New stylized facts of financial exuberance periods*

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December 9, 2022

Abstract

Stylized facts of asset returns are widely established. Among these are e.g. non-normality, volatility clustering and high persistence. Another important recurring aspect is the existence of financial exuberance, often interpreted as an explosive price bubble. Exuberance periods consist of two parts, (i) the temporary explosive price period and (ii) the mean-reverting reverse period of market correction. We provide a comprehensive analysis of exuberance periods by analysing 30 markets from different asset classes over a time period of fifty years. We cover international stock markets, the US housing market, Gold, Silver and Oil as well as the Bitcoin prices. Overall, we find 143 exuberance phases and document evidence on important characteristics like (i) durations of explosive phases, (ii) collapse duration and behaviour during market correction phases, (iii) magnitude of autoregressive parameters during exuberance and market correction and (iv) distributional characteristics like fat tails and shifts in the innovation variance. We classify the cross-sectional results on 143 exuberance phases into relatively low, middle and high values. We test a number of common beliefs in the literature and provide new insights into typical empirical properties of explosive prices and their collapse. Our results indicate significant discrepancies with typical settings in the literature. Empirical explosiveness is much milder and collapse phases are in most cases smooth rather than abrupt. Moreover, prices do not revert back to the initial value, but stay significantly above. The simplified view that prices are strongly exploding with a full collapse in short time is not supported by our results. Duration dependence modelling reveals that the length of the explosive phase is positively affected by economic growth, while the collapse duration is only driven by the length of the preceding explosive phase in a positive way. Finally, we offer empirically relevant parametrizations for data generating processes and study the consequences for the empirical performance of popular bubble detection and date-stamping procedures.

*__This is preliminary work in progress.__ The authors would like to thank George Tauchen, Niklas Johan Christoffer Ahlgren, Karim Abadir, Eric Girardin, Daan Opschoor, Vasyl Golosnoy, Yuze Liu, Pascal Goemans, Maxime Nicolas and participants of the Statistical Week (2021, Kiel), 15th International Conference Computational and Financial Econometrics (2021, London), 6th UA RuhrMetrics Seminar (2022, Duisburg), RCEA Conference on Recent Developments in Economics, Econometrics and Finance (2022, The Rimini Centre for Economic Analysis), 29th Annual SNDE Symposium (2022, Florida), 6th International Workshop on Financial Markets and Nonlinear Dynamics (2022, Paris), 4th Quantitative Finance and Financial Econometrics Conference (2022, Marseille), 27th Forecasting Financial Markets Conference (2022, Milano), and the 28th Annual Meeting of the German Finance Association (2022, Marburg) for helpful comments and discussions. Kruse-Becher gratefully acknowledges financial support from CREATES funded by the Danish National Research Foundation (DNRF78).

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Keywords: Explosive prices, financial turmoil, stylized facts, duration, market recovery, volatility shifts

JEL classification: C15, C22, C41, C58, G01

1 Introduction

Due to the extensive number of financial exuberances and crises that took place during the last 30 years (e.g., dot-com bubble, sub-prime mortgage crisis, European debt crisis) and their serious consequences on whole economies and societies as well as their contagion effects, there is an ever-growing interest in how to detect and deal with such extreme situations. Many authors investigate bubbles and exuberance periods in financial time series. Phillips, Wu, and Yu (2011) have developed a popular procedure to date stamp periods of financial exuberance. In order to be able to capture multiple explosive periods in a single financial time series, they offer an extended approach which is known as the Generalized Supremum Augmented Dickey Fuller (GSADF) Test in 2015 (Phillips, Shi, and Yu, 2015a; Phillips, Shi, and Yu, 2015b).

The goal of our research is to investigate the behaviour of daily real log-prices during the two phases of financial exuberance, namely the time from start to peak of the exuberance period (explosive period) and from the first day after the peak until the end of exuberance (mean-reverting period). Naturally, the question emerges what we understand under the term "exuberance" and more specifically, how it is related to the often used word "bubble". First of all, both of them are no synonyms. While exuberances show explosive behaviour - which is also true for bubbles - not each exuberance period is a bubble. Thus, one can assume that bubbles imply exuberance but the reverse is not true. Kindleberger and Aliber (2015) define a bubble as "[...] a generic term for the increases in the prices of securities or currencies in the mania phase of the cycle that cannot be explained by the changes in the economic fundamentals." We do not account for fundamentals because they cannot be estimated in a reasonable and most importantly reliable way (Siegel, 2003; Rosser, 2013). Using dividends to obtain the fundamental value is problematic because not each company pays dividends, they are not smooth, their prediction is hardly possible, more and more firms nowadays engage in share buyback systems rather than paying dividends and dividends are heavily influenced by firm decisions and are thus no reliable indicator for company health and performance (Basse, Klein, Vigne, and Wegener, 2021). Using more modern proxies like future/forward prices comes with its own problems. Therefore, the best guess we can make is to rely on real prices (a.k.a. inflation adjusted prices). So, using statistical procedures we identify periods of exuberance which can then be further analysed by macroeconomic techniques and judgement to determine if it is a bubble or not.

While stylized facts[1] of financial time series in general are well-investigated (Pagan, 1996; Cont, 2001), there is not much known about the behaviour in different states of a financial time series. Therefore, we start by examining the empirical features of explosive phases and their corresponding mean-reverting market correction phase from a meta-analytical viewpoint.

The following general stylized facts have been demonstrated in the literature: log-returns of many financial time series do not show autocorrelation but instead autocorrelation is present in squared and absolute returns (Pagan, 1996; Cont, 2001). Furthermore, many time series show power-law distribution similarities but when increasing the latency of the data, e.g., from daily to yearly, the distribution becomes more and more like a normal distribution (Mandelbrot, 1963; Fama, 1965; Mandelbrot, 1967). Another finding is a gain/loss asymmetry as well as a phenomenon called "volatility clustering" (Engle, 1982; Bollerslev, 1986). In such a case, periods of low volatility tend to be followed by low volatility and periods of high volatility by high volatility. Another important finding are heavy tails, so that the emergence of extreme events is much more likely than in the case of a normal distribution (Mandelbrot, 1963; Fama, 1965; Mandelbrot, 1967). Besides these findings, also a leverage effect has been identified. It states that many volatility measures are negatively correlated with returns (Glosten, Jagannathan, and Runkle, 1993; Zakoian, 1994).

We strive to close the gap between the literature on explosive price periods, bubble tests (and related econometric (monitoring) procedures) and stylized facts and subsequently, to develop new stylized facts. We are doing this by analysing the identified exuberance periods returns concerning their distributional and dynamic properties for both the explosive and reverse period. Additionally, we analyze some general financial exuberance characteristics. Therefore, we look into how much value is typically gained during an explosive period and how much is lost in the reverse. Besides, we also investigate the duration of an exuberance period and the ratio of the duration of the explosive period compared to the reverse. Furthermore, we provide specific values for the autoregressive parameter in dynamic time series models during the explosive and reverse phase. Typical perceptions on these quantities might differ significantly from empirical features. Our whole analysis is done - in contrast to most of the existing literature on financial exuberance - based on real daily data.[2]

Based on our analysis of 143 identified exuberance periods, we are able to extract the subsequent

[1]Characteristics that are shared by many different kinds of financial time series are called "stylized facts" (Cont, 2001).

[2]In line with Whitehouse (2019), we use daily data because they provide more information than monthly data about the underlying data generating process. Furthermore, the investigation of exuberance characteristics and stylized facts, i.e., persistence and distribution, results in a serious small sample problem if one uses monthly instead of daily data.

stylized facts which show major differences to assumed theoretical DGP settings in the literature: (i) Empirical explosiveness illustrated by autoregressive model parameters, is much milder and collapse phases are in most cases smooth rather than abrupt. Especially, assumed one period reverses like in Evans (1991) are hardly observable. (ii) Prices do not revert back to the initial value, but stay significantly above. So, theoretical models based on the believe that the first day before and after the exuberance is over have equal prices is not empirically justified. Thus, the simplified view that prices are strongly exploding with a full collapse in short time is not supported by our data-driven results. (iii) Duration dependence modelling reveals that the length of the explosive phase is positively affected by economic growth, while the collapse duration is only driven by the length of the preceding explosive phase in a positive way. (iv) Finally, we offer empirically relevant parametrizations for data generating processes. We extract median values for the DGP's parameters and additionally generate low- and high-setting values. So, our DGPs are able to account for different types of financial exuberances. Based on them, we study the consequences for the empirical performance of popular bubble detection and date-stamping procedures and clearly illustrate that they suffer in power by applying data-driven rather than theoretical assumed DGP parametrizations. With our research we thus contribute to the existing literature of stylized facts by enlarging it and we additionally provide empirical-reliable, data-driven parametrizations of DGPs, so that new developed exuberance and bubble detection procedures can be tested based on realistic data generating processes rather than on some solely theoretically driven processes.

The structure is as follows: Section two gives a literature overview and part three describes the used data set. Section four deals with the testing and identification procedure for financial exuberance periods and in the upcoming fifth section, the identified price exuberance periods are described and additionally, we provide basic characteristics and stylized facts of those periods. The sixth section consists of Monte Carlo simulation results for the power of popular unit root tests against explosive alternatives based on empirically relevant specifications of the DGPs. Conclusions are drawn in section seven.

2 Literature Review

Due to the severe systematic and societal consequences of financial turmoil and crisis (e.g., tulip mania, south sea bubble, great depression, dotcom bubble, global financial crisis), there is a rich history of research which deals with such situations. Nevertheless, the starting point is quite recently in 1978. Before, there was a common believe that financial crisis cannot be modelled mathematically. Kindleberger (1978)[3] set the basis by describing the theoretical aspects and consequences of manias, panics and crashes. Amongst his work, there have been published other famous books about the general structure of financial crisis, most prominently "This time is different: Eight centuries of financial folly" by Reinhart and Rogoff (2009). Next to this more qualitative books, there is a great strand of literature about econometric and time series based bubble detection procedures. The first bubble test has been proposed by Flood and Garber (1980). Based on this research, Blanchard (1979)[4] showed that speculative bubbles do not collide with the rationality assumption. Then there is the category of variance bounds tests which has been proposed by Shiller (1981) and LeRoy and Porter (1981). Initially, they have not been developed for bubble detection but they are used by many authors for this purpose. Then in 1987, West introduced a two-step procedure (West, 1987; West, 1988). During the same time, Diba and Grossman (1987), Diba and Grossman (1988a), and Diba and Grossman (1988b) applied standard stationarity- and cointegration-based tests for bubbles. Their approach has been famously criticised by Evans (1991) who especially pointed out that the above tests have poor power issues in detecting periodically collapsing bubbles. A detailed overview of econometric exuberance detection procedures up to the beginning of the 21st century can be found in Gürkaynak (2008). Another more recent category of bubble detection procedures is based on fractional integration tests, see e.g., Cunado, Gil-Alana, and Gracia (2005) and Frömmel and Kruse (2012).

Beginning with Phillips, Wu, and Yu (2011) (PWY), the area of recursive unit root testing for bubbles gained popularity. They propose a right-tailed unit root test (supremum augmented Dickey-Fuller (SADF) test) which is not only able to detect exuberance periods but is also able to estimate the start and endpoint of a bubble. Further simulation results of the SADF test under different DGP settings is provided by Phillips, Shi, and Yu (2014). In the meantime, Homm and Breitung (2012) (HB) tested the power of different statistical procedures which have not been applied to exuberance detection so far and benchmarked them against the SADF test. They were able to show that two tests, namely one Chow-type Dickey Fuller test and a modified test of Busetti and Taylor (2004) show higher power than the SADF test. Furthermore, they successfully applied CUSUM tests for date stamping bubbles. A generalization of the CUSUM procedure has been done by Astill, Harvey,

[3] Today, his book is available as the 7th edition (Kindleberger and Aliber, 2015). His work has been hold to life by Robert Z. Aliber after Kindleberger past away in 2003. So, for the 5th to 7th edition, Aliber has been responsible.

[4] Interestingly, nevertheless Blanchard based his analysis on Flood and Garber (1980), Blanchard's article has been published earlier.

Leybourne, Taylor, and Zu (2021). In 2015 then, the SADF test has been enlarged by Phillips, Shi, and Yu (2015a) (PSY) and Phillips, Shi, and Yu (2015b) to solve the power issues if there are multiple periodically collapsing bubbles within a time series, which is typically the case. So, they propose a generalization of the SADF test, namely the GSADF test for explosiveness testing and the backward SADF (BSADF) test for date stamping. Both approaches are nowadays considered as the market standard and have been applied to numerous markets, e.g., Anundsen, Gerdrup, Hansen, and Kragh-Sorensen (2016) for housing, Corbet, Lucey, and Yarovaya (2018) for cryptocurrencies, Brunnermeier, Rother, and Schnabel (2020) for stock markets and systematic risk and Contessi, De Pace, and Guidolin (2020) for fixed income markets.

There are many different studies which consider different aspects that are not accounted for in PWY and PSY. Harvey, Leybourne, and Sollis (2015) compare the power of PWY and HB and finally propose a unions of rejection strategy. Harvey, Leybourne, Sollis, and Taylor (2016) investigate the effect of non-stationary volatility on the performance of the PWY procedure and proposed a wild bootstrap procedure which Phillips and Shi (2020) later used in the PSY setting. Nowadays, the wild bootstrap approach has emerged as the standard procedure to conduct robust inference. Phillips and Shi (2018) address the empirically highly relevant issue of smooth collapses in the context of bubble detection and date-stamping. In the mean time, Harvey, Leybourne, and Zu (2019) and Whitehouse (2019) developed GLS based versions of PWY and a unions of rejections strategy based on OLS PWY and GLS PWY. Moreover, Hafner (2020) investigates the effect of a time-varying volatility, consisting of a deterministic long-term component and a stochastic short-run element, on the PWY procedure, Pedersen and Schütte (2020) consider the performance of PWY and PSY in the case of autocorrelated innovations and propose a sieve bootstrap procedure while Kurozumi, Skrobotov, and Tsarev (2021) consider the case of time-varying non-stationary volatility for PSY. Recently, Monschang and Wilfling (2021) test different exuberance detection procedures in the case of leverage effects by introducing a TGARCH model into their DGP.

Next to this research branch, Harvey, Leybourne, Sollis, and Taylor (2016) have proposed a BIC procedure to date stamp a single bubble and Harvey, Leybourne, and Whitehouse (2020) enlarged the procedure to date stamp multiple bubbles. Furthermore, Astill, Harvey, Leybourne, and Taylor (2017) provide a new procedure to test for end of sample bubbles, Guo, Sun, and Wang (2019) introduces a new test of explosiveness and Harvey, Leybourne, and Zu (2020) propose a sign-based version of PSY.[5]

[5] An extensive overview of bubble detection procedures of the 'newer' generation can be found in Skrobotov (2022).

Furthermore, there is the literature on co-explosiveness of bubbles in financial time series, see e.g., Evripidou, Harvey, Leybourne, and Sollis (2022). It is important to mention that this list is by no means complete due to the rich amount of literature published about financial crisis and bubbles.

3 Data

During the analysis, a diverse range of financial time series is analysed. This include equity market indices, precious metals, oil, cryptocurrencies and real estate indices. The advantage of this high diversity of financial time series is to search for stylized facts and not just for some asset specific findings.

All time series are downloaded as price indices from REFINITIV Datastream (formerly known as Thomson Reuters Datastream). In each case, we use daily observations which start at 2nd January 1970[6] or if data are not available from this point in time, the longest available history is used. Equity indices are chosen in such a way that not only the most important indices of the world are considered but also emerging and frontier markets as well as indices from countries which are spread all around the world. This is done to avoid a bias towards developed countries and to the biggest financial markets, especially towards the United States. Therefore, we use a total of 24 indices from Europe (AEX, CAC 40, DAX 30, FTSE 100, OMXH and SMI), America (Mexico IPC, NASDAQ, S&P 500 and S&P TSX Composite), Asia (Hang Seng, IDX Composite, KOSPI, NIFTY 500, NIKKEI 225, Shanghai SE A Share, Straits Times Index L and TOPIX), Africa (FTSE South Africa, HRMS, MASI and TUNINDEX) and the two intercontinental countries Israel (Israel TA 125) and Russia (MOEX). An overview of all applied stock market indices is available in Table 1.

In addition to stock market indices, we also consider the two precious metals gold and silver which are often used and assumed as safe heaven assets due to their low correlation to stock and bond market indices and they are also seen many times as assets which can protect against inflation (Hillier, Draper, and Faff, 2006; Bauer and B. M. Lucey, 2010; Bauer and McDermott, 2010; Bampinas and Panagiotidis, 2015). Besides, to get a broader picture, also the two oil indices Crude Oil WTI[7]

[6]Start is not on the 1st January 1970 because in most countries, stock exchanges were closed on this day.

[7]To be able to apply the logarithm, the negative value of USD 37.63 at 20th April 2020 is replaced by USD 0.0001. This negative value occurred due to the lag of storage of oil producers due to a lag of demand for WTI oil and was a unique event never happened before (Corbet, Goodell, and Günay, 2020).

