on shocks to the CIV factor. We see that the results regarding CIQ(τ) betas still hold both qualitatively and quantitatively similar to the case of univariate regressions. Moreover, CIV risk is priced as well; especially strong is the relationship when we control for CIQ(τ) betas with τ from the right part of the distribution. These results suggest that both common idiosyncratic volatility and quantile risk are priced and do not convey the same pricing information.

[18] A stock is identified as available, if it posses all the return observations during the last 60-month window up to time t and also an observation at time $t+1$.

Table 5: *Fama-MacBeth regressions using monthly CIQ(τ) factors.* The table contains estimated prices of risk and t-statistics from the Fama-MacBeth predictive regressions. Each segment contains prices of risk of ΔCIQ(τ) betas while controlling for various risk measures. The coefficients are standardized by the cross-sectional standard deviations of the corresponding betas and annualized. Data contain the period between January 1963 and December 2015. In each window, we use all the CRSP stocks that have all 60 monthly observations, and we exclude penny stocks with prices less than 1$.

τ	0.1	0.15	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.85	0.9
$\beta^{CIQ}(\tau)$	1.327	1.318	1.315	1.204	1.056	0.993	0.585	0.729	0.458	0.354	0.180
	2.527	2.534	2.543	2.556	2.336	1.538	0.977	1.203	0.632	0.450	0.216
$\beta^{CIQ}(\tau)$	1.219	1.168	1.169	1.074	0.773	0.352	0.337	0.370	0.063	-0.073	-0.238
	2.248	2.172	2.194	2.238	1.763	0.604	0.597	0.650	0.091	-0.097	-0.296
β^{CIV}	-1.208	-1.276	-1.343	-1.390	-1.512	-1.697	-1.778	-1.565	-1.530	-1.561	-1.578
	-1.953	-2.075	-2.194	-2.296	-2.556	-3.006	-3.192	-2.824	-2.822	-2.887	-2.888
$\beta^{CIQ}(\tau)$	1.327	1.411	1.341	1.259	1.052	1.157	0.739	0.848	0.700	0.465	0.168
	2.693	3.060	2.958	2.946	2.554	2.058	1.454	1.886	1.380	0.777	0.247
$\beta^{Mean-SQ}$	0.097	0.118	0.020	-0.118	-0.364	-0.472	-0.430	-0.556	-0.571	-0.382	-0.187
	0.102	0.129	0.023	-0.135	-0.428	-0.579	-0.515	-0.674	-0.703	-0.477	-0.236
$\beta^{CIQ}(\tau)$	1.352	1.311	1.286	1.162	0.926	0.766	0.272	0.548	0.307	0.143	0.041
	2.574	2.537	2.510	2.523	2.227	1.269	0.493	0.960	0.437	0.186	0.049
β^{PC-SQ}	0.039	0.018	0.003	0.015	-0.050	0.086	0.311	-0.013	-0.065	0.030	0.026
	0.102	0.048	0.008	0.039	-0.137	0.248	0.809	-0.038	-0.205	0.095	0.084
$\beta^{CIQ}(\tau)$	0.862	0.941	0.899	0.857	0.677	0.818	0.593	0.616	0.429	0.210	-0.008
	1.871	2.052	1.973	1.973	1.709	1.752	1.339	1.719	1.153	0.535	-0.018
β^{MKT}	0.056	0.109	0.102	0.095	0.022	-0.044	-0.059	-0.121	-0.139	-0.085	-0.061
	0.064	0.125	0.118	0.111	0.026	-0.055	-0.074	-0.147	-0.167	-0.100	-0.071
β^{SMB}	0.424	0.493	0.506	0.499	0.424	0.536	0.479	0.315	0.276	0.332	0.443
	0.451	0.524	0.540	0.536	0.467	0.601	0.534	0.354	0.312	0.373	0.491
β^{HML}	1.510	1.503	1.479	1.457	1.513	1.471	1.556	1.570	1.649	1.722	1.792
	1.856	1.855	1.837	1.819	1.928	1.954	2.064	2.094	2.231	2.341	2.415
$\beta^{CIQ}(\tau)$	0.984	0.974	0.968	0.905	0.817	0.708	0.327	0.648	0.432	0.357	0.192
	2.083	2.076	2.082	2.081	1.996	1.216	0.603	1.196	0.672	0.512	0.258
β^{TR}	0.437	0.453	0.483	0.507	0.551	0.368	0.388	0.539	0.565	0.575	0.602
	0.633	0.652	0.691	0.709	0.762	0.520	0.547	0.765	0.831	0.865	0.921
$\beta^{CIQ}(\tau)$	1.421	1.449	1.453	1.375	1.159	0.926	0.686	0.774	0.571	0.433	0.205
	2.894	2.958	2.944	2.984	2.504	1.489	1.157	1.324	0.843	0.594	0.265
β^{LIQ}	0.118	0.145	0.143	0.169	0.083	0.075	0.115	0.102	0.243	0.291	0.417
	0.195	0.240	0.233	0.268	0.128	0.123	0.189	0.168	0.427	0.532	0.789
$\beta^{CIQ}(\tau)$	1.295	1.315	1.332	1.261	1.190	1.220	0.899	1.013	0.692	0.561	0.340
	2.404	2.479	2.537	2.653	2.637	1.945	1.504	1.606	0.920	0.691	0.399
β^{Skew}	-1.362	-1.336	-1.374	-1.381	-1.382	-1.452	-1.462	-1.391	-1.372	-1.384	-1.372
	-2.946	-2.901	-3.014	-3.046	-3.054	-3.373	-3.415	-2.865	-2.803	-2.869	-2.847
$\beta^{CIQ}(\tau)$	1.132	1.165	1.176	1.096	1.018	0.978	0.645	0.843	0.595	0.497	0.317
	2.098	2.186	2.216	2.268	2.250	1.567	1.090	1.386	0.813	0.625	0.378
β^{Kurt}	-0.077	-0.030	-0.037	-0.064	-0.093	-0.236	-0.222	-0.207	-0.197	-0.196	-0.201
	-0.155	-0.060	-0.073	-0.126	-0.187	-0.510	-0.486	-0.415	-0.395	-0.395	-0.406
$\beta^{CIQ}(\tau)$	1.498	1.511	1.524	1.402	1.227	1.110	0.798	0.783	0.488	0.348	0.137
	3.110	3.144	3.178	3.193	2.926	1.890	1.431	1.402	0.735	0.478	0.177
β^{DR1}	0.516	0.536	0.559	0.535	0.479	0.612	0.673	0.393	0.428	0.460	0.499
	0.880	0.903	0.938	0.883	0.787	1.071	1.184	0.682	0.792	0.879	0.982
$\beta^{CIQ}(\tau)$	1.329	1.370	1.410	1.342	1.200	0.954	0.707	0.816	0.593	0.430	0.225
	2.914	2.968	3.040	3.100	2.818	1.592	1.294	1.466	0.945	0.631	0.312
β^{DR2}	0.746	0.789	0.829	0.810	0.738	0.615	0.624	0.537	0.549	0.585	0.592
	0.929	0.971	1.014	0.975	0.878	0.757	0.777	0.657	0.715	0.784	0.816

