Co-movement dynamics and disruptions of the major stock markets

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Abstract

We apply three approaches to investigate the evolution of the time varying (TV) multivariate cointegration of the G7 stock markets from 1970 to 2022. Our work contributes to the empirical literature on cointegration by illustrating the extent to which markets equilibrium relations are fallen apart during periods of time. The evolution of the markets interdependence is illustrated by investigating the instability and the strength of the markets co-movement over time. We find that the growing globalization of the world economies restrains markets segmentation during recent economic crises in particular the global financial and the covid-19 crises. We also find that during the period 1980s the cointegration of G7 stock markets is destabilized far more than during the following decades. The growing China is found to negatively impact 7-variate cointegration of the G7 in the recent decade whereas the 8-variate cointegration is strengthened. We also find that the quadrivariate cointegration of the European stock markets becomes steady from the late 1990s to date.

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1 Introduction

International stock markets cointegration has been subject of a wide range of empirical investigations. Empirical evidence suggests that market linkages do not follow a static linear relation over time. Many types of findings imply a rather complicated evolution of the linkage among stock markets. This complexity can be a result of continuous as well as concurrent changes in the technological, economic or political circumstances.

The linkage among market indices is evidenced in the literature to bear either abrupt structural breaks or to be continuously varying. In a number of studies, it is assumed that the linear relation can be broken up following a shift in the economic circumstances[1]. A regime shift or an unknown sudden shift in the cointegration relation has been also widely used in the literature to explore the dynamics in the markets co-movement[2]. Another set of of studies assume a continuously varying cointegration and adopt a rolling time window approach or a recursive estimation to explore the dynamics of the cointegration[3].

One major takeaway of the empirical findings in the literature is that markets cointegration undergoes periods of intensity and weakness. It can also bear periods of stability and instability. The globalization and integration of the world economies on one hand, and the political as well as the economic uncertainties associated with the persistence of the integration process on the other hand, can intermittently disrupt and boost the integration process. For example, markets liberalization and abolition of the exchange rate control is found to leave positive impact on markets cointegration (see Taylor & Tonks (1989)). Furthermore, the uncertainties associated with the trade war between China and the US can potentially leave negative impact on markets cointegration.

To date, the econometric tools that have been developed and employed in order to study these TV relations have not enabled empirical researchers to fully uncover the nature of these linkage dynamics. The empirical works on TV cointegration of stock markets suffer from two major drawbacks. First, the results depend on the underlying time window in all

[1] The papers that address a sudden pre-identified moment for the shift in the cointegrating vectors, include Taylor & Tonks (1989), Arshanapalli & Doukas (1993), Bachman et al. (1996), Kanas (1998), Aggarwal & Kyaw (2005), Aladesanmi et al. (2019) and Patel (2019).

[2] See for example Voronkova (2004), Davies (2006), Lucey & Voronkova (2008) and Menezes et al. (2012).

[3] For example see Crowder & Wohar (1998), Pascual (2003), Gilmore et al. (2008), Awokuse et al. (2009), Mylonidis & Kollias (2010) and Chien et al. (2015).

the approaches, ranging from the recursive estimation to the sub-period analysis. Second, they present no TV measure standing for the strength of the linkage among markets. The few works aiming at investigating the integration of the stock markets use the trace statistic of the recursive cointegration or of the rolling time window to illustrate the integration progress (see for example Mylonidis & Kollias (2010) and Chien et al. (2015)).

We investigate the dynamics of the cointegration by exploring periods of intensification and weakening of the linkage among the G7 stock market indices. We address the question of the dynamic process of convergence among the G7 stock market indices and introduce two measures of the cointegration strength and instability that can capture the convergence process. A methodological innovation in the area of cointegration enables us to estimate a TV measure representing the evolution of the intensity in the multivariate cointegration. In order to tackle the question of varying intensity and weakness of the long run equilibrium among stock market indices, we aim at estimating TV "lambdas" which, in the static form introduced by Johansen (1988), represent the strength of cointegration. As a measure of instability, the shifts in the normal vector of the spanning space of the TV cointegrating vectors fulfill our objectives.

Our main empirical objective is to study the evolution of the multivariate cointegration among G7 stock market indices and the potential impact of the growing China on the cointegration dynamics of the world's major economies. To this end, we focus on the instability and the strength of the long run markets interdependence over the past half century. The state-space modeling of the Vector Error Correction Model (VECM) allows us to estimate the TV eigenvalues which also paves the way to investigate the evolution of the strength of multivariate cointegration. Moreover, the TV cointegrating vectors estimated by state-space formulation make it possible to study the instability of the spanning space of the cointegrating vectors.

We believe that the approach advocated in this paper is suitable in order to faithfully estimate the cointegration dynamics. To the best of our knowledge, the theoretical developments in the literature do not provide a satisfactory way to estimate TV lambdas of the VECM. For example in the work by Bierens & Martins (2010), the cointegrating vectors are smoothly varying, whereas the lambdas are assumed constant. The recent work by Eroğlu et al. (2022) uses a single equation model to estimate TV cointegration which lacks the possibility to estimate a TV measure of cointegration strength. Our work contributes to the literature by providing a way to estimate not only the TV cointegrating vectors, but also the TV lambdas. This allows us to explore how the degree of markets cointegration is evolved over time. We further address the impact of the surge of China on the evolution of the cointegration degree among the world's most developed stock markets. This approach can shed more light on the evolution of the extent to which G7 and China's stock markets have been cointegrated over the past decades starting from 1969 to 2022, as it has been evidenced in the literature that the long run equilibrium relation can be broken up by a number of political, technological end economic events. In particular, we address the question whether globalization has contributed to stronger and more stable cointegration over time. We also document to what extent the potential segmenting events of the markets lead to any adjustment in the long run equilibrium relations.

We find that globalization has contributed to stronger interdependence among the G7 stock markets until 2010. The economic uncertainties in the 1980s and during the beginning of the 1990s are found to weigh on markets co-movement. Since then, the cointegration of the G7 stock markets has intensified until shortly after the global financial crisis. We also find that the growing integration of China to the world economy is a source that has weakened the linkage of the G7 markets in the last decade. The uncertainties concerning the future of China's economic growth seems to have negatively impacted the international markets cointegration. The impact of the economic crises on markets segmentation seems to have been diminished over time. A number of events including the technological innovation in the production of crude oil, shift in the monetary policy and the uncertainties concerning the trade war between China and US, are found to disrupt markets linkages.

This paper is organized as follows. Section 2 presents a review on the literature with a focus on empirical findings on the cointegration of world major stock markets. In section 3 our approach to estimate TV cointegration is explained. Section 4 discusses empirical findings on the evolution of the multivariate cointegration of G7 stock markets and section 5 concludes.

2 Literature review

2.1 *Theoretical developments on TV cointegration*

The concept of cointegration and its relevancy for the actual economic variables was first introduced by Granger (1981). The representation theorem introduced by Engle & Granger (1987) illustrated the theoretical links between the vector autoregressive (VAR), the VECM and the cointegration relation of the underlying series. The work also provided a two-step method to estimate and test for cointegration. The landmark work by Johansen (1988) provided a formulation to estimate the independent vectors spanning the space of the cointegrating vectors. The work also provided a rank test with the asymptotic theory to test for the number of the cointegrating vectors[4].

The empirical findings supporting TV cointegration relation motivated theoretical researchers to develop a methodology to estimate TV cointegration. Gregory & Hansen (1996) consider sudden shift in the cointegration relation and develop a test for identifying the possible break point. Park & Hahn (1999) developed an approach to estimate TV cointegration with smoothly evolving coefficients. Saikkonen & Choi (2004) use the smooth transition model to estimate single equation TV cointegration. Hansen & Johansen (1999) fix the parameters associated with the short term dynamics to the full sample and recursively estimate the long run parameters of the VECM to identify possible structural breaks.

Bierens & Martins (2010) developed an approach to estimate multivariate TV cointegration in which the smoothly varying cointegrating vectors are expressed in terms of the Chebyshev time polynomials. Their estimation theory is founded on Johansen (1988) with the TV cointegrating vector being estimated by an augmented form of VECM in which Y_{t-1} is replaced by a vector whose components are products of the Chebychev time polynomials and Y_{t-1}.

Eroğlu et al. (2022) develop a single equation cointegration model in which the cointegrating coefficients and the stationary residual follow distinct autoregressive processes. As-

[4]A number of the works developed asymptotic theory for the distribution of the estimated parameters in cointegration relations (see the general asymptotic theory developed by Phillips & Durlauf (1986), the asymptotic properties of the OLS and the non-linear least squares developed by Stock (1987) and the single equation bias corrected estimator known as the fully modified developed by Phillips & Hansen (1990)).

suming non-Gaussian distribution for the residuals, they develop a semi-parametric procedure to test the null hypothesis of static cointegration against the alternative of TV cointegration.

2.2 *Empirical studies*

Since our objective is to study the evolution of the cointegration among the G7 stock markets, we focus on the empirical studies on the most developed stock markets. Given that the developments in the technological environment and changes in economic policies can lead to adjustments in the long run relation among stock market indices, a number of works evidenced variation in the cointegration relation among stock market indices.

A series of works consider abrupt structural breaks in the cointegration relation and test for identifying the possible break point. Taylor & Tonks (1989) focus on the abolition of the UK exchange rate control in 1979 on the bivariate cointegration between UK and either of the US, Germany, Japan and the Netherlands from 1973 to 1986. The null hypothesis of no cointegration in the pre-abolition period is not rejected, whereas during the post-abolition period, supporting evidence for cointegration with other markets else than the US is found. Arshanapalli & Doukas (1993) study the bivariate cointegration between the US stock market and either of the four markets of the UK, Germany, France and Japan from 1980 to 1990 and focus on the stock market crash in 1987. They find no evidence of cointegration in the pre-crash subperiod, whereas the analysis supports cointegration during the post-crash subperiod in three bivariate relations. Bachman et al. (1996) examine multivariate cointegration of the G7 countries from 1970 to 1989 and find two cointegration in each of the two decade-long subperiods. The European countries are found to have one cointegration in each of the two decade-long subperiods. The entire period provides weaker evidence of cointegration. Kanas (1998) investigates bivariate cointegration between the US stock market and that of the either of the European major markets including the UK, France, Germany, Italy, Netherlands and Switzerland over the period ranging from 1983 to 1996. They find no significant cointegration in neither of the pre-crash nor the post-crash sub-periods. The analysis on the entire period reveals no evidence of significant cointegration either. Aladesanmi et al. (2019) study the bivariate cointegration between the US and UK stock markets in three subperiods representing three monetary regimes from 1935 to 2015 and find stronger evidence of cointegration during the post Bretton Woods regime.

A number of the works do not consider a priori known breaks in the long rune co-movements, rather investigate variations in the relation or switching between equilibrium regimes. Crowder & Wohar (1998) perform the recursive cointegration analysis on the 5-variate cointegration, including stock market indices of the US, UK, Germany, Canada and Japan. The results of the Johansen rank test on the empirical data from 1974 to 1990 supports weak evidence of time invariant (TI) cointegration; nevertheless, the constancy test statistic of the cointegrating vectors decline until the end of the period. Assuming two equilibrium regimes, Davies (2006) finds strong evidence of 7-variate TV cointegration among stock markets of the US, UK, Germany, Japan, Canada, Australia and Switzerland. Using a 2-year rolling time window, Awokuse et al. (2009) apply Johansen's multivariate cointegration test to three developed stock markets (i.e. US, UK and Japan) and nine Asian markets. Their findings support the positive impact of financial markets liberalization during early 1990s on markets integration.

