Price Discovery with Divergence of Opinion, Institutional Ownership, and Short Selling

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December 2023

This is a preliminary version. Please do not quote or circulate without permission from the author. I thank Heitor Almeida, Charlie Kahn, Dermot Murphy (Discussant, DePaul University Conference 2015) and UIUC Friday Seminar participants for their comments on an earlier version of the paper. I also thank Zhe Liu for excellent programming assistance.

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Abstract

We investigate how competitive dealers set equilibrium bid and ask prices and how those prices adjust towards full information value in a securities market with short selling and divergence of opinion among investors. We model a competitive dealership market in which information is released to traders, who arrive in a probabilistic fashion to trade a single risky security for cash with a market maker. The traders fall into two classes, pessimists and optimists, who disagree about the future value of a tradable security in spite of observing an identical information signal indicating high or low future value of the security. A security has either a high or low institutional ownership. Traders choose to trade (buy, sell, or short) or not trade at all. Derived results from the model suggest that while buy or sell reveals private information, although asymmetrically, no-trade may provide information about the signal from public information depending on the fraction of optimists (pessimists) among incoming traders/investors, which typifies bullish (bearish) market conditions. Empirical results from simulated data confirm asymmetry between bid and ask prices and an inverse 'J' shaped liquidity premium. We incorporate high frequency trading in our model by arbitrarily increasing the rate of trade arrivals and find the bid, ask, and spread functions while retain their shape show reduced cross sectional volatility. Security prices adjust to their full information value at a faster rate when discretionary short selling constraints are low but not zero. However, the speed and the nature of adjustment of security prices depend on the market conditions denoted by the interaction between the fraction of pessimists in the market and the level of institutional ownership in the security.

I. Introduction

One of the basic tenets of market microstructure theory is that trades reveal information about future prices of securities and hence frictions such as no trade intervals and short constraints impede price discovery. Easley and O'Hara (1992a) consider no trade intervals arising due to discrete trading times and find those do not contain information about future values of securities but indicate the absence of relevant information. On the contrary, Diamond and Verrecchia (DV later) (1987) show that short constraints, which include regulatory constraints for example, margin restrictions and market frictions such as the limited availability of short stocks and the rebate fees on securities lending slow price discovery but do not cause overvaluation of stocks. Nevertheless, Miller (1977), Allen et al. (1993), Lim (2011), and Hong and Stein (2003) offer contrary predictions as to whether in a finite horizon model short constraints may lead to overvaluation and bubble that may potentially end in a market crash.

While little empirical research exists on the price impact of no trade intervals, an extensive body of empirical evidence finds consistent excess return and slow price adjustments for shorted stocks, which deepen as short constraints intensify fueling the argument that short constraints lead to bubbles.[1] On the other hand, securities market regulators fear and scattered anecdotal evidence indicates unconstrained shorting leads to grossly inefficient prices and high volatility caused by large drops in securities prices followed by a market wide panic and selloff as observed during bear raids, a term used to indicate aggressive short selling for the purpose of price manipulation.[2]

Those conflicting empirical evidence on the economics of short constraints aside, over the past 30 years, short selling has more than doubled as a proportion of outstanding shares in the market while the US equity market structure and the associated trading environment have changed drastically due to the growing institutionalization of the equity market (Friedman [1996], Gompers and Metrick [2001], Stein [2009]), which impacts multiple aspects of equity trading but disproportionately short selling since institutions dominate both the buy and the sale sides of equity lending market. Boehmer

[1] A huge literature documents the consistent predictability of the returns on short stocks. Ibbotson et al. (2011) reports that for the period 1995-2009, long short equity hedge funds, which target generating returns from both long and short positions earned the highest compounded annual returns (9.9%) as well as alpha (4.79%) among all hedge fund categories; during this period, the return on long only funds. Lynch et al. (2014) report that increase in aggregate short volume leads to a decline in future market return. Empirical evidence also exists that when short constraints are more severe, there is less arbitrage, more overpricing, and greater return predictability (Jones and Lamont 2002; Lamont and Thaler 2003; Ofek, Richardson, and Whitelaw 2004; Ali and Trombley 2006; Bris, Goetzmann, and Zhu 2007; Cohen, Diether, and Malloy 2007;Greenwood 2009). Kraus and Rubin (2003) examine the role of short sale constraint removal by introducing index options on market volatility while Arnold et al (2000), Biais (2001), and Danielson and Sorescu (2001) provide empirical evidence that mitigating short selling constraints by way of introducing options facilitate price discovery. Nevertheless, the continued persistence of short profits indicates shorting as a unique strategy that is not easily replicable.

[2] Cross sectional analyses find short constraints hinder price discovery and result in a systemic overvaluation of those securities. Besides short sale restrictions, trading halts and suspensions are also forms of institutional frictions on trading imposed by exchanges. Lamont (2012) finds short squeeze- a mechanism by which directors constrain short selling, has a similar effect of lowering negative abnormal return for firms. Jain (2013) provides evidence on the effect of short sale constraints in a multi country setting. For evidence on bear raids, please refer to SEC communications regarding Lehman Bros., Bear Stearns, and Adventix. Flash crash is a similar market failure phenomenon in a high frequency trading environment, although its causes are not clear.

et al. (2008) report that 75 (< 2) percent of short sellers are institutions (individuals) while D'Avolio (2002) reports that large custodian banks working on behalf of mostly passive institutional investors, e.g., pension funds and endowments are the primary and most reliable lenders of stocks. Further, D'Avolio (2002), Asquith et al. (2005), Nagel et al. (2009), Anderson et al. (2012), and Lynch et al. (2014) point to multiple trading motives- information, liquidity, speculation, and arbitrage associated with short sellers.

In light of this growing and predominant role of institutions in equity ownership and trading including short selling and the recognition of the diverse trading motives of traders including short sellers, we re-parameterize DV (1987) and Easley and O'Hara (1992a) in order to address the following questions:[3] First, how do no-trade intervals affect price formation in a competitive market making model for securities with heterogeneous traders, institutional ownership, and short selling? Second, in equilibrium, are the adverse effects of short constraints on security prices nuanced and asymmetric with respect to over and undervalued stocks and their respective paths to convergence to equilibrium prices? Third, can unregulated short constraints be self-enforcing- promoting temporary overvaluation while also preventing bubble formation?

We model a competitive dealership market for securities as in DV (1987) and Easley and O'Hara (1992a) with four distinct features- optimist and pessimist traders; securities belong to a high or low institutional ownership category; discretionary short

[3] Gompers and Metrick (2001) note that institutional holding of equities doubled between 1980 and 1985 while Madhavan and Cheng (1994) report that large commonly institutional trades comprise of 57% of trading at NYSE and Hendershott et al. () indicate how institutions engage in sophisticated hi-frequency and algorithmic trading to minimize their cost of trading. Besides the emergence of institutions as the leading investor in the equity market, the availability of large scale trade level data has allowed intensive and robust empirical research to uncover diverse trading motives much beyond just informed vs. uninformed. In particular, the evidence in Anderson et al. (2012) on short selling in family controlled firms suggests an interaction between ownership and motive for shorting and implies that non-information motive drives shorting demand for securities with high institutional ownership most commonly observed in diversely held corporations.

selling; and finally, no trade intervals, which arise due to discrete trade time and limit trading activity including shorts. In our competitive making model, two groups of risk neutral informed traders, pessimists (bears) and optimists (bulls) observe a private information signal and arrive sequentially to buy, sell, or short a single risky security for cash with the market maker; a trader may also choose not to trade. The traders are identically informed but due to their predisposition as pessimists vs. optimists differ in their opinions as to the impact of an information signal on the future value of a security. A security has either a high or low institutional ownership that denotes availability of securities for lending and liquidity. In addition to the information signal, those two unique parameters in our model, divergence of opinion among investors and institutional ownership of securities influence the trading decision (buy, sell, or short) of the trader. On receiving an order, competitive market makers infer the information content in the order and set bid and ask prices as the expected value of the asset conditional on the order (buy, sell, or no trade) received.

Working with the above model, we determine how competitive market makers set equilibrium bid, ask, and spread for securities. Further, we derive how absent regulations, discretionary short selling determined by demand and supply in the equity lending market affects price adjustment. We contend that investor, security, and market characteristics denoted by divergence of opinion (investor), institutional ownership (security), and discrete trading times (market) respectively are responsible for curbing discretionary trading including shorts and limiting information transmission while also preventing bubbles and crashes. This tradeoff is economically efficient since it ensures security prices *almost surely* converge in finite time to their full information values.[4]

Our derived results confirm that although buy and sell reveal good (high future value of a security) and bad (low future value of a security) news respectively, albeit asymmetrically, whether 'no trade' reveals any future prices related information depends on the fraction of pessimists in the market and the extent of institutional ownership in the firm. Barclay and Hendershott (2008) provide compelling empirical evidence of asymmetric price discovery during trading and non-trading (after hours) hours and in short constrained vs. unconstrained securities and markets. Note that in our model institutional ownership denotes liquidity and demand supply constraints in equity lending. Further, the results provide a foundation for the asynchronous time series properties of bid ask prices and spread for high and low price stocks observed by Hausman et al. (1992), Hasbrouck (1991), and Pascual and Pascual-Fuster (2014).

Analyzing simulated data, we find convincing evidence of asymmetric information content in buy vs. sale orders denoted by bid and ask prices and observed in Easley and O'Hara (1992b), Chan and Lakonishok (1993), Koski and Michaely (2000), Chang et al. (2013), and Pascual and Pascual-Fuster (2014) among others, for all discrete prices and a steeply declining inverse 'J' shaped relation between return and relative spread indicating high liquidity premium for only those stocks, which already enjoy the lowest level of spread but negligible (almost zero) premium for all remaining stocks.

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[4] Theoretically speaking, we contend that there may exist an informationally efficient and socially optimal interior point shorting level for each traded security. Although we do not explicitly solve for an optimal shorting level since that is beyond the purview of this paper, it's clearly doable within the present model structure. Further, the concept of information efficiency in this paper is different from that in Rappaport (2005), who considers a fundamental efficiency where future prices cannot be determined from past or current prices. Instead, we consider price efficiency as the fastest path to certain convergence to full information prices. Saffi and Sigurdsson (2010) derive several measures of empirical efficiency from return correlations.

Further, by arbitrarily increasing trading intervals and hence relaxing the discrete trade time constraint on trading activity, we investigate price discovery in a simulated high frequency trading environment. Results from our experiments show that high frequency trading does not change the nature of asymmetry between bids and asks for low and high price stocks; however, the quote functions turn smoother and less volatile for high frequency trades. In a comprehensive study of US listed stocks, Conrad et al. (2015) report pricing efficiency gains due to high frequency trades. The asymmetries between bid and ask functions conspicuously change between low and high institutional ownership stocks. Since institutions typically invest in high liquidity stocks and trade frequently often for the purpose of splitting large trades, the evidence in Conrad et al. (2015) may also be related to the asymmetry in stocks with high vs. low institutional ownership.

Our results also find that while the adjustment of security prices to their respective full information values is certain, its rate and the nature are asymmetric between high and low ending values as empirically observed in Chen and Rhee (2010). Specifically, the rates of convergence are asymmetric for over and undervalued securities and those with high and low institutional ownerships and hence by proxy, securities with high and low short constraints; however, the asymmetry is magnified for low values of institutional ownership. Our theoretical and simulated results confirm that security prices indeed converge to full information prices, albeit asymmetrically and hence refute the notion of short constraints as a catalyst for bubble formation. Instead, our results validate the joint role of heterogeneous investors and institutional ownership in securing market efficiency and stability while averting bubbles and crashes likely from unrestricted shorting. Empirically, Boulton and Braga-Elves (2010), Leece et al. (2012), and Beber and Pagano (2013) find investor, security, and market characteristics as factors in curbing discretionary trading including shorts and limiting information transmission while preventing bubbles and crashes.

Our research extends the limited theoretical literature on price formation in securities markets by blending in market frictions such as discrete trading time and short selling demand and supply constraints with heterogeneous traders and private information in a tractable competitive market making model. Theoretically, we build on Miller (1977) who contends that shorting constraints affect only pessimists and hence no trades due to shorting restrictions lead to overpricing and price bubbles in short constrained stocks. Scheinkman and Xiong (2003) and Hong and Stein (2003) show how overconfidence develops divergence of opinion among investors, which combined with short sales constraints, causes bubbles in security prices. In contrast, DV (1987) introduce no trade friction in a dynamic price discovery model through short sale restrictions, which they show do not bias prices upwards but only slow down the process of private information revelation via trade prices.[5] Yet, Allen et al. (1993) show the existence of price bubbles in a private information model when rational traders have heterogeneous belief about the time when a private information will be realized. Lim (2011) confirms that in a private information model, there may be short term bubbles and that while security price over time reflects its full information value, short constraints hinder the price adjustment process. Empirically, D'Avolio (2000) and Boehmer et al.