Table 1: Stock market indices applied in the analysis

index	Datastream	currency	country	start date	end date
AEX	AMSTEOE	EUR	The Netherlands	1983-01-03	2021-04-30
CAC 40	FRCAC40	EUR	France	1990-01-01	2021-04-30
DAX 30	DAXINDX	EUR	Germany	1970-01-02	2021-04-30
FTSE 100	FTSE100	GBP	United Kingdom	1988-01-01	2021-04-30
FTSE South Africa	JSEOVER	ZAR	South Africa	1995-06-30	2021-04-30
Hang Seng	HNGKNGI	HKD	Hong Kong	1980-10-01	2021-04-30
HRMS	EGHFINC	EGP	Egypt	1995-01-02	2021-04-30
IDX Composite	JAKCOMP	IDR	Indonesia	1996-01-01	2021-04-30
Israel TA 125	ISTA100	ILS	Israel	1987-04-23	2021-04-30
KOSPI	KORCOMP	KRW	Republic of Korea	1974-12-31	2021-04-30
MASI	MASIIDX	MAD	Morocco	2007-01-01	2021-04-30
Mexico IPC	MXIPC35	MXN	Mexico	1988-01-04	2021-04-30
MOEX	RSMICEX	RUB	Russian Federation	1997-09-22	2021-04-30
NASDAQ	NASCOMP	USD	United States	1971-02-05	2021-04-30
NIFTY 500	ICRI500	INR	India	2011-01-03	2021-04-30
NIKKEI 225	JAPDOWA	JPY	Japan	1970-01-02	2021-04-30
OMXH	HEXINDX	EUR	Finland	1987-01-02	2021-04-30
S&P 500	S&PCOMP	USD	United States	1970-01-02	2021-04-30
S&P TSX Composite	TTOCOMP	CAD	Canada	1970-01-02	2021-04-30
Shanghai SE A Share	CHSASHR	CNY	China	1992-01-02	2021-04-30
SMI	SWISSMI	CHF	Switzerland	1988-06-30	2021-04-30
Straits Times Index L	SNGPORI	SGD	Singapore	1999-08-31	2021-04-30
TOPIX	TOKYOSE	JPY	Japan	1970-01-02	2021-04-30
TUNINDEX	TUTUNIN	TND	Tunisia	1997-12-31	2021-04-30

This table provides an overview of all applied stock market indices in the analysis. They are sorted alphabetically based on their first letter. The first column provides the name of each index and the second the Datastream symbol that has been used to obtain the data. In the third column, the ISO 4217 currency codes for the indices are provided. The next column illustrates the country which is covered by the specific index and the last two columns provide the starting and end date of the index which is used in the analysis section.

and Europe Brent are investigated.[8] On top of this, Bitcoin is considered as a representative of the new asset class cryptocurrencies. Especially against the background of the global financial crisis of 2007/09, also the DJ US Real Estate Index is investigated. Table 2 provides an overview of the used assets other than stock market indices.

Table 2: Indices of other assets than stocks applied in the analysis

index	**Datastream**	**currency**	**asset category**	**start date**	**end date**
Bitcoin	BTCTOU$	USD	cryptocurrency	2011-08-18	2021-04-30
Crude Oil WTI	CRUDOIL	USD	oil	1983-01-10	2021-04-30
DJ US Real Estate	DJUSRE$	USD	real estate	1992-01-02	2021-04-30
Europe Brent	EIAEBRT	USD	oil	1987-05-20	2021-04-30
Gold	GOLDBLN	USD	precious metal	1980-01-01	2021-04-30
Silver	SILVERH	USD	precious metal	1980-01-01	2021-04-30

This table provides an overview of all applied assets in the analysis which are no equity indices. They are sorted alphabetically based on their first letter. The first column provides the name of each asset/index and the second the Datastream symbol that has been used to obtain the data. In the third column, the ISO 4217 currency codes for the indices are provided. The next column states the asset category which is covered by the specific index and the last two columns illustrate the starting and end date of the index which is used in the analysis section. Due to the reliability of data, Gold and Silver are only considered from 1st January 1980 and not prior.

We perform our analysis based on real log prices, so that price increases simply driven by inflation are cancelled out. This is done by dividing the assets'/indices' price by the consumer price index (CPI) of the corresponding country/region:[9]

$$p_{t_{real}} = \ln\left(\frac{P_t}{CPI_t}\right). \tag{1}$$

4 Econometric methods

In the literature, there are many different tests for identifying periods of financial exuberance. The most often applied models are based on the idea of rational bubbles. In this chapter, the well-known Generalized Supremum Augmented Dickey Fuller (GSADF) technique of Phillips, Shi, and Yu (2015a) and Phillips, Shi, and Yu (2015b) and their date stamping procedure (Backward Supremum Augmented Dickey Fuller (BSADF) Test) are described.

[8]Because gold, silver and oil are considered as homogenous assets, we decide to focus on a few hand selected time series of these assets because the use of more time series would not result in an additional information gain.

[9]Due to the fact that CPIs are only available on a monthly basis, the monthly CPI corresponding to each index's/asset's currency is used as a proxy for the daily price level. Some time series are shorten because of the later availability of CPI values. Furthermore, all price data are scaled by 100 to avoid situations where the logarithm of the real price is negative. This is done to do not have problems with situations where the start of exuberance is with a negative price and its peak is positive because in such situations, one cannot reasonably calculate growth rates.

4.1 Testing for multiple explosive prices

The idea is to apply a recursive regression procedure based on ADF tests because it has been demonstrated in the literature that standard unit root and cointegration tests are not able to detect multiple collapsing bubbles within the same time series (Flood and Garber, 1980; Flood and Hodrick, 1986; Evans, 1991). The right-tailed unit root test is constructed based on the following ADF-regression which is simply estimated by ordinary least squares (OLS):

$$\Delta y_t = \mu_{r_1,r_2} + \rho_{r_1,r_2} y_{t-1} + \epsilon_t. \tag{2}$$

The first difference of log real prices Δy_t (which are log returns) is regressed on the sum of a slope μ_{r_1,r_2}, a ρ_{r_1,r_2} weighted first order lag of the log real price y_{t-1}, and the error term ϵ_t. This ADF regression is estimated multiple times by using different subsets of the sample data. The null hypothesis of a unit root is then tested against the alternative hypothesis of a (mildly) explosive process:[10]

$$H_0 : \rho_{r_1,r_2} = 0 \text{ (unit root)},$$
$$H_1 : \rho_{r_1,r_2} > 0 \text{ (mildly explosive behaviour)}.$$

The normalized subset's start and end values r_1 and r_2 are both defined in such a way that they are allowed to grow. The minimum value of r_1 is 0. So, r_1 starts with the first observation of the applied data set. Its maximum is set to the difference between the value of r_2 and the minimum window size r_0. In contrast, r_2 runs from r_0 to the latest (most new) observation (1) in the data set:

$$r_1 \in [0, r_2 - r_0],$$
$$r_2 \in [r_0, 1].$$

The minimum window size r_0 is defined as $0.01 + 1.8/\sqrt{T}$, where T illustrates the number of observations. The size of the subsample is increased by one observation until the limit is reached - the last observation of the used data set.

[10] Equivalently, often the following version of the ADF regression is used: $y_t = \mu_{r_1,r_2} + \rho_c y_{t-1} + \epsilon_t$. In this setting $\rho_c = \rho_{r_1,r_2} + 1$ and consequently, the null hypothesis $\rho_c = 1$ is tested against the alternative of $\rho_c > 1$ (Contessi, De Pace, and Guidolin, 2020).

The GSADF test is then stated as:

$$GSADF(r_0) = \sup_{\substack{r_2 \in [r_0, 1] \\ r_1 \in [0, r_2 - r_0]}} \{ADF_{r_1}^{r_2}\}. \tag{3}$$

To identify if financial exuberances are in the sample, the calculated GSADF statistic is compared to its critical value of the distribution under the null hypothesis. This critical value is determined by a bootstrapping procedure proposed by Phillips and Shi (2020) which accounts for potential heteroskedasticity issues. This procedure consists of five steps. First, the ADF regression model is estimated under the hypothesis that ρ is 0 based on the total available data set. In the second step, a bootstrap sample is constructed and after it, the PSY test statistic series and based on it, the maximum value is calculated. Next, these two steps are repeated n times (in this research, 300 times). In the last step, the 95% quantile of the series of maximum values is calculated and this is the critical value of the test. If there are no exuberance periods found in the time series, the time series is considered to do not show financial exuberant prices at any time and therefore drops out of the following date stamping and analysis.

4.2 Identification of explosive periods

The previously described approach is able to detect the existence of exuberant prices but it does not allow for date stamping. To locate the begin and end of exuberance, the so-called backward SADF (BSADF) test of Phillips, Shi, and Yu (2015a) and Phillips, Shi, and Yu (2015b) is applied.[11] It performs ADF tests based on a backward expanding sample which has a fixed endpoint r_2 but varying starting points r_1:

$$BSADF_{r_2}(r_0) = \sup_{r_1 \in [0, r_2 - r_0]} \{ADF_{r_1}^{r_2}\}. \tag{4}$$

[11] We apply these procedures due their vast popularity in applied work. Thereby, we enable comparison to existing results in the empirical literature. However, it might be interesting to compare the outcomes to those obtained from CUSUM-based and BIC procedures in future work. The latter ones are typically performing somewhat better, see Homm and Breitung (2012), Breitung and Kruse (2013), Harvey, Leybourne, and Sollis (2017), Whitehouse (2019) and Harvey, Leybourne, and Whitehouse (2020).

The initiation date $\hat{r}_e$ is the first time, where the test statistic exceeds its critical value ($scv_{r2}(\beta_T)$) and the termination date $\hat{r}_f$ is the date at which the test statistic first lies below its critical value:

$$\hat{r_e} = \inf_{r_2 \in [r_0, 1]} \{r_2 : BSADF_{r_2}(r_0) > scv_{r_2}(\beta_T)\}, \tag{5}$$

$$\hat{r_f} = \inf_{r_2 \in [\hat{r_e}, 1]} \{r_2 : BSADF_{r_2}(r_0) < scv_{r_2}(\beta_T)\}. \tag{6}$$

To make the identification technique of Phillips, Shi, and Yu (2015a) and Phillips, Shi, and Yu (2015b) more robust, to account for the fact that no econometric model is perfect and to produce better results, we make an adjustment. We only consider periods of financial exuberance which have a duration of at least 66 business/trading days.[12] Shorter periods are excluded to reduce the influence of noise and to account for the fact that no exuberance detection model is perfectly designed. On top of this, small duration periods are hardly considered as being a financial exuberance period.[13]

5 Identified periods of exuberance and stylized facts

In this section we intensively discuss the identified exuberance periods.[14] Based on the BSADF test, we have identified positive exuberances in 29 out of all 30 investigated time series. Only WTI Oil does not show such behaviour. In these 29 time series, we detect 146 positive exuberances.[15] In contrast, we identify negative exuberance periods in only 23 out of all 30 assets/indices and the total amount of negative exuberances is 63.[16]

First, we provide basic characteristics of financial exuberance periods. Duration is defined as the

[12]The duration is set based on Phillips and Shi (2018). They use monthly data and require three subsequent observations that are detected by the BSADF procedure to be considered as a financial exuberance period. Because we are using daily data, we scale the number of required observations to 66 trading days which is the approximate number within a three month period.

[13]All financial exuberance periods with a duration of at least 66 trading days are used to analyse the basic characteristics of exuberance periods like duration, starting value, peak value, end value etc. But in the upcoming analysis of stylized facts, the minimum duration of both financial exuberance parts (explosive and reverse period) is slightly adapted because otherwise, the applied models have not enough observation data and could lead to false/biased results. This is a trade-off which automatically emerges while combining the areas of financial exuberance and stylized facts.

[14]A detailed overview with all 143 identified exuberance periods can be found in the appendix in Table 15, Table 16, Table 17, Table 18, Table 19.

[15]Three exuberance periods are dropped out because they are not fully burst at the end of our observation sample. These exuberances are in BITCOIN, NASDAQ and S&P 500. Using them would bias our results because we would analyse exuberance periods which are not over. So, we have in total 143 positive exuberance periods.

[16]All identified positive exuberance periods are shown with some details in the appendix. In the following, when using the term exuberance period we always refer to the positive ones. A further future research could deal with negative exuberances which consists of a crash and a following recovery phase rather than first an explosive period and then a crash. Literature dealing with such type of exuberance are e.g., Fry and Cheah (2016), Goetzmann and Kim (2018), and Acharya and Naqvi (2019).

number of trading days a financial exuberance period consists of. The mean value in our sample is 307.06 days with a median value of 150. The shortest exuberances are only 66 days and they have been identified in the Brent Oil time series for the period running from 2008-05-02 to 2008-08-01 and in the DAX30 time series running from 1989-07-14 to 1989-10-13. In contrast, the longest identified exuberance periods were observable in TOPIX (1983-10-27 to 1991-11-29, 2112 trading days) and TUNINDEX (2005-04-05 to 2013-05-27, 2125 trading days). The mean duration is mainly driven by the duration of the explosive period which is on average 2.52 (219.90/87.16) times longer than that of the reverse period. Looking on the average duration ratio rather then dividing the mean explosive duration by the mean reverse duration, the ratio is even bigger (4.90). Similar but smaller results are obtained by using the median value instead. In this case, the explosive period is more than three times as high (3.08, respectively 3.10) than the reverse period. Another finding is the much higher increase during the explosive period compared to the decline in the reverse. The increase is approximately 41% (based on mean) or 22% (based on median) while the decrease is 19% (mean) or 11% (median) (Table 3).

Table 3: Basic characteristics of financial exuberance periods

	mean	median	sd	min	max	5%Q	95%Q
duration	307.06	150	407.46	66	2125	69.10	1338.50
explosive duration	219.90	114	299.49	16	1603	41.00	948.50
reverse duration	87.16	37	125.89	2	692	8.00	394.90
duration ratio	4.90	3.10	5.82	0.21	47.00	0.51	15.11
increase explosive	0.41	0.22	0.56	0.05	4.42	0.07	1.30
decrease reverse	-0.19	-0.11	0.22	-1.52	-0.02	-0.58	-0.03

This table provides an overview of the basic characteristics of the identified financial exuberance periods. Mean, standard deviation (sd), 5% quantile (5%Q) and 95% quantile (95%Q) are round to two digits while median, minimum value (min) and maximum value (max) are stated in integers for duration, explosive duration and reverse duration. Duration ratio is defined as the quotient of explosive duration and reverse duration.

This overview clearly illustrates that the often made assumption of a 1-period-crash [17] is in general not justified for the well-known financial exuberance periods. Additionally, after the exuberance is over, the real asset price does not reach the level which it had before the exuberance started. The new value is most often much higher compared to the initial value. To statistically justify both observations, we run hypothesis tests. First, we test the common belief that prices collapse within a single period. As the minimal reverse duration is equal to two days, a test based on daily periods

[17] Evans (1991) bubble DGP which is based on monthly data, assumes that there is a "hard" crash within one month.

can only reject such a null hypothesis. However, most econometric procedures are simulated on a monthly basis, such that we might instead test the one-sided null hypothesis that the collapse duration is smaller or equal to 22 trading days (which corresponds to a single period on a monthly frequency). Based on normality of log-durations[18], we obtain a t-statistic of 7.07.[19] Hence, the null hypothesis is clearly rejected in favor of the alternative that durations exceed 22 days on average.

Second, we test the hypothesis of a full collapse (irrespective of its duration). A full collapse is characterized by a complete reversion of the price to the initial pre-explosive value. In other words, the market correction brings the price back to its value before the exuberance period started. This might take place within a single period (sudden collapse) or within a prolonged phase (disturbing collapse). The former possibility is already rejected based on our previous findings. What remains is the possibility of a disturbing or smooth collapse. One-sided testing of the null hypothesis of a full collapse leads to a heteroskedasticity-robust t-statistic of 4.46. Taking the evidence together, the form of a smooth collapse — as innovated in Phillips and Shi (2018) — is clearly supported on average.

Next, we further analyse the relation between returns in the explosive and reverse period. The following return scatter plot (see Figure 1) which is similar to Etienne, Irwin, and Garcia (2014) shows a clear linear relationship between the cumulated returns for the phase (i) from peak to burst (y-axis) and (ii) start to peak (x-axis) ($R^2 = 0.822$). The returns from start to peak are significantly larger in magnitude as the reversions from peak to burst. An interesting outlier is one of the Bitcoin cases (2013-01-08 to 2014-09-17) where the log-return in the explosive phase exceeds 400%, while its reversion is less than 100%.

Keeping this in mind, we will later construct new data generating processes (DGP) which are much more realistic and backed by empirical results compared to the DGP of Evans (1991). Especially, we account for the diversity of financial exuberance periods, so that we develop more than one DGP.

5.1 AR parameter estimation

Another important information when dealing with often applied AR(1) models is the value of the AR parameter ρ.[20] Therefore, we provide evidence that in the explosive period, ρ is, like expected,

[18] The distributional behaviour of durations is shown in some detail in the subsection Duration testing and modelling.

[19] Throughout the analysis, we entertain Huber-Eicker-White robust standards errors against cross-sectional heteroskedasticity (Eicker, 1967; Huber, 1967; White, 1980).

[20] To avoid estimation issues due to very small periods of explosiveness or reverse, we eliminate all periods with less than 10 trading days.

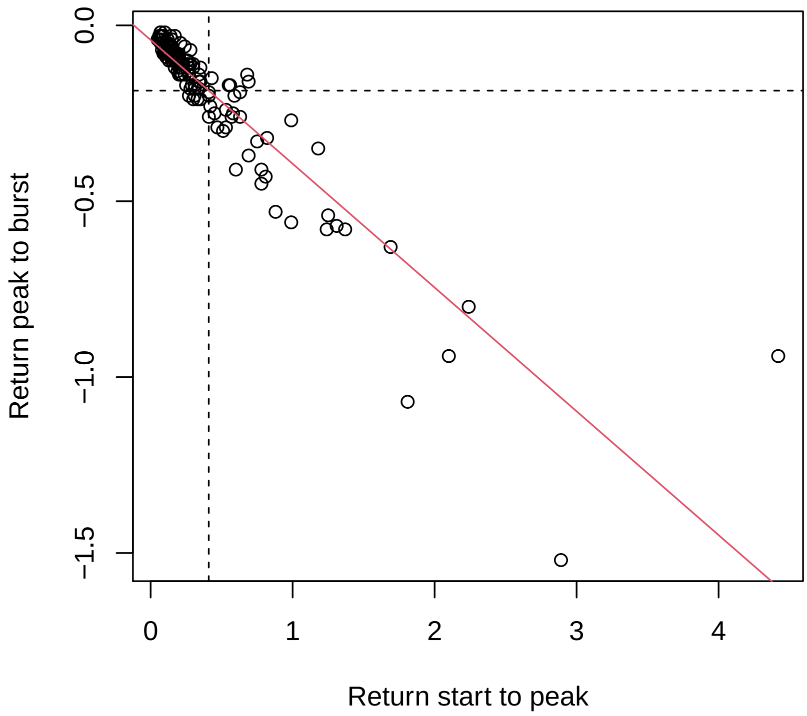

Figure 1: Returns during explosive and reverse period

slightly greater than 1. In all investigated exuberance periods, it is always greater than 1. The lowest observed value is 1.00008 while the highest one is 1.01627. Its average value is 1.00049 based on mean and 1.00023 based on median. In contrast, in each reverse period, ρ lies between 0.99771 and 0.99996. Thus, it is close to unity but slightly smaller than 1 and has an average value of 0.99953 (mean) or 0.99967 (median) (Table 4). Knowing these specific values is essential to be able to state an empirical justified data generating process.