3 Multivariate TV cointegration

Johansen (1988) uses canonical correlation analysis to estimate the linearly independent cointegrating vectors from the vector error correction model (VECM). Canonical correlations are considered as measures standing for the strength of the linear relations, and the TV canonical correlations can then be used to investigate the evolution of the cointegration relations. Suppose Y_t is a vector containing n stock indices. The VECM can be represented by equation 1:

$$\Delta Y_t = C + \alpha\beta' Y_{t-1} + \sum_{i=1}^{k} \Gamma_i \Delta Y_{t-i} + \varepsilon_t \tag{1}$$

with $E[\varepsilon_t] = 0$ and $E[\varepsilon_t \varepsilon'_\tau] = \Omega$ when $t = \tau$ and $E[\varepsilon_t \varepsilon'_\tau] = 0$ otherwise. β is a $(n \times r)$ matrix composed of the TI cointegrating vectors and α is a $(n \times r)$ matrix of adjustment coefficients. Johansen's approach to estimate independent cointegrating vectors involves two sets of regressions, the first of which is to regress the first differenced vector ΔY_t on a constant and its lagged vectors i.e. $\Delta Y_{t-1}, \ldots, \Delta Y_{t-k}$, and the second is to regress the lagged level variable

Y_t on a constant and the lagged first differenced variables $\Delta Y_{t-1}, \ldots, \Delta Y_{t-k}$. The residuals of these two regressions are stored in u_t and v_t respectively. Let Σ_{uu}, Σ_{uv}, Σ_{vv} and Σ_{vu} represent the mutual variance-covariance matrices associated with u and v. Let $\hat{\beta}$ denote the eigenvectors of the matrix $\Sigma_{vv}^{-1}\Sigma_{vu}\Sigma_{uu}^{-1}\Sigma_{uv}$ normalized by $\hat{\beta}'\Sigma_{vv}\hat{\beta} = I$ with the corresponding eigenvalues $\lambda = (\lambda_1, \ldots, \lambda_n)$ ordered as $\lambda_1 \geq \cdots \geq \lambda_n$. Johansen (1988) proved that the normalized eigenvectors $\hat{\beta}$ and the matrix $\hat{\alpha} = \Sigma_{vv}\hat{\beta}$ maximize the likelihood function of the VECM in equation 1.

As the λ_is equal the squared canonical correlations between the first differenced variables and the lagged level ones, each λ_i represents the fit of the corresponding equilibrium error associated with a certain cointegrating vector and a stationary series. Thus the TV λ_is provide us with the information on the evolution of the fit associated with each equilibrium and a stationary series. We aim at the estimation of the $\lambda_{i,t}$s by state-space representation of the VECM. As it will be explained in the next subsections, the state-space approach provides us with the $\lambda_{i,t}$s associated with the TV cointegrating vectors.

3.1 *Time-Varying VECM*

We base our estimation of TV cointegration on state-space formulation of the VECM in equation 1. Particularly we aim at estimating TV impact matrix by the following specification of the VECM, assuming that the impact matrix follows a random walk:

$$\Delta Y_t = C + \Pi_t Y_{t-1} + \sum_{i=1}^{k} \Gamma_i \Delta Y_{t-i} + \varepsilon_t \qquad \text{(2a)}$$

$$\Pi_t = \Pi_{t-1} + \delta_t \qquad \text{(2b)}$$

where $\Pi_t = \alpha_t \beta_t'$ is a $(n \times r)$ matrix composed of the TV cointegrating vectors and α_t is a $(n \times r)$ matrix of TV adjustment coefficients. $E[\varepsilon_t] = 0$ and $E[\varepsilon_t \varepsilon_\tau'] = \Omega$ when $t = \tau$ and $E[\varepsilon_t \varepsilon_\tau'] = 0$ otherwise. The intercept in equation 2a is fixed to full sample whereas Π_t is varying. Having Fixed part of the parameters to the full sample, and letting the other parameters to vary, is an approach used in the literature to investigate TV cointegration (see Hansen & Johansen (1999) and Hendry & Ericsson (1991)).

The well-known estimation procedure of Johansen ensures the estimation of the independent vectors spanning the space of the cointegrating vectors. Since in this paper our objective is to investigate the sources of markets segmentation and the evolution of the interdependence of the markets in light of the globalization, the state-space modeling of the VECM meets our objectives. In other words, the evolution of the Π_t can shed light on the periods of instability in the cointegrating vectors, which allows us to identify sources of the shocks to the long run equilibrium relation.

We apply Kalman filter independently to each row of the TV VECM to estimate the state variables in equation 2b. Given that this approach involves no binding condition to estimate independent cointegrating vectors, the rank of the impact matrix Π_t, at each time, can vary from 1 to n. This fact can be more illustratively explained in terms of the common trend representation by Stock & Watson (1988). Stock & Watson (1988) prove that the vector Y_t can be represented as follows:

$$Y_t = Y_0 + A\tau_t + a_t \tag{3a}$$

$$\tau_t = \pi + \tau_{t-1} + \nu_t \tag{3b}$$

where τ_t is a $(n \times 1)$ vector of random walks, A is a $(n \times n)$ matrix, a_t is a $(n \times 1)$ vector of stationary processes, Y_0 and π are the intercepts and v_t is the vector of the innovations. The rank of the matrix A represents the number of the common trends. If the matrix A is assumed to vary, then there is, by construction, no constraint on the rank of the matrix and thus it can also change over time taking any value from 1 to n.

The Kalman filter yields the optimal values of the state-variables conditional on the prior realization. In other words, it represents the forecasted value of the TV cointegrating vectors one period ahead based on the actual value of the estimated state variables. Performing a Kalman smoothing can yield the unconditional ones. We obtain the smoothed values of the impact matrix to investigate the evolution of the unconditional space of the cointegrating vectors.

Johansen normalized the impact matrix by $\hat{\beta}_i' = \hat{\Pi}_i/(\hat{\Pi}_i\hat{\Sigma}_{vv}\hat{\Pi}_i')$ in which $\hat{\Pi}_i$ represents the i-th row of the impact matrix $\hat{\Pi}$, to obtain the cointegrating vectors satisfying $\hat{\beta}'\Sigma_{vv}\hat{\beta} = I$. Our primary objective to study the evolution of the cointegrating vectors can be fulfilled by considering the evolution of the space spanned by the impact matrix $\hat{\Pi}_t$, because the space remains invariant by the normalization.

The state-space modeling of the VECM allows us to examine the evolution of the extent to which markets are cointegrated in periods of economic and political instability. The TV cointegrating vectors span a space having the dimension equal to their rank. Let $rank(\hat{\beta}_t) = p$, then the space spanned by $\hat{\beta}_t$ forms a hyperplane in $\mathbb{R}^{p+1}$. This hyperplane is characterized by its normal vector which can serve as an indicator signaling any shift in the space of the cointegrating vectors. We take the shifts in the normal vector of the hyperplane as a proxy signaling the instability of the spanned space of the TV cointegrating vectors. In the n-variate case, without loss of generality we take $(n-1)$ number of the TV cointegrating vectors to investigate the variability of the spanned space. The angle between two consecutive normal vectors associated with the spanned space of the cointegrating vectors illustrates how sever the space is destabilized.

In addition to the instability, we estimate time-varying $\lambda_{i,t}$ as a set of measures representing the strength of markets co-movement. Johansen (1988) proved that the matrix β composed of the r cointegrating vectors and the corresponding eigenvalues λ_i satisfy equation 4:

$$\frac{|\beta'(\Sigma_{vv} - \Sigma_{vu}\Sigma_{uu}^{-1}\Sigma_{uv})\beta|}{|\beta'\Sigma_{vv}\beta|} = \prod_{i=1}^{n}(1 - \lambda_i) \tag{4}$$

where λ_is are the eigenvalues of $\Sigma_{vv}\Sigma_{uu}^{-1}\Sigma_{uv}$ with respect to Σ_{vv} ordered as $\lambda_1 \geq \cdots \geq \lambda_n$. The matrix β minimizing the left hand side of equation 4 together with the matrix $\alpha = \Sigma_{vv}\beta$ are the best fit matrices of the VECM in equation 1. The minimizing matrix of the quotient is composed of the eigenvectors associated with the eigenvalues λ_i. Moreover, given an eigenvector β_i, the value of the quotient equals $1 - \lambda_i$ in which the λ_i is the i-th squared canonical correlation of equation 1.

The quotient in equation 4 is the generalized Rayleigh quotient and can simply be reduced to a Rayleigh quotient $|x'Ax|/|x'x|$ where $A = H^{-1}(\Sigma_{vv} - \Sigma_{vu}\Sigma_{uu}^{-1}\Sigma_{uv})(H')^{-1}$, $\Sigma_{vv} = HH'$ and $x = H'\beta$. For a given vector x, the Rayleigh quotient of a covariance matrix can be expressed as the weighted sum of the eigenvalues in which the weights are the squared coordinates of x in the eigenbasis. It can also be expressed as the weighted sum of the squared cosines of the angles between x and the eigenvectors of the covariance matrix in which the weights are the eigenvalues. Moreover the Rayleigh quotient lies between the minimum and the maximum eigenvalues of the underlying matrix.

Let $\beta_{i,t}$ denote the i-th TV cointegrating vector estimated by the state-space modeling of the VECM in equation 2b. We calculate the TV parameter $\lambda_{i,t}$ by virtue of the TV $\beta_{i,t}$ in equation 5:

$$\lambda_{i,t} = 1 - \frac{|\beta'_{i,t}(\Sigma_{vv} - \Sigma_{vu}\Sigma_{uu}^{-1}\Sigma_{uv})\beta_{i,t}|}{|\beta'_{i,t}\Sigma_{vv}\beta_{i,t}|} \tag{5}$$

Note that the whole sample variance-covariance matrices are used in equation 5. The reason is that the quotient $\frac{|\Sigma_{vv} - \Sigma_{vu}\Sigma_{uu}^{-1}\Sigma_{uv}|}{|\Sigma_{vv}|}$ converges regardless of the fact that there exists significant cointegration relation or not. Now consider multiplying the numerator and the denominator of the quotient by $1/T$. The matrix $\Sigma_{vv} - \Sigma_{vu}\Sigma_{uu}^{-1}\Sigma_{uv}$ in the numerator is the variance-covariance matrix of the residuals of an orthogonal projection. Divided by T, it converges even in case of a spurious regression. The matrix $\Sigma_{vv} = 1/T\sum_{i=1}^{n} v_t v'_t$ in the denominator can also be written as the random variable $1/T^2\sum_{i=1}^{n} v_t v'_t$ which converges in distribution to $\Sigma^{1/2}\int_0^1 W_t W'_t\, dt\Sigma^{1/2}$ where W_t is a Wiener process and Σ is the long run variance-covariance matrix of v_t (for a detailed discussion see Phillips & Durlauf (1986)).

The measure $\lambda_{i,t}$ introduced in equation 5 lies between λ_1 and λ_n and serves us to examine the strength of the markets long run link over time. Note that the maximum value attained by $\lambda_{i,t}$ equals λ_1 because the variance-covariance matrices in 5 are the whole sample ones. The use of the whole sample variance-covariance matrices involves the implicit assumption that the components of the vector v_t follow unit root processes in which there is no structural break. Such a structural break in the evolution of the v_t, if exists, is reflected in the evolution of β_t.

3.2 *Single equation TV cointegration*

We also estimate the single equation TV cointegration model developed by Eroğlu et al. (2022), which allows us to test the null hypothesis of no TV cointegration against the alternative of TV cointegration.

$$y_t = \alpha + X'_t\beta_t + w_t \qquad t = 1, \ldots, T \tag{6a}$$

$$\beta_t = \phi\beta_{t-1} + \eta_t \tag{6b}$$

$$w_t = \theta w_{t-1} + \sum_{i=1}^{k} \delta_i \Delta w_{t-i} + \epsilon_t \tag{6c}$$

where y_t is the stock market index chosen as the dependent variable, and X_t is a $(p \times 1)$ vector of regressors. The TV cointegrating vector which can be written as $[1 \quad -\beta_{1,t} \quad \ldots \quad -\beta_{p,t}]'$, depends on the choice of the dependent variable. α, ϕ, θ and δ_is are scalars. The error vector η_t follows jointly *i.i.d.* normal distributions having zero mean and standard deviations σ_i for $i = 1, \ldots, p$. The error terms ϵ_t follow *i.i.d.* normal distributions having zero mean and standard deviations σ_w. We also assume that all the error terms have zero covariances.