Third, to investigate whether the quantile factors provide different priced information beyond conventional approximate factor models, we construct and control for two related factors. In both cases, we proceed similarly as in the construction of the quantile factors –

using the 60-month moving window, we extract the idiosyncratic returns and then square them. We use a simple cross-sectional average of the squared residuals in the first case and denote it as Mean-SQ. In the second case, we perform principal component analysis on those squared residuals and take the first principal component that explains the most common time variation across the squared residuals, and we denote it as PC-SQ. In both cases, we then difference the factors and use their increments as control factors. From the results, we can conclude that the quantile factors extract very different information regarding the expected returns, as both specifications based on the factors extracted from the squared residuals turn out not to be significant predictors in the cross-section of stock returns. One has to look deeper into the common distribution if he wants to identify priced information regarding the common distributional movements.

Next, we focus on various less related risk measures previously proposed in the literature. We control for the three factors of Fama and French (1993). This specification decreases the effect and significance of the CIQ(τ) betas the most among all the discussed specifications. Still, the CIQ(τ) betas for the lower tail ($\tau \leq 0.3$) possess t-statistics above 1.8. The other significant predictor from these regressions is the HML factor which performs well, especially when we include CIQ(τ) betas for higher τ.

As another related control, we use the tail risk factor of Kelly and Jiang (2014). As we can see, TR betas do not drive out the CIQ(τ) betas' effect, which remains significant, similarly to the univariate specification. We also control for the impact of liquidity betas of Pastor and Stambaugh (2003),[19] which do not alter the results regarding the CIQ(τ) betas, neither.

