State-dependent Intra-day Volatility Pattern and Its Impact on Price Jump Detection - Evidence from International Equity Indices

By Ping Chen TSAI [a] and Cheoljun EOM [b]

[a] National Sun Yat-sen University
No.70 Lien-hai Rd., Kaohsiung 80424
Taiwan (Corresponding author)
vincenttsai@mail.nsysu.edu.tw

[b] Pusan National University,
2, Busandaehak-ro 63beon-gil,
Geumjeong-gu, Busan, 46241
South KOREA shunter@pusan.ac.kr

Abstract

Current price jump tests assume a constant intra-day volatility pattern (IVP) over sample period. We test this assumption by allowing IVP to depend on the sign of returns from day t-1 or overnight period. Estimation results from 5-minute GARCH for four equity indices show that squared-return-based IVP weights increase in early morning hours when previous returns are negative, suggesting an asymmetric IVP. For a jump-robust IVP estimator, strong response is found for days with Realized Variance increasing from the previous day. Our results are consistent with and complement recent studies on time-varying IVP. Jumps test results using the state-dependent IVP are more prudent, show lower degree of clustering and are less concentrated over trading hours.

Key words: Intra-day Volatility Pattern, Jump, Realized Volatility, Leverage Effect, GARCH, High-frequency Data

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1. Introduction

The U-shape intraday volatility pattern (IVP) is a well-established stylized fact for high-frequency data (Andersen and Bollerslev 1997; Taylor and Xu 1997). Thus, many jump tests propose to remove the deterministic IVP before identifying intraday jumps (Andersen et al. 2007; Lee and Mykland 2008; Boudt et al. 2011; Caporin 2022). Boudt et al. (2011) further show that it is important to use a jump-robust IVP estimator, as opposed to the squared-return-based estimator by Taylor and Xu (1997), hereafter TX, in implementing intraday jump tests. The Weighted Standard Deviation (WSD) estimator of Boudt et al. (2011) has since become a standard approach for finding intraday jumps[1].

Both the TX and WSD estimators of IVP give a static U-shape pattern over time. More recently, however, there have been studies that find evidence on time-varying IVP (Christensen et al. 2018; Andersen et al. 2019)[2]. Specifically, in Christensen et al. (2018) they show that a deterministic IVP does not fully capture the intraday variation of volatility. Andersen et al. (2019) also provide evidence that IVP weights are not constant – their Figure 3 show two shapes of IVP for days with and without FOMC announcements. The IVP weights become lower in the morning but increase sharply in

[1] See, for example, Gilder et al. (2014), Jondeau et al. (2015), Piccotti (2018), Sun and Gao (2020), Caporin and Poli (2022) and Marcaccioli et al. (2022).

[2] Müller et al. (2011) and Gabrys et al. (2013) consider functional data analysis for IVP, which explicitly specify that IVP is time-varying.

the afternoon hours for those days with FOMC. Andersen et al. (2019) further show that IVP weights present different patterns depending on the level of volatility as measured by the VIX index.

In this paper, we follow the developments in Andersen et al. (2019) and allow IVP weights to depend on some state variables or events. Specifically, the state variables are chosen to reflect the so-called leverage effect or asymmetric volatility, a phenomenon in which volatility tends to increase after negative returns than positive ones; see the extensive results in Yu (2005) and Catania (2022). Therefore, we classify days in our sample into those that see previous negative returns versus positive ones, and investigate whether IVP weights should be adjusted in response to these different states. Intuitively, a previous negative return will not have uniform or homogenous impact on all IVP weights, but the effect will decrease toward the end of a trading day. Therefore, we use an exponential decay function to describe the relative increase in IVP. This state-dependent structure of IVP and the associated parameters are then estimated using an intraday GARCH (1, 1) model, and a formal test on the significance of the state events can be done by a likelihood ratio test.

Our empirical results are obtained from four equity indices sampled at 5-minute frequency during Jan 2019 to March 2021. First, we find that the TX IVP weights on average increase in the early morning hours when previous daily returns or overnight returns are negative. This finding confirms leverage effect in the TX IVP estimator. On the other hand, the jump-robust WSD estimator of Boudt et al. (2011) is more resilient to negative previous returns; however, when the states are chosen such that Realised Variance (RV) is relatively large, or it increases from the previous day, i.e., $RV_t > RV_{t-1}$, we find strong evidence in rejecting constant WSD weights.

Our findings of asymmetric or state-dependent IVP weights have important implications on intraday jump detection. To illustrate the consequence of using fixed IVP weights versus state-dependent ones, we implement the jump test by Andersen et al. (2007). The jumps detected using our approach are less concentrated during the trading hours, and also display a smaller degree of over-dispersion across the whole sample period, when compared with the standard WSD approach. These properties are relevant in determining whether and to what extent the arrivals of price jumps cluster in time. As a result, we show that ignoring the state-dependent IVP weights in intraday jump tests could lead to spurious estimation of jump intensity.

The paper is organized as follows. Section 2 describe our intraday equity indices data and Section 3 develops our methodology and models. Section 4 reports the empirical results with a discussion, and Section 5 concludes.

2. Intraday Equity Indices Data

Our data includes the index values of KOSPI200, NI225, FTSE100 and ESTX50 from firstratedata.com, with sample period 2019/01/03 - 2021/03/31. We use data sampled at 5-minute frequency to mitigate the effect of micro-structure noise. After removing incomplete trading days[3], we obtain and summarize the four indices in Table 1. First, note that we have made some adjustments on the data. Specifically, we delete the last 10-minutes in KOSPI200 index, as trading is halted and saved for auction before market close at 15:00 in Korean Stock Exchange. For NI225, there is a lunch break during 11:30-12:30 in Tokyo Stock Exchange, and therefore we remove the 10-minute period 12:30-12:40 to avoid the abrupt increase of trading activities after lunch break. For both FTSE100 and ESTX50, we observe very erratic price movements in the first 10-minute interval of a trading day, and following Gilder et al. (2014) and Ferriani and Zoi (2020) we delete this interval from our sample. As a result, the number of 5-minute intervals per day M are 70, 58, 100 and 100 for the four indices respectively.

In the bottom of Table 1, we report summary statistics of 5-minute returns for the four indices. It can be seen that that 5-min returns do not have a normal distribution but display very large values of kurtosis, which is a typical result from high-frequency data.

