2. On Recursive Aspects of Quantum Dynamics

A great research challenge is the search for the underlying principles that govern the behaviour of the Universe, from the extremely small quantum particles to the immense objects of the Cosmos. I have just finished reading the excellent and enormously stimulating book *The Road to Reality* by Sir Roger Penrose (2004), Emeritus Rouse Ball Professor of Mathematics at the Mathematical Institute, University of Oxford, and was intrigued by Figure 22.1 of his book, a copy of which is shown here in Figure 1. In the plot, “The time-evolution of the state ψ for a physical system, according to the accepted tenets of quantum mechanics, alternates between two completely different procedures: unitary Schrödinger evolution **U** (continuous and deterministic); and ‘state reduction’ **R** (discontinuous, probabilistic)”. The state vector, in this case, chatacterizes the ‘wave function’, as shown in Figure 21.5 of Professor Penrose's book and copied in Figure 2, whose evolution is described by the deterministic Schrödinger wave equation, until a measurement is performed on the system. For interest, the Schrödinger equation takes the following, deceptively simple, form:

$$i\hbar\frac{\partial\psi}{\partial t} = \mathcal{H}\psi$$

where the lower case *h* with a bar is Dirac's form of Planck's constant and the upper case script *H* is the Hamiltonian. This may well be unfamiliar to the reader and is shown here merely to illustrate the kind of mathematics involved: indeed, if I have managed to excite anyone into knowing more about this subject, I strongly urge them to read Penrose's challenging but beautifully crafted and illustrated book.

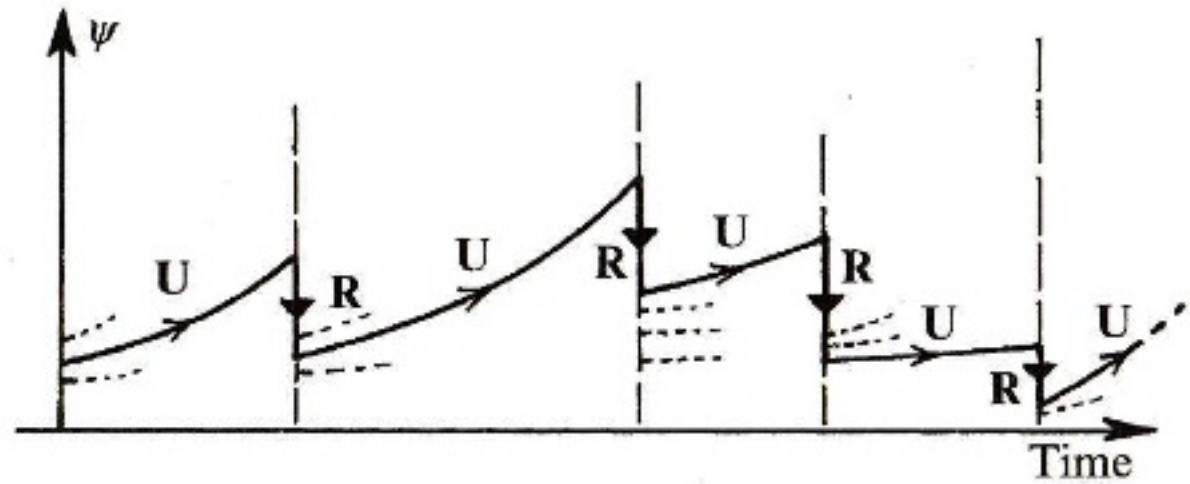

Figure 1. Time evolution of the wave function state (from Penrose, 2004)