3.3 *Smoothly varying cointegration*

Bierens & Martins (2010) develop a testing approach for the multivariate TV cointegration in which the cointegrating vectors are expressed in terms of Chebyshev time polynomials. The VECM can then be represented by:

$$\Delta Y_t = C + \alpha\left(\sum_{i=0}^{m} \xi_i P_{i,T(t)}\right)' Y_{t-1} + \sum_{i=1}^{k} \Gamma_i \Delta Y_{t-i} + \epsilon_t \tag{7}$$

where Y_t is a $(n \times 1)$ vector and the parameters C, α, Γ_i and ϵ_t are as explained in equation (1). $P_{i,T(t)}$ is the i-th Chebyshev time polynomial and defined by $P_{0,T(t)} = 0$ and $P_{i,T(t)} = \sqrt{2}\cos(i\pi(t-0.5)/T)$ for $i = 1, 2, \ldots$ where T is the sample size, m is the Chebyshev polynomial order and ξ_i is a $(n \times r)$ matrix. They generalize the limiting distribution of the ordered eigenvalues to a more general asymptotic theory in which the Wiener process is replaced by an augmented vector process composed of the multiplication of the Chebyshev polynomials and Wiener processes. Though their approach estimates TV cointegrating vectors, the eigenvalues do not vary with time and therefore the evolution of the strength of the multivariate cointegration remains unexplored.

4 Empirical analysis

4.1 *Data*

We use the monthly values of the MSCI indices of G7 stock markets, namely US, UK, France, Germany, Italy Japan and Canada, downloaded from Datastream. The MSCI indices are adjusted for the effect of cross listing. The analysis period starts from 1969 until June 2022 covering 631 months. All the indices are in terms of the US dollars and are transformed by the natural logarithms. In order to further analyze the impact of the Growing China on the multivariate cointegration of the G7 stock markets, we add MSCI index of China's stock market to the analysis. Figure 1 exhibits the normalized G7 and China stock market indices.

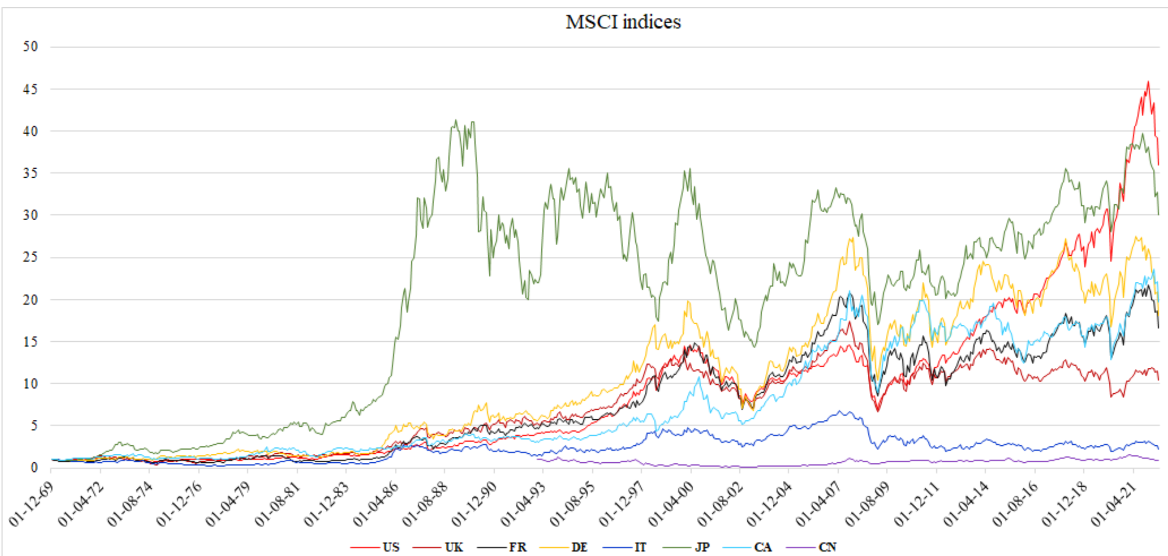

Figure 1: MSCI indices of G7 and China stock markets. The indices are in terms of the US dollars and are transformed by the natural logarithms

The descriptive statistics of the indices transformed by the natural logarithms is exhibited in Table 1. As the China stock market index starts from the December of 1992, the observation number of the index is 355 months. Since the indices are reported in terms of the US dollars, that of the Japan takes negative values after having transformed by the natural logarithms.

Table 1: Descriptive statistics of the G7 and China sotck market indices

Descriptive statistic	MSCI indices							
	US	UK	FR	DE	IT	JP	CA	CN
mean	6.21	7.04	6.43	5.77	6.31	1.35	6.13	1.94
std.	1.19	1.00	1.17	1.11	0.82	1.08	0.99	0.61
median	6.41	7.44	6.73	6.05	6.58	1.86	5.97	2.18
Kurtosis	-1.36	-1.08	-1.40	-1.32	-0.70	-0.24	-1.39	-0.44
Skewness	-0.08	-0.65	-0.43	-0.43	-0.66	-1.08	-0.03	-0.85
Min	4.17	4.60	4.34	3.68	4.39	-1.45	4.37	0.50
Max	8.43	8.34	7.97	7.21	7.66	2.45	7.70	2.89
Sample No.	631	631	631	631	631	631	631	355

4.2 *Unit root test*

The results of the unit root test are shown in Table 2. The augmented Dickey-Fuller test is applied twice for each index in two specifications, assuming an intercept with no trend and an intercept coupled with a trend. Only in the single case of the Japan, the null is rejected at 8% significance level.

Table 2: Augmented Dickey-Fuller test

	Intercept			Intercept and trend		
	t-statistic	p-value	Lag lengths	t-statistic	p-value	Lag lengths
US	0.02	0.96	0	-2.35	0.40	0
UK	-1.35	0.61	0	-1.30	0.89	0
FR	-0.89	0.79	0	-1.80	0.71	0
DE	-1.41	0.58	0	-1.76	0.72	0
IT	-1.20	0.68	0	-1.52	0.82	0
CA	-0.95	0.77	0	-3.13	0.10	0
JP	-2.68	0.08	0	-1.53	0.82	0
CN	-1.45	0.56	0	-2.20	0.49	0

The augmented Dickey-Fuller test is applied to the G7 and China stock market indices. The first columns assumes an intercept with no time trend in the model. The second column include both an intercept and a time trend in the model. The lag length is selected by BIC.

4.3 *Evidences on static cointegration and TV cointegration tests*

We also perform a TI cointegration test using Johansen trace as well as the maximum eigenvalue test statistics. The test is inclusive, accounting for a number of specifications in the cointegration equation and in the corresponding VAR model. The evidence supporting static 7-variate cointegration in Table 3 is quite weak, showing only one significant relation at 5%

significance level and no significant relation at 1% significance level. The results of the trace and maximum eigenvalue statistics at 10% significance level show one significant relations for only three model specifications. Given that the underlying time period is long enough, covering almost half a century, these results are not surprising. A number of events, potentially impacting markets co-movement, are realized in the past half century, some of which are floating exchange rate regime following Bretton woods, crude oil crisis in the aftermath of the political instability in the middle east, foundation of OPEC, introduction of Euro, liberalization of free capital flow and growing foreign investment.

Table 3: The number of cointegrating vectors by applying Johansen TI cointegration test

1% significance level					
Trend specification	None	None	Linear	Linear	Quadratic
Cointegration relation	No intercept	Intercept	Intercept	Intercept	Intercept
VAR	No intercept	No Intercept	No Intercept	No Intercept	Intercept
	No trend	No trend	No trend	Trend	Trend
Trace	0	0	0	0	0
Max-Eigenvalue	0	0	0	0	0
5% significance level					
Trend specification	None	None	Linear	Linear	Quadratic
Cointegration relation	No intercept	Intercept	Intercept	Intercept	Intercept
VAR	No intercept	No Intercept	No Intercept	No Intercept	Intercept
	No trend	No trend	No trend	Trend	Trend
Trace	1	0	0	0	0
Max-Eigenvalue	1	0	0	0	0
10% significance level					
Trend specification	None	None	Linear	Linear	Quadratic
Cointegration relation	No intercept	Intercept	Intercept	Intercept	Intercept
VAR	No intercept	No Intercept	No Intercept	No Intercept	Intercept
	No trend	No trend	No trend	Trend	Trend
Trace	1	0	0	0	0
Max-Eigenvalue	1	1	1	0	0

Johansen rank test is applied to MSCI market indices of G7 countries from Dec 1969 to Jun 2022. Both trace and maximum eigenvalue test statistics are estimated in five specifications. Each specification differs the others in assuming a time trend in the cointegration relation and an intercept in the cointegration relation or in the VAR.

To gain insight into the variability of the cointegrating vectors, we estimate the multivariate TV cointegration developed by Bierens & Martins (2010) to investigate parameter instability of the 7-variate cointegration relation. The corresponding null hypothesis is TI cointegration and the alternative hypothesis is TV cointegration. As Table 4 indicates, the null of TI cointegration is strongly rejected, given any number of cointegrating vectors and

Chebyshev polynomial order m.

Table 4: Likelihood ratio test of multivariate TV cointegration

	$m=1$	$m=2$	$m=3$	$m=4$	$m=5$
			$lags=1$		
$r=1$	20.94	36.02	58.23	94.71	125.85
	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)
$r=2$	47.18	81.00	131.71	175.26	228.77
	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)
$r=3$	67.62	117.20	188.80	246.30	317.80
	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)
$r=4$	94.75	161.35	239.31	309.67	399.11
	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)
$r=5$	110.43	198.84	284.09	370.24	469.82
	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)
$r=6$	121.86	226.88	329.26	420.03	536.69
			$lags=2$		
$r=1$	30.68	47.53	69.98	96.60	126.80
	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)
$r=2$	53.19	97.55	140.12	185.27	241.27
	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)
$r=3$	76.19	135.83	208.35	267.80	342.07
	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)
$r=4$	101.82	176.58	258.66	329.73	426.04
	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)
$r=5$	116.03	212.54	301.96	390.67	496.81
	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)
$r=6$	129.08	238.54	343.47	438.87	561.98
	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)

The likelihood ratio test of TV cointegration, developed by Bierens and Martins (2010), is applied to MSCI stock market indices of the G7 countries from Dec 1969 to Jun 2022. The parameters m and r represent Chebyshev polynomial order and the number of TV cointegrating vectors respectively. The test statistic follows Chi-squared distribution with mrk degrees of freedom where k stands for the number of the variables. The numbers in the parentheses represent the p-values.

We also estimate the TV single equation cointegration model developed model by Eroğlu et al. (2022) to test for TV cointegration in the 7-variate and 4-variate models. The results are presented in Table 5. The results of this model should depend on the choice of the dependent variable. Therefore, we perform the model in equation 6 to the 7-variate cointegration depending on the choice of the dependent variable. The results of the test are highly consistent with the results of the TV model developed by Bierens & Martins (2010) presented in Table 4. The autoregressive coefficient of the stationary residual series, i.e. the parameter θ, determines the existence of the cointegration. The $|\theta| < 1, T = 1$ and small standard deviations σ_i imply a smoothly varying cointegration relation. The coefficient of the autoregressive series w_t are estimated to be less than 1 in 6 relations out of the seven cointegration relations, whereas the null hypothesis of $\theta = 1$ is rejected in 5 cointegration relations. The autoregressive coefficient of the cointegrating vector i.e. $\hat{\phi}$ is significantly different than 1 in the case of the relation having Italy as the dependent variable. The estimated standard deviation of the $\hat{\phi}$ equals 0.0008 which is the lowest among all the cointegration relations.