Table 4: AR(1) model parameters of exuberances

	mean	median	sd	min	max	5%Q	95%Q
AR1 expl	1.00049	1.00023	0.00145	1.00008	1.01627	1.00010	1.00126
AR1 rev.	0.99953	0.99967	0.00044	0.99771	0.99996	0.99848	0.99993

This table provides an overview of the AR(1) model parameter for both the explosive and reverse period. Provided are the mean, median, standard deviation (sd), minimum value (min), maximum value (max), 5% (5%Q) and 95% quantile (95%Q). All values are rounded to five digits.

For the comparison of daily explosive autoregressive parameters (ρ_d) to monthly counterparts (ρ_m) — as typically considered in the related literature — we apply the simple conversion $\rho_m = \rho_d^{22}$. Our results suggest monthly explosive autoregressive parameters of 1.004 (low); 1.008 (mid) and 1.046

(high). They clearly indicate that typical autoregressive parameters used in Monte Carlo simulations are far too high and lead to too optimistic results as the econometric procedures clearly benefit from large autoregressive roots exceeding unity. Testing (one-sided) the null hypothesis that the average autoregressive parameter we find empirically equals 1.02 on a monthly basis (usually the minimal value for explosive roots, see e.g. Homm and Breitung (2012) and many others) leads to $t = 3.37$. Hence, the null hypothesis is clearly rejected in favour of smaller autoregressive parameters. In our Monte Carlo simulations, we investigate the impact of these findings on the power of the popular ADF-type tests against explosiveness.

5.2 Grouping and parameter estimation

For our later developed data generating processes (DGPs) we group our data into three different categories (low, mid, high) (see Table 5).

Table 5: Plain vanilla DGP parameter values

		group 1	group 2	group 3
1	p_0	4.51	6.76	8.94
2	T_e;T_r	(81;28)	(226;99)	(815;350)
3	ν	4	5	6
4	σ_e^2;σ_r^2	(0.000057;0.000096)	(0.000156;0.000329)	(0.001008;0.001278)
5	ρ_e	1.000163	1.000352	1.002027
6	ρ_r	0.998638	0.999480	0.999804

This table provides an overview of the eight variables/parameters set for the DGP. While the starting price p_0 is rounded to two digits, both variances and AR coefficients are round to six digits due to their smallness. Duration and variances for both periods are stated in tuples because empirically mixing these parameters from different groups is not observable.

5.3 Duration testing and modelling

Logarithmic durations (explosive expansion and mean-reverting collapse phases) are investigated in terms of their distributional characteristics. We find that the normality hypothesis for both durations cannot be rejected at any conventional significance level. The p-values for the Jarque and Bera (1980) test are equal to 0.51 and 0.26 for explosive and mean-reverting durations, respectively. This result motivates the use of a log-normal distribution for the survival regressions involving the durations in level.

Turning to the duration dependence modelling (see e.g. McQueen and Thorley (1994) and Lunde and Timmermann (2004)), we investigate the role of annual US real interest and growth rates on the durations of explosive and collapse phases. The macroeconomic data is obtained from the FRED and matched to the beginning of the explosive phase, similar to Kennan (1985) for the modelling of strike durations. Based on our previous investigations, we apply a parametric log-normal survival model. Contrary to Lunde and Timmermann (2004), we do not find evidence for the effect of interest rates ($t = 1.33$). However, it must be noted that the authors consider bull and bear markets which are different from the phases we investigate here, albeit clearly related in concept. We find real GDP growth to have a positive and significant impact on the length of explosive regime durations ($t = 2.14$). Moreover, collapse durations are not driven by economic measures, but strongly depend on the length of the preceding explosive phase ($t = 9.60$) (see Figure 2). In short, long explosive phases do not collapse quickly, but rather take their time ($R^2 = 0.302$).

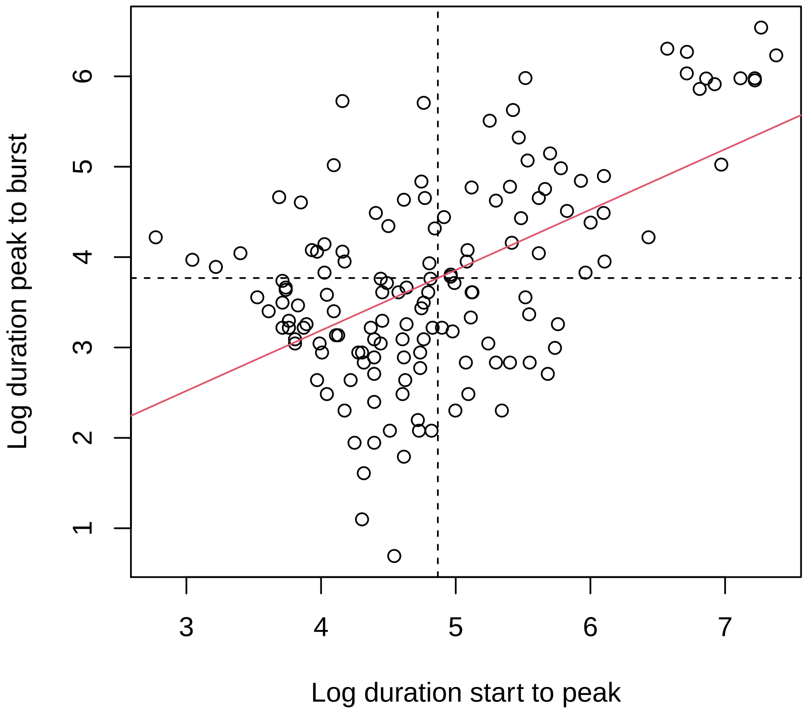

Figure 2: Log durations during explosive and reverse period

5.4 Distribution

Since Mandelbrot (1963), Fama (1965), and Mandelbrot (1967) there is evidence that most financial time series exhibit heavy tails and thus, the log returns do not follow an often assumed normal distribution. Instead it has been shown that the process looks more like a Pareto or power law distribution. In the following, we investigate if this finding can be confirmed for exuberance periods and both its explosive and reverse part. The first step in our analysis is therefore to create QQ-plots to visually analyse this task. Next we estimate the sample skewness and kurtosis and finally, we determine the tail-index.

The QQ-plots clearly illustrate that the great majority of investigated exuberance periods as well as both subperiods strongly differ from a normal distribution. Due to the high number of QQ-plots (3×143 QQ-plots), they are not printed here and are available on request. Next, we calculate the skewness (s) and kurtosis (k) to determine if they differ from the Gaussian distribution (McNeil, Frey, and Embrechts, 2015):

$$s = \frac{\frac{1}{T}\sum_{i=1}^{T}(X_i - \bar{X})^3}{\left[\frac{1}{T}\sum_{i=1}^{T}(X_i - \bar{X})^2\right]^{1.5}}, \tag{7}$$

$$k = \frac{\frac{1}{T}\sum_{i=1}^{T}(X_i - \bar{X})^4}{\left[\frac{1}{T}\sum_{i=1}^{T}(X_i - \bar{X})^2\right]^{2}}. \tag{8}$$

The skewness is found to be negative on average for all three periods which is a well-known stylized fact of exuberance periods (Cont, 2001). Based on both mean and median, the skewness is most negative for the complete exuberance (-0.45 and -0.46) compared to its subperiods (-0.23 and -0.23 for explosive period and -0.33 and -0.25 for reverse period). Comparing the skewness of the explosive and reverse period, the mean of the reverse period is more negative but based on median, there are hardly differences between both parts. Based on the standard deviation, minimum and maximum value as well as on the 5% and 95% quantile, it is obvious that there is a high fluctuation between the skewness of the investigated exuberances. This is not surprising due to the fact that many exuberances and crisis are driven by different causes like greed, moral hazard, strong currency depreciations/appreciations, credit and housing markets turmoil etc. The kurtosis also differs for all three periods. The mean and median values are both higher for the complete exuberance (6.03 and 5.91) rather than for both subperiods (explosive: 5.91 and 3.99, reverse: 4.22 and 3.21). Comparing both subperiods, the kurtosis is higher during the explosive period. The following table provides an overview of these two moments (Table 6):[21]

In the last step, to further confirm if there are heavy tails, we apply the weighted Hill estimator of Huisman, Koedijk, Kool, and Palm (2001). This estimator fixes the small sample issue and the

[21] The four moments are calculated for all exuberance, explosive and reverse periods which consist of at least 10 trading days.

Table 6: Skewness and kurtosis of exuberance, explosive and reverse periods

		mean	median	sd	min	max	5%Q	95%Q
	exub	-0.45	-0.46	0.65	-2.38	2.58	-1.47	0.60
skew	expl	-0.23	-0.23	0.75	-3.07	2.93	-1.24	0.93
	rev	-0.33	-0.25	0.64	-3.31	0.98	-1.39	0.52
	exub	6.03	4.54	4.72	2.50	32.14	2.90	14.48
kurt	expl	5.91	3.99	7.78	2.21	66.54	2.54	15.16
	rev	4.22	3.21	2.82	1.46	18.88	2.05	10.33

challenge of choosing the threshold value k which emerges in the original Hill estimator (Hill, 1975). The Hill estimator for the largest positive returns is defined as:

$$\xi(k) = \frac{1}{k}\sum_{i=1}^{k}[\ln(r_{T-i+1}) - \ln(r_{T-k})]. \tag{9}$$

It is applied to increasing sorted returns, such that $r_1 \leq r_2 \leq \cdots \leq r_T$. But because risk management is mainly concerned with the most negative returns and the Hill estimator is only able to handle positive values, all returns are multiplied by (-1) to obtain losses, so, e.g., a -5% return is a 5% loss. Now, Huisman, Koedijk, Kool, and Palm (2001) use the Hill estimator definition but instead of only calculating the Hill estimator for one specific chosen k, the idea is to calculate the Hill estimator for the threshold values $k \in \{1, 2, \ldots, \kappa\}$ and then, the weighted Hill estimator is defined as:

$$\xi^m(\kappa) = \sum_{k=1}^{\kappa} w(k)\xi(k). \tag{10}$$

The weighted hill estimator is calculated based on weighted least squares (WLS) and it is the first element of the vector b_{WLS}:

$$b_{WLS} = (Z'W'WZ)^{-1}Z'W'W\xi^*. \tag{11}$$

Z is a $\kappa \times 2$ matrix with 1's in its first column and $k \in \{1, 2, ..., \kappa\}$ in its second column. W is a $\kappa \times \kappa$ weighting matrix with $\{\sqrt{1}, \sqrt{2}, \ldots, \sqrt{\kappa}\}$ on its diagonal and Os everywhere else. Additionally, ξ^* is a vector containing all Hill estimates up to the maximum threshold κ. The maximum threshold value is then chosen in line with Huisman, Koedijk, Kool, and Palm (2001) who suggest $\delta = T/2$. Estimating the parameters using OLS would result in two major issues, namely neglecting heteroscedasticity and correlation between the variables $\gamma(k)$. Therefore, a weighted least squares

(WLS) approach is applied. Based on the weighted Hill estimator the tail index α is defined as:

$$\alpha = \frac{1}{\xi}. \tag{12}$$

Applying this procedure to our data, we obtain that on average and based on median as well, the tail index is about 4 to 6 what means that the underlying stochastic process has a finite fourth moment. Furthermore, with only a handful of exceptions, each exuberance period and its subperiods have a finite variance. Furthermore, we find support of the gain/loss asymmetry because the tail indexes for losses are smaller and thus, more extreme than the tail estimates for the largest returns (Table 7).[22]

Table 7: Tail estimator of largest returns and losses

		mean	median	sd	min	max	5%Q	95%Q
	exub	6.44	5.48	4.25	2.50	25.07	2.91	13.26
positive tail	expl	6.14	5.61	2.92	2.19	18.06	2.82	10.44
	rev	5.94	4.50	4.05	2.47	16.59	2.54	13.63
	exub	5.38	4.44	3.92	1.84	23.31	2.44	11.83
negative tail	expl	4.50	4.21	1.72	2.28	7.67	2.45	7.40
	rev	5.48	4.80	3.22	1.98	14.19	2.19	11.85

To summarize, most exuberance, explosive and reverse periods show signs of non-normality and of underlying stochastic processes which have finite variance.

Finally, we test the null hypothesis of no structural break in the innovation variance during the explosive and the collapse phase. The resulting t-statistic equals 7.61 and thus confirms the existence of a structural change in the unconditional variance. Moreover, we are able to quantify typical break sizes. These are 1.27 (low); 1.69 (mid) and 2.11 (high). Compared to the existing literature, these break sizes are relatively small.

6 Monte Carlo simulations

This subsection features Monte Carlo simulation results regarding the empirical power of popular tests. Based on our main findings, we construct data generating processes (DGPs) which include the stylized facts of explosive and reverse periods. In contrast to the well-known data generating

[22] In line with Huisman, Koedijk, Kool, and Palm (2001) their weighted Hill estimator is applied to all exuberance, explosive and reverse periods which have at least 100 positive or negative returns. Otherwise, the estimator would be biased.

process of e.g. Evans (1991), our processes are able to model a long-lasting reverse rather than a one period reverse which in most cases cannot be empirically confirmed. Moreover, they do not rely on the assumption of a full collapse, i.e. a sudden or disturbing collapse. In particular, we are able to model a smooth collapse.

First, we start with a 'plain vanilla DGP' which is later increased in its complexity. The DGP is split into one part which models the explosive behaviour of the exuberance and one part which captures the reverse period. p_0, our initial real log-price is set based on empirical observations. All following prices p_t with $t \in \{1, 2, \dots, T_e\}$ - where T_e is the duration of the explosive period - are calculated based on an AR(1) process. The same is true for the reverse period. So, both parts of the DGP are defined as:

$$p_t = \rho_e p_{t-1} + \sigma_e u_t, \qquad t = 1, ..., T_e \tag{13}$$

$$p_t = \rho_r p_{t-1} + \sigma_r u_t, \qquad t = T_e + 1, ..., T. \tag{14}$$

We have $\rho_e > 1$ and $\rho_r < 1$. The innovations u_t are drawn from a standardized t-distribution with ν degrees of freedom. A variance shift is captured by the structural break in the scaling parameters σ_e and σ_r. We start (and end) with a random walk regime of 50 observations which has the same innovation variance as the proceeding (preceding) explosive (mean-reverting) regime. The respective end (starting) value is matched to with the simulated trajectory in order to exclude artificial jumps in the simulated data.

We run the SADF test of Phillips, Wu, and Yu (2011) with zero lag augmentation. Hence, we provide the test with the information about the first-order lag structure and the maximum of one single explosive phase. To do so, we implement 729 different value combinations (3^6) of the parameters stated in Table 8, Table 9 and Table 10. However, we do not impose further simplifications. In further considerations, currently under investigation, we remove these information.

Some results are already available: Empirical power for the average case (group 2 for all relevant parameters, see exemplary Table 8) leads to an empirical power of 52.9%. A larger starting value leads to higher power. Clearly, a longer duration of the explosive regime increases power. Also, the empirical power rises (as expected) with an increasing explosive autoregressive coefficient. Finally, an increased innovation variance reduces power as the signal-to-noise ratio worsens. The degrees of freedom ν controlling the excess kurtosis does not impact power in any noticeable way, therefore, no results are reported here but they are available on request.

Table 8: Empirical power - ceteris paribus analysis for group 1

p_0	T_e	T_r	ν	σ_e^2	σ_r^2	ρ_e	ρ_r	Power
4.51	81	28	5	0.00100805	0.00127795	1.00016253	0.999804	0.051
6.76	81	28	5	0.00100805	0.00127795	1.00016253	0.999804	0.065
4.51	226	99	5	0.00100805	0.00127795	1.00016253	0.999804	0.044
4.51	81	28	5	0.00015589	0.00032906	1.00016253	0.999804	0.072
4.51	81	28	5	0.00100805	0.00127795	1.00035159	0.999804	0.058
4.51	81	28	5	0.00100805	0.00127795	1.00016253	0.999480	0.056
8.94	81	28	5	0.00100805	0.00127795	1.00016253	0.999804	0.068
4.51	815	350	5	0.00100805	0.00127795	1.00016253	0.999804	0.084
4.51	81	28	5	0.00005670	0.00009570	1.00016253	0.999804	0.080
4.51	81	28	5	0.00100805	0.00127795	1.00202689	0.999804	0.206
4.51	81	28	5	0.00100805	0.00127795	1.00016253	0.998638	0.060

Table 9: Empirical power - ceteris paribus analysis for group 2 (baseline)

p_0	T_e	T_r	ν	σ_e^2	σ_r^2	ρ_e	ρ_r	Power
6.76	226	99	5	0.00015589	0.00032906	1.00035159	0.999480	0.529
4.51	226	99	5	0.00015589	0.00032906	1.00035159	0.999480	0.263
8.94	226	99	5	0.00015589	0.00032906	1.00035159	0.999480	0.771
6.76	81	28	5	0.00015589	0.00032906	1.00035159	0.999480	0.105
6.76	815	350	5	0.00015589	0.00032906	1.00035159	0.999480	0.910
6.76	226	99	5	0.00005670	0.00009570	1.00035159	0.999480	0.901
6.76	226	99	5	0.00100805	0.00127795	1.00035159	0.999480	0.099
6.76	226	99	5	0.00015589	0.00032906	1.00016253	0.999408	0.141
6.76	226	99	5	0.00015589	0.00032906	1.00202689	0.999480	1.000
6.76	226	99	5	0.00015589	0.00032906	1.00035159	0.998638	0.533
6.76	226	99	5	0.00015589	0.00032906	1.00035159	0.999804	0.517

Table 10: Empirical power - ceteris paribus analysis for group 3

p_0	T_e	T_r	ν	σ_e^2	σ_r^2	ρ_e	ρ_r	Power
8.94	815	350	5	0.00005670	0.00009570	1.00202689	0.998638	1.000
6.76	815	350	5	0.00005670	0.00009570	1.00202689	0.998638	1.000
8.94	226	99	5	0.00005670	0.00009570	1.00202689	0.998638	1.000
8.94	815	350	5	0.00015589	0.00032906	1.00202689	0.998638	1.000
8.94	815	350	5	0.00005670	0.00009570	1.00035159	0.998638	1.000
8.94	815	350	5	0.00005670	0.00009570	1.00202689	0.999480	1.000
4.51	815	350	5	0.00005670	0.00009570	1.00202689	0.998638	1.000
8.94	81	28	5	0.00005670	0.00009570	1.00202689	0.998638	1.000
8.94	815	350	5	0.00100805	0.00127795	1.00202689	0.998638	1.000
8.94	815	350	5	0.00005670	0.00009570	1.00016253	0.998638	0.859
8.94	815	350	5	0.00005670	0.00009570	1.00202689	0.999804	1.000

Now, we also provide four examples which will serve throughout the analysis as 'running examples' (Table 11).