[5] In Kyle (1985) and Glosten and Milgrom (1985) time is exogenous to price discovery and hence has no information content. Diamond and Verrecchia (1987) skillfully convert time domain to frequency domain, number of steps required to attain efficiency.

(2008) examine the overall efficiency of the US equity lending market after noting that speculative institutions interested in exploiting transient mispricing are the primary short sellers while Saffi and Sigurdsson (2010) provide evidence on the price efficiency of short supply from a study of 12,000+ shorted securities around the world.

Methodologically, we build on DV (1987) and Easley and O'Hara (1992) extending Lim (2011) in a significant way; however conceptually, our work is most closely related to Chen et al. (2002) in which price formation is subject to both differences of opinion among investors and short sale constraints. Our model predictions are somewhat similar to those in Bai et al. (2008) but differ in many fundamental ways from those of Easley and O'Hara (1992a), Saar (2001), Kraus and Rubin (2003), and Lim (2011). The results of our model yield several equity market design and welfare (prevention of market crashes) implications and are particularly relevant in the aftermath of the securities market collapse and global financial crisis in 2008. Specifically, the results point to the welfare implications of limited and focused short sale regulations in securities and markets with limited liquidity and those in countries where investor heterogeneity and institutional ownership of public equities are low.

I. A Competitive Dealers Market Making Model

The security pricing model in this paper builds upon a sequential competitive market making model with informed and uninformed traders, no trade intervals, and/or short selling as in Easley and O'Hara (1992a) and DV (1987). However, we deviate from the commonplace notion of information asymmetry between informed and uninformed traders and advance the idea in Harris and Raviv (1993) that it's not information *per se* but the divergence of opinion among identically informed investors as to the effect of information is what motivates trading and causes trading volume surge. Baker and Wurgler (2009) and Garfinkel (2007) refer to time varying investor sentiment denoted by the fraction of optimists or pessimists as an indicator of divergence of opinion, which Chen et al. (2002) and Hong and Stein (2003) contend leads to overpricing and finite horizon bubbles in short constrained stocks.[6]

We model orders submitted by identically informed, pessimist (bears) and optimist (bulls) traders to trade a security with high or low institutional holding.[7] We introduce two parameters, μ (1- μ) and θ (1- θ) denoting the proportions of pessimist (optimist) traders and high (low) institutional holding respectively. The parameters μ and θ central to our model are motivated by a growing literature that contends and finds traders' psychology and institutional ownership impact trading including short selling and the consequent price adjustments towards or away from full information prices.[8]

[6] Refer to Hong and Stein (2007) for a literature review on disagreement due to psychological predisposition. Volume and volatility inducing disagreement may also be the effect of 'agreeing to disagree' as in Banerjee and Kremer (2010).
[7] The notion of replacing informed and liquidity traders by pessimists and optimists is attractive for several reasons. First, it's impossible to disentangle information and liquidity motives for trading and thus identify traders exclusively with one motive or the other. On the contrary, traders self-identified as bulls (optimists) and bears (pessimists) are pretty common. For example, during CNBC Fast Money program, Amazon: Bull vs. bear, Wednesday, 10 Sep 2014 | 5:40 PM ET, FM traders Jon Najarian and Dan Nathan put forward their contrary positions regarding how to play their hands with respect to Amazon stocks. Second, this classification allows traders with multiple motives, for example, liquidity or speculation to trade (buy, sell, or short) and hence opens up short selling by informed traders with liquidity and speculation motives (Campbell et al. [1993], Blau and Wade (2010), Engelberg et al. [2014]). Regarding speculation motive behind shorting, the financial press reports indicate multiple hedge funds were complicit in a trading strategy involving naked shorting along with deep out of the money put options with respect to Bear Stearns and Lehman Bros., which aggravated the crisis and contributed to their sudden demise. In addition, please refer to SEC investigation files related to Adventix on a similar issue.
[8] Note the implications of the parameters, μ and θ in the context of a noisy rational expectations model a la Glosten and Milgrom (1985) with short selling and no trade options. In Glosten and Milgrom (1985), an informed trader observing a low (high) signal always sells (buys) and thus absent an uninformed liquidity trader, who buys or sells in an uncorrelated fashion, the informed trader's trade becomes fully revealing. In our model, all traders are informed; they observe the same signal and yet choose different, sometimes contrary trade options- buy, sell, short, or no trade due to disagreement regarding the impact of information on security prices and liquidity conditions associated with a security. Thus security prices are not instantly fully revealing with certainty (probability=1).

Let $\mu \in (0,1)$ be the probability that an incoming informed trader is a pessimist; hence, $(1 - \mu)$ denotes the probability that an incoming trader is an optimist. Note that the probabilities μ and $(1\text{-}\mu)$ are estimates of the fractions of pessimists and optimists traders respectively at the beginning of each trading day. There are finitely many trading intervals during the day when traders trade. We assume the number of traders to be sufficiently large that a trader's exit from the market after each trading interval is not going to alter the fraction of pessimist or optimist traders, μ during the trading day. It is expected that μ is low in a bull market and high in a bear market.

All traders, pessimists and optimists alike are informed and rational. They agree on the high and low payoffs but differ on the priors associated with the high and low payoffs due to their *a priori* disposition as pessimists vs. optimists and use an expectation model to determine the expected value of a security (Hong and Stein [2007]).[9] Specifically, on receiving an information signal about the future payoff related to a security, pessimists (optimists) assign a lower (higher) probability to the realization of a high value signal and conversely, a higher (lower) probability to a low value signal and hence rationally arrive at different expected values and consequently choose different actions for trading.

A traded security has a high or low institutional ownership that denotes its liquidity among other things.[10] Let $\theta \in (0, 1)$ denote the probability that the traded security has a high institutional ownership; hence, $(1\text{-}\theta)$ denotes a security with a low

[9] Hong and Stein (2007) refer to those as disagreement due to heterogeneous priors. They refer to other forms of disagreements as due to limited attention and lagged or slow information.

[10] One could operationalize θ, the probability of high institutional holding in a stock from the fraction of securities in the market which have institutional ownership above any arbitrary cutoff. Dey and Radhakrishna (2015) report that during 1990-91, the mean institutional trading in a stratified sample of NYSE stocks is 18 percent while Madhavan and Cheng (1994) report 57% of equity trades at NYSE are large institutional trades.

institutional ownership. Further, empirical research by D'Avolio (2002) and Boehmer et al. (2008) finds institutional ownership affects both demand and supply of shorts while Nagel (2009), Asquith et al. (2005), and Anderson et al. (2012) have uncovered diverse trading motives of short sellers.[11] A high (low) institutional ownership because of its liquidity and stock lending implications may produce an asymmetric effect on traders' buying, selling, or shorting decisions. Nature chooses whether a security has high or low institutional ownership.

Figure 1 graphically portrays the first trade of a typical trading day in this market. At the beginning of a trading day, nature determines if there is an information event. An information event occurs with probability ε that denotes event uncertainty.[12] The information event may be a firm specific event e.g., earnings announcement as in Kim and Verrecchia (1991) or a broad economy wide event e.g., interest rate change as in Harris and Raviv (1993) with implications for a high or low future value of the security.[13]

[11] D'Avolio (2002) reports that large custodian banks working on behalf of mostly passive institutional investors, e.g., pension funds and endowments, which typically hold more large cap and liquid stocks than other institutions like mutual funds and hedge funds are the primary and most reliable lenders of stocks besides discount brokerage houses, which often pick up a small slice of the stock lending supply. Boehmer et al. (2008) report that 75 (< 2) percent of short sellers are institutions (individuals), which D'Avolio (2002) notes include specialists and market makers (portfolio balancing), derivatives traders (hedging), hedge funds (arbitrage), and speculators (outright shorting). Anderson et al. (2012) report high levels of informed short selling in concentrated family firms but Asquith et al. (2005) and Nagel (2009) find short interests uncorrelated with information. Campbell et al. (1993) find linkages between liquidity motivated short sales and buy and sell order imbalances as the driver of asymmetric price effects whereas Cohen et al. (2009) find evidence of behavioral bias and profit making by short sellers who exhibit short term overreaction and increase trading following positive returns (return chasing). Blau and Wade (2010) look into short selling around earnings announcements and observe that short sellers are not 'informed' and most likely speculators. By contrast, Engelberg et al. (2014) conclude that short sellers do not possess private information but are skilled in processing public information. Easley et al. (1998) make similar observations about the universe of traders and Jain et al. (2006) find intraday short trading volume in the US as 'U' shaped similar to 'smile' observed in market trading volume implying that as a class, short sellers are just typical investors with multiple diverse trading motives.

[12] Event uncertainty, the uncertainty about whether an information event has occurred is a key element in the price discovery process in Easley and O'Hara (1987, 1992). This feature also distinguishes their model from Glosten and Milgrom (1985) in which an information event is presumed to exist. In Dey and Kazemi (2008) where there are three types of traders, informed, liquidity and institutions as information-liquidity traders, event uncertainty becomes redundant in the price discovery process.

[13] Vega (2006) argues that whether information is private or public is irrelevant for stock price reaction; the arrival rates of traders determine the reaction.

If an information event occurs, a signal revealing the future value of a security is released to all traders in the market. The future value of a security is a random variable v with two possible values, 1 or 0 corresponding to a high (good news) or low (bad news) value signal respectively. The signal set also contains a null denoting neither high nor low values. Hence, the signal set, $s \in (L,H,\phi)$ denotes low, high, or no signal respectively. Nature determines whether the future value is high or low. A low value occurs with probability δ that denotes information uncertainty. It follows then that the future value of the security follows a Bernoulli distribution, $v \sim \text{Bernoulli}(\delta)$ with the unconditional expected value and variance of the security as follows:

$E(v) = \delta * 0 + (1-\delta) * 1 = (1-\delta)$ ....1a and

$Var(v) = \delta(1-\delta) = \sigma_v^2$ (say) ....1b.

Following the release of an information signal at the beginning of a trading day, trading continues as during each trading interval within a trading day, an informed optimist or pessimist trader arrives to trade (buy, sell, or short) one unit of a risky security with high or low institutional ownership with a competitive market maker; she may also choose not to trade for strategic reasons, for example, a pessimist (bearish) investor receives a buy signal or a potential short seller faces constraints in the form of limited availability of loanable stocks and high rebate fees and hence decides to forego a trading opportunity.[14] If an information event does not occur, traders may still submit an order

[14] Almazan et al. (2003) report that approximately 70 percent of US mutual funds are explicitly prohibited from shorting by their charters. No trade may also be due to lack of 'relevant' news. Chan (2003) finds asymmetric stock price reaction between portfolios with 'news' and 'no news', where a differential return drift is observed only for those portfolios subject to 'bad news' and that the drift is strongest for illiquid small stocks.

(buy or sell) for other strategic and non-information reasons like momentum, liquidity, and information cascade.[15]

A trading day is divided into an increasing ordered sequence of discrete trade times denoted by the time of the arrival of a trader, $t(n) = t(1)....t(N)$ where $n = 1....N$ denotes the order of traders. The arrival times of traders, $t(n)$ are separated by predetermined intervals of equal length such that each trading interval, the difference between two successive arrivals, $\Delta t = t(n) - t(n-1)$ is just long enough to submit exactly one trade. For example, a 6.5 hours trading day at the NYSE would translate into 390 one-minute trading intervals, with $t(n)$ = 9:30 9:31…15:59 16:00 further implying that with a one minute trading interval, 390 is the highest trade count or maximum number of likely order submissions in a day. Note that Δt denotes the inverse of trading frequency or trade counts. Clearly, as $\Delta t \rightarrow 0$, trading frequency increases without bound approaching high frequency and continuous time trading.[16]

There are finitely many trading intervals within a trading day during which traders opt to trade or not trade. In terms of an incoming trader's feasible actions, she chooses to trade with probability γ and hence does not trade with probability $(1-\gamma)$; sell with probability α, buy with probability β, leaving the probability $(1-\alpha-\beta)$ for her to short. A unique feature of the trading options in this paper is that a trader's probability to short is conditional on the values of $\alpha > 0$ and $\beta > 0$. Conditional on $\gamma > 0$, when $\alpha = 0$ or $\beta = 0$,

[15] Unlike Glosten Milgrom (1985) and Easley and O'Hara (1992a), in our model all traders receive an information signal when there is an information event. When there is no information event, traders pursue non-information reasons e.g. information cascade or liquidity for trade. Note that in Harris and Raviv (1993) traders having the same information may trade differently strictly because of divergence of opinion but not due to information cascade.