[3] See an online Appendix which is available from the authors upon request.

3. Methodology

First, an intraday return $r_{t,j}$ on the j-th 5-minute interval of day t is defined as:

$$r_{t,j} = \ln(P_{t,j}) - \ln(P_{t,j-1}), \tag{1}$$

where $P_{t,j}$ is the index value. We consider two non-parametric IVP estimators. The first is the $r_{t,j}^2$-based estimator λ_j^{TX} by Taylor and Xu (1997):

$$\lambda_j^{TX} = \frac{\sum_{t=1}^{T} r_{t,j}^2}{\sum_{t=1}^{T}\sum_{j=1}^{M} r_{t,j}^2}, \tag{2}$$

for $j = 1, \dots, M$ and $\sum_{j=1}^{M} \lambda_j^{TX} = 1$. The second is the jump-robust Weighted Standard Deviation (WSD) estimator by Boudt et al. (2011):

$$\lambda_j^{WSD} = \frac{WSD_j^2}{\sum_{j=1}^{M} WSD_j^2}, \tag{3}$$

where

$$WSD_j^2 = 1.081 \frac{\sum_{t=1}^{T} w_{t,j} \bar{r}_{t,j}^2}{\sum_{t=1}^{T} w_{t,j}}, \tag{4}$$

with $w_{t,j} = \mathbf{1}_{\{Z_{t,j}^2 \leq 6.635\}}$ is an indicator function for $Z_{t,j} = r_{t,j}\left(BPV_t \lambda_j^{SH}\right)^{-1/2}$, $r_{t,j}$ is standardized by the Bipower Variation (BPV) of Barndorff-Nielsen and Shephard (2004, 2006):

$$\bar{r}_{t,j} = \frac{r_{t,j}}{\sqrt{BPV_t(1/M)}}, \tag{5}$$

$$BPV_t = \left(\frac{\pi}{2}\right)\left(\frac{M}{M-1}\right)\sum_{j=2}^{M} |r_{t,j-1}||r_{t,j}|, \tag{6}$$

and in $Z_{t,j}$ the shortest half scale estimator λ_j^{SH} is given by Rousseeuw and Leroy (1988)[4]:

[4] An alternative way can be given by the Q_n estimator of Rousseeuw and Croux (1993); we thank Peter Rousseeuw for pointing out this alternative.

$$\lambda_j^{SH} = \frac{ShortH_j^2}{\sum_{j=1}^{M} ShortH_j^2}, \tag{7}$$

where $ShortH_j = 0.741 \min\{\bar{r}_{(h),j} - \bar{r}_{(1),j}, \dots, \bar{r}_{(T),j} - \bar{r}_{(T-h+1),j}\}$, $h = \lfloor T/2 \rfloor + 1$, $\lfloor x \rfloor$ takes the integer part of x and the order statistics are defined as $\bar{r}_{(1),j} < \bar{r}_{(2),j} \dots < \bar{r}_{(T),j}$. The WSD estimator is robust to price jumps because firstly the shortest half scale estimator λ_j^{SH} avoids tail values in $\bar{r}_{t,j}$ across days in the sample, and secondly the indicator function $w_{t,j}$ truncates extremely large $Z_{t,j}$.

The two IVP estimators will be used in estimating an intraday GARCH model for 5-minute returns:

$$r_{t,j} = \mu + \sqrt{h_{t,j}} z_{t,j},\ z_{t,j} \sim i.i.d. N(0,1), \tag{8}$$

$$\frac{h_{t,j}}{\lambda_j M} = \omega + \frac{\alpha e_{t,j-1}^2}{\lambda_{j-1} M} + \frac{\beta h_{t,j-1}}{\lambda_{j-1} M}, \tag{9}$$

for $t = 1, \dots, T$, $j = 1, \dots, M$ and by definition, $\lambda_0 \equiv \lambda_M$. The IVP weights λ_j in (9) can be either the conventional λ_j^{TX} or the jump-robust λ_j^{WSD}. Estimation results based on (8) and (9) are our baseline model, i.e., the IVP weights λ_j^{TX} or λ_j^{WSD} are constant over time.

To develop the relevance of state-dependent IVP, we first allow λ_j to depend on the following two events:

$$A = \{r_{t-1}^{opop} < 0\} \text{ and } B = \{r_{t-1}^{ovn} < 0\},$$

where r_{t-1}^{opop} is the open-to-open return and r_{t-1}^{ovn} is the overnight return from day $t-1$. Specifically, when the state A or B is present, we conjecture the IVP weights will see higher values than otherwise, which is consistent with the leverage effect or asymmetric volatility. However, a previous negative return $r_{t-1}^{opop} < 0$ or $r_{t-1}^{ovn} < 0$ is unlikely to impose the same impact on all the IVP weights throughout a trading day, and we use an exponential function to describe this potentially decaying pattern. The structure of state-dependent IVP weights when, for example, event A is true, is thus given as:

$$\lambda_j^A = \frac{\lambda_j + c_1 \mathbf{1}_A \exp\left(-(1+c_2)\frac{j-1}{M}\right)}{\sum_{j=1}^{M}\left(\lambda_j + c_1 \mathbf{1}_A \exp\left(-(1+c_2)\frac{j-1}{M}\right)\right)}, \qquad (10)$$

where $c_1 > 0$ and $c_2 \geq 0$ are two constant parameters, $\mathbf{1}_A$ is the indicator function for event A and importantly by construction, $\sum_{j=1}^{M} \lambda_j^A = 1$. Therefore, the first IVP weight λ_1^A will increase – by an amount proportional to c_1 – and this increase will decay at a rate controlled by $(1 + c_2)$ as j counts from 1, 2, … to M. Thus, a rejection of $c_1 = 0$ and $1 + c_2 = 0$ will suggest that IVP weights are not constant over time, but show asymmetric responses to the states considered. The specification of λ_j in (9) now becomes $\lambda_j(1 - \mathbf{1}_A) + \lambda_j^A \mathbf{1}_A$.