Table 11: Running examples

index/asset	p_0	T_e	T_r	ν	σ_e^2	σ_r^2	ρ_e	ρ_r	Power
Gold (GFC)	5.65	376	127	5	0.000102	0.000324	1.000233	0.999604	0.323
NASDAQ (dot-com)	6.18	1368	386	5	0.000181	0.000914	1.000196	0.999625	0.533
DJ US RE (GFC)	4.95	127	75	5	0.000060	0.000154	1.000396	0.999565	0.667
Brent Oil	3.94	45	21	5	0.000413	0.000539	1.001300	0.998218	0.209

The first one is the Gold price during the Great Financial Crisis (GFC). The second one is the prominent NASDAQ index during the famous dot-com bubble. The other two examples are the housing market (DJ US RE) and Brent Oil during the GFC. Typical trajectories of the simulated prices are given in the figures below (see Figure 3, Figure 4, Figure 5, Figure 6). The simulated power shows that the Gold bubble was more difficult to detect in relative terms when compared to the NASDAQ, given the differences in empirical power. The GFC period in the housing market is detected with quite a high power which is due to the relatively low innovation variance in both the explosive and reverse period. In contrast, identifying the GFC in the Brent Oil index shows a low power of 0.209 which is mainly due to the short duration and the relatively high innovation variance in the explosive period.

Overall, the empirical power is not extensively high for three of the given empirical cases. As the simulations already contain a number of simplifications, the simulated power can be seen as some kind of an upper bound as the power will be reduced by search for the optimal lag length via the BIC and especially when considering a supremum statistic with an unknown timing of the bubble. Furthermore, the possibility of having more than one single explosive phase in the sample further reduces power. These issues are currently under consideration by the authors as well.

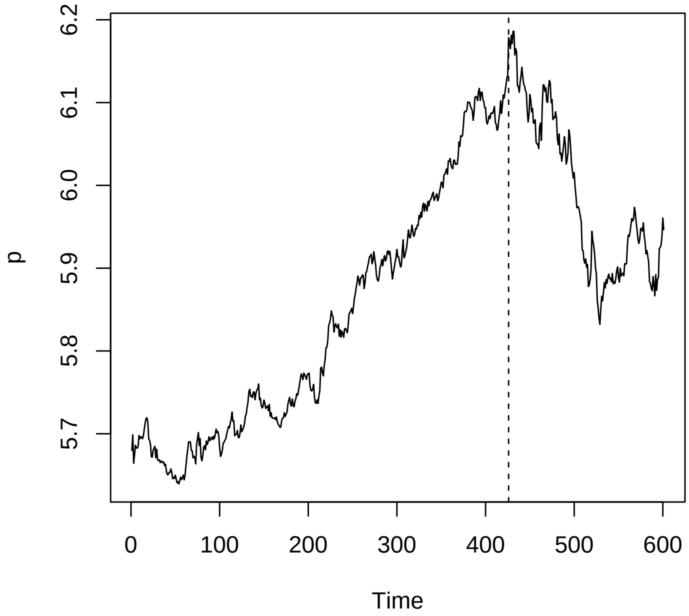

Figure 3: Typical trajectory of explosive Gold prices during the GFC.

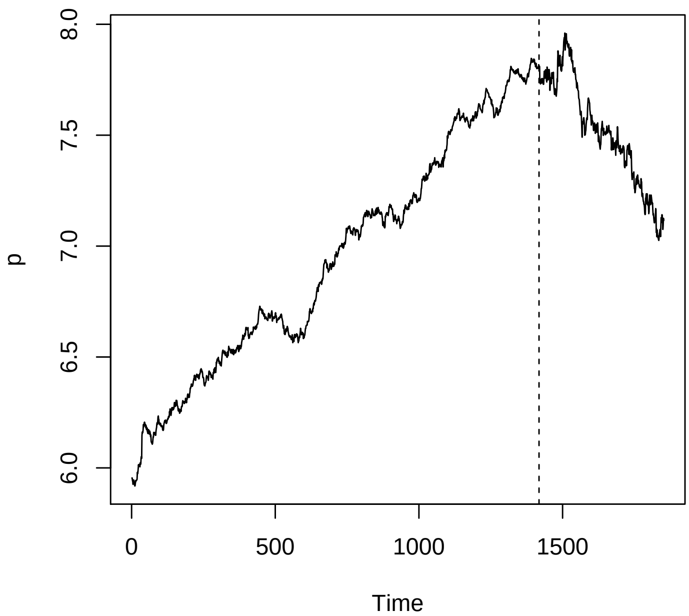

Figure 4: Typical trajectory of explosive Nasdaq prices during the dot-com bubble.

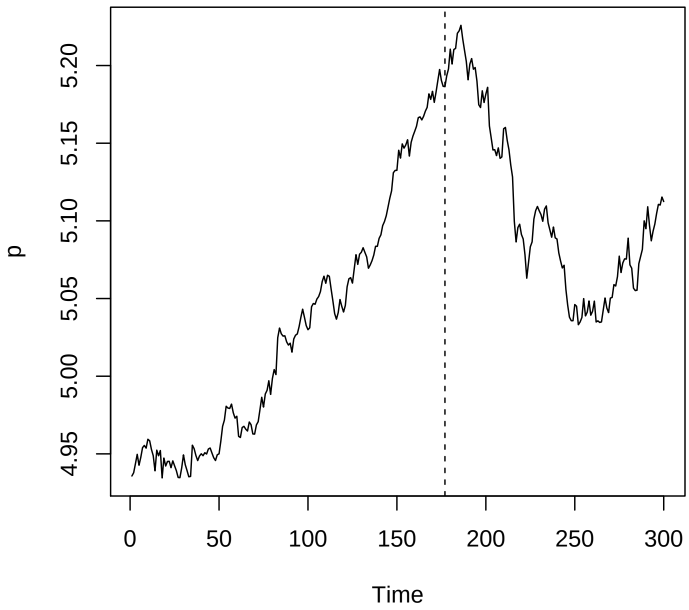

Figure 5: Typical trajectory of explosive housing prices during the GFC.

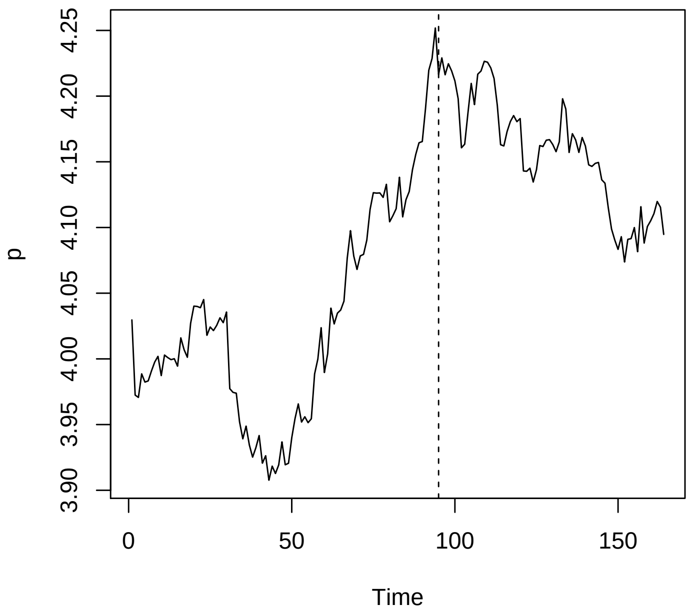

Figure 6: Typical trajectory of explosive oil prices during the GFC.

7 Conclusions

We provide new and comprehensive evidence on explosive phases in market prices. We establish a couple of new stylized facts which are useful for the understanding of these outstanding periods, often interpreted as bubbles. They are also useful for the specification of empirically relevant parameter settings in simulations for the performance of popular tests and monitoring procedures. In fact, our results on thirty different markets over a period of fifty years yields 143 explosive phases. Their characteristics deviate significantly from commonly entertained data generating processes in the following ways: (i) explosiveness is typically remarkably mild, (ii) collapse phases are smooth and (iii) market prices do not fully drop back to the initial pre-explosive value, but stay way above. We also find quite heterogeneous durations of explosive and mean-reverting phases. In addition, we confirm the existence of volatility shifts in the innovations, with an increased variance in the collapse period. Moreover, innovation distributions are fat-tailed and almost symmetric. However, the idealistic view that market prices are strongly exploding and fully collapsing in very short time is definitely not supported by our results. These stylized facts have important implications for the empirical power of tests and the performance of monitoring procedures typically conducted in practice. In particular, we find that most explosive phases of financial exuberance are much harder to detect than expected, at least given what the literature has suggested so far.

A natural extension of our research is the investigation of the multivariate perspective because during major financial exuberances, many different indices are influenced. This can e.g., be seen for the dot-com-bubble or the global financial crisis 2007/09. In the context of regulation and the subsequent avoidance of spillover and contagion effects, a knowledge of the behaviour of multiple time series (portfolio context) is essential. On top of this, the financial exuberance monitoring technique of Phillips, Shi, and Yu (2015a) and Phillips, Shi, and Yu (2015b) could likely be enhanced by including other macroeconomic variables or by applying more advanced techniques like deep neural networks.

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Appendix

DGPs and parameter settings

We provide an overview of the DGPs, assumptions and parameters which are applied during finite sample Monte Carlo power simulations of financial exuberance detection and date stamping procedures in the literature. Therefore, we first briefly describe the used frameworks of more than 20 research papers and in the end, we provide an extensive overview of the applied parameter settings:

Phillips, Wu, and Yu (2011)

The following DGP (Evans (1991) type model) is assumed (pp. 218-221):

$$
\begin{aligned}
P_t &= P_t^f + 20B_t, \\
P_t^f &= \mu_D(1+g)g^{-2} + \frac{D_t}{g}, \\
D_t &= \mu_D + D_{t-1} + \epsilon_{d,t}, \\
B_{t+1} &= \begin{cases} (1+g)B_t\epsilon_{b,t+1} & \text{if } B_t \leq \alpha \\ \left[\zeta + \pi^{-1}(1+g)\theta_{t+1}(B_t - (1+g)^{-1}\zeta)\right]\epsilon_{b,t+1} & \text{if } B_t > \alpha \end{cases}
\end{aligned}
$$

with $g > 0, \epsilon_{d,t} \sim NID(0, \sigma_d^2), \epsilon_{b,t} = \exp(y_t - \tau^2/2)$, $y_t \sim NID(0, \tau^2)$, θ follows a Bernoulli process that takes the value 1 with probability π and 0 with $1-\pi$, and $E(B_{t+1}) = (1+g)B_t$.
The following parameter settings are used: $\mu_d = 0.0373, \sigma_d^2 = 0.1574, D_0 = 1.3, T = 100, g = 0.05, \alpha = 1, \zeta = 0.5, B_0 = 0.5, \tau = 0.05, \pi = \{0.999, 0.99, 0.95, 0.85, 0.75, 0.5, 0.25\}$.

Homm and Breitung (2012)

The following DGP is assumed (pp. 212-222):

$$
\begin{aligned}
y_t &= \rho_t y_{t-1} + \epsilon_t, \\
&E(\epsilon_t) = 0, E(\epsilon_t^2) = \sigma^2, y_0 = c < \infty.
\end{aligned}
$$

The hypothesis $H_0 : \rho_t = 1, t = 1, 2, \ldots, T$ is tested against $H_1 : \rho_t = \begin{cases} 1 & \text{for } t = 1, \ldots, [\tau^* T] \\ \rho^* > 1 & \text{for } t = [\tau^* T] + 1, \ldots, T \end{cases}$

The following parameter settings are used: $y_0 = 0, \epsilon_t$ is Gaussian white noise, $T \in \{100, 200, 400\}, \tau^* \in \{0.7, 0.8, 0.9\}, \rho^* \in \{1.02, 1.03, 1.04, 1.05\}$. Additionally, the break date estimators are testes on the following parameter settings: $\tau^* \in \{0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}, T \in \{200, 400\}, \rho^* = 1.05$.

Also the case of randomly starting bubbles is investigated. Therefore, a different DGP/model is applied:

$$
\begin{aligned}
P_t =& P_t^f + B_t \\
& P_t^f = \frac{1+R}{R^2}\mu + \frac{1}{R}D_t, \\
& D_t = \mu + D_{t-1} + u_t, \\
& B_t = \begin{cases} B_{t-1} + \frac{RB_{t-1}}{\pi}\theta_t & \text{if } B_{t-1} = B_0 \\ (1+R)B_{t-1} & \text{if } B_{t-1} > B_0 \end{cases} \quad \text{for } t = 1, \dots, T
\end{aligned}
$$

θ is a Bernoulli process which takes on the value of 1 with probability π and on 0 with $1 - \pi$.

The following parameter settings are used: $R = 0.05, \mu = 0.0373, D_0 = 1.3, u_t \sim$ identical normally distributed$(0, 0.1574), (B_0, \pi) \in \{(B_0 = 2, \pi \in \{\text{no bubble}, 0.02, 0.05, 0.10, 0.25, 0.50, 1.00\}),$ $(B_0 = 0.05, \pi \in \{\text{no bubble}, 0.01, 0.02, 0.05, 0.10\})\}, T \in \{100, 200\}$.

Another DGP is applied for periodically collapsing bubbles, namely the Evans (1991) DGP. The equations for the fundamental value P_t^f and the dividend process D_t remain the same as before, but the composition of the bubble component changes:

$$
\begin{aligned}
& P_t = P_t^f + 20B_t, \\
& B_{t+1} = \begin{cases} (1+R)B_t u_{t+1} & \text{if } B_t \leq \alpha \\ [\delta + \pi^{-1}(1+R)\theta_{t+1}(B_t - (1+R)^{-1}\delta)]u_{t-1} & \text{otherwise} \end{cases}
\end{aligned}
$$

For δ and α it holds: $0 < \delta < (1+R)\alpha$, $\{u_t\}_{t=1}^{\infty} i.i.d., u_t \geq 0, E_t(u_{t+1}) = 1 \forall t, \{\theta_t\}_{t=1}^{\infty}$ iid Bernoulli process.

The following parameters are used: $u_t = \exp(\zeta_t - \frac{1}{2}\tau^2), \zeta_t \sim iidN(0, \tau^2), \alpha = 1, \delta = 0.5, \tau = 0.05, R = 0.05, T = 100, \pi \in \{\text{no bubble}, 0.999, 0.990, 0.950, 0.850, 0.750, 0.500, 0.250\}$.

Breitung and Kruse (2013)

The following DGP is assumed (pp. 918-927):

$$
\begin{aligned}
y_t &= B_t + P_t^f \\
B_t &= \varrho B_{t-1} + u_t \\
P_t^f &= P_{t-1}^f + \epsilon_t
\end{aligned}
$$

with $\varrho \in \{\frac{1+r}{\pi}, 1 + \frac{c}{T^{0.75}}\}, \epsilon_t \sim iidN(0,1), u_t \sim iidN(0, \sigma_u^2), \lambda^2 = \sigma_u^2/\sigma_\epsilon^2, T_0 = [\tau_0 T]$.

The following parameter settings are used: $T \in \{69, 100, 121\}, \tau \in [0.4, 0.8]$ as the searching interval for the bubble crash, $r = 0.05, c \in \{0.5, 1, 2, 3, 4, 5\}, \pi \in \{0.90, 0.92, 0.94, 0.96, 0.98\}, \varrho = [1.02, 1.2088]$. Only bubbles longer than $[0.4T]$ periods are considered. The bubble size is measured by the variance ratio $\lambda^2, \lambda \in \{0.1, 1, 10\}$.

Phillips, Shi, and Yu (2014)

The following DGP is assumed (pp. 325-327):

$$
\begin{aligned}
P_t &= P_t^f + \kappa B_t, \\
P_t^f &= \frac{\mu\rho}{(1-\rho)^2} + \frac{\rho}{1-\rho} D_t, \\
D_t &= \mu + D_{t-1} + \epsilon_{Dt}, \\
B_{t+1} &= \begin{cases} \rho^{-1} B_t \epsilon_{B,t+1} & \text{if } B_t < b \\ \left[\zeta + (\pi\rho)^{-1}\theta_{t+1}(B_t - \rho\zeta)\right] \epsilon_{B,t+1} & \text{if } B_t \geq b \end{cases}
\end{aligned}
$$

with $\rho^{-1} > 1, \epsilon_{Dt} \sim iidN(0, \sigma_d^2), \epsilon_{B,t} = \exp(y_t - \tau^2/2)$, $y_t \sim NID(0, \tau^2)$, θ follows a Bernoulli process that takes the value 1 with probability π and 0 with $1 - \pi$, and $E(B_{t+1}) = \rho^{-1} B_t$.
The following parameter settings are used: $T \in \{100, 200, 400\}, P_0^f \in \{41.195, 94.122\}, \kappa \in \{20, 150\}, \rho \in \{0.952, 0.985\}, B_0 = 0.50, \pi = 0.85, \zeta = 0.5, \tau = 0.05, b = 1, \mu = 0.0020, \sigma_D^2 = 0.0034$.