Table 5: The estimation of the 7-variate cointegration using TV single equation model

Model parameters	US	UK	FR	DE	IT	JP	CA
$\hat{\beta}_{1,0}$	-0.1856	-0.2461	-0.5225	0.0362	-0.3862	3.3921	0.0502
$\hat{\sigma}_1$	0.100	0.0057	0.0067	0.0065	0.0042	0.0053	0.0048
$\hat{\beta}_{2,0}$	0.0698	0.0831	-0.3470	-0.7488	-3.3794	-1.2168	-0.0428
$\hat{\sigma}_2$	0.0052	0.0052	0.0047	0.0044	0.0037	0.0049	0.0042
$\hat{\beta}_{3,0}$	0.1961	0.0948	0.5775	0.5296	0.2860	-1.7654	-0.0029
$\hat{\sigma}_3$	0.0054	0.0049	0.0077	0.0062	0.0041	0.0063	0.0046
$\hat{\beta}_{4,0}$	-0.0310	-0.0132	0.2584	0.1102	1.2393	0.9587	0.0089
$\hat{\sigma}_4$	0.0041	0.0052	0.0062	0.0055	0.0052	0.0063	0.0051
$\hat{\beta}_{5,0}$	0.2835	0.1042	-0.2304	1.6516	-5.7239	-0.4058	-0.0105
$\hat{\sigma}_5$	0.0104	0.0127	0.0092	0.0182	0.0155	0.0059	0.0047
$\hat{\beta}_{6,0}$	0.2344	0.1582	0.1404	0.6712	1.4826	-0.4450	-0.1457
$\hat{\sigma}_6$	0.0045	0.0049	0.0069	0.0050	0.0045	0.0050	0.0165
$\hat{w}_0$	0.0205	0.0096	0.0028	0.0014	-0.0024	0.0088	0.0147
$\hat{\sigma}_w$	0.0182	0.0213	0.0116	0.0112	0.0427	0.0324	0.0268
Intercept	3.8887***	5.2908***	4.5160***	3.7416***	5.6022***	-0.8768***	4.3957***
	(0.1293)	*(0.1030)*	*(0.0771)*	*(0.0854)*	*(0.0009)*	*(0.1196)*	*(0.0927)*
$\hat{\phi}$	1.0010	1.001	1.0021	1.0027	0.9940***	1.0042	1.002
	(0.0494)	*(0.0510)*	*(0.0343)*	*(0.0369)*	*(0.0008)*	*(0.0484)*	*(0.0463)*
$\hat{\theta}$	0.7372	0.6285***	0.1415***	0.2503***	1.1167	0.6462***	0.4962***
	(0.2464)	*(0.1286)*	*(0.3135)*	*(0.2608)*	*(0.1004)*	*(0.0562)*	*(0.1373)*
max Log likelihood	839.23	842.22	813.68	861.04	701.59	787.66	896.26
BIC	-1568.85	-1574.84	-1517.76	-1612.48	-1293.57	-1465.71	-1682.91

The 7-variate cointegration is estimated for the G7 stock market indices using TV single equation in equation 6. The estimated relations are exhibited in the columns, each of which corresponds to different choices of the dependent variable. The $\hat{\beta}_{i,0}$s for $i = 1, \ldots, 6$ stand for the initial values of the cointegrating vectors in which the first component is normalized to one. The $\hat{w}_0$ stands for the initial value of the stationary component w. The $\hat{\sigma}_i$s for $i = 1, \ldots, 6$ and $\hat{\sigma}_w$ stand respectively for the standard deviation of the cointegrating vector and of the stationary residual series. The numbers in parentheses represent the standard deviations. The number of the lagged terms Δw_{t-i} are chosen according to BIC. *, ** and *** respectively stand for the significance at 1%, 5% and 10% significance levels. The test on ϕ is performed on the null hypothesis of $\phi = 1$ against the alternative of $\phi \neq 1$. The test on θ is performed on the one sided null hypothesis of $\theta = 1$ against the alternative of $\theta < 1$.

We estimate the quadrivariate single equation TV cointegration among European countries of the G7 in Table 6. The null hypothesis of $\theta = 1$ against the alternative hypothesis

$\theta < 1$ is strongly rejected in four models differing on the choice of the dependent variable. The autoregressive coefficient $\hat{\phi}$ of the cointegrating vectors is not significantly different than unity. The stationarity of the residuals, as evidenced in the parameter $\hat{\theta}$, and the presence of the unit root in the cointegrating vector imply smoothly varying cointegration in the quadrivariate relation.

Table 6: The estimation of the 4-variate cointegration using TV single equation model

Model parameters	UK	FR	DE	IT
$\hat{\beta}_{1,0}$	0.0459	-0.7370	-0.6129	-1.8998
$\hat{\sigma}_1$	0.0083	0.0084	0.0083	0.0083
$\hat{\beta}_{2,0}$	0.0340	0.3844	0.3976	-0.1007
$\hat{\sigma}_2$	0.0093	0.0102	0.0092	0.0090
$\hat{\beta}_{3,0}$	-0.0295	0.4978	0.3033	2.8282
$\hat{\sigma}_3$	0.0088	0.0094	0.0093	0.0101
$\hat{w}_0$	0.0180	0.0001	0.0005	0.0018
$\hat{\sigma}_w$	0.0466	0.0391	0.0446	0.0421
Intercept	5.2466***	4.4434***	3.5858***	5.5367***
	(0.0875)	*(0.0808)*	*(0.0877)*	*(0.0546)*
$\hat{\phi}$	0.9999	1.0021	1.0024	1.0040
	(0.0426)	*(0.0350)*	*(0.0358)*	*(0.0486)*
$\hat{\theta}$	0.3598***	-0.2371***	-0.0197***	0.0390***
	(0.1343)	*(0.1484)*	*(0.1799)*	*(0.0853)*
max Log likelihood	722.72	690.20	706.77	663.49
BIC	-1335.83	-1270.79	-1303.94	-1217.37

The 4-variate cointegration is estimated for the G7 stock market indices using TV single equation in equation 6. The estimated relations are exhibited in the columns, each of which corresponds to different choices of the dependent variable. The $\hat{\beta}_{i,0}$s for $i = 1, \ldots, 3$ stand for the initial values of the cointegrating vectors in which the first component is normalized to one. The $\hat{w}_0$ stand for the initial value of the stationary component w. The $\hat{\sigma}_i$s for$i = 1, \ldots, 3$ and $\hat{\sigma}_w$ stand respectively for the standard deviation of the cointegrating vector and of the stationary residual series. The numbers in parentheses represent the standard deviations. The number of the lagged terms Δw_{t-i} are chosen according to BIC. *, ** and *** respectively stand for the significance at 1%, 5% and 10% significance levels. The test on ϕ is performed on the null hypothesis of $\phi = 1$ against the alternative of $\phi \neq 1$. The test on θ is performed on the one sided null hypothesis of $\theta = 1$ against the alternative of $\theta < 1$.

4.4 *Empirical findings on $\lambda_{i,t}$s*

The theoretical developments applied so far, provide us with the evidence on the variation of the cointegrating vectors. Though the shifts take place in the markets linkages, the fact that these linkages are intensified or not is unexplored. The TV measures introduced in equation 5 illustrate the process of convergence of the markets interdependence. As discussed earlier, we apply Kalman smoothing to obtain the TV impact matrix of the multivariate cointegration relations. Then we focus on the $\lambda_{i,t}$s and the instability measure to gain more in depth understanding of the dynamics in the markets interdependence..

Figures 2, 6 and 9 exhibit respectively the smoothed TV impact matrix in equation 2b for the 7-variate, 4-variate and 3-variate cointegration relations. Though the TV cointegrating vectors exhibit periods of smoothly varying cointegrating vectors interrupted by periods of high variability, the evolution of the orthogonal vector on the spanning space of all the cointegrating vectors can reveal the instability of the long run relation of the underlying markets.

4.5 *The 1970s*

During the decade of 1970s, the TV cointegrating vectors look relatively stable across all the cointegration models. The stability is more apparent in the 7-variate case in Figure 2. Nevertheless, a closer look at the angular shift of the normal vector of the space of the cointegrating vectors in Figure 3, shows a sever instability during the oil crisis and the stock market crash of 1973-1974. The Short period of the bull market following the crash exhibits a rather stable cointegration relation until the inception of the stagflation in late 1970s.

4.6 *The 1980s*

During the early years of the 1980s, the cointegrating vectors suffer from sever instability as evidenced in Figure 3. Although the economies recover by mid 1980s, the stable 7-variate cointegration does not last long until the end of the decade. This finding is consistent with the work by Menezes et al. (2012) where they find a level and a regime shift in the long run equilibrium relation of the G7 markets during the decade. The shift identified by two distinct methodological approaches in the work by Menezes et al. (2012) happen respectively on black Monday in 1987 and in 1984-1985. The instability during this period of time is also evidenced in the literature in the bivariate cointegration relations. Kanas (1998) finds no evidence of bivariate cointegration between the US and either of the European members of the G7 in the period from 1986 to 1996. This finding is consistent with the work by Arshanapalli & Doukas (1993), who find no bivariate cointegration between the US stock market and either of France, Germany, Japan and the UK in the period starting from 1980 until 1987.

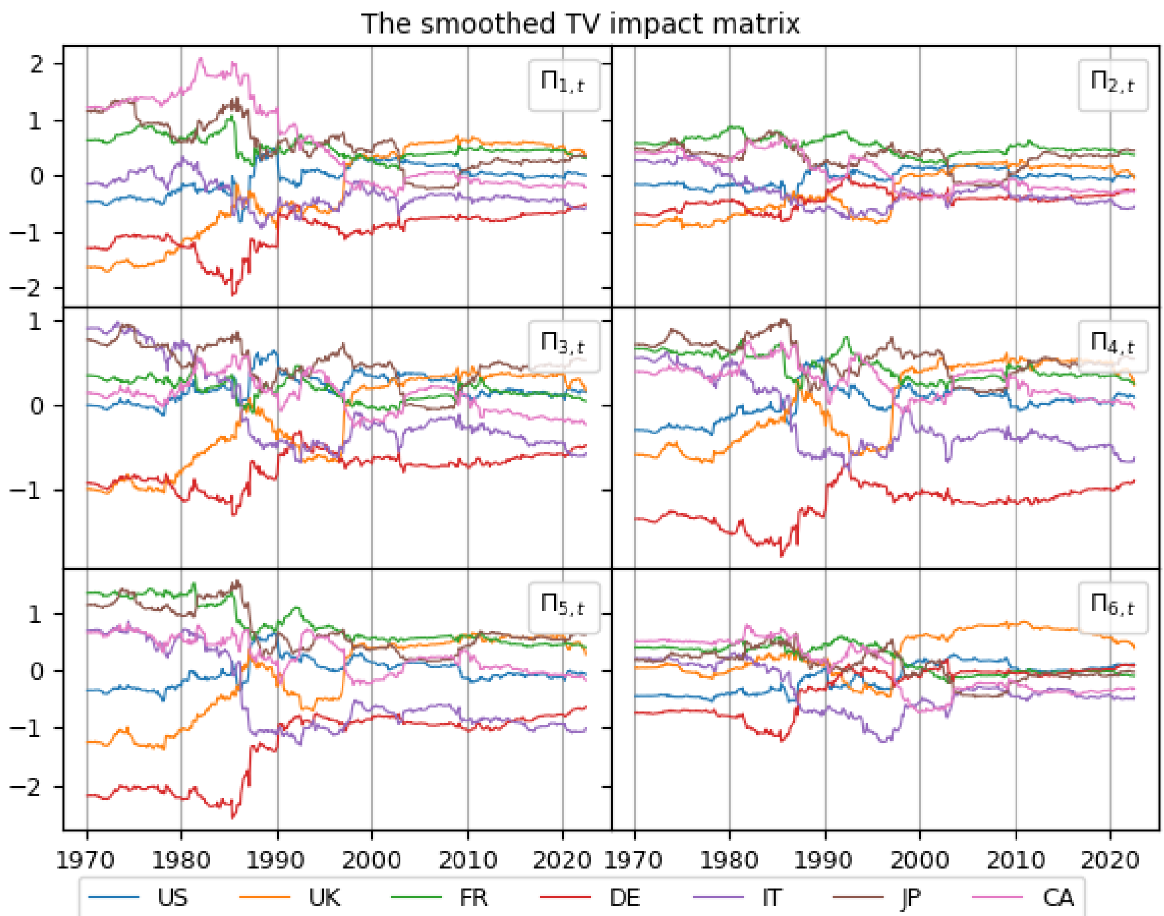

Figure 2: The smoothed TV impact matrix of the 7-variate cointegration model. The 7-variate TV cointegrating vectors of the G7 stock market indices are estimated by the state-space modeling of the VECM. Each subplot corresponds to an equation of the VECM or in other words to $\Pi_{i,t}$ which represents the i-th row of the impact matrix. Kalman smoothing is performed on the filtered state-space.