The dependence of IVP weights on previous returns in (10) reflects the leverage effect or asymmetric volatility, and to our knowledge is the first mechanism in the literature which links IVP with leverage effect. The two states A and B represent conditional information, as they look at the direction of returns from day $t-1$. In the intraday GARCH model (8) and (9), however, the use of λ_j implies the information set is not necessarily limited to day $t-1$, because the TX and WSD IVP weights in (2) and (3) are calculated from the whole sample. Therefore, we further consider two states:

$$C = \{RV_t > median(RV_{t=1...T})\} \text{ and } D = \{RV_t > RV_{t-1}\},$$

where RV_t is the Realized Variance on day t. These two states represent a relatively higher RV, or when RV increases from the previous day, in an *ad hoc* sense. We emphasize the same *ad hoc* classification is used in Andersen et al. (2019), who find that time-varying IVP weights depend on the level of VIX index. Thus, our states C and D will serve to confirm the findings in Andersen et al. (2019).

4. Results

4.1 Test for State-dependent IVP

In Table 2 the estimation results of GARCH (1, 1) for 5-minute returns of four equity indices show typical α and β estimated values, with the stationary condition $\alpha + \beta < 1$ satisfied. The log-likelihood values L_0 will be used to conduct a likelihood ratio (LR) test for the state-dependent parameters c_1 and c_2. The obtained L_0 of KOSPI200, NI225 and FTSE100 is smaller when the terms of the intraday conditional variance in (9) are scaled by the Taylor-Xu estimator λ_j^{TX} than by the WSD estimator λ_j^{WSD}. The reverse is true for ESTX50.

In Table 3, we see the introduction of c_1 and c_2 in the Taylor-Xu estimator λ_j^{TX} works quite well when event $A = \left\{r_{t-1}^{opop} < 0\right\}$ is true. An LR test with test statistic

$2(L_1 - L_0) \sim \chi^2_{d.f=2}$ rejects $c_1 = 0$ and $1 + c_2 = 0$ with small p values for *all* indices. On the other hand, however, such a specification does not attain significantly higher log-likelihood when the jump-robust λ_j^{WSD} is used to account for IVP in (9), except for ESTX50. Therefore, there appears to be some leverage effect in Taylor-Xu IVP weights, but not in jump-robust IVP.

The performance of state-dependent IVP weights improves as we use information set $B = \{r_{t-1}^{ovn} < 0\}$, $C = \{RV_t > median(RV_{t=1...T})\}$ and $D = \{RV_t > RV_{t-1}\}$. To save space, in Table 4 we only report the estimated c_1 and c_2 values, and the LR test results. In the top panel, when we allow IVP weights to depend on event $B = \{r_{t-1}^{ovn} < 0\}$, the increases in log-likelihood values are all significant for the Taylor-Xu estimator λ_j^{TX}[5]. For jump-robust λ_j^{WSD}, the information $B = \{r_{t-1}^{ovn} < 0\}$, like the event $A = \left\{r_{t-1}^{opop} < 0\right\}$, only gives marginal improvement in log-likelihood values for KOSPI200, NI225 and FTSE100. In the bottom panel, however, the estimation results on the state-dependent structure improve greatly when IVP weights can react to event $C = \{RV_t > median(RV_{t=1...T})\}$. For *both* λ_j^{TX} and λ_j^{WSD}, the introduction of c_1 and c_2 is highly significant for days with relatively large RV_t, with p values from the LR test smaller than 1%.

We obtain our most significant results in Table 5 when IVP weights can react

[5] With the exception of FTSE100, which is due to the fact that most overnight returns of FTSE100 are zero (98.94%).

differently to the event $D = \{RV_t > RV_{t-1}\}$. The increases in log-likelihood values are usually in hundreds for both λ_j^{TX} and λ_j^{WSD} and this can be seen across the four equity indices. This result suggests that both IVP weights λ_j^{TX} and λ_j^{WSD} are markedly different for days which see RV increasing from the previous day, which is consistent with the empirical findings of Andersen et al. (2019). Figure 1 plots the constant (blue circle) and the state-dependent IVP (green line) for two IVP estimators using the estimation results in Table 5. In Figure 1A, we see a clear leverage effect in Taylor-Xu IVP – when RV increases from the previous day, the IVP weights become higher in the morning hours than the constant case. In Figure 1B, on the other hand, the jump-robust WSD IVP weights are initially lower after market open, then higher during the noon and finally become lower again in the afternoon than their static counterpart[6].

4.2 Consequence on Intraday Jump Detection

To illustrate the consequence of ignoring state-dependent IVP weights and using a set of constant ones, we conduct the intraday jump test by Andersen, Bollerslev and Dobrev (2007)[7]:

$$\frac{|r_{t,j}|}{\sqrt{BPV_t \lambda_j}} > \Phi_{1-\gamma/2}, \tag{11}$$

[6] This behaviour can be due to the construction of WSD IVP estimator, which will dampen very large weights in the opening and toward the market close.

[7] Our state-dependent IVP can also be applied to the jump test by Lee and Mykland (2008) and those in Maneesoonthorn et al. (2020).

where $\Phi_{1-\gamma/2}$ is a $N(0,1)$ critical value at significance level $\gamma = 1-(0.99)^{1/M}$. Hence, we use 1% daily significance level of the jump test in (11), and given our sample of about 550 days, the nominal size is about 5.50. We only use the jump-robust WSD IVP weights in (11), and compare the test results with those given by its state-dependent version, since the Taylor-Xu IVP estimator is not robust to jumps.

In Table 6, we report the summary statistics of detected jumps using static WSD IVP and its responsive version. In the top panel, both methods give much more jumps than the nominal size 5.50, indicating the jumps are a genuine feature of price process. In between the two IVP methods, the responsive weights unanimously give fewer detections than the static approach. In addition, the responsive IVP weights give larger average jump size as measured by absolute jump returns, except for ESTX50. These results suggest jump test conducted with the state-dependent IVP weights could deliver a more prudent outcome.

In the bottom panel of Table 6, we look at the inter-arrival times, measured in hour, between two consecutive jumps. When these time durations have a standard deviation larger than their sample mean, the data is said to display over-dispersion which is a common feature for clustering count data. Moreover, the value of dispersion ratio, defined as the standard deviation of inter-arrival times divided by their sample mean, is closely related to the branching ratio of a self-exciting point process; see Hardiman and

Bouchaud (2014) and the review in Tsai (2021). We find that both static and responsive IVP weights produce jump times with the dispersion ratios larger than 1, except for NI225; however, the degree of over-dispersion is lower for the jumps given by the responsive, state-dependent IVP weights. This result suggests that ignoring the state-dependent nature of IVP and using static ones in intraday jump tests could over-estimate the degree of price jumps clustering.