Finally, we control for another related group of risk measures, which consider the nonlinear relationship between asset and market returns. By following the specifications of Harvey and Siddique (2000) and Ang et al. (2006), respectively, we control for coskewness and cokurtosis defined as

$$CSK_{t,i} = \frac{\frac{1}{60}\sum_{k=1}^{60}(r_{t,k,i,} - \bar{r}_{t,i})(f_{t,k} - \bar{f}_t)^2}{\sqrt{\frac{1}{60}\sum_{k=1}^{60}(r_{t,k,i,} - \bar{r}_{t,i})^2}\frac{1}{60}\sum_{j=1}^{60}(f_{t,j} - \bar{f}_t)^2} \tag{13}$$

$$CKT_{t,i} = \frac{\frac{1}{60}\sum_{k=1}^{60}(r_{t,k,i,} - \bar{r}_{t,i})(f_{t,k} - \bar{f}_t)^3}{\sqrt{\frac{1}{60}\sum_{k=1}^{60}(r_{t,k,i,} - \bar{r}_{t,i})^2}\frac{1}{60}\left(\sum_{j=1}^{60}(f_{t,j} - \bar{f}_t)^2\right)^{3/2}} \tag{14}$$

where $f_{t,j}$ is a return of the market factor with time average $\bar{f}_t$, and $\bar{r}_{t,i}$ denotes the time average of the asset return. Although the coskewness is highly significant with the expected sign, it does not drive out the significance of the CIQ(τ) betas, and both

[19] We obtained the liquidity factor data from the Lubos Pastor's personal website.

measures simultaneously predict stock returns. On the other hand, the cokurtosis does not exhibit any predictive power, and the CIQ(τ) betas remain significant.

Another approach to capturing non-linear dependence is via downside risk (DR) betas, which describe conditional covariance below some threshold level. We entertain two specifications of the DR betas, which differ in the threshold value. We use the specification of Lettau et al. (2014) – DR1, which uses as a threshold value average market return minus standard deviation of the market return, and the specification of Ang et al. (2006) – DR2, which sets the threshold value equal to the average market return. More specifically, DR betas are estimated using

$$\beta_i^{DR1} = \frac{\mathbb{C}ov(r_i, f | f < \mu_f - \sigma_f)}{\mathbb{V}ar(f | f < \mu_f - \sigma_f)}, \tag{15}$$

$$\beta_i^{DR2} = \frac{\mathbb{C}ov(r_i, f | f < \mu_f)}{\mathbb{V}ar(f | f < \mu_f)}. \tag{16}$$

In application, we employ the empirical counterparts of the measures. As we can see, neither of the specifications turns out to drive out or even be a significant predictor of future returns.

To summarize this subsection, we have shown that the CIQ(τ) results from the Fama-MacBeth regressions support the results obtained from the portfolio sorts. Namely, the exposure to the idiosyncratic left tail common events is priced in the cross-section of stock returns, and that none of the discussed risks drives out the significance of these results.

6 Conclusion

We investigate the pricing implications of the exposures to the common idiosyncratic quantile factors. These factors capture non-linear common movements in various parts of the distributions across a large panel of stocks. Similarly, as the quantile regression extends the classical linear regression, our quantile factor model of asset returns extends the approximate factor models used in empirical asset pricing literature. We observe that the expected returns are associated with the exposures to the common movements in various parts of the left tail of the distributions in contrast to the right tail. We perform various robustness checks to show that these results are not attributable to other previously proposed risk factors.

References

Aït-Sahalia, Y., J. Jacod, and D. Xiu (2020, November). Inference on risk premia in continuous-time asset pricing models. Working Paper 28140, National Bureau of Economic Research.

Allen, L., T. G. Bali, and Y. Tang (2012, 09). Does Systemic Risk in the Financial Sector Predict Future Economic Downturns? *The Review of Financial Studies 25*(10), 3000–3036.

Amengual, D. and E. Sentana (2020). Is a normal copula the right copula? *Journal of Business & Economic Statistics 38*(2), 350–366.

Amihud, Y. (2002). Illiquidity and stock returns: cross-section and time-series effects. *Journal of Financial Markets 5*(1), 31–56.

Andersen, T. G., N. Fusari, and V. Todorov (2020). The pricing of tail risk and the equity premium: Evidence from international option markets. *Journal of Business & Economic Statistics 38*(3), 662–678.

Ando, T. and J. Bai (2020). Quantile co-movement in financial markets: A panel quantile model with unobserved heterogeneity. *Journal of the American Statistical Association 115*(529), 266–279.

Ang, A., J. Chen, and Y. Xing (2006, 03). Downside Risk. *The Review of Financial Studies 19*(4), 1191–1239.