[16] Note that holding trade size and time constant, trading frequency and no trade intervals are the respective inversely related frequency and time domains of trades, i.e., as number of trades during any unit time interval increases, 'no trade' intervals decrease.

the probability of shorting turns into *(1-β)* and *(1-α)* respectively; when $\alpha = \beta = 0$, the probability is 1 implying certain shorting.[17]

Competitive market makers face no inventory constraint and set equilibrium bid and ask prices as $b_t^* = E(v_t \mid Sell_t)$ and $a_t^* = E(v \mid Buy_t)$ respectively where $E_t(v \mid Q) = (1 - \delta(Q_t))$ to earn zero profit from each trade based on $\delta(Q)$, the conditional probability of a low value signal given a trade/no trade. Before trading begins, a competitive market maker sets $\delta = \delta^u$ (we drop the u superscript later for notational convenience) as the uninformed prior. During each trading interval, she observes an incoming order from a trader belonging to the set $Q \in (S/SS, B, N)$ but not the information signal and hence updates δ, that is, she computes $\delta(Q)$, the conditional probability of a low signal given an order from the set Q (for notational convenience, we drop the '*t*' subscript) as below:

$$\begin{aligned}\delta(Q) &= \Pr(v=0 \mid Q) \\ &= 1.\Pr(s=L \mid Q) + 0.\Pr(s=H \mid Q) + \delta \Pr(s=\phi \mid Q)\end{aligned} \quad \text{........2a}$$

Further, by Bayes' rule

$$\Pr(s=x \mid Q) = \frac{\Pr(Q \mid s=x)\Pr(s=x)}{\sum_{s \in (L,H,\phi)} \Pr(Q \mid s)\Pr(s)} \quad \text{........2b.}$$

The purpose of the game is to determine if competitive equilibrium prices exist for any/all possible values of the parameters and also how equilibrium prices change with

[17] Note that in Diamond and Verrecchia (1987), investors face different levels of costs of short selling including zero cost. Hence, they assume that without short constraints, a trader will always prefer shorting to any other action including no trade. In our model, however, a trader may choose not to trade with probability *(1-γ)* and hence γ**(1-α-β)* denotes the conditional probability that a trader who chooses to trade, shorts. Evidently in our model, short selling in specific securities is conditioned on the probability of trade and further the conditional probabilities of buy or sell given a trader's intention to trade. Thus a current owner of a security may strategically short instead of selling.

respect to changes in the parameter values, particularly those parameters μ and θ denoting investor heterogeneity (pessimist vs. optimist) and differences in security characteristics (high vs. low institutional ownership) respectively. We derive equilibrium bid ask prices and spread and discuss those results in Section II. Finally, in section III, we derive a few dynamic properties of the bid and ask quotes and the convergence of prices to their full information values as the proportion of short sales varies.

II. Effect of Trades on Equilibrium Bid Ask Prices: Cross sectional Analysis

Before we discuss the equilibrium price formation and the effects of no trade intervals on price discovery, we examine and reconcile the occurrence of no trades between DV (1987) and Easley and O'Hara (1992a). This discussion is important since our model combines elements of both models, which yields contradictory results with respect to the effect of no trades on quotes.

In DV (1987), both informed and uninformed traders may opt out of trading due to multiple reasons. First, a fraction of the universe of traders, $(1-g)$ arbitrarily decide not to trade; second, a fraction of informed traders, (ga) and a fraction of uninformed traders $(g(1-a))$, who wish to sell but do not own the security and face excessive cost of short selling exit the market without trading. In Easley and O'Hara (1992a), on the other hand, only $(1-\varepsilon_S)$ or $(1-\varepsilon_B)$ fraction of uninformed sellers or buyers respectively opt out of trading while the informed traders always trade (buy or sell) based on their private information signal. Consequently, no trades in DV (1987) may be informative whereas in Easley and O'Hara (1992a) only uninformed traders do not trade and hence no trades do not reveal information about the future value of the security, rather they reveal the antecedent condition around an information release.

In the graphical representation of our model (Graph 1), $(1-\gamma) > 0$ fraction of traders may choose to arbitrarily opt out of trading that resembles the model structure in DV (1987); however, by allowing $1 \geq \gamma > 0$ and setting $\gamma = 1$ in nodes 1 and 5 (denoted by two step lines instead of slanted lines), we embrace the model structure in Easley and O'Hara (1992). In essence, by disallowing the no trade option in those two nodes $(\gamma = 1)$, we compel the traders to sell or short, if there is bad news, and buy or short, if there is good news. Since all traders are informed, absent this no trade option, informed traders always trade- buy, sell/short as in Easley and O'Hara (1992). Specifically, a short option allows a pessimist trader, who doesn't own the security to sell short during bad news (node 1) and dodge buying when she doubts a good news signal (node 5). For the purpose of this paper, we mostly follow the model structure in Easley and O'Hara (1992a) but present differences in findings, wherever those arise based on the model structure in DV (1987).

Proposition 1: No information event increases the likelihood of no-trades. The probability of no trade is higher when there is no information event than when an information event exists.

Define $\eta_0 = \Pr(N_t \mid \varepsilon = 0)$ and $\eta_1 = \Pr(N_t \mid \varepsilon = 1)$ respectively as the conditional probability of no trades given no information event exists or an information event that leads to a signal has occurred. Derivations show $\eta_0 > \eta_1$ thus confirming similar findings in Easley and O'Hara (1992a), where ε, the probability of an information event leading to noisy signal (a signal contains a buy, a sale, and a null value implying neither buy nor sell) is a priced risk factor.

Further, the result that the difference between the probability, (η_0/η_1) is inversely related to (1- $\mu\theta$) points to the important role that the correlation, $\rho_{\mu\theta}$ between μ (denotes investor type, pessimist or optimist) and θ (indicates high or low liquidity via high or low institutional ownership) plays in determining the impact of news events on the incidence of trades or no-trades. Specifically, for $\rho_{\mu\theta} > 0$, $1< \eta_0/\eta_1 < \infty$ is increasing; for $\rho_{\mu\theta} < 0$, $1< \eta_0/\eta_1 <1.33$ is increasing for $0 < \mu\theta < 0.25$, decreasing for $0.25 < \mu\theta <1.00$, and has an inflection point $\mu\theta = 0.25$; while for $\rho_{\mu\theta} = 0$, $\eta_0/\eta_1 = 1$. That is, during a bear (bull) market with low (high) μ, a high (low) liquidity stock denoted by high (low) institutional ownership, θ will experience a monotonically increasing likelihood of a no event associated with no trade; however, as market conditions, bull vs. bear market and a security's liquidity, high vs. low go against each other, the likelihood of a no event associated with no trade turns non-linear increasing for $0 < \mu\theta < 0.25$ and decreasing for $0.25 < \mu\theta <1.00$ with an inflection point at $\mu\theta = 0.25$. Note that the inflection point is at $\mu=\theta=0.5$.

As we switch to the model structure in DV (1987) and assume that a proportion of traders, namely $(1-\gamma)>0$ always opt out of trading, we find $\eta_0 = \eta_1$, which implies that an information event is redundant for a trader's decision to trade or opt out of trading. In other words, a fraction of traders, perhaps noise traders arbitrarily choose to opt out of the market. From the graphical representation of the game in Graph 1, it's clear why that is true. In DV (1987) framework, no trades are equally likely irrespective of whether there is an information event or not.

Proposition 2a: Information content in trades. Trades (sell/short or buy) reveal private information and hence impact bid and ask prices; a sale order prompts the market maker to lower the bid while a buy order prods her to move the ask price up. No trades do not reveal information and therefore do not affect bid or ask prices.

Proposition 2b: Implicit cost of short selling (to the short seller) is higher in a bear market (high μ) than in a bull market (low μ).

The market maker observes either an order (buy or sell) or the absence of an order. As she observes an order or its absence thereof, she updates the probability of low value conditional on its status, buy, sale, or no order. The probability of a security's low value conditional on a sale (buy) order is higher (lower) than the unconditional probability of a low value. Thus a sale (buy) order signals bad (good) news and consequently the market maker reduces (increases) the bid (ask) price. A no trade does not alter the probability of low value signal and hence has no effect on prices.

The equilibrium bid and ask prices for a buy and a sale during a trading interval are computed as follows:

$$a_1 = 1 - \delta\left[\frac{\varepsilon(1-\mu)(1-\theta)\gamma\beta+(1-\varepsilon)(1+\mu\theta-\mu)\gamma\beta}{\varepsilon\delta(1-\mu)(1-\theta)\gamma\beta+\varepsilon(1-\delta)\beta(\mu\theta(1-\gamma)+\gamma)+(1-\varepsilon)\gamma\beta(1+\mu\theta-\mu)}\right] \text{......3a}$$

and

$$b_1 = 1 - \delta\left[\frac{\varepsilon\mu\alpha(\theta+(1-\theta)\gamma)+\epsilon\theta(1-\alpha-\beta)(\mu+(1-\mu)\gamma)+(1-\epsilon)\gamma\alpha(\mu+\theta(1-\mu))}{\epsilon\delta\mu\theta\alpha+\varepsilon\mu\theta(1-\alpha-\beta)(1-\delta)+\varepsilon\delta\mu\gamma\alpha(1-\theta)+\varepsilon\delta\theta\gamma(1-\mu)(1-\alpha-\beta)+(1-\varepsilon)\gamma\alpha(\mu+\theta(1-\mu))}\right] \text{....3b}$$

Note that a buy or sale order diverts the bid and the ask prices away from the pre-order price; in particular, the first bid and ask quotes of the day move in opposite directions farther away from the unconditional price, $(1-\delta)$. Easley and O'Hara (1992a) find similar evidence of initial price movements with an incoming sell or buy order; however, unlike in Easley and O'Hara (1992a), where the price movements are symmetric, in the present instance, the movements in the bid and the ask prices are asymmetric with respect to pre-order price level and the direction (buy or sale) of the order. Saar (2001) provides an explanation for this observed asymmetry in the price effect between buy and sell based on institutions' dynamic (tactical!) portfolio rebalancing.

Transitioning to the price effect of no trades, in Easley and O'Hara (1992a) model setup, a 'no trade' prompts no change in the conditional probability of a low value signal and hence the bid and ask prices remain unchanged. Based on the computed conditional probability of bad news, we also find no price update following a no trade based on DV (1987), where $1 > \gamma > 0$.

Our results on the zero price effect of no trades corroborate similar findings in Easley and O'Hara (1992a) but contradict those in DV (1987), in which no trade is perceived as bad news and hence prices are revised downward. Easley and O'Hara (1992a) rationalize that the DV (1987) model structure incentivizes trading by allowing traders with multiple motives to trade (buy, sell, or short) and hence no trades point to hidden or unidentifiable risk implicit in trading. Therefore, in DV (1987), no trades are bad news. In our model, no trades are due to informed traders arbitrarily opting out of trading in a random fashion and hence do not reveal lack of information.

Overall, our results that trades reveal information while no trades do not, find mixed empirical support in Barclay and Hendershott (2008) who compare pre-open and opening prices for NASDAQ stocks and note that "Considering the emphasis that has been placed on trading in the price discovery process, it is surprising that for most stocks the opening price reflects the same amount of information with or without trading." However, they also report asymmetric price discovery between pre-open price and opening prices, albeit after a threshold trading volume. On the contrary, Campbell and Hentschel (1992) document asymmetric impact of public 'news' and 'no news' events on volatility in monthly and quarterly CRSP index returns and confirm that volatility increases during news events. By contrast, Hopewell and Schwartz (1976) find security specific but no market (bull vs. bear) effects due to temporary suspensions of trading at NYSE and Jiang et al. (2010) observe that information flow parameters such as liquidity, depth, and spread of 'information linked' securities are significantly and adversely impacted during NYSE announced halts.

We simulate the parameters of bid and ask functions in equations 1a and 1b above and plot those in Figures 2A-2E with a follow up discussion. We generate random numbers from several discrete uniform distributions $U(0, 1)$ each denoting a specific parameter of the bid and ask functions. Since in our model, the parameters denoting probabilities of an information event and a low value signal ε and δ, respectively for a given value of μ and θ are updated daily, while those denoting probabilities associated with trading actions, γ, α, are β are updated every trading interval, many times during a day, we run the above simulation arbitrarily fixing the values of ε and δ and randomly drawing the other intraday parameters from 100 to 500 intervals, in increments of 100, during a day. In order to allow a strictly positive level of short sales, we set $\alpha+\beta < 1$. We repeat the simulation for 100 times for different values of ε and δ and record the mean bid and ask prices.