Harvey, Leybourne, and Sollis (2015)

The following DGP is assumed (pp. 174-180):

$$y_t = \begin{cases} y_{t-1} + v_t & t = 2, \ldots, \lfloor \tau_1 T \rfloor \\ (1+\delta) y_{t-1} + v_t & t = \lfloor \tau_1 T + 1 \rfloor, \ldots, \lfloor \tau_2 T \rfloor \\ y_{t-1} + v_t & t = \lfloor \tau_2 T + 2 \rfloor, \ldots, T \end{cases}$$

with $\delta \geq 0, y_1 = v_1, y_{\lfloor \tau_2 T \rfloor + 1} = y_{\lfloor \tau_2 T \rfloor} + v_{\lfloor \tau_2 T \rfloor + 1} + y^* \mathbf{1}(\delta > 0), v_t =$ mds with $\sigma^2, \sup_t E(\epsilon_t^4) < \infty, v_1 = o_p(T^{-1/2})$. Two cases for $y^* : y^* = 0$ RW directly after explosive period vs. $y^* = y_{\lfloor \tau_1 T \rfloor} - y_{\lfloor \tau_2 T \rfloor}$ hard crash, then RW after explosive period.

The following parameter settings are used: $T = 300$ in combination with $\delta \in \{0, 0.001, 0.002, \ldots, 0.080\}$, $T = 600$ in combination with $\delta \in \{0, 0.001, 0.002, \ldots, 0.040\}, v_t \sim iidN(0,1), (\tau_1, \tau_2) \in \{(0.45, 0.55), (0.40, 0.60), (0.90, 1.00), (0.80, 1.00)\}$. For robustness testing, $T = 300$ is used in combination with $v_t \sim t_5$. To investigate the effect of the explosive period's location, the following parameter settings are used: $v_t \sim iidN(0,1), T = 300, \delta = 1.05, (\tau_1, \tau_2) = (0.70, 0.80), E = \lfloor \tau_1 T \rfloor \in \{210, 211, \ldots, 300\}$. So, the sample end is shrunk. This is also done for an early bubble with $(\tau_1, \tau_2) = (0.20, 0.30)$ and the start date $S = \lfloor \tau_2 T + 1 \rfloor \in \{1, 2, \ldots, 91\}$ which results in effective sample sizes T^* of 210 to 300.

Phillips, Shi, and Yu (2015a)

The following DGP is assumed (pp. 1059-1065):

$$P_t = P_t^f + \kappa B_t,$$

$$P_t^f = \frac{\mu \rho}{(1-\rho)^2} + \frac{\rho}{1-\rho} D_t$$

$$D_t = \mu + D_{t-1} + \epsilon_{D_t}, \epsilon_{D_t} \sim N(0, \sigma_D^2),$$

$$B_{t+1} = \begin{cases} \rho^{-1} B_t \epsilon_{B,t+1} & \text{if } B_t < b \\ \left[\zeta + (\pi \rho)^{-1} \theta_{t+1} (B_t - \rho \zeta) \right] \epsilon_{B,t+1} & \text{if } B_t \geq b \end{cases}$$

with $\kappa > 0, \epsilon_{B,t} = \exp\left(y_t - \frac{\tau^2}{2} \right), y_t \sim N(0, \tau^2), \theta_t$ is a Bernoulli process which is 1 with probability π and 0 with $1 - \pi$.

The following parameter settings are used: $\mu = 0.0024, \rho \in \{0.975, 0.980, 0.985, 0.990\}, D_0 = 1.0, \sigma_D^2 = 0.0010, \kappa = 20, \zeta = 0.50, \pi \in \{0.25, 0.75, 0.85, 0.95\}, B_0 = 0.50, b = 1, \tau = 0.05, T \in \{100, 200, 400, 800, 1600\}$.

The following DGP for exactly one bubble is considered:

$$\begin{aligned} X_t =& X_{t-1}\mathbf{1}\{t < \tau_e\} + \delta_T X_{t-1}\mathbf{1}\{\tau_e \leq t \leq \tau_f\} \\ &+ \left(\sum_{k=\tau_f+1}^{t} \epsilon_k + X^*_{\tau_f}\right)\mathbf{1}\{t > \tau_f\} + \epsilon_t \mathbf{1}\{t \leq \tau_f\} \end{aligned}$$

with $\delta_T = 1 + cT^{-\alpha}, c > 0, \alpha \in (0,1), \epsilon_t \sim_{iid} (0, \sigma^2), X^*_{\tau_f} = X_{\tau_e} + X^*, \tau_e = \lfloor Tr_e \rfloor$.

The following parameter settings are used: $x_0 = 100, \sigma = 6.79, c = 1, \alpha = 0.6, T = 100, \tau_e = \{\lfloor 0.2T \rfloor, \lfloor 0.4T \rfloor, \lfloor 0.6T \rfloor\}$ and the bubble length $\tau_f - \tau_e = \{\lfloor 0.10T \rfloor, \lfloor 0.15T \rfloor, \lfloor 0.20T \rfloor\}$.

The following DGP for exactly two bubbles is considered:

$$\begin{aligned} X_t =& X_{t-1}\mathbf{1}\{t \in N_0\} + \delta_T X_{t-1}\mathbf{1}\{t \in B_1 \cup B_2\} \\ &+ \left(\sum_{k=\tau_{1f}+1}^{t} \epsilon_k + X^*_{\tau_{1f}}\right)\mathbf{1}\{t \in N_1\} \\ &+ \left(\sum_{l=\tau_{2f}+1}^{t} \epsilon_l + X^*_{\tau_{2f}}\right)\mathbf{1}\{t \in N_2\} \\ &+ \epsilon_t \mathbf{1}\{t \in N_0 \cup B_1 \cup B_2\}. \end{aligned}$$

with $N_0 = [1, \tau_{1e}), B_1 = [\tau_{1e}, \tau_{1f}], N_1 = (\tau_{1f}, \tau_{2e}), B_2 = [\tau_{2e}, \tau_{2f}], N_2 = (\tau_{2f}, \tau]$.

The following parameter settings are used: $x_0 = 100, \sigma = 6.79, c = 1, \alpha = 0.6, T = 100, \tau_{1e} = \lfloor 0.2T \rfloor, \tau_{2e} = \lfloor 0.6T \rfloor, \tau_{1f} - \tau_{1e} = \{\lfloor 0.10T \rfloor, \lfloor 0.15T \rfloor, \lfloor 0.20T \rfloor\}, \tau_{2f} - \tau_{2e} = \{\lfloor 0.10T \rfloor, \lfloor 0.15T \rfloor, \lfloor 0.20T \rfloor\}$.

Harvey, Leybourne, Sollis, and Taylor (2016)

The following DGP is assumed (pp. 558-560):

$$y_t = \mu + u_t$$

$$u_t = \begin{cases} u_{t-1} + \epsilon_t & t = 2, \ldots, \lfloor \tau_{1,0} T \rfloor \\ (1 + \delta_{1,T})u_{t-1} + \epsilon_t & t = \lfloor \tau_{1,0} T \rfloor + 1, \ldots, \lfloor \tau_{2,0} T \rfloor \\ (1 - \delta_{2,T})u_{t-1} + \epsilon_t & t = \lfloor \tau_{2,0} T \rfloor + 1, \ldots, \lfloor \tau_{3,0} T \rfloor \\ u_{t-1} + \epsilon_t & t = \lfloor \tau_{3,0} T \rfloor + 1, \ldots, T \end{cases}$$

with $\delta_{1,T} \geq 0, \delta_{2,T} \geq 0, u_1 = o_p(T^{1/2})$ and for $\{\epsilon_t\}$ the following holds: $\epsilon_t = \sigma_t z_t, z_t \sim iid(0,1), E|z_t|^r <$

$K < \infty$ for $r \geq 4, \sigma_t = \omega(t/T), \omega(.) \in D$ is non-stochastic and strictly positive. There are four different models of $\omega(s)$ considered:

$$\omega(s) = \sigma_0 + (\sigma_1 - \sigma_0)\mathbf{1}(s > \tau_\sigma) \qquad \text{single volatility shift}$$

$$\omega(s) = \sigma_0 + (\sigma_1 - \sigma_0)\mathbf{1}(0.4 < s \leq 0.6) \qquad \text{double volatility shift}$$

$$\omega(s) = \sigma_0 + (\sigma_1 - \sigma_0)\frac{1}{1 + \exp\{-50(s - 0.5)\}} \qquad \text{logistic smooth transition in volatility}$$

$$\omega(s) = \sigma_0 + (\sigma_1 - \sigma_0)s \qquad \text{trending volatility}$$

with $s \in [0, 1]$.

The following parameter settings are used: $\mu = 0, u_1 = \epsilon_1, z_t \sim iidN(0, 1), T = 200, \delta_{1,T} = \delta_1 \in \{0.02, 0.04, 0.06, 0.08\}, \delta_{2,T} = 0, \tau_{1,0} = 0.4, \tau_{2,0} = 0.6, \sigma_1/\sigma_2 \in \{1/6, 1/3, 1, 3, 6\}$. For the case of a single volatility shift, $\tau_\sigma \in \{0.3, 0.5, 0.7\}$.

Astill, Harvey, Leybourne, and Taylor (2017)

The following DGP is assumed (pp. 655-664):

$$y_t = \mu + u_t \quad t = 1, \ldots, T + m$$

$$u_t = \begin{cases} u_{t-1} + \epsilon_t & t = 1, \ldots, T \\ \phi u_{t-1} + \epsilon_t & t = T + 1, \ldots, T + m \end{cases}$$

with $\{\epsilon_t\}$ having mean 0, is stationary and ergodic.

The following parameter settings are used: $\mu = 0, u_0 = 100, \epsilon_t \sim iidN(0, 1)$, bubble length $m \in \{2, 5, 10\}, T \in \{100, 200\}, \phi \in [1, \phi_{max}], \phi_{max} = 1.05$ for $m = 2$ and $\phi_{max} = 1.02$ for $m \in \{5, 10\}$ with a grid of 50. To measure the speed of detection a bubble length $m = 20$ is considered and the sample is defined as $1, \ldots, E$ with $E \in \{181, 182, \ldots, 200\}, \phi \in \{1.01, 1.02\}$. Additionally, the case of a variance break is considered:

$$\epsilon_t \sim \begin{cases} iidN(0, 1) & t = 1, \ldots, T_\sigma \\ iidN(0, \sigma^2) & t = T_\sigma + 1, \ldots, T^* \end{cases}$$

with $\sigma^2 \in \{\frac{1}{10}, \frac{1}{5}, 1, 5, 10\}, T_\sigma \in \{\frac{T}{2}, T^* - 5\}$.

Harvey, Leybourne, and Sollis (2017)

The following DGP is assumed (pp. 126-129):

$$y_t = \mu + u_t,$$

$$u_t = \begin{cases} u_{t-1} + \upsilon_t & t = 2, \dots \lfloor \tau_{1,0} T \rfloor \\ (1+\delta_1) u_{t-1} + \upsilon_t & t = \lfloor \tau_{1,0} T \rfloor + 1, \dots, \lfloor \tau_{2,0} T \rfloor \\ (1-\delta_2) u_{t-1} + \upsilon_t & t = \lfloor \tau_{2,0} T \rfloor + 1, \dots, \lfloor \tau_{3,0} T \rfloor \\ u_{t-1} + \upsilon_t & t = \lfloor \tau_{3,0} T \rfloor + 1, \dots T \end{cases}.$$

$\delta_1 \geq 0, \delta_2 \geq 0, u_1 = O_p(1)$. Furthermore, the innovation process $\{\upsilon_t\}$ is given by:

$$\upsilon_t = C(L)\eta_t, C(L) := \sum_{j=0}^{\infty} C_j L^j$$

with $C(1)^2 > 0, \sum_{i=0}^{\infty} i|C_i| < \infty, \{\eta_t\} \sim i.i.d.(0,1)$ and finite fourth moment. The short run variance of υ_t is given by $\sigma_\upsilon^2 = \sum_{j=0}^{\infty} C_j^2$. Based on the general DGP setting introduced, 4 different kinds of DGPs are applied during the analysis; these are:

	unit root	bubble	collapse	unit root	mathematics
DGP 1	X	X			$0 < \tau_{1,0} < 1, \tau_{2,0} = 1$
DGP 2	X	X		X	$0 < \tau_{1,0} < \tau_{2,0} < 1, \tau_{2,0} = \tau_{3,0}$
DGP 3	X	X	X		$0 < \tau_{1,0} < \tau_{2,0} < 1, \tau_{3,0} = 1$
DGP 4	X	X	X	X	$0 < \tau_{1,0} < \tau_{2,0} < \tau_{3,0} < 1$

The following parameter settings are used: $T = 200, \delta_1 \in \{0.0400, 0.0425, \dots, 0.1000\}$, fraction of minimum duration for bubble $s = 0.1$, tolerance $k \in \{0, 1, 5\}$, duration bubble regime $\lfloor 0.2T \rfloor$, duration reversion process in GDP 4 $\lfloor 0.1T \rfloor$, magnitude of stationary parameter in GDP 3 and 4: $\delta_2 \in \{\delta_1, \delta_1/2\}$, y_t with $\mu = 0, \upsilon_t \sim iidN(0,1)$.

Phillips and Shi (2018)

The following DGP is assumed (pp. 719-725):

$$X_t = \begin{cases} cT^{-\eta} + X_{t-1} + \epsilon_t & t \in N_0 \cup N_1 \\ \delta_T X_{t-1} + \epsilon_t & t \in B \\ \gamma_T X_{t-1} + \epsilon_t & t \in C \end{cases}$$

with $B = [T_e, T_c]$ as the bubble episode, $C = (T_c, T_r]$ as the collapse period with T_r as the recovery date and the normal market periods are $N_0 \cup N_1 = [1, T_e) \cup (T_r, T]$, $\delta_T = 1 + c_1 T^{-\alpha}$ with $c_1 > 0, \alpha \in [0, 1), \gamma_T = 1 - c_2 T^{-\beta}$ with $c_2 > 0, \beta \in [0, 1), \epsilon_t \sim iid(0, \sigma^2)$.

The following parameter settings are used: $X_0 = 100, c = c_1 = c_2 = 1, T = 100, \eta \in \{0.6, 1, 2\}, \sigma = 6.79, \alpha = 0.60, \beta = 0.1, d_{CT} = \lfloor 0.01T \rfloor, d_{BT} = \lfloor 0.20T \rfloor, f_e = 0.4$ (bubble start date). Furthermore, the collapse type sudden (which is already implemented $\beta = 0.1, d_{CT} = \lfloor 0.01T \rfloor$) is compared to disturbing collapse ($\beta = 0.5, d_{CT} = \lfloor 0.1T \rfloor$) and smooth collapse ($\beta = 0.9, d_{CT} = \lfloor 0.2T \rfloor$). For robustness testing, in the case of disturbing collapse, additional parameter settings are applied: $\beta \in [0.3, 0.7], d_{CT} \in [0.05T, 0.15T]$.

For the delay evaluation of market recovery, the following settings are used: $X_0 = 100, \sigma = 6.79, c = c_1 = c_2 = 1, \alpha = 0.6, d_{BT} = \lfloor 0.20T \rfloor, f_e = 0.4, T = 100$ and $T = 200$ is additionally used.

Guo, Sun, and Wang (2019)

The following DGP is assumed (pp. 85-87):

$$y_t = \mu_T + \rho y_{t-1} + u_t, t = 1, 2, \dots T$$

with $\rho = 1 + \frac{c}{k_T}, c = 1, k_T = T^{\alpha}, \alpha \in (0, 1)$.

The following parameter settings are used: $\mu_T \in \{0, T^{-\alpha/2}, T^{-\alpha/4}, 1\}, y_0 = \mu_T, u_t \in \{\sim iidN(0, 1), \sim iidU(-\sqrt{3}, \sqrt{3})\}, T = 100, \alpha = 0.5$.

Moreover, simulations are run considering weakly dependent errors (but only with $\mu_T = T^{-\alpha/4}$): AR case: $u_t = \theta u_{t-1} + \sqrt{1-\theta^2} e_{1,t}, e_{1,t} \sim iidN(0, 1)$. MA case: $u_t = \theta e_{2,t-1} + \sqrt{1-\theta^2} e_{2,t}, e_{2,t} \sim iidN(0, 1), \theta \in \{0.00, 0.25, 0.50, 0.75\}, \alpha = 0.5, T = 100$.

Harvey, Leybourne, and Zu (2019)

The following DGP is assumed (pp. 1140-1143):

$$y_t = \mu + x_t$$
$$x_t = (1 + \rho_t) x_{t-1} + u_t$$
$$u_t = \sigma_t \epsilon_t.$$

with $E(\epsilon_t | F_{t-1}) = 0, E(\epsilon_t^2 | F_{t-1}) = 1, \{u_s\}_{s \geq 1}, E(\epsilon_t^4) < \infty, \sigma_t = \sigma(t/T), \sigma(.) \in D[0, 1], \rho_t = 0$ for $t = 2, \dots, [\tau^* T], \rho_t = c/T$ for $t = [\tau^* T] + 1, \dots, T$.

The following parameter settings are used: $T = 200, \mu = 0, x_1 = 0, \epsilon_t \sim iidN(0,1), \tau^* \in \{0.6, 0.8\}, c \in \{0, 1, \ldots, 8\}$. 9 different functions of volatility $\sigma(r)$ are considered:

$\sigma(r) = 1 \forall r$ constant volatility

$\sigma(r) = 1 + 5\mathbf{1}(r \geq 0.3)$ early upward shift

$\sigma(r) = 1 + 5\mathbf{1}(r \geq 0.8)$ late upward shift

$\sigma(r) = 1 + 5\mathbf{1}(r < 0.3)$ early downward shift

$\sigma(r) = 1 + 5\mathbf{1}(r < 0.8)$ late downward shift

$\sigma(r) = 1 + 5r$ upward trend

$\sigma(r) = 6 - 5r$ downward trend

$\sigma(r) = 1 + 5\mathbf{1}(0.4 < r \leq 0.6)$ double shift

$\sigma(r) = d\sigma^2(r) = 0.03(0.25 - \sigma^2(r))dr + 0.1\sqrt{\sigma^2(r)}dB(r)$ stochastic volatility.