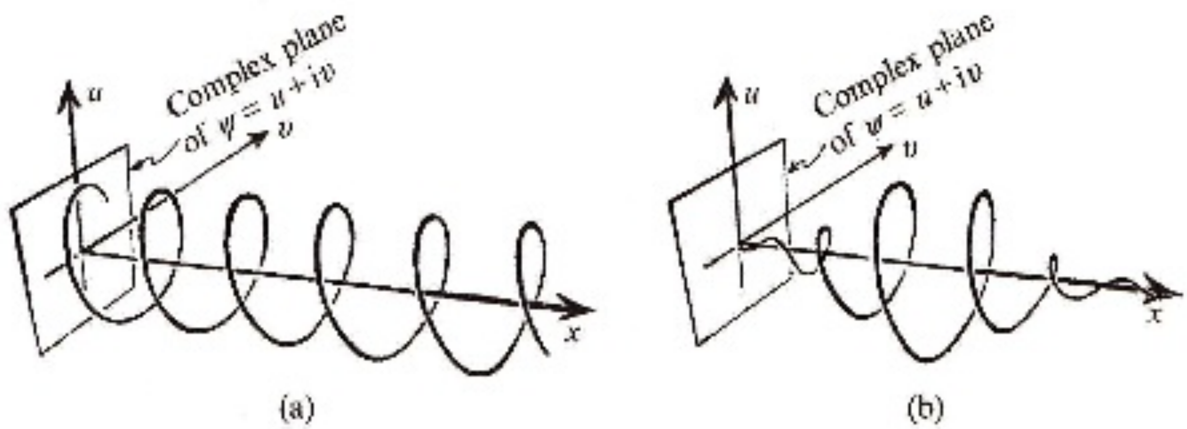

Figure 2. The quantum particle wave function as a complex function of position *x* : (a) the momentum state, depicted as a corkscrew; (eigenfunction of momentum *p*); (b) a wave packet (from Penrose, 2004).

This famous Schrödinger equation is actually linear but complex valued, the latter arising from the presence of the complex number on the left hand side of the equation (represented here by *i* rather than the *j* used in most of my publications), which is associated with the obvious periodicity of the wave function shown in Figure 2 (see the discussion in section of Appendix in Young, 2011, on operators and their interpretation in frequency response terms).

Within quantum mechanics, the measurement process is described mathematically in an entirely different manner from the Schrödinger evolution and such measurements (e.g. of position and momentum) lead to the ‘jumps’ seen at the vertical lines marked **R** in Figure 1; this represents a complex process that quantum physicists refer to as ‘state vector reduction’ or the ‘collapse of the wave function’. The result of this collapse is that the state is no longer deterministic, it jumps to some eigenstate of an operator **Q** (called an ‘observable’) which is defined probabilistically (shown in the plot as the dashed lines). Here, the jump is uncertain but the associated probabilities are specified by a set of rules that are defined precisely by the associated quantum mathematics.