Figure 2A plots bid and ask prices as functions of the unconditional pre-trade prices of the security. A visual inspection of the plotted bid and ask price functions establishes the asymmetry between the bid and ask prices based on the conditional probability of a low value signal given a buy or sell order.

Evidently, the asymmetry in bid and ask prices persist over the entire range of security prices, low, middle, and high. The most interesting aspect of the asymmetric bid ask prices is that while ask prices show a concave function over pre-trade prices, which tapers off at the top right corner indicating a slowing down of growth for high price stocks, bids show a convex function over those exact pre-trade prices for which the slope attains its maximum (minimum) value for the high (low) price stocks respectively. Indeed, only the ask prices in our model show a concave price function similar to the transaction price function demonstrated by Kraus and Rubin (2003), while the quasi-concave mid quote function implies embedded stochastic volatility implied in Black Scholes call option price [Hull and White (1987)]. Empirically, such asymmetry in the bid and ask prices is documented in Chan and Lakonishok (1993), Campbell et al. (1993), and Koski and Michaely (2000). Chiyachantanya et al. (2004) find evidence of the bid ask price asymmetry related to differences in liquidity conditions in bull/bear market.

Next, we address quotes formation in a high frequency trading setting. By many estimates, high frequency trading already accounts for a significant and growing proportion of equity trading in many major markets in the world and hence we want to investigate if more trades lead to higher efficiency in security prices. Since in our model, the probabilities of an information event and a low value signal ε and δ, respectively for a given value of μ and θ are updated daily, while the probabilities associated with trading actions, γ, α, are β are updated every trading interval, many times during a day, we run the above simulation arbitrarily fixing the values of ε and δ and randomly drawing the other intraday parameters for 500 to 10,000 intervals, in increments of 500, during a day. By increasing the number of orders/quotes, we introduce high frequency trading in the market. We repeat the simulation for 100 times for different values of ε and δ and record the mean bid and ask prices.

Figure 2B plots bid, ask, and spread against pre-trade price of a security in a high frequency trading environment. Evidently, by comparison with Figure 2A, the bid, ask, and spread functions retain their respective convex, concave, and quadratic (inverse 'U') nature over pre-trade prices; however, the functions appear to have become much smoother. The reason is most likely statistical. Since the simulations are draws from identical independent distributions, the bid and ask functions denote the computed means of each point of those functions. However, based on the law of large numbers, as price changes become more granular, the estimated means get closer to the true means while the sample variance tends towards zero. Thus high levels of trading seem to reduce the variability of bid and ask prices, an indirect measure of spread rather than improving liquidity directly by way of reducing bid ask spread. Intuitively, that is the consequence of the disparity between a random information arrival and a predetermined trade arrival and pricing processes. While in theory, high frequency trades may enable fast incorporation of information into trading, it's not clear whether and how the physical trading process can catch up to that speed and further incorporate the relevant information into pricing for the following reasons. First, information occurs at different points in time and space and thus it is inconceivable how such information would reach different traders precisely at the same time; second, it's unlikely that heterogeneous traders will have similar reaction to the information content while many may prefer to mask their information under different trading strategies, for example, split trading; and finally, for every few informed trading, there may be hundreds/thousands of liquidity trades, which by definition do not contribute to price discovery. Broggard et al. (2011) confirm that algorithmic trades mostly exploit macro news events and limit book imbalances, are liquidity motivated, and further find asymmetric liquidity effect for liquidity demanding vs. liquidity supplying algorithmic trades where improvement in liquidity occurs only for selected large stocks after crossing a threshold trading volume barrier. Hagstromer and Norden (2013) observe market making activities by high frequency traders reduce volatility.

Next, in order to investigate whether the above bid ask price functions are robust with respect to different ranges of values of μ and θ, besides fixing ε and δ, we also fix quartile intervals of μ and θ to generate random numbers from discrete beta distributions along with randomly generated γ, α, are β from discrete uniform distributions for 10,000 trading intervals as above. We repeat the simulations 1000 times for each ε and δ and record the mean bid and ask prices. We plot those bid, ask, mid-quote, and spread functions for separate ranges of μ and θ. Figures 2C and 2D show how the first bid and ask prices of the day are formed for different ranges of μ and θ respectively. Figure 2E plots the bid, ask, mid quote, and spread for different combinations of μ and θ. The most striking observation in Figure 2E is that the ask function changes dramatically for certain combinations of μ and θ; for example, for $\mu<0.25$ and $\theta<0.25$, the ask price function turns into almost a 45^0 line with no visible change in the bid price function.

Having discussed the asymmetry in price effect between high and low price stocks and also bid and ask for those stocks, we now move on to discuss how spread develops in this market, how no trades affect spread, and the relation between spread and return, liquidity premium. First, the bid ask spread is inverted 'U' shaped and closely tracks a volatility function that peaks for midprice stocks confirming the information risk component in both volatility and spread. The respective convex and concave bid and ask functions indicate that market makers cross subsidize execution costs of high and low price stocks charging the highest spreads for midcap stocks confirming evidence of stealth trading and gaping information asymmetry in midcap stocks in Chakravarty (2001).

Second, in our model, no-trade retains the bid and ask at their current price points and as such spread remains unchanged at its pre no trade value. Thus no trade flattens the spread function while a trade widens the spread. Recent empirical studies, for example, Broggard et al. (2014) find improved market quality due to lower bid ask spread associated with high frequency trades, albeit in a relatively small number of stocks, but also point to increased volatility and high trading volume as drivers of market inefficiency and that highly volatile markets are more susceptible to crash.

Third, Figure 3 shows liquidity premium, the inverse return spread relation first hypothesized by Amihud and Mendelson (1986). Compared to an approximately linear inverse relation between return and relative spread proposed by Amihud and Mendelson (1986), we find a non-linear, reverse elongated 'J' relation between return and relative spread, much like intraday spread in a competitive dealership market similar to NASDAQ, in which securities with the lowest relative spread enjoy the highest rate of return. Lin et al (1995) report a similar nonlinear relation between spread and trade size. Such nonlinear relations perhaps justify intense high level of trading and further reduction in spread for selected mid and large cap liquid securities with already low relative spread. Chordia et al. (2011) find evidence of large difference (almost 1/3rd) in spreads of large vs. small stocks following decimalization. Thus high frequency trading concentrated in mostly liquid large cap stocks do little to increase market liquidity and hence large scale inefficiency continues in those markets where there are a disproportionately large number of illiquid and thinly traded securities.

Finally, we discuss the implied price effect of sale vs. short in this market. While the market maker cannot distinguish between a short and a sale, in our model, the probability of short, $(1\text{-}\alpha\text{-}\beta)$ is inversely related to α and β, the probabilities associated with a sale and a buy respectively. By recognizing this inverse relation between the probabilities of short vs. sale and with the help of a few comparative statics, we can determine that for any given value of $\beta = \beta^*$, the price effect of sale vs. short. A testable implication of this result is that the cost of securities lending and the average level of short interest is higher in a bullish (more optimists than pessimists) market than in a bear (more pessimists than optimists) market. Karpoff (1988) tests and finds supporting evidence that short sales are costlier when volume and return are correlated, typically observed in a bullish market.

III. Convergence of Prices to Full Information Value

Until this point, for simplicity of exposition, we have focused only on the first trade or no trade of the day and its impact on the opening bid, ask, and spread. The evolution of intraday quotes and transaction prices (mid quotes) over a trading day, however, requires an order flow during a day and hence it is important to determine the conditional probabilities associated with a random sequence of buy, sell, and no orders. Therefore, as a first step, we theoretically determine how a sequence of trades and no trades alters the conditional probabilities of a low, high, or no signal. Specifically, how and whether no trades affect a future information signal outcome is important, albeit complicated by the fact that we find that a no trade is more likely when there is no information event and therefore no signal, despite the findings that no trades seem to have no effect on security price updates. Those conditional probabilities determine the dynamic properties of bid and ask and consequently the transaction price of a security.

Proposition 3: Dynamic evolution of quotes following no trades.

Proposition 3a: The conditional probability of no signal given a no trade is higher than the conditional probability of no signal given a trade occurs.

Proposition 3b: The forecast conditional probability of a signal or no signal at time t+1 given a history of trades and no trades until t-1 and a no trade at time t varies. The forecast conditional probabilities for high and low signals rise while for no signal, the probability falls. The magnitude dependent on the respective values and the correlation between the parameters, μ (1- μ) fraction of pessimists (optimists) among traders denoting bearish (bullish) market conditions and θ, whether a security has high or low institutional ownership.

Proposition 3a states that an information signal (not information event) is more likely when a trade occurs confirming our earlier related results - 'no trade is more likely when there is no information event.' As before, this finding correlates the likelihood of an information event and that of a trade but goes a step further in separating an information event from a signal and asserting that it's not an event but a signal following from the event is the primary driver for trades in the model. An event causes a signal and hence an information motivated trade is more likely when there is an event; however, a signal contains a null, neither buy nor sell, which neutralizes the information content in a signal and acts as ‘no event’. The higher likelihood of an information motivated trade after an information release is inversely related to $\mu\theta$, the covariance between the measures of divergence of opinion and liquidity.

Proposition 3b states the impact of a no trade preceded by a random sequence of buy, sale, and no trades on the conditional probability of a low, high, or no signal. The relevant proofs in the appendix for the above propositions are based on the conditional probabilities of trade signals at time $t+1$ given a history (quotes until time $t-1$) and no trade at time t. Given a history of trades and no-trades concluding in a no trade at time t, the probability of no information event, a high, or a low signal at time $t+1$ falls; on the contrary, the probability of no information event signal at time $t+1$ increases. Consequently, as the market maker’s belief about the probabilities of low and high signal fall, the bid and ask too fall.

The market maker learns from recent trades and no trades. Note that in proposition 2, the conditional probability of a low signal given the first trade of the day is a no trade remains unchanged as the unconditional probability of bad news. In contrast, the conditional probability of bad news given a no-trade increases during a bull market (low μ) and declines during a bear market (high μ) but only for a stock with given liquidity (fixed θ). Unlike the first no-trade of the day, the market maker learns from the history of trades and no trades and uses no trade as a signal for contrary security (liquidity) and market (bull vs. bear) conditions.

Proposition 3 states that no-trade at time t causes the probability of low signal at $t+1$ to decline during market pessimism and the probability of high signal at $t+1$ to decline during market optimism. Thus bid and ask get closer to the unconditional expected value under different market conditions and thus spread may diminish more in one period that the other due to the asymmetry between price movements due to buy or sell. This result is different from those in Easley and O'Hara (1992) and Saar (2001).

In the remainder of the paper, we focus on the convergence of prices to full information value and investigate how security prices adjust to the full information value, 0 or 1. First, in Proposition 4, we show as in Easley and O'Hara (1992) that indeed prices converge, with probability tending to 1 to full information value; next, we show some simulated sample converging price paths; finally, in Proposition 5, we follow DV (1987) and compute the expected number of steps necessary for prices as functions of the frequency of short sales to adjust to full information value, 0 or 1.

Proposition 4: Convergence to full equilibrium: Prices almost surely converge to full information value, 0 or 1.

The price of the security defined as the conditional expected value, $E(v|Q|s) = 1- \delta(Q|s)$ where $\delta(Q|s)$, the conditional probability of a low signal is bounded by 0 and 1 and therefore $E(v|Q|s)$ is bounded by 0 and 1. Since the information signal, $s \epsilon (L,H,\phi)$ is released once at the beginning of the day and remains in place throughout the day, with every new order, security price, $E(v|Q|s) \rightarrow E(v|Q|s*)$, where s* is the realized value of the signal and $E(v|Q|s*) = 0 \text{ or } 1$, the low and high values respectively. Further, during any trading interval, when there is no trade, $0 < \delta(Q|s*) < 1$ and hence the expected value of the security, $E(v|Q|s*) \geq 0 \text{ or } 1,$ unchanged from the prior trade. Since the discrete signal set, $s \epsilon (L,H,\phi)$ is complete and the probabilities associated with the set sum up to 1, $E(v)$ almost surely (in probability) converges to 1 or 0.

In Easley and O’Hara (1992), the buy and the sell sides are symmetric and thus the rates of convergence from both sides are the same; in Saar (2001), the rate of convergence from the buy and the sell sides are different due to the asymmetry between the buy and the sell sides. In the present model, asymmetry in convergence is due to both buy vs. sell, and optimistic vs. pessimistic market conditions.

Using simulated data, we construct sample paths of price convergence to full information values, 1 and 0, high and low values respectively. Due to the relatively detailed and complicated action set and the conditional probability structure of low value that determines prices, compared to that in Easley and O’Hara (1992b), we need an involved logistics for the simulation to generate the relevant price histories. We do so in the following steps.