Ang, A., R. J. Hodrick, Y. Xing, and X. Zhang (2006). The cross-section of volatility and expected returns. *The Journal of Finance 61*(1), 259–299.

Annaert, J., M. De Ceuster, and K. Verstegen (2013). Are extreme returns priced in the stock market? european evidence. *Journal of Banking & Finance 37*(9), 3401–3411.

Bai, J. (2003). Inferential theory for factor models of large dimensions. *Econometrica 71*(1), 135–171.

Bai, J. and S. Ng (2002). Determining the number of factors in approximate factor models. *Econometrica 70*(1), 191–221.

Bali, T. G., N. Cakici, and R. F. Whitelaw (2011). Maxing out: Stocks as lotteries and the cross-section of expected returns. *Journal of Financial Economics 99*(2), 427–446.

Bali, T. G., K. O. Demirtas, and H. Levy (2008). Nonlinear mean reversion in stock prices. *Journal of Banking & Finance 32*(5), 767–782.

Bali, T. G., S. Gokcan, and B. Liang (2007). Value at risk and the cross-section of hedge fund returns. *Journal of Banking & Finance 31*(4), 1135–1166. Bricks versus Clicks:

The Changing Nature of Banking in the 21st Century.

Barro, R. J. (2006, 08). Rare Disasters and Asset Markets in the Twentieth Century*. *The Quarterly Journal of Economics 121*(3), 823–866.

Bollerslev, T., A. J. Patton, and R. Quaedvlieg (2021). Realized semibetas: Disentangling "good" and "bad" downside risks. *Journal of Financial Economics*.

Bollerslev, T., G. Tauchen, and H. Zhou (2009, 02). Expected Stock Returns and Variance Risk Premia. *The Review of Financial Studies 22*(11), 4463–4492.

Bollerslev, T. and V. Todorov (2011). Tails, fears, and risk premia. *The Journal of Finance 66*(6), 2165–2211.

Campbell, J. Y. and S. B. Thompson (2007, 11). Predicting Excess Stock Returns Out of Sample: Can Anything Beat the Historical Average? *The Review of Financial Studies 21*(4), 1509–1531.

Carhart, M. M. (1997). On persistence in mutual fund performance. *The Journal of Finance 52*(1), 57–82.

Chabi-Yo, F., M. Huggenberger, and F. Weigert (2022). Multivariate crash risk. *Journal of Financial Economics 145*(1), 129–153.

Chabi-Yo, F., S. Ruenzi, and F. Weigert (2018). Crash sensitivity and the cross section of expected stock returns. *Journal of Financial and Quantitative Analysis 53*(3), 1059–1100.

Chamberlain, G. and M. Rothschild (1983). Arbitrage, factor structure, and mean-variance analysis on large asset markets. *Econometrica 51*(5), 1281–1304.

Chen, L., J. J. Dolado, and J. Gonzalo (2021). Quantile factor models. *Econometrica 89*(2), 875–910.

Connor, G. and R. A. Korajczyk (1986). Performance measurement with the arbitrage pricing theory: A new framework for analysis. *Journal of Financial Economics 15*(3), 373–394.

de Castro, L. and A. F. Galvao (2019). Dynamic quantile models of rational behavior. *Econometrica 87*(6), 1893–1939.

Fama, E. F. and K. R. French (1993). Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics 33*(1), 3–56.

Farago, A. and R. Tédongap (2018). Downside risks and the cross-section of asset returns. *Journal of Financial Economics 129*(1), 69–86.

Giglio, S., B. Kelly, and D. Xiu (2022). Factor models, machine learning, and asset pricing. *Annual Review of Financial Economics 14*.

Giglio, S., D. Xiu, and D. Zhang (2021, July). Test assets and weak factors. Working Paper 29002, National Bureau of Economic Research.

Harvey, C. R. and A. Siddique (2000). Conditional skewness in asset pricing tests. *The Journal of Finance 55*(3), 1263–1295.

Herskovic, B., B. Kelly, H. Lustig, and S. Van Nieuwerburgh (2016). The common factor in idiosyncratic volatility: Quantitative asset pricing implications. *Journal of Financial Economics 119*(2), 249–283.

Hou, K. and R. K. Loh (2016). Have we solved the idiosyncratic volatility puzzle? *Journal of Financial Economics 121*(1), 167–194.

Huang, W., Q. Liu, S. Ghon Rhee, and F. Wu (2012). Extreme downside risk and expected stock returns. *Journal of Banking & Finance 36*(5), 1492–1502.