Whitehouse (2019)

The following DGP is assumed (pp. 35-37):

$$y_t = \mu + \beta t + u_t,$$

$$u_t = \begin{cases} T^{1/2}\sigma\alpha & t = 1 \\ u_{t-1} + v_t & t = 2, \ldots, \lfloor \tau_1 T \rfloor \\ (1+\delta)u_{t-1} + v_t & t = \lfloor \tau_1 T \rfloor + 1, \ldots, \lfloor \tau_2 T \rfloor \\ u_{t-1} + v_t & t = \lfloor \tau_2 T \rfloor + 1, \ldots, T \end{cases}$$

with $\delta = c/T, c \geq 0$. For v_t (mds), the following assumptions are made: $E(v_t^2) = \sigma^2 < \infty$, $\frac{1}{T}\sum_{t=1}^T v_t^2 \rightarrow_p \sigma^2, \sup_t \sum_{t=1}^T E(v_t^2) < \infty$. For every $\epsilon > 0$ it is: $\lim_{T\to\infty} \sigma^{-2}T^{-1}\sum_{t=1}^T \int_{v^2 > \epsilon T\sigma^2} v^2 dF_t(v) = 0$ with F_t as the distribution function of v_t. To test for the effects of heteroskedasticity also a GARCH version of the innovation term v_t is used: $v_t = \eta_t\sqrt{h_t}$ with $\eta_t \sim iidN(0,1)$ and $h_t = \omega + \gamma v_{t-1}^2 + \phi h_{t-1}$.

The following parameter settings are used: $T = 150, \alpha \in \{0, 10\}, [\tau_1, \tau_2] \in \{[0.45, 0.55], [0.2, 0.8], [0.15, 0.25], [0.75, 0.85]\}$ as the starting and end value of the bubble, $c \in \{\{0, 1, \ldots 56\}, [0, 3.00]\}, \omega = 30, \gamma = 0, \phi = 0.6$.

Hafner (2020)

The following DGP is assumed (pp. 244-245):

$$y_t = (1 + \delta J_t)y_{t-1} + \epsilon_t$$

with $J_t = I(t \geq T/2), \delta = 0$ for first sample part, $\delta > 0$ for second part. $\epsilon_t = g(t/T)\sqrt{h_t}\xi_t, \xi_t \sim iid(0,1)$ is a centered normalized negative $\log \chi^2$ distribution.

The following parameter settings are used: $\delta \in \{0, 0.01, 0.02, 0.03\}, g(t/T) = 0.05(1+c\cos(\pi t/T)^2), c \in \{1,2\}, h_t = \omega + \alpha\frac{\epsilon_{t-1}^2}{g(\frac{t-1}{T})^2} + \beta h_{t-1}, \alpha = 0.1, \beta = 0.85, \omega = 1 - \alpha - \beta, T \in \{100, 200, 500\}$.

Harvey, Leybourne, and Whitehouse (2020)

The following DGP is assumed (pp. 232-234):

$$y_t = \mu + x_t + u_t,$$
$$u_t = (1 + \rho_t)u_{t-1} + v_t,$$
$$\rho_t = \sum_{j=1}^{N}\{\rho_{j1}^*\mathbf{1}\left(\lfloor\tau_{j1}^*T\rfloor < t \leq \lfloor\tau_{j2}^*T\rfloor\right) + \rho_{j2}^*\mathbf{1}\left(\lfloor\tau_{j2}^*T\rfloor < t \leq \lfloor\tau_{j3}^*T\rfloor\right) - \mathbf{1}(t = \lfloor\tau_{j3}T\rfloor + 1)\},$$
$$x_t = \sum_{j=1}^{N} u_{\lfloor\tau_{j3}^*T\rfloor}\mathbf{1}\left(t > \lfloor\tau_{j3}^*T\rfloor\right)$$

with $\rho_{j1}^* \geq 0, \rho_{j2}^* \leq 0\ \forall j = 1, \ldots, N, \tau_{11}^* > 0, \tau_{N3}^* \leq 1, \tau_{(j+1)1} > \tau_{j3}^*$.

The following parameter settings are used: $(N,T) \in \{(2,200),(3,300)\}, v_t \sim iidN(0,1), \mu = 0$ and then, the following six cases are run:

N	T	$(\tau_{11}^*, \tau_{12}^*, \tau_{13}^*)$	$(\rho_{11}^*, \rho_{12}^*)$	$(\tau_{21}^*, \tau_{22}^*, \tau_{23}^*)$	$(\rho_{21}^*, \rho_{22}^*)$	$(\tau_{31}^*, \tau_{32}^*, \tau_{33}^*)$	$(\rho_{31}^*, \rho_{32}^*)$
2	200	(0.2, 0.3, 0.4)	(0.1, -0.05)	(0.6, 0.7, 0.8)	(0.1, -0.05)		
2	200	(0.3, 0.5, 0.55)	(0.05, -0.05)	(0.75, 0.85, 1.0)	(0.075, -0.05)		
2	200	(0.2, 0.3)	0.075	(0.6, 0.7, 0.75)	(0.075, -0.075)		
2	200	(0.4, 0.5)	0.05	(0.95, 1.0)	0.05		
3	200	(0.2, 0.35, 0.4)	(0.075, -0.075)	(0.6, 0.7, 0.75)	(0.075, -0.075)	(0.9, 1)	0.075
3	200	(0.3, 0.4, 0.5)	(0.075, -0.05)	(0.6, 0.7, 0.75)	(0.075, -0.05)	(0.85, 0.95, 1)	(0.075, -0.05)

The following situations are modelled by the DGPs (i) first explosive regime with collapse, second explosive regime with collapse, (ii) first explosive regime with collapse, second explosive regime with collapse running to sample end, (iii) first explosive regime without collapse, second explosive regime with collapse, (iv) first explosive regime without collapse, second explosive regime running to sample end, (v) first explosive regime with collapse, second explosive regime with collapse, third explosive regime running to sample end, (vi) first explosive regime with collapse, second explosive regime with collapse, third explosive regime with collapse running to sample end.

Harvey, Leybourne, and Zu (2020)

The following DGP is assumed (pp. 139-144):

$$y_t = \mu + u_t$$

$$u_t = \begin{cases} u_{t-1} + \epsilon_t & t = 2, \ldots, \lfloor \tau_1 T \rfloor \\ (1+\delta_{1,T})u_{t-1} + \epsilon_t & t = \lfloor \tau_1 T \rfloor + 1, \ldots, \lfloor \tau_2 T \rfloor \\ (1-\delta_{2,T})u_{t-1} + \epsilon_t & t = \lfloor \tau_2 T \rfloor + 1, \ldots, \lfloor \tau_3 T \rfloor \\ u_{t-1} + \epsilon_t & t = \lfloor \tau_3 T \rfloor + 1, \ldots, T \end{cases}$$

with $\delta_{1,T} \geq 0, \delta_{2,T} \geq 0, \delta_{i,T} = c_i T^{-1}, c_i > 0, i = 1, 2, u_1 = o_p(T^{1/2}), E(\epsilon_t) = 0, \epsilon_t = \sigma_t z_t, z_t \sim iid(0,1), E(|z_t|^r) < \infty$ for $r \geq 4, \sigma_t = \sigma(t/T), \sigma(.) \in D$ is non-stochastic and strictly positive. Based on the general DGP setting introduced, 4 different kinds of DGPs are applied during the analysis; these are:

	unit root	bubble	collapse	unit root	mathematics
DGP 1	X	X			$0 < \tau_1 < 1, \tau_2 = \tau_3 = 1$
DGP 2	X	X		X	$0 < \tau_1 < \tau_2 < 1, \tau_2 = \tau_3$
DGP 3	X	X	X		$0 < \tau_1 < \tau_2 < 1, \tau_3 = 1$
DGP 4	X	X	X	X	$0 < \tau_1 < \tau_2 < \tau_3 < 1$

The following parameter settings are used: $T = 100, \mu = 0, u_1 = \epsilon_1, z_t \sim iidN(0,1), \sigma_1 \in \{1/6, 1/3, 1, 3, 6\}, c_1 \in \{2, 4, 6, 8\}$. For DGP 1: $\sigma(s) = \mathbf{1}(0 \leq s \leq \tau_1) + \sigma_1 \mathbf{1}(\tau_1 < s \leq 1), \tau_1 \in \{0.4, 0.8\}$. For DGP 2: $\sigma(s) = \mathbf{1}(0 \leq s \leq \tau_1) + \sigma_1 \mathbf{1}(\tau_1 < s \leq \tau_2) + \mathbf{1}(\tau_2 < s \leq 1), \tau_2 = 0.7, \tau_1 \in \{0.1, 0.5\}$. DGP3 and 4 are not applied.

Kurozumi (2020)

The following DGP is assumed (pp. 520-524):

$$\begin{aligned} X_t =& \mu + (X_{t-1} + \epsilon_t)\mathbf{1}(t \leq m + k_e) + (\delta_m X_{t-1} + \epsilon_t)\mathbf{1}(m + k_e + 1 \leq t \leq m + k_c) \\ &+ \left(X_{m+k_e} + X^* + \sum_{j=m+k_c+1}^{t} \epsilon_j \right) \mathbf{1}(t \geq m + k_c + 1) \end{aligned}$$

with $\delta_m = 1 + \frac{c}{m^\alpha}, c > 0, \alpha \in (0.5, 1)$.

The following parameter settings are used: $\mu = 0, m \in \{50, 100, 200\}$, and the monitoring procedure is stopped at $\bar{m} \in \{2m, 4m, 6m\}$, $X^* = 0, \delta_m = 1 + c/100, c \in \{0.5, 1.0, \ldots 5.0\}, k_e \in \{0, 0.5m, 1m\}$, size of the bubble regime: $10, 25, 50, 75, 100\,(k_c = k_e + 10, k_e + 25, \ldots, k_e + 100)$. The following three cases are considered:

(i) $X_0 = 100$ and $\{\epsilon_t\} \sim_{iid} N\left(0, \sigma^2\right), \sigma = 6.79$

(ii) $X_0 = 100, \{\epsilon_t\} \sim_{iid} t(5)$ scaled to a standard deviation of 6.79,

(iii) $X_0 = 376.8, \epsilon_t = v_t\sqrt{h_t}, \{v_t\} \sim_{iid} N(0,1), h_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}, \omega = 30.69, \alpha = 0, \beta = 0.61$.

Pedersen and Schütte (2020)

The following DGP is assumed (pp. 216-220):

$$y_t = \begin{cases} y_{t-1} + v_t & t = 1, \ldots, \tau_e - 1 \\ \delta_1 y_{t-1} + v_t & t = \tau_e, \ldots, \tau_c - 1 \\ \delta_2 y_{t-1} + v_t & t = \tau_c, \ldots, \tau_x - 1 \\ y_{t-1} + v_t & t = \tau_x, \ldots, T \end{cases}$$

with $\delta_1 = 1 + T^{-\alpha}, \delta_2 = 1 - T^{-\alpha}, v_t = \phi_1 v_{t-1} + \epsilon_t + \vartheta_1\epsilon_{t-1} + \vartheta_2\epsilon_{t-2} + \vartheta_3\epsilon_{t-3}, \epsilon_t \sim_{iid} N(0, \sigma_v)$.

The following parameter settings are used: $y_0 = 100, \sigma_v = 6.79, \alpha = 0.6, (T, \delta_1, \delta_2) \in \{(100, 1.06, 0.94),$ $(200, 1.04, 0.96), (400, 1.03, 0.97)\}$. For the innovation term, different models are applied: White noise, MA(1) ($\vartheta_1 = 0.5$), MA(3) ($\vartheta_1 = \vartheta_2 = \vartheta_3 = 0.5$) and AR(1) ($\phi_1 = 0.5$). Additionally, $(r_e = 0.4, r_c = 0.6, r_x = 0.7), (r_e = 0.4, r_c = 0.7, r_x = 0.8), (r_e = 0.2, r_c = 0.4, r_x = 0.5), (r_e = 0.8)$. For their base case $r_e = 0.4, r_c = 0.6, r_x = 0.7$ with $T \in \{100, 200, 400\}$, local power curves are constructed with $y_0 = 0, \sigma_v = 1, \delta_1 = 1 + c/T, \delta_2 = 1 - c/T, c \in \{1, 2, \ldots, 24\}$.

Next, the previously applied DGP with one bubble is allowed to have more bubbles (DGP is not stated in their paper!). Here, $T = 200$, the innovation models are the same as before. Three combinations of bubbles are considered: Short-long bubble ($r_{e1} = 0.2, r_{c1} = 0.3, r_{x_1} = 0.35, r_{e2} = 0.6, r_{c2} = 0.8, r_{x_2} = 0.85$), long-short bubble ($r_{e1} = 0.2, r_{c1} = 0.4, r_{x_1} = 0.45, r_{e2} = 0.6, r_{c2} = 0.7, r_{x_2} = 0.75$), long-long bubble ($r_{e1} = 0.2, r_{c1} = 0.4, r_{x_1} = 0.45, r_{e2} = 0.6, r_{c2} = 0.8, r_{x_2} = 0.85$).

Astill, Harvey, Leybourne, Taylor, and Zu (2021)

The following DGP is assumed (pp. 18-25):

$$Y_t = \mu + u_t \quad t = 1, \ldots, \gamma T$$

$$u_t = \begin{cases} u_{t-1} + e_t & t = 1, \ldots, T \\ u_{t-1} + e_t & t = T+1, \ldots, \lfloor \tau_1 T \rfloor \\ (1+\delta)u_{t-1} + e_t & t = \lfloor \tau_1 T \rfloor + 1, \ldots, \lfloor \tau_2 T \rfloor \\ u_{t-1} + e_t & t = \lfloor \tau_2 T \rfloor + 1, \ldots, \lfloor \gamma T \rfloor \end{cases}$$

with $1 \leq \tau_1 \leq \tau_2 \leq \gamma, \gamma > 1, u_0 = O_p(1), \epsilon_t \sim iidN(0,1), \mu = 0$. Remark: The observations from $t = 1, \ldots, T$ are used as the training sample for CUSUM, thus the DGP structure looks a little different to the other DGPs.

The following parameter settings are used: $\lfloor \tau_1 T \rfloor \in \{220, 230\}, \lfloor \tau_2 T \rfloor = \lfloor \tau_1 T \rfloor + 25, \delta \in \{0.004, 0.006$ $, 0.008, 0.010\}$. Additionally to the homoscedastic error term assumption, a smooth transition is considered: $\sigma_t = 1 + a[1 + \exp(-\theta(t - T_b))]^{-1}$ with $\theta = 0.25, T_b = \lfloor \tau_1 T \rfloor, a \in \{0, \sqrt{2} - 1, \sqrt{3} - 1, \sqrt{4} - 1\}$. All parameter remain the same but $\delta = 0.007$ is applied. Additionally, the authors consider the case where an explosive period is present in the training sample (for brevity, this is not stated here.)

Kurozumi (2021)

The following DGP is assumed (pp. 320-325):

$$X_t = \mu + (\delta_m X_{t-1} + \epsilon_t)\mathbf{1}(t \leq m + k_e) + (\delta_m X_{t-1} + \epsilon_t)\mathbf{1}(t \geq m + k_e + 1)$$

with $t = 1, \ldots, m, \ldots, \bar{m}, \mu = 0, \delta_m = 1 + \frac{c}{m^\alpha}, k_e = \lfloor m^\beta \rfloor, \epsilon_t \sim_{iid} N(0, \sigma^2)$.

The following parameter settings are used: $\alpha = 0.75, c \in \{2, 4, 6\}, m \in \{100, 200, 400\}$ (monitoring period), $\bar{m} = 4m$ (end of monitoring period), thus, the length of the monitoring period is $3m$, $\beta \in \{0, 0.27, 0.33, 0.375, 0.5, 0.55, 0.67, 0.75, 0.77, 0.83, 0.875, 1\}, X_0 = \sqrt{m}x_0, x_0 = 10/6.79$.

Kurozumi, Skrobotov, and Tsarev (2021)

The following DGP is assumed (pp. 11-14):

$y_t = \mu + u_t$

$$u_t = \begin{cases} u_{t-1} + \epsilon_t & t = 1, \ldots, \lfloor \tau_{1,0} T \rfloor \\ (1+\delta_1) u_{t-1} + \epsilon_t & t = \lfloor \tau_{1,0} T \rfloor + 1, \ldots, \lfloor \tau_{2,0} T \rfloor \\ (1-\delta_2) u_{t-1} + \epsilon_t & t = \lfloor \tau_{2,0} T \rfloor + 1, \ldots, \lfloor \tau_{3,0} T \rfloor \\ u_{t-1} + \epsilon_t & t = \lfloor \tau_{3,0} T \rfloor + 1, \ldots, T \end{cases} \qquad \epsilon_t = \sigma_t e_t$$

with $\delta_1 \geq 0, \delta_2 \geq 0, 0 \leq \tau_{1,0} < \tau_{2,0} \leq \tau_{3,0} \leq 1, \mu = 0, \{e_t\}$ is a mds with $E(e_t^2|F_{t-1}) = 1, E(|e_t|^p|F_{t-1}) < \infty$ almost surely for $p > 6, \sigma_t = \omega(t/T), \omega(s) \in D[0,1]$ for $s \in [0,1]$ is a non-stochastic and strictly positive function.