First, we decide on the target full information value, 1 or 0. Second, we count the fraction of each action from the action set, (sell/short, buy, no trade) available to traders given the information signal is good or bad denoting the full information value of 1 and 0 respectively. Third, we generate a sequence of 100 actions retaining the theoretical proportion of each action in the action set. Fourth, we generate a set of random numbers from independent discrete uniform distributions to denote the realizations for the parameters of the model. Fifth, using the first set of those random draws including δ, we update the conditional probability measure, $\delta(Q)$ and further use $\delta(Q)$ to generate a sequence of prices corresponding to the random sequence of actions. We stop the sequence when the price reaches 1 or 0, as the case maybe. We repeat the above steps, 1-5, 10,000 times and compute the mean number of steps, which were necessary for the convergence to occur in those 10,000 trials. For robustness, we increased the sequence of actions to 200 and repeated the trials 10,000 times again recording the mean number of steps necessary for convergence to occur.

Figures 4A and 4B provide histograms of the convergence steps for the 10,000 trials for convergence to zero (low value) and one (high value) respectively. The convergence results indicate a slower convergence to low value, 0, than to high value 1.

Proposition 4 and the accompanying simulation results on *almost sure* convergence confirms the notion that a securities market with short opportunities leads to systemic overpricing but rejects bubble formation and market crashes.

IV. Conclusion

We investigate if order time and short sales have any bearing on how prices are formed, and how prices adjust to their full information value in a market with multiple dealers in an order driven market. Price adjustment and convergence properties are of common interest to a wide variety of financial economists. We model a trading system where two groups of traders, pessimists and optimists trade a risky security with a competitive dealer for cash. A security has a high or low institutional ownership. All traders receive a signal indicating a high or low future value of a security; however, because of their predisposition as pessimists or optimists, traders do not agree on the impact of the signal. Traders arrive sequentially and trade (buy, sell, or short) a single risky security with a competitive market maker; a trader may choose to not trade.

The derived results from the model find asymmetric price effects for buy and sell orders at all prices. In particular, plots of simulated bid and ask show that the slope of those functions are directly opposite to each other. Said, differently, while ask prices rise steeply for low price stocks and flatten as they reach high price stocks, bid prices rise most steeply for the high price stocks. No trade intervals may lead to an increase or decrease in bid or ask depending on the fraction of pessimists in the market. Bid ask spread peaks for mid-price stocks and the relation between return and relative spread is nonlinear with the steepest slope observed at the lowest level of relative spread.

The adjustment of security prices to their full information value is asymmetric between low and high values of the security. The speed of adjustment to low value is an increasing (decreasing) function of the probabilities of short (no trade); however, while the speed of adjustment to high value is increasing with respect to the probability of short, it is both increasing/decreasing with respect to the probability of no trade depending on the fraction of pessimists and the level of institutional ownership.

Bibliography

Aitken, Michael, Alex Frino, Michael McCorry, and Peter Swan, 1998, Short sales are almost instantaneously bad news: Evidence from the Australian Stock Exchange, *Journal of Finance*, 53, 2205-23

Anderson, R., D. Reeb, and W. Li, 2012, Family controlled firms and informed trading: Evidence from short sales, *Journal of Finance*, 67, 351-85

Allen F., S. Morris, and A. Postlewaite, 1993, Finite bubbles with short sales constraints and asymmetric information, *Journal of Economic Theory*, 61, 206-29

Almazan, A., M. Carlson, C. B. Keith, and D. Chapman, 2003, Why constrain your mutual fund manager? *Journal of Financial Economics*, 76,

Arnold, Tom, Alexander Butler, Timothy Crack, and Yan Zhang, 2005, The information content of short interest: A natural experiment, *Journal of Business,* 78, 1307-36

Asquith, Paul, and Lisa Meulbroek, 2001, An empirical investigation of short interest, *Journal of Financial Economics*, 74

Asquith, P., P. Pathak, and J. Ritter, 2005, Short interest, institutional ownership, and stock return, *Journal of Financial Economics*, 78, 243-76

Bai, Yang, E. Chang, and J. Wang, 2008, Asset prices under short-sale constraints, *Working paper*

Barberis, N., A. Shleifer, and R. Vishny, 1998, A model of investor sentiment, *Journal of Financial Economics*, 49, 307-43

Barclay, M. and T. Hendershott, 2008, A comparison of trading and non-trading mechanisms for price discovery, *Journal of Empirical Finance*, 15, 839-49

Beber, A. and M. Pagano, 2013, Short selling bans around the world: Evidence from 2007-09 crises, *Journal of Finance*, 68, 343-381

Biais, Bruno, Christophe Bisiere, and Jean-Paul Decamps, 1999, Short sales constraints, liquidity, and price discovery: An empirical analysis on the Paris Bourse, *European Financial Management*, 5, 395-409

Bikchandani, S., D. Hirshleifer, and I. Welch, 2002, Learning from the behavior of others: Conformity, fads, and information cascades, *Journal of Economic Perspectives*,

Boehmer, E., C. Jones, and X. Zhang, 2008, When shorts are informed? *Journal of Finance*, 63, Boulton, T. and M. Braga-Elves, 2010, The skinny on the 2008 naked short sale restrictions, *Journal of Financial Markets*, 13, 397-421

Bris, A., W. Goetzman, and N. Zhu, 2007, Efficiency and the bear: Short sales and markets around the world, *Journal of Finance*, 62, 1029-79

Campbell, J. and L. Hentschel, 1992, No news is good news: An asymmetric model of changing volatility of stock returns, *Journal of Financial Economics*, 31, 281-318

Chan, L., and J. Lakonishok, 1993, Institutional trades and intraday stock price behavior, *Journal of Financial Economics*, 33, 173-99

Chan, W., 2003, Stock price reaction to news and no-news: Drift and reversal after headlines, *Journal of Finance*, 58, 223-60

Chang, E., J. Cheng, and Y. Yu, 2006, Short sales constraints and price discovery: Evidence from the Hong Kong market, *Journal of Finance*, 61,

Chen, Joseph, Harrison Hong, and Jeremy Stein, 2002, Breadth of ownership and stock returns, *Journal of Financial Economics*, 66, 171-205

Chen, H. and V. Singal, 2003, Rise of speculative short sales in price formation: the case of weekend effect, *Journal of Finance*, 58, 685-705

Chiyachantanya, C., P. Jain, C. Jiang, R. Wood, 2004, International evidence on institutional trading behavior and price impact, *Journal of Finance*, 59, 869-98

Chordia, T., A. Sarkar, and A. Subrahmanyam, 2011, Liquidity dynamics and cross-autocorrelations, *Journal of Financial and Quantitative Analysis*, 46, 709-36

Cohen, L., K. Diether, and C. Malloy, 2007, Supply and demand shifts in the shorting market, *Journal of Finance*, 62, 2061-96

D'Avolio, Gene, 2002, The market for borrowing stock, *Journal of Financial Economics*, 66, 271-306

Danielson, Bartley, and Sorin Sorescu, 2001, Why do option introductions depress stock prices? A study of diminishing short sale constraints, *Journal of Financial and Quantitative Analysis*, 36, 451-84

Dechow Patricia, Amy Hutton, Lisa Meulbroek, and Richard Sloan, 2001, Short sellers, fundamental analysis, and stock returns, *Journal of Financial Economics*,

Diamond, Douglas, and Robert Verrecchia, 1987, Constraints on short-selling and asset price adjustment to private information, *Journal of Financial Economics*, 18, 277-311

Diether, Karl, K. Lee, and I. Werner, 2008, Short sale strategies and return predictability, *Review of Financial Studies*, 21,

Duffie, D., N. Garleanu, and L.H. Pedersen, 2002, Securities lending, shorting, and pricing, *Journal of Financial Economics*, 66, 307-339

Easley, David, and Maureen O'Hara, 1992a, Time and the process of security price adjustment, *Journal of Finance*, 47, 577-605

Easley, David, and Maureen O'Hara, 1992b, Adverse selection and large trading volume: The implications for market efficiency, *Journal of Financial and Quantitative Analysis*, 27, 185-208

J. Farenilla, E. Graham and C. McDonald, 2001, Does high short interest lead to under performance," *Journal of Investing*,

Figlewski, Stephen, 1981, The informational effects of restrictions on short sales: Some empirical evidence, *Journal of Financial and Quantitative Analysis*, 16, 463-76

Figlewski, Stephen, and G. Webb, 1993, Options, short sales, and market completeness, *Journal of Finance*, 48, 761-77

Friedman, B., 1996, Economic implications of changing share ownership, *Journal of Portfolio Management*, 22, 59-70

Glosten, Larry, and Paul Milgrom, 1985, Bid, ask, and transaction prices in a specialist market with heterogeneously informed traders, *Journal of Financial Economics*, 14, 71-100

Hasbrouck, Joel, 1991, Measuring the information content of stock trades, *Journal of Finance*, 46, 179-207

Harris, Milton and Artur Raviv, 1993, Differences of opinion make a horse race, *Review of Financial Studies*, 6, 473-506

Hausman, Jerry, A MacKinlay, and Andrew Lo, 1992, An ordered probit analysis of transaction stock prices, *Journal of Financial Economics*, 31, 319-79

Hong, H. and J. Stein, 2003, Differences of opinion, short sales constraints, and market crashes, *Review of Financial Studies*, 16, 487-525

Hu, G., G. Meng, and M. Potter, 2008, Opinion divergence among professional investment managers, *Journal of Business Finance and Accounting*, 35, 679-703

Hull, J. and A. White, 1987, The pricing of options on assets with stochastic volatilities, *Journal of Finance*, 42, 281-300

Jarrow, R., 1980, Heterogeneous expectations, restrictions on short sales, and equilibrium asset prices, *Journal of Finance*, 35, 1105-13

Jones, C. and O. Lamont, 2002, Short sale constraints and stock returns, *Journal of Financial Economics*, 66, 207-39

Kim, O, and Verrecchia, 1991, Trading volume and price reactions to public information, *Journal of Accounting Research*, 29, 302-21

Lim,

Koski, J, and R. Michaely, 2000, Prices, liquidity, and the information content of trades, *Review of Financial Studies*, 13, 659-96

Kraus, A. and Amir Rubin, 2003, The effect of short sales constraint removal on the volatility in the presence of heterogeneous beliefs, *International Review of Finance*, 4, 171-88

Kyle, Albert, 1985, Continuous auctions and insider trading, *Econometrica*, 53, 1315-35

Miller, E., 1977, Risk, uncertainty, and divergence of opinion, *Journal of Finance*, 32, 1151-68

Nagel, Stefan, 2005, Short sales, institutional investors, and the cross section of stock returns, *Journal of Financial Economics*, 78, 277-309

Rubinstein, M. E., 2004, Great moments in financial economics III: Short sales and stock prices, *Journal of Investment Management*, 2,

Saar, Gideon, 2001, Price impact asymmetry of block trades: An institutional trading explanation, *Review of Financial Studies*, 14, 1153-82