Another marked difference between jumps given by static versus responsive IVP weights is their distribution over trading hours. Figure 2 plots the intraday distribution of jumps detected during days with $RV_t > RV_{t-1}$. The blue bars are numbers of jumps obtained using the static WSD IVP weights, and the green ones are given by the responsive WSD IVP weights. It can be seen that whilst most jump detections given by the static IVP weights are concentrated in the middle of trading hours, the state-dependent, responsive IVP weights discover more jumps in the beginning and toward the end of trading hours. Figure 3 plots the daily counts of detected jumps from the static and responsive IVP weights.

4.3 Discussion

In this subsection, we discuss some limitations and potential extensions of our approach. First, we can consider different structures for the dependence of IVP on state variables in (10). Secondly, the 5-minute returns of equity indices are highly non-Gaussian, and so it is useful to see if the LR test results in Section 4.1 still hold when the GARCH model is estimated with fat-tail densities such as t distribution. One can also add intraday leverage effect in (9) as in Tsai and Eom (2022).

In terms of the state variables, the four events we consider in constructing the state-dependent IVP either reflect leverage effect in volatility, with A and B as conditional information, or represent a relatively higher volatility level, with C and D two *ad hoc* measure of volatility level. It is certainly possible to use an *ex ante* measure of volatility level, such as:

$$E = \{E[RV_t|I_{t-1}] > RV_{t-1}\},$$

and let the IVP weights depend on this event. The conditional expectation $E[RV_t|I_{t-1}]$ can be given by some suitable volatility models such as the HAR regression of Corsi (2009) and many of its extensions (see Tsai and Eom 2022).

In addition, the WSD IVP weights can be modified such that the shortest half scale estimator in (7) can be replaced by the alternative Q_n estimator of Rousseeuw and Croux (1993)[8]. We note that a similar approach is given by Yeh, Wang and Kuan (2013).

Last but not the least, our proposed state-dependent IVP can be classified according to many market measures which are relevant for price jumps; these include measures on price impact, liquidity, the degree of asymmetric information and market

[8] This is directly suggested by Peter Rousseeuw through personal communication.

sentiment; see the review in Ahn and Tsai (2021). One can test if IVP weights depend on these measures, and investigate how such a dependence affect subsequent jump test results. We will leave this interesting task for future research.

5. Conclusion

In the current literature, standard approaches in finding intraday price jumps assume constant IVP weights over the test sample. We show that IVP weights can depend on some state variables or events, and propose to test the significance of such dependencies within an intraday GARCH framework. For four stock equity indices sampled at 5-minute frequency, we find strong evidence on asymmetric responses of IVP weights to the event $RV_t > RV_{t-1}$, and thus reject the constant IVP assumption. This result is consistent with the recent study by Andersen et al. (2019); it also suggests the state-dependent IVP weights should be used in finding intraday price jumps on those days with $RV_t > RV_{t-1}$.

In our empirical illustration, we show that accounting for the state-dependent nature of IVP weights could result in potentially more prudent jump detections, a lower degree of jumps clustering and a less concentrated intraday distribution of detected jumps over the trading hours. As these properties are fundamental to the nature of price jumps, we highlight the importance of state-dependent IVP weights and advocate the use of them in relevant applications.

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Table 1. Summary Statistics of Four Equity Indices

	KOSPI200	NI225	FTSE100	ESTX50
T	551	543	565	569
Trading hours (local time)	09:00-15:00	09:00-11:30 & 12:30-15:00	08:00-16:30	09:00-17:30
Period deleted	14:50-15:00	12:30-12:40	08:00-08:10	09:00-09:10
M	70	58	100	100
Mean	-2.5e-7	4.7e-6	6.4e-7	3.4e-6
S.D.	0.0013	0.0011	9.5e-4	0.0010
Min.	-0.0219	-0.0313	-0.0211	-0.0175
Max.	0.0240	0.0212	0.0202	0.0188
Skewness	-0.2849	-0.9391	0.0164	0.0041
Kurtosis	27.758	61.163	31.302	27.352

Note: Table 1 reports the summary statistics of four equity indices data. The top panel shows the trading hours in local time, as well as the periods deleted and the number of 5-min returns M in a day. The bottom panel shows that 5-min returns do not have a normal distribution.

Table 2. Estimation of GARCH (1, 1) for 5-minute Returns

	KOSPI200		NI225		FTSE100		ESTX50	
	λ_j^{TX}	λ_j^{WSD}	λ_j^{TX}	λ_j^{WSD}	λ_j^{TX}	λ_j^{WSD}	λ_j^{TX}	λ_j^{WSD}
$\mu \cdot 10^3$	0.0072	0.0061	0.0076	0.0085	0.0034	0.0028	0.0105	0.0109
	(0.0044)	(0.0039)	(0.0030)	(0.0051)	(0.0021)	(0.0017)	(0.0020)	(0.0023)
$\sigma \cdot 100$	0.1102	0.1097	0.1207	0.2818	0.0923	0.0960	0.1221	0.1733
	(0.0006)	(0.0006)	(0.0026)	(0.1108)	(0.0010)	(0.0014)	(0.0033)	(0.0140)
α	0.0361	0.0353	0.0964	0.0833	0.0505	0.0463	0.0619	0.0416
	(0.0024)	(0.0020)	(0.0033)	(0.0019)	(0.0021)	(0.0021)	(0.0021)	(0.0014)
$\alpha + \beta$	0.9928	0.9935	0.9931	0.9991	0.9967	0.9973	0.9976	0.9993
	(0.0009)	(0.0008)	(0.0016)	(0.0012)	(0.0006)	(0.0006)	(0.0006)	(0.0004)
L_0	213102.9	213280.8	183103.9	183105.9	331659.7	331790.8	331560.4	331417.1

Note: we report parameter $\sigma = \sqrt{\omega/(1-\alpha-\beta)}$ and standard errors inside parenthesis. The GARCH (1, 1) is estimated for 5-minute returns by assuming a conditional normal density.