The variability during the late 1980s and early 1990s, evidenced in Figure 3, is notable because firstly, the global financial crisis segments markets to a less degree, and secondly, the period is the long-lasting period of unsteady markets linkage across the whole time interval. The long-lasting period of the frail linkage among G7 stock markets can be explained by a set of pivotal economic policies across G7 countries. During the early years of 1980s, the economic recession was coincident with raging inflation, causing then an unprecedented stagflation and encouraging monetary contraction in all the G7 countries. The decade is also concurrent with a number of economic legislation to liberalize the economy and also to overcome the double-dip recession. A number of new economic policies were also put in force by the US in an effort to bolster economic growth. The money market of the US was further liberalized during the decade by lifting the ceiling on deposit interest rate and legalizing mortgage lending. Deregulation acts in the banking sector of the US, which liberated deposit interest rates and legalized the adjustable-rate mortgage loans, are among the liberalization measures which were sought to encourage economic recovery. The two well-known acts on the deregulation of the banking sector, though perceived as deregulating measures, led financial sector to collapse including numerous bank failures. The Saving and Loan Crisis (S&L Crisis) in the second half of 1980s could have also affected the linkage.

The decade of 1980s is also identified as a turning point in macroeconomic policy of the states in which the policy of budget deficit is abandoned by the countries and state intervention is mostly concentrated on decreasing public spending, rising tax and broad privatization of the state-owned firms. The decade is generally marked by the evolution of the G7 economies from state-owned firms and regulated economies into the regime of private ownership and liberalized integrated markets. The UK and France considerably reduced budget deficit by raising tax revenue and decreasing public spending to overcome the inflation. France admitted during Mitterrand's administration the policy of "turn of rigor"as opposed to the "Keynesian stimulus policy"during which the public spending is substantially decreased, whereas a series of new taxes were put in force. Given the tax increase in most of the G7 countries, the US tax policy was heterogeneous vis-à-vis the other G7 members in that two bills of tax reduction were passed as a measure of economic incentive. That was in 1990s that the two bills targeted a tax raise in the US.

The 1980s decade is also distinguished by the contractionary monetary policy as opposed to the expansionary policy in the other economic recessions. All the G7 countries responded to the inflation by rising interest rates. The heterogeneous interest rates of G7 countries in particular that of the UK during late 1980s and early 1990s until the UK currency crisis could have adjusted expectations against investment in the UK market. The US interest rate follow a downward trend in this period, whereas those of the other G7 countries remain upward trending. The heterogeneous trending of the interest rate persists across the

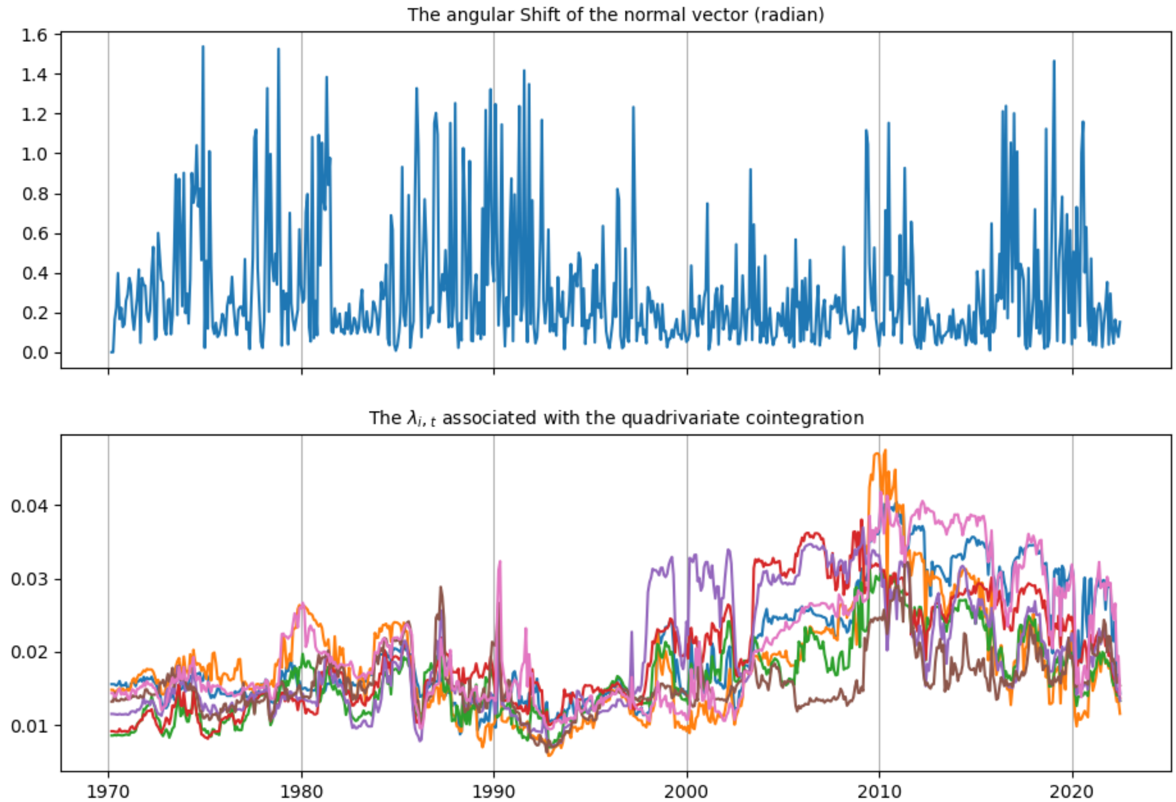

Figure 3: The instability and the strength of 7-variate cointegration. The normal vector at each time is the vector orthogonal to all the cointegrating vectors. The shift in two consecutive normal vectors is expressed as the angle between two vectors in terms of radian.

Table 7: The sub-period regressions of the $\lambda_{i,t}$s

Sub-period	coefficients	TV $\lambda_{i,t}$s						
		$\lambda_{1,t}$	$\lambda_{2,t}$	$\lambda_{3,t}$	$\lambda_{4,t}$	$\lambda_{5,t}$	$\lambda_{6,t}$	$\lambda_{7,t}$
1969 − 2022	c	0.0102	0.0141	0.0100	0.0098	0.0001	0.0134	0.0115
		(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)
	Trend	0.00003	0.00004	0.00002	0.00001	0.00003	0.00000	0.00003
		(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)
1992 − 2010	c	-0.0168	-0.0264	-0.0072	-0.0219	-0.0213	0.0010	-0.0219
		(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)
	Trend	0.00010	0.00012	0.00007	0.00012	0.00012	0.00004	0.0001
		(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)
2010 − 2022	c	0.0787	0.1192	0.0697	0.0562	0.0573	0.0385	0.1000
		(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)
	Trend	-0.00009	-0.00017	-0.00009	-0.00006	-0.00005	-0.00004	-0.00010
		(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)

The $\lambda_{i,t}$s of the 7-variate cointegration model are regressed on a constant and a trend variable. The numbers in the parentheses represent the p-values.

decade of the 1970s. These results are comparable to the findings by Hansen & Johansen (1999) who find that the 1 to 4 month US treasury yields bear a structural break in the early 1980s. The work confirms the influence of the changing procedures of the federal reserve on the interest rates.

4.7 *The 1990s*

The early years of 1990s are also characterized to a high degree by instability in the cointegrating vectors. The recession at the beginning of the decade destabilizes stock market co-movements of the G7 countries, not less than the faltering period of the preceding decades. Overall, it seems that the turmoil period of the unsteady cointegration is not present after the UK and Italy currency crisis in 1992.

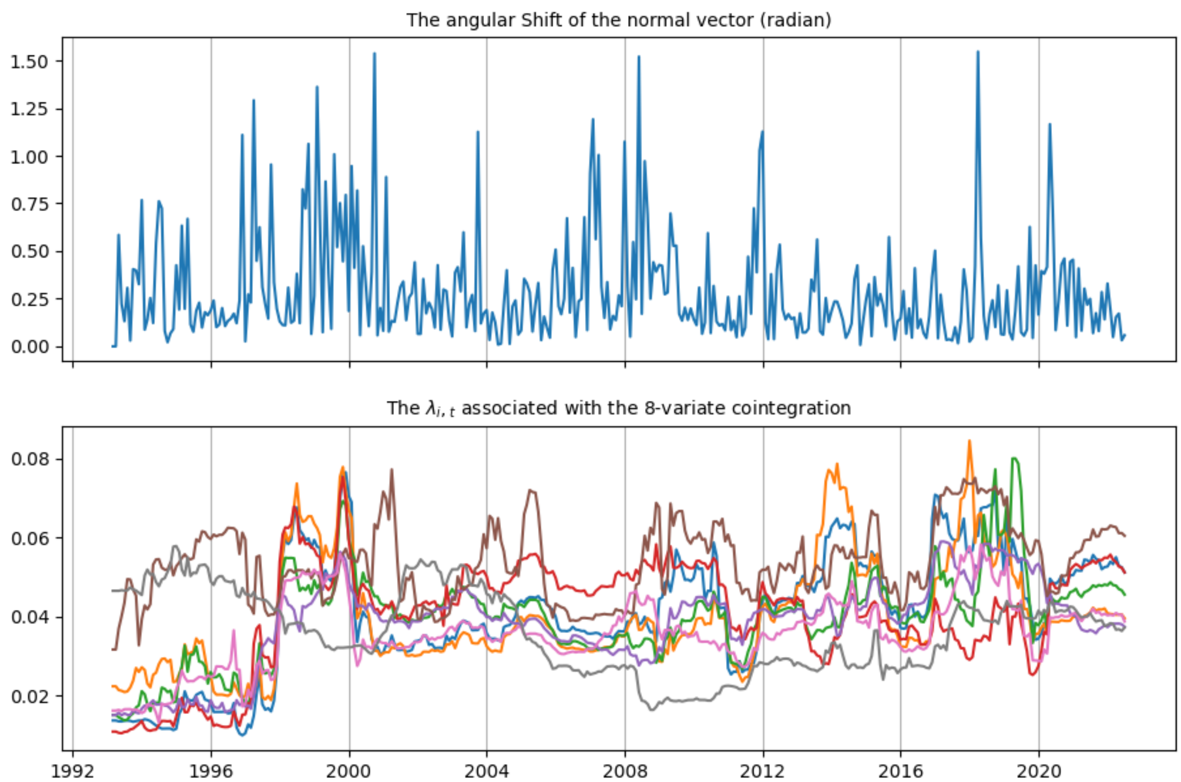

Figure 4: The instability and the strength of 8-variate cointegration of the G7 and China. The normal vector at each time is the vector orthogonal to all the cointegrating vectors. The shift in two consecutive normal vectors is expressed as the angle between two vectors in terms of radian.