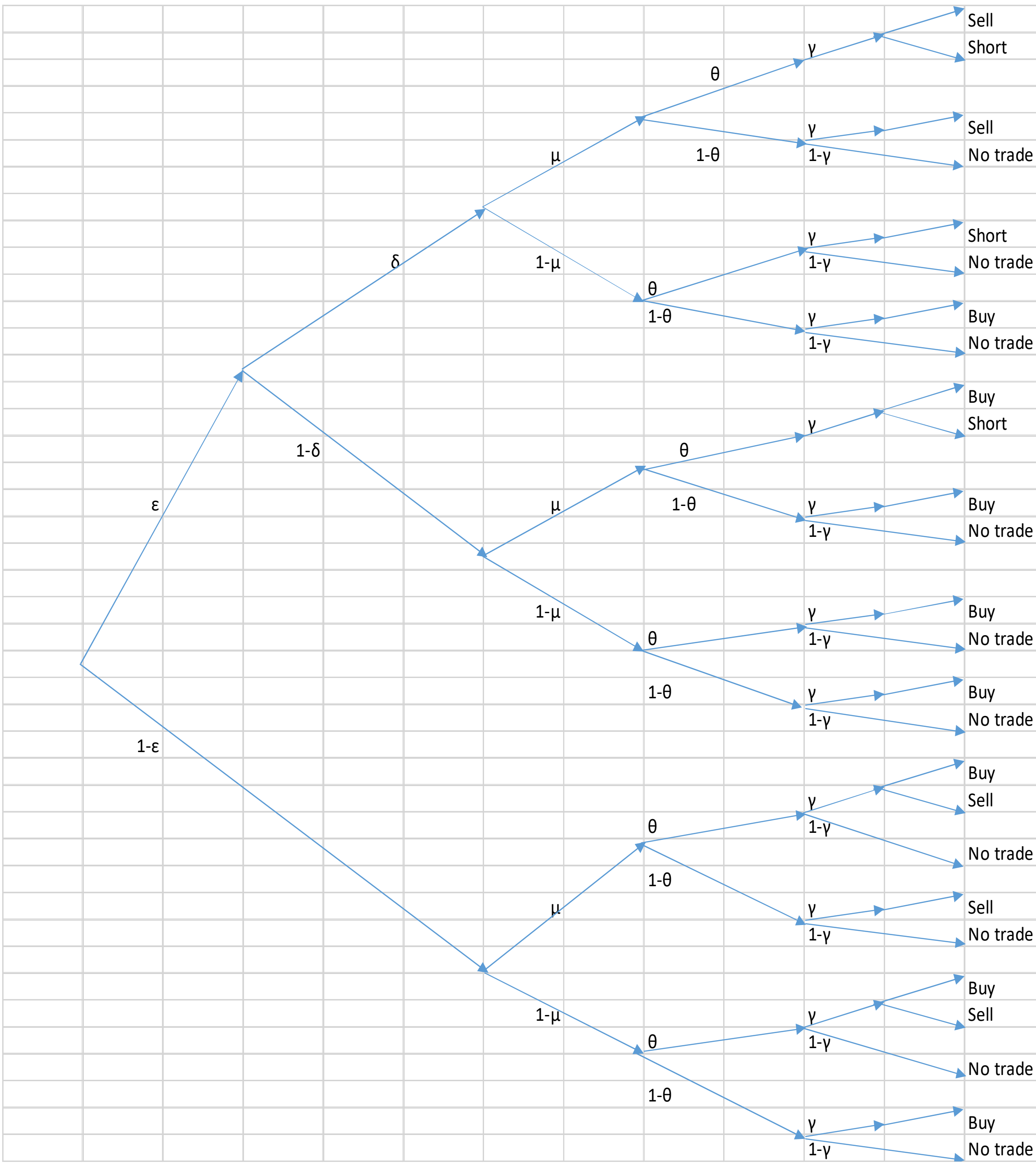
Saffi, 2011, Price efficiency and short selling, 2011, *Review of Financial Studies*, 24, 1153-82

Stein, J., 2009, Sophisticated investors and market efficiency, *Journal of Finance*, 64, 1517-48

Table 1: A summary of notations used in Figure 1 and elsewhere in the paper

Notations	Definition
v	Value of a traded security, 0 with probability δ; 1 with probability $(1-\delta)$
δ	Probability that the value of the security is 0, low value next period
ε	Probability that an information event happens leading to a signal being generated about the value of the security next period
μ	Probability that an incoming trader is a pessimist; the probability that the incoming trader is an optimist is (1- μ)
Θ	Probability that the tradable security has high institutional ownership. This probability represents the fraction of securities with high institutional ownership and therefore (1- θ) denotes the fraction of securities with low institutional ownership. High institutional ownership is associated with high liquidity, low cost of lending, and high short interests and thus may induce more short sales from traders who do not face constraints.
γ	Probability that a trader chooses to trade
α	Probability that a trader who chooses to trade, sells
β	Probability that a trader who chooses to trade, buys
(1-α-β)	Probability that a trader short sells
$t = 1, 2.., T$	Time of trade or quote denoting a trading interval within a day
S	Signal set: includes L, H, and ϕ denoting low, high, and no signal respectively

Figure 1: Securities market trading game with heterogeneous traders. Competitive market makers set bid ask prices at expected value conditional on buy or sell orders.



A trade occurs sequentially as follows. An information event occurs with probability ε. In case there is an information event, bad news are likely with probability δ. An incoming trader is likely to be a pessimist with a probability of μ while she is likely to be an optimist with a probability of (1-μ). The probability that a tradable stock has high (low) institutional ownership is θ (1-θ). A trader trades with probability γ; hence, the probability of no trade is (1-γ). A trader sells with probability α, buys with probability β, and hence shorts with probability (1-α-β).

Appendix

Proof of Proposition 1:

Define $\eta_0 = \Pr(N_t \mid \varepsilon = 0)$ and $\eta_1 = \Pr(N_t \mid \varepsilon = 1)$. Derivation finds $\eta_0 = (1-\gamma)$ and $\eta_1 = (1-\gamma)(1-\mu\theta)$. Since all parameters are strictly positive and $\mu < 1$, $\theta < 1$, $\eta_0 > \eta_1$.

Solution for the limiting case when $1 > \gamma > 0$ based on the model structure in DV (1987) finds $\eta_0 = \eta_1 = (1-\gamma)$.

Proofs of Corollaries1-2 and Proposition 2:

Corollary 1: Conditional probability of a low signal,

a) $Pr(s=L|S_1) > \delta^u$
b) $Pr(s=L|B_1) < \delta^u$
c) $Pr(s=L|N_1) = \delta^u$

By applying Bayes' rule, the conditional probability of a low value signal given a trade (buy or sell) or no trade is computed as follows. For a sale/short-sale, the probability of low value conditional on a sale order, $\delta\,(S_1)$,

$\delta\,(S_1) > \delta^u$ » $\varepsilon(1-\delta)\,[\mu\theta\alpha + \mu(1-\theta)\,\gamma\alpha + (1-\mu)\theta)\gamma(1-\alpha-\beta)] > 0$. Since all parameters are strictly positive, $\delta\,(S_1) > \delta^u$.

Similarly, the probability of low value conditional on a buy order, $\delta\,(B_1)$,

$\delta\,(B_1) < \delta^u$ » $-\varepsilon(1-\delta)\,\beta\,[(1-\mu)\theta\gamma + \mu\theta + \mu(1-\theta)\gamma] > 0$. Since all parameters are strictly positive, $\delta\,(B_1) < \delta^u$.

Further, the probability of low value conditional on a no order, $\delta\,(N_1) = \delta^u$.

For the special case based on the model structure in DV (1987):

$$Pr(s=L|N_1) = \delta^u$$

Finally, note that $\varepsilon\delta\left(1-\delta\right) = \Omega$ (say) is the conditional variance and hence

$\frac{\partial s_1}{\partial \Omega} > 0, \frac{\partial s_1}{\partial \alpha} > 0$, and $\frac{\partial s_1^2}{\partial \Omega \partial \alpha} > 0$.

Proposition 2: Bid or Ask offers

The market maker's bid or ask quote for the opening trade, a sale or a buy order, is computed as:

$b_1 = E(v|sell) = 1 - \delta(S_1)$ and $a_1 = E(v|buy) = 1 - \delta(B_1)$. It follows then

$$a_1 = 1 - \delta\left[\frac{\varepsilon(1-\mu)(1-\theta)\gamma\beta+(1-\varepsilon)(1+\mu\theta-\mu)\gamma\beta}{\varepsilon\delta(1-\mu)(1-\theta)\gamma\beta+\varepsilon(1-\delta)\beta(\mu\theta(1-\gamma)+\gamma)+(1-\varepsilon)\gamma\beta(1+\mu\theta-\mu)}\right]$$

and

$$b_1 = 1 - \delta\left[\frac{\varepsilon\mu\alpha(\theta+(1-\theta)\gamma)+\epsilon\theta(1-\alpha-\beta)(\mu+(1-\mu)\gamma)+(1-\epsilon)\gamma\alpha(\mu+\theta(1-\mu))}{\epsilon\delta\mu\theta\alpha+\varepsilon\mu\theta(1-\alpha-\beta)(1-\delta)+\varepsilon\delta\mu\gamma\alpha(1-\theta)+\varepsilon\delta\theta\gamma(1-\mu)(1-\alpha-\beta)+(1-\varepsilon)\gamma\alpha(\mu+\theta(1-\mu))}\right]$$

The opening spread for the first round trip, a buy and a sell orders,

Proof of Proposition 3:

Let the conditional probabilities of a low, high, or no-signal conditional on a series of sale/short, buy, and no-trade be defined as follows:

$$\rho_{0,t} = \Pr(s = \phi \mid Q^{t-1})$$
$$\rho_{L,t} = \Pr\left(s = L \mid Q^{t-1}\right)$$
$$\rho_{H,t} = \Pr\left(s = H \mid Q^{t-1}\right)$$

Derivation finds

$$Pr(s = \phi \mid N_t) = \frac{1}{2-\mu\theta} > Pr(s \neq \phi \mid N_t) = \frac{1-\mu\theta}{2-\mu\theta}$$

Further, by extending and applying Bayes' law, the forecast probabilities of a low, high or no-signal conditional on a history, Q^{t-1} and N_t, no trade at time t are computed as follows:

$$\rho_{o,t+1} = \frac{\rho_{o,t}}{\rho_{o,t}+(1-\mu\theta)(1-\rho_{o,t})} > \rho_{o,t}$$
$$\rho_{L,t+1} = \frac{\rho_{L,t(1-\mu\theta)}}{\rho_{o,t}+(1-\mu\theta)(1-\rho_{o,t})} < \rho_{o,t}$$

$$\rho_{H,t+1} = \frac{\rho_{H,t(1-\mu\theta)}}{\rho_{o,t}+(1-\mu\theta)(1-\rho_{o,t})} < \rho_{o,t}$$

Proof of Proposition 4:

Consider the following probability density function (PDF) of v, the security value based on the discrete domain values of the signal, *s ϵ (L,H,ϕ).*

$$v = \begin{cases} 1 \text{ if } s = H \\ 0 \text{ if } s = L \\ 1 < v < 0 \text{ if } s = \phi \end{cases}$$

The price of the security defined as the conditional expected value, *E(v|Q|s) = 1- δ(Q|s)* where *δ(Q|s),* the conditional probability of a low signal is bounded by 0 and 1 and therefore *E(v|Q|s)* is bounded by 0 and 1. Since the random event, in this case the signal set, is discrete, mutually exclusive and exhaustive, the conditional probability associated with each outcome, s ∈ (H, L, ϕ) is bounded by [0,1] and the probabilities sum up to 1. Hence, by standard Bayesian results, E(*v|s)* ∈ [0,1] *almost surely* converges (Sveshnikov, 1968).

Proof of Proposition 5:

The details of this proof follow a similar proof in Diamond and Verecchhia (1987). We first compute the expected number of periods until the price of a security reaches its full information price, either 0 or 1. In fact though, instead of 0 and 1, we work with two hypothetical boundaries $\log(\Phi)$, and $\log(\Psi)$ where $\Psi > \Phi$. The threshold $\log(\Phi)$ and $\log(\Psi)$ are the possible values of the posterior log likelihood ratios and is similar to the Wald's sequential ratio test, $\log\left(\frac{p_t}{1-p_t}\right)$ for the hypothesis *v=0* against *v=1*, where p_t is the price or conditional expectation of *v* given low value of the security.

Let $\tilde{N}$ be a random variable denoting the number of periods until the price first reaches either the lower boundary $\log(\Phi)$ or the upper boundary $\log(\Psi)$.

$$E\left(\tilde{N}\right)=\frac{E\left[Log\left(\tilde{\Lambda}_N\right)\right]}{E\left[\tilde{Z}\right]}\text{ where}$$

$$\Lambda_N=\frac{p_n}{1-p_n}\text{ and }Z^A=\log\left(\frac{q_1^A}{q_0^A}\right).$$

Further, we define the expected number of periods till the price reaches a boundary, high or low true value of the security, as follows:

$$\tilde{N}_0=E\left(\tilde{N}\mid v=0\right)=\frac{\left(\frac{1-\Phi}{\Psi-\Phi}\right)\log(\Psi)+\left(\frac{\Psi-1}{\Psi-\Phi}\right)\log(\Phi)}{\sum_{A\in(Q)}q_0^A\log\left(\frac{q_1^A}{q_0^A}\right)}\text{ and}$$

$$\tilde{N}_1=E\left(\tilde{N}\mid v=1\right)=\frac{\Psi\left(\frac{1-\Phi}{\Psi-\Phi}\right)\log(\Psi)+\Phi\left(\frac{\Psi-1}{\Psi-\Phi}\right)\log(\Phi)}{\sum_{A\in(Q)}q_1^A\log\left(\frac{q_1^A}{q_0^A}\right)}.$$

Since the numerators are fixed, we consider the probability of short-sell *(1-α-β)* indirectly via the effect of α and β on the denominator. Let the denominators be called π_0 and π_1 corresponding with N_0 and N_1 respectively. We derive explicitly the denominators as follows:

$$\pi_0=\sum_{A\in Q}q_0^A\,log\left(\frac{q_1^A}{q_0^A}\right)=q_0^B\,ln(1+y)-q_0^S\,ln(1+x)+q_0^N\,ln(z)\text{ and}$$

$$\pi_1=\sum_{A\in Q}q_1^A\,log\left(\frac{q_1^A}{q_0^A}\right)=(1+y)q_0^B\,ln(1+y)-q_0^S\,ln(1+x)/(1+x)+zq_0^N\,ln(z)$$

Where,

$$\frac{q_1^B}{q_0^B}=\frac{\mu\theta\gamma\beta+\mu(1-\theta)\gamma\beta+(1-\mu)\theta\gamma\beta}{(1-\mu)(1-\theta)\gamma\beta}+1=y+1$$

$$\frac{q_0^S}{q_1^S} = \frac{\mu\theta\alpha+\mu(1-\theta)\gamma\alpha+(1-\mu)\theta\gamma(1-\alpha-\beta)}{\mu\theta(1-\alpha-\beta)} + 1 = x+1$$

$$\frac{q_1^N}{q_0^N} = \frac{(1-\mu\theta)(1-\gamma)}{(1-\mu\theta)(1-\gamma)} = 1$$

Our interest is to show the effect of *(1-α-β)* conditional on $\gamma > 0$ on N_0 and N_1 and therefore we postulate the following relationships in terms of π_0 and π_1, the absolute rates of convergence to low and high values respectively.