Table 3. Estimation of GARCH (1, 1) for 5-minute Returns with State-dependent IVP on $r_{t-1}^{opop} < 0$

	KOSPI200		NI225		FTSE100		ESTX50	
	λ_j^{TX}	λ_j^{WSD}	λ_j^{TX}	λ_j^{WSD}	λ_j^{TX}	λ_j^{WSD}	λ_j^{TX}	λ_j^{WSD}
$\mu \cdot 10^3$	0.0069	0.0061	0.0072	0.0083	0.0032	0.0028	0.0103	0.0108
	(0.0072)	(0.0454)	(0.0032)	(0.0090)	(0.0022)	(0.0064)	(0.0016)	(0.0025)
$\sigma \cdot 100$	0.1098	0.1097	0.1305	0.2521	0.0919	0.0960	0.1201	0.1677
	(0.0005)	(0.0098)	(0.0034)	(2.1876)	(0.0010)	(0.0032)	(0.0024)	(0.0140)
α	0.0369	0.0353	0.0927	0.0822	0.0498	0.0464	0.0592	0.0409
	(0.0022)	(0.1250)	(0.0038)	(0.1025)	(0.0022)	(0.0042)	(0.0020)	(0.0018)
$\alpha + \beta$	0.9925	0.9935	0.9948	0.9989	0.9967	0.9973	0.9977	0.9993
	(0.0009)	(0.0219)	(0.0014)	(0.0433)	(0.0006)	(0.0029)	(0.0005)	(0.0004)
c_1	0.0049	3.99E-4	0.0207	7.8E-4	0.0023	9.9E-7	0.0047	7.7E-4
	(0.0009)	(2.9127)	(0.0019)	(1.2745)	(0.0005)	(1.8830)	(0.0005)	(0.0634)
c_2	1.4545	2.25E-4	4.0379	3.4E-6	2.1426	6.4E-4	3.5971	5.5E-3
	(0.0313)	(1.1257)	(0.1491)	(7.2629)	(0.0641)	(1.0001)	(0.0068)	(2.9107)
L_1	213119.1	213280.1	183194.6	183106.8	331675.8	331790.8	331599.4	331422.6
LR test	32.4	-1.4	181.4	1.8	32.2	0.0	78.0	11.0
p value	9.2E-08***	NA	4.1E-40***	0.407	1.0E-07***	1.000	1.2E-17***	0.004***

Note: Table 3 reports the estimation of GARCH (1,1) for 5-min returns when IVP weights depend on the sign of previous open-to-open returns in (10). The LR test is conducted by $2(L_1 - L_0) \sim \chi^2_{d.f=2}$, with L_0 given by the baseline GARCH (1, 1) in Table 2. The symbols "*", "**" and "***" indicate significant at 10%, 5% and 1% level respectively.

Table 4. Estimation of GARCH (1, 1) for 5-minute Returns with Asymmetric IVP weights for $r_{t-1}^{opop} < 0$ and $RV_t > median(RV_{t=1...T})$

	KOSPI200		NI225		FTSE100		ESTX50	
	λ_j^{TX}	λ_j^{WSD}	λ_j^{TX}	λ_j^{WSD}	λ_j^{TX}	λ_j^{WSD}	λ_j^{TX}	λ_j^{WSD}
State-dependent on: $B = \{r_{t-1}^{ovn} < 0\}$								
c_1	0.0084	0.0017	0.0276	0.0031	0.0468	0.0429	0.0091	0.0016
	(0.0011)	(0.0013)	(0.0037)	(0.0013)	(0.0248)	(0.0215)	(0.0019)	(0.0003)
c_2	1.2795	0.2888	3.0500	1.0609	12.001	12.300	4.2758	1.2108
	(0.0372)	(0.9569)	(0.1777)	(0.0929)	(1.4669)	(1.9483)	(0.2840)	(0.2346)
L_1	213141.1	213283.8	183254.3	183110.5	331665.0	331794.5	331674.4	331427.6
LR test	76.4	6.0	300.8	9.2	10.6	7.4	228.0	21.0
p value	2.6E-17***	0.050*	4.8E-66***	0.010**	0.005***	0.025*	3.1E-50***	2.8E-05***
State-dependent on: $C = \{RV_t > median(RV_{t=1...T})\}$								
c_1	0.0159	0.0062	0.0197	0.0127	0.0030	0.0023	0.0040	0.0055
	(0.0023)	(0.0014)	(0.0018)	(0.0018)	(0.0006)	(0.0004)	(0.0008)	(0.0005)
c_2	2.1850	1.7467	2.9846	1.42E-4	2.1476	3.6E-4	11.498	2.1E-7
	(0.5787)	(0.1987)	(0.2852)	(0.0979)	(1.1103)	(1.2875)	(0.0899)	(0.3274)
L_1	213200.9	213294.2	183202.7	183164.9	331681.2	331811.4	331573.2	331550.2
LR test	196.0	26.8	197.6	118.0	43.0	41.2	25.6	266.2
p value	2.7E-43***	1.5E-06***	1.2E-43***	2.4E-26***	4.6E-10***	1.1E-09***	2.8E-06***	1.6E-58***

Table 5. Estimation of GARCH (1, 1) for 5-minute Returns with State-dependent IVP weights on $RV_t > RV_{t-1}$