Jondeau, E., Q. Zhang, and X. Zhu (2019). Average skewness matters. *Journal of Financial Economics 134*(1), 29–47.

Kelly, B. and H. Jiang (2014, 06). Tail Risk and Asset Prices. *The Review of Financial Studies 27*(10), 2841–2871.

Kelly, B. T., S. Pruitt, and Y. Su (2019). Characteristics are covariances: A unified model of risk and return. *Journal of Financial Economics 134*(3), 501–524.

Kozak, S., S. Nagel, and S. Santosh (2018). Interpreting factor models. *The Journal of Finance 73*(3), 1183–1223.

Lettau, M., M. Maggiori, and M. Weber (2014). Conditional risk premia in currency markets and other asset classes. *Journal of Financial Economics 114*(2), 197–225.

Lettau, M. and M. Pelger (2020). Factors that fit the time series and cross-section of stock returns. *The Review of Financial Studies 33*(5), 2274–2325.

Lu, Z. and S. Murray (2019). Bear beta. *Journal of Financial Economics 131*(3), 736–760.

Ma, S., O. Linton, and J. Gao (2021). Estimation and inference in semiparametric quantile factor models. *Journal of Econometrics 222*(1, Part B), 295–323. Annals Issue:Financial Econometrics in the Age of the Digital Economy.

Meligkotsidou, L., E. Panopoulou, I. D. Vrontos, and S. D. Vrontos (2014). A quantile regression approach to equity premium prediction. *Journal of Forecasting 33*(7), 558–576.

Pastor, L. and R. F. Stambaugh (2003). Liquidity risk and expected stock returns.

Journal of Political Economy 111(3), 642–685.

Renault, E., T. van der Heijden, and B. J. Werker (2019). Arbitrage pricing theory for idiosyncratic variance factors. *Available at SSRN 3065854*.

Rietz, T. A. (1988). The equity risk premium a solution. *Journal of Monetary Economics 22*(1), 117–131.

Ross, S. A. (1976). The arbitrage theory of capital asset pricing. *Journal of Economic Theory 13*(3), 341–360.

Routledge, B. R. and S. E. Zin (2010). Generalized disappointment aversion and asset prices. *The Journal of Finance 65*(4), 1303–1332.

Van Oordt, M. R. and C. Zhou (2016). Systematic tail risk. *Journal of Financial and Quantitative Analysis 51*(2), 685–705.

Welch, I. and A. Goyal (2007, 03). A Comprehensive Look at The Empirical Performance of Equity Premium Prediction. *The Review of Financial Studies 21*(4), 1455–1508.

A Appendix

Figure 6: *Average pairwise correlations.* The figure captures yearly average pairwise time-series correlations between monthly excess returns or FF3 residuals of the CRSP stocks. Figures partially replicate results of Herskovic et al. (2016).

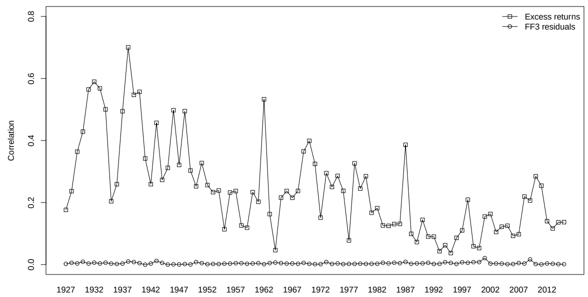
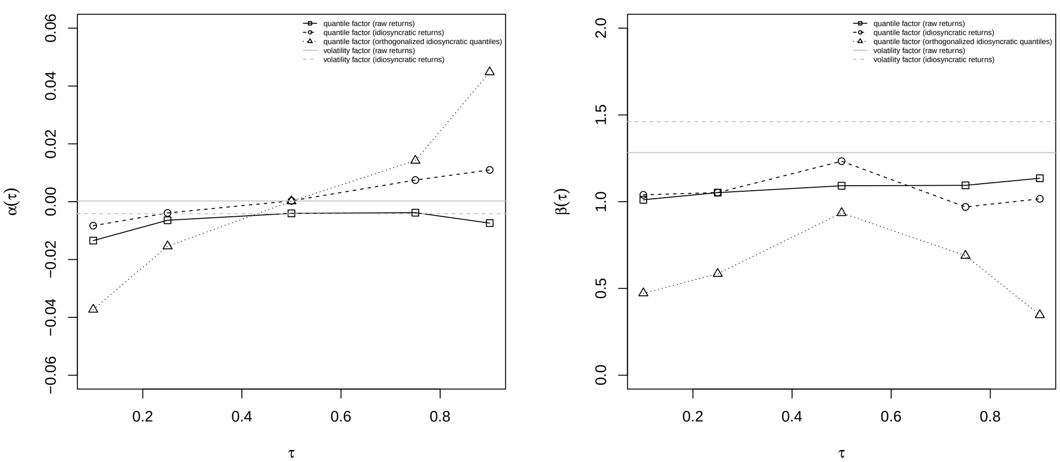