Further to determine any asymmetric effect of $\beta|\gamma_{>0}$ on the relative convergence rate, $R(.) = -\frac{\pi_1}{\pi_0}$ we need to show that $\frac{\partial^2\pi_0}{\partial\gamma\partial\beta} < 0$ and $\frac{\partial^2\pi_1}{\partial\gamma\partial\beta} > 0$.

Analytical solutions find the signs of the derivatives as follows.

$$\frac{\partial\pi_0}{\partial(1-\alpha-\beta)} = -\left\{\theta\, ln(1+x) + q_1^s\left(\frac{1 - q_0^s\mu^2\theta(1-\alpha-\beta)}{\mu(1-\alpha-\beta)}\right)\right\} < 0$$

$$\frac{\partial\pi_1}{\partial(1-\alpha-\beta)} =$$

$$-\left\{\theta\, ln(1+x)\,(1+x)^{-1} + q_1^s\left(\frac{1 - q_0^s\mu^2\theta(1-\alpha-\beta)}{\mu(1-\alpha-\beta)}\right)\left(1 - \frac{1}{(1+x)}\right)\right\} < 0$$

The signs of the above derivatives translate into $\frac{\partial N_0}{\partial(1-\alpha-\beta)} > 0$ and $\frac{\partial N_1}{\partial(1-\alpha-\beta)} < 0$ respectively. And finally, using L'Hospital's rule, $\frac{\partial R(.)}{\partial(1-\alpha-\beta)} > 0$.

Figure 2A: Bid, Ask, Mid-quote, and Spread from moderate trading interspersed with no trade intervals. The simulations data are the means of 100 random draws from discrete uniform distributions between 0 and 1 for each parameter of the bid (b1) and ask (a1) prices.

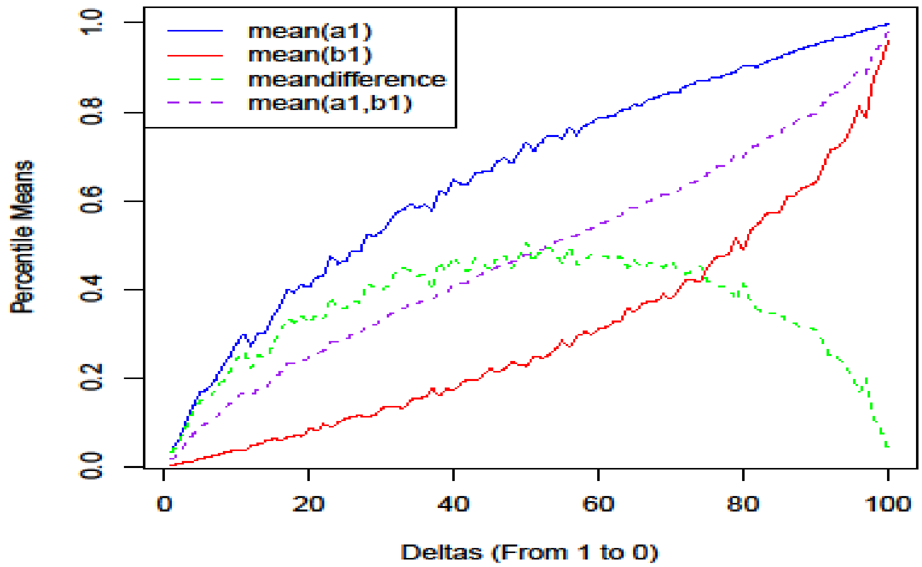


Figure 2B: Bid, Ask, and Spread as functions of pre-trade prices with high trading, low no trade intervals. The simulations data are the means of 1000 random draws from discrete uniform distributions between 0 and 1 for each parameter of the bid (b1) and ask (a1) prices.

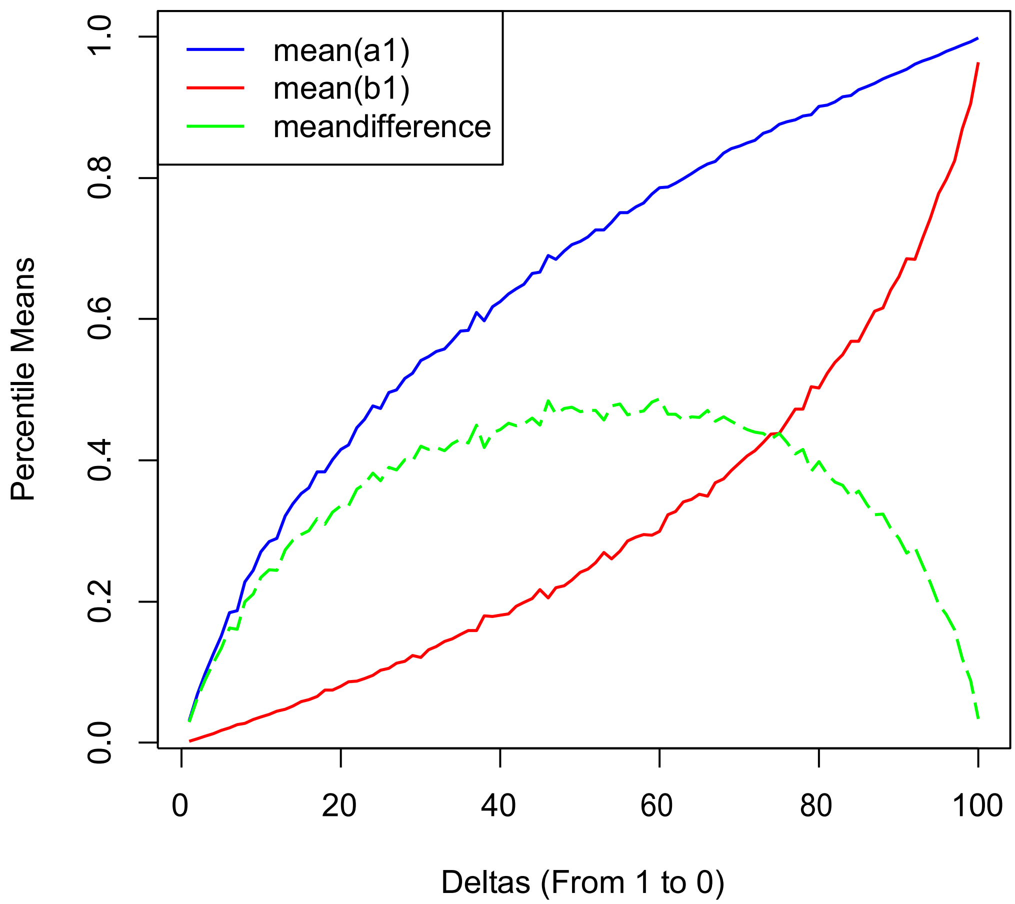
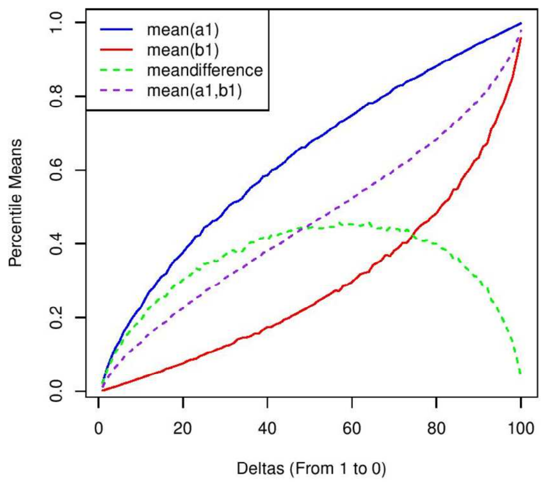
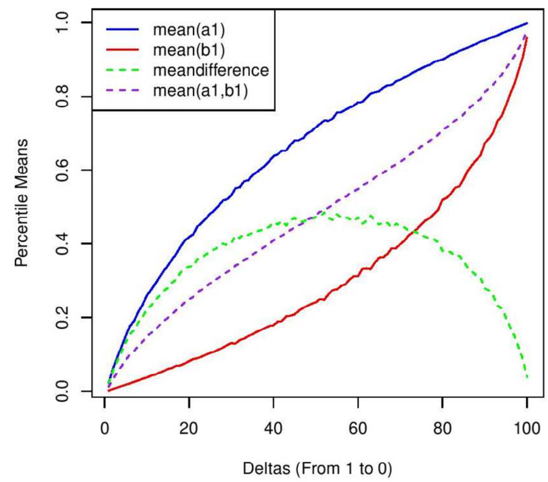


Figure 2C: Bid, Ask, Mid quote, and Spread as functions of the unconditional price for different values of μ, fraction of pessimists among traders. The simulations data are the means of 1000 x 10,000 sets of the parameters of the pricing model.

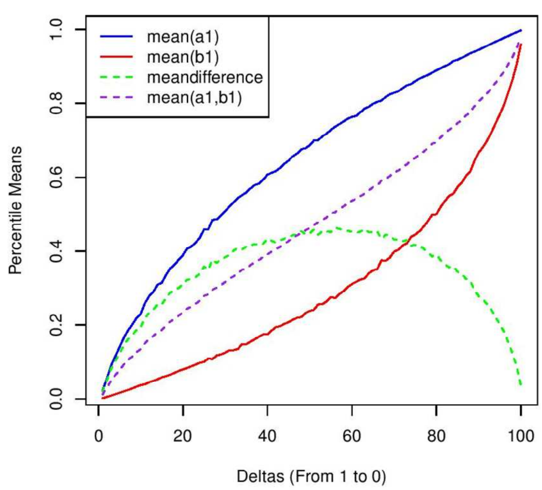
Mu: 0 – 0.25



Mu: 0.5 – 0.75



Mu: 0.25 – 0.5



Mu: 0.75 – 0.1

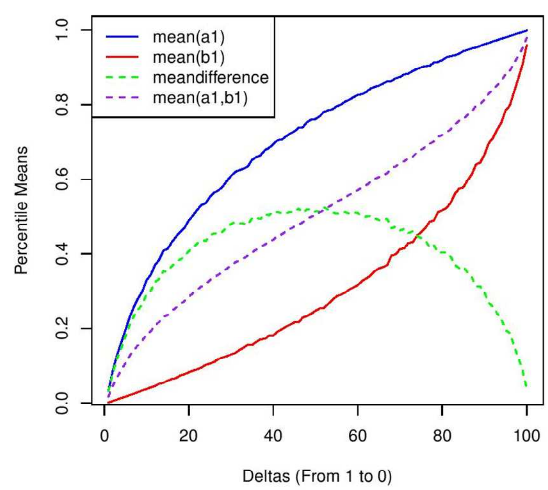
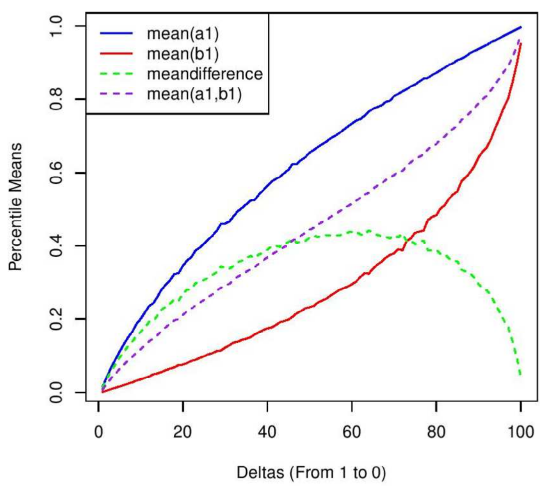
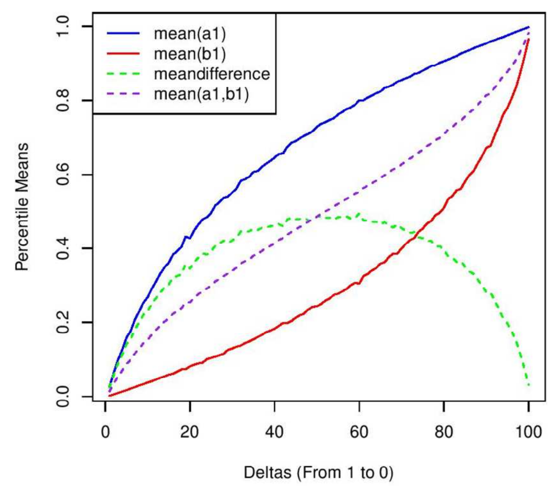


Figure 2D: Bid, Ask, Mid quote, and Spread as functions of the unconditional price for different values of θ, proportion of institutional ownership. The simulations data are the means of 1000 x 10,000 sets of the parameters of the pricing model.