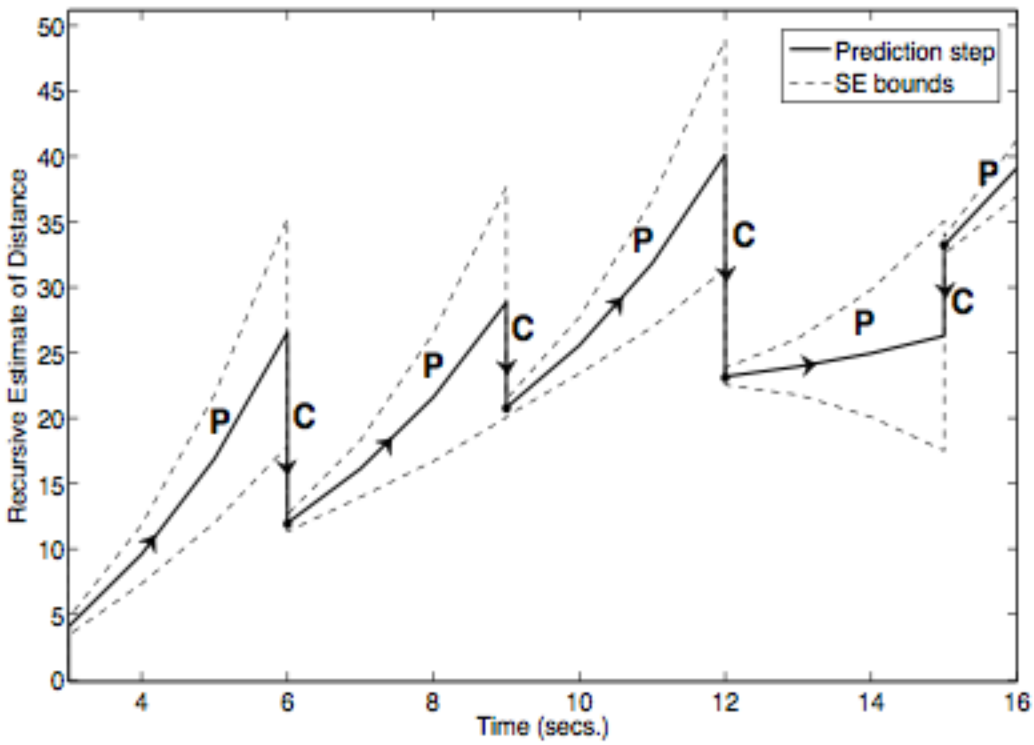

Figure 3. Moving body example: Kalman filter recursive estimate of the distance showing the standard errors uncertainty bounds (dashed).

Figure 3 shows the recursive *Kalman Filter* (KF) estimates of the state travelled by a moving body, based on a simple, stochastic, state space model of this moving body and irregular, noisy measurements of the distance it has travelled. The plot shows that the position estimates are being continually updated, in a discontinuous fashion, as measurements of the distance are made and so provide more information on the true state (position and velocity) of the moving body as it traverses space. In order to compare this plot with the plot in Figure 1, I selected a random realization that led to qualitatively similar variations in the estimated position. Also, the standard error uncertainty bounds are shown dashed; arrows have been put on the curves; and the letters **P** and **C** have been added to show the prediction and correction steps, respectively. The latter relate qualitatively to the **U** and **R** steps in Figure 1 and, as in the measurement step **R** of this figure, the corrective measurement step **C** is vertical. This is because the probability distribution, which has diffused because of the increasing uncertainty about the position of the body since the last measurement was taken, suddenly becomes much more sharply peaked as the new distance measurement provides information about where the body is now located. And this process is continued recursively, each time with the uncertainty about the position growing until the next measurement is taken.

The plots in Figures 1 and 3 are certainly qualitatively very similar and both show the evolution of a state variable in a multi-state system. Of course, it is obviously impossible draw a direct analogy between the probabilistic estimates of a quantum particle and the behaviour of a body governed by Newtonian dynamics and I certainly am not trying to do this here. Rather I am pointing out that my knowledge of recursive estimation, here coupled with a knowledge of operators and their links to the dynamic system's frequency response via the use of complex numbers, helped me to better understand the Penrose book. I was struck by the similarities: an underlying linear dynamic system, albeit one complex valued, being used to predict the evolution of the state vector; the intervention of a measurement that leads to a ‘jump’ with a probabilistic outcome; and then the continued prediction from the initial conditions defined by this probability distribution. And this stimulated me to read the book in more detail and so enhance my knowledge of quantum theory. Even to speculate on whether it might not be possible to develop some form of ‘Quantum Kalman Filter’ that is able to track the evolution of the quantum particle!

As in the brain/vision example considered previously, I am not trying to suggest that my observations necessarily have any credibility in scientific terms: they are just thoughts that occurred to me when reading the Penrose book. I draw attention to the qualitative similarity between Figures 1 and 3 simply because it highlights, once again, the potential importance of recursive estimation in a stochastic setting. Considered in these terms, it seems to me that we live in an inherently stochastic world; one where everything we observe can be described only by probability distributions or some other suitable form of uncertainty measure. Moreover, this brief discussion of quantum dynamics, together with the brain/vision example, show how a knowledge of recursive estimation, as well as its application to DBM modelling with the help of the CAPTAIN routines, leads one to ask wider questions about the nature of reality and uncertainty in the physical and biological world. Perhaps these thoughts may help to stimulate research which tries to consider the application of recursive estimation, in all its forms, within the strange stochastic worlds of the neural networks in the brain and particle dynamics in a quantum field system (e.g. Johnson and Hu, 2002).

P. R. Johnson and B. L. Hu. Stochastic theory of relativistic particles moving in a quantum field: Scalar abraham-lorentz-dirac-langevin equation, radiation reaction, and vacuum fluctuations. *Physical Review D*, 65, 2002.

R. Penrose. *The Road to Reality: A Complete Guide to the Laws of the Universe*. Jonathon Cape, UK, 2004.

P. C. Young. *Recursive Estimation and Time Series Analysis.* Springer-Verlag, Berlin, 1984 (revised, enlarged edition due for publication in 2011).