In the aftermath of the currency crisis during the early 1990s, the 7-variate co-movement is intensified. As Table 7 shows, the deterministic trend in period from 1992 until 2010 is strongly significant. This is a major result which also finds support in the work by Awokuse et al. (2009) who investigate 12-variate cointegration and find evidence for stronger cointegration in the aftermath of the financial liberalization policies. The findings by Taylor & Tonks (1989) also implies positive impact of markets liberalization, in particular floating exchange rate regime, on markets co-movement. The markets linkage during the two decades of 1990s and 2000s enjoys a steady intensification. As it is apparent in Figure 3, the strength of the 7-variate cointegration takes a downward trend in the last decade after the peak in the post-crisis period. There is also a remarkable rise in the instability, couple of years before the covid-19 crisis. This finding is surprising in particular because the liberalization of the markets and international collaboration have reached a maturity in the recent decade.

4.8 *The integration of China and the particular sub-period of 2010-2022*

We conduct a 8-variate cointegration including China to bring this finding into more scrutiny. As exhibited in Figure 4, following the global financial crisis, the strength measure is upward sloping until the innovation in crude oil production which significantly lowered energy prices. The results in Table 8 evidence significant deterministic trend in most of the $\lambda_{i,t}$s during the sub-period of 2010-2022 as well as in the whole period. From 2017 on, when the crude price stabilizes, the strength measure is restored until the trade wars between the US and China which lasts almost until the covid-19 crisis. As a proxy for the overall evaluation of the the 7-variate and 8-variate cointegration relations, we compare the summation of the TV $\lambda_{i,t}$s in Figure 5. It is evident that China has enjoyed a steady integration into the world economy from the early 1990s. During the late 1990s, the summation of the 8-variate $\lambda_{i,t}$s spikes, and during the following decade it returns to the long run trend. The downward trend of the 7-variate model after 2010 is accompanied by the rising trend of the 8-variate model in Figure 5.

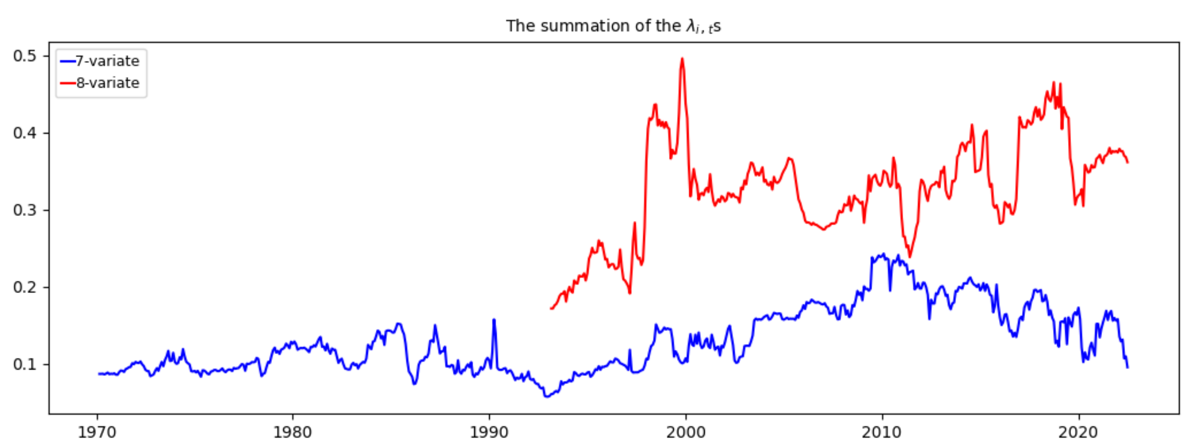

Figure 5: The Summation of the TV $\lambda_{i,t}$s.

Table 8: The sub-period regressions of the $\lambda_{i,t}$s

Sub-period	coefficients	TV $\lambda_{i,t}$s							
		$\lambda_{1,t}$	$\lambda_{2,t}$	$\lambda_{3,t}$	$\lambda_{4,t}$	$\lambda_{5,t}$	$\lambda_{6,t}$	$\lambda_{7,t}$	$\lambda_{8,t}$
1993 − 2022	c	-0.0027	0.0159	0.0136	0.0213	0.0084	0.0394	0.0159	0.0549
		(0.35)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)
	Trend	0.00010	0.00005	0.00006	0.00004	0.00007	0.00003	0.000041	-0.00004
		(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)	(<0.01)
2010 − 2022	c	-0.0010	0.0389	-0.0115	0.0204	0.0221	0.0217	0.0050	-0.0483
		(0.93)	(<0.01)	(0.22)	(0.02)	(<0.01)	(0.02)	(0.48)	(0.48)
	Trend	0.00009	0.00001	0.0001	0.00004	0.0004	0.00006	0.00006	0.0001
		(<0.01)	(0.59)	(<0.01)	(0.02)	(<0.01)	(<0.01)	(<0.01)	(0.48)

The $\lambda_{i,t}$s of the 8-variate cointegration model are regressed on a constant and a trend variable. The numbers in the parentheses represent the p-values.

4.9 *The dynamic interdependence of the European countries*

There is also remarkable evidence that the long run linkage among the European countries faces instability in the period of the late 1980s and early 1990s, whereas it remains relatively stable during the decade of 1970 (Figures 6 to 7). The quadrivariate TV impact matrix of the European members of the G7, as exhibited in Figure 6, shows higher instability in the short period of time preceding the currency crisis. The onset of the currency crisis of 1992 which led to the UK and Italy withdrawal from the European monetary system, provokes sharp variability in the quadrivariate cointegrating vectors. In the aftermath, during the late 1990 and after the introduction of Euro, the quadrivariate TV cointegration remains considerably stable. The small shifts in the cointegrating vectors made by the global financial crisis of 2008 and the Covid-19 crisis seem far less extensive than the earlier ones in particular the one triggered by the currency crisis of 1992. The European level cointegration enjoys a tightening linkage from the mid-1990s to 2022, so that the economic recessions or the political rifts among the countries in particular Brexit do not impact the quadrivariate co-movement of the four European countries (Figure 7).

As Figure 7 exhibits, the strength measure of the quadrivariate co-movement is downward sloping after 2005. Two events of the sovereign debt crisis and the technological innovation in the production of crude oil negatively impact the strength measure, though it is not restored to the levels earlier than 2005. We explore again the impact of China's economy on

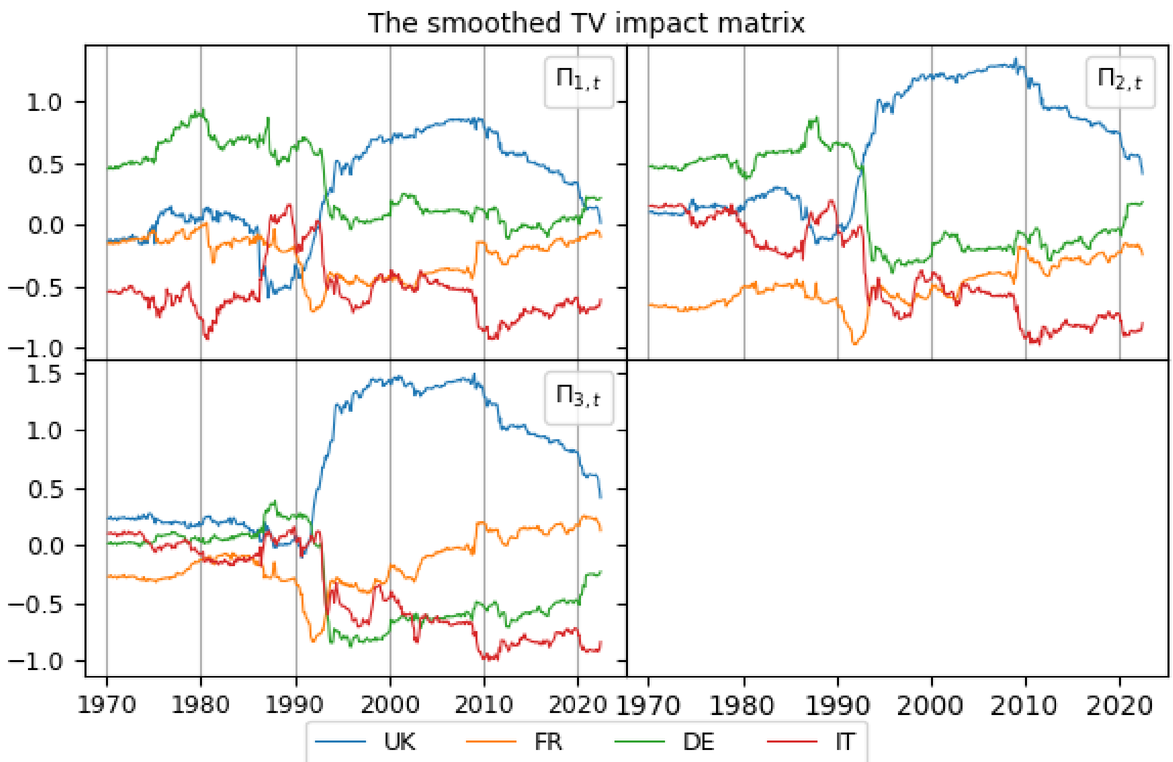

Figure 6: The smoothed TV impact matrix of the quadrivariate cointegration model. The quadrivariate TV cointegrating vectors of the European countries of the G7 are estimated by the state-space modeling of the VECM. Each subplot corresponds to an equation of the VECM or in other words to $\Pi_{i,t}$ which represents the i-th row of the impact matrix. Kalman smoothing is performed on the filtered state-space.

quadrivariate cointegration of the European markets. It is evident that China is integrated to the European economies from 2006 onwards when the country enjoyed repeated record of unprecedented economic growth. The crude oil innovation only impacts the strength of two cointegrating vectors, whereas the other one becomes stronger until after 2016.

The 5-variate cointegration of the European markets and China as shown in Figure 8 seems to have been adversely impacted in 2018, possibly by Trumpism policy in which higher tariffs were imposed on some European and Chinese industries. The US-Europe and US-China trade frictions have shocked the linkage of the European and Chinese markets in particular after the bear market of 2018 in China when the fears of negative impacts of the trade friction were escalated.