The following parameter settings are used: $u_0 = e_0, e_t \sim iidN(0,1), T \in \{100, 200\}, \delta_1 \in \{0, 0.02, 0.04, 0.06, 0.08, 0.10\}, \delta_2 = 0, \lfloor \tau_{1,0} T \rfloor = 0.4T, \lfloor \tau_{2,0} T \rfloor = 0.6T$. For the volatility, four different functions are applied:

$\omega(s) = \sigma_0 + (\sigma_1 - \sigma_0)\mathbf{1}(s > \tau_\sigma)$ single shift in volatility

$\omega(s) = \sigma_0 + (\sigma_1 - \sigma_0)\mathbf{1}(0.4 < s \leq 0.6)$ double shift in volatility

$\omega(s) = \sigma_0 + (\sigma_1 - \sigma_0)\frac{1}{1+\exp\{-50(s-0.5)\}}$ logistic smooth transition in volatility

$\omega(s) = \sigma_0 + (\sigma_1 - \sigma_0)s$ trending volatility

with $\tau_\sigma \in \{0.3, 0.5, 0.7\}, \sigma_1/\sigma_2 \in \{\frac{1}{6}, \frac{1}{3}, 1, 3, 6\}$.

Lui, Xiao, and Yu (2021)

The following DGP is assumed (pp. 526-527):

$$y_t = \mu_n + \rho y_{t-1} + u_t, t = 1, 2, \ldots, n.$$

with $y_0 = 0, \mu_n = \frac{\mu}{n^\vartheta}, \rho = (1 + cm/n), c > 0, u_t = (1-L)^d \epsilon_t, \epsilon_t \sim_{iid} N(0,1)$.

The following parameter settings are used: $(n, m) \in \{(100, 10), (500, 15), (1000, 20)\}, d \in \{-0.45, -0.4, -0.3, -0.2, -0.1, -0.01\}, c \in \{0.5, 1\}, \mu = 1, \vartheta = \frac{1}{2} - d + 0.1$.

Monschang and Wilfling (2021)

The following DGP (Rotermann and Wilfling (2018) model) is assumed (pp. 156-165):

$$
\begin{aligned}
P_t = P_t^f + B_t \\
&P_t^f = \frac{1+r}{r^2}\mu + \frac{1}{r}D_t, \\
&D_t = \mu + D_{t-1} + e_t, \\
&B_t = \begin{cases} \frac{\alpha}{\psi\pi} B_{t-1}u_t & \text{with probability } \pi \\ \frac{1-\alpha}{\psi(1-\pi)} B_{t-1}u_t & \text{with probability } 1-\pi \end{cases}
\end{aligned}
$$

with $\psi := (1+r)^{-1}, \alpha \in (0,1), \pi \in (0,1], \frac{\alpha}{\pi} > 1, \frac{1-\alpha}{\psi(1-\pi)} < 1, \{u_t\}_{t=1}^{\infty} \sim iid$ lognormally distributed $u_t \sim LN(\frac{-l^2}{2}, l^2)$. Another time, the same DGP is applied with TGARCH(1,1) errors:

$$
\begin{aligned}
\epsilon_t = s_t\sqrt{h_t}, \\
&s_t \sim iidN(0,1), \\
&h_t = \omega + \gamma\epsilon_{t-1}^2 + \beta h_{t-1} + \phi\epsilon_{t-1}^2 \mathbf{1}(\epsilon_{t-1} < 0).
\end{aligned}
$$

The following parameter settings are used: $\mu = 0, D_0 = 1.6942, \alpha = 0.9675, \psi = 0.9840, \pi = 0.9595, B_0 = 10.1925, l^2 = 0.0061, \sigma_e^2 \in \{0.4476, TGARCH(\omega = 0.4387, \gamma = 0, \beta = 0.9319, \phi = 0.1306)\}, T \in \{100, 200, 400, 800, 1600\}$. The robustness is tested by varying some parameters on a ceteris-paribus basis (but only for $T = 400$): $\mu \in \{0.000, 0.001, 0.002, 0.003\}, \psi \in \{0.975, 0.980, 0.985, 0.990\}, (\pi, \alpha) \in \{(0.35, 0.3675), (0.55, 0.5775), (0.75, 0.7875), (0.95, 0.9975)\}$, π and α are changed in such a way that the mean bubble growth rate is still 1.06.

To evaluate the performance of date stamping procedures, they apply a modified version of the DGP suggested by Phillips and Shi (2018):

$$
y_t = \begin{cases} aT^{-\eta} + y_{t-1} + \epsilon_t & t \in N_0 \cup N_1 \\ \delta_T y_{t-1} + \epsilon_t & t \in B \\ \gamma_T y_{t-1} + \epsilon_t & t \in C \end{cases}
$$

with $\epsilon_t \sim TGARCH(1,1), \delta_T = 1 + c_1 T^{-\tilde{\alpha}}, \gamma_T = 1 - c_2 T^{-\tilde{\beta}}, B = [\lfloor Tr_e \rfloor, \lfloor Tr_f \rfloor], C = (\lfloor Tr_f \rfloor, \lfloor Tr_r \rfloor], N_0 \cup N_1 = [1, \lfloor Tr_e \rfloor) \cup (\lfloor Tr_r \rfloor, T]$

The following parameter settings are used: $TGARCH : \omega = 0.4387, \gamma = 0, \beta = 0.9319, \phi =$

$0.1306, T = 100, a = \eta = c_1 = c_2 = 1, y_0 = 100, \tilde{\alpha} = 0.6, r_e = 0.4, r_f = 0.6, d_B = \lfloor 0.2T \rfloor$. For the collapse types sudden, disturbing and smooth, the following three settings are applied: $(\tilde{\beta}, r_r, d_c) \in \{(0.1, 0.61, \lfloor 0.01T \rfloor), (0.5, 0.7, \lfloor 0.1T \rfloor), (0.9, 0.8, \lfloor 0.20T \rfloor)\}$.

Parameter overview

In the first column, the authors of the investigated paper are stated, in the second and third columns are the autoregressive parameters for the explosive (ρ_e) and reverse/crash period (ρ_c). In the columns 4 to 6 are the duration of the complete process simulated (T), the explosive (T_e) and reverse/crash duration (T_c). In the next column is the distribution of the innovation terms ($dist_{innov}$) and the two upcoming columns show the variance of the innovations during the explosive ($var(innov)_e$) and reverse/crash period ($var(innov)_c$). The last column states the initial value of the process (y_0) that is simulated. In the tables, the parameters are labelled in the same way, so the names/symbols applied are not necessarily the same as in the investigated papers. For comparability purposes, e.g., the autoregressive parameter is always stated as the parameter itself and not as, e.g., $1 + \delta, 1 + c/T$ are something else. If the applied framework is too far away from the autoregressive DGP setting, then it is labelled as 'not comparable'. If for a specific parameter/variable, there is no value given (but maybe an integration order), it is labelled as 'no specific value stated'. In the case, that something like the reverse/crash period is not modelled, then 'not considered' is stated.

For convenience and comparability purposes, only the AR setting of studies like Homm & Breitung (2012) and Monschang & Wilfling (2021) are provided in the tables. Their additional settings which are too far away from the autoregressive explosive world are not stated.

Table 12: Parameter settings applied in DGPs I

author/paper	ρ_e	ρ_c	T	T_e	T_c	$dist_{innov}$	$var(innov)_e$	$var(innov)_c$	y_0
Phillips, Wu & Yu (2011)	1.05	not specified	100	not comparable	not comparable	not comparable	not comparable	not comparable	not comparable
Homm & Breitung (2012)	$\{1.02, 1.03, 1.04, 1.05\}$	not considered	$\{100, 200, 400\}$	$\{10, 20, 30, 40, 60, 80, 100, 120, 160, 200, 240\}$	no crash in the AR setting, just in the Evans and Blanchard like case	Gaussian WN	no specific value stated	no specific value stated	0
Breitung & Kruse (2013)	$[1.02, 1.2088]$	not considered	$\{69, 100, 121\}$	$[28, 97]$	no specific value stated	$iidN(0, \sigma^2)$	variance ratios are given, but no specific values for the variance	variance ratios are given, but no specific values for the variance	no specific value stated
Phillips, Shi & Yu (2014)	$\{1.0152, 1.0504\}$	not specified	$\{100, 200, 400\}$	not comparable	not comparable	not comparable	not comparable	not comparable	not comparable
Harvey, Leybourne & Sollis (2015)	$\{1.001, 1.002, \ldots, 1.08\}$	{hard crash, then immediately RW; no crash, immediately RW}	$\{300, 600\}$	$\{30, 60, 120\}$	{no crash, 1}	$\{iidN(0,1), t_5\}$	$\{1, 1.6667\}$	$\{1, 1.6667\}$	v_0
Phillips, Shi & Yu (2015)	1.0631	hard crash, then immediately RW	100	$\{10, 15, 20\}$	1	$iid(0, \sigma^2)$	6.79^2	6.79^2	100
Harvey, Leybourne, Sollis & Taylor (2016)	$\{1.02, 1.04, 1.06, 1.08\}$	1.0 - no hard collapse, just switch to RW	200	40	no crash period	$iid(0,1)$ term multiplied by a volatility function	$iid(0,1)$ term multiplied by a volatility function	$iid(0,1)$ term multiplied by a volatility function	no specific value stated
Astill, Harvey, Leybourne & Taylor (2017)	$\{[1, \ldots, 1.02], [1, \ldots, 1.05]\}$	no crash	$\{100, 200\}$	$\{2, 5, 10, 20\}$	no crash considered	$iidN(0,1)$	$\{1/10, 1/5, 1, 5, 10\}$	no crash	100
Harvey, Leybourne & Sollis (2017)	$\{1.0400, 1.0425, \ldots, 1.1000\}$	{no collapse, no collapse and immediate RW, $\{0.95, 0.95125, \ldots, 0.98\}$, $\{0.9, 0.925, \ldots, 0.96\}$}	200	40	not applicable for 3 cases, for one case, it is 20	$iidN(0,1)$	1	1	no specific value stated

Table 13: Parameter settings applied in DGPs II

author/paper	ρ_e	ρ_c	T	T_e	T_c	$dist_{innov}$	$var(innov)_e$	$var(innov)_c$	y_0
Phillips & Shi (2018)	$\{1.0416, 1.0631\}$	{0.3690, 0.4113, 0.7488, 0.7960, 0.9000, 0.9293, 0.9602, 0.9755, 0.9842, 0.9915 }	$\{100, 200\}$	$\{20, 40\}$	{ 1, 2, 5, 10, 15, 20, 30, 40}	$iid(0, \sigma^2)$	6.79^2	6.79^2	100
Guo, Sun & Wang (2019)	1.1	not considered	100	100	not considered	{$iidN(0, 1)$, iidU $(-\sqrt{3}, \sqrt{3})$, AR(1), MA(1)}	$\{1, \sqrt{3}\}$	not considered	$\{0, T^{-\alpha/2}, T^{-\alpha/4}, 1\}$
Harvey, Leybourne & Zu (2019)	$\{1.005, 1.01, \ldots, 1.04\}$	no collapse	200	$\{40, 80\}$	no crash period considered	$iidN(0, 1)$ term multiplied by a volatility function	{constant, (early/late) upward/downward shift, upward/downward trend, double shift, stochastic volatility}	no crash	0
Whitehouse (2019)	$[1.00001, 1.3733]$	1.00 - no hard collapse, just switch to RW	150	$\{15, 90\}$	no crash, just RW	{ mds, GARCH-type}	{ not specified, GARCH-type}	no crash	no specific value stated
Hafner (2020)	{ 1.01, 1.02, 1.03 }	1.00 - no hard collapse, just switch to RW	{100, 200, 500}	{50, 100, 250}	no crash, just RW	GARCH-type	GARCH-type	no crash	not specified
Harvey, Leybourne & Whitehouse (2020)	$\{1.05, 1.075, 1.1\}$	{no collapse, $0.925, 0.95\}$	$\{200, 300\}$	$\{10, 20, 30, 40\}$	$\{0, 10, 20, 30\}$	$iidN(0, 1)$	1	1	no specific value stated
Harvey, Leybourne & Zu (2020)	{1.02, 1.04, 1.06, 1.08}	{0.92, 0.94, 0.96, 0.98}	100	$\{20, 60\}$	{no collapse, 30}	$iidN(0, 1)$ term multiplied by a volatility function	$iidN(0, 1)$ term multiplied by a volatility function	$iidN(0, 1)$ term multiplied by a volatility function	no specific value stated
Kurozumi (2020)	{1.005, 1.01, $\ldots 1.05\}$	1.00 - no hard collapse, just switch to RW	$\{50, 100, 200\}$	{10, 25, 50, 75, 100}	no crash, just RW	{ $iidN(0, 6.79^2)$, $iidt(5)$, GARCH-type}	{6.79^2, GARCH-type}	no crash	$\{100, 376.8\}$

Table 14: Parameter settings applied in DGPs III

author/paper	ρ_e	ρ_c	T	T_e	T_c	$dist_{innov}$	$var(innov)_e$	$var(innov)_c$	y_0
Pedersen & Schütte (2020)	$[1.0025, 1.24]$	$[0.76, 0.9975]$	$\{100, 200, 400\}$	{20, 30, 40, 60, 80, 120}	{10, 20, 40}	WN, MA(1), MA(3), AR(1)	6.79^2	6.79^2	$\{0, 100\}$
Astill, Harvey, Leybourne, Sollis & Zu (2021)	{1.004, 1.006, 1.007, 1.008, 1.010}	no crash, just RW	$\{245, 255\}$	25	no crash considered	{$iidN(0,1)$, time-varying volatility}	{1, based on a time-varying volatility function}	no crash	100
Kurozumi (2021)	{1.0224, 1.0376, 1.0447, 1.0632, 1.0671, 1.0752, 1.1128, 1.1265, 1.1897}	not considered	$\{400, 800, 1600\}$	not clear	not considered	$iidN(0, \sigma^2)$	not specified	no crash	$\sqrt{m}10/6.79$
Kurozumi, Skrobotov & Tsarev (2021)	{1.00, 1.02, 1.04, 1.06, 1.08, 1.10}	1.00 - no hard collapse, just switch to RW	$\{100, 200\}$	$\{20, 40\}$	no crash, just RW	$iidN(0,1)$ term multiplied by a volatility function	volatility evolves based on a single or double shift, smooth transition, or is trending	no crash	initial error term
Lui, Xiao & Yu (2021)	{1.01, 1.015, 1.02, 1.03, 1.05, 1.10 }	not considered	$\{100, 500, 1000\}$	$\{100, 500, 1000\}$	not considered	$(1-L)^d \epsilon_t$, $\epsilon_t \sim iidN(0,1)$	$(1-L)^d \epsilon_t$, $\epsilon_t \sim iidN(0,1)$	no crash	0
Monschang & Wilfling (2021)	1.0631	{0.3690, 0.9000, 0.9842}	100	20	$\{1, 10, 20\}$	TGARCH-based	TGARCH-based	TGARCH-based	100

Table 15: Financial exuberance characteristics - part I

	start date	start value	peak date	peak value	burst date	burst value	duration	duration expl.	duration crash	duration ratio	increase expl.	decrease crash
AEX	1985-11-19	5.25	1986-01-08	5.4	1986-02-19	5.32	67	37	30	1.23	0.15	-0.08
AEX	1993-10-05	5.52	1994-01-31	5.72	1994-03-31	5.63	128	85	43	1.98	0.2	-0.09
AEX	1995-12-27	5.78	1996-06-06	5.93	1996-07-23	5.85	150	117	33	3.55	0.15	-0.09
AEX	1996-07-25	5.84	2000-08-24	6.83	2001-03-26	6.56	1218	1066	152	7.01	0.99	-0.27
AEX	2005-12-01	6.22	2006-04-21	6.32	2006-05-11	6.3	116	102	14	7.29	0.1	-0.02
BITCOIN	2012-07-13	1.2	2012-08-16	1.76	2012-10-24	1.59	74	25	49	0.51	0.56	-0.17
BITCOIN	2013-01-08	1.77	2013-12-04	6.19	2014-09-17	5.25	442	237	205	1.16	4.42	-0.94
BITCOIN	2017-03-27	6.06	2017-12-18	8.95	2018-11-28	7.42	438	191	247	0.77	2.89	-1.52
BITCOIN	2019-05-02	7.65	2019-06-26	8.53	2019-11-21	7.99	146	40	106	0.38	0.88	-0.53
Brent Oil	2008-05-02	3.94	2008-07-03	4.18	2008-08-01	4.04	66	45	21	2.14	0.24	-0.14
CAC 40	1998-01-28	8.29	1998-07-17	8.63	1998-09-16	8.47	166	123	43	2.86	0.35	-0.16
CAC 40	1998-12-21	8.49	2000-09-04	9.06	2001-03-09	8.8	580	446	134	3.33	0.57	-0.26
CAC 40	2006-01-26	8.62	2006-05-09	8.69	2006-05-12	8.66	77	74	3	24.67	0.07	-0.03
DAX 30	1983-03-10	7.01	1983-07-07	7.18	1983-08-29	7.1	123	86	37	2.32	0.17	-0.08
DAX 30	1983-09-19	7.11	1984-02-03	7.24	1984-02-21	7.19	112	100	12	8.33	0.12	-0.05
DAX 30	1985-05-06	7.29	1986-04-17	7.87	1987-10-23	7.67	645	249	396	0.63	0.59	-0.2
DAX 30	1989-07-14	7.81	1989-09-08	7.88	1989-10-13	7.83	66	41	25	1.64	0.07	-0.05
DAX 30	1989-12-04	7.86	1990-07-18	8.04	1990-08-03	7.96	175	163	12	13.58	0.18	-0.08
DAX 30	1997-01-08	8.23	2000-03-07	9.22	2001-10-11	8.66	1242	825	417	1.98	0.99	-0.56
DAX 30	2001-11-05	8.67	2002-01-04	8.77	2002-02-05	8.7	67	45	22	2.05	0.1	-0.08
DAX 30	2006-01-25	8.74	2006-05-09	8.86	2006-05-16	8.81	80	75	5	15	0.12	-0.05
DAX 30	2007-03-29	8.96	2007-06-20	9.11	2008-01-17	9.01	211	60	151	0.4	0.15	-0.1
DJ US RE	1993-01-13	4.36	1993-03-31	4.46	1993-06-03	4.41	102	56	46	1.22	0.09	-0.05
DJ US RE	1993-06-30	4.41	1993-10-07	4.52	1993-11-03	4.47	91	72	19	3.79	0.11	-0.05
DJ US RE	1996-08-15	4.51	1997-10-07	4.81	1998-06-04	4.69	471	299	172	1.74	0.3	-0.12
DJ US RE	2003-11-24	4.62	2004-04-01	4.76	2004-04-05	4.7	96	94	2	47	0.14	-0.06
DJ US RE	2006-08-15	4.95	2007-02-07	5.2	2007-05-23	5.03	202	127	75	1.69	0.25	-0.17
FTSE SA	2005-05-26	10.2	2007-10-11	10.89	2008-01-15	10.73	689	621	68	9.13	0.69	-0.16
FTSE 100	1997-04-29	8.76	1997-10-03	8.93	1997-10-27	8.83	130	114	16	7.12	0.17	-0.1
FTSE 100	1997-12-01	8.85	1998-07-20	9.07	1998-08-27	8.93	194	166	28	5.93	0.22	-0.14

This table provides a detailed overview of the identified positive financial exuberance periods within the 30 applied financial time series. For each period, the start, peak and end value/date are provided. On top of this, the duration of the explosive part, the reverse period and the complete financial exuberance period are given. The stated duration ratio is the duration of the explosive part divided by the reverse. Last, the increase and decrease during both periods are given.