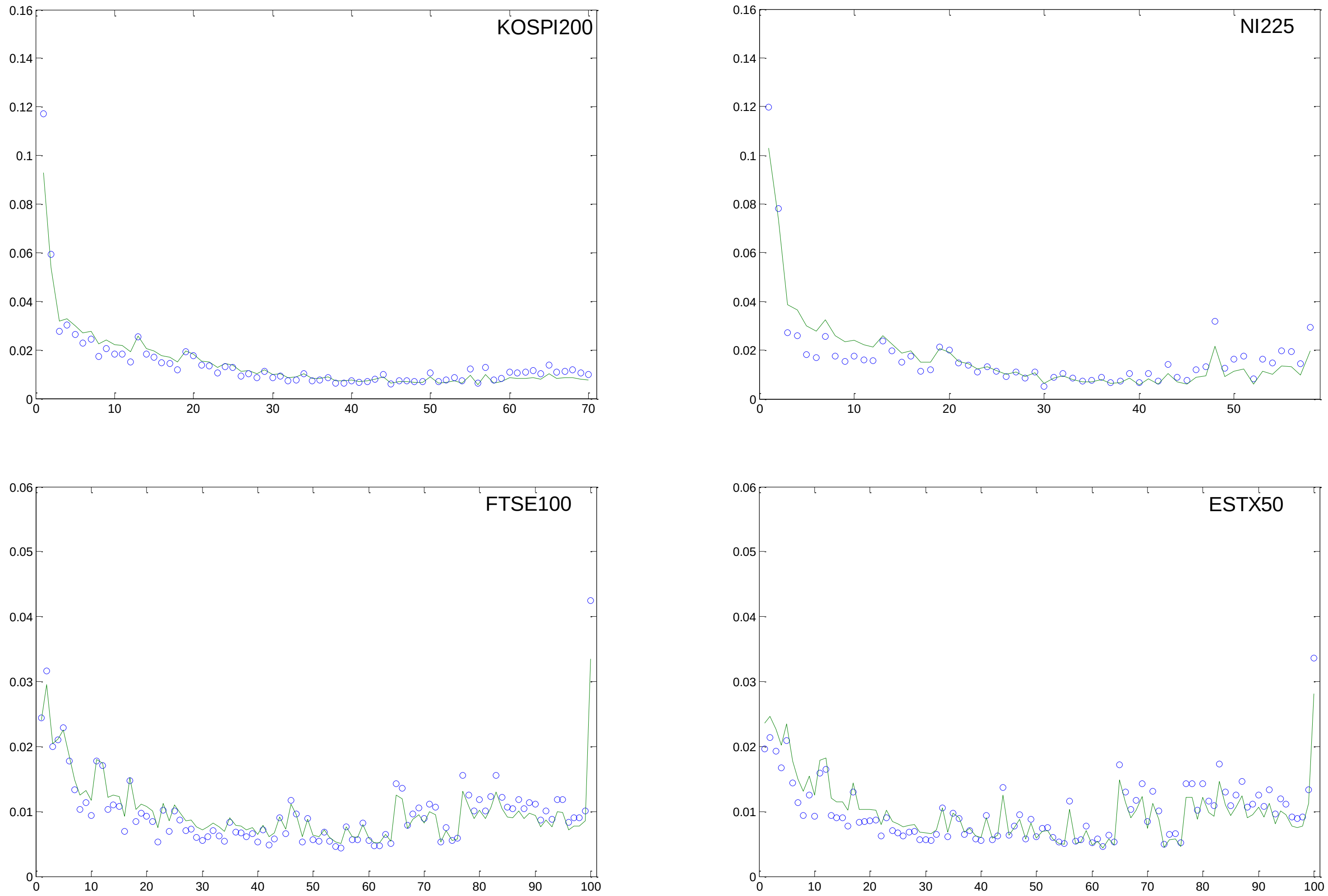
	KOSPI200		NI225		FTSE100		ESTX50	
	λ_j^{TX}	λ_j^{WSD}	λ_j^{TX}	λ_j^{WSD}	λ_j^{TX}	λ_j^{WSD}	λ_j^{TX}	λ_j^{WSD}
$\mu \cdot 10^3$	0.0068	0.0058	0.0071	0.0066	0.0031	0.0025	0.0102	0.0102
	(0.0040)	(0.0031)	(0.0029)	(0.0035)	(0.0024)	(0.0024)	(0.0021)	(0.0023)
$\sigma \cdot 100$	0.1129	0.1116	0.1600	0.1606	0.0992	0.1030	0.1336	0.1748
	(0.0011)	(0.0008)	(0.0076)	(0.0262)	(0.0012)	(0.0035)	(0.0055)	(0.0097)
α	0.0234	0.0242	0.0601	0.0515	0.0328	0.0274	0.0487	0.0279
	(0.0012)	(0.0012)	(0.0035)	(0.0037)	(0.0015)	(0.0015)	(0.0022)	(0.0011)
$\alpha + \beta$	0.9977	0.9974	0.9985	0.9985	0.9988	0.9992	0.9987	0.9997
	(0.0004)	(0.0004)	(0.0006)	(0.0017)	(0.0003)	(0.0003)	(0.0004)	(0.0001)
c_1	0.0215	0.0161	0.0367	0.0128	0.0068	0.0060	0.0085	0.0079
	(0.0014)	(0.0011)	(0.0022)	(0.0012)	(0.0005)	(0.0004)	(0.0007)	(0.0004)
c_2	1.9814	1.6148	3.1695	8.8E-7	1.0178	4.1E-4	3.3944	2.3E-5
	(0.1479)	(0.0137)	(0.1003)	(0.6766)	(0.1465)	(0.7099)	(0.4409)	(2.0012)
L_1	213320.8	213388.9	183361.8	183194.9	331793.6	331935.6	331674.1	331734.9
LR test	435.8	216.2	515.8	178.0	267.8	289.6	227.4	635.6
p value	2.3E-95***	1.1E-47***	9.9E-113***	2.2E-39***	7.0E-59***	1.3E-63***	4.2E-50***	9.6E-139***

Table 6. Summary Statistics of Sizes and Inter-arrival Times of Price Jumps

	KOSPI200		NI225		FTSE100		ESTX50	
λ_j^{WSD}	Static	Responsive	Static	Responsive	Static	Responsive	Static	Responsive
				Jump size (in absolute value)				
No. of Obs.	85	73	119	110	140	131	198	179
Mean	0.0044	0.0048	0.0036	0.0038	0.0036	0.0037	0.0037	0.0036
S.D.	0.0044	0.0047	0.0033	0.0034	0.0028	0.0028	0.0027	0.0025
Min	0.0011	0.0011	7.7e-4	7.7e-4	9.7e-4	9.7e-4	8.3e-4	8.3e-4
Max	0.0240	0.0240	0.0212	0.0212	0.0202	0.0202	0.0188	0.0188
				Inter-arrival time (in hour)				
Mean	37.95	44.28	22.06	23.88	32.95	35.57	23.27	25.76
Median	20.75	25.21	16.50	18.25	19.75	22.67	13.92	16.58
S.D.	60.22	65.23	19.81	19.76	37.13	37.80	26.27	28.14
Min	0.17	0.17	0.08	0.17	0.08	0.08	0.08	0.08
Max	390.50	390.50	107.17	80.00	178.50	195.67	162.58	154.75
Dispersion	1.5869	1.4733	0.8981	0.8275	1.1270	1.0626	1.1290	1.0924