Table 6: *Correlations between the monthly data CIQ(τ) factors.* We present the results of the unconditional correlations between estimated monthly data CIQ(τ) factors. We estimate the factors using FF3 residuals of the CRSP stocks' returns. Data contain the period between January 1963 and December 2015.

	τ	0.1	0.15	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.85	0.9
	0.1	1.000	0.981	0.951	0.853	0.640	0.165	0.041	-0.196	-0.426	-0.503	-0.554
	0.15	0.981	1.000	0.984	0.912	0.725	0.229	0.106	-0.110	-0.354	-0.441	-0.504
	0.2	0.951	0.984	1.000	0.955	0.799	0.278	0.160	-0.014	-0.265	-0.357	-0.430
	0.3	0.853	0.912	0.955	1.000	0.921	0.391	0.284	0.206	-0.049	-0.150	-0.240
	0.4	0.640	0.725	0.799	0.921	1.000	0.533	0.458	0.489	0.266	0.171	0.075
Level	0.5	0.165	0.229	0.278	0.391	0.533	1.000	0.860	0.558	0.443	0.403	0.346
	0.6	0.041	0.106	0.160	0.284	0.458	0.860	1.000	0.616	0.522	0.494	0.442
	0.7	-0.196	-0.110	-0.014	0.206	0.489	0.558	0.616	1.000	0.940	0.891	0.824
	0.8	-0.426	-0.354	-0.265	-0.049	0.266	0.443	0.522	0.940	1.000	0.983	0.948
	0.85	-0.503	-0.441	-0.357	-0.150	0.171	0.403	0.494	0.891	0.983	1.000	0.980
	0.9	-0.554	-0.504	-0.430	-0.240	0.075	0.346	0.442	0.824	0.948	0.980	1.000
	0.1	1.000	0.972	0.944	0.875	0.757	0.411	0.321	0.327	0.140	0.037	-0.053
	0.15	0.972	1.000	0.980	0.931	0.831	0.473	0.376	0.417	0.225	0.116	0.017
	0.2	0.944	0.980	1.000	0.965	0.881	0.509	0.421	0.491	0.302	0.192	0.091
	0.3	0.875	0.931	0.965	1.000	0.953	0.582	0.498	0.618	0.440	0.331	0.224
	0.4	0.757	0.831	0.881	0.953	1.000	0.683	0.596	0.754	0.610	0.515	0.413
Increments	0.5	0.411	0.473	0.509	0.582	0.683	1.000	0.894	0.664	0.579	0.542	0.482
	0.6	0.321	0.376	0.421	0.498	0.596	0.894	1.000	0.655	0.574	0.544	0.492
	0.7	0.327	0.417	0.491	0.618	0.754	0.664	0.655	1.000	0.943	0.888	0.810
	0.8	0.140	0.225	0.302	0.440	0.610	0.579	0.574	0.943	1.000	0.975	0.924
	0.85	0.037	0.116	0.192	0.331	0.515	0.542	0.544	0.888	0.975	1.000	0.970
	0.9	-0.053	0.017	0.091	0.224	0.413	0.482	0.492	0.810	0.924	0.970	1.000

Figure 7: *Factor structure in quantiles.* The figure depicts average $\alpha_i(\tau)$ and $\beta_i(\tau)$ from time-series regression of CRSP stock-level quantiles (volatilities) on the averaged value of quantile (volatility) across all stocks. The quantile (or volatility) factor is defined as the equal-weighted cross-sectional average of firm quantiles (volatilities) within a year computed on monthly data. The estimated factor models take form: $q_{i,t}^{\tau} = \alpha_i(\tau) + \beta_i(\tau)\bar{q}_t^{\tau}(x) + v_{i,t}^{\tau}$ and $\sigma_{i,t}^2 = \alpha_i + \beta_i\bar{\sigma}_t(x) + v_{i,t}$. We report $\alpha_i(\tau)$ and $\beta_i(\tau)$ for a factor model of (i) raw return quantile on average return quantile in black line (ii) idiosyncratic quantiles on average idiosyncratic quantiles in black dashed line (iii) idiosyncratic quantiles on average idiosyncratic quantiles orthogonalized by average volatility in black dotted line across all $\tau \in (0, 1)$ quantile levels. In addition, a factor model of (iv) raw volatility on average volatility in gray line, (v) idiosyncratic volatility on average idiosyncratic volatility in gray dashed line. Note that results for factor model on volatilities are constant across quantiles.