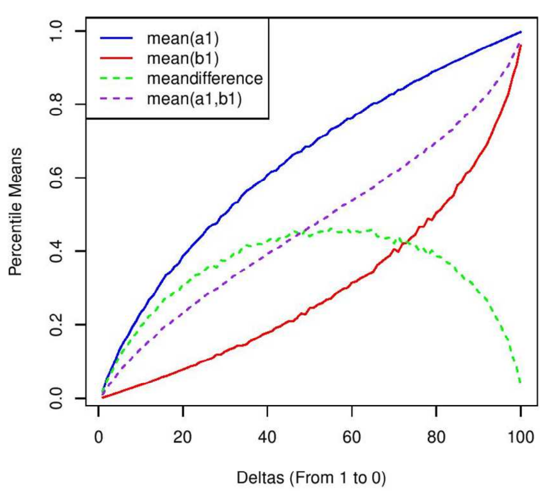
Theta: 0 – 0.25



Theta: 0.5 – 0.75



Theta: 0.25 – 0.5



Theta: 0.75 – 1

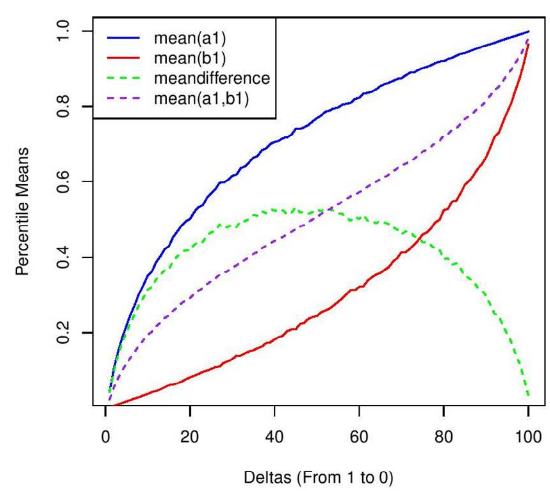
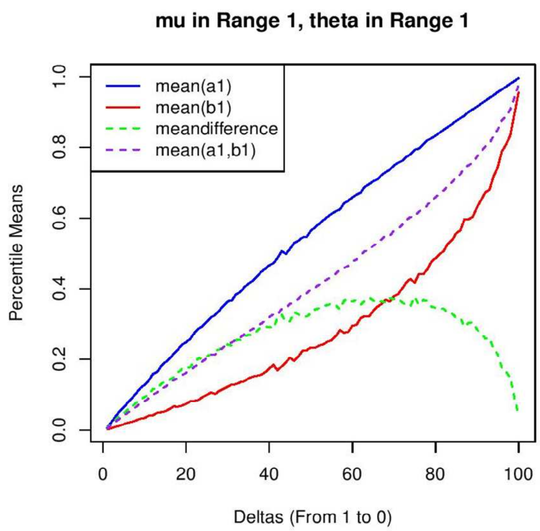
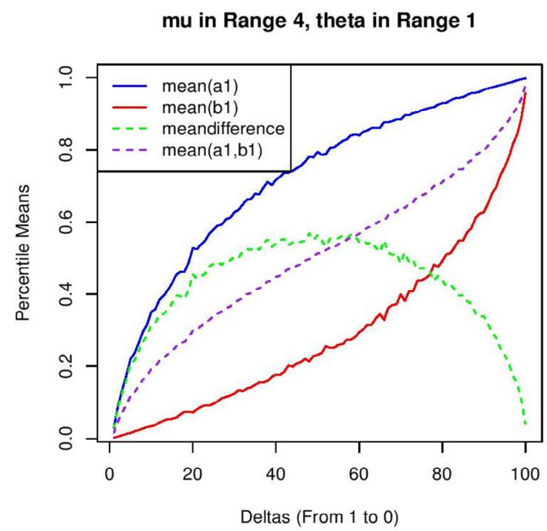


Figure 2E: Bid, Ask, Mid quote, and Spread as functions of the unconditional price for different combination of values of μ and θ, proportions of pessimists and institutional ownership respectively. The simulations data are the means of 1000 x 10,000 sets of the parameters of the pricing model.

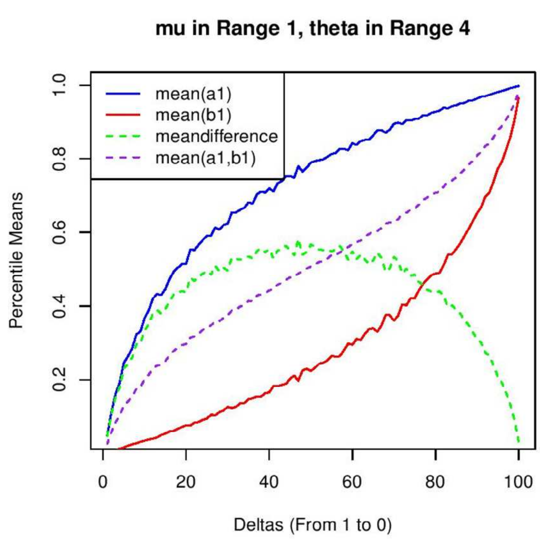
Mu: 0 – 0.25; Theta: 0 – 0.25



Mu: 0.75 – 1; Theta: 0 – 0.25



Mu: 0 – 0.25; Theta: 0.75 – 1



Mu: 0.75 – 1; Theta: 0.75 – 1

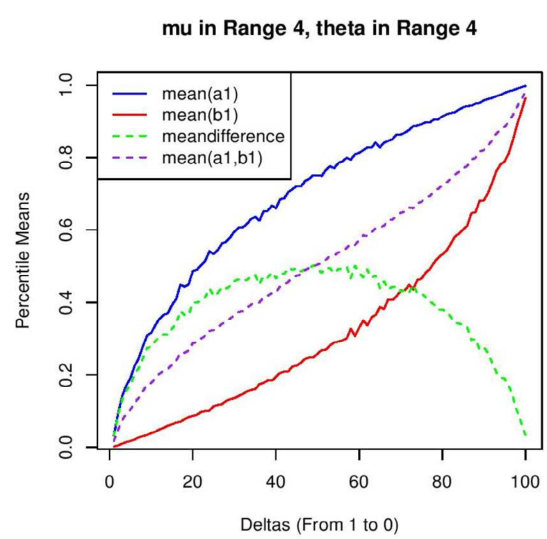


Figure 3: Return vs. relative spread based on high trading low no trade intervals simulated bid ask prices. The simulations data are the means of 100 random numbers from discrete uniform distributions between 0 and 1 for each parameter of the bid (b1) and ask (a1) prices.

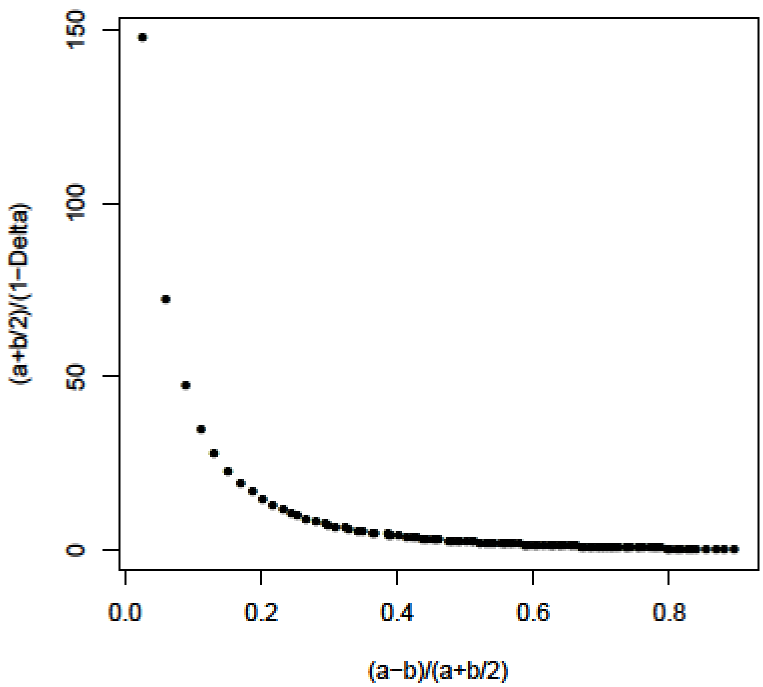


Figure 4A: Convergence to '0' statistics

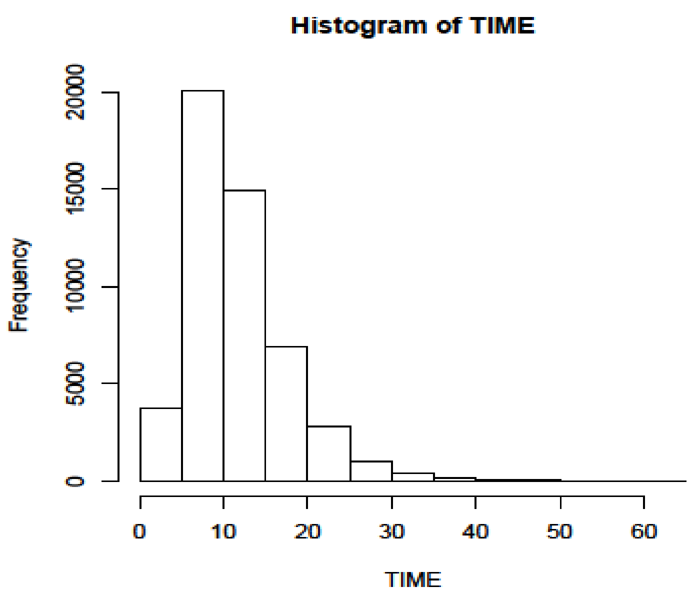


Figure 4B: Convergence to '1' statistics

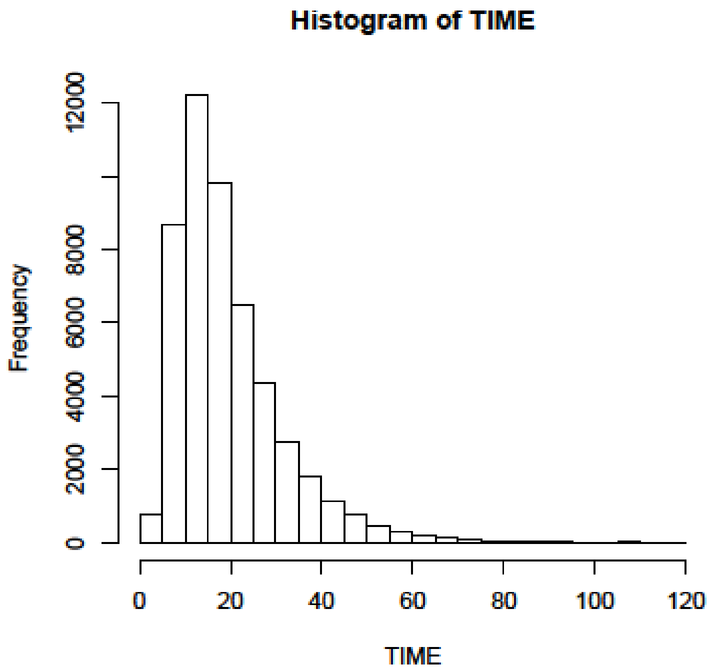


Figure 4C: Ratios of mean convergence to ‘0’ and ‘1’ for quartiles of μ and θ