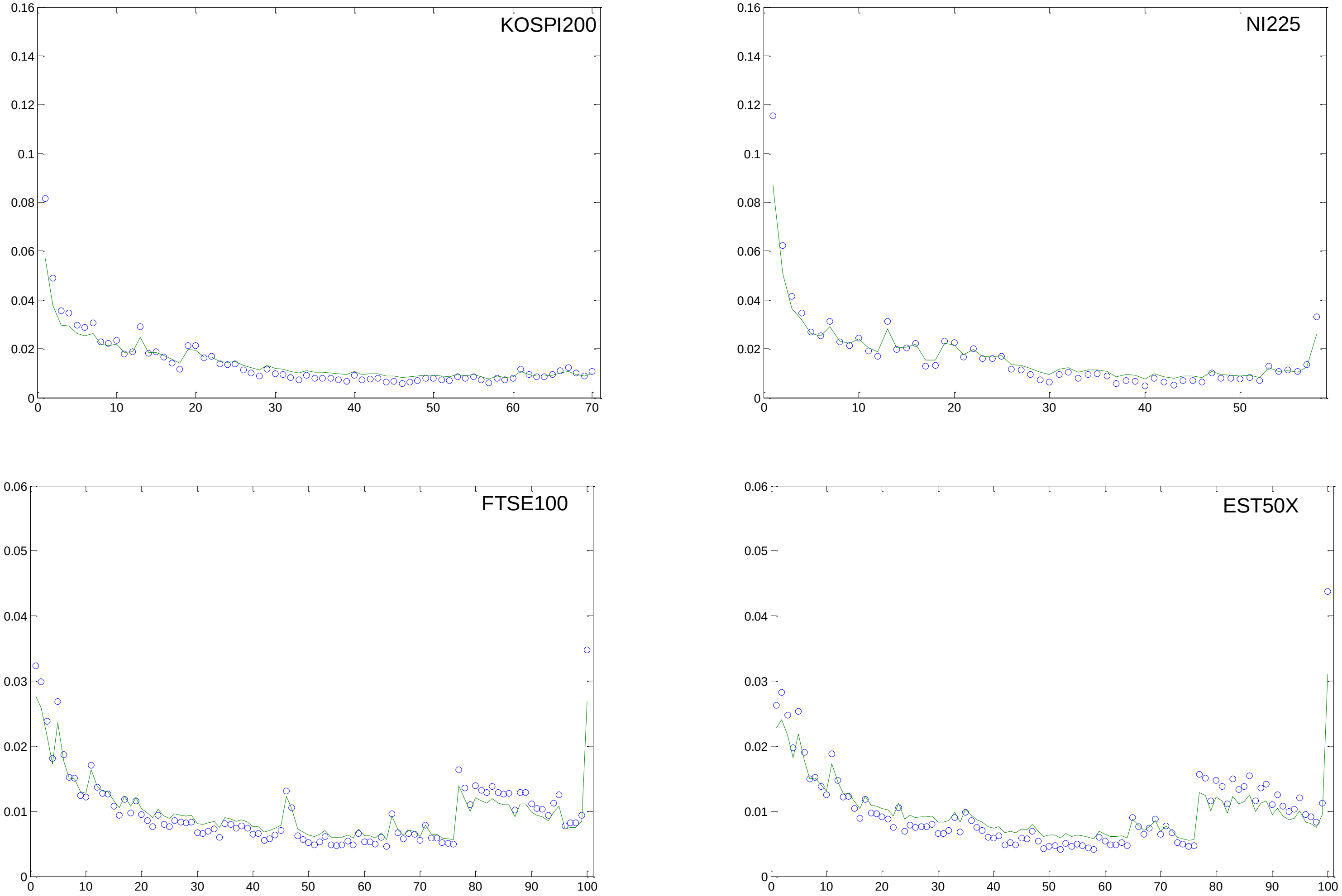
Note: The dispersion ratio is given by S.D./Mean of the inter-arrival times between jumps. It is equal to 1 for a Poisson process, and a value larger than one indicates clustering in the arrivals.

Figure 1A. Static and State-dependent Taylor-Xu IVP Weights on $RV_t > RV_{t-1}$



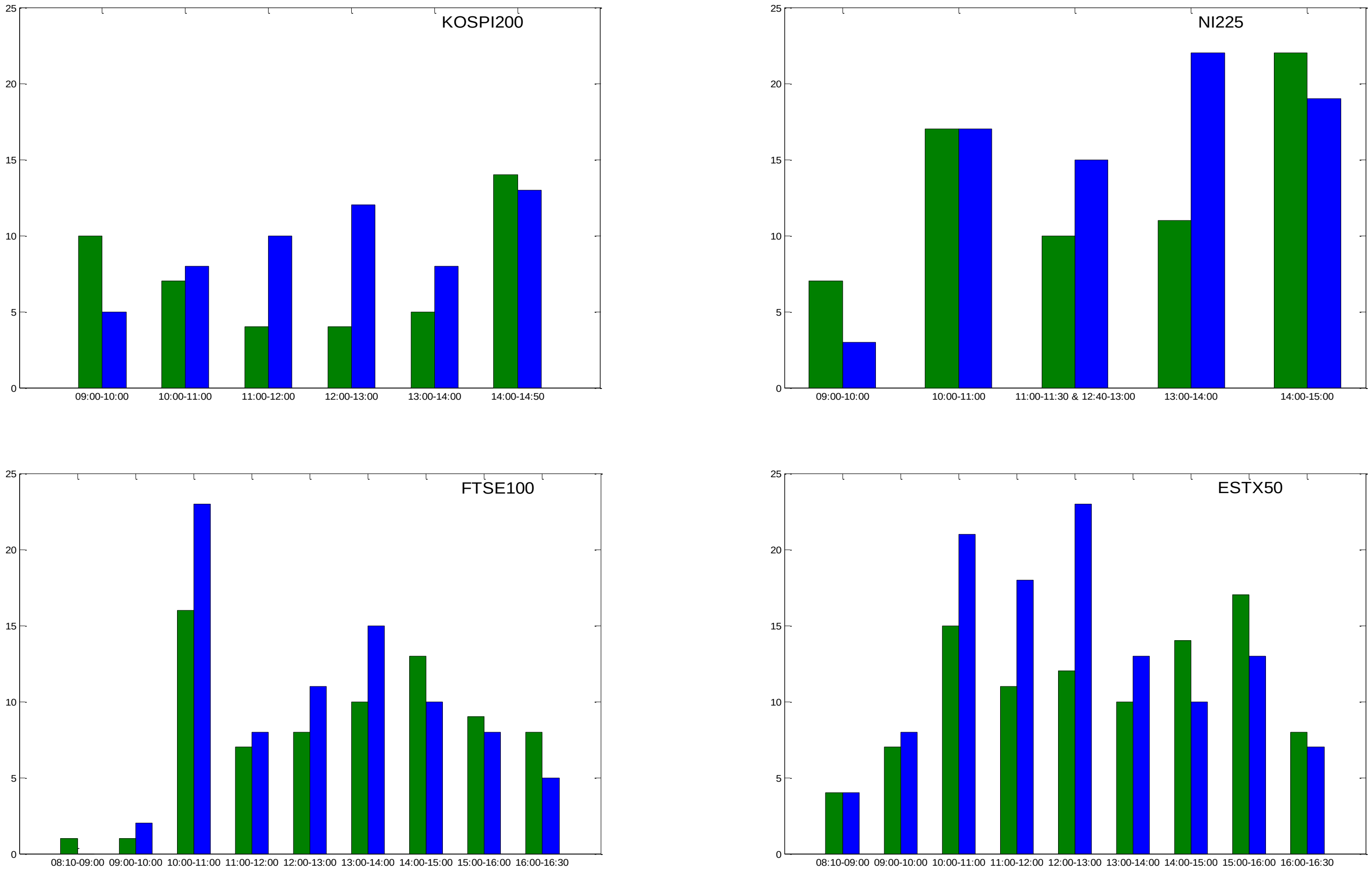
Note: When RV increases from day $t-1$ to day t, the state-dependent IVP (green line) has larger weights in the morning session than the static case (blue circle). This change of pattern is consistent with the results in Table 1 and 2, and thus confirming leverage effect in Taylor-Xu IVP.