We also conduct a trivariate TV cointegration among three European countries other than the UK. The trivariate cointegration among France, Germany and Italy confirms again the fact that the decade of 1980s is the paramount period across the underlying time hori-

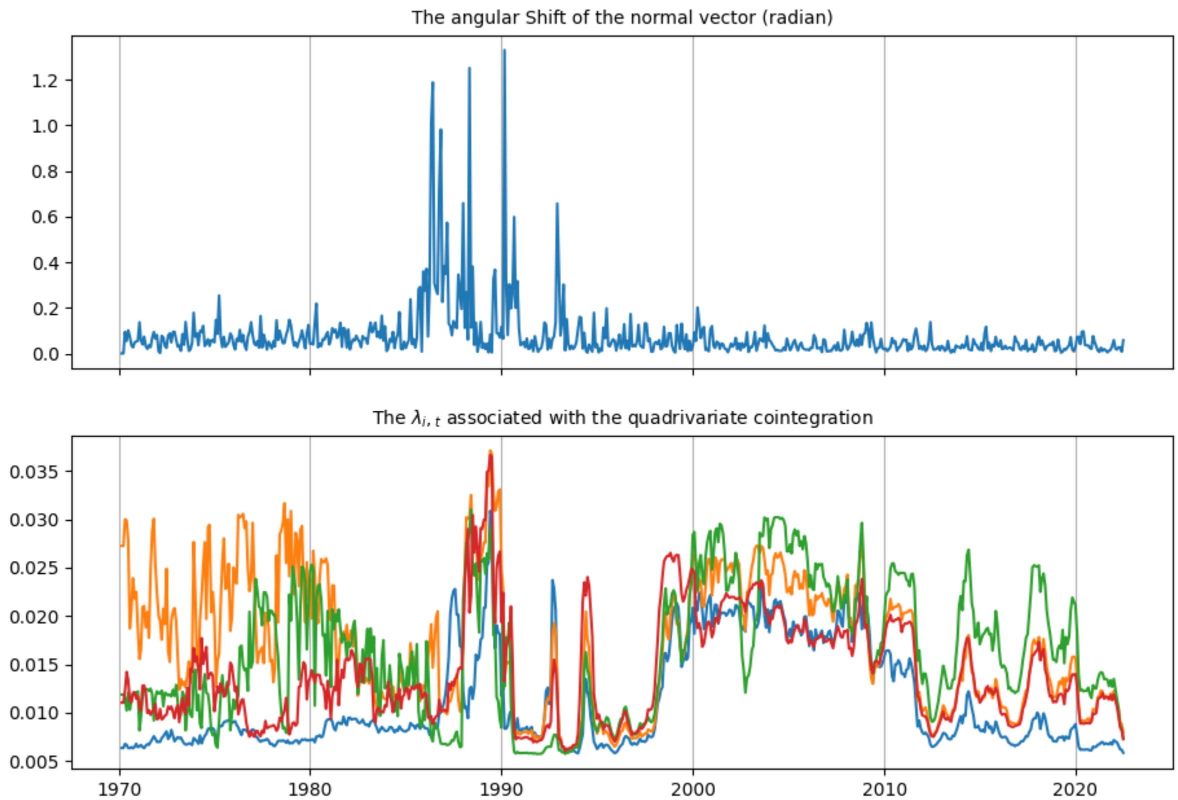

Figure 7: The instability and the strength of quadrivariate cointegration of the European countries of the G7. The normal vector at each time is the vector orthogonal to all the cointegrating vectors. The shift in two consecutive normal vectors is expressed as the angle between two vectors in terms of radian.

zon. Although the quadrivariate cointegration among the European members of G7 becomes unsteady from the second half of the decade, it seems that the trivariate cointegration is destabilized from the beginning of the period. This fact evidences the disrupting role of the UK market on the trivariate cointegration relation of the other European countries during crucial period of the economic reversal of the western countries. Both trivariate and quadrivariate cointegration at European level become steady from the second half of the 1990s, with the trivariate relation more stable than the quadrivariate one, and seemingly the UK market has no longer the disruptive effect on the trivariate relation.

We see the impact of the Brexit on neither of the multivariate cointegration relations. The quadrivariate as well as the trivariate cointegration relations are stabilized in the early 1990s after the constitution of the European union. One cannot certainly reject the inference that the liberalization of capital flow in the European countries, enforced in 1988, promoted markets co-movement. The fact that notable crises in the era following these dates do not

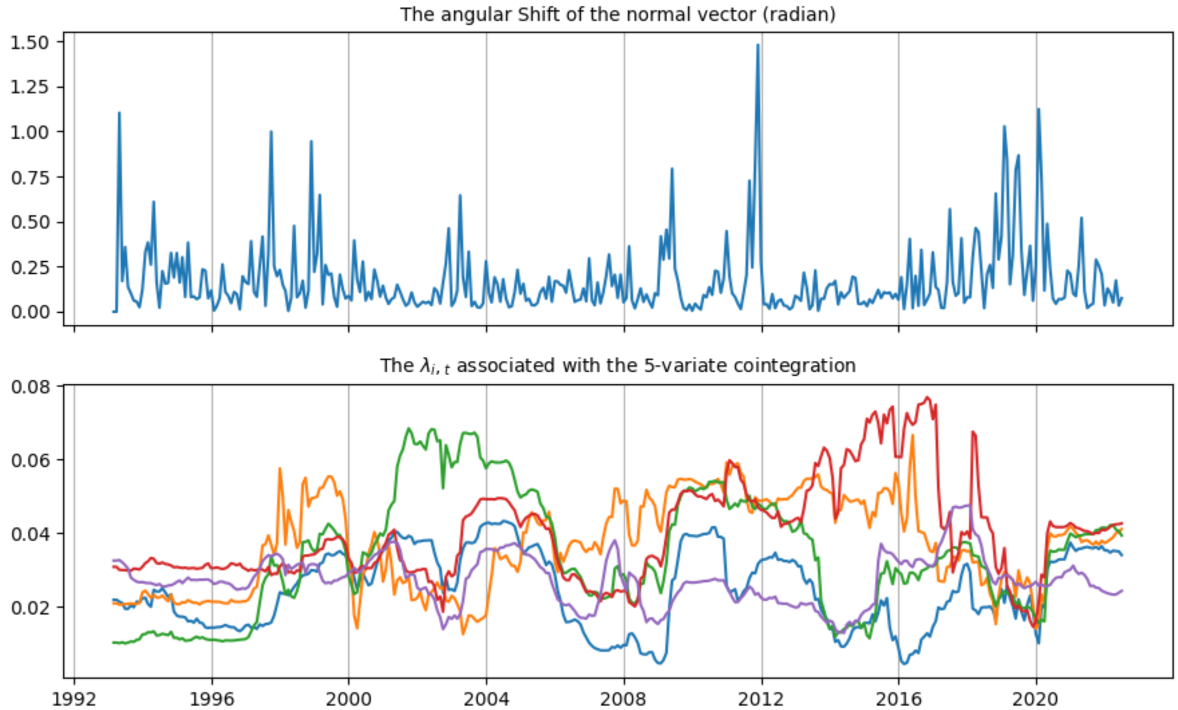

Figure 8: The instability and the strength of 5-variate cointegration of the European countries and China. The normal vector at each time is the vector orthogonal to all the cointegrating vectors. The shift in two consecutive normal vectors is expressed as the angle between two vectors in terms of radian.

affect adversely the stability of the cointegrating vectors, could be the consequence of both free capital flow and institution of the European union.

From the standpoint of the theory of the interest rate parity, the persistent instability in the equilibrium relation can be attributed to the varying disparities. The differential interest rates, ceteris paribus, do not ensue any shift in the cointegration relation, whereas the varying disparities between onshore and offshore interest rates can shift the cointegration relation. The disparities between interest rates reflect exchange rate or other economic uncertainties caused by political risks. The results of Figure 10, which evidence high instability in the period prior to the 1990 and the few years afterwards, indicate the high uncertainty associated with the future exchange rates of the European countries. The uncertainty associated with the economic reversals at state level and the uncertain future of the talks to establish the European union are among the potential factors causing repeated adjustments in the equilibrium relation among European members of the G7.

In order to further bring the evolution of the TV cointegration estimated by the single equation model under scrutiny, we investigate the instability in the cointegrating vectors as in

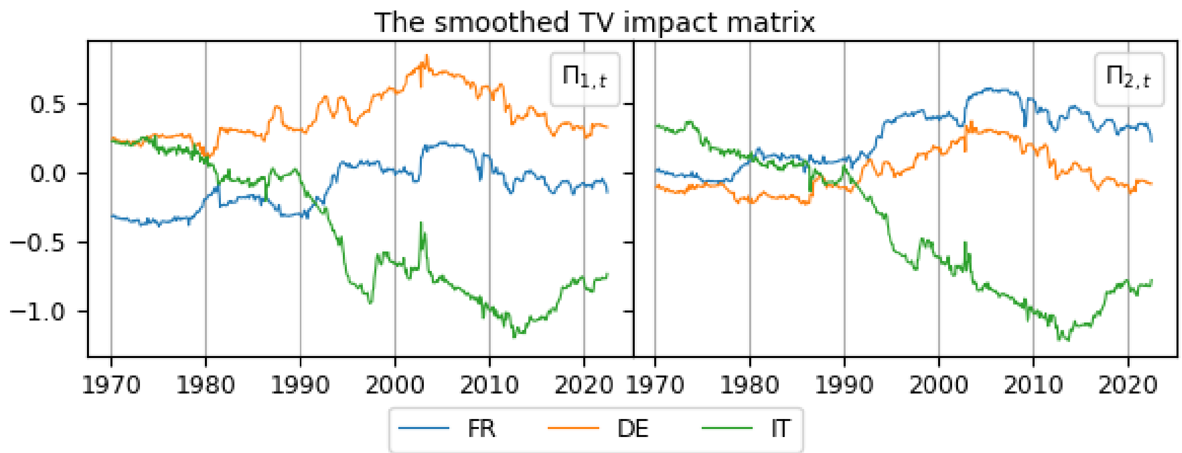

Figure 9: The smoothed TV impact matrix of the trivariate cointegration model. The trivariate TV cointegrating vectors of the Germany, France and Italy are estimated by the state-space modeling of the VECM. Each subplot corresponds to an equation of the VECM or in other words to $\Pi_{i,t}$ which represents the i-th row of the impact matrix. Kalman smoothing is performed on the filtered state-space.

the model estimated by the TV VECM in equation 2. We measure the shift in two consecutive cointegrating vectors by the angle between two vectors.

Figure 11 represents the shift in the consecutive 7-variate cointegrating vectors measured as the angle between two consecutive vectors in terms of radian with different choices of the dependent variable. One can conclude that in 6 cointegration models, the cointegrating vector is more volatile during 1980s decade and to some extent during 1990s. Only in one model having Japan as the dependent variable, the results of the model is not in compliance to the other models. The conclusion on the 7-variate TV cointegration is the fact that the cointegrating vectors incur lower instability during the global financial crisis in 2008. The covid-19 crisis leaves significantly less instability on the 7-variate cointegration among G7 countries. In overall, the positive impact of the globalization on the markets co-movement is in line with the conclusions made on the TV cointegrating vectors estimated by the VECM.

The results of the cointegration instability exhibited in Figure 12, in the cases having the UK and France stock markets as the dependent variables, evidence high variability of the cointegrating vector during the period from 1970 to 1990. The positive impact of the globalization on the steadiness of the markets co-movement is also evident in the relations with the UK and France as the dependent variables. The cointegration equation having the German stock market as the dependent variable also evidences, to a less degree, the positive impact of the globalization on markets convergence. The Global financial crisis and the Covid-19 crisis

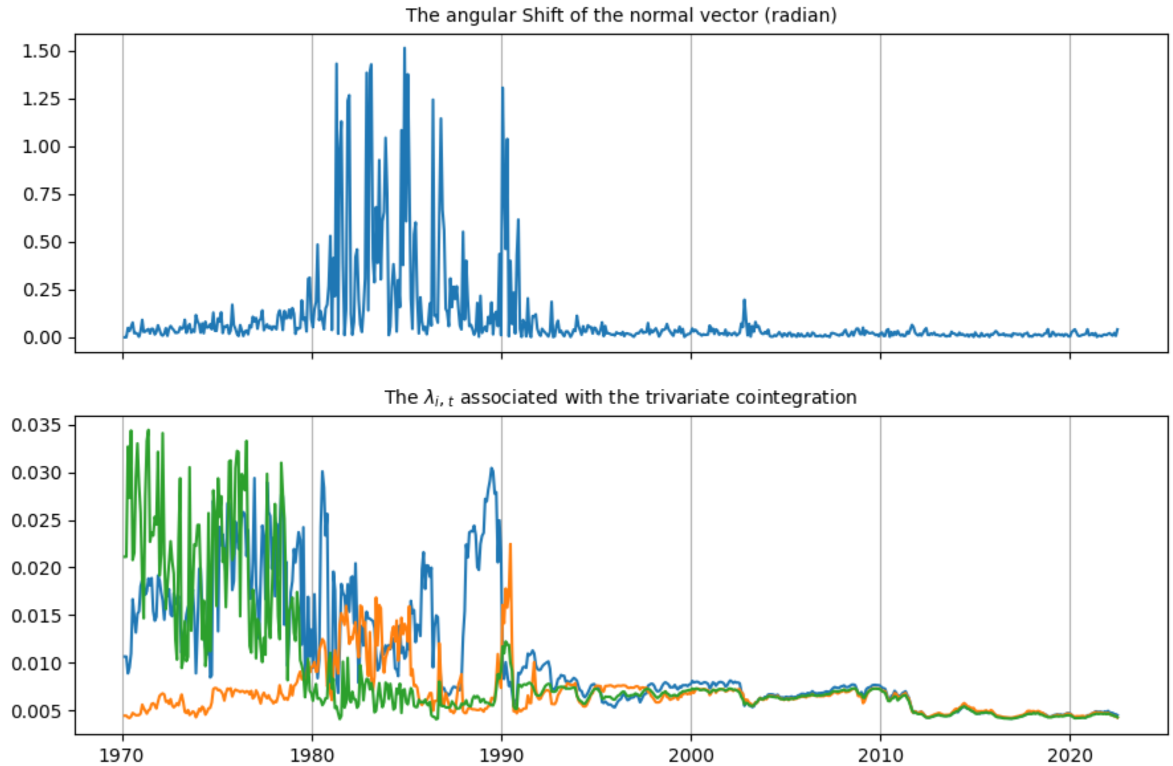

Figure 10: The instability and the strength of trivariate cointegration of FR, DE and IT. The normal vector at each time is the vector orthogonal to all the cointegrating vectors. The shift in two consecutive normal vectors is expressed as the angle between two vectors in terms of radian.

segment European markets far less than the earlier crises, in particular those of the decades 1970s and 1980s.