(a) **(b)**

Table 7: *Five univariate sorted portfolios using monthly data.* The table contains annualized out-of-sample excess returns of five portfolios sorted on the exposure to the ΔCIQ(τ) factors computed from the monthly data. Panel A reports results based on the one-month returns following the formation period, Panel B results for the twelve-month period. We use all the CRSP stocks that have all 60 monthly observations in each window. We exclude penny stocks with prices less than 1$. We report returns of the high minus low (H - L) portfolios, their t-statistics, and annualized 5-factor alphas with respect to the four factors of Carhart (1997) and CIV shocks of Herskovic et al. (2016). We also report t-statistics for these alphas. Data contain the period between January 1963 and December 2015.

τ	Low	2	3	4	High	H - L	t-stat	α	t-stat
Panel A: One-month returns									
				Equal-weighted					
0.1	8.020	9.467	9.672	9.737	11.629	3.608	2.442	5.066	3.922
0.15	7.971	9.519	9.563	10.238	11.232	3.261	2.210	5.051	4.049
0.2	7.999	9.484	9.902	9.918	11.221	3.222	2.244	5.039	4.166
0.3	7.884	9.893	9.778	9.824	11.147	3.263	2.529	5.101	4.483
0.4	8.138	9.845	9.670	9.861	11.008	2.870	2.310	4.321	3.704
0.5	8.344	9.542	9.661	10.317	10.630	2.286	1.361	2.569	1.481
0.6	8.760	9.827	9.537	9.939	10.434	1.674	1.094	1.090	0.755
0.7	9.122	9.123	9.841	9.368	11.045	1.923	1.206	1.027	0.738
0.8	9.567	8.872	9.726	9.883	10.442	0.875	0.457	-0.405	-0.269
0.85	9.463	9.241	9.448	9.968	10.364	0.901	0.429	-0.586	-0.369
0.9	9.495	9.345	9.742	9.906	9.996	0.501	0.227	-1.225	-0.771
				Value-weighted					
0.1	5.309	6.422	6.348	6.193	8.905	3.597	1.700	3.482	2.197
0.15	4.842	6.707	6.469	6.302	8.345	3.503	1.650	4.396	2.874
0.2	5.259	6.794	6.009	6.850	7.660	2.400	1.147	3.745	2.521
0.3	5.673	6.447	6.312	6.351	6.582	0.908	0.501	2.871	1.999
0.4	5.492	6.390	6.733	6.080	6.156	0.664	0.358	2.995	1.619
0.5	5.494	5.639	6.101	6.590	5.878	0.384	0.194	0.851	0.406
0.6	5.621	5.967	5.823	6.601	5.860	0.239	0.117	0.721	0.345
0.7	6.185	6.294	6.064	5.899	7.232	1.047	0.538	0.837	0.429
0.8	6.043	5.934	6.452	6.296	6.266	0.223	0.097	-0.243	-0.122
0.85	5.984	5.625	6.588	6.201	6.948	0.964	0.402	0.093	0.047
0.9	5.631	5.975	6.629	6.695	6.214	0.583	0.231	-0.601	-0.327
Panel B: Twelve-month returns									
				Equal-weighted					
0.1	10.456	11.023	11.082	11.250	13.384	2.929	2.905	3.726	3.550
0.15	10.565	11.150	10.952	11.242	13.285	2.720	2.732	3.463	3.344
0.2	10.659	11.128	10.883	11.289	13.235	2.575	2.638	3.079	2.936
0.3	10.782	11.090	10.963	11.106	13.252	2.471	2.723	2.895	2.540
0.4	11.021	10.917	10.884	11.041	13.326	2.305	2.887	2.545	2.188
0.5	10.645	10.938	10.873	11.442	13.289	2.643	2.945	2.534	2.412
0.6	10.501	11.113	10.827	11.442	13.303	2.803	3.068	1.436	1.174
0.7	11.364	10.925	10.845	10.982	13.073	1.709	1.555	-0.444	-0.334
0.8	11.712	10.603	10.992	11.133	12.748	1.035	0.758	-1.752	-1.089
0.85	11.622	10.631	11.046	11.286	12.602	0.980	0.670	-1.832	-1.123
0.9	11.453	10.776	11.067	11.387	12.505	1.052	0.699	-1.437	-0.904
				Value-weighted					
0.1	6.174	7.127	6.648	6.954	8.986	2.812	2.068	3.214	2.597
0.15	6.369	7.411	6.672	6.840	8.482	2.113	1.577	2.712	2.254
0.2	6.724	7.336	6.702	6.872	8.052	1.329	1.023	1.953	1.630
0.3	6.860	7.370	6.684	6.935	7.060	0.200	0.178	1.205	0.941
0.4	7.019	7.151	6.787	6.563	6.823	-0.196	-0.183	0.763	0.535
0.5	6.006	6.599	6.644	6.998	7.005	0.999	0.894	0.725	0.693
0.6	5.561	6.346	6.799	7.060	7.228	1.668	1.492	1.242	1.111
0.7	6.732	6.400	7.112	6.606	7.438	0.707	0.547	-0.843	-0.743
0.8	6.579	6.604	6.960	6.739	7.317	0.737	0.494	-1.013	-0.759
0.85	6.393	6.391	7.363	6.619	7.376	0.983	0.608	-0.955	-0.701
0.9	6.373	6.504	7.297	6.858	7.128	0.755	0.450	-1.438	-1.002