Figure 1B. Static and State-dependent Jump-robust WSD IVP Weights on $RV_t > RV_{t-1}$



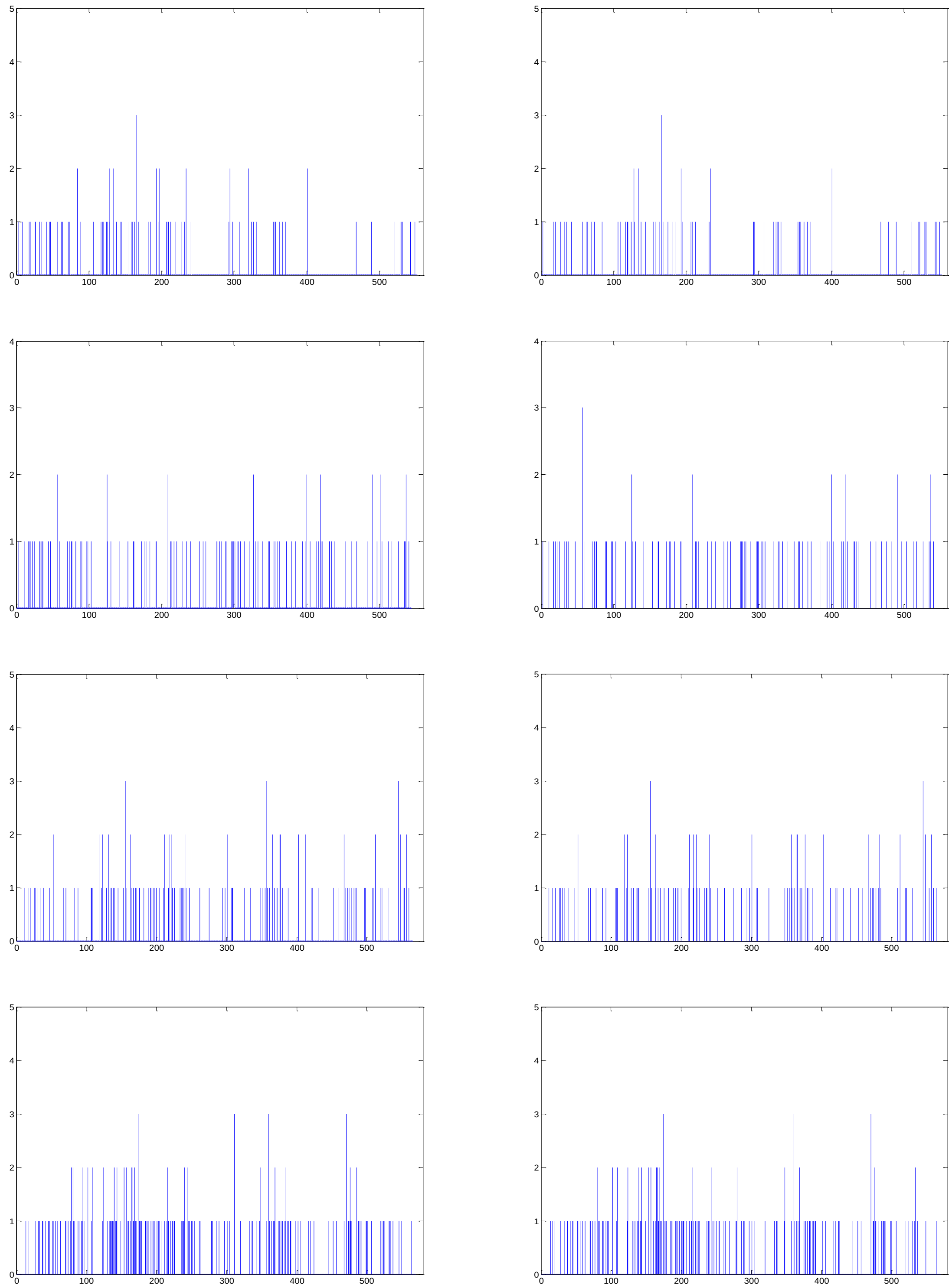
Note: When RV increases from day $t-1$ to day t, the state-dependent IVP (green line) first has lower weights than the static case (blue circle), then shows higher weights in the noon session and finally lower weights again in the afternoon session.

Figure 2. Intraday Distribution of Price Jumps Detected during Days with $RV_t > RV_{t-1}$



Note: Blue bars are the number of jumps obtained from static WSD IVP, and green bars are number of jumps obtained from the state-dependent, responsive WSD IVP.

Figure 3. Daily Count of Detected Jumps



Note: From top to down: KOSPI200, NI225, FTSE100 and ESTX50. Left panel: jumps obtained with static WSD IVP; right panel: jumps obtained with state-dependent WSD IVP.