Table 16: Financial exuberance characteristics - part II

	start date	start value	peak date	peak value	burst date	burst value	duration	duration expl.	duration crash	duration ratio	increase expl.	decrease crash
FTSE 100	1998-12-16	8.97	1999-07-06	9.13	1999-08-09	9.05	169	145	24	6.04	0.16	-0.08
Gold	2005-12-22	5.54	2006-05-11	5.87	2006-10-03	5.67	204	101	103	0.98	0.33	-0.21
Gold	2006-10-09	5.65	2008-03-17	6.16	2008-09-10	5.86	503	376	127	2.96	0.51	-0.3
Gold	2008-12-10	5.95	2011-09-05	6.73	2013-10-10	6.32	1262	714	548	1.3	0.78	-0.41
Gold	2020-04-03	6.45	2020-08-06	6.67	2020-11-23	6.56	167	90	77	1.17	0.22	-0.11
Hang Seng	1986-10-07	8.76	1987-09-30	9.3	1987-10-23	9.13	274	257	17	15.12	0.55	-0.17
Hang Seng	1992-04-22	9.13	1992-07-16	9.29	1992-08-18	9.21	85	62	23	2.7	0.16	-0.08
Hang Seng	1993-10-06	9.45	1994-01-04	9.86	1994-03-17	9.59	117	65	52	1.25	0.41	-0.26
Hang Seng	2003-10-02	9.68	2004-02-18	9.87	2004-03-19	9.78	122	100	22	4.55	0.19	-0.09
Hang Seng	2006-10-05	10.08	2007-02-22	10.24	2007-03-02	10.15	107	101	6	16.83	0.16	-0.09
Hang Seng	2007-03-19	10.14	2007-10-30	10.62	2008-01-21	10.33	221	162	59	2.75	0.47	-0.29
HRMS	1996-09-13	6.35	1997-02-24	7.04	1998-04-21	6.67	418	117	301	0.39	0.69	-0.37
HRMS	2003-04-29	5.98	2008-01-11	8.22	2009-07-17	7.42	1624	1229	395	3.11	2.24	-0.8
HRMS	2009-07-21	7.45	2009-10-26	7.58	2009-11-04	7.48	77	70	7	10	0.13	-0.1
HRMS	2016-11-10	7.2	2017-01-17	7.36	2017-02-22	7.29	75	49	26	1.88	0.16	-0.08
HRMS	2018-01-08	7.41	2018-04-26	7.57	2018-05-31	7.48	104	79	25	3.16	0.16	-0.09
IDX	2003-11-21	7.31	2004-04-27	7.57	2004-05-07	7.46	121	113	8	14.12	0.26	-0.1
IDX	2004-10-22	7.58	2005-07-28	7.85	2005-08-22	7.74	217	200	17	11.76	0.26	-0.1
IDX	2006-09-14	7.91	2007-12-11	8.49	2008-07-02	8.24	470	324	146	2.22	0.58	-0.25
IDX	2010-09-15	8.48	2010-11-10	8.59	2011-01-07	8.54	83	41	42	0.98	0.11	-0.05
IDX	2011-03-23	8.52	2011-07-27	8.67	2011-08-08	8.58	99	91	8	11.38	0.15	-0.09
IDX	2013-02-14	8.69	2013-05-20	8.81	2013-06-07	8.73	82	68	14	4.86	0.12	-0.08
ITA 125	1992-06-19	5.82	1993-02-08	6.23	1993-07-22	6.03	285	167	118	1.42	0.41	-0.2
ITA 125	1993-07-26	6.03	1994-01-14	6.31	1994-02-18	6.17	150	125	25	5	0.27	-0.14
ITA 125	1999-11-12	6.36	2000-03-03	6.67	2000-04-04	6.5	103	81	22	3.68	0.31	-0.17
ITA 125	2003-10-15	6.34	2004-07-05	6.61	2004-08-03	6.54	210	189	21	9	0.28	-0.07
ITA 125	2004-11-11	6.57	2006-05-09	7	2006-07-12	6.85	435	389	46	8.46	0.43	-0.15
ITA 125	2006-07-14	6.83	2007-10-31	7.25	2008-03-06	7.06	430	339	91	3.73	0.41	-0.19
ITA 125	2015-03-16	7.24	2015-04-13	7.31	2015-06-25	7.28	74	21	53	0.4	0.07	-0.03
KOSPI	1986-01-31	6.23	1989-03-31	7.91	1991-04-11	7.29	1355	826	529	1.56	1.69	-0.63

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Table 17: Financial exuberance characteristics - part III

	start date	start value	peak date	peak value	burst date	burst value	duration	duration expl.	duration crash	duration ratio	increase expl.	decrease crash
KOSPI	1999-05-31	7.03	1999-07-09	7.37	1999-09-28	7.23	87	30	57	0.53	0.34	-0.14
KOSPI	2005-11-02	7.34	2006-01-16	7.49	2006-02-14	7.42	75	54	21	2.57	0.15	-0.07
KOSPI	2007-08-20	7.65	2007-10-31	7.82	2007-11-20	7.72	67	53	14	3.79	0.17	-0.1
MASI	2016-10-28	9.26	2017-01-10	9.47	2017-03-31	9.35	111	53	58	0.91	0.21	-0.12
MASI	2017-06-16	9.38	2017-09-13	9.45	2017-12-04	9.42	122	64	58	1.1	0.08	-0.03
MASI	2018-01-12	9.43	2018-02-02	9.49	2018-05-09	9.45	84	16	68	0.24	0.05	-0.04
Mexico IPC	1989-05-18	8.34	1990-07-25	8.97	1990-08-22	8.78	330	310	20	15.5	0.63	-0.19
Mexico IPC	1991-03-08	8.86	1992-03-26	9.68	1992-08-20	9.36	380	275	105	2.62	0.82	-0.32
Mexico IPC	1993-11-16	9.66	1994-02-08	9.93	1994-03-11	9.79	84	61	23	2.65	0.27	-0.14
Mexico IPC	2003-08-18	9.53	2007-07-06	10.84	2008-12-05	10.27	1385	1015	370	2.74	1.31	-0.57
Mexico IPC	2009-11-04	10.63	2010-04-15	10.75	2010-05-17	10.68	139	117	22	5.32	0.12	-0.07
Mexico IPC	2010-09-09	10.7	2010-12-31	10.85	2011-05-05	10.76	171	82	89	0.92	0.15	-0.09
Mexico IPC	2012-12-03	10.87	2013-01-28	10.95	2013-03-14	10.88	74	41	33	1.24	0.08	-0.07
MOEX	2005-11-02	6.06	2006-05-08	6.59	2006-06-12	6.3	159	134	25	5.36	0.53	-0.29
MOEX	2006-06-19	6.31	2006-08-16	6.51	2006-09-22	6.38	70	43	27	1.59	0.21	-0.14
MOEX	2006-09-26	6.38	2007-12-12	6.66	2008-01-17	6.55	343	317	26	12.19	0.28	-0.11
NASDAQ	1978-07-10	5.2	1978-09-13	5.34	1978-10-18	5.26	73	48	25	1.92	0.14	-0.09
NASDAQ	1982-12-17	5.46	1983-06-24	5.8	1983-10-21	5.62	221	136	85	1.6	0.34	-0.18
NASDAQ	1985-11-21	5.65	1986-07-03	5.93	1986-09-15	5.75	213	161	52	3.1	0.28	-0.18
NASDAQ	1986-09-17	5.75	1986-12-04	5.8	1986-12-22	5.76	69	57	12	4.75	0.05	-0.04
NASDAQ	1987-01-05	5.78	1987-08-26	5.99	1987-10-16	5.86	205	168	37	4.54	0.2	-0.12
NASDAQ	1991-12-27	6.02	1992-02-12	6.14	1992-04-01	6.07	69	34	35	0.97	0.13	-0.08
NASDAQ	1992-11-10	6.09	1993-01-26	6.21	1993-04-23	6.13	119	56	63	0.89	0.11	-0.08
NASDAQ	1993-04-27	6.12	1994-01-31	6.31	1994-06-22	6.18	302	200	102	1.96	0.19	-0.13
NASDAQ	1994-07-07	6.17	1994-09-15	6.26	1994-12-07	6.2	110	51	59	0.86	0.09	-0.06
NASDAQ	1994-12-14	6.18	2000-03-10	7.99	2001-09-03	6.92	1754	1368	386	3.54	1.81	-1.07
NASDAQ	2017-02-10	7.76	2018-08-29	8.08	2018-12-19	7.88	484	404	80	5.05	0.31	-0.2
NASDAQ	2019-01-04	7.89	2020-02-19	8.24	2020-03-11	8.03	309	294	15	19.6	0.35	-0.21
NIFTY 500	2014-05-12	8.46	2014-09-08	8.61	2014-10-15	8.57	113	86	27	3.19	0.15	-0.04
NIFTY 500	2014-10-20	8.57	2015-03-03	8.72	2015-04-23	8.65	134	97	37	2.62	0.15	-0.06

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Table 18: Financial exuberance characteristics - part IV

	start date	start value	peak date	peak value	burst date	burst value	duration	duration expl.	duration crash	duration ratio	increase expl.	decrease crash
NIKKEI 225	1972-03-16	9.07	1973-01-24	9.6	1973-04-24	9.36	289	225	64	3.52	0.53	-0.24
NIKKEI 225	1983-06-22	9.29	1983-10-12	9.35	1983-10-21	9.33	88	81	7	11.57	0.07	-0.02
NIKKEI 225	1983-12-01	9.34	1984-05-04	9.5	1984-05-17	9.42	121	112	9	12.44	0.16	-0.08
NIKKEI 225	1984-10-04	9.44	1989-12-29	10.68	1991-07-05	10.1	1762	1367	395	3.46	1.24	-0.58
NIKKEI 225	2005-10-31	9.55	2006-04-07	9.8	2006-05-22	9.7	146	115	31	3.71	0.26	-0.11
NIKKEI 225	2013-01-30	9.36	2013-05-22	9.69	2013-06-12	9.53	96	81	15	5.4	0.34	-0.16
NIKKEI 225	2015-02-20	9.82	2015-06-24	9.94	2015-08-20	9.9	130	89	41	2.17	0.12	-0.04
OMXH	1993-03-25	7.25	1994-02-04	7.92	1995-03-01	7.78	505	227	278	0.82	0.68	-0.14
OMXH	1995-04-25	7.78	1995-09-14	8.07	1995-10-20	7.9	129	103	26	3.96	0.29	-0.17
OMXH	1996-09-12	7.97	2000-03-06	10.06	2001-07-10	9.12	1259	908	351	2.59	2.1	-0.94
OMXH	2001-11-02	9.15	2002-01-04	9.32	2002-02-19	9.19	78	46	32	1.44	0.17	-0.12
OMXH	2005-12-02	9.15	2006-04-21	9.36	2006-05-17	9.25	119	101	18	5.61	0.21	-0.11
OMXH	2007-04-03	9.39	2007-07-13	9.51	2007-08-09	9.46	93	74	19	3.89	0.13	-0.06
Silver	2008-01-08	2	2008-03-05	2.27	2008-04-28	2.07	80	42	38	1.11	0.27	-0.2
Silver	2010-11-17	2.47	2011-04-29	3.07	2011-09-23	2.66	223	118	105	1.12	0.6	-0.41
SMI	1993-10-05	7.96	1994-01-31	8.19	1994-03-01	8.08	106	85	21	4.05	0.23	-0.11
SMI	1996-02-28	8.22	1996-04-26	8.31	1996-05-31	8.28	68	43	25	1.72	0.09	-0.03
SMI	1996-11-01	8.32	1998-07-21	9.13	1998-10-01	8.7	500	448	52	8.62	0.81	-0.43
SMI	1998-10-09	8.69	1999-01-06	9.04	2000-03-10	8.91	371	64	307	0.21	0.35	-0.12
SMI	2000-03-16	8.95	2000-08-23	9.1	2001-02-15	9.04	241	115	126	0.91	0.16	-0.07
SMI	2005-10-31	8.88	2006-03-22	9.02	2006-05-16	8.98	142	103	39	2.64	0.14	-0.03
S&P TSX	1978-12-19	8.12	1979-10-05	8.39	1979-10-19	8.27	219	209	10	20.9	0.27	-0.12
S&P TSX	1979-11-09	8.25	1980-02-29	8.55	1980-03-26	8.35	99	81	18	4.5	0.3	-0.21
S&P TSX	1980-05-14	8.36	1980-11-28	8.56	1981-01-30	8.47	188	143	45	3.18	0.2	-0.1
S&P TSX	1983-04-20	8.31	1983-09-26	8.4	1983-10-21	8.32	133	114	19	6	0.09	-0.08
S&P TSX	1987-01-27	8.51	1987-08-13	8.69	1987-10-14	8.59	187	143	44	3.25	0.18	-0.1
S&P TSX	1993-05-18	8.41	1994-03-23	8.59	1994-04-15	8.51	239	222	17	13.06	0.19	-0.08
S&P TSX	1996-03-11	8.63	1996-05-31	8.68	1996-07-12	8.64	90	60	30	2	0.06	-0.04
S&P TSX	1996-08-08	8.64	1998-04-22	9.06	1998-08-25	8.83	534	445	89	5	0.42	-0.23
S&P TSX	1999-10-28	8.93	2000-09-01	9.38	2001-02-15	9.13	341	222	119	1.87	0.45	-0.25

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Table 19: Financial exuberance characteristics - part V

	start date	start value	peak date	peak value	burst date	burst value	duration	duration expl.	duration crash	duration ratio	increase expl.	decrease crash
S&P TSX	2005-07-05	9.15	2005-10-03	9.24	2005-10-17	9.19	75	65	10	6.5	0.09	-0.05
S&P TSX	2005-11-02	9.19	2006-04-19	9.34	2006-06-09	9.25	158	121	37	3.27	0.15	-0.09
S&P TSX	2006-10-12	9.28	2007-10-31	9.48	2008-01-18	9.34	332	275	57	4.82	0.2	-0.14
S&P TSX	2008-03-24	9.36	2008-05-20	9.48	2008-07-14	9.38	81	42	39	1.08	0.13	-0.1
S&P 500	1986-02-03	5.28	1986-08-27	5.44	1986-09-10	5.41	158	148	10	14.8	0.17	-0.03
S&P 500	1987-01-05	5.42	1987-08-25	5.68	1987-10-15	5.55	204	167	37	4.51	0.26	-0.13
S&P 500	1995-06-13	5.86	1996-05-24	6.07	1996-07-12	6.02	284	249	35	7.11	0.21	-0.05
S&P 500	1996-07-31	6.01	2000-03-24	6.79	2001-09-27	6.35	1347	953	394	2.42	0.78	-0.45
S&P 500	2001-10-02	6.38	2001-12-05	6.5	2002-04-24	6.41	147	47	100	0.47	0.11	-0.09
SSA	2006-09-08	7.45	2007-10-16	8.7	2008-03-26	8.16	404	288	116	2.48	1.25	-0.54
SSA	2014-11-03	7.83	2015-06-12	8.58	2015-07-07	8.25	177	160	17	9.41	0.75	-0.33
Straits	2006-10-19	8.12	2007-10-11	8.46	2007-11-21	8.3	285	256	29	8.83	0.35	-0.16
TOPIX	1972-02-07	6.43	1973-01-24	7.06	1973-09-04	6.8	412	253	159	1.59	0.63	-0.26
TOPIX	1983-06-16	6.67	1983-09-28	6.74	1983-10-21	6.71	92	75	17	4.41	0.07	-0.02
TOPIX	1983-10-27	6.71	1989-12-18	8.08	1991-11-29	7.5	2112	1603	509	3.15	1.37	-0.58
TOPIX	2005-09-15	7.22	2006-04-07	7.52	2006-06-05	7.4	188	147	41	3.59	0.3	-0.11
TOPIX	2013-01-30	6.88	2013-05-22	7.19	2013-06-06	7.01	92	81	11	7.36	0.31	-0.18
TOPIX	2015-02-18	7.31	2015-08-10	7.43	2015-08-20	7.39	132	124	8	15.5	0.12	-0.04
TUNINDEX	1999-08-23	7.61	2000-07-24	7.85	2000-11-17	7.79	325	241	84	2.87	0.24	-0.06
TUNINDEX	2005-04-05	7.7	2010-09-30	8.88	2013-05-27	8.52	2125	1433	692	2.07	1.18	-0.35
TUNINDEX	2015-04-07	8.6	2015-06-24	8.66	2015-08-13	8.63	93	57	36	1.58	0.07	-0.04
TUNINDEX	2017-06-15	8.6	2017-08-30	8.67	2017-09-26	8.63	74	55	19	2.89	0.06	-0.03
TUNINDEX	2018-02-12	8.66	2018-07-31	8.88	2018-10-10	8.75	173	122	51	2.39	0.22	-0.12

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