Table 8: *Five bivariate sorted portfolios.* The table contains annualized out-of-sample excess returns of ten portfolios double sorted on the exposure to the ΔCIQ(τ) factors and ΔCIV. First, we perform sorts based on the ΔCIV betas, then, within each ΔCIV portfolio, we sort on the ΔCIQ(τ) betas, and then we collapse all the ΔCIV portfolios for a given ΔCIQ(τ) portfolio into one. The obtained portfolios vary in their ΔCIQ(τ) betas but not in their ΔCIV betas. We report returns of the high minus low (H-L) portfolios, their t-statistics, and annualized 5-factor alphas with respect to the four factors of Carhart (1997) and CIV shocks of Herskovic et al. (2016). We also report t-statistics for these alphas. Data contain the period between January 1963 and December 2015.

τ	Low	2	3	4	High	H - L	t-stat	α	t-stat
				Equal-weighted					
0.1	8.250	9.507	9.656	10.331	10.761	2.511	1.898	3.565	3.380
0.15	8.068	9.751	9.932	10.162	10.586	2.518	1.883	3.837	3.712
0.2	8.048	9.702	10.132	10.097	10.523	2.475	1.901	3.835	3.772
0.3	8.064	9.891	10.020	10.044	10.487	2.423	2.017	3.806	3.785
0.4	8.468	10.113	9.877	9.725	10.313	1.845	1.728	3.117	3.161
0.5	9.122	10.141	10.087	9.730	9.403	0.282	0.208	0.538	0.408
0.6	8.938	10.117	10.171	9.890	9.365	0.427	0.328	0.374	0.319
0.7	9.331	9.556	9.749	9.753	10.093	0.762	0.545	-0.041	-0.034
0.8	9.546	9.729	9.667	9.493	10.037	0.491	0.291	-0.554	-0.431
0.85	9.632	9.819	9.531	9.717	9.769	0.137	0.073	-1.002	-0.729
0.9	9.758	9.830	9.541	9.749	9.597	-0.161	-0.083	-1.389	-0.954
				Value-weighted					
0.1	5.647	7.704	7.826	7.466	8.090	2.443	1.321	2.618	2.027
0.15	5.975	7.311	8.273	7.662	7.396	1.421	0.799	1.951	1.693
0.2	5.876	7.564	7.967	7.968	6.999	1.123	0.648	1.741	1.495
0.3	6.121	7.684	7.827	7.908	6.554	0.433	0.261	1.433	1.139
0.4	6.169	7.940	7.688	7.038	6.659	0.490	0.322	1.762	1.234
0.5	6.411	7.439	7.574	7.432	6.419	0.008	0.004	0.422	0.225
0.6	6.158	7.433	7.470	7.499	6.686	0.528	0.294	0.610	0.342
0.7	6.187	7.738	7.760	6.990	7.380	1.193	0.694	0.551	0.342
0.8	6.298	7.900	7.310	6.903	7.418	1.120	0.567	0.608	0.360
0.85	6.527	7.680	7.477	7.131	6.960	0.433	0.199	-0.217	-0.123
0.9	7.124	7.462	7.582	7.388	6.797	-0.327	-0.143	-0.903	-0.541