4.10 *The role of states and central banks*

The results of applying two methodologies to multivariate cointegration of the G7 stock markets reveal a number of landmark facts in the evolution of the stock markets. Both methodological approaches evidence that segmenting impact of the economic crises on the markets co-movement has considerably declined over time. Markets deregulation and extensive privatization, following the recession of 1982-83 that marked the decade as a transition to a new regime of market economy, are accompanied in the 1990 decade by more capital availability and free capital movements across markets. The capital availability is promoted by a number of events including macroeconomic and technological achievements. The US budget policy shifts into tax raising during president Clinton administration that stepped further back from the deficit policy. The lower deficit reduced state borrowing and enhanced capital availability

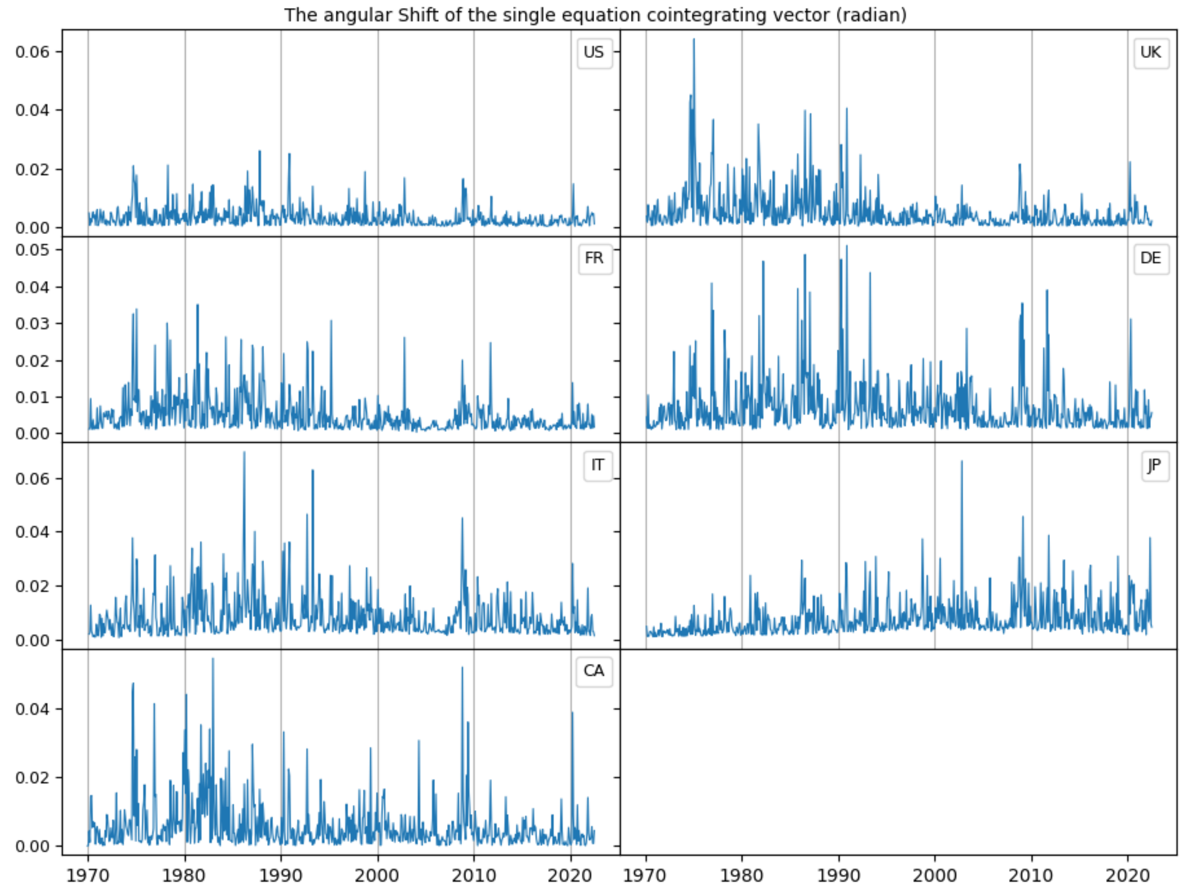

Figure 11: The angular shift of the cointegrating vector of the 7-variate cointegration model. The shift in two consecutive cointegrating vectors is expressed as the angle between two vectors in terms of radian.

in the financial markets. The innovation in the information technology in the second half of the decade created new jobs and produced accumulated capital during the dot-com bubble. The revolution in the information technology provoked further capital mobility across markets. Despite the positive advancements, it seems the vectors are not considerably stable until the second half of the 2000s.

The prevalent high uncertainty during the period 1980 to mid-1990, as the empirical investigation strongly confirms, seems to have calmed down in particular after 2000s. The possible political reasons retracting the rifts in markets co-movement comprise a number of the momentous political strategies and technological innovations including extensive liberalization of the markets, constitution of the European union, free movement of labor in the European union, free capital flow across G7 countries, introduction of the Euro currency and the technological revolution in the information technology. The political reversals and the

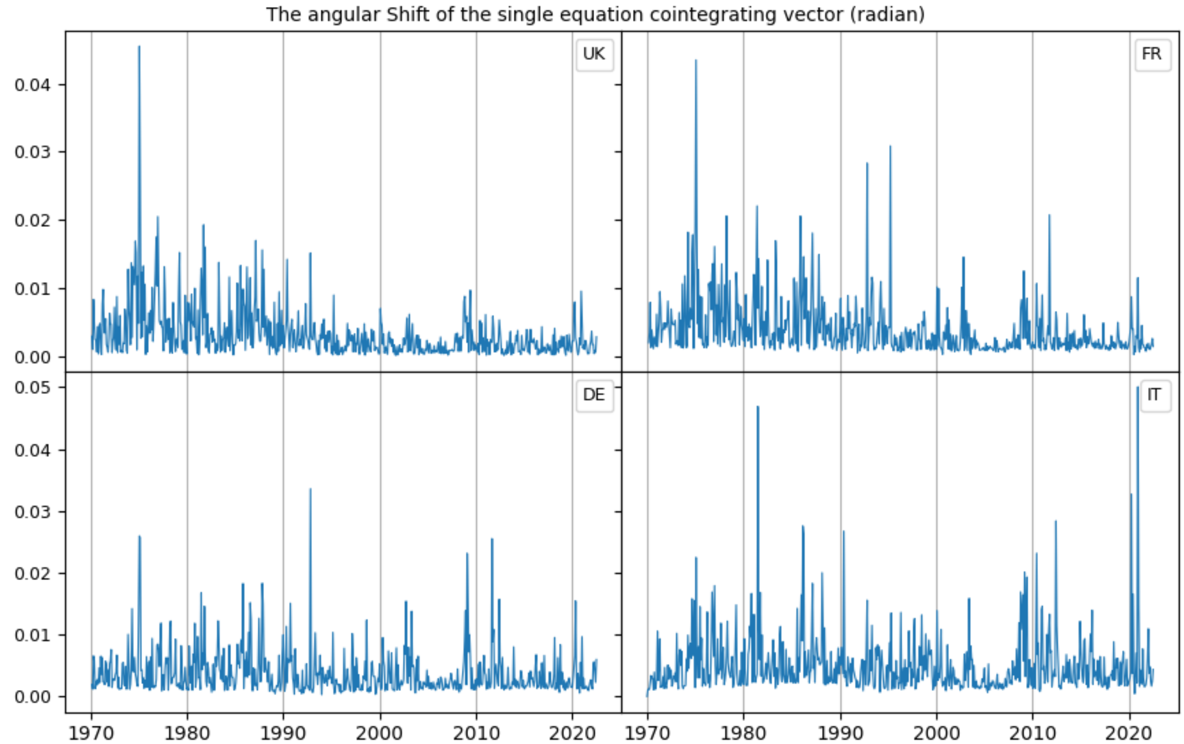

Figure 12: The angular shift of the cointegrating vector of the 4-variate cointegration model. The shift in two consecutive cointegrating vectors is expressed as the angle between two vectors in terms of radian.

tightening political and economic ties among G7 countries paved the way for more integrated world economies and substantially mitigated the uncertainty associated with the economies.

The smaller segmentation during serious economic depressions in the recent decades can be also attributed to the more active role of the central banks in taking crucial courses of action to bring the economic turmoil into control. The central banks interventions during the stagflation of the 1980s was limited to the conventional monetary policy of increasing interest rates to fight inflation. Higher interest rates spawn uncertainty to the market by making a gloomy future for the aggregate demand. Overall, markets co-movement suffered from the uncertain economic future and low globalization during the decade of 1980s. It seems that from 2000 onwards, the uncertainty associated with the interest rates vanished as a result of more active role of the central banks in particular in taking expansionary policy of the quantitative easing. This happened during the global financial crisis of 2008, when central banks of the G7 resorted to the quantitative easing policy to stimulate the economy. During covid-19 crisis, world central banks promptly reacted to the crisis by asset buying programs and pumping unprecedented volumes of liquidity into the economy.

5 Conclusions

The development of Johansen's estimation procedure of multivariate cointegration paved the way to investigate multi-variate long run equilibrium relation among stock markets. State-space modeling of the VECM, though not formulated to estimate independent cointegrating vectors, can reveal the variation in the space of the TV cointegrating vectors. We analyzed the instability of the spanning space by examining the shifts in the normal vector of the space. As Johansen's λs proxy the strength of the equilibrium relations, we introduced TV λs to investigate the TV strength of the equilibrium relations. In an effort to further support our empirical conclusions, we also estimated the two existing approaches in the literature including the single equation cointegration model developed by state-space modeling and the smoothly varying approach developed by the Chebyshev time polynomials.

The influential events in favor of the markets long run co-movement can be cited as the momentous economic policies to deregulate markets and liberalizing capital flow leading to more integrated capital markets. The pivotal period of the economic policy reversal from the deficit policy and state-owned economies to the liberalized economies during 1980s ensues high variability of the markets co-movements. The availability of capital in the aftermath of the right wing economic policies of the states and the developments in the information technology and free cross border information flow, higher foreign trade liberalization, and easier international financial transaction have strengthened markets co-movements.

The more active role of the central banks has overshadowed the uncertainty arising from the economic crisis. The extensive expansionary measures in terms of quantitative easing during the global financial crisis have dominated the gloomy outlook and motivated the investment and therefore it has limited the extent that markets co-movements are destabilized during the